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*Partial Current Information and Signal Extraction in a
Rational Expectations Macroeconomic Model: A
Computational Solution.*

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Partial Current Information and Signal Extraction in a Rational Expectations
Macroeconomic Model: A Computational Solution.

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Abstract

Previous attempts at modelling current observed endogenous financial variables in a
macroeconomic model have concentrated on only one observed endogenous variable
– namely the short-term rate of interest. The solution method for dealing with more
than one observed endogenous variable has thus far been computationally intractable.
This paper applies a general search algorithm to a macroeconomic model with an
observed interest rate and exchange rate to solve the signal extraction problem. The
informational advantage of applying the signal extraction algorithm to all the current
observed endogenous variables is examined in terms of the implication for policy
from the misperceptions of specific macroeconomic shocks.

Keywords: Rational Expectations, Partial Current Information, Signal Extraction,
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1. Introduction

The implication of partial current information in rational expectations models was first demonstrated by Lucas (1972) in his "islands story" of spatially separated markets. There is no doubt that the insights unveiled by Lucas (1972) and Lucas (1973), have had a major impact on the modern macroeconomic thinking. The assumption of partial information alters the solution of rational expectations (RE) models since, current observed variables contain partial information about current disturbances. The inferences made from this information will in turn, influence the current state of the system, and hence the observation themselves.

While Lucas's articles have been widely cited there has been little subsequent empirical work on macroeconomic models that embed the signal extraction assumption. Arguably, the most important reason why these papers have not generated more empirical and numerical results is because specifying and determining an equilibrium within which agents extract signals from endogenous variables have proved to be technically complex[2].

The method of solving forward-looking RE macro-models conditional on full past information is widely practised by a variety of computer algorithms. The problem emerges when the agents' information set contains all past data and partial current data. Previous attempts[3] at modelling current observed endogenous financial variables in a macroeconomic model have concentrated on only one observed endogenous variable – namely the short term rate of interest (Matthews et. al 1994a, b). The solution method for dealing with more than one observed endogenous variable has so far been computationally intractable.

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[2] For some previous attempts on modelling the signal extraction in macro models see for example, Thomas Sargent (1991), Neil Wallace (1992), Jean-Pascal Benassy (1999,2001), Pearlman et al. (1986).

[3] Methods, using state-space representations and the Kalman filter, for solution in linear models have been proposed by Pearlman et al. (1986) and Sargent (1991). However, these methods cannot be used for non-linear models unless linearised around a particular path, which could prove to be computationally expensive and costly in research time.

This paper applies a general search algorithm to a macroeconomic model with an observed interest rate and exchange rate to solve the signal extraction problem. The algorithm is tested against a linear model with a known analytical solution. The model is solved numerically and examines the implication of signal extraction for the interpretation of shocks in a simulation framework. Observation of the current values of macroeconomic variables is shown to offer a possible explanation of why the economy might respond 'paradoxically' to shocks.

The exposition is organised as follows. The conceptual framework is set out in section 2. Section 3 describes a stylised open economy macro model with partial information and shows how the analytical solution is obtained. The algorithm is outlined in section 4. Section 5 applies the algorithm to a numerical version of the model and examines the implication of partial current information for the interpretation of shocks to the model. Section 6 concludes.

2. The Conceptual Framework

The application of partial current information can be viewed as a solution to the "ragged edge" problem of forecasting, where the forecaster is aware of the values of some endogenous variables only with a lag but can observe other current endogenous variables at the time of forecast. A typical example is the observation of current interest rates and exchange rates. The framework for the use of observed endogenous variables in forecasting with a linear model is examined in Wallis (1986), which applies the properties of the multivariate normal distribution in order to obtain the optimal forecast.

Consider a general structural form of a linear stochastic econometric model:

$$Fy_t + Gx_t = \varepsilon_t \tag{2.1}$$

where y_t is a vector of endogenous variables, x_t is a vector of pre-determined variables, ε_t is a vector of stochastic disturbance terms, and $\mathbf{F}$ and $\mathbf{G}$ are appropriate matrices of coefficients of the known structural parameters. The stochastic disturbances are assumed to be normally distributed with mean zero and the covariance matrix $\mathbf{E}(\varepsilon_t \varepsilon_t^T) = \mathbf{\Sigma}$, where superscript T denotes transpose. Also, each

stochastic disturbance term is assumed to be uncorrelated with any stochastic disturbance term at any other point in the sample.

If matrix F is non-singular the structural form (2.1) can be solved for the endogenous variables as explicit functions of all exogenous variables and stochastic disturbance terms. Pre-multiplying (2.1) by $\mathbf{F}^{-1}$ and solving for y_t yields the reduced form:

$$y_t = \Pi x_t + \omega_t \qquad (2.2)$$

where $\Pi = -F^{-1}G$ and $\omega_t = F^{-1}\varepsilon_t$.

The covariance matrix of ω_t is Ω and is given by:

$$E(\omega_t \omega_t^T) = F^{-1}E(\varepsilon_t \varepsilon_t^T)(F^{-1})^T = F^{-1}\Sigma(F^{-1})^T = \Omega \qquad (2.3)$$

The reduced form (2.2) uniquely determines the probability distributions of the endogenous variables, given the exogenous variables, the coefficients, and the probability distributions of the stochastic disturbance terms.

The equality in (2.3) implies that

$$\Sigma = \mathbf{F}\Omega\mathbf{F}^{\mathbf{T}} \qquad (2.4)$$

showing the relationship between the covariance matrix of the structural form Σ and that of the reduced form Ω. The assumptions related to the elements of Σ imply that the forecasting equations are of the form:

$$\hat{\mathbf{y}}_{\mathbf{t}} = \mathbf{\Pi}\mathbf{x}_{\mathbf{t}} \qquad (2.5)$$

From (2.2) and (2.5) it immediately follows that, if all future exogenous variables are treated as known, the one-step-ahead forecast errors $y_t - \hat{y}_t$ coincide with the reduced formed disturbances ω_t.

To differentiate between the variables that are known at the time of the forecast and those that are not, it is useful to partition the reduced form equation (2.2) as follows:

$$\begin{bmatrix} \mathbf{y}_{1t} \\ \mathbf{y}_{2t} \end{bmatrix} = \begin{bmatrix} \mathbf{\Pi}_1 \\ \mathbf{\Pi}_2 \end{bmatrix} \mathbf{x}_t + \begin{bmatrix} \omega_{1t} \\ \omega_{2t} \end{bmatrix} \tag{2.6}$$

With the elements of the sub-vector y_{1t} assumed to be known, the optimal forecast of the unobserved vector y_{2t} is given by its conditional expectation,

$$E(y_{2t} \mid y_{1t}) = \Pi_2 x_t + \Omega_{21}\Omega_{11}^{-1}(y_{1t} - \Pi_1 x_t) \tag{2.7}$$

where the Ω matrix is partitioned as

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix}$$

Thus, the required forecast is given by the unconditional forecast $\Pi_2 x_t$ to which it is added the observed forecast errors $y_{1t} - \Pi_1 x_t$, with coefficients $\Omega_{21}\Omega_{11}^{-1}$.

