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Would price-level targeting destabilise the economy?*

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Abstract

When indexation is endogenous price level targeting slightly adds to economic stability, contrary to widespread fears to the contrary. The aggregate supply curve flattens and the aggregate demand curve steepens, increasing stability in the face of supply shocks.

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In recent years there has been a rising interest in price level targeting. (for example, Vestin, 2000; Nessen and Vestin, 2000; Smets, 2000; Williams, 1999) By this is meant that monetary policy aims after any shock to restore prices to some pre-set path (which might be rising steadily, at for example a 2% per annum inflation rate). It is different from inflation targeting whose aim is to restore inflation to some pre-set rate (like the same 2%). Under price-level targeting therefore prices are stationary around the target path whereas under inflation targeting they are non-stationary.

There are some obvious general attractions of such stationarity. Under non-stationarity the variance surrounding future prices rises the further in the future the period. This uncertainty affects in an extreme way the issuing of long-term nominal contracts, such as long term bonds or wage contracts. One can say in reply that the absence of such nominal contracts does not obviously matter since indexation can allow people to deal with real variables directly and since these are what they care about, there is just as much market completeness. However indexation has imperfections, both of timing and of exactness; hence indexed contracts seem unlikely to be as efficient as nominal ones in effecting a swap of future for present purchasing power .

In addition to this argument from uncertainty and markets there is the practical question of the ‘zero bound’ on nominal interest rates. In this era of low inflation it has worried central bankers considerably that a serious recession could require large interest rate cuts; yet these might be limited by the zero bound at which the demand for money becomes essentially indeterminate, so that we are caught in a liquidity trap and unable to help the economy recover- as recently with deflation in Japan This concern has led policy authorities to choose inflation targets away from zero — typically 2% as we know — so

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that there is ‘room’ for nominal interest rates to fall. Price-level targeting can be helpful in this respect as expected future inflation equals the target path plus the present deviation of prices from the path.¹

The main objection to price level targeting has been that it would create instability in output and inflation and so reduce welfare. As noted by Svensson (1999a), the consensus of earlier authors (Hall, 1984; Duguay, 1994; Bank of Canada, 1994; Fischer, 1994) was that targeting prices rather than inflation (and presumably also by implication money rather than its growth rate) would lower long-term price variance at the expense of higher short-term inflation and output variance. As he puts it ‘The intuition is straightforward: in order to stabilize the price level under price-level targeting, higher-than-average inflation must be succeeded by lower-than-average inflation... (apparently implying)...higher inflation variability...(which)...via nominal rigidities...would then seem to result in higher output variability’.

This result however has been questioned in various ways. One has been to note that if commitment cannot be achieved, then price-level targeting introduces an element of commitment which can improve welfare. Thus Svensson gives an example where discretionary monetary policy facing a Phillips Curve with persistence in output actually raises inflation variability (without benefiting output stability) if it targets inflation rather than the price level — in effect the price level target is preventing some of the ‘useless’ inflationary discretion (useless because private agents fully anticipate it so that it has no effect on output). This result plainly depends on there being no commitment that would rule out such useless discretion by other means — with such commitment price-targeting is inferior to inflation-targeting. Kiley (1998) also notes that Svensson’s set-up is one of ‘policy ineffectiveness’ on output (as in Sargent and Wallace, 1975) and hence no trade-offs arise with output stability; there is, as Svensson puts it, a free lunch in the model as price-level targeting reduces the inflation bias with no cost in output stability. Svensson’s approach has been followed up by others, including Vestin (2000) and most recently Nessen and Vestin (2000); they find that within a Phillips Curve with forward- and backward-looking expectations (where there is no such free lunch) average inflation targeting (a weighted average of a price level and inflation target) can improve on both inflation and price level targeting, again because of the effect that such targeting has in limiting discretion. This approach is one we eschew here on the grounds that commitment is generally available (e.g. by legislation about the proper behaviour of the central bank) in these days of consensus about the undesirability of inflation. Minford (1995) noted that the principal in all such policy frameworks is the electorate itself; it is reasonable to assume that sooner or later appropriate institutions for achieving its best interests, which in this context would include commitment, will be found. Hence the argument seems somewhat fragile to likely institutional change.

A second approach operates within models with backward-looking agents, as for example adaptive agents in learning situations. There it is found that there may well be gains from integral control of inflation, that is reacting to cumulative inflation, viz the price level. An example is Aoki and Nikolov (2004) where there is central bank learning about the economy’s parameters. They find that price-level targeting rules ‘automatically undo past policy mistakes’. Such integral control was found in an earlier literature before rational expectations to be generally helpful for stabilisation in complex dynamic structures. This set of models are attractive for dealing with situations where policy is in flux; however here we are arguing about a settled long term policy regime, evolving rather easily and naturally from our current inflation targeting regime, and for this experiment learning does not seem appropriate.

¹An arithmetic example is useful to show this. If normal real interest rates are 3%, then normal nominal interest rates would be $3 + 2 = 5\%$. Under inflation targeting an interest rate at the zero bound would then be a real interest rate of -2% , the minimum it could reach. However under price level targeting the implied future inflation is the deterministic inflation path element, again 2% , plus any current deviation of prices below the path. Hence taking the worst case, if an economy in recession suffered from deflation and current prices fell 2% in the past year against a target rise of 2% , then at the zero bound the real interest rate would be -6% . It is worth dwelling on this point in the context of the gold standard. One should be able to find useful data bearing on the effects of price-level stability from this era. However it is pretty clear that the nineteenth century was a period of some fairly large booms and slumps, in which prices both fell and rose for a decade or more at a time, price level stability was there in the long term but it was very long term (Bordo and Filardo, 2004).. Price-level stability in the sense intended here is much shorter term; indeed in the experiments we will report later it is immediate. So at this stage it is hard to know what lessons we can extract from the era of gold.

In a third approach, a number of authors have examined optimal monetary policy under commitment within models embodying substantial nominal rigidity. Smets (2000) uses a model with a Calvo-style Phillips Curve to examine the optimal horizon for bringing inflation or the price-level back to their targets; he finds that the optimal length becomes shorter the more forward-looking are the price expectations and the steeper is the Phillips Curve. Williams (1999) evaluates a variety of such rules in the FRB large-scale model of the US in which there is a forward/backward-looking Phillips Curve and inertial pricing dynamics as in Fuhrer and Moore (1995). He finds that multi-period inflation targeting ranks highly and that price-level targeting only causes minor output instability. With this family of models, interest rate rules of the Taylor(1993)-Henderson-McKibbin(1993) type give good results, with the optimal degree of inertia in response (the lagged interest rate coefficient) depending on the degree of forward-lookingness. In this commitment context the addition of price level targeting at a suitably long horizon has little effect on the optimal trade-off — as such it is an innocuous optional extra for policy-makers desiring to anchor the long-run price level. Commitment thus basically removes the benefits of price-level targeting but also makes it innocuous.

The main argument of this paper will be that insufficient attention has been paid to the endogenous response of indexation (or equivalent variations in contract length) to the monetary policy regime. Indexation acts as an additional element loosening the nominal rigidity of contracts, over and above expectations at the time they are signed. We argue that when this element is allowed for rigorous price-level targeting (far more immediate than that considered by Smets and Williams, *op cit*) has a benign effect on the behaviour of the economy, essentially because it reduces the need to rely on indexation. Hence price stationarity is an objective which can be attained not merely without a stabilisation cost but even with some modest gain.

The issue of price-level targeting is one among a number that arise in determining optimal monetary policy. However, we will mostly abstract from these other issues, on the assumption that they are orthogonal to the one in hand. Thus, the optimal inflation tax issue (going back to Friedman, 1969) essentially relates to the choice of the inflation path, which needs to be determined regardless of whether the price level or inflation is stabilised. A further issue is that distortions in the economy could interact with monetary policy; however we will assume as our benchmark case that such distortions either are eliminated by other policies or are unaffected by monetary policy choice. Hence our focus in this paper is on the stabilisation aspects of price-level targeting.

