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The Credit Risk Premium in a Disaster-Prone World

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Abstract

The seminal Barro (2006) closed-economy model of the equity risk premium in the presence of extreme events ("disasters") allowed for leverage in the form of risky corporate debt which defaulted only in states when the Government defaulted on its debt. The probability of default was therefore exogenous and independent of the degree of leverage. In this paper, we take the model a step closer reality by assuming that, on the one hand, the Government never defaults, and on the the other hand, that the "corporate sector" in the form of the Lucas tree owner pays its debts in full if and only if its asset value is sufficient, which is always the case in non-crisis states. Otherwise, in exceptionally severe crises, it defaults and hands over the whole "firm" to its creditors. The probability of default by the tree owner is thus endogenous, dependent both on the volume of debt issued (taken as exogenous) and on the uncertain value of output. We show, using data from both Barro (2006) and Barro and Ursua (2008), that the model can generate values of the riskless rate, equity risk premium and credit risk spread broadly consistent with those typically observed in the data.

JEL Classification: F3, G1

Keywords: equity risk premium, default risk, credit spread, leverage, corporate debt

1 Introduction

For practitioners and academics alike, the size of the credit risk spread is a long-standing puzzle, whose solution seems more urgent than ever against the background of the crisis in global financial markets which started in mid-2007. In this paper, we concur with the view taken in Bhamra et al (2007), Chen et al (2008) and Chen (2007) among others, that the issue needs to be addressed alongside the equity premium puzzle, and moreover in a setting which takes explicit account of the vulnerability of all economies to occasional extreme, invariably negative shocks. In this spirit, we follow the macrofinance approach pioneered by Rietz (1988) and extended in the seminal work by Barro (2006), whose model of the equity risk premium in the presence of extreme events has already generated a new literature on crises and their implications for asset pricing. Most of the papers modify the basic framework in one way or another in order to generalise the model and make its assumptions more realistic, with varying conclusions regarding the robustness of Barro (2006)'s claim to have resolved the famous equity premium puzzle.

Thus, Gourio (2007) examines the evidence on how quickly countries typically recover from catastrophic falls in the level of economic activity and finds that incorporating a recovery probability into the model makes it again impossible to explain the stylized facts. Starting from a different point of view, Copeland and Zhu (2007) show that if we allow for a second tree (a "foreign country"), then anything less than perfect correlation between the output of the two trees implies the existence of diversification opportunities, making it almost impossible to reconcile the observed 6% risk premium with the parameter values derived in Barro (2006) from a survey of twentieth century experience.

On the other hand, a number of developments on the empirical research front make the problem somewhat more tractable. First, Dimson et al (2006) suggest that, viewed from a global perspective, the equity premium is not quite as large as for the original US dataset of Mehra and Prescott (1985). Secondly, after substantially broadening the scope of their analysis of the historical data, Barro and Ursua (2008) conclude that the probability of a crisis is in fact double the figure given in Barro (2006), which would be likely to boost estimated risk premia for most calibrations, other things being equal.[1] On the theoretical front, Gabaix (2008) introduces a time-varying intensity of disasters into the model and claims to explain the risk premium along with a number of other stylized facts in the asset pricing literature.

The contribution made by this paper relates to the submodel of the debt market. Barro (2006) allows for leverage in the form of risky corporate debt which defaults only in states when the Government also defaults on its debt. The latter event occurs with exogenously given probability in some but not all crisis states. In this setting, the probability of default by the tree-owner is therefore exogenous and independent of the degree of leverage.

Here, we take the model a step closer to reality by assuming that, on the one hand, the Government never defaults,[2] and on the other hand, that the "corporate sector" in the form of the Lucas (1978) tree owner pays its debts in full if and only if its exogenous harvest is

[1] See also Brown, Goetzmann and Ross (1995), Jorion and Goetzmann (1999) who make essentially the same point in drawing attention to the survivorship bias implicit in ignoring the losses inflicted on investors by the wars and revolutions of the Twentieth Century, especially when these disasters resulted in stock market closure, expropriation or wipe-out.

[2] Government defaults on domestic currency unindexed debt are quite rare, at least if we exclude developing countries, though not apparently as rare as we thought prior to reading Reinhart and Rogoff (2008).

sufficient, which is always the case in non-crisis states. In some, but not all crisis states, however, output is inadequate to cover debt service, in which case it hands over the whole harvest to its creditors. The probability of default by the tree-owner is thus endogenous, dependent both on the volume of debt issued (taken as exogenous) and on the uncertain value of output. By deriving a closed-form solution for the critical value of output below which the firm is unable to repay its borrowing, we are able to find expressions for the riskless rate, equity risk premium and credit risk spread as well as the (endogenous) probability of default. Calibrations with parameter values based on data from both Barro (2006) and Barro and Ursua (2008), show that the model specified in this fashion can generate results consistent with the broad outline of the facts, even with reasonably low levels of risk aversion.

In the following sections, we give a brief survey of related literature on the credit spread, followed by an overview of the original Barro (2006) crisis model. We then proceed to set out the submodel of corporate debt which is to be embedded in the crisis model (Section 4). Solving the model for the rates of return on the three types of security (equity, private sector debt and Government debt) in Section 5, we are able to derive expressions for the credit risk spread and equity premium (Section 6). The results of our calibration are discussed in Sections 7 and 8.

2 Literature Review

If investors are risk-averse, the credit spread between the yield on corporate and Government debt should reflect the sum of the expected default loss and the default risk premium. Various models of credit risk have been proposed to explain the size of the credit risk premium, but

the consensus remains that models calibrated to match the observed default data predict a far lower credit spread than is consistent with the data, a failure often called the credit spread puzzle in the published literature.

Researchers have explored a number of different avenues in attempting to explain this anomaly. Elton et al (2001) showed that the favourable treatment of government bonds relative to corporate debt under the US tax regime accounts for some of the spread. However, this can hardly be the end of the story, since De Jong and Driessen (2006) show that the credit spread is puzzlingly high in Europe too, even though European corporate bonds are not subject to tax at state level.

More importantly as far as the present paper is concerned, Elton et al (2001) found that the credit risk premium i.e the residual spread after allowing for expected default cost and tax effects, was related to the stock market risk premium associated with the Fama-French 3-factor Model, thereby making a direct empirical link between the equity risk premium and the credit spread.

Another obvious possibility is that the spread may be due to the convertibility options frequently attached to corporate debt. On closer investigation, however, the conversion feature turns out to be able to account for only around 10 basis points of the spread (Crabbe (1991)). Similarly, the liquidity premium to compensate for the fact that corporate debt markets are far thinner than Government debt markets should only be about 25 points (Ericsson (2005), Perraudin and Taylor (2003) and Amato and Remolona (2005)). Given that the spread is around 200 basis points (Huang and Huang (2003)), the two together can only account for a very small proportion.

