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Inflation, Human Capital and Tobin's q

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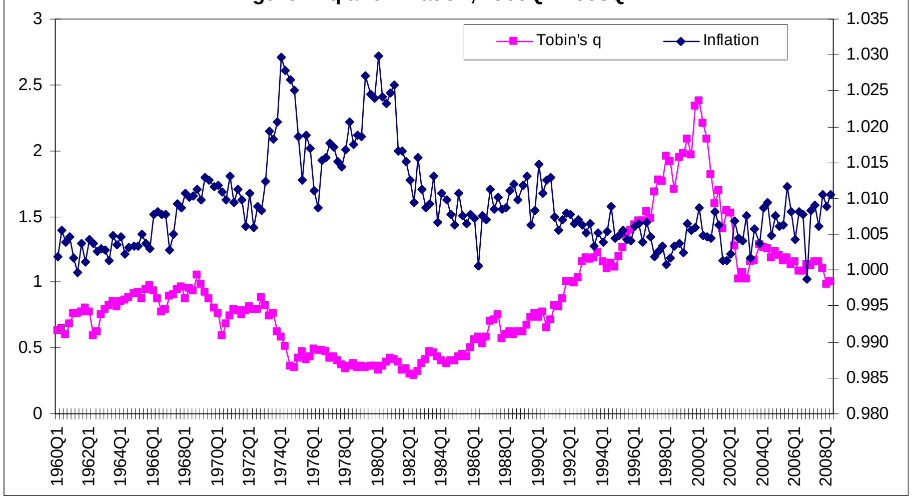
Abstract

A pervasive empirical finding for the US economy is that inflation is negatively correlated with the normalized market price of capital (Tobin's q) and growth. A dynamic stochastic general equilibrium model of endogenous growth is developed to explain these stylized facts. In this model, human capital is the principal driver of self-sustained growth. Long run comparative statics analysis suggests that inflation diverts scarce time resource to leisure which lowers human capital utilization. This impacts growth adversely and modulates capital adjustment cost downward resulting in a decline in Tobin's q. For the short run, a Tobin effect of inflation on growth weakens the negative association between inflation and q.

1 Introduction

The negative association between the stock prices and inflation in general equilibrium has been of the focus of work at least since Danthine and Donaldson (1986), who use a money-in-the-utility function with an endowment economy. However, the negative relation between inflation and Tobin's q normalization of the market price of capital, as seen in Figure 1 postwar US data, apparently still remains to be explained within a calibrated dynamic stochastic general equilibrium (DSGE) monetary economy.[1]

Figure 1: q and Inflation, 1960Q1-2008Q2



The Figure 1 negative correlation is particularly pronounced after 1965Q1 (the 1960:1 to 2008:2 correlation coefficient is -.55). The Tobin's q bottoms out around early 1980s when inflation peaks. The subsequent remarkable rise of q coincides with an era of a steady disinflation. The q then reaches an all time high of 2.5 before the stock market crash when inflation reaches about 1%. Since then the high frequency relationship is less clear cut although q is on a declining path while is inflation on the rise. McGrattan

[1] The Figure 1 data for q comes from the Smithers & Co (http://www.smithers.co.uk/). The Figure 1 negative correlation holds alternatively using Hall's (2001) Tobin's q series which goes up to 1999, or by constructing a q series from the S&P market index by dividing it by the physical capital stock (which is derived by aggregating investment in fixed capital using linear and nonlinear depreciation rules).

and Prescott (2005) argue that the rise until 2000 of the stock price to GDP ratio is due to lower taxes on capital. In this paper, we offer a monetary counterpart to the results of McGrattan and Prescott (2005). We argue that inflation acts as a tax on human capital and it reduces the capacity utilization rate of human capital by reducing the amount of time employed productively. This inflation tax on human capital contributes to a fall in the value of the firm just as the capital income tax on physical capital in McGrattan and Prescott (2005) lowers the value of the firm.[2]

The paper models the effect of inflation on Tobin's q by including within a real business cycle model the standard physical capital adjustment costs of Lucas and Prescott (1971), and the implicit human capital adjustment costs of Becker (1975) and Lucas (1988); the role of the latter is taken by the diversion of productive human capital to further human capital creation. This double adjustment cost combination allows the negative effect of the inflation tax on growth, which occurs through a reduced utilization rate of human capital, to translate directly into a reduced magnitude of Tobin's q. Such an approach using endogenous growth is reasonable given the empirical support for endogenous growth models that goes back as far as Kocherlakota and Yi (1996).

The paper starts with the previously modeled effect of how the inflation tax reduces endogenous growth, as in Gomme (1993) and Gillman and Kejak (2005). Inflation within a cash-only exchange economy induces the representative agent to substitute from consumption towards leisure, thereby reducing the return on human capital and the growth rate. The cost of adjusting the physical capital stock depends upon the growth rate as in Lucas and Prescott (1971), and as specifically modeled in Basu (1987) and Hercowitz and Sampson (1991), except that in those papers the depreciation rate is 100% and here it is more generally specified. The paper develops a simple closed form relationship for Tobin's q and growth demonstrating

[2] While McGrattan and Prescott (2005) look at the behavior of stock price/GDP ratio, we examine the Tobin's q. The Tobin's q is the ratio of stock price index to the capital stock which can be alternatively written as stock price/GDP multiplied by GDP/capital stock. Given that the output/capital ratio is stable, as pointed out by McGrattan and Prescott (2005), the stock price/GDP also reflects the behavior of Tobin's q.

the long run relationship between inflation, human capital utilization and q. These long run relationships form the baseline for the subsequent short run analysis and calibration.

Both long run and short run forces have a role in the inflation transmission effect on the market price of capital through the human capital channel. In the short run, the relationship between inflation, q and growth depends on the shocks driving the correlation. The model has real and monetary shocks. Real shocks are two productivity shocks, namely in each of the goods and human capital investment sectors. The monetary shock is purely a money supply shock. The productivity shocks tend to induce a negative correlation between inflation and q as well as between inflation and growth, while q and growth move inversely with respect to these productivity shocks. The monetary shock, on the other hand, induces a Tobin (1965) increase in physical capital accumulation thus weakening the negative growth-inflation correlation and negative q-inflation correlation. The overall correlations between q, inflation and growth depends on the relative strengths of real and monetary shocks.

The paper is organized as follows. The following section lays out the model. Section 3 analyzes the steady state and comparative statics. Section 4 presents the short run analysis and calibration. Section 5 performs sensitivity analysis. Section 6 concludes.

2 The Representative Household

The representative household allocates time between leisure (x_t) and work in the goods sector (l_{Gt}) at a nominal wage W_t, and work in the human capital investment (l_{Ht}) . Households own the human capital (h_t) and augment it through human capital investment only. Firms own the physical capital (k_t) and accumulate it through investment (i_t^k).

Sequencing of markets is as follows. Households trade in goods first with the cash in advance (M_t) and then they visit the asset markets to trade in stocks at the ex-dividend prices V_t and nominal bonds at the price P_t^b . Each such nominal bond pays 1 unit of currency with certainty in the following periods; B_t is the number of bonds held at date t. Cash is only used for transaction in goods, and augmented by the Central Bank through lump-sum transfer.[3]

At date t the source of funds of the household are nominal dividends D_t from the shares, fractional claims z_t to physical capital ownership of the firm, proceeds from nominal bonds, B_t, the cash carried over from the previous period (M_{t-1}), and the lump sum transfer from the Central Bank, $\mu_t \bar{M}_{t-1,}$ where μ_t is the rate of growth of the money supply and $\bar{M}_{t-1}$ is the money supply at date t-1.

