CARDIFF BUSINESS SCHOOL
WORKING PAPER SERIES

Cardiff Economics
Working Papers

Eric Scheffel

A Credit-Banking Explanation of the Equity Premium, Term Premium, and Risk-Free Rate Puzzles

E2008/30

Cardiff Business School
Cardiff University
Colum Drive
Cardiff CF10 3EU
United Kingdom
t: +44 (0)29 2087 4000
f: +44 (0)29 2087 4419
www.cardiff.ac.uk/carbs

ISSN 1749-6101
December 2008

A Credit-Banking Explanation of the Equity Premium, Term Premium, and Risk-Free Rate Puzzles

Eric Scheffel [a]

[a] *Cardiff Business School, Economics Section, Aberconway Building, Colum Rd, Cardiff, CF10 3EU*

Abstract

Micro-founded de-centralized financial intermediation in a cash and costly-credit model(see Gillman and Kejak, 2008) results in a cost-distortion of returns implying a lower average nominal and real risk-free rate when compared to standard cash-in-advance RBC models. Failure of both short-run and long-run Fisher equation relationships based on *observable* real and nominal rates and inflation are obtained. The cost-distortion also leads to an unconditionally upward-sloping average yield curve of interest rates which is also convex in shape. The model is capable of producing a positive correlation between the nominal rate and velocity, and a negative correlation between the ex-post real rate and inflation. More importantly, the model also predicts a negative correlation between the *ex-ante* real rate and the *ex-ante* expected rate of inflation. Finally, the conditional spread between the usual CCAPM rate as defined by Canzoneri and Diba (2005) and the model-implied money market rate is positively correlated with the stance of monetary policy, offering a new perspective on this systematic link recently studied empirically by Canzoneri et al. (2007a) and theoretically by Canzoneri and Diba (2005).

JEL Classification: E4,E44
Keywords: Business Cycles, Money, Term Structure of Interest Rates

Preliminary and Incomplete *Current Version 15 November 2008*

1 Introduction

Ever since the development of the consumption-based general equilibrium version of the CAPM model (Merton, 1971; Breeden, 1979; Lucas, 1978), the majority of contributions to the literature studying asset prices within this framework have focused on determining ex-ante expected asset returns in terms of risk premia derived from undiversifiable systematic risk. This approach of studying risk premia implied by the covariance of an asset's return with a typically preference-based stochastic discount factor (SDF) - or alternatively, the beta representation involving a correlation coefficient times a unique market price of risk [1] - has established itself as the standard way of studying asset prices in general equilibrium, not exclusively due to, but also because of the striking resemblance to earlier approaches in finance, most notably the standard CAPM (Sharpe, 1964; Lintner, 1969) of pricing assets, which instead of using some consumption-related measure, typically uses the market-portfolio's return as a way of proxying current marginal value of wealth.

In spite of the relative success of general equilibrium models in explaining the behaviour of aggregate quantities, it has proven immensely difficult to accomplish the same regarding asset prices, where any such failure of matching up theory with financial data stylized facts has typically been labelled a "puzzle". Two of such puzzles are the equity premium puzzle (Mehra and Prescott, 1985) and the closely related risk-free rate puzzle (Weil, 1989), on the one hand, and the term premium puzzle (Backus et al., 1989; Donaldson et al., 1990), on the other. Whereas the equity premium puzzle documents the *quantitative* failure of the consumption beta model to explain the observed excess return risky stocks earn over the risk-free rate, the low risk-free rate puzzle asks why, given a historical long-term annual growth rate of consumption in U.S. data of roughly 2%, the observed historical average real risk-free rate has been only approximately 1%. In a recent empirical study based on various utility specifications and using data on consumption and inflation, Canzoneri et al. (2007a) compare the theory-implied (i.e. consumption Euler equation-implied) CCAPM rates with the observed ex-post money market rates, only to find that they typically bear little resemblance conditionally and that they exhibit a positive spread unconditionally, the low risk-free rate again. They also find that the spread between the two rates is directly related to the stance of monetary policy. Kocherlakota (1996) emphasises how the low-risk free rate can really be viewed as a puzzle arising from the tension which emerges from explaining the two phenomena of the low risk-free and the high equity returns *simultaneously.* Also, in contrast to much of literature's recent emphasis placed

[1] Using Cochrane's notation, $p = E(mx)$ can always be expressed as $E(R^i) = R^f + \left(\frac{cov(R^i,m)}{var(m)}\right)\left(\frac{var(m)}{E(m)}\right)$ which is just $E(R^i) = R^f + \beta_{i,m}\lambda_m$, the beta representation (Cochrane, 2005)

on unconditional excess returns typically derived from first-order conditions using log-normal distributional assumptions about returns, Giovannini and Labadie (1991) show how in dynamic simulations of theoretical ex-ante bond and stock returns obtained from a monetary endowment economy with standard power utility, both rates move conditionally almost in identical fashion together, leading to the striking result that only fluctuations in the SDF (or equivalently the marginal utility of wealth) represent the underlying common factor driving movements in both rates, whereas conditional movements of risk premia appear to play little or no significance in this regard, the equity premium puzzle again. There now exists a sizeable literature trying to explain the high equity premium, whose review would be beyond the scope of this paper. A recent and very comprehensive survey of the equity premium literature is provided by Mehra and Prescott (2003). Other good discussions of the equity premium are also contained in Cochrane (2007), Cochrane (2005) and Campbell (2000).

The term premium puzzle, on the other hand, pertains to the unconditional yield curve of government-issued bonds, which in post-war U.S. data is upward-sloping, both for the real[2] and the nominal yield curve (see Fama, 1990; den Haan, 1995). What is also of importance is that the unconditional yield curve is typically much steeper at the short- than the long-end, so it is also highly convex on average[3]. In contrast to this, bond yields derived from standard general equilibrium models obey a generalised, risk-adjusted version of the pure expectations hypothesis of the term structure of interest rates (see Backus et al., 1989; Donaldson et al., 1990; den Haan, 1995). Risk-neutral investors or deterministic settings imply a completely flat yield curve "on average", risk-averse agents facing uncertainty and a *positively* autocorrelated process for the stochastic discount factor, imply an unconditionally downward-sloping yield curve. Within the latter set of assumptions, Backus et al. (1989) also show that an independently evolving stochastic discount factor also implies a flat yield curve on average. Typically, general equilibrium as well as atheoretical "affine" one-factor models approximate the nominal yield curve by simply modeling it's real counterpart, in order to make valuation of yields tractable and to avoid theoretical concerns over how money demand ought to be motivated on theoretical grounds. Labadie (1994) and den Haan (1995) also show

[2] Using evidence from UK inflation-indexed bonds, Seppala (2000) recently argued that the *real* term structure for the UK is downward-sloping, so he asserts the standard RBC model's predictions are correct. However, in a series of studies, Mishkin (1982, 1990b,a, 1992) found that real and nominal interest rates move in tandem, contradicting this view.

[3] Campbell et al. (1997), using the McCulloch and Kwon (1993) U.S term structure data base, find an average spread of the 10-year zero-coupon log yield over the one-month TB yields of 137 basis points. Also, the average yield spread over the one-month TB is 33 BP at three months, 77 BP at one year and 96 BP at two years. There is very little further change in average yields after two years.

that care needs to be taken in specifying the endowment process *in levels* in a simple Lucas exchange economy. Difference-stationary specifications lead to small persistence in expected consumption growth and positively autocorrelated consumption growth (leading to the counterfactual downward-sloping term structure on average), whereas trend-stationary specification leads to the opposite (an upward-sloping term structure on average, but with a counterfactual dynamic behaviour of the SDF using power utility [4]). Also, regardless what type of autocorrelation for the SDF is either assumed or endogenously obtained from within a model, for standard power utility with low risk aversion, the slope would be relatively constant and thus not exhibit the stronger curvature effects at the short end seen in the data [5] , as well as quantitatively small thus in an approximate sense leading to a practically flat yield curve unconditionally [6] . Related to this last point, Hansen and Jagannathan (1991) argue that the *observed* slope of the term structure must imply high volatility of the stochastic discount factor due to high Sharpe ratios in the bond market which result from small average bond term premia coupled with low term premia volatility. However, Campbell (2000) points out that "high Sharpe ratios of this sort [...] are of course highly sensitive to transactions costs or *liquidity services* [emph. added] provided by Treasury bills".

A large majority of explanations put forth in an attempt to resolve the equity premium or term premium puzzle are typically derived from simple *endowment* economies, in which output is perishable and governed by an exogenously specified process, thus through market clearing also determining the level of consumption in each period. The habit persistence literature (see Constantinides, 1990; Abel, 1990) has been a particular focus of attention, as habit persistence in consumption can potentially alter the stochastic discount factor in ways to induce more volatility in marginal utility, and thus lower the degree of risk-aversion required to obtain sufficiently large risk premia. Equally, other utility function specifications, such as Epstein-Zin preferences (Epstein and Zin, 1991) have also been explored, as they allow disentangling of the elasticity of intertemporal substitution from the coefficient of relative risk aversion. However, as recently emphasised by Cochrane (2007) and Mehra and Prescott (2003), the current state of affairs is such that none of the contributions made

[4] However, if the discount factor is equal to some power of expected consumption growth, both Donaldson et al. (1990) and den Haan (1995) discuss how observed consumption growth can be either positive or *negative*, depending on whether quarterly or monthly consumption data is analysed, thus raising concerns over aggregation bias.

[5] This is a typical feature of simple *one-factor* affine term structure models to which standard general equilibrium models with power utility typically reduce to. Backus et al. (1998) provide a good survey of discrete-time term structure models demonstrating the lack of convexity in one-factor models.

[6] This fact is also emphasised by (Bansal and Coleman, 1996, p.1148) who also call the theoretical term structure in standard models "essentially flat".

hitherto have reduced macro-finance's reliance on assumptions of fairly large levels of risk aversion in order to explain the equity premium, whereas a broad consensus view on the parameter of relative risk-aversion appears to place this value at a plausible maximum of five, and perhaps closer to one, i.e. logarithmic specification (see Kocherlakota, 1996, p.52). Indeed, to draw an analogy to another popular literature, just as research in the New Keynesian literature for a period of time has asked "How much rigidity [in price and wage contracts] do we need?", apparently the general equilibrium asset pricing literature has and still remains asking itself "How much risk-aversion do we need?"

Assuming sufficiently high levels of risk-aversion and adopting new utility functions has resulted in some degree of success in explaining stylized asset pricing facts from within *endowment* economies. However, regarding both the equity premium and the term premium, den Haan (1995) and Jermann (1998) demonstrate how fully specified general equilibrium models with non-trivial production and a physical capital storage technology (in which consumption and dividends are endogenously determined), allow the representative agent to more successfully implement her consumption smoothing objective (i.e. allowing consumption to react endogenously in order to smooth the volatility in the marginal value of wealth), thus eliminating many positive results obtained from simple endowment economies, in particular such which have been obtained in combination with habit persistence. In fact, Jermann (1998) is only able to preserve a sufficiently large equity premium by reducing physical capital's effectivity as a storage technology by introducing adjustment costs to investment (and thus making the supply of physical capital inelastic) in addition to incorporating habit persistence in consumption. Similarly, Boldrin et al. (2001) also combine habit formation with real rigidities in the productive sector, by adding a capital-goods production sector with decreasing returns (leading to an inelastic supply of capital that way) and disallowing labour to react to current-period shocks.

One problem which is shared by both of the aforementioned production-economy based explanations of the equity premium (and indeed in general with other models using habit persistence in consumption), is that the increased volatility in the marginal value of wealth, which stems from the non-seperable nature of such utility specifications, also raises the volatility of real interest rates to implausibly high levels and as a result also alters the behaviour of aggregate quantities in non-trivial ways. [7] . Tallarini (2000) modifies an environment similar to the production general equilibrium model studied

[7] Within an *endowment* economy framework, Campbell and Cochrane (1999) have recently proposed a solution to the interest rate volatility problem, by specifying a nonlinear habit utility function, in which the "intertemporal substitution" effect - which is the culprit for implausibly high variation in interest rates - is just offset by a "precautionary savings" effect.

by Jermann to include Epstein and Zin non-expected utility. Although he cannot account for the equity premium, he improves on the risk-free rate puzzle and the market price of risk (or equivalently, the Sharpe ratio). But the main result of his paper is to show that there is a real possibility to modify simple general equilibrium models such as to improve asset pricing predictions, leaving aggregate quantity dynamics *practically unaltered*, something which could not be said for Jermann (1998) and Boldrin et al. (2001) [8].

The present paper follows the tradition of the above-mentioned literature on asset pricing within fully specified production economies, embodied by Jermann (1998); Boldrin et al. (2001); Tallarini (2000), whose focus is primarily on "second-order" *risk-induced* arguments related to undiversifiable systematic risk, and McCallum and Goodfriend (2007); Canzoneri and Diba (2005); Canzoneri et al. (2008), whose focus is instead on "first-order" certainty-equivalent arguments typically related to implicit liquidity returns of short-term debt. Building on previous work by Gillman and Kejak (2008) and Benk et al. (2005), this paper develops, calibrates and dynamically analyses a monetary general equilibrium model, which is essentially of cash-in-advance type, but is modified by the addition of a de-centralised micro-founded banking sector, whose credit production specification is motivated by the financial intermediation literature (Clark, 1984; Hancock, 1985).

The banking sector acts as a financial intermediary, using labour and deposits to produce a credit service which can be used in conjunction with money to pay for the consumption good. Further, the bank is also holding government debt of all maturities, which are translated one-for-one into equivalent bond-backed saving deposits of various maturity held in turn by the representative household. Closely resembling an argument related to the liquidity-providing role of short-term government debt developed by Bansal and Coleman (1996), credit is assumed to be produced subject to a collateral requirement, meaning that a *share* of the economy-wide supply of short-term (nominally) riskless nominal debt is retained on the banking sector's balance sheet in order to back up the aggregate amount of credit produced.

In contrast to Bansal and Coleman, the banking sector's payout on it's collateral is re-distributed back to the household in form of a dividend payment equal in value to the total cost of using credit. *Instead*, the total payout [9] on the credit-backing collateral *share* of economy-wide short-term debt (and thus the total payout on the equivalent share of the one-period saving deposit) is *replaced* by the total revenue from credit production *minus* the banking

[8] However, (Cochrane, 2007, p. 297) argues against the possibility of ever obtaining a pure "separation theorem" of quantity and price dynamics.

[9] A discussion of the "total payout" on (a share) of debt is possible in this model, as short-term debt is assumed, as in Bansal and Coleman (1996) to be in net *positive* supply.

wage bill, which upon normalising by credit (or equivalently by the share of economy-wide short-term debt equal in value to credit) implies a return on that share equal to the price (or average revenue) of credit minus the average product (or average cost) paid out to the consumer in form of the credit-normalised banking wage bill. The lower than usual residual return on the collateral share of debt which equals the per-credit normalised revenue share of deposits in credit production constitutes a no-arbitrage equilibrium vis-a-vis the higher (nominal) return on capital represented by the CCAPM rate as defined in Canzoneri and Diba (2005), as the household is compensated in form of the banking wage bill equal to that difference.

The model presented here thus offers a micro-founded financial intermediation approach, driving a cost-wedge derived from liquidity (credit) production between the CCAPM and the money market rate, which is in contrast to Canzoneri and Diba (2005), who obtain a similar return wedge by placing (a function of) bonds in an ad-hoc way directly into the cash-in-advance constraint, and McCallum and Goodfriend (2007), by specifying a loan-management function, in which bonds are more effective as collateral vis-a-vis physical capital, obtaining the interest rate wedge that way. Also, McCallum and Goodfriend (2007) cannot account for variations in velocity, since in their model loans and the monetary base (i.e. two distinct "high-powered" exchange bases) are lumped together through the financial accelerator framework (see Bernanke et al., 1999) to give a measure for broad money entering the cash-in-advance constraint as the only effective means of exchange. In contrast, the model presented here is capable of exhibiting endogenous variation in velocity, as a unique exchange equilibrium between money and produced credit is obtained through diminishing returns to labour only in the credit sector and the resulting intersection between the cost of holding money and a convex upward-sloping marginal cost schedule in credit production (see Gillman and Kejak, 2008).

Through the key mechanism described above, the model is qualitatively capable of simultaneously explaining the low risk-free rate, the high equity premium and a term structure which is on average upward-sloping and convex in shape. Novel asset pricing results are obtained by distorting *deterministic mean* returns directly in a certainty-equivalent framework sense through endogenous cost-driven effects of the banking sector, rather than through the usual *risk-induced* variations of ex-ante returns, related to the undiversifiability of systematic risk. However, as I will discuss further below, "first-order" certainty-equivalent "market-driven" asset price distortions (such as those in Bansal and Coleman (1996), Canzoneri and Diba (2005) and Canzoneri et al. (2008) and the model presented here) can potentially be combined trivially with second-order risk-induced asset price distortions stemming from undiversifiable systematic risk, as long as the former distort returns (or prices) of traded assets such as to be *visibly* different in equilibrium from the usual CCAPM rates in ways which are directly compensated (or hedged) elsewhere, leading to distortions which do not constitute undiversifiable systematic risk but are completely hedged *by construction* [10] .

The model improves on Bansal and Coleman (1996) along two dimensions. First, the model's results are more transparently driven by the distortive effects of a micro-founded banking sector, instead of appealing to an essentially ad-hoc transactions-cost function (McCallum, 1983). Secondly, combined with the usual expectations hypothesis of the term structure present in such models, the term structure results do no require multi-period bonds to provide liquidity services and - directly related to this last point - the model is capable of explaining a term structure which is steeper at the short-end than the long-end. Further, simulations reveal a positive correlation between the nominal rate and velocity and a negative correlation between the ex-post real risk-free rate and inflation. Finally, in line with recent theoretical (Canzoneri and Diba, 2005; Canzoneri et al., 2007b) and empirical (Canzoneri et al., 2007a) evidence of a systematic link between monetary policy and the spread between a theory-implied CCAPM and the observed ex-post money market rate [11] , the model is capable of linking monetary tightening to a fall in this spread and vice-versa.

In order to permit a direct comparison with Bansal and Coleman (1996), I present theoretical asset pricing results using two different modeling frameworks, one which follows Canzoneri et al. (2008) in which the nominal CCAPM rate of interest is in the usual way *endogenously* determined through the standard Fisher relationship and the money supply process is *exogenously* specified, and Bansal & Coleman's modeling technique, in which the nominal rate of interest is assumed to be an essentially fixed state-contingent government target (or alternatively, exogenously specified process) and the money supply process is, for a given endogenous real rate of interest determined from the model, *endogenously* implied through the Fisher equation. Using the latter approach, I show how the credit-banking model presented here is capable of producing asset pricing results which are functionally equivalent to those in Bansal & Coleman, and in particular how this permits simultaneously lowering the risk-free rate and raising the equity premium.

I show how the cost-driven distortive effects of the banking sector are es-

[10] The model presented here satisfies this condition, as endogenous cost-driven variations of the low risk-free rate are always perfectly hedged by equivalent compensating variations in the banking wage bill. Related to this, Coeurdacier et al. (2007) explain the equity home bias puzzle through the hedging function of the domestic goods production wage bill, when households face re-distributive shocks.

[11] In similar fashion to Canzoneri and Diba (2005), the model explains this spread by providing a *theoretical* framework in which both the CCAPM and the money market rates (and thus their spread) are determined from within a model.

sentially driven by a partitioning of the return on a fraction of short-term debt into a lower than usual risk-free rate and a compensating payout in form of the banking wage bill paid out to the representative household, which constitutes a no-arbitrage position vis-a-vis the standard CCAPM return, as the former two always add up to the latter. Further, I argue that standard uncertainty-induced risk premia results (typically embodied by covariances of the stochastic discount factor and the cash flow of an asset in terms of the consumption good) stemming from the undiversifiability of systematic risk in stochastic environments are preserved, since the only additional uncertainty the representative household faces is the ex-ante ignorance over how much of the return he receives will be in form of his short-term saving deposit and how much residually in form of the banking wage bill. Since the two always add up to the standard CCAPM rates (both real and nominal), this type of return uncertainty is not of systematic risk type and in fact perfectly hedged or insured and thus will not upset standard stochastic ex-ante *effective* returns, once the return in form of the banking wage bill is added back towards the *visibly* lower return on the short-term saving deposit.

The remainder of the paper is structured as follows. Section 2 outlines the setup of the model which is populated by a household, a firm and and a credit-banking sector. Section 3 discusses how the model is capable of altering asset pricing results obtained from standard (monetary) real business cycle models. A direct comparison with Bansal and Coleman (1996) is presented and in particular, the model's *stochastic* asset pricing implications are discussed using Bansal&Coleman's modeling technique. In preparation for dynamic analysis of the first-order approximate solution of the model contained in section 5, section 4 studies the steady state properties of the model in it's entirety, instead of exclusively focusing on asset pricing results. In similar spirit to Canzoneri et al. (2008), the dynamic analysis in the section thereafter focuses on impulse responses as well as correlations obtained from simulations of the model solved for it's recursive law of motion. Section 6 provides a discussion of the results and finally, section 7 concludes.