Structural models which treat debt and equity as contingent claims on the asset value of

the firm, following Merton (1974) and Black and Cox (1976), are also incapable of fully explaining the spread (Jones et al (1984)). Moreover, extensions to the original model intended to solve the problem have had limited success. For example, a number of papers make the default boundary endogenous. In Leland (1994), (1998) and Leland and Toft (1996), the capital structure of a firm is chosen by equity holders to maximize their utility. To avoid bankruptcy, firms issue more debts whenever possible. Default occurs when equity value drops to zero. On the other hand, in Anderson and Sundaresan (1996), Anderson et al (1996) and Mella-Barral and Perraudin (1997), equity owners choose to default in order to extract concessions from bondholders. Bondholders therefore demand a premium for holding corporate bonds. Fischer et al (1989) and Goldstein et al (2001) show that the value of the firm is maximized when the leverage ratio is kept within a small band, while Collin-Dufresne and Goldstein (2001) proposed a model in which the capital structure is set to generate a stationary leverage ratio.

While these authors claim their models generate a credit spread close to what we observe in the data, Huang and Huang (2003) show that, when calibrated to match the default frequency and recovery rate, they are all inadequate in explaining the credit spread. Likewise, the introduction of a jump process for the underlying value can be consistent with a larger credit spread (Zhou (2001) and Delianedis and Geske (2001)), but only with unrealistic jump parameters.

Another possible route to a wider credit spread involves nonstandard assumptions regarding utility. For example, Chen et al (2008) use Campbell and Cochrane (1999) habit formation to generate a counter-cyclical time-varying risk premium. However, calibration results showed that habit formation could only generate a sufficiently high credit spread in combination with an exogenous countercyclical default boundary or idiosyncratic volatility.

3 The Barro (2006) Setting

The broad framework of our analysis follows Barro (2006), insofar as we start from an economy populated by a representative agent with time-additive utility and an initial endowed income in the form of the fruit of a Lucas-tree, with equity claims on the time $t+1$ stochastic endowment ("dividends") traded at time t. Specifically, agents maximise a power utility function[3] of the standard form:

$$E_t(U) = U(C_t) + \sum_{s=1}^{\infty} e^{-\rho s} E_t \left[U(C_{t+s})\right] \tag{1}$$

where:

$$U(C_{t+s}) = \frac{C_{t+s}^{1-\theta}}{1-\theta} \tag{2}$$

while the (log of the) endowment process, A_t, is a random walk with drift γ, and subject to two types of disturbance at any time, $t+1$:

$$g_{t+1} = \ln A_{t+1} - \ln A_t = \gamma + u_{t+1} + v_{t+1} \tag{3}$$

The first disturbance, u_{t+1}, is normally distributed with zero mean and constant variance, σ^2. The crucial component is the nonnormal shock v_{t+1} which takes the value zero (i.e no crisis) with probability e^{-p} and the value $\ln d_{t+1}$ with probability $(1-e^{-p})$, where p is approximately the probability of a crisis (a "disaster"), and $0 < d_{t+1} < 1$ is a random

[3] Barro and Ursua (2008), Gourio (2007), Chen (2007) use the Epstein-Zinn utility function, which has the advantage of incorporating separate parameters for the marginal rate of intertemporal substitution and coefficient of risk aversion, but has the disadvantage of making it even harder to arrive at closed-form solutions. It could also be argued that using a nonseparable utility function, as we do here, makes the task of matching the observed data more of a challenge.

variable representing the level to which output falls in a disaster scenario. The distribution of d_{t+1} is approximated empirically by the frequency found in the Barro (2006) and Barro and Ursua (2008) research on the economic history of the past century.[4] It is important to note that u_{t+1} and v_{t+1} are assumed to be independent identically-distributed shocks. Even in crisis states, the economy is still assumed to be subject to the normal zero-mean shock process (albeit tiny relative to d_{t+1}), so we find it convenient to write crisis output before allowing for growth as:

$$w_{t+1} \equiv e^{u_{t+1}} d_{t+1}$$

4 Debt Markets

We deviate from Barro (2006) in our specification of the market for the two types of fixed-income instrument traded in the model. First, we assume the Government issues a bill or bond in the form of a claim paying a fixed return, R. Government borrowing is completely riskless, in the sense that in this model the Government never defaults in any state of the world. This specification is in contrast to Barro (2006), who assumes that in extreme crisis states, the Government and private sector both default, an event which occurs with an exogenously given probability conditional on an output disaster.

In our model, unlike Government debt, the privately-issued fixed interest security is subject to default risk. The firm which owns the tree issues δ_t one-period securities [5] promising

[4] Note that Barro (2006) writes his equations in terms of the size of the fall in output in the crisis state, whereas we find it convenient to deal with the level of output in a crisis i.e our d is equivalent to his $(1-b)$.

[5] For the sake of consistency, we can think of the equityholder issuing the debt in order to finance the purchase of shares. In a sense, therefore, the tree is not part of the initial endowment, but needs to be

a face return, fixed in advance at time t for payment at $t+1$. In noncrisis situations and in all but the most severe crises, output is adequate to cover the cost of paying the face return, R^F_{t+1}, leaving the residue as a dividend to the equityholders, but in the event of a severe crisis, it may fall short, so that the firm is forced to default on its debt. In this extreme scenario, the bondholder receives the whole output, and the equity gets nothing. However, in the aftermath of a default, the firm is reconstituted in the next period with the same degree of leverage, an assumption required in order to preserve the IID property of cashflows. It follows that the realized return on corporate debt, R^B_{t+1} depends on realized output at time $t+1$. If output is sufficient to cover the contractual payment $\delta_t R^F_{t+1}$, the realized return equals the face return. Otherwise, the entire output is paid to bondholders, so that, in the event of default, the realized bond return is A_{t+1}/δ_t.

When the period length is small, the price of the claim on the entire output at time $t+1$ approximates output at time t. Hence, the debt-equity (leverage) ratio of the firm can be written as:

$$\lambda_t = \frac{\delta_t}{A_t - \delta_t}$$

We find it easier to work with the asset-debt ratio:

$$\Lambda_t = \frac{A_t}{\delta_t} \tag{4}$$

Given these assumptions, we can derive the critical value of output, A^*_{t+1}, below which default occurs:

$$A^*_{t+1} = \delta_t R^F_{t+1}$$

purchased at the outset with the proceeds of the bond sale, as in Barro (2006).