The household thus solves:

$$Max \quad E_0 \sum_{t=0}^{\infty} \beta^t \left\{ U(c_t) + \psi \Gamma(x_t) \right\} \tag{1}$$

subject to the flow budget constraint,

$$P_t c_t + V_t (z_{t+1} - z_t) + P_t^b B_{t+1} + M_t = D_t z_t + W_t l_{Gt} h_t + B_t + M_{t-1} + \mu_t \bar{M}_{t-1}, \tag{2}$$

the time allocation constraint,

$$1 = x_t + l_{Gt} + l_{Ht}, \tag{3}$$

and the human capital accumulation constraint and exchange constraint. Here, human capital investment is linear in effective labor time $l_{Ht} h_t$ as in Lucas (1988), with a depreciation rate of δ_h and with A_{Ht} the exogenous sectoral total factor productivity (TFP), giving the accumulation equation of

$$h_{t+1} = (1 - \delta_h) h_t + A_{Ht} l_{Ht} h_t. \tag{4}$$

[3]The money supply is assumed to be passive in the sense that it is independent of economic growth and inflation. A more sophisticated model can endogenize money supply by either connecting the monetary policy to fiscal financing or add a Taylor type interest rate rule. Since the thrust of the paper is to understand how real and monetary shocks transmit to the market price of capital via human capital channel, we abstract from these complications in the present paper.

The exchange constraint is that consumption purchases require money:

$$P_t c_t \leq M_{t-1} + \mu_t \bar{M}_{t-1}. \tag{5}$$

The household first order conditions are found in Appendix A.1.

The stochastic discount factor facing the household is given by

$$m_{t+1} \equiv \frac{\beta E_{t+1}\left[U'(c_{t+2})\frac{c_{t+2}}{c_{t+1}}\frac{1}{1+\mu_{t+2}}\right]}{E_t\left[U'(c_{t+1})\frac{c_{t+1}}{c_t}\frac{1}{1+\mu_{t+1}}\right]}, \tag{6}$$

which reflects the anticipated inflation tax effects via the monetary growth terms. Using the equations (A.13) and (A.14) in Appendix A.1, the stock price and bond price equations can be written in a compact form as

$$1 = E_t m_{t+1}\left\{\frac{v_{t+1}+d_{t+1}}{v_t}\right\}, \tag{7}$$

and

$$1 = E_t m_{t+1}\{p_t^{b^{-1}}\}, \tag{8}$$

where v_t is the real share price, $v_t \equiv V_t/P_t$ and $p_t^b \equiv \frac{P_t^b}{P_t}$.

2.1 Firm Problem

A firm produces goods only with the production function $A_{Gt}F(k_t, l_{Gt}h_t)$, with A_{Gt} as the date t total factor productivity (TFP). It accumulates physical capital (k_t), and employs workers, and then distributes dividends to households. Note that both the investment good and labor are credit goods meaning the firm is not subject to any exchange constraint. The firm is subject to an investment adjustment cost technology which makes an increase in physical investment (i_t^k) incur a cost that rises with the investment rate. The firm maximizes the discounted stream of dividends for the household using the household's perceived intertemporal marginal rates of substitution as stochastic discount factor. With λ_t (characterized in (A.8) in Appendix A) as the shadow price of the flow nominal income of the household in (2), the firm solves

$$Max \quad E_0 \sum_{t=0}^{\infty} \lambda_t \left[P_t A_{Gt} F(k_t, l_{Gt} h_t) - W_t l_{Gt} h_t - P_t i_t^k \right] \tag{9}$$

subject to the cost of physical capital accumulation relation which relates the physical investment to the capital stock through the adjustment cost technology $\gamma(\cdot)$:

$$k_{t+1} = k_t \gamma \left(\frac{i_t^k}{k_t} \right). \tag{10}$$

Further, $\gamma(\cdot)$ is a monotonically increasing, strictly concave function with $\gamma(0) > 0$ and where the inverse function $\delta_k = \gamma^{-1}(1)$ exists for $\delta_k \in (0, 1)$.[4]

We use a Cobb-Douglas specifications for the production function,

$$F(k_t, l_{Gt} h_t) = k_t^{\alpha} (l_{Gt} h_t)^{1-\alpha}, \tag{11}$$

with $\alpha \in (0, 1)$. For the adjustment cost function, similar to Basu (1987) and Hercowitz and Sampson (1991), we assume

$$\gamma \left(\frac{i_t^k}{k_t} \right) = \left[1 - \delta_k + \frac{i_t^k}{k_t} \right]^{\theta}, \tag{12}$$

with $\theta \in (0, 1)$. The parameter θ represents the extent of adjustment cost. For $\theta = 1$, the investment technology reduces to a standard linear depreciation rule.

Note that an approximation of (12) around a steady state investment capital ratio (denoted by η) yields

$$\frac{k_{t+1}}{k_t} = (1-\delta+\eta)^{\theta} + \frac{\theta}{(1-\delta+\eta)^{1-\theta}} \left\{ \frac{i_t}{k_t} - \eta \right\} - \frac{\theta(1-\theta)}{(1-\delta+\eta)^{2-\theta}} \left\{ \frac{i_t}{k_t} - \eta \right\}^2.$$

[4] Alternatively as in Basu (1987), the firm can maximize the present value of real cash flows:

Max $E_0 \sum_{t=0}^{\infty} \prod_{i=0}^{t} m_i \left[A_{Gt} F(k_t, l_{Gt} h_t) - (W_t/P_t) l_{Gt} h_t - i_t^k \right]$ s.t. (10) and $\{m_i\}$ as given by (6). This is equivalent to (9).

This approximation resembles the quadratic level form adjustment cost function as in Christiano et al (2007). However, one major difference is that the adjustment cost in Christiano et al is zero in the stationary state while in contrast equation (12) implies a positive adjustment cost along the balanced growth path equilibrium.

2.2 Characterization of Equilibrium

(E.1): Given the sequence $\{P_t\}$, $\{W_t\}$, $\{Q_t\}$, $\{P_t^b\}$, and the money growth rates $\{\mu_t\}$, the household maximizes utility in equation (1) subject to equations (2) to (5).

(E.2): Given the sequence $\{P_t\}, \{W_t\}, \{A_{Gt}\}, \{A_{Ht}\}$, the goods producer maximizes profit in equation (9) subject to equations (10) to (12).

(E.3) : Spot assets and goods markets clear, whereby $z_t = 1$, $B_t = 0$, and $M_t = \bar{M}_t$.[5]

2.2.1 Tobin's q

Using the first order condition with respect to physical capital investment and equation (A.17) of Appendix A.2, one immediately gets the following relation which we will define as Tobin's q (call it q_t hereafter):

$$q_t \equiv \frac{\omega_t}{P_t \lambda_t} = \frac{1}{\theta}\left[1 - \delta_k + \frac{i_t^k}{k_t}\right]^{1-\theta}. \tag{13}$$

This defines Tobin's q in a standard way as the shadow price of physical capital investment, ω_t, relative to the shadow price of consumption $P_t\lambda_t$.[6] As a result, the Tobin's q of equation (13) is the marginal cost of investment in terms of output. With $\theta = 1$, $q = 1$.[7]

[5] There is also an implicit labor market equilibrium condition which we omit for brevity. In principle, one may distinguish between labor supply to the goods sector (say l_{Gt}^s) and the corresponding labor demand (say l_{Gt}^d). In equilibrium $l_{Gt}^d = l_{Gt}^s = l_{Gt}$. To avoid notational burden, we use l_{Gt} to represent both labour supply and demand.

[6] The shadow price of consumption is the shadow price of nominal income in (2) of the household multiplied by the nominal price level P_t.

[7] Equivalently, Tobin's q can be defined in terms of the asset pricing equation (7) as v_t/k_{t+1}. In Appendix A.4 this equivalence is established.

2.2.2 Exogenous Forcing Processes

The exogenous variables A_{Gt} , A_{Ht}, μ_t follow the processes :

$$A_{Gt} - \bar{A}_G = \rho_G(A_{Gt-1} - \bar{A}_G) + \epsilon_t^G \tag{14}$$

$$A_{Ht} - \bar{A}_H = \rho_H(A_{Ht-1} - \bar{A}_H) + \epsilon_t^H \tag{15}$$

$$\mu_t - \bar{\mu} = \rho_\mu(\mu_{t-1} - \bar{\mu}) + \epsilon_t^\mu \tag{16}$$

where $\epsilon_t^G, \epsilon_t^H\ \ \epsilon_t^\mu$ are white noises with a variance covariance matrix Σ. Letters with a bar represent steady state values.

