CARDIFF BUSINESS SCHOOL
WORKING PAPER SERIES

Cardiff Economics Working Papers

Kent Matthews, David Meenagh, Patrick Minford and Bruce Webb

Monetary regimes: is there a trade-off between consumption and employment variability?

E2006/12

Cardiff Business School
Cardiff University
Colum Drive
Cardiff CF10 3EU
United Kingdom
t: +44 (0)29 2087 4000
f: +44 (0)29 2087 4419
www.cardiff.ac.uk/carbs

ISSN 1749-6101
February 2006

Monetary regimes: is there a trade-off between consumption and employment variability?*

Kent Matthews David Meenagh Patrick Minford and Bruce Webb

(Cardiff University)

January 2004

Abstract

Macro models generally assume away heterogeneous welfare in assessing policies. We investigate here within two aggregative models — one with a representative agent, the other a long-used forecasting model of the UK — whether allowing for differences in welfare functions (specifically between those in continuous employment and those with frequent unemployment spells) alters the rankings of monetary policies. We find that it does but that a set of policies (money supply targeting implemented by money supply control) can be found that are robust in the sense of avoiding very poor outcomes for either of the two groups.

Keywords: Robustness, heterogenous welfare, money supply rules, interest rate setting, price level targeting

JEL codes: E52

*We are grateful for useful comments to Sylvester Eiffinger and other participants at the Bank of Finland and CEPR workshop, Heterogeneous information and modelling of monetary policy, in Helsinki in October 2003.

1 Introduction

In this paper we ask whether the choice of monetary regimes affects the balance of volatility across consumption and employment; our reason for doing so is to investigate whether business cycle volatility carries rather larger and more diverse costs than are usually captured in the welfare of a representative agent. To build up the fiction of this representative agent, it is usually assumed that there is perfect pooling across agents of all shock effects (Lewis, 2003a,b) In particular, 'unemployment' is achieved by a lottery, so that some people are allocated leisure but perfect compensation across agents ensures that they are no better or worse off than the others who work. Yet this contradicts the basic facts of the labour market and the impact of business cycles on households.

We would argue that the key feature perhaps of a macro economy is the difference in situation between the unemployed and the employed. In a highly flexible labour market, unemployment will purely consist of people spending short amounts of search time between jobs; in such a case it seems reasonable to assume that those unemployed are essentially no different from the employed — unemployment is an occasional state all, or most, experience. Here the pooling assumption is quite appropriate.

However most, maybe all, economies have not so flexible a labour market: various forms of social intervention ensure that a significant number of unemployed remain so for a long period because the jobs they can find are less rewarding than their unemployment package. Such unemployed will be on the margin of unemployment benefits and employment and hence will tend to be relatively unskilled (in benefit systems, the majority, where benefit/earnings replacement ratios rise markedly as one moves down the wage distribution). It is natural in such economies therefore to group the adult working population into those relatively unskilled for whom the risk of long-term unemployment is substantial and the rest. The labour economics literature has adopted the terms 'outsider' and 'insider' respectively for these two groups: we will generally use 'unskilled' and 'skilled' instead, indicating the overlap with income distribution differences.

It follows that unemployment variation will impact most heavily on the former groups, whereas consumption variation will impact most heavily on the latter. Thus the typical representative agent welfare function will tend to underestimate the costs of the business cycle to the former group. We treat the welfare of the second group as dependent only on consumption, that of the first only on unemployment. (We will also show averages of the two groups' welfare; but these have no status as welfare measures unless we are willing to give the two groups weights based on some political or other ranking).

A full treatment of heterogeneity would involve separating households into two diverse sets of agents with different constraints and behaviour (for examples of models where heterogeneity is embedded in the structure, see Storesletten et al, 2004 and Heathcote, 2005). Such a treatment, while undoubtedly correct in principle, involves a huge investment in complexity for the macro model itself; we doubt in fact whether the likely difference in behaviour of the main macro functions which go to determine overall business cycle behaviour, justifies such an investment. Aggregate behaviour could be quite robust to heterogeneity between employed and unemployed; after all the latter predominate hugely over the former. It is surely in their welfare function evaluation of business cycle developments that we should see big differences. These evaluations are recursive to the model determining the business cycle — or so the above argument would suggest.

There is a further practical reason for investigating these different evaluations within an aggregative model. Among well-developed models, whether calibrated or in the empirical domain (as e.g. forecasting models), we only have aggregative models available.

We would argue therefore that there is a case for investigating the differential effects on welfare of business cycle variation before we can get around to building models of the economy with explicit attention to the effects of heterogenity. We look below at measures of differential impact and argue that they are of some importance in assessing the robustness of different monetary policies. One might then argue, armed with such prima facie evidence, that the effort of building heterogeneity into the model itself would be justified. It is in this spirit of inquiry that we investigate the effects of heterogeneity in welfare using available models which ignore such heterogeneity in their business cycle determination.

We will be evaluating monetary regimes on the assumption that they produce the same mean behaviour in response to shocks; hence differences between them purely concern higher moments, of which we only examine the variance (using 2nd order Taylor series expansions). Our method will be to compare welfare rankings of monetary regimes for the unskilled and the skilled. If rankings are different, we will conclude that heterogeneity matters and attempt some assessment of how much and whether robustness can be achieved towards it.

Much analysis of monetary policy assumes either a lack of indexation altogether or a fixed inexation scheme such a lagged indexing. Yet Minford et al (2003) showed that the reaction of indexation to monetary policy is important in determining what type is optimal. In particular it argued that monetary policy targeting nominal *levels* of variables (such as prices or money) could be superior to those targeting rates of change of those variables (inflation or money supply growth). For analysing this issue they developed a model in which indexation was endogenous, chosen to optimise their welfare by skilled agents who were assumed to be continuously employed. This model is well suited to considering the issue here; in that paper the welfare measure used was a weighted average of the two groups' welfare — here we use the heterogeneous measures.

That model can be considered as a calibrated macro model with a moderate degree of nominal rigidity (from overlapping contracts) related to the extent of indexation. The resulting model is close to the 'New Classical' end of the modelling spectrum. However, to test the robustness of this stylisation, we also look at an estimated forecasting model of the same general type, the Liverpool Model of the UK (Minford, 1980). It would be interesting to know how far the results we obtain would generalise to other models with nominal rigidity and endogenous indexation; but we (and as far as we know others) have been unanble so far to do this work and so it must await future research.

The monetary regimes we consider are:

1. various sorts of (exact) targets for expected (one-period-ahead) outcomes. Each is implemented via setting the expected money supply for the coming period; the actual money supply is subsequently delivered with an independent stochastic error (a 'trembling hand' which can be interpreted in various ways — e.g. a banking system supply error or an error in current-period setting of a supply instrument such as bank reserves). The targets we consider are for: inflation, money supply growth, the price level and the money supply level. A burgeoning literature has grown up (a partial list is: Bank of Canada, 1994; Berg and Jonung, 1999; Casares, 2002; Duguay, 1994; Fischer, 1994; Hall, 1984; Kiley, 1998; Nessen and Vestin, 2000; Smets, 2000; Svensson, 1999a and b; Vestin, 2000; and Williams, 1999) around the issue of whether prices or inflation (money or its growth rate) should be targeted; under level targeting the level is stationary, under rate of change targeting the level is non-stationary.

2. alternative ways of organising current-period responses to shocks. Specifically, we consider a rigorous interest-rate-control regime where an interest rate target is chosen for the coming period and then exactly adhered to; against a money-supply-control regime as above where the money error is random. This is often referred to as the issue of 'operating procedure'; in a quarterly model this is not quite exact but it is helpful. The issue was first addressed by Poole; and since then there have been a large number of papers assuming that it is interest rates that are controlled by central banks, for example in the manner of the Henderson-McKibbin-Taylor rules (Henderson and McKibbin, 1993; Taylor, 1993). Our reason for reopening this old issue is simply that it acquires a new dimension when indexation is endogenous and also when there are two sets of agents.

1.1 The Models used

Our two models both assume:

1. competitive markets in labour and output

2. overlapping wage contracts, with a variable indexation parameter chosen optimally by workers; however a marginal labour supply is always provided to the auction market at an auction supply price.

3. Cobb-Douglas production functions.

4. Monetary policy is implemented via money supply feedback rules (i.e. with the expected money supply being set for the next period in response to current information; actual money supply is then determined by the impact of shocks under the assumed operating procedure, initially taken to be that of money-supply-control). However this assumption is a convenience only; the same rules could be expressed as a feedback from current information to expected future interest rates, with actual interest rates being determined by shocks, again under the assumed operating procedure.

Therefore both models share the familiar labour supply curve (based on a combination of contracted and free labour) and labour demand curve (based on marginal labour productivity); and the aggregate supply curve (from production function and employment) interacts with an aggregate demand curve (from the interplay of money markets, LM curve, and an IS curve).

Two key differences should be mentioned. The calibrated model (henceforth CM) does not allow consumers to access the credit markets; the reason is to create a strong incentive to smooth consumption via the wage contract. The Liverpool Model (henceforth LPM) is an open economy model — here using floating exchange rates — so that, in addition to the relationships already mentioned, it embodies efficient international bond markets (which imply the uncovered interest parity condition, forcing the real interest rate differential into equality with expected real exchange rate change — a constant can also be added for the risk-premium arising from model covariances but this does not affect simulation properties) and a current account equation related to home and world output and the real exchange rate. LPM also assumes that inflows of foreign capital occur flexibly in response to investment needs so that the production function treats the capital stock as endogenous. Hence it can be seen that the two models differ in detail and in the extent of theoretical abstraction rather than in basic approach.

LPM is in essence a less restricted, open economy, version of CM. It is a rational expectations IS-LM model, such as can be derived from a micro-founded model by suitable approximations (McCallum and Nelson, 1999) — thus for example in LPM the IS curve has the expectation of future output in it, the hallmark of this approximation. The model's Phillips or Supply curve assumes overlapping wage contracts as in CM. The labour market underpinning it is explicit and the model solves for equilibrium or natural rates of output, unemployment and relative prices. Thus from a theoretical viewpoint the model could be considered reasonably protected against Lucas' (1976) critique. From the empirical viewpoint, we have found the model's parameters to be rather stable. In recent work a new FIML algorithm developed in Cardiff University (Minford and Webb, 2005) has been used to re-estimate the model parameters: it turns out that the new estimates are little different from the model's original ones, based partly on single-equation estimates, partly on calibration from simulation properties. In terms of forecasting tests, as we discuss next, the model has performed fairly well across a variety of regime changes, not merely on the monetary but also on the supply side of the economy.