2 The economic environment

In what follows I am going to write down an essentially standard cash-in-advance real business cycle model (Lucas, 1982; Stokey and Lucas, 1987), which is only modified by adding a further means-of-exchange, credit, which is costly produced by a decentralised financial intermediary (FI) by use of a two factor CRS Cobb-Douglas production function, whose specification is motivated by the financial intermediation literature (see Clark, 1984; Hancock, 1985). Following Gillman and Kejak (2008), deposits are created from the total exchange liquidity used in the model for carrying out consumption both in terms of money and credit, which means that consumption and (real) deposits can be used interchangeably:

$$c_t \equiv d_t \tag{1}$$

Further, in line with Canzoneri and Diba (2005) [12], I will refer to the standard derivations of prices for real and nominal bonds as (where my derivations assume a constant rate of economic growth equal to γ):

$$p_{1,t}^{b,N} = E_t\left[\frac{\beta\lambda_{t+1}}{\gamma\lambda_t\left(1+\pi_{t+1}\right)}\right] = m_{t,N}^{t+1} \tag{2}$$

and

$$p_{1,t}^{b} = E_t\left[\frac{\beta\lambda_{t+1}}{\gamma\lambda_t}\right] = m_{t}^{t+1} \tag{3}$$

as the CCAPM prices of short-term nominal and real bonds, respectively. $m_{t,N}^{t+1}$ and m_t^{t+1}, on the other hand, are the the corresponding one-period nominal and real CCAPM stochastic discount factors from period t to $t+1$. Notice also, that the above expressions can also be expressed in terms of CCAPM *real* returns on nominal and real bonds, where the real return on the real bond is simply the inverse of the real discount factor:

$$1 + r_{1,t} = \left[E_t\left(\frac{\beta\lambda_{t+1}}{\gamma\lambda_t}\right)\right]^{-1} \tag{4}$$

[12] They only define CCAPM *returns*, but returns can of course always alternatively be expressed in terms of prices as well.

and the real-valued return on the nominal bond, given by $1 + i^r_{1,t}$, is defined as:

$$\begin{aligned} 1 + i^r_{1,t} &= (1 + i_{1,t})\, E_t\left[(1 + \pi_{t+1})^{-1}\right] \\ &= (1 + r_{1,t}) - (1 + i_{1,t})\, \frac{Cov\left(\frac{\lambda_t}{\lambda_{t+1}}, (1 + \pi_{t+1})^{-1}\right)}{E_t\left(\frac{\lambda_t}{\lambda_{t+1}}\right)} \end{aligned} \tag{5}$$

where the covariance term reflects the inflation risk-premium driving a wedge between the pure Fisher equation relationship (see Giovannini and Labadie, 1991). Notice that it may sometimes be convenient in analytical computations to set this covariance between the expected marginal value of wealth and inflation equal to zero, so that for given inflation expectations, modeling of nominal rates of returns can be approximated by modelling real counterparts [13] . Refering to the above *standard* concepts in this explicit fashion is necessary, as the present model will provide an alternative definition for the equivalent risk-free real and nominal price for bonds, given by:

$$\tilde{p}^b_{1,t} = \tilde{m}^{t+1}_t \tag{6}$$

and

$$\tilde{p}^{b,N}_{1,t} = \tilde{m}^{t+1}_{t,N} \tag{7}$$

respectively, which will generally differ from the standard definitions given above. The derivation of these distorted short-term real and nominal discount factors (and their corresponding rates) and analysis of their endogenous behaviour with respect to various factors, such as monetary policy, is the key contribution of this paper. Further, in order to aid derivations of the CCAPM (or purely intertemporal) rates, the household's budget constraint will also contain net balances of one-period nominal *virtual* bonds, which are not thought to be traded in reality and correspond to the usual nominal bonds in standard cash-in-advance models in which the absence of an FI eliminates distortions to the analogous risk-free payout on the corresponding short-term saving deposit. The net balances of these virtual nominal short-term bonds as they appear on the consumer's budget constraint are given by:

$$\frac{(1 + i_{t-1})}{(1 + \pi_t)} b^*_{t-1} - b^*_t \tag{8}$$

In similar fashion to Bansal and Coleman (1996), I assume that the financial intermediary needs to retain a share of the short-term government debt equal in value to credit as collateral on it's balance sheet. Instead of paying out

[13] see Gibbons and Ramaswamy (1993) for an application of this assumption in the Cox-Ingersoll-Ross model of the term structure.

the usual return on that share (i.e. the net nominal CCAPM rate), the FI redistributes the earnings on it's collateral back to the representative household in form of a dividend equal in value to the original cost of purchasing the credit service. However, as I will show, the representative household still obtains the same *effective* return on the short-term saving deposit, as it would in an undistorted cash-in-advance model. However, since part of this return is redistributed back in form of the banking wage bill, the *visibly* obtained residual return on the short-term saving deposit will reflect this cost and thus be lower.

It is this arrangement which leads to the result of a lower than usual risk-free rate (and a convex upward-sloping average term structure), which I will discuss in more detail further below. The model is set up such as to allow only trade in nominally-denominated government debt, however shadow prices and returns of inflation-indexed bonds can of course be derived in the usual way. Before specifying the various sectors characterising the economic environment, I will first discuss how short-term debt is modeled to be in positive net supply, in terms of a government-targeted debt ratio, similar to Bansal and Coleman (1996) and Canzoneri et al. (2008) [14].

2.1 Modeling the supply of short-term debt

The results derived below require short-term government debt to be modeled in positive net supply, as a share of the short-term government bond will affect asset pricing results in the economy. Here, I will briefly discuss my strategy of doing so, which employs the idea of a proportional supply of short-term debt, relative to deposits. Therefore, I define the variable:

$$\tilde{\eta}_{t-1} = \frac{\tilde{b}_{1,t-1}/\left(1+\pi_t\right)}{d_t} \tag{9}$$

to be the *pre-transfer* beginning-of-period proportional amount of debt, relative to real deposits, before any fiscal government adjustment has taken place and $\tilde{b}_{1,t-1}$ as the corresponding *pre-transfer* pre-determined beginning-of-period amount of short-term (one-period) government nominal debt (in terms of beginning-of-period prices). As will be shown, the model's results are derived from some notion of how a share of this proportional debt will be retained by the FI and issued instead as a dividend payment on retained collateral. Since credit in the model is an endogenously determined variable, the share of retained government debt is also endogenously determined. In order to keep the analysis tractable and the intuition simple, my strategy will be to keep the supply of proportional debt fixed in all time periods.

[14] The former specify an exogenous process for the value of total bond issue, the latter also work with debt ratios.

Notice though that debt is pre-determined at the beginning of the period $t-1$, but deposits (or consumption) and inflation are endogenously determined from within the model at the end of the period t. In order to keep the proportional supply of debt fixed in all time periods, the following timing convention will hold. After the revelation of all shocks and the resolution of uncertainty, the government can move first and, knowing the full structure of the economy (and thus the outcome of the competitive equilibrium prices and quantities), and thus also the level of end-of-period deposits and the level of inflation, uses fiscal transfers to adjust the level of pre-determined debt such as to perfectly obtain a fixed debt-to-consumption ratio. After that all other agents in the economy move and the competitive equilibrium is obtained. This timing convention amounts to the following specification of the proportional supply of short-term government debt:

$$\eta_{t-1} = \bar{\eta} = \frac{b_{1,t-1}/\left(1+\pi_t\right)}{d_t} \tag{10}$$

where $b_{1,t-1}$ is the corresponding beginning-of-period amount of short-term debt *after* the fiscal adjustment to keep in line with the endogenously determined level of deposits and inflation has taken place[15] . This simplifying assumption allows me to focus only on the way a *share* of this fixed proportional supply of debt influences relevant results in the model. In particular, as I will show below, the share of consumption paid for in credit - where the absolute level of credit is given by f_t and the corresponding share by f_t^* - is a well-defined (production) function bounded between $[0,1]$ and is defined as:

$$f_t^* = \frac{f_t}{d_t} \tag{11}$$

Therefore, as long as the government incurs sufficient short-term proportional (relative to deposits) liabilities relative to the proportional (relative to deposits as well) production of credit[16] , i.e

$$\bar{\eta} = \frac{b_{1,t-1}/\left(1+\pi_t\right)}{d_t} > \frac{f_t}{d_t} \tag{12}$$

for fixed $\bar{\eta}$ it is then always possible to define a share s_t^b, which as long as the above fiscal liability condition holds is bounded between $[0,1]$ and defines the proportional production of credit relative to the proportional amount of short-term debt. Because the proportionality factor is given by deposits for both supply of debt and production of credit, s_t^b defines the proportion of

[15] The main idea here is to adjust the absolute size of debt before agents trade, so that after they willingly hold the debt, they still incur the end-of-period erosion through inflation.

[16] Canzoneri et al. (2008) also require sufficient debt to be issued relative to demand in order for an equilibrium to be attained, see footnote 11 in their paper.

credit relative to debt directly, i.e.

$$s_t^b = \frac{f_t^*}{\eta} = \frac{f_t/d_t}{\frac{b_{1,t-1}}{1+\pi_t}/d_t} = \frac{f_t}{b_{1,t-1}/\left(1+\pi_t\right)} \tag{13}$$

Notice that s_t^b represents the share of short-term debt which is retained by the financial intermediary as collateral for credit, and I may sometimes wish to refer to it as the *debt utilisation rate* due to credit production or simply the banking sector's debt utilisation rate. Next, I will discuss the optimisation problem of the financial intermediary, which acts on behalf of the representative household and thus discounts current and future items on it's balance sheet using the household's stochastic discount rate.

2.2 The financial intermediary

The financial intermediary acts in a decentralised fashion as a producer of the credit exchange service demanded by the representative household and is also assumed to be profit maximising, sharing the economy-wide discount factor, given by $\frac{\beta^k \lambda_{t+k}}{\lambda_t}$. It produces credit using a CRS technology in labour and deposits created by the household, which is given by:

$$f_t = e^{v_t} A_f \left(\kappa_{t-1} n_{f,t}\right)^{\rho} \left(d_t\right)^{1-\rho} \tag{14}$$

where κ_t is a labour-augmenting exogenously specified parameter evolving according to:

$$\kappa_t = \gamma \kappa_{t-1} \tag{15}$$

thus determining the exogenously specified growth rate of the economy. Notice that it will often be convenient to express the credit-production function as a deposit- (or consumption-) normalised equivalent credit-share production function, which exhibits decreasing returns in deposit-normalised augmented labour and is given by:

$$f_t^* = \frac{f_t}{d_t} = e^{v_t} A_f \left(\frac{\kappa_{t-1} n_{f,t}}{d_t}\right)^{\rho} \tag{16}$$

Notice that this specification - assuming appropriately parametrised values for ρ, implies a convex marginal cost schedule in credit production (see Benk et al., 2005; Gillman and Kejak, 2008), which given a certain price of credit (which will turn out to be the net nominal CCAPM rate) leads to a unique exchange equilibrium between money and credit. Further, in a stochastic environment with shocks to credit productivity (and thus to credit production's marginal cost schedule) and shocks leading to variation in the (net) nominal CCAPM rate of interest, the economy will exhibit variation in (money-consumption) velocity, which will be positively correlated with the nominal CCAPM rate (or the price of credit). The FI is assumed to be the conduit for all liquidity supplied to the consumer. Besides providing the produced credit, the FI is also assumed to carry out optimal portfolio decisions on behalf and as instructed by the representative household. This means that the FI holds beginning-of-period money balances and receives instructions over how much much of current wealth be used to acquire end-of-period money balances, i.e.

$$\frac{m_{t-1}}{(1+\pi_t)} - m_t \tag{17}$$

where m_t represent end-of-period t real money balances. Government taxes or transfers are not modeled explicitly. However, given the government's endogenously determined fiscal policy to hit the fixed debt-to-deposit ratio and the corresponding interest payment obligations, appropriate helicopter-money lump-sum taxation can always be chosen independently in state-contingent fashion such as to implement a steady state money growth rate Θ^* with some random component embodied by the shock parameter u_t. The stochastic nominal money growth rate is thus given by

$$\Theta_t = (M_t/M_{t-1} - 1) = (\Theta^* + e^{u_t} - 1) \tag{18}$$

where Θ_t is the growth rate of money and Θ^* is it's stationary counterpart. Since the economy is growing at the steady state growth rate γ, in order to obtain a particular steady state target level of *inflation*, the monetary authority has to set the growth rate of money above the exogenous economic level of growth by that inflation target, so in steady state:

$$1 + \pi = \frac{1+\Theta^*}{\gamma} \tag{19}$$

which for a positive steady state target inflation rate $(1+\pi) > 1$ implies setting $1 + \Theta^* > \gamma$. The money balances on the bank's balance sheet are part of it's liabilities, as the representative household (frictionlessly) sources money balances from ATM machines with equivalent electronic balances (which are in turn linked to non-interest paying current account balances appearing on the representative household's budget constraint). These ATM electronic balances are sourced from current accounts (D_t^c) and therefore appear on the bank's balance sheet as follows:

$$-\frac{d_{t-1}^c}{(1+\pi_t)} + d_t^c \tag{20}$$

where the bank's liquidity restriction given by $m_t = d_t^c \quad \forall t$ implies:

$$\frac{m_{t-1}}{(1+\pi_t)} - m_t - \frac{d_{t-1}^c}{(1+\pi_t)} + d_t^c = 0 \tag{21}$$

where d_t^c are current-period current account balances in terms of the consumption good, from which withdrawals from ATM machines are sourced. Moreover, The FI is also the holder of nominal government debt balances of all maturity, which as I will show will translate into one-for-one nominal balances of equivalent saving accounts, which will be part of the representative household's budget constraint. The receipts of government nominal debt balances net of new purchases will therefore appear in the FI's balance sheet as follows:

$$\frac{1+i_{1,t-1}}{(1+\pi_t)} b_{1,t-1} + \sum_{j=2}^{n} \frac{1+i_{j,t-1}}{(1+i_{j-1,t})(1+\pi_t)} b_{j,t-1} - b_{1,t} - \sum_{j=2}^{n} b_{j,t} \tag{22}$$

Notice that the FI willingly holds all government debt on behalf of the representative household at prices implied by the (nominal) stochastic discount factor consistent with the one from the representative household. Instead of viewing the FI as the portfolio optimiser acting on behalf of the representative agent one may also view the FI as the channel through which the government "floats" it's debt structure, thus commissioning the FI to convert bond holdings into equivalent saving deposits held in turn by the representative household. From this perspective, and as a specific example only focusing on the one-period nominal short-term debt (which is modeled in net positive supply), the FI has an obligation to the monetary authority (or the government), to float *all of the supply* and to pay the interest as implied by the equilibrium nominal stochastic discount factor. This perspective will be relevant in further discussions related to the derivation of the risk-free (nominal and real) rate. Finally, closely resembling a liquidity argument originally developed by Bansal and Coleman (1996), credit balances, given by f_t have to be backed up by a retained share of the short-term government debt equal to that value on it's balance sheet, which will serve as collateral for credit and will be defined as Ω_t^c. Since the collateral has to be retained by the FI, the net CCAPM return on this collateral multiplied by the amount of collateral (equal to credit) is re-distributed back to the representative household as a dividend payment from the financial intermediary. This implies:

$$f_t = s_t^b \frac{b_{1,t-1}}{(1+\pi_t)} \equiv \Omega_t^c \tag{23}$$

where Ω_t^c is the collateral. This implies for the re-distributed dividend Π_t^f:

$$\Pi_t^f = \Omega_t^c i_{t-1} = \Omega_t^c \left(\frac{\mu_t}{\lambda_t}\right) = f_t \left(\frac{\mu_t}{\lambda_t}\right) = f_t p_t^f \tag{24}$$

Where the last equality emphasises the fact that the cost of credit is re-distributed back to the household in lump-sum fashion in form of a dividend payment from the FI. Notice also that equation (23) can alternatively be expressed in terms of proportions (relative to deposits) and then solved for the share variable, s_t^b:

$$s_t^b = \frac{f_t/d_t}{\frac{b_{1,t-1}}{(1+\pi_t)}/d_t} = \frac{f_t^*}{\bar{\eta}} \tag{25}$$

which emphasises the endogenous determination of this share and how this affects the (proportional) share of short-term government debt which is retained by the FI on it's balance sheet and whose payout is finally re-distributed in terms dividend payment. Of course the discussion thus far then begs the question: If the FI has the obligation to pay interest on *all* of the amount of short-term debt as commissioned by the government (and thus also the equivalent short-term saving deposit held by the representative agent), but retains some share of this short-term bond as collateral for credit production whose value (i.e. collateral time the net CCAPM return) is re-distributed as a dividend payment, how is it going to fulfil this obligation? It can only do so by paying out the revenue from credit production instead (which is price times quantity, i.e. the net nominal CCAPM rate times the amount of credit equal to the retained debt-share as collateral, or $i_{t-1}f_t = i_{t-1}s_t^b\frac{b_{1,t-1}}{(1+\pi_t)}$), whose amount exactly equals that share of short-term government debt which is retained as collateral. However, part of this revenue is paid out directly as the banking wage bill, which upon normalising by $f_t = s_t^b\frac{b_{1,t-1}}{(1+\pi_t)}$ to convert it into a return, equals the average product of banking labour time[17]:

$$\rho\left(\frac{\mu_t}{\lambda_t}\right) = \rho i_{t-1} = \frac{w_t n_{f,t}}{s_t^b\frac{b_{1,t-1}}{(1+\pi_t)}} \tag{26}$$

Thus implying a proportional banking time cost defined over the entire supply of short term debt given by[18]:

$$s_t^b\rho\left(\frac{\mu_t}{\lambda_t}\right) = s_t^b\rho i_{t-1} = \frac{w_t n_{f,t}}{\frac{b_{1,t-1}}{(1+\pi_t)}} \tag{27}$$

Since the return on the retained share of short-term debt (and the equivalent share of the short-term saving deposit) is partially paid out in terms of the average product of banking labour time, it follows that the residual *visible* earned rate of return on the short-term saving deposit itself, net of that banking labour cost, has to be lower and, given a fixed deposit-proportional supply of short-term nominal debt $\bar{\eta} = \frac{b_{1,t-1}}{(1+\pi_t)}/d_t$, that endogenous variation in the share of credit used in purchasing consumption $f_t^* = f_t/d_t$ affecting s_t^b,

[17] To better understand the validity of this expression, recall that $f_t = s_t^b\frac{b_{1,t-1}}{(1+\pi_t)}$. Therefore the above can also be written as $i_{t-1}\rho f_t = w_t n_{f,t}$ which results from Cobb-Douglas specification of credit production.

[18] Notice that in my derivations I typically abstract from including the stationary growth rate κ and assume that all growing variables have already been normalised by κ and thus converted to stationary equivalents.

will cause variations in this cost distortion. The FI's balance sheet solvency restriction that assets equal liabilities is given by:

$$P_t f_t + M_t = P_t d_t \tag{28}$$

The balance sheet liquidity constraint is that money withdrawn by the consumer is covered by deposits:

$$P_t d_t \geq M_t^h \tag{29}$$

which would hold with strict equality if no credit were produced. The FI's objective is to maximise it's discounted stream of current and future profits:

$$\begin{aligned}
\max_{n_{f,t}} E_0 \sum_{k=0}^{\infty} \frac{\beta \lambda_{t+k}}{\lambda_t} \Bigg\{ \\
& p_{t+k}^f f_{t+k} - w_{t+k} n_{f,t+k} \\
& + (1 + i_{1,t+k-1}) \left(1 - s_{t+k}^b\right) \frac{b_{1,t+k-1}}{(1 + \pi_{t+k})} + \sum_{j=2}^{n} \frac{1 + i_{j,t+k-1}}{(1 + i_{j-1,t+k})(1 + \pi_{t+k})} b_{j,t+k-1} \\
& - b_{1,t+k} - \sum_{j=2}^{n} b_{j,t+k} + \Omega_{t+k}^c i_{t+k-1} - \Pi_{t+k}^f \\
& - \frac{1 + \tilde{i}_{1,t+k-1}}{(1 + \pi_{t+k})} d_{1,t+k-1}^s - \sum_{j=2}^{n} \frac{1 + \tilde{i}_{j,t+k-1}}{\left(1 + \tilde{i}_{j-1,t+k}\right)(1 + \pi_{t+k})} d_{j,t+k-1}^s \\
& + d_{1,t+k}^s + \sum_{j=2}^{n} d_{j,t+k}^s \\
& + \frac{m_{t+k-1}}{(1 + \pi_{t+k})} - m_{t+k} \\
& - \frac{d_{t+k-1}^c}{(1 + \pi_{t+k})} + d_{t+k}^c \Bigg\}
\end{aligned} \tag{30}$$

Notice therefore that the earnings on the retained share of debt as collateral equal in amount to credit are given by:

$$\left(\frac{\mu_t}{\lambda_t}\right) f_t = i_{t-1} f_t = s_t^b \frac{b_{1,t-1}}{(1 + \pi_t)} i_{t-1} \equiv \Omega_t i_{t-1} = \Pi_t^f \tag{31}$$

The return on the share of debt which is retained is then simply replaced by the revenue from credit production, which upon normalising by that share of debt (or alternatively by credit, since they are the same in amount), results in a partitioned payout in form of the banking wage bill and a residually visibly lower return on that share of the short-term saving deposits, i.e.:

$$\frac{MP_{n_{f,t}} \times n_{f,t}}{f_t} = \frac{w_t n_{f,t}}{f_t} = \frac{w_t n_{f,t}}{s_{1,t-1}^b \frac{b_{1,t-1}}{(1+\pi_t)}} = p_t^f \rho = i_{t-1} \rho \tag{32}$$

where $MP_{n_{f,t}}$ is the marginal product of banking labour which upon multiplication with the amount of banking labour, represents the return in form of the banking wage bill and

$$\frac{MP_{d,t} \times d_t}{f_t} = \frac{MP_{d,t} \times d_t}{s^b_{1,t-1} \frac{b_{1,t-1}}{(1+\pi_t)}} = p^f_t \left(1-\rho\right) = i_{t-1} \left(1-\rho\right) \tag{33}$$

represents the residual payout on the short-term saving deposit, which relates to the revenue creation from deposits in credit production. $MP_{d,t}$ is the marginal product of deposits in credit production and $MP_{d,t} \times d_t$ therefore the total revenue share of credit production due to deposits. This makes clear how the share of the short-term saving deposit thus commands a return which equals the net nominal CCAPM rate (or the price of credit) *minus* the average cost paid out in form of the banking wage bill. Therefore, the model predicts a money market rate paid out to the representative household's short-term saving deposit which equals the usual CCAPM pure intertemporal rate, *minus* the salary the household takes home from his activity as a banker, whose business it is to produce an exchange credit service evading the exchange cost.

