

**Cardiff Economics
Working Papers**

Patrick Minford and Naveen Srinivasan

*Determinacy in New Keynesian models: a role for money after
all?*

E2009/21

Cardiff Business School
Cardiff University
Colum Drive
Cardiff CF10 3EU
United Kingdom
t: +44 (0)29 2087 4000
f: +44 (0)29 2087 4419
www.cardiff.ac.uk/carbs

ISSN 1749-6101

Created October 2009, updated September 2010

This working paper is produced for discussion purpose only. These working papers are expected to be published in due course, in revised form, and should not be quoted or cited without the author's written permission. Cardiff Economics Working Papers are available online from: <http://www.cardiff.ac.uk/carbs/econ/workingpapers>
Enquiries: EconWP@cardiff.ac.uk

Determinacy in New Keynesian models: a role for money after all?

Patrick Minford

Cardiff Business School and CEPR, UK

Naveen Srinivasan*

Indira Gandhi Institute of Development Research, Mumbai, India

Abstract

The New-Keynesian Taylor-Rule model of inflation determination with no role for money is incomplete. As Cochrane (2007a, b) argues, it has no credible mechanism for ruling out bubbles (**or deal with the non-uniqueness problem that arises when the Taylor principle is violated**) and as a result fails to provide a reason for private agents to pick a unique stable path. We propose a way forward. Our proposal is in effect that the New-Keynesian model should be formulated with a money demand and money supply function. It should also embody a terminal condition for money supply behaviour. **If indeterminacy of stable (or unstable paths) occurred** the central bank would switch to a money supply rule explicitly designed to stop it via the

*Corresponding Author: Naveen Srinivasan, IGIDR, Gen A. K. Vaidya Marg, Goregoan (E), Mumbai – 65, India Email: naveen@igidr.ac.in

terminal condition. This would be therefore a ‘threat/trigger strategy’ complementing the Taylor Rule — only to be invoked if inflation misbehaved. Thus we answer the criticisms levelled at the Taylor Rule that it has no credible mechanism for **dealing with these issues**. However it does imply that money cannot be avoided in the new Keynesian set-up, contrary to Woodford (2008).

JEL classifications: E31, E52, E58

Keywords: New-Keynesian; Taylor rule; Determinacy; terminal condition; money supply

1 Introduction

The New-Keynesian Taylor-Rule (NK, henceforth) approach to monetary economics provides the current standard model of inflation determination. By linking interest rate decisions directly to inflation and economic activity, Taylor Rules offered a convenient tool for studying monetary policy while abstracting from a detailed analysis of the demand and supply of money.¹ This change in the standard analytics is an understandable reflection of how most central banks now make monetary policy: by setting a short-term nominal interest rate, with little if any explicit role for money (see Friedman, 2003).² Furthermore, econometric evidence supporting the stabilization properties of this rule (see Taylor, 1999) and its usefulness for understanding historical monetary policy (see Clarida, Gali, and Gertler, 2000) explains its popularity.³

While the NK approach has been remarkably successful, there are reasons to be uneasy about the lack of modelling of money markets. For example, Cochrane (2007a) argues that the way standard “New Keynesian” models work to discipline inflation is in fact incredible: In effect, the Fed threatens to raise inflation and interest rates without limit should inflation deviate from the stable path. That is, the Fed threatens hyperinflation or deflation, unless

¹Woodford (2008) describes a class of New-Keynesian models and draws attention to the fact that interest rates transmit directly to intertemporal spending decisions and that monetary policy need not be framed in terms of monetary aggregates. For an account of the origins of the Taylor Rule in early work by Henderson and McKibbin (1993), see Minford (2008).

²There are exceptions of course. For example, the European Central Bank (ECB) continues to assign a prominent role to money in its monetary policy strategy. In what the ECB calls its “two-pillar strategy,” one pillar is “economic analysis,” which “assesses the short-to-medium-term determinants of price developments.” In addition, a second pillar, “monetary analysis,” assesses the medium- to long-term outlook for inflation, exploiting the long-run link between money and prices.

³These developments have greatly influenced monetary policy research and teaching. This allowed the development of simpler models (see the survey in Clarida, Gali, and Gertler, 1999) and the replacement of the “LM curve” with a Taylor Rule in textbook treatments of the Hicksian IS-LM apparatus (see Taylor (2000) and Romer (2000)).

inflation jumps to one particular value on each date. This is true: if inflation takes off along a bubble path in this model what is there to stop it? The NK answer seems to be: just the horrifying thought that this might happen! Essentially, the government threatens to ‘blow up the (monetary) world’ to use Cochrane’s phrase should any but one equilibrium occur. Because people believe this threat, inflation goes to this unique path. But would people really avoid deviant paths fearing this nuclear option? And would they believe that the Fed would stick with such a rule under such circumstances?

