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Panayiotis M. Pourpourides

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Cardiff Business School
Cardiff University
Colum Drive
Cardiff CF10 3EU
United Kingdom
t: +44 (0)29 2087 4000
f: +44 (0)29 2087 4419
www.cardiff.ac.uk/carbs

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Implicit Contracts and the Cyclical-ity of the Skill-Premium*

Panayiotis M. Pourpourides (Cardiff Business School)[†]

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Abstract

To examine the cyclical behavior of the skill-premium, this paper introduces implicit labor contracts in a DSGE model where production is characterized by capital-skill complementarity and the utilization of capital is endogenous. It is shown that this model can reproduce the observed cyclical patterns of wages and the skill-premium. The feature of capital-skill complementarity coupled with variable capital utilization rates does not come at odds with the acyclical behavior of the skill-premium. The paper argues that the skill-complementarity of capital is not a quantitatively significant factor at high frequencies. The key aspects are the contracts and the capital utilization margin.

JEL Classification Codes: E13, E24, E32

Keywords: Implicit Contracts; Wages; Skill-Premium; Business Cycles; Capital-Skill Complementarity

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[†]Correspondence: Cardiff Business School, Cardiff University, Aberconway Building, Colum Drive, Cardiff, CF10 3EU, United Kingdom. Tel: +44(0)29 2087 0591. E-mail: pourpouridesp@cardiff.ac.uk

1 Introduction

Recently, there has been a revival of interest in the behavior of the aggregate skill-premium, or the relative wage of skilled (college) and unskilled (no college) workers, at low frequencies. Krusell, Ohanian, Rios-Rull and Violante (2000, hereafter Krusell *et al.*) find that when technology exhibits the feature of capital-skill complementarity, changes in capital and labor inputs can account for nearly all the low frequency variation of the US skill-premium.¹ Despite the success of Walrasian equilibrium models in explaining economic growth facts they fail to account for the weak contemporaneous correlation between real wages and the business cycle. This is due to the fact that Walrasian real wages respond only to the marginal product of labor which is strongly positively correlated with the business cycle. Furthermore, in Walrasian markets, the coexistence of capital-skill complementarity and variable capital utilization generates strongly procyclical skill-premia which are at odds with the empirical findings.² This paper examines the extent to which the feature of capital-skill complementarity is important in accounting for the cyclical behavior of the skill-premium from the perspective of a real business cycle model with a non-Walrasian labor market. It also examines the role of endogenous capital utilization and shows that the coexistence of the latter and capital-skill complementarity can be reconciled with the acyclical behavior of the skill-premium.

The paper introduces implicit labor contracts in a dynamic stochastic general equilibrium

¹Capital-skill complementarity means that the elasticity of substitution between capital equipment and unskilled labor is higher than that between capital equipment and skilled labor. Evidence of capital-skill complementarity can also be found in previous studies. Hamermesh (1993) presents an extensive survey.

²With variable capital utilization and capital-skill complementarity present in the production process, the marginal productivity of skilled labor responds much more to technology improvements than the marginal productivity of unskilled labor because the response of the utilization of capital is strongly positive. Since technology shocks are the dominant source of business cycle fluctuations, a model which assumes a Walrasian labor market (i.e real wages equal marginal labor productivities) generates a strongly procyclical skill-premium which is at odds with the empirical findings. This implication of the Walrasian model is problematic because empirical evidence supports that both capital-skill complementarity and variable capital utilization are present in the production process. The importance of variable capital utilization is stressed, among others, by Bils and Cho (1994), Burnside and Eichenbaum (1996), Shapiro (1996), Basu and Kimball (1997) and King and Rebelo (1999).

(DSGE) model where production is characterized by capital-skill complementarity and the utilization of capital is endogenous. The model economy is populated by risk averse workers (skilled and unskilled) and less risk averse entrepreneurs that possess the capital stock of the economy. Following Boldrin and Horvath (1995), I consider the existence of one-period contracts that provide insurance against business cycle fluctuations to workers of both skill-types. The model is an otherwise standard neoclassical model where business cycles are driven by two types of shocks, Harrod-neutral and investment-specific.

As long as the workers are substantially more risk averse than the entrepreneurs, the Pareto optimal allocation implied by the model is quantitatively consistent with empirical findings on wages and the skill-premium while preserving basic business cycle properties of macroeconomic aggregates.³ The quantitative analysis indicates that the feature of capital-skill complementarity coupled with variable capital utilization rates does not come at odds with the observed cyclical variation of the skill-premium. Capital-skill complementarity has a significant role at low frequencies due to the fact that the trend of real wages can be well approximated by marginal productivities. The current study shows that short-run fluctuations of the skill-premium may not necessarily correspond only to fluctuations of the relative marginal productivity. The latter is consistent with the empirical findings of Cooley and Ogaki (1996). The relative wage fluctuations in the model are substantially affected by the variation of an insurance component which is embodied in real wages.

The empirical analysis shows that the steady state level of wage inequality has a significant role in short-run dynamics. Sensitivity analysis demonstrates that changing the steady state level of the skill-premium produces significant changes in the cyclical behavior of the latter. In particular, a higher level of wage inequality implies less procyclical (or even more countercyclical) skill-premia. This is due to the fact that as unskilled labor becomes cheaper than skilled labor, the firms have a bigger margin to increase relatively more the wages of unskilled workers in response to a technology improvement. The role of variable capital uti-

³The contracts are Pareto optimal in terms of the expected utility. See footnote 16.

lization is essential in the model because not only it magnifies and propagates the effects of shocks in the economy but also affects the responses of wages to the shocks in a way that the mixture of responses generates an acyclical skill-premium. The analysis indicates that when the utilization of capital is constant the wages and the skill-premium become considerably procyclical and in sharp contrast with the empirical findings.

Lindquist (2004) examines the cyclical behavior of the skill-premium by employing a DSGE model. Specifically, he finds the social planner's solution to a DSGE model with capital-skill complementarity in production and a fixed capital utilization rate. He shows that the model produces skill-premia which are uncorrelated with the business cycle and that a model without capital-skill complementarity bears no resemblance to the data. In either case, the model fails to replicate the observed cyclicity of real wages. The social planning solution corresponds to a decentralized Walrasian equilibrium because the welfare theorems hold. In Walrasian models however, wages always equal the marginal product of labor and hence, are strongly correlated with output. Furthermore, Young (2003) shows that if the utilization of capital is allowed to vary over the business cycle then, the feature of capital-skill complementarity generates a strongly procyclical skill-premium. The latter is attributable to the fact that the ratio of effective capital to skilled labor becomes strongly procyclical, dominating the effect of relative labor supply. The current study addresses those issues.

The remainder of this paper is organized as follows. Section 2 provides a brief discussion on implicit contracts and the cyclical behavior of real wages. Section 3 displays the model economy and section 4 analyzes the model quantitatively. Concluding remarks are contained in section 5.

2 Implicit Contracts and Real Wages

Beaudry and DiNardo (1991) show that when past market conditions are taken into consideration, in a manner consistent with models of labor contracting, the data suggest that real wages do not move systematically over the business cycle.⁴ Cooley and Ogaki (1996), find that the real wage equals marginal productivity only in the long run and that the time series properties of real wages are better explained by an optimal labor contract model. Ham and Reilly (2002), model the marginal product of labor as a function of observable demand variables and find that the implicit contracts model cannot be rejected by the data while the Walrasian model is rejected in all the cases considered.⁵

Keane and Prasad (1993) were among the first who studied the cyclical variation of wage differentials by classifying skilled and unskilled workers according to education. Their estimates indicate that, at the aggregate level, skilled and unskilled workers face essentially the same degree of cyclical variation in their wages. In other words, the wage premium paid to skilled workers is found to be acyclical. This observation is confirmed by evidence provided by Young (2003), Lindquist (2004) and Castro and Coen-Pirani (2008) who conduct their analyses using various aggregate measures of real wages.

The current study incorporates a mechanism of labor relations in which wages are decoupled from marginal productivity.⁶ Specifically, the wage rate and hours of work are

⁴The acyclicity of various aggregate measures of real wages is documented in many studies (eg Lindquist (2004) and Castro and Coen-Pirani (2008)).

⁵The implicit contracts theory has performed well in testing, even under the assumption that workers simply consume their labor earnings (eg Beaudry and DiNardo (1991, 1995) and Cooley and Ogaki (1996)). This assumption is also made in the current paper. Although this restrictive assumption is made for tractability it has ample empirical support. Campbell and Mankiw (1989) find that the permanent income hypothesis is not satisfied for about 50% of the population or, in other words, for about half the population consumption equals labor earnings. Guvenen (2006) finds that households in the top 20% of the wealth distribution hold about 90% of capital and land and virtually all financial assets but account for only 30% of aggregate consumption. Mankiw and Zeldes (1991) and Cagetti and De Nardi (2005, *b*) report similar results. Finally, several econometric studies estimate euler equations and find less evidence of asset pricing anomalies when only data on stockholders are employed (eg Ait-Sahalia, Parker and Yogo (2001), Attanasio, Banks and Tanner (2002), Brav, Constantinides and Ceczy (2002) and Vissing-Jørgensen (2002)).

⁶An alternative approach is to use a model of search and matching frictions. In such model, the frictions articulate an endogenous need for surplus sharing between workers and firms As Hall (2005) shows, any desired level of decoupling of wages from marginal product can be achieved by employing various wage-setting

determined prior to the realization of shocks via a perfectly enforceable labor contract. The contract is implicit in the sense that it specifies a wage bill and labor hours for each possible realization of a vector of shocks.⁷ The idea is based on the proposition that workers prefer relatively smooth predictable patterns to their income and thereby, are willing to buy insurance against business cycle fluctuations. Assuming that the workers are restricted from accessing capital markets, the labor contract is the only insurance device available to them. Consequently, wages are not only a function of productivity but also a function of an insurance component which breaks the one-to-one relationship between the wage and the marginal product of labor. The implication is that the insurance component prevents the wage from declining sharply during recessions and increasing substantially during expansions.