Using (3) and (4), we can rewrite the critical output value (as a proportion of trend) in terms of the underlying shocks as follows:

$$w^*_{t+1} = \frac{R^F_{t+1}}{\Lambda_t e^{\gamma}} \tag{5}$$

so that w^*_{t+1} is the critical point on the distribution of the composite random variable $e^{u_{t+1}} d_{t+1}$, determined by the joint distribution of the normal shock, u_{t+1}, and the crisis shock, d_{t+1}.

These changes to the model have far-reaching implications. First, instead of being an event occuring with an exogenously fixed probability, default happens whenever output falls below the endogenously-determined critical level, w^*_{t+1}, for which we solve below. For the moment, note that w^*_{t+1} is determined by a number of factors, most importantly the leverage ratio since, other things being equal, the more debt the tree-owner issues, the greater the burden of repayments and therefore the smaller the output contraction sufficient to cause a default. Associated with this critical level of w_{t+1} is a probability which, for convenience in writing out the model equations, we denote π:

$$\pi_{t+1} = \Pr(w_{t+1} < w^*_{t+1})$$

which is the probability of default conditional on a crisis occurring. Note that, in spite of the shorthand, π_{t+1} is not exogenous, but is determined by the model parameters and, in particular, by the initial leverage ratio.

5 The Model Solution[6]

As a first step to solving the model, we compute the stochastic discount factor (SDF) in each of the three states of the world with which we are concerned here. In the no-disaster scenario, when output is subject only to normally-distributed shocks, the SDF is derived straightforwardly from the first-order conditions as:

$$M_{t+1} = e^{-\rho-\gamma\theta-u_{t+1}\theta} \tag{6}$$

Now consider the two disaster states. In the first, output falls disastrously, but not enough to cause a default i.e. $w^*_{t+1} \leq w_{t+1} < 1$. In the second, the negative shock is so great that output falls below the critical value at which the firm goes into default. However, since the SDF depends on the level of output and hence consumption, and not on its distribution between equity and bondholders, it is the same function of d whether the outcome is above or below the critical value. The crisis SDF is therefore simply:

$$d^{-\theta}_{t+1} M_{t+1} \tag{7}$$

Note that since d_{t+1} is the level of output in the crisis state as a proportion of its noncrisis level, and it is usually assumed that $\theta >> 1$, it follows that the discount factor in these states is a multiple of its normal size, a factor which will be critical in explaining why the model generates substantial risk premia, since it implies that the marginal utility of consumption increases sharply as it falls. It follows that assets which either deliver reduced payoffs (like corporate debt) or possibly no payoff at all (equity) in disaster scenarios are valued far lower, other things being equal, than those which give a return in all possible states (Government debt).

[6] Most of the derivations are given in the Appendix to the paper.

As far as the payoff on the corporate debt is concerned, the expected value of its return in each state valued by the corresponding SDF has to satisfy the condition: $1 = E(SDF_{t+1} \cdot R^B_{t+1})$. In the two nondefault states it pays the face return originally promised, R^F_{t+1}, whereas when $w_{t+1} < w^*_{t+1}$, the situation is more complicated. In this case, bondholders receive the total output, so that the relationship between the face return and w^*_{t+1} depends on the degree of leverage.

Given the value of the SDF in each of the three relevant states, and the payoff on the debt in each state, we can compute the expected return for $t+1$, conditioned on information at time t, as:[7]

$$\ln ER^B_{t+1} \approx (1 - p\pi_{t+1}) \ln R^F_{t+1} + p\pi_{t+1} \left[\Lambda_t e^{\gamma + \frac{1}{2}\sigma^2} \cdot E(d_{t+1} \mid w_{t+1} < w^*_{t+1}) - 1 \right] \qquad (8)$$

This says that the (log of) the expected return offered by this security is a weighted average of two components, the face return (i.e. assuming no default), R^F_{t+1}, and the return in the default scenario, the latter being the bondholder's claim on the normal growth in output, $\Lambda_t e^{\gamma + \frac{1}{2}\sigma^2}$, scaled down by the proportionate fall in output that brings about the default. Note that the impact of the asset-to-debt ratio, Λ_t, is complicated. In the first place, a higher value of Λ_t (lower leverage) makes the return greater in the event of default, and also raises the size of the contraction needed to trigger a default, reducing w^*_{t+1}, and consequently also the default probability, π_{t+1}. At the same time, the greater security associated with lower leverage (higher Λ_t) lowers the equilibrium nominal return, other things being equal.[8]

[7] As all expectations in the model are for time $t+1$ conditional on time t information, we drop the subscripts on the expectation operator from now on.

[8] In the Appendix, we give the explicit solution for the face return and also the proof that both it and the critical output value are increasing in p and in the leverage.

The return on default-free government debt, which is the riskless rate in this model, is simply:[9]

$$\ln R_{t+1} = \rho + \theta\gamma - \frac{1}{2}\theta^2\sigma^2 + p\left[1 - Ed_{t+1}^{-\theta}\right] \tag{9}$$

This says that the riskfree rate is the sum of the rate of time preference, ρ, and the marginal value of output (and consumption), $\theta\gamma$, less components relating to risk in the normal and abnormal states. The first is the familiar convexity adjustment, while the second is the expected value of incremental consumption in the disaster state. Note that, since $0 < d_{t+1} < 1$, the term in square brackets is negative, so that the riskless rate is unambiguously decreasing in p.

6 The Credit Spread and Equity Risk Premium

In our model, the credit spread is equivalent to the difference between the face return and the risk-free rate i.e. the gap between the rates promised by corporate and government riskless debt. Comparing (8) and (9), we get:

$$\begin{aligned}\ln R_{t+1}^F - \ln R_{t+1} \quad = \quad & \\ & p(1-\pi_{t+1})\left[Ed_{t+1}^{-\theta} - \left(Ed_{t+1}^{-\theta} \mid w_{t+1} \geq w_{t+1}^*\right)\right] \\ & +p\pi_{t+1}\left[Ed_{t+1}^{-\theta} - \Lambda_t e^{-\rho+(1-\theta)\gamma+\frac{1}{2}(1-\theta)^2\sigma^2} \cdot E(d_{t+1}^{1-\theta} \mid w_{t+1} < w_{t+1}^*)\right]\end{aligned} \tag{10}$$

which can be viewed as the sum of the required compensation for the expected loss in default and the premium associated with this type of systematic risk. As Elton et al (2001) emphasise, the latter component will be present as long as investors are risk-averse and, in

[9] as in Barro (2006), in the special case when the default probability (denoted q in his model) is zero.

fact, in the light of the evidence they and others have produced, actually accounts for much of the spread.