One problem is that these threats are not credible. The reason is that, once inflation or deflation happens, carrying through on the threat is a disastrous policy. As a result self-destructive threats are less likely to be carried out *ex-post*, and thus less likely to be believed *ex-ante*. A second problem with these threats is that even if they were credible and did actually happen, there seems to be nothing to stop people following the implied paths. While undesirable from a social viewpoint, they do not appear to be *impossible*. Thus no transversality conditions on real variables appear to be violated for reasonable versions of NK models.

Another reason to be uneasy with this model is that its account of the high inflation episode of the 1970s is vacuous. The main claim of the NK Taylor rule view of inflation determination is that inflation got out of control in the 1970s because the Fed reacted insufficiently to inflation (Clarida, Gali, and Gertler, 2000). Yet, as Cochrane (2007a, b) and Minford (2008) have argued, if the Fed in the 1970s had such a Taylor rule in place, then inflation would have been indeterminate. So, in what sense does this account for any inflation path at all?

What this shows is that the Taylor Rule is an incomplete description of monetary policy,

at least within a NK model; it cannot account for determinate inflation before 1980, and after 1980 it lacks a clear mechanism for ruling out unstable paths. One has to assume that the authorities have some additional tool in their locker to **deal with these issues**.

Our proposal is in effect that the NK model should be formulated with a money demand and money supply function. It should also embody a terminal condition for money supply behaviour. If **indeterminacy** occurred the central bank would switch to a money supply rule explicitly designed to stop it via say a terminal condition. This would be therefore a threat or trigger strategy complementing the Taylor Rule — only to be invoked if inflation misbehaved. Of course if the strategy is credible it would never be observed and you would just get the Taylor Rule. Thus we achieve a determinate solution without appealing to the notion that the unstable paths are ruled out by an extreme threat to wreck the monetary economy⁴; and also answer the criticisms levelled at the Taylor Rule that it has no credible mechanism **for dealing with bubbles or the non-uniqueness problem** — we do this via our threat strategy.⁵ However it does imply that money cannot be avoided in the NK set-up, contrary for example to Woodford (2008). There has to be a money supply rule operating in emergency at least. Thus in summary we reinterpret the nature of monetary policy under Taylor Rules used in NK models. Monetary policy is in effect not fully revealed by simply writing down a Taylor Rule; ‘behind it’ lies various implied commitments — viz to the provision of money according to a long-term (terminal) condition that limits undesirable

⁴Cochrane (2007b) argues this can be a non-Ricardian fiscal policy. This is a possible route but here we maintain the usual NK assumptions: that the Taylor Principle applies and that fiscal policy is Ricardian. Our objective is to show that the NK model can work in its own terms, by adding a ‘background condition’ relating to money supply policy.

⁵We agree that the ‘NK set-up’ is modified, as it must be given its flawed nature on which there is wide agreement; but our point is that the modification allows the model to work exactly as intended and hence removes this flaw. We would maintain that these supplements are not damaging to the NK approach in practical macro modelling terms, though they do prevent any claim to there being a ‘cashless world’.

behaviour of inflation with an override of the money supply rule implicit in the Taylor Rule.

The article is organized as follows. In Section 2 we construct a frictionless NK model, and uncover the general properties of this model. We also study determinacy in the standard three-equation NK model. We verify that the issues are the same, and the Fed **after 1980** does in fact determine inflation by threatening hyperinflation, not by stabilizing past inflation. **In contrast, inflation is indeterminate before the 1980s in this model.** Section 3 explains how we deal with **both** explosive solution paths **and the non-uniqueness problem** in traditional macro models of the 1970s. Section 4 explain how we deal with **these issues** in a frictionless NK model. We also answer the criticisms levelled at the Taylor Rule that it has no credible mechanism for ruling out bubbles **or dealing with the problem of indeterminacy** — we do this via a terminal condition for money supply behaviour. Section 5 provides concluding remarks.

2 Determinacy in New-Keynesian model

2.1 Determinacy in a frictionless New-Keynesian Model

The basic points do not require the Phillips - IS curve features of NK models, and thus they do not need any frictions. This might come as a surprise to those of us who have been brought up to think that in the Keynesian framework the Phillips curve pins down the inflation rate given output supply, which is demand determined. But in the NK literature it is routine to discuss inflation determination without mentioning the Phillips curve (see Woodford 2008 for example). Following Cochrane (2007a) we start with a very simple model

consisting only of a Fisher equation and a Taylor Rule describing monetary policy:⁶

$$i_t = r + E_t \pi_{t+1} \quad (1.1)$$

$$i_t = r + \pi^* + \phi (\pi_t - \pi^*) \quad (1.2)$$

where i_t =nominal interest rate, π_t =inflation and r =constant real rate. The coefficient $\phi > 0$ measures how sensitive the central bank's interest rate target is to inflation. We can solve this model by substituting out the nominal interest rate, leaving only inflation,

$$E_t \pi_{t+1} = \phi (\pi_t - \pi^*) + \pi^*,$$

or

$$E_t \pi_{t+i+1} = \phi (E_t \pi_{t+i} - \pi^*) + \pi^*, \quad (\text{for } i \geq 0) \quad (1.3)$$

where we have a first order expectational difference equation in π_t . The general solution for this first order difference equation can be expressed as

$$E_t \pi_{t+i+1} = \pi^* + (\pi_t - \pi^*) (\phi)^i, \quad (\text{for } i \geq 0). \quad (1.4)$$