Labor contracts of this nature were previously introduced in dynamic real business cycle models by Horvath (1994) and Boldrin and Horvath (1995). Both papers assume that the economy is populated by a single type of risk averse workers and relatively less risk averse entrepreneurs. Horvath assumes that entrepreneurs simply organize production whereas Boldrin and Horvath introduce entrepreneurial labor effort. They allow for risk sharing between workers and entrepreneurs which takes the form of a utility contract structured as a state contingent wage and labor hours menu. Both show that the model with contracts not only replicates all the features by which the standard real business cycle model is deemed to fit the data well but also replicates the behavior of real wages. Gomme and Greenwood (1995) obtain similar results by introducing labor contracts in a slightly different way. First, the contracts are not derived from an incentive compatibility constraint. Instead, they assume that the real wage consists of two additively separable components, the marginal product of labor and a mixture of state contingent claims via which workers and entrepreneurs share

mechanisms or even simply varying parameters of a given wage-setting mechanism (i.e, Nash bargaining). Nevertheless, features such as workers of different skill types, capital-skill complementarity, various technology shocks and endogenous capital utilization are considerably difficult to model within a search/matching framework. The results of the current study are particularly useful for such an extension.

⁷The theoretical background of implicit contracts originates in the work of Bailey (1974), Azariadis (1975, 1976) and Gordon (1974).

aggregate risk. Second, hours of work are not determined a priori by the contract but are merely set to satisfy the model's efficiency conditions.⁸

3 The Benchmark Economy

Consider an economy where there are three types of infinitely lived agents: the skilled, the unskilled and the capitalists. The agents within each group are identical and in numbers such that every agent perceives his influence on aggregate quantities to be insignificant. The capitalists own all the capital stock of the economy which consists of capital structures, K_s , and capital equipment, K_e . The workers are restricted from accessing capital markets so that their current labor earnings equal their current consumption. Furthermore, there is a fixed number s of skilled workers and a fixed number u of unskilled workers for each capitalist.

The capitalists, first observe the realization of technology shocks, denoted by the vector $f \in \mathfrak{S}$, and then choose the level of output they produce, pay the workers, and retain the residual output to be either consumed or invested in future capital stock. The capitalists' output at time t is denoted by Y_t , and is given by the following four-factor constant returns to scale production function:

$$Y_t = e^{A_t} F(K_{st}, U_t K_{et}, L_{st}, L_{ut}) \quad (1)$$

where A_t is a Harrod-neutral technology shock that follows a stationary Markov process, L_{st} and L_{ut} are total skilled and unskilled hours, respectively, U_t is the utilization rate of capital equipment and $F(\cdot)$ is the following CES aggregator:

$$F(K_{st}, U_t K_{et}, L_{st}, L_{ut}) = K_{st}^\theta \left[\mu L_{ut}^\nu + (1 - \mu) [\lambda (U_t K_{et})^\varphi + (1 - \lambda) L_{st}^\varphi]^\frac{\nu}{\varphi} \right]^\frac{1-\theta}{\nu}$$

⁸Danthine and Donaldson (1992) incorporate labor contracting in a real business cycle model that is populated by young and old workers. The role of the contract is simply to guarantee full employment to all old workers. The model is successful in replicating the observed volatility in hours.

where θ is the income share of capital structures. Parameters μ and λ determine the income share of unskilled labor and the income share of capital equipment relative to skilled labor, respectively. The elasticity of substitution between capital equipment and unskilled labor is $1/(1 - \nu)$ while the elasticity of substitution between capital equipment and skilled labor is $1/(1 - \varphi)$. Whenever $\nu > \varphi$, the production function is said to exhibit capital-skill complementarity. Capital structures can be produced from final output on one-to-one basis. The stock of structures evolves according to the following law of motion:

$$K_{st+1} = (1 - \delta_s) K_{st} + I_{st} \quad (2)$$

where I_{st} is investment in structures and δ_s is the corresponding depreciation rate. The accumulation equation for capital equipment is the following:

$$K_{et+1} = (1 - \delta_e(U_t)) K_{et} + e^{Z_t} I_{et} \quad (3)$$

where I_{et} is investment in equipment and Z_t is an investment specific shock that follows a stationary Markov process which is independent of that of A_t . Let the vector of technology shocks be denoted by $f_t = [A_t, Z_t]'$. Then, f follows a stationary Markov process summarized by the transition function $P(f, f')$ - where the prime, $'$, denotes next period.

The depreciation rate of capital equipment is assumed to change with the utilization rate of capital and is defined as

$$\delta_e(U_t) = b \frac{U_t^{1+\omega}}{1+\omega} \quad (4)$$

where $b > 0$ and $\omega > 0$.⁹ Installing new capital involves adjustment costs $\alpha_t = \alpha_{st} + \alpha_{et}$, where α_{st} is the cost for structures and α_{et} is the cost for equipment. These costs have the

⁹As noted by Greenwood *et al.* (2000), equipment, unlike structures, has variable rates of utilization and depreciation because it has a more active role in production. There is also evidence that new technologies are mainly embodied in capital equipment rather than in capital structures.

following functional form:

$$\alpha_i = \frac{\xi_i}{2} (I_{it} - \delta_i K_{it})^2 \text{ for } i = s, e \quad (7)$$

where $\xi_i > 0$.¹⁰ Notice that at the steady state adjustment costs are zero by construction.

The capitalists act only as entrepreneurs and simply organize production. The utility of a capitalist depends only on the level of his consumption, C_{mt} , and it is assumed to be logarithmic:

$$v(C_{mt}) = \ln C_{mt} \quad (8)$$

Given that the workers are restricted from accessing the capital markets, the choice of their utility function has a particular importance. I assume that workers are homogeneous in terms of their preferences and that both types have the following nonseparable utility function:

$$u(c_{it}, T - l_{it}) = \frac{x(c_{it}, T - l_{it})^{1-\eta}}{1-\eta} \quad (9)$$

where

$$x(c_{it}, T - l_{it}) = [\varkappa c_{it}^\gamma + (1 - \varkappa)(T - l_{it})^\gamma]^{\frac{1}{\gamma}}, \text{ for } i = s, u$$

c_{it} and l_{it} are consumption and individual working hours for a worker of type i , T is the total number of nonsleeping hours, $\eta > 1$, $0 \leq \gamma < 1$ and $0 < \varkappa < 1$. The term $\bar{\gamma} = 1/(1 - \gamma)$ is the elasticity of substitution of leisure for consumption and will play a role later in the paper. The reasons for choosing the specific form of utility function are twofold. First, separability is usually rejected by the data. In particular, Ham and Reilly (2002) perform tests and reject additive within-period preferences within an implicit contract model.¹¹ Second, with separable preferences the Walrasian labor supply would have a positive slope only if the workers had risk aversion below one. Since the empirical evidence suggests that an individ-

¹⁰Parameter δ_e denotes the steady state depreciation rate of capital equipment.

¹¹See also Browning, Deaton and Irish (1985), Altonji and Ham (1990) and Basu and Kimball (2002).

ual's risk aversion is decreasing with his wealth, the capitalist would have risk aversion even further below one.¹² The latter would imply an extremely large elasticity of intertemporal substitution (*EIS*) which is at odds with empirical findings.¹³

Given that the agents within each group are identical, the resource constraint of the economy can be written as:

$$Y_t \geq C_{mt} + C_{st} + C_{ut} + I_{st} + I_{et} + \alpha_t \quad (10)$$

where $C_{st} = sc_{st}$ and $C_{ut} = uc_{ut}$

3.1 The Walrasian Alternative

The Walrasian equilibrium is defined in order to compute the workers' reservation utilities.¹⁴ To distinguish individual choices from equilibrium outcomes the latter are denoted with a superscript star, *. When workers trade only in spot markets, decisions follow after f is observed. The problem of a worker of type i can be written as

$$\begin{aligned} \max_{q_{sw}} \sum_{t=0}^{\infty} \beta^t \int_{\mathfrak{S}} u(c_{it}, T - l_{it}) P_t(f_t, df_{t+1}) \\ \text{subject to } 0 < c_{it} \leq W_{it} \equiv w_{it}l_{it} \end{aligned} \quad (11)$$

where w_{it} is the wage per hour, $q_{sw} = \{c_{it}, l_{it}\}$ and $0 < \beta < 1$. The first-order conditions are summarized by the following condition:

$$w_{it}^S = \frac{u_2(W_{it}, T - l_{it})}{u_1(W_{it}, T - l_{it})} \quad (11a)$$

¹²Among others, see Ogaki and Atkeson (1997).

¹³Empirical findings suggest that the *EIS* should be around 0.1 – 0.2 for an average consumer and around 1 for the wealthy stockholding minority (see Guvenen (2006)).

¹⁴The Walrasian market will always constitute an alternative market for the workers. The employment contract however is designed in such a way that eventually none of the workers will trade in this market.

where w_{it}^S denotes the inverse labor supply. Using (9), the condition above delivers the labor supply:

$$l(w_{it}^S) = \frac{T}{\bar{\varkappa}^{\frac{1}{\gamma-1}} [w_{it}^S]^{\frac{\gamma}{\gamma-1}} + 1} \quad (11b)$$

or simply $l_{it} = l(w_{it}^S)$ where $\bar{\varkappa} = \varkappa / (1 - \varkappa)$. Notice that the response of wages to changes in labor supply is larger the smaller the elasticity of substitution between consumption and leisure.

Given an initial state vector $\Omega_0 = [f_0, K_{s0}, K_{e0}]'$, and a stochastic sequence of wage rates $\{w_{it}\}_{t=0}^{\infty}$ for $i = s, u$, the dynamic programming problem facing the representative capitalist can be written as:

$$\begin{aligned} V(\Omega_0) &= \max_{q_{sc}} \sum_{t=0}^{\infty} \beta^t \int_{\mathfrak{S}} v(C_{mt}) P_t(f_t, df_{t+1}) \\ &\text{subject to } e^{At} F(K_{st}, U_t K_{et}, L_{st}, L_{ut}) \geq C_{mt} \\ &\quad + s w_{st} l_{st} + u w_{ut} l_{ut} + I_{st} + I_{et} + \alpha_t, \\ K_{st+1} &= (1 - \delta_s) K_{st} + I_{st}, \quad K_{et+1} = (1 - \delta_e(U_t)) K_{et} + e^{Z_t} I_{et} \end{aligned} \quad (12)$$

where $q_{sc} = \{C_{mt}, l_{st}, l_{ut}, U_t, K_{st+1}, K_{et+1}\}$, $L_{st} = s l_{st}$ and $L_{ut} = u l_{ut}$. The problem of the capitalist delivers the inverse labor demand, w_{it}^D :

$$w_{it}^D = MPL_{it}$$

where MPL_{it} denotes the marginal product of labor of a worker of type i . In equilibrium, $w_{it}^S = w_{it}^D$ which implies the following intratemporal efficiency condition:¹⁵

$$\frac{u_2(W_{it}^*, T - l_{it}^*)}{u_1(W_{it}^*, T - l_{it}^*)} = MPL_{it}^* \quad (13)$$

Definition of Spot Equilibrium: A spot equilibrium is a vector of initial conditions $\Omega_0 = [f_0, K_{s0}, K_{e0}]'$, a set of allocation rules $C_{mt}^* = C_m^s(\Omega_t)$, $K_{st+1}^* = K_s^s(\Omega_t)$, $K_{et+1}^* =$

¹⁵Equilibrium labor hours, l_{it}^* , are obtained by evaluating $l(w_{it})$ at $w_{it} = MPL_{it}$.

