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Welfare in the Nash Equilibrium in Export Taxes under Bertrand Duopoly*

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Abstract

In the Eaton and Grossman (1986) model of export taxes under Bertrand duopoly, it is shown that welfare in the Nash equilibrium in export taxes is always higher than welfare under free trade for both countries.

Keywords: Trade Policy, Imperfect Competition, Oligopoly.

JEL Classification: F12, F13, L13.

*We thank Gene Grossman for his comments, but any errors that remain are the responsibility of the authors.

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1. Introduction

Textbook analysis of trade policy games used to model 'trade wars' usually assumes symmetry so that both countries are identical or similar in size. Then, with both the terms of trade argument for a tariff and the profit-shifting argument for export subsidies, the trade policy game is a prisoners' dilemma where both countries are worse off in the Nash equilibrium in trade policies than under free trade. However, with the terms of trade argument for a tariff, Johnson (1953-54) showed that one country could be better off in the Nash equilibrium in tariffs than under free trade if its elasticity of import demand is sufficiently larger than that of the other country. Kennan and Riezman (1988) and Syropoulos (2002) showed that it is the larger country that may win a trade war. With the profit-shifting argument for an export subsidy as in Brander and Spencer (1985), Collie (1993) has shown that a country may be better off in the Nash equilibrium in export subsidies than under free trade if its firm has sufficiently lower costs than the firm in the other country. Thus, when there are asymmetries between countries, one country may win a trade war.

The Eaton and Grossman (1986) model of export taxes under Bertrand duopoly is different as both exporting countries will be better off in the Nash equilibrium in export taxes than under free trade in a symmetric trade policy game. This has been shown diagrammatically by Helpman and Krugman (1989, pp. 111-112) and algebraically, with linear demand functions and constant marginal costs, by Rivera-Batiz and Oliva (2003, pp. 270-271). An obvious question is whether one country may lose the 'trade war' in the Eaton and Grossman (1986) model when there are asymmetries between the countries in terms of demand or cost functions. This letter presents a simple proof to show that both countries will always be better off in the Nash equilibrium in export taxes than under free trade whatever the demand and cost asymmetries and whatever the functional form of the demand and cost functions.

2. The Model

Two countries *A* and *B* each have one firm that competes in a Bertrand duopoly in a third-country market as in Eaton and Grossman (1986). The demand in the third-country market for the product of firm *A* is $y^A\left(p^A, p^B\right)$ and for the product of firm *B* is $y^B\left(p^A, p^B\right)$ where p^A is the price set by firm *A* and p^B is the price set by firm *B*. Demand functions are decreasing in a firm's own price, $y_A^A \equiv \partial y^A / \partial p^A < 0$ and $y_B^B < 0$, and increasing in the price of its competitor, $y_B^A \equiv \partial y^A / \partial p^B > 0$ and $y_A^B > 0$, so the products are substitutes. The own-price effects on demand are assumed to dominate the cross-price effects so $\left|y_A^A\right| > \left|y_B^A\right|$ and $\left|y_B^B\right| > \left|y_A^B\right|$. There is no consumption of the products of the two firms in the markets of countries *A* and *B*. The total costs of firm A are $c^A\left(y^A\right)$ with $c_y^A \equiv \partial c^A / \partial y^A > 0$ and the total costs of firm B are $c^B\left(y^B\right)$ with $c_y^B \equiv \partial c^B / \partial y^B > 0$. The government in country *A* imposes an export tax e^A and the government in country *B* imposes an export tax e^B per unit of exports. The profits of the two firms from exports to the third-country market are:

$$\pi^A = p^A y^A - c^A\left(y^A\right) - e^A y^A \qquad \pi^B = p^B y^B - c^B\left(y^B\right) - e^B y^B \tag{1}$$

Assuming an interior solution where both firms export to the third-country market, the first-order conditions for the Nash equilibrium in prices (the Bertrand-Nash equilibrium) are:

$$\begin{aligned}
\pi_A^A &\equiv \frac{\partial \pi^A}{\partial p^A} = \left(p^A - c_y^A - e^A\right) y_A^A + y^A = 0 \\
\pi_B^B &\equiv \frac{\partial \pi^B}{\partial p^B} = \left(p^B - c_y^B - e^B\right) y_B^B + y^B = 0
\end{aligned} \tag{2}$$