Equation (1.4) has many solutions, and this observation forms the classic doctrine that inflation is indeterminate with an interest rate target. However, if $\phi > 1$ (Taylor Principle), all of these solutions except one eventually explode. This example makes it crystal-clear that

⁶Cochrane (2007a) takes the standard three-equation NK model and simplifies it by assuming full price flexibility so that output equals the natural rate in each period. This eliminates the Calvo Phillips curve and the output gap term in the standard Taylor rule. He also assumes that the natural rate of output is a constant which yields the relation (1.1), where r is a constant real rate of interest.

inflation determination comes from a threat to increase future inflation if current inflation gets too high. If inflation takes off along a bubble path what is there to stop it in this model? The NK answer is: just the dreadful thought that this might happen. This is because in this model the monetary authority is absolutely committed to raising interest rates more than one for one with inflation, for all values of inflation. For only one value of inflation today will we fail to see inflation that either explodes or, more generally, eventually leaves a local region. Ruling out non-local equilibria, NK modellers conclude that inflation today jumps to the unique value that leads to a locally-bounded equilibrium path.

In contrast, if $\phi < 1$ (the Taylor Principle is violated), one could choose any value for π_t different from π^* , and the solution to (1.4) describes a path that eventually takes the system back to steady state (i.e., $\pi_t \rightarrow \pi^*$, as $i \rightarrow \infty$). Because there is an uncountable number of such paths, each of which follows a path back to steady state, it follows that there is a multiplicity of stable equilibria- the non-uniqueness problem. In principle any of these stable paths could be selected. Thus, as Cochrane (2009b) argues, the NK Taylor rule model has absolutely nothing to say about inflation in the 1970s, other than “inflation is indeterminate”, so any value can happen.

2.2 Determinacy in the three-equation model

Now let us consider a standard NK model with frictions (for example, see Clarida et al., (1999) and Woodford (2003)). For determinacy questions, we can work with a stripped-

down model without constants or shocks.

$$y_t = E_t y_{t+1} - \sigma r_t, \quad \sigma > 0 \tag{2.1}$$

$$i_t = r_t + E_t \pi_{t+1}, \tag{2.2}$$

$$\pi_t = \beta E_t \pi_{t+1} + \gamma y_t, \quad \beta, \gamma > 0 \tag{2.3}$$

where where y_t = output and r_t = real interest rate. This representation can represent deviations from a specific equilibrium of a model with shocks (see Cochrane (2007b)). The first two equations derive from consumer first order conditions for consumption today vs. consumption tomorrow.⁷ The first equation is a log-linear approximation to an Euler equation for the timing of aggregate expenditure, sometimes called an “intertemporal IS relation.” This is the one that indicates how monetary policy affects aggregate expenditure: the expected short-term real rate of return determines the incentive for intertemporal substitution between expenditure in periods t and $t + 1$. The last equation is the NK Phillips curve. It is derived from the first order conditions of intertemporally-optimizing firms that set prices subject to costs.⁸ The remaining equation required to close the system is a specification of monetary policy. We might, for example, specify policy by a rule of the kind proposed by Taylor (1993) for the central bank’s operating target for the short-term nominal interest

⁷See Woodford (2003, chaps. 3–5) for discussion of the microeconomic foundations underlying equations (2.1) and (2.2). Woodford (2008) refers to models of this kind “neo-Wicksellian,” to draw attention to the fundamental role in such models of a transmission mechanism in which interest rates affect intertemporal spending decisions, so that monetary policy need not be specified in terms of an implied path for the money supply, but the terminology “NewKeynesian” for such models has become commonplace, following Clarida et al. (1999), among others.

⁸This equation represents a log-linear approximation to the dynamics of aggregate inflation in a model of staggered price-setting of the kind first proposed by Calvo (1983).

rate,

$$i_t = \phi\pi_t, \quad \phi > 0. \tag{2.4}$$

2.3 Can Such a Model Explain the Rate of Inflation?

A first question about this model is whether such a model which has thus far made no reference to the economy's supply of money has any implication for the rate of inflation. Woodford (2008) argues that while a model like this does not determine the inflation rate independently of monetary policy, it does determine the inflation rate without any reference to money growth and without any need to specify additional relations beyond those listed above. He goes on to argue that there is nothing “conceptually incoherent” about a model of inflation determination that involves no role whatsoever for measures of the money supply. Using (2.4) to substitute for i_t in (2.2), the model (2.1)–(2.3) can be written in the form,

$$\begin{bmatrix} E_t y_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} = \frac{1}{\beta} \begin{bmatrix} \beta + \sigma\gamma & -\sigma(1 - \beta\phi) \\ -\gamma & 1 \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \end{bmatrix}.$$

The eigenvalues of matrix A that is, λ_1 and λ_2 , are computed by setting $\det(A - \lambda I) = 0$.