3 Balanced Growth Path Equilibrium

The balanced growth rate depends positively on the steady state marginal product of physical capital and the adjustment cost of capital, as well as on the return to human capital. From these relations, Tobin's q can be written as a function of the growth rate. Following this, a concept of the adjustment cost wedge between the returns to physical and human capital q can be formulated.

3.1 Growth and Tobin's q

Proposition 1 *The balanced growth rate in this economy is given by*

$$1 + g = \left[\frac{\beta\theta\left(\bar{A}_G F_1 + 1 - \delta_k\right)}{1 - \beta\left(1 - \theta\right)} \right]^{\theta}, \tag{17}$$

in terms of the physical capital marginal product F_1, and by

$$1 + g = \beta[1 - \delta_h + \bar{A}_H\left(1 - x\right)]. \tag{18}$$

in terms of the human capital return of $\bar{A}_H\left(1 - x\right)$.

Proof: Appendix B.

Corollary 2 *The balanced growth path Tobin's q is a function of the growth rate and the adjustment cost parameter θ, as given by the following relation:*

$$q = \frac{1}{\theta}(1+g)^{\frac{1-\theta}{\theta}}; \tag{19}$$

a lower θ for a positive g thus means a higher q.

Proof: Equation (19) follows directly from equations (13), (17) and (B.3) in Appendix B. Note that $\frac{\partial q}{\partial \theta} < 0$ for $\theta \in (0,1)$.

Note that solving for g in equation (18), and then substituting this in for g in equation (19), yields an alternative expression for q :

$$q = \frac{1}{\theta}.[\beta\{1 - \delta_h + \bar{A}_H(1-x)\}]^{\frac{1-\theta}{\theta}}. \tag{20}$$

The implication is that a lower human capital utilization rate of $1-x$, or greater leisure use, lowers Tobin's q. The intuition for this is that a lesser utilization rate of human capital, by which is meant a lower amount of the time $1-x$ that is spent productively, lowers growth and modulates the adjustment cost of capital downward. This means a lower q.

The adjustment cost of capital and human capital utilization crucially interact through the growth rate to have an effect on q. This interaction is lost in cases when there is no physical capital cost adjustment or when there is zero growth. To see this, shut down the adjustment cost by setting $\theta = 1$, and then from (19) it follows that $q = 1$, which means no human capital effect on q. For the second special case, when $\theta \in (0,1)$, if the household invests just enough time to keep the human capital constant, so that the balanced growth rate is zero, then the growth effect on q in equation (19) disappears, and q equals $1/\theta$.

3.2 The Physical Capital Adjustment Cost Wedge

Generally, with $\theta \in (0,1)$, the physical capital adjustment cost drives a wedge between the returns to physical capital and to human capital. This wedge depends on the human capital investment. To see this note that the gross return to human capital can be defined from equation (18) as

$$R^h \equiv 1 + \bar{A}_H(1-x) - \delta_h.$$

Using (17), (18), and defining $1/(1+\rho) \equiv \beta$, the equivalence between the returns to human and physical capital is given by

$$R^h = (1+\rho)\left[\frac{\theta}{\rho+\theta}\right]^{\theta}\left[1+\bar{A}_G F_1 - \delta_k\right]^{\theta}. \tag{21}$$

In the benchmark case of no adjustment cost ($\theta = 1$), it follows from equation (17) that the traditional Euler equation holds, meaning

$$1 + g = \beta(\bar{A}_G F_1 + 1 - \delta_k), \tag{22}$$

From equations (18) and (22) note that the returns on human and physical capital are equal, in that

$$\bar{A}_H(1-x) - \delta_h = \bar{A}_G F_1 - \delta_k. \tag{23}$$

In the present setting, the adjustment cost wedge or the user cost of capital depends non-trivially on the long run growth rate. To see this, use (21) and (17) to obtain the following expression for the user cost of capital:

$$\frac{1+\bar{A}_H(1-x)-\delta_h}{1+\bar{A}_G F_1 - \delta_k} = \left\{(1+\rho).\frac{\theta}{\rho+\theta}\right\}.\frac{1}{(1+g)^{(1-\theta)/\theta}} \tag{24}$$

For any growing economy, the right hand side of (24) is always a positive fraction. This means that $\bar{A}_H(1-x) - \delta_h < \bar{A}_G F_1 - \delta_k$. This inequality result can be interpreted as implying that the physical capital adjustment cost creates a user cost wedge that causes a lower physical capital to effective labor ratio in equilibrium than when $\theta = 1$. And this is consistent with our notion that accumulating physical capital is more costly in the presence of adjustment cost, as in Lucas (1967). What is novel in the present setting is the interaction between this user cost wedge and the investment in human capital via the long run growth rate, g. For example, we demonstrate later in the comparative statics section 4.3.1 that a higher inflation tax adversely impacts the human capital investment and through this channel it reduces this physical investment wedge by lowering q. Moreover, a higher productivity of human capital could widen this user cost wedge by driving the growth rate up and thus raising q.

3.3 The Human Capital q^h and Tobin's q

Consider defining a human capital q^h in the same way as the physical capital q : as the ratio of the shadow price of the (human) capital investment η_t to the shadow price of output $P_t\lambda_t$. Using (A.6), (A.7) and (A.16) of Appendix A.1, write this ratio as the real wage normalized by $\bar{A}_H$:

$$q_t^h \equiv \frac{\eta_t}{P_t\lambda_t} = \frac{W_t}{A_{Ht}P_t} = \frac{A_{Gt}}{A_{Ht}}(1-\alpha)\left(\frac{k_t}{l_{Gt}h_t}\right)^{\alpha}. \tag{25}$$

The human capital q^h is the cost of the foregone time that is devoted to human capital investment instead of goods production which is simply the ratio of the marginal product of labor in goods production $\left(\frac{W_t}{P_t}\right)$ to the marginal product of labor in human capital investment production A_{Ht}. As more time is devoted to the human capital sector, real wage rises due to relative scarcity of raw labor in the goods sector. This raises the opportunity cost of human capital investment that is reflected by a higher q^h. This is an implicit adjustment cost of human capital.[8]

Tobin's q reflects marginal cost of physical investment while the q^h reflects the human capital adjustment cost of shifting scarce time from the goods sector to the human capital sector. These two adjustment costs move in opposite directions in response to change in fundamentals. For example,

[8] Our way of treating human capital adjustment cost differs from Merz and Yashiv (2007) who assume that there is a common adjustment cost for both human and physical capital in a one sector model. We have a two sector model in which the human capital adjustment cost is measured in terms of the opportunity cost of diverting time from goods to the human capital sector while in Merz and Yashiv, this adjustment cost is the opportunity cost of diverting output from consumption to investment.

in response to an increase in the human capital TFP, A_H, the human capital cost of adjustment reflected by q^h falls, while from (20), the Tobin's q rises if the amount of productively employed time, $1-x$ rises.[9]

With inflation q and q^h also go in opposite directions. Should inflation cause the real wage to rise and the output growth rate to fall, as is standard in such models with human capital (see Gillman and Kejak, 2005, 2008), this "Tobin inflation effect" results in an increase in the human capital q^h. With the growth rate also falling, q falls as the capital user cost falls.

4 Model Simulation

4.1 Calibration

Three issues that arise in the calibration are the frequency of data, the sample period, and the particular data series to be used. For the frequency, we use annual data to calculate the baseline model in order to target the output growth rate, inflation, q, leisure and the investment rate. Using the baseline deep parameter estimates from the annual data calibration, we use the quarterly data to calibrate the second moments of the forcing processes to match the second moments for q, growth and inflation.

The sample period for the baseline is 1960-1999, for which the average q exceeds unity. Note that from 1999 onward q is below unity following the stock market crash. Thus rather than make the 1960-2008 period the baseline, the additional subperiod of 1999-2008 is added to the baseline period of 1960-1999 as an extension below. And this allows a view of the effect of the rise and fall of stock prices during the 1999-2008 period.

