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Consumption Velocity in a Cash Costly-Credit Model

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Abstract

In a seminal study Hodrick et al. (1991) evaluate the ability of a simple cash-credit model to produce realistic variability in *consumption* velocity while at the same time successfully explaining other key statistics. Sufficient variability in the latter is found to be associated with far too volatile interest rate behaviour. Introducing habit-formation in consumption into a production-based cash costly-credit model (see Gillman and Benk, 2007) makes the evolution of deposits more rigid relative to credit. The same deposit rigidity leads to a more volatile price of credit, causing credit production overshooting relative to deposits. But only by introducing adjustment costs to investment in addition to habit persistence does credit production overshoot sufficiently to produce realistic variability in consumption velocity. The model succeeds in capturing sufficient variability in consumption velocity without obtaining too volatile interest rates. Also, this model of endogenous velocity does not suffer from indeterminacy problems discussed in Auray et al. (2005). In contrast to Gillman and Benk (2007), the present study examines the role of the price-channel of credit production at business cycle frequency, ignoring or holding fixed the marginal cost channel stemming from credit productivity shocks.

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1 Introduction

A discussion of velocity within the canonical real business cycle model framework(see Kydland and Prescott, 1982; Hansen, 1985; King et al., 1988) requires a theory explaining why the representative agent would want to hold a return-dominated asset in the first place, ideally by specifying a *purposeful* and *plausible* role for money, without sacrificing the tractability of such models, while perhaps making some concessions to the level of depth of microfoundations attained to preserve that tractability. While research into modeling seriously the microfoundations of money demand in general equilibrium is still an ongoing, very controversial and inconclusive agenda, the last couple of decades have seen the development and popularisation of arguably three "first-generation" theories of money demand in general equilibrium, the Sidrauski money-in-the-utility-function approach (Sidrauski, 1967), the interrelated shopping-time and transactions cost approaches (Baumol, 1952; Tobin, 1956; Barro, 1976; McCallum, 1983; Bansal and Coleman, 1996), and the Clower-Lucas cash-in-advance approach (Clower, 1967; Lucas, 1982; Svensson, 1985; Stokey and Lucas, 1987; Cooley and Hansen, 1989).

While the recent rise in popularity of the new neoclassical synthesis generation of GE models (see Lawrence J. Christiano and Evans, 2005; Canzoneri et al., 2007a; Smets and Wouters, 2007) has seen a concurrent de-emphasising of the significance of modeling money in some purposeful way at all[1], it is also interesting to observe how of all of the three mentioned "first-generation" theories of GE money demand, arguably the most plausible and theoretically robust - the cash-in-advance role of money - to some it may seem is perhaps closest in failing one of the toughest tests of all, the test of time, as some of it's predictive shortcomings have led many to pursue the alternatives or to devise new ways of modeling money demand in general equilibrium alltogether.

One of such shortcomings of early formulations of the cash-in-advance model, typically spelled out in a simple Lucas-exchange endowment economy framework, was it's prediction that *consumption*-money velocity is always fixed at unity. While a subtle modification of the information set available to the representative agent (see Svensson, 1985) opened up the possibility of a precautionary money demand component, meaning that the cash-in-advance constraint in theory would not always bind and money balances beyond those required for consumption would be held, this avenue was quickly dismissed, as in simulation-based experiments, the CIA constraint was found to be binding almost always in practice (see Hodrick et al., 1991). As a consequence of this finding, cash-in-advance models are now routinely analysed and discussed

[1] Either money is completely absent, or MIUF in combination with a Taylor Rule implies a corresponding money supply rule *residually.*

assuming a strictly binding cash-in-advance constraint.

In order to make possible a realistic modeling of the *average* velocity of *consumption* velocity within the cash-in-advance paradigm, Prescott (1987) and Stokey and Lucas (1987) developed the *cash-credit* model, in which *preferences* over a cash-in-advance and a credit good were specified (thus leading to a multi-good barter economy), and the relative price of the cash good vis-a-vis the credit good was in the usual way related to the opportunity cost of holding money, thus making this relative price equal the net nominal rate of interest. In such a model, the average level of consumption velocity is therefore fully characterised by the optimality condition of setting the marginal rate of substitution between the cash and the credit good equal to the relative price between the two:

$$\frac{\partial U\left(c_{m,t}, c_{c,t}\right)/\partial c_{m,t}}{\partial U\left(c_{m,t}, c_{c,t}\right)/\partial c_{c,t}} = 1 + i_{t-1} \tag{1}$$

where I have defined $c_{c,t}$ to be current level of the credit good and $c_{m,t}$ to be current level of the money (or cash) good. While this allowed such models - for suitably calibrated preference parameters - to correctly match the empirically observed *average* velocity of consumption, it became quickly apparent that matching the observed *volatility* of consumption velocity required interest rates to be implausibly volatile at the same time (see Hodrick et al., 1991), either by assuming too high a level of relative risk aversion or by adopting habit persistence in consumption, both of which lead to too volatile interest rates *in endowment economies* [2] . It is interesting to note that this specific shortcoming of the predictive failure regarding velocity in particular has led many to adopt either shopping time or transaction cost functions to motivate money demand in their models instead (see inter alia Marshall, 1992; den Haan, 1995; Bansal and Coleman, 1996; Auray et al., 2005).

Intuitively, as the average level of consumption velocity is characterised by the point of tangency between a relatively smooth utility function and a downward-sloping relative price schedule given by the nominal rate of interest, dramatic volatility in the slope of that price schedule is needed to attain significantly different and dispersed loci of tangency. Indeed for the comparatively small perturbations seen in both the real and nominal rates in practice, period-by-period loci of tangency are all contained within some small neighbourhood and thus consumption velocity does not vary sufficiently through time. Although the setup of the model therefore essentially amounts to too small an interest rate elasticity of consumption velocity, this elasticity is typically not independent of the *level* of interest rates, which means that conducting such analysis by calibrating the model at business cycle frequency (quarterly) or

[2] The same is typically not the case in production-based RBC models, in which the representative agent can use saving and labour to smooth marginal valuation.

a more medium- to long-term frequency (annually) can lead to differences in results obtained.

This sensitivity of results in relation to velocity due to the chosen time horizon is also examined by Hodrick et al. (1991), who report results based on both quarterly and annual specification of models. Gillman and Kejak (2007), on the other hand, whose model is a fully specified production-based RBC model with physical and human capital, also calibrate their cash costly-credit model using a quarterly time horizon, but include a shock to credit production productivity which serves as a further channel to explain variability in *income* velocity within their framework - they do not discuss any numerical results pertaining to the variability of *consumption* velocity directly [3] . The importance of *money shocks* in explaining velocity within their framework is possibly also a result of specifying credit innovations to be highly contemporaneously correlated with money shock innovations. This suggests that their results are predominantly driven by the obtained high credit-shock elasticity of velocity and the influence of variation in this credit shock *alone*.

Closely related to this last point, an approximate conceptual analogy can be drawn between the role of *credit shocks* in the *technology-based* cash costly-credit model (see Gillman, 1993; Benk et al., 2005; Gillman and Benk, 2007) on the one hand, and how the analogous counterpart of the same shock could be viewed as a *preference shock* in the desirability of the credit good relative to the cash good in the seminal *preference-based* cash-credit model (see Stokey and Lucas, 1987), on the other. To my knowledge, the latter approach has never been explored, and I would find it surprising if it ever had been, as equipping a simple preference-based cash-credit model with preference shocks to the credit and cash goods would not *explain* the variability in velocity, but through the exogenous specification of such preference shocks, instead essentially amount to assuming it trivially.

This analogy is however only approximate, as in the former case the level and variability of velocity depend on the intersection of a horizontal price schedule (the net nominal rate of interest) with a convex upward-sloping marginal cost schedule in credit production (see Gillman and Kejak, 2008), and variability in both the price and the marginal cost schedule affect the variability of velocity, whereas in the latter case the above-discussed changes in loci of tangency matter. What will be of importance in the present study is also the distinction that can be drawn between the two cases with regards to the interest rate elasticity of consumption velocity, and how this elasticity varies with the level of the interest rate, and generally differs in the cash costly-credit from the cash credit model.

[3] Although, they do of course point out that variability in consumption velocity is a component of of the overall variability in income velocity.

On the other hand, a discussion of *income* or *output* velocity within the cash-in-advance paradigm and a fully specified production-based real business cycle model with physical capital and investment presents less of a challenge, as it is typically only consumption which is modeled subject to the liquidity constraint. As demonstrated theoretically by Cooley and Hansen (1989) and re-emphasised by Gillman and Kejak (2007), in such models *income* velocity is therefore trivially different from unity and also exhibits the observed pro-cyclicality seen in U.S. data. Intuitively, the Friedman permanent income-implied consumption-smoothing property (see Friedman, 1957) also leads to smooth behaviour of money demand, whereas productivity shocks and endogenous variation in the leisure-labour trade-off lead to much more volatile *income* fluctuations around a smooth *consumption* (money demand) trend, resulting in the pro-cyclicality of income velocity.

Here, money supply shocks are of little significance (for explaining the variability of velocity measures) and interest-rate implied means-of-exchange switching is either absent (if there is no credit good), or for the above-discussed reasons quantitatively unimportant in explaining much of the volatility seen in income velocity, as interest rates in *production-based* fully specified real business cycle models are typically even smoother than in endowment economies, since the representative household has a larger menu of choice variables (saving, leisure-labour) at his disposal to smooth his marginal valuation through time (see den Haan, 1995; Jermann, 1998).

