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Positive and Zero Nominal Interest Rate*

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Fiscal Policy Multipliers in a New Keynesian Model under Positive and Zero Nominal Interest Rate[*]

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Abstract

This paper uses a simple new-Keynesian model (with and without capital) and calculates multipliers of four types. That is, we assume either an increase in government spending or a cut in sales/labor/capital tax that is financed by lump-sum taxes (Ricardian evidence holds). We argue that multipliers of a temporary fiscal stimulus for separable preferences and zero nominal interest rate results in lower values than what is obtained by Eggertsson (2010). Using Christiano *et al.* (2009) non-separable utility framework which they used to calculate spending multipliers we study tax cuts as well and find that sales tax cut multiplier can be well above one (joint with government spending) when zero lower bound on nominal interest binds. In case of a permanent stimulus we show in the model without capital and assuming non-separable preferences that it is the spending and wage tax cut which produce the highest multipliers with values lower than one. In the model with capital and assuming that the nominal rate is fixed for a one-year (or two-year) duration we present an impact multiplier of government spending that is very close to the one in Bernstein and Romer (2009) but later declines with horizon in contrast to their finding and in line with the one of Cogan *et al.* (2010). We also demonstrate that the long-run spending multiplier calculated similarly to Campolmi *et al.* (2010) implies roughly the same value for both types of preferences for particular calibrations. For comparison, we also provide long-run multipliers using the method proposed by Uhlig (2010).

JEL codes: E45

Keywords: New-Keynesian model, fiscal multipliers, zero lower bound, monetary policy, government spending, tax cut, permanent, transitory.

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1 Introduction

The American Recovery and Reinvestment Act was passed at the beginning of 2009 in order to help the US economy recover from the financial crises started in 2008. Bernstein and Romer (2009) provided a document that gives a detailed picture of the estimated effects of this stimulus package[1]. However, there is wide disagreement in the economics profession on the value of the fiscal multipliers[2] listed in their paper[3]. As Cogan *et al.* (2009) assert, it is not straightforward what kind of model Bernstein and Romer (2009) used to obtain multipliers above one[4] for a permanent increase in government spending under the assumption that the nominal interest rate is held constant[5] for the time interval of their simulation. Furthermore, Cogan *et al.* (2009) argues that the Bernstein and Romer (2009) model can't be a New Keynesian model as the model's setup would imply explosive dynamics.

This paper proposes a standard New Keynesian dynamic stochatic general equilibrium (DSGE) model (with and without capital) used widely in academic literature and in central banks for supporting decision-making to investigate into the effects of various fiscal stimuli on output both for zero and non-zero nominal interest rate. The DSGE model used here is basically a stochastic growth (or RBC) model enriched with monopolistic competition on the product market and staggered price setting in Calvo-style (Calvo, 1983). Being aware that fiscal policy is not constrained to variations only in spending but can also operate with various taxes to achieve its goal, we consider three possible sources of a fiscal stimulus separately: an increase in non-productive (that is not creating investment opportunities in the economy) government spending, a sales tax cut and a cut in payroll tax.

Of course, this is not the first paper using a New Keynesian model that studies fiscal multipliers. Two recent contributions of the topic are Christiano *et al.* (2010) and Eggertsson (2010). Christiano *et al.* (2010) discuss government spending both for a model with and without capital when the zero bound on interest rate is binding and not-binding by using non-separable preferences in consumption and leisure. Their most interesting finding is that the spending multiplier is more than three times higher when the nominal interest rate is zero compared to the case when it is positive. Eggertsson (2010) calculates fiscal multipliers (payroll tax cut, profit tax cut, sales tax cut, capital tax cut and an increase in government

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[1]For example, in their Table 5, they provide numbers on the jobs created in each industry in 2010Q4 as a result of the Recovery Package.

[2]This is the change in output due to a change in government spending, dY_{t+k}/dG_t. For $k = 0$ we get back the impact multiplier.

[3]In particular, in their Appendix 1 they consider "output effects of a permanent stimulus of 1% of GDP (percent)"

[4]That is, a dollar spent by the government increases output by more than one dollar.

[5]Note that the federal funds rate at the time of the introduction of the recovery package was almost zero and this is a fact that a model has to take into consideration.

spending) for the case of separable preferences with special attention to the case of binding zero bound on nominal interest rate. His most interesting finding is that the multiplier associated with a payroll tax cut is negative.

In this paper we argue that the multipliers associated with a temporary fiscal stimulus using separable preferences and assuming that the nominal interest rate is zero are lower than the same ones calculated by Eggertsson (2010)[6]. Moreover, we show by using non-separable preferences that we can obtain multipliers of magnitude similar to the ones in Eggertsson (2010) when interest is zero. Our model is based on the one by Christiano *et al.* (2010). However, we extend the model of Christiano *et al.* (2010)—who restrict the analysis to government spending in their models without capital—by taxes of three types.

Furthermore, we check the robustness of the findings of a permanent fiscal stimulus by Eggertsson (2010)—who uses separable utility—for non-separable utility and conclude that the long-run multipliers (when the Taylor rule is in action and nominal interest rate is allowed to be positive) associated with a permanent stimulus can be of somewhat higher magnitude than the ones reported by him. In particular, we found that a permanent rise spending or a permanent labour tax cut lead to a positive multiplier that is much smaller than one.

After extending the new-Keynesian model with capital we emphasize *three* more findings. **Firstly**, the difference between separable and non-separable preferences concerning the size of the multiplier diminishes *on impact*—that is, with certain calibration they are roughly the same—if we consider *long-run* multipliers (calculated similarly to the one in Campolmi *et al.*, 2010). **Secondly**, we present that the *impact* multiplier associated with a *transitory and anticipated* increase in government spending—under the assumption that utility is non-separable and the nominal interest is held fixed for two years—is equal to one in line with Bernstein and Romer (2009). **Thirdly**, multipliers of longer horizon are *definitely lower* than the ones reported by Bernstein and Romer (2009) and somewhat higher than the ones by Cogan *et al.* (2010) who, unlike us, assumed permanent stimulus[7].

This paper shows by using non-separable preferences that we can match the stylised fact of rising consumption in response to a positive government spending shock. As it is well-known, when Ricardian equivalence holds, the use of non-separable preferences mitigates the negative wealth effect associated with the fact that consumers expect a rise in future taxes when there is an increase in government spending (or a tax cut) in the present. In the

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[6]The difference between the results of Eggertsson (2010) and ours—with respect to the government spending multiplier—comes from the fact that he assumed zero steady-state government spending (quite unusual to most of the literature but similar to Woodford (2010)) while we assume a steady-state government spending-output ratio of 20% consistent with most of the post-war experience as discussed by Baxter and King (1993).

[7]Multipliers from a new-Keynesian model under a permanent stimulus must be lower than the same ones under a transitory experiment as we show below. That is, it is a question how Bernstein and Romer (2009) obtained numbers higher than ours knowing that we used a transitory stimulus.

following we provide some empirical evidence on the weakness of this negative wealth effect (Monacelli and Perotti, 2008).

There is extensive empirical literature on the effect and size of fiscal multipliers (see, e.g., Blanchard and Perotti, 2002 and Gali *et al.,* 2007). For example, Gali *et al.* (2007) reports some VAR evidence that government spending multiplier i.e. the change in output with respect to a change in government spending, is 0.78 on impact, and 1.74 at the end of the second year. Interestingly they also found that **consumption, working hours and wages** respond to increased government purchuses **positively** in small and large (including a complete list of explanatory variables) VAR models on many subsamples. Is is also important that the magnitude of the response in consumption, working hours and wages are quantitatively large. In case of consumption the change is usually close to or larger than one in the 4th and the 8th quarter but definitely not on impact after a rise in government spending. However, not all the empiricial VAR literature is consistent with the positive connection between consumption and government spending. For example, the identification strategy applied by Ramey (2008) implies that shortly after increases in government spending consumption declines. The latter one is based on capturing news about government spending hikes, instead of relying on the delayed effect as in standard VAR.

The outline of the thesis is as follows. In Section 2 and 3, we formulate a simple New-Keynesian model and derive analytical short-run (or impact) multipliers of four cases (for both separable and non-separable preferences, respectively). In section 4, we modify the baseline model with separable preferences to investigate into the case of zero nominal interest rate. Section 5 summarise and discuss multipliers from the models without capital. Next, in Section 6, we calculate and briefly assess multipliers of permanent stimuli. Section 7 contains the baseline model augmented with capital to assess the robustness of the findings of the models without capital. Finally, we conclude with the main results.

2   A simple DSGE model without capital

The setup of the model used here builds strongly upon Christiano *et al.* (2010). The idea of tax rates (labour, sales and capital tax) are introduced into the model following Eggertsson (2010). However, Eggertsson (2010) use only separable preferences, while here **both** separable and non-separable preferences are used and discussed. Christiano *et al.* (2010) use non-separable preferences and refers to their results—without reporting them—on separable preferences. As we will see, the optimality conditions can always be characterised by the intratemporal condition, the intertemporal Euler equation, the New Keynesian Phillips curve (NKPC), the exogenous shock process and the Taylor rule.

2.1 The household's problem

The household maximises the following utility that is separable in consumption and leisure:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma} - 1}{1 - \sigma} - \frac{N_t^{1+\varphi}}{1 + \varphi} + v(G_t^N) \right]$$

with respect to its budget constraint

$$(1 - \tau_t^A)(1 + R_t)B_t + \int_0^1 profit_t(i)di + (1 - \tau_t^W)P_tW_tN_t = B_{t+1} + (1 + \tau_t^S)P_tC_t + T_t$$

where τ_t^A, τ_t^W and τ_t^S denote, respectively, tax on capital, labour and consumption. B_t denotes the amount of one-period riskless bonds, R_t is the net nominal one-period rate of interest that pays off in period t. N_t is the sum of all labour types i, that is, $N_t \equiv \int_0^1 N_t(i)di$ and $P_tW_tN_t$ denotes the 'mass' of nominal wages (with the real wage rate W_t). T_t denotes lump-sum taxes net of transfers. $profit_i$ denotes the profit of firm i. The transversality condition, $\lim_{t\to\infty} B_{t+1}/\left[(1 + R_0)(1 + R_1)...(1 + R_t)\right] \geq 0$, is also satisfied.

The household has separable preferences in consumption (C_t), leisure $(1 - N_t)$ and government spending $(v(G_t))$. We do not specify v here as it is not needed for the optimality conditions. Throughout the whole paper we assume that $\sigma > 1$ and $\varphi \geq 0$.

2.2 The firms' problem

2.2.1 Final good sector

The competitive firms produce a single final good using the following technology:

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}, \theta > 1$$

where $Y_t(i)$, $i \in [0, 1]$ denotes the intermediate good i. The profit-maximisation problem of competitive firms results in the demand equation for $Y_t(i)$:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\theta} Y_t \tag{1}$$

where $P_t(i)$ denotes the price of the intermediate good i and P_t is the price of the homogenous final good.

2.2.2 Intermediary sector

The intermediate good i, $Y_t(i)$, is produced by a monopolist i^{th} using a linear technology:

$$Y_t(i) = N_t(i)$$

where $N_t(i)$ denotes the hours used by monopolist i to produce intermediate good i. To be able to calculate multipliers analytically, we later abstract from capital formation in this section. However, in Section 5 we introduce capital into the production function as well. We assume that there is no entry or exit into the industry that produces the ith intermediate good. Furthermore, we have Calvo-price setting that means that a *random* fraction of firms are allowed to re-optimise its price every period with probability $1 - \xi$. With probability ξ a fraction of firms cannot re-optimise their price and uses their previous period price:

$$P_t(i) = P_{t-1}(i).$$

The discounted profit of the ith intermediary firm can be written as:

$$E_t \sum_{T=0}^{\infty} \beta^{t+T} v_{t+T} \left[P_{t+T}(i) Y_{t+T}(i) - (1-\nu) W_{t+T} N_{t+T}(i) \right],$$

where we assume that the subsidy is set such ($\nu = \frac{1}{\theta}$) that corrects for the steady-state distortion induced by the presence of monopoly power and v_{t+T} is the Lagrange multiplier on the budget contraint in the household's optimisation problem.