For calibrating the long run q we use the series in Hall (2001), which gives an annual average estimate of q exceeding unity. For short run calibration, we use more up to date estimates of q constructed by Smithers and Co (2008). Short run second moment properties of both Hall and Smithers q series are similar. Leisure is estimated at 0.55 by using the annual average

[9] The comparative statics reported in the following section confirms that $1-x$ indeed rises in response to increase in $\bar{A}_H$.

weekly hours of work based on Bureau of labor Statistics (BLS) data; total daily time is 16 hours excluding 8 hours of sleeping time; daily leisure hours is then [16-(average weekly hours of work/5)]/16 assuming a 5-day working week. Note also that this BLS data start only in 1964 instead of 1960 as in the other data series.

4.2 Baseline Growth Model

Table 1 presents the target variables based on the US annual data for $1960-1999$: average GDP growth rate of 3.4%, an average inflation rate of 4.3%, an average q of 1.25, an investment rate of 0.16, and a leisure usage rate of 0.55**,** which is similar to that of the Gomme and Rupert (2007) calibration value of 0.505**.** The model performs reasonably well to match these targets.

Table 1: Values of the Target Variables: Actual and Model

Target Variables, $1960-1999$	Data	Model
GDP Growth	3.4%	3.26%
Rate of Inflation	4.3%	4.20%
q	1.25	1.26
i/y	0.16	0.20
Leisure x	0.55	0.52

Table 2 gives the baseline model parameters values. Standard values are chosen for β, α, and ψ. The mean money supply growth rate, μ is chosen to target the 4.3% inflation rate. The human capital technology parameters $\bar{A}_H$ and δ_h are fixed to target the 3.4% GDP growth rate and a human capital utilization rate $1-x$ equal to 0.45 based on equations (4) and (18). The physical capital depreciation rate is fixed at 0.03 in line with Benk et al (2009).

Table 2: Baseline Parameter Values

β	α	δ_k	δ_h	ψ	θ	$\bar{A}_G$	$\bar{A}_H$	μ
0.96	0.36	0.03	0.024	1.84	0.8	1.2	0.21	0.076

The adjustment cost parameter θ in our specification is novel. Hercowitz and Sampson (1991) find an estimate of θ at 0.44 but assume 100% depreciation of physical capital while we allow partial depreciation of capital. To get the baseline θ, we use the steady state solution for q in (19). Plugging a long run average value of q of 1.26, and the target growth rate, g into (19) we obtain an estimate of θ equal to 0.80.

4.3 Results

The model with endogenous growth has two distinct components that comprise its equilibrium over time : (i) long run balanced growth and (ii) short run transitional dynamics in response to shocks. First the balanced growth results are examined with comparative static sensitivity analysis using the baseline calibration based on annual data for the 1960-1999 period. Second, using the same baseline parameter estimates we analyze the short run properties over an extended sample period, 1960Q1-2008Q2, which encompasses the recent financial volatility. Here the impulse responses are derived along with the model's simulated correlations as compared to the data, in a real business cycle tradition. This is followed with a sensitivity analysis with respect to the adjustment cost and shock specification.

4.3.1 Comparative Statics

Table 3 reports comparative statics effects of a change in money growth rates from the baseline level. The balanced growth path equilibrium effect of an inflationary monetary policy is as follows. Agents tend to switch to leisure which is not subject to inflation tax. Human capital investment and human capital utilization decline and so does growth, as well as q. The rise in leisure induces a rise in real wage, a fall in the real interest rate, a rise in the physical to human capital ratio, as human capital is more labor intensive in its production.

Table 4 reports the comparative statics of a change in $\bar{A}_H$. A small increase in human capital TFP has significant effects on agent's allocation of time between three activities. Agents cut back on leisure and reallocate

Table 3: Comparative Statics of a Change in μ

	$\mu = 0.076$	$\mu = 0.086$	$\mu = 0.096$
Growth	3.26%	3.15%	3.04%
x	0.52	0.53	.54
l_H	0.269	0.26	0.259
Inflation	4.2%	5.28%	6.36%
q	1.2600	1.2597	1.2594
i/y	0.20	0.20	0.20
k/h	1.42	1.44	1.47

time to human capital investment. The Tobin's q rises, while inflation falls, growth rises and so does the physical investment rate. An increase in human capital investment explains why k/h falls.[10]

Table 4: Comparative Statics of a Change in $\bar{A}_H$

	$\bar{A}_H = 0.21$	$\bar{A}_H = 0.22$	$\bar{A}_H = 0.23$
Growth	3.26%	4.29%	5.30%
x	0.52	0.50	0.47
l_H	0.269	0.30	0.33
Inflation	4.2%	3.16%	2.17%
q	1.2600	1.2632	1.2662
i/y	0.20	0.22	0.23
k/h	1.42	1.16	0.97

Table 5 reports the effects of a change in adjustment cost parameter θ. A higher adjustment cost (lower θ), lowers growth, and raises inflation. Two opposing effects are at work on q : (i) higher adjustment cost drives q up and (ii) a lower growth drives it down. The former effect dominates the latter.

These comparative statics effects provide insights about the long run association between inflation and q. Inflation is driven by fundamentals which include monetary policy (μ), human capital TFP ($\bar{A}_H$) and the adjustment

[10] A rise in $\bar{A}_G$ is offset by a fall in MPK via a rise in k/h. This leaves the balanced growth g in (17) unaffected which means q and inflation do not change. However, shocks to A_G have short run effects which we analyze in the next section.

Table 5: Comparative Statics of a Change in θ

	$\theta = 0.8$	$\theta = 0.7$	$\theta = 0.6$
Growth	3.26%	3.18%	3.09%
x	0.52	0.53	0.53
l_H	0.269	0.26	0.26
Inflation	4.2%	4.27%	4.36%
q	1.26	1.45	1.70
i/y	0.20	0.20	0.19
k/h	1.42	1.22	1.01

cost (θ). How inflation impacts q depends on the underlying cause of inflation. If inflation is purely a monetary phenomenon (driven by monetary policy), it has little long run effects on q. On the other hand, if inflation is driven by real fundamentals such as adjustment cost or human capital investment TFP, it has a pronounced effect on q.

4.3.2 Impulse Responses

For the same baseline parameters, the short run analysis is summarized by the impulse responses with respect to orthogonalized shocks to A_G, A_H and μ based on the loglinearized version of the short run equation system (A.19) through (A.25) in Appendix A.3. Figures 2 through 4 plot these responses respectively.

A positive productivity shock in the goods sector makes agents substitute time away from human capital investment and leisure to goods production. Such a surge in current goods production temporarily lowers inflation ($infl$). The growth rate, however, falls due to lesser time devoted to human capital sector. A temporary positive shock to the marginal product of physical capital drives investment:GDP ratio (i/y) up. This rise in investment rate modulates adjustment cost upward which explains the rise in q. Increase in saving due to decreased leisure and increased investment gives rise to an intertemporal substitution in leisure which explains a subsequent rise in x.