This discussion therefore makes clear how within this framework, *income* velocity varies sufficiently and pro-cyclically[4], due to the *investment* velocity component alone (investment jumps, but money demand due to consumption demand hardly moves at all), primarily driven by productivity shocks and permanent income-implied consumption smoothing. Clearly then, attempts of modeling *consumption* velocity successfully within this framework would run into the same difficulties already discussed above, related to insufficiently volatile substitution between alternative means-of-exchange.

The present study is complementary to Gillman and Kejak (2007) and related to Hodrick et al. (1991) in the sense that it tries to examine to what extent a de-centralised cash costly credit based on Gillman and Kejak (2008), exhibiting the same upward-sloping marginal cost schedule in credit production as in Gillman and Kejak (2007), is capable of explaining the variability of *consumption* velocity but without resorting to either credit or money growth rate shocks. The present study therefore ignores or holds fixed the direct effects from credit shocks causing shifts in the position of the marginal cost schedule,

[4] But as pointed out and improved on by Gillman and Kejak (2007) within their framework, the model discussed by Cooley and Hansen (1989) shows *too much* procyclicality of velocity.

and instead focuses exclusively on the price- (interest rate-) channel affecting velocity in this class of models. It is the focus on this last point which relates the present study to Hodrick et al. (1991). Holding *both* the credit *and* the money growth rate shocks fixed, the present study thus emphasises a purely goods-productivity driven Wicksellian determination of consumption velocity, in which endogenous money demand and it's response to the real rate of interest matters.

There is a strong motivation for conducting such an experiment, since in practice velocity varies also sufficiently at business cycle frequency (quarterly), which begs to be explained using models calibrated and simulated, and with structural shocks mattering most *at business cycle frequency.* In as far as the quantitative analysis conducted by Gillman and Kejak (2007) - although also based on a calibration using a *quarterly* time horizon, and where positive results are obtained predominantly through the inclusion of credit production shocks - can be understood as an analysis focusing on *institutional* shocks embodied by episodes of financial deregulation which may matter less at business cycle frequency, then the observed volatility of consumption velocity measured *quarterly* still appears to pose a theoretical challenge, as financial-deregulatory credit shocks arguably play a lesser role at shorter frequencies, whereas productivity shocks do. It is this last point which justifies a more in-depth study of the extent to which the goods productivity shock-driven *price-channel* alone is capable of explaining the variability in *consumption* velocity, as nominal interest rate volatility induced through goods productivity shocks may matter more at business cycle frequency[5] .

To this end, a baseline decentralised credit model is presented exhibiting the same convex upward-sloping marginal cost schedule in credit production as in Gillman and Kejak (2007). I abstract from human capital and endogenous growth, which is however crucial to Gillman and Kejak's analysis to identify credit and money shocks using data, as they affect growth in opposite ways. I show how the baseline model, calibrated and simulated *without* either exogenous credit or money growth rate shocks and with a realistically low steady state nominal rate, cannot account for the observed variability in *consumption* velocity, as too little variability in the real and nominal rate (through little variable inflation expectations) leaves the price channel ineffective and the endogenous share of credit in consumption insufficiently variable. I demonstrate, using appropriate simulation graphs, how in the baseline model, credit production moves almost one-for-one with consumption over the business cycle, producing too little variability in the credit share and thus money velocity .

[5] Gillman and Kejak's focus is on the variability of *income* velocity, instead. But novel results are primarily obtained by endogenizing the variability of the *consumption* velocity component.

Then, I add habit persistence in consumption, as in Constantinides (1990) (which is of internal relative habit type), which immediately results in some degree of disentanglement of consumption (deposits) from credit (and therefore also more variable consumption velocity). Essentially, simulation graphs reveal that strong habit in consumption introduces a smooth hump-shaped response of consumption to productivity shocks, while preserving and enhancing a strong contemporaneous endogenous switch between means-of-exchange, leading to a *phase shift* in the frequency domain between credit and consumption (deposits) [6] .

This effect alone is however not strong enough to account for the variability of consumption velocity seen in the data. Only by adding adjustment costs to investment can the model both disentangle consumption (deposits) from credit production (through the habit-induced phase shift) on the one hand, and do so quantitatively sufficiently through an increased volatility in the real rate of interest, on the other, to make consumption velocity vary sufficiently enough so as to match observed variability. A key finding is that the required volatility in real rates is however nowhere near as unrealistically dramatic as in Hodrick et al. (1991), who report interest rate volatility figures of around 30% (in standard deviations) in order to obtain realistic velocity variability, quite the contrary, variability of real and nominal rates is still below the level of volatility observed in the data.

Introducing habit persistence *and* adjustment costs to investment into a cash-in-advance (or alternatively, here, an exchange-in-advance) model has, to my knowledge, not been done before, whereas the introduction of habit only into such models is not new. In particular, Auray et al. (2005) is very close in spirit to the approach taken here, in that they also study Cooley and Hansen's prototypical monetary RBC model, also add relative habit but introduce endogenous variation in velocity through a transactions cost function as in Marshall (1992) and Carlstrom and Fuerst (2001) instead of using costly credit, as in Gillman (1993); Benk et al. (2005); Gillman and Benk (2007). They show that such types of models suffer from indeterminacy (no stable saddle-path solution) for already fairly low values of the relative habit parameter *and* an increasing net real resource cost of using money, introduced through the transactions cost function.

A surprising - but given the aforementioned authors result, very intuitive - complementary result I obtain in the present cash costly-credit model, is that indeterminacy disappears alltogether, regardless of the strength of relative habit chosen. The reason for this is that the present model provides the repre-

[6] Aggregate consumption now turns in to a smoothly evolving endogenous state variable, whereas endogenous credit-money switching retains it's "jump variable" nature.

sentative household with an alternative means-of-exchange to escape the *cash component* of the exchange-in-advance constraint. Crucial to obtaining global determinacy in the cash costly-credit model, is that - although using credit also distorts the margin between consumption and leisure - the representative household's net cost of using credit is zero, as the cost of credit is re-distributed back in terms of the banking wage bill and the return on it's deposits, which feature in credit production [7] .

The contribution of the paper is therefore twofold. Firstly, in as far as credit shocks are of institutional nature and should matter less at quarterly horizons, I explore to what extent the price-channel alone, driven by business cycle frequency shocks to the goods sector productivity alone, can explain the variability seen in consumption velocity. This investigation is thus complementary to Gillman and Kejak (2007), whose results are also driven by their model's high consumption velocity elasticity of credit shocks. Using a similar argument as in Jermann (1998), Hornstein and Uhlig (2000) and Boldrin et al. (2001) [8] , I show how a combination of habit persistence *and* adjustment costs to investment is required to make the price (interest rate) channel sufficiently variable enough so as to induce sufficient variability in credit production relative to consumption (deposits).

Credit production overshooting relative to more autocorrelated and smoothly evolving consumption (deposits) is obtained [9] , where the latter feature combined with adjustment costs increases the volatility of the real rate and thus (for given inflation expectations) of the price of credit leading to the former phenomenon. Assuming strong habit persistence is important, as it introduces a *phase shift* in the frequency domain between credit and consumption, as consumption responds more sluggishly to productivity shocks than credit.

Solving and simulating the model over a whole range of habit persistence parameters, I also demonstrate that indeterminacy is not a problem, as credit is a costless means-of-exchange in terms of net wealth. Secondly, in contrast to Hodrick et al. (1991), whose cash-credit model exhibits a very low interest elasticity of consumption velocity for the reasons discussed above (thus requiring extremely high interest rate volatility to explain volatility in veloc-

[7] Auray et al. (2005) prove that equipping the representative household with a costless means-of-exchange alternative, makes indeterminacy disappear. The present model provides such a costless alternative in terms of credit, thus exhibiting global determinacy.

[8] Interest Rates in canonical RBC models exhibit very little volatility. The three references provide theoretical frameworks using habit persistence *and* inelastic "q-theory" supply of physical capital, to raise the volatility of interest rates.

[9] "Overshooting" here is meant in a percentage change from steady state sense, *not* in an absolute sense, as credit - being a means of exchange for consumption (deposits), can never overshoot beyond consumption in absolute terms.

ity, close to 30% standard deviation), I demonstrate how the present model requires much less variability in the interest rate in order to induce enough variability in consumption velocity so as to match up with the data.

The remainder of the paper is structured as follows. Section 2 describes the baseline de-centralised credit-banking model, and using representative simulations from the solved model, illustrates it's inability to capture the observed variability in *consumption* velocity. Section 3 extends the model to include habit persistence and illustrates how this changes the behaviour of the baseline model. Section 4 extends the baseline-habit model to also include adjustment costs to investment, which raises the volatility in both real and nominal rates, as demonstrated by Jermann (1998). In similar fashion to Hodrick et al. (1991), a sensitivity analysis is conducted, based on simulations of the final extended model using combinations of range of parameter values related to habit persistence and investment adjustment costs. Section 5 discusses the results obtained, section 6 concludes.

2 Decentralised Credit-Banking: A Baseline Model

The representative agent economy is a standard monetary cash-in-advance real business cycle model (Lucas, 1982; Stokey and Lucas, 1987), which is only modified by adding a further means-of-exchange, credit, which is costly produced by a decentralised financial intermediary (FI) by use of a two factor CRS Cobb-Douglas production function. Following Gillman and Kejak (2008), deposits are created from the total exchange liquidity used in the model for carrying out consumption both in terms of money and credit, which means that consumption and (real) deposits can be used interchangeably:

$$d_t \equiv c_t \tag{2}$$

The same amount of deposits (or equivalently the level of consumption) is then used as an input factor to credit production in combination with banking time, which is a credit production specification motivated by the financial intermediation literature (see Hancock, 1985; Clark, 1984). In principle, physical capital could also feature as another input factor in credit production, but is omitted for simplicity.