Our choice of model for this purpose is guided by both theoretical and empirical considerations. Theoretically we seek a model with nominal rigidity (so that money matters for output) whose Phillips Curve does not violate the strong natural rate hypothesis whereby a fully anticipated monetary policy cannot affect output. Empirically the model's responses should fit the facts of not only the business cycle but also of indexation. These considerations have caused us to use a simple overlapping wage contracts model in which wages are set 4 quarters in advance with a sliding quarterly response to expected inflation and a lagged indexation rule; this is embedded in a rather simple representative agent model where the indexation response is chosen by employed agents to smooth their real wages which are also their consumption.

The widely-used Calvo and Taylor contracts models are ruled out here by their failure to satisfy the strong natural rate hypothesis (Minford and Peel, 2003). Our contracts model can be regarded as a Taylor wage contract model adapted to satisfy the hypothesis (Minford and Peel, 2002). The Calvo model could also be adapted to satisfy it (Le and Minford, 2005); it is an interesting question how that would affect the price stationarity issue but that must be deferred to future work. We may note in passing that the large literature of optimal price targeting using the Calvo model which generally finds the optimality of fixing the price level is effectively arguing for a zero inflation target, not a price level target.²

²Numerous people have written about optimal monetary policy within the so-called 'New neo-Keynesian Synthesis' model of nominal rigidity in prices or wage — see most recently Woodford, 2003; previous papers include Collard and Dellas, 2003; Goodfriend and King, 2001; Khan et al, 2002; Clarida et al, 1999. The general conclusion from these models, which appears to be robust to a variety of enrichments of the basic model structure, is that it is optimal to stabilise the

We discuss the model in general terms in the first section that follows. We then use a linearised and simplified version to understand the likely effects of price-level targeting on the economy and welfare within an approximate closed form. In the succeeding section we use stochastic simulations on the full non-linear model, before some final concluding remarks.³

1 The Model: A representative agent model of indexation

Our model assumes a risk-averse representative household with a cash-in-advance constraint facing a financial marketplace in which the costs of continuous borrowing to smooth consumption in the short term (within some notional ‘borrowing period’) are excessive; thus it attempts to smooth its consumption through wage arrangements (i.e. ‘contracts’). These may make wages be fixed nominally in advance, or indexed to prices, or indexed to auction wages; a contract specifies that wages react in some proportion to each of these elements (proportions add to unity to preserve homogeneity). Indexation is imperfect in two senses: the index used is biased in the short term because of fixed weights and it is paid retrospectively. These two features imply that 100% indexation is not automatic. Minford et al (2003) explored how households will maximize their expected utility via a (coordinated) choice of contract proportions. The household with the power over contract structure is assumed to be fully employed, in line with the well-known facts of bargaining, whereas the average representative household in the model does have unemployed time; it follows that contract structure effectively minimizes the variance of the real wage, given that the expected real wage and employment are invariant to contract structure.

We embed our representative household in an environment of profit-maximizing competitive firms which on a large proportion of their capital stock face a long lag before installation (a simple time-to-build set-up) and a government that levies taxes and pays unemployment benefits (which distort households’ leisure decisions and introduce a ‘social welfare’ element into monetary policy). Firms and governments use the financial markets costlessly and settle mutual cash demands through index-linked loans; since there is no binding cash constraint on these agents, these loans are assumed to be unaffected by the imperfections of the price index which are short term in nature.

The resulting dynamic stochastic general equilibrium (DSGE) model is set out in detail in Appendix A. A linearised and simplified version is also set out there.

The government is assumed to smooth both the tax rate and the growth rate of the money supply by borrowing (from firms). Nevertheless it cannot avoid noise in its money supply setting — the source of this could be its inability to monitor the money supply quickly or even at all (for example in the USA the use of dollars by foreigners around the world makes it impossible to know what the domestic issue of dollars is). Throughout we will use money supply rules to carry out our targeting objectives; many recent papers use interest rate rules to do this but in principle a suitably designed interest rate rule can mimic the behaviour of a money supply rule and vice versa. Within the model here where money plays a central role via cash in advance it is more transparent to use a money rule to express monetary behaviour; but its operation can readily be translated into interest rate terms.

price level at the last available price level. The reason for this is that price disturbances in this model cause relative price distortions; thus to eliminate the latter one must eliminate the former. It has been usual within these works to refer to this optimal policy as that of fixing the price level or price-level targeting. However this is a misnomer as many have recognised. Prices in practice will vary because of shocks beyond the control of policy fixed in an earlier period (for example, one may think of this price level targeting as set one quarter in advance for the following quarter’s price). When they do, it will prove optimal each period to set future prices at the new price level, thus avoiding future disturbances. This of course means that the policy actually sets the change in prices to zero: it is setting an inflation target of zero. It is therefore best described as an inflation-targeting, not a price-level-targeting, policy.

³In all these results we find — in conformity with Lucas (1987) — that the variations in the equivalent consumption loss due to different monetary stabilisation policies are extremely small. However we have continued to treat them as of interest, on the grounds that business cycle variation remains a matter of acute concern to policymakers in spite of the smallness of these numbers, which must be regarded therefore as a puzzle.

A further point about the rules we will consider is that they are extreme — ‘nutter’ rules in central bank parlance whereby a ‘target’ is achieved uncompromisingly in the next feasible period. The reason for this is merely simplicity and clarity so as to maximise the chance of a clear conclusion. Loosening the rules off in a practical way is something that can be done later if something promising emerges with these tight rules.

In deterministic simulations, a permanent money supply shock raises prices in the long run, and in the short run also raises output, with persistence extending up to 15 quarters but with most effect over after 10. In the high-indexed case there is less real effect and less persistence than in the high-nominal case.

These fairly standard properties stem from the model’s deliberate drawing on elements that have been shown by past work to be useful in explaining the business cycle and also natural rates as discussed for example by Parkin (1998). The elements here include: time-to-build investment, cash-in-advance, nominal contracting (as noted above), household liquidity constraints, and (on the natural rate side) the influence of unemployment benefits on labour supply. With suitable country-by-country calibration one would expect to be able to model OECD countries’ business cycle and natural rate experience with at least some modest success. One example of a model with many of these elements is the Liverpool Model of the UK (Minford, 1980), which can be regarded as an IS-LM approximation of a DSGE model as suggested by McCallum and Nelson (1999b); this has had reasonable success in explaining and forecasting UK macroeconomic behaviour in the past two decades (Andrews et al, 1996).

In this model, given stationary productivity and money supply shocks, indexation would be minimal with only a slight tendency to rise as the variance of money shocks rises. However when shocks to either become highly persistent indexation to prices or to their close competitor, auction wages, (which together we term ‘real wage protection’) become large, becoming largest when both shocks are persistent. The reason is that productivity shocks would disturb prices and so the real worth of nominal wage contracts; indexation is of little use in remedying this disturbance if it is temporary because by the time the indexation element had been spent the shock would have disappeared, but with a permanent disturbance indexation can help offset it with a lag. If into this already-indexed world of persistent productivity shocks, monetary persistence is also injected, indexation rises further, to help alleviate the increased disturbance to real wages. This higher indexation also helps to alleviate the instability in unemployment which accompanies the greater shock persistence of money — the point being that this persistence induces persistence in the economy’s departure from its baseline and so disturbs unemployment too for longer.

In the OECD in the 1970s it is well-known that real wage protection was substantial; the calibrated model, when estimated variances and persistence of money and productivity shocks are fed into it, predicts high protection in all countries covered, apparently in line with the facts. Also, contrary to much casual comment, there was little evidence of any diminution of real wage protection in the 1990s; the model also predicts as much, for even though the variance of money supply shocks fell in the 1990s, their persistence remained essentially unchanged. Since it is persistence in both shocks (monetary and real) that is inducing both the greater instability of real wages and the increasing indexation this produces, price level targeting is a natural avenue in which to investigate improvement in monetary policy. (Price level targeting can be considered as ‘cumulative inflation targeting’ where any past deviation from the inflation target is offset with an opposite deviation the following period). The reason is that such a policy will eliminate the effect of persistence in shocks from the price level and so from the real value of a nominal wage contract.