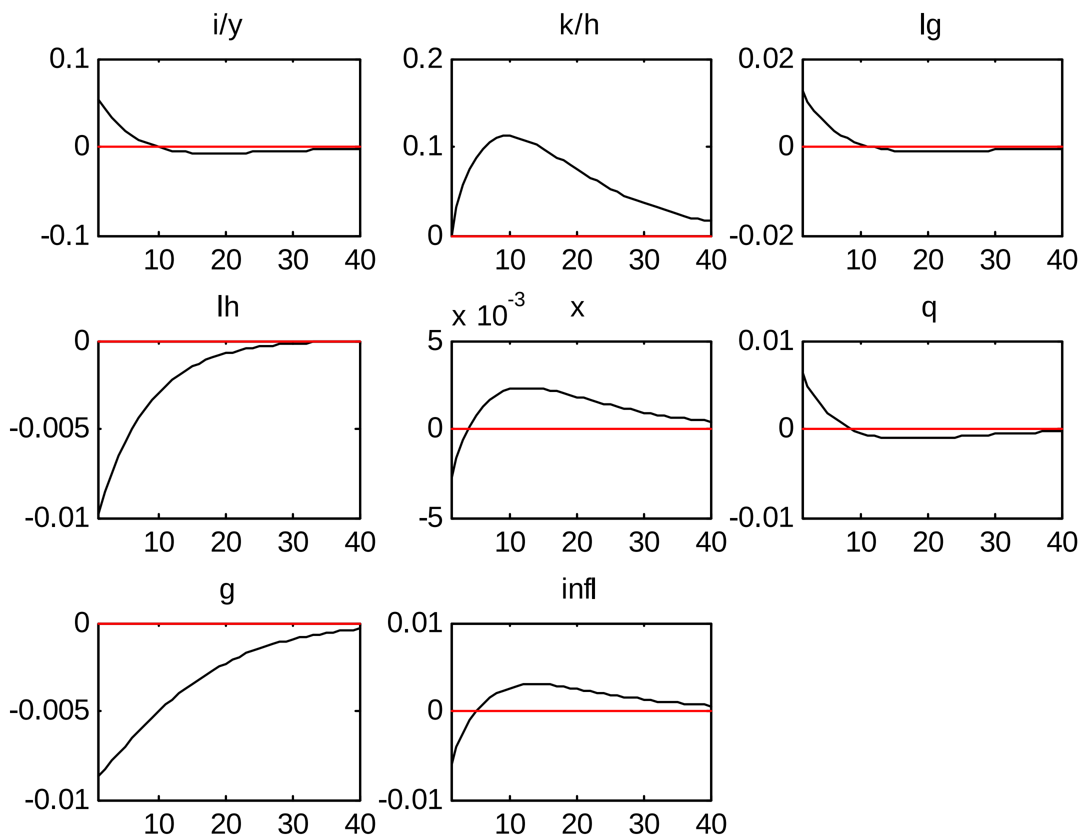


Figure 2: Impulse Responses with respect to A_G

In response to a positive shock to A_h, agents switch from goods production and leisure to human capital investment. Physical investment also declines also since agents switch resources to human capital investment. Despite this the growth rises due to the long term effect of human capital on GDP. Inflation initially rises slightly due to shortage of consumption goods when agents switch time to human capital investment. The physical capital investment rate thus declines reflected by the lower growth of the physical capital stock. Consequently adjustment cost declines which explains the decline in q.

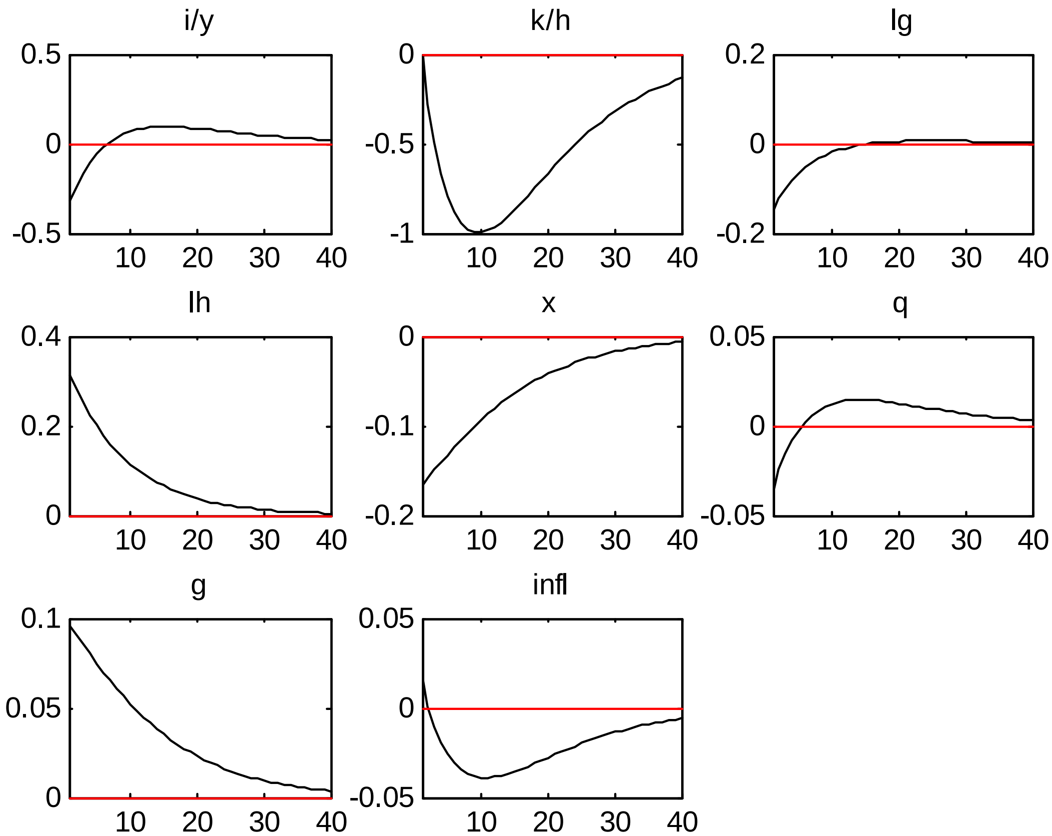


Figure 3: Impulse Responses with respect to A_H

A positive monetary shock via raising inflation induces agents to switch time from goods production which is subject to inflation tax to leisure and human capital investment. Consumption also falls due to this persistent inflation tax. The consequent rise in physical investment rate resembles a Tobin effect reflected by the growth of the capital stock. Adjustment cost thus rises which explains the rise in q. Growth of output reflects the growth of physical and human capital stocks.

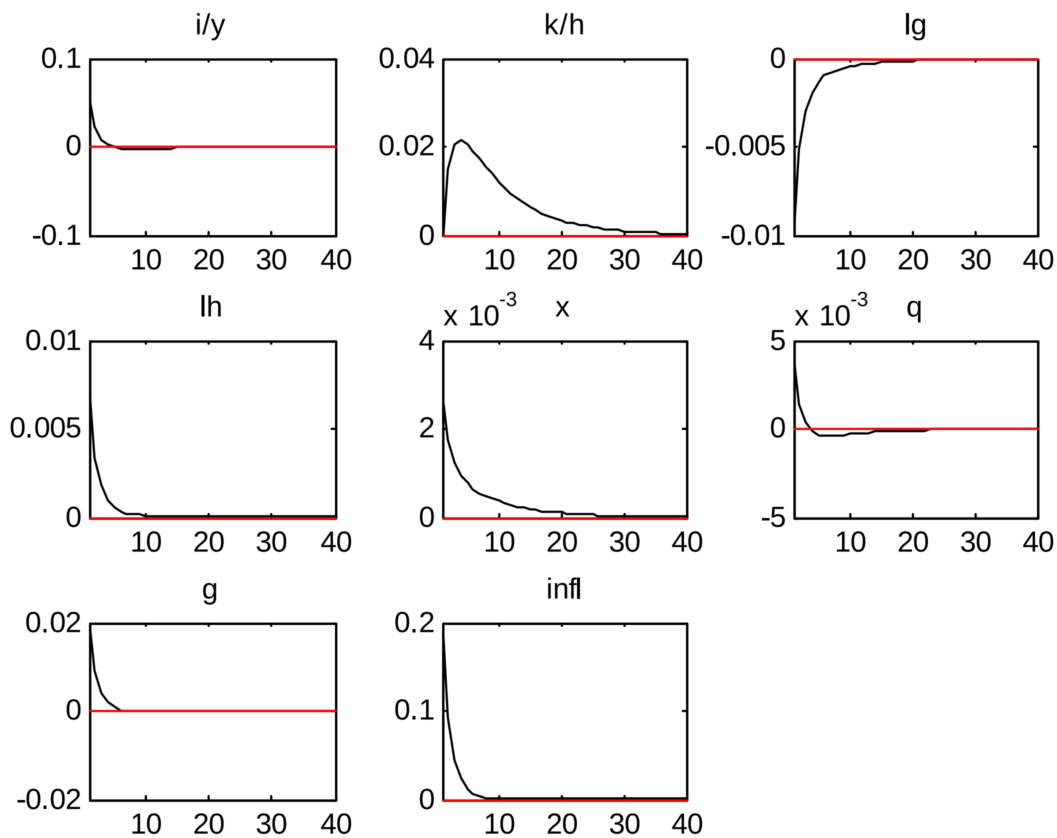


Figure 4: Impulse Responses with respect to μ

The upshot of this short run analysis is that the correlation between Tobin's q and inflation depends on the relative strengths of productivity and monetary shocks. If the predominant shock is either a goods sector TFP, A_G or a human capital TFP, A_H, they will contribute to a negative association between q and also negative correlation between q and growth. If the predominant shock emanates from monetary sources, the Tobin effect of inflation will weaken the negative relation between q and inflation as well as inflation and growth; it will strengthen the positive correlation between q and growth.