2.1 The financial intermediary

In contrast to Gillman and Kejak (2008) but similar to Benk et al. (2005), physical capital is only used in the goods production sector, whereas credit production is CRS Cobb-Douglas in labour only and deposits. As in Benk et al. (2005), the economy is modeled such as to assume zero growth. The credit production function is therefore given by:

$$f_t = e^{v_t} A_f \left(n_{f,t}\right)^{\rho} \left(d_t\right)^{1-\rho} \tag{3}$$

where $n_{f,t}$ represents the fraction of labour time spent in the credit production sector, v_t is the credit shock (which throughout the paper is held fixed at it's steady state value) and A_f is the total factor productivity parameter in credit production, which may generally differ in steady state from the same parameter in the goods production sector, analogously given by A_g. The credit production function in the *level* of credit can alternatively be written as a decreasing returns-to-scale deposit-normalised credit share production function (see Gillman and Kejak, 2008) given by:

$$f_t^* = \frac{f_t}{d_t} = e^{z_t} A_f \left(\frac{n_{f,t}}{d_t}\right)^{\rho} \tag{4}$$

Gillman et al. (2007) show in an endogenous growth, Yeoman-version of the same credit economy, how appropriate parametrisation of the diminishing returns parameter ρ leads to a convex upward-sloping marginal cost schedule, which for a given price of credit (equal to the net nominal interest rate), translates into elasticities of money demand of variable size (depending on the *level* of calibrated variables) with respect to key variables, such as the nominal (net) rate of interest and the shock to credit TFP. The properties of the credit production function leading to this and other results is also discussed in more depth in the decentralised steady state discussion provided in Gillman and Kejak (2008). Preference-based cash-credit models, on the other hand, as discussed in Cooley and Hansen (1995) or Stokey and Lucas (1987) exhibit a uniformly much lower interest rate elasticity and abstract from a credit-production sector which may be subjected to shocks. This same low interest elasticity of preference-based cash-credit models is also documented quantitatively in Hodrick et al. (1991).

As in Gillman and Kejak (2008), the financial intermediary is assumed to operate competitively and sets the price of deposits before the household decides how much of the deposits to hold, as with mutual banks. The bank has to obey a solvency restriction, where assets have to be equal to liabilities, given by:

$$P_t f_t + M_t = P_t d_t \tag{5}$$

Also, the liquidity constraint implies that cash sourced from the bank for shopping has to be backed by deposits:

$$P_t d_t \geq P_t c_t \tag{6}$$

The above two constraints collapse into a single one when credit production is zero and deposits only consist of cash balances held with the FI. The financial intermediary is assumed to be profit-maximising, and it's labour-demand can therefore be obtained by solving the problem:

$$\max_{n_{f,t}} R_{f,t} d_t = p_t^f f_t - w_t n_{f,t} \tag{7}$$

where in particular p_t^f is the price in terms of the consumption good the household is paying to the FI per unit of credit used, which in an equilibrium has to equal the cost of otherwise using money, which is the net nominal rate of interest, or $p_t^f = i_{t-1}$. The optimisation problem results in the standard first-order condition of setting the real wage equal to the marginal (revenue) product of labour in credit-production:

$$w_t = p_t^f e^{v_t} A_f \rho \left(n_{f,t}\right)^{\rho-1} \left(d_t\right)^{1-\rho} \tag{8}$$

since the model determines the price of credit endogenously by setting it equal to the opportunity cost of using the alternative means of exchange (money), given by the net nominal rate of interest, the above condition can also be re-stated as:

$$w_t = i_{t-1} \frac{f_t}{n_{f,t}} \tag{9}$$

The value of credit production due to deposits (consumption) is then re-distributed back to the representative household in form of a dividend per-unit of deposits (which are equal to consumption). Since the unnormalized value (i.e. the total revenue share due to deposits in credit production) is given by $R_{f,t} = f_t \left(1-\rho\right) i_{t-1}$, the normalised dividend or return is thus given by:

$$\tilde{R_{f,t}} = \frac{R_{f,t}}{d_t} = \left(\frac{f_t}{d_t}\right) \left(1-\rho\right) i_{t-1} = f_t^* \left(1-\rho\right) i_{t-1} \tag{10}$$

Since this term is paid out per unit of deposits and therefore per unit of consumption, the model will exhibit an exchange cost of consumption different from standard cash-in-advance models, which is an average exchange cost distorted by the cost of producing credit (see Gillman and Kejak, 2008), to be discussed in more detail in the following section describing the representative household's optimisation problem.

2.2 The goods firm

The firm producing aggregate output y_t is spelled out in decentralised fashion and is also assumed to be the owner of the stock of physical capital. It maximises the net present value of cash flows remitted back to the household in form of dividend payments made on equity holdings, where discounting is carried out such as to respect the stochastic discount factor of the household. The firm's problem is therefore formulated as:

$$\max_{n_{g,t},k_t} E_t \sum_{k=0}^{\infty} \frac{\beta^k \lambda_{t+k}}{\lambda_t} \left\{ y_{t+k} - w_{t+k} n_{g,t+k} - i^k_{t+k} \right\} \tag{11}$$

where $n_{g,t}$ is the fraction of time spent in goods production and i^k_t is the amount of investment into physical capital, which the firm pays for exclusively from retained earnings. The technology employed in producing aggregate output is given by a standard constant returns-to-scale Cobb-Douglas production function, given by:

$$y_t = e^{z_t} A_g \left(n_{g,t}\right)^{\alpha} \left(k_{t-1}\right)^{1-\alpha} \tag{12}$$

where A_g is the (steady state) goods sector total factor productivity parameter, z_t the corresponding productivity shock, and k_{t-1} the predetermined level of physical capital employed in production. Investment in physical capital i^k_t satisfies the following constraint:

$$i^k_t = k_t - (1-\delta) k_{t-1} \tag{13}$$

Maximisation of the firm's problem then yields the usual first-order conditions of employing up to the point at which the wage rate equals the marginal product of goods labour, and of installing more physical capital up to the point where the marginal cost today is equal to the discounted future return in terms of the future marginal product of capital net of depreciation. The former condition implies:

$$w_t = \alpha e^{z_t} A_g \left(n_{g,t}\right)^{\alpha-1} \left(k_{t-1}\right)^{1-\alpha} = \alpha \frac{y_t}{n_{g,t}} \tag{14}$$

the latter condition related to the optimal amount of physical capital implies (the consumption Euler equation):

$$\begin{aligned} \lambda_t =& \beta E_t \lambda_{t+1} \left[(1-\alpha) e_{z_{t+1}} A_g \left(n_{g,t+1}\right)^{\alpha} \left(k_t\right)^{-\alpha} + (1-\delta) \right] \\ =& \beta E_t \lambda_{t+1} \left[(1-\alpha) \frac{y_{t+1}}{k_t} + (1-\delta) \right] \\ =& \beta E_t \lambda_{t+1} \left[r^k_{t+1} + (1-\delta) \right] \end{aligned} \tag{15}$$

where r^k_{t+1} is the future expected marginal product (or marginal net return exclusive of depreciation) of the current level of installed units of physical capital, decided upon optimally in period t.

The representative household receives it's only non-financial income from selling it's labour endowment to the goods and credit sector at the equilibrium real wage. Since physical capital is assumed to be owned by the goods firm and the optimal investment decision left to the latter, besides receiving it's wage bill, the household also holds money and a vector of financial assets, which may include a risky stock in the firm and inflation-indexed real bonds, but possibly also other assets [10] . Coupled with the vector of assets are corresponding price and dividend vectors, given by p_t^a and d_t^a, respectively, where as an example the "dividend" of the inflation-indexed (or real) bond is just equal to 1, i.e. paying the representative household one unit of the consumption good. Utility is derived from following function:

$$U_t\left(c_t, l_t\right) = \log c_t + \Psi \log l_t \tag{16}$$

where I have assumed the representative household's utility to be separable in it's two arguments, consumption c_t and leisure l_t, and logarithmic in both consumption and leisure. The household's budget constraint is therefore given by:

$$w_t\left(n_{g,t} + n_{f,t}\right) + a'_{t-1}\left(p_t^a + d_t^a\right) + \frac{m_{t-1}}{(1+\pi_t)} + v_t \tag{17}$$

$$+ \tilde{R}_{f,t} c_t \geq c_t + a'_t p_t^a + m_t + p_t^f f_t \tag{18}$$

where w_t is the real wage, $n_{g,t}$ the amount of labour time spent in goods production and $n_{f,t}$ the amount of time spent in credit production. Notice that the total time endowment of the representative household is normalised to one, translating into the following time constraint:

$$1 - l_t = n_{g,t} + n_{f,t} \tag{19}$$

Using credit incurs a cost in terms of a price charged per unit of credit, so that the total cost from using credit f_t is given by $p_t^f f_t$. The household also receives a dividend payment from the financial intermediary in form of a return per unit of deposit, translating into a total payout due to deposits held with the FI given by $\tilde{R}_{f,t} c_t$. This dividend distorts the usual marginal rate of substitution between consumption and leisure so as to be different from an otherwise standard CIA model. Using the first-order conditions with respect to consumption and leisure results in the steady state relationship:

$$MRS_{c,l} = \frac{\Psi c_t}{l} = \frac{1+\tilde{i}}{w} \tag{20}$$

[10] I follow the notation chosen by Jermann (1998).