2.3 Monetary Policy

The monetary policy is assumed to follow the following simple rule:

$$R_{t+1} = \max(Z_{t+1}, 0) \tag{2}$$

where

$$Z_{t+1} = (1/\beta)(1 + \pi_t)^{\phi_\pi(1-\rho_R)} (Y_t/Y)^{\phi_Y(1-\rho_R)} [\beta(1 + R_t)]^{\rho_R} - 1 \tag{3}$$

where Y denotes the steady-state value of Y_t[8]. π_t is the time-t rate of inflation. As usual, we assume that $\phi_\pi > 1$ and $\phi_Y \in (0,1)$. The main implication of the rule in equation (2) is that whenever the nominal interest rate becomes negative, the monetary policy set it equal to zero, otherwise it is set by the Taylor rule specified in equation (3). The parameter ρ_R

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[8] And for the rest of the paper, a variable without a time subscript denotes steady-state value.

measures how quickly monetary policy reacts to changes in inflation and output and we assume that $0 < \rho_R < 1$. Furthermore, we also assume that the inflation in steady state is zero which implies that steady-state net nominal interest rate is $1/\beta - 1$.

2.4 Fiscal policy

We have an exogenous AR(1) process for government spending (and the same could be written for labour tax and sales tax as well):

$$G_{t+1} = (G_t)^{\rho_G} \exp(\varepsilon_{t+1}^G) \tag{4}$$

where ρ_G measures persistence of government spending process and ε^G is an *i.i.d.* shock with zero mean and constant variance. We assume in this simple model that the government spending, the labour tax cut, the sales tax cut and the employment subsidy to restore efficiency in steady-state is financed through *lump-sum* taxes. That is, the Ricardian equivalence holds under our assumptions and the exact timing of taxes is irrelevant and we don't have to take into consideration the government budget constraint. The implications of fiscal policy when the nominal rate is zero is discussed in Section 4.

2.5 Equilibrium

Definition 1 *A monetary equilibrium is a collection of stochastic processes which contains endogenously determined quantitites $\{Y_t(i), Y_t, N_t, N_t(i), B_{t+1}\}$, prices $\{P_t(i), P_t, \pi_t, W_t, R_t, v_t\}$ and an exogenous process $\{G_t\}$ with initial condition G_0. The equilibrium can be characterised by five equations which we rewrite for their log-linear form. The Intratemporal, Euler, NKPC, the shock process and the Taylor equations are listed below together with market clearing. The real marginal cost that appears in the NKPC coincides with W_t due to the linear technology (i.e. it is model-specific). Variables with a hat, $\,\widehat{}\,$, denote percentage deviations from their steady-states: $\widehat{N}_t \equiv \log(N_t/N)$, $\widehat{C}_t \equiv \log(C_t/C)$ with the exception of taxes $\widehat{\tau}_t^i \equiv \tau_t^i - \tau^i, i = \{A, S, W\}$, which are already expressed in percentages and $\widehat{G}_t$ which is defined as the percentage deviation $(G_t - G)$ from steady-state output[9] (Y): $\widehat{G}_t \equiv (G_t - G)/Y$. For inflation, π_t, and nominal interest rate, R_t, we consider their deviation from steady-state.*

Intratemporal condition (in linearised form)

$$\widehat{MC}_t = \widehat{W}_t = \varphi\widehat{N}_t + \sigma\widehat{C}_t + \frac{1}{1-\tau^W}\widehat{\tau}_t^W + \frac{1}{1+\tau^S}\widehat{\tau}_t^S$$

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[9]Thus, a percentage increase in $\widehat{G}_t$ is comparable with the percentage change in output $(\widehat{Y}_t)$ because both variables start from the same steady-state (Y).

Euler equation (in linearised form)[10]

$$E_t\left\{-\sigma\widehat{C}_{t+1} - \frac{1}{1+\tau^S}\widehat{\tau}^S_{t+1} - \frac{\varrho}{1-\tau^A}\widehat{\tau}^A_{t+1} + \beta(R_{t+1} - R)\right\} = -\sigma\widehat{C}_t - \frac{1}{1+\tau^S}\widehat{\tau}^S_t + E_t\pi_{t+1}$$

The New Keynesian Phillips curve (in linearised form)

$$\pi_t = \beta E_t\pi_{t+1} + \kappa\widehat{MC}_t \tag{5}$$

where $\kappa \equiv (1-\xi)(1-\beta\xi)/\xi$ and ξ is the Calvo parameter. We also need the linearised version of the Taylor-rule in equation (3)

$$R_{t+1} - R = \rho_R(R_t - R) + \frac{1-\rho_R}{\beta}(\phi_\pi\pi_t + \phi_Y\widehat{Y}_t), \tag{6}$$

and the linear shock process for public expenditures in equation (4) to describe the equilibrium:

$$\ln G_t = \rho_G \ln G_{t-1} + \varepsilon^G_t.$$

The market clearing condition is also satisfied:

$$\widehat{Y}_t = (1-g)\widehat{C}_t + \widehat{G}_t \tag{7}$$

where $g \equiv 1 - C/Y = G/Y$.

2.6 Parametrisation

Parameters of the model are given in Table 1 for separable and non-separable preferences separately. Most of the parameters, like $\beta, \rho_G(= \rho_{\tau^W} = \rho_{\tau^S} = \rho_{\tau^A})$ and ϕ_π are standard in economics literature. The value of φ is taken from Gali *et al.* (2007). The values of σ and γ are from Christiano *et al.* (2010). To guarantee stability, the inflation coefficient, ϕ_π in the Taylor rule must be greater than one. The steady-state values of payroll tax, τ^W, sales tax, τ^S and the government spending to GDP ratio, g is taken from Uhlig (2010). The value of τ^A is taken from Eggertsson (2010). The value of the Calvo parameter, ξ, is usually chosen to be 0.67 (or 0.75) implying that firms that cannot determine prices optimally use their last price for three quarters (or for a year) on average. However, we choose here ξ somewhat

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[10]Following Eggertsson (2010) we scale capital tax, $\widehat{\tau}^A_{t+1}$, so that it remains to be comparable to percent deviation in annual capital income taxes in steady-state. That is, the scaling parameter on capital tax is $\varrho \equiv 1 - \beta$.

larger (0.85) for reasons asserted in the following sections[11]. The standard deviation of the noise term (σ_{ε^G}, $\sigma_{\varepsilon^{\tau W}}$, $\sigma_{\varepsilon^{\tau S}}$ and $\sigma_{\varepsilon^{\tau A}}$) of the shock process in equation (4) for all four types of stimulus is one percent.

Table 1: Parametrisation of the New Keynesian Model without Capital

Parameters	Separable	Non-separable
σ	2	2
φ	0.2	na
β	0.99	0.99
γ	na	0.29
$\rho_G = \rho_{\tau W} = \rho_{\tau S} = \rho_{\tau A}$	0.8	0.8
ϕ_π	1.5	1.5
ϕ_Y	0	0
ρ_R	0	0
ξ	0.85	0.85
$G/Y (\equiv g)$	0.18	0.18
τ^W	0.28	0.28
τ^A	0.00	0.00
τ^S	0.05	0.05
$\sigma_{\varepsilon^G} = \sigma_{\varepsilon^{\tau W}} = \sigma_{\varepsilon^{\tau S}} = \sigma_{\varepsilon^{\tau A}}$	0.01	0.01
Implied parameters		
κ	0.03	0.03
N	na	1/3

Remark to Table 1: na=non applicable. The parameters φ and γ are applicable for separable and non-separable preferences, respectively.

2.7   Multipliers for Separable Preferences

There are three important requirements for being able to solve the model analytically by methods of undetermined coefficients: (1) linear production function, (2) no interest rate smoothing in Taylor rule ($\rho_R = 0$) and (3) the assumption that government spending and changes in distortionary taxes are financed through lump-sum taxes (in other words Ricardian equivalence holds). The parametrisation for the separable case can be found in the first column of Table 1. The exact formulas for the multipliers (by assuming that the zero

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[11]The Calvo parameter, ξ, should be greater than 0.82 for two reasons: (1) we can achieve a government spending multiplier that is larger than one for *non-separable* preferences in the model with *positive* nominal interest rate and (2) we can meet the algebraic requirement in the model of Section 4 for the zero bound to bind.

bound does not bind in this section) presented here are derived under the above three main assumptions.

We solve the model *analytically* by using the method of undetermined coefficients. That is, we guess that output and inflation is some function of $\widehat{G}_t$ (and similarly for $\widehat{\tau}_t^W, \widehat{\tau}_t^S$ and $\widehat{\tau}_t^A$) and can be expressed as:

$$\pi_t = A_\pi \widehat{G}_t, \tag{8}$$

$$\widehat{Y}_t = A_Y \widehat{G}_t. \tag{9}$$

Moreover, we can eliminate forward-looking variables, like $E_t G_{t+1}$, if we assume an exogenous AR(1) process for government spending as it is in equation (4).

2.7.1 Multipliers for separable preferences

First, we discuss when the government spending multiplier is larger than one. For this purpose, take the total derivative of the linear version of the aggregate resource constraint in equation (7):

$$\frac{dY_t}{dG_t} = \frac{d\widehat{Y}_t}{d\widehat{G}_t} = \frac{d\left[(1-g)\widehat{C}_t + \widehat{G}_t\right]}{d\widehat{G}_t} = 1 + (1-g)\frac{d\widehat{C}_t}{d\widehat{G}_t}. \tag{10}$$

This formula implies that the size of the spending multiplier depends on how consumption reacts to government spending. For separable preferences the latter one in equation (10) is negative: $d\widehat{C}_t/d\widehat{G}_t < 0$. Thus, the spending multiplier is smaller than one and this can also be seen on Figure 1 where consumption falls and the multiplier is smaller than one on impact (0.97).
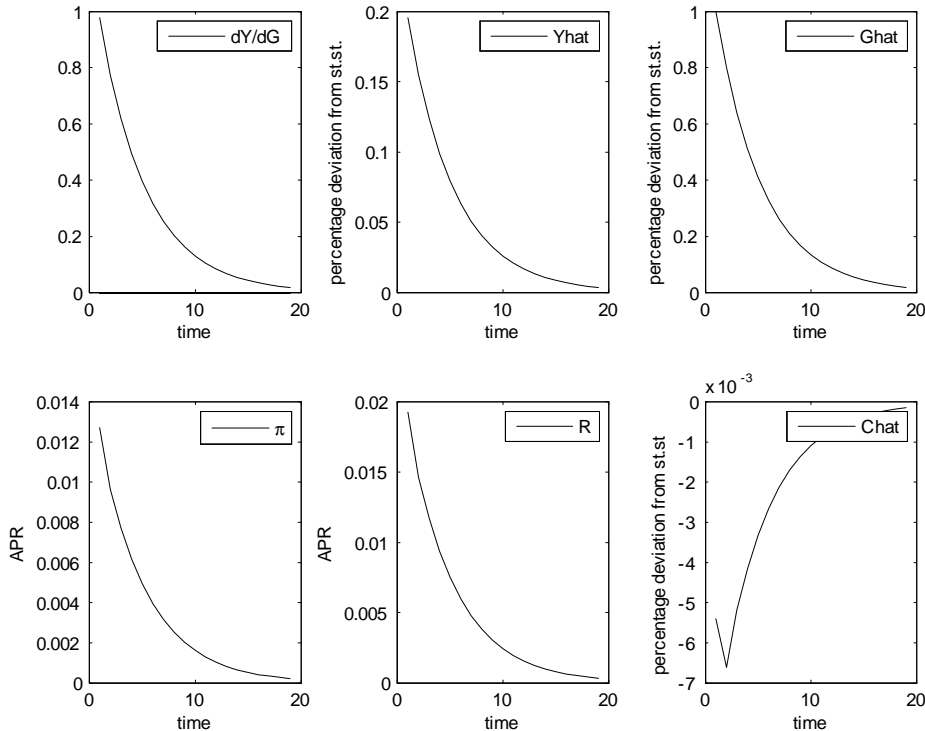
The government spendig, payroll tax cut, sales tax cut and capital tax cut multipliers for separable preferences are given, respectively, by the following formulas:

$$\frac{d\widehat{Y}_t}{d\widehat{G}_t} = \sigma \frac{(1-\rho) + (1-g)(\phi_\pi - \rho)\frac{\kappa}{1-\beta\rho}}{\sigma(1-\rho) + \phi_Y(1-g) + (1-g)(\phi_\pi - \rho)\frac{\kappa}{1-\beta\rho}\left(\varphi + \frac{\sigma}{1-g}\right)},$$

$$\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^W} = \frac{(\rho - \phi_\pi)\frac{\kappa}{1-\beta\rho}\frac{1}{1-\tau^W}}{\sigma - (\rho - \phi_\pi)\frac{\kappa}{1-\beta\rho}(\varphi + \sigma) - (\sigma\rho - \phi_Y)},$$

$$\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^S} = \frac{\frac{1}{1+\tau^S}\left[(\rho - 1) - (\phi_\pi - \rho)\frac{\kappa}{1-\beta\rho}\right]}{(\sigma + \phi_Y) + (\phi_\pi - \rho)\frac{\kappa}{1-\beta\rho}(\varphi + \sigma) - \sigma\rho},$$