$K_e^s(\Omega_t)$, $U_t^* = U^s(\Omega_t)$, $l_{st}^* = l_s^s(\Omega_t)$ and $l_{ut}^* = l_u^s(\Omega_t)$, a set of pricing functions $w_{st}^* = w_s^s(\Omega_t)$ and $w_{ut}^* = w_u^s(\Omega_t)$, such that: (i) Workers solve problem (11), taking as given the aggregate state of the world Ω and the form of functions $w_s^s(\cdot)$ and $w_u^s(\cdot)$, with the equilibrium solution to this problem satisfying $l_{st}^* = l_s^s(\Omega_t)$ and $l_{ut}^* = l_u^s(\Omega_t)$. (ii) Capitalists solve problem (12), given Ω and the functions $w_s^s(\cdot)$ and $w_u^s(\cdot)$, with the equilibrium solution to this problem satisfying $L_{st}^* = sl_s^s(\Omega_t)$, $L_{ut}^* = ul_u^s(\Omega_t)$, $C_{mt}^* = C_m^s(\Omega_t)$, $K_{st+1}^* = K_s^s(\Omega_t)$, $K_{et+1}^* = K_e^s(\Omega_t)$ and $U_t^* = U^s(\Omega_t)$.

The spot skill-premium is expressed as the ratio of skilled to unskilled wage. Given (1), the logarithm of the skill-premium can be written as a function of input ratios, that is

$$\ln \pi_t^{sp} = \vartheta + \frac{\nu - \varphi}{\varphi} \ln \left[\lambda \left(\frac{U_t^* K_{et}^*}{L_{st}^*} \right)^\varphi + (1 - \lambda) \right] + (\nu - 1) \ln \left[\frac{l_{st}^*}{l_{ut}^*} \right] \quad (14)$$

where ϑ is a constant. Notice, that there are two effects driving the skill-premium: the ratio of effective capital equipment to skilled hours and the ratio of skilled to unskilled hours. Krusell *et al.* (2000), call the latter the *relative supply effect* and the former the *capital-skill complementarity effect*. If capital-skill complementarity is present in the production process increases in the equipment-skill ratio tend to increase the skill-premium.

3.2 Equilibrium with Contracts

In this section, I assume that at any time t there is a competitive market for one period ahead contracts. At the end of each contractual period ($t-1$) capitalists offer the skilled and the unskilled workers menus $\{W_s(f_{t-1}, f_t), L_s(f_{t-1}, f_t)\}$ and $\{W_u(f_{t-1}, f_t), L_u(f_{t-1}, f_t)\}$ of possible labor earnings and working hours which will be in effect the following period (t). As in Beaudry and DiNardo (1995), wages and hours depend on both the market conditions at the time the contract is signed and the market conditions at the time the contract will be in effect, and thus are functions of both f_{t-1} and f_t . These contracts are perfectly enforceable

at no observable cost to either party (i.e workers and employers are expected to honor the agreement). To avoid dealing with issues of asymmetric information I assume that the realization of shocks is public information. The contract is designed so that the capitalist and the worker always reach an agreement.

As noted by Boldrin and Horvath (1995), both workers and capitalists have an incentive to sign labor contracts. When there is a recession and the labor market is driven by a Walrasian mechanism the workers experience a severe fall in their utility. Consequently, workers are willing to accept a contract scheme that prevents such big declines. On the other hand, the capitalists exhibit higher tolerance for aggregate risk and thereby, desire to exploit the higher gains during expansions in exchange of undergoing a larger fraction of losses during recessions. As stated by Rosen (1985), implicit contracts embody implicit payments of insurance premiums by workers in favorable states of nature and receipt of indemnities in unfavorable states.

Every period, both skilled and unskilled workers have reservation utilities denoted by \bar{u}_{st} and \bar{u}_{ut} , respectively. The reservation utility is the lower bound of the expected utility that workers require in order to sign a contract. An optimal contract guarantees that the expected utility is at least the same as the reservation utility over the lifetime of the contract.¹⁶ If the expected utility from signing a contract is less than the acceptable lower bound then, the workers will trade in the spot market. The incentive compatibility constraint (ICC) or commitment constraint for a worker of type i can be written as:

$$\bar{u}_{it} \leq E_t [u(W_{it+1}, T - l_{it+1})] \equiv \int_{\mathfrak{S}} u(W_i(f_t, f_{t+1}), T - l_i(f_t, f_{t+1})) P_t(f_t, df_{t+1}) \quad (15)$$

¹⁶As noted by Hart and Holmström (1987), optimality is not to be understood in a first-best sense, but rather in a constraint or second-best sense because of informational restrictions. These restrictions arise due to the uncertainty about future realizations of f .

where

$$\begin{aligned} \bar{u}_{it} &= E_t \left[u \left(\tilde{w}_{it+1} \tilde{l}_{it+1}, T - \tilde{l}_{it+1} \right) \right] \\ &\equiv \int_{\mathcal{L}} u \left(\tilde{w}_i(\Omega_{t+1}) \tilde{l}_i(\tilde{w}_i(\Omega_{t+1})), T - \tilde{l}_i(\tilde{w}_i(\Omega_{t+1})) \right) Q(\Omega_t, d\Omega_{t+1}) \end{aligned}$$

where the space $\mathcal{L} = \mathfrak{S} \times \mathbb{k}$ denotes the set of feasible Ω 's and $Q(\Omega_t, d\Omega_{t+1})$ is the equilibrium transition function (see Boldrin and Horvath (1995)). Variables W_i and l_i are income and working hours under the contract for a worker of type i and \tilde{w}_i and \tilde{l}_i are the equilibrium wage and working hours for a worker of type i that trades in a spot market while all other workers in the group signed a contract. The spot-market wage is the marginal product of labor evaluated at the labor supply of the workers under contract. The individual spot-market labor supply is the choice of labor hours defined in equation (11b). The ICC implies that a contract which is agreed in favorable market conditions will generally be superior than a contract agreed in unfavorable market conditions because the reservation utility will tend to be higher.

The capitalist needs to offer contracts, $\{W_s(f_t, f_{t+1}), L_s(f_t, f_{t+1})\}$ and $\{W_u(f_t, f_{t+1}), L_u(f_t, f_{t+1})\}$, to workers of both skill types and simultaneously choose future capital stocks, the utilization rate of capital and his consumption level. As shown by Boldrin and Horvath (1995), the equilibrium in the contracts economy can be derived from two separate problems. In the first problem, the capitalist takes parametrically C_{mt+1}^* , U_{t+1}^* , K_{st+1}^* , K_{et+1}^* , K_{st+2}^* and K_{et+2}^* and chooses labor contracts for both types of workers which maximize his expected

utility:¹⁷

$$\begin{aligned}
& \max_{q_{cc}} \int_{\mathfrak{S}} v(C_{mt+1}) P_t(f_t, df_{t+1}) \quad \text{subject to} \\
& \bar{u}_{it} \leq \int_{\mathfrak{S}} u(W_i(f_t, f_{t+1}), T - l_i(f_t, f_{t+1})) P_t(f_t, df_{t+1}) \quad \text{for } i = s \text{ and } u, \\
& e^{A_{t+1}} F(K_{st+1}^*, U_{t+1}^* K_{et+1}^*, L_s(f_t, f_{t+1}), L_u(f_t, f_{t+1})) \geq C_{mt+1} \\
& \quad + sW_s(f_t, f_{t+1}) + uW_u(f_t, f_{t+1}) + I_{st+1}^* + I_{et+1}^* + \alpha_{t+1}^*, \\
& K_{st+2}^* = (1 - \delta_s) K_{st+1}^* + I_{st+1}^*, \quad K_{et+2}^* = (1 - \delta_e (U_{t+1}^*)) K_{et+1}^* + e^{Z_{t+1}} I_{et+1}^*
\end{aligned} \tag{16}$$

where $q_{cc} = \{W_s(\cdot), L_s(\cdot)\}, \{W_u(\cdot), L_u(\cdot)\}$. Competition in the market of contracts ensures that in equilibrium all capitalists offer identical menus of wages and labor hours. In the second problem, the capitalist chooses future capital stocks, the utilization rate of capital and his consumption to maximize expected utility subject to the budget constraint and capital accumulation equations, taking as given the optimal contracts:

$$\begin{aligned}
& \Upsilon(\Omega_t; W_s^c(\cdot), L_s^c(\cdot), W_u^c(\cdot), L_u^c(\cdot)) = \max_{\bar{q}_{cc}} \{v(C_{mt}) \\
& + \beta \int_{\mathfrak{S}} \Upsilon(\Omega_{t+1}; W_s^c(\cdot), L_s^c(\cdot), W_u^c(\cdot), L_u^c(\cdot)) P_t(f_t, df_{t+1})\} \\
& \text{subject to } e^{A_t} F(K_{st}, U_t K_{et}, L_s^c(\cdot), L_u^c(\cdot)) \geq C_{mt} + sW_s^c(\cdot) \\
& + uW_u^c(\cdot) + K_{st+1} - (1 - \delta_s) K_{st} + e^{-Z_t} K_{et+1} - (1 - \delta_e (U_t)) e^{-Z_t} K_{et} \\
& + \frac{\xi_s}{2} (K_{st+1} - K_{st})^2 + \frac{\xi_u}{2} (e^{-Z_t} K_{et+1} - (1 - \delta_e (U_t)) e^{-Z_t} K_{et} - \delta_e K_{et})^2
\end{aligned} \tag{17}$$

where $W_i^c(\cdot), L_i^c(\cdot)$ denote the equilibrium solution to (16) as a function of the state and of other equilibrium variables, and $\bar{q}_{cc} = C_{mt}, U_t, K_{st+1}, K_{et+1}$.