3. The Theoretical Model

3.1 The Structure of the Model

Consider the following open economy version of the Minford and Peel (1983) model. The model assumes the existence of a global capital market and partial information:

$$Y_t = -\alpha(R_t - E_t P_{t+1} + E_t P_t) - \gamma(S_t + P_t) + u_t \tag{3.1}$$

$$Y_t = \beta(P_t - E_t P_t) + (1-\mu)Y^* + \mu Y_{t-1} + v_t \tag{3.2}$$

$$M_t^d = P_t + Y_t - \delta R_t \tag{3.3}$$

$$M_t^s = (1-\theta)M^* + \theta M_{t-1} + \varepsilon_t \tag{3.4}$$

$$R_t = S_t - E_t S_{t+1} \tag{3.5}$$

where $\alpha, \beta, \mu, \delta, \gamma$ and θ are all positive real numbers and u, v *and* ε are random shocks normally independently distributed with known variances $\sigma_u^2, \sigma_v^2, \sigma_\varepsilon^2$. Here Y denotes output, R is the nominal interest rate, P represents the price level, S is the exchange rate, M^d is money demand and M^s denotes money supply. Long -run equilibrium values of output and money are denoted by Y^* and M^* respectively. Following the usual tradition all variables except the interest rate are in logarithms.

The subscript *t* indicates time and the mathematical operator E_t denotes the expectation conditional on information available at time t. Consequently, E_tP_t represents the current expectation conditional on last period's full data and this period's partial data.

Equation (3.1) is an open economy version of the IS curve, (3.2) is a Sargent and Wallace type supply curve which allows for the persistence of shocks, (3.3) is a conventional money demand function, (3.4) is a money supply rule with a feedback response $\theta(M_{t-1} - M^*)$, and (3.5) is the uncovered interest parity condition where the foreign rate of interest is assumed to be zero for convenience.

The effects of the shocks on the model solutions depend on the signal extraction agents make. Because they know what the model is, after observing current endogenous variables R_t and S_t, agents form an expectation regarding the shocks, which will be a function of the model parameters and the known variances of the shocks. To solve the model we first need to get the expressions for the expected shocks.

3.2 Extracting Signals about Unexpected Shocks

Rational expectations models with expectations based on information available in the current period assume that agents are aware of all relevant information, including current innovations. However, in the case of partial current information their task is more difficult. Agents are limited in their current knowledge of the economy and they face a signal extraction problem having to estimate the unobserved current innovations from the observed variables in the system. The conditional expectations of the innovations will turn out, as discussed below, to be a linear combination of the two pieces of current information. The first piece of information is contained in the observation of the interest rate R_t and the second is contained in the observation of the exchange rate S_t.

The observed vector of current endogenous variables, $\mathbf{Z_t}$, is assumed to be a function of a deterministic component plus a linear combination of reduced form shocks. In matrix form it can be written as:

$$Z_t = \Psi^T X_{t-1} + K^T U_t \qquad (3.6)$$

where,

$$Z_t = \begin{bmatrix} R_t \\ S_t \end{bmatrix}, \quad \Psi = \begin{bmatrix} \Psi_{11} & \Psi_{21} \\ \Psi_{12} & \Psi_{22} \\ \Psi_{13} & \Psi_{23} \\ \Psi_{14} & \Psi_{24} \end{bmatrix}, \quad X_{t-1} = \begin{bmatrix} Y^* \\ M^* \\ Y_{t-1} \\ M_{t-1} \end{bmatrix}, \quad K = \begin{bmatrix} A & A' \\ B & B' \\ C & C' \end{bmatrix},$$

$$U_t = \begin{bmatrix} u_t \\ v_t \\ \varepsilon_t \end{bmatrix}$$

Although the assumption of linearity regarding the current effects on current one-period shocks is made to make the solution more tractable, as experience shown, it is also a good approximation for most non-linear macro models.

The elements in matrix X_{t-1}, which consist of the long-run equilibrium values of output and money and their lagged values, are assumed throughout to be contained in the current individual information set so that $\Psi^T X_{t-1} = E(Z_t \mid \Phi_{t-1})$. Matrices Ψ and K contain elements that are constants derived from the model parameters[4]. Taking expectations of (3.6) and noting that $Z_t = E_t Z_t$, we get:

$$Z_t = E(Z_t \mid \Phi_{t-1}) + K^T E_t(U_t) \qquad (3.7a)$$

Subtracting 3.6 from 3.7a yields

$$K^T E_t(U_t) - K^T U_t = 0 \qquad (3.7b)$$

The conditional expectation of the vector $E_t(U_t)$ contains information, which is revealed in the observation of both R_t and S_t, that is:

$$E_t(U_t) = \Gamma^T U_{RS} \qquad (3.8)$$

where

$$\Gamma = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \end{bmatrix}, \quad U_{RS} = \begin{bmatrix} R_t - E(R_t \mid \Phi_{t-1}) \\ S_t - E(S_t \mid \Phi_{t-1}) \end{bmatrix}$$

---

[4] This result is valid as long as the variances of the shocks are stable as it is the present case.

It can be seen that equation 3.8 above has the same interpretation as the equation 2.7. Intuitively the elements of the $\boldsymbol{\Gamma}$ matrix can be seen as least-squares estimates - using the known population variances and covariances - obtained by regressing the elements of $\mathbf{U_t}$ on the vector of unobserved structural form shocks given by $\mathbf{U_{RS}}$. Under the assumption of independence and normality of current disturbances u_t, v_t, and ε_t, the elements of the $\boldsymbol{\Gamma}$ matrix can be easily determined using the conditional probability properties for the multivariate normal distribution (see also Graybill, 1962). Thus,

$$\boldsymbol{\Gamma} = (\mathbf{K^T \Omega K})^{-1} \mathbf{K^T \Omega} \tag{3.9}$$

where $\boldsymbol{\Omega}$ is a diagonal (3x3) variance-covariance matrix[5] given by:

$$\Omega = \begin{bmatrix} \sigma_u^2 & 0 & 0 \\ 0 & \sigma_v^2 & 0 \\ 0 & 0 & \sigma_\varepsilon^2 \end{bmatrix}$$

More explicitly, the general linear solution for the rate of interest and the exchange rate is of the form:

$$R_t = \Psi_{11} Y^* + \Psi_{12} M^* + \Psi_{13} Y_{t-1} + \Psi_{14} M_{t-1} + A u_t + B v_t + C \varepsilon_t \tag{3.10}$$

$$S_t = \Psi_{21} Y^* + \Psi_{22} M^* + \Psi_{23} Y_{t-1} + \Psi_{24} M_{t-1} + A' u_t + B' v_t + C' \varepsilon_t \tag{3.11}$$

and the expected shocks are given by:

$$E_t u_t = \frac{\alpha_1}{A} [R_t - E(R_t \mid \Phi_{t-1})] + \frac{\beta_1}{A'} [S_t - E(S_t \mid \Phi_{t-1})] \tag{3.12}$$

$$E_t v_t = \frac{\alpha_2}{B} [R_t - E(R_t \mid \Phi_{t-1})] + \frac{\beta_2}{B'} [S_t - E(S_t \mid \Phi_{t-1})] \tag{3.13}$$

$$E_t \varepsilon_t = \frac{\alpha_3}{C} [R_t - E(R_t \mid \Phi_{t-1})] + \frac{\beta_3}{C'} [S_t - E(S_t \mid \Phi_{t-1})] \tag{3.14}$$

---

[5] The covariances between the innovations were all set to zero in order to make the solution more tractable.