2.3 The household

The representative household derives utility in standard fashion from a momentary utility function in consumption and leisure:

$$U_t = U\left(c_t, l_t\right) \tag{34}$$

where later on, I typically may want to consider a specific parametrisation which is additively separable and logarithmic in both consumption and leisure. An important reason for doing so is to emphasise that significant asset price distortions can be obtained as in Canzoneri and Diba (2005) and Canzoneri et al. (2008) in terms of "first-order" market-driven effects instead of the usual "second-order" risk-induced arguments which typically rest on assumption of high risk aversion and/or non-standard utility function specifications. Throughout the paper I will therefore assume the following specification for utility:

$$U_t = \log c_t + \Psi \log l_t \tag{35}$$

The household's only non-financial endowment is labour time, which she can supply to both the goods producing firm producing the consumption good and the financial intermediary producing exchange credit, or partially use up by taking leisure. This endowment is normalised to one and therefore translates into the following constraint:

$$1 - l_t = n_{g,t} + n_{f,t} \tag{36}$$

where $n_{f,t}$ is the amount of time spend in producing the credit exchange service and $n_{g,t}$ is the amount of time spend working in the goods-producing sector, both of which are remunerated by paying the household the equilibrium wage rate. Deciding on the optimal level of consumption, the household needs to obey an exchange condition in form of a cash-in-advance constraint, which is modified to allow not only money balances to provide the required liquidity services, but also a costly credit exchange service. The total amount of nominal liquidity translates into an equivalent nominal value of deposits held with the FI. Expenditure will be sourced from the total deposits, either by withdrawal of the household's cash balances from an ATM machine, which are in turn connected to corresponding current accounts, given by D_t^c or residually by using costly credit exchange services, where the money (or, residually, credit) velocity measures will crucially also depend on the price of credit. The exchange constraint is thus given by:

$$P_t d_t \equiv D_t^c + P_t f_t \geq P_t c_t \tag{37}$$

or, by dividing through by the current price level, defining $d_t^c = D_t^c / P_t$ and with a positive CCAPM nominal rate resulting strict equality we have:

$$d_t \equiv d_t^c + f_t = c_t \tag{38}$$

Using the definition of the per unit of deposits credit production function, f_t^*, which is the inverse of credit-deposit velocity (or alternatively, the inverse of credit-consumption velocity), enables me to re-write the exchange constraint in implied money demand form, as a function of the inverse of deposit-credit velocity and total deposits (being identically equal to consumption):

$$d_t^c = (1 - f_t^*)\, d_t \equiv (1 - f_t^*)\, c_t \tag{39}$$

On choosing the optimal level of consumption, the household creates (real-valued) deposits with a financial intermediary, which are then taken as a given input factor to producing credit.

$$d_t \equiv c_t \tag{40}$$

Finally, as the credit exchange service is produced by a decentralised financial intermediary, the price of this credit service is explicitly spelled out (see Gillman and Kejak, 2008), and in a unique no-arbitrage exchange equilibrium between money and credit, has to be equal to the cost of using money in carrying out transactions, which is the usual CCAPM net nominal market interest rate, as defined in Canzoneri and Diba (2005). The household's budget constraint is thus given by:

$$
\begin{aligned}
& w_t\left(n_{g,t}+n_{f,t}\right)+\Pi_t^f+\frac{d_{t-1}^c}{\left(1+\pi_t\right)} \\
& +\frac{\left(1+i_{1,t-1}\right)}{\left(1+\pi_t\right)} b_{1,t-1}^*+\sum_{j=2}^n \frac{1+i_{j,t-1}}{\left(1+i_{j-1,t}\right)\left(1+\pi_t\right)} b_{j,t-1}^* \\
& +\frac{\left(1+\tilde{i}_{1,t-1}\right)}{\left(1+\pi_t\right)} d_{1,t-1}^s+\sum_{j=2}^n \frac{1+\tilde{i}_{j,t-1}}{\left(1+\tilde{i}_{j-1,t}\right)\left(1+\pi_t\right)} d_{j,t-1}^s \\
& \geq c_t+p_t^f f_t+\sum_{j=1}^n b_{1,t+j}^*+\sum_{j=1}^n d_{1,t+j}^s
\end{aligned}
\tag{41}
$$

where in particular, $D_{1,t}^s$ equals the amount of dollars held in a nominal short-term debt (or saving) deposit with the financial intermediary (which the financial intermediary in turn backs up one-for-one by an equal amount of nominal government bonds), earning the household the current period nominal risk-free interest rate of $1+\tilde{i}$, which, as will be shown, is different from the usual nominal CCAPM interest rate, $1+i_t$, due to the way short-term debt is partially used as credit-backing collateral. Notice that $d_t^c = D_t^c/P_t$ represents the corresponding level of the real-valued saving deposit. As in Canzoneri and Diba (2005), I have included a virtual nominal bond, given by $B_{1,t-1}^*$, in order to aid the derivation of the standard CCAPM nominal return from the household's side[19]. As they do, I do not assume that such bonds are actually held or traded by the household directly in this economy (although they *will* be traded or held by the financial intermediary). As mentioned above, the household needs to pay a price for using the credit exchange service in conjunction with money balances, which is given by $p_t^f = \mu_t/\lambda_t = i_{t-1}$. Subject to her budget and exchange constraint, the representative household maximises her life-time utility over an infinite horizon by choosing an optimal recursive policy function in order to maximise:

$$
V\left(m_{t-1}, s_t\right)=\max_{c_t, m_t} E_0\left\{U\left(c_t, l_t\right)+\beta V'\left(m_t, s_{t+1}\right)\right\} \tag{42}
$$

where s_t is some vector of structural shocks, whose exogenously specified law of motion will be specified further below.

2.4 The goods-producing sector

The discussion of the goods-producing firm is standard. The goods-producing firm is maximising the present discounted value of current and future dividend

[19] One may think of these virtual nominal bond holdings shadowing the equivalent nominal government bond holdings of the financial intermediary.

streams, whereby it only has to optimally decide on labour demand. The production technology of the firm is given by a standard CRS production function, which is typically assumed to be of Cobb-Douglas specification (or in absence of physical capital, just linear in labour):

$$y_t = e^{z_t} A_g F(\kappa_{t-1} n_{g,t}) = e^{z_t} A_g \kappa_{t-1} n_{g,t} \tag{43}$$

where A_g is some stationary total productivity factor and κ_t is the same labour-augmenting exogenously specified technological progress specified in credit production, governing the steady state trending growth path of the economy. The firm's objective is maximised using a discount factor equivalent to that of the representative household and is given by:

$$\max_{n_{g,t}} \sum_{k=0}^{\infty} \frac{\beta^k \lambda_{t+k}}{\lambda_t} \left\{ y_{t+k} - w_{t+k} n_{g,t+k} \right\} \tag{44}$$

where λ_t is the current period multiplier on the household's budget constraint. Optimising with respect to goods labour leads to the usual (after de-trending) condition of optimality equal to:

$$w_t = e^{z_t} A_g \tag{45}$$

2.5 Equilibrium, Government Financing constraint, Shocks

After netting out financial asset positions on the one hand, and the price of credit times credit minus the re-distributed dividend payment on collateral from the financial intermediary, on the other, we can write the social resource constraint as follows:

$$c_t = n_{g,t} w_t = y_t \tag{46}$$

Also, as already discussed above, the government implements a steady state growth rate of the money supply equal to Θ^* which also contains a random component. Further, the government is assumed not to engage in any Ponzi-game regarding the management of it's debt. The government financing constraint is given by:

$$M_t + V_t - M_{t-1} = (1 + i_{t-1}) B_{1,t-1} - B_{1,t} \tag{47}$$

Notice that since the proportional amount of nominal debt is adjusted at the beginning of the period such as to implement a constant real-valued debt-to-consumption ratio $\bar{\eta}$, the level of debt varies in state-dependent fashion, and so do the debt interest payment obligations of the government. However, given this circumstance, the government can always vary V_t independently in state-dependent fashion such as to implement any desirable money growth rate with some random component, given by Θ_t. All shocks behave according to some log-normal autoregressive process of order one. The vector of shocks is given by $s_t = [z_t, u_t, v_t]'$, where the shocks are goods productivity, money growth rate and credit productivity, respectively, whose law of motion can be summarised in VAR form as:

$$s_{t+1} = \Phi s_t + \epsilon_{t+1} \tag{48}$$

where Φ is a 3x3 matrix with the autocorrelation parameters specified along the diagonal of Φ and $\epsilon \sim (0, \Omega)$.

3 Asset Pricing in the Credit-Banking Model

This section is going to describe how the distortive effects of the banking sector (embodied by the average cost in terms of the banking wage bill) will affect asset pricing results in the model. The key intuition underlying the derived asset pricing formulae (and in particular the low risk-free rate) is that the usual return on a *share* of short-term debt equal to the banking sector's debt-utilisation rate (or collateral requirement relative to debt supply) is instead paid out as a dividend. The household's return on the equivalent share of it's short-term saving deposit is instead equal to (or replaced by) the price of credit minus the average cost of producing credit, which is being paid out in form of the banking wage bill. First, I am going to demonstrate how, using Bansal and Coleman's modeling technique, - they *exogenously* specify a nominal interest rate target (see Coleman, 1996; Bansal and Coleman, 1996) - which allows for simple closed-form derivations of *stochastic* asset pricing results, their and my results regarding the risk-free rate and the equity premium are functionally equivalent. Following this, I am going to follow Canzoneri and Diba (2005) and show how using the usual modeling strategy applied to standard monetary general equilibrium models (in which money supply is exogenously modelled and the nominal rate endogenously determined), the banking sector's distortive effects creates a wedge between the CCAPM and the risk-free rate (as defined by Canzoneri and Diba) and how this, through the usual expectations hypothesis of the term structure of interest rates, produces an upward-sloping term structure, which is steeper at the short-end.

3.1 A comparison with Bansal and Coleman

Bansal and Coleman's theoretical as well as numerical results obtained from simulations of their estimated model are based on a modelling technique, which essentially amounts to holding *fixed* (or alternatively, *exogenously* specifying the law of motion of) the *nominal* rate of interest. In order to preserve a convincing general equilibrium framework, they then proceed by assuming that, given a *fixed* nominal interest rate target, the monetary authority *endogenously* delivers a state-contingent money supply growth rate, such as to produce an (expected) state-contingent inflation rate which is such as not to be in violation with the usual Fisher equation derived from within the model. Therefore in Bansal and Coleman, the nominal rate is *exogenously*, the real rate in the usual way endogenously and the money supply also *endogenously* determined such as to satisfy the Fisher equation[20]. This is in contrast with dynamic treatments of monetary general equilibrium models such as standard cash-in-advance models described in textbooks such as Walsh (2003), in which typically the real rate is endogenously determined from within the model, the (expected) inflation rate is essentially driven by the exogenously specified money supply growth rate and the nominal rate is also endogenously obtained from the Fisher relationship between the real rate and expected inflation. As will be clear, Bansal and Coleman's modeling strategy regarding the interplay between the real, nominal and inflation rate coupled with a simplified assumption about the distribution of shocks allows closed-form solutions for the risk-free rate and equity premium under uncertainty. Notice that Bansal and Coleman introduce a role for money (as well as other means of exchange, such as credit and checkable deposits) through an ad-hoc transaction cost function, which extends the transaction cost literature (Baumol, 1952; Tobin, 1956; Barro, 1976; McCallum, 1983). They also show how their results (regarding velocity), which are driven by the *technology* specification of this function (and thus the marginal rate of transformation between cash and credit goods), closely resemble results obtained from Stokey & Lucas' cash-credit model, in which results are instead driven by the *preference* specification of the utility function over the cash and credit goods (and thus the marginal rate of substitution between cash and credit goods). Velocity and asset pricing results in the model presented here are also technology-driven in the sense of being dependent on the specification of the credit-production function and the resulting stable money-credit equilibrium driven by the convex marginal cost schedule. Bansal and Coleman's key result regarding the risk-free rate stems from equation (22) in their paper, which I reproduce here:

$$\begin{aligned}\frac{1-\xi_2(pc)+\xi_3(pc)}{1-\xi_2(pc)}\frac{u_1(c)}{1+\xi_1(pc)}\left[1-\xi_2(pc)\right]q \\ =\beta E_s\left[\frac{u_1(c')}{1+\xi_1(p'c')}\frac{1-\xi_2(p'c')}{\Pi'}\right]\end{aligned} \tag{49}$$

[20] This technique was first employed in Coleman (1996) to theoretically explore reverse causation from output to money, hence the necessity for a framework in which money is endogenous.

where the ξ_i are derivatives of the transaction cost function with respect to different means of exchange. What is most important in this expression is the term:

$$\frac{1-\xi_2(pc)+\xi_3(pc)}{1-\xi_2(pc)} \tag{50}$$

which is smaller than one as long as bond-backed checkable deposits are used in purchasing the consumption good, where $\xi_3(pc)<0$ is the marginal product in the "production" of transactions services due to bond-backed checkable deposits and pc is the consumption velocity of cash. Notice that, similar to Stokey and Lucas's cash-credit model (Stokey and Lucas, 1987), in which velocity is determined by equating the nominal interest rate with the marginal rate of subsitution derived from the preference specification of the cash and credit goods, velocity in Bansal & Coleman's model is also pinned down in current and future periods, once the nominal rate of interest is assumed to be held *fixed* at some target level. Using this modelling strategy and by assuming that consumption growth $\varrho'=\frac{c'}{c}$ is identically and independently distributed, that the (gross) inflation rate is fixed at some state-contingent target $\bar{\Pi}$ (implying an expected inflation rate which does not violate the Fisher relationship), and that utility is of CRRA type and given by $U(c)=\frac{c^{1-\tau}}{1-\tau}$, Bansal and Coleman's equation (22) can be simplified to give:

$$\frac{1-\xi_2(pc)+\xi_3(pc)}{1-\xi_2(pc)}q=\frac{\beta}{\bar{\Pi}}E\left[\varrho^{-\tau}\right] \tag{51}$$

implying a *real* risk-free rate of interest equal to:

$$\frac{1}{\beta E\left[\varrho^{-\tau}\right]}\frac{1-\xi_2(pc)+\xi_3(pc)}{1-\xi_2(pc)} \tag{52}$$

which, as long as short-term debt is providing liquidity service, embodied by the term $\xi_3<0$, implies a real risk-free rate which is lower than the standard rate, given by $1/\beta E\left[\varrho^{-\tau}\right]$. Notice that the model presented in this paper permits an equivalent representation of the risk-free rate, but instead of relying on an argument based on the *marginal* product in (the production of) transactions costs due to short-term debt (which backs up checkable deposits), here it is the *average* cost paid out in form of the banking wage bill which drives down the risk-free rate. Also, in contrast to Bansal and Coleman, in the model presented, measures of relative supply of short-term debt (given by the debt-to-deposits ratio η), and the credit demand-linked (and thus inflation-dependent) debt utilisation rate $s^b_{1,t}$, which essentially represents the banking sectors collateral demand backing up the produced credit service, matter. In general therefore, for a fixed relative supply, inflation-induced increases in the use of credit (and thus increases in money velocity as less money is used), lead to an increase of the debt utilisation rate $s^b_{1,t}$, which will also affect asset price results. The derivation of the nominal risk free rate in the model is therefore as follows:

$$1+\tilde{i}_t = E_t\left\{1+\left[s^b_{1,t+1}(1-\rho)+\left(1-s^b_{1,t+1}\right)\right]i_t\right\} \tag{53}$$

which shows that the one-period nominal risk-free rate consists of (an expectation over) an endogenously moving average, in which the share of short-term debt (or the equivalent share of the short-term saving deposit) backing up credit, given by $s^b_{1,t+1}$ commands a rate of return which is net of the average cost paid out in terms of the banking wage bill[21] , whereas the residual share of economy-wide debt, which does not serve as collateral, given by $\left(1-s^b_{1,t+1}\right)$ commands the usual undistorted net nominal CCAPM rate. To simplify notation, let me define:

$$\Upsilon^a_t \equiv s^b_{1,t}\rho\left(\frac{\mu_t}{\lambda_t}\right) \equiv s^b_{1,t}\rho p^f_t \tag{54}$$

and

$$\Upsilon^b_t \equiv \frac{1}{1+s^b_{1,t}\rho\left(\frac{\mu_t}{\lambda_t}\right)} \equiv \frac{1}{1+s^b_{1,t}\rho p^f_t} \tag{55}$$

Notice then, since $i_t = p^f_t$ is of the order of a (quarterly) net rate and that $0 < s^b_{1,t} < 1$ and $0 < \rho < 1$, thus making $s^b_{1,t}\rho p^f_t$ of the order of a (quarterly) net rate, it follows that, for some variable of the order of a gross (quarterly) rate $1+i_t$:

$$(1+i_t) - \Upsilon^a_t \approx (1+i_t)\,\Upsilon^b_t \tag{56}$$

Using the above definitions, the gross nominal risk-free rate can thus be written as:

$$\begin{aligned} 1+\tilde{i}_t & \\ &= E_t\left\{1+\left(1-\rho s^b_{1,t+1}\right)i_t\right\} \\ &= (1+i_t) - E_t\left(s^b_{1,t+1}\rho p^f_{t+1}\right) \\ &= (1+i_t) - E_t\left(\Upsilon^a_{t+1}\right) \\ &\approx (1+i_t)\,E_t\left(\Upsilon^b_{t+1}\right) \end{aligned} \tag{57}$$

[21] To understand that this is an (expected) average cost, recall that in the model $E_t(\mu_{t+1}/\lambda_{t+1}) = i_{t+1} = E_t\left(p^f_{t+1}\right)$ meaning that the net nominal CCAPM rate is also equal to the expected price, and thus expected total average product, of credit.

Within Bansal and Coleman's modeling framework, within which I wish to place my results in order to allow for a direct comparison, both the nominal CCAPM and the inflation rate are assumed to be fixed state-contingent targets, which in turn implies (using some further restrictions placed on the model economy presented here, which I will discuss below) that current and future Υ^a and Υ^b are fixed and thus known in advance. Following Bansal and Coleman, I therefore get:

$$\begin{aligned}1+\tilde{i}_t &= \\ &= \frac{1+i}{1+\rho s_1^b p^f} \\ &= \frac{\gamma\left(1+\bar{\pi}\right)}{\beta} E_t\left[\frac{\lambda_t}{\lambda_{t+1}}\right]\left[\frac{1}{1+\rho s_1^b p^f}\right] \\ &= \frac{\gamma\left(1+\bar{\pi}\right)}{\beta} E_t\left[\frac{\lambda_t}{\lambda_{t+1}}\right]\left[\Upsilon^b\right] \qquad (58)\end{aligned}$$

implying in particular a real risk-free rate given by:

$$\frac{\gamma}{\beta} E_t\left[\frac{\lambda_t}{\lambda_{t+1}}\right]\left[\Upsilon^b\right] \qquad (59)$$

which after substituting out for the marginal utility of wealth in terms of marginal utility and the usual cash-in-advance relative cost of consumption (the gross nominal interest rate), gives:

$$\frac{\gamma}{\beta} E_t\left[\frac{\left(1+i\right)U_c\left(c,l\right)}{\left(1+i'\right)U_c\left(c',l'\right)}\right]\left[\Upsilon^b\right] \qquad (60)$$

If I now appeal to the same modeling strategy of Bansal & Coleman and assume fixed nominal interest rate and inflation rate targets delivered by the monetary authority, specify consumption growth $\varrho' = \frac{c_{t+1}}{c_t}$ to be identically and independently distributed and utility to be of CRRA type in consumption (and additively separable in consumption and leisure), such as:

$$U\left(c,l\right) = \frac{c_t^{1-\tau}-1}{1-\tau} + A\log\left(l_t\right) \qquad (61)$$

the above formulae reduces to:

$$\frac{\gamma}{\beta^* E\left[\varrho^{-\tau}\right]}\left[\Upsilon^b\right] \qquad (62)$$

where $\beta^* = \beta\gamma^{1-\tau}$ is the growth-adjusted impatience factor (see Jermann, 1998). For an economy with no growth (as was assumed by Bansal and Coleman), the above reduces to:

$$\frac{1}{\beta E\left[\varrho^{-\tau}\right]}\left[\Upsilon^b\right] \qquad (63)$$

where $\Upsilon^b < 1$, which shows the functional equivalence between their and my results for the low risk-free rate, when viewed from their modeling assumption. Notice that in order to fully emulate an environment equivalent to Bansal and Coleman's (which is, in contrast to the model here, a simple exchange economy), assuming a fixed nominal CCAPM rate of interest and fixed rate of inflation is not quite sufficient, as this will not restrict the credit-banking model enough to make next period's debt utilisation rate s_1^b (and therefore next period's value of Υ^b) known with perfect certainty. This share's future value will only be known with certainty, as long as future credit production is known with certainty. Fixing the nominal CCAPM rate already accomplishes fixing the *price* of credit, but two more conditions have to be imposed in order to make next period's level of credit production known with certainty. First, it has to be assumed that there are no shocks to credit production (which would shift the marginal cost schedule of credit production) and secondly, that labour between the credit and goods sector is completely immobile. This last requirement is necessary, since shocks to the goods production sector alone would lead to labour movement between sectors in order to equate the marginal products and thus the wage rate. As I have shown above, the debt utilisation rate can also be equivalently expressed as a function of the credit-banking cost (in terms of the banking wage bill) relative to the economy-wide short term debt in the economy (which, relative to deposits or consumption is always fixed). Therefore, the above additional restrictions essentially amount to holding next period's banking wage bill fixed, thus making next-period's debt-utilisation rate constant. Finally, to replicate Bansal and Coleman shock distribution for the discount factor (which they use to illustrate their analytic results, in numerical exercises shock processes are autoregressive of order one), the productivity shock on the goods production function would have to be identically and independently distributed.

