Sectoral Technology and Structural Transformation*

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We assess how the properties of technology affect structural transformation, i.e. the reallocation of production factors across the broad sectors agriculture, manufacturing, and services. To this end, we estimate sectoral CES and Cobb–Douglas production functions on postwar US data. We find that differences in technical progress across the three sectors are the dominant force behind structural transformation whereas other differences across sectoral technology are of second–order importance. Our findings imply that Cobb–Douglas sectoral production functions that differ only in technical progress capture the main technological forces behind the postwar US structural transformation.
JEL: O11; O14
Keywords: capital share; CES production function; Cobb–Douglas production function; elasticity of substitution; structural transformation

The reallocation of production factors across the broad sectors agriculture, manufacturing, and services is one of the important stylized facts of growth and development. As economies develop agriculture shrinks, manufacturing first grows and then shrinks, and services grow. A growing recent literature has studied this so–called structural transformation and has shown that it has important implications for the behavior of aggregate variables such as output per worker, hours worked, and human capital.[1] The current paper is part of a broader research program that asks what economic forces are behind structural transformation. Herrendorf, Rogerson and Valentinyi (2013) addressed the preference aspect of this question and quantified the importance of the effects of changes in income and relative prices for changes in the composition of households' consumption bundles. In the current paper, we focus on the technology aspect and ask how important for structural transformation are differences across sectors in technical progress and the technology parameters, including the capital share and the elasticity of substitution between capital and labor.

There are two different views in the literature about this question. Many papers on structural transformation use sectoral production functions of the Cobb–Douglas form,

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[1] The recent literature started with Echevarria (1997) and Kongsamut, Rebelo and Xie (2001); see Herrendorf, Rogerson and Valentinyi (2014) for a review.

which have equal exponents and differ only in technical progress. The advantage of this way of proceeding is that under the additional assumptions of perfect competition and profit maximization, the exponent on capital equals the share of capital in aggregate income (capital share for short), which is easily calculated. The potential disadvantage of this way of proceeding is that it restricts our attention to forces behind structural transformation that result directly from technical progress.

Some contributions to the recent literature on structural transformation suggest that sectoral differences in the capital share and the substitutability between capital and labor also have important implications for structural transformation. To see how these features of technology may matter for structural transformation, suppose first that labor–augmenting technical progress is even across sectors and compare two sectoral production functions that are of the Cobb Douglas form and only differ in the capital share. If GDP per capita is relatively low, then capital is relatively scarce compared to labor, the rental price of capital relative to the rental price of labor is relatively high, and the relative price of the output of the sector with the higher capital share is relatively high. As technical progress takes place, GDP per capita increases, capital becomes less scarce compared to labor, and the relative price of the output of the sector with the higher capital share falls. Given standard preferences, this leads to the reallocation of resources towards this sector. Acemoglu and Guerrieri (2008) emphasized this economic force behind structural transformation.

Suppose instead that the sectoral production functions only differ in the substitutability between capital and labor (and that technical progress again is even). If GDP per capita is relatively low, then capital is scarce and the relative price of the output of the sector with the low substitutability between capital and labor is relatively high. As technical progress takes place, GDP per capita increases and the relative price of the output of the sector with low substitutability falls. Given standard preferences, this again leads to the reallocation of resources towards this sector. Alvarez-Cuadrado, Long and Poschke (2013) emphasized this economic force behind structural transformation.

The goal of this paper is to assess how important these different features of sectoral technology are quantitatively for structural transformation. We will estimate CES production functions for agriculture, manufacturing, and services on postwar US data. To have a reference point, we will also estimate Cobb–Douglas production functions with sector–specific capital shares and Cobb–Douglas production functions with a common capital share. Before we proceed to the details of the estimation, we need to decide whether the sectors produce gross output or value added. The difference between the two, of course, is that gross output counts everything that the sector produces whereas value added counts only what the sector produces beyond the intermediate inputs that it uses. While the literature typically uses value–added production functions, it is not clear that they exist in general. We start with a production function for gross output and derive conditions under which a production function for value added exists. We then provide evidence suggesting these conditions are met in the data and focus on value added production functions.

Turning now to the details of the estimation, one contribution of this paper is to derive a normalization of value–added production functions that considerably simplifies the estimation. Essentially the normalization implies that the weights on capital and labor in the CES production function equal the average income shares of capital and labor. We may therefore proceed as researchers typically do for the special case of Cobb–Douglas production functions and calibrate the weights of the CES production functions before we estimate the other parameters. While León-Ledesma, McAdam and Willman (2010) also normalize the CES production function, there is a subtle but important difference between the two papers: their normalization involves an approximation and may not be accurate far away from the point of approximation; in contrast, our normalization is exact and applicable everywhere. We combine our normalized production function with the first–order conditions for the optimal choices of capital and labor, assuming that there is exogenous exponential capital– and labor augmenting technical progress. We estimate the resulting three–equation system for each sector and for the aggregate economy via a non–linear version of the method by Cochrane and Orcutt (1949).

The estimation of the sectoral CES production functions yields the following results. First, labor–augmenting technical progress is quantitatively much more important than capital–augmenting technical progress, and at the aggregate level, capital–augmenting technical progress is not statistically different from zero; labor–augmenting technical progress is fastest in agriculture and slowest in services, and the differences in the growth rates are sizeable. Second, agriculture has the highest capital share, services have the second–highest capital share, and manufacturing has the lowest capital share. The finding that services have a higher capital share than manufacturing reflects the fact that services include owner–occupied housing. Third, capital and labor are most substitutable in agriculture and least substitutable in services; moreover, in agriculture capital and labor are more substitutable than in the Cobb–Douglas case and in manufacturing and services they are less substitutable. The finding that in agriculture capital and labor are more substitutable than in the Cobb–Douglas case is consistent with the view that after the second world war a wave of mechanization led to massive substitution of capital for labor in US agriculture; see for example Schultz (1964).

In order to assess how quantitatively important the different features of the estimated sectoral production functions are for structural transformation, we endow competitive stand–in firms in each sector with the estimated technologies and ask how well their optimal choices replicate structural transformation, taking as given the observed prices for value added, the observed rental prices for capital and labor, and the estimated technical progress at the sectoral level. We focus on two important features of structural transformation: the observed allocation of labor across sectors, which is the most widely available measure of sectoral activity, and the changes in sectoral relative prices, which determine the sectoral composition of consumption at the household side. We find quantitatively that uneven technical progress is the dominant force behind these features, whereas sectoral differences in the capital shares and substitution elasticities have second–order implications only. This implies that Cobb–Douglas sectoral production functions that differ only in technical progress capture the main forces behind the postwar US structural transformation that operate on the technology side. Perhaps somewhat surprisingly, this statement holds even in agriculture where the capital share and the elasticity of substitution are largest. The reason why this does not give the CES production function a notable advantage over the other specifications is that the effects on structural transformation of the large capital share and the large elasticities in agriculture work in opposite directions and largely cancel each other, leaving the effects of uneven labor–augmenting technical progress as the dominating force. All three production functions capture that force similarly well.

Our findings lend support to a key assumption in the seminal paper on structural transformation by Ngai and Pissarides (2007), who used Cobb–Douglas production functions with equal capital shares. Nonetheless, our findings should not be interpreted to imply that always and everywhere the Cobb–Douglas production function with equal factor shares is the best modeling choice. For example, if one is interested in the *level* of employment in agriculture instead of secular *changes* in employment, then it is important to model that the labor share in agriculture is much lower than in the other sectors. A Cobb–Douglas production function with equal shares would overpredict the level of employment in agriculture considerably, even though it does capture the main changes in employment. Moreover, one should keep in mind that our results are obtained for the postwar period, during which the US was fairly developed and agriculture had a relatively small share in overall employment and value added. It would therefore be premature to conclude that Cobb–Douglas sectoral production functions will do a good job at capturing structural transformation also in less developed economies in which agriculture has much larger employment and value added shares than in the US economy.

Our work belongs to a large literature that estimates production functions at the aggregate level, the industry level, or the firm level. Antràs (2004), Klump, McAdam and Willman (2007) and León-Ledesma, McAdam and Willman (2010) are the contributions to this literature which are most closely related to our work. They asked the question how substitutable capital and labor are at the level of the aggregate US economy and found that they are less substitutable than in the Cobb–Douglas case. In contrast, we focus on the disaggregate level of the three broad sectors that are relevant in the context of structural transformation. We stress that for the aggregate US economy our exercise yields very similar findings to those obtained by the above papers. Our work is also related to that of Oberfield and Raval (2014), who develop a framework to estimate the aggregate elasticity of substitution between capital and labor by aggregating the choices of individual plants. Using micro data for the US manufacturing sector, they come up with a value for the aggregate elasticity of substitution of the manufacturing sector that is fairly similar to our estimate.

The remainder of the paper is organized as follows. In Section I we introduce the concept of value–added production functions. Section II derives the system of equations that we estimate, including the normalization of the production function, and it discusses the issues that arise in the estimation. Section III describes the data that we use. In Section IV, we present the estimation results and in Section V we compare the performance of CES production functions with the performance of Cobb–Douglas production functions. Section VI discusses the implications of our results for building multi–sector models and

Section VII concludes.