where $1+\tilde{i}$ is given by:

$$\begin{aligned}\left(1+\tilde{i}\right) &= 1+i-f^{*}\left(1-\rho\right)i \\ &= 1+\left(1-f^{*}\right)i+f^{*}\rho i \end{aligned} \tag{21}$$

which shows that the distorted exchange cost affecting the marginal rate of substitution between consumption and leisure equals an average exchange cost which endogenously varies with the share of credit used in consumption (see Gillman and Kejak, 2008, 2005). Further, m_{t-1} is the real value of predetermined money balances held at the beginning of the period and v_t represents some real-valued lump-sum tax governing the growth rate of the money supply. The household purchases the consumption good subject to an exchange constraint, given by [11] :

$$\frac{m_{t-1}}{(1+\pi_t)}+v_t+f_t \geq c_t \tag{22}$$

where both money and the credit exchange service can be used in conjunction to pay for the consumption good. For a positive nominal rate of interest, the constraint always binds in a strict sense, which is assumed throughout. Notice that by defining the multiplier on the budget constraint to be equal to λ_t and the multiplier on the liquidity constraint to be μ_t, taking first-order conditions with respect to credit f_t, results in:

$$p_t^f=\left(\frac{\mu_t}{\lambda_t}\right)=i_{t-1} \tag{23}$$

demonstrating that in the decentralised credit-banking model, in equilibrium the price of credit has to equal the opportunity cost of otherwise using money, which equals the net nominal rate of interest, i_{t-1}. The growth rate of the money supply has a deterministic and could in principle also be given some random component, and could thus be defined as:

$$m_t=\Theta_t\frac{m_{t-1}}{(1+\pi_t)}=\left(\Theta^{*}+e^{u_t}-1\right)\frac{m_{t-1}}{(1+\pi_t)} \tag{24}$$

where u_t represents the stochastic component of the money supply growth rate, which is modeled assuming a log-normally distributed autoregressive process of order one, as is the productivity shock in the goods sector. However, throughout I am going to set this shock equal to it's steady state value, and thus assume that:

$$m_t=\frac{\Theta^{*}}{1+\pi_t}m_{t-1} \tag{25}$$

[11] Throughout I am going to assume a *fixed* "k-percent" Friedman-type growth rate rule of the money supply, meaning that $v_t=\bar{v} \quad \forall \quad t$

where Θ^* represents the exogenously specified deterministic gross growth rate of the nominal money supply. Therefore, I am going to assume a Friedman-type "k-percent" deterministic growth rate of the nominal supply of money Friedman (1960). Notice also that although, for purposes of comparison, the present model has been described as also containing a shock to the productivity of the credit sector, as in Gillman and Kejak (2007), throughout this paper I am going to set this shock equal to it's steady state value and keep it fixed in simulations. The vector of shocks can thus be summarised as $s_t = [z_t, u_t, v_t]$, where in simulations this vector assumes the shocks to be modeled in logs and to be autoregressive of order one with innovations normally distributed and zero off-diagonal variance-covariance matrix. Therefore, formally, the structural shocks affecting the stochastic off steady state characteristics of the model can be summarised in VAR form as:

$$s_{t+1} = \Phi s_t + \epsilon_{t+1} \tag{26}$$

where Φ is a $3x3$ matrix containing the autocorrelation parameters specified along the diagonal of Φ and $\epsilon \sim (0, \Omega)$. Although coefficients of autocorrelation for the structural credit productivity shock as well as the money growth rate shock may be formally be specified, they will not matter in practice, as the structural credit and money shock innovations $\epsilon_{v,t}$ and $\epsilon_{u,t}$ will always be set equal to zero. Since the economy contains no growth trend, and all variables have been expressed normalised by dividing by the relative price of money P_t, the definition of the equilibrium can be set up in recursive form. Denoting the state of the economy as $s_t = [k_{t-1}, m_{t-1}, z_t, u_t, v_t]$, and with $\beta \in (0, 1)$, the representative household's optimisation problem can be written in recursive form as:

$$V(s) = \{\log c_t + \Psi \log l_t + \beta EV(s')\} \tag{27}$$

The model is solved by symbolically differentiating the first-order conditions, market-clearing identities and exogenously specified laws of motion with respect to current endogenous states, future exogenous states and future endogenous jump (control) variables, as well as with respect to pre-determined past period endogenous states, current exogenous states and current period jump variables, thus obtaining the Jacobian of the system. Before differentiating, all variables (except for rates) will have been expressed in logs. The Jacobian can then be evaluated at the (log) steady state, split into matrices A and B, which can be used to solve for the recursive law of motion using the Schur decomposition as documented in Klein (2000) and Klein and Gomme (2008). I therefore solve for the recursive law of motion using a first-order perturbation method, where the local approximation is taken around the log steady state of the system (except for rates, which are in levels), so that the matrices describing the solution to the system typically contain elasticities and thus percentage changes. The resulting stationary recursive laws of motion are expressible as:

$$\mathbf{X}_t = P\mathbf{X}_{t-1} \tag{28}$$
$$\mathbf{Y}_t = F\mathbf{X}_{t-1} \tag{29}$$

where the matrices P and F contain the elasticities and describe the solution to the system and vectors $\mathbf{X}$ and $\mathbf{Y}$ contain the endogenous and exogenous states, and the endogenous control (jump) variables, respectively. A stable (non-explosive) solution requires all elements in P to lie within the unit circle, so as to make the evolution of the endogenous and exogenous states behave according to some stationary process.

2.4 Credit Production: The Source of Variability in Money Velocity

Before discussing the choice of calibrated values for relevant variables determining the steady state and off-steady state locally approximated dynamics, it will be instructive to discuss the source of variability in consumption velocity within the decentralised credit-banking model. A similar analysis is also carried out in Gillman and Kejak (2007), who discuss and derive the interest rate elasticity of velocity using the same credit production function. Here, instead of focusing on consumption-money velocity directly (given by $M_t/P_t c_t = m_t/c_t$, I conduct my discussion using the inverse of relevant velocity measures (which are the means-of-exchange *shares* in consumption), as volatility in the inverse of a velocity measure translates into volatility of that velocity measure itself. Therefore, substituting the implied labour demand in the credit production sector back into the credit production function, I obtain

$$f_t^* = (A_f e^{v_t})^{\frac{1}{1-\rho}} \left(\frac{\rho i_{t-1}}{w_t}\right)^{\frac{\rho}{1-\rho}} \tag{30}$$

which can be log-linearised around the log of steady state variables, to give:

$$\hat{f}_t^* \equiv \hat{f}_t - \hat{c}_t = \left(\frac{1}{1-\rho}\right)\hat{v}_t + \left(\frac{\rho}{1-\rho}\right)\hat{i}_{t-1} - \left(\frac{\rho}{1-\rho}\right)\hat{w}_t \tag{31}$$

Setting the credit shock equal to zero, which is assumed throughout the present paper, and recalling that $\hat{i}_{t-1} = \hat{p}_t^f = \hat{\mu}_t - \hat{\lambda}_t$, the above equation can be written as:

$$\begin{aligned} \hat{f}_t^* \equiv \hat{f}_t - \hat{c}_t &= \left(\frac{\rho}{1-\rho}\right)\left(\hat{p}_t^f - \hat{w}_t\right) \\ &= \left(\frac{\rho}{1-\rho}\right)\left(\hat{\mu}_t - \hat{\lambda}_t - \hat{w}_t\right) \end{aligned} \tag{32}$$

Notice then, since $\hat{f}_t^*$ is the log-deviation of the share of credit used in purchasing consumption, then $\hat{m}_t^* \equiv \hat{m}_t - \hat{c}_t = \left(1 - \frac{\bar{c}}{\bar{m}}\right) \hat{f}_t^*$ represents the log-deviation of the share of money used in purchasing the consumption good. Sufficient variability in the latter defined share, $\hat{m}_t^*$, translates into sufficient variability of it's inverse $(\hat{m}_t^*)^{-1} = V_t^m$ which is equal to money velocity.

This means that the credit share has to be sufficiently variable in order to obtain sufficient variability in money consumption velocity. The choice of calibrating the labour share parameter ρ in credit production, which matters in the present discussion as it affects the elasticity of the credit share with respect to $\hat{v}_t$, $\hat{i}_{t-1}$ and $\hat{w}_t$, has varied in studies conducted thus far. For instance, Benk et al. (2005) base their calibration of $\rho = 0.21$ on a time series estimate conducted by Gillman and Otto (2005). Gillman and Kejak (2007) obtain a lower calibrated value at $\rho = 0.13$, and show how it can be obtained using financial industry data, Scheffel (2008) also calibrates the credit labour share value to 0.21 so as to match the model's predictions of asset prices. Also, related to this Gillman and Kejak (2008) prove how a value of $\rho < 0.5$ is required for the marginal cost schedule to be convex.

In the present study, I will calibrate $\rho = 0.18$, so as to be comparable to previously chosen calibrations. As previous studies have indicated that a realistic range of this value appears to be $0.1 < \rho < 0.25$, this leads to the direct consequence that credit shocks can potentially have much stronger effects on money velocity than either changes in the price of credit or the wage rate. Using $\rho = 0.18$ as a benchmark case, leads to a credit share elasticity with respect to $\hat{v}_t$ equal to $\eta_v = \left(\frac{1}{1-\rho}\right) = 1.22$, whereas the same elasticity for the price of credit and the wage rate is $\eta_i = \eta_w = \left(\frac{\rho}{1-\rho}\right) = 0.22$, much lower. Disregarding credit (and money growth rate) shocks alltogether and only focusing on how goods productivity shocks can affect money velocity through the price (and indirectly also the wage) channel, requires the price-wage (or interest-wage) ratio to be volatile enough, which as I will show can be achieved by making the real (an for given inflation expectations) and thus also the nominal rate more volatile.