Although the risk-free rate thus obtained is functionally equivalent to Bansal & Coleman's low risk-free rate, the intuition is of a different sort. Whereas in Bansal & Coleman the term $\frac{1-\xi_2(pc)+\xi_3(pc)}{1-\xi_2(pc)}$ is responsible for driving down the risk-free rate, which is directly related to the production specification of the ad-hoc transaction cost function, in the model presented here it is the partitioning (into the banking wage bill and residually the return on the saving deposit) of the payout received on the collateral share of short-term debt which leads to a lower return. Notice that both here and in Bansal & Coleman, variations in inflation, velocity and asset prices are linked together. Also, in contrast to Stokey and Lucas (1987) whose velocity results depend on preference specifications, both here and in Bansal & Coleman velocity results are essentially technology-driven, with the difference that here the technology is transparently modelled in terms of a micro-founded theory of financial intermediation, whereas Bansal & Coleman's specification is based on an ad-hoc transaction cost function. Notice that the obtained "reduced" risk-free rate can exist in the absence of arbitrage, as the household is compensated in return in form of the banking wage bill. The degree to which this banking time cost can affect the risk-free rate depends on the relative production of credit to deposits on the one hand, and the relative supply of debt to deposits, which has been assumed to be a government target which is held fixed in each period. As in Bansal and Coleman (1996), whenever such considerations of proportional supply of debt matter, the standard fashion of modeling debt to exist in net zero supply have to abandoned and replaced with some notion of specific supply-modeling. Notice also that in the model presented here, the banking wage bill generated by credit production drives down the return on the short-term saving deposit. This production function was given by:

$$f_t = e^{v_t} A_f \left(\kappa_{t-1} n_{f,t}\right)^{\rho} c_t^{1-\rho} \qquad \text{where} \qquad c_t \equiv d_t \tag{64}$$

Bansal & Coleman's specification of their *transactions cost* function, which may be thought of as the transactions cost literature's exchange cost analogue to the exchange cost (given by the net nominal interest rate) in the cash-in-advance literature, is given by:

$$\Psi\left(c, c_1, c_2\right) = \bar{\Psi} c^{\alpha} l^{1-\alpha} \qquad \text{where} \qquad l = \left(c_1^{\omega} + \kappa c_2^{\omega}\right)^{1/\omega} \tag{65}$$

Comparing the two functions and noticing that c_2 is the fraction of consumption goods paid for with the *bond-backed* checkable deposits in Bansal & Coleman, this demonstrates the close analogy between their approach and the approach taken here. In their specification, bond holdings also yield a transactions service return in terms of marginally adding to the total value of such transaction services, which could equivalently expressed in terms of labour foregone [22] (which they hint at, by defining the transaction cost function in that particular way, using $l = \left(c_1^{\omega} + \kappa c_2^{\omega}\right)^{1/\omega}$). In contrast, in the model presented here, the transaction cost share due to the use of credit is *equal* to the value of some share of beginning-of-period short-term bond holdings, whose return paid out in terms of a dividend from it's use as collateral, and instead replaced by the total return of credit production, which is partitioned into the banking wage bill and *residually* into the return on the short-term saving deposit. Also, in contrast to Bansal & Coleman, here proportional supply and the credit-induced banking sector's demand of debt matter, as they define the bond utilisation rate, $s_{1,t}^{b}$, whereas in their approach, the total value of the bond issue is always equal to checkable deposits.

After demonstrating, within their specific modelling strategy, how the risk-free rate is affected by the liquidity role of short-term debt, Bansal & Coleman proceed by including in standard fashion a risky asset into their model and show

[22] On page 1140, Bansal & Coleman state that purchasing the consumption good incurs a transaction cost in terms of foregone output or in terms of *time devoted* to the *production* of consumption goods. In the credit-banking model, the credit share of consumption is also the outcome of a productive process, involving labour.

how the same term affecting the risk-free rate, $\frac{1-\xi_2(pc)+\xi_3(pc)}{1-\xi_2(pc)}$, also affects the equity premium and the non-parametric HJ bound (Hansen and Jagannathan, 1991). Because of the functional equivalence between their and my results, I refer the reader to the straightforward derivation provided in their paper and state for completeness the equity premium and the modified HJ bound [23] . Assuming an identically and independently distributed growth rate of dividends given by χ, the equity premium (defined as the ratio of the expected gross return to equities over the gross return to bonds) is thus modified to be defined as follows:

$$\frac{E\left[\gamma^{-\varrho}\right]E\left[\chi\right]}{E\left[\gamma^{-\varrho}\chi\right]}\left[1+\rho s_1^b p^f\right] \tag{66}$$

which depends in the usual way on the covariance of the growth in marginal utility with the growth in dividends, but rises proportionately with the term $\left[1+\rho s_1^b p^f\right]$. Therefore, the same term which lowers the risk-free, also raises the equity premium. Similarly, Bansal & Coleman show how the HJ-bound given by:

$$\frac{\sigma\left(k\right)}{E\left(k\right)} \geq \frac{E\left(\zeta\right)}{\sigma\left(\zeta\right)} \tag{67}$$

where k is equal to the intertemporal marginal rate of substitution and ζ equals any excess return, is easier to satisfy (for equity) once the liquidity-providing role of the short-term bond is taken into account. In the model presented here, the same argument can be made by taking into account how here the effect of the banking wage bill distorts the risk-free rate. Analogously to Bansal & Coleman, it can thus be shown that, with regard to equities, the HJ bound imposes restrictions on the excess return according to:

$$\zeta_e' = R_e' - \left[1+\rho s_1^b p^f\right](1+i) \tag{68}$$

Therefore, as long as credit is produced and the bond-utilisation rate s_1^b is different from zero, the average excess return $E\left(R_e'\right)$ is smaller than the observed equity premium, given by $R_e' - (1+i)$, by an amount related to the average cost in producing the debt-backed credit service. This feature tends to lower the Sharpe ratio on ζ_e' and therefore makes it easier for the above bound to be satisfied.

[23] Another reason for placing less emphasis on this derivation is the suspicion that B&C's derivation of the high equity premium is an artifact of their peculiar modeling assumption based on reverse causation and endogenous money.

This section is going to explore the implications for stochastic as well as steady state asset pricing results in the credit-banking model. Notice that from here on onwards, I will take the dynamics of the model to be based on the usual interpretation of the Fisher relationship, and not the one employed by Bansal and Coleman (1996) which was motivated by a study of reverse causation from output to money. Also, for the discussion of stochastic asset pricing results, it will be convenient to re-state the problem first in terms of prices and then to use continuous time formulae to convert back to net returns[24]. Further, notice that for a given ex-post realised rate of inflation, the cost-term driving down the nominal risk-free rate would also imply a reduction in the short-term real rate by the same amount. Therefore, in order to abstract from inflation and reduce cluttering of my analytical results with products or sums of log inflation rates, I will only consider shadow prices (and returns) of inflation-indexed bonds, also aiding comparison with previous results from the literature focusing on asset pricing results of real bonds. Of course, having said that, it is important to highlight the fact that no matter whether one considers real or nominal rates of return, the credit-banking related cost-wedge in form of the proportional banking wage bill over total short-term debt is always a function of the net *nominal* CCAPM rate, as the price of credit equals the opportunity cost of money holding money.

3.2.1 The low risk-free rate

The low nominal risk-free rate obtained above implies for a short-term inflation-indexed bond's real return, or the equivalent inflation-indexed short-term saving deposit's return:

$$1+\tilde{r}_{1,t}=(1+r_{1,t})\,E_t\left(\Upsilon^b_{t+1}\right)=\frac{\gamma}{\beta}\frac{E_t\lambda_{t+1}}{\lambda_t}\left[E_t\left(\Upsilon^b_{t+1}\right)\right] \tag{69}$$

The above return derivation of the low real risk-free rate, implies the following for the price of the same financial asset:

$$\tilde{p}^b_{1,t}=(1+\tilde{r}_{1,t})^{-1}=\frac{\beta}{\gamma}\frac{\lambda_t}{E_t\lambda_{t+1}}\left[E_t\left(\Upsilon^b_{t+1}\right)\right]^{-1} \tag{70}$$

Defining $\Lambda_{t+1}=\frac{\lambda_t}{E_t\lambda_t}=\frac{c_{t+1}(1+i_t)}{c_t(1+i_{t-1})}$ and assuming this variable to be log-normally distributed (which it would be, if consumption and nominal interest rates are log-normal, an assumption also used by Bohn (1991)), then using continuous

[24] This technique is also used by den Haan (1995). Cochrane (2005, p. 15) emphasises the interchangeability of price-based and return-based asset price derivations.

time formulae as in den Haan (1995) implies:

$$
\begin{aligned}
\tilde{r}_{1,t} &= -\ln\left(\tilde{p}^{b}_{1,t}\right) \\
&= \left[-\ln\beta^{*} + \ln\left(E_t\Upsilon^{b}_{t+1}\right)\right] - \ln\left[E_t\Lambda_{t+1}\right] - \frac{1}{2}Var\left[\ln\left(\Lambda_{t+1}\right)\right] \\
&= \left[-\ln\beta^{*} - E_t\Upsilon^{a}_{t+1}\right] - \ln\left[E_t\Lambda_{t+1}\right] - \frac{1}{2}Var\left[\ln\left(\Lambda_{t+1}\right)\right] \\
&= \left[\bar{r} - E_t\Upsilon^{a}_{t+1}\right] - \ln\left[E_t\Lambda_{t+1}\right] - \frac{1}{2}Var\left[\ln\left(\Lambda_{t+1}\right)\right] \qquad (71)
\end{aligned}
$$

where $\ln\beta^{*} = \ln\left(\frac{\beta}{\gamma}\right)$. Upon substituting out for the marginal value of wealth, the above expression can be written as:

$$
\begin{aligned}
\tilde{r}_{1,t} = &\left[\bar{r} - E_t\Upsilon^{a}_{t+1}\right] + \left[E_t\Delta c_{t+1} + \Delta i_t\right] \\
&- \frac{1}{2}Var\left(E_t\Delta c_{t+1}\right) - \frac{1}{2}Var\left(\Delta i_t\right) - Cov\left(E_t\Delta c_{t+1}, \Delta i_t\right) \qquad (72)
\end{aligned}
$$

This expressions is equal to the *conditional* value of the risk-free rate as implied by conditional variations in the stochastic discount factor. Observing that the model's first-order conditions have been divided through by κ_{t-1} such as to make all variables stationary, taking unconditional expectations implies $E_{\infty}\Delta c_{t+1} = 0$ and $E_{\infty}\Delta i_t = 0$, and the *unconditional* value of the same expression is equivalent to:

$$
E_{\infty}\tilde{r}_{1,t} = \left[\bar{r} - E_{\infty}\Upsilon^{a}_{t+1}\right] - \frac{1}{2}Var\left[\ln\left(\Lambda_{t+1}\right)\right] \qquad (73)
$$

The above conditional and unconditional expressions for the risk-free rate generalise standard derivations of the this rate in barter economies along two dimensions. First of all, since exchange-in-advance (in form of either credit or money) translates into the usual exchange cost of the net CCAPM nominal rate of interest, the stochastic discount factor given by the marginal rate of substitution in consumption reflects this cost in marginal utilities of consumption in the current and next period. This of course then means that upon expansion of the expectation of the log-normally distributed SDF, the variance of the net CCAPM interest rate and it's covariance with consumption growth matter as well (see Bohn, 1991). The intuition for the negative effect of the volatility of the SDF on the unconditional return of a risk-free bond is well known and relates the the increased demand for savings today when valuation of tomorrow's state is relatively volatile (see Jermann, 1998). Secondly, and more importantly, there is a "first-order" market-driven cost effect in form of the partitioning of the *effective* return of the bond into the residually lower return on the short-term saving deposit, given by $\tilde{r}^{b}_{1,t}$ and the banking wage bill. This effect is embodied by the conditional expectation term:

$$
E_t\Upsilon^{a}_{t+1} = E_t\left(s^{b}_{t+1}\rho p^{f}_{t+1}\right) = E_t\left(s^{b}_{t+1}\rho i_t\right) = \frac{w_{t+1}n_{f,t+1}}{b_{1,t}/E_t\left(1+\pi_{t+1}\right)} \qquad (74)
$$

which is subtracted from the *pure deterministic* rate given by $\bar{r} = -\log(\beta^*)$. Credit-banking asset price distortions do not produce "second-order" risk-induced effects in form of covariances embodying systematic risk. The intuition for this is that although the investor does not know ex-ante how much of his payout on the short-term saving deposit will be subtracted and paid out instead in form of his wage bill he receives in his activity as a banker, the variation of the low-risk free rate due to that effect is perfectly hedged by construction. All that is required to establish the valuation of the deterministic part of his short-term saving deposit's return, is to subtract the expected payout in form of the expected proportional cost given by the future expected banking wage bill (which is directly related to the future expected level of credit production). Of course, ex-post this valuation may turn out to be wrong, say, because of an unexpectedly large level of credit production (implying a higher banking wage bill and an ex-post lower return on the short-term saving deposit), but such unexpected outcomes and the resulting unexpected variation in the deterministic return component of the risk-free rate is perfectly offset by compensating unexpected variations in the ex-post realised banking wage bill. Finally, in line with Canzoneri and Diba (2005), the model still allows the definition of the purely intertemporal rate affecting the consumption Euler equation, which is defined in the usual way as:

$$r_{1,t} = \bar{r} + [E_t \Delta c_{t+1} + \Delta i_t] - \frac{1}{2} Var\left[\ln\left(\Lambda_{t+1}\right)\right] \tag{75}$$

with an unconditional mean of:

$$E_\infty r_{1,t} = \bar{r} - \frac{1}{2} Var\left[\ln\left(\Lambda_{t+1}\right)\right] \tag{76}$$

Of course, the only element distinguishing this intertemporal rate from it's low-risk free return counterpart which the representative household earns on the short-term saving deposit is the cost-distortion term due to credit production.

Notice that we could also think of a risky asset (equity) whose valuation would depend on the uncertain flow of future dividend payments. In the credit-banking model presented here, this dividend could be endogenously determined as the the firm's revenue minus it's wage bill, i.e. $div_t = e^{z_t} y_t - w_t n_{g,t}$ [25] or proxied by simply setting it equal to consumption, $div_t = c_t$. Since utility is assumed to be logarithmic and additively separable, it is then a well-known fact that the price of this risky asset, given by p_t^{eq}, is proportional to the current dividend payout in the following way:

$$p_t^{eq} = \frac{\beta}{1-\beta} div_t \tag{77}$$

[25] This would cause the dividend to be equal to the productivity innovation in the goods sector (see Rouwenhorst, 1995; Jermann, 1998).

Defining the gross real return of this risky asset in the usual way as as the expected future price and dividend payment divided by the current purchase price:

$$r_t^{eq} = \frac{E_t\left(p_{t+1}^{eq} + div_{t+1}\right)}{p_t^{eq}} \tag{78}$$

the model would then also produce a condition of optimality for this asset in the usual way as:

$$1 = E_t\left[\frac{\gamma}{\beta}\Lambda_{t+1} r_t^{eq}\right] \tag{79}$$

which leads to the well-known derivation of an excess return of the risky asset over the *purely intertemporal CCAPM rate* given by:

$$r_t^{eq} - r_{1,t} = -\frac{Cov\left(\Lambda_{t+1}, r_t^{eq}\right)}{E_t\Lambda_{t+1}} \tag{80}$$

This means that the conditional excess return of the risky asset over the *low risk-free* rate on the short-term saving deposit is therefore given by[26]:

$$r_t^{eq} - \tilde{r}_{1,t} = E_t\Upsilon_{t+1}^{a} - \frac{Cov\left(\Lambda_{t+1}, r_t^{eq}\right)}{E_t\Lambda_{t+1}} \tag{81}$$

This result shows how the equity premium in the credit-banking model defined as the excess return of the risky asset over the cost-distorted risk-free rate obtained on the short-term saving deposit is given in the usual way by the risk-adjustment due to systematic risk of receiving a low dividend in times of already low consumption plus the expected payout in form of the future expected banking wage bill. The latter factor will crucially depend on the expectation of future credit production *and* the future expected price of credit, $p_{t+1}^{f} = \mu_{t+1}/\lambda_{t+1}$.Given the stochastic specification of the low risk-free rate and the excess return of a risky asset over this rate, the steady state real risk-free rate ignores risk-induced adjustments between the risky and the purely

[26] The covariance between stock returns and some measure of the marginal value of wealth, like consumption, is negative. Consumption typically rise (lowering marginal valuation) when stock markets rise. Therefore $-Cov\left(\Lambda_{t+1}, r_t^{eq}\right) > 0$

intertemporal rate and is thus given by:

$$\begin{aligned}
1+\tilde{r} =& \frac{\gamma}{\beta}\left[\frac{1}{1+s^b\rho p^f}\right] \\
=& \frac{\gamma}{\beta}\Upsilon^b \\
=& \frac{\gamma}{\beta} - \Upsilon^a \\
=& \frac{\gamma}{\beta} - s^b\rho p^f
\end{aligned} \tag{82}$$

which is different from the usual steady state real rate in standard models given by γ/β, and as long as credit production is positive, *lower* than the standard rate, due to average cost incurred from producing the debt-backed credit. Therefore, the ratio of the real CCAPM over the real risk-free rate (and thus the steady state equity premium) is given by:

$$\frac{1+r}{1+\tilde{r}} = 1 + \rho s^b p^f \tag{83}$$

or

$$r - \tilde{r} = \rho s^b p^f = \frac{w_t n_f}{b_1/(1+\pi)} \tag{84}$$

which again demonstrates the simple intuition that the wedge driven between the real risk-free and the real CCAPM rates is related to the proportional banking time cost in terms of the banking wage bill over the total amount of short-term debt.