I. Value–added Production Functions

We start with the question of whether to write production functions in gross–output form or in value–added form. Since gross output equals the sum of value added and intermediate inputs (all expressed in current prices), the difference between the two possibilities lies in whether one counts everything that the sector produces ("gross output") or whether one counts only what the sector produces beyond the intermediate inputs that it uses ("value added"). To appreciate the difference between the two possibilities, it is useful to start with the aggregate production function. In a closed economy, GDP equals value added by definition. Therefore, GDP G is ultimately produced by combining domestic capital K and labor L. Many authors therefore specify the aggregate production function as a value–added production function:

$$G = F(K, L)$$

In an open economy, GDP is in general not equal to domestic value added because some intermediate inputs are not produced domestically but are imported from other countries. Therefore, GDP is produced with domestic capital, labor, and imported intermediate inputs Z:

$$G = H(K, L, Z)$$

While imported intermediate inputs are often abstracted from, they can be quantitatively important, in particular in small open economies that import most of the resources and many of the agricultural and manufactured intermediate goods that they use.

Turning now to sectoral production functions, the question of which type of production functions to use arises even in a closed economy. The reason for this is that a typical sector uses intermediate inputs from other sectors, and so sectoral output does not equal sectoral value added even in a closed economy. Therefore, it is natural to start with a production function for gross output and ask under what conditions a production function for value added exists.

Denoting the sector indexes for agriculture, manufacturing, and services by $i \in \{a, m, s\}$, the production function for sectoral gross output can be written as:

$$G_i = H_i(K_i, L_i, \mathbf{Z}_i)$$

where $\mathbf{Z}_i$ denotes the vector of intermediate inputs in sector i that are produced by all sectors including sector i: $\mathbf{Z}_i = [Z_{i1}, ..., Z_{in}]$. The question we ask here is under which conditions do value–added production functions $F_i(K_i, L_i)$ exist such that sectoral value added is given by:

$$(1) \qquad Y_i \equiv \frac{P_{gi} H_i(K_i, L_i, \mathbf{Z}_i) - \sum_j P_{ij} Z_{ij}}{P_{yi}} = F_i(K_i, L_i)$$

where P_{gi}, P_{ij}, and P_{yi} denote the prices of gross output, intermediate input Z_{ij}, and value added (all expressed in current dollars).

Sato (1976) showed that a value added production function exists if there is perfect competition, if firms behave optimally, and if the other input factors are separable from intermediate inputs, that is, the gross–output production function is of the form

$$G_i = H_i(F_i(K_i, L_i), \mathbf{Z}_i) \tag{2}$$

where H_i and F_i satisfy the usual regularity conditions, that is, they are positive, finite, twice continuously differentiable, monotonically increasing in both arguments, strictly concave, homogeneous of degree one, and satisfy the Inada conditions. To understand Sato's argument, consider the problem of a stand–in firm that takes prices and gross output as given and chooses capital, labor, and intermediate inputs to minimize its costs subject to the constraint that it produces the given output:

$$\min_{K_i, L_i, \mathbf{Z}_i} R_i K_i + W_i L_i + \sum_j P_{ij} Z_{ij} \quad \text{s.t.} \quad H_i(F_i(K_i, L_i), \mathbf{Z}_i) \geq G_i \tag{3}$$

where R_i and W_i denote the rental rates for capital and labor, both expressed in current dollars. The first–order conditions for an interior solution to this problem imply:[2]

$$P_{yi} = \lambda_i \frac{\partial H_i(F_i(K_i, L_i), \mathbf{Z}_i)}{\partial Y_i} \tag{4}$$

$$R_i = \lambda_i \frac{\partial H_i(F_i(K_i, L_i), \mathbf{Z}_i)}{\partial Y_i} \frac{\partial F_i(K_i, L_i)}{\partial K_i} \tag{5}$$

$$W_i = \lambda_i \frac{\partial H_i(F_i(K_i, L_i), \mathbf{Z}_i)}{\partial Y_i} \frac{\partial F_i(K_i, L_i)}{\partial L_i} \tag{6}$$

where λ_i is the multiplier on the constraint. Substituting the first equation into the second

[2]To obtain (4), we start with the interior first–order condition for the optimal choice of Z_{ij}:

$$P_{ij} = \lambda_i \frac{\partial H_i}{\partial Z_{ij}}$$

The assumption that H is homogeneous of degree one implies that

$$G_i = Y_i \frac{\partial H_i}{\partial Y_i} + \sum_j Z_{ij} \frac{\partial H_i}{\partial Z_{ij}}$$

Solving this equation for $\partial H_i / \partial Z_{ij}$, substituting the result into the first–order condition for Z_{ij}, and rearranging gives:

$$Y_i = \frac{\lambda_i G_i - \sum_j P_{ij} Z_{ij}}{\lambda_i \partial H_i / \partial Y_i}$$

Using that via the envelope theorem, $\lambda_i = P_{gi}$, (4) follows by comparing the denominator of the previous equation with the denominator of (1).

and third equations gives:

$$R_i = P_{yi} \frac{\partial F_i(K_i, L_i)}{\partial K_i} \tag{7}$$

$$W_i = P_{yi} \frac{\partial F_i(K_i, L_i)}{\partial L_i} \tag{8}$$

These conditions are the first–order conditions for an interior solution to the problem of a stand–in firm that takes prices and value added as given and chooses capital and labor to minimize its costs subject to the constraint that it produces the given value added:

$$\min_{K_i, L_i} R_i K_i + W_i L_i \quad \text{s.t.} \quad F_i(K_i, L_i) \geq Y_i \tag{9}$$

where we have used that by the envelope theorem the multiplier on the new constraint equals the price of value added P_{yi}.

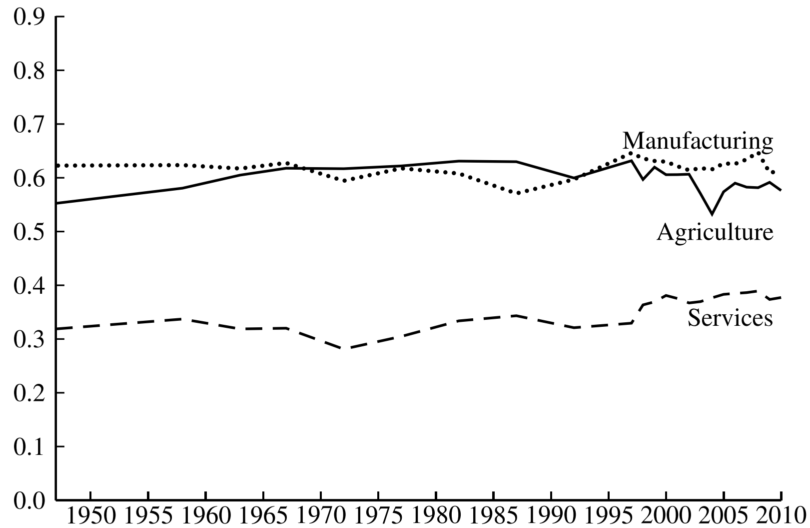
The question remains if condition (2) holds for the postwar US economy. A sufficient (but not necessary) condition is that the sectoral production function is of the Cobb–Douglas form between value added and an aggregator X_i of all intermediate inputs:

$$G_i = [F_i(K_i, L_i)]^{\eta_i} [X_i(\mathbf{Z}_i)]^{1-\eta_i} \tag{10}$$

In this case, perfect competition and firm optimization imply that the share of intermediate inputs is constant over time. Figure 1 plots the intermediate good shares for the postwar US economy. We can see that while they are not strictly constant none of them has a pronounced long–run trend, which is consistent with the Cobb–Douglas form (10). An additional piece of evidence in favor of the Cobb–Douglas form is that when we regress the changes in the intermediate good share of a given sector on the changes in the price of intermediate goods relative to value added in that sector, the regression coefficient is not significant.[3] We interpret these pieces of evidence to mean that the functional form (10) is a reasonable starting point when one is interested in long–run secular trends that the literature on structural transformation focuses on. We will therefore proceed under the assumption that sectoral value–added production functions exist. In the next section, we will discuss the issues involved in estimating them.

[3]For this exercise we use postwar US data from WorldKLEMS. We thank an anonymous referee for suggesting to do this.

Figure 1. Intermediate Inputs Shares in the US



Source: Input–Output Tables for the United States, Bureau of Economic Analysis

II. Estimating Value Added Production Functions

We focus on the class of CES production functions that was introduced to economics by Arrow et al. (1961):[4]

$$F_i(K_{it}, L_{it}) = \left[\alpha_i \left[\exp(\gamma_{ik} t) K_{it}\right]^{\frac{\sigma_i - 1}{\sigma_i}} + (1 - \alpha_i) \left[\exp(\gamma_{il} t) L_{it}\right]^{\frac{\sigma_i - 1}{\sigma_i}}\right]^{\frac{\sigma_i}{\sigma_i - 1}} \tag{11}$$

where $i \in \{a, m, s\}$ denotes the sector, $\sigma_i \in [0, \infty)$ is the (constant) elasticity of substitution between capital and labor, γ_{ik} and γ_{il} are the growth rates of capital– and labor–augmenting technical progress, and α_i is the relative weight on capital. For $\sigma_i \to 1$, the CES production function (11) converges to the Cobb–Douglas production function:

$$F_i(K_{it}, L_{it}) = \left[\exp(\gamma_{ik} t) K_{it}\right]^{\alpha_i} \left[\exp(\gamma_{il} t) L_{it}\right]^{1-\alpha_i} \tag{12}$$

[4]In contrast, Jorgenson, Gollop and Fraumeni (1987) estimated translog production functions for 45 disaggregate US industries during 1948–79. Since a translog is a Taylor–series approximation of the unknown production function, the parameters of translogs are not "deep" and the elasticity of substitution is not constant. Translogs are therefore not very useful for general equilibrium models that require calibration, although they are often preferred when flexibility is valued in empirical work.