This gives a second-order polynomial in λ :

$$\frac{1}{\beta} [\lambda^2 - (1 + \beta + \sigma\gamma)\lambda + \beta(1 + \sigma\gamma\phi)] = 0$$

where $\lambda_1 + \lambda_2 = (1 + \beta + \sigma\gamma)$ and $\lambda_1\lambda_2 = \beta(1 + \sigma\gamma\phi)$.

Proposition 1 *If the number of eigenvalues of A outside the unit circle is equal to the*

number of non-predetermined variables (or forward-looking variables), then there exists a unique stable solution. Blanchard and Kahn (1980)

Proposition 2 *Let λ_1, λ_2 lie in the complex plane, then: the λ_i 's ($i = 1, 2$) are both outside the unit circle if and only if the following conditions are satisfied:*

$$|\lambda_1 + \lambda_2| < |1 + \lambda_1\lambda_2|$$

$$|\lambda_1\lambda_2| > 1.$$

For the usual parameter values in NK models ($\beta \cong 1, \sigma > 0, \gamma > 0$ and $\phi > 1$) the system guarantees both eigenvalues are greater than one. Thus the general solution for $E_t\pi_{t+1}$ can be expressed as

$$E_t\pi_{t+i+1} = \pi^* + A_1 (\lambda_1)^i + A_2 (\lambda_2)^i, \quad (\text{for } i \geq 0). \quad (2.5)$$

How does the Fed plan to stabilise inflation in this model? In this model, $E_t y_{t+i}$ and $E_t \pi_{t+i}$ explode in any equilibrium other than $y = 0, \pi = 0$. According to NK modellers, $\phi > 1$ (the Taylor Principle), would stabilize inflation. But how does it rule out the unstable path? Here NK authors become vague, saying that such paths would be ‘inconceivable’ and hence ‘ruled out by private agents’.

Thus for example King (2000, p. 58–59, cited in Cochrane, 2007a) writes: “*By specifying $[\phi > 1]$ then, the monetary authority would be saying, ‘if inflation deviates from the neutral level, then the nominal interest rate will be increased relative to the level which it would be at under a neutral monetary policy.’ If this statement is believed, then it may be enough to convince the private sector that the inflation and output will actually take on its neutral level.*”

Ruling out such non-local equilibria, the NK tradition concludes that output and inflation are again determinate. According to Cochrane (2007a), in effect if current inflation misbehaves the Fed threatens to implement such paths (hyperinflation or hyperdeflation). Thus the threat is to ‘blow up the world’ — and this threat is supposed to be so terrifying that private agents expect the stable path instead. No economic consideration rules out the explosive solutions. With $\phi > 1$, the explosive solutions are legitimate solutions of the model, just as the multiple solutions are legitimate with $\phi < 1$.⁹

This interpretation of the ruling-out of unstable paths raises many questions. Consider what is being said. 1) if inflation rises (falls), it will be forced into a hyperinflation (hyperdeflation) by the Fed. 2) if inflation remains on target, then the Fed will maintain it at that target. So we need to establish how this enables private agents to choose the stable path. Clearly they will prefer the stable path; but how can they be sure it will happen, given that all the paths are feasible. The first question is: is the threat in statement 1) credible? People know that hyperinflation (hyperdeflation) is costly for the central bank/government too. If we think of inflation as a tax chosen by the government on optimising grounds then plainly the government will be thrown away from its optimum, obtaining excessive (inadequate) revenue etc. Thus if the central bank carries out this threat, the government’s and society’s

9

Proposition 3 *If the number of eigenvalues outside the unit circle is less than the number of non-predetermined variables, there is an infinity of stable solutions. Blanchard and Kahn (1980)*

Proposition 4 *Let λ_1, λ_2 lie in the complex plane, then: the λ_i ’s ($i = 1, 2$) are one inside and one outside the unit circle if and only if the following condition is satisfied:*

$$|\lambda_1 + \lambda_2| > |1 + \lambda_1\lambda_2|.$$

For the usual parameter values in NK models and $\phi < 1$ this condition is met. That is, there are infinity of stable paths — the ‘non-uniqueness’ problem (Taylor, 1977).

interests would be badly damaged. So people would conclude that the central bank would simply not follow up on its threat in society's best interests. That is, they would expect the central bank to accommodate rising inflation ($\phi < 1$). So clearly the implicit threat in NK models (the 'Taylor Principle') is simply not credible in equilibrium.