According to the model, it is indexation that varies in response to changes in persistence except in quite unusual circumstances where the auction element also varies; and it is therefore on indexation that we focus here. >From society’s viewpoint reducing indexation improves the economy’s stability in the face of supply shocks because it both flattens the AS (Phillips Curve) and steepens the AD curve, as illustrated in Figure 1.

The resulting intersections for a supply shock as shown at **A** (high indexation) and **B** (low indexation). Thus the drop in indexation is stabilising to both employment and prices in the face of a supply shock.

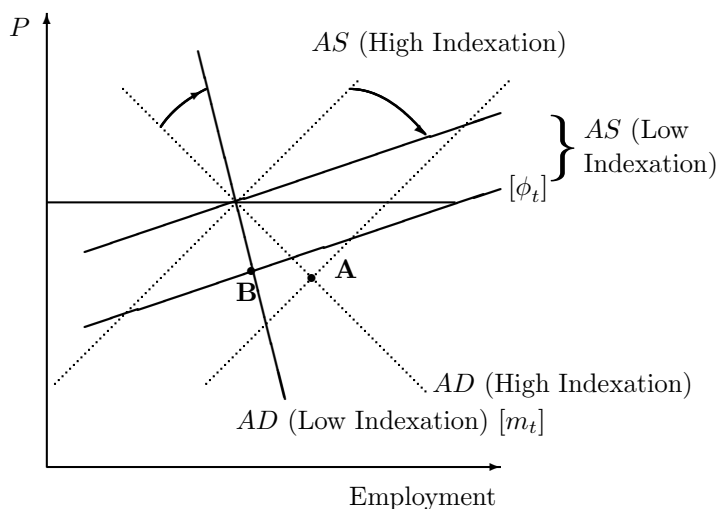


Figure 1: The effect of reduced indexation on slopes of AS and AD curves [ϕ_t = productivity shock; m_t = monetary shock]

For a money (demand) shock the result is greater employment instability, though probably less price instability; however, in principle at least, money shocks can be avoided by monetary policy while demand shocks can be neutralised by it.

In order to assess the welfare of society from different monetary policy rules we give the average household the standard Constant Relative Risk Aversion utility function with Cobb-Douglas preferences across consumption and leisure. The resulting welfare function for the policy-maker to maximize is therefore:

$$EU_t = E \sum_{\tau=t}^{\infty} \delta^{\tau-t} \left\{ \frac{(c_{\tau}^{\nu} [1 + a_{\tau}]^{1-\nu})^{1-\rho} - 1}{1 - \rho} \right\} \quad (1)$$

where

$$c_t = \frac{\bar{W}_{t-1}}{p_t} (1 - a_{t-1});$$

c = consumption per head;

a = unemployed time per head as a proportion of normal working time;

\bar{W} = nominal wages;

p = prices;

and we have normalised leisure time to be equal to normal working time, so that $1 + a$ = leisure time as a proportion of normal working time. We set $\nu = 0.7$, based on the marginal valuation of leisure at wages net of unemployment benefit. Because households get unemployment benefit on their spells of eligible unemployment, a_t , this implies that their choice is distorted; they choose leisure in response to the differential between wages and benefits. But then of course they must pay for the benefit burden via taxes; the present discounted value of this tax burden is the same as this benefit bill and so the value of benefits is not included in consumption.

We begin by considering how a monetary rule may be optimised within this model; this discussion is conducted entirely in terms of the simplified linear model. We then consider the performance of various forms of targeting rules within the same linearised model. Finally we use stochastic simulations of the full non-linear model to derive the accurate optimality results. We conclude with some policy implications.

2 Finding Optimal policy rules within the linearised model

In calibrating our welfare function (1) we have set the value of $\rho = 1.5$. For simplicity (to set cross-partial derivatives to zero) we now set it to unity, so that the welfare function becomes logarithmic:

$$EU_t = \sum_{\tau=t}^{\infty} \beta^{\tau-t} \{ \nu (\ln \bar{W}_{\tau-1} - \ln p_{\tau}) + \nu \ln(1 - a_{\tau-1}) + (1 - \nu) \ln(1 + a_{\tau}) \} \quad (2)$$

Here \bar{W} is the contracted nominal wage (as partially indexed); p is the consumer price level; a is leisure.

Taking a second-order Taylor series expansion around the expected values of these variables, we can rewrite this as

$$EU_t = \frac{-0.5}{1 - \beta} \{ \nu \text{Var}(\bar{W} - p) + \text{Var}(a) \} \quad (3)$$

since the second derivatives $\left(\frac{1}{(W-p)^2}; \frac{1}{(1-a)^2}; \frac{1}{(1+a)^2} \right)$ are each approximately unity under the base run normalisation we have used.

In order to compute the optimal monetary policy we use the money supply, M_t , as the instrument. This has the advantage that we may realistically assume inflation cannot be set precisely and also that current shocks cannot be precisely observed.

We write monetary policy as:

$$m_t = \bar{m} + \sum_{i=0}^4 z_i \eta_{t-i} + \epsilon_t \quad (4)$$

where η_t is the innovation in the productivity shock ϕ_t (we will set $\bar{m} = 0$). Here we are allowing policy-makers in general to react to the current productivity shock but to have incomplete control of current money (a ‘trembling hand’) so that there is a pure demand shock, ϵ_t , as before; the idea is that by responding to current economy-wide information (such as interest rates) the central bank can engineer such a current response but with error. For example $z_0 = 0$ would imply that the current productivity shock either cannot be observed or is not responded to (as in a fixed rule for the money supply). The variance of ϵ_t measures the precision of policymakers’ targeting. Notice that we assume no moving average element in ϵ_t : this amounts to ruling out inflation or money growth targeting (where $m_t = m_{t-1} + \epsilon_t = \sum_{i=0}^4 \epsilon_{t-i}$, truncated at four lags by contract length). Introducing such additional persistence in the pure error term seems unlikely to be beneficial — as we later checked and found to be the case with our calibration.

Using our general solutions (see the appendix for parameter notation and derivations) we obtain

$$\begin{aligned} \bar{W}_{t-1} - p_t &= \alpha [lV \phi_{t-1} + (1-l)Qm_{t-1}] + v' \pi (Qm_{t-1} - V \phi_{t-1}) - \pi (Qm_t - V \phi_t) \\ &= \alpha \sum_{i=1}^4 [lV + (1-l)Qz_{i-1}] \eta_{t-i} + v' \pi \sum_{i=1}^4 (Qz_{i-1} - V) \eta_{t-i} - \\ &\quad \pi \sum_{i=0}^3 (Qz_i - V) \eta_{t-i} + \alpha(1-l)Q\epsilon_{t-1} + v' \pi Q\epsilon_{t-1} - \pi Q\epsilon_t \quad (5) \end{aligned}$$

and

$$a_t = \psi (lV \phi_t + (1-l)Qm_t) = \psi \sum_{i=0}^3 (lV + (1-l)Qz_i) \eta_{t-i} + \psi(1-l)Q\epsilon_t \quad \text{where } \psi = a_0 \sigma \quad (6)$$

where $\alpha, l, V, Q, \pi, \psi, \sigma$ are model parameters or combinations of them; $v' = v(1+c)$ is the indexation parameter v adjusted for an upward bias $(1+c)$ in the official CPI; α is the weight given by the contract to the auction wage.