For $\sigma_i \rightarrow 0$, the CES production function (11) converges to the Leontief production function:[5]

$$F_i(K_{it}, L_{it}) = \min\{\exp(\gamma_{ik}t)K_{it}, \exp(\gamma_{il}t)L_{it}\}$$

One might worry that assuming constant technical progress is overly restrictive, in particular since recent papers like León-Ledesma, McAdam and Willman (2010) also considered different specifications. However, as we will see below, the estimated production functions do a good job at fitting the secular trends of sectoral capital, employment and relative prices. It is important to realize that to identify the parameters of the model one needs to put some structure on technical progress. To see why, consider the case where γ_i may change freely over time. We could then fit the data irrespective of the values of σ_i. Even in the extreme case of a Leontief production function that allows for no substitutability between capital and labor, we could rationalize years with low capital–to–labor ratios by choosing high γ_{ik}/γ_{il} and years with high capital–to–labor ratios by choosing low γ_{ik}/γ_{il}. As a result, σ_i would not be identified.

We assume that there is perfect competition in product and factor markets and that each sector has a stand–in firm that minimizes costs.[6] The firm takes as given value added, Y_{it}, the price of value added, P_{yit}, and the rental prices for the production factors, R_{it} and W_{it}, and chooses capital and labor to minimize its costs subject to the constraint that it produce at least the given value added. We denote the rental prices in term of sector i's value added by r_{it} and w_{it}:

$$r_{it} \equiv \frac{R_{it}}{P_{yit}}$$
$$w_{it} \equiv \frac{W_{it}}{P_{yit}}$$

The problem of the stand–in firm can then be written as:

$$\min_{K_{it}, L_{it}} r_{it}K_{it} + w_{it}L_{it} \quad \text{s.t.} \quad F_i(K_{it}, L_{it}) \geq Y_{it} \tag{13}$$

[5]Some authors raise the weights in the CES function to the power $1/\sigma_i$. The reason for this is that for $\sigma_i \rightarrow 0$ the limit of the CES is the generalized Leontief production function

$$F_i(K_{it}, L_{it}) = \min\{\alpha_i \exp(\gamma_{ik}t)K_{it}, (1-\alpha_i)\exp(\gamma_{il}t)L_{it}\}$$

This is relevant if one wants to use unequal weights in the limiting Leontief case; see for example Herrendorf, Rogerson and Valentinyi (2013). Since we are not interested in the Leontief case here, we continue with the functional form (11) for simplicity.

[6]We don't let the stand–in firm maximize profits here because with constant returns profit maximization results in an indeterminate scale of production. In contrast, cost minimization results in a determinate scale of production even with constant returns to scale. Otherwise profit maximization and cots minimization are equivalent.

The first–order conditions for an interior solution to this problem are:

$$r_{it} = \alpha_i \exp(\gamma_{ik} t)^{\frac{\sigma_i - 1}{\sigma_i}} \left(\frac{Y_{it}}{K_{it}}\right)^{\frac{1}{\sigma_i}} \tag{14}$$

$$w_{it} = (1 - \alpha_i) \exp(\gamma_{il} t)^{\frac{\sigma_i - 1}{\sigma_i}} \left(\frac{Y_{it}}{L_{it}}\right)^{\frac{1}{\sigma_i}} \tag{15}$$

For future reference, we note that these first–order conditions imply that the income shares of the production factors are given as:

$$\theta_{it} \equiv \frac{r_{it} K_{it}}{Y_{it}} = \alpha_i \left[\exp(\gamma_{ik} t) \frac{K_{it}}{Y_{it}}\right]^{\frac{\sigma_i - 1}{\sigma_i}} \tag{16}$$

$$1 - \theta_{it} \equiv \frac{w_{it} L_{it}}{Y_{it}} = (1 - \alpha_i) \left[\exp(\gamma_{il} t) \frac{L_{it}}{Y_{it}}\right]^{\frac{\sigma_i - 1}{\sigma_i}} \tag{17}$$

where we have used the fact that constant returns and perfect competition imply that the income shares add up to one. In the Cobb–Douglas case, $\sigma_i = 1$ and $\theta_{it} = \alpha_i$, implying the well known result that the capital share equals the exponent of capital.

For estimation purposes it is advantageous to normalize the CES production function (11) so that the relative weights on capital and labor equal the averages of the income shares. To achieve this, we divide and multiply each variable other than time by its geometric average:

$$F_i(K_{it}, L_{it}) = \overline{Y}_i \left[\alpha_i \left[\frac{\exp\left(\gamma_{ik} \bar{t}\right) \overline{K}_i}{\overline{Y}_i}\right]^{\frac{\sigma_i - 1}{\sigma_i}} \left(\frac{\exp(\gamma_{ik} t) K_{it}}{\exp(\gamma_{ik} \bar{t}) \overline{K}_i}\right)^{\frac{\sigma_i - 1}{\sigma_i}} + (1 - \alpha_i) \left[\frac{\exp\left(\gamma_{il} \bar{t}\right) \overline{L}_i}{\overline{Y}_i}\right]^{\frac{\sigma_i - 1}{\sigma_i}} \left(\frac{\exp(\gamma_{il} t) L_{it}}{\exp(\gamma_{il} \bar{t}) \overline{L}_i}\right)^{\frac{\sigma_i - 1}{\sigma_i}} \right]^{\frac{\sigma_i}{\sigma_i - 1}}$$

where $\overline{Y}_i$, $\overline{K}_i$ and $\overline{L}_i$ are the geometric averages of output, capital and labor over the sample period and $\bar{t}$ is the arithmetic average of the time index. Using (16)–(17), we have:

$$\alpha_i \left[\exp\left(\gamma_{ik} \bar{t}\right) \frac{\overline{K}_i}{\overline{Y}_i}\right]^{\frac{\sigma_i - 1}{\sigma_i}} = \bar{\theta}_i \tag{18}$$

$$(1-\alpha_i)\left[\exp\left(\gamma_{il}\bar{t}\right)\frac{\bar{L}_i}{\bar{Y}_i}\right]^{\frac{\sigma_i-1}{\sigma_i}} = \overline{1-\theta_i} \tag{19}$$

where $\bar{\theta}_i$ and $\overline{1-\theta_i}$ are the average income shares of the sample period. Since the income shares are observed, their geometric averages are readily calculated and we can substitute their values into the production function prior to estimating the other parameters:

$$F_i(K_{it}, L_{it}) = \bar{Y}_i\left[\bar{\theta}_i\left(\frac{\exp(\gamma_{ik}t)K_{it}}{\exp(\gamma_{ik}\bar{t})\bar{K}_i}\right)^{\frac{\sigma_i-1}{\sigma_i}} + (\overline{1-\theta_i})\left(\frac{\exp(\gamma_{il}t)L_{it}}{\exp(\gamma_{il}\bar{t})\bar{L}_i}\right)^{\frac{\sigma_i-1}{\sigma_i}}\right]^{\frac{\sigma_i}{\sigma_i-1}} \tag{20}$$

In other words, our normalization generalizes to CES production functions the property of Cobb–Douglas production functions that the exponents equal the income shares, given the maintained assumption of perfect competition and firm optimization.

León-Ledesma, McAdam and Willman (2010) normalized the CES production function in a similar way and demonstrated that the resulting two–step procedure reduces the numerical complexity of the estimation procedure. There is a subtle but important difference between their and our normalization: they used *arithmetic* averages of the variables and the income shares whereas we use *geometric* averages. This implies that in their paper the normalized CES is an approximation to the actual CES, which may not be accurate far away from the point of approximation. In contrast, in our paper (20) is an identity that holds everywhere.

The first–order conditions (14)–(15) can be rewritten in terms of normalized variables:

$$r_{it} = \frac{\bar{\theta}_i\bar{Y}_i}{\bar{K}_i}\exp\left(\frac{\sigma_i-1}{\sigma_i}\gamma_{ik}(t-\bar{t})\right)\left(\frac{Y_{it}/K_{it}}{\bar{Y}_i/\bar{K}_i}\right)^{\frac{1}{\sigma_i}} \tag{21}$$

$$w_{it} = \frac{(\overline{1-\theta_i})\bar{Y}_i}{\bar{L}_i}\exp\left(\frac{\sigma_i-1}{\sigma_i}\gamma_{il}(t-\bar{t})\right)\left(\frac{Y_{it}/L_{it}}{\bar{Y}_i/\bar{L}_i}\right)^{\frac{1}{\sigma_i}} \tag{22}$$

Our goal is to estimate the parameter values in (20)–(22). To this end, we multiply each equation with an error term, which we think of as productivity shocks or measurement error that may be correlated over time. Taking logs and rearranging gives:

$$\log\left(\frac{Y_{it}}{\bar{Y}_i}\right) = \frac{\sigma_i}{\sigma_i-1}\log\left[\bar{\theta}_i\left(\exp(\gamma_{ik}(t-\bar{t}))\frac{K_{it}}{\bar{K}_i}\right)^{\frac{\sigma_i-1}{\sigma_i}} + (\overline{1-\theta_i})\left(\exp(\gamma_{il}(t-\bar{t}))\frac{L_{it}}{\bar{L}_i}\right)^{\frac{\sigma_i-1}{\sigma_i}}\right] + \epsilon_{yit} \tag{23}$$

$$\log(r_{it}) = \log\left(\frac{\bar{\theta}_i \bar{Y}_i}{\bar{K}_i}\right) + \frac{\sigma_i - 1}{\sigma_i}\Big[\gamma_{ik}(t - \bar{t})\Big] + \frac{1}{\sigma_i}\log\left(\frac{Y_{it}/K_{it}}{\bar{Y}_i/\bar{K}_i}\right) + \epsilon_{rit} \tag{24}$$

$$\log(w_{it}) = \log\left(\frac{(\overline{1-\theta_i})\bar{Y}_i}{\bar{L}_i}\right) + \frac{\sigma_i - 1}{\sigma_i}\Big[\gamma_{il}(t - \bar{t})\Big] + \frac{1}{\sigma_i}\log\left(\frac{Y_{it}/L_{it}}{\bar{Y}_i/\bar{L}_i}\right) + \epsilon_{wit} \tag{25}$$

where $(\epsilon_{yit}, \epsilon_{rit}, \epsilon_{wit})$ denote the errors.[7]