LPM has been used in forecasting continuously since 1979, and is now one of only two in that category. The other is the NIESR model, which however has been frequently changed in that 20-year period: the only changes in LPM were the introduction of the explicit natural rate supply-side equations in the early 1980s and the shift from annual data to a quarterly version in the mid-1980s. In an exhaustive comparative test of forecasting ability over the 1980s, Andrews et al (1996) showed that out of three models extant in that decade — LPM, NIESR, and LBS — the forecasting performance of none of them could 'reject' that of the others in non-nested tests, suggesting that LPM during this period was, though a newcomer, at least no worse than the major models of that time. For 1990s forecasts no formal test is available, but forecasting with the LBS model stopped and in annual forecasting post-mortem contests the NIESR came top in two years, LPM in three. Thus we would suggest that LPM has a respectable forecasting record, at least on a par with the only other model available of the general type we seek — viz. micro-founded and suitably estimated. Comparative work on the NIESR model would also be of interest; so far it has not been possible. There are also models in the public sector — those of the Treasury and the Bank of England — but they are not easily accessible as yet with the required back-up of micro-foundations and forecasting record.

Lastly, in respect of simulation properties and use of these for policy analysis, we note that LPM has been extensively used in policy analysis in support of the 'monetarist' and 'supply-side' reforms of the 1980s and 1990s. which are generally considered to have been broadly successful. We therefore suggest that the LPM could be regarded as a suitable vehicle for checking the 'realism' of the policy conclusions we will initially derive from CM.

In order to clarify the two models' structures we list each of them below opposite the categories already used:

Equation category	CM equation nos[1]	LPM equation nos[2]
Labour supply/wage contract	A1, A4, A5	B8
Labour demand	A7	B5
Production function, cost equations	A3	B9
IS curve	A6	B14
Investment	A2	B22, B6
Consumption	(=M_{t-1}/p_t via no-credit constraint)	B7
Other	none	B1, B19
Money Demand and supply	A10, A9, A8	B4, exog money supply eq, B11, B12, B13
Open economy:	n.a.	
Capital Account (UIP)		B10
Current account		B2, B3

[1]Full listing in Appendix A

[2]Full listing in Appendix B

2 Targeting within a calibrated model (CM):

The CM model can be organised most simply in terms of aggregate supply (shifted by the productivity shock, ϕ) and aggregate demand (shifted by the money supply shock, m). Monetary targeting then moves planned next period's m so as to produce the target price or money in that period; however the actual m is then delivered with a random error. The model's behaviour in response to the two shocks is standard. The key innovation in the model is the finding that from society's viewpoint reducing indexation improves the economy's stability in the face of supply shocks because it both flattens the AS (Phillips Curve) and steepens the AD curve, as illustrated in Figure 1 below. The indexation response depends in turn on the *persistence* of the two shocks(Minford et al, 2003): this is in contrast to the usual Fischer-Gray result which depends on the *size* of the shock variances. The reason persistence matters is that indexation is spent with a delay (from the cash-in-advance lag); hence in the presence of temporary shocks to prices it does not pay to index wages because the shock effect on consumption only lasts one period and indexation will produce extra consumption in the following period, which *adds* to the overall variance of consumption. On the other hand if shocks persist, the indexation payment will offset the effects on consumption that persist beyond the first period; hence indexation becomes desirable. Rising indexation steepens the Phillips Curve in a well-known way, in that price surprises now clearly will induce (after period 1) less of an effect on real wages and hence on employment and output. It also flattens the Aggregate Demand Curve because the greater responsiveness of wages to prices implies that, given the fixed available money supply (which is required by employers and the government to pay the bill for wages and unemployment benefits), rising prices induces a greater reduction in employment.

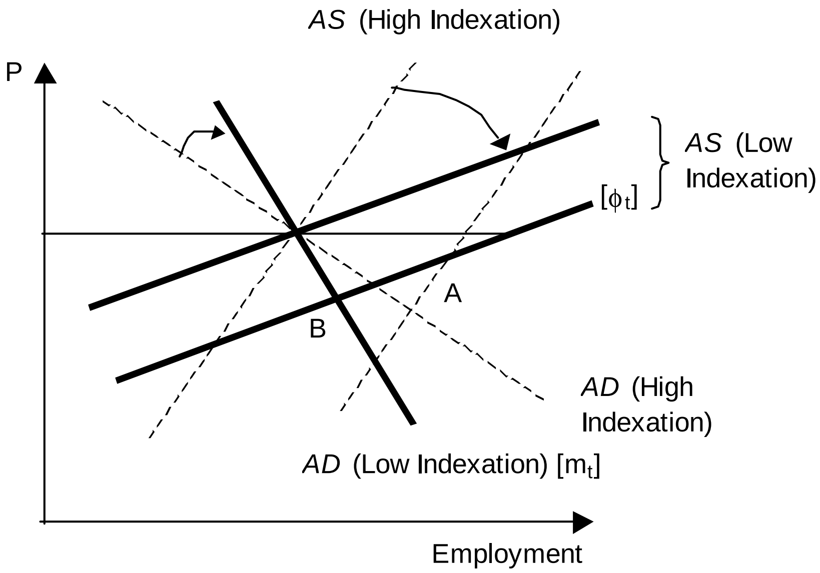

Figure 1: The effect of reduced indexation on slopes of AS and AD curves [ϕ_t = productivity shock; m_t = monetary shock]

The resulting intersections for a supply shock as shown at A (high indexation) and B (low indexation). Thus the drop in indexation is stabilising to both employment and prices in the face of a supply shock. Of course for a money (demand) shock the result is greater employment instability, though probably less price instability; however money is a policy-controlled variable and if the policy error in setting it can be kept within limits then supply shocks will matter most.

Though the focus in this paper is on the separate welfares of our two groups, it is useful in the discussion to refer also to aggregate measures; for example when both groups gain it is helpful to measure by how much on average — arbitrary as that average of course must be, it is like an index. For this purpose we use two measures. The first, Welfare #1, is the standard measure used in representative agent models, the Constant Relative Risk Aversion utility function with Cobb-Douglas preferences across consumption and leisure:

$$U_t = \sum_{\tau=t}^{\infty} \delta^{\tau-t} \left\{ \frac{(c_\tau^{\nu}[\lambda + a_\tau]^{1-\nu})^{1-\rho} - 1}{1-\rho} \right\}$$

where

$$c_t = \frac{\overline{W}_{t-1}}{p_t}(1 - a_{t-1}); \lambda = 1$$

implying that leisure time is equal to working time when unemployment a_t is zero. We set $v = 0.7$, based on the marginal valuation of leisure at wages net of unemployment benefit. Because households get unemployment benefit on their spells of eligible unemployment, a_t, this implies that their choice is distorted; they choose leisure (l, which we in practice set at unity, is the ineligible part of leisure) in response to the differential between wages and benefits. But then of course they must pay for the benefit burden via taxes; the present discounted value of this tax burden is the same as this benefit bill and so we deduct this from their consumption to obtain total private utility.

The second measure, Welfare #2, is simply the inverse of a weighted average of the two variances, of consumption and unemployment, with similar weights, 0.7 for consumption and 0.3 for unemployment.

We examine the relative merits of various forms of monetary targeting. The main current targeting choice of central banks is inflation targeting; we therefore make this the benchmark regime against which to measure alternatives. The first with which we compare it is price-level targeting. A target rule chooses a money supply for next period that forces the expected inflation, or price-level respectively, to be on target in this next period; this money supply plan is however executed with an error, the model's 'monetary shock' (which can in practice be interpreted as a shock on either the supply or demand side of the money market; it is the model's demand shock). There is thus no current response of money supply to shocks; nor any implied interest rate smoothing in the current period — we defer such issues to the next section.

Our results can be summarised simply. Inflation-targeting generates a high degree of indexation. When price-level targeting is undertaken but indexation is assumed constant, welfare falls, because the variability of unemployment rises sharply. But when indexation is allowed to change endogenously, it drops to nil and the result is a rise in welfare, with the variance of consumption down markedly and that of unemployment down substantially.

Table 1: Price-level and inflation targeting in CM

standard error in parenthesis^{+}

Inflation-target = 100	Indexation (%)	Welfare #1	Welfare #2	Var (cons.)	Var (unemp.)
Inflation-targeting	71	100	100	100	100
			(3)	(3)	(3)
Price-level targeting (holding indexation fixed)	71	98	96	99	119**
Price-level targeting (indexation endogenous)	0	102	125	96	69**

$^{+}$standard error of Montecarlo sample variance = est. variance $\times\sqrt{\left(\frac{2}{n}\right)}$ where n is the number of sample observations (here 2000) — source Wallis, 1995

Definition: #1 is the standard CRRA formula in the text; #2 is the weighted average (weight on consumption, of employed $= 0.7$, on unemployment $= 1.0$;the latter weight includes . the effect of unemployment on consumption) of the two (inverted) variances

* significant at 10% level;** significant at 1% level

What we notice in Table 1 comparing inflation and price-level targeting is that both variances fall as we move to price-level targeting, once allowance is made for the endogenous response of indexation. This response eliminates indexation which means that the Phillips Curve flattens causing unemployment to respond little to the current productivity shock.

However, it is worth noticing that if indexation for some reason does not respond, then there is a marked difference in the two variances: unemployment variance shoots up while consumption variance falls on the move to price targeting (naturally as real wages are smoother with the price level being held to its expected trajectory). Hence what we see from Table 1 that CM's properties are very much in line with the usual views of macroeconomists under the usual assumption that indexation is constant: viz. that targeting the price level would destabilise employment and output, even if the stability of prices would indeed yield benefits to those with long term, nominal, or partly nominal, contracts, as here exemplified by workers with wage contracts that are not fully indexed. (The details of how CM generates this result are unravelled to a reasonable extent in Appendix C, using a simplified linear version of CM).

Of course the endogenous response of indexation should occur; but it could take some time to occur (especially if the shift of regime is not at all clearly communicated). What we are seeing here therefore is a potential conflict of interest between the skilled and unskilled groups. The skilled welcome price targeting because it smooths consumption; the unskilled do not because it worsens employment variability.

Table 2: Money-level and Money-growth targeting in CM

standard error in parenthesis^{+} Inflation-target = 100	Indexation (%)	Welfare #1	 #2	Var (cons.)	Var (unemp.)
Money–growth-targeting (=inflation targeting in this model)	71	100	100 (3)	100 (3)	100 (3)
Money-level targeting (holding indexation fixed)	71	100.6	102.8	102	94**
Money-level targeting (indexation endogenous)	37	104	135**	100	56**

$^{+}$standard error of Montecarlo sample variance = est. variance $\times\sqrt{\left(\frac{2}{n}\right)}$ where n is the number of sample observations (here 2000) — source Wallis, 1995
Definition: #1 is the standard CRRA formula in the text; #2 is the weighted average (weight on consumption, of employed $= 0.7$, on unemployment $= 1.0$;the latter weight includes . the effect of unemployment on consumption) of the two (inverted) variances
* significant at 5% level;** significant at 1% level

Money targeting is not helpful to real wage smoothing; the reason is that unlike price level targeting it does not remove the persistent effect on prices (and so on real wages) of the productivity shock (in fact when indexation remains fixed it slightly increases the effect of productivity persistence on the real wage). But it does reduce the variance of unemployment even when there is no response of indexation. The reason is that the persistence in the money supply shock is eliminated and hence the aggregate demand curve is less variable. As indexation falls and the Phillips Curve flattens this effect becomes more important. Also the effect of productivity shocks is dampened on both employment and prices. Hence the additional move to lower indexation makes the money-level rule more stabilising, just as it did with the price-level rule.