Definition of the Contracts Equilibrium: An equilibrium for the contracts economy

is a vector of initial conditions $[f_0, K_{s0}, K_{e0}]$ and a set of functions $W_s^c(\cdot), L_s^c(\cdot), W_u^c(\cdot), L_u^c(\cdot), C_m^c(\cdot), K_s^c(\cdot), K_e^c(\cdot)$ and $U^c(\cdot)$ depending on the state vector $\Omega_t = [f_{t-1}, f_t, K_{st}, K_{ut}]'$ such that:¹⁸ (i) $W_s^c(\cdot), L_s^c(\cdot), W_u^c(\cdot)$ and $L_u^c(\cdot)$ solve problem (16)

¹⁷The capitalist is choosing a pair of $\{W_i, L_i\}_{i=s,u}$ for each possible realization of the vector f_{t+1} , conditional on the current realization of f_t .

¹⁸Decisions for next period's wage bill and hours of work are made in the current period (i.e conditional

for all Ω_t given $C_m^c(\cdot)$, $K_s^c(\cdot)$, $K_e^c(\cdot)$ and $U^c(\cdot)$. (ii) $C_m^c(\cdot)$, $K_s^c(\cdot)$, $K_e^c(\cdot)$ and $U^c(\cdot)$ solve problem (17) for all Ω_t given $W_s^c(\cdot)$, $L_s^c(\cdot)$, $W_u^c(\cdot)$ and $L_u^c(\cdot)$.

It is important to note that the optimal contract is described by the ICC and the following intratemporal efficiency condition (see Beaudry and DiNardo (1995)):¹⁹

$$\frac{u_2(W_{it+1}^c, T - l_{it+1}^c)}{u_1(W_{it+1}^c, T - l_{it+1}^c)} = MPL_{it+1}^* \quad (18)$$

Condition (18) differs from condition (13) in that the hourly wage in (18) is not equal to marginal productivity. This has different implications on the dynamics of the model. Condition (18) implies that while an increase in MPL tends to increase labor hours, an increase in the hourly wage tends to decrease hours. In other words, changes in wages induce only income effects on hours.²⁰

In an economy with contracts wages are not perfectly correlated with the marginal product of labor. Other than the effect of MPL , contract wages and the skill-premium are directly affected by the elasticities of substitution of consumption for leisure and the ratios of leisure to labor. Using the functional forms for preferences and technology, (18) is reduced to an explicit function of the equilibrium hourly wage. The logarithm of the contract wage can be expressed as:

$$\ln w_{it}^{co} = \tau + \bar{\gamma} \ln w_{it}^{sp} + \ln \left[\frac{T - l_{it}}{l_{it}} \right], \text{ for } i = s, u \quad (19)$$

on the current realization of shocks, the capitalists offer a contract which specifies a wage bill and working hours for each possible future realization of shocks). Therefore, the level of equilibrium variables not only depends on current period shocks but also on previous period shocks. Beaudry and DiNardo (1991), show that this feature of implicit contracts is empirically significant.

¹⁹As pointed out by Beaudry and DiNardo, the addition of the commitment constraint, (15), does not create any trade-off between ex post efficiency and optimal risk sharing between workers and capitalists.

²⁰Beaudry and DiNardo (1995), Cooley and Ogaki (1996) and Ham and Reilly (2002) find strong evidence in favor of the implicit contract theory by testing the intratemporal condition.

and the logarithm of the skill-premium can be expressed as:

$$\ln \pi_t^{co} = \bar{\gamma} \ln \pi_t^{sp} + \ln \left[\frac{(T - l_{st}) / l_{st}}{(T - l_{ut}) / l_{ut}} \right] \quad (20)$$

where w^{sp} is the spot wage or *MPL*, and τ is a constant. As shown in equation (19), the harder it is to substitute consumption for leisure (the smaller $\bar{\gamma}$ is) the less responsive the contract wage is to variations in marginal product fluctuations. Likewise, $\bar{\gamma}$ controls the elasticity of the contract skill-premium to variations in the spot skill-premium. The existence of real rigidities takes the form of risk sharing between workers and capitalists. Changes in hours induce significant opposite effects on wages via the third term of equation (19). An increase (decrease) in hours tends to decrease (increase) the ratio of leisure to labor which is positively related to the wage. For instance, during expansions where labor hours increase the worker receives a wage below his marginal productivity whereas during recessions where labor hours decrease he receives a wage above his marginal productivity. This prevents the worker's utility from dropping a lot during recessions and enables the capitalist to reap most of the benefit during expansions. The risk-sharing component has an impact on the skill-premium via the second term of equation (20).

4 Quantitative Analysis of the Model

4.1 Numerical Solution and Calibration

To solve the model numerically, the stochastic processes of technology shocks are parameterized. Both Harrod-neutral and investment specific shocks are assumed to behave according to the following independent *AR*(1) processes:

$$\begin{aligned} A_t &= \rho_A A_{t-1} + \varepsilon_{At} \\ Z_t &= \rho_Z Z_{t-1} + \varepsilon_{Zt} \end{aligned} \quad (21)$$

where $\varepsilon_{At} \sim iidN(0, \sigma_{\varepsilon A}^2)$, $\varepsilon_{Zt} \sim iidN(0, \sigma_{\varepsilon Z}^2)$ with $0 < \rho_A < 1$, $0 < \rho_Z < 1$ and $E(\varepsilon_{At}\varepsilon_{Zt}) = 0$.²¹ Following Horvath (1994) and Boldrin and Horvath (1995), the solution is obtained by linearizing the first-order conditions of problems (16) and (17) around the non-stochastic steady state values. The optimal rules are obtained by expressing all variables as functions of the state vector Ω_t using the method of undetermined coefficients.²²

Notice that the values of the non-stochastic steady states are the same for both the Walrasian and the contract economy because the existence of contracts is due to the uncertainty about the realization of shocks. In other words, in a deterministic environment the Walrasian and the contract economy are equivalent. There are twenty three parameters to be calibrated : five preference parameters, $\{\eta, \eta_m, \varkappa, \gamma, \beta\}$, eleven parameters specifying the production process, $\{\theta, \mu, \lambda, \nu, \varphi, \delta_s, \delta_e, \omega, b, \xi_s, \xi_e\}$, four parameters pertaining to the stochastic processes, $\{\rho_A, \rho_Z, \sigma_{\varepsilon A}, \sigma_{\varepsilon Z}\}$ and three parameters determining the time endowment and employment, $\{T, s, u\}$. These parameters imply steady state values for all the variables, $\{c_m, k_s, k_e, l_s, l_e, U, w_s, w_e\}$. The model is calibrated such that the parameters lie within a range consistent with the existing literature and implying steady states that are consistent with patterns observed in the data.

First, I normalize the total number of nonsleeping hours per average person, T , to unity. Then, the following parameters are chosen based on a priori information:

(i) $\delta_e = 0.027$. Cummins and Violante (2002) back out the appropriate annual physical depreciation rates for capital equipment using the BEA (Bureau of Economic Analysis) measures of economic depreciation and their measure for the Z -process. The depreciation rate δ_e is set to match the average of the annualized depreciation rates.

(ii) $\delta_s = 0.014$. The depreciation rate for structures is set to match the annualized value calibrated by Greenwood et al. (1997).²³

²¹For the sake of simplicity, and given that the attention is focused on business cycles, I assume that the stochastic processes are stationary.

²²The elements of the state vector are percentage deviations from steady state values.

²³There is no evidence that the depreciation rates changed significantly over the recent years.

(iii) $\theta = 0.117$, $\nu = 0.401$, $\varphi = -0.495$. The share of capital structures in output and the parameters determining the elasticities of substitution between inputs were estimated in Krusell *et al.* (2000).²⁴

(iv) $\eta = 10$. Attanasio, Banks and Tanner (2002) and Vissing-Jørgensen (1998) estimate euler equations and find that the *EIS* of nonstockholders differs widely from that of stockholders which is close to unity. Following Guvenen (2006), the workers' coefficient of risk aversion is chosen to be 10 so that the workers are substantially more risk averse than the capitalists. In my sensitivity analysis, I also investigate the effects of reducing η , by studying the case $\eta = 5$.

Based on averages of US data and information from previous empirical studies I impose several restrictions on the steady state equations. As noted by Cagetti and De Nardi (2006, *a*), there is a tight relationship between being an entrepreneur and being rich. Cagetti and De Nardi (2005, *b*) use data from the Survey of Consumer Finances (SCF) and report that households in the top 20% of wealth distribution hold 81% of the wealth in the US economy. They also find that a significant fraction of the population holds little or no wealth at all. Likewise, Guvenen (2006) uses data from the Panel Study of Income Dynamics (PSID) and finds that 20% of the wealthiest households hold about 90% of capital and land and nearly all financial assets.²⁵ For those reasons, I choose the share of capitalists in total population to be 25%.²⁶ The calibrated model produces similar results even when the share of capitalists is set to 20%. The fraction of skilled workers in aggregate employment is about 30%.²⁷ Following Young (2003), I assume that at the steady state the average worker takes 30% of his time endowment as labor and that skilled workers work, on average, 20% more than unskilled workers. The former is consistent with the American Time Use Surveys and

²⁴The results do not change significantly for values around the Krusell *et al.* estimates.

²⁵In an earlier paper, Mankiw and Zeldes (1991) present evidence that no more than 25% of households own all the equity in US.

²⁶Changes in wealth distribution, and especially changes in the right tale of the distribution are likely to be slow.

²⁷This number is consistent with estimates from CPS data (source: Bureau of Labor Statistics).

the latter with estimates by Welch (1997). The steady state ratios of investment to output are set to match the average ratios in the data. I set the steady state utilization of capital equipment, U , equal to 80% using the average capacity utilization in Industrial Production reported by the Federal Reserve Board as a proxy.²⁸

The functional form for the labor supply implies that the elasticity of labor supply decreases as the wage increases. In other words, workers with higher wages have a lower elasticity of labor supply. The long-run trend of the average measure of wages the period 1979-2003 indicates that skilled wages are about 75% higher than unskilled wages.²⁹ It follows that the steady state elasticity of unskilled labor supply, $\epsilon_{u,w}$, is greater than that of skilled labor, $\epsilon_{s,w}$. In particular, the former is 8.57% higher than the latter. This steady state implication is consistent with recent empirical findings. Among others, Kimball and Shapiro (2008), estimate labor supply elasticities for individuals with different educational attainment (high school diploma; some college; college degree) using survey data from the Health and Retirement Study (HRS). They find that individuals with college degrees have substantially lower labor supply elasticities than individuals with some college education or no college education.³⁰ Blau and Kahn (2007), report similar results by focusing on the labor supply behavior of married women using data from CPS. The role of the elasticity of labor supply will be examined in the following section.