We now turn to the details of estimating the system (23)–(25). As mentioned above, $\bar{\theta}_i$ and $\overline{1-\theta_i}$ equal the geometric averages of the income shares in sector i. The first step therefore is to calculate the geometric averages of the income shares from the data according to the method of Gollin (2002). Given the values of $\bar{\theta}_i$ and $\overline{1-\theta_i}$, the second step is to estimate σ_i, γ_{ik}, and γ_{il} for the three sectors. Note that for the special case of a Cobb–Douglas production function, this two–step procedure boils down to the usual way of proceeding: first calibrate the exponents by setting them equal to the average factor income shares; then estimate the Solow residuals. In order to tie our work to the literature, we also estimate σ, γ_k, and γ_l for the aggregate economy and compare the results with those in the literature. For the aggregate economy this results in a three–equation system and for the sectoral estimation in a nine–equation system with three equations for each of the three sectors. By estimating the equations for the three sectors together, we allow for the possibility that the error terms across equations and sectors are correlated.

We employ a non–linear version of the method of Cochrane and Orcutt (1949). More specifically, we use the non–linear, feasible, generalized three–stage least squares estimation routine offered by Eviews. The first stage obtains the instruments by running a linear least squares regression of the endogenous right–hand side variables on the one–period lags of all variables plus time trends. Fair (1970) showed that the Cochrane–Orcutt approach delivers consistent estimates in a system of simultaneous equations with autocorrelated errors if one includes all lagged variables as instruments. The second stage is a non–linear least squares regression with the instruments as the right–hand side variables. This stage takes into account the AR(1) structure of the error terms via the Cochrane–Orcutt procedure.[8] The third stage uses the estimated error terms from the previous stage to correct for heteroscedasticity and cross–equation correlation of the error terms using the non–linear, feasible, generalized least square estimator. Since the estimation in the second stage is non–linear, the results are obtained numerically and may depend on the starting values. We vary the starting values widely. If different starting values result in different parameter estimates (that is, the procedure converges to a local rather than a global maximum), then we choose the one with the smallest log determinant of the residual covariance matrix.

The choice of instruments warrants further discussion. As we have just described, the

[7]To avoid confusion, note that there will be a problem with estimating σ_i if it exactly equals one, because then the first equation converges to the log of the Cobb–Douglas limit of (20) and σ_i disappears from it. However, this is not an issue in the current context because our estimates of σ_i are significantly different from one.

[8]Using boldfaced symbols to denote vectors and matrices, the system (23)–(25) can be written as $\boldsymbol{y}_t = h(\boldsymbol{x}_t) + \boldsymbol{\epsilon}_t$. Eviews estimates the system as $\boldsymbol{y}_t = h(\boldsymbol{x}_t) + \boldsymbol{\rho}[\boldsymbol{y}_{t-1} - h(\boldsymbol{x}_{t-1})] + \boldsymbol{\nu}_t$.

non–linear Cochrane–Orcutt method uses one–period lagged values (appropriate to each sector or the aggregate economy) of the log rental rates of capital and labor, log normalized value added, log normalized capital, log normalized labor and the time trend. To be valid instruments, the lags need to satisfy two conditions: (i) they are correlated with the variable for which they instrument; (ii) they are uncorrelated with the unobserved determinants of the dependent variable. If the one–period lagged values are correlated with the contemporaneous values, then condition (i) is met. This is the case in the data. However, the one–period lagged values are also correlated with the contemporaneous error terms, because, as it will turn out, the error terms follow an AR(1) process:

$$\epsilon_{jit} = \rho_{ji}\epsilon_{jit-1} + \nu_{jit}$$

where $\rho \in (-1, 1)$ and ν is a current–period innovation. However, since the instruments are lagged values of the right–hand side variables, they are predetermined in the current period and are uncorrelated with ν_{jit} if ν_{jit} is i.i.d. with mean zero and finite variance, which we test for via Box–Ljung Test. Appendix D reports that the test does not reject the null hypothesis that the innovations to the errors are not serially correlated after estimating our equation with an AR(1) process. Condition (ii) is then also met. In other words, this specification gives no reason to reject the validity of the lagged values of the right–hand side variables as instruments.

A potential problem arises from the fact that system (23)–(25) features several non stationary variables (Y_{it}, K_{it}, L_{it}, and $\log(w_{it})$) and two trends governed by γ_{ik} and γ_{il}. Standard asymptotic theory does not in general apply anymore and it is not clear whether the parameter estimates are consistent. However, in our context the parameter estimates will be consistent if the error terms ν_{jit} are white noise and the estimated autocorrelated error processes are stationary, that is, $|\rho_{ji}| < 1$; see Chang, Park and Phillips (2001) for further discussion. We addressed the former in the previous paragraph. We test for the latter via the Augmented–Dickey–Fuller Test. As Appendix D reports, the test rejects the null hypothesis that the errors are not stationary.

III. Data

We use annual US data for the period 1947–2010. We start in 1947 because before hours worked by sector are not available. We use the North American Industrial Classification (NAICS) to the extent possible and define the three broad sectors in the obvious way: agriculture comprises farms, fishing, forestry; manufacturing comprises construction, manufacturing, and mining; services comprise all other industries (i.e. education, government, real estate, trade, transportation, etc.).[9]

We obtain nominal and real value added from the BEA's "GDP–by–Industry" tables. An issue arises in agriculture because NIPA reports "Rent paid to nonoperator landlords" as value added in the real estate industry although conceptually it is value added gener-

[9]Although industry might seem a better term for the sector comprising construction, manufacturing, and mining, we use the term manufacturing because industry typically refers to a generic production category.

ated in agriculture. We therefore add "Rent Paid to Nonoperator Landlords" (as reported by the BEA in NIPA Table 7.3.5 "Farm Sector Output, Gross Value Added, and Net Value Added") to the value added of agriculture and subtract it from the value added of services. Since the BEA does not publish the quantity of value added at the level of our broad sectors, we have to construct the sectoral quantities from the underlying BEA data ourselves. An additional complication arises when doing this because the reported real quantities are constructed according to the chain–weighted method, implying that they are not additive. We use the so–called cyclical expansion procedure to calculate real quantities of sectoral aggregates; see Appendix A for the details.

We calculate the capital stocks by sector from the BEA's "Fixed Asset" tables, which contain the year–end current costs and quantity indexes in 2005 prices of the net stock of fixed assets. The capital stocks during year t are the geometric averages of the year–end capital stocks in $t-1$ and t, again using the cyclical expansion procedure to aggregate real capital stocks to the sectoral level. Since the BEA does not include agricultural land in its fixed assets, we construct capital in agriculture by aggregating capital and land following the methodology of Jorgenson and Griliches (1967). The data for the quantity of agricultural land in acres are from "Land in Farms" and "Farm Real Estate Values" tables of the "U.S. and State Farm Income and Wealth Statistics" tables from the U.S. Department of Agriculture (USDA). To aggregate capital and land, we use the rental rates for their services. Note that this does not require them to be perfect substitutes, but requires that aggregate capital in agriculture K_a is separable from labor: $Y_a = F_a(K_a, L_a)$ where $K_a = f(K_{1a}, K_{2a})$, f is a production function with the standard regularity conditions (differentiability, constant returns etc), and K_{1a} and K_{2a} denote reproducible capital and land in agriculture. If K_{1a} and K_{2a} are paid their marginal products, which is implied by our maintained assumptions of perfect competition and cost minimization, then constant returns imply that $K_a = R_1 K_{1a} + R_2 K_{2a}$ where R_i are the corresponding rental rates. Appendix B contains a more detailed discussion of how we obtain aggregate capital in agriculture.

We calculate sectoral labor inputs as hours worked by persons engaged. The principal data sources are the BEA's "Income–and–Employment–by–Industry" Tables, which contain information about hours worked by full– and part–time employees by industry, full–time equivalent employees by industry, self–employed persons by industry, and persons engaged in production by industry. Unfortunately, the industry classification system changes in these tables: SIC72 applies to 1948–1987, SIC87 to 1987–1997, and NAICS to 1998–2010. Fortunately, the "GDP–by–Industry Tables" tables report full– and part–time employees by industry consistently according to NAICS throughout the whole period. We merge the two data sources using the GDP–by–Industry Tables for full– and part–time employees by industry and using the 'Income–and–Employment–by–Industry" Tables for all other statistics. Appendix C contains the details.