To understand (10), note that when the probability of a crisis, p, is zero, there is no gap between the returns on Government and private sector debt, since default can only occur in crises. Another limiting case is the "superprime borrower" for whom w^*_{t+1} is near 0% i.e. the borrower who would only be forced into default if income fell almost to zero. In this case, the conditional and unconditional expectations in the first square bracket are almost the same thing, so that this component is very small. Likewise, the second term involves $p\pi_{t+1}$, which will be extremely small (the probability of a crisis multiplied by the probability of default, which is also tiny in this case). In sum, the credit spread will be small, as intuition would lead us to expect, for a good quality borrower.

From (10), it is clear that the effect of leverage on the credit spread is complex. To see that this is the case, note that the final term in square brackets will be substantially greater than unity if $\theta > 1$, because the mean of the distribution of d_{t+1} in default scenarios will be small. Since higher levels of leverage increase π_{t+1}, the effect on the spread through this component is positive. Moreover, this effect will be reinforced because higher leverage (lower Λ_t) makes the firm more vulnerable to a downturn. In other words, it raises w^*_{t+1}, making $E(d_{t+1} \mid w_{t+1} < w^*_{t+1})$ larger and therefore $E(d^{1-\theta}_{t+1} \mid w_{t+1} < w^*_{t+1})$ smaller, thereby further widening the credit spread. On the other hand, the term in the first square bracket on the right hand side must be positive, and increasing in leverage. But higher leverage also increases π_{t+1} and hence reduces $p(1 - \pi_{t+1})$, so that the effect on the expression before the plus sign as a whole is ambiguous. In the Appendix, we show that the net effect is positive, as might be expected i.e. a higher leverage is associated with a wider spread.

As far as equity is concerned, leverage means that the payoff to shareholders is simply the residue of output after the bondholders have received a sum no greater than the face value of the debt plus the accrued interest. In the event of default, the return on equity is, of course, zero. Hence the return on the levered equity is just:

$$R^L_{t+1} = \frac{A_{t+1} - \delta_t R^B_{t+1}}{P_t - \delta_t} \approx \frac{A_{t+1} - \delta_t R^B_{t+1}}{A_t - \delta_t} \tag{11}$$

which is related to the unlevered return by the familiar weighted average cost of capital formula:

$$R^E_{t+1} = \frac{\Lambda_t - 1}{\Lambda_t} R^L_{t+1} + \frac{1}{\Lambda_t} R^B_{t+1} \tag{12}$$

where R^E is the return on the underlying asset (i.e. on the unlevered equity), given by:

$$\ln R^E_{t+1} = \rho + \theta\gamma - \frac{1}{2}\theta^2\sigma^2 + \theta\sigma^2 - p\left(Ed^{1-\theta}_{t+1} - Ed_{t+1}\right) \tag{13}$$

and $R^L$and R^B are the returns on the levered equity and on the firm's bonds respectively. The equity risk premium can then be derived from (11) and (9). Although our main concern in this paper is with the pricing of debt, we also generate results for equities as an additional check on the plausibility of our results, given that the data are more plentiful and more accurate than they are for bond markets, a point emphasised by Huang and Huang (2003).

We now turn to the question of how far it is possible to explain the stylised facts within the framework of the model set out here, using parameter values taken from Barro (2006) and the more extensive data collected in Barro and Ursua (2008).

7 Calibration

The key element in calibrating the model is the annual crisis probability, p, which was estimated as 1.7% in Barro (2006) and 3.7% in Barro and Ursua (2008), and the frequency distribution of the output contraction, $1-d_{t+1}$. In view of the problematic nature of historic data, and the fact that the value of p is critically dependent on the definition of a crisis, we calibrate for a number of different values in the range indicated by the facts for the last century. It must be emphasised however that, while the equity risk premium puzzle as originally posed by Mehra and Prescott (1985) was originally based on the experience of the twentieth century, the credit risk spread has largely been considered in the context of datasets which were shorter and hence possibly less representative. In particular, in comparing our calibration results with current levels of the credit spread, we need to bear in mind that the market may be anticipating a very different crisis frequency in the future than in the past.

In view of the importance of the crisis probability, we should make clear the channels through which it impacts on the credit spread. On the one hand, the higher is the probability of a crisis, p, the wider must be the credit spread, for the obvious reason that, in a more uncertain world, risk-free assets must be more and risky assets less valuable, other things being equal. On the other hand, a greater value for the riskless security and consequently lower riskless rate, R, reduces the cost of debt service and thereby raises $1-w^*_{t+1}$, the critical size of output contraction needed to trigger a default. This latter mechanism, taken on its own, reduces the conditional probability of default, π_{t+1}. The credit spread is determined by the unconditional probability of default, which is the product of p and π_{t+1}. Higher p raises this probability directly, but indirectly (via a fall in π_{t+1}) reduces it. The net effect is

therefore complex, but as will be seen from our results, invariably positive in practice.

In Tables 1 and 2, we take as given the estimates of the rate of time preference, ρ, the trend growth rate, γ, and the standard deviation of normal shocks, σ, and show how the results are affected by changes in the crisis probability for different values of the leverage ratio, and θ, the inverse of the elasticity of intertemporal substitution. Note that, as far as the last parameter is concerned, we only take the values 3.0, 3.5 and 4.0, all well within the range usually considered reasonable.

It can be seen that, for both parameter sets, the model generates plausible values of the riskless rate (in the zero to 3% range), for the return on leveraged equity (the 7% to 11% range) and for the equity risk premium (5% to 10%).

As far as the credit spread is concerned, note that the model generates values that are as high as those actually observed, and even greater in some cases. For example, taking the 3.7% probability from Barro and Ursua (2008), we get spreads of 39, 74 or 147 points for the three values of the risk aversion parameter, $\theta = 3.0$, 3.5 and 4.0 respectively, with a leverage ratio of only 50%. At higher levels of leverage, the credit spread widens dramatically, to 99, 185 and 342 points at 75%, for example.

Note that the reason why this model yields such a large credit spread is to be found in the correlation it generates between the discount factor and the loss in default, so that corporate debt inflicts heavy losses on investors in precisely those states where consumption is most valuable. By contrast, Government debt is riskless and therefore benefits from the "flight to quality", so that its equilibrium return is reduced by the possibility of a severe contraction in output.