Indeed, as I will demonstrate, combining habit persistence in consumption with adjustment costs to investment as in Jermann (1998), leads to a more volatile behaviour of the representative household's marginal valuation, $\hat{\lambda}_t$, a consequently more volatile stochastic discount factor and thus also a more volatile behaviour of the real rate of interest. As the volatility of marginal valuation also affects the volatility of the credit share (and thus money velocity), this - for some given conditional behaviour of the wage rate - can potentially enhance the effect of the price channel alone on consumption velocity. Before turning to the baseline model's simulation evidence, and later one similar evidence from extended version of the baseline model, I will summarise the calibration of the model in the section which follows.

2.5 Steady State Calibration

Table of benchmark calibrated Parameters			
β =0.99	discount factor	ρ =0.18	credit labour param.
α =0.64	goods labour param.	f^*=0.31	credit-to-cons ratio
A_g=1.0	TFP goods	A_f=1.204	TFP credit
l =0.7	leisure	n_f=0.00044	credit labour
Θ =1.0125	money g.	n_g=0.29956	goods labour
b =0.8	habit pers.	κ =3.0	cap. adj. cost
ϕ_u=0.70	AR money g. shock	ϕ_z=0.95	AR goods shock
ϕ_v=0.95	AR credit shock	ϵ_z =0.0075	s.d goods shock
ϵ_u =0.0(set to 0)	s.d. moneyg shock	ϵ_v =0.00(set to 0)	s.d credit shock

Table 1
Baseline Calibration

The above table summarises the chosen baseline calibration - where in anticipation of extensions to the baseline model using habit in consumption and adjustment costs to investment - the table already contains calibrated values for parameter values relevant for the extensions as well. Turning attention to the calibrated parameters, first of all, the discount factor β, the labour share in goods production α, the steady state total labour-leisure trade-off l and $n_f + n_g$ and the steady state growth rate of money, are all set to standard values familiar from calibration exercises of similar models conducted elsewhere. Specifically then, labour in goods production receives approximately 2/3 of the value of production in form of the goods production wage bill and leisure l is 70% of the total time endowment. Also, by calibrating the discount factor $\beta = 0.99$, I obtain an annualised steady state real rate of 4%, and by setting the quarterly gross growth rate in the money supply $\Theta = 1.0125$, I obtain an annualised steady state growth rate of inflation of 5%.

For the credit-banking sector, I chose calibrated values for the steady state share of credit $f^* = 0.31$ to be very close to the same value chosen by Benk et al. (2005) (who set this equal to 0.3) and the labour share parameter $\rho = 0.18$, which is slightly less than what the same authors choose (they set this equal to 0.21). Given the TFP value in the goods sector of $A_g = 1.0$, finding the root of a non-linear system of equations residually determines $n_f = 0.00044$ (which is very close to Benk et al.'s obtained 0.00049) and $A_f = 1.204$ (compared to Benk et al.'s obtained value of 1.422). Structural shock autocorrelation parameters are also chosen in standard fashion and are set to $\pi_u = 0.70$ and $\phi_z = 0.95$, with standard deviations of $\epsilon_u = 0.01$ and $\epsilon_z = 0.0075$, respectively, all of which are as in Benk et al. (2005).

Notice that, although the autocorrelation parameters on the credit and money growth shocks are formally calibrated, the standard deviation of the innovation is set to zero for both cases, so as to effectively eliminate those shocks in simulations of the solved model. The habit persistence parameter was calibrated to $b = 0.8$ as in Jermann (1998) and Constantinides (1990). Finally, using a specification of the adjustment cost function to investment from Canzoneri et al. (2007a), the relevant parameter $\kappa = 3.0$, which is much smaller than Canzoneri et al.'s chosen value of 8.0, thus calibrating the model on the conservative side regarding investment adjustment costs. Next, I am going to discuss simulation results obtained from the baseline model.

2.6 Simulation results

In this section I am going to present simulation evidence from the calibrated baseline model. Evidence is presented in two different ways. First of all, graphs of a representative simulation run are provided so as to allow visual inspection and verification of key properties. All graphs also show standard errors (of percentage deviations) [12] - and autocorrelation coefficients - from the representative one-off simulation, so as to convey clearly the volatility of various simulated time series. To emulate a typical quarterly post-war sample size, the simulation length is always fixed at $n = 200$, all simulated series are hp-filtered prior to graphing and computing relevant statistics. Also, shocks to productivity have been drawn once and then kept fixed in graphs across *all* extended versions of the baseline model (inclusive of the baseline model itself), so as to make simulation graphs directly comparable across model versions.

Secondly, tables with key statistical measures computed from simulations of the model with habit persistence and adjustment costs to investment are provided, which are not based on one, but 1000 simulations so as to obtain expected values and standard errors of statistical measures. The simulation length is, as before, held fixed at $n = 200$. Tables are only computed for the final extended version of the baseline model, as for appropriate choices of the parameter space, this model version nests all other models versions. The main focus throughout is to examine if sufficient variability in (money-) consumption velocity can be obtained without requiring too volatile interest rate series (both real and nominal), but other key statistics are also examined. Regarding the definition of "sufficient" variability in consumption velocity, I use the

[12] where interest rates are quarterly deviations.

reported sample means calculated by Hodrick et al. (1991), who measure variability in velocity using the coefficient of variation statistic to capture this measure [13].

For annual data, they obtain a correlation of variation approximately equal to 0.46 with a standard error of 0.0097, and for quarterly data corresponding values of approximately 0.4 with standard error 0.006. A secondary concern, also examined by Hodrick et al. (1991), is whether this can be achieved with statistics of other key variables lying within plausible ranges as well, where my approach is more focused here mainly emphasising the joint volatility of interest rates. So with regards to this, I take the computed sample value of the percentage standard deviation of the real risk-free rate in Mehra and Prescott (1985) as a benchmark, which is also reported in Jermann (1998), and is equal to 5.76%. So a model calibrated quarterly needs to obtain a value for the coefficient of variation in consumption velocity of 0.4 or more *and* a standard deviation of the real rate in the neighbourhood of 5.76% [14] in order to successfully jointly capture the second moments of consumption velocity and real interest rates.

Figure (1) documents the behaviour of various key variables of the baseline credit-banking model described in the theoretical section, using graphs of one representative simulation run. First of all, as is usual for standard (cash-in-advance monetary) real business cycle models containing no additional frictions, the stochastic discount factor (and thus the real rate) is conditionally relatively high whenever (expected) consumption is higher than on average. As usual, since this coincides with lower current period marginal valuation vis-a-vis expected future marginal valuation (thus leading to a conditionally low intertemporal marginal rate of substitution), the conditional real interest rate is high, so as to prevent the household from borrowing against the expected future rise in consumption [15].

More importantly, the top and bottom left-hand quadrants illustrate that the baseline model's variability in the nominal rate of interest is very low, thus also leading to very little variability in the price of credit. This in turn leaves the variability in the share of credit in consumption very low, thus resulting in a very low variability of consumption velocity. The nominal rate turns out to be so dampened in its movement relative to the real rate, as increases in consumption (and thus liquidity demand) is not entirely covered

[13] This is defined as the ratio of the standard deviation to the mean, so it is essentially equal to a % standard deviation, which has the advantage of being scale-free.
[14] Actually, in principle assuming a much lower variability in real rates is permissible, as the latter reported figure of Mehra and Prescott (1985) is based on the variability of *ex-post* realised real rates, as opposed to their *ex-ante* expected counterparts, using *expectations* of inflation.
[15] (see Uhlig, 1995, p.15).

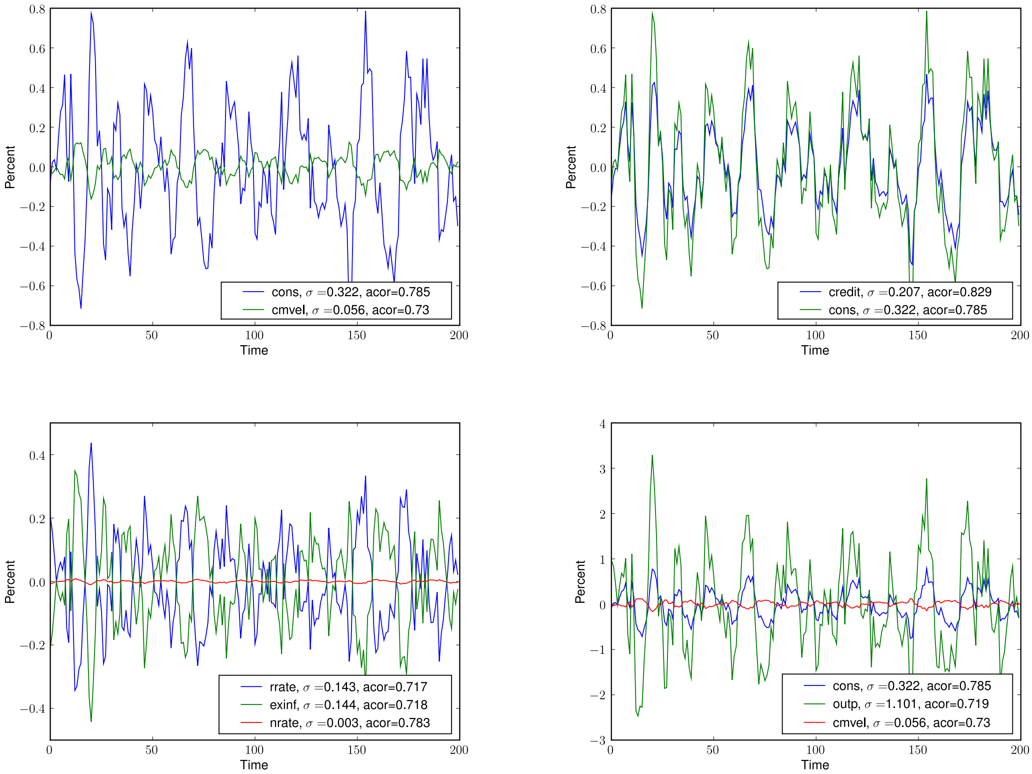

Fig. 1. Baseline Model

by increases in credit, and thus has to be partially also met by increasing real money balances via *a fall* in (expected) inflation. In this particular case, inflation expectations moving in opposite direction to the real rate, leave the nominal rate fairly invariant. A similar situation occurs in the model with habit persistence, where I discuss this property in more detail.

