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Remote angular displacement sensor based on Faraday effect: Experiment and modeling

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Recently, we have reported a magneto-optic angular displacement (MOAD) sensor where both the incident and reflected laser beams pass through a magneto-optic (MO) film. In this letter, we report a modified MOAD sensor where only a reflected laser beam passes through the MO film. With the modified configuration, the modified MOAD sensor is a truly remote sensor such that the MO film can be located close to the detector and far from the sample. Furthermore, the modified sensor system can measure angular displacements with an improved resolution of 1×10^{-3} deg, which is ten times better than that previously reported. © *2006 American Institute of Physics*.
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Research and development for an accurate monitoring of angular and spatial displacements have been increased due to the increase of sophisticated automatic operation.[1–3] For example, the monitoring of the satellite operation with high resolution is critically important in order to diagnose and correct the motion of the satellite.[3] Recently, we have developed a magneto-optic angular displacement (MOAD) sensor to measure small angular displacements and spatial displacements of the surfaces of a material.[4] This MOAD sensor, consisting of a laser, a magneto-optic (MO) film, two permanent magnets, an ac coil, and a photodetector, bases its function on the periodic motion of a magnetic domain wall (DW) in a MO film where both the incident and reflected laser beams pass through the MO film. Two opposite permanent magnets create two domains and a domain wall in the MO film, and an ac coil causes a domain wall to set a periodic motion.

From observation of the detected light intensity pattern as a result of periodic movements of the magnetic domain wall, information about the angular displacement of the surface of a material with a resolution of about 0.01° can be extracted.[4] In this MOAD sensor system, both the incident and reflected beams were set to pass through the MO film.[4] Such configuration limited the distance between the MO film and the testing sample due to the finite surface size of the MO film. To eliminate this restriction, we changed the geometrical configuration of the sensor system such that only the reflected light beam can pass through the MO film by setting the MO film close to the photodetector and away from the testing sample. In addition to the construction of a practical remote sensing system, we improved the resolution of the experimental data ten times compared to data obtained from the previous MOAD sensor system. This was possible due to the elimination of errors associated with the incident beam passing through the MO film in the previous MOAD sensor system.

The schematic configuration of the modified MOAD sensor system is shown in Fig. 1. As the incident laser beam is reflected from a sample, it goes through a sensing part, which is represented by a MO film, an analyzer, and a photodetector. An aluminum mirror was used as a sample for this study. We employed a MO film made of a bismuth-doped iron garnet $(Bi,Tm)_3(Fe,Ga)_5O_{12}$ (thickness ~3 μm) grown on a thin substrate of gadolinium gallium garnet (GGG) with a thickness of 0.5 mm. The bismuth-doped iron garnet film has a large specific Faraday rotation θ_F up to 2.3°/μm of thickness. The domain walls in the bismuth-doped garnet MO film can be activated at a threshold magnetic field of 1–3 G. In order to produce the special magnetic structure of two domains and a domain wall between them, two hard ferrite magnets (remanence $B_R=0.350$ T and coercive field $H_C=260$ kA/m) with opposite polarities were positioned 5 mm apart from the MO film. These two magnets generate a uniform magnetic field gradient in the MO garnet film. A solenoid coil with a radius of 13 mm and a resistance of 4.6 Ω was employed to derive an ac magnetic field to the MO film by applying a current having a sawtooth wave form of frequency $f=10$ Hz and $V_{max}=10$ V as shown in Fig. 2(a). A convex lens with a focal length of 600 mm was used to reduce the size of the laser beam spot on the MO film in order to reduce the error in data as demonstrated in the MO linear displacement sensor.[5] Distance L between the MO film

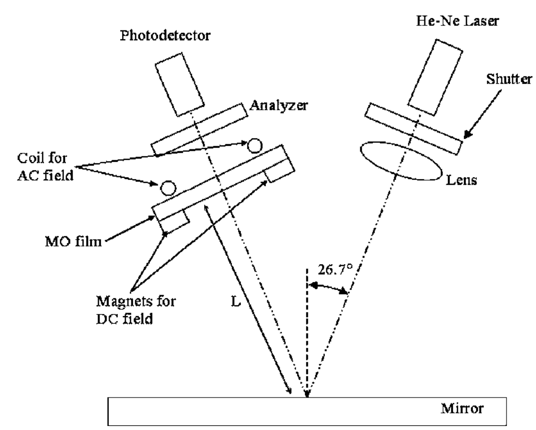

FIG. 1. The configuration of the new remote MOAD sensor.