3.2.2 The Term Structure of Interest Rates

Restating the representative household's choice of (inflation-indexed) bonds (and undistorted virtual bonds) in her budget constraint in terms of prices rather than returns, i.e.

$$\begin{aligned}
&\ldots + d^s_{1,t-1} + \sum_{j=2}^{n} \tilde{p}^b_{j-1,t} d^s_{j,t-1} - \sum_{j=1}^{n} \tilde{p}^b_{j,t} d^s_{j,t} \ldots \\
&\ldots + b^*_{1,t-1} + \sum_{j=2}^{n} p^b_{j-1,t} b^*_{j,t-1} - \sum_{j=1}^{n} p^b_{j,t} b^*_{j,t} \ldots
\end{aligned} \tag{85}$$

Then, the first-order conditions for subsequent inflation-indexed bonds imply the following formula for the price of the j-th period inflation-indexed bond:

$$
\begin{aligned}
\tilde{p}^b_{j,t} &= \left(\frac{\beta}{\gamma}\right)^j \left[\frac{E_t\lambda_{t+j}}{\lambda_t}\right] \left[E_t\Upsilon^b_{t+j}\right]^{-1} \\
&= \left(\frac{\beta}{\gamma}\right)^{j-1} \left[\frac{\beta}{\gamma}\left(E_t\Upsilon^b_{t+j}\right)^{-1}\right] \left[\frac{E_t\lambda_{t+j}}{\lambda_t}\right]
\end{aligned} \tag{86}
$$

The above expression thus shows how the cost-distortion in form of the banking wage bill only affects the expected tail-end one-period return of any j-period bond. For bonds with shorter maturity, the distortion will have a disproportionately larger effect on the average yield than for bonds with very high maturity. This argument clearly explains the intuition behind the convexity of the term structure in the credit-banking model. Using the same logic as for the one-period real risk-free rate, taking the negative log of the above price for a j-th period bond, the expression can can be written in terms of the net *holding period* return for a j-th period bond:

$$
\begin{aligned}
\tilde{r}_{j,t} &= (j-1)\,\bar{r} + \left(\bar{r} - E_t\Upsilon^a_{t+j}\right) \\
&\quad + \left[E_t\Delta c_{t+j} + E_t\Delta i_{t+j-1}\right] - \frac{1}{2}Var\left[\ln\left(\frac{\lambda_t}{E_t\lambda_{t+j}}\right)\right]
\end{aligned} \tag{87}
$$

by dividing through by j we can find the average yield of a j-period bond:

$$
\begin{aligned}
\hat{\tilde{r}}_{j,t} &= \left(\frac{j-1}{j}\right)\bar{r} + \left(\frac{1}{j}\right)\left(\bar{r} - E_t\Upsilon^a_{t+j}\right) \\
&\quad + \frac{\left[E_t\Delta c_{t+j} + E_t\Delta i_{t+j-1}\right] - \frac{1}{2}Var\left[\ln\left(\frac{\lambda_t}{E_t\lambda_{t+j}}\right)\right]}{j}
\end{aligned} \tag{88}
$$

The above expression for the *conditional* yield of inflation-indexed bonds of various maturity embeds the derivation of the low risk-free rate by setting $j = 1$. Also, the *unconditional* or average yield is given by:

$$
E_\infty\hat{\tilde{r}}_{j,t} = \left(\frac{j-1}{j}\right)\bar{r} + \left(\frac{1}{j}\right)\left(\bar{r} - E_\infty\Upsilon^a_{t+j}\right) - \frac{\frac{1}{2}Var\left[\ln\left(\frac{\lambda_t}{E_t\lambda_{t+j}}\right)\right]}{j} \tag{89}
$$

Notice that the analogous *virtual* term structure related to the undistorted fictitious bonds again ignores the distortive cost term and it's implied conditional yield expression is therefore given by:

$$
\hat{r}_{j,t} = \bar{r} + \frac{\left[E_t\Delta c_{t+j} + E_t\Delta i_{t+j}\right] - \frac{1}{2}Var\left[\ln\left(\frac{\lambda_t}{E_t\lambda_{t+j}}\right)\right]}{j} \tag{90}
$$

and it's unconditional or average yield expression by:

$$E_\infty \hat{r}_{j,t} = \bar{r} - \frac{\frac{1}{2}Var\left[\ln\left(\frac{\lambda_t}{E_t\lambda_{t+j}}\right)\right]}{j} \tag{91}$$

What is important here is that the cost-distorted lower return of the tail end on any j-period bond has a disproportionately larger effect on bonds with smaller maturity than bonds for which j is very large. Indeed for a bond for which $j \to \infty$, the effect of the reduced tail-end return asymptotically disappears, meaning that the steady state average yield for the limiting bond is equal to to the steady state real CCAPM rate:

$$\begin{aligned}
&(1+\hat{r}_j)_{j\to\infty} \\
&= \lim_{j\to\infty}\left\{\left[\frac{\gamma}{\beta}\right]^{\frac{j-1}{j}}\left[\frac{\gamma}{\beta} - E_t\Upsilon^a_{t+j}\right]^{\frac{1}{j}}\right\} \\
&= \frac{\gamma}{\beta}
\end{aligned} \tag{92}$$

Notice however the following drawback implied by this result. Calibrating the CCAPM rate (by choosing a sufficiently low enough impatience factor) such as to match it with the average return obtained on equity as observed in the data, means that one automatically also pins down the average yield of the long-term (limiting) bond to the same average return. However, in the data, the real return on a short-term bond (i.e. the risk free money market rate) is roughly equal to 1%, whereas the premium of a long-term bond above the risk-free rate is typically only equal to 1% (thus earning an average yield of roughly 2%). Mehra and Prescott (2003) raise exactly the same concern, by concluding that "[Bansal & Coleman's] model implies that there should be a significant yield differential between T-bills and long term government bonds that presumably do not have a significant transaction service component". However, in Bansal & Coleman's particular modeling framework, they clearly show how this argument may not be valid, as roughly the same liquidity argument responsible for lowering the usual stochastic risk-free also simultaneously raises the equity premium by proportionately more. But this argument may indeed be an artifact of their particular modeling strategy, in which money supply is endogenously determined. In steady state the above expressions regarding the term structure can be simplified to give:

$$1 + r_j = \left(\frac{\gamma}{\beta}\right)^j \tag{93}$$

which is the undistorted steady state holding period return for the *virtual* term structure implying an undistorted per-period average yield which is just equal to the steady state real interest rate:

$$1 + \hat{r}_j = (1 + r_j)^{1/j} = \frac{\gamma}{\beta} \tag{94}$$

where $1+\hat{r}_j$ represents the j-period real bond's average yield. Including multi-period saving deposits into the household's budget constraint which are backed up by corresponding multi-period government bonds and taking into account the distortive effect of the banking sector on the short-term risk free rate, implies the following steady state average yield on a j-period bond:

$$1 + \hat{r}_j = \left\{ \left[\frac{\gamma}{\beta}\right]^{j-1} \left[\frac{\gamma}{\beta}\left(\Upsilon^b\right)\right] \right\}^{1/j} \tag{95}$$

where $\Upsilon^b < 1$ for as long as credit is produced in the economy.

4 The Steady State

In the following section I am going to describe the steady state levels of endogenous variables, that result after de-trending all growing variables by dividing through by κ_{t-1} and thus obtaining a non-trending stationary equilibrium. The steady state can be summarised by the following set of equations:

$$1 + \pi = (1 + \Theta)/\gamma \tag{96}$$

$$1 + r = \frac{\gamma}{\beta} \tag{97}$$

$$1 + i = \frac{\gamma}{\beta}(1 + \pi) \tag{98}$$

$$p^f = \left(\frac{\mu}{\lambda}\right) = i \tag{99}$$

$$\frac{f}{c} = f^* = A_f^{\frac{1}{1-\rho}} \left(\frac{\rho}{w}\right)^{\frac{\rho}{1-\rho}} i^{\frac{\rho}{1-\rho}} \tag{100}$$

$$\frac{m}{c} = m^* = (1 - f^*) \tag{101}$$

$$MRS_{c,l} = \frac{l}{\Psi c} = \frac{1+i}{w} \tag{102}$$

$$A_g = i\rho\frac{f}{n_f} = w \tag{103}$$

$$1 + \tilde{i} = 1 + \left[\left(1 - s_1^b\rho\right) i\right] \tag{104}$$

$$1 + \tilde{r} = \frac{1 + \tilde{i}}{1 + \pi} \tag{105}$$

$$s_1^b = \frac{f^*}{\bar{\eta}} \tag{106}$$

First of all, equation (96) in the usual way sets the steady state rate of inflation equal to the growth rate of the money supply adjusted for the exogenously specified economy-wide economic growth rate. The steady state real CCAPM rate is just equal to the inverse of the pure impatience factor, adjusted for growth, which is given by equation (97). Then, given some calibrated values of the discount factor β and the exogenously specified growth rate of the economy γ, using the Fisher equation (98), I residually obtain the standard CCAPM nominal rate of interest. Equation (99) shows that the price of using the credit exchange service in equilibrium has to be equal to the cost of otherwise holding money, which is the net CCAPM interest rate. Given this price of credit and the first-order conditions of optimality of the FI with respect to labour, substituting the implied labour factor demand back into the credit production function gives equation (100), which is the steady state value of the inverse of credit-deposit (or credit-consumption) velocity. Residually from this, the inverse of the money-consumption velocity is thus also defined and given by equation (101). As shown by equation (102), the usual exchange cost embodied by the (net) CCAPM nominal rate will thus affect the marginal rate of substitution between consumption and leisure in the usual way. Turning attention to the productive sectors in the economy, perfectly mobile labour between the two sectors results in a condition given by equation (103), which just means that the marginal *revenue* products of labour in each sector have to be equal to one common equilibrium wage rate. Equations (104), (105) and (106) embody the key results of this paper and show how the nominal (and inflation-adjusted real) risk-free rates paid out on the short-term saving deposit held by the representative household are below the usual CCAPM nominal (and real) rates, due to the average cost (or average product) in collateral-backed credit production paid out in form of the banking wage bill, which crucially depends on the debt utilisation rate s_t^b *and* the price of credit, which in turn defines the proportional banking time cost over the total amount of short-term debt available in the economy.

5 Calibration & Dynamic Analysis

In the following section, I am going to motivate and describe the baseline calibration on which steady state and dynamic analyses obtained from the solved model are based. As the mechanics underlying the de-centralised credit-banking model presented here are very similar to Benk et al. (2005), in which credit is directly produced by the household, similar steady state results are obtained. However, reflecting the asset pricing approach taken in this paper, the calibration to follow differs from standard treatments such as in Benk et al. (2005) in the way the usual real steady state interest rate (which in this model is equal to the real CCAPM rate) is chosen. Typically, standard calibration

Table of benchmark calibrated Parameters			
$1+r$=$1.03^{(1/4)}$	real. CCAPM rate	ρ =0.21	credit labour param.
$\frac{f}{c}$ =0.25	credit-to-cons ratio	A_g=1.0	TFP goods
γ =$1.02^{(1/4)}$	g. rate	l =0.7	leisure
n_f =0.0003	credit labour	n_g=0.2992	goods labour
Θ =$1.05^{(1/4)} \times \gamma$	money g.	ϕ_z=0.95	AR goods shock
ϕ_u =0.60	AR money g. shock	ϕ_v=0.95	AR credit shock
η_y =0.40	debt-to-deposit ratio	ϵ_z =0.65	s.d goods shock
ϵ_u =0.01	s.d. moneyg shock	ϵ_v =0.75	s.d credit shock

Table 1
Baseline Calibration

exercises of models with zero growth set the impatience factor $\beta = 0.99$, thus obtaining a *quarterly* real steady state risk-free rate equal to 1% (and thus an annual of 4%). However, the risk-free rate in the model discussed here is going to be below the usual CCAPM rate, where in particular the cost term $\frac{1}{1+s^b \rho p^f}$ affecting the money market rate received on the short-term saving deposit is going to play a key role in this respect. Owing to this latter argument, I calibrate the real CCAPM rate to be a somewhat lower annualised 3%[27] The steady state money supply growth rate is calibrated such as to imply an annual inflation rate of 5%, the economy's steady state exogenous growth rate to equal 2%, implying an annualised nominal CCAPM rate of 8% and an impatience factor $\beta = 0.9975$. The goods sector's steady state total productivity term is set to $A_g = 1.0$ and leisure and *total* labour are in the usual way calibrated to $l = 0.7$ and $n_f + n_g = 0.3$. Then I proceed in similar fashion to Benk et al. (2005) and Gillman and Kejak (2005) and calibrate the degree of diminishing returns in the credit sector $\rho = 0.21$, which is the U.S. time-series estimate obtained for this parameter in Gillman and Otto (2005). Also, similar to Benk et al. (2005), the steady state share of credit used in purchasing the consumption good (i.e. the inverse of credit-consumption velocity) is fixed at a value of $\frac{f}{c} = 0.25$, which is somewhat lower than the chosen 0.3 by the aforementioned authors. My choice is motivated by Canzoneri et al. (2007b), who use a calibrated value for the *debt-to-deposit ratio* in U.S. banking institutions (that is debt which is held by those institutions relative to deposits) equal to 25%. Assuming, as is the case in the model presented here, that the banking sector's holdings of debt is equal in value to the amount of credit produced and that deposits are equal to the level of consumption, the calibrated value

[27] Tallarini (2000) argues in the same way when calibrating the impatience factor in his Epstein-Zin production general equilibrium model, where he also calibrates β so as to imply a theoretical risk-free rate which is closer to the one observed in the data.

for $f^* = \frac{f}{d} = \frac{f}{c} = 0.25$ is obtained. Although Benk et al. (2005) base their calibration of the share of credit used in purchasing consumption on observable long-run velocity of some monetary aggregate, my choice based on the level of intra-bank debt reflecting the collateral requirement assumption of credit in the model, roughly results in the same calibrated value and thus makes this calibration more robust, as both perspectives yield roughly the same value. The calibrated values residually imply a leisure utility preference parameter $\Psi = 2.29$, and steady state banking time share of $n_f = 0.0003$, which is within close range of values obtained by Benk et al. (2005), who obtain a value of 0.00049. Also residually obtained then are a goods sector labour share equal to $n_g = 0.2993$ and a banking sector total productivity term $A_f = 1.05$, where the latter differs from Benk et al. (2005) obtained value of 1.422, which is likely due to their inclusion of physical capital into their model, which is absent here, but also their higher calibrated value for f^*. Based on the chosen baseline calibration, figures (1) and (2) show the theoretical steady state average yield curve, which has been calculated based on the steady state formulae provided in the preceding asset pricing section. The periods are in quarters and the yields represent annualised values. The reduced one-period return, due to the cost-distortion term $\frac{1}{1+s_1^b \rho p_f}$, is perpetuated throughout the entire term structure, thus implying a decreasing effect on yields of higher maturity, as only their tail-end one-period return is affected by this. Notice therefore,

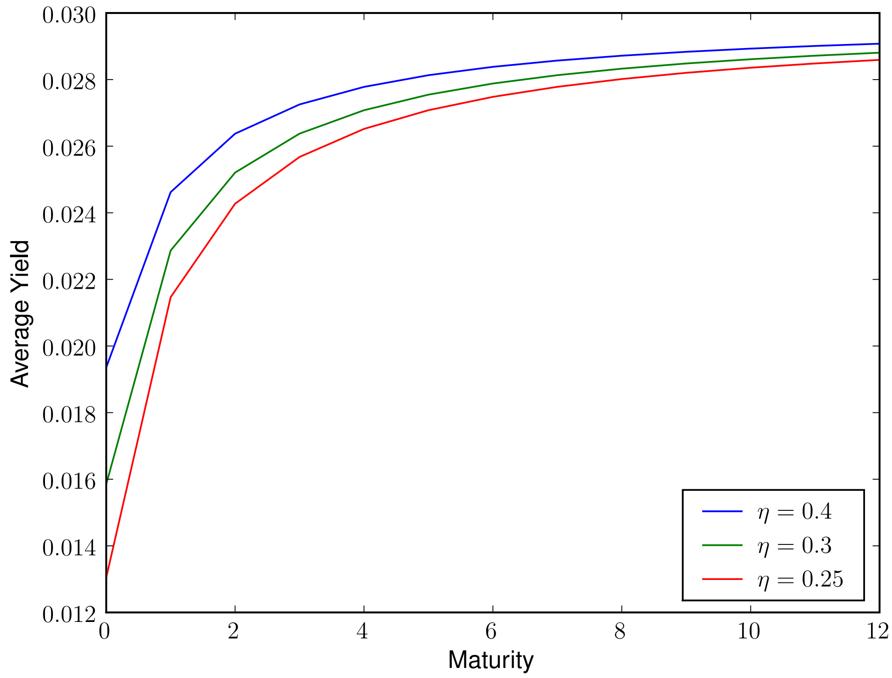

Fig. 1. Theoretical Average Yield Curve (for $\rho = 0.21$)

how the two graphs show how the average yield structure also depends on the calibration of the degree of diminishing returns in credit production, ρ, but also on the calibration of the amount (or supply) of short-term debt relative to consumption (or in this model, deposits). Clearly, as figure (1) demonstrates, increasing the amount of relative short-term debt available in the economy, makes the distortive cost-effect of the banking sector matter relatively less (since the steady state share of credit is calibrated at a fixed level of 0.25). In contrast, increasing the degree of diminishing returns in the credit sector, for a given price of credit, increases the banking wage bill and thus lowers the residual payout received on the short-term saving deposit net of that cost.

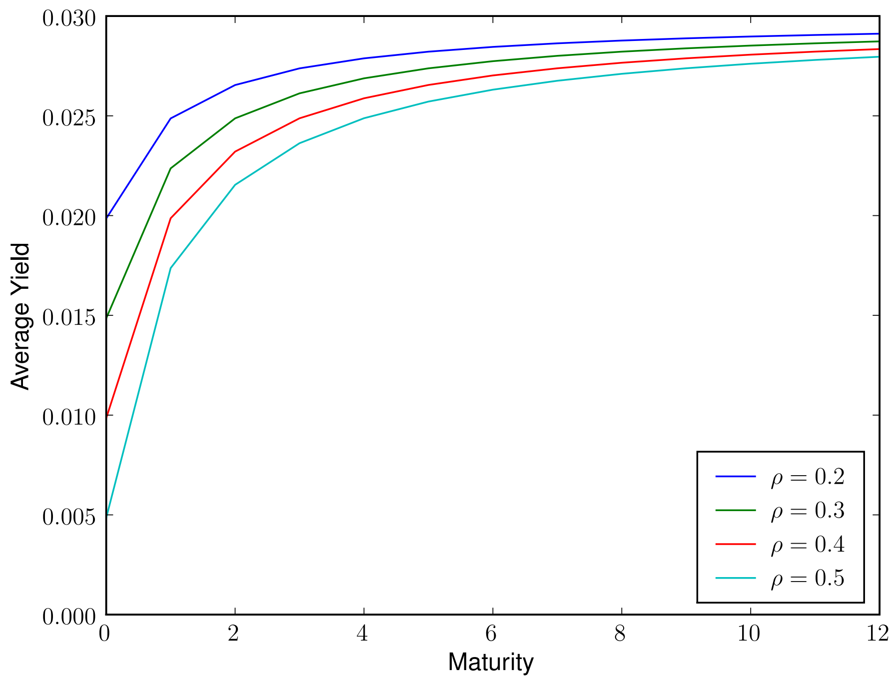

Fig. 2. Theoretical Average Yield Curve (for $\eta = 0.40$)

5.1 Dynamic Analysis

In this section I am going to present a dynamic analysis of the model, based on impulse response graphs as well as calculated contemporaneous correlations obtained from simulations of the solved model. A competitive equilibrium for this economy consists of a set of allocations $\{c_t, l_t, n_{g,t}, n_{f,t}, \pi_t, fv_t, mv_t, k_t, M_t\}_{t=0}^{\infty}$, a set of prices $\{w_t, , icap_t, rcap_t, ib_t, rb_t\}_{t=0}^{\infty}$, exogenous shock processes $\{z_t, u_t, v_t\}$, money supply process and initial condition M_{-1} such that given the prices, shocks and government transfers, the allocations solve the consumers utility maximization problem, solve the firms profit maximisation problem and the goods, labour and money markets clear. By log-linearizing the de-trended non-linear first-order conditions around the steady state I obtain a (singular) linear rational expectations system of equations, where variables will typically be in terms of log-deviations from steady state (but rates will remain in deviations from levels) [28] . De-trending implies expressing all variables in stationary form by first dividing nominal variables through by the price level P_t to express all variables in real terms and secondly by dividing through by κ_{t-1}, the labour augmenting factor driving the exogenously specified growth rate in the economy. Pre-determined endogenous and exogenous state variables are summarised in the vector $\mathbf{X}_t = [\mathbf{z}_t, \mathbf{x}_t]' = [\hat{z}_t, \hat{u}_t, \hat{v}_t, \hat{m}_t]'$ and control variables similarly summarised in the vector $\mathbf{Y}_t = [\hat{c}_t, \hat{l}_t, \hat{n_g}_t, \hat{n_f}_t, \hat{w}_t, \hat{\pi}_t, \hat{icap}_t, \hat{rcap}_t, \hat{ib}_t, \hat{rb}_t, \hat{fv}_t, \hat{mv}_t]'$. I proceed to solve for the first-order accurate solution of the policy function using the Schur decomposition method to find the stable saddle-path of the model (Klein, 2000). The resulting stationary recursive laws of motion are expressible as:

$$\mathbf{X}_t = P\mathbf{X}_{t-1} \tag{107}$$
$$\mathbf{Y}_t = F\mathbf{X}_{t-1} \tag{108}$$

which are used to produce impulse-responses to describe relevant effects and also later on to simulate the model and analyse contemporaneous correlations between variables. The solved system is thus given by:

$$\begin{bmatrix} \hat{z}_t \\ \hat{u}_t \\ \hat{v}_t \\ \hat{m}_t \end{bmatrix} = \begin{bmatrix} 0.95 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.60 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.95 & 0.00 \\ 1.04 & -3.35 & -1.97 & 0.00 \end{bmatrix} \begin{bmatrix} \hat{z}_{t-1} \\ \hat{u}_{t-1} \\ \hat{v}_{t-1} \\ \hat{m}_{t-1} \end{bmatrix} \tag{109}$$

[28] I do not log-linearize explicitly, but actually symbolically differentiate the set of first-order, market clearing conditions and exogenous laws of motion (which are all modified to express variables in logs) w.r.t. to future and current states and controls to obtain the Jacobian of the system, which I evaluate at the (log) steady state. The Jacobian can then be split into matrices A containing partial derivatives w.r.t. future controls and current states, and B containing partial derivatives w.r.t. current controls and past pre-determined states, which can be solved for the recursive laws using the Schur decomposition (see Klein and Gomme, 2008).

and

$$\begin{bmatrix} \hat{c}_t \\ \hat{l}_t \\ \hat{n_g}_t \\ \hat{n_f}_t \\ \hat{w}_t \\ \hat{\pi}_t \\ \hat{icap}_t \\ \hat{rcap}_t \\ \hat{ib}_t \\ \hat{rb}_t \\ \hat{fv}_t \\ \hat{mv}_t \end{bmatrix} = \begin{bmatrix} 0.994 & -0.461 & 0.030 & 0.000 \\ 0.003 & 0.180 & -0.012 & 0.000 \\ -0.006 & -0.461 & 0.038 & 0.000 \\ 0.353 & 40.69 & -1.452 & 0.000 \\ 1.000 & 0.000 & 0.000 & 0.000 \\ -1.033 & 4.264 & 0.195 & 1.000 \\ 0.007 & 0.583 & -0.035 & 0.000 \\ -0.231 & -3.350 & -0.016 & 0.000 \\ 0.005 & 0.417 & -0.027 & 0.000 \\ -0.232 & -0.181 & 0.010 & 0.000 \\ 0.133 & -8.511 & -0.678 & 0.000 \\ -0.039 & 2.821 & 0.225 & 0.000 \end{bmatrix} \begin{bmatrix} \hat{z}_{t-1} \\ \hat{u}_{t-1} \\ \hat{v}_{t-1} \\ \hat{m}_{t-1} \end{bmatrix} \tag{110}$$

The above recursive laws of motion for the endogenous states and control variables are used in the usual way to compute impulse-response graphs and correlations between variables based on the simulated time-series obtained from subjecting the model to goods sector productivity, money growth rate, and credit productivity shocks.