In (3.12) – (3.14) the terms of the unexpected components of the interest rate, $R_t - E(R_t \mid \Phi_{t-1})$, and exchange rate, $S_t - E(S_t \mid \Phi_{t-1})$, have an obvious regression interpretation.

3.3 Model Solution

By solving the model given by equations (3.1) – (3.5), the solutions for the exchange rate and interest rate turn out, as shown in the Appendix A, to be

$$
\begin{aligned}
E_t S_t (= S_t) = E(S_t \mid \Phi_{t-1}) + \frac{1}{(\alpha + \gamma)(1 + \delta)} E_t u_t + \\
\frac{1}{1 + \delta - \delta\mu}\left(1 - \frac{1}{\alpha + \gamma - \alpha\mu}\right) E_t v_t - \frac{1}{1 + \delta - \delta\theta} E_t \varepsilon_t
\end{aligned}
\tag{3.15}
$$

$$
\begin{aligned}
E_t R_t (= R_t) = E(R_t \mid \Phi_{t-1}) + \frac{1}{(\alpha + \gamma)(1 + \delta)} E_t u_t + \\
\left[\frac{1 - \mu}{1 + \delta - \delta\mu}\left(1 - \frac{1}{\alpha + \gamma - \alpha\mu}\right)\right] E_t v_t - \frac{1 - \theta}{1 + \delta - \delta\theta} E_t \varepsilon_t
\end{aligned}
\tag{3.16}
$$

The solutions for the exchange rate - given by (3.15) - and the interest rate – given by (3.16) – are a function of the expected shocks which, in turn depend on the value of the coefficients A, B, C, A', B', C'. To obtain these values we first substitute the expected values of the innovations given by equations (3.12) - (3.14) into (3.15) and (3.16). Using the method of undetermined coefficients we obtain:

$$
A = -S(1 - \theta)\left(\alpha_3 \frac{A}{C} + \beta_3 \frac{A'}{C'}\right) + U(1 - T)(1 - \mu)\left(\alpha_2 \frac{A}{B} + \beta_2 \frac{A'}{B'}\right) + V(\alpha_1 + \beta_1)
\tag{3.17}
$$

$$
A' = -S\left(\alpha_3 \frac{A}{C} + \beta_3 \frac{A'}{C'}\right) + U(1 - T)\left(\alpha_2 \frac{A}{B} + \beta_2 \frac{A'}{B'}\right) + V(\alpha_1 + \beta_1)
\tag{3.18}
$$

$$
B = -S(1 - \theta)\left(\alpha_3 \frac{B}{C} + \beta_3 \frac{B'}{C'}\right) + U(1 - T)(1 - \mu)(\alpha_2 + \beta_2) + V\left(\alpha_1 \frac{B}{A} + \beta_1 \frac{B'}{A'}\right)
\tag{3.19}
$$

$$
B' = -S\left(\alpha_3 \frac{B}{C} + \beta_3 \frac{B'}{C'}\right) + U(1 - T)(\alpha_2 + \beta_2) + V\left(\alpha_1 \frac{B}{A} + \beta_1 \frac{B'}{A'}\right)
\tag{3.20}
$$

$$
C = -S(1 - \theta)(\alpha_3 + \beta_3) + U(1 - T)(1 - \mu)\left(\alpha_2 \frac{C}{B} + \beta_2 \frac{C'}{B'}\right) + V\left(\alpha_1 \frac{C}{A} + \beta_1 \frac{C'}{A'}\right)
\tag{3.21}
$$

$$C' = -S(\alpha_3 + \beta_3) + U(1-T)\left(\alpha_2 \frac{C}{B} + \beta_2 \frac{C'}{B'}\right) + V\left(\alpha_1 \frac{C}{A} + \beta_1 \frac{C'}{A'}\right) \qquad (3.22)$$

where for simplicity we denote:

$$S = (1+\delta - \theta\delta)^{-1}, \ T = (\alpha + \gamma - \mu\alpha)^{-1}, \ U = (1+\delta - \mu\delta)^{-1}, \ V = (\alpha + \gamma)^{-1}(1+\delta)^{-1}$$

Unfortunately the solution for the constants A, B, C, A', B', C' cannot be written as closed-form expressions because the α_i and β_i (i=1,2,3) are themselves non-linear in the constants. However, the system of equations given by (3.17) – (3.22) can be solved numerically.

4. The Solution Algorithm

This section describes a general search algorithm which, applied to a macroeconomic model with an observed interest rate R and exchange rate S, solves the signal extraction problem. The algorithm according to Minford and Webb (2002) searches over the parameter space for the undetermined coefficients relating the observed endogenous variables to the unobserved current shocks in a way similar to a hill-climbing search. The superiority of this method over other search algorithms resides in its relative simplicity of implementation. The convergence process is also achieved relatively quickly without a loss in accuracy[6].

The solution algorithm searches for the coefficients for which the absolute differences $[R_t - E(R_t | \Phi_{t-1})]$ and $[S_t - E(S_t | \Phi_{t-1})]$ are both less than a tolerance level, taking the following steps:

Initialisation. Choose a set of shocks $u_0, v_0, and \varepsilon_0$, a set of initial guesses for the parameters to be determined, the step variations, and specify a tolerance limit λ. Construct a base run conditional on information set Φ_{t-1} from which $E_t(R | \Phi_{t-1}) and E_t(S | \Phi_{t-1})$ are obtained.

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[6] It is well known that, with large numbers of parameters both grid and normal multidimensional hill-climbing search, where combinations of parameters are varied, are computationally difficult to implement.

Step 1. Using the signal extraction formula given by equation (3.8) compute a set of expected shocks $Eu_0, Ev_0, and E\varepsilon_0$.

Step 2. The model is solved conditional on the expected shocks from *step 1* and the solution values obtained for interest rate ($E_t R_t$) and exchange rate ($E_t S_t$) are retained.

Step 3. With expectations held constant from *step 2* the model is simulated for the actual shocks. Again the solutions for the interest rate - call this $\mathbf{R}^*$ - and exchange rate – call this $\mathbf{S}^*$ are retained.

Step 4. Check if the sum of the absolute differences $|R^* - E_t R_t| + |S^* - E_t S_t| \leq \lambda$ [7]. If the inequality holds, the process stops if it does not, go to *step 5*.

Step 5. Each initial parameter is varied in turn by plus and minus some percentage of its initial value and for each of these changes a new set of pairs ($\mathbf{R}^*$, $E_t R_t$) and ($\mathbf{S}^*$, $E_t S_t$) is calculated. Whichever parameter's movement generates a sum of the absolute differences that is closer to λ is adopted.