At the same time, our results confirm the consensus view (e.g. Huang and Huang (2003),

Elton et al (2001), Bhamra et al (2007)) that the credit spread overwhelmingly reflects the default risk premium rather than the expected loss. In fact, the cost of default makes a negligible contribution for almost all values of the parameters. One possible explanation for this result is that the recovery rates we generate look high relative to observed levels. There are two reasons for this. First, we make no allowance here for the substantial costs lenders have in reality to pay for recovering their loans. Secondly, as Huang and Huang (2003) explicitly recognise in their continuous time model, in practice lenders are usually unable or unwilling to call in loans as soon as the value of the entreprise falls to the face value of the debt (in fact, Huang and Huang (2003) explicitly assume foreclosure at the 60% boundary). In the context of our model, however, lenders have no reason to delay foreclosure, so they capture a higher proportion of the assets. This in turn makes the expected loss in default small.

8 The Credit Spread Time Series

Calibrations can only answer the question: how well does the model explain the average credit spread and equity risk premium over the data period? We would ideally like to apply a more demanding test of its validity, the problem of course being lack of data. However, a kind of time series approach is possible based on a simulation of equation (10), using quarterly values of the leverage ratio on the right hand side to predict the levels of the credit spread on the left. Unfortunately, we are unable to cover anywhere near as long a period as Barro and Ursua (2008). Instead, we are restricted to quarterly data from 1953 until early 2008, a total of 220 observations for the leverage ratio and the observed spread. Using the

same parameter values as in the calibrations (i.e. derived from 120-year averages) makes the test even more challenging.

Before examining the outcomes, it is worth noting that the relationship between the leverage and the spread is not linear, as might appear at first glance. One nonlinearity is that the probability of default, π_{t+1}, is itself a function of the leverage ratio, as also is w^*_{t+1}, the threshold output level. The latter relationship is summarised in (5), where w^*_{t+1} and the leverage ratio are inversely related for any given value of the face return, R^F_{t+1}, which is endogenous, but in practice cannot be very far above 1.0. In fact, both w^*_{t+1} and the spread are concave functions of the leverage ratio. The net effect, however, must be that the leverage ratio increases the credit spread (see Appendix).

Figure 1 shows the simulated spread (solid line) and the difference between the yield on Moodys Baa Index and the US Treasury benchmark 10-year yield (broken line). The correlation coefficient is 0.43. Note that the two series diverge in the mid-1990's, when leverage ratios appear to have been driven down by the bull market in equities. Note also that the predicted spread fell to zero at a number of points during the data period, as leverage ratios were low enough to generate a smaller value of w^*_{t+1} than was ever observed in the twentieth century, making the probability of default at these times zero.

9 Conclusions

We have shown that a generalised version of the Barro (2006) crisis model to allow for defaultable corporate debt can account for the broad facts about the credit spread for a range of plausible values of the parameters and for two alternative estimates of the frequency dis-

tributions of the large output contractions experienced in the last century. However, like most other authors, we find that very little of the credit spread appears to be explained by expected default loss; it appears to be almost all risk premium. It is quite possible that this result is due to sampling error of the peso-problem type. On the one hand, during our data period in the USA there was no recession sufficiently grave to qualify as a crisis in the Barro and Ursua (2008) definition of the word. However, the relevant probability distribution is the unobservable one on which the rational investor bases his or her behaviour. Even with rationality, this subjective distribution may well deviate from the observed frequency in the case of events far into the tail (the Peso Problem). Specifically, it is unlikely to have given a zero weight to the probability of a crisis, but on the other hand it is unlikely to have been as pessimistic as the Barro and Ursua (2008) frequency distribution, which covers both industrialised and developing countries, and in any case includes data from the first half of the twentieth century spanning two world wars and the interwar global slump. Insofar as the spread data we observe may understate the probability distribution of a crisis occurring, our estimates of the expected return on corporate debt (and hence the spread) may be biased upwards. Equally, basing our simulated series on an excessively pessimistic frequency distribution may also have resulted in upward bias. There is no way of knowing which of these two biases is the greater. Future research will need to be directed to this issue.

Moreover, like the rest of this literature, our work has nothing to say about events like the 1929 or 1987 global stock market crashes nor the current financial crisis. In all these cases, the initial shock struck the financial markets, with any impact on the real economy following from it, so that the causal direction was the reverse of the one we model in this paper. Accounting for this type of event will require substantial further work.

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10 Appendix

10.1 Riskless rate (equation (9))

$$\begin{aligned} R_{t+1} &= \frac{1}{E(SDF_{t+1})} \\ &= \left[(1-p)E(M_{t+1}) + pE(M_{t+1} \cdot d_{t+1}^{-\theta})\right]^{-1} \\ &= \left[\Psi\left(1 - p + pEd_{t+1}^{-\theta}\right)\right]^{-1} \end{aligned}$$

where

$$\begin{aligned} M_{t+1} &= e^{-\rho-\theta\gamma-\theta u_{t+1}} \\ \Psi &= E(M_{t+1}) = e^{-\rho-\theta\gamma+\frac{1}{2}\theta^2\sigma^2} \end{aligned}$$

so:

$$\ln R_{t+1} \approx \rho + \theta\gamma - \frac{1}{2}\theta^2\sigma^2 + p\left(1 - Ed_{t+1}^{-\theta}\right)$$

The approximation follows as p is close to zero.

10.2 Face return of the bond

$$1 = E(SDF_{t+1} \cdot R^B_{t+1})$$

$$= (1-p)E(M_{t+1})R^F_{t+1} + p(1-\pi_{t+1})E[M_{t+1} \cdot d^{-\theta}_{t+1} \mid w_{t+1} \geq w^*_{t+1}]R^F_{t+1}$$

$$+p\pi_{t+1}E[\Lambda_t \cdot M_{t+1} \cdot e^{\gamma+u_{t+1}} \cdot d^{1-\theta}_{t+1} \mid w_{t+1} < w^*_{t+1}]$$

$$= (1-p)E(M_{t+1})R^F_{t+1}$$

$$+p(1-\pi_{t+1})E(M_{t+1})R^F_{t+1} \cdot E[d^{-\theta}_{t+1} \mid w_{t+1} \geq w^*_{t+1}]$$

$$+p\pi_{t+1}\Lambda_t E(M_{t+1}e^{\gamma+u_{t+1}}) \cdot E[d^{1-\theta}_{t+1} \mid w_{t+1} < w^*_{t+1}]$$

$$= (1-p)\Psi R^F_{t+1}$$

$$+p(1-\pi_{t+1})\Psi R^F_{t+1} \cdot E[d^{-\theta}_{t+1} \mid w_{t+1} \geq w^*_{t+1}] + p\pi_{t+1}\Lambda_t\Phi \cdot E[d^{1-\theta}_{t+1} \mid w_{t+1} < w^*_{t+1}]$$

$$= \Psi \cdot \left[(1-p)R^F_{t+1} + p(1-\pi_{t+1})R^F_{t+1} \cdot E\left(d^{-\theta}_{t+1} \mid w_{t+1} \geq w^*_{t+1}\right)\right]$$

$$+p\pi_{t+1}\Lambda_t\Phi \cdot E\left(d^{1-\theta}_{t+1} \mid w_{t+1} < w^*_{t+1}\right)$$

where:

$$\begin{aligned}
\pi_{t+1} &= pr(w_{t+1} < w^*_{t+1}) \\
\Lambda_t &= \frac{A_t}{\delta_t} \\
\Phi &= E(M_{t+1}e^{\gamma+u_{t+1}}) = e^{-\rho+(1-\theta)\gamma+\frac{1}{2}(1-\theta)^2\sigma^2}
\end{aligned}$$