3 Adding habit persistence

In this section I am going to investigate the model's properties which are obtained by adding habit persistence in consumption, which is of relative habit type as in Constantinides (1990), and not of "keeping-up-with-the-Joneses" type, as in Abel (1990). Doing so amounts to a modification of the utility function of the representative household, thus affecting the stochastic as well as the steady state expression for the marginal utility of consumption. The modified utility function including habit persistence is thus given by:

$$U_t = \log\left(c_t - bc_{t-1}\right) + \Psi \log l_t \tag{33}$$

The marginal utility with respect to current consumption will therefore also include a term taking into account how the choice of current consumption affects next period's marginal utility. Therefore:

$$U_{c,t} = \frac{\partial U_t}{\partial c_t} = \frac{1}{(c_t - bc_{t-1})} - b\beta \frac{1}{(E_t c_{t+1} - bc_t)} \tag{34}$$

Habit persistence eliminates the time-separability in consumption, as current marginal valuation not only depends on current consumption, but on current consumption relative to some fraction of last period's level of consumption. Notice that although this creates a dynamic smoothing objective for consumption [16], the steady state value of marginal valuation *and* the marginal rate of substitution between consumption and leisure are practically unaltered. This follows from writing down the steady state version of the marginal utility of consumption:

$$\bar{U}_c = \frac{(1-\beta b)}{(1-b)\,\bar{c}} \approx = \frac{1}{\bar{c}} \tag{35}$$

which for $\beta = 0.99$ can be approximated by the steady state of the marginal utility of consumption for the logarithmic case

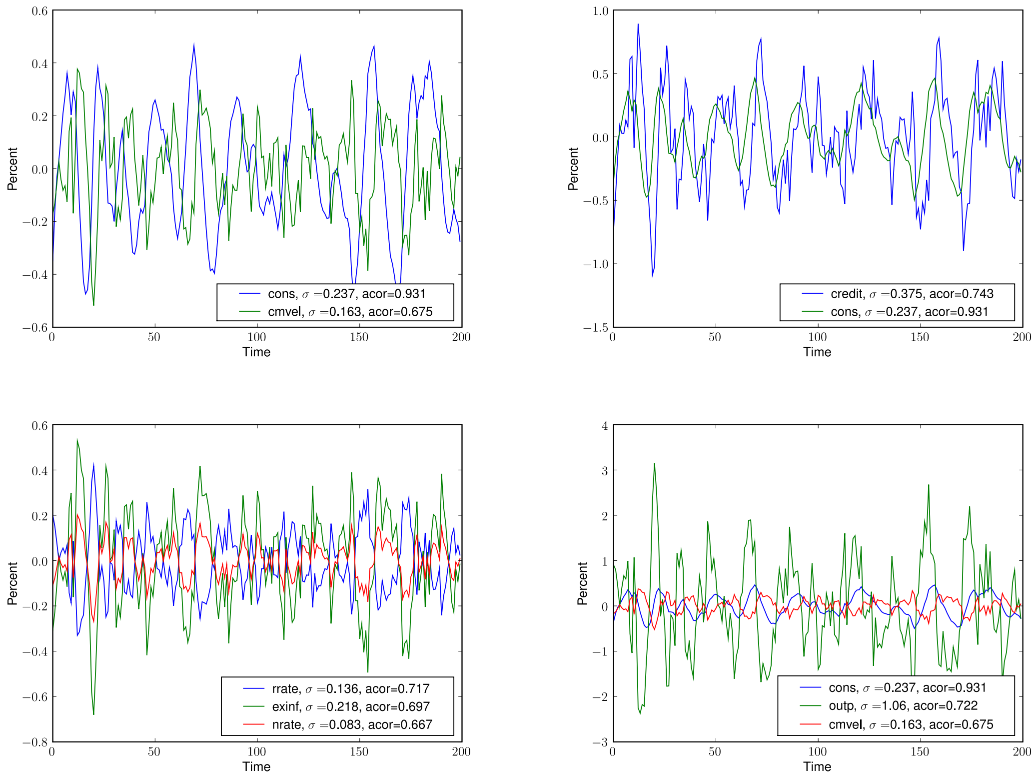

Fig. 2. Baseline Model with habit persistence

[16] i.e. the evolution of consumption will optimally be more autocorrelated (see den Haan, 1995).

Figure (2) illustrates the conditional behaviour of key variables of the credit-banking model with habit persistence in consumption. The top right-hand quadrant diagram documents how the representative household's smoothing objective (relative to the previous level of consumption) leads to an endogenous consumption process which is highly autocorrelated and much smoother compared to the baseline model's prediction, in which consumption responds in proportional fashion to productivity shocks hitting the goods production sector. Due to habit persistence, the same quadrant also reveals how consumption is less volatile, whereas credit becomes more volatile, in particular relative to consumption. In general, compared to the baseline model, the level of credit becomes to some extent disentangled from the endogenous consumption process, and since past consumption works as a "drag" on current consumption, current consumption reacts less responsively to current period productivity shocks.

The top left-hand quadrant shows consumption-money velocity graphed against the level of consumption. In spite of the already mentioned inertia displayed in the endogenous consumption process, which results in velocity leading consumption slightly, the graph still displays that consumption-money velocity is counterfactually counter-cyclical with both consumption (but less so than with output, due to consumption's habit-induced inertia) and output. The counter-cyclicality of velocity displayed by the baseline model with habit persistence only thus runs counter evidence from the U.S. documented by Hansen (1985) and Gillman and Kejak (2007). At the same time both the top left-hand and bottom right-hand quadrants document how adding habit persistence to the baseline model results in a much more volatile consumption velocity series, without leading to too volatile interest rate series. Along this dimension, adding habit persistence improves the production-based baseline model, increasing consumption velocity variability to reach a coefficient of variation equal to 0.173, close to trebling the same measure obtained in the baseline model without habit persistence.

The bottom left-hand quadrant shows how the volatility of the real rate is not significantly different from the baseline model, whereas the nominal rate is now much more volatile and generally moves in opposite direction to the real rate. This means in particular that following a positive shock to goods productivity, the economy expands and the return on physical capital rises (thus also leading to a rise in the real rate of interest). The higher productivity in the goods sector relative to the credit production sector leads to a movement of labour to the former, thus resulting in a drop of credit production.

Less credit produced means more money demanded for some level of consumption purchases, which requires pre-determined money balances to be adjusted upwards by a drop of inflation below it's steady state value and convergence of the latter from below. But this implies a fall in inflation expectations which -

through the Fisher equation - is strong enough so as to result in a nominal rate moving in opposite direction to the real rate. In order to illustrate this last point better, figure (3) shows the co-movement of consumption, the nominal and real rate, and expected inflation, which through the Fisher relationship will move so as to equate the two rates in real terms, adjusting the nominal rate by inflation.

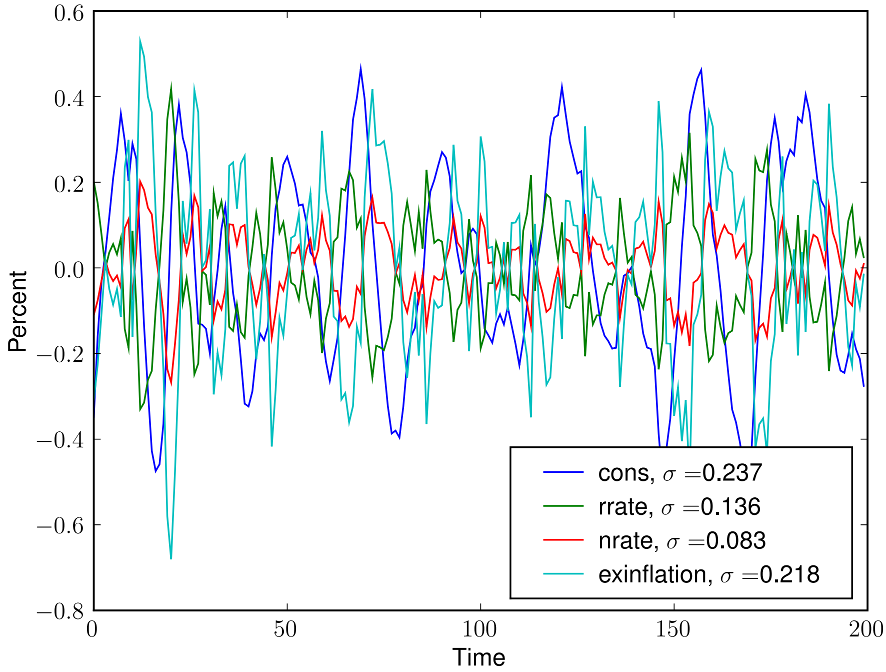

Fig. 3. Baseline Model with habit persistence, prod. shock only

4 Adding adjustment costs to investment

In this section I am going to investigate the model's properties which are obtained by adding also adjustment costs to investment to the baseline model in addition to habit persistence in consumption. Adjustment costs have been studied by Eisner and Strotz (1963), Prescott and Lucas (1971), Hayashi (1982), Baxter and Crucini (1993) and Jermann (1998). They typically also feature in many models of the new neoclassical synthesis type, such as Lawrence J. Christiano and Evans (2005) and Canzoneri et al. (2007b). This modification requires a discussion of the modified problem of the decentralised aggregate output producing firm. First of all, notice that I use an adjustment cost function as in Canzoneri et al. (2007a). This function is given by:

$$\zeta\left(\frac{i_t^k}{k_{t-1}}\right) = \frac{\kappa}{2}\left(\frac{i_t^k}{k_{t-1}} - \delta\right)^2 k_{t-1} \tag{36}$$