5.2 Impulse Responses

In this section I am going to present the model's behaviour in response to one-off shocks, where I particularly wish to focus on monetary growth rate as well as credit productivity shocks. The impulse responses obtained from money supply growth innovations demonstrate how monetary policy shocks are capable of conditionally increasing the wedge between the nominal CCAPM and nominal risk-free rates, given by:

$$\Upsilon_t^b = \frac{1}{1 + s_t^b \rho p_t^f} \tag{111}$$

but only in as far as such shocks lead to higher expected inflation, thus raising the nominal CCAPM rate and therefore also the price of credit. For money shocks, this also leads to a rise in the share of credit, increasing the banking wage bill overall, due to price and quantity effects. It is well-known from standard cash-in-advance models (see Walsh, 2003), that the price effect (i.e. primarily through changes in expected inflation) is only possible when the exogenous process for the money growth rate shock is modeled with some degree of persistence. White noise money growth shocks, on the other hand, would never change inflation expectations, but only lead to one-off variations in the unexpected component of inflation (i.e. inflation forecast errors), leaving inflation expectations unaltered.

Also, analysis of credit productivity innovations reveals that increases in the share of credit alone do not necessarily lead to an increase of the wedge between the nominal CCAPM and the nominal risk-free rates, since a fall in the price of credit (through a fall in inflation expectations, thus lowering the nominal CCAPM rate) may offset the quantity effect sufficiently enough in order to lead to a conditional fall in the banking wage bill, closing the conditional gap between the nominal CCAPM and the nominal risk-free rate. This case is obtained for the one-off credit productivity innovation.

5.2.1 Monetary Shocks

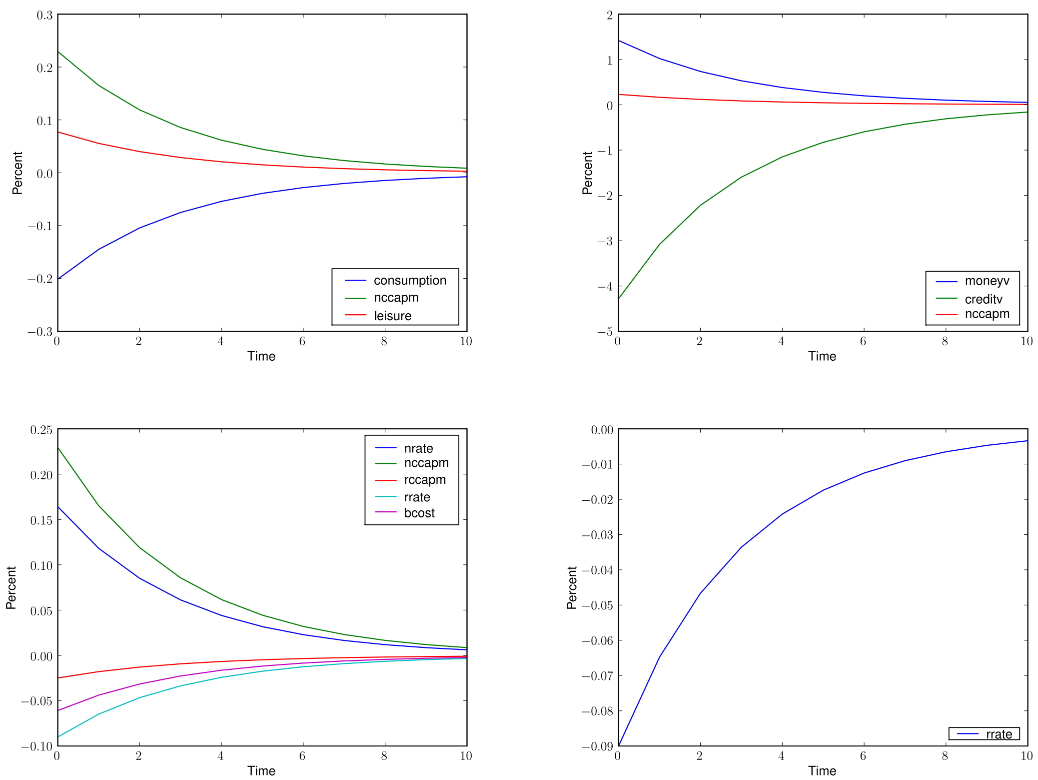

Fig. 3. Response to 1 percent innovation in money growth

When shocked with 1% standard error innovation in the money supply growth rate process, regarding the nominal CCAPM rate and consumption, the model exhibits a behaviour which is equivalent to a standard cash-in-advance model explained in standard textbook treatments, such as Walsh (2003). Increases in the money supply growth rate lead to an increase in (expected) inflation, thus raising the nominal CCAPM rate through the Fisher relationship. This increases the exchange cost of consumption in the usual inflation-tax way (Cooley and Hansen, 1989), thus lowering the level of consumption and leading to a substitution effect towards more leisure. Notice that, although not shown here, although total labour residually has to fall, there is a shift of labour from the goods to the credit sector, thus leading to decrease in goods labour and an increase in banking labour. The top right hand quadrant in figure (3), shows how the increase in the nominal CCAPM rate lowers credit-consumption velocity (defined as $\frac{c}{f}$) and residually increases money-consumption velocity. As the money shock raises the ratio of the current period marginal utility of liquidity services over the marginal utility of wealth, $\frac{\mu_t}{\lambda_t} = p_t^f$, which also equals the current-period price of credit, the convex upward-sloping marginal cost schedule in credit-production thus implies a larger share of the consumption good paid for in credit instead of cash, thus leading to a substitution in means of exchange from money to credit. Using the (first-order approximated) intertemporal Euler equation, one can see that a low current-period marginal value λ_t of wealth coupled with a higher expected marginal value of wealth $E_t \lambda_{t+1}$, implies a low (or fall in the) real CCAPM interest rate[29] :

$$\begin{aligned} &E_t \left[r_{1,t} + E_t \Delta \lambda_{t+1} \right] \approx 0 \Leftrightarrow \\ &E_t \left[r_{1,t} - \Delta c_{t+1} - \Delta i_t \right] \approx 0 \end{aligned} \tag{112}$$

The bottom-right quadrant of figure (3) illustrates how a lower marginal valuation today vis-a-vis a higher expected marginal valuation in future periods, through the dynamic Euler equation implies a modest fall in the *real* CCAPM rate. Although following the shock, expected consumption growth is *positive*, implying a fall in the real CCAPM rate, nominal interest rates are falling making the term $\Delta i_t < 0$. As the latter effect dominates the former, marginal valuation turns out to be low today and expected to rise, thus leading to a fall in the real rate. The bottom-left quadrant illustrates the response of return measures as well as the cost-distortion term $\frac{1}{1+s_{1,t}^b \rho p_t^f}$ (called bcost in the graph), responsible for driving a wedge between the nominal CCAPM and nominal bond rate, thus for a given (expected) rate of inflation, residually implying a lower real return on the bond-backed saving deposit. Notice that since both the current and expected future price of credit is high and the current and future share of credit (or inverse of credit-consumption velocity) is high as well relative to steady state, following the money growth rate shock, this implies a further fall in Υ_t^b, leading to a further decrease of the real risk-free rate paid out on the short-term saving deposit, thus dynamically increasing the equity premium (the gap between the real risk-free and the real CCAPM rates) further beyond the steady-state wedge (recall that this wedge was defined as $\left(1+\tilde{i}_t\right) = (1+i_t) E_t \left(\Upsilon_{t+1}^b\right)$). In particular, this means that the model implies

[29] see Uhlig (1995)

(holding other shocks fixed) that money supply increases (decreases) lead to corresponding dynamic increases (decreases) of the deterministic component of the equity premium, embodied by Υ^b_{t+1}. Such a systematic conditional relationship between the stance of monetary policy and the size of the equity premium[30] has recently been established in a VAR analysis by Canzoneri et al. (2007a). Whereas their analysis defines the behaviour of the equity premium as the conditional behaviour of the difference between a model-implied CCAPM rate and the observed risk-free money market rate, the model presented here explains endogenous variation in this gap theoretically through the distortive (cost-driven) behaviour of a micro-founded banking sector. In summary, a money growth rate shock raises current and expected inflation, translating into increases of the current and expected future price of credit. This leads to an expansion of the credit sector, an increase in money velocity and a residual fall in credit velocity, as the representative household reacts to the increased inflation tax by using more credit instead of money balances, which are now taxed more heavily. As both the price and the share of credit rise, the proportional cost-driven distortive effect on the short-term saving deposit rises, thus increasing the wedge between the nominal CCAPM and the nominal bond rate, for given inflation expectations, implying a fall in the ex-ante *real* risk-free rate. As consumption falls, more leisure is taken and although *total* labour falls, banking time actually increases at the expense of less time spent in the goods production sector.

5.3 Credit Shocks

Focusing first on the top-right hand quadrant of figure (4), which summarises the responses to a 1% innovation in credit productivity, the responses of credit and money velocity are qualitatively similar to those obtained from a money growth rate shock. But whereas the shock to money growth increased the price of credit, thus leading to a higher credit share that way, here increasing the productivity of the credit sector lowers the marginal cost of producing credit for any given level of credit (and for any given price of credit), thus leading to a higher use of credit that way. Where the two figures differ, is shown in the top-left hand quadrant, which shows how the nominal CCAPM rate (and thus price of credit) falls, thus leading to a higher level of consumption and a substitution effect away from labour towards leisure. Notice that, although the bottom-right hand quadrant shows how the real CCAPM rises, the fall in the nominal CCAPM rate is due to a fall in the expected rate of inflation, which

[30] In their empirical analysis, they do not explicitly call this the equity premium, but just an interest rate spread between the CCAPM and money market rate. However, the idea of calling and calibrating this according to the equity premium is entertained in Canzoneri and Diba (2005).

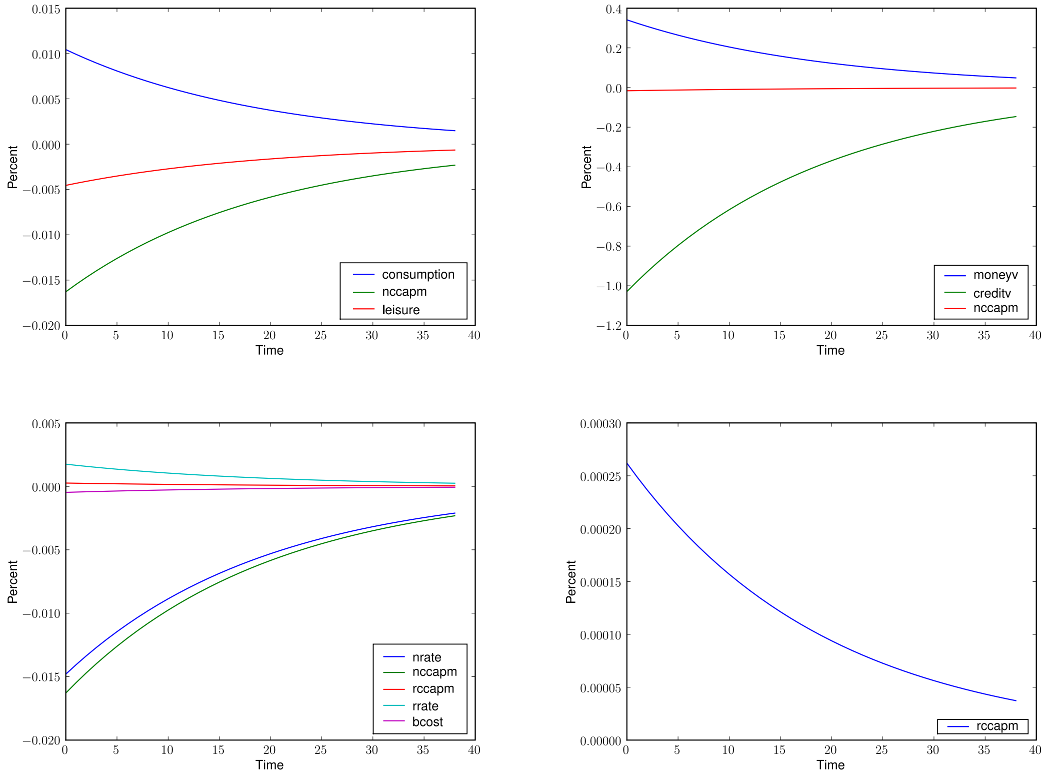

Fig. 4. Response to 1 percent innovation in credit productivity

follows from an initially large spike in current inflation, due to the sharp fall in the demand for money balances. This large spike in inflation after a credit shock, followed by convergence from below it's steady state value (implying a fall in the expected rate of inflation), has also been documented by Benk et al. (2005) in their analysis of a similar credit model. Notice that although leisure increases and thus total labour has to fall residually, the labour spent in the goods sector actually increases, whereas banking time falls. This movement of labour from the banking to the goods sector is primarily due to the falling relative price of credit. Recall that the labour market equilibrium condition between the two sectors was given by:

$$w_t = i_{t-1}\rho\frac{f_t}{n_{f,t}} = \frac{y_t}{n_{g,t}} \tag{113}$$

Therefore, is the fall in the price of credit, given by the net nominal CCAPM rate i_{t-1} is stronger relative to the increase in credit production, then following the shock, the marginal revenue product of labour in the credit sector falls below the one in the goods sector. Therefore, labour will move from the former to the latter sector until the marginal products are equalised at some common wage rate. In spite of the fall in banking time, more credit relative to deposits (consumption) can be produced, due to the boost in credit productivity alone. Regarding return measures, the responses in the risk-free rate paid on the short-term saving deposit are quite different when compared to those obtained from the money growth shock. Notice that although the share of credit used in purchasing the consumption good has increased, typically implying a fall in the term responsible for lowering the risk-free rate below the CCAPM rate, given by $\Upsilon_t^b = \frac{1}{1+s_t^b \rho p_t^f}$, as the top-left hand quadrant shows, the term actually rises (so it's denominator must be falling). This is because the falling price (cost) of credit, p_t^f, more than outweighs the increase in the debt utilisation rate s_t^b through higher credit production, thus leading to an effective increase in this term. This means that following a credit shock, the conditional gap between the CCAPM and risk-free rate paid on the short-term saving deposit actually falls. In summary, a positive shock to credit productivity increases the share of credit, but also lowers the price of credit through the falling nominal CCAPM rate. Due to the falling exchange cost, consumption rises and less leisure is taken. The negative price effect of credit is strong enough to induce a shift of labour from the credit to the goods sector, implying a fall in banking time and an increase in goods production labour. Also, regarding the conditional determination of return measures, the same price effect is strong enough to outweigh the velocity effect on the distortionary cost term, thus for a given expected inflation rate, leading to a temporary increase in the real risk-free rate, and a temporary fall in the steady state gap between the CCAPM and the risk-free rate paid out on the saving deposit.

5.4 Goods Shocks

Following a 1% standard error to the goods sector productivity, the top left-hand quadrant shows a modest increase in the nominal CCAPM rate as well as a quantitatively similarly small substitution effect towards leisure (Consumption therefore rises almost one-for-one with the rise in goods productivity, but is omitted in the graph, in order to better illustrate the modest increases in the other two variables). In spite of the increase in the nominal CCAPM rate, leisure is taken such as to imply a fall in both goods and labour time. As revealed by the top right-hand quadrant, the fall in banking labour time leads to a fall of the share of credit used in consumption and a residual rise in the money share, thus implying a corresponding increase in credit and decrease in money velocity, respectively. The bottom left-hand quadrant illustrates how in spite of the modest increase in the price of credit, the fall in credit production is the dominant effect, implying a conditional rise of the distortionary cost term above it's steady state value, implying a conditional fall in the wedge between the CCAPM and the risk-free rate. Also, a rise in the real CCAPM rate again shows how marginal valuation is again dominated by the change in the nominal CCAPM interest rate, changes in consumption appear to play an insignificant role in this regard. What is not shown in the diagrams, is the behaviour of the inflation rate, which behaves conversely to the credit shock

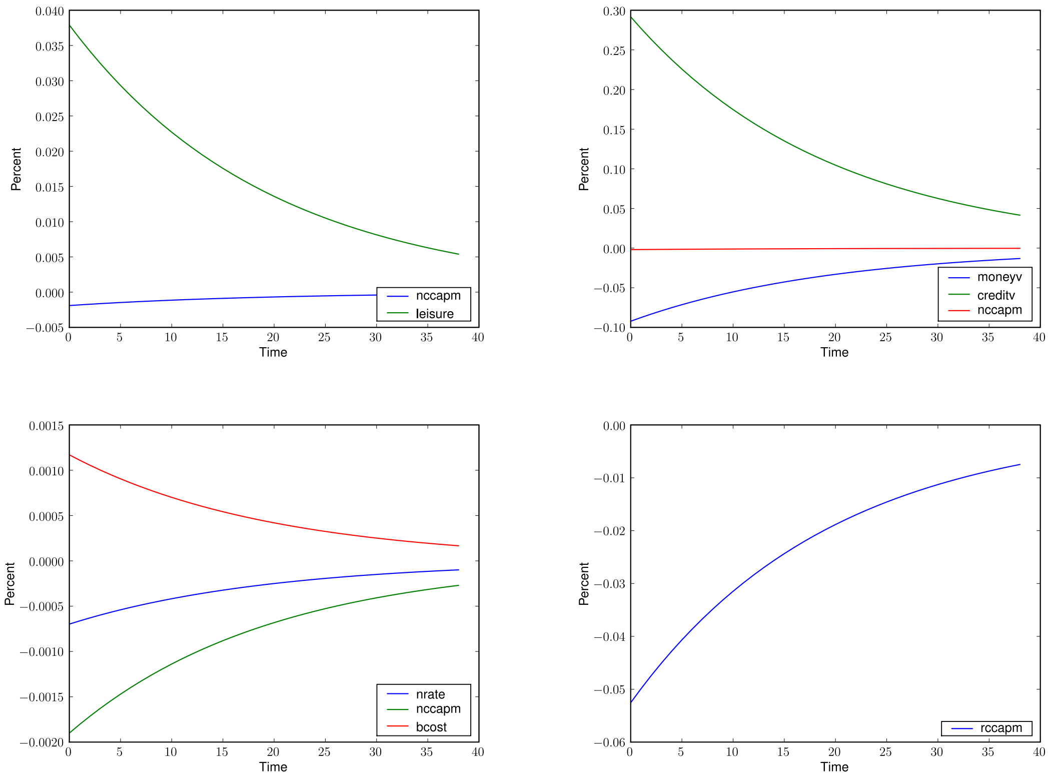

Fig. 5. Response to 1 percent innovation in goods productivity

case. Although credit increases, the increase in consumption also leads to a higher demand for money balances in the period when the shock occurs, thus implying an initial sharp fall in inflation to adjust the given pre-determined money balances upwards. As consumption begins to fall again in subsequent periods, inflation jumps above it's steady state in the period after the shock only to converge to it's long-run level from above. In summary, a goods sector productivity shock leads to a modest increase in the real and nominal CCAPM rate, implying a modest increase in the price of credit. There is a modest substitution effect towards leisure, implying a modest fall in total labour, where this time both banking time and goods production time fall. This leads to less credit being produced and more money being held. The distortionary cost term rises, thus leading to an fall in the gap between the CCAPM rate and the risk free rate, conditionally reducing the equity premium.