If we look at the two policies from the viewpoint of average welfare, then we find that money targeting is superior before indexation has adjusted — confirming the majority macroeconomist viewpoint that price targeting is too rigid in driving prices back to their target track. Money targeting however is flexible enough to deliver some benefit overall compared with inflation targeting; it slightly destabilises consumption but markedly stabilises employment. After indexation has adjusted, it turns out that the two policies deliver rather similar improvements in general welfare.

But we also need to compare money with price level targeting from the viewpoint of different agents. First, we see that under conditions where indexation has not yet changed, money targeting is preferred by unskilled workers, whereas price targeting is preferred by skilled workers. Second, even once indexation has changed, money targeting is still preferred by the unskilled, while price targeting is still preferred by the skilled. The fact that money targeting is better liked by those most vulnerable to the economic cycle underlines its 'compromise status' between pure price stabilisation and pure employment stabilisation; in line with this it induces the elimination of only half the indexation we start off with under inflation targeting.

3 Targeting within the Liverpool forecasting model of the UK (LPM)

As part of our robustness check, we now turn to LPM. Our method is as with CM to run our monetary rules in LPM under stochastic simulation. We shock the full range of endogenous and exogenous errors, exactly as in the model specification. We adopt the same expressions to evaluate welfare, the only difference being that in LPM we use non-durable consumption in place of total consumption, since LPM has no measure of the latter (in it durable consumption is included with other investment). The model's wage equation is written in terms of the real wage reacting to the real benefit rate and to unemployment, which are the auction wage components (implicitly the auction wage element has a weight of 0.2), and negatively to the difference of the price level from the average forecast of it at the times of wage contracting, then positively to this difference lagged.

For our purposes here we adapt it as follows:

$$\overline{W_t} = vP_t + wE_{t-j}P_t + \alpha(P_t) + w^* \ldots = (\alpha + v)(P_t - E_{t-j}P_t) + E_{t-j}P_t + w^* \ldots$$

Now we lag the auction and indexed elements two periods because of a delay due to the firm's internal checking procedures in adjusting pay to the unexpected change in the price level and obtain the real wage as:

$$\overline{W_t} - P_t = -(P_t - E_{t-j}P_t) + (\alpha + v)(P_{t-2} - E_{t-j-2}P_{t-2}) + w^* \ldots$$

Hence the two-period-lagged term carries the extent of real wage protection. It is this part that is adjusted endogenously by the employed to minimise the variance of their real wage.

The results within LPM are about as favourable to price-level targeting as in the representative-agent model. Again, Table 3 shows that under inflation-targeting there is a high degree of indexation and that this would drop to nil under price-level targeting. Similarly, too, they show that if indexation is assumed endogenous, welfare will rise significantly if price-level targeting is introduced; in LPM the variance of consumption falls more and that of unemployment falls less than in the representative agent model but both fall significantly.

However there is a crucial difference: when indexation is held fixed, the variances behave very much the same in LPM as when indexation is endogenous. There is still a substantial gain over inflation targeting, revealing that the mechanisms at work cannot be the ones in CM, whereby the Aggregate Demand and Supply Curves' slopes are changed by indexation.

The first reason appears to be that the great improvement in stability both for consumption and unemployment between rows 1 and 2 comes about because price-level targeting greatly reduces the variability of private *wealth (* it reduces the variance of wealth by 8%, regardless of the degree of indexation): price-level targeting makes the real price of bonds more stable because it makes the variability of the price level so much smaller (thus future bond prices are set in nominal terms by the nominal rate of interest which in turn depends on the real rate plus the expected rate of inflation; but future *real* bond prices are additionally dependent on the future price level). In LPM private wealth has strong demand effects on private consumption and investment; thus dampening its variability dampens an important demand shock. We can demonstrate this by redoing the stochastic simulations without wealth effects; we find that under price level targeting with the same 80% indexation as inflation targeting the variance of consumption is 10.2% higher than under inflation targeting and the variance of unemployment 6.2% higher, very much in line with the familiar macroeconomists' intuition that having to reverse inflation shocks subsequently to hit a price level target is destabilising to the economy. In LPM this intuition is overridden by the destabilising wealth effects of prices, which price level targeting reduces.

Second, the indexation mechanism in LPM does indeed work (a little) to dampen the effects of supply shocks which are the main source of shocks in LPM; thus if we simulate the price-level target regime with 80% indexation and then again with zero indexation for supply shocks only, we find that the zero indexation reduces the variance of consumption by 2% and that of unemployment by 1%. But this plainly is not a powerful effect. (For demand shocks too the move to indexation is slightly favourable — comparing zero with 80% indexation there is no difference for consumption variance and a 2.5% reduction in unemployment variance. But within LPM, demand shocks also have a 'supply' element because via the exchange rate effect they enter the Phillips Curve so the sharp distinction of CM is not present.) Thus in effect indexation as such (the difference between rows 2 and 3 for Table 3) has very little effect within LPM — confirming that LPM is a model very much at the 'New Classical' end of the spectrum, that is with little effect of nominal rigidity. It does however have powerful wealth effects (notably of government bonds), thus it is highly 'non-Ricardian', that is it does not exhibit Ricardian equivalence under which a bond-financed tax cut would raise savings by the tax cut, leaving consumption unchanged.

In sum what the Liverpool Model shows pre-eminently is the importance of wealth shocks to demand and the way in which price-level targeting helps to make the economy more stable by dampening these.

Table 3: Inflation and price-level targeting in LPM

standard error in parenthesis+ Inflation-target = 100	Indexation (%)	Welfare #1	#2	Var (cons.)	Var (unemp.)
Inflation-targeting	80	100	100	100	100
			(1.3)	(1.3)	(1.3)
Price-level targeting (holding indexation fixed)	80	102.3	119**	81**	90**
Price-level targeting (indexation endogenous)	0	102.4	120**	81**	89**

+standard error of Montecarlo sample variance = est. variance $\times\sqrt{\left(\frac{2}{n}\right)}$ where n is the number of sample observations (here 12078) — source Wallis, 1995

Definition: #1 is the standard CRRA formula in the text; #2 is the weighted average (weight on consumption, incl. unemployed's = 0.7, on unemployment = 0.3) of the two (inverted) variances.

* significant at 5% level

* significant at 1% level

When we turn to a comparison of money targeting with price targeting, (Table 4) we find that money targeting relatively stabilises nominal and real interest rates, real investment and total wealth, but it relatively destabilises prices and inflation and hence the real value of financial wealth (nominal bonds). The variance of unemployment and output rise as we switch from price- to money-targeting while that of consumption falls. The key to these differences lies in the behaviour of the expected future price level which in an IS/LM framework enters the IS curve in terms of its percentage difference from the current price level. Under price-level targeting expected future prices do not move whereas under money-level targeting they are positively correlated with current prices because any permanent supply shock will continue to affect prices in the same direction in the next period. Hence under money-targeting expected inflation varies less which disturbs the IS curve less. This reduced IS variability under money-targeting implies less interest rate variability, real and nominal too; and also less variability in the real capital stock. However, this also leads to a reduction in real exchange rate variability via uncovered interest parity; this means less dampening of net trade volume variability (consider a rise in world trade, the main such source of variance; under money-level targeting, it shifts IS less rightwards, generating less of a rise in real interest rates and so in the real exchange rate which would counteract the rise in net trade). Thus of the three exogenous sources of output demand variability, consumption's and investment's fall because of reduced interest rate and wealth volatility but net trade's goes up by more, as does thus also output and employment volatility rise.

Aggregate welfare under price- and money-targeting are roughly the same. However when we turn to the welfare of the two groups, we find that under LPM the preference ordering of both between price and money level targeting is reversed compared with CM: now the employed/skilled prefer money targeting while the unemployed/unskilled prefer price targeting.

Table 4: Price- and money-targeting in LPM

standard error in parenthesis^{+} Inflation-target = 100	Indexation (%)	Welfare #1	Welfare #2	Var (cons.)	Var (unemp.)
Price-level-targeting (indexation endogenous)	0	102.4	120**	81**	89**
Money-level-targeting (indexation endogenous)	40	102.7	122**	68**	97*

$^{+}$standard error of Montecarlo sample variance = est. variance $\times\sqrt{\left(\frac{2}{n}\right)}$ where n is the number of sample observations (here 12078) — source Wallis, 1995

Definition: #1 is the standard CRRA formula in the text; #2 is the weighted average (weight on consumption, incl. unemployed's $= 0.7$, on unemployment $= 0.3$) of the two (inverted) variances.

* significant at 5% level** significant at 1% level

Contrasting LPM with CM, we find that the major role of wealth effects in LPM, entirely absent in CM, gives a rather different perspective on monetary rules. In LPM the endogeneity of wage contracting is of little importance; even if it does change one transmission mechanism, it is minor in its overall impact on variances. Instead we find that the key source of macro variability is the variability in nominal variables, themselves primarily controlled by monetary policy; when monetary policy increases the stability of these nominal variables, the macro economy too is less variable.

4 Should interest rates or the money supply be controlled as the operating instrument?

We come last to a well-worn issue of monetary policy — whether monetary policy should control (i.e. keep fixed) interest rates in the very short run, operating, period (of say a month ahead) or should control the money supply. The seminal work of Poole (1970) noted that the answer depended, within the IS/LM model, on the relative variances of the IS and LM shock. The issue is how stable the Aggregate Demand curve can be kept in response to the 'nuisance' shocks in the IS and LM curves. Instability in the AD curve will spill over into prices and output and so into the welfare function used here.

We can analyse both CM and LPM in these terms. Notice that because the IS curve responds to $(E_tp_{t+1} - p_t)$ it potentially matters which *targeting* regime is being followed: thus under inflation targeting $(E_tp_{t+1} - p_t)$ will not move whereas under price-level targeting E_tp_{t+1} will not move so that $(E_tp_{t+1} - p_t)$ moves by the full amount of $-p_t$. Under money-level targeting $(E_tp_{t+1} - p_t)$ moves by less than this amount because E_tp_{t+1} will vary directly with but less than p_t. So each targeting regime must be considered separately.