Step 6. With the newly altered parameter set the procedure from *step 5* continues until either the convergence criteria specified at *step 4* is achieved, in which case the algorithm stops, or there is no improvement in the minimisation criteria from *step 4*, in which case go to *step 7*.

Step 7. Once a step variation is exhausted, the search process continues, repeating *steps 4 - 6* for all pre-defined step variations. The algorithm stops if no improvement in the minimisation criteria is obtained for the last pre-defined step variation.

The starting values for the parameters were obtained as follows. Given a set of initial conditions and known fixed exogenous variables a base run on lagged information was constructed. Next, the model was shocked by a hundred drawings of innovations – $u, v, \varepsilon \approx N(0, 0.1)$ - for each of the three behavioural equations. Finally,

the model deviation from its base value for each of the two observed endogenous variables were regressed on the set of drawings of the shocks.

It is worth noting that the values from the regressions obtained in this way are taken to be only indicative starting values. This is because we assume that the variance of all three sets of shocks is the same. If the variances were different and, for example, a noise in one of the innovations predominated, the agents would misinterpret the effects of the shocks. Such an imperfect signal perception implies that the actual coefficients could, in such a case, be different. In practice it might be useful not to impose a tolerance limit but to leave the algorithm to find a minimum. Thus, if the algorithm finds more than one set of parameters that satisfy the minimisation criteria from *step 4* it will choose the one which is the closest to zero.

Given the initial starting values for the parameters a certain minimisation number - representing the left-hand side of the inequality from *step 4* - is obtained. Then, according to the pre-defined step variation each of the parameters are sequentially increased and decreased by the corresponding amount. Thus, if the number of the parameters to be determined was '*n*' the programme would generate '*2n+1*' sets of parameters - including the initial starting values - and for each set a minimisation criteria would be computed. Each set will contain '*n-1* 'of the initial coefficients and a coefficient changed by the pre-specified percentage, either up or down. The set corresponding to the lowest absolute value given by the minimisation criteria is then used for the next iteration replacing the initial starting values in the input file.

The search process begins with +/– 50% variations in parameter values. The algorithm uses 6 pre-defined step variations: {50, 25, 10, 5, 2, and 1}%. Once there is no improvement in the minimisation criteria for a given percentage change the programme 'jumps' to the next one until a minimum is reached for the last change. The above values were arbitrarily chosen. They can be easily modified and is

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[7] It is the sum of the absolute differences that is minimised and not each absolute difference in part due to massive computational difficulties that arise from number manipulations in the later case.

advisable to start with a larger percentage change especially when there is no indication about the area where the final solution set may be.

As with all algorithms there is no guarantee it will find the global maximum. With a single set of errors the algorithm achieves convergence relatively quickly. But, there is still a possibility that the parameters obtained in this way represent a local solution. In order to reduce the likelihood of such an event occurring we used 100 different sets of errors as input files[8].

On average, with a 100 sets of shocks the time length varies between 12-15 minutes per iteration and it could take 45-50 iterations. Obviously, the length of the convergence process is sensible to the initial starting values, the pre-defined percentage changes, and the load of the system.

5. A Numerical Model

This section describes the solution of the model given by equations (3.1) – (3.5) for a particular numerical example[9]. Our objective is to find the unique set of constants represented by the elements of **K** that satisfy equation (3.7b).

A constant term (Yc = 10.02) was added to equation (3.1), and the remaining parameter values of the model were set as:

$\alpha = 0.2$ $\qquad \beta = 0.3$ $\qquad \mu = 0.8$ $\qquad \delta = 0.5$ $\qquad \gamma = 0.5$ $\qquad \theta = 0.6$

The exogenous variables were set as $Y^* = 10$, $M^* = 1$ and the initial conditions $M_0 = 0.9$, $Y_0 = 9.5$, and $S_0 = -0.9$. The model was solved for 15 periods for forward rational expectations with the tolerance for successive iterates being arbitrarily set to 0.005. Given the initial conditions and the known set of fixed exogenous variables[10]

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[8] Now, for one iteration, in the case of 'n' parameters, the programme computes (2n+1)x100 minimisation criteria corresponding to each set of shocks for each of the '$2n+1$'sets of coefficients. For the same set of coefficients a resulting minimisation criteria is calculated which is taken to be an average across 100 shocks corresponding values. The computational burden increases somewhat, nevertheless the time length in which the convergence is achieved is still fairly reasonable.

[9] The calculations of this section were executed using an algorithm that solves the rational expectations models by dynamic programming using the method of terminal conditions as in Minford et al. (1980).

[10] The exogenous variables for the base run were fixed to their long–run equilibrium values. The money supply was held fixed for all simulations.

we first constructed a base run that is also consistent with the lagged information expectation.

The second step was to generate the expected innovations using equations (3.12) – (3.14). The starting values for the elements of matrix **K** were obtained from the least squares regression of $[R - E(R \mid \Phi_{t-1})]$ and $[S - E(S \mid \Phi_{t-1})]$ on $u, v, and\ \varepsilon.$ For a randomly selected sample size of 100 sets of shocks, the regression coefficients turned out to be:

$$K^{regression} = \begin{bmatrix} 0.98555 & 0.942497 \\ -0.218797 & -0.754112 \\ -0.658378 & -0.670439 \end{bmatrix}$$

The model was shocked by the expected innovations to generate the expected outcome given by a set of data Y, P, R, S, and r, where r is the real rate of interest.

The third step was to shock the model by the actual innovations keeping the expectations from the previous step fixed. This is because the expectations have already being formed and cannot be changed in the current period. Of course they will be altered in the following period once the true nature of the shocks is revealed.

The true values of the elements of **K** were obtained by algorithmic solution[11].

$$K^{true} = \begin{bmatrix} 0.944714 & 0.931200 \\ -0.128436 & -0.707906 \\ -0.361813 & -0.902263 \end{bmatrix},$$

and the corresponding values of the constants α_i and β_i (i=1,2,3) are

$$\Gamma^{true} = \begin{bmatrix} 1.374271 & -0.139589 & -0.234682 \\ -0.407781 & 0.686915 & 0.720866 \end{bmatrix}$$

The coefficients corresponding to the lower minimisation criteria obtained using the solution algorithm were

$$K^{algorithm} = \begin{bmatrix} 0.944675 & 0.932712 \\ -0.128313 & -0.707855 \\ -0.361822 & -0.902401 \end{bmatrix}$$

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[11] It is well known that the Gauss-Seidel method needs good initial guesses otherwise the convergence may not achieved. And there is still the question regarding the uniqueness of the solution set. We tried different starting values for which the solution algorithm converged either to the same coefficient values presented here or to a solution set that was a multiple of them. Multiple solution sets would also make little economic sense (see also Barro 1980).

Comparing the two sets of coefficients it can be seen that the algorithm settles on a set of parameters in the neighbourhood of the true values.