5.5 Simulation Analysis

This section is going to analyse the simulated time series from the solved model and, similar to Bansal and Coleman (1996), in particular focus on correlations of velocity and ex-post asset returns with measures of monetary policy. In order to make simulations comparable to historical post-war quarterly time series data, a simulation length of 200 was chosen, where each time series is hp-filtered. Standard errors (where applicable) are generated by repeating simulations 1000 times. The obtained correlations are then compared to equivalent measures obtained from U.S. data, which have been taken from Bansal and Coleman (1996). Table 1 shows the credit-banking model's behaviour

Table 1: Velocity

Statistic	Data	Model
Velocity:		
Average	1.20	1.33
Std.	0.11	0.021
Autocorrelation	0.97	0.56
Std.	(-)	0.06
Correlation:		
Velocity and Nom. Risk-Free Rate	0.74	0.97
Std.	(-)	0.006

of consumption-money velocity. Based on the steady state calibration, the model's implied average value of velocity measure compare favourably with the equivalent measure observed in U.S. data and represents an improvement over standard cash-in-advance models which typically exhibit a velocity value of unity. Also, in contrast to the credit-cash model by Stokey and Lucas (1987), in which velocity measures different from unity and a positive relationship to the nominal interest rate is obtained and whose approach is based on a simple preference specification argument (in which cash and credit goods are imperfectly substitutable), velocity in the credit-banking model is primarily determined by variations in the *price* (equalling the net nominal rate) of credit on the one hand, and the credit-productivity induced *shifts* in the convex marginal cost schedule of the credit sector, on the other. Therefore, velocity is not preference-, but instead technology-driven and credit is purchased in a de-centralised market in which the intersection of the price of credit (which in a unique exchange equilibrium between use of cash and credit has to equal the opportunity cost of cash, the net nominal CCAPM rate i_t) and the upward-sloping marginal cost curve determined by the degrees of diminishing return parameter ρ, determine velocity.

The model does fairly well in capturing the autocorrelation of velocity as well as the contemporaneous correlation with the nominal rate of interest. Notice that in simulation results not reported here, the autocorrelation of velocity is strongly linked to how persistently money supply growth rate shocks are modeled. The model is less successful in capturing the observed volatility of velocity, which is not surprising given the findings of Hodrick et al. (1991), who study the variability of velocity in a preference-based cash-credit model, only to find that high levels of risk-aversion are needed, in order to make interest rates more volatile, leading to sufficient variability in velocity that way. The model presented here also exhibits low variability in the nominal rate, due to low variability of the stochastic discount factor (or the real rate) coupled with low variability in inflation expectations, which is common for flex-price models, in which a large proportion of money supply growth innovations directly translate into unexpected inflation forecast errors in the period of the shock, leaving little left to be captured by inflation expectations. Notice that, in spite of the credit-banking model's second potential channel affecting volatility - shocks to credit productivity - it appears that given the baseline calibration, variability in the price of credit seems to matter far more for the determination of the variability of velocity.

Table 2: Real Risk-Free Rate

Statistic	Data	Model
Ex-Post Real Rate:		
Average	1.12%	1.95%
Std.	3.27	2.31
Correlation:		
Ex-Post Real Rate and Inflation	-0.68	-0.99
Std.	(-)	0.001
Ex-Ante Real Rate and Exp. Inflation	-0.34	-0.98
Std.	(-)	0.001

Note:The ex-post risk-free rate is defined as $1 + \tilde{i}_{1,t} - \pi_{t+1}$

Table 2 illustrates the model's time series characteristics of the ex-post low risk-free rate and compares this to historical equivalent measures from the U.S. Through the credit-banking cost distortion, the low risk-free rate obtained on short-term saving deposits can be calibrated such to to be much closer to the historically observed low risk-free rate of approximately 1%. The model does very well in capturing the observed standard deviation of the ex-post real return on the short-term saving deposit. More importantly, there is a strong negative correlation between this real rate and the realised rate of inflation. But this results is hardly surprising, as the ex-post rate is constructed by subtracting the ex-post realised rate of inflation from the ex-ante nominal risk-free rate. Therefore, all of the inflation forecast errors (which are very large) are contained in the ex-post rate, such as to produce a very high correlation of this rate with the ex-post inflation rate (which contains the same inflation forecast errors).

What is more interesting however, is that the model is capable of producing a strongly negative correlation between the *ex-ante* real rate and the *ex-ante* expected rate of inflation, which is also seen in U.S. data and has been found to be robust through various studies (see Huizinga and Mishkin, 1984; Summers, 1984). The intuition for why this is the case is straightforward. As inflation expectations rise, so does the current and future price of credit (through the rise in the current and future expected nominal CCAPM rate which is largely driven by inflation expectations), leading to a current and future expected expansion of the credit sector. This however increases the future expected proportional payout of the short-term saving deposit in form of the future expected banking wage bill, leading to a residually lower real risk-free return. Notice that although this implies an apparent unconditional as well as conditional violation of the Fisher equation as measured by *observable* cost-distorted money market returns and inflation, once the banking wage bill is taken into account, the *effective* Fisher relationship is not violated unconditionally (naturally, it will however never hold exactly ex-post conditionally, because of errors in inflation expectations, but also never ex-ante unconditionally, because of the inflation risk-premium).

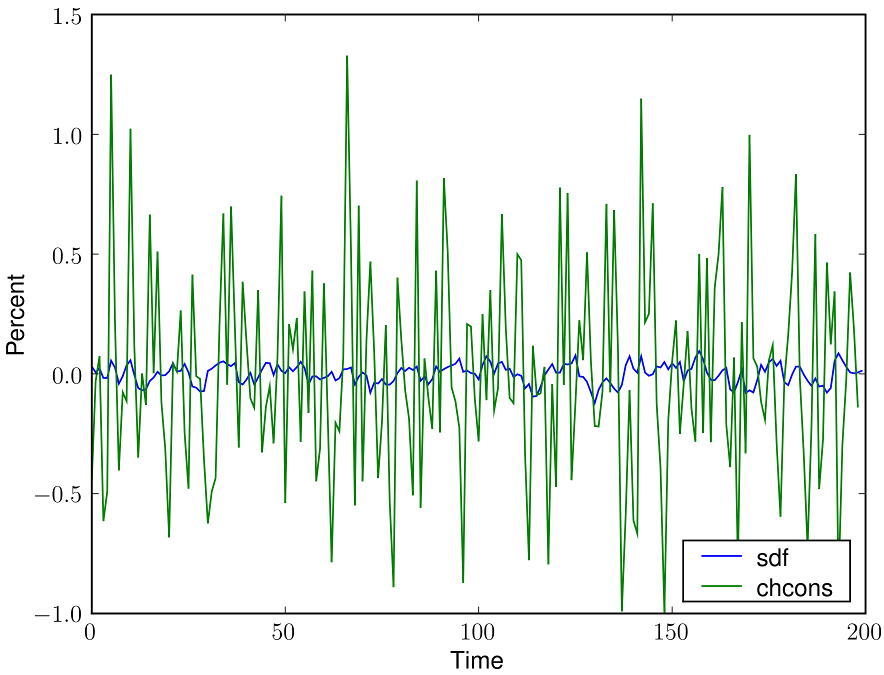

Fig. 6. %Δ of Log Stochastic Discount Factor and Consumption

Figure (6) illustrates a representative simulation for the endogenously determined (from within the model) % change in consumption *growth* on the one hand, and the % change in the log stochastic discount factor of the credit-banking model, on the other. Notice that the latter is given by:

$$\log(\Lambda_{t+1}) = \log E_t \left[\frac{\gamma}{\beta} \frac{\lambda_{t+1}}{\lambda_t} \right] \tag{114}$$

which differs from the usual log discount factor of endowment barter economies, which is just equal to the (expected) change in consumption, whereas here marginal valuation also depends on the nominal CCAPM rate of interest. Fitting an AR1 process to a representative simulation of consumption growth results in an autocorrelation coefficient equal to -0.16 with a standard error of 0.45, making this measure close to i.i.d. The relevant log stochastic discount factor for the economic environment discussed here has a positive autocorrelation coefficient equal to 0.62 with a standard error of 0.03. Regarding the stochastically implied average yield curve of the term structure of interest rates, this implies a slightly downward-sloping yield curve due to the cumulative effect of positive Jensen's inequality terms (or variance terms), which are subtracted from the deterministic mean return (see Backus et al., 1989; den Haan, 1995; Cochrane, 2005). However, it is well-known that this risk-adjustment of yield returns due to the hedging role of long-term bonds when the representative household faces growing future volatility of valuation decreases with ever less risk-averse representative agents, making this effect quantitatively very small for logarithmic specification of preferences (see den Haan, 1995). In any case, the quantitative effect of the "first-order" cost-distortion leading to the downward-sloping convex-shaped of the *deterministic* component of average yields will outweigh the previously mentioned "second-oder" risk-induced effect causing the yield curve to be slightly downward-sloping, leading overall to a downward-sloping convex-shaped yield curve, stochastically[31] .

Another feature of the credit-banking model which sets it somewhat apart from standard cash-in-advance models is that inflation forecast errors, though of course through rational expectations on average zero, will however generally be much larger on average. This is illustrated in figure (7). The reason for this lies in the endogenous variation of velocity measures (through the endogenous variation in credit production). Since money balances are pre-determined, shocks leading to an imbalance between nominal money supply and money demand, require an endogenous response in actual inflation in order to restore monetary equilibrium in real terms. If, during the same time (say, following an unexpected money growth shock), credit expands as well, then nominal pre-determined money balances have to experience and even stronger adjustment in real terms through inflation in order to establish an equilibrium between

[31] Cochrane (2005, ch.19,p.361) shows how a log discount factor with autocorrelation coefficient of $\rho = 0.9$ and a standard error of $\sigma_\epsilon = 0.02$ results in a downward-sloping yield curve with a quantitatively very small slope, leading to an essentially flat yield curve.

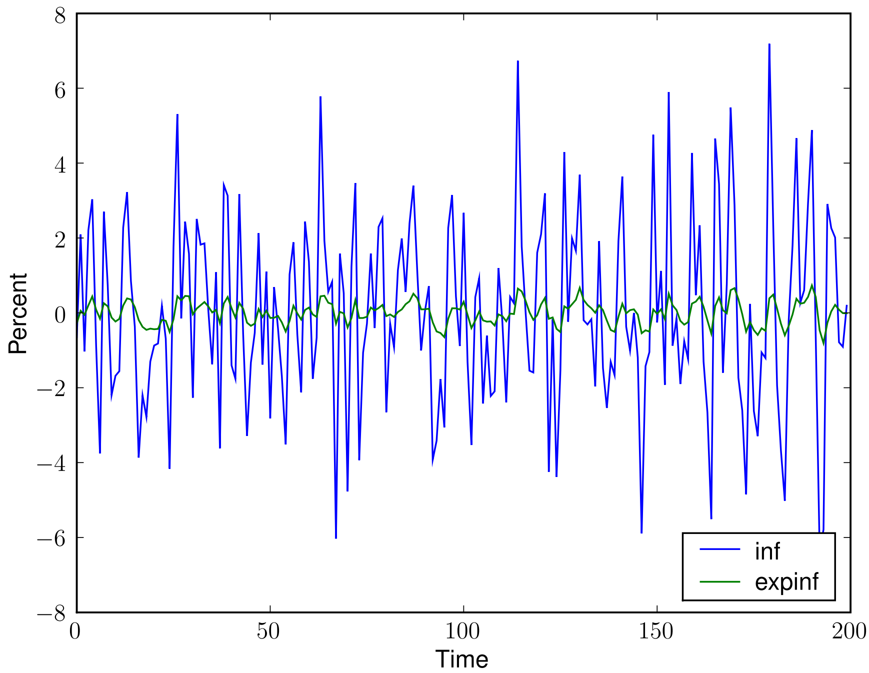

Fig. 7. Ex-Ante Expected and Ex-Post Realised Inflation

the total supply of exchange means (i.e. money *and* credit) and money demand, given by the current level of consumption. Therefore, adding credit supply to a liquidity market (given by the cash-in-advance constraint) leads to much stronger variation in actual inflation relative to expected inflation than otherwise found in standard CIA models. Of course, additional shocks to credit productivity further increase the uncertainty about future expected inflation[32].

Figure (8) illustrates how variations in monetary policy indicators affect the cost-wedge between the nominal CCAPM and the nominal risk-free rate (and thus the equity premium) over the business cycle and how the stance of monetary policy is positively correlated with this wedge. The top left and right hand, and the bottom right hand quadrant graphs are essentially all linked through the stochastic money supply growth process and corresponding changes in inflation expectations, also affecting the nominal CCAPM rate through the Fisher relationship. The same positive relationship between the equity premium and actual inflation also holds, however with a less stronger association for the reasons discussed above regarding larger inflation forecast errors. The simulations confirm the results derived in the theoretical section and clearly show the link between unexpected shocks to money growth, correspond

[32] This point is also discussed in Gillman et al. (2007), section 3.5.2: Effects of Shocks on Inflation.

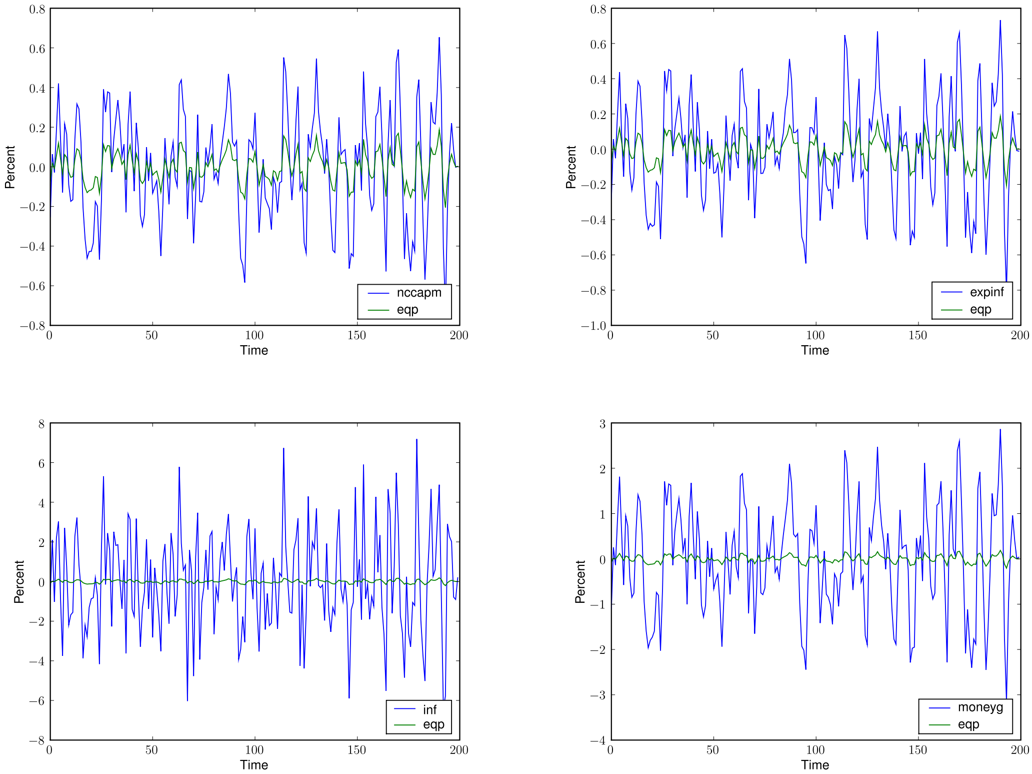

Fig. 8. Simulated conditional $\%\Delta$ in equity premium ($eqp_t = i_{1,t} - \tilde{i}_{1,t}$)

changes in the price of credit (via the CCAPM nominal rate) leading to variations in velocity measures driving the distortive cost distortion. In particular, shocks to money growth are persistently modeled and thus lead to an increase in *inflation expectations*, therefore increasing the price of credit which, ceteris paribus, leads to an expansion of the credit sector. This in turn increases the banking wage bill, primarily through higher banking time, but also through a slight increase in the overall wage rate in general. The credit-banking model therefore offers a new theoretical perspective on the positive association between the stance of monetary policy and the spread between the nominal CCAPM and money market rate, as illustrated empirically by Canzoneri et al. (2007a) and demonstrated theoretically by Canzoneri and Diba (2005). The latter mentioned authors explain this systematic link through a falling ad-hoc modelled bond liquidity premium as the issuing of bonds increases in an open-market operation reducing the amount of money. The credit-banking model, on the other hand, links increases in money growth, through their effect on inflation expectations, to a rise in the price of credit and credit production, thus resulting in a larger proportion of short-term debt's return to be paid out in form of the wage the representative household takes home in his activity as a banker. The model therefore provides a micro-founded theoretical explanation of this effect based on de-centralised credit production motivated by the financial intermediation literature.

6 Discussion

Before concluding, this section's purpose is to briefly related the results obtained from the credit-banking model to relevant themes of the existing literature. In particular, I will discuss how the distortive cost-effect due to credit production (and directly related to the banking wage bill) driving a wedge between the nominal CCAPM and nominal risk-free rate (and thus for given inflation expectations, also between the relevant real rates), can also be interpreted as a tax (or a subsidy, depending on whether one refers to returns or prices of a bond). Secondly, the results relevancy regarding the failure of Euler consumption equations will be discussed, and finally, a more detailed discussion of the equity premium in the credit banking model will be provided.

6.1 The low risk-free rate: A banking time tax on the price of bonds?

Thus far the discussion of the low risk-free rate (or money market rate) has been spelled out in terms of a partitioned payout on a short-term saving deposit, which was backed up one-for-one by an equivalent amount of short-term government debt by the financial intermediary. The relevant result describing this was given by:

$$\begin{aligned}&(1+\tilde{r}_{1,t})\\&=(1+r_{1,t})-E_t\left[\Upsilon^a_{t+1}\right]\\&\approx(1+r_{1,t})\,E_t\left[\Upsilon^b_{t+1}\right]\end{aligned}\tag{115}$$

where $E_t\left[\Upsilon^b_{t+1}\right]=E_t\left[1/1+s^b_{t+1}\rho p^f_{t+1}\right]$ is the term responsible for driving down the real risk-free rate. Therefore, writing the above results out in full, it is clear that one could alternatively view this as a banking time distortion of real CCAPM rate, thus resulting in the lower risk-free rate obtained on the short-term saving deposit:

$$(1+\tilde{r}_{1,t})=(1+r_{1,t})\,E_t\left[\frac{1}{1+s^b_{t+1}\rho p^f_{t+1}}\right]\tag{116}$$

where the distortion is equal to the future expected proportional payout of the return on the short-term bond in terms of the banking wage bill:

$$\begin{aligned}&E_t\left[\frac{1}{1+s^b_{t+1}\rho p^f_{t+1}}\right]\approx E_t\left[-s^b_{t+1}\rho p^f_{t+1}\right]\\=&E_t\left[-\frac{i_{t+1}\rho f^*_{t+1}}{\bar{\eta}}\right]=E_t\left[-\frac{w_{t+1}n_{f,t+1}}{b_{1,t}/\left(1+\pi_{t+1}\right)}\right]\end{aligned}\tag{117}$$

Alternatively, the distortion of the CCAPM real return related to the future banking wage bill can also be viewed as a *tax* on the price of the financial asset commanding that return:

$$\begin{aligned}\tilde{p}^b_{1,t} = (1+\tilde{r}_{1,t})^{-1} = p^b_{1,t}\left[E_t \Upsilon^b_{t+1}\right]^{-1} \\ p^b_{1,t} E_t \left(1 + s^b_{t+1}\rho p^f_{t+1}\right)\end{aligned} \qquad (118)$$

Besides the usual Tobin effects which are often cited as one of the factors responsible for affecting the Fisher relationship, the banking time tax on short-term debt implies a distortion of the Fisher equation implied by *observable* money market rates and measures of expected inflation affecting this relationship both in a steady state long-run, but also conditionally over the business cycle. The credit-banking model discussed here makes this relationship crucially depend on two factors: firstly, the *price* of credit embodied by the net nominal CCAPM rate which typically also directly influences the level of production of credit[33], and secondly $\bar{\eta} = [b_{1,t-1}/(1+\pi_t)]/c_t$ the proportional amount or *supply* of short-term debt circulating in the economy and how this relates to the proportional amount of debt distorted, which is captured by the proportional *supply* of credit $f^*_t = f_t/c_t$. Therefore, both the unconditional average of $s^b_t = f^*_t/\bar{\eta}$ and it's conditional behaviour over the business cycle are crucial in understanding the degree to which the banking time tax can affect the return (or price of) on short-term debt.

6.2 Euler Equation Rates and Money Market Rates

As pointed out by Canzoneri et al. (2007a), there exists a sizeable literature documenting the empirical failure of consumption Euler equation regressions based on the behaviour of aggregate consumption and observable money market rates. This is problematic for models discussing optimal monetary policy in a fashion implying *equivalence* of the observed nominal money market rate and the CCAPM rate integral to the consumption Euler equation. Further more, Canzoneri et al.'s empirical study suggests that there exists a *systematic* link between the spread of the two rates and monetary policy, and how this fact confirms certain central banker's as well as academics unease about models of monetary policy embodied by the new neoclassical synthesis in which the role of money has been marginalised (and monetary policy is modeled by empirical Taylor rules) and how financial intermediaries are not modeled at all. Figure (9) clearly shows how changes in expected inflation (thus raising the nominal

[33] Although, I have shown in the impulse response analysis, that a shock to credit productivity can lead to a fall in the price and a rise in the quantity of credit, where the former has outweighed the latter in it's effect on the conditional equity premium.