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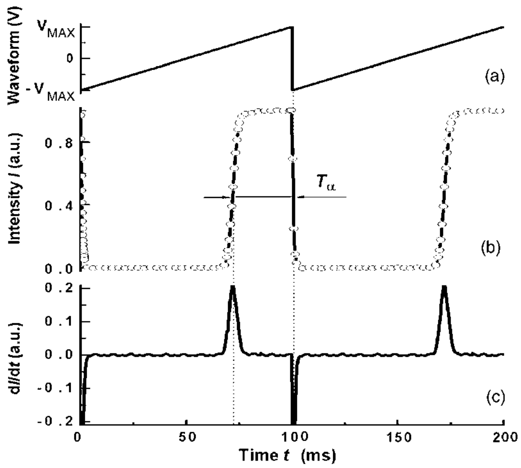

FIG. 2. (a) Sawtooth wave form of the applied voltage. (b) The typical response of the remote MOAD sensor (intensity I vs time t) under the sawtooth ac wave form with the $V_{\max}$ amplitude. T_α is the temporal characteristic of the signal at the reference angle $\alpha=26.7°$. (c) The derivative of the intensity dI/dt. The changes of the intensity around 0, 100, and 200 ms are due to changes of the wave form direction (the frequency is 10 Hz); the changes of the intensity close to 75 and 175 ms reveal the overlapping of the domain wall and the impingement position of the laser beam.

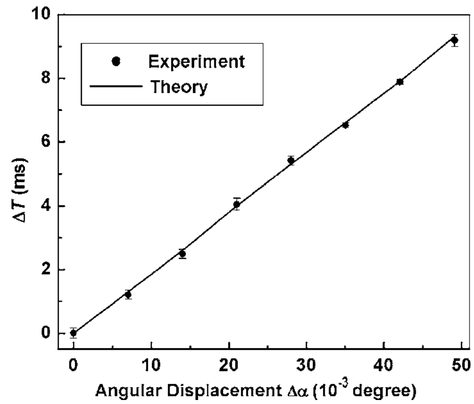

FIG. 3. Experimental and theoretically computed dependence of $\Delta T=T_{\alpha+\Delta\alpha}-T_\alpha$, where $\alpha=26.7°$ on the angular displacement $\Delta\alpha$.

and the Al mirror was set to 435 mm, and the distance between the lens and the Al mirror was set to 170 mm.

As a reference angle, an incident angle of 26.7° was chosen, and its intensity was recorded [see Fig. 2(b)]. The intensity was shifted and normalized such that it takes two stable values, 0 and 1, with a transition region between them. The shape of this intensity can be explained as follows. As a cycle of the sawtooth wave form begins, the reflected laser beam lies in the "magnetization-down" domain; therefore the Faraday rotation is $-\theta_F$; this state is represented by the value 0 in the intensity graph shown in Fig. 2(b). As the ac magnetic field increases with time, the domain wall moves toward the laser beam spot on the MO film. The transition region occurs when the domain wall meets the reflected laser beam spot; at this moment the spot is in both domains and the value of the Faraday rotation depends on the relative portion of the two domains. Finally, at the end of the cycle the reflected laser beam is in the ″magnetization-up″ domain and its Faraday rotation is $+\theta_F$; this state is represented by the value 1 in the intensity graph.

For a particular incident angle α and for a fixed distance L between the MO film and the mirror, the position (in time) when the domain wall reaches the laser beam spot is unique and can be used to identify the time interval of either the state of positive or that of negative polarization rotation angles from the intensity plot. In order to determine this time interval accurately, we performed the first derivative of the intensity and the difference, T_α, between two extrema can be easily identified, as shown in Fig. 2. For example, as in the case of Fig. 2, where the reference angle $\alpha=26.7°$, the time interval at this reference angle is $T_\alpha=28.2$ ms.

As the mirror is rotated on some angle $\Delta\alpha$, the time interval $T_{\alpha+\Delta\alpha}$ changes as well. Introducing the difference between two time characteristics as $\Delta T=T_{\alpha+\Delta\alpha}-T_\alpha$, we find that ΔT increases linearly as a function of $\Delta\alpha$, as shown in Fig. 3. In this experiment we rotated the mirror with a step of 0.007° from the reference angle $\alpha=26.7°$. For an error estimation, $T_{\alpha+\Delta\alpha}$ was measured ten times at each $\alpha+\Delta\alpha$, and the maximum error of ΔT was about 0.19 ms. As the relationship between ΔT and $\Delta\alpha$ is linear, ΔT can be used as an indication of the amount of rotation (angular displacement). From the slope of the ΔT vs the $\Delta\alpha$ curve and the maximum error of ΔT, the resolution of the angular displacement was estimated to be 1×10^{-3} deg.