The informational advantage of applying the signal extraction algorithm to all the current observed endogenous variables is examined in terms of the implication for a forecasting exercise and the response of the model to various shocks. The paper proceeds as an exercise on simulation, which identifies some paradoxical responses due to misperceptions of specific macroeconomic shocks. As an example we examine the effects on inflation and output of an unanticipated temporary shock to the IS schedule, a shock to the supply curve, and an expansionary monetary shock. However, we also examine the implication for forecasting accuracy as discussed in Appendix C. If the elements of matrix **K** are the true values of the searched coefficients then we should observe a gain in the forecasting efficiency under the assumption of current partial information. The error statistics reported in tables C1 and C2 in Appendix C confirm the improvement in forecasting efficiency.

Tables 5.1 – 5.3 present the results. The first column reports the base run values, which are of course, the same for all three simulations. Armed with the information on the two global variables, the interest and exchange rate, and all the equations of the (global) macro-model the agents form a view of the shocks driving the observed global variables, which must be consistent with what the model would produce. This is the expected outcome presented in column 2. But it must also be the case that the actual shocks, which will in general differ from what they expect, must via the model produce the same interest rate and exchange rate. This is the actual outcome shown in column 3. Column 4 reports what happens when the values of R_t and S_t are not known in the current period. The results presented in columns 3 and 4 represent the response of output and the price level in the first period, when the shock occurs. Charts 1 – 6 in Appendix B show the behaviour of output and inflation for the whole simulation period for the three shocks considered.

All shocks exhibit conventional effects. A positive shock to the IS schedule raises output and the price level (and thereby inflation). Across the expected and actual outcome the values of interest and exchange rate are the same. However, as it

can be seen from Table 5.1 output expands by less and the price level increases by more than under the assumption of lagged information.

Table 5.1. A Shock to the IS schedule

Actual Shocks[12]: u = 0.0434, v = 0, ε = 0, Expected Shocks: Eu = 0.0419, Ev = 0.0053, Eε = -0.0057

	Base Run	Partial Information	Partial Information	Lagged Information
		Expected Outcome	Actual Outcome	
R	0.1508	0.2087	0.2088	0.1997
Y	9.5995	9.6048	9.6020	9.6046
P	1.3833	1.3937	1.4022	1.4006
R	0.0651	0.1080	0.1081	0.1098
S	-0.603	-0.5634	-0.5634	-0.5630

In this case the magnitude of the expected shock to the IS curve, 0.0419, is close to the magnitude of the actual shock, 0.0434. In addition agents expect a small positive supply shock and a negative monetary shock which have an overall effect of dampening the expansion in output – shown in Chart 1 as fraction of a difference from base - under the assumption of partial information.

Table 5.2 shows the results for an unanticipated negative aggregate supply shock. The shock is misinterpreted as a combination of negative shocks, with almost similar weights being assigned to the expected aggregate supply and monetary shocks. The contraction in output shown in Chart 3 is marginally higher in the partial information case. Also inflation (Chart 4) raises by slightly more under the partial information.

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[12] The shocks used here were randomly chosen from three independent N ~ (0,0.1)

Table 5.2. Aggregate Supply Shock

Actual Shocks: u = 0, v = -0.0239, ε = 0, Expected Shocks: Eu = -0.0029, Ev = -0.0131, Eε = -0.0115

	Base Run	Partial Information	Partial Information	Lagged Information
		Expected Outcome	Actual Outcome	
r	0.1508	0.1545	0.1545	0.1431
Y	9.5995	9.5861	9.5805	9.5820
P	1.3833	1.3892	1.4064	1.4048
R	0.0651	0.0744	0.0745	0.0729
S	-0.603	-0.5888	-0.5888	-0.5861

The implication of signal extraction is not so obvious in this case. The negative supply shock is interpreted as a combination of negative demand, supply and monetary shocks that broadly produces a similar overall outcome to the lagged information case.

The results for the current period for an unanticipated increase in the money supply are shown in Table 5.3. Again the shock is misinterpreted as a combination of supply, demand, and monetary shocks.

Table 5.3. Money Supply Shock

Actual Shocks: u = 0, v = 0, ε = 0.0298, Expected Shocks: Eu =-0.0039, Ev = 0.0144, Eε = 0.0145

	Base Run	Partial Information	Partial Information	Lagged Information
		Expected Outcome	Actual Outcome	
r	0.1508	0.1346	0.1355	0.1326
Y	9.5995	9.6134	9.6058	9.6043
P	1.3833	1.3741	1.3971	1.3991
R	0.0651	0.0464	0.0473	0.0468
S	-0.603	-0.6227	-0.6220	-0.6214

However, the size of the expected monetary shock, 0.0145, is half the size of the actual shock. Moreover the agents expect a relatively large positive aggregate supply shock. As a consequence, under partial information output increases by more than under lagged information (Chart 5). Due to the presence of the output persistence term in the aggregate supply equation it takes 9 quarters for output to come back to its

long-run equilibrium in the partial information case compared to 6 quarters under lagged information.

Clearly the knowledge of current partial information can have strong implications for the short–term properties of the model. Because the expected shocks are a function of the model parameters and the variances of the shocks, a different combination of these is likely to impact differently on the behaviour of output and inflation. A decomposition of the historical shocks could yield useful information regarding the expectations formation.

The existence of lagged effects in the model implies that the influence of the expected shocks will persist even after agents have discovered the true nature of the shocks. This shows the striking complexity in the contemporaneous response of the economy to shocks. Thus, the global signal extraction process can be viewed as a further contemporaneous transmission mechanism of shocks, over and above their direct transmission mechanism.

6. Conclusion

We have presented a simple algorithm for the solution of a rational expectations model with an observed interest rate and exchange rate which was used to solve the signal extraction problem. Its implementation is extremely useful in non-linear models where an analytical solution is difficult to obtain. The algorithm was tested on a theoretical model with a known analytical solution. Convergence was achieved without a loss in accuracy in a short number of iterations but this came at no surprise given the linearity of the model.

There are two potential applications for the use of the algorithm. Firstly, it may be used to explain the apparent peculiar responses of the economy in some circumstances. The effects of the shocks being misinterpreted are well known at the level of everyday comment. A suggestive example would be the behaviour of the UK economy in 1980 when people misinterpreted the monetary shock as a predominantly supply shock; an interpretation that may have seriously worsened the recession[13].

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[13] See also Matthews and Minford (1986).

As our simulation exercise has shown, there are potential implications for policy from the misperceptions of specific macroeconomic shocks. As long as people do not have full information, the course of the economy is contemporaneously influenced by what they think are the shocks driving it. Because of the existence of the lagged effects in the model the persistence of shocks applies not only to their direct effects but also to their indirect effects due to signal extraction.

Secondly, the signal extraction method could be used to improve the forecasts of the unobserved endogenous variables over and above that produced by a 'pure' model forecast. The results shown here support this view. However, it has to be borne in mind that in reality the forecasts contain a certain amount of judgement in the form of residual adjustments to equations. For this reason a good forecast team will always beat a mechanical method of forecasting[14]. Even so, the use of partial current information may be useful as a 'benchmark' for gauging relative performance.

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[14] Matthews et al. (2002) apply the algorithm for the solution of partial current information presented here to a macroeconomic model of the UK for the period 1992q4-2001q4. Their results validate the conclusion that the algorithm does not add too much to the forecasts made by the forecasting team.