The second question is: assume the threat is credible; then if it were to be carried out is there anything to stop the unstable path continuing to infinity? One possible way that the path could be stopped is by violating real variable transversality conditions. In the NK model this is not the case, as noted by Cochrane. In models with a demand for real balances, McCallum (2009a) notes that transversality conditions on real money demand cannot rule out hyperinflations for reasonable preferences. Obstfeld and Rogoff (1983) reached the same conclusion when money enters the utility function, suggesting that the government could rule out hyperinflation by backing the currency at some fractional value. This is a policy suggestion, which acts in a similar way to our suggestion below, as we will explain.

Thus we find that a) the rule implies an incredible threat; and that b) even were it to be credible, it would imply that unstable paths would continue to infinity were they to occur. Under a) the Taylor Rule defaults to an accommodative rule under which there is indeterminacy of stable paths. Under b) unstable paths would carry on for ever were they to occur. Hence there is nothing to make them infeasible. Thus effectively we have two possible NK models; either one with indeterminacy of stable paths or one with indeterminacy of unstable paths. Notice we are not attacking the NK model as such but we are arguing that it fails to provide a reason for private agents to pick a unique stable path.¹⁰

¹⁰McCallum (2009a,b) further proposes to rule out unstable paths in the NK model by a 'learnability condition'. That this condition does rule them out is disputed by Cochrane (2009). We also note that there may be situations where agents already know, or learn in some direct way, the models that exist. In such

3 Traditional Macro Model and Inflation Determinacy

How do we deal with **these issues** in traditional macro models of the 1970s? Our objective here is twofold. We show that the solution of this model is similar to the NK model discussed above. Moreover, we shall show how **we deal with indeterminacy** in these models. We illustrate this with the aid of the Minford and Peel (2002) version of Cagan's (1956) hyperinflation model, described by the equation system (3.1)–(3.2).

$$m_t = p_t - \alpha (E_t p_{t+1} - p_t), \quad \alpha > 0 \tag{3.1}$$

$$m_t = \bar{m}, \tag{3.2}$$

where m_t and p_t are the natural logarithms of money supply and the price level, respectively; and \bar{m} is a monetary target, and E is the rational expectations operator. The first equation is a money demand function, specifying that the demand for money responds negatively to expected price level changes. The second equation is a money supply function where the government has a monetary target, \bar{m} . The above model is an example of RE models involving a future variable, and the main problem in solving the model comes from the presence of $E_t p_{t+1}$ in the first equation.

situations learnability (or at least least squares learnability) does not arise and the criterion cannot therefore work. Our proposal here is offered as an alternative criterion that is bound to work in all models of this type.

Substituting ([?]) in (3.1) for m_t yields;

$$E_t p_{t+1} = \left(\frac{1 + \alpha}{\alpha} \right) p_t - \frac{\bar{m}}{\alpha} \quad (3.3)$$

or

$$E_t p_{t+i+1} = \left(\frac{1 + \alpha}{\alpha} \right) E_t p_{t+i} - \frac{\bar{m}}{\alpha} \quad (\text{for } i \geq 0). \quad (3.4)$$

The solution of this first-order non-homogenous difference equation is:

$$\begin{aligned} E_t p_{t+i+1} &= p^* + A(\lambda)^i, \quad (\text{for } i \geq 0) \\ E_t p_{t+i+1} &= \bar{m} + (p_t - \bar{m}) \left(\frac{1 + \alpha}{\alpha} \right)^i, \quad (\text{for } i \geq 0). \end{aligned} \quad (3.5)$$

where \bar{m} is the equilibrium of p_t (the ‘particular solution’), $\frac{1+\alpha}{\alpha}$ is the unstable root and $p_t - \bar{m}$ is the constant (determined by the initial value p_t) in the ‘general’ solution. Notice that the general solution for $E_t p_{t+i+1}$ has the same form as (1.4) above. Equation (3.5) gives an infinite number of solution paths for $E_t p_{t+i+1}$ ($i \geq 0$). For we are free to choose any value of p_t we like; the model does not restrict our choice. Another way of looking at (3.5) is to say that we can choose any future value for $E_t p_{t+i+1}$ we wish and work back from that to a solution for p_t . Any view of this future will then compel a present which is consistent with it; any set of expectations is therefore self-justifying i.e., anything can happen provided it is expected, but what is expected is arbitrary. Worse still, as (3.5) illustrates, these paths for events can be unstable; in fact, our model here implies that all paths for prices except that for which $p_t = \bar{m}$, explode monotonically. Thus the model would assert that only by accident would an equilibrium price level be established, otherwise prices would be propelled

into either ever-accelerating hyperinflation or ever-deepening hyperdeflation, even though money supply is held rigid!