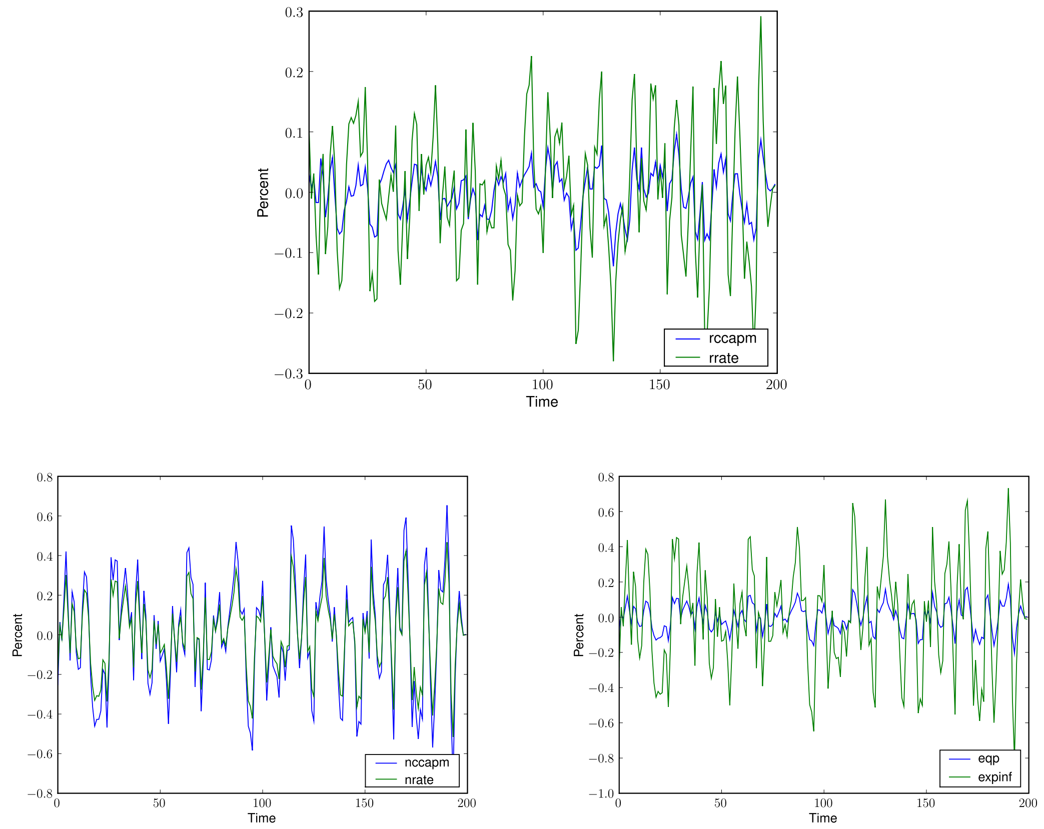

Fig. 9. Simulated conditional %Δ in interest rate spreads (real and nominal)

CCAPM rate and therefore the price of credit) lead to a discrepancy (due to the banking time tax implied by endogenous credit production) between the observed nominal money market rate paid on short-term saving deposits and the underlying purely intertemporal nominal CCAPM rate. The crucial point is that it is the *expected* inflation rate channel, driven by persistence in the money supply growth rate process, which influence the spread between the two rates. Therefore, the conditional spread between the two rates can only vary systematically with monetary policy in as far as the former is capable of varying inflation expectations. The graph can be best understood by recalling that an increase in the *nominal* CCAPM rate (and thus the price of credit) causes and expansion of credit production and thus an endogenous increase in the banking time tax levied on short-term deposits. This means that the *nominal* market rate is "buffered" compared to the underlying nominal CCAPM rate, it moves less in either way as the nominal CCAPM either falls or rises, due to the banking time tax. As the top graph reveals, for a given inflation expectation as implied by the Fisher relationship between the real and nominal CCAPM rates, the ex-ante *real* rate on the short-term saving deposit typically move more than the purely real intertemporal rate and sometimes they even appear to be de-linked in their movements. Notice that, although not pursued in this paper, this de-linked nature of the real money market and the real CCAPM rate and the uncertainty over it could even be increased by modeling a government debt-to-deposit target which is only imperfectly met each period, such as to make $\eta_t = \rho_\eta \eta_{t-1} + \epsilon_{\eta,t}$ an exogenous state variables as well, incorporating expected and unexpected variation in the proportional supply of debt as well.

6.3 The decline of the Equity Premium (Puzzle) ?

The credit-banking model creates an interest rate distortion (or differential) between the T-bill rate and a limiting long-term bond of approximately 1.05%, both in real and nominal terms and by doing so - through the expectations theory of the term structure - propagates this distortion in a cross-sectional fashion across bonds of various maturity and thus produces the convexly shaped term structure of average yields seen in U.S. data. Similarly, for a given growth-adjusted deterministic discount factor β, the model is thus capable of producing a theoretically low risk-free rate at 1.95% to be much closer to the one observed in post-war U.S. data. The model-theoretic premium return on long-term bonds over T-bills (i.e. the money market or risk-free rate) is also seen in the data to be approximately equal to 1.05%, but regarding the high return on equity, the model is only capable in contributing towards the resolution of this puzzle in as far as it has been successful in reducing the low risk-free rate by that same 1.05%. The steady state calibration of the model reflects a compromise between fitting unconditional returns of equity and bonds. In particular, I have chosen to slightly over-estimate both the average real return on the T-bill rate (at 1.95% versus the roughly 1% seen in U.S. data) and the average real return on a long-term bond (at approximately 3% versus the roughly 2.3% seen in U.S. data). However, the theoretical *spread* between short and long rates is correctly fitted. Notice therefore that in steady state, the return of a long-term limiting bond quickly approaches the CCAPM rate implied by the Euler equation, where the latter ought to be understood as the model's approximate counterpart of the return on equity. This implies a steady state return on equity equal to 3%, and a steady state return on the short-term saving deposit of 1.95% [34] . Based on early results of the equity premium literature, this may seem only a small contribution towards explaining the return differential between risky and (cost-distorted) money market rates. However, the current asset pricing literature appears more and more in favour of a view claiming an initial over-estimate of this return differential and it is not uncommon to encounter views which place the value of the equity premium to be as low as 2%−3%. One reason why the *true population* equity risk premium as sampled from many different stock and bond exchanges may be

[34] The quarterly spread is calculated as $i\rho s^b = 0.02 \times 0.21 \times (0.25/0.4) = 0.002625$. This implies an annual spread of 0.0105 or 1.05%

lower, is due to survivorship bias implied by observable U.S. stock and bond returns (see Brown et al., 1995). Also, some authors have argued that, given the large historical fluctuations in stock returns and the relatively short amount of data available, one may view the post-war experience as an unrepresentative spell of luck (in terms of high equity returns vis-a-vis the risk-free rate) and indeed, given the sample's variability, an equity premium of $2\% - 3\%$ is still within range of a 95% confidence interval[35]. Indeed, stronger than expected *economic growth* and thus also *dividend growth* affecting stock returns may explain the unusually high and unexpected equity risk-premium observed over such a long period of time (see Cochrane, 2007, p.266). Therefore, contrary to the view put forth in Kocherlakota (1996), the direction the current consensus appears to take is to de-emphasise excessively high stock market returns of the past. Indeed, in spite of recent downturns, recently observed stock market prices well above historical levels, broader stock market participation and the corresponding decline in the return on equity could be taken as evidence supporting the view that much of the historically observed excess return of equity over the risk-free rate was unexpectedly and unrepresentatively high. However, calibrating the real CCAPM rate at values of $3\% - 4\%$ and viewing this as the model's approximate return on capital (i.e. the risky rate), still leaves both the unconditional as well as *systematic* conditional variation of the low risk-free rate to be explained. The model presented here puts forth a theory of the risk-free or money market rate earned on short-term savings deposits, which is based on the endogenous variation of a banking time tax.

[35] Lettau et al. (2006) examine the role of a fall in macroeconomic risk leading to the fall of the equity premium in the 1990s.

7 Conclusion

The equity premium and the complementary risk-free rate puzzle (Mehra and Prescott, 1985; Weil, 1989), the related failure of theory-implied consumption Euler equation regressions and the apparent inequality of observed money market rates and theory-implied Euler consumption equation rates (Canzoneri et al., 2007a), as well as the term premium puzzle (Backus et al., 1989), are all indicative of a hole in our understanding of how such return measures ought to be derived within a general equilibrium framework. A promising avenue contributing towards filling this gap is to devise ways of modeling financial intermediation more explicitly (see McCallum and Goodfriend, 2007; Canzoneri et al., 2008; Gillman and Kejak, 2008), thus opening up possibilities to distort such return measures by better understanding what roles such financial intermediaries may play and what implications for relevant measures might ensue as a result.

Building on previous steady state analysis work conducted by Gillman and Kejak (2008) and stochastic dynamic analysis by Benk et al. (2005), I have described and solved a model of essentially cash-in-advance nature, which was modified by incorporating a de-centralised credit-banking sector, serving the dual role of conduit for liquidity in terms of money and a produced credit exchange service, and of being the sole point-of-sales outlet for saving deposits of various maturity held by the representative household (which are internally backed up one-for-one by corresponding government bonds). The model is capable of driving a cost-related wedge (in form of the banking wage bill) between both the nominal and real CCAPM rates and the corresponding nominal and real rates obtained on the short-term saving deposit, thus lowering the *deterministic* component of the stochastic risk-free rate beyond the usual (growth-adjusted) real rate defined by the inverse of the representative household's impatience factor. The mechanism underlying the derivation of this wedge is motivated by the distortive cost effects produced by a micro-founded banking sector based on the theory of financial intermediation (Hancock, 1985; Clark, 1984). In contrast to Bansal & Coleman, I show how the reduced short-term money market rate is perpetuated through the term structure via the expectations theory, thus leading to a convex upward-sloping term structure with a much steeper slope at the short- than the long end, as only any j-period bond's tail-end return is affected by the banking sector's cost distortion, implied by the banking wage bill.

The key mechanism driving steady state, as well as dynamic results asset pricing results, is that some share of economy-wide short-term debt equal in value to the credit exchange service is retained as credit-backing collateral within the banking sector and instead re-distributed in form of a dividend payment back to the household at the end of the period. Instead, the representative household receives on this share of the short-term saving deposit the per unit-of-credit normalised revenue generated by the deposits (or average product of deposits, equal to $(1-\rho)\, i_t$), which equals the price of credit residual of the average product (cost) paid out to banking labour in terms of the banking wage bill. Since asset pricing results depend on the magnitude of the bond utilisation rate $s_{1,t}^{b}$, relative supply and relative credit-production induced demand of short-term debt matters. In as far as positive money supply growth rate innovations can lead to an increase in the nominal CCAPM rate (primarily by affecting the expected rate of inflation) and thus the price of credit, the credit-banking model experiences an expansion of the credit sector and thus a proportionately larger cost distortion in form of a banking time tax on the return of the short-term saving deposit, as the bank's debt-utilisation rate rises. This leads to a conditional widening of the gap between the nominal CCAPM and the nominal market rate. The paper therefore puts forth a new perspective on the systematic link between the stance of monetary policy and the spread between the money market and the CCAPM rate as implied by the consumption Euler equation, as described empirically by Canzoneri et al. (2007a) and explored theoretically by Canzoneri and Diba (2005). Finally, the model is also capable of generating velocity above unity and a positive correlation of this with the nominal rate of interest (both the intertemporal nominal CCAPM and the nominal market rate), and a negative correlation between the ex-post real rate and inflation, but more importantly, also a negative correlation between the *ex-ante* real rate and the *ex-ante* expected rate of inflation.

References

Abel, A. B., May 1990. Asset prices under habit formation and catching up with the joneses. American Economic Review 80 (2), 38–42.

Backus, D., Foresi, S., Telmer, C., Sep. 1998. Discrete-time models of bond pricing. NBER Working Papers 6736, National Bureau of Economic Research, Inc.

Backus, D. K., Gregory, A. W., Zin, S. E., November 1989. Risk premiums in the term structure : Evidence from artificial economies. Journal of Monetary Economics 24 (3), 371–399.

Bansal, R., Coleman, W. J. I., December 1996. A monetary explanation of the equity premium, term premium, and risk-free rate puzzles. Journal of Political Economy 104 (6), 1135–71.

Barro, R. J., March 1976. Integral constraints and aggregation in an inventory model of money demand. Journal of Finance 31 (1), 77–88.

Baumol, W. J., November 1952. The transactions demand for cash: An inventory theoretic approach. Quarterly Journal of Economics 66.

Benk, S., Gillman, M., Kejak, M., July 2005. Credit shocks in the financial deregulatory era: Not the usual suspects. Review of Economic Dynamics 8 (3), 668–687.

Bernanke, B. S., Gertler, M., Gilchrist, S., 1999. The financial accelerator in a quantitative business cycle framework. In: Taylor, J. B., Woodford, M. (Eds.), Handbook of Macroeconomics. Elsevier.

Bohn, H., May 1991. On cash-in-advance models of money demand and asset pricing. Journal of Money, Credit and Banking 23 (2), 224–42.

Boldrin, M., Christiano, L. J., Fisher, J. D. M., March 2001. Habit persistence, asset returns, and the business cycle. American Economic Review 91 (1), 149–166.

Breeden, D. T., September 1979. An intertemporal asset pricing model with stochastic consumption and investment opportunities. Journal of Financial Economics 7 (3), 265–296.

Brown, S. J., Goetzmann, W. N., Ross, S. A., July 1995. Survival. Journal of Finance 50 (3), 853–73.

Campbell, J. Y., 08 2000. Asset pricing at the millennium. Journal of Finance 55 (4), 1515–1567.

Campbell, J. Y., Cochrane, J. H., 1999. By force of habit: A consumption-based explanation of aggregate stock market behaviour. Journal of Political Economy 107, 205–251.

Campbell, J. Y., Lo, A. W., MacKinlay, C. A., 1997. The Econometrics of Financial Markets. Princeton University Press.

Canzoneri, M. B., Cumby, R., Diba, B., López-Salido, J. D., May 2008. Monetary aggregates and liquidity in a neo-wicksellian framework. CEPR Discussion Papers 6813, C.E.P.R. Discussion Papers.

Canzoneri, M. B., Cumby, R. E., Diba, B. T., October 2007a. Euler equations and money market interest rates: A challenge for monetary policy models.

Journal of Monetary Economics 54 (7), 1863–1881.
Canzoneri, M. B., Cumby, R. E., Diba, B. T., Lopez-Salido, D., 2007b. Monetary and fiscal policy coordination with bonds provide transactions services. working paper .
Canzoneri, M. B., Diba, B. T., March 2005. Interest rate rules and price determinacy: The role of transactions services of bonds. Journal of Monetary Economics 52 (2), 329–343.
Clark, J. A., February 1984. Estimation of economies of scale in banking using a generalized functional form. Journal of Money, Credit and Banking 16 (1), 53–68.
Cochrane, J. H., 2005. Asset Pricing, 2nd Edition. Princeton University Press.
Cochrane, J. H., 2007. Financial markets and the real economy. working paper .
Coeurdacier, N., Kollmann, R., Martin, P., Sep. 2007. International portfolios with supply, demand and redistributive shocks. CEPR Discussion Papers 6482, C.E.P.R. Discussion Papers.
Coleman, W. J. I., March 1996. Money and output: A test of reverse causation. American Economic Review 86 (1), 90–111.
Constantinides, G. M., June 1990. Habit formation: A resolution of the equity premium puzzle. Journal of Political Economy 98 (3), 519–43.
Cooley, T. F., Hansen, G. D., September 1989. The inflation tax in a real business cycle model. American Economic Review 79 (4), 733–48.
den Haan, W. J., 1995. The term structure of interest rates in real and monetary economies. Journal of Economic Dynamics and Control 19 (5-7), 909–940.
Donaldson, J. B., Johnsen, T., Mehra, R., October 1990. On the term structure of interest rates. Journal of Economic Dynamics and Control 14 (3-4), 571–596.
Epstein, L. G., Zin, S. E., April 1991. Substitution, risk aversion, and the temporal behavior of consumption and asset returns: An empirical analysis. Journal of Political Economy 99 (2), 263–86.
Fama, E. F., January 1990. Term-structure forecasts of interest rates, inflation and real returns. Journal of Monetary Economics 25 (1), 59–76.
Gibbons, M. R., Ramaswamy, K., 1993. A test of the cox, ingersoll, and ross model of the term structure. Review of Financial Studies 6 (3), 619–58.
Gillman, M., Benk, S., Kejak, M., 2007. Volatility cycles of output and inflation: A good shock, bad shock story. working paper .
Gillman, M., Kejak, M., 01 2005. Inflation and balanced-path growth with alternative payment mechanisms. Economic Journal 115 (500), 247–270.
Gillman, M., Kejak, M., 2008. Inflation, investment and growth: A banking approach. Tech. rep., forthcoming, Economica.
Gillman, M., Otto, G., 2005. Money demand in a banking time economy.
Giovannini, A., Labadie, P., Dec. 1991. Asset prices and interest rates in cash-in-advance models. The Journal of Political Economy 99 (6), 1215–1251.
Hancock, D., October 1985. The financial firm: Production with monetary and

nonmonetary goods. Journal of Political Economy 93 (5), 859–80.
Hansen, L. P., Jagannathan, R., April 1991. Implications of security market data for models of dynamic economies. Journal of Political Economy 99 (2), 225–62.
Hodrick, R. J., Lucas, D., Kocherlakota, N. R., April 1991. The variability of velocity in cash-in-advance models. Journal of Political Economy 99 (2), 358–84.
Huizinga, J., Mishkin, F. S., July 1984. Inflation and real interest rates on assets with different risk characteristics. Journal of Finance 39 (3), 699–712.
Jermann, U. J., April 1998. Asset pricing in production economies. Journal of Monetary Economics 41 (2), 257–275.
Klein, P., September 2000. Using the generalized schur form to solve a multivariate linear rational expectations model. Journal of Economic Dynamics and Control 24 (10), 1405–1423.
Klein, P., Gomme, P., 2008. Second order approximation of dynamic models without the use of tensors. working paper .
Kocherlakota, N. R., March 1996. The equity premium: It's still a puzzle. Journal of Economic Literature 34 (1), 42–71.
Labadie, P., 1994. The term structure of interest rates over the business cycle. Journal of Economic Dynamics and Control 18 (3-4), 671–697.
Lettau, M., Ludvigson, S., Wachter, J. A., Mar. 2006. The declining equity premium: What role does macroeconomic risk play? CEPR Discussion Papers 5519, C.E.P.R. Discussion Papers.
Lintner, J., May 1969. The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets: A reply. The Review of Economics and Statistics 51 (2), 222–24.
Lucas, R. E., November 1978. Asset prices in an exchange economy. Econometrica 46 (6), 1429–45.
Lucas, R. E., 1982. Interest rates and currency prices in a two-country world. Journal of Monetary Economics 10 (3), 335–359.
McCallum, B. T., 1983. The role of overlapping-generations models in monetary economics. Carnegie-Rochester Conference Series on Public Policy 18, 9–44.
McCallum, B. T., Goodfriend, M. S., July 2007. Banking and interest rates in monetary policy analysis: A quantitative exploration. Journal of Monetary Economics 54 (5), 1480–1507.
McCulloch, H. J., Kwon, H., 1993. U.s. term structure data, 1947-1991. Tech. rep., Ohio State University.
Mehra, R., Prescott, E. C., March 1985. The equity premium: A puzzle. Journal of Monetary Economics 15 (2), 145–161.
Mehra, R., Prescott, E. C., 2003. The equity premium in retrospect. In: Constantinides, G., Harris, M., Stulz, R. M. (Eds.), Handbook of the Economics of Finance. Elsevier.
Merton, R. C., December 1971. Optimum consumption and portfolio rules in

a continuous-time model. Journal of Economic Theory 3 (4), 373–413.
Mishkin, F. S., Jan. 1982. The real interest rate: An empirical investigation. NBER Working Papers 0622, National Bureau of Economic Research, Inc.
Mishkin, F. S., August 1990a. The information in the longer maturity term structure about future inflation. The Quarterly Journal of Economics 105 (3), 815–28.
Mishkin, F. S., January 1990b. What does the term structure tell us about future inflation? Journal of Monetary Economics 25 (1), 77–95.
Mishkin, F. S., November 1992. Is the fisher effect for real? : A reexamination of the relationship between inflation and interest rates. Journal of Monetary Economics 30 (2), 195–215.
Rouwenhorst, K., 1995. Asset Pricing Implications of Equilibrium Business Cycle Models. Princeton University Press.
Seppala, J. I., Aug. 2000. The term structure of real interest rates: Theory and evidence from the u.k. index-linked bonds. Econometric Society World Congress 2000 Contributed Papers 0245, Econometric Society.
Sharpe, W. F., 1964. Capital asset prices: A theory of market equilibrium under conditions of risk. Journal of Finance 19 (3), 425–442.
Stokey, N. L., Lucas, R. E., May 1987. Money and interest in a cash-in-advance economy. Econometrica 55 (3), 491–513.
Summers, L. H., Sep. 1984. The nonadjustment of nominal interest rates: A study of the fisher effect. NBER Working Papers 0836, National Bureau of Economic Research, Inc.
Tallarini, T. D., June 2000. Risk-sensitive real business cycles. Journal of Monetary Economics 45 (3), 507–532.
Tobin, J., August 1956. The interest elasticity of the transactions demand for cash. Review of Economics and Statistics 38.
Uhlig, H., 1995. A toolkit for analyzing nonlinear dynamic stochastic models easily. Discussion Paper 97, Tilburg University, Center for Economic Research.
Walsh, C. E., 2003. Monetary Theory and Policy, 2nd edition. MIT Press.
Weil, P., November 1989. The equity premium puzzle and the risk-free rate puzzle. Journal of Monetary Economics 24 (3), 401–421.