Output can be written as a function of capital structures and other parameters using the euler equation for capital structures:

$$y = \frac{1 - \beta(1 - \delta_s)}{\beta\theta} k_s \quad (22)$$

²⁸This value is within the range of values that appear in the literature. The model is not very sensitive around the chosen value.

²⁹Notwithstanding the standard average measure might not be the best measure for the cyclical properties of real wages, it is a reasonable indicator of the level of the skill-premium during the sample period.

³⁰The survey question which was designed by Kimball and Shapiro asks respondents to imagine what they would do if they won a sweepstakes. Their findings suggest that labor supply elasticities of college graduates are not only smaller relative to individuals with little education but can become negative due to the fact that the income effect dominates the substitution effect.

Then, the steady state ratio of aggregate investment to output is

$$\frac{i}{y} = \frac{\beta\theta\delta_s(1+i_e/i_s)}{1-\beta(1-\delta_s)} \quad (23)$$

Using (22) and the values for i_s/y and i_e/y I pin down the discount factor β . Furthermore, the following equations must hold at the steady state:

$$y = AF(k_s, Uk_e, L_s, L_u) \quad (24)$$

$$1 = \beta UAF_2(k_s, Uk_e, L_s, L_u) + \beta(1-\delta_e) \quad (25)$$

$$\omega\delta_e = UAF_2(k_s, Uk_e, L_s, L_u) \quad (26)$$

where (24) is the production function, (25) is the euler equation for capital equipment and (26) is the optimality condition for capital utilization. Substituting out y from (24) using (22), equations (24)-(26) along with the restriction that the skill-premium equals 1.75 comprise a system of four equations in four unknowns, $\{\lambda, \mu, k_s, \omega\}$. The choice for ω pins down the value of b using (4). Even though the literature does not provide any guide for assigning magnitudes to γ and \varkappa the two labor supply equations imply unique values for the two parameters, given the steady state values of wages and hours. In section 4.2.2, I examine the sensitivity of the results by allowing the utility parameters to be different across skilled and unskilled workers, i.e γ_s and \varkappa_s for skilled workers and γ_u and \varkappa_u for unskilled workers. It is shown that each pair of (γ_s, γ_u) corresponds to a unique value of relative labor supply elasticity $\epsilon_{u,w}/\epsilon_{s,w}$. In the analysis of section 4.2.2, I examine a wide range of values for the latter.

Adjustment costs are zero at the steady state and thereby, parameters ξ_s and ξ_e do not appear in the equations at the steady state. Moreover, there is no data on adjustment costs and hence, there is no a priori information to pin down the corresponding parameters. Consequently, ξ_s and ξ_e are chosen so that the volatilities of each type of investment equal

those observed in the data over the sample period.

Finally, I am left with the parameters pertaining to the stochastic processes, $\{\rho_A, \rho_Z, \sigma_{\varepsilon_A}, \sigma_{\varepsilon_Z}\}$. Following Greenwood *et al.* (1997, 2000), the stochastic process of investment specific shocks is identified by the inverse of the real price of investment in equipment. The real price of equipment is defined as the ratio of the equipment deflator and the consumption deflator. The reference for measurement of investment specific technologies is Gordon (1990). Gordon's series are annual and cover only the period 1947-1983.³¹ Cummins and Violante (2002) estimate the quality bias implicit in the NIPA price indexes using Gordon's series and then extrapolate the quality bias in the period 1984-2000. In this way, they extend Gordon's series by constructing annual quality-adjusted series through 2000. For the purpose of the current study, a further issue is that there are no quality adjusted quarterly series for the equipment deflator. Following Fisher (2003), I use the Gordon-Cummins-Violante annual series and the quarterly series from NIPA data to obtain quarterly quality-adjusted measures. To interpolate the adjusted series I use Denton's (1971) method that minimizes the distortion of the original NIPA series via a penalty function and satisfies the condition that the average of the adjusted series, each year, equals the annual quality adjusted deflator of Gordon-Cummins-Violante.³² In practice, the logarithm of the inverse of the real price of investment is a nonstationary process and exhibits a unit root.³³ I identify the Z -process by estimating the following regression:

$$\ln \left(\frac{P_c}{P_i} \right)_t = \varpi_1 + \varpi_1 \ln \left(\frac{P_c}{P_i} \right)_{t-1} + Z_t$$

³¹Several studies (eg Gordon (1990), Cummins and Violante (2002)) report that the quarterly NIPA data series for the price of equipment is a biased measure of investment specific technological progress.

³²I employ a penalty function based on the proportionate differences between the first differences of the NIPA and adjusted series. The Gordon-Cummins-Violante series was extended for the years 2001-2003 by adjusting the annual measures based on the pattern of NIPA data.

³³The null hypothesis of a unit root is decidedly not rejected by the Phillips-Perron test which takes into account serially correlated disturbances. Fisher (2003) allows for a unit root in a standard RBC model with a Cobb-Douglas production function and shows how all variables can be transformed in order to be expressed as stationary. In the present model I ignore the growth part of the process and deal only with the transitory shocks that induce the desired business cycles.

where P_c is the consumption deflator, P_i is the price of investment goods, c and d are constants and Z_t follows (21). Using the Cochrane-Orcutt estimation procedure and the quarterly 1979:1-2003:4 sample, the estimated parameters are³⁴

$$\varpi_1 = \frac{0.013}{(0.0019)}, \varpi_2 = \frac{1.001}{(0.0031)}, \rho_Z = \frac{0.6015}{(0.0827)}, \sigma_Z = 0.0047 \text{ with DW statistic} = 2.02$$

where the numbers in parentheses are standard deviations. It is more difficult to estimate the A -process because the utilization rate of capital equipment is unobserved. For this reason I set $\rho_A = 0.95$, which is the value typically used in RBC literature, and then assume that the two shocks are jointly responsible for the whole variation of output. Thus, I set σ_{ε_A} so that the volatility of output in the model matches the volatility observed in the data. The calibrated model demonstrates that investment specific shocks account for 16% of output fluctuations.³⁵ It is also worth noting that the specific parameterization implies that the steady state of labor share in income is 72% which is about the same as that observed in the data. The assigned parameter values are displayed in table 1. Table 3 displays parameter values for different calibration exercises which are conducted to examine the sensitivity of the model to various parameter values and structural features.

4.2 Model vs Data

This section evaluates the business cycle properties of the model. The predictions of the model are assessed by comparing the generated standard deviations and cross-correlations between output and the other variables with corresponding statistics from US data.³⁶ Par-

³⁴The estimates of interest are robust to different estimation methods (eg. Hildreth-Lu and maximum likelihood grid search procedures).

³⁵Greenwood *et al.* (2000) find that investment specific shocks is the source of about 30% of output fluctuations. Fisher (2006) finds that investment specific shocks account for 42 – 67% of output fluctuations while both shocks jointly account for 44 – 80% of output variability.

³⁶The measures of wages and labor hours are expressed in efficiency units and were constructed by Castro and Coen-Pirani (2008). These measures control for the cyclical change in demographic composition as well as various aggregation effects. In their analysis, Castro and Coen-Pirani find that the volatility of the working hours of skilled labor increased substantially after 1984. In contrast, the cyclical properties of unskilled hours

ticularly, I compare the moments of HP filtered series from the model with moments of HP filtered series from the data. Table 2 displays statistics from US data and the benchmark model while tables 4 and 5 display statistics from various calibration exercises, including a model where technology is not characterized by capital-skill complementarity and a model where capital utilization is constant.

4.2.1 The Benchmark Model

I. Statistical Moments

The benchmark model reasonably captures the properties of basic macroeconomic regularities. Investment is more volatile than output which, in turn, is more volatile than consumption. The correlations generated by the model indicate that the comovement of variables is fairly close to those of US data. Wages and the skill-premium do not move systematically over the business cycle and exhibit similar behavior to the data. Even though the utilization of capital is strongly procyclical and the production is characterized by capital-skill complementarity, the correlation between the skill-premium and output is close to zero as in the data. The analysis demonstrates that variable capital utilization is not only consistent with the cyclical behavior of the skill-premium but it is also a necessary mechanism in the model. The latter is illustrated in the following sections. The analysis indicates that investment-specific shocks alone explain up to 16% of output's fluctuations. In contrast to standard RBC models, labor productivity does not appear to be perfectly correlated with output. Nevertheless, low skilled workers have more volatile wages than high skilled workers.³⁷

as well as the cyclical properties of real wages remained essentially unchanged over the pre-1984 and the post-1984 periods. Since the main focus of the paper is on the behavior of real wages and not on the change in the volatility of skilled hours, I do not examine the pre-1984 period separately from the post-1984 period. Details about the rest of the data are provided in the Appendix.

³⁷Differences in the volatilities of wages and hours among the two types of workers could be related to differences in the duration of contracts. If skilled workers hold, on average, contracts of a longer duration then, one would expect skilled wages to be less volatile than unskilled wages. This issue is not examined

II. Impulse Response Functions

While a Harrod-neutral shock increases the marginal efficiency of capital and labor inputs, an investment-specific shock increases the marginal efficiency of investment in equipment. This section analyzes the different effects of the two shocks on the macroeconomic aggregates. Impulse response functions (IRFs) are displayed in figures 1 and 2.

Output, investment, the utilization of capital equipment, labor productivity and labor hours exhibit an immediate and positive response to both Harrod-neutral and investment-specific shocks. Consumption responds positively only after a Harrod-neutral shock and declines a little after an investment specific shock. The latter is due to the fact that the workers' consumption does not react much to the shock whereas the capitalist's consumption decreases substantially. As pointed out by Barro and King (1984) and discussed by Greenwood, Hercowitz and Huffman (1988), shocks that improve the marginal efficiency of newly produced capital induce individuals to lower consumption. Not surprisingly this behavior holds true for the capitalist's consumption.³⁸

The wages for both types of workers have an immediate and negative response to both Harrod-neutral and investment specific shocks. The induced income effects increase work effort substantially for both types of workers and thereby, production. The shocks affect the workers' reservation utilities whose effects are evident in next period's wages and hours. Thus, the highest impact on wages occurs during the quarter that follows a shock reflecting the strong influence of market conditions at the time the contract is signed. Lagakos and Ordonez (2007) find that wages in low-wage industries respond more to productivity shocks than wages in high-wage industries. Since the wage of skilled workers is higher than the wage of unskilled workers we expect a larger response of the latter to a productivity shock. Indeed, as shown in figure 1, the unskilled wage responds more to a neutral productivity shock than

here.