To prevent these unstable paths, we appeal to an optimising government, choosing the inflation tax. Having chosen its optimum target — which here for simplicity we set at zero — we assume it sets a money supply target designed to achieve it, provided unstable paths do not occur. It then needs, in order to achieve this optimum, to prevent these unstable paths from occurring. It turns out that a commitment on its part to put an end to any inflation (deflation) bubble paths at some point, by decreasing (increasing) the money supply sufficiently to force prices off this path, will do the trick. For if people expect that inflation will stop at some period $t + N$ (at which the bank will ‘step in’), then desired real money balances in $t + N$ will now be higher and inflation would fall at $t + N$. If inflation falls in $t + N$ then people would postpone consumption at $t + N - 1$ and inflation would fall at $t + N - 1$ too. And so on. By backward induction the whole process gets invalidated. We can show this formally by imposing the terminal condition

$$E_t p_{t+i+1} - E_t p_{t+i} = 0 \text{ for } i \geq N.$$

Substituting the terminal condition in (3.4) yields

$$\begin{aligned} E_t p_{t+N+1} &= \left(\frac{1 + \alpha}{\alpha} \right) E_t p_{t+N} - \frac{\bar{m}}{\alpha} \\ E_t p_{t+N+1} &= \bar{m}. \end{aligned}$$

This implies from (3.5) that, $p_t = \bar{m}$. **It can be seen that a terminal condition**

has the effect in the model of selecting the unique stable path for the price level. Another way of describing our terminal condition would be as a ‘side’ or ‘transversality’ condition: all these express the same idea, that there is an additional restriction on the model, here coming from government or central bank behaviour designed to rule out what is from their (or society’s) viewpoint an undesirable outcome, in this instance for the monetary environment.

Dealing with the Non-uniqueness problem

Now let us suppose for some reason α in (3.5) is negative and < -0.5 . Now we have multiplicity of stable paths since $|\frac{1+\alpha}{\alpha}| < 1$: there is no unique stable path. Now all the paths in (3.5) are stable, because we have rigged it so that $|\frac{1+\alpha}{\alpha}| < 1$. This problem was first pointed out by Taylor (1977). It turns out that a terminal condition ($E_t p_{t+N} - E_t p_{t+N+1}$) does in fact impose a unique stable solution in this case too (see Minford and Peel, 2002). Our terminal solution implies:

$$\bar{m} + (p_t - \bar{m}) \left(\frac{1 + \alpha}{\alpha} \right)^{N+1} = \bar{m} + (p_t - \bar{m}) \left(\frac{1 + \alpha}{\alpha} \right)^N,$$

which is strictly valid for finite N only when $A = 0$. Thus also: $E_t p_{t+N+1} = p_t = \bar{m}$. Thus the terminal condition helps us to select a unique stable path even in this case. The justification for such a condition is that non-uniqueness must cause quite as serious a problem as bubbles. For the endogenous variables may in each period jump by unpredictably large (strictly unbounded) amounts; even though they will subsequently be expected to return to equilibrium. Such uncertainty would in all likelihood provoke changes in behaviour sufficient to

create an incentive for the government or money issuer to make a commitment such as is set out in the terminal condition. This commitment would then limit the uncertainty as we have seen, to that associated with the ‘most stable’ path – a result much in the spirit of Taylor (1977) that the minimum variance path will be selected by ‘collective rationality’.

4 The New Keynesian model with terminal condition

An analogous argument can be constructed for the frictionless NK model discussed above. Suppose that the economy is described by (1.1) and (1.2) plus a demand for money function given by (3.1) and the identity $p_t = \pi_t + p_{t-1}$ (money supply being endogenous under the Taylor rule, money is supplied as demanded in 3.1). Now we add to this model the terminal condition, $E_t\pi_{t+i+1} = E_t\pi_{t+i}$ (for $i \geq N$). This terminal condition is produced by a supplementary feature in the overall monetary policy framework, which states that if this terminal condition were not satisfied under the Taylor rule then policy would switch to a money supply rule that would force the satisfaction of this condition. It is easy to see that the model of 3.1 (plus 1.1 for interest rates, now no longer controlled by the Taylor rule) and such a rule would yield a solution for $t + N$ of $E_t\mu_{t+N} = (1 + \alpha)E_t\pi_{t+N} - \alpha E_t\pi_{t+N+1}$: this monetary equilibrium condition tells us what money supply growth (μ) must be at $t + N$ for given $E_t\pi_{t+N}$ in order to implement the terminal condition, viz $E_t\mu_{t+N} = E_t\pi_{t+N}$. In effect money supply growth must be whatever is needed to force the demand for money and the supply of money into equality at a constant inflation rate from $t + N$.¹¹

¹¹It must be stressed that this solution for the money supply growth required is simply the mathematical implication for the money supply growth that would emerge from this policy. In practical terms one would

Now returning to the model with the Taylor rule but adding this terminal condition, we find that substituting it in (1.4) yields $E_t\pi_{t+N+1} = \pi^*$. Thus, we have $\pi_t = E_t\pi_{t+1} = \pi^*$. We have ruled out unstable paths by appealing to a terminal condition on inflation implemented via the money supply (or money growth) rule. Is this terminal condition credible? As we have argued, the government has an incentive to prevent a bubble path because it is the stable path that maximises its objectives. There is a lot at stake here for it and people understand this. Therefore a commitment to stop these bubble paths is credible. This implies that money cannot be avoided in NK models, contrary to the ‘cashless world’ invoked by Woodford et al.¹²

It can be seen that this government commitment acts to disrupt unstable paths in just the same way that Obstfeld and Rogoff’s (1983) suggestion for government fractional backing of the currency prevents hyperinflation paths (while a transversality condition on real balances not tending to infinity because they would be swapped for consumption goods rules out hyperdeflation). Our suggestion can be thus thought of as a practical way of implementing the same idea.