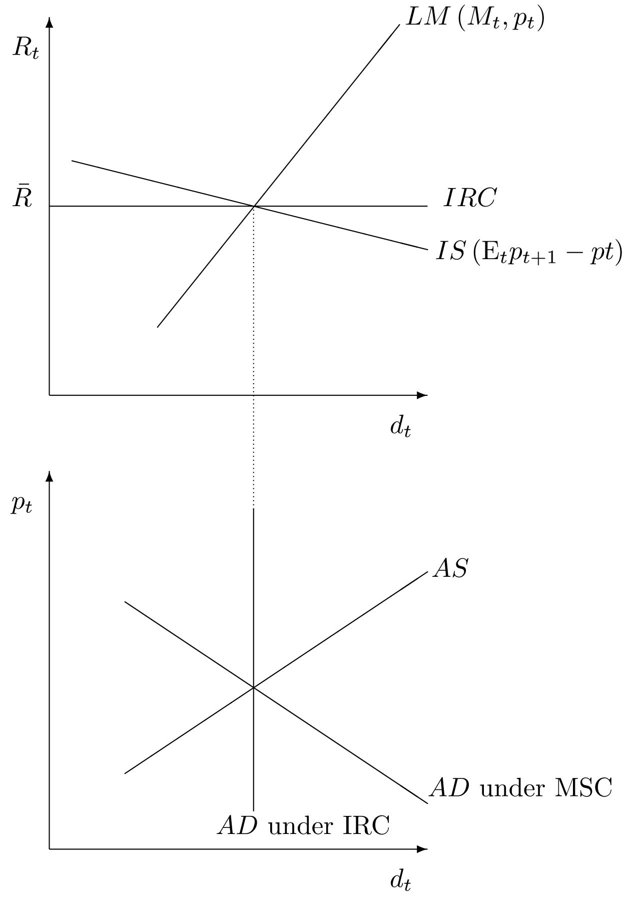

Figure 2: The Poole set-up illustrated for CM under inflation-targeting; MSC= money supply control, IRC= interest rate control.

4.1 Interest-rate-control within CM

Within CM, an important feature is a very flat IS curve. This can be seen from the large size of the parameter on the real interest rate in equation A2 (for the capital stock). In linearised form this equation (see Appendix C) is:

$$K_t = 1.11 \left\{ \frac{k(1-\mu)(1-T_o)}{r_0} \right\} \left\{ (d_t - \left[\frac{r_t}{r_o} \right] \right\} \tag{A2}$$

The relevant parameter is therefore $-1.11 \left\{ \frac{k(1-\mu)(1-T_o)}{r_0} \right\} \left\{ [\frac{1}{r_o}] \right\}$Notice the denominator is $(\frac{1}{r_0})^2$ which is a very large number owing to the small size of r, the real interest rate, whose units are fractions per period. Now note that $r_t = R_t - (E_t p_{t+1} - p_t)$.

Under the inflation targeting regime, which we consider first, $(E_t p_{t+1} - p_t)$ is kept at the target level by the monetary rule. This implies that fixing nominal interest rates, R_t, will also fix real interest rates, r_t. Also movements in the price level will not shift the IS curve. Thus the Aggregate Demand Curve will slope vertically under inflation targeting.and interest-rate control (IRC). Figure 2 illustrates CM under inflation targeting. However under money supply control (MSC) the AD curve will slope normally because the LM curve will react to prices. Finally we note that the IS curve is shifted in CM by productivity shocks in a real business cycle manner, since investment reacts sharply to productivity prospects.

IRC implies that IS curve shocks are unaffected by movements in prices because these are offset by equal movements in future prices; nevertheless the IS curve is so flat that only a very small interest rate change is produced and thus effectively the IS barely shifts at given interest rates; thus shifts in the AD curve are very small. By contrast with MSC the LM curve shifts with random movements in money supply which introduces larger shocks to the economy. In Poolean terms, here shocks to the money supply dominate IS shocks in their impact on the economy. Table 5 below shows IRC or MSC under CM with inflation targeting

Table 5: IRC or MSC under CM with inflation targeting

standard error in parenthesis^{+} Index: Money = 100	Indxtn	Welfare #1	 #2	Var (cons.)	Var (unemp.)
Money Supply Control, MSC	71	100	100	100	100
			(3)	(3)	(3)
Interest Rate Control, IRC	71	101.5	107.4*	96	91

$^{+}$standard error of Montecarlo sample variance = est. variance $\times\sqrt{\left(\frac{2}{n}\right)}$ where n is the number of sample observations (here 2000) — source Wallis, 1995

Definition: #1 is the standard CRRA formula in the text; #2 is the weighted average (weight on consumption, of employed $= 0.7$, on unemployment $= 1.0$;the latter weight includes . the effect of unemployment on consumption) of the two (inverted) variances

* significant at 5% level;** significant at 1% level

Table 6: IRC or MSC under RAM with price-level targeting

standard error in parenthesis^{+} Index: Money = 100	Indxtn	Welfare #1	 #2	Var (cons.)	Var (unemp.)
Money Supply Control, MSC	92	100	100	100	100
			(3)	(3)	(3)
Interest Rate Control, IRC	92	100.1	97	98	107**

$^{+}$standard error of Montecarlo sample variance = est. variance $\times\sqrt{\left(\frac{2}{n}\right)}$ where n is the number of sample observations (here 2000) — source Wallis, 1995

Definition: #1 is the standard CRRA formula in the text; #2 is the weighted average (weight on consumption, of employed $= 0.7$, on unemployment $= 1.0$;the latter weight includes . the effect of unemployment on consumption) of the two (inverted) variances

* significant at 5% level;** significant at 1% level

When the monetary rule is a price-level one, $E_t p_{t+1}$ is now fixed; thus any movement in p_t represents an equal movement in the real interest rate with a very large effect shifting the IS curve. Thus the AD curve here is very flat mirroring this effect of prices on the IS curve. Now IS curve shocks, combined with a flat AS curve because of zero indexation, will produce large swings in output but prices will be heavily stabilised (as they must be to keep the real, and so the nominal, interest rate constant). Hence IS shocks dominate in the Poole sense. Table 6 above reveals that the variance of unemployment rises markedly while price variability falls, stabilising the real wage and so consumption. Overall welfare is

Table 7: IRC or MSC under CM with money-level targeting

standard error in parenthesis^{+} Index: Money = 100	Indxtn	Welfare #1	#2	Var (cons.)	Var (unemp.)
Money Supply Control, MSC	37	100	100	100	100
			(3)	(3)	(3)
Interest Rate Control, IRC	78	99.3	68**	95	182**

$^{+}$standard error of Montecarlo sample variance = est. variance $\times\sqrt{\left(\frac{2}{n}\right)}$ where n is the number of sample observations (here 2000) — source Wallis, 1995

Definition: #1 is the standard CRRA formula in the text; #2 is the weighted average (weight on consumption, of employed $= 0.7$, on unemployment $= 1.0$;the latter weight includes . the effect of unemployment on consumption) of the two (inverted) variances

* significant at 5% level;** significant at 1% level

reduced, as is that of the unskilled while that of the skilled rises.

Essentially the same occurs under money-level targeting — Table 7. The main difference from price-level targeting is that now expected future prices only change with productivity shocks; under IRC money shocks have no effect and so only productivity shocks matter. These, being entirely permanent, induce far greater persistence in prices and so far greater indexation, which in turn sharply increases the effect of productivity shocks on output and employment..

Summarising, we could say that interest rate control also controls prices more. This is good for the skilled, smoothing their real wages. But it destabilises employment and output, which is bad for the unskilled.on the margins of the labour market.

4.2 Interest rate control in LPM

The interpretation of LPM results in Poolean terms is similar, except that it has a more standard IS curve with a much more modest interest-rate elasticity, simply because it does not have the fierce real business cycle reaction of CM but rather a looser stock-adjustment reaction of the investment to demand and monetary conditions. Hence LPM is less sensitive to whether the monetary rule is for inflation or price-level targeting. Within LPM IS shocks have a higher variance than LM shocks (remembering the model uses the monetary base rather than any wider definition of money, because of the concern with financial deregulation). It follows in the standard manner of Poole that there will be greater AD instability which will also show up in greater instability of output and prices (and so of both real wages and financial wealth). However the model tends to stabilise consumption since price and output rises are positively correlated; thus a rise in output lowers financial wealth, with the effects of the latter on consumption more than offsetting those of output. This is the pattern that shows up in the simulations of Interest Rate Control below; unemployment, output and price variability all rise but consumption variability falls.

The one exception to this is the case of money-supply-targeting, where LM shocks have a higher variance than IS shocks. In this case when the money supply is 'controlled' any independent growth rate of the money supply above its target rate is immediately followed by a growth rate below the target by an equal amount; this variability appears to dominate that from the IS shocks. Hence here output and unemployment are more variable under Money Supply Control, while consumption is less variable because of the same mechanism as above.

Table 8: Interest rates or money setting in LPM

standard error in parenthesis^{+} LPM under inflation targeting					
Index: Money = 100	Index	Welfare #1	#2	Var (cons.)	Var (unemp.)
Money Supply Setting	80	100	100 (1.3)	100 (1.3)	100 (1.3)
Interest Rate setting	80	100.8	100.3	97*	106**

standard error in parenthesis^{+} LPM under price-level targeting					
Index: MSSetting = 100	Index	Welfare #1	#2	Var (cons.)	Var (unemp.)
Money Supply Setting	0	100	100 (1.3)	100 (1.3)	100 (1.3)
Interest Rate setting	0	99.8	98.8	100	104**
standard error in parenthesis^{+} LPM under money-level targeting					
Index: MSSetting = 100	Index	Welfare #1	#2	Var (cons.)	Var (unemp.)
Money Supply Setting	40	100	100 (1.3)	100 (1.3)	100 (1.3)
Interest Rate setting	40	95.4**	93.0**	112**	97*

$^{+}$standard error of Montecarlo sample variance = est. variance $\times\sqrt{\left(\frac{2}{n}\right)}$ where n is the number of sample observations (here 10824; note lesser number because some runs would not solve under both MSC and IRC) — source Wallis, 1995 Definition: #1 is the standard CRRA formula in the text; #2 is the weighted average(weight on consumption, incl. unemployed's = 0.7, on unemployment = 0.3) of the two (inverted) variances.
* significant at 5% level** significant at 1% level

When we review the effects on different groups we find that the effect on each group depends on whether the target regime is price/inflation or money-level. The employed prefer IRC under the first but MSC under the second, and vice versa for the unemployed. This leads finally into a discussion of how we might use these results to guide the choice of regime.

4.3 Regime choices under robustness criteria:

In robustness studies it is often suggested that one should avoid policies that produce extreme bad results in any model (Kilponen and Salmon, 2004). The principle — a descendant of Roy's 'safety first principle' — is a way of knocking out a 'potentially dangerous' policy. In this context we are concerned about heterogeneity both of models and of social groups. One can think of this in political terms, from the viewpoint of a policy-maker: the two groups represent the two main sets of voters on whom the macro economy has effects; while the two models represent the possible spectrum of model uncertainties. One can also think of it in welfare terms as a way of seeking to satisfy a practical version of the Pareto principle — that there should be no (serious) losers. This indicates we should look for regimes that badly affect either group under either model, using the existing regime (inflation-targeting under interest-rate-control) as the benchmark. Table 9 shows all regime/model combinations relative to this benchmark.