Since in steady state I obtain $\bar{i^k} = \delta\bar{k}$, adjustment costs will not matter for steady state calculations, as they will be equal to zero[17]. However stochastically (or off steady state), inclusion of this adjustment cost function means that the equation describing the evolution of physical capital is modified to give:

$$i_t^k - \zeta\left(\frac{i_t^k}{k_{t-1}}\right) = k_t - (1-\delta)\, k_{t-1} \tag{37}$$

The firm's problem is then setup differently, where first-order conditions with respect to capital *and* investment have to be taken. The firm's modified problem is thus stated as:

$$\max_{n_{g,t+k},k_{t+k}} E_t \sum_{k=0}^{\infty} \frac{\beta^k \lambda_{t+k}}{\lambda_t} \Bigg\{ y_{t+k} - w_{t+k} n_{g,t+k} - i_{t+k}^k + \frac{\xi_{t+k}}{\lambda_t}\left[i_{t+k}^k - \zeta\left(\frac{i_{t+k}^k}{k_{t+k-1}}\right) - k_{t+k} + (1-\delta)\, k_{t+k-1} \right] \Bigg\} \tag{38}$$

where the multiplier on the firm's capital accumulation constraint is equal to marginal utility in steady state only, i.e. $\bar{\lambda} = \bar{\xi}$. Notice that the ratio of the marginal value of installed physical capital to the marginal value of one extra unit of wealth is equal to Tobin's q, i.e. $q_t = \xi_t/\lambda_t$. Taking first-order conditions with respect to investment i_t^k and the end-of-period physical capital stock k_t, results in the following conditions of optimality, which differ from the baseline model:

$$\lambda_t = \xi_t \left[1 - \zeta_i'\left(\frac{i_t^k}{k_{t-1}}\right)\right] \tag{39}$$

which is the first-order condition with respect to investment, where $\zeta_i'\left(\frac{i_t^k}{k_{t-1}}\right)$ is the derivative of the adjustment cost function with respect to investment. Optimality with respect to physical capital yields:

$$\xi_t = \beta E_t \left\{ \lambda_{t+1} r_{t+1}^k + \xi_{t+1}\left[(1-\delta) - \zeta_k'\left(\frac{i_{t+1}^k}{k_t}\right)\right]\right\} \tag{40}$$

Figure (4) summarises key results of the baseline model with habit and adjustment costs to investment using graphs of a representative simulation run.

[17] This property does not only hold for the average adjustment costs, but also the derivatives with respect to capital and investment.

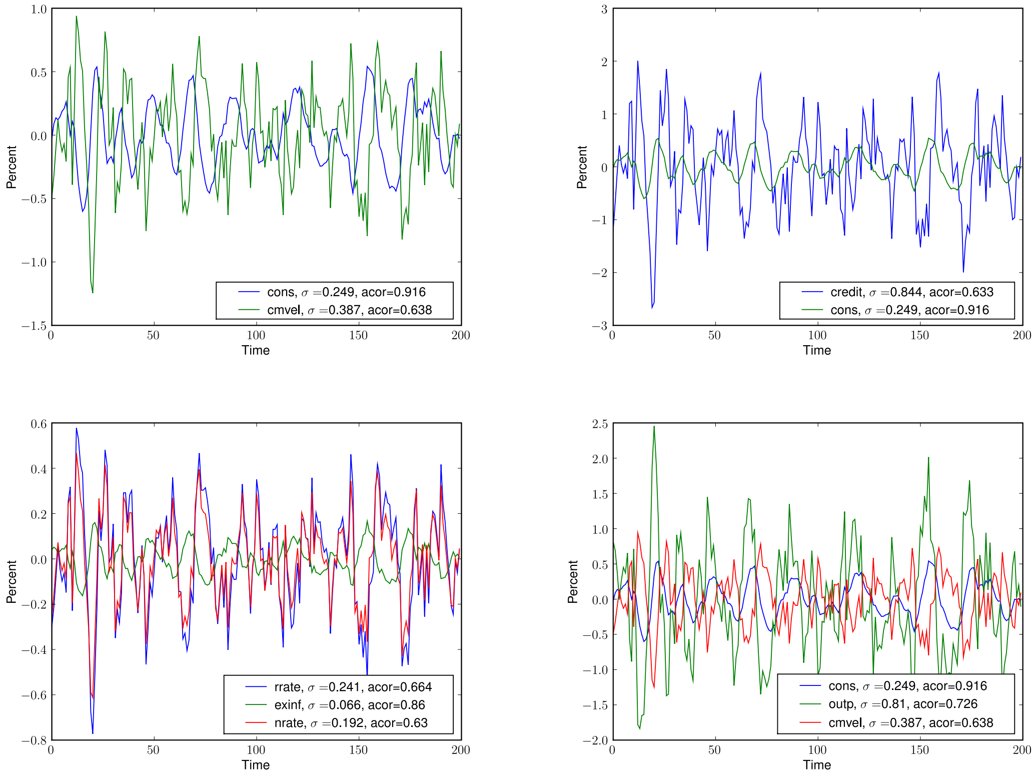

Fig. 4. Baseline Model with habit persistence & adjustment costs

Adding adjustment costs to investment to the model which already contains habit in consumption has two well-known effects documented by Jermann (1998). Firstly, it raises the volatility of consumption in comparison to the habit-only model, as the representative household faces a more inelastic supply of the physical capital storage technology, i.e. higher demand in savings (consumption smoothing) goes hand in hand with higher adjustment costs incurred from equating savings to investment, thus resulting in consumption smoothing to be less successfully implemented (or to be frustrated by high values of q_t in times when the household would want to save more). Secondly, combining habit persistence (which makes the representative household care more about volatility in the *absolute level* of consumption) with the inelastic supply of investment opportunities leads to a more volatile series of the marginal value of wealth (or marginal valuation) thus increasing the volatility in real interest rates.

For this version of the baseline model, the interplay between the goods and liquidity markets (which are linked through the Fisher equation) happen to be such as to result in relatively little volatility in inflation expectations, thus causing most of the *real* interest rate volatility to directly translate into an equivalent volatility in the *nominal* rate of interest as well. Also, in contrast to the habit-only model, real and nominal rates move in tandem, which is an improvement as this has been found to be the case in various studies (see Mishkin, 1982, 1990b,a, 1992). Real and nominal interest rate volatility, which is almost doubled for the former and trebbled for the later in comparison to the habit-only model, thus leads to an increased volatility of the *price-channel* in credit production, which is the main focus of the present study.

For a given fixed upward-sloping marginal cost schedule in credit production, the increased volatility in the price of credit leads to a larger variability in the production of credit relative to consumption (or deposits) and thus also to a more volatile consumption velocity process, which now possesses a coefficient of variation close to 0.4, thus making this version of the original baseline model capable of successfully explaining the variability in consumption velocity observed in the data. The key difference between the baseline model and this version incorporating habit and adjustment costs, can be discovered by comparing the respective model variants' top right-hand quadrants, which graph consumption and the level of credit.

Whereas the volatility of consumption is approximately the same, credit production now overshoots [18] consumption in percentage terms. As in Jermann (1998), here the very rigidity or inertia in consumption coupled with adjustment costs of investment leads to an increased volatility in the price of credit, which causes the latter to be much more volatile. In Jermann (1998), the *real* rate becomes more volatile as a consequence of habit formation and adjustment costs, but here the same volatility also carries over into the nominal rate, via the Fisher equation and the inflation rate behaviour implied by *counter-cyclical* credit production.

In contrast to Gillman and Kejak (2007), the successful modeling of consumption velocity is obtained without any shocks to credit productivity at all, but exclusively by raising the volatility of the price of credit. Improving on Hodrick et al. (1991), the model only requires modest real and nominal rate volatility, which are much less volatile than the aforementioned authors' reported interest rate volatility of approximately 30%. Finally, the habit-adjustment-cost variant also reverses the negative finding of the habit-only model, in which velocity was found to be counter-cyclical, which is now found to be pro-cyclical again, as observed in the data.

[18] actually, more precisely it *undershoots* following a positive shock to productivity, and it is real money balances which are now procyclical and overshooting, as observed in the data. But due to the inert behaviour of consumption, relative to this series, overshooting sometimes is apparent.