Lastly, we calculate the rental prices of the factors of production by sector according to

$$r_{it} = \frac{\theta_{it} Y_{it}}{K_{it}}$$

$$w_{it} = \frac{(1 - \theta_{it})Y_{it}}{L_{it}}$$

where, as before, θ_i denotes the share of capital income in sector i's value added. Given that we have already described the construction of Y, K and L, we only need to describe the calculation of the factor shares. We split value added reported in the BEA's "Components–of–Value–Added–by–Industry" Tables in the standard way: "Compensation of Employees" is labor income; "Gross Operating Surplus minus Proprietors' Income" is capital income; proprietors' income is split into capital and labor income using the above shares. In the case of agriculture, we add "Rent Paid to Nonoperator Landlords" to "Gross Operating Surplus minus Proprietors' Income" since it is capital income. An issue arises because again the industry classification in these tables changes twice. We calculate the sectoral capital shares for each subperiod during which the classification remains unchanged and assume that the same share applies to the corresponding NAICS classifications as well. Since our three sectors are fairly broad, this is unlikely to affect our results in a quantitatively important way.

IV. Estimation Results

Table 1— Estimation Results

	Aggregate	Agriculture	Manufacturing	Services
σ	0.84***	1.58***	0.80***	0.75***
	(0.041)	(0.068)	(0.015)	(0.020)
γ_k	−0.010	0.023***	−0.045***	−0.002
	(0.006)	(0.003)	(0.009)	(0.004)
γ_l	0.022***	0.050***	0.044***	0.016***
	(0.003)	(0.004)	(0.007)	(0.002)
$\bar{\theta}$	0.33	0.61	0.29	0.34

Standard errors in parentheses; *** Significant at the 1 percent level.

We are now ready to report the estimation results, which are summarized in Table 1. We find that capital and labor are most substitutable in agriculture and least substitutable in services. In agriculture capital and labor are more substitutable than in the Cobb–Douglas case, which is consistent with the observation that a mechanization wave led to massive substitution of capital for labor in agriculture after World War II. In manufacturing and services capital and labor are less substitutable than Cobb–Douglas. Our estimate of 0.80 for the elasticity of substitution in manufacturing is close to that Oberfield and Raval (2014) obtain from micro data for manufacturing plants. Aggregating the actions of individual plants, they find elasticities of 0.67 in 1970 and 0.75 in 2007 for the manufacturing sector. On the aggregate, we find that capital and labor are less substitutable than Cobb–Douglas, which is consistent with the previous results of Antràs (2004), Klump, McAdam and Willman (2007) and León-Ledesma, McAdam and Willman (2010).[10]

Labor–augmenting technical progress is fastest in agriculture and slowest in services and the differences in technical progress are sizeable: while the growth rates were 5.0% per year in agriculture and 4.4% in manufacturing, they were only 1.6% in services; these growth rates result in an average of 2.2% annual growth of aggregate labor–augmenting technical progress. The fact that technical progress is slowest in services while the share of value added produced in services is growing is sometimes referred to as Baumol "disease". Baumol (1967) was the first to point out that these two facts imply decreasing growth rates of real GDP. Moreover, if the current trends of structural transformation continue, then services will dominate the economy in the limit and aggregate labor–augmenting technical progress will fall to the low technical progress in services.

We find mixed results regarding capital–augmenting technical progress. At the aggregate it is negative but not significant.[11] At the sectoral level, capital–augmenting technical progress is significantly different from zero in agriculture and manufacturing and not significantly different from zero in services. Moreover, in agriculture capital–augmenting technical progress is positive and in manufacturing it is negative and the negative growth rate in manufacturing is relatively large. At first sight, negative technical progress in manufacturing is challenging to interpret. However, if one thinks of the decline of sizeable parts of US manufacturing during the postwar period, then negative technical progress in manufacturing may just reflect that the BEA underestimated the depreciation of manufacturing capital. Since this issue is not central to our study, we leave further investigation of it for future research.

The last row of Table 1 reports $\bar{\theta}$, that is, the average capital share in the post war period. We can see that the aggregate capital share comes out as the standard value of $1/3$. The sectoral capital shares differ from the aggregate capital share: while the agricultural capital share is considerably larger than the aggregate capital share, the capital shares in manufacturing and services are fairly close to the aggregate capital share. The capital share in agriculture is much larger than the other two capital shares because agriculture is intensive in both physical capital and land, which have income shares in agricultural value added equal to 0.54 and 0.07, respectively. The capital share in services is larger than in manufacturing because owner–occupied housing is part of services and is very capital intensive.

[10] Appendix D contains further information. In particular, it shows that the fit is good. It also reports an Augmented–Dickey–Fuller Test, which tests for the stationarity of the autocorrelated error processes, and the multivariate Ljung–Box Adjusted Q–statistics, which test for autocorrelation in the residuals. The null hypothesis of no higher–order autocorrelation is not rejected. To conserve space we only report the test statistics for the second lag, but the results for higher order autocorrelation are similar.

[11] Antràs (2004) studies the aggregate US production function during the period 1948–1998 and also finds that capital–augmenting technical progress was negative.

V. Sectoral Technology and Structural Transformation

A. CES versus Cobb–Douglas production functions

In this section, we evaluate the implications of the different features of sectoral production functions for structural transformation. To this end, we compare the unrestricted CES production functions that we have estimated above with two Cobb–Douglas production functions which we estimate now. They result when we impose that $\sigma_i = 1$ and that the exponents on capital equal the arithmetic average of the sector–specific capital shares, $\alpha_i = \tilde{\theta}_i$, or the arithmetic averages of the aggregate capital shares, $\alpha_i = \tilde{\theta}$.[12] It is convenient to rewrite (20) in the three cases as follows:

$$F_i(K_{it}, L_{it}) = \left[\bar{\theta}_i \left(A_{ikt}K_{it}\right)^{\frac{\sigma_i-1}{\sigma_i}} + \left(\overline{1-\theta_i}\right)\left(A_{ilt}L_{it}\right)^{\frac{\sigma_i-1}{\sigma_i}}\right]^{\frac{\sigma_i}{\sigma_i-1}} \tag{26}$$

$$F_i(K_{it}, L_{it}) = \left(A_{ikt}K_{it}\right)^{\alpha_i}\left(A_{ilt}L_{it}\right)^{1-\alpha_i} \tag{27}$$

$$F_i(K_{it}, L_{it}) = \left(A_{ikt}K_{it}\right)^{\alpha}\left(A_{ilt}L_{it}\right)^{1-\alpha} \tag{28}$$

where A_{ikt} and A_{ilt} are defined as:

$$A_{ikt} \equiv \exp(\gamma_{ik}[t-\bar{t}])\frac{\overline{Y}_i}{\overline{K}_i} \text{ and } A_{ilt} \equiv \exp(\gamma_{il}[t-\bar{t}])\frac{\overline{Y}_i}{\overline{L}_i} \text{ if } F_i \text{ is CES} \tag{29}$$

$$A_{ikt} = \exp(\gamma_i[t-\bar{t}])\frac{\overline{Y}_i}{\overline{K}_i} \text{ and } A_{ilt} = \exp(\gamma_i[t-\bar{t}])\frac{\overline{Y}_i}{\overline{L}_i} \text{ if } F_i \text{ is Cobb Douglas} \tag{30}$$

The reason for the difference between the two rows is that in the Cobb–Douglas case it is not possible to identify γ_{ik} and γ_{il} separately so that we are left with just the growth factors of TFP γ_i. In contrast, for the CES production function, γ_i cannot be obtained because the rates of capital– and labor–augmenting technical progress cannot be translated into an observationally–equivalent rate of TFP growth.

We calculate the values of A_{ijt} according to the expressions in (29) and (30). We obtain the geometric averages $\overline{Y}_i$, $\overline{K}_i$, and $\overline{L}_i$ directly from the data. While we use the values from Table 1 for γ_{ik} and γ_{il} in the CES case, we estimate γ_i from the output equations (23) for the special case of the Cobb–Douglas production function jointly for the three sectors given the values of the exponents and again assuming AR(1) error terms. Table 2 reports the resulting average annual growth rates of TFP. They are somewhat larger than what other studies like Jorgenson, Gollop and Fraumeni (1987) tend to find. The most likely reason for this is that we have not taken into account improvements in the quality of sectoral labor (e.g., through increases in years of schooling), which therefore show up as technical progress in our estimation.

[12]In case of the Cobb–Douglas function we use arithmetic instead of geometric averages because we want the capital and labor shares to add up to one so that the Cobb–Douglas production function has constant returns to scale. Geometric averages of the capital and labor shares do not add up to one in general.

To obtain the exponents of the Cobb–Douglas production function, we use that under our maintained assumptions of perfect competition in factor and product markets and cost minimization the exponent on capital equals the capital share. This is the case in each period and for the arithmetic average. We calibrate the exponents on capital by setting them equal to the observed arithmetic average of the sectoral capital share or the observed arithmetic average of the aggregate capital shares.

Table 2— Average Annual Growth Rates of TFP (in percent)

	Aggregate	Agriculture	Manufacturing	Services
CD with α_i	1.1	3.3	1.5	1.0
CD with α	1.1	3.9	1.4	1.0

B. Sectoral labor allocations

We now turn to the sectoral labor allocations that result from the optimal choices of stand–in firms which are endowed with the production functions (26)–(28). Solving the first-order conditions to the firm problem for sectoral labor, we obtain for the different functional forms:

$$L_{it} = \left(\bar{\theta}_i \left(\frac{\bar{\theta}_i}{\overline{1-\theta_i}} \frac{A_{ikt} w_{it}}{A_{ilt} r_{it}}\right)^{1-\sigma_i} + (\overline{1-\theta_i})\right)^{-\frac{\sigma_i}{1-\sigma_i}} \frac{Y_{it}}{A_{ilt}} \tag{31}$$

$$L_{it} = \left(\frac{\alpha_i}{1-\alpha_i} \frac{A_{ikt} w_{it}}{A_{ilt} r_{it}}\right)^{-\alpha_i} \frac{Y_{it}}{A_{ilt}} \tag{32}$$

$$L_{it} = \left(\frac{\alpha}{1-\alpha} \frac{A_{ikt} w_{it}}{A_{ilt} r_{it}}\right)^{-\alpha} \frac{Y_{it}}{A_{ilt}} \tag{33}$$

Note that to derive these expressions we did not impose that the marginal product of labor be equalized across sectors. While that is a common assumption in multi–sector models, it may not hold in the data and, in any case, it is not required for studying the labor allocations that are implied by the different production functions. In section VI.A we will discuss further how our results relate to multi–sector models that assume that marginal products are equalized across sectors.