4.3.3 Second Moments

We now turn to the second step of our calibration, that is calibrating the second moments. Using the same baseline parameter estimates obtained from calibrating the growth model, we pose the question whether the same growth model can replicate the short run properties of the financial data. Since the focus of the paper is on understanding the relationship between q, inflation and growth, for short run calibration we focus only on these three key endogenous variables. For this second step calibration, we need to calibrate the baseline parameters of the forcing processes. The values of these parameters are chosen through a process of iterative trial and error to come closest to the sample correlations between q, inflation and growth. Table 6 reports these baseline parameter estimates.

Table 6: Baseline Parameters for the Second Momemnts of the Forcing Processes

ρ_G	ρ_H	ρ_μ	σ_G	σ_H	σ_μ
.9	.9	.49	.08	.08	.25

Table 7 reports the second moments as found in the data for two sample periods: $1960Q1-1999Q4$ and $1960Q1-2008Q2$. For both sample periods, the model's q-inflation correlation and growth-inflation correlation are not too much at odds with the data. The model fits closely the $1960Q1-2008Q2$ sample correlation between output growth and q. The volatility of q is generally higher in the data than in the model, while the model's volatility of the growth rate and inflation is higher than in the data. Such a failing of the volatility is familiar in consumption-based CAPM models.

5 Sensitivity Analysis

To consider the robustness of the results, different values of the adjustment cost parameter θ are considered, and additional shocks are introduced.

Table 7: Correlations of q, Growth, Inflation and Leisure

	Data : $1960Q1 - 1999Q4$	Data: $1960Q1 - 2008Q2$	Model
corr(q,infl)	-0.56	-0.55	-0.40
corr(infl,g)	-0.22	-0.21	-0.35
corr(q,g)	0.15	0.05	0.04
sd(q)	0.42	0.45	0.07
sd(g)	0.009	0.008	0.26
sd(infl)	0.007	0.006	0.26

5.1 Variations in θ

Table 8 reports the performance of the model against the data for various adjustment cost economies (different θ). The model performs better in predicting the volatility of q for economies with higher adjustment cost (lower θ).

Table 8: Correlations of q, Growth, Inflation and Leisure

	Model $\theta = 0.8$	Model $\theta = 0.7$	Model $\theta = 0.6$
corr(q,infl)	-0.40	-0.37	-0.35
corr(infl,g)	-0.35	-0.36	-0.37
corr(q,g)	0.04	0.05	0.06
sd(q)	0.07	0.14	0.24
sd(g)	0.26	0.26	0.25
sd(infl)	0.26	0.26	0.26

5.2 Additional Shocks

To check for further robustness, we introduce two additional shocks to the model, namely to (i) to the preference, (ii) adjustment cost technology. The instantaneous utility function now changes to:

$$\ln c_t + \exp(\xi_t^x)\psi \ln x_t$$

and the physical investment technology (12) changes to:

$$\frac{k_{t+1}}{k_t} = \exp(\xi_t^k)\left[1 - \delta_k + \frac{i_t^k}{k_t}\right]^{\theta}$$

We assume the following stochastic processes for ξ_t^x and ξ_t^k :

$$\begin{aligned} \xi_t^x &= \rho_x \xi_{t-1}^x + v_t^x \\ \xi_t^k &= \rho_K \xi_{t-1}^k + v_t^k \end{aligned}$$

where $v_t^x \sim N(0, \sigma_x)$ and $v_t^k \sim N(0, \sigma_k)$.

As before, the calibrated values of ρ_x, ρ_k, σ_x, σ_k reported in Table 9 are chosen by a trial and error process to minimize the difference between model and actual second moments. Table 10 reports the model performances for various adjustment cost scenarios based on these parameters .

Table 9: Baseline Parameters for the Second Moments of the Forcing Processes of the 5-Shock Model

ρ_x	ρ_K	σ_x	σ_k
0.9	0.9	0.56	0.01

Table 10: Correlations of q, Growth, Inflation and Leisure: 5 Shock Model and Actual: 1960-2006

	Model $\theta = 0.8$	Model $\theta = 0.7$	Model $\theta = 0.6$
corr(q,infl)	-0.46	-0.44	-0.42
corr(infl,g)	-0.41	-0.42	-0.43
corr(q,g)	0.11	0.12	0.13
sd(q)	0.08	0.16	0.28
sd(g)	0.30	0.29	0.29
sd(infl)	0.28	0.28	0.28

The model predicts the q-growth correlations better in this environment. The variability of q increases in high adjustment cost economies. This increase in volatility of q is primarily due to the shock to leisure preference and has very little do with the shock to adjustment cost technology.[11] However, this is accomplished at a cost. In order to match the q-growth and q-inflation correlations well, the leisure shock has to have an implausibly large standard error. Chari et al (2008) criticize the role of leisure shocks in DSGE model.

The upshot of this quantitative analysis is as follows. Given the stylized nature of the model, it performs very well in matching the long run target variables. For the short run, the model predicts the three-way correlations between q, inflation and growth reasonably well. As in any standard consumption CAPM, the quantity side of the model (e.g. growth and inflation in our context) is too volatile compared to the data.

6 Conclusion

The paper contributes an explanation of the empirical stylized negative correlation between market price of capital and inflation. It does this in an endogenous growth environment where human capital is a major driver of growth. A DSGE endogenous growth model identifies plausible fundamentals that determine the q-inflation-growth relationship. The importance of this is that while there is an emerging literature that shows how monetary policy affects the stock market boom-bust through sticky wages and inflation targeting (Christiano et al, 2007), little is known of the effect of monetary policy over the stock market via human capital-driven growth. The paper develops a closed form solution for Tobin's q with physical adjustment cost to understand the long run relation between inflation, q and human capital utilization. The long run comparative statics and the short run impulse response analysis help us understand the transmission mechanism of real and monetary shocks via a "human capital channel", which is novel in the literature.

[11] In fact, turning off the adjustment cost shock make the model overshoot the q-growth correlation. An adjustment cost shock makes q and growth move in opposite direction and thus dampens the correlation between q and growth and make the model q-growth correlation come in line with the data.

The results provide an explanation simultaneously of two particular stylized facts: a negative correlation between (i) the market price of capital and inflation, and (ii) the output growth rate and inflation. Here inflation affects the value of the stock market via its distortionary effect on human capital utilization, which causes agents to divert time from productive activities towards leisure that is not subject to inflation tax. This lowers the growth rate and at the same time the Lucas and Prescott (1971) cost of adjustment of physical capital accumulation, Tobin's q falls along with the growth rate.

The model can be enriched in several dimensions, with an aim to predict better the volatilities. An introduction of research and development (R&D) sector in the firm could bring additional realism to the model, as an alternative or supplement to the human capital channel for productivity increases. For example this might build upon the intangible capital of McGrattan and Prescott (2008). A human capital externality can be introduced as an additional engine of growth that would bring in convergence towards the aggregate human capital level and physical capital can be added to the human capital production process.