10

Figure 1: Impulse responses of the simple new-Keynesian model with separable preferences to a one percent spending shock. We can see that $dC/dG < 0$.

and

$$\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^A} = \frac{\rho\varrho}{\left((\phi_Y + \sigma) - \frac{\kappa}{1-\beta\rho}(\varphi + \sigma)(\rho - \phi_\pi) - \rho\sigma\right)(1 - \tau^A)}.$$

As we have discussed it previously the *spending multiplier* (which is 0.97 under the parametrisation used in Table 1) is always smaller than one with separable preferences. It can be also be noted that with certain parametrisation it can be very close to one but never goes beyond one. Under baseline parametrisation in Table 1 we assumed that monetary policy reacts to only inflation through the Taylor rule ($\phi_\pi > 1, \phi_Y = 0$). Instead, we can assume that the monetary policy reacts to changes in the output gap as well (where ϕ_Y can take the positive value originally proposed by Taylor (1993)). In the latter case the multiplier is lower than the one under pure inflation targeting (results for the case when we have positive coefficient on output gap—in particular it is set to $\phi_Y = 0.5/4$—can be seen in Table 8 in Appendix). We discuss these results in the section when we summarise temporary fiscal multipliers.

The value of the *labour tax cut multiplier* (with the baseline calibration) is 0.23, which is similar to those found in new-Keynesian literature (see, e.g., Eggertsson (2010)). The reason

why this multiplier is lower than the spending and sales tax ones is because it stimulates output only indirectly through an outward shift in the labour supply.

The value of the *sales tax cut multiplier* (0.44) is generally lower than the one for government spending as this variable has a coefficient ($\frac{1}{1+\tau^S}$) in the Euler equation that generally downscales its value. The baseline idea behind sales tax decrease is that people consume more goods with a lower tax on them. However, we know that VAT-type taxes (like sales tax) are comparatively lower in developed countries than in less-developed ones. Thus, the potential stimulative effects of a huge sales tax decrease may turn out to be small for a country like the US.

3 Non-separable preferences

The household maximises the following utility that is non-separable in consumption (C_t) and leisure ($1 - N_t$):

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{[C_t^{\gamma}(1 - N_t)^{1-\gamma}]^{1-\sigma} - 1}{1 - \sigma} \right]$$

with respect to its budget constraint

$$(1 - \tau^A)(1 + R_t)B_t + \int_0^1 profit_t(i)di + (1 - \tau_t^W)P_tW_tN_t = T_t + B_{t+1} + (1 + \tau_t^S)P_tC_t.$$

3.1 Equilibrium conditions

Again, the equilibrium conditions can be described by the intratemporal condition, the Euler equation, the new-Keynesian Phillips curve, the Taylor rule and the shock process. We list only those equilibrium conditions which change after imposing the non-separable preferences assumption.

The intratemporal condition

$$\widehat{W}_t = \widehat{C}_t + \frac{N}{1 - N}\widehat{Y}_t + \frac{1}{1 - \tau^W}\widehat{\tau}_t^W + \frac{1}{1 + \tau^S}\widehat{\tau}_t^S, \tag{11}$$

where steady-state hours, N, depends on g (or, in case of tax cut multipliers, it depend on steady-states taxes) and preference parameter, γ.

The Euler equation

$$E_t \left\{ \beta \left(R_{t+1} - R \right) + [(1 - \sigma)\gamma - 1]\widehat{C}_{t+1} - (1 - \gamma)(1 - \sigma)\frac{N}{1 - N}\widehat{N}_{t+1} - \frac{1}{1 + \tau^S}\widehat{\tau}^S_{t+1} \right\}$$

$$= [(1 - \sigma)\gamma - 1]\widehat{C}_t - (1 - \gamma)(1 - \sigma)\frac{N}{1 - N}\widehat{N}_t - \frac{1}{1 + \tau^S}\widehat{\tau}^S_t + \frac{\varrho}{1 - \tau^A}E_t\widehat{\tau}^A_{t+1} + E_t\pi_{t+1} \quad (12)$$

The New Keynesian Phillips curve, the Taylor rule and the shock process is the same.

In case of non-separable preferences the general form of NKPC is the same as in equation (5) with only real marginal cost, $\widehat{MC}_t$, being different from the one for separable case. When using linear production function the real marginal cost coincides with real wage, $\widehat{MC}_t = \widehat{W}_t$, which latter is given the intratemporal condition in equation (11). We also need a Taylor rule (in equation (6)) and an exogenous shock process (see equation (4)) to close the system.

3.2 The role of non-separable preferences

In the New Keynesian model used here we have infinitely-lived agents, complete asset markets, monopolistic competition, lump-sum taxation and sticky prices. One of our major finding is that the size of the government spending multiplier depends largely on the preference specification of the representative household. In order to generate a government spending multiplier that is larger than one we have to assume complementarity between consumption and hours worked, that is, non-separable preferences in consumption and leisure has to be used.

In the New Keynesian model with separable preferences, a rise in government spending induce a negative wealth effect as the consumer expects a rise in future lump-sum taxes and, as a consequence, he/she consumes less and works more. The negative wealth effect implies an outward shift in the labour supply curve leading to higher hours worked and *lower* real wages while the labour demand curve remains unchanged. The negative Hicksian wealth effect induced by government spending leads to a rise in output and a fall in consumption and real wages.

However, there is little empirical evidence on the strength of this negative wealth effect (see, e.g., Gali *et al.*, 2007). Monacelli and Perotti (2008) revisits the so-called Greenwood-Hercowitz-Huffmann (GHH) preferences which implies a very low Hicksian wealth effect and concludes by using non-separable preferences of GHH type that we can generate a case when the labour supply curve does not shift, but stays still, in reaction to a rise in government spending (that is, the wealth effect is zero).

If there was a shift in the labour supply, the real wage would decrease and the consumer

would substitute consumption for hours worked (negative substitution effect). Thus, to generate a rise in consumption we need the real wage to increase that can be only achieved by a positive outward shift in the labour demand curve. To make this happen we have to introduce sticky prices into the model. Under the presence of sticky prices, not all the firms can change its prices when the demand for their products, due to an increase in government purchases, increase. Thus, those firms who cannot change price will satisfy new demand by an increase in production which can be achieved by hiring extra workers. When hiring extra workers, labour demand shifts out and the rising real wage as a necessary condition for rising consumption after a spending spree is satisfied (Monacelli and Perotti, 2008).

3.3 Multipliers for Non-separable Preferences

It remains true also in case of non-separable preferences that we can solve for the multipliers (see necessary assumptions at the separable case) analytically by the methods of undetermined coefficients. Figure 2 shows the response of variables (and the multiplier) to a temporary 1% spending shock under non-separable preferences. We can observe two things: (1) the multiplier is slightly larger than one (1.05) on impact and (2) $dC/dG > 0$.

Figure 2: Impulse responses of the simple new-Keynesian model with non-separable preferences to a one percent spending shock. We can observe that $dC/dG > 0$.

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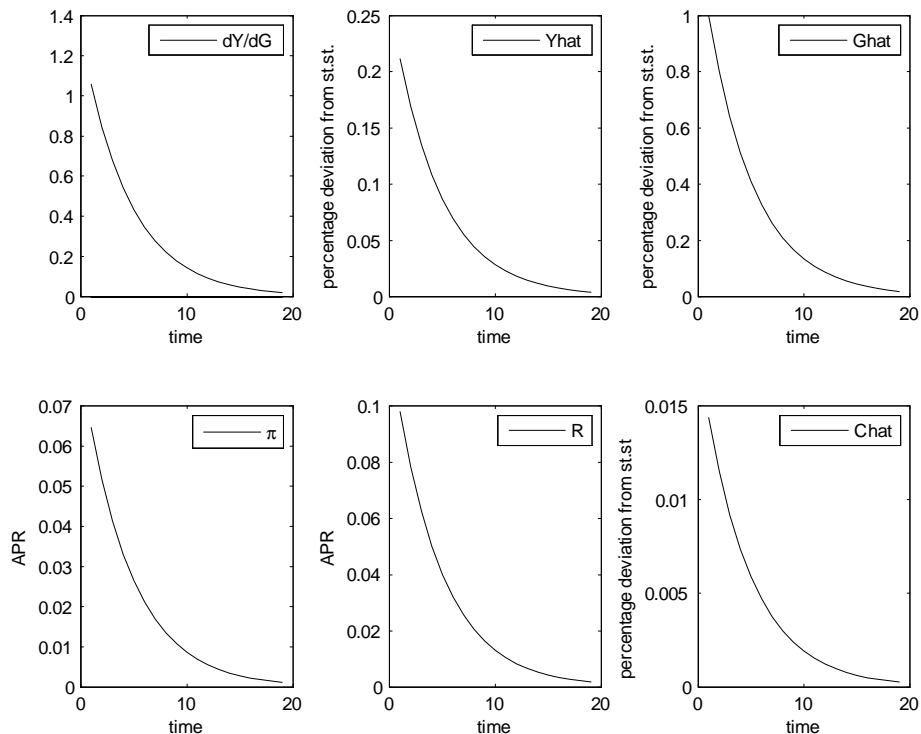
The government spendig, payroll tax cut, sales tax cut and capital tax cut multipliers for non-separable preferences are given, respectively, by the following formulas:

$$\frac{d\widehat{Y}_t}{d\widehat{G}_t} = \frac{(\rho - \phi_\pi)\kappa - [\gamma(\sigma - 1) + 1](1 - \rho)(1 - \beta\rho)}{(1 - \beta\rho)[\rho - 1 - (1 - g)\phi_Y] + (1 - g)(\rho - \phi_\pi)\kappa\left(\frac{1}{1-g} + \frac{N}{1-N}\right)},$$

$$\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^W} = \frac{(\phi_\pi - \rho)\frac{1}{1-\tau^W}\frac{\kappa}{1-\beta\rho}}{[(1 - \sigma)\gamma\tau^W - 1](1 - \rho) - \phi_Y - (\phi_\pi - \rho)\frac{\kappa}{1-\beta\rho}\left(1 + \frac{N}{1-N}\right)},$$

$$\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^S} = \frac{1}{1+\tau^S}\left[\frac{(\phi_\pi - \rho)\frac{\kappa}{1-\beta\rho} - (\rho - 1)}{\frac{\gamma(1-\sigma)\tau^S - (1+\tau^S)}{1+\tau^S}(1 - \rho) - \phi_Y - (\phi_\pi - \rho)\frac{\kappa}{1-\beta\rho}\frac{1}{1-N}}\right],$$

and

$$\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^A} = \frac{\rho\varrho}{(1 - \tau^A)\left[1 - (\rho - \phi_\pi)\frac{\kappa}{1-\beta\rho}\frac{1}{1-\gamma} - \rho + \phi_Y\right]}.$$

As previously argued in detail, the *government spending* multiplier is generally larger than the one corresponding to separable preferences due to positive reaction of consumption to the spending shock (see Figure 2). However, it is important to note that a multiplier that is larger than one can be obtained by assuming a high value for average price stickiness, that is a value of at least $\xi = 0.8$ (firms that cannot change price holding their last price for longer than a year) or larger which means that κ is around at most 0.03.

Note again that a *labour tax cut* has only indirect effect on output (that is modifying only the economy's AS curve leaving the AD unaffected) as it modifies the household's labour supply decision which is given implicitly by the intratemporal condition. A labour tax cut has smaller effect in case of non-separable preferences because the output coefficient in the Euler equation are multiplied by the steady-state of payroll tax, τ^W which latter is smaller than one. In case of separable preferences there is no such "discount term" on output (see more on this term at the sensitivity analysis). Based on this fact, the labour tax multiplier is rather small (roughly 0.17) for both types of preferences.