Hence:

$$\begin{aligned}
R^F_{t+1} &= \left[1 - p\pi_{t+1}\Lambda_t\Phi \cdot E\left(d^{1-\theta}_{t+1} \mid w_{t+1} < w^*_{t+1}\right)\right] \\
&\quad \times \left\{e^{-\rho-\theta\gamma+\frac{1}{2}\theta^2\sigma^2} \cdot \left[1 - p + p(1-\pi_{t+1}) \cdot E\left(d^{-\theta}_{t+1} \mid w_{t+1} \geq w^*_{t+1}\right)\right]\right\}^{-1}
\end{aligned}$$

and:

$$\ln R^F_{t+1}$$

$$\approx \rho + \theta\gamma - \frac{1}{2}\theta^2\sigma^2 + p - p\pi_{t+1}\Lambda_t \Phi E\left(d_{t+1}^{1-\theta} \mid w_{t+1} < w^*_{t+1}\right) - p(1-\pi_{t+1})E\left(d_{t+1}^{-\theta} \mid w_{t+1} \geq w^*_{t+1}\right)$$

$$= \rho + \theta\gamma - \frac{1}{2}\theta^2\sigma^2$$

$$+p\left[1 - (1-\pi_{t+1})E\left(d_{t+1}^{-\theta} \mid w_{t+1} \geq w^*_{t+1}\right) - \pi_{t+1}\Lambda_t e^{-\rho+(1-\theta)\gamma+\frac{1}{2}(1-\theta)^2\sigma^2} E\left(d_{t+1}^{1-\theta} \mid w_{t+1} < w^*_{t+1}\right)\right]$$

where the approximation again uses the fact that p is close to zero.

10.3 Expected return of the bond (equation (8))

$$\begin{aligned} ER^B_{t+1} &= (1-p\pi_{t+1})R^F_{t+1} + p\pi_{t+1}\Lambda_t \cdot E[e^{\gamma+u_{t+1}} \cdot d_{t+1} \mid w_{t+1} < w^*_{t+1}] \\ &= (1-p\pi_{t+1})R^F_{t+1} + p\pi_{t+1}\Lambda_t e^{\gamma+\frac{1}{2}\sigma^2} \cdot E(d_{t+1} \mid w_{t+1} < w^*_{t+1}) \\ &= (1-p\pi_{t+1})(R^F_{t+1}-1) + p\pi_{t+1}\left[\Lambda_t e^{\gamma+\frac{1}{2}\sigma^2} \cdot E(d_{t+1} \mid w_{t+1} < w^*_{t+1}) - 1\right] + 1 \end{aligned}$$

so:

$$\ln ER^B_{t+1} \approx (1-p\pi_{t+1}) \ln R^F_{t+1} + p\pi_{t+1}\left[\Lambda_t e^{\gamma+\frac{1}{2}\sigma^2} \cdot E(d_{t+1} \mid w_{t+1} < w^*_{t+1}) - 1\right]$$

where the approximation uses the fact that $(R^F_{t+1}-1)$ and $p\pi_{t+1}$ are close to zero.

10.4 Proof that Credit Spread is Positively Related to Leverage

Since the riskless rate given by (9) is not dependent on the leverage ratio, we need only concern ourselves with the determination of the nominal return on the corporate debt.

Note that w^*_{t+1} on the RHS of the previous equation is unknown and π_{t+1} depends on w^*_{t+1}. Since we impose no analytical distribution on the disaster size, and instead rely on the

empirical frequency, we cannot derive a closed-form analytical solution for the face return. It has to be solved numerically, as shown in the calibration section. However, combining (5) in the text with the last equation in the Appendix, we can deduce the nature of the qualitative relationship between w^*_{t+1}, the leverage ratio and R^F_{t+1}. The critical issue is the direction of the change in w^*_{t+1} when leverage increases.

Consider two scenarios, involving quantities of debt δ_1 and δ_2, with $\delta_2 > \delta_1$. Let the critical output levels associated with δ_1 and δ_2 be $w^*_{1,t+1}$ and $w^*_{2,t+1}$ respectively. Which is greater: $w^*_{1,t+1}$ or $w^*_{2,t+1}$?

Case 1: $w^*_{2,t+1} < w^*_{1,t+1}$ In this case, as the debt burden becomes larger, the output fall required to trigger default actually increases, and thus the probability of default gets smaller. Consider what this means in an intermediate state when output is w'_{t+1} where $w^*_{2,t+1} < w'_{t+1} < w^*_{1,t+1}$ i.e. when a disaster occurs that would previously have caused default but no longer does so, even though the firm is more leveraged. This is clearly impossible because, whereas previously the bondholders would have received the whole output w'_{t+1},they can now only claim a part of it, with the remainder being paid out as a a dividend to the equity. Given that, by construction, the upfront cost of the debt (its face value) is greater, the result has to be a fall in the return on bonds when output is $w'_{t+1}$and in all similar intermediate states. As far as severe disaster states are concerned, when output is below $w^*_{2,t+1}$, the return also falls because the payoff is unchanged while the upfront cost increases.

These conclusions imply that, in order to preserve the pricing condition $1 = E(SDF_{t+1} \cdot R^B_{t+1})$, the returns in non-disaster and mild disaster states have to rise, which can only be achieved by a rise in the face return of the bond. However, as equation (5) shows, the face

return is increasing in the product of w^*_{t+1} and the asset-to-debt ratio. Therefore it cannot rise when both factors have fallen. Hence, this is an impossible scenario.

Case 2: $w^*_{2,t+1} > w^*_{1,t+1}$ The reverse scenario is more complex.

Case 2A: When output is $w''_{t+1}$where $w^*_{1,t+1} < w''_{t+1} < w^*_{2,t+1}$, the total payoff to bond holders increases because previously some output would have been paid as a dividend to shareholders. However, there are now an increased number of bonds so that the return might be higher or lower than the previous face return (though it is, by definition, lower than the new face return).