To obtain the comparative static results for the effects of the export taxes of the two countries on prices, totally differentiate the first order-conditions:

$$\begin{pmatrix} \pi_{AA}^{A} & \pi_{AB}^{A} \\ \pi_{BA}^{B} & \pi_{BB}^{B} \end{pmatrix} \begin{pmatrix} dp^{A} \\ dp^{B} \end{pmatrix} = \begin{pmatrix} y_{A}^{A} de^{A} \\ y_{B}^{B} de^{B} \end{pmatrix} \tag{3}$$

where the second-order derivatives are:

$$\begin{aligned} \pi_{AA}^{A} &= 2y_{A}^{A} + \left(p^{A} - c_{y}^{A} - e^{A}\right) y_{AA}^{A} - \left(y_{A}^{A}\right)^{2} c_{yy}^{A} \\ \pi_{AB}^{A} &= y_{B}^{A} + \left(p^{A} - c_{y}^{A} - e^{A}\right) y_{AB}^{A} - y_{A}^{A} y_{B}^{A} c_{yy}^{A} \\ \pi_{BB}^{B} &= 2y_{B}^{B} + \left(p^{B} - c_{y}^{B} - e^{B}\right) y_{BB}^{B} - \left(y_{B}^{B}\right)^{2} c_{yy}^{B} \\ \pi_{BA}^{B} &= y_{A}^{B} + \left(p^{B} - c_{y}^{B} - e^{B}\right) y_{BA}^{B} - y_{B}^{B} y_{A}^{B} c_{yy}^{B} \end{aligned} \tag{4}$$

The second-order conditions for profit maximisation imply that $\pi_{AA}^{A} < 0$ and $\pi_{BB}^{B} < 0$ while the usual assumption in Bertrand duopoly models is that $\pi_{AB}^{A} > 0$ and $\pi_{BA}^{B} > 0$ so that the prices of the two firms are strategic complements. The own-price effects on (marginal) profits are assumed to dominate the cross-price effects so $\left|\pi_{AA}^{A}\right| > \left|\pi_{AB}^{A}\right|$ and $\left|\pi_{BB}^{B}\right| > \left|\pi_{BA}^{B}\right|$, and this implies that the determinant of the matrix in (3) is positive: $\Delta = \pi_{AA}^{A}\pi_{BB}^{B} - \pi_{AB}^{A}\pi_{BA}^{B} > 0$. Solving (3) yields the comparative static results for the effects of the export taxes on the prices set by the two firms:

$$\begin{aligned} \frac{\partial p^{A}}{\partial e^{A}} &= \frac{y_{A}^{A}\pi_{BB}^{B}}{\Delta} > \frac{\partial p^{B}}{\partial e^{A}} = -\frac{y_{A}^{A}\pi_{BA}^{B}}{\Delta} > 0 \\ \frac{\partial p^{B}}{\partial e^{B}} &= \frac{y_{B}^{B}\pi_{AA}^{A}}{\Delta} > \frac{\partial p^{A}}{\partial e^{B}} = -\frac{y_{B}^{B}\pi_{AB}^{A}}{\Delta} > 0 \end{aligned} \tag{5}$$

An export tax increases the price set by both firms. The comparative static results for the effects of the export tax on the exports of the two firms are:

$$\begin{aligned} \frac{\partial y^{A}}{\partial e^{A}} &= y_{A}^{A}\frac{\partial p^{A}}{\partial e^{A}} + y_{B}^{A}\frac{\partial p^{B}}{\partial e^{A}} = \frac{y_{A}^{A}}{\Delta}\left[y_{A}^{A}\pi_{BB}^{B} - y_{B}^{A}\pi_{BA}^{B}\right] < 0 \\ \frac{\partial y^{B}}{\partial e^{B}} &= y_{A}^{B}\frac{\partial p^{A}}{\partial e^{B}} + y_{B}^{B}\frac{\partial p^{B}}{\partial e^{B}} = \frac{y_{B}^{B}}{\Delta}\left[y_{B}^{B}\pi_{AA}^{A} - y_{A}^{B}\pi_{AB}^{A}\right] < 0 \end{aligned} \tag{6}$$

The expression in square brackets is positive for firm A since it is assumed that $\left|y_A^A\right| > \left|y_B^A\right|$ and $\left|\pi_{BB}^B\right| > \left|\pi_{BA}^B\right|$, and analogous assumptions hold for firm B. Therefore, an export tax will reduce the exports of the country that imposes the export tax.