We have argued in the separable case that the *sales tax cut* (0.8) multiplier is lower than the one of government spending because the *direct effect* of sales tax cut on output (that is increasing aggregate spending) is generally *lower* than the one of government spending. In case of non-separable preferences this direct effect is even weaker. That is, the stimulative effect is even more muted due to a composite term—which contains deep parameters and steady-state sales tax—multiplying output that is lower than one (see equation (12)).

4 When zero lower bound on interest rate binds

4.1 The two-state process

In accordance with Christiano *et al.* (2010) we assume that the zero bound on nominal interest rate binds due to an exogenous increase in the discount rate (people's propensity toward savings increases). To be able to model zero bound we modify the discount factor in the household's problem to become time dependent and is given by the cumulative product of interest rates. Christiano *et al.* (2010) considers non-separable utility but, now, we consider the separable case. That is, the household maximises its utility which is separable in consumption and leisure:

$$U = E_0 \sum_{t=0}^{\infty} d_t \left[\frac{C_t^{1-\sigma} - 1}{1 - \sigma} + \frac{(1 - N_t)^{1+\varphi}}{1 + \varphi} \right]$$

with respect to its budget constraint:

$$(1-\tau^A)(1+R_t)B_t + \int_0^1 profit_t(i)di + (1-\tau_t^W)\int_0^1 P_tW_t(i)N_t(i)di = T_t + B_{t+1} + (1+\tau_t^S)P_tC_t$$

where the discount factor, d_t is given by (r_{t+1} denotes the real rate of interest at time t that will be actual in $t+1$).

The time-varying discount factor is given by:

$$d_t = \begin{cases} \frac{1}{1+r_1}\frac{1}{1+r_2}...\frac{1}{1+r_t}, & t \geq 1 \\ 1, & t = 0. \end{cases}$$

Initially the economy is in the steady-state. Then, in the first period $r_1 = r_l$. Then, for $t \geq 1$, r_t evolves as follows. The discount factor remains high with probability p:

$$\Pr(r_{t+1} = r_l | r_t = r_l) = p.$$

Or, the discount factor jump back to its steady-state value with probability $1 - p$:

$$\Pr(r_{t+1} = r | r_t = r_l) = 1 - p.$$

We assumed that initially we are in the zero bound and thus:

$$\Pr(r_{t+1} = r_l | r_t = r) = 0.$$

In period $0 < t \le T$ zero bound binds while in period $t > T$ zero bound ceases to bind. We assume that the shock to the discount factor is high enough to make the zero bound binding. Moreover the following holds in steady-state (the steady-state value of r_{t+1} is denoted as r): $\beta(1+r) = 1$. When zero bound binds inflation, output and government spending at time t is denoted by π_l, $\widehat{Y}_l$, and $\widehat{G}_l$ respectively.

In period t and $t+1$ variable $\widehat{X}_i = \{\widehat{G}_i, \widehat{Y}_i, \pi_i\}$, for $i \in \{t, t+1\}$ are taking, respectively, the following values:

$$\widehat{X}_t = \begin{cases} \widehat{X}_t = \widehat{X}_l, 0 < t \le T, \text{ zero bound binding,} \\ \widehat{X}_t = 0, t > T, \text{ zero bound not binding,} \end{cases}$$

and

$$\widehat{X}_{t+1} = \begin{cases} (1-p)\widehat{X}_t = 0, \text{ with probab. } 1-p \text{ variable } X \text{ reverts back to steady-state,} \\ p\widehat{X}_l, \text{ with probab. } p \text{ zero bound continues to bind.} \end{cases}$$

In summary, the relevant cases are: $\widehat{G}_t = \widehat{G}_l$, $E_t(\widehat{G}_{t+1}) = p\widehat{G}_l$, $E_t(\pi_{t+1}) = p\pi_l$, $\widehat{Y}_t = \widehat{Y}_l$, and $E_t(\widehat{Y}_{t+1}) = p\widehat{Y}_l$.

4.2   Solution and calibration of the model

The equilibrium is characterised by two values for each variable: one value when the zero bound binds and one when it does not. When zero bound binds output and inflation are given, respectively, by the following closed form equations:

$$\begin{aligned} \widehat{Y}_l &= \left[\frac{\sigma(p-1)(1-\beta p) - \sigma p\kappa}{\Omega}\right]\widehat{G}_l \\ &+ \left[\frac{(1-g)\left[(1-\beta p)(p-1) + p\kappa\right]}{\Omega(1+\tau^S)}\right]\widehat{\tau}_l^S \\ &+ \frac{(1-g)(1-\beta p)\varrho}{\Omega(1-\tau^A)}\widehat{\tau}_l^A + \frac{(1-g)(1-\beta p)}{\Omega}\beta r_l \\ &+ \frac{(1-g)p\kappa}{\Omega(1-\tau^W)}\widehat{\tau}_l^W \end{aligned} \tag{13}$$

and

$$
\begin{aligned}
\pi_l =\ & \frac{\kappa\left(\varphi(1-g)+\sigma\right)}{(1-\beta p)(1-g)}\left[\frac{\{\sigma(p-1)(1-\beta p)-\sigma p\kappa\}\left(\varphi(1-g)+\sigma\right)-\sigma\kappa\Omega}{\Omega\left(\varphi(1-g)+\sigma\right)}\right]\widehat{G}_l \\
& +\frac{\kappa\left(\varphi(1-g)+\sigma\right)}{(1-\beta p)(1-g)}\left[\frac{(1-g)\left[(1-\beta p)(p-1)+p\kappa\right](1-\beta p)+\Omega\kappa}{\Omega(1+\tau^S)}\right]\widehat{\tau}_l^S \\
& +\frac{\kappa\left(\varphi(1-g)+\sigma\right)\varrho}{(1-\beta p)\Omega(1-\tau^A)}\widehat{\tau}_l^A \\
& +\frac{\kappa\left(\varphi(1-g)+\sigma\right)}{\Omega}\beta r_l \\
& +\left[\frac{\left(\varphi(1+g)+\sigma\right)p\kappa^2+\kappa\Omega}{(1-\beta p)\Omega(1-\tau^W)}\right]\widehat{\tau}_l^W
\end{aligned}
\tag{14}
$$

with $\Omega \equiv \sigma(1-p)(1-\beta p)-p\kappa(\varphi(1-g)+\sigma)$. The algebraic requirement for the zero lower bound to bind is $\Omega > 0$, which is satisfied, ceteris paribus, for $0.028 \leq \kappa \leq 0.0465$ (or, equivalently, $0.81 \leq \xi \leq 0.85$). That is the Calvo parameter, ξ, should be sufficiently large.

Why does the zero bound bind in equilibrium?

Christiano *et al.* (2010) has an appealing interpretation for market clearing when zero bound binds. In this simple model without investment the savings has to be zero in equilibrium. A possible way to curb peoples' desire to save more is through a reduction in the real interest rate. According to the Fisher rule we know there are two possible ways to decrease real interest rate: a decrease in the nominal rate or an increase in expected inflation. However, we know that the decrease in the nominal rate is limited by its natural zero lower bound. We also know that the inflation cannot accelerate when there is a discount factor shock (if we look at equation (14), and, at the same time, assuming that fiscal variables do not change, we can see there is deflation due to $r_l < 0$). Otherwise, positive inflation in our sticky prices model is accompanied by increasing output that can induce people to save more. Thus, the reduction in real interest rate may not be enough to deter people from further saving. If the discount rate shock is big enough the real interest rate cannot fall by enough to reduce savings because the zero bound becomes binding prior to the point that would re-establish equilibrium. Therefore, the only possible way for savings to become zero in equilibrium is a **large** transitory fall in output and an accompanying deflation as it can be seen on the element (1,2) and (1,3) of Figure 3, respectively.

The government spending multiplier when the zero bound binds is given by the coefficient on $\widehat{G}_l$ in equation (13) as:

$$
\frac{d\widehat{Y}_l}{d\widehat{G}_l}=\frac{\sigma(p-1)(1-\beta p)-\sigma p\kappa}{\Omega}.
$$

The zero bound in case of government spending binds if $\Omega > 0$, which is satisfied for $0.02 \leq \kappa \leq 0.036$ and $0.75 \leq p \leq 0.82$. This range of values of κ implies a Calvo parameter that is $\xi \geq 0.82$ (and this is true for each of the multipliers considered here).

Why is the spending multiplier so high when the nominal rate is zero?

When there is an increase in spending the marginal cost, the inflation and the output rises and the markup falls. If the zero bound binds, the nominal interest is zero and the Taylor rule is inact. Because of the zero nominal rate, the rise in inflation will not coincide with an increase in the nominal rate (which in normal circumstances would react to inflation by larger than one due to the coefficient on inflation in the Taylor rule) and therefore lead to a fall in the real interest rate that encourages people to consume more today (note that we have no investment channel in this model). Higher consumption implies higher output, higher inflation and even lower real rate that again leads to a rise in output and the process replicates. The result is a large multiplier. The (1,1) element of Figure 3 shows the government spending multiplier (where '□' indicates the benchmark value based on the parameter configuration in Table 1). As κ rises, we have more flexible prices (i.e. the Calvo parameter, ξ, is lower) and the value of the multiplier rises. The (1,2) and (1,3) elements of Figure 3 show the value inflation and output, respectively for zero nominal interest rate in the absence of a change in government spending. It can be inferred that the more flexible prices are (i.e. the higher is κ) the larger transitory fall in output (and a corresponding deflation) is needed to restore savings to zero in equilibrium. The second row of Figure 3 shows the longer the economy is in the zero bound state (i.e. a higher is p), the higher is the value of the multiplier and the bigger is the deflation and contraction in the economy to restore equilibrium level of savings.

4.3   The case of negative labour tax multiplier

This is the most important finding of Eggertsson (2010). A cut in labour tax makes the AS curve shift to the right as one additional unit of hours worked incurs less taxes that creates the incentive for people to work more (i.e. providing more labour). As Eggertsson (2010) argues the outward shift in labour supply reduce real wages, firms are willing to supply more goods at a lower price leading to deflationary pressures. However, when the zero bound becomes binding the negative slope of AD in the *output-inflation* space changes to positive. This seems to be counterintuitive but let us discuss what happens. Technically speaking, we can infer from the equations we got for $\widehat{Y_l}$ and π_l that a cut in payroll tax in the deflationary state l leads to a fall both in output and inflation. Now let us discuss the intuition behind this. To gain insight we start with the case of positive nominal rate.

Figure 3: Sensitivity analysis of government spending multipliers in case of zero nominal interest rate for parameters κ and p.
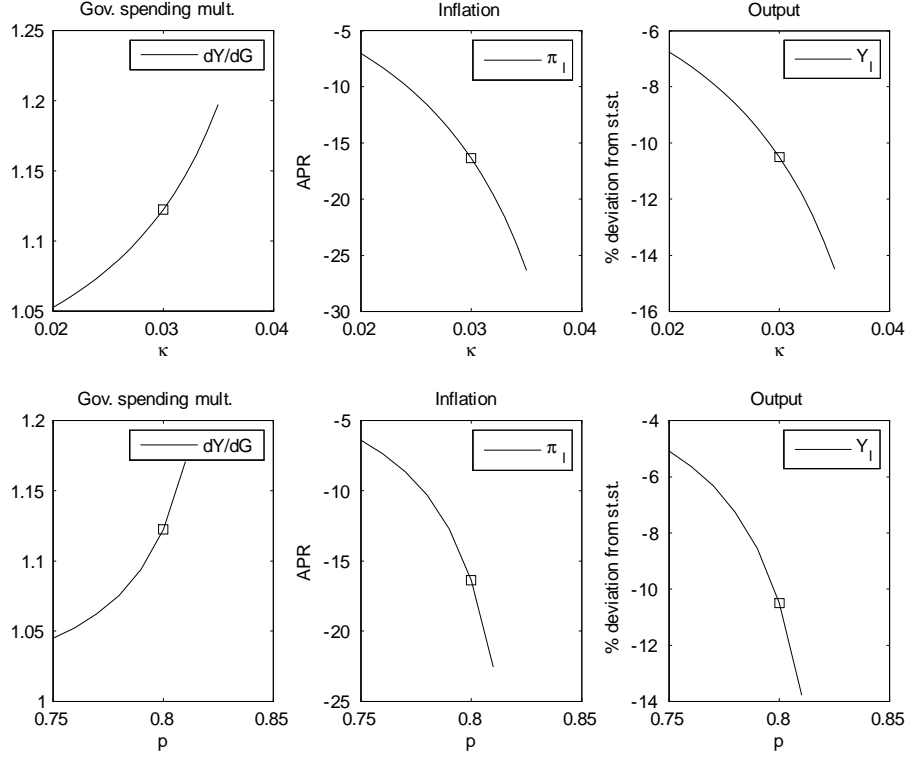
In the absence of zero nominal interest, the reaction of the central bank to deflation is a cut in the nominal interest rate by more than one-to-one with inflation (this is the famous $\phi_\pi > 1$ requirement in the Taylor rule). If the inflation speeds up then the answer of the central bank is an increase in the nominal rate by more than one-to-one with inflation. Thus, in case of deflationary pressures the real interest rate will decline as the central bank will cut nominal interest rate by more than one in proportion to inflation.