It is clear that if we take the safety principle literally, we must rule out all changes of regime except one. The price-level regime would raise the variance of unemployment sharply under CM if indexation did not adjust or did so very slowly. Similarly money-targeting would raise it sharply under CM under interest rate control. Inflation targeting if it shifted to money supply control would do the same under CM. This only leaves money-targeting under money supply control.

If one is willing to assume that indexation will adjust in the manner predicted by the models then one reaches a similar result but by a different argument. Price-level targeting would then be disaster-free under both MSC and IRC, while money targeting would be disaster-free under MSC. Deciding between these two turns out to depend on which model one uses: under CM the employed prefer price-targeting with IRC the unemployed money-targeting (with MSC) but under LPM the unemployed prefer price-level targeting with MSC, the employed money-targeting (with MSC). Hence neither group has a clear preference for either regime. One might in these circumstances take the disaster-free regime which produces the best average welfare across both models: this (again) is money-targeting with MSC.

This regime represents in this context a compromise between price-level targeting and inflation targeting, in that price targeting produces too stark a contrast with the existing benchmark. It also represents a shift away from interest-rate control to money supply control; this occurs because of the switch in targeting regime, in the sense that MSC is dangerous under inflation targeting whereas IRC is dangerous under money-targeting for reasons discussed earlier to do with the different ways expected inflation is formed.Thus we find a 'back to the future' result here: monetary policy should revert to money supply control under a regime of targeting the path of the money supply.

Table 9: Inflation-targeting under Interest-rate-control=100: comparing welfare and variances for various combinations (figures taken from sample of 2000 for RAM and 10824 for LPM)

			CM(se=3)				LPM(se=1.3)	
	Inflation	P-level		M-level		Inflatn	P-level	M-level
		optim. indxn	fix indexn	optim. indxn	fix indexn.			
Total welfare:								
MSC	93	117	83	125	96	100	110	124
IRC	**100**	114	83	93	93	**100**	106	115
Var Unemp								
MSC	110^+	75	131^+	62	103	94	84	90
IRC	**100**	81	137^+	113^+	113^+	**100**	87	88
Var Cons:								
MSC	105	100	104	105	107	103	94	77
IRC	**100**	98	96	99	99	**100**	97	86

$^+$ denotes extreme bad variance, interpreted as 10% or more over the benchmark.

5 Conclusions:

We looked at the operation of monetary rules both within a calibrated model and within a 'live' forecasting model of the UK, and in both models we distinguished between the welfare of two groups of agents, the (usually) employed and the (often) unemployed, whom we identified respectively with skilled and unskilled workers. Our aim was to see how far allowing for such a difference of interests could matter for the choice of monetary regime. We found that it did matter, in the sense that certain model-policy combinations could cause harm particularly to the unskilled group and therefore would prudently be avoided either from a welfare or a political viewpoint We concluded that targeting the level of the money supply within an operating system of money-supply-control is the dominant monetary regime. It is both the only regime that strictly avoids a disaster to any group and it is also the one that delivers the highest average welfare across both models and groups.

One possible limitation of our results is that we have tackled the modelling issue by retaining aggregate functions in the models while identifying group welfare recursively — thus the groups may be differently affected, but aggregate behaviour is unaffected by this difference. We argued at the beginning that this limitation should not be a serious one; but it should in principle be investigated nevertheless.

There are of course other limitations: we have not investigated possible variations in policy (for example our targeting has all been assumed to be strict, that is no temporary deviations are allowed for in future plans) or in modelling issues (for example money velocity is stable, apart from the current random error) or in models (many other models can be considered). Thus our results should be considered as preliminary in content and methodologically illustrative.

References

[1] Andrews, M.J., P. Minford and J. Riley (1996) 'On comparing macroeconomic models using forecast encompassing tests', Oxford Bulletin of Economics and Statistics, Vol. 58, No. 2, 1996, pp. 279-305.

[2] Bank of Canada (1994) *Economic behaviour and Policy Choice under Price stability*, Ottawa.

[3] Berg, C. and L. Jonung (1999) 'Pioneering price level targeting: the Swedish experience 1931-37', *Journal of Monetary Economics*, 43(3), June 1999, pp525-551.

[4] Casares, M. (2002) 'Price setting and the steady-state effects of inflation', European Central Bank working paper no.140, May 2002, ECB, Frankfurt.

[5] Duguay, Pierre (1994) 'Some thoughts on price stability versus zero inflation', working paper, Bank of Canada — presented at the conference on Central Bank Independence and Accountability, Universita Boccoini, Milan, March 1994.

[6] Fischer, S. (1994) 'Modern central banking', in Capie, F. et al *The Future of Central Banking.*

[7] Hall, Robert E. (1984) 'Monetary strategy with an elastic price standard', in *Price Stability and Public Policy*, Federal Reserve Bank of Kansas City, Kansas City, 137-159.

[8] Heathcote, J. 'Fiscal policy with heterogeneous agents and incomplete markets,' *Review of Economic Studies*, 72(1), 161-188.

[9] Henderson, D.W., and McKibbin, W.J. (1993) 'An assessment of some basic monetary-policy regime pairs: analytical and simulation results from simple multi-region macroeconomic models', in R.C. Bryant, P. Hooper and C.L. Mann (eds.), *Evaluating policy regimes — new research in empirical macroeconomics*, Washington DC: Brookings Institution.

[10] Kiley, M. T. (1998) 'Monetary policy under neoclassical and new-Keynesian Phillips Curves, with an application to price level and inflation targeting', *Finance and Economics Discussion Series* 1998-27, Federal reserve Board, Washington, D.C.

[11] Kilponen, J. and M. Salmon (2004) 'Model misspecification, robustness and monetary policy', in Money Matters — essays in honour of Alan Walters, ed. P. Minford, forthcoming, Edward Elgar, January 2004.

[12] Lewis, J.M. (2003a) 'When does aggregation work?' mimeo, discussion paper,, Cardiff University.

[13] Lewis, J.M. (2003b) 'Attack of the clones' mimeo, discussion paper,, Cardiff University.

[14] Lucas, R.E. Jr (1976) ‘Econometric policy evaluation : A critique’, in K. Brunner and A.H. Meltzer (eds), *The Phillips Curve and Labour Markets*, Carnegie Rochester Conference Series on Public Policy No. 1, Supplement to the Journal of Monetary Economics.

[15] McCallum, Bennett T. and Edward Nelson (1999) ‘An optimising IS-LM specification for monetary policy and business cycle analysis’, Journal of Money, Credit and Banking, 31(3), August 1999, Part 1, pp. 296-316.

[16] Matthews, K., D. Meenagh, P. Minford and B. Webb (2003) ‘Stochastic simulations of monetary policy using the Liverpool Model of the UK’, mimeo, Cardiff University.

[17] Minford, A.P.L. (1980) ‘A rational expectations model of the United Kingdom under fixed and floating exchange rates’, in K. Brunner and A.H. Meltzer (eds) *On the State of Macroeconomics*, Carnegie Rochester Conference Series on Public Policy, **12**, Supplement to the Journal of Monetary Economics.

[18] Minford, A.P.L., E. Nowell and B. Webb (2003) ‘Nominal contracting and monetary targets — drifting into indexation’ *Economic Journal*, January, **113**, 65-100..

[19] Minford, P. and E. Nowell (2003) ‘Optimal monetary policy with endogenous wage contracts: is there a case for price level targeting?’ revised version of CEPR discussion paper no. 2616, 2000; mimeo, Cardiff University, latest version available http://www.cf.ac.uk/carbs/econ/webbbd/pm.html.

[20] Minford, P. and B. Webb (2005) ’Estimating large rational expectations models by FIML-some experiments using a new algorithm with bootstrap confidence limits’, *Economic Modelling*, vol. 22(1), January 2005, pp 187-205 (also appendices, Small Model Annex and Large Model Annex available at http://www.cf.ac.uk/carbs/econ/webbbd/pm.html)

[21] Nessen, M. and D. Vestin (2000) ‘Average inflation targeting’, mimeo, December 2000, Sveriges Riksbank, Stockholm.

[22] Parkin, J.M. (1998) ‘Unemployment, inflation and monetary policy’, *Canadian Journal of Economics*, 31(5), 1003-32.

[23] Poole, William (1970) ‘Optimal Choice of Monetary Policy Instruments in a Simple Stochastic Macro Model’ Quarterly Journal of Economics 84(2) May 1970, pp 197-216

[24] Smets, F. (2000) ‘What horizon for price stability?’ *European Central Bank working paper* no. 24, July 2000, ECB, Frankfurt.

[25] Storesletten, K., C.I. Telmer and A. Yaron (2004) ’Consumption and risk sharing over the life cycle’, *Journal of Monetary Economics*, 51 (3), 609-633.

[26] Svensson, L. E.O. (1999a) 'Price level targeting versus inflation targeting: a free lunch?' *Journal of Money Credit and Banking*, 31(3), pp277-295, August, 1999.

[27] Svensson, L. E.O. (1999b) 'Price stability as a target for monetary stability: defining and maintaining price stability', discussion paper no. 2196, August 1999, Centre for Economic Policy research, London.

[28] Taylor, J.B. (1993) 'Discretion versus policy rules in practice', *Carnegie-Rochester Series on Public Policy* 39, pp. 195-214.

[29] Vestin, David (2000) 'Price-level targeting versus inflation targeting in a forward-looking model', mimeo, IIES, Stockholm University.

[30] Wallis, K.F. (1995) 'Large-scale macroeconometric modelling' in H.Pesaran and M.Wickens (eds.) *Handbook of Applied Econometrics*, Blackwell.

[31] Williams, J. C. (1999) 'Simple rules for monetary policy', mimeo, Federal Reserve Board, February 1999, Washington D.C.

Appendix A The calibrated model (CM)

The model has two exogenous shocks driving it, a monetary (demand) shock, m_t, to the money supply presumed to originate from monetary policy, and a supply (productivity) shock, ϕ_t. The productivity shock is (rather naturally) modelled as a random walk throughout. Of course whether the money supply shock is transitory or permanent depends on the monetary rule; if it targets for example the level of money it will be transitory, if it targets the money supply growth rate, it will be turned into a random walk The authors then asked whether the monetary regime should target the growth rate or the level of the money supply; or of prices? They suggested that these choices appear in an unfamiliar light when indexation is endogenous. When the monetary regime moves to a price level rule with exogenous stationary money supply shocks, the aggregate supply curve flattens (as we have seen already above in our Phillips Curve set-up) and the aggregate demand curve steepens, generating a high degree of macro stability (i.e. in the face of supply shocks) provided that money supply shocks themselves are low-variance and stationary.

The representative household is assumed to be entirely liquidity-constrained; this assumption emphasises the importance of the contract choice, since a choice that minimises the variance of the spendable real wage is therefore identical with one minimising the variance of the employed agent's consumption. In a more realistic model with consumption smoothing this motive would have been implemented by including some transactions cost on smoothing, thus providing a motive for smoothing the real wage itself; however, this involves greater complexity than the stark assumption made that the transactions costs are in effect insuperable.