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Appendix A

The derivation of the analytical expressions for the expected interest rate and exchange rate.

In order to solve for the expected values of P_t, R_t and S_t, we first equate (3.3) with (3.4). Substituting the resulting expression for Y_t into (3.2) and then taking expectations yields:

$$E_t P_t + (1-\mu)Y^* + \mu Y_{t-1} + E_t v_t - \delta E_t R_t = (1-\theta)M^* + \theta M_{t-1} + E_t \varepsilon_t \qquad (A1)$$

Equating (3.1) with (3.2) and taking expectations gives us:

$$-\alpha(E_t R_t - E_t P_{t+1} + E_t P_t) - \gamma(E_t S_t + E_t P_t) + E_t u_t = (1-\mu)Y^* + \mu Y_{t-1} + E_t v_t \ (A2)$$

Taking expectations of (3.5) and then substituting the expression of $E_t R_t$ into (A1) and (A2) respectively yields:

$$E_t P_t + (1-\mu)Y^* + \mu Y_{t-1} + E_t v_t - \delta(E_t S_t - E_t S_{t+1}) - \\ (1-\theta)M^* - \theta M_{t-1} - E_t \varepsilon_t = 0 \qquad (A3)$$

$$\alpha(-E_t S_t + E_t S_{t+1} + E_t P_{t+1} - E_t P_t) - \gamma(E_t S_t + E_t P_t) + \\ E_t u_t - (1-\mu)Y^* - \mu Y_{t-1} - E_t v_t = 0 \qquad (A4)$$

Equations (A3) and (A4) provide the solutions for the $E_t P_t$ and $E_t S_t$. Using the backward operator, B[15], equation (A4) can be written as:

$$E_t S_t + E_t P_t = \frac{(1-\mu)Y^* + \mu Y_{t-1} + E_t v_t - E_t u_t}{-(\alpha+\gamma)\left(1 - \frac{\alpha}{\alpha+\gamma}B^{-1}\right)} = \frac{E_t Y_t - E_t u_t}{-(\alpha+\gamma)\left(1 - \frac{\alpha}{\alpha+\gamma}B^{-1}\right)} \quad (A5)$$

Noting that an expression of the form $\dfrac{1}{1-\lambda B^{-1}}$ can be expanded into an infinite series (given that $\lambda<1$)

$$\frac{1}{1-\lambda B^{-1}} = 1 + \lambda B^{-1} + \lambda^2 B^{-2} + ... + \lambda^N B^{-N} + ... \qquad ,$$

then, the right hand side of (A5) generates an infinite forward expansion. Imposing the stability condition, the remainder term of the expansion is forced to zero as $N \rightarrow \infty$. Thus, equation (A5) becomes:

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[15] The backward operator B instructs us to lag only the expected variable but not the date of expectations, that is $B(E_t P_t)=E_t P_{t-1}$. For a more detailed explanation of how to solve RE models using the backward operator B see Sargent (1980).

$$E_t S_t + E_t P_t = -\left(\frac{1}{\alpha+\gamma}\right)\left[\sum_{j=0}^{\infty}\left(\frac{\alpha}{\alpha+\gamma}\right)^j E_t Y_{t+j} - \sum_{j=0}^{\infty}\left(\frac{\alpha}{\alpha+\gamma}\right)^j E_t u_{t+j}\right] =$$
$$-\left(\frac{1}{\alpha+\gamma}\right)\sum_{j=0}^{\infty}\left(\frac{\alpha}{\alpha+\gamma}\right)^j E_t Y_{t+j} + \left(\frac{1}{\alpha+\gamma}\right)E_t u_t \tag{A6}$$

since $E_t u_{t+j} = 0$ for any $j > 0$.

Noting that $E_t Y_{t+j} = \mu^{j+1} Y_{t-1} + \mu^j E_t v_t + (1-\mu)Y^* \sum_{k=0}^{j}\mu^k$ equation (A6) can be re-written as:

$$E_t S_t + E_t P_t = \frac{-\mu}{\alpha+\gamma-\alpha\mu}Y_{t-1} - \frac{(1-\mu)(\alpha+\gamma)}{\gamma(\alpha+\gamma-\alpha\mu)}Y^* -$$
$$\frac{1}{\alpha+\gamma-\mu\alpha}E_t v_t + \frac{1}{\alpha+\gamma}E_t u_t \tag{A7}$$

Substituting for $E_t P_t$ given by (A7) into (A3) the expression for the exchange rate turns out to be:

$$E_t S_t = \frac{\mu\left(1-\frac{1}{\alpha+\gamma-\alpha\mu}\right)Y_{t-1} + (1-\mu)\left[1-\frac{\alpha+\gamma}{\gamma(\alpha+\gamma-\alpha\mu)}\right]Y^* - (1-\theta)M^*}{(1+\delta)\left(1-\frac{\delta}{1+\delta}B^{-1}\right)} +$$

$$+ \frac{-\theta M_{t-1} + \frac{1}{\alpha+\gamma}E_t u_t - E_t\varepsilon_t + \left(1-\frac{1}{\alpha+\gamma-\mu\alpha}\right)E_t v_t}{(1+\delta)\left(1-\frac{\delta}{1+\delta}B^{-1}\right)} \tag{A8}$$

Using the properties of the backward operator once again and writing each term of equation (A8) separately we obtain:

Term in Y_t:

$$\frac{\mu}{1+\delta}\left(1-\frac{1}{\alpha+\gamma-\alpha\mu}\right)\frac{Y_{t-1}}{\left(1-\frac{\delta}{1+\delta}B^{-1}\right)} = \frac{\mu}{1+\delta}\left(1-\frac{1}{\alpha+\gamma-\alpha\mu}\right)$$
$$\left[\frac{1+\delta}{1+\delta-\delta\mu}Y_{t-1} + \frac{\delta}{1+\delta-\delta\mu}E_t v_t + \frac{\delta(1-\mu)(1+\delta)}{1+\delta-\mu\delta}Y^*\right] \tag{A9}$$

Term in Y^*:

$$\frac{1-\mu}{1+\delta}\left[1-\frac{\alpha+\gamma}{\gamma(\alpha+\gamma-\alpha\mu)}\right]\frac{Y^*}{\left(1-\dfrac{\delta}{1+\delta}B^{-1}\right)}=$$

$$\frac{1-\mu}{1+\delta}\left[1-\frac{\alpha+\gamma}{\gamma(\alpha+\gamma-\alpha\mu)}\right]Y^*\sum_{i=0}^{\infty}\left(\frac{\delta}{1+\delta}\right)^i=(1-\mu)\left[1-\frac{\alpha+\gamma}{\gamma(\alpha+\gamma-\alpha\mu)}\right]Y^* \tag{A10}$$

Term in M^*:

$$-\frac{1-\theta}{1+\delta}\frac{M^*}{\left(1-\dfrac{\delta}{1+\delta}B^{-1}\right)}=-\frac{1-\theta}{1+\delta}\sum_{i=0}^{\infty}\left(\frac{\delta}{1+\delta}\right)^i M^*=-(1-\theta)M^* \tag{A11}$$