The welfare of each country is given by the sum of its firm's profits and its export tax revenue:

$$\begin{aligned} W^A\left(e^A, e^B\right) &= \pi^A + e^A y^A = p^A y^A - c^A\left(y^A\right) \\ W^B\left(e^A, e^B\right) &= \pi^B + e^B y^B = p^B y^B - c^B\left(y^B\right) \end{aligned} \tag{7}$$

In the Nash-equilibrium in export taxes each country maximises its welfare given the export tax set by the other country. The first-order conditions for the Nash equilibrium in export taxes are:

$$\begin{aligned} \frac{\partial W^A}{\partial e^A} &= \left[\left(p^A - c_y^A\right) y_A^A + y^A\right] \frac{\partial p^A}{\partial e^A} + \left(p^A - c_y^A\right) y_B^A \frac{\partial p^B}{\partial e^A} = 0 \\ \frac{\partial W^B}{\partial e^B} &= \left[\left(p^B - c_y^B\right) y_B^B + y^B\right] \frac{\partial p^B}{\partial e^B} + \left(p^B - c_y^B\right) y_A^B \frac{\partial p^A}{\partial e^B} = 0 \end{aligned} \tag{8}$$

Substituting the first-order conditions for the Bertrand-Nash equilibrium (2) into (8) and using the comparative static results from (5) and (6) yields the Nash-equilibrium export taxes:

$$e_N^A = \frac{-y^A y_B^A \pi_{BA}^B}{y_A^A\left(y_A^A \pi_{BB}^B - y_B^A \pi_{BA}^B\right)} > 0 \qquad e_N^B = \frac{-y^B y_A^B \pi_{AB}^A}{y_B^B\left(y_B^B \pi_{AA}^A - y_A^B \pi_{AB}^A\right)} > 0 \tag{9}$$

From (6) the denominator is negative while the numerator is negative since the products are assumed to be substitutes ($y_B^A > 0$ and $y_A^B > 0$) and prices are strategic complements ($\pi_{BA}^B > 0$ and $\pi_{AB}^A > 0$). Therefore, the Nash-equilibrium export taxes are positive for both countries.

To compare the welfare of country *A* in the Nash equilibrium in export taxes, $W^A\left(e_N^A, e_N^B\right)$, with welfare under free trade, $W^B\left(0,0\right)$, note that the difference in welfare can be rewritten as:

$$W^A\left(e_N^A, e_N^B\right) - W^B\left(0,0\right) = \left[W^A\left(e_N^A, e_N^B\right) - W^A\left(0, e_N^B\right)\right] + \left[W^A\left(0, e_N^B\right) - W^A\left(0,0\right)\right] \qquad (10)$$

Since, in a Nash equilibrium, the government maximises its welfare given the export tax of the other country, $W^A\left(e_N^A, e_N^B\right) > W^A\left(0, e_N^B\right)$, so the first expression in square brackets must be positive. The second expression in square brackets can be written as follows using the mean value theorem:

$$W^A\left(0, e_N^B\right) - W^A\left(0,0\right) = \frac{\partial W^A\left(0, \overline{e}^B\right)}{\partial e^B} e_N^B \qquad (11)$$

where $\overline{e}^B \in \left[0, e_N^B\right]$.

Differentiating the welfare of country *A* with respect to the export tax of country *B*, and noting that the welfare of country *A* is equal to the profits of firm *A* when the export tax of country *A* is zero, yields:

$$\frac{\partial W^A\left(0, \overline{e}^B\right)}{\partial e^B} = \frac{\partial \pi^A}{\partial e^B} = \left[\left(p^A - c_y^A\right) y_A^A + y^A\right] \frac{\partial p^A}{\partial e^B} + \left(p^A - c_y^A\right) y_B^A \frac{\partial p^B}{\partial e^B} \qquad (12)$$

Note that the first-order condition for the Bertrand-Nash equilibrium for firm *A* implies that the expression in square brackets is zero when the export tax of country *A* is zero. Hence, (12) becomes:

$$\frac{\partial W^A\left(0, \overline{e}^B\right)}{\partial e^B} = \left(p^A - c_y^A\right) y_B^A \frac{\partial p^B}{\partial e^B} > 0 \qquad (13)$$

This is unambiguously positive so (11) is also positive and therefore both the expressions in square brackets in (10) are positive so the welfare of country *A* in the Nash equilibrium in export taxes is greater than welfare under free trade. Exactly the same argument can be used for country *B*. This leads to the following proposition:

Proposition*: Welfare in the Nash equilibrium in export taxes is always higher than welfare under free trade for both countries under Bertrand duopoly.*

Both countries are better off in the Nash equilibrium in export taxes than under free trade even if there are cost or demand asymmetries. The result has been derived for general demand and cost functions. Of course, consumers in the third-country export market are worse off and their losses exceed the gains of the exporting countries.

3. Conclusion

It has been shown that both exporting countries are better off in the Nash equilibrium in export taxes than under free trade. This is the case whatever the demand and cost asymmetries and whatever the functional forms for cost and demand functions. This contrasts with the results in other trade policy games where one country may be better off and one country may be worse off in the Nash equilibrium in trade policies.

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