However, this is no longer true when the zero bound binds and the central bank cannot cut interest rates to mitigate deflationary shock. As the zero bound becomes binding the deflationary spiral will induce a rise in the real rate which, as a consequence, lead to a fall in output. That is, the downward-sloping AD curve in the inflation-output space becomes upward sloping when the zero bound becomes binding. Accordingly, we can say that a simple New Keynesian model with Calvo pricing implies that labour tax is contractionary in an environment of zero policy rate (Eggertsson, 2010).

The payroll tax multiplier for separable preferences and under the assumption that the

nominal rate is zero is given by coefficient on $\widehat{\tau}_l^W$ in equation (13):

$$\frac{d\widehat{Y}_l}{-d\widehat{\tau}_l^W} = \frac{(1-g)p\kappa}{\Omega(1-\tau^W)},$$

where $\Omega > 0$ is needed for the zero bound to bind. This is satisfied for the the same κ and p parameter intervals as in case of government spending.

The payroll tax multiplier is depicted on the element $(1,1)$ of Figure 4 for a range of κ. The elements $(1,2)$ and $(1,3)$ of Figure 4 show the deflation and contraction in output associated with the zero bound state is increasing in κ in the absence of a change in payroll tax. As we can see on $(2,1)$ element of Figure 4 the longer the economy is in the zero bound state (i.e. the higher is p) the smaller is the payroll tax multiplier (i.e. it is more negative) and the bigger is the associated deflation and contraction in output needed to decrease savings to zero level, shown, respectively on $(2,2)$ and $(2,3)$ elements of Figure 4.

Figure 4: Sensitivity of labour tax multiplier for the case of zero nominal interest rate for parameters κ and p.
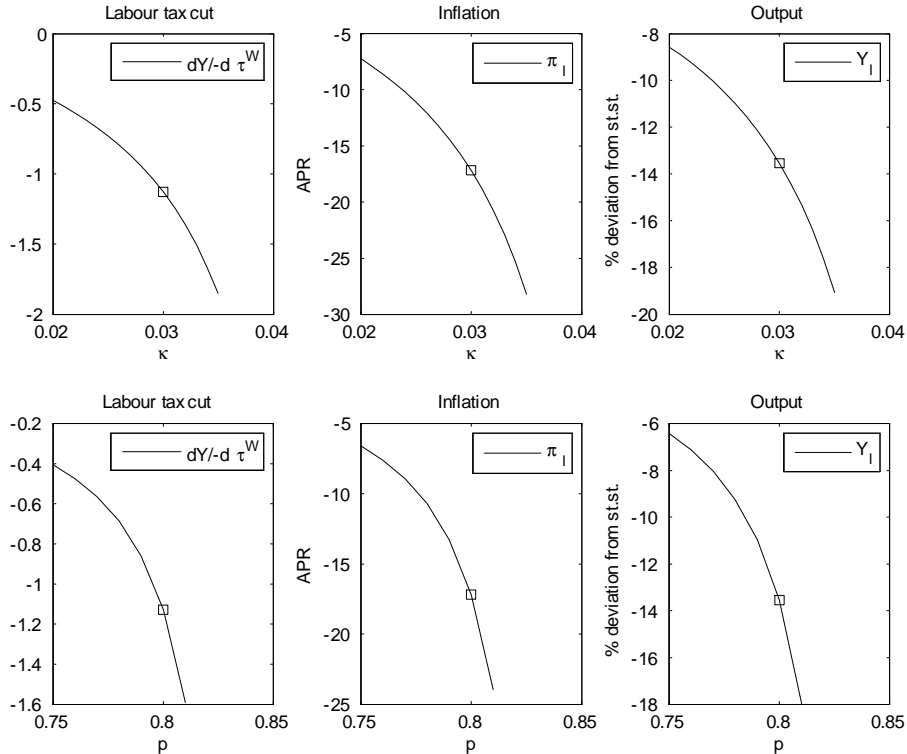
The working of sales tax cut and capital tax cut multiplier for zero nominal rate is rather similar to the government spending and the corresponding discussion and graphs are omitted here.

5   Summary of temporary fiscal multipliers

As we discussed previously multipliers can be higher or equal to one if we have non-separable preferences or if the economy is in the zero lower bound state. The summary of the analytic multipliers obtained under the assumption of positive or zero interest rates can be seen in Table 2. With separable preferences the government spending multiplier is very close to one although never bigger than one. With certain parametrisation—e.g., choosing coefficient on output gap, ϕ_Y, zero in the Taylor rule in equation (6)—we can obtain a multiplier that is slightly larger than one for the non-separable case.

We can see that the multiplier for non-separable preferences is nearly as large as the multiplier in the separable case with zero interest rate $(R = 0)$. *Eggertsson (2010) report high numbers (with values above two) for government spending and sales tax cut multiplier for zero interest rate (for separable preferences). But, now, I argue, that the government spending and sales tax multiplier are lower for separable preferences when the zero bound binds (the spending multiplier, 1.10, is above one while the sales sales tax cut multiplier, 0.54, is lower than one in the third column of Table 2) than the ones reported by him.* When we use non-separable preferences to model the zero lower bound—as Christiano *et al.* (2010) did—then we can see that the multipliers in the last column of Table 2 are well above one (these are values 3.7 and 1.63 for spending increase and sale tax cut, respectively).

The labour tax multipliers under zero interest rate are negative irrespectively of the preference specifications, which is the most interesting finding of Eggertsson (2010). In the previous section we argued in detail why labour tax multipliers are negative when the nominal interest rate is zero (it is mainly due to the AD curve, which has a negative slope under positive nominal interest rate, becomes positively-sloped under zero interest rate).

As we can see in the last row capital tax cut is not a good way to stimulate output as the multipliers are negative irrespectively whether we are in or out of the zero bound state.

We can discuss how sensitive these results to the underlying parameter values. Table 8 in Appendix use a parametrisation that differs from the one in Table 1 only in parameter choice for ϕ_Y—originally set to zero implying no response of monetary policy to changes in output gap—which is set to the value $0.5/4$[12] as in Taylor (1993). We can observe that this change—quite counter-intuitively—results in a lower spending multiplier for non-separable preferences compared to the separable one when interest rate is positive. However, the sales tax cut multiplier remains to be higher for non-separable preferences after the change in ϕ_Y but the difference in magnitude between the separable and non-separable case fell considerably.

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[12]We have quarterly data, thus, we have to divide the value of ϕ_2 by four.

Table 2: Summary of Multipliers—Temporary fiscal policy

Multipliers	Separable		Non-separable	
	$R > 0$	$R = 0$	$R > 0$	$R = 0$
Gov. spending, $\frac{d\widehat{Y_t}}{d\widehat{G_t}}$	0.9675	1.1027	1.052	3.7
Payroll tax, $\frac{d\widehat{Y_t}}{-d\widehat{\tau}_t^W}$	0.2318	**-0.941**	0.1691	**-2.38**
Sales tax, $\frac{d\widehat{Y_t}}{-d\widehat{\tau}_t^S}$	0.4398	0.5389	0.796	1.63
Capital tax, $\frac{d\widehat{Y_t}}{-d\widehat{\tau}_t^A}$	-0.0121	-6.1227	-0.0282	-0.7692

Remarks to Table 2: here we used the parametrisation given in Table 1. When $R > 0$, The multipliers are obtained by using the method of of undetermined coefficients. When $R = 0$ these are given by coefficients on variables $\widehat{G}_t, \widehat{\tau}_t^W, \widehat{\tau}_t^S$ and $\widehat{\tau}_t^A$ in equation (13).

6   Permanent changes in fiscal policy

Suppose now that the change in fiscal policy—though slightly unrealitically—is permanent similar to the experiment by Cogan *et al.* (2010). Eggertsson (2010) also considered permanent fiscal action in his new-Keynesian framework using separable preferences. Now let us assess the robustness of the results of Eggertsson (2010) when we employ the non-separable preferences setting of Christiano *et al.* (2010) to obtain multipliers of a permanent change.

When modelling a permanent change in policy we assume that the variables take on their long-run(permanent) values immediately with no implementation lag. Similarly to Eggertsson (2010) we can distinguish between **three** cases (let us assume that at time $t = T$ the zero bound ceases to bind): 1) in the long run, that is, in $t \geq T$ when the zero bound does not bind; 2) in the short run, $t < T$, when the zero bound doesn't bind and a Taylor rule is in action (in that case the algebraic condition [see below] for a binding zero lower bound is not satisfied) and 3) short run, $t < T$, when the zero bound binds and nominal interest is zero.

In *case 1* when all variables take their long-run values (output, inflation and fiscal ones are denoted subscript L) at the time of announcement and $\widehat{Y}_L$ and π_L can be expressed,

respectively, by the following closed form equations[13]:

$$\widehat{Y}_L = -\frac{\kappa(1-g)(\phi_\pi - 1)(1-\gamma)}{\Lambda(1-\tau^W)}\widehat{\tau}_L^W - \frac{\kappa(1-g)(\phi_\pi - 1)(1-\gamma)}{\Lambda(1+\tau^S)}\widehat{\tau}_L^S$$
$$+\frac{\kappa(\phi_\pi - 1)(1-\gamma)}{\Lambda}\widehat{G}_L + \frac{\varrho(1-\beta)(1-\gamma)(1-g)}{\Lambda(1-\tau^A)}\widehat{\tau}_L^A,$$

with $\Lambda \equiv b\kappa(\phi_\pi - 1) + (1-g)(1-\beta)(1-\gamma)\phi_Y$ and

$$\pi_L = \frac{\kappa b\varrho}{[\Xi\phi_Y + (\phi_\pi - 1)](1-\tau^A)}\widehat{\tau}_L^A \tag{15}$$
$$-\frac{\kappa\Xi\phi_Y}{[\Xi\phi_Y + (\phi_\pi - 1)\kappa b](1-\beta)}\left[\frac{1}{1-g}\widehat{G}_L - \frac{1}{1-\tau^W}\widehat{\tau}_L^W - \frac{1}{1+\tau^S}\widehat{\tau}_L^S\right]$$

with $\Xi \equiv (1-g)(1-\gamma)(1-\beta), a \equiv \frac{1-\tau^W}{1+\tau^S}$ and $b \equiv 1 - \gamma + a\gamma$.