Term in M_{t-1}:

$$-\frac{\theta}{1+\delta}\frac{M_{t-1}}{\left(1-\dfrac{\delta}{1+\delta}B^{-1}\right)}=-\frac{\theta}{1+\delta}M_{t-1}\sum_{i=0}^{\infty}\left(\frac{\theta\delta}{1+\delta}\right)^i -$$

$$\frac{\theta\delta}{(1+\delta)^2}E_t\varepsilon_t\sum_{i=0}^{\infty}\left(\frac{\theta\delta}{1+\delta}\right)^i-\frac{\delta\theta(1-\theta)}{(1+\delta-\theta\delta)}M^*=-\frac{\theta}{1+\delta-\theta\delta}M_{t-1}- \tag{A12}$$

$$\frac{\theta\delta}{(1+\delta)(1+\delta-\theta\delta)}E_t\varepsilon_t-\frac{\delta\theta(1-\theta)}{(1+\delta-\theta\delta)}M^*$$

Term in $E_t u_t$: $\qquad\qquad [(\alpha+\gamma)(1+\delta)]^{-1}$ $\qquad\qquad\qquad$ (A13)

Term in $E_t\varepsilon_t$: $\qquad\qquad -(1+\delta)^{-1}$ $\qquad\qquad\qquad\qquad$ (A14)

Term in $E_t v_t$: $\qquad\qquad (1+\delta)^{-1}[1-(\alpha+\gamma-\mu\alpha)^{-1}]$ $\qquad$ (A15)

Substituting equations (A9)-(A15) into (A8) we get the solution for the exchange rate given by equation (3.15) in the paper.

To obtain the expression for $E_t R_t$ we first have to derive a solution for $E_t P_t$ which can be obtained by substituting the expression of $E_t S_t$, given by (3.15), into (A6):

$$E_t P_t = \left[-\frac{\mu}{\alpha + \gamma - \alpha\mu} - \frac{\mu}{1 + \delta - \delta\mu} \left(1 - \frac{1}{\alpha + \gamma - \alpha\mu} \right) \right] Y_{t-1} + \frac{\theta}{1 + \delta - \theta\delta} M_{t-1} +$$

$$+ \frac{(1-\theta)(1+\delta)}{(1+\delta-\theta\delta)} M^* - \left\{ (1-\mu) + \frac{\mu\delta(1-\mu)}{(1+\delta-\mu\delta)} \left(1 - \frac{1}{\alpha+\gamma-\alpha\mu} \right) \right\} Y^* +$$

$$+ \frac{\delta}{(\alpha+\gamma)(1+\delta)} E_t u_t - \left[\frac{1}{\alpha+\gamma-\alpha\mu} + \frac{1}{1+\delta-\delta\mu} \left(1 - \frac{1}{\alpha+\gamma-\alpha\mu} \right) \right] E_t v_t +$$

$$+ \frac{1}{1+\delta-\delta\theta} E_t \varepsilon_t$$

(A16)

Taking expectations of (3.3) and (3.4) and then substituting for $E_t Y_t$ – given by (3.2) - yields the solution for the interest rate as described by (3.17) in the paper. Using equation (3.9), the constants α_i and β_i (i=1,2,3) which link the expected shocks with the unobserved components of interest and exchange rates can be expressed as:

$$\alpha_1 = A \frac{A\zeta_1 - A'\zeta_2}{\zeta_3\zeta_1 - \zeta_2^2} \sigma_u^2$$

(A17)

$$\beta_1 = A' \frac{A'\zeta_3 - A\zeta_2}{\zeta_3\zeta_1 - \zeta_2^2} \sigma_u^2$$

(A18)

$$\alpha_2 = B \frac{B\zeta_1 - B'\zeta_2}{\zeta_3\zeta_1 - \zeta_2^2} \sigma_v^2$$

(A19)

$$\beta_2 = B' \frac{B'\zeta_3 - B\zeta_2}{\zeta_3\zeta_1 - \zeta_2^2} \sigma_v^2$$

(A20)

$$\alpha_3 = C \frac{C\zeta_1 - C'\zeta_2}{\zeta_3\zeta_1 - \zeta_2^2} \sigma_\varepsilon^2$$

(A21)

$$\beta_3 = C' \frac{C'\zeta_3 - C\zeta_2}{\zeta_3\zeta_1 - \zeta_2^2} \sigma_\varepsilon^2$$

(A22)

where $\zeta_1, \zeta_2, \zeta_3$ are given by:

$$\zeta_1 = AA'\sigma_u^2 + BB'\sigma_v^2 + CC'\sigma_\varepsilon^2$$

(A23)

$$\zeta_2 = A'^2\sigma_u^2 + B'^2\sigma_v^2 + C'^2\sigma_\varepsilon^2$$

(A24)

$$\zeta_3 = A^2\sigma_u^2 + B^2\sigma_v^2 + C^2\sigma_\varepsilon^2$$

(A25)

Appendix B

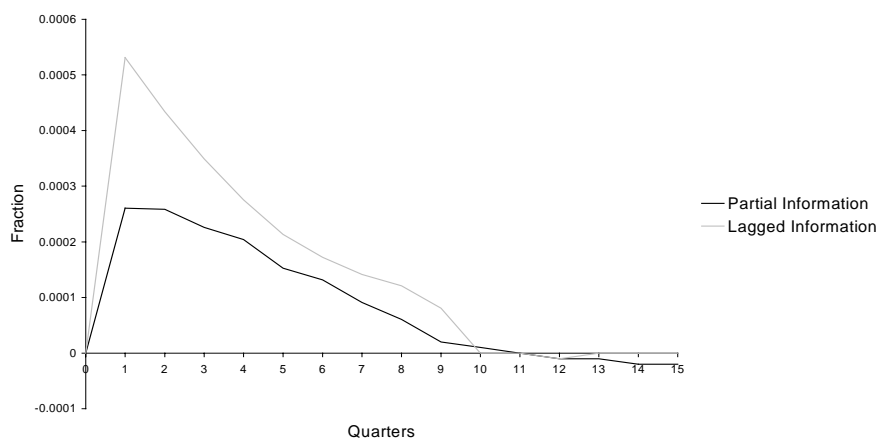### Chart 1: Output response to a shock to IS schedule