³⁸The Walrasian model predicts consumption to be lower than the steady state along the entire impulse response function (see Lindquist (2004), p532, figure 2).

the skilled wage does. The following section illustrates that the latter is reversed when the utilization of capital is held constant over time. Variable utilization of capital equipment has two significant roles in the model. First, it magnifies and propagates the shocks over the business cycle and second, it differentiates the effects of shocks on labor variables.

A neutral technology shock induces an increase in production and also an increase in the worker's reservation utility. The increase in the wage of unskilled workers is larger because the increase in their reservation utility is greater than that of skilled workers. As I show in section 4.2.2, the latter is due to the fact that low-skilled labor is much cheaper than high-skilled labor. As a result, there is a margin to increase unskilled wages relatively more than skilled wages in response to a neutral shock. The income effect on hours for unskilled workers the period after the shock is strong enough to make hours fall below their steady state. The income effect on skilled workers is not as strong as to cause a similar response for skilled hours. Even though unskilled hours decrease after an improvement in technology, the capitalists can still exploit the higher gains by increasing the utilization of capital. As it will be shown in the following section, the response of unskilled hours to the shock changes when the mechanism of endogenous capital utilization is not available.

The responses of wages to investment-specific shocks are different from those associated with neutral technology shocks. The decline in equipment's replacement value lowers the marginal utilization cost. This leads to an increase in the utilization of capital and next period's capital stock. The latter motivates an increase in total hours for both types of workers. In order to increase unskilled hours which are, on average, twice as many as total skilled hours a small income effect is necessary. Consequently, entrepreneurs offer unskilled workers a lower wage. It turns out that the latter is not necessary for skilled workers. The skill-premium does not peak until the period following an investment-specific shock and decreases after a delay of one quarter following a Harrod-neutral shock. Thus, Harrod-neutral shocks tend to decrease wage inequality whereas investment-specific shocks tend to increase it. The mixed response of wages to the shocks produces a contemporaneous correlation

between the skill-premium and output that is close to zero.

The fact that the skill-premium behaves oppositely to relative hours is an indication that it is mainly driven by those than the ratio of effective capital to skilled labor. Hence, it seems that the relative supply effect dominates the capital-skill complementarity effect. This result is not surprising since the contract skill-premium is a function of three terms of relative hours and only one term of the ratio of effective capital to skilled labor.³⁹ The dominance of the relative supply effect is confirmed in the sensitivity analysis of section 4.2.2.

III. Autocorrelations

Auto-correlograms of output, consumption and investment are displayed in figure 7. Although the autocorrelation functions are not as smooth as they appear to be in the data they follow a similar pattern. As shown in figure 2, the impact of shocks on investment is not very persistent because only a small fraction of aggregate consumption is smoothed through the savings channel. This means that investment decisions in the model are irrelevant for a large fraction of the population for whom consumption smoothing is achieved only via labor contracts. Apparently, this property of the model affects the behavior of investment. The lack of persistency in investment also has an impact on the low autocorrelations of output, partly due to changes in capital. The lack of smoothness in the correlation functions might be related to the fact that actual contracts usually have a longer duration and are negotiated in various time periods. The present model successfully captures the idea that decisions in the labor market are made prior to the realization of shocks and practically gives us a better understanding of the role of contracts in the dynamics of wages and the skill-premium.

4.2.2 Sensitivity Analysis

In this section, I examine the sensitivity of the model to changes in certain parameter values and different model specifications. Each time a parameter value or a model specification is

³⁹Impulse response functions of the capital-skill ratio and relative hours are displayed in figure 6.

changed, the rest of the parameters are re-calibrated according to the procedure described in section 4.1. The parameter values for the various calibration exercises can be found in tables 1 and 3.

First, I examine the sensitivity of the baseline model in the case where the difference in risk aversion between capitalists and workers decreases. Specifically, I examine the case where η equals 5. As shown in table 2, most of the results do not significantly change. Wages become slightly more volatile and moderately countercyclical while labor productivity becomes much less procyclical.⁴⁰ Second, I examine the sensitivity of the results to the feature of capital-skill complementarity. In particular, I set the elasticities of substitution between capital equipment and the two labor inputs to be equal so that the skill-premium is only driven by relative hours.⁴¹ The statistics in table 4 (columns 1), show that the results are nearly identical to the benchmark case. Contrary to the case of the Walrasian model (see Lindquist, 2004), the model with contracts is not sensitive to the degree of substitutability between capital equipment and labor of different skill types.

In order to examine the importance of variable utilization of capital equipment I set the utilization rate equal to unity and hold it constant over time. In addition, I set the depreciation rate of equipment equal to its steady state value. Corresponding statistics can be found in table 4 (columns 2). The model's performance is somewhat different in this case. When a neutral productivity shock occurs the production can be increased further only if work effort increases. As a result, in the absence of endogenous capital utilization the capitalists also need to increase unskilled labor the period that follows the shock. In order to cause an increase in unskilled hours, through the income effect, the capitalists offer unskilled workers a relatively lower wage. Then, both types of shocks tend to increase wage inequality and thereby, the mixed response produces a procyclical skill-premium. The results do not

⁴⁰Unreported impulse response functions indicate that the decrease in workers' risk aversion causes a greater immediate and negative response of wages to both types of shocks.

⁴¹The experiment is similar to that conducted by Krusell *et al.* (2000). Specifically, I set $\nu = \varphi = 1/3$ which yields an elasticity of substitution between skilled and unskilled labor that is consistent with estimates reported by Johnson (1997).

change substantially when capital-skill complementarity does not characterize the production function.

A property of variable capital utilization is that it magnifies and propagates shocks over the business cycle. In the absence of this mechanism larger neutral shocks are required to match the observed volatility of output. Calibration exercises and impulse response functions signify that technology neutral shocks tend to produce procyclical wages whereas investment specific shocks are associated with countercyclical wages. Since neutral technology shocks are larger, the new mix of shocks generates procyclical wages.⁴²

To examine the sensitivity of the model to differences in labor supply elasticities across skilled and unskilled workers I relax the assumption that the utility parameters are the same across workers of different skill types. Specifically, I assume that the utility parameters γ_i and \varkappa_i change with the skill level of the worker (i.e $i = s, u$). Then, given the steady state values of w_i and l_i , for any value of γ_i there is a corresponding unique value of \varkappa_i . It follows that for each pair (γ_s, γ_u) there is a unique steady state relative elasticity of labor supply, $\epsilon_{u,w}/\epsilon_{s,w}$. Experimenting with different pairs (γ_s, γ_u) enables me to examine the sensitivity of the model across a wide range of values for $\epsilon_{u,w}/\epsilon_{s,w}$. The surface plots of figure 3, show the latter for different pairs (γ_s, γ_u) . Table 5 provides statistics from the model, calibrated for indicative values of $\epsilon_{u,w}/\epsilon_{s,w}$; 0.65 (columns 3), 1.60 (columns 4) and 60 (columns 5).⁴³ The statistics designate that changes in model dynamics which occur in response to different steady state values for the relative labor supply are very minor. As a result, this dimension of heterogeneity across workers is not so significant.

Next, I fix $\epsilon_{u,w}/\epsilon_{s,w}$ to its benchmark value and experiment with different values for the steady state level of the skill-premium. Even though the benchmark value for the latter is the average observed in the data, analysis of how the model responds to different values can provide a better understanding of the extent to which this value impacts the dynamics of the

⁴²The new mix of shocks indicates that investment specific shocks can explain only 5 – 6% of output fluctuations.

⁴³The steady state level of the skill-premium is held to its benchmark value of 1.75.

model. Table 5 displays model statistics for the cases where w_s/w_u is 1.60 (columns 6) and 1.85 (columns 7). Unreported impulse response functions for the case where w_s/w_u equals 1.60 indicate that a narrower wage gap between skilled and unskilled workers induces bigger immediate decreases for the wages of both skill types in response to a Harrod-neutral shock. This follows from (19) in which the effect of the insurance component dominates the increase in marginal productivity. As a result, wages become more countercyclical compared to the benchmark case. This experiment also demonstrates that for lower levels of wage inequality, the skill-premium becomes more procyclical. The immediate decrease of the unskilled wage is relatively bigger which makes the sign of the contemporaneous correlation between the skill-premium and output positive. On the other hand, increasing the steady state value of the skill-premium to 1.85 makes the initial response of wages to the neutral shock positive and thus, the contemporaneous correlations with output positive. In addition, as steady state wage inequality increases, the instant wage response of unskilled labor becomes relatively bigger. This induces a negative contemporaneous correlation between the skill-premium and output. In other words, cheaper unskilled labor relative to skilled labor allows a higher margin for firms to increase relatively more the wages of unskilled workers in response to an improvement in technology.⁴⁴

5 Conclusion

The finding that capital-skill complementarity is an important determinant of the long-run behavior of the skill-premium influenced the recent research on wage inequality. Over the business cycle neither real wages nor the skill-premium exhibit a systematic pattern. The latter raises doubts about whether the labor market is purely Walrasian. Furthermore, a previous study shows that if the labor market is Walrasian and firms are allowed to vary

⁴⁴Note that apart from the changes in the sign of initial responses of w_{st} and w_{ut} as well as the scale of the responses, the pattern of the shape of the responses remains roughly unchanged, with either $w_s/w_u = 1.60$ or $w_s/w_u = 1.85$. Finally, in the latter case, unskilled hours become acyclical.

the utilization of capital then, capital-skill complementarity generates strongly procyclical skill-premia which are at odds with the empirical findings. Labor contracts constitute an alternative explanation for the movement of real wages. This paper analyzes the behavior of the skill-premium in a DSGE model where agents trade labor contracts and the utilization of capital equipment is endogenous.

The model economy is populated by risk averse workers and less risk averse entrepreneurs that own the capital stock of the economy. Since the workers cannot access capital markets to shed income risk the labor contracts are the only insurance device available to them. The model generates the weak contemporaneous correlation of wages and the skill-premium with output while performing well in matching basic macroeconomic regularities. Contrary to previous findings, it is shown that the feature of capital-skill complementarity is not central to the behavior of the skill-premium at high frequencies. The risk-sharing component of real wages reinforces the effect of the relative labor supply which dominates the capital-skill complementarity effect. The analysis also illustrates that the level of wage inequality plays a significant role in short-run dynamics. Finally, the paper demonstrates that variable capital utilization rates are not only consistent with the behavior of the skill-premium but constitute a necessary aspect of the model. In the absence of endogenous capital utilization, wages and the skill-premium become considerably procyclical.