So

$$EU_t = \left(\frac{-0.5}{1-\beta} Q^2 Var\eta \right) \left\{ \nu \left[\begin{aligned} &\pi^2 \left(\frac{V}{Q} - z_0 \right)^2 + \sum_{i=0}^2 \left\{ \alpha \left[l \frac{V}{Q} + (1-l)z_i \right] - v'\pi \left(\frac{V}{Q} - z_i \right) + \pi \left(\frac{V}{Q} - z_{i+1} \right) \right\}^2 \\ &\quad + \left\{ \alpha \left[l \frac{V}{Q} + (1-l)z_3 \right] - v'\pi \left(\frac{V}{Q} - z_3 \right) \right\}^2 \\ &\quad + \psi^2 \sum_{i=0}^3 \left(l \frac{V}{Q} + (1-l)z_i \right)^2 \\ &\quad + \frac{Var\epsilon}{Var\eta} \left\{ \nu[\pi]^2 + \nu \{ \alpha(1-l) + v'\pi \}^2 + \psi^2(1-l)^2 \right\} \end{aligned} \right] \right\} \quad (7)$$

If we treat v' and α as parametric (the Nash non-cooperative strategy), then we can obtain the optimal policy parameters from the first order conditions as:

$$z_0 = \left\{ \frac{\left(\frac{V}{Q} \right) [\nu\pi^2 - \nu\{\alpha(1-l) + v'\pi\}\{\alpha l + \pi(1-v')\} - \psi^2 l(1-l)] + \nu\pi\{\alpha(1-l) + v'\pi\}z_1}{\nu\pi^2 + \nu\{\alpha(1-l) + v'\pi\}^2 + \psi^2(1-l)^2} \right\} \quad (8)$$

$$z_1 = \left\{ \frac{\left(\frac{V}{Q} \right) [\nu\pi\{\alpha l + \pi(1-v')\} - \nu\{\alpha(1-l) + v'\pi\}\{\alpha l + \pi(1-v')\} - \psi^2 l(1-l)] + \nu\pi\{\alpha(1-l) + v'\pi\}(z_0 + z_2)}{\nu\pi^2 + \nu\{\alpha(1-l) + v'\pi\}^2 + \psi^2(1-l)^2} \right\} \quad (9)$$

$$z_2 = \left\{ \frac{\left(\frac{V}{Q} \right) [\nu\pi\{\alpha l + \pi(1-v')\} - \nu\{\alpha(1-l) + v'\pi\}\{\alpha l + \pi(1-v')\} - \psi^2 l(1-l)] + \nu\pi\{\alpha(1-l) + v'\pi\}(z_1 + z_3)}{\nu\pi^2 + \nu\{\alpha(1-l) + v'\pi\}^2 + \psi^2(1-l)^2} \right\} \quad (10)$$

$$z_3 = \left\{ \frac{\left(\frac{V}{Q} \right) [\nu\pi\{\alpha l + \pi(1-v')\} - \nu\{\alpha(1-l) + v'\pi\}\{\alpha l + \pi(1-v')\} - \psi^2 l(1-l)] + \nu\pi\{\alpha(1-l) + v'\pi\}(z_2)}{\nu\pi^2 + \nu\{\alpha(1-l) + v'\pi\}^2 + \psi^2(1-l)^2} \right\} \quad (11)$$

In the model, employed people choose their contracts by minimising the variance of $(\bar{W}_{t-1} - p_t)$ — the terms in (7) above that are multiplied by ν — given monetary reactions. We find that their first order condition for v' of employed people yields the following contract share (that for α will yield very small values for any values of z_i as we have found throughout when productivity shocks are random

walks as here):

$$v' = \left(\frac{\frac{var\epsilon}{var\eta} \{-\alpha[\pi - \alpha]\} + [\pi - \alpha] \sum_{i=0}^2 \left[\frac{V}{Q} - z_i \right] \left[\alpha z_i + \pi \left(\frac{V}{Q} - z_{i+1} \right) \right] + [\pi - \alpha] \left\{ \frac{V}{Q} - z_3 \right\} [\alpha z_3]}{[\alpha - \pi]^2 \left\{ \frac{Var\epsilon}{Var\eta} + \sum_{i=0}^3 \left[\frac{V}{Q} - z_i \right]^2 \right\}} \right) \quad (12)$$

This is in practice as far as we can take the analytical solution. We now therefore explore the solution of these conditions numerically using our benchmark calibration of the model (see appendix). Consider how monetary reaction changes as v' rises. We find numerically that at $v' = 0$ the average z, \bar{z} , is just below 0.2; as v' rises \bar{z} falls becoming negative just over $v' = 0.1$. It becomes steadily more negative until around $v' = 0.7$. From here as v' rises \bar{z} also rises, reaching around 0.3 when $v' = 1.0$. Figure 2 shows this reaction function of \bar{z} to v' .

Figure 2 also shows the reaction of v' to the movement of \bar{z} (on the simplifying assumption that all the z s are equal); this rises steadily from 0 as \bar{z} declines from just below 1.1., reaching 1.0 when \bar{z} reaches around 0.3. Hence we find the Nash non-cooperative solution is around this point at:

$$\begin{aligned} z_0 &= 0.52 \\ z_1 &= 0.39 \\ z_2 &= 0.26 \\ z_3 &= 0.13 \\ \bar{z} &= 0.325 \\ v' &= 1.0 \\ EU_t &= -0.0547 \\ CE &= 0.078\% \end{aligned}$$

where CE, the equivalent cost in average annual consumption, is 0.078% (this corresponds to the assumed 1% standard deviation in the z shock).

The social second-best optimum is found when the monetary authority acts like a Stackelberg leader and takes into account the employed people's contract decision-making. Here it maximises (7) subject to the optimising choice of v' as a constraint. When we investigate this numerically (setting $\alpha = 0.1$) we find that the optimum is:

$$\begin{aligned} z_0 &= -0.6 \\ z_1 &= 1.0 \\ z_2 &= 0.4 \\ z_3 &= 0.2 \\ v' &= 0.36 \\ EU_t &= -0.034 \\ CE &= 0.048\% \end{aligned}$$

reducing the consumption equivalent cost by 0.03%.

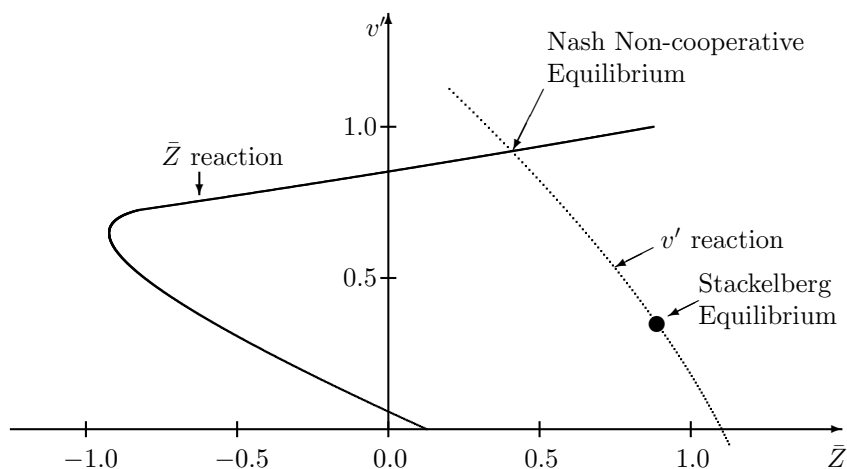


Figure 2: Strategic behaviour with no restrictions placed on Z s

If we constrain the $z_i (i = 1-3)$ to be equal the welfare is only slightly lower and we obtain:

$$\begin{aligned} z_0 &= -0.6 \\ z_i &= 0.8 (i = 1 - 3) = z \\ v' &= 0.36 \\ EU_t &= -0.035 \\ CE &= 0.050\% \end{aligned}$$

with a similar consumption cost.