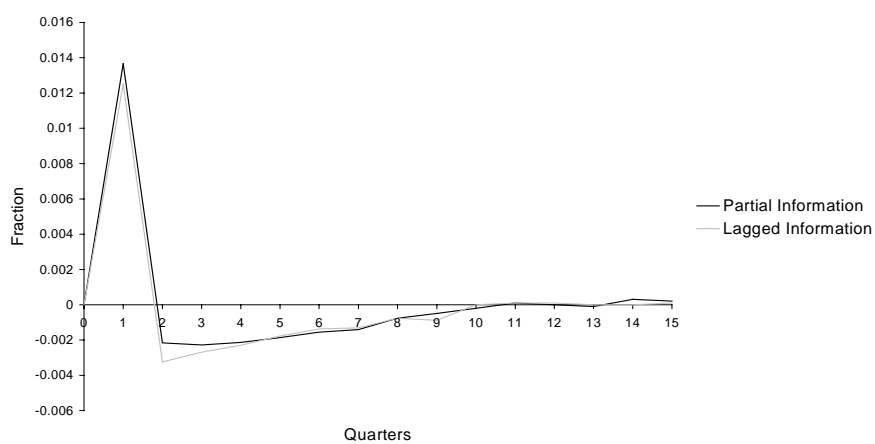### Chart 2: Inflation response to a shock to IS schedule
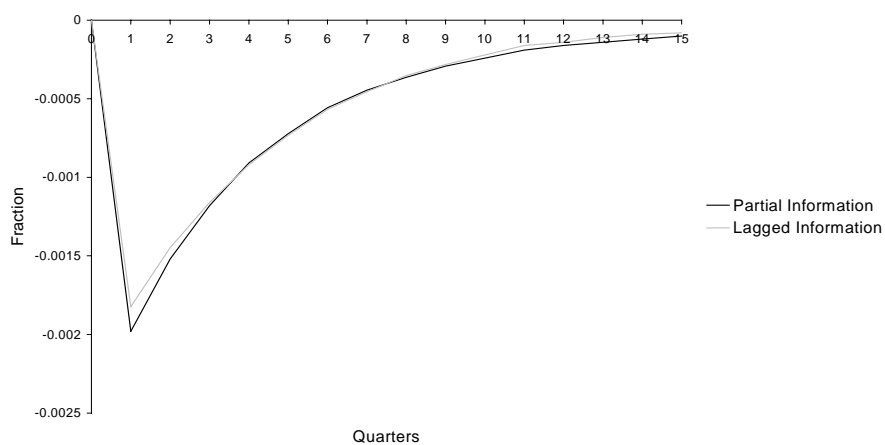

Chart 3: Output response to a supply shock
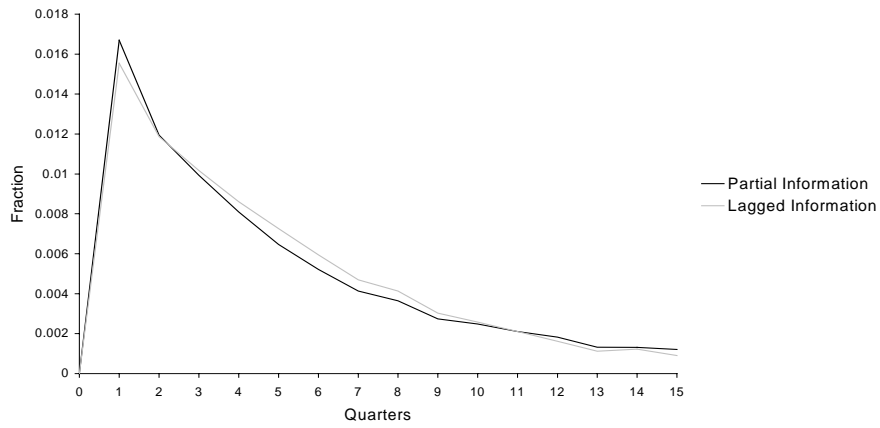
Chart 4: Inflation response to a supply shock

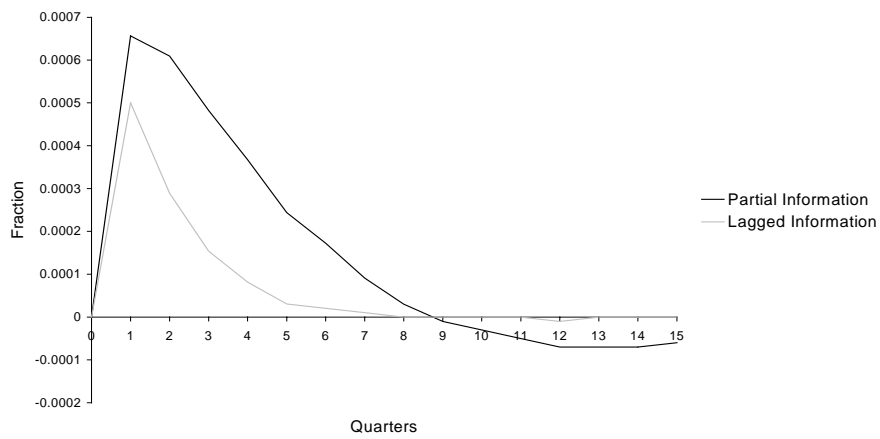

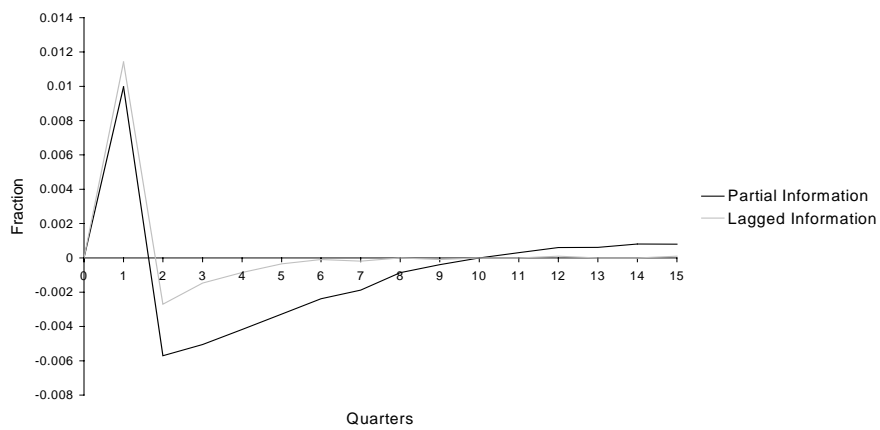**Chart 5: Output response to a monetary shock**

Chart 6: Inflation response to a monetary shock

Appendix C

This appendix presents the results of an exercise that reinforce the predictive superiority of the use of current partial information. We first generated a base run on lagged information. Next, a set of data consistent with the assumption on expectations based on lagged information was obtained. The model was then shocked by a randomly selected 11 sets of innovations to generate the model solved endogenous variables. The error statistics are reported in Table C1.

Table C1

Variable	Mean Error	RMSE	Theil
Error Statistics – Lagged Information Case
r	0.0609	0.1178	0.3057
Y	-0.0084	0.0853	0.0044
P	-0.0336	0.1358	0.0496
R	0.0681	0.1115	0.4957
S	0.0723	0.1344	0.1173

The next step was to use the signal extraction method to check if it improves the forecasts of the unobserved endogenous variables over and above that produced by the model under the assumption of lagged information. The results are shown in Table C2 below.

Table C2

Variable	Mean Error	RMSE	Theil
Error Statistics – Current Partial Information Case
r	-0.0002	0.0007	0.0016
Y	-0.0181	0.0461	0.0024
P	0.0550	0.1399	0.0505
R	-0.0002	0.0006	0.0019
S	-0.0001	0.0004	0.0003

There is a significant improvement in prediction efficiency in the use of current partial information. Apart from inflation, which is marginally worse off, there is a clear gain in the forecasts of all other endogenous variables. This proves the fact that the use of superior information reduces the expectational errors.