The terminal condition also solves the non-uniqueness problem that arises

imagine that were a bubble situation of ‘inflation disorder’ to break out the central bank would have to deploy open market operations that convinced market participants of the impossibility of any alternative inflation path being permitted. For example these could include a severe contraction of money designed to give a downward shock to rising inflationary expectations. The point is that the central bank does whatever is needed to enforce its commitment against bubble paths. Success in forcing inflation to be constant would then imply the money supply growth shown. Of course such success in turn implies that we will only ever observe money supply growth of, π^* .

¹²We offer a similar argument incidentally as a possible interpretation of the European Central Bank’s second pillar. The second pillar could be regarded as a commitment device designed to anchor inflation expectations in the face of explosive paths for credit and broad money. Thus the ECB uses the Taylor Rule as a practical short term device but always stands ready to override this Rule should the behaviour of the banking sector threaten to produce unsatisfactory longterm inflation behaviour. Neumann (2006), in a review of monetary targeting by the Bundesbank, stresses the desire to influence public expectations of inflation as a central motivation for the strategy and a key element in its success.

when the Taylor principle is violated in this model, $\phi < 1$, and rules out all stable paths except the original saddlepath. This is the case if the terminal condition is selected in finite time as it is with a terminal condition. What this suggests is that one could have had a Taylor rule pre-1980 that did not satisfy the Taylor principle and yet produced determinate inflation in line with the inflation target. If so why was inflation so high in the 1970s? If one agrees that a terminal condition is indispensable in the context of this model then the only plausible explanation for changes in inflation dynamics is time-variation in inflation targets.¹³ In sum, we agree with Cochrane (2007a, b) that the central empirical argument for the NK Taylor rule view of inflation determination that inflation got out of control in the 1970s because the Fed reacted insufficiently to inflation is inadequate.

Finally, we would also note that the NK model behaves in its usual way with a Taylor rule since we never observe the unstable paths or the **non-uniqueness problem** under which the Taylor rule is not binding; **the terminal condition's mere existence therefore stops bubbles or non-uniqueness, so that the policy switch is never observed, only the Taylor rule.** Thus in a very practical way we have here the ordinary NK model; all we have done is to support the validity of its operation by a terminal condition implemented by a monetary policy switch that never in practice needs to be used because its mere existence makes its use unnecessary; **we have also rendered the Taylor Principle inoperative, its role being taken over by the terminal condition.** This latter is the

¹³We would note that this is implicitly what Ireland (2007) is saying. His NK model has a Taylor rule that satisfies the Taylor principle for both the pre- and post-1970 period. The dynamics of inflation in his model is explained by time-variation in inflation target.

‘Big Stick’ in the closet whose existence ensures orderly behaviour under the normal Taylor rule. Thus the terminal condition acts like a special sort of monetary rule in which a variable (money) is used to implement a target for another variable (prices) by taking whatever value is necessary. Examples of such rules are fixed exchange rates in which reserves (thus money) are varied as much as necessary to hit an exchange rate target; or fixed interest rate rules in which money is varied as much as necessary to hit an interest rate target. Under the terminal condition in the NK model, money becomes endogenously set at whatever growth rate is needed to achieve an exogenous price target path.

5 Conclusion

The New Keynesian model of inflation determination has no effective mechanism for ruling out explosive bubbles **or to deal with indeterminacy of stable paths**. It fails to provide a reason for private agents to pick a unique stable path. We propose a way forward. Our proposal is in effect that the NK model should be formulated with a money demand and money supply function, as above in the ‘traditional’ model; as there too, it should embody a terminal condition on inflation. This is implemented by an override on the money supply rule explicitly designed to stop **unstable paths or the non-uniqueness problem** at some point should they occur. To replicate the Taylor Rule we simply specify the money supply rule to imply the Taylor Rule being followed; thus the Taylor Rule has the interpretation of an operational money supply rule — as intended by Taylor in his original paper. It is apparent from our reformulation that nothing changes in the NK model. **The escape clause strategy in which the monetary authority commits to deviating from the**

Taylor rule to a policy of controlling the money supply in the event that bubbles or indeterminacy of stable paths occurred eliminates this equilibrium in NK model; its mere existence therefore stops bubbles/non-uniqueness, so that the policy switch is never observed, only the Taylor rule. Hence the NK model behaves exactly as usual. Note however that the Taylor Principle is no longer operative; in effect its role is superseded by the terminal condition.