It is natural to think of monetary policy as responding in this way solely to the most recent data on productivity rather than attempting to respond finely to all the lagged productivity innovations over the period of the contract lag. This allows us to write the money rule conveniently as

$$m_t = \bar{m} + z_0 \eta_t + z \phi_{t-1} + \epsilon_t$$

To make sense of these numbers let us then distinguish the two dimensions in this monetary reaction on our assumption that current information cannot be observed and acted upon (in a discretionary manner) for at least one quarter:

- a) feedback onto available information: this necessarily occurs with a lag and therefore concerns z . Here we have already identified two main types of feedback: targeting the price-level ($z = \frac{V}{Q}$) and the level of money ($z = 0$).
- b) current response: this necessarily implies that the response is an ‘automatic’ one — that is, one that is built into operating procedures that react instantly to market data or to micro data the authorities directly observe. z_0 and the inaccuracy element ϵ_t summarise this element. It ranges at one extreme from fixing interest rates in the current period (which implies $z_0 = 1.3$ with a price-level feedback target and $z_0 = 0.2$ with money-level feedback) in which case we might naturally wish to think of this as potentially 100% accurate ($\epsilon_t = 0$); to, at the other extreme, money-level targeting, which would imply $z_0 = 0$ and a degree of inaccuracy (ϵ_t non-zero) since such targeting involves the possibility of shocks to velocity or the banking system.

If we examine the Stackelberg reaction in the light of this discussion, we see that the negative value of z_0 is ‘perverse’ in that it involves monetary policy tightening in response to a productivity increase, instead of accommodating it or at worst not responding. Such a perverse immediate response would drive prices even lower than they would naturally go in response to the rise in productivity in the absence of a monetary response. Even if one could think of a plausible ‘automatic’ mechanism that could deliver such a response, we can discount it because of the sensitivity of welfare to the precise assumptions; were v' to be smaller than the assumed level then welfare deteriorates sharply under this response, with a large destabilisation of employment as well as real spendable wages. If we constrain $z_0 \geq 0$, as well as keeping the other z s equal, then the Stackelberg optimum becomes

$$\begin{aligned} z_0 &= 0.0 \\ z_i &= 1.0 \quad (i = 1-3) = z \\ v' &= 0.35 \\ EU_t &= -0.038 \\ CE &= 0.054\% \end{aligned}$$

with a consumption cost that is only a little higher.

We now consider in more detail this latter result. First, we hold constant the zero automatic response $z_0 = 0$ and the assumption that the z_i are all the same; we then reconstruct Figure 2 for v' and z in the region of the Stackelberg solution and fill in the social indifference curves. This is done in Figure 3. What we find is that social utility increases as we move south-east along the v' reaction function on Figure 3, but that this increase comes to a halt short of full price-level targeting. The reason for this turns out to be that though further movement to full price-level targeting would minimise the variance of the spendable real wage with zero indexation, the variance of employment increases as indexation gets so low. This can be seen from the terms in ψ^2 in EU_t which become large as $l = v'$ gets close to 0 and z gets close to $\frac{V}{Q}$. Intuitively, what is happening is that as indexation tends to zero the AD curve tends to become vertical and shift purely with money shocks; but money shocks entirely accommodate productivity shocks so that employment fluctuates substantially with this accommodation. This effect starts at this point to overwhelm the dampening of real spendable wage variance.

One potential departure from this optimum would be to raise z_0 from 0 (money supply setting); in particular when interest rates are set, which would imply a z_0 of 1.1. In this case welfare deteriorates to -0.04825 ($CE = 0.069\%$) and $v' = 1$. The reason is that the setting of interest rates removes stationary monetary noise ($\text{var}\epsilon = 0$). This implies that the persistent productivity error dominates which induces full indexation and this in turn reduces welfare. Any setting of interest rates has this effect, whatever the values of the other z s.

3 Money Supply Targeting and feedback rules — a stochastic Simulation analysis for the full model:

We now proceed to consider some specific targeting rules, along the lines of our opening discussion. Our aim is to relate our results back if possible to the preceding discussion of optimality. Our main results use stochastic simulation; but we relate our results as we go to the linearised model in order to shed light on them. In the stochastic simulations we treat each period outcome as a stochastic experiment of equal likelihood in every future period. Thus the expected utility in every future period is simply the average of these stochastic outcomes. Discounting such a constant expected utility would merely multiply it by a constant, which we ignore. Our stochastic simulations are done for 50 sets of 40 overlapping shocks; thus if as above we treat each quarterly outcome in all 50 runs as a separate observation, we have a sample of 2000 observations from which to derive the variances and utility that interest us.

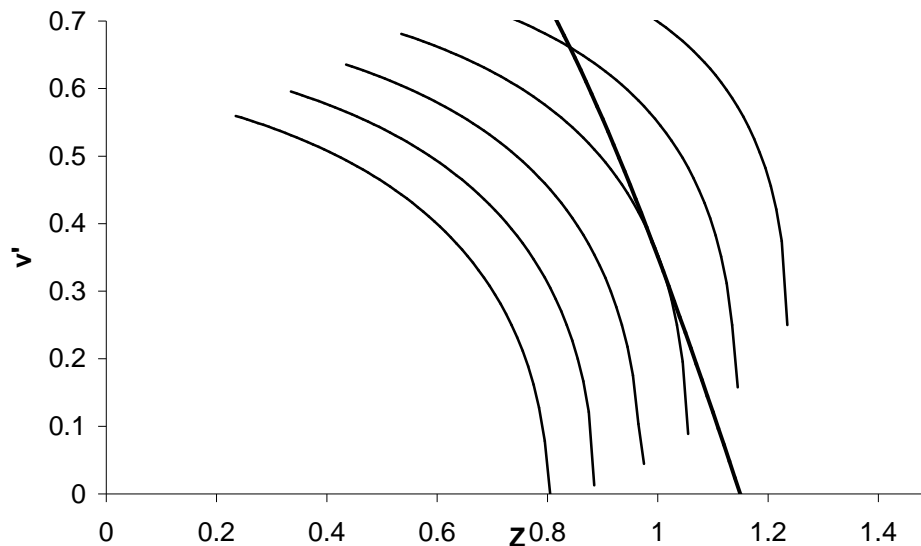


Figure 3: Stackelberg equilibrium (showing v' reaction and social indifference curves) assuming $z_0 = 0$ and $z_i = z$ ($i = 1 - 3$)

There is one small cautionary point to note. Certain sets of non-stationary productivity shocks generated runs that did not converge under particular monetary policy regimes; these sets were not used in what follows. There were 22 of these; we do not fully understand why they did not converge — it could possibly be that our algorithm was not powerful enough to find the solution or it could be that there was no solution. For what it is worth, out of these 22 runs 14 failed to converge with all three money rules (viz. the rule targeting inflation/money-supply-growth, the rule targeting the price level, and the money-level-targeting rule). A further 4 failed with both the money growth/inflation target rule and the price-level rule. And a further 3 also failed with the price level rule. In so far as this indicates anything, it might suggest that the price level rule had a rather greater tendency to promote instability than the other two in ‘difficult’ (perhaps highly non-stationary) samples. But our main reaction is simply to exclude these samples from consideration in comparing the rules, since we do not properly understand why most of them fail to converge for all the rules.

3.1 Are any targeting rules optimal?

We found that on the linearised model, using the Taylor series approximation to second order, the Stackelberg optimum was approximately $z_0 = 0$, $z_1 = 1$ with $v' = 0.35$. When a monetary rule with these values was injected into the full model, the resultant full welfare value was -0.0363 ($CE = 0.052\%$ is the percentage cost in equivalent consumption in terms of present value). This represents a small movement away from full price-level targeting, where we would obtain zero indexation ($v' = 0$; $w = 0.92$) and $z_0 = 0$; $z_1 = 1.3$ with welfare = -0.0375 ($CE = 0.054\%$).

These results emerge from the full model when the rule is expressed as a linear function of shocks. However, the full model is non-linear. In these circumstances, the optimal rule may also be non-linear; furthermore the optimal rule as discovered in the linear approximative model may not be optimal on the full model. Unfortunately analytic methods cannot get us any further in investigating non-linear

responses. So what we do now is to use these linear results as a guide to a search over various classes of monetary rule that have in practice been suggested. Our linear optimal rule lies close to price-level targeting but falls a little short of it; and it sets the money supply not interest rates. This suggests that we should primarily explore price-level targeting with the money supply as the short-term instrument.