Appendix: Data

The measures of wages are expressed in efficiency units. These measures were constructed by Castro and Coen-Pirani (2008) by drawing data from the Current Population Survey (CPS).⁴⁵ Nominal consumption $\bar{c} = \bar{c}_m + s\bar{c}_{st} + u\bar{c}_{ut}$ is measured as the sum of per-

⁴⁵Mean and median wages produce similar statistics.

sonal consumption expenditures for durable and nondurable goods and services, government consumption expenditures and investment in residential structures. Nominal investment in structures \bar{i}_s is measured as the sum of producer's investment in nonresidential structures and government's investment in structures. Nominal investment in equipment \bar{i}_e is measured as the sum of producer's and government's investment in equipment and software. Nominal output, \bar{y} , is measured as $\bar{c} + \bar{i}_s + \bar{i}_e$ plus net exports and the change in private inventories. All variables are measured in consumption units, as they are expressed in the resource constraint. They are deflated using a common implicit consumption deflator, deseasonalized using the Census Bureau's seasonal adjustment program, logged and Hodrick-Prescott (HP) filtered before computing statistics.

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Table 1 - Values of Calibration Parameters for the benchmark case

s	0.9	λ	0.545	θ	0.117			
u	2.1	ν	0.401	ω	0.6482		$\eta = 10$	$\eta = 5$
T	1	δ_s	0.014	μ	0.4503	ξ_s	2.935	6.36
γ	0.321	δ_e	0.027	ρ_Z	0.6015	ξ_e	2.42	5.14
\varkappa	0.3991	φ	-0.495	$\sigma_{\varepsilon Z}$	0.0047	$\sigma_{\varepsilon A}$	0.0069	0.0052
β	0.9828	b	0.0643	ρ_A	0.9500			

Table 2 - Statistical Moments: Quarterly US data, 1979:1-2003:4,
and the benchmark model

x	σ_x (%)			$corr(x_t, y_t)$		
	US data	Model		US data	Model	
		$\eta = 10$	$\eta = 5$		$\eta = 10$	$\eta = 5$
Output	1.39	1.39	1.39	1.00	1.00	1.00
Aggregate consumption	1.22	1.19	1.02	0.83	0.93	0.95
Investment in structures	4.36	4.36	4.36	0.36	0.69	0.86
Investment in equipment	4.23	4.23	4.23	0.87	0.73	0.90
Aggregate investment	3.73	4.04	4.22	0.76	0.76	0.90
Average labor productivity	0.50	1.04	0.91	0.50	0.72	0.26
Capitalist's consumption	...	2.94	4.37	...	0.88	0.92
Capital Utilization	2.51	0.87	0.87	0.84	0.94	0.94
Skilled wage	1.12	0.84	0.92	0.34	0.22	-0.54
Unskilled wage	0.80	1.23	1.50	0.30	0.18	-0.51
Aggregate wage	0.89	1.13	1.35	0.31	0.18	-0.52
Skill-premium	0.60	0.49	0.77	0.09	-0.09	0.35
Skilled labor hours	0.99	0.68	0.84	0.69	0.95	0.97
Unskilled labor hours	1.19	1.18	1.86	0.73	0.54	0.71
Aggregate labor hours	1.00	0.96	1.45	0.82	0.67	0.79
$(\sigma_y^{model} / \sigma_y^{data}) _{\sigma_A^{model}=0}$...	0.16	0.11			

Table 3 - Values of parameters for different calibration exercises

	(1)	(2a)	(2b)	(3)	(4)	(5)	(6)	(7)
$\frac{w_s}{w_u}$	1.75	1.75	1.75	1.75	1.75	1.75	1.6	1.85
$\frac{\epsilon_{u,w}}{\epsilon_{s,w}}$	1.0857	1.0857	1.0857	0.65	1.6	60	1.0857	1.0857
γ_s	0.321	0.321	0.321	0.3215	0.2429	0.0415	0.3602	0.3007
γ_u	0.321	0.321	0.321	0.221	0.321	0.7053	0.3602	0.3007
\varkappa_s	0.4164	0.3916	0.3916	0.3992	0.3843	0.3471	0.4097	0.3937
\varkappa_u	0.4164	0.3916	0.3916	0.3609	0.3991	0.5532	0.4097	0.3937
η	10	10	5	10	10	10	10	10
μ	0.4153	0.4684	0.4684	0.4503	0.4503	0.4503	0.4717	0.437
λ	0.2021	0.584	0.584	0.545	0.545	0.5450	0.5584	0.5368
ξ_s	3.23	2.285	6.67	3.01	3.02	2.6260	4.73	1.5
ξ_e	3.02	1.282	3.22	2.46	2.46	2.21	3.84	1.358
$\sigma_{\varepsilon A}$	0.0068	0.0076	0.0068	0.00715	0.0071	0.00585	0.0054	0.00825

*The columns of the table correspond to the following cases: (1) model without capital-skill complementarity, i.e $\nu = \varphi = 1/3$, (2) model with constant capital utilization, i.e $U_t = 1$ and $\delta_e(U_t) = \delta_e$ with (a) $\eta = 10$ and (b) $\eta = 5$, (3) $\epsilon_{u,w}/\epsilon_{s,w} = 0.65$ and $w_s/w_u = 1.75$, (4) $\epsilon_{u,w}/\epsilon_{s,w} = 1.60$ and $w_s/w_u = 1.75$ and (5) $\epsilon_{u,w}/\epsilon_{s,w} = 60$ and $w_s/w_u = 1.75$, (6) $w_s/w_u = 1.60$ and $\epsilon_{u,w}/\epsilon_{s,w} = 1.0857$, (7) $w_s/w_u = 1.85$ and $\epsilon_{u,w}/\epsilon_{s,w} = 1.0857$. The values of the parameters not specified in the table are the same as the ones in the benchmark case.

Table 4 - Statistical Moments: The model without capital-skill complementarity
and the model with constant capital utilization*

x	σ_x (%)			$corr(x_t, y_t)$		
	(1)	(2)		(1)	(2)	
	$\eta = 10$	$\eta = 10$	$\eta = 5$	$\eta = 10$	$\eta = 10$	$\eta = 5$
Output	1.39	1.39	1.39	1.00	1.00	1.00
Aggregate consumption	1.15	1.14	1.04	0.94	0.95	0.95
Investment in structures	4.36	4.36	4.36	0.70	0.75	0.91
Investment in equipment	4.23	4.23	4.23	0.77	0.76	0.86
Aggregate investment	3.96	3.88	4.23	0.80	0.83	0.89
Average labor productivity	1.07	0.84	0.65	0.67	0.99	0.99
Capitalist's consumption	3.07	0.73	0.55	0.86	0.91	-0.14
Utilization	0.99	0.92
Skilled wage	0.73	0.89	0.89	0.16	0.72	0.70
Unskilled wage	1.31	0.82	0.59	0.13	0.72	0.64
Aggregate wage	1.13	0.82	0.66	0.12	0.71	0.64
Skill-premium	0.69	0.09	0.32	-0.08	0.53	0.79
Skilled labor hours	0.67	0.45	0.44	0.94	0.91	0.89
Unskilled labor hours	1.31	0.66	0.93	0.52	0.98	0.99
Aggregate labor hours	1.04	0.58	0.75	0.64	-0.85	0.99
$(\sigma_y^{model}/\sigma_y^{data}) _{\sigma_A^{model}=0}$	0.18	0.04	0.03			

*The columns of the table correspond to the following cases: (1) model without capital-skill complementarity, i.e $\nu = \varphi = 1/3$, (2) model with constant capital utilization, i.e $U_t=1$ and $\delta_e(U_t) = \delta_e$.

Table 5 - Statistical Moments: Models of different average relative labor supply elasticities, and levels of the skill-premium*

x	σ_x (%)					$corr(x_t, y_t)$				
	(3)	(4)	(5)	(6)	(7)	(3)	(4)	(5)	(6)	(7)
Output	1.39	1.39	1.39	1.39	1.39	1.00	1.00	1.00	1.00	1.00
Aggregate consumption	1.20	1.19	1.15	1.07	1.35	0.92	0.92	0.93	0.94	0.94
Investment in structures	4.36	4.36	4.36	4.36	4.36	0.68	0.70	0.75	0.82	0.48
Investment in eq.	4.23	4.23	4.23	4.23	4.23	0.73	0.73	0.75	0.87	0.49
Aggregate invesment.	4.04	4.05	3.99	4.18	3.43	0.75	0.76	0.80	0.87	0.60
Labor productivity	1.09	1.07	0.80	0.82	1.28	0.53	0.72	0.53	0.44	0.88
Capitalist's consumption	3.05	3.02	2.48	3.12	3.09	0.74	0.89	0.81	0.92	0.81
Capital utilization	0.88	0.86	0.80	0.86	0.89	0.94	0.94	0.92	0.94	0.93
Skilled wage	0.86	0.90	0.91	0.79	0.94	0.24	0.22	0.07	-0.31	0.67
Unskilled wage	1.27	1.24	0.98	1.21	1.34	0.21	0.19	0.02	-0.29	0.61
Aggregate wage	1.17	1.16	1.00	1.11	1.24	0.21	0.18	-0.05	-0.32	0.64
Skill-premium	0.51	0.47	0.45	0.52	0.55	-0.11	-0.07	0.10	0.21	-0.35
Skilled hours	0.68	0.65	0.59	0.78	0.56	0.95	0.93	0.85	0.98	0.93
Unskilled hours	1.15	1.20	1.50	1.58	0.88	0.47	0.53	0.79	0.74	0.15
Aggregate hours	0.93	0.96	1.18	1.27	0.67	0.62	0.64	0.81	0.81	0.40
$(\sigma_y^{model} / \sigma_y^{data}) _{\sigma_A^{model}=0}$	0.15	0.15	0.19	0.13	0.19					

*The columns of the table correspond to the following cases: (3) $\epsilon_{u,w} / \epsilon_{s,w} = 0.65$ and $w_s / w_u = 1.75$, (4) $\epsilon_{u,w} / \epsilon_{s,w} = 1.60$ and $w_s / w_u = 1.75$ and (5) $\epsilon_{u,w} / \epsilon_{s,w} = 60$ and $w_s / w_u = 1.75$, (6) $w_s / w_u = 1.60$ and $\epsilon_{u,w} / \epsilon_{s,w} = 1.0857$, (7) $w_s / w_u = 1.85$ and $\epsilon_{u,w} / \epsilon_{s,w} = 1.0857$.