It is worth taking a moment to build intuition for how the different features of technology affect the allocation of labor across the three broad sectors. The term Y_{it}/A_{ilt} is common to the right–hand sides because more labor–augmenting technical progress implies that less labor is needed to produce the given quantity Y_{it} of sectoral value added. The other right–hand–side terms differ among the different functional forms. It is easiest to start with the Cobb–Douglas cases. The term $[\alpha_i/(1-\alpha_i)]^{-\alpha_i}$ has a local maximum at $\alpha_i = 0.22$, and so it is decreasing for $\alpha_i \in (0.22, 1)$ which includes the standard value $1/3$. This captures that for empirically relevant values of the capital share, a sector with a larger capital share receives less labor than a sector with a smaller capital share. The term $[(A_{ikt}w_{it})/(A_{ilt}r_{it})]^{-\alpha_i}$ captures that an increase in the relative rental price of capital to labor (where both rental rates are expressed relative to the relevant A) leads to a decrease in the sectoral capital–labor ratio and an increase in sectoral labor. These effects are larger when the sectoral capital share is larger.

To see how these features of technology may matter for structural transformation, recall the intuition developed earlier in the introduction. Acemoglu and Guerrieri (2008) showed that sectoral differences in the capital share could drive structural transformation. To see how, consider two sectors that have production functions which differ only in their capital share. When technical progress occurs at equal rates in both sectors, GDP per capita increases, "capital deepening" occurs, and the relative price of value added output in the sector with the higher capital share decreases. This prompts a reallocation of resources toward the sector with a falling relative price of output under standard preferences.

Turning now to the case of the CES production functions, we have an additional substitution effect: if the elasticity of substitution is larger than one, a higher rental rate of capital relative to labor leads to a larger reduction of the capital–labor ratio than in the Cobb–Douglas case; if the elasticity of substitution is smaller than one, a higher rental rate of capital relative to labor leads to a smaller reduction of the capital-labor ratio than in the Cobb-Douglas case. If we consider two sectors that differ only with respect to their capital–labor substitutability, then we observe the force described by Alvarez-Cuadrado, Long and Poschke (2013), notably, as technical progress occurs at equal rates in both sectors and GDP per capita increases, the relative price of value added in the sector with low capital–labor substitutability falls. Given standard preferences, this again leads to the reallocation of resources towards this sector.

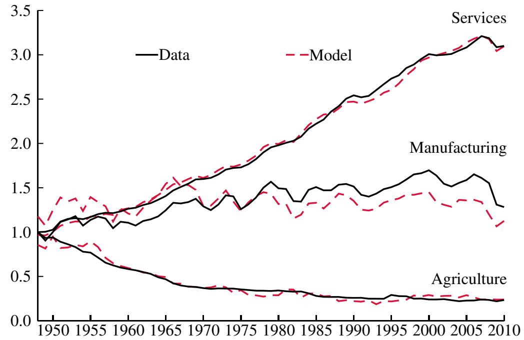
Figure 2 plots the labor allocations that are implied by equations (31)–(33) when we plug in the estimated parameter values for σ_i, $\bar{\theta}_i$, A_{ikt}, and A_{ilt} and the data values of the exogenous variables of Y_{it}, r_{it}, and w_{it}. Note that we have divided the hours series from the data and from the model by the hours worked in the data in 1948. This implies that in each sector hours worked in the data in 1948 are equal to one, but hours implied by the model in 1948 are equal to one only if the model gets the level in 1948 right.

All three functional forms do a reasonable job at capturing the secular changes in sectoral hours worked. The main differences between them are that the CES form does marginally better at mimicking the short–run fluctuations in the service sector whereas the Cobb–Douglas production function with unequal shares does somewhat better in mimicking the labor allocations in agriculture and manufacturing.[13] The Cobb–Douglas production function with equal shares performs similarly to the one with unequal shares

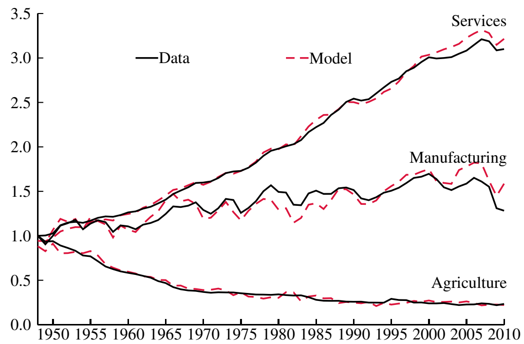
[13]To avoid confusion, we should emphasize that there is nothing strange about the finding that a Cobb–Douglas production function outperforms CES production function regarding sectoral employment. The reason for this is that when we estimated the production function, we did not target the labor allocations.

Figure 2. Hours Worked (Data=1 in 1948)

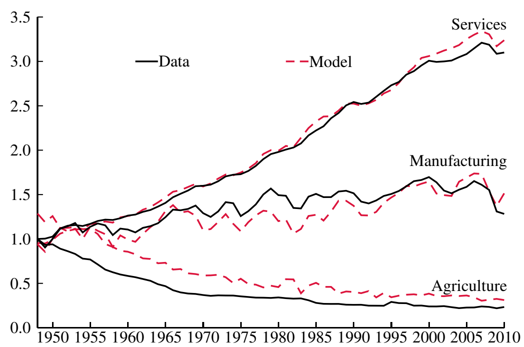
CES



Cobb Douglas with Different Capital Shares



Cobb Douglas with Same Capital Shares



in the service and manufacturing sector. Moreover, it does a reasonable job at capturing the secular change in agriculture, but overpredicts the level of employment in agriculture. The root–mean–square percentage deviations in Table 6 in Appendix D confirm these observations.

The reason for the differences in the performance of the two Cobb–Douglas production functions in agriculture is that the one with equal shares misses that agriculture has a much smaller labor share than the aggregate, and so it systematically allocates too much labor to agriculture. Nonetheless, even the Cobb–Douglas production function with equal shares captures the main changes in hours worked.[14] The reason why the CES production function does not outperform the other production functions in agriculture is that it has both by far the largest capital share and the largest elasticity of substitution. Hence, the effects on structural transformation of the large capital share and the large elasticities in agriculture work in opposite directions and largely cancel each other, leaving the effects of uneven labor–augmenting technical progress as the dominating force. All three production functions capture that force.

C. *Relative prices*

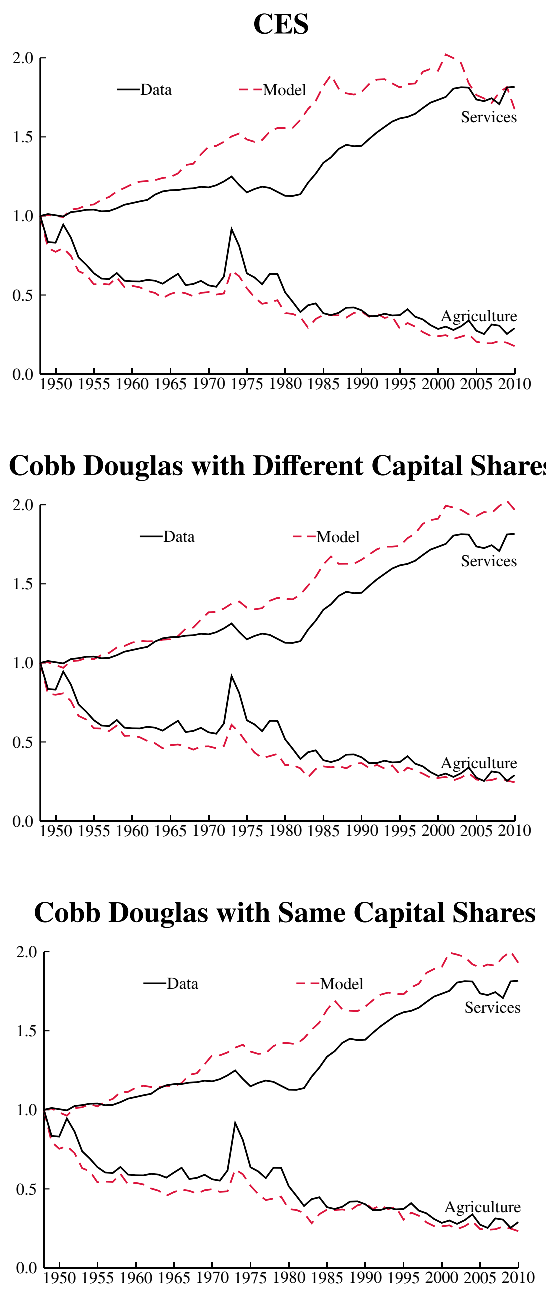
We continue by assessing how well we can match relative prices of sectoral value added with the three production functions. Relative prices are of interest in the context of structural transformation because they influence not just the sectoral composition of value added, but also that of employment. To be concrete, if preferences were homothetic, then the observed decrease in the relative price of agriculture to manufacturing during the postwar period would have led to an increase in the share of agricultural value added in total consumption. The fact that share decreased in the data suggests that preferences are not homothetic and that the income elasticity of agricultural consumption is smaller than one. Herrendorf, Rogerson and Valentinyi (2013) assessed in detail the preferences aspect of structural transformation, in particular the importance of the effects of changes in relative prices and in income on the composition of household consumption. The goal of this subsection is to assess how well each of the three functional forms does in terms of the implied prices of agriculture and services relative to manufacturing compared to those in the data. We proceed under the maintained assumption that the sectoral stand–in firm behaves competitively and minimizes its costs subject to a production constraint.