In *case 2* when output and inflation take their short run values—while fiscal variables jump to their long-run levels at the time of announcement—we have a closed form solution with a positive nominal interest rate[14] that is governed by the Taylor-rule in equation (6). Hence, the nominal interest rate in the short is positive ($R_l > 0$) and the closed form expressions for $\widehat{Y}_l$ and π_l can be written, respectively, as:

$$\widehat{Y}_l = \left(\frac{\mathcal{B}(1-g)}{\mathcal{A}(1-\tau^A)}\right)\widehat{\tau}_L^A + \frac{\mathcal{B}}{\mathcal{A}}\left(\frac{(1-g)(p-\phi_\pi)\kappa}{(1+\tau^S)(1-p\beta)}\right)\widehat{\tau}_L^S$$
$$+\frac{\mathcal{B}(1-g)(p-\phi_\pi)\kappa}{\mathcal{A}(1-\tau^W)(1-p\beta)}\widehat{\tau}_L^W + \frac{\mathcal{B}}{\mathcal{A}}\left(\frac{(1-g)(1-p)(1-p\beta) + (1-g)p\beta(1-p)}{1-p\beta}\right)\pi_L$$
$$+\frac{\mathcal{B}(1-p)c}{\mathcal{A}}\widehat{Y}_L - \frac{\mathcal{B}(p-\phi_\pi)\kappa}{\mathcal{A}(1-p\beta)}\widehat{G}_L$$

with $\frac{\mathcal{A}}{\mathcal{B}} \equiv \frac{\{c(1-p)+(1-g)\phi_Y\}(1-g)(1-\gamma)(1-p\beta) - (1-g)(p+\phi_\pi)\kappa b}{(1-g)(1-\gamma)(1-p\beta)}$ and

$$\pi_l = \frac{\kappa\varrho}{\Theta(1-\tau^A)}\widehat{\tau}_L^A + \frac{\kappa(1-p)}{\Theta}\pi_L + \frac{\kappa(1-p)c}{\Theta(1-g)}\widehat{Y}_L$$
$$-\frac{\kappa\Gamma(1-\gamma)(1-\beta)}{\Theta(1-\beta)}\left[\frac{1}{1-g}\widehat{G}_L - \frac{1}{1-\tau^W}\widehat{\tau}_L^W - \frac{1}{1+\tau^S}\widehat{\tau}_L^S\right]$$

---

[13]We calculate tax cuts or spending increase separately. That is, when there is a labour tax cut, $d\widehat{Y}_l/d\widehat{\tau}_l^W$, there is no change in government spending, $\widehat{G}_l = 0$, and in other taxes ($\widehat{\tau}_l^S = \widehat{\tau}_l^A = 0$) as well as their steady-state is zero, i.e. $\tau^S = \tau^A = 0, g = 0$ but $\tau^W > 0$. Otherwise, we should specify a budget rule that governs the relationship between the timing of spending and taxes. E.g. we can imagine, though quite unrealistically, a balanced budget rule which says that current taxes should finance current spending.

[14]The algebraic condition for a binding zero lower bound fails to be satisfied in this case.

with $\Theta \equiv \Gamma(1-\gamma)(1-\beta) + \kappa(\phi_\pi - p)$, $\Gamma \equiv c(1-p) + (1-g)\phi_Y$, and $c \equiv -[(1-\sigma)\gamma(1-a) - 1]$.

In *case 3* output and inflation are, again, determined on both short run ($\widehat{Y}_l$ and π_l) and long-run ($\widehat{Y}_L$ and π_L) while fiscal variables take their permanent (or long-run) level. The algebraic condition for the zero lower bound is satisfied (see below). Hence, nominal interest rate is zero, $R_l = 0$. Consequently, $\widehat{Y}_l$ and π_l can be written, respectively, as:

$$\widehat{Y}_l = \frac{\mathcal{D}(1-g)\beta}{\mathcal{C}} r_l + \frac{\mathcal{D}(1-p)c}{\mathcal{C}} \widehat{Y}_L + \frac{\mathcal{D}}{\mathcal{C}} \left[\frac{(1-g)p\beta(1-p) + (1-g)(1-p)(1-\beta p)}{1-\beta p} \right] \pi_L$$

$$+ \frac{\mathcal{D}}{\mathcal{C}} \left(\frac{1-g}{1-\tau^A} \right) \varrho \widehat{\tau}_L^A + \frac{\mathcal{D}\kappa(1-g)p}{\mathcal{C}(1-\tau^W)(1-\beta p)} \widehat{\tau}_L^W$$

$$+ \frac{\mathcal{D}}{\mathcal{C}} \left(\frac{(1-g)p\kappa}{(1-\beta p)(1+\tau^S)} \right) \widehat{\tau}_L^S - \frac{\mathcal{D}(1-g)p\kappa}{\mathcal{C}(1-\beta p)} \widehat{G}_L,$$

with $\frac{\mathcal{C}}{\mathcal{D}} \equiv \frac{c(1-p)(1-g)(1-\gamma)(1-\beta p) - (1-g)p\kappa b}{(1-g)(1-\gamma)(1-\beta p)}$ and

$$\pi_l = \frac{\beta c(1-p)^2(1-\gamma)}{\Upsilon} \pi_L - \frac{\kappa c(1-\gamma)(1-p)}{\Upsilon} \left[\frac{1}{1-g} \widehat{G}_S - \frac{1}{1-\tau^W} \widehat{\tau}_S^W - \frac{1}{1+\tau^S} \widehat{\tau}_S^S \right]$$

$$+ \frac{\kappa b \varrho}{\Upsilon(1-\tau^A)} \widehat{\tau}_L^A + \frac{\kappa(1-p)b}{\Upsilon} \pi_L + \frac{\kappa(1-p)cb}{\Upsilon(1-g)} \widehat{Y}_L + \frac{\kappa b}{\Upsilon} \beta r_l$$

with $\Upsilon \equiv c(1-\gamma)(1-\beta p)(1-p) - \kappa pb$. The algebraic requirement for the zero bound to bind is $\mathcal{C} > 0$ which is satisfied for $0.028 \leq \kappa \leq 0.0365$ (or, equivalently, $0.83 \leq \xi \leq 0.85$). That is the Calvo parameter, ξ, should be sufficiently large.

In case of permanent fiscal policy—in contrast to the temporary one—we have to take into consideration the long-run disinflationary effect (the effect of lower long-run inflation expectations) of the increase (decrease) in spending (taxes). This effect is present due to the central bank's commitment of inflation stabilisation (in the form of the Taylor rule). Therefore, the long-run multipliers (in column 4 of Table 3) have to be corrected by deflationary pressures (in column 5 of Table 3) i.e. long multipliers result as the sum of column four and two times column five for each row. The disinflationary effect is quite small for government spending and capital tax cut (they might as well be disregarded). If we suppose that the central bank focuses exclusively on inflation stabilisation—which is a reasonable assumption—then disinflationary effect of the fiscal policies disappears (that is, the coefficients on some of the fiscal variables in equation (15) are zero as now we have $\phi_Y = 0$). In column 5 we can see the net multipliers after taking account of the disinflationary expectations. We have the highest values for government spending and labour tax cut.

In column 2 of Table 3 we can see that the sales and labour tax multipliers in the short

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Table 3: Summary of Multipliers—Permanent fiscal policy

Multipliers	Short-run	Short-run	Long-run $(d\widehat{Y}_L/d\widehat{X}_L)\dagger$	Long-run $(d\pi_L/d\widehat{X}_L)\dagger$	Total effect $\left(\frac{d\widehat{Y}_L}{d\widehat{X}_L} + \sigma\frac{d\pi_L}{d\widehat{X}_L}\right)\dagger$
	$R > 0$	$R = 0$	$R > 0$	$R > 0$	Col.4+σ*Col.5
Gov. spending, $\frac{d\widehat{Y}_t}{d\widehat{G}_t}$	0.0194	-1.7752	0.6750	-0.0051	0.6648
Sales tax, $\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^S}$	0.193	-1.921	0.6442	-0.1814	0.2814
Payroll tax, $\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^W}$	0.2863	-1.9438	1.0387	-0.2647	0.5093
Capital tax, $\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^A}$	-0.0221	-0.2063	0.0000	-0.0192	-0.0384

Remarks to Table 3: the results here are derived using non-separable preferences. Eggertsson (2010, pp. 31) has a similar table where he used separable utility. However, his results and ours are not directly comparable because he used different calibration. †In the last column we can see the total effect of a permanent increase in spending or tax cut, $X_L = \{\widehat{G}_L, \widehat{\tau}_L^S, \widehat{\tau}_L^W, \widehat{\tau}_L^A\}$.

run when interest is positive have some non-trivial positives values (smaller than one), while government spending increase and capital tax cut have negligible effects.

In column 3 of Table 3 we realise that all fiscal policy multipliers are large negative number under permanent fiscal. However, this was not the case under temporary policy when government spending and sales tax cut was positive while labor tax cut and capital tax cut was negative (see column 3 and 5 in Table 2).

It is useful to discuss why permanent increase in spending in the zero bound state (column 3, $R = 0$) is contractionary. First, observe that there is no direct effect of spending on the long run when interest is positive because G terms drop out from the Euler equation (both G_t and G_{t+1} take on their long run value G_L). Let us turn to the case in column 3 when the zero bound binds. In this case expected output and inflation takes their long-run values, $\widehat{Y}_L, \pi_L$, with probability $1 - p$. Previously, we claimed that the only thing that mitigates the downturn of the economy is a spending spree in the zero bound state. The AD curve is positively-sloped in the zero bound as explained in the previous section. The rise in spending generates a decrease in consumption due to the wealth effect and shifts out the labor supply curve which causes recession when the AD curve is positively sloped. However, when we are out of the zero bound state (and the Taylor rule is in action) the net effect of spending hinges on the deflationary term $\left(\frac{d\pi_L}{d\widehat{G}_L}\right)$ that is due to the commitment of monetary policy (see results in column 2).

7 Adding capital to the New Keynesian model

In this section we assume that both households and firms can also use some of their resources to invest into capital. The government is still supposed to make purchases that are financed by lump-sum taxes. Moreover, we also include capital adjustment costs into the model to be able to match the observed slugishness of real variables to shocks. Accordingly, both household's and intermediate goods firms' problems change after the inclusion of capital. Again, following the notations of Christiano *et al.* (2010), we start with the household's optimisation problem.

7.1 The household's problem

The household maximises the following non-separable utility in consumption (C_t) and leisure ($1 - N_t$):

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{[C_t^\gamma (1 - N_t)^{1-\gamma}]^{1-\sigma} - 1}{1 - \sigma} \right] \tag{16}$$

with respect to its budget constraint

$$(1 + R_t)B_t + \int_0^1 profit_t(i)di + \int_0^1 P_t W_t N_t(i)di + \int_0^1 P_t R_t^k K_t(i)di$$
$$= T_t + B_{t+1} + P_t C_t + P_t I_t \tag{17}$$

where R_t^k denotes the real rental rate of capital which serves as an income for the household and I_t denotes investment as a further way of spending.

There is an equation that describes the accumulation of capital. According to this equation, investment is the change in capital stock from time t to time $t + 1$:

$$K_{t+1} = I_t + (1 - \delta)K_t - \frac{\sigma_I}{2} \left(\frac{I_t}{K_t} - \delta \right)^2 K_t \tag{18}$$

where the last term on the RHS is the capital adjustment cost that is now specified as quadratic. The parameter $\sigma_I > 0$ governs the magnitude of adjustment costs to capital accumulation. That is, the household's problem is to maximise its utility in equation (16) subject to its budget constraint in equation (17) and the capital accumulation equation (18).

7.2 The final and intermediary goods' producers problem

The final good producers' problem remains the same while the intermediary firms' problem can be written as follows. Intermediaries set their prices in Calvo manner as it is in model

in section one. The ith intermediary maximises its discounted profit:

$$E_t \sum_{T=0}^{\infty} \beta^{t+T} v_{t+T} \left[P_{t+T}(i) Y_{t+T}(i) - (1 - \nu) \left[P_{t+T} W_{t+T} N_{t+T}(i) + P_{t+T} R^k_{t+T} K_{t+T}(i) \right] \right], \quad (19)$$

where $N_t(i)$ and $K_t(i)$ denotes the value of labour and capital used by i^{th} intermediary, respectively. As we can see from the above formulation the costs are made up of two parts: labour and capital rental costs, respectively. The output of the ith is produced by:

$$Y_t(i) = [K_t(i)]^{\alpha} [N_t(i)]^{1-\alpha}. \quad (20)$$

Similarly to the model in the first section we assume that the monopolist markup in the steady-state is eliminated by a fiscal subsidy, that is, $\nu = 1/\theta$. Also note that v_{t+T} corresponds to the Lagrange multiplier on the budget constraint in the household's optimisation problem. Accordingly, the ith intermediary maximises the expression in equation 19 with respect to the production function in equation (20) and the demand function for $Y_t(i)$ in equation (1).

The conduct of monetary and fiscal policy is not affected by the inclusion of capital into the baseline model.