5 Sensitivity analysis of perturbation of the parameter space

In what follows, I present a sensitivity analysis based on a large number of simulations and over some range of calibrated parameters of interest, where I wish to focus in particular on the degree of habit persistence (the parameter b), and the extent to which adjustment costs to investment matter (the parameter κ). In particular, I choose a parameter space for b equal to $b = [0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9]$ and for κ equal to $\kappa = [0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0, 5.5, 6.0]$ which amounts to simulating the model over a 9×12 grid. For each grid point, expected values and standard errors of relevant statistics are computed based on 1000 simulations of the model, setting the simulation length equal to a typical postwar quarterly sample size of $n = 200$. All simulated series are filtered using the Hodrick-Prescott filter prior to calculating the statistics. The first row in each cell is always the coefficient of variation of simulated consumption-money velocity (with standard errors), while the second row is always the corresponding volatility of the real rate[19] of interest (with standard errors). It is clear that - unlike in Jermann (1998), who does not incorporate a labour-leisure choice - although real rate volatility rises as physical capital supply becomes more inelastic *and* habit persistence increases, it stays well below the unrealistically high values reported in Hodrick et al. (1991). This is because endogenous responses in labour help to dampen the volatility of the stochastic discount factor (or equivalently the marginal valuation of wealth).

[19] The interest rate volatility has been *annualised*.

Adj. Cost \ Habit	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.5	0.071 (.005)	0.091 (.005)	0.114 (.007)	0.140 (.009)	0.168 (.010)	0.193 (.013)	0.218 (.015)	0.228 (.020)	0.209 (.021)
	0.051 (.004)	0.024 (.004)	0.049 (.004)	0.044 (.004)	0.040 (.004)	0.038 (.004)	0.034 (.003)	0.038 (.003)	0.06 (.002)
1.0	0.077 (.006)	0.103 (.006)	0.134 (.007)	0.169 (.010)	0.204 (.012)	0.237 (.015)	0.271 (.020)	0.288 (.024)	0.271 (.024)
	0.011 (.001)	0.016 (.001)	0.022 (.002)	0.028 (.002)	0.035 (.003)	0.041 (.004)	0.048 (.005)	0.053 (.005)	0.057 (.005)
1.5	0.082 (.006)	0.114 (.006)	0.150 (.008)	0.191 (.011)	0.231 (.014)	0.274 (.018)	0.311 (.024)	0.338 (.027)	0.322 (.029)
	0.045 (.004)	0.054 (.005)	0.063 (.005)	0.075 (.005)	0.085 (.006)	0.096 (.007)	0.106 (.008)	0.110 (.009)	0.117 (.010)
2.0	0.085 (.006)	0.121 (.007)	0.163 (.009)	0.208 (.012)	0.254 (.016)	0.300 (.020)	0.344 (.025)	0.375 (.031)	0.358 (.032)
	0.051 (.005)	0.082 (.006)	0.098 (.007)	0.112 (.007)	0.127 (.008)	0.140 (.010)	0.154 (.012)	0.164 (.014)	0.162 (.015)
2.5	0.088 (.006)	0.127 (.008)	0.172 (.010)	0.221 (.012)	0.273 (.017)	0.321 (.021)	0.371 (.027)	0.408 (.036)	0.395 (.036)
	0.090 (.008)	0.105 (.008)	0.123 (.008)	0.142 (.008)	0.162 (.011)	0.177 (.013)	0.194 (.015)	0.206 (.019)	0.202 (.019)
3.0	0.090 (.006)	0.132 (.008)	0.180 (.010)	0.233 (.014)	0.286 (.016)	0.343 (.021)	0.395 (.026)	0.433 (.032)	0.420 (.040)
	0.105 (.009)	0.124 (.009)	0.145 (.010)	0.168 (.011)	0.189 (.012)	0.211 (.014)	0.228 (.016)	0.24 (.018)	0.232 (.023)
3.5	0.092 (.006)	0.136 (.008)	0.186 (.010)	0.241 (.014)	0.300 (.019)	0.356 (.022)	0.415 (.030)	0.459 (.037)	0.444 (.038)
	0.117 (.009)	0.139 (.010)	0.164 (.010)	0.189 (.012)	0.215 (.014)	0.238 (.015)	0.260 (.019)	0.273 (.022)	0.266 (.022)
4.0	0.094 (.006)	0.140 (.008)	0.191 (.011)	0.247 (.014)	0.311 (.020)	0.376 (.024)	0.431 (.030)	0.476 (.038)	0.460 (.038)
	0.128 (.010)	0.152 (.011)	0.179 (.011)	0.208 (.012)	0.238 (.014)	0.267 (.018)	0.286 (.020)	0.300 (.024)	0.278 (.023)
4.5	0.095 (.006)	0.141 (.008)	0.197 (.010)	0.256 (.015)	0.319 (.020)	0.384 (.023)	0.445 (.030)	0.493 (.040)	0.480 (.041)
	0.135 (.011)	0.162 (.010)	0.195 (.012)	0.226 (.015)	0.258 (.018)	0.286 (.018)	0.308 (.022)	0.321 (.027)	0.299 (.026)
5.0	0.095 (.006)	0.145 (.008)	0.200 (.012)	0.262 (.015)	0.326 (.020)	0.393 (.025)	0.456 (.035)	0.503 (.040)	0.495 (.044)
	0.140 (.010)	0.173 (.011)	0.206 (.013)	0.242 (.016)	0.276 (.018)	0.306 (.020)	0.329 (.026)	0.339 (.028)	0.316 (.028)
5.5	0.096 (.006)	0.145 (.009)	0.203 (.011)	0.266 (.015)	0.335 (.022)	0.403 (.026)	0.475 (.035)	0.521 (.038)	0.505 (.044)
	0.146 (.012)	0.180 (.013)	0.217 (.013)	0.254 (.014)	0.294 (.020)	0.325 (.022)	0.354 (.030)	0.360 (.026)	0.33 (.036)
6.0	0.010 (.006)	0.148 (.009)	0.208 (.010)	0.272 (.016)	0.340 (.021)	0.412 (.026)	0.481 (.034)	0.530 (.044)	0.518 (.047)
	0.154 (.012)	0.189 (.014)	0.229 (.013)	0.270 (.016)	0.308 (.020)	0.343 (.022)	0.369 (.027)	0.375 (.032)	0.344 (.032)

Table 2. Parameter Sensitivity Analysis of Variability of Velocity and Real Rates

6 Conclusion

The observed variability of consumption velocity, modeled using a simple preference-based cash-credit model (see Stokey and Lucas, 1987), has been found to be impossible to explain jointly with plausible variability of *real* interest rates as implied by the same model (see Hodrick et al., 1991). In that model, sufficient variability in the former requires too high variability in the latter. Therefore, successfully modeling the second moments of the two variables jointly appears to be an impossibility in that particular framework. Since velocity is determined by a point of tangency between a downward-sloping relative price schedule (determined by the *nominal* rate of interest) and a smooth utility function in the cash and credit good, large fluctuations in that relative price are needed in order to induce sufficient variability in velocity by sufficiently dispersing that locus of tangency through time. This theoretical failure is therefore a direct consequence of the low nominal interest rate elasticity of velocity in the preference-based cash-credit model. With very little variability in expected inflation - a typical outcome of flex-price models - sufficient variability in the *nominal* rate is - through the Fisher relationship - induced primarily through variability in the *real* rate, which is required to be too high in comparison with empirical evidence on observed rates to explain consumption velocity.

A decentralised version of a cash costly-credit model (see Gillman, 1993; Benk et al., 2005; Gillman and Benk, 2007) determines the *average* of and variability in consumption velocity instead through the intersection (and for variability through the dispersion of that point of intersection) of a convex upward-sloping marginal cost schedule in credit production and the price of credit, which equals the net nominal rate of interest. This results in a different interest-rate elasticity of consumption velocity, compared to the preference-based cash-credit model, which is based on the technology specification of credit production. In contrast to Gillman and Kejak (2007), who focus on modeling the variability in *income* velocity primarily through the marginal cost channel (the credit shock channel) and the resulting high *credit shock* elasticity of consumption velocity, the present study has exclusively emphasised the variability of the *price channel* in determining *consumption* velocity variability. It is this focus which likens the present study to Hodrick et al. (1991).

The primary focus is to examine quantitatively using simulation evidence, whether the model is capable of exhibiting sufficient variability in consumption velocity without relying on too volatile interest rate behaviour, both real and nominal. As in Jermann (1998), increasing interest rate volatility is induced in a standard monetary RBC model using a combination of habit persistence in consumption *and* adjustment costs to investment. The second moment of consumption velocity are well-matched, which constitutes a significant improvement over earlier findings of Hodrick et al. (1991) using a simple cash-credit model. Both the second moments of the real and nominal rates of interest - although increased through the habit-adjustment-cost framework - remain relatively low, and are nowhere as unrealistically high as in Hodrick et al. (1991)[20].

Finally, successfully obtained stable saddle-path solutions over a large range of habit-persistence parameters indicates global determinacy of the model, making the cash costly-credit model superior in this regard vis-a-vis simple cash-only or cash-credit models, which have both been found to exhibit real indeterminacy (see Auray et al., 2005). The reason for this is that costly credit provides the representative household with an alternative means-of-exchange to escape the cash-only component of the exchange constraint *costlessly* in a net wealth sense, as the cost of using credit is re-distributed back in form of the banking wage bill and the dividend return on deposits held with the financial intermediary.

[20] One reason why in the present model, interest rate volatility remains low compared to Jermann (1998), is because a labour-leisure choice is incorporated, which is absent in Jermann (1998). See also Lettau and Uhlig (2000) on how endogenous variation in labour can dampen fluctuations in the real rate, in spite of other rigidities such as habit formation and capital adjustment costs.

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