Thus in summary we reinterpret the nature of monetary policy under Taylor Rules used in NK models. Monetary policy is in effect not fully revealed by simply writing down a Taylor Rule; ‘behind it’ lies various implied commitments — viz to the provision of money according to a long-term (terminal) condition that limits undesirable behaviour of inflation with an override of the money supply rule implicit in the Taylor Rule. These commitments enable the NK model with a Taylor rule to operate in the usual way. However it does imply that money cannot be avoided in the NK set-up, contrary to the ‘cashless world’ invoked by Woodford (2008).

References

- [1] Blanchard, O., Kahn, C.M., (1980). “The Solution of Linear Difference Models under Rational Expectations,” *Econometrica*, 48, 1305–1313.
- [2] Cagan, P., (1956). “The Monetary Dynamics of Hyperinflation,” In: *Studies in the Quantity Theory of Money*. Friedman, M. (Ed.), Chicago: University of Chicago Press, 25– 117.
- [3] Calvo, G.A. (1983). “Staggered Prices in a Utility-Maximizing Framework,” *Journal of Monetary Economics*, 12, 383–98.
- [4] Clarida, R., Gali, J., Gertler, M., (1999). “The Science of Monetary Policy: A New Keynesian Perspective,” *Journal of Economic Literature*, 37 (4), 1661–1707.
- [5] Clarida, R., Gali, J., Gertler, M., (2000). “Monetary policy rules and macroeconomic stability: evidence and some theory,” *The Quarterly Journal of Economics*, CXV, 147–180.
- [6] Cochrane, J.H. (2007a). “Inflation Determination with Taylor Rules: A Critical Review,” NBER Working Paper 13409.
- [7] ————— (2007b). “Identification with Taylor Rules: A Critical Review,” NBER Working Papers 13410.
- [8] ————— (2009) “Can learnability save new-Keynesian models?” *Journal of Monetary Economics*, 56(8), 1109-13.

- [9] Friedman, B. M. (2003). “The LM Curve: A Not-so-fond Farewell,” NBER Working Paper 10123.
- [10] Henderson, D.W., Mckibbin, W.J. (1993). “A Comparison of Some Basic Monetary Policy Regimes for Open Economies: Implications of Different Degrees of Instrument Adjustment and Wage Persistence,” *Carnegie-Rochester Conference Series on Public Policy*, 39, 221–317.
- [11] Ireland, P. N. (2007). “Changes in the Federal Reserve’s Inflation Target: Causes and Consequences,” *Journal of Money, Credit and Banking*, 39, No. 8, 1851–82.
- [12] King, R.G. (2000). “The New IS-LM Model: Language, Logic, and Limits,” *Federal Reserve Bank of Richmond Economic Quarterly*, Volume 86/3, 45–103.
- [13] McCallum, B. T. (2009a) “Inflation determination with Taylor Rules: Is New Keynesian analysis critically flawed?” NBER working paper 14534, December 2008, published *Journal of Monetary Economics*, 56(8), 1101-8.
- [14] ————— (2009b) “Rejoinder to Cochrane”, *Journal of Monetary Economics*, 56(8), 1114-5.
- [15] Minford, P. (2008). “Commentary on Economic Projections and Rules of Thumb for Monetary Policy” (by Athanasios Orphanides and Volker Wieland). *Federal Reserve Bank of St. Louis Review*.
- [16] Minford, P., Peel, D (2002). “Advanced Macroeconomics: A Primer,” Edward Elgar Publishing Ltd.

- [17] Neumann, M.J.M. (2006). “Pre-commitment and Guidance: Lessons from the Bundesbanks History,” A paper presented at the ECB colloquium Monetary Policy: A Journey from Theory to Practice. Frankfurt.
- [18] Obstfeld, M., Rogoff, K. (1983). “Speculative Hyperinflations in Maximizing Models: Can We Rule Them Out?,” *Journal of Political Economy*, 91, 675–87.
- [19] Romer, D. (2000). “Keynesian Macroeconomics without the LM Curve,” *Journal of Economic Perspectives*, 14(2), 149–169.
- [20] Taylor, J.B. (1977) “Conditions for unique solutions to macroeconomic models with rational expectations,” *Econometrica*, 47, 1267-86.
- [21] Taylor, J.B. (1993) “Discretion Versus Policy Rules in Practice,” *Carnegie-Rochester Conference Series on Public Policy*, 39, 195-214.
- [22] Taylor, J.B. (Ed), (1999). “Monetary Policy Rules,” Chicago: University of Chicago.
- [23] Taylor, J.B. (2000). “Teaching Modern Macroeconomics at the Principles Level,” *American Economic Review*, 90(2), 90–94.
- [24] Woodford, M.(2003). “Interest and Prices: Foundations for a Theory of Monetary Policy,” Princeton University Press.
- [25] Woodford, M. (2008). “How Important is Money in the Conduct of Monetary Policy?” *Journal of Money, Credit and Banking*, 40, 1561–98.