The first–order conditions to the firm problem (13) imply that the real wage w_{it} expressed in units of sector i's value added equals the marginal product of labor. Hence, the price of sector i's value added relative to manufacturing is given by:

$$\frac{P_{it}}{P_{mt}} = \frac{W_{it}}{W_{mt}}\frac{MPL_{mt}}{MPL_{it}} \tag{34}$$

[14]In a different context, Herrendorf and Valentinyi (2012) obtained a similar finding: conducting a development accounting exercise at the sectoral level, they found that Cobb–Douglas production functions with equal capital shares imply similar gaps of sectoral TFP compared to the US as Cobb–Douglas production functions with sector–specific capital shares.

Figure 3. Sectoral Prices Relative to Manufacturing (Data and Model =1 in 1948)



We observe the nominal wages W_{it} and W_{mt} in the data. The model implies the values of the marginal products MPL_{it} and MPL_{mt} as functions of the observed factor prices. Given these, it is straightforward to calculate the implied relative prices from equation (34).

Figure 3 reports the implied relative prices for the three functional forms. In plotting the figure, we have chosen 1948 as the base year, and so by construction the relative prices equal one in the data as well as in the model. We can see that a similar conclusion as for the labor allocation emerges: all three functional forms do reasonably well with respect to changes in the relative prices. In particular, there is little difference among the implied prices of agricultural value added relative to manufacturing value added. The root–mean–square percentage deviations reported in Table 6 of Appendix D confirm this impression. Interestingly, they are largest for the CES and smallest for the Cobb–Douglas production function with equal capital shares. A similar picture emerges for the implied relative prices of service value added relative to manufacturing value added, except that now all functional forms overpredict the relative price of services after 1970. The root–mean–square percentage deviations reported in Table 6 of Appendix D show that again the CES does slightly worse than the two Cobb–Douglas production functions which are now very close. Moreover, the Cobb Douglas with unequal capital shares slightly outperforms the one with equal capital shares.[15]

These findings confirm our conclusion that a Cobb Douglas production function with equal capital shares captures the main technological forces behind structural transformation in the postwar US economy.

VI. Implications for Building Multi–sector Models

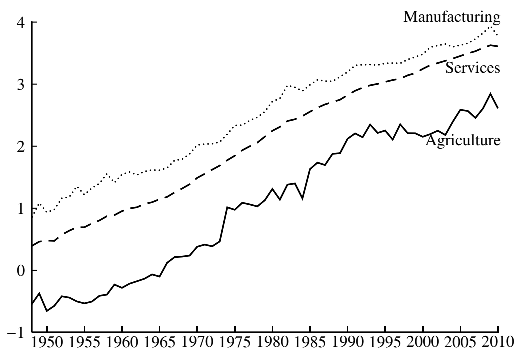
A. *Equalizing marginal value products*

Many builders of multi–sector models assume that the marginal value products of each primary factor of production are equalized across sectors. A set of assumptions that implies this in multi–sector models is: (i) competitive firms rent each factor of production in a common factor market at a common nominal rental rate; and (ii) each factor of production can be moved across sectors without frictions or costs. Unfortunately, it turns out that in the US the nominal rental rates are not equalized across sectors. Figure 4 shows that the marginal value product of labor is somewhat higher in manufacturing than in services, and is much lower in agriculture than in the other two sectors. Given this evidence, our estimation strategy of system (23)–(25) has been to use the *observed* nominal rental rates and prices of sectoral value added instead of imposing that nominal rental rates are equalized across sectors.

The previous paragraph raises the question, in which way our estimated sectoral production functions may be used for building multi–sector models that equalize marginal value products across sectors. In order to incorporate our estimated production func-

[15]Note that there is nothing strange about the finding that a Cobb Douglas production function outperforms the CES production function also regarding relative prices. The reason again is that we did not target the relative prices either.

Figure 4. Sectoral Marginal Value Products of Labor (in logs)



tions in such models, one needs to add a reason for the difference in the marginal value products across sectors. In the case of labor, the most obvious reason is that there are difference in sectoral human capital as in Jorgenson, Gollop and Fraumeni (1987) or Herrendorf and Schoellman (2014). The latter paper, for example, found that average sectoral human capital is lower in agriculture than in the rest of the US economy, and that the difference accounts for almost all of the difference in nominal wages. This implies that per efficiency unit of labor the average nominal wages were roughly equal in agriculture and the rest of the US economy during the last thirty years. In the case of capital, the reasons for the difference in the marginal value products across sectors include unmeasured quality differences in the measured stock of sectoral capital and unmeasured parts of the stock of capital; see Jorgenson, Gollop and Fraumeni (1987) and McGrattan and Prescott (2005) for further discussion.

B. Value–added versus final–expenditure production functions

So far, we have focused on value–added production functions. While this is a natural starting point when one studies the forces behind structural transformation on the technology side, Herrendorf, Rogerson and Valentinyi (2013) pointed out that one can also interpret the sectoral outputs as final goods that are consumed or invested. In this subsection we discuss the implications of our results for models of structural transformation that interpret sectoral outputs as final goods, instead of as value added.

Before we delve into the details, an example may be helpful. Consider a household that derives utility from the three consumption categories: agriculture, manufacturing, and services. Herrendorf, Rogerson and Valentinyi (2013) pointed out that one can take two different perspectives on what these categories are: the value–added perspective and the final–goods perspective. The value–added perspective breaks the household's consumption into the value–added components from the three sectors and assigns each value–added component to a sector. For example, if the household consumes a cotton shirt, then the value added of producing raw cotton goes to agriculture, the value added of processing to manufacturing, and the value added of distribution to services. This means that the consumption categories in the utility function of the household are the value added that is produced in the three sectors agriculture, manufacturing, and services. In contrast, the final–goods perspective assigns each consumption good to one of the three consumption categories. The cotton shirt, for example, would typically be assigned to manufacturing. This means that the consumption categories in the utility function of the household become final–goods categories. This changes the meaning of the three sectors, as the manufacturing sector now produces the entire cotton shirt, implying that it combines the value added from the different industries that is required to produce the cotton shirt.

Although the sectoral production functions under the two perspectives are very different objects, we emphasize that they are two representations of the *same* underlying data, which are linked through intricate input–output relationships. To see the implications of this, it is useful to think of the sectoral output under the final–goods perspective as a weighted average of the sectoral value added from the value–added perspective. This implies that the production functions under the final–goods perspective are a weighted average of the production functions under the value–added perspective. Valentinyi and Herrendorf (2008) showed that as a result the capital shares of industry gross output tend to be closer to the aggregate capital share than the capital shares of industry value added. This should imply that the sectoral capital shares under the final–goods perspective are closer to the aggregate capital share than the sectoral capital shares under the value–added perspective. Following the same logic, we conjecture that the differences among the elasticities of substitution of the different sectoral production functions are smaller under the final–goods perspective than under the value–added perspective.

These arguments suggest that under the final–goods perspective the sectoral production functions are at least as close to the Cobb–Douglas production function with a common capital share as they are under the value–added perspective. Since we have shown above that the Cobb–Douglas production functions with a common capital share do a reasonable job at capturing sectoral employment and relative prices under the value–added perspective, this suggests that they will also do a reasonable job under the final–goods perspective. Note that since the aggregate capital share is the same under both perspectives, it is straightforward to parameterize the Cobb–Douglas production functions with a common capital share under the final–goods perspective.

VII. Conclusion

In this paper, we have assessed the technological forces behind the reallocation of production factors across agriculture, manufacturing, and services. In particular, we have asked how important for structural transformation are sectoral differences in capital– and labor–augmenting technical progress, the capital share, and elasticity of substitution between capital and labor. We have estimated CES and Cobb–Douglas production functions for agriculture, manufacturing, and services on postwar US data and have compared their implications for labor allocations and relative prices. We have found that differences in technical progress are the predominant force behind structural transformation and that sectoral Cobb–Douglas production functions with equal capital shares (which by construction abstract from differences in the elasticity of substitution and in capital shares) do a reasonably good job of capturing the main trends of US structural transformation.

We have restricted our attention to the postwar US economy. It is also of interest to extend this analysis to a larger set of countries, in particular to situations which feature a larger range of real incomes and a higher share of agricultural employment and value added. This will be useful in assessing the extent to which one can account for the process of structural transformation with stable sectoral technologies.