Case 2B: When output is $w''_{t+1}$where $w''_{t+1} < w^*_{1,t+1}$, the bondholders receive the whole output, as before, but given their increased number, the realised returns must again be lower.

Case 2C: When output is $w''_{t+1}$where $w''_{t+1} > w^*_{2,t+1}$, the bonds are paid their face return.

However, both Cases 2A and 2C were no-default scenarios prior to the leverage increase. Hence the return in both cases was the face return. After the leverage increase, the average of the return in these two cases is less than the new face return, but higher than it was previously. Hence the face return has risen.

Since the average return in Case 2B is lower after the leverage increase, the average return in Case 2A and 2C must increase in order to maintain the pricing relationship $1 = E(SDF_{t+1}R^B_{t+1})$. Since both Cases 2A and 2C were no-default scenarios prior to the leverage increase, the return in both cases was the face return. After the leverage increase, Case 2A brings default where the return is less than the new face return. Hence, the average return

of Case 2A and 2C after the increase in leverage is less than the new face return, but it is greater than the previous face return. Therefore, the face return must have risen. *Proved.*

FIGURE 1: SIMULATED vs ACTUAL CREDIT SPREAD 1953-2008

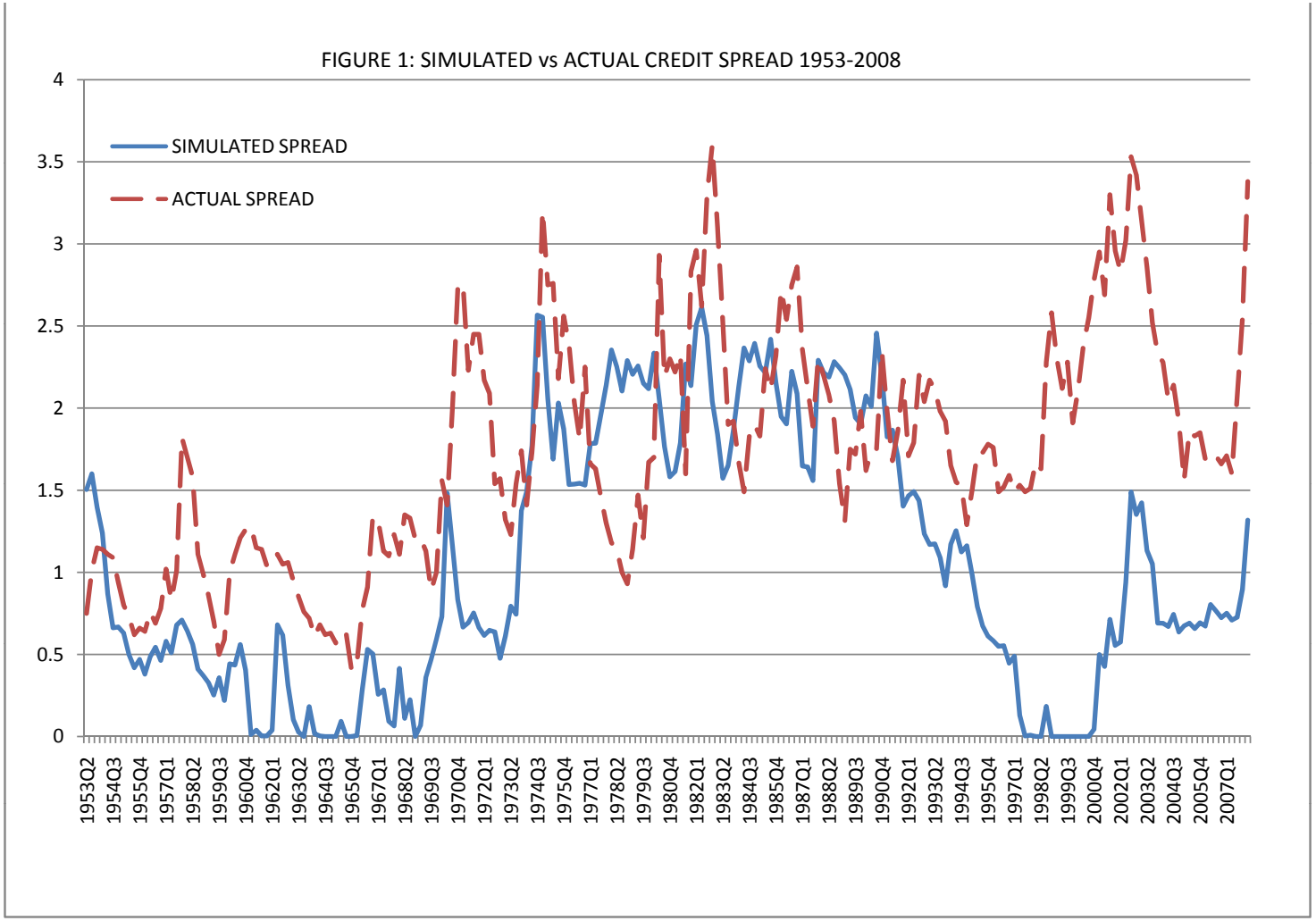

TABLE 1: CALIBRATIONS WITH BARRO (2006) PARAMETERS (ρ = 3% γ = 2.5% σ = 2%)

range found in historic data […,...]

Highlighted data in normal (bold italic) font match histroric data for single target range (all four target ranges)