The household is embedded in an environment of profit-maximizing competitive firms which on a large proportion of their capital stock face a long lag before installation (a simple time-to-build set-up) and a government that levies taxes and pays unemployment benefits (which distort households' leisure decisions and introduce a 'social welfare' element into monetary policy). Firms and governments use the financial markets costlessly and settle mutual cash demands through index-linked loans; since there is no binding cash constraint on these agents, these loans are assumed to be unaffected by the imperfections of the price index which are short term in nature. This model is too simplified in many ways to match the data of a modern economy whether in trend or dynamics; however its focus is purely on the wage contract decision and its simplicity is justified in terms of its ability to match the OECD facts about wage contracts.

In calibrating the model the authors chose parameters perceived as plausible for modern OECD economies. The contract length is set at 4 quarters; the elasticity of leisure supply (σ) at 3; the share of stocks and other 'short-term' capital (k) at 0.3; the average life of other capital at 20 quarters; the share of labour income in value-added (μ) at 0.7 (the production function is Cobb-Douglas); the elasticity of the official price index to unanticipated inflation (c) at 0.2 (implying that a 1% unexpected rise in inflation would result in a 0.2% temporary overstatement of the price level faced by the representative consumer). The initial values assume 10% unemployment; a capital-output ratio of 6; an average (=marginal) tax rate of 0.10; a real interest rate of 5%.

The government is assumed to smooth both the tax rate and the growth rate of the money supply by borrowing (from firms). Nevertheless it cannot avoid noise in its money supply setting — the source of this could be its inability to monitor the money supply quickly or even at all (for example in the USA the use of dollars by foreigners around the world makes it impossible to know what the domestic issue of dollars is).

Money supply raises prices in the long run, and in the short run also raises output, with persistence extending up to 15 quarters but with most effect over after 10. In the high-indexed case there is less real effect and less persistence than in the high-nominal case.

These fairly standard properties stem from the model's deliberate drawing on elements that have been shown by past work to be useful in explaining the business cycle and also natural rates as discussed for example by Parkin (1998), though he notes we are still some way from building dynamic stochastic general equilibrium models that can fully explain the business cycle. The elements here include: time-to-build investment, cash-in-advance, nominal contracting (as noted above), household liquidity constraints, and (on the natural rate side) the influence of unemployment benefits on labour supply. With suitable country-by-country calibration one would expect to be able to model OECD countries' business cycle and natural rate experience with at least some modest success.

Minford et al found that in the face of stationary productivity and money supply shocks indexation would be minimal with only a slight tendency to rise as the variance of money shocks rose dramatically. However when shocks to either became highly persistent indexation to prices or to their close competitor, auction wages, (which together we term 'real wage protection') become large, becoming largest when both shocks are persistent. The reason was that productivity shocks would disturb prices and so the real worth of nominal wage contracts; indexation was of little use in remedying this disturbance if it was temporary because by the time the indexation element had been spent the shock would have disappeared, but with a permanent disturbance indexation can help offset it with a lag. If into this already-indexed world of persistent productivity shocks, monetary persistence is also injected, indexation rises further, to help alleviate the increased disturbance to real wages. This higher indexation also helps to alleviate the instability in unemployment which accompanies the greater shock persistence of money — the point being that this persistence induces persistence in the economy's departure from its baseline and so disturbs unemployment too for longer.

The authors looked at experience in the OECD in the 1970s where it is well-known that real wage protection was substantial; their calibrated model, when estimated variances and persistence of money and productivity shocks were fed into it, predicted high protection in all countries they could cover, apparently in line with the facts. They also found, contrary to much casual comment, that there was little evidence of any diminution of real wage protection in the 1990s; the model also predicted as much, for even though the variance of money supply shocks fell by then, their persistence remained essentially unchanged.

A1 Supply of work

$$a_t = a_c \cdot \left(W_t / \left(b_t \cdot P_{t-4}\right)\right)^{-\sigma} \cdot \varepsilon_t$$

A2 Demand for capital goods

$$K_t = (1-k) \cdot (1-\mu) \cdot E_{t-20}\left[d_t \cdot (1/R_t) \cdot (1-T_t)\right] + k \cdot (1-\mu) \cdot d_t \cdot (1-T_t) \cdot (1/r_t)$$

A3 Output function

$$d_t = \phi_t \cdot K_t^{(1-\mu)} \cdot \{(1-a_t) \cdot N\}^{\mu}$$

A4 Wage rate, solved for W_t

$$\overline{W_t} = (1-v-w) \cdot W_t + v \cdot E_{t-4}\left[W_t/P_t\right] \cdot P_t + w \cdot E_{t-4}\left[W_t\right]$$

A5 Official price index

$$\ln(P_t) = ln\left(p_t\right) + c \cdot \left(\ln\left(p_t\right) - \ln\left(E_{t-1}\left[p_t\right]\right)\right)$$

A6 Goods market clearing, solved for r_t after substituting for K_t from Eqn.2

$$d_t = M_{t-1}/p_t + K_t - K_{t-1}$$

A7 Labour market clearing, solved for p_t

$$N \cdot (1-a_t) = \left(\mu \cdot d_t \cdot (1-T_t) \cdot p_t\right) / \overline{W_t}$$

A8 Money market clearing, solved for $\overline{W_t}$

$$M_t = N\{\overline{W_t} \cdot (1-a_t) + b_t \cdot P_{t-4} \cdot a_t\}$$

A9 Efficiency

$$R_t = E_t\left[f(r)\right]^{1/20} - 1 \; ; \; f(r) = \Pi_{i=1}^{20}\left(1 + \tfrac{r_{t+i}}{4}\right)$$

A10 Money supply

$$M_t = \overline{M_t} + m_t$$

A11 Government budget constraint

$$b_t^g = \left(M_{t-1} - M_t + N \cdot B_t \cdot P_{t-4} \cdot a_t - d_t \cdot p_t \cdot T_t\right) / p_t + \left(1 + \tfrac{r_{t-1}}{4}\right) \cdot b_{t-1}^g$$

A12 Firm's budget constraint

$$d_t \cdot (1-T_t) = K_t - K_{t-1} + \left(\overline{W_t} \cdot (1-a_t) \cdot N\right) / p_t + b_{t-1}^p \cdot \left(1 + \tfrac{r_{t-1}}{4}\right) - b_t^p$$

Notes: 1. By Walras's Law the bond market clearing equation, $b_t^p + b_t^g = 0$, is redundant.

2. To normalise the variables d_t, K_t, r_t, p_t and $\overline{W_t}$ to their base run values constant factors were applied to the right-hand sides of the following equations in their solved form: A2 1.11 (multiplicative); A3 0.629 (multiplicative); A6 +0.0135 (additive); A7 0.7 (multiplicative); A8 0.9574 (multiplicative).

A.1 Variables and coefficients for CM

A.1.1 Endogenous variables : base run values

a_t	Supply of work	0.10
K_t	Demand for capital goods	6.00
d_t	Output function	1.00
W_t	Wage rate	1.00
P_t	Official price index	1.00
r_t	Real interest rate (fraction per annum)	0.05
p_t	Price level	1.00
$\overline{W_t}$	Average wage	1.00
R_t	Long term real interest rate (fraction per annum)	0.05
M_t	Money supply	1.00
b_t^g	Government bonds outstanding	0.00
b_t^p	Firms' bonds outstanding	0.00

A.1.2 Exogenous variables : base run values

B_t	Benefits	0.60
ε_t	$N(1.0, 0.01)$	1.00
ϕ_t	$N(1.0, 0.1)$	1.00
$\overline{M_t}$	Money supply target	1.00
m_t	Money shock	0.00
T_t	Tax rate	0.10

A.1.3 Coefficients

$$a_c = 0.46$$

$$\sigma = 3.00$$

$$k = 0.30$$

$$\mu = 0.70$$

$$N = 1.00$$

$$c = 0.20$$

Appendix B The Liverpool Model – Listing of equations

B.1 Behavioural equations

$$\log(EG_t) = \log\left(EGSTAR_t\right) + A39\log\left(Y_t/YSTAR_t\right) \tag{1}$$

$$\begin{aligned} XVOL_t = {} & A40YSTAR_t\{A27\log(WT_t) + A28\log(Y_t) + A47 + \\ & A29\left\{ESTAR_t + 0.6\left\{RXR_t - ESTAR_t\right\}\right\} + \\ & A30\left\{XVOL_t - 1/\left\{A40YSTAR_t - 1\right\}\right\}\} \end{aligned} \tag{2}$$

$$\begin{aligned} XVAL_t = {} & XVAL_{t-4} + \{XVOL_t - XVOL_{t-4}\} + A31 \\ & \{0.32YSTAR_t\{RXR_t - RXR_{t-4} - ESTAR_t + ESTAR_{t-4}\}\} + \\ & A32XVALres_{t-1} \end{aligned} \tag{3}$$

$$\begin{aligned} \log(M0_t) = {} & A44 + A13\log(M0_{t-1}) + A14\{\log(Y_t) + \\ & \log(1 - TAX_{t-1})\} + A16TREND_t + A17NRS_t + A18VAT_t \end{aligned} \tag{4}$$

$$\begin{aligned} \log(U_t) = {} & A42 + A3\log(Y_t) + A4\{\log(RW_t) + \log(1.0 + BO_t) + \\ & \log(1.0 + VAT_t)\} + A5TREND_t + A6\log(U_{t-1}) + A36Ures_{t-1} \end{aligned} \tag{5}$$

$$\begin{aligned} \log(G_t) = {} & A45 + A19RL_t + A20\{\log(G_{t-1}) - \log(FIN_{t-1})\} + \\ & A21\{\log(G_{t-1}) - \log(Gt-2)\} + \log(G_{t-1}) \end{aligned} \tag{6}$$

$$\begin{aligned} \log(CON_t) = {} & A46 + A22RL_t + A23\log(W_t) + A24QEXP_t + \\ & A25\log(CON_{t-1}) \end{aligned} \tag{7}$$

$$\begin{aligned} \log(RW_t) = {} & A43 + A7UNR_t + A8\{\log(UB_t) + \log(1.0 + LO_t)\} + \\ & A9\log(U_t) + A37\log(RW_{t-1}) + \{.095\}UNR_t\{-A10\} + \\ & A10\log(RW_{t-2}) + A11ETA_t + A12ETA_{t-1} \end{aligned} \tag{8}$$

$$\begin{aligned} RXR_t = {} & A41 + 0.000 + A1\{\log(RW_t) + \log(1.0 + BO_t)\} + \\ & A53\{\log(P_t) - \log(P_{t-4})\} + \{1. + A1\}\log(1. + VAT_t) + \\ & A2TREND_t + A35RXRres_{t-1} \end{aligned} \tag{9}$$