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Appendix

Appendix A: Aggregation of Chained Quantity Indices according to the Cyclical Expansion Method

Chain indices relate the value of an index number to its value in the previous period. In contrast, fixed–base indices relate the value of an index number to its value in a fixed base period. While chain indices are preferable to fixed–base indices when relative prices change considerably over time, using them leads to the problem that real quantities are not additive, that is, the real quantity of an aggregate does not equal the sum of the real quantities of its components except in the base year or if relative prices don't change. In practice, this becomes relevant when one needs to calculate the real quantity of an aggregate, but the statistical agency only reports the real quantities of the components of this aggregate. This appendix explains how to construct the real quantity of the aggregate according to the so called cyclical expansion procedure.[16]

Let Y_{it} be the nominal value, y_{it} the real value, Q_{it} the chain–weighted quantity index, and P_{it} the chain–weighted price index for variable $i \in \{1, \ldots, n\}$ in period t. Let $t = b$ be the base year for which we normalize $Q_{ib} = P_{ib} = 1$. The nominal and real values of variable i in period t are then given by:

$$Y_{it} = P_{it}\frac{Q_{it}}{Q_{ib}}Y_{ib} = P_{it}Q_{it}Y_{ib},$$
$$y_{it} = \frac{Y_{it}}{P_{it}} = Q_{it}Y_{ib}.$$

Let $Y_t = \sum_{i=1}^{n} Y_{it}$ and suppose that the statistical agency reports y_{it}, Q_{it} and P_{it} for all components i but not y_t, Q_t and P_t. Since in general $y_t \neq \sum_i y_{it}$, we need to find a way of calculating y_t.

We start by constructing Q_t using the "chain–summation" method:[17]

$$\frac{Q_t}{Q_{t-1}} = \sqrt{\frac{\sum_i P_{it-1}y_{it}}{\sum_i P_{it-1}y_{it-1}}\frac{\sum_i P_{it}y_{it}}{\sum_i P_{it}y_{it-1}}}.$$

Using this expression iteratively, we obtain Q_t as:

$$Q_t = \frac{Q_t}{Q_{t-1}}\frac{Q_{t-1}}{Q_{t-2}}\ldots\frac{Q_{b+1}}{Q_b}Q_b = \frac{Q_t}{Q_{t-1}}\frac{Q_{t-1}}{Q_{t-2}}\ldots\frac{Q_{b+1}}{Q_b},$$

where the last step used the normalization $Q_b = 1$. The real value and the price in period

[16]For a more detailed discussion of the practical issues arising from the non–additivity of chain indexes, see the excellent discussion in Whelan (2002).

[17]Conceptually this formula is exact. In practice, it is an approximation because the statistical agency typically uses more disaggregate categories when calculating sums like $\sum_i P_{it-1}y_{it}$ than are available to us.

t then follow as:

$$y_t = Q_t Y_b,$$
$$P_t = \frac{Y_t}{Q_t Y_b}.$$

Appendix B: Aggregating Reproducible Capital and Land

To aggregate reproducible capital and fixed capital (i.e., land) to total capital in agriculture, we use the rental rates for the services that they provide. The rental rates of services from reproducible and fixed capital equal the real rates of return. To calculate them, we calculate the income shares of agricultural value added that are paid to reproducible capital and fixed capital, θ_{1t} and θ_{2t}. We only observe $\theta_{1t} + \theta_{2t} = 1 - \theta_{Lt}$. To calculate θ_{1t} and θ_{2t}, we will impose that a risk–neutral investor be indifferent between holding the two assets. Assuming that both assets face the same tax treatments, we obtain:

$$(1 - \delta_{1t} + r_{1t})\frac{P_{1t}}{P_{1t-1}} = (1 - \delta_{2t} + r_{2t})\frac{P_{2t}}{P_{2t-1}}$$

where r_{1t} and r_{2t} denote the rates of return on and P_{1t} and P_{2t} are the price levels of reproducible and fixed capital. Using the factor incomes and the asset stocks, this condition can be rewritten as:[18]

$$\left(1 - \delta_{1t} + \theta_{1t}\frac{P_{Yt}Y_t}{P_{1t}K_{1t}}\right)\frac{P_{1t}}{P_{1t-1}} = \left(1 - \delta_{2t} + \theta_{2t}\frac{P_{Yt}Y_t}{P_{2t}K_{2t}}\right)\frac{P_{2t}}{P_{2t-1}}$$

where K_{1t} denotes the stock of reproducible capital in real terms, K_{2t} denotes the stock of fixed capital measured in acres, Y_t denotes agricultural value added in real terms, and P_{1t}, P_{2t} and P_{Yt} denote the price levels of reproducible capital, fixed capital, and agricultural value added. Using the fact that $\theta_{2t} = 1 - \theta_{Lt} - \theta_{1t}$, we can rewrite the above equation as

$$\theta_{1t}\left(\frac{P_{Yt}Y_t}{P_{1t}K_{1t}}\frac{P_{1t}}{P_{1t-1}}\frac{P_{2t-1}}{P_{2t}} + \frac{P_{Yt}Y_t}{P_{2t}K_{2t}}\right) = \left(1 - \delta_{2t} + \frac{(1-\theta_{Lt})P_{Yt}Y_t}{P_{2t}K_{2t}}\right) - (1 - \delta_{1t})\frac{P_{1t}}{P_{1t-1}}\frac{P_{2t-1}}{P_{2t}}$$

implying

$$\theta_{1t} = \frac{1 - \delta_{2t} + \frac{(1-\theta_{Lt})P_{Yt}Y_t}{P_{2t}K_{2t}} - (1 - \delta_{1t})\frac{P_{1t}}{P_{2t}}\frac{P_{2t-1}}{P_{1t-1}}}{\frac{P_{Yt}Y_t}{P_{2t}K_{2t}} + \frac{P_{1t}}{P_{2t}}\frac{P_{2t-1}}{P_{1t-1}}\frac{P_{Yt}Y_t}{P_{1t}K_{1t}}}$$

We calculate $P_{1t}K_{1t}$ as the current–cost fixed assets in agriculture from the BEA Standard Fixed Assets tables, $P_{2t}K_{2t}$ as the value of land from USDA calculated as the price of

[18] We drop the index for agriculture to economize on notation, keeping in mind that everything in Appendix C refers to agriculture.

land per acre multiplied by the quantity of land measured in acres, and θ_{Lt} as labor share. We assume that $\delta_{1t} = 0.06$ and $\delta_{2t} = 0$, which are standard values.

Since in the real world the previous indifference condition holds only in expectations, we cannot assume that it holds ex post in all periods. To deal with this problem, we replace the price of capital relative to land P_{1t}/P_{2t} with its HP filtered version where the prices both of capital and land are normalized to 1 in 2005. We use a relatively high value of 1800 for the smoothing parameter so as to ensure that the nominal land rent per acre implied by our model is close to the one observed in the data.

Appendix B: Construction of Hours by Persons Engaged

In this appendix we describe how we combine the "Income–and–Employment–by–Industry" tables with the "GDP–by–Industry Tables" in order to obtain hours by persons engaged by sector. Recall that the 'Income–and–Employment–by–Industry" contain information about full–time equivalent employees, self–employed persons, and persons engaged in production but change classification from SIC to NAICS; the "GDP–by–Industry Tables" tables contain only full– and part–time employees by industry, but use NAICS throughout the whole period. We combine them as follows:

$$\text{full–time–equiv empl} = \frac{\text{full–time equiv empl}_{SIC}}{\text{part \& full–time empl}_{SIC}} \text{ part \& full–time empl}_{NAICS}$$

$$\text{hours full–time equiv empl} = \frac{\text{hours full–time equiv empl}_{SIC}}{\text{full–time equiv empl}_{SIC}} \text{ full–time–equiv empl}$$

$$\text{self–empl} = \frac{\text{self–empl}_{SIC}}{\text{part \& full–time empl}_{SIC}} \text{ part \& full–time empl}_{NAICS}$$

$$\text{hours persons engaged} = \text{hours full–time equiv empl} + \frac{\text{hours full–time equiv empl}}{\text{full–time equiv empl}} \text{ self–empl}$$

Appendix D: Estimation Results

Table 3— Standard Errors of Regression Equations (23)–(25)

	Agr	Man	Ser
Specification		(23)	
CES	0.078	0.026	0.010
CD (unequal)	0.078	0.025	0.010
CD (equal)	0.077	0.026	0.010
		(24)	
CES	0.038	0.050	0.018
		(25)	
CES	0.052	0.025	0.001

The standard error of an equation is the unbiased estimator of the root–mean–square error of the equation.

Table 4— Augmented Dickey Fuller Test Statistics (23)–(25)

	Agr	Man	Ser
Specification		(23)	
CES	−2.714	−1.793	−2.287
CD (unequal)	−2.741	−1.719	−2.365
CD (equal)	−4.822	−1.717	−2.341
		(24)	
CES	−3.730	−2.375	−2.013
		(25)	
CES	−2.660	−1.996	−1.911

[a] H_0: The errors are not stationary
[b] Critical values are −2.603 (p = 0.01), −1.946 (p = 0.05), and −1.613 (p = 0.10)

Table 5— Multivariate Ljung–Box Q–Statistics

Specification	# of Lags	Degrees of freedom	Adj. Q–stat	p–value
CES	2	162	186.662	0.090
CD (unequal)	2	18	25.058	0.123
CD (equal)	2	18	20.079	0.328

[a] H_0: The errors are not autocorrelated
[b] Note that if one does not reject H_0 for lag 2, one will not reject it for higher–order lags either.

Table 6— Root–Mean–Squared Percentage Deviations

	Labor Allocation			Relative Prices	
Specification	Ag	Man	Ser	Ag	Ser
CES	0.123	0.140	0.025	0.180	0.196
CD (unequal)	0.091	0.074	0.028	0.174	0.124
CD (equal)	0.500	0.098	0.029	0.163	0.128