7.3   Equilibrium

Definition 2 *A monetary equilibrium is a collection of stochastic processes which contains endogenously determined quantitites $\{Y_t(i), Y_t, N_t, N_t(i), K_t, I_t, \ B_{t+1}, MC_t, M_t, F_t\}$, (shadow) prices $\{P_t(i), P_t^*, P_t, \pi_t, W_t, R_t, R^k_t, Q_t, v_t, \vartheta_t, \Delta_t\}$ and an exogenous process $\{G_t\}$ with initial conditions K_0 and G_0. The Intratemporal condition in equation (11) and the Euler equation (12)[15] are exactly the same in the model with capital and they are not listed again here. However, after taking the derivative of the households' problem with respect to I_t and K_{t+1} we get two additional equilibrium conditions (see equation 21 and 22 below). The aggregate resource constraint—which, now, contains investment as well—is satisfied. Furthermore, there is market clearing in goods, labor and capital markets.*

Firstly, there is a connection between capital and investment that also involves Tobin's Q i.e. the consumption value of an additional unit of capital[16]:

$$1 = Q_t \left[1 - \sigma_I \left(\frac{I_t}{K_t} - \delta \right) \right], \quad (21)$$

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[15]However, the NKPC should be written in recursive form instead of the loglinear one. And, of course, the real marginal cost changes after including capital into the model.

[16]This first order condition is obtained by taking the derivative of the Lagrangian associated with the houshold's problem with respect to I_t.

which formula implies the mean reversion of I_t/K_t toward its steady-state value, δ. In Christiano *et al.* (2010) interpretation, the latter equation implies that an increase in investment by one unit raises K_{t+1} by $1 - \sigma_I \left(\frac{I_t}{K_t} - \delta \right)$ unit i.e., due to capital adjustment cost K_{t+1} rises by less than one unit.

Secondly, there is another equilibrium condition describing the dynamics of Tobin-Q that can be derived from the household's problem[17]:

$$\frac{\vartheta_t}{\beta \vartheta_{t+1}} = \frac{1}{Q_t} \left\{ R_t^k + Q_{t+1} \left[(1 - \delta) + \frac{\sigma_I}{2} \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right)^2 - \frac{\sigma_I}{2} \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right) \frac{I_{t+1}}{K_{t+1}} \right] \right\} \qquad (22)$$

where $\vartheta_t \equiv \gamma \left[C_t^\gamma (1 - N_t)^{1-\gamma} \right]^{-\sigma} C_t^{\gamma-1} (1 - N_t)^{1-\gamma}$. As there is no money in the model we measure capital in consumption units as well. One unit of consumption good worth $1/Q_t$ units of installed capital. In order to understand the intuition behind equation (22) we have to observe that the LHS equals the real interest rate based on the Euler equation in (12)[18]. Thus, the LHS equals real return on one-period bonds. The real return of installed capital (the RHS of equation 22) is composed of the following terms: the first term on the RHS of equation 22 is the marginal product of capital, the second term is the undepreciated capital in consumption units, $(1 - \delta)Q_{t+1}$ and the third and fourth terms capture the reduction in adjustment costs as the value of installed capital increase from time t to time $t + 1$. As a result, equation (22) can be interpreted as a no-arbitrage condition (Christiano *et al.*, 2009).

When we analyse the case of binding zero bound we assume that the monetary authority holds the interest rate at constant level. In order to be able to analyse the zero lower bound in Dynare we have to compute the equations in their original form (that is, not in log-linear form)[19]. However, the NKPC is a log-linear equilibrium condition. Alternatively, we can express NKPC in recursive form. For this purpose let us express the ratio of optimal price, P_t^*, and the economy-wide price index, P_t, recursively as:

$$\frac{P_t^*}{P_t} = \frac{M_t}{F_t},$$

where M_t and F_t are given, recursively, by:

$$M_t = \vartheta_t MC_t + \xi \beta E_t \left\{ \pi_{t+1}^\theta M_{t+1} \right\}$$

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[17] Note that, in equilibrium, R_t^k equals the real marginal product of capital, i.e. $R_t^k = \alpha E_t \left[K_{t+1}^{\alpha-1} N_{t+1}^{1-\alpha} \right]$.

[18] The Euler is given by: $1 = E_t \left(\beta \frac{\vartheta_{t+1}}{\vartheta_t} \frac{1+R_{t+1}}{P_{t+1}/P_t} \right)$ with corresponding stochastic discount factor, ϑ_t.

[19] When equations are computed into the Dynare in log-linear form they are not allowed to contain constants. When zero bound on nominal interest rate binds, nominal interest rate in the Taylor rule is held at constant level and this is the reason why the log-linear setup in Dynare is not suitable for the analysis of the model when the nominal interest rate is zero.

and

$$F_t = \vartheta_t + \xi\beta E_t\left\{\pi_{t+1}^{\theta-1}F_{t+1}\right\}.$$

Accordingly, the expression for the real marginal cost changes to:

$$MC_t = \varsigma \left(R_t^k\right)^\alpha W_t^{1-\alpha},$$

with $\varsigma \equiv \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}$.

The labour and capital demand of intermediate goods firms are given, respectively, by

$$N_t = (1-\alpha)\frac{MC_t}{W_t}Y_t\int_0^1\left(\frac{P_t(i)}{P_t}\right)^{-\theta}di \equiv (1-\alpha)\frac{MC_t}{W_t}Y_t\Delta_t,$$

and

$$K_t = \alpha\frac{MC_t}{R_t^k}Y_t\int_0^1\left(\frac{P_t(i)}{P_t}\right)^{-\theta}di \equiv \alpha\frac{MC_t}{R_t^k}Y_t\Delta_t,$$

where we have $N_t \equiv \int_0^1 N_t(i)di$, $K_t \equiv \int_0^1 K_t(i)di$ and $\Delta_t \equiv \int_0^1\left(\frac{P_t(i)}{P_t}\right)^{-\theta}di$. The price dispersion, Δ_t, can be written recursively as:

$$\Delta_t \equiv \int_0^1\left(\frac{P_t(i)}{P_t}\right)^{-\theta}di = (1-\xi)\left(\frac{P_t^*}{P_t}\right)^{-\theta} + \xi\pi_t^{-\theta}\Delta_{t-1}.$$

The economy's resource constraint after including investment, I_t, modifies to:

$$Y_t = C_t + I_t + G_t.$$

The government spending shock in the economy is the same as specified by equation (4).

7.4   Calibration

The parametrisation of the model with capital can be found in Table 4. After including capital, the resource constraint will contain investment as well. We calibrate the share of capital, α, in the production function by using the values of I/Y, δ and β. The value of β and δ are standard in economics literature. We know from Uhlig (2010) that G/Y is 0.18, I/Y is around 0.22 (the latter is slightly lower than the one in Uhlig (2010) who has international asset position as well) which implies that C/Y is 0.6. The parameter of the convex adjustment cost of capital, σ_I, can be found in Christiano *et al.* (2010)[20]. The parameter θ is calibrated

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[20]It can be interesting to note that if we use the linearised version of the equation that describes the dynamics of Tobin Q — which is not the case here — then the σ_I parameter is not needed as we do not need to specify the capital adjustment cost function with a certain functional form that contains σ_I.

to 6 so that we can have a steady-state markup of 20% as in Gali *et al.* (2007). The value of ϕ_Y is set to 0.5/4 which is the one originally proposed by Taylor (1993). Please also note that now we consider—more realistically—an environment in which price rigidity is lower (in the models without capital ξ was quite high (0.85) but here we set it to 2/3 which implies that firms set new prices, on average, every three quarters). The remaining parameter values are the same as in Table 1.

Table 4: Parametrisation of the New Keynesian Model with Capital

Parameters	Separable & capital	Non-separable & capital
σ	2	2
φ	0.2	na
β	0.99	0.99
γ	na	0.29
ρ_G	0.8	0.8
δ	0.025	0.025
θ	6	6
ϕ_π	1.5	1.5
ϕ_Y	0.5/4	0.5/4
ρ_R	0	0
ξ	2/3	2/3
$G/Y (\equiv g)$	0.18	0.18
I/Y	0.22	0.22
σ_I	17	17
σ_{ε^G}	0.01	0.01
Implied Parameters		
C/Y	0.6	0.6
κ	0.17	0.17
N	na	1/3

7.5 Experiments

As we already said in the Introduction (in Section 1) the Bernstein and Romer (BR) (2009) numbers are based on a permanent fiscal stimulus. In the following we restrict the analysis to government spending multiplier only and study—more realistically—the effects of a temporary, anticipated increase in spending to test the robustness of the numbers of BR (2009). However, we know that the results are not comparable because BR (2009) assumed a permanent stimulus in contrast to our temporary one which we think is more realistic. As we have a forward looking model we have to specify explicitly the assumptions about firms' and households' expectations. Here the main assumption is that people expect a temporary increase in spending that is initially financed by issuing debt. Later, the debt is reduced by levying lump-sum taxes that do lower the after tax income earnings and thereby wealth (Cogan *et al.*, 2009). In the following we consider *three types* of multipliers (an impact and two types of long-run multiplier) under the assumption of fixing the nominal interest rate at a constant level for one and two years in line with recent empirical evidence on US[21].

7.5.1 When nominal interest rate is positive

In Table 5 we can see short and long run multipliers from the model with capital. The impact multipliers are calculated similarly to the ones in section 2 and 3. The idea of long-run multiplier is borrowed from Campolmi *et al.* (2009). It is calculated as the sum of discounted output changes divided at each time t by the sum of discounted spending changes. The impact multipliers of spending in the model with capital are generally lower than the corresponding ones in the model without capital because in the former one an increase in spending leads to a rise in real interest rate that crowds out private investment (see findings in Table 2 and Table 5). As we can also observe that the impact multiplier is a bit lower for separable preferences that is in line with our findings of section 2. However, the long-run multipliers, which are generally lower than the impact ones, tell that the distinction coming from the assumption on preferences disappear on the long run if we assume that there is no response to changes in output gap in the Taylor rule (i.e. these values are roughly the same; see numbers with an asterisk Table 5).

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[21]At the time of the introduction of the American Recovery and Reinvestment Package in 2009Q1 it was not known for how many quarters the Federal Funds Rate will stay around the zero level. Accordingly, we consider two experiments: in case 1 we assume that the nominal rate is zero for two years (i.e. the nominal interest is held fixed on its steady-state level for eight quarters) or case 2 it is held fixed for one year.

Table 5: Impact and Long-Run Multipliers of a temporary 1 % spending shock for separable and non-separable preferences

	Impact Multiplier($dY/dG|_{t=1}$)	Long-run Multiplier
Separable	0.7260	0.6884 (0.8726*)
Non-separable	0.6966	0.5230 (0.8487*)

Remarks to Table 5: the long-run multiplier is defined as dividing discounted output changes by discounted changes in spending at each time t.
*If we assume $\phi_Y = 0$ in the Taylor-rule then the long-run multipliers are roughly the same.

7.5.2 When nominal interest rate is held constant[22]

Table 6 shows the response of real GDP to a transitory, anticipated[23] increase in government purchases of 1 per cent of steady-state GDP assuming that the nominal interest rate is held constant for a duration of **two years** starting in the first quarter of 2009[24]. The latter means that the Fed can start to increase interest rate in 2012Q1 at earliest (technically, it means that the Taylor rule will be put back into practice in 2012Q1). The first and second row shows the findings of BR (2009) and this paper, respectively. As we can see the impact multiplier of the BR (2009) are generally in line with the finding of ours. However, the latter is not true for longer horizons. As we can see our multipliers are generally around one at one, two or even at three years horizon. However, the BR (2009) numbers are much larger than the ones of ours. Here we can confirm the findings of Cogan *et al.* (2009) who find using a more elaborate (i.e. containing more frictions) version of the type of model used here that the spending multiplier should decline with the horizon. However, in contrast to Cogan *et al.* (2009) who find that multipliers decline sharply with the horizon (with values below 0.5), we show here that the multipliers can remain over 0.5 (but still below one) even for longer horizons. The difference between results of Cogan *et al.* (2009) and ours comes from the fact that while they assumed permanent stimulus we used a transitory one here. We have seen in the previous section— using models without capital—that a permanent increase in spending results in middle-sized multipliers with non-separable preferences (the only exception is labour cut which is around one). Of course, Eggertsson (2010), who uses separable utility report lower multipliers than

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[22] Here we assume in line with the most elaborate model of Christiano *et al.* (2009) that the nominal rate is held constant at the natural rate of interest.

[23] Here we assumed that people anticipate the recovery package by two quarters before it is taken into action. However, for example Uhlig (2009) assumed four quarters.