3.1.1 Inflation and price-level targeting

Under price-level targeting (when ϕ_t is a random walk as we shall assume throughout)

$$m_t = \frac{V}{Q}\phi_{t-1} + \epsilon_t \quad (13)$$

whence

$$W_t = (1-l)Q\epsilon_t + V(l\phi_t + [1-l]\phi_{t-1}) \quad (14)$$

and using (B27), (13) and (14)

$$\overline{W}_{t-1} - p_t = \pi V[\eta_t - v'\eta_{t-1}] + \alpha V[l\eta_{t-1} + \phi_{t-2}] - \pi Q[\epsilon_t - v'\epsilon_{t-1}] + \alpha Q(1-l)\epsilon_{t-1} \quad (15)$$

As we saw earlier, minimisation of the variance of real wages leads to a low degree of real wage protection ($v = 0, w = 90, \alpha = 10$).

Under inflation-targeting, the rule sets (we assume for simplicity the target is zero inflation)

$$E_t p_{t+1} = p_t \quad (16)$$

From this it follows that

$$E_t m_{t+1} = m_t \quad (17)$$

whence m_t should also be a random walk. Thus this is the equivalent of our original I(1) money supply rule, or pure money growth targeting. In our simulations in the full model we assume that inflation targeting is the money growth rule.

Comparing price- and inflation-targeting in the full model (Table 1) we find that for *given* contract structure the variance of real wages goes down while that of unemployment goes up as one moves to price-level away from inflation targeting. As contract structure is optimised (with the consequent near-elimination of real wage protection) real wage variance drops further while unemployment variance drops below its original level under inflation targeting. Again the substantial gains come with the shift in contract structure. We also note that the exact (non-linear) price and inflation target rules achieve better stability than our linear ones.

3.1.2 Money growth and money level targeting

We noted earlier the results of Svensson and others comparing price-targeting with inflation-targeting under fixed contract structures: while the previous consensus suggested that price-targeting would create a cost in short-term macro variability, these more recent studies are ambivalent. Our results, repeated in Table 2 below show (col.3 versus col.1) that money-level targeting is superior to money-growth targeting. Our focus here has been on contract structure shift as the main factor behind the superiority of level-over growth-targeting for nominal variables. Could it however be that our particular model or welfare criteria are important factors, contract structure held constant? Table 2 also compares the two types of targeting with contract structure given (col.2 versus col.1).

Table 1: price- and inflation-targeting: stochastic simulation results

	Inflation target*	Price level target	Price level target
	contract structure optimal	contract structure optimal	with contracts
	for inflation target	for inflation target	optimised
variance of real	0.00762	0.00755	0.00729
consumed wage	(0.00024)	(0.00024)	(0.00023)
variance of	0.00199	0.00237	0.00137
unemployment	(0.00006)	(0.00006)	(0.00004)
h'hold welfare	-0.0359	-0.0367	-0.0353
CE %	0.0510	0.0524	0.0504
	(0.0015)	(0.0015)	(0.0015)
wage contract shares (%)			
nominal share	22	22	92
indexed share	71	71	0
auction share	7	7	8

(Standard deviations of shocks: $st.dev. of \epsilon = st.dev.\eta = 0.01$)

(productivity process: $\phi_t = \phi_{t-1} + \eta_t$)

(numbers in parentheses are stochastic simulation standard errors, as per Wallis, 1995)

* inflation target rule is the same as a money growth target rule — see text.

Table 2: money-level and money-growth targeting: stochastic simulation results targeting

	Money growth rule	Money level rule	Money level rule
	contract structure optimal	contract structure optimal	with contracts
	for growth rule	for growth rule	optimised
variance of real	0.00762	0.00779	0.00764
consumed wage	(0.00024)	(0.00025)	(0.00025)
variance of	0.00199	0.00186	0.00112
unemployment	(0.00006)	(0.00006)	(0.00004)
h'hold welfare(a)	-0.0359	-0.0358	-0.0345
CE %	0.0512	0.0511	0.0492
	(0.0015)	(0.0015)	(0.0015)
wage contract shares (%)			
nominal share	22	22	59
indexed share	71	71	37
auction share	7	8	4

(Standard deviations of shocks: $st.dev. of \epsilon = st.dev.\eta = 0.01$)

(productivity process: $\phi_t = \phi_{t-1} + \eta_t$)

(numbers in parentheses are stochastic simulation standard errors, as per Wallis, 1995)

What we see here is in fact that in a heavily indexed world with exogenous contracts changes in money supply persistence have virtually no effect on welfare. As we can see below from the linearised model the variance of worker consumption in such a world is positively related to the variance of inflation: this goes up with money-level targeting. The variance of the auction wage (and so unemployment) is related (slightly) to the variance of the money shock: this falls as persistence is reduced by money-level targeting.

From (B19) we can see that the real spendable wage, with α close to zero and v' close to 1, approximates to minus the inflation rate. From (B18) we can see that for the auction wage, with l close to 1, the term in the money shock is small but its persistence will raise its variance.

What is striking is that the changes in variances are quite small as money's persistence changes between columns one and two with given contract structure. It is the third column, where contract structure is allowed to be reoptimized, that shows a large reduction in unemployment variance while consumption variance falls back somewhat. The implication is clear: it is that the change in contract structure, not the model, that delivers the gains from targeting the level of money. Here the gains are larger than they are in the move to price level targeting and more than one standard error.

3.2 Comparing money- and price-level targeting

Table 3: stochastic simulation results for the two nominal level targets

	Pure money rule	Price level rule
variance of real	0.00764	0.00729
consumed wage	(0.00024)	(0.00023)
variance of	0.00112	0.00137
unemployment	(0.00004)	(0.00004)
h'hold welfare	-0.0345	-0.0353
CE %	0.0492	0.0504
	(0.0015)	(0.0015)
wage contract shares (%)		
nominal share	59	92
indexed share	37	0
auction share	4	8

(Standard deviations of shocks: $st.dev. of \epsilon = st.dev.\eta = 0.01$)

(productivity process: $\phi_t = \phi_{t-1} + \eta_t$)

(numbers in parentheses are stochastic simulation standard errors, as per Wallis, 1995)

Our stochastic simulation results for the two nominal level targets are compared in Table 3. According to the full nonlinear model indexation falls as the rule moves from pure money to a price rule. The net result is that the variance of real wages falls, while the variance of employment rises. Overall welfare is slightly higher under money level targeting

What seems to be happening is that the greater nominal rigidity induced by the price level target reduces the effect of productivity shocks on auction wages and employment as suggested in Figure 1. But it increases the effect of money shocks; and these are now more persistent and hence have higher cumulative variance.

With regard to real spendable wages, money level targeting includes the effect of the lagged price level times the indexation parameter. Price level targeting, by eliminating indexation, removes this effect, leaving only the temporary shock of the current price level.

Using the relevant equations above, and for simplicity approximating $\alpha = 0$, then under the money

rule, $m_t = \epsilon_t$, we have the following variances:

$$\text{Var}W_t = 4(lV)^2\sigma_\eta^2 + [(1-l)Q]^2\sigma_\epsilon^2 \quad (18)$$

and

$$\text{Var}(\overline{W}_{t-1} - p_t) = \pi^2 \{Q^2(1+v'^2)\sigma_\epsilon^2 + V^2(1+3(1-v')^2\sigma_\eta^2)\} \quad (19)$$

where $l \approx 0.7$, $v' \approx 0.5$. Whereas under the price rule we have approximating $v' = l = 0$:

$$\text{Var}W_t = 3(V)^2\sigma_\eta^2 + Q^2\sigma_\epsilon^2 \quad (20)$$

and

$$\text{Var}(\overline{W}_{t-1} - p_t) = \pi^2 \{Q^2\sigma_\epsilon^2 + V^2\sigma_\eta^2\} \quad (21)$$

What we see is that (21) is less than (19) and (20) more than (18), as in our simulations.

In terms of our representative household's welfare, the pure money rule is optimal; no intermediate (weighted average) combination of price-level and money-level targeting dominates this. The reason is that the variance of unemployment falls by slightly more than the (weighted) variance of consumption rises. They are only just distinguishable in welfare terms; nevertheless, the full model does confirm the finding of the linearised model, that something less than full price-level targeting, implying a modest degree of indexation, is optimal, because this is helpful to unemployment stability.