B.2 Identities and calibrated relationships

$$RS_t = \{RXR_t - E_t RXR_{t+1}\} + RSUS_t \quad (10)$$

$$NRS_t = E_t INFL_{t+1} + RS_t \quad (11)$$

$$RL_t = \{RXR_t - E_t RXR_{t+20}\}/5.0 + RLUS_t) \quad (12)$$

$$NRL_t = RL_t + E_t \left(\sum_{i=1}^{5} INFL_{t+i}/5 \right) \quad (13)$$

$$Y_t = GINV_t + CON_t + EG_t + XVOL_t - AFC_t \quad (14)$$

$$\begin{aligned} INFL_t &= \log(MON_t) - \log(MON_{t-4}) - \log(M0_t) \\ &\quad + \log(M0_{t-4}) \end{aligned} \quad (15)$$

$$\log(P_t) = \log(P_{t-4}) + INFL_t \quad (16)$$

$$W_t = FIN_t + G_t \quad (17)$$

$$BDEF_t = EG_t - 2.0 \times TAX_t \times Y_t + TAX_0 \times Y_0 \quad (18)$$

$$\begin{aligned} AFC_t &= Y_t\{0.6588318\{AFC_{t-1}/Y_{t-1}\} + 0.1966416\{AFC_{t-3}/Y_{t-3}\} + \\ &\quad 0.1454006\{AFC_{t-4}/Y_{t-4}\} + \} \end{aligned} \quad (19)$$

$$PSBR_t = BDEF_t + RDI_t \quad (20)$$

$$\begin{aligned} RDI_t &= -.5\{NRL_{t-1}/4.0\}FIN_{t-1}\{\{\{Pt/P_{t-1}\}^{0.66}\} - 1.0\} + \\ &\quad PSBR_t\{.32\{NRS_t/4.0\} + .5\{NRL_t/4.0\}\} + \\ &\quad 0.32\{NRS_t/4.\}FIN_{t-1} - .32\{NRS_{t-1}/4.\}FIN_{t-1} + RDI_{t-1} \end{aligned} \quad (21)$$

$$GINV_t = G_t - G_{t-1} + A38G_{t-1} \quad (22)$$

$$\begin{aligned} FIN_t &= EG_t - Y_t * \{TAX_t\} + XVAL_t + A54 * FIN_{t-1} + \\ &\quad \{1. - A54\} * \{FIN_{t-1} * \{\{P_{t-1}/P_t\}^{0.66}\}\} \\ &\quad \{1.0 - 0.155 * \{\{NRL_t/NRL_{t-1}\} - 1.0\}\} + res\_FIN_t + RDI_t \end{aligned} \quad (23)$$

B.3 Equilibrium variables (-star):

The -star variables YSTAR, USTAR, ESTAR and WSTAR are the equilibrium values of Y, U, RXR and RW respectively, found by solving equations 2,5,8 and 9 under the conditions that XVOL=0 and exogenous variables maintain their current values; EGSTAR is the value of EG that would produce a constant debt/GDP ratio with Y=YSTAR.

B.4 Coefficient values in order A1–56:

```
1.528 -0.003 -2.150 0.792 0.010 0.804 0.470 0.210 -0.018 -0.224
-0.290 0.189 0.870 0.150 0.000 -0.002 -0.349 0.839 -0.016 -0.004
0.640 -0.215 0.056 0.153 0.870 0.000 0.529 -1.205 -0.388 0.429
0.103 0.193 0.000 0.000 0.931 0.271 1.000 0.012 -0.125 0.320
0.170 25.262 0.102 -0.337 0.013 0.666 11.503 -0.016 -0.011 0.017
0.011 0.750 -0.750 0.300 -1.000 -1.000
```

(Exogenous variables — e = error term)

$RSUS_t = c + 0.899RSUS_{t-1} + e_t$

$EUNRS_t = c + 0.977EUNRS_{t-1} + e_t$

$\Delta \log WT_t = c + e_t$

$\Delta BO_t = c + e_t$

$\Delta VAT_t = c - 0.286\Delta VAT_{t-1} + e_t$

$\Delta UNR_t = c + 0.869\Delta UNR_{t-1} + e_t$

$\Delta UB_t = c + e_t$

$\Delta LO_t = c + e_t$

$\Delta TAX_t = c - 0.365\Delta TAX_{t-1} + e_t$

$\Delta \log EURXR_t = c + 0.235\Delta \log EURXR_{t-1} + e_t$

$\Delta \log EUCPI_t = c + 0.503\Delta \log EUCPI_{t-1} + e_t$

$\Delta \log MON_t = PEQ_t + MTEM_t + e_t$

B.5 Model notation:

Endogenous Variables

Y	GDP at factor cost
P	Consumer Price Level
INFL	Percentage growth rate of P (year-on-year)
MON	Nominal Money Stock (M0)
RW	Real wages (Average Earnings/Price)
U	Unemployment
Q	Output deviation from trend (Y/YSTAR)
AFC	Adjustment to factor cost
EG	real government spending on goods and services
BDEF	interest-exclusive budget deficit (deflated by CPI)
PSBR	public sector borrowing requirement (deflated by CPI)
XVAL	real current account of balance of payments
XVOL	same, at constant terms of trade
RS(RL)	real short term (log term) interest rate
NRS (NRL)	nominal short term (long term) interest rate
M0	real money balances (M0)
G	real private stock of durable goods, including inventories
W	real private stock of wealth
FIN	real private stock of financial assets (net)
CON	real private non-durable consumption
RXR	real exchange rate (relative CPI, UK v. ROW)
RDI	real debt interest
GINV	gross private investment in durables plus stock building

Exogenous Variables

MTEM	Temporary growth of money supply
PEQ	Growth of money supply
BO	Employers national insurance contributions
UNR	Trade Unionisation rate
LO	Average amount lost in taxes and national insurance
TREND	Time trend
WT	World Trade
TAX	Overall tax rate
UB	Unemployment benefit rate (in constant pounds)
EUNRS	Euro nominal short-term interest rates
EURXR	Euro real exchange rate index
EUCPI	Euro CPI
RSUS	US real short-term interest rate

Appendix C Targeting rules and their effects within CM

To examine how targeting works within the CM which though small is nevertheless non-linear and not analysable therefore in its original form (hence our use of stochastic simulations to discover its properties), it is necessary to linearise the model and simplify it into a form where we can derive its key analytical properties. The following lists the linearised equations (the numbering corresponds to that of the full model of Appendix A). The numbers shown are the effect of the normalising constants referred to in Appendix A. In order the equations are: (1) marginal labour supply which reacts to the auction wage; (2) the demand for capital; (3) the production function; (4) the actual nominal wage, a weighted average of auction (weight of α), indexed (weight of v), and nominal.; (5) the over-reaction of the official price index to the true price index; (6) goods market-clearing; (7) labour market-clearing; (8) money market-clearing.; and (9) the real spendable wage (wages are paid with a 1-period lag so the real spendable wage is the lagged one, deflated by the current price level).

$$a_t = -a_0 \sigma W_t \tag{A1}$$

$$K_t = 1.11 \left\{ \frac{k(1-\mu)(1-T_o)}{r_0} \right\} \left\{ (d_t - [\frac{r_t}{r_o}] \right\} \tag{A2}$$

$$d_t = \frac{(1-\mu)}{K_0} K_t + \phi_t - \frac{\mu}{(1-a_0)} a_t \tag{A3}$$

$$\overline{W}_t = \alpha W_t + v P_t \tag{A4}$$

$$P_t = (1+c) p_t \tag{A5}$$

$$d_t = m_{t-1} - p_t + K_t - K_{t-1} \tag{A6}$$

$$a_t = \frac{-[\mu(1-T_o)]}{0.7} (d_t + p_t - \overline{W}_t) \tag{A7}$$

$$m_t = \frac{(1-a_o)}{0.96} \overline{W}_t - \left(\frac{\overline{W}_o}{0.96} - B_o \right) a_t \tag{A8}$$

$$\overline{W}_{t-1} - p_t = \alpha W_{t-1} + v(1+c) p_{t-1} - p_t \tag{A9}$$

where $\alpha = (1 - v - w)$; equation numbers correspond to Appendix A.

For simplicity we have omitted all price and wage expectations from the wage-setting equation, (5); these are all dated at t-4. Similarly from the labour supply equation, (1), we omit the 4-quarter lagged price level which indexes unemployment benefits. Hence in effect the model solves in terms of the news occurring between t-4 and t, and in the case of (9), the real wage available for spending, because wages are paid with a 1-period lag, news between t-5 and t. The very long lag (20 quarters) terms determining the demand for capital are similarly omitted. We now explain the model's structure in terms of supply and demand.

Equation (A9) is the implied behaviour of the employed consumer's living standard, whose uncertainty is being minimised by the contract structure. We can progressively reduce the simultaneous block of equations (A1)–(A8) to three as follows. We can use equation (A3), the production function, and (A6), the supply of savings from goods market clearing, while also using (A1) to eliminate a_t, to obtain:

$$\Delta d_t = Z(d_t - m_{t-1} + p_t) + \Delta\phi_t + \frac{a_o \mu\sigma}{(1-a_o)}\Delta W_t \tag{A10}$$

where $Z = \frac{1-\mu}{K_o}$

This is the output supply made available by savings (and so capital) and by labour supply; the first terms in Z emerge from equation (A6) as the amount of savings (i.e. the output not devoted to consumption which is $m_t - p_t$). Note in passing that we can solve equation (A2) for r_t conditional on d_t, m_{t-1}, and p_t: since the latter determine available savings, the interest rate has to force the demand for capital to equal this availability. Hence equation (A2) and the interest rate are in a second, recursive block, and can therefore be ignored.

Equations (A1) and (A7), labour supply and demand, yield with (A4) and (A5), defining wages and the price index,

$$W_t = \frac{\mu(1-To)/0.7}{[\alpha\mu(1-T_o)/0.7] + a_o\sigma}\,[d_t + (1-v')p_t] \tag{A11}$$

where $v' = v(1+c)$

(A11) therefore specifies the free wages that would clear the labour market, given output and the price level. (A10) and (A11) between them constitute the supply-side of the model, augmented to include the market for savings (which depend on last period's money supply).

Finally, using the money market equation (A8) together with labour supply (A1) (which defines the split between benefits and wage payments) we obtain:

$$W_t = Q[m_t - (v'(1-a_o)/0.96)p_t] \tag{A12}$$

where

$$Q = \frac{1}{\alpha(1-a_o)/0.96 + (\overline{W}_o/0.96 - B_o)a_o\sigma}$$

(A12) is reminiscent of Robertson's 'wages fund'; there is a certain stock of money available to pay wages and benefits and given the structure of contracts, it determines free (auction) wages.