[24] In a simple New-Keynesian model with capital Christiano *et al.* (2010) shows that we can endogenously determine—depending on the properties of the discount factor shock—the date t_1 at which zero bound becomes binding and another date, t_2, $t_2 > t_1$ at which the zero bound ceases to bind.

ours in case of a permanent stimulus.

Figure 5 show a type of long run multiplier calculated similarly to Uhlig (2010). This multiplier is defined as the discounted sum of output changes until each horizon is divided by the sum of discounted spending changes until the same horizon (Uhlig, 2010). Even on very long horizons (e.g. after one hundred periods), the multiplier is still around one.

Table 6: Spending multiplier calculated by assuming that the nominal rate is held constant for two-year duration from 2009Q1 on

|	Percentage increase in real GDP					
	2009Q1	2009Q4	2010Q4	2011Q4	2012Q4	Long run
BR (2009)	1.05	1.44	1.57	1.57	1.55	na
Our findings	1.06	0.96	0.93	0.79	0.59	0.85

Remarks to Table 6: BR (2009)= Bernstein-Romer (2009) paper. We used non-separable preferences to perform the calculations in Table 6 and 7.

We re-did our calculations assuming that nominal rate is held constant for a duration of **one year**. The results can be observed in Table 7 and Figure 6. As we can see in the second row of Table 7, the multipliers are generally lower if nominal interest rate is fixed at a constant number for a shorter period of time (here it is one instead of two years). Now, the impact multiplier does not coincide with the one in BR (2009). Also, the longer horizon findings of ours depart even further from the results of BR (2009) while at the same time approach more the ones of Cogan *et al.* (2009). However, it has to be pointed out that our results concerning multipliers of a temporary measure on a time horizon longer than one year are higher than the ones reported by Cogan *et al.* (2009) who assumed permanent stimulus exercise. This finding is favored by the results obtained from models without capital in previous sections. Figure 6 shows that the long run multiplier is clearly less than one when the nominal rate is fixed at constant level for one year.

Table 7: Spending multiplier calculated by assuming that the nominal rate is held constant for one-year duration from 2009Q1 on

	Percentage increase in real GDP					
	2009Q1	2009Q4	2010Q4	2011Q4	2012Q4	Long run
BR (2009)	1.05	1.44	1.57	1.57	1.55	na
Our findings	0.94	0.78	0.71	0.59	0.55	0.68

Figure 5: Uhlig (2010)-type long-run spending multiplier calculated by assuming that the nominal rate is held constant for **two-year** duration from 2009Q1 on
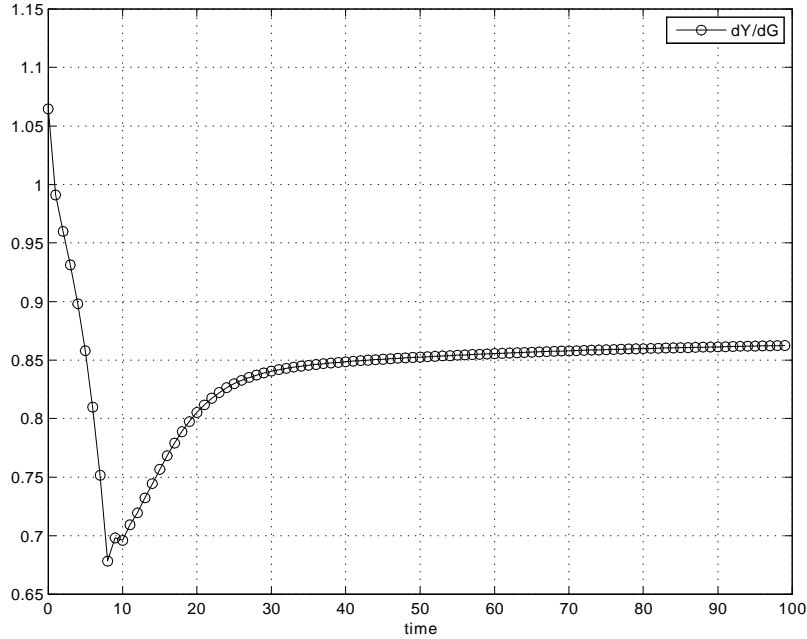

7.6 Further extensions and shortcomings of these types models

The models considered thus far assume that the government deficit is financed through lump-sum taxes. However, in a recent paper, Uhlig (2010) considers a simple RBC model with capital, distortionary taxation (that is, labour, capital and sales tax) and a budget rule. In the budget rule of Uhlig (2010) a certain part of the deficit plus a random amount is financed by current labour taxes. In the latter case the value of the multiplier is influenced meaningfully by a parameter called budget balance speed defining the share of the deficit financed through labour taxes. If the budget balance speed is sufficiently low then the positive effect of a government spending on output will last longer. However, the most important finding of Uhlig (2010) is that the long run multiplier associated with a spending shock is always negative even if its impact on GDP is positive in the first couple of years. Future research may test the robustness of the findings of Uhlig (2010) in a reacher structure including frictions[25] that are popular now in leading New Keynesian models such as the Smets and Wouters (2007) model.

Hall (2010) points out that the biggest shortcoming of the new-Keynesian models is the countercyclical markup which is procyclical empirically. Of course, we also emphasized that

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[25]Such frictions include but not restricted to sticky prices and wages, variable capital utilisation, investment adjustment costs, habit formation and non-Ricardian (i.e. credit constrained) households.

Figure 6: Uhlig (2010)-type long-run spending multiplier calculated by assuming that the nominal rate is held constant for **one-year** duration from 2009Q1 on
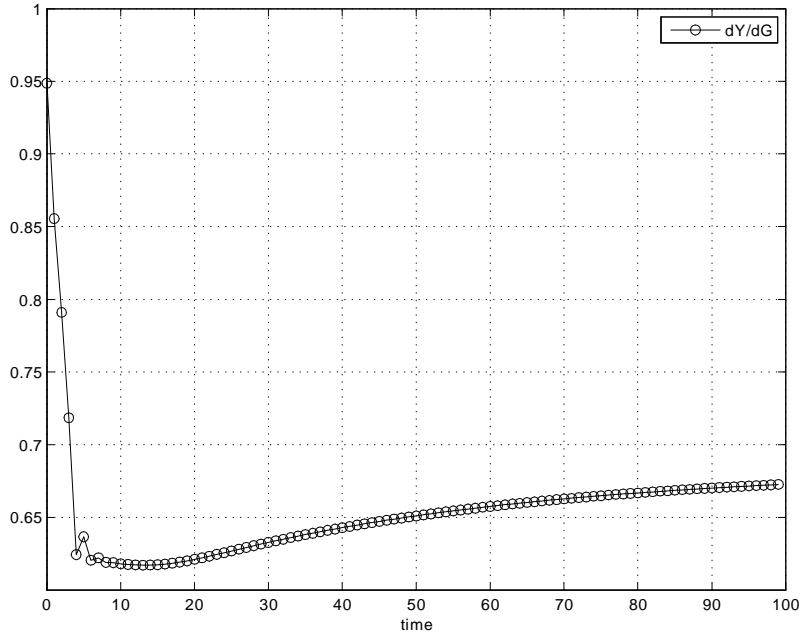
the endogenous fall in the mark-up after an increase in spending is a key element of these type of models. As prices are sticky the the price over marginal cost—which is the definition of the mark-up—falls after an increase in aggregate demand due to an increase in spending.

The estimation of the parameters of the models used here is, of course, also a matter of future research. For example, Denes and Eggertsson (2009) who estimated the model of Eggertsson (2010) with Bayesian methods. In particular, Denes and Eggertsson (2009) calibrated their model parameters to data prevailing under the Great Depression by maximising the posterior distribution of his model to match a 30 percent decline in output and a 10 percent deflation at the first quarter of 1933 when the zero lower bound became to be binding on the nominal interest rate.

8 Conclusion

Even if the models presented here are too simple (i.e. they contain only few frictions) for providing sufficient background for policy decisions, still we can obtain a *fair* picture on the outcome of possible fiscal policy measures. Initially, in a model without capital, we consider the effects of three different fiscal policy measures that can be used for stimulating the economy: a temporary and unexpected rise in non-productive government spending, a cut in sales tax, a cut in payroll tax or a cut in capital tax. Each of them are considered separately (that is, when government spending increases there is no change in sales or payroll taxes) and financed by lump-sum taxes. Thus, Ricardian equivalence holds. The paper used separable and non-separable preferences in consumption and leisure as well. Most of VAR evidence point out that increased government spending lead to a rise in consumption. To model this stylised fact, we use non-separable preferences in consumption and leisure which imply low negative Hicksian wealth effect emerging after a rise in government spending in contrast to separable preferences where negative wealth effect on consumption is high. However, in the same model with capital we show that the long-run multiplier we borrowed here from Campolmi *et al.* (2010) to calculate the long-run effects of a government spending shock produces roughly the same result for both types of preference specifications (these results depends largely on the underlying parametrisation—e.g., we found, surprisingly, that by having output targeting in the Taylor rule results in a lower impact and long-run multipliers for non-separable preferences compared to the separable ones whereas the opposite is true when we consider inflation targeting only).

During the recent financial crises the nominal interest rate in the U.S. was almost zero, that is the zero lower bound on nominal interest rate was binding. Based on this stylised fact we consider the above multipliers in a model with *separable* preferences when the zero bound binds and *found that the multipliers are smaller than the ones reported by Eggertsson (2010)* and are more close in magnitude to values that we obtain using non-separable preferences with positive nominal interest rate (the only exception is the labour tax cut which is negative under zero nominal rate irrespectively of preference specifications). However, we found, in line with Christiano *et al.* (2010), that a temporary spending stimulus can be turn out to be very high when holding the nominal interest rate at zero level and assuming non-separable utility. Moreover, we employed Christiano *et al.* (2010) non-separable framework to calculate tax cut multipliers and found that the sales tax cut multiplier is well above one for non-separable preferences when interest is zero. We show that the most important finding of Eggertsson (2010) which says that labour tax cut multiplier is negativ under the separable case for zero nominal interest, remains true for non-separable utility as well.

Some influential papers like Cogan *et al.* (2009) considered permanent stimulus. So we did and found that it is the government spending and labour tax cut multipliers which take the highest values under non-separable preferences in case of a permanent stimulus in contrast to Eggertsson (2010) who found lower values than ours under separable preferences for the same type of permanent stimulus. Our results show that multipliers of permanent action *must be* lower than the same of a transitory one.

Finally, we augmented our model with capital and used three types of multipliers (impact, long run and Uhlig (2010) type) to compare our findings to the ones of Bernstein and Romer (BR) (2009) and draw the following conclusions. Firstly, it is possible to obtain multipliers around one *on impact* similar in magnitude to the ones in BR (2009). Secondly, our temporary multipliers decline with the horizon similarly to the findings of Cogan *et al.* (2009)—who calculated multipliers of a permanent stimulus—and in sharp contrast with the results of BR (2009). Futhermore, the reason why multipliers in Cogan *et al.* (2010) are lower than ours is that they consider a permanent stimulus opposed to the temporary ones of ours.

9   Appendix

We explained that spending multipliers in the non-separable case are generally higher than in the separable one (or they are roughly the same), which is true only when the coefficient on output gap, ϕ_Y, in the Taylor rule is chosen to be zero for both positive and zero nominal interest rates. However, when ϕ_Y is chosen to be positive this is not the case any more (see Table 8) under positive interest rates. Note that the latter finding is not necessarily true for sales and labour tax cut multipliers as can be seen in the second and fourth column.

Table 8: Summary of Multipliers—Temporary fiscal policy

Multipliers	Separable		Non-separable	
	$R > 0$	$R = 0$	$R > 0$	$R = 0$
Gov. spending, $\frac{d\widehat{Y}_t}{d\widehat{G}_t}$	0.8382*	1.1027	0.8133*	3.7
Payroll tax, $\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^W}$	0.1931*	**-0.941**	0.1982*	**-2.38**
Sales tax, $\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^S}$	0.3662*	0.5389	0.5837*	1.63
Capital tax, $\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^A}$	-0.0102*	-6.1227	-0.0196*	-0.7692

Remarks to Table 8: calibration for these result is the
same as in Table 1 with the only exception
here $\phi_Y = 0.5/4$ as in Taylor (1993) instead of zero. Only
the multipliers under positive interest rate are affected by
this change (these numbers are denoted with an asterisk).

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