A Appendix

A.1 Equilibrium Conditions

A.1.1 Household

Define the lagrange multipliers associated with the flow budget constraint (2) as λ_t, the human capital technology (4) as η_t and the cash-in-advance constraint (5) as γ_t. The first order conditions facing the household are:

$$c_t : \beta^t U'(c_t) - P_t(\lambda_t + \gamma_t) = 0 \tag{A.1}$$

$$M_t : -\lambda_t + E_t\{\lambda_{t+1} + \gamma_{t+1}\} = 0 \tag{A.2}$$

$$z_{t+1} : -\lambda_t V_t + E_t \lambda_{t+1}\{V_{t+1} + D_{t+1}\} = 0 \tag{A.3}$$

$$B_{t+1:} - P_t^b \lambda_t + \lambda_{t+1} = 0 \tag{A.4}$$

$$h_{t+1}: -\eta_t + E_t\lambda_{t+1}l_{Gt+1}W_{t+1} + E_t\eta_{t+1}(1-\delta_h + A_{Ht+1}l_{ht+1}) = 0 \quad \text{(A.5)}$$

$$l_{Gt}: -\psi\Gamma'(1-l_{Gt}-l_{Ht})\beta^t + \lambda_t W_t h_t = 0 \quad \text{(A.6)}$$

$$l_{Ht}: -\psi\Gamma'(1-l_{Gt}-l_{Ht})\beta^t + A_{Ht}\eta_t h_t = 0 \quad \text{(A.7)}$$

Using (A.1) and (A.2)

$$\lambda_t = \beta^{t+1}E_t\frac{U'(c_{t+1})}{P_{t+1}} \quad \text{(A.8)}$$

which upon substitution in (A.3) and (A.4) yields

$$V_tE_t\left[\frac{U'(c_{t+1})}{P_{t+1}}\right] = \beta E_t\left[E_{t+1}\left[\frac{U'(c_{t+2})}{P_{t+2}}\right]\{V_{t+1}+D_{t+1}\}\right] \quad \text{(A.9)}$$

$$P_t^bE_t\left[\frac{U'(c_{t+1})}{P_{t+1}}\right] = \beta E_t\left[E_{t+1}\left[\frac{U'(c_{t+2})}{P_{t+2}}\right]\right]. \quad \text{(A.10)}$$

A binding cash in advance constraint means that (5) reduces to

$$\frac{\bar{M}_t}{P_t} = c_t, \quad \text{(A.11)}$$

which implies that

$$\frac{P_t}{P_{t+1}} = \frac{c_{t+1}}{c_t}\frac{1}{1+\mu_{t+1}}. \quad \text{(A.12)}$$

Upon substitution into (A.9) and (A.10) it results that

$$v_tE_t\left[U'(c_{t+1})\frac{c_{t+1}}{c_t}\frac{1}{1+\mu_{t+1}}\right] = \beta E_t\left[E_{t+1}\left[U'(c_{t+2})\frac{c_{t+2}}{c_{t+1}}\frac{1}{1+\mu_{t+2}}\right]\{v_{t+1}+d_{t+1}\}\right], \quad \text{(A.13)}$$

and

$$p_t^b E_t \left[U'(c_{t+1}) \frac{c_{t+1}}{c_t} \frac{1}{1+\mu_{t+1}} \right] = \beta E_t \left[E_{t+1} \left[U'(c_{t+2}) \frac{c_{t+2}}{c_{t+1}} \frac{1}{1+\mu_{t+2}} \right] \right] \tag{A.14}$$

where v_t = real share price(=V_t/P_t), $p_t^b = \frac{P_t^b}{P_t}$, w_t real wage (=W_t/P_t).

Using (A.11) and (6), one obtains the following compact expression for m_{t+1} :

$$\frac{\lambda_{t+1} P_{t+1}}{\lambda_t P_t} = m_{t+1} \tag{A.15}$$

A.2 Firm

Define ω_t as the Lagrangian multiplier associated with the adjustment cost technology (10). Firms' first order conditions are

$$l_{Gt}^f : \frac{W_t}{P_t} = A_{Gt} F_2(k_t, l_{Gt}^f h_t) \tag{A.16}$$

$$i_t^k : \lambda_t P_t = \theta \omega_t \left(1 - \delta_k + \frac{i_t^k}{k_t} \right)^{\theta - 1} \tag{A.17}$$

$$k_{t+1} : -\omega_t + E_t \left(P_{t+1} \lambda_{t+1} A_{Gt+1} F_{1t+1} \right) +$$

$$E_t \omega_{t+1} \left[(1-\theta) \left\{ 1 - \delta_k + \frac{i_{t+1}^k}{k_{t+1}} \right\}^{\theta} + \theta(1-\delta_k) \left\{ 1 - \delta_k + \frac{i_{t+1}^k}{k_{t+1}} \right\}^{\theta - 1} \right] = 0 \tag{A.18}$$

A.3 Summary of the Equilibrium Conditions

Based on the first order conditions, the model can be summarized by the following equations.

Tobin's q equation:

$$q_t = E_t m_{t+1} \left[\alpha A_{Gt+1} l_{Gt+1}^{1-\alpha} \left(\frac{k_{t+1}}{h_{t+1}} \right)^{-(1-\alpha)} + 1 - \delta_k + (1-\theta)\theta^{\theta/(1-\theta)} q_{t+1}^{1/(1-\theta)} \right] \tag{A.19}$$

l_G equation:

$$\begin{aligned} & \frac{A_{Gt}}{A_{Ht}} . l_{Gt}^{-\alpha} \cdot \left[\frac{k_t}{h_t} \right]^{\alpha} && \text{(A.20)} \\ = \quad & E_t \left[m_{t+1} A_{Gt+1} l_{Gt+1}^{1-\alpha} \left(\frac{k_{t+1}}{h_{t+1}} \right)^{\alpha} \right] + \\ & E_t \left[m_{t+1} l_{Gt+1}^{-\alpha} \left(\frac{k_{t+1}}{h_{t+1}} \right)^{\alpha} (1 - \delta_h + A_{Ht+1} l_{ht+1}) \frac{A_{Gt+1}}{A_{Ht+1}} \right] \end{aligned}$$

x equation:

$$\frac{\psi}{x_t} - (1-\alpha)\beta E_t \left[\frac{1}{1+\mu_{t+1}} A_{Gt} l_{Gt}^{-\alpha} \left(\frac{k_t}{h_t} \right)^{\alpha-1} \left(\frac{c_t}{k_t} \right)^{-1} \right] = 0 \tag{A.21}$$

k/h equation:

$$\frac{k_{t+1}}{h_{t+1}} = \frac{\{(1-\delta_k)(k_t/h_t) + A_{Gt} l_{Gt}^{1-\alpha} (k_t/h_t)^{\alpha} - (c_t/k_t).(k_t/h_t)\}^{\theta} (\frac{k_t}{h_t})^{1-\theta}}{1 - \delta_h + A_{Ht}(1 - l_{Gt} - x_t)} \tag{A.22}$$

Output growth equation:

$$\frac{y_{t+1}}{y_t} = \left[\frac{A_{Gt+1}}{A_{Gt}} \right] \left[\frac{k_{t+1}/h_{t+1}}{k_t/h_t} \right]^{\alpha} \{A_{Ht} l_{Ht} + 1 - \delta_h\} \cdot \left[\frac{l_{Gt+1}}{l_{Gt}} \right]^{1-\alpha} \tag{A.23}$$

Inflation equation:

$$\frac{P_{t+1}}{P_t} = \frac{1+\mu_{t+1}}{\{(c_{t+1}/k_{t+1})/(c_t/k_t)\}\{(k_{t+1}/h_{t+1})/(k_t/h_t\}\{A_{Ht}l_{Ht}+1-\delta_h\}} \tag{A.24}$$

The discount factor equation:

$$m_{t+1} = \beta\frac{\{1+(1+\rho)\bar{\mu}-\rho\mu_{t+1}\}}{\{1+(1+\rho)\bar{\mu}-\rho\mu_t\}}\frac{(c_t/k_t)}{(c_{t+1}/k_{t+1})}\cdot\frac{(k_t/h_t)}{(k_{t+1}/h_{t+1})}\cdot\frac{1}{1-\delta_h+A_{Ht}l_{Ht}} \tag{A.25}$$