Figure 1 - IRFs of wages , skill-premium, labor hours, and relative hours to a Harrod-neutral shock (—) and an investment-specific shock (- -) in the benchmark model

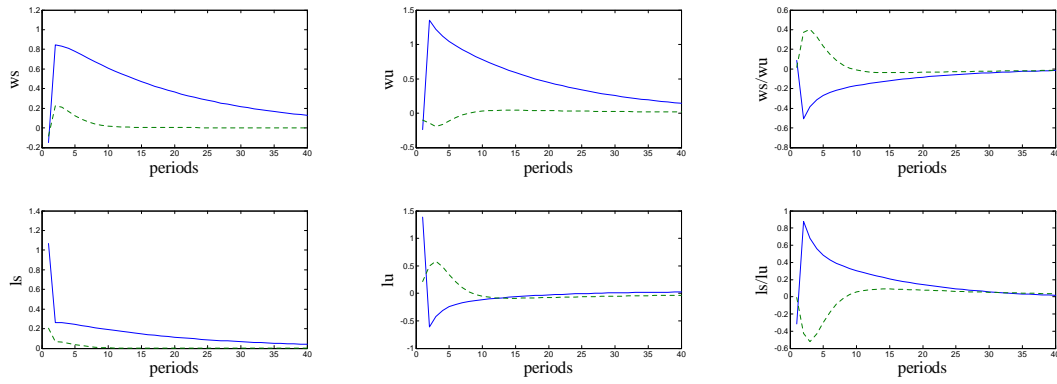


Figure 2 - IRFs of aggregate variables to a Harrod-neutral shock (—) and an investment-specific shock (- -) in the benchmark model

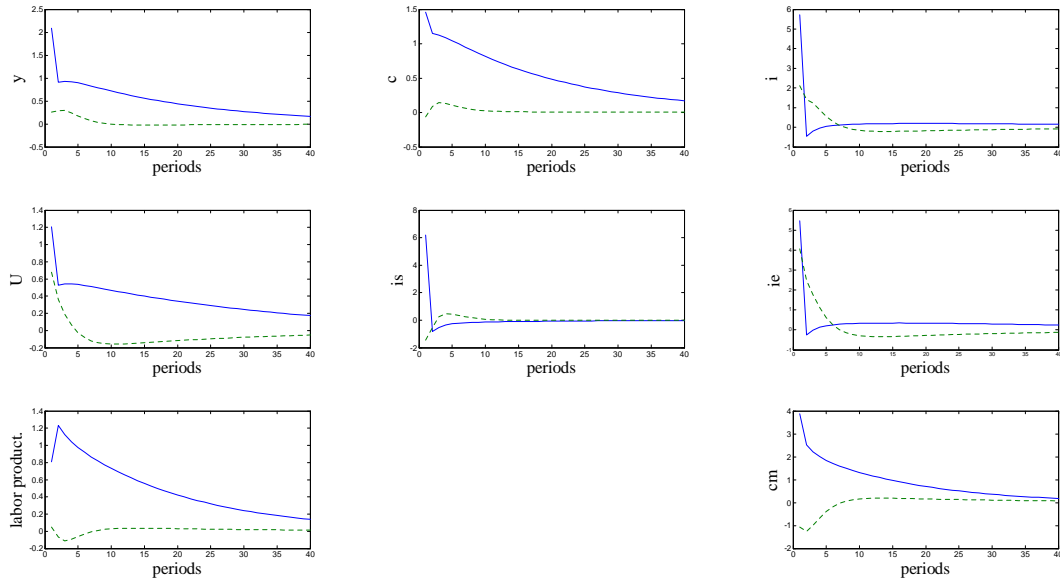


Figure 3 - Surface plots of steady state relative elasticity of unskilled labour supply ($\epsilon_{u,w}/\epsilon_{s,w}$)

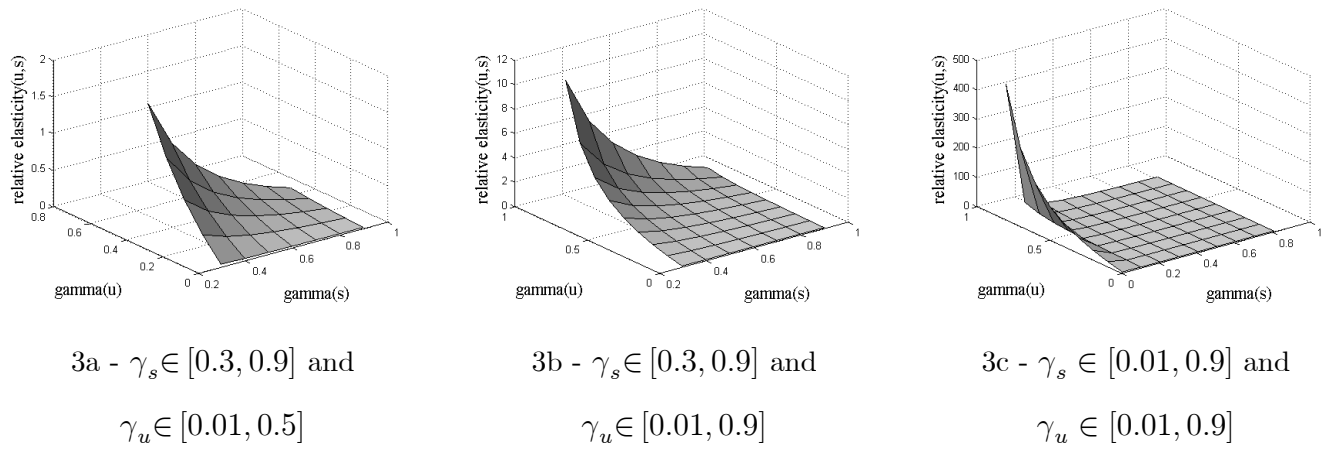


Figure 4 - IRFs of wages, skill-premium, labor hours, and relative hours to a Harrod-neutral shock (—) and an investment-specific shock (- -) in the model with constant capital utilization

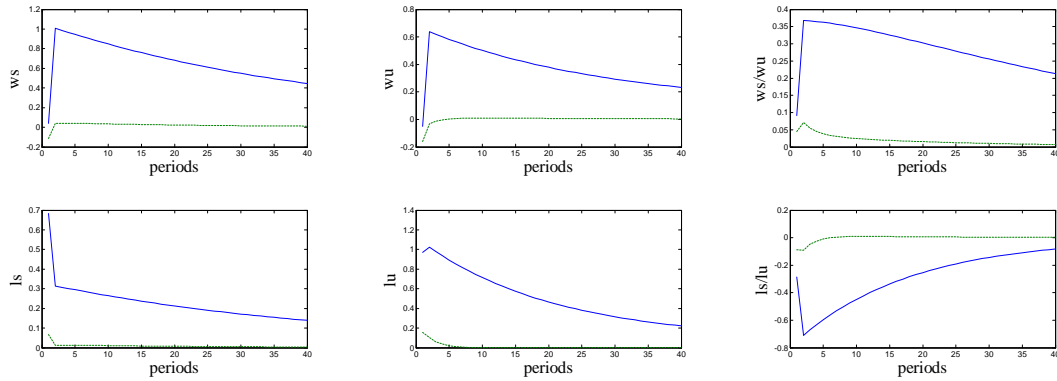


Figure 5 - IRFs of aggregate variables to a Harrod-neutral shock (—) and an investment-specific shock (- -) in the model with constant capital utilization

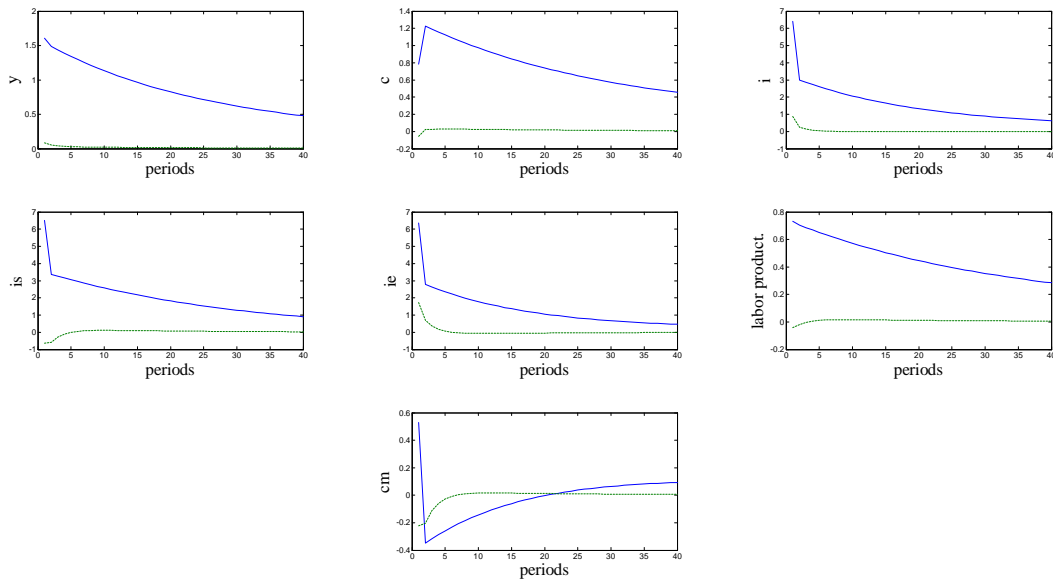


Figure 6 - IRFs of the capital-skill ratio (—) vs relative hours (- -). 1st row: benchmark model. 2nd row: model with constant capital utilization. 1st column: A -shock. 2nd column: Z -shock

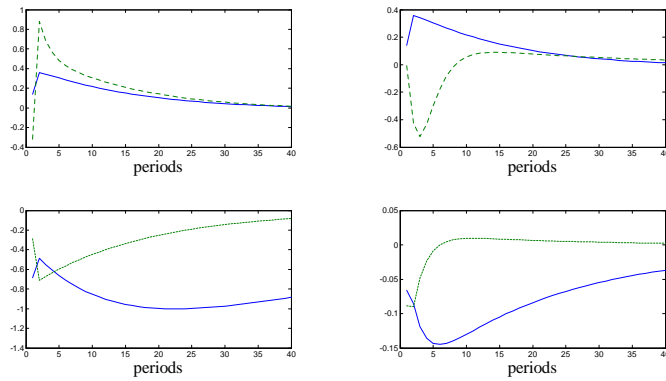


Figure 7 - Auto-correlogram, of macroeconomic aggregates from the benchmark model (- -) and US data (—)