		p = 0.015 θ			p = 0.017 θ			p = 0.020 θ		
	leverage	3	3.5	4	3	3.5	4	3	3.5	4
risk-free										
	0.5	5.32%	3.91%	1.06%	4.66%	2.89%	-0.49%	3.66%	1.37%	-2.81%
[-1%,3%]	0.75	5.32%	3.91%	***1.06%***	4.66%	***2.89%***	-0.49%	3.66%	***1.37%***	-2.81%
	1	5.32%	3.91%	1.06%	4.66%	***2.89%***	-0.49%	3.66%	***1.37%***	-2.81%
	1.5	5.32%	3.91%	1.06%	4.66%	***2.89%***	-0.49%	3.66%	***1.37%***	-2.81%
leveraged-equity		3	3.5	4	3	3.5	4	3	3.5	4
return	0.5	9.13%	9.86%	10.44%	8.95%	9.62%	10.13%	8.68%	9.25%	9.73%
[7%,11%]	0.75	9.39%	10.20%	***10.85%***	9.23%	***9.99%***	10.57%	9.00%	***9.68%***	10.20%
	1	9.52%	10.36%	11.04%	9.38%	***10.17%***	10.78%	9.17%	***9.89%***	10.43%
	1.5	9.66%	10.53%	11.20%	9.53%	***10.35%***	10.94%	9.33%	***10.07%***	10.56%
leveraged -risk premium		3	3.5	4	3	3.5	4	3	3.5	4
	0.5	3.81%	5.96%	9.38%	4.29%	6.73%	10.62%	5.02%	7.88%	12.54%
[5%,10%]	0.75	4.07%	6.30%	***9.79%***	4.58%	***7.10%***	11.06%	5.35%	***8.31%***	13.01%
	1	4.19%	6.45%	9.98%	4.72%	***7.28%***	11.27%	5.51%	***8.52%***	13.24%
	1.5	4.33%	6.62%	10.14%	4.87%	***7.45%***	11.43%	5.67%	***8.70%***	13.37%
credit spread		3	3.5	4	3	3.5	4	3	3.5	4
	0.5	0.09%	0.17%	0.29%	0.10%	0.19%	0.29%	0.12%	0.21%	0.14%
[0.4%, 3.6%]	0.75	0.56%	0.99%	***1.74%***	0.64%	***1.12%***	1.97%	0.75%	***1.32%***	2.25%
	1	0.93%	1.58%	2.70%	1.05%	***1.79%***	3.05%	1.23%	***2.10%***	3.54%
	1.5	1.37%	2.27%	3.79%	1.55%	***2.58%***	4.30%	1.82%	***3.03%***	5.06%
expected default loss		3	3.5	4	3	3.5	4	3	3.5	4
	0.5	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	0.75	0.02%	0.02%	0.02%	0.02%	0.02%	0.02%	0.03%	0.02%	0.02%
	1	0.04%	0.04%	0.04%	0.05%	0.04%	0.04%	0.05%	0.05%	0.04%
	1.5	0.08%	0.08%	0.08%	0.09%	0.09%	0.08%	0.10%	0.10%	0.09%
recovery rate		3	3.5	4	3	3.5	4	3	3.5	4
	0.5	91.30%	92.24%	94.72%	91.87%	93.13%	96.19%	92.51%	94.50%	98.63%
	0.75	81.98%	82.75%	84.37%	82.44%	83.45%	85.43%	83.14%	84.49%	86.39%
	1	79.53%	79.88%	80.48%	79.86%	80.29%	80.94%	80.13%	80.59%	81.00%
	1.5	69.40%	69.32%	69.65%	69.39%	69.41%	70.26%	69.52%	69.86%	71.23%

TABLE 2: CALIBRATIONS WITH BARRO (2008) PARAMETERS (ρ = 3% γ = 2.5% σ = 2%)

range found in historic data […,...]

Highlighted data in normal (bold italic) font match histroric data for single target range (all four target ranges)

		p = 0.027 Θ			p = 0.037 Θ			p = 0.047 Θ		
	leverage	3	3.5	4	3	3.5	4	3	3.5	4
risk-free										
	0.5	4.87%	3.17%	-0.32%	2.85%	***0.08%***	-5.14%	***0.82%***	-3.01%	-9.95%
[-1%,3%]	0.75	4.87%	3.17%	-0.32%	***2.85%***	***0.08%***	-5.14%	***0.82%***	-3.01%	-9.95%
	1	4.87%	3.17%	-0.32%	***2.85%***	***0.08%***	-5.14%	***0.82%***	-3.01%	-9.95%
	1.5	4.87%	3.17%	-0.32%	***2.85%***	***0.08%***	-5.14%	***0.82%***	-3.01%	-9.95%
leveraged-equity return		3	3.5	4	3	3.5	4	3	3.5	4
	0.5	8.69%	9.34%	9.85%	8.02%	***8.49%***	8.72%	***7.37%***	7.62%	7.58%
[7%,11%]	0.75	8.95%	9.68%	10.24%	***8.36%***	***8.92%***	9.31%	***7.79%***	8.19%	8.36%
	1	9.11%	9.88%	10.47%	***8.58%***	***9.18%***	9.56%	***8.04%***	8.49%	8.71%
	1.5	9.29%	10.08%	10.65%	***8.79%***	***9.42%***	9.76%	***8.29%***	8.76%	8.96%
leveraged -risk premium		3	3.5	4	3	3.5	4	3	3.5	4
	0.5	3.82%	6.17%	10.17%	5.18%	***8.41%***	13.85%	***6.54%***	10.63%	17.53%
[5%,10%]	0.75	4.08%	6.51%	10.56%	***5.52%***	***8.84%***	14.44%	***6.96%***	11.20%	18.31%
	1	4.25%	6.71%	10.79%	***5.73%***	***9.10%***	14.70%	***7.22%***	11.50%	18.66%
	1.5	4.43%	6.91%	10.98%	***5.95%***	***9.34%***	14.90%	***7.47%***	11.77%	18.91%
credit spread		3	3.5	4	3	3.5	4	3	3.5	4
	0.5	0.30%	0.59%	1.08%	0.39%	***0.74%***	1.47%	***0.47%***	0.93%	1.87%
[0.4%, 3.6%]	0.75	0.73%	1.37%	2.57%	***0.99%***	***1.85%***	3.42%	***1.24%***	2.30%	4.29%
	1	1.03%	1.89%	3.47%	***1.41%***	***2.58%***	4.73%	***1.79%***	3.26%	5.92%
	1.5	1.45%	2.56%	4.57%	***1.98%***	***3.50%***	6.24%	***2.51%***	4.43%	7.86%
expected loss in default		3	3.5	4	3	3.5	4	3	3.5	4
	0.5	0.00%	0.00%	0.00%	0.01%	0.00%	0.00%	0.01%	0.01%	0.00%
	0.75	0.02%	0.02%	0.02%	0.02%	0.02%	0.02%	0.03%	0.02%	0.02%
	1	0.03%	0.03%	0.03%	0.04%	0.04%	0.04%	0.05%	0.05%	0.04%
	1.5	0.07%	0.07%	0.06%	0.09%	0.08%	0.07%	0.10%	0.09%	0.08%
recovery rate		3	3.5	4	3	3.5	4	3	3.5	4
	0.5	87.43%	86.77%	80.05%	85.04%	80.20%	82.21%	80.56%	80.88%	86.17%
	0.75	78.90%	78.98%	79.67%	79.70%	79.29%	77.39%	78.91%	78.03%	77.75%
	1	77.63%	77.47%	77.12%	77.24%	77.72%	77.94%	77.46%	77.53%	78.00%
	1.5	75.07%	75.08%	75.27%	75.09%	75.54%	76.40%	75.46%	76.10%	74.98%