Equation (A.19) follows from (A.18), (13) and (A.15). Equation (A.20) follows from (A.5), (A.6), (A.7), (A.8), A.15) and (A.16). Equation (A.21) follows from (A.6), (A.8) and (A.16). Equation (A.22) follows by combining (4) (11) and (12). The growth equation (A.23) follows from (4) and (11). To obtain the inflation equation (A.24) rewrite the cash-in-advance constraint (5) using (4) as:

$$\frac{P_{t+1}}{P_t} = \frac{(1+\mu_{t+1})(A_{Ht}l_{Ht}+1-\delta_h)^{-1}}{\frac{c_{t+1}}{k_{t+1}}\frac{k_{t+1}}{h_{t+1}}\left(\frac{c_t}{k_t}\frac{k_t}{h_t}\right)^{-1}}$$

Regarding (A.25) use the log utility specification and (4) to rewrite this as:

$$m_{t+1} = \frac{\beta\frac{c_t}{k_t}\frac{k_t}{h_t}}{\frac{c_{t+1}}{k_{t+1}}\frac{k_{t+1}}{h_{t+1}}}\cdot\frac{E_{t+1}\left[\frac{1}{1+\mu_{t+2}}\right]}{E_t\left[\frac{1}{1+\mu_{t+2}}\right]}\cdot\frac{1}{1-\delta_h+A_{Ht}l_{Ht}}$$

Next take a first order approximation around the steady state and use the forcing process for money supply growth (16) to get the expression in (A.25).

A.4 An Asset Pricing Based Formulation of Tobin's q

In this appendix, we show that defining $q_t = v_t/k_{t+1}$ yields the same equilibrium q relationship (A.19) and the steady state q-growth relation as in Corollary 2.

Rewrite (7) as

$$\frac{v_t}{k_{t+1}} = E_t m_{t+1} \left[\frac{v_{t+1}}{k_{t+2}} . \frac{k_{t+2}}{k_{t+1}} + \frac{d_{t+1}}{k_{t+1}} \right] \tag{A.26}$$

which can be rewritten by using (12) as

$$q_t = E_t m_{t+1} \left[\theta^{\frac{\theta}{1-\theta}} q_{t+1}^{\frac{1}{1-\theta}} + \frac{d_{t+1}}{k_{t+1}} \right] \tag{A.27}$$

Next using the constant returns to scale property of the production function d_{t+1}/k_{t+1} can written as:

$$\frac{d_{t+1}}{k_{t+1}} = A_{Gt+1} F_{1t+1} + 1 - \delta_k - (\theta q_{t+1})^{\frac{1}{1-\theta}} \tag{A.28}$$

which upon substitution in (A.27) yields (A.19).

To prove the Tobin's q equation (19), use (A.9) and (A.11) and log-utility specification to get

$$\frac{\xi_t v_t}{c_t} = \beta E_t \left[\frac{\xi_{t+1}(v_{t+1} + d_{t+1})}{c_{t+1}} \right] \tag{A.29}$$

where

$$\xi_t = E_t \left[\frac{1}{1 + \mu_{t+1}} \right] \tag{A.30}$$

Along a balanced growth path money growth rate is constant and thus (A.29) reduces to:

$$v_t = \frac{\beta}{1-\beta} d_t \tag{A.31}$$

which can be further simplified by plugging in $d_t = \bar{A}_G F(k_t, l_{Gt} h_t) - (W_t/P_t) l_{Gt} h_t - i_t^k$.

Dividing through by k_{t+1} and imposing the balanced growth condition

$$\frac{v_t}{k_{t+1}} = \frac{\beta}{1-\beta}\left[(\bar{A}_G F_1 + 1 - \delta_k - \left(\frac{k_{t+1}}{k_t}\right)^{1/\theta}\right](1+g)^{-1} \tag{A.32}$$

Finally using the adjustment cost function (12) and using (17) one gets the same Tobins'q formula as in (19).

A.5 Balanced Growth Equilibrium

Based on (4), (17), the resource constraint, time constraint, (A.20) and (A.21), the steady state can be represented as

$$\begin{aligned}
1+g &= 1-\delta_h + \bar{A}_H l_h = (1-\delta_k + \frac{i_k}{k})^{\theta} && \text{(A.33)}\\
1+g &= \beta(1-\delta_h + \bar{A}_H(1-x)) && \text{(A.34)}\\
\frac{c}{k} + \frac{i_k}{k} &= \frac{y}{k} = \bar{A}_G\left(\frac{l_G h}{k}\right)^{1-\alpha} && \text{(A.35)}\\
\frac{c}{k}\frac{\psi}{x} &= \frac{(1-\alpha)\beta}{(1+\bar{\mu})}\frac{y}{k}\frac{1}{l_g} && \text{(A.36)}\\
\beta\theta(\alpha\frac{y}{k} + 1 - \delta_k) &= [1-\beta(1-\theta)](1+g)^{\frac{1}{\theta}} && \text{(A.37)}\\
1 &= x + l_G + l_H && \text{(A.38)}
\end{aligned}$$

Equating the $1+g$ terms in the first equality of (A.33) and (A.34) yields (with the help of (A.38)) a linear relationship between l_G in terms of x emerges as follows:

$$(1-\delta_h)(1-\beta) = \bar{A}_H[l_G - (1-\beta)(1-x)] \tag{A.39}$$

From (A.36) and the first part of (A.35) we can obtain $\frac{y}{k}$ in terms of $\frac{i_k}{k}$ and $\frac{x}{l_g}$. Substituting this into (A.37) and then writing $\frac{i_k}{k}$ in terms of g from (A.33) yields a further expression for g in terms of $\frac{x}{l_g}$. Finally we replace l_G

by its representation in terms of x, and g in terms of x from (A.34) to get an equation solely in x:

$$\frac{\theta(1-\beta)}{\bar{A}_H}(1-\delta_k)(1-\alpha)(1-\delta_h+\bar{A}_H(1-x))\psi - \frac{\theta(1-\delta_k)(1-\alpha)\beta}{1+\bar{\mu}}x$$
$$+\frac{(1-\beta(1-\theta))(1-\alpha)}{1+\bar{\mu}}\beta^{\frac{1}{\theta}}(1-\delta_H+\bar{A}_H(1-x))^{\frac{1}{\theta}}x \tag{A.40}$$
$$-\frac{(1-\beta+\beta\theta(1-\alpha))(1-\beta)}{\bar{A}_H}\psi\beta^{\frac{1}{\theta}-1}(1-\delta_h+\bar{A}_H(1-x))^{1+\frac{1}{\theta}}=0$$

Once x is solved from (A.40), l_G can be solved from (A.39). The remaining endogenous variables which are just functions of l_G and x can be computed.

B Proof of Proposition 1

Note first from (6) and (A.8) that along the balanced growth path

$$m_{t+1}=\frac{P_{t+1}\lambda_{t+1}}{P_t\lambda_t}=\frac{\beta}{1+g}. \tag{B.1}$$

Using (13), (A.19) , and (B.1), and imposing the balanced growth condition, $\frac{i_t^k}{k_t}=\frac{i_{t+1}^k}{k_{t+1}}$ one obtains that

$$\left(1-\delta_k+\frac{i_t^k}{k_t}\right)^{1-\theta}=\frac{\beta\theta}{1+g}\bar{A}_G F_1+\frac{\beta}{1+g}\left[(1-\theta)\left(1-\delta_k+\frac{i_t^k}{k_t}\right)+\theta(1-\delta_k)\right]. \tag{B.2}$$

Use the adjustment cost function (12) to write

$$\frac{i_t^k}{k_t}=(1+g)^{1/\theta}-1+\delta, \tag{B.3}$$

which after plugging into equation (B.2) yields the proposition result of equation (17). Also it is straightforward to verify from equation (A.20) of Appendix A.1 the standard result in such Lucas (1988) human capital models with leisure, that $1+g=\beta[1-\delta_h+A_H(1-x)]$.

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