The full solution is complex. However, we can represent the model's main workings by reducing equations (A10) and (A11) to a 'supply curve' between free wages and the price level; and juxtapose it with the 'demand curve' given by (A12), the wages fund equation. We neglect terms in Z as of small magnitude, which conveniently allows us to rewrite (A10) in levels form as

$$d_t = \phi_t + \frac{a_o\mu\sigma}{(1-a_o)}W_t + c_0$$

where c_0 is a constant, ignored in what follows, reflecting the initial values of d_t, ϕ_t and W_t. In this case the supply curve from (A10) and (A11) becomes:

$$W_t = V[\phi_t + (1 - v')p_t] \text{ where } V = \frac{1}{\alpha + \frac{0.7a_o\sigma}{\mu(1-T_o)} - \frac{a_0\mu\sigma}{1-a_o}} \tag{A13}$$

It follows that:

$$p_t = \pi(Qm_t - V\phi_t) \text{ where } \pi = \frac{1}{[Qv'(1-a_0)/0.96] + V'(1-v')} \tag{A14}$$

and

$$W_t = lV\phi_t + (1-l)Qm_t \text{ where } l = \frac{[Qv'(1-a_0)/0.96]}{[Qv'(1-a_0)/0.96] + V'(1-v')} \tag{A15}$$

and the spendable real wage is

$$\overline{W}_{t-1} - p_t = \alpha W_{t-1} + v'p_{t-1} - p_t \text{ where } \alpha = 1 - v - w \tag{A16}$$

For $0 < v' < 1$ the resulting demand and supply picture is familiar. Figure 2 shows the model in p_t, W_t space; since labour supply $(1 - a_t)$ varies directly with the auction wage, this is also price level, employment space (output depends also on the capital stock, but is closely related to employment, and so this is also effectively familiar price, output space.) As v' tends to 0, DD (eq. A12) steepens and the SS (eq. A13) flattens.

We begin by considering within this model the nature of various basic monetary rules to which our discussion of optimality will be related; we then show a monetary rule may be optimised within that model; this discussion is conducted entirely in terms of the simplified linear model. We then consider the performance of various forms of targeting rules within the model in terms of the same model. Finally we use stochastic simulations of the full non-linear model to derive the accurately-calibrated optimality results. We conclude with some policy implications.

C.1 Targeting inflation and the level of money or prices — some mechanics of Monetary rules

If we take the linearised version of our model, we find the following solutions in general:

$$p_t = \pi(Qm_t - V\phi_t) \tag{C1}$$

where

$$\pi = \frac{1}{Qv'(1-a_0) + V(1-v')}$$

and

$$W_t = lV\phi_t + (1-l)Qm_t \tag{C2}$$

where

$$l = \frac{Qv'(1-a_0)}{Qv'(1-a_0) + V'(1-v')}$$

Both V and Q vary inversely with the share of auction contracts,α:

$$\begin{aligned} Q &= \frac{1}{\alpha(1-a_o)/0.96 + (\overline{W}_o/0.96 - B_o)a_o\sigma} \\ V &= \frac{1}{\alpha + \frac{0.7a_o\sigma}{\mu(1-T_o)} - \frac{a_0\mu\sigma}{1-a_o}} \end{aligned}$$

Recall that W_t (the auction wage, and the shadow price of labour supply) also directly determines employment through the labour supply function. Thus we can take it and employment as the same subject to some linear transformation.

Real (consumed) wages are:

$$\overline{W}_{t-1} - p_t = \alpha W_{t-1} + v'p_{t-1} - p_t \tag{C3}$$

Suppose that

$$\phi_t = \phi_{t-1} + \eta_t \tag{C4}$$

that is productivity follows a random walk. As we have seen, households raise their real wage protection (of their real consumed wage), the more persistent are price level shocks. Thus if there was zero protection ($\alpha = v' = 0$) they would be wide open to the variability of p_t. The more persistent the price shocks, the higher that variability, because the shocks would cumulate.

If we now compare a money supply rule that eliminates money shock persistence with one that eliminates price shock persistence, the first plainly eliminates one independent source of persistence in price shocks. Thus we would expect to find, and do, that protection falls. The second takes the process one stage further, eliminating all price shock persistence. Thus we should find that protection falls further still. (In the full non-linear model there are other sources of persistence, and these keep some incentive to protection alive; hence it does not disappear altogether.) An inflation targeting rule by contrast with both the money and price level targeting rules ensures that prices are expected to be a random walk, entirely persistent; and therefore in this regime indexation is high.

A price level rule is one that sets

$$0 = E_t p_{t+1} = \pi(QE_t m_{t+1} - VE_t \phi_{t+1}) \tag{C5}$$

and hence

$$E_t m_{t+1} = \frac{V}{Q} E_t \phi_{t+1} = \frac{V}{Q} \phi_t \tag{C6}$$

whence the 'price level rule' is

$$m_t = \frac{V}{Q} \phi_{t-1} + \epsilon_t \tag{C7}$$

whereas the ('pure money') rule that eliminates money shock persistence is simply

$$m_t = \epsilon_t \tag{C8}$$

Notice that under the price level rule money supply accommodates known past productivity shifts; this removes persistence from price shocks, though at the cost of persistence in money shocks.

An inflation-targeting rule sets

$$E_{tP_{t+1}} = \pi(QE_t m_{t+1} - VE_t \phi_{t+1}) = p_t = \pi(Qm_t - V\phi_t)$$

and hence (remembering that $E_t \phi_{t+1} = \phi_t$) $\quad$:$E_t m_{t+1} = m_t$ so that the behaviour of the money supply becomes:

$$m_t = m_{t-1} + \epsilon_t$$

When these are substituted into (C2) we obtain

$$\begin{aligned} \text{(price level rule) } W_t &= (1-l)Q\epsilon_t + V(l\phi_t + [1-l]\phi_{t-1}) && \text{(C9)} \\ \text{(pure money rule) } W_t &= (1-l)Q\epsilon_t + V(l\phi_t) && \text{(C10)} \\ \text{(inflation rule) } W_t &= (1-l)Q[\epsilon_t + m_{t-1}] + V(l\phi_t) && \text{(C11)} \end{aligned}$$

and when into (C3) we obtain:

$$\begin{aligned} \text{(price level rule) } \overline{W}_{t-1} - p_t = \pi V[\eta_t - v'\eta_{t-1}] - \pi Q[\epsilon_t - v'\epsilon_{t-1}] \\ + \alpha V[l\eta_{t-1} + \phi_{t-2}] + \alpha Q(1-l)\epsilon_{t-1} \quad \text{(C12)} \end{aligned}$$

and

$$\text{(money level rule) } \overline{W}_{t-1} - p_t = \pi V[\phi_t - v'\phi_{t-1}] - \pi Q[\epsilon_t - v'\epsilon_{t-1}]$$
$$+ \alpha V[l\phi_{t-1}] + \alpha Q(1-l)\epsilon_{t-1}$$
$$\text{(inflation rule)} \qquad \overline{W}_{t-1} - p_t = \pi V[\phi_t - v'\phi_{t-1}] - \pi Q[m_t - v'm_{t-1}]$$
$$+ \alpha V[l\phi_{t-1}] + \alpha Q(1-l)[\epsilon_{t-1} + m_{t-2}] \quad \text{(C13)}$$

For the nominal wage, W_t, which is directly related to employment (supply) and hence to output, the stability ranking is, with indexation at a high level as optimal under inflation-targeting, (from the most stable down), money rule>inflation rule>price-level rule; with indexation endogenous, money>price>inflation. For the spendable real wage (consumption), the stability ranking is, with indexation high as for inflation targeting, price>inflation>money; and for endogenous indexation, price>inflation=money. But in fact for spendable real wages all regimes deliver very similar stability; only for price-level targeting with endogenous indexation is the stability gain statistically significant.

Using the relevant equations above, and assuming that the wage contract length is 4 periods, then under the price rule we have:

$$\text{Var}W_t = \left(\frac{V}{Q}\right)^2 [3 + l^2]\sigma_\eta^2 + [(1-l)]^2\sigma_\epsilon^2 \quad \text{(C14)}$$

and

$$\text{Var}(\overline{W}_{t-1} - p_t) = \left[\frac{V}{Q}\right]^2 [\pi^2 + (\pi v' - \alpha l)^2 + 2\alpha^2]\sigma_\eta^2 + \{[\alpha(1-l) + \pi v']^2 + \pi^2\}\sigma_\epsilon^2 \quad \text{(C15)}$$

whereas under the money rule, $m_t = \epsilon_t$, we have the following variances (all divided for presentational convenience by Q^2):

$$\text{Var}W_t = 4\left(l\frac{V}{Q}\right)^2 \sigma_\eta^2 + [(1-l)]^2\sigma_\epsilon^2 \quad \text{(C16)}$$

and

$$\text{Var}(\overline{W}_{t-1} - p_t) = \left\{\pi^2 + [\pi v' + \alpha(1-l)]^2\}\sigma_\epsilon^2 + \left(\frac{V}{Q}\right)^2 \{\pi^2 + 3[\pi v' - \alpha l]^2\}\sigma_\eta^2\right\} \quad \text{(C17)}$$

and under the inflation rule we have:

$$\text{Var}W_t = 4\left(l\frac{V}{Q}\right)^2 \sigma_\eta^2 + 4[(1-l)]^2\sigma_\epsilon^2 \quad \text{(C18)}$$

and

$$\text{Var}(\overline{W}_{t-1} - p_t) \quad = \quad \left(\frac{V}{Q}\right)^2 \{(\pi)^2 + 3[\pi(1-v') - \alpha l]^2\}\sigma_\eta^2 + \quad \text{(C19)}$$
$$\{(\pi)^2 + [\pi(1-v') - \alpha(1-l)]^2 + 2[\pi v' - \alpha(1-l)]^2\}\sigma_\epsilon^2 \quad \text{(C20)}$$

Table 10: Table of Variances with CM as calibrated in this linearised and simplified version*:

VarW_t	Indexation as for inflation target (Indexation parameters: $v' = 0.5; l = 0.7$)	Indexation endogenous
Inflation rule	100	100 ($v' = 0.5; l = 0.7$)
Money rule	93	35 ($v' = 0..26; l = 0.36$)
Price rule	163	165 ($v' = l = 0$)
Var$(\overline{W}_{t-1} - p_t)$	(Indexation parameters: $v' = 0.5; l = 0.7$)	
Inflation rule	100	100 ($v' = 0.5; l = 0.7$)
Money rule	102	96 ($v' = 0..26; l = 0.36$)
Price rule	126	82 ($v' = l = 0$)

*calibration: $\pi = 0.2$ (for $v' = 0.5$);$= 0.15$(for $v' = 0$) and $= 0.175$ (for $v' = 0.26$); $\alpha = 0.1; \frac{V}{Q} = 1.3; \sigma_\eta^2 = \sigma_\epsilon^2 = 0.0001$

While these calculations show that the simplified linear model in our calibrations is somewhat adrift of the full model (especially with respect to the price-level rule for employment), they do show that if indexation is fixed at high inflation-targeting levels the money rule is good for employment stability, while the price rule is bad for it; and that with endogenous indexation the money rule is good for employment stability while the price rule is good for consumption stability.