4 Conclusions

In this paper we have investigated the nature of optimal monetary policy. Rather than the widely-used New neo-Keynesian synthesis model, which we argued had some unresolved difficulties in this context, we used a model of overlapping wage contracts where an employed representative agent chooses an optimal degree of wage indexation (to prices and the auction wage) in response to the monetary regime. We found that monetary rules targeting the level of a nominal variable, whether money or prices, can do so without increasing macro instability, compared with monetary rules that target rates of change of the nominal variables; indeed they slightly decrease macro instability (i.e slightly increase the welfare of the representative agent). The reason is the strong shift in contract structure away from indexation to nominal. Thus this work is mildly encouraging to an idea that is of some interest to monetary authorities: that they can afford to aim for stationarity in nominal variables.

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Appendix A The Model — equations and calibration

Supply of work

$$a_t = a_c (W_t / (b_t P_{t-4}))^{-\sigma} \quad (\text{A1})$$

Demand for capital goods

$$K_t = (1 - k) (1 - \mu) E_{t-20} [d_t (1/R_t) (1 - T_t)] + k (1 - \mu) d_t (1 - T_t) (1/r_t) \quad (\text{A2})$$

Output Function

$$d_t = \phi_t K_t^{(1-\mu)} \{(1 - a_t) N\}^\mu \quad (\text{A3})$$

Wage rate, solved for \bar{W}_t

$$W_t = (1 - v - w) W_t + v E_{t-4} [W_t / P_t] P_t + w E_{t-4} [W_t] \quad (\text{A4})$$

Official Price Index

$$\ln P_t = \ln p_t + c (\ln p_t - \ln (E_{t-1} p_t)) \quad (\text{A5})$$

Goods market clearing, solved for r_t after substituting for K_t from (A2)

$$d_t = M_{t-1} / p_t + K_t - K_{t-1} \quad (\text{A6})$$

Labour market clearing, solved for p_t

$$N (1 - a_t) = (\mu d_t (1 - T_t) p_t) / \bar{W}_t \quad (\text{A7})$$

Money market clearing, solved for \bar{W}_t

$$M_t = N \{ \bar{W}_t (1 - a_t) + b_t P_{t-4} a_t \} \quad (\text{A8})$$

Efficiency

$$R_t = E_t [f(r)]^{1/20} - 1; f(r) = \prod_{i=1}^{20} \left(1 + \frac{r_{t+i}}{4} \right) \quad (\text{A9})$$

Money Supply

$$M_t = \bar{M}_t + m_t \quad (\text{A10})$$

Government budget constraint

$$b_t^g = (M_{t-1} - M_t + N B_t P_{t-4} a_t - d_t p_t T_t) / p_t + \left(1 + \frac{r_{t-1}}{4} \right) - b_{t-1}^g \quad (\text{A11})$$

Firm's budget constraint

$$d_t (1 - T_t) = K_t - K_{t-1} + (\bar{W}_t (1 - a_t) N) / p_t + b_{t-1}^p \left(1 + \frac{r_{t-1}}{4} \right) - b_t^p \quad (\text{A12})$$

Notes:

1. By Walras's Law the bond market clearing equation, $b_t^p + b_t^g = 0$, is redundant.
2. To normalise the variables d_t , K_t , r_t , p_t and \bar{W}_t to their base run values constant factors were applied to the right-hand sides of the following equations in their solved form: A2 1.11 (multiplicative); A3 0.629 (multiplicative); A6 +0.0135 (additive); A7 0.7 (multiplicative); A8 0.9574 (multiplicative).

A.1 Variables and coefficients for the CGE model

A.1.1 Endogenous variables : base run values

a_t	Supply of work	0.10
K_t	Demand for capital goods	6.00
d_t	Output function	1.00
W_t	Wage rate	1.00
P_t	Official price index	1.00
r_t	Real interest rate (fraction per annum)	0.05
p_t	Price level	1.00
\bar{W}_t	Average wage	1.00
R_t	Long term real interest rate (fraction per annum)	0.05
M_t	Money supply	1.00
b_t^g	Government bonds outstanding	0.00
b_t^p	Firms' bonds outstanding	0.00

A.1.2 Exogenous variables : base run values

B_t	Benefits	0.60
ϕ_t	$N(1.0, 0.01)$	1.00
\bar{M}_t	Money supply target	1.00
m_t	Money shock; $N(1.0, 0.01)$	0.00
T_t	Tax rate	0.10

A.1.3 Coefficients

Calibration is in line with assumed OECD experience. The contract length (m) is set at 4 quarters; the elasticity of leisure supply (σ) at 3; the share of stocks and other 'short-term' capital (k) at 0.3; the average life of other capital at 20 quarters; the share of labour income in value-added (μ) at 0.7 (the production function is Cobb-Douglas); the elasticity of the official price index to unanticipated inflation (c) at 0.2 (implying that a 1% unexpected rise in inflation would result in a 0.2% temporary overstatement of the price level faced by the representative consumer). The initial values assume 10% unemployment; a capital-output ratio of 6; an average (= marginal) tax rate of 0.10; and a real interest rate of 5%.

a_c	=	0.46
σ	=	3.00
k	=	0.30
μ	=	0.70
N	=	1.00
c	=	0.20

A.2 The model linearised and simplified

The following lists the linearised equations (the numbering corresponds to that of the full model of Appendix A). The numbers shown are the effect of the normalising constants referred to in Appendix A. In order the equations are: (A1) marginal labour supply which reacts to the auction wage; (A2) the demand for capital; (A3) the production function; (A4) the actual nominal wage, a weighted average of auction (weight of α), indexed (weight of v), and nominal.; (A5) the over-reaction of the official price index to the true price index; (A6) goods market-clearing; (A7) labour market-clearing; (A8) money market-clearing.; and (A9) the real spendable wage (wages are paid with a 1-period lag so the real spendable wage is the lagged one, deflated by the current price level).

$$a_t = -a_0\sigma W_t \quad (\text{A1})$$

$$K_t = 1.11 \left\{ \frac{k(1-\mu)(1-T_o)}{r_0} \right\} \left\{ (d_t - [\frac{r_t}{r_o}]) \right\} \quad (\text{A2})$$

$$d_t = \frac{(1-\mu)}{K_0} K_t + \phi_t - \frac{\mu}{(1-a_0)} a_t \quad (\text{A3})$$

$$\bar{W}_t = \alpha W_t + v P_t \quad (\text{A4})$$

$$P_t = (1+c)p_t \quad (\text{A5})$$

$$d_t = m_{t-1} - p_t + K_t - K_{t-1} \quad (\text{A6})$$

$$a_t = \frac{-[\mu(1-T_o)]}{0.7} (d_t + p_t - \bar{W}_t) \quad (\text{A7})$$

$$m_t = \frac{(1-a_o)}{0.96} \bar{W}_t - \left(\frac{\bar{W}_o}{0.96} - B_o \right) a_t \quad (\text{A8})$$

$$\bar{W}_{t-1} - p_t = \alpha W_{t-1} + v(1+c)p_{t-1} - p_t \quad (\text{A9})$$

where $\alpha = (1 - v - w)$; equation numbers correspond to Appendix A

For simplicity we have omitted all price and wage expectations from the wage-setting equation, (A4); these are all dated at $t - 4$. Similarly from the labour supply equation, (A1), we omit the 4-quarter lagged price level which indexes unemployment benefits. Hence in effect the model solves in terms of the news occurring between $t - 4$ and t , and in the case of (A9), the real wage available for spending, because wages are paid with a 1-period lag, news between $t-5$ and t . The very long lag (20 quarters) terms determining the demand for capital are similarly omitted. We now explain the model's structure in terms of supply and demand.

Equation (A9) is the implied behaviour of the employed consumer's living standard, whose uncertainty is being minimised by the contract structure. We can progressively reduce the simultaneous block of equations (A1)–(A8) to three as follows. We can use equation (A3), the production function, and (A6), the supply of savings from goods market clearing, while also using (A1) to eliminate a_t , to obtain:

$$\Delta d_t = Z(d_t - m_{t-1} + p_t) + \Delta \phi_t + \frac{a_o \mu \sigma}{(1 - a_o)} \Delta W_t \quad (\text{A10})$$

where $Z = \frac{1-\mu}{K_o}$

This is the output supply made available by savings (and so capital) and by labour supply; the first terms in Z emerge from equation (A6) as the amount of savings (i.e. the output not devoted to consumption which is $m_t - p_t$). Note in passing that we can solve equation (A2) for r_t conditional on d_t , m_{t-1} , and p_t : since the latter determine available savings, the interest rate has to force the demand for capital to equal this availability. Hence equation (A2) and the interest rate are in a second, recursive block, and can therefore be ignored.

Equations (A1) and (A7), labour supply and demand, yield with (A4) and (A5), defining wages and the price index,

$$W_t = \frac{\mu(1 - T_o)/0.7}{[\alpha\mu(1 - T_o)/0.7] + a_o\sigma} [d_t + (1 - v')p_t] \quad (\text{A11})$$

where $v' = v(1 + c)$

(A11) therefore specifies the free wages that would clear the labour market, given output and the price level. (A10) and (A11) between them constitute the supply-side of the model, augmented to include the market for savings (which depend on last period's money supply).

Finally, using the money market equation (A8) together with labour supply (A1) (which defines the split between benefits and wage payments) we obtain:

$$W_t = Q[m_t - (v'(1 - a_o)/0.96)p_t] \quad (\text{A12})$$

where

$$Q = \frac{1}{\alpha(1 - a_o)/0.96 + (\overline{W}_o/0.96 - B_o)a_o\sigma}$$

(A12) is reminiscent of Robertson's 'wages fund'; there is a certain stock of money available to pay wages and benefits and given the structure of contracts, it determines free (auction) wages.

The full solution is complex. However, we can represent the model's main workings by reducing equations (A10) and (A11) to a 'supply curve' between free wages and the price level; and juxtapose it with the 'demand curve' given by (A12), the wages fund equation. We neglect terms in Z as of small magnitude, which conveniently allows us to rewrite (A10) in levels form as

$$d_t = \phi_t + \frac{a_o\mu\sigma}{(1 - a_o)}W_t + c_0$$

where c_0 is a constant, ignored in what follows, reflecting the initial values of d_t , ϕ_t and W_t . In this case the supply curve from (A10) and (A11) becomes:

$$W_t = V[\phi_t + (1 - v')p_t] \text{ where } V = \frac{1}{\alpha + \frac{0.7a_o\sigma}{\mu(1 - T_o)} - \frac{a_o\mu\sigma}{1 - a_o}} \quad (\text{A13})$$

It follows that:

$$p_t = \pi(Qm_t - V\phi_t) \text{ where } \pi = \frac{1}{[Qv'(1 - a_o)/0.96] + V'(1 - v')} \quad (\text{A14})$$

and

$$W_t = lV\phi_t + (1 - l)Qm_t \text{ where } l = \frac{[Qv'(1 - a_o)/0.96]}{[Qv'(1 - a_o)/0.96] + V'(1 - v')} \quad (\text{A15})$$

and the spendable real wage is

$$\overline{W}_{t-1} - p_t = \alpha W_{t-1} + v'p_{t-1} - p_t \text{ where } \alpha = 1 - v - w \quad (\text{A16})$$

For $0 < v' < 1$ the resulting demand and supply picture is familiar. Figure 1 shows the model in price level, employment space; since labour supply $(1 - a_t)$ varies directly with the auction wage, this is also price level, auction wages space (output depends also on the capital stock, but is closely related to employment, and so this is also effectively familiar price, output space.) As v' tends to 0, DD (eq. A12) steepens and the SS (eq. A13) flattens.

The calibration emerging from the main model's chosen parameter values is: $\frac{V}{Q} = 1.3$; $Q = 3.85$; $\beta = 0.99$; $\alpha = 0.1$; $\pi = 0.2$; $l = v'$; $\psi = 0.3$, $\nu = 0.7$. We set $Vare = Var\eta = 0.0001$ as in our simulations.

Appendix B Targeting the level of money or prices — some mechanics of Monetary rules

If we take the linearised version of our model, we find the following solutions in general:

$$p_t = \pi(Qm_t - V\phi_t) \quad (\text{B17})$$

where

$$\pi = \frac{1}{Qv'(1 - a_0) + V(1 - v')}$$

and

$$W_t = lV\phi_t + (1 - l)Qm_t \quad (\text{B18})$$

where

$$l = \frac{Qv'(1 - a_0)}{Qv'(1 - a_0) + V'(1 - v')}$$

Recall that W_t (the auction wage, and the shadow price of labour supply) also directly determines employment through the labour supply function. Thus we can take it and employment as the same subject to some linear transformation.

Real (consumed) wages are:

$$\bar{W}_{t-1} - p_t = \alpha W_{t-1} + v'p_{t-1} - p_t \quad (\text{B19})$$

Suppose that

$$\phi_t = \phi_{t-1} + \eta_t \quad (\text{B20})$$

where η is an i.i.d. error; that is productivity follows a random walk. As noted above, the model implies that households raise their indexation level (v), the more persistent are price level shocks; clearly the persistence of productivity will create some persistence in price level shocks which will raise indexation.

If we now compare a money supply rule that eliminates money shock persistence with one that eliminates price shock persistence, the first plainly eliminates one independent source of persistence in price shocks. Thus we would expect to find, and do, that indexation falls. The second takes the process one stage further, eliminating all price shock persistence. Thus we should find that indexation disappears.

A price level rule is one that sets

$$0 = E_t p_{t+1} = \pi(QE_t m_{t+1} - VE_t \phi_{t+1}) \quad (\text{B21})$$

and hence

$$E_t m_{t+1} = \frac{V}{Q} E_t \phi_{t+1} = \frac{V}{Q} \phi_t \quad (\text{B22})$$

whence the ‘price level rule’ is

$$m_t = \frac{V}{Q} \phi_{t-1} + \epsilon_t \quad (\text{B23})$$

whereas the (‘pure money’) rule that eliminates money shock persistence is simply

$$m_t = \epsilon_t \quad (\text{B24})$$

Notice that under the price level rule money supply accommodates known past productivity shifts; this removes persistence from price shocks, though at the cost of persistence in money shocks.

When these are substituted into (B18) we obtain

$$\text{(price level rule)} \quad W_t = (1-l)Q\epsilon_t + V(l\phi_t + [1-l]\phi_{t-1}) \quad (\text{B25})$$

$$\text{(pure money rule)} \quad W_t = (1-l)Q\epsilon_t + V(l\phi_t) \quad (\text{B26})$$

and when into (B19) we obtain:

$$\begin{aligned} \text{(price level rule)} \quad \bar{W}_{t-1} - p_t = \pi V[\eta_t - v'\eta_{t-1}] + \alpha V[l\eta_{t-1} + \phi_{t-2}] - \\ \pi Q[\epsilon_t - v'\epsilon_{t-1}] + \alpha Q(1-l)\epsilon_{t-1} \end{aligned} \quad (\text{B27})$$

and

$$\begin{aligned} \text{(money level rule)} \quad \bar{W}_{t-1} - p_t = \pi V[\phi_t - v'\phi_{t-1}] - \pi Q[\epsilon_t - v'\epsilon_{t-1}] + \\ \alpha V[l\phi_{t-1}] + \alpha Q(1-l)\epsilon_{t-1} \end{aligned} \quad (\text{B28})$$

Both V and Q vary inversely with the share of auction contracts, α :

$$Q = \frac{1}{\alpha(1-a_o)/0.96 + (\bar{W}_o/0.96 - B_o)a_o\sigma}$$

$$V = \frac{1}{\alpha + \frac{0.7a_o\sigma}{\mu(1-T_o)} - \frac{a_o\mu\sigma}{1-a_o}}$$