In order to support the observed experimental results, theoretical modeling of the sensor was performed. The response of the sensor can be analyzed by means of a Jones matrix representation. The intensity I of the laser beam passing through all the optical elements of the modified MOAD sensor system (the light beam reflects from the mirror, and the reflected light beam passes through the MO film and the analyzer) can be computed as

$$I=|E_f|^2=\tfrac{1}{2}|[\text{analyzer}][\text{MOPR}]_{\text{refl}}[\text{mirror}][\text{laser}]|^2 = \tfrac{1}{2}[1-\sin(2\theta_{\text{refl}})], \tag{1}$$

where the vector

$$[\text{laser}]=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$$

represents a 45° polarized laser beam, so that the photodetector with an analyzer could distinguish between Faraday rotations in opposite directions, the matrix

$$[\text{MOPR}]_{\text{refl}}=\begin{bmatrix}\cos\theta_{\text{refl}} & \sin\theta_{\text{refl}}\\ -\sin\theta_{\text{refl}} & \cos\theta_{\text{refl}}\end{bmatrix}$$

represents the magneto-optic Faraday rotation of the light beam on a specific angle θ_{refl}, and the matrix [mirror] and [analyzer] are 2×2 Jones matrices representing the mirror and the analyzer in the system. In Eq. (1), normalized quantities for the intensity and the electrical field of the laser beam are used.

If d is the position of the impingement of the laser beam spot of radius R, the total Faraday rotation angle θ_{refl} can be computed as

$$\theta_{\text{refl}}(x) = \begin{cases} -\theta_F & \text{if } x<(d-R)\\ \dfrac{\theta_F}{\pi}\{2\phi(x)+\sin[2\phi(x)]\} & \text{if } (d-R)\leqslant x\leqslant(d+R)\\ +\theta_F & \text{if } x>(d+R), \end{cases} \tag{2}$$

where θ_F is the Faraday rotation in the MO film, x is the position of the domain wall at time t, $\phi(x)=\arcsin[(x$

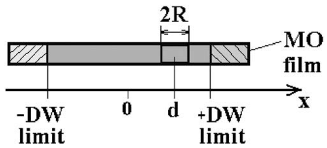

FIG. 4. Schematic representation of the relative positions of the limits for the domain wall movement (±DW limit) and the position of impingement (d) of the laser beam (radius R) on the MO film. The rotation angle (i.e., the measured intensity) depends on the relative position of the domain wall (not shown here) and the position of impingement [see Eq. (2)]. The thickness of the domain wall was assumed to be negligible.

$-d)/R]$, and the system of coordinates was chosen as in Fig. 4. Here, we assume that for Eq. (2) to be correct θ_F should be relatively small and that the thickness of the domain wall should be negligible compared with the radius of the laser beam spot. If we also assume that the domain wall moves without damping, the position x of the domain wall at time t is connected with the periodical ac magnetic field (sawtooth wave form in this study) produced by the excitation coil, and, therefore, the domain wall moves within some limits. The schematic representation of the relative positions of the limits for the domain wall movement and the position of impingement d of the laser beam on the MO film is shown in Fig. 4.

Considering the geometrical arrangements of the sensor as shown in Fig. 1, if the surface of the sample rotates by $\Delta\alpha$, the change in the impingement position will be

$$\Delta d = \frac{L^2 \tan(2\Delta\alpha)}{\sqrt{L^2 - d^2} - d\tan(2\Delta\alpha)}, \tag{3}$$

where d is the position of the impingement at the reference angle ($\Delta\alpha=0°$ at $\alpha=26.7°$). As we use $L \gg d$ ($L=435$ mm and d is less than the DW limit amplitude of 4 mm) and $\Delta\alpha$ which is relatively small ($\Delta\alpha \ll 1°$), Eq. (3) can be written as

$$\Delta d \cong 2L\Delta\alpha. \tag{4}$$

As we assume that the domain wall moves with a constant speed due to the linear gradient of the dc magnetic field and due to a sawtooth wave form of the ac magnetic field, it is obvious that ΔT is directly proportional to Δd with a fixed coefficient. Therefore, from Eq. (4) there will be a linear relationship between the time characteristic ΔT and the angular displacement $\Delta\alpha$. It is worth noting that the reference angle α is not included into Eq. (3) or Eq. (4) directly, and, therefore, the time characteristic will not be affected by the reference angle. This is an additional advantage of the proposed MOAD sensor.

For the intensity calculation, the Faraday rotation of the laser beam, θ_{refl}, is computed by Eq. (2) with the position of the impingement d modified according to Eq. (4). As an example of the theoretical calculation, Fig. 5 shows the intensities for the reference angle ($\alpha=26.7°$ and $\Delta\alpha=0°$) and for $\Delta\alpha=0.049°$, where the intensity was shifted and normalized such that it takes values between 0 and 1 with a transition region. An excellent agreement is clearly seen between experimental and theoretical intensities. In calculations above we used a frequency $f=10$ Hz, a radius of the laser beam $R=0.3$ mm, a DW limit amplitude of 4 mm, the position of "zero" impingement $d=1.76$ mm, and the distance between the sample and the MO film $L=435$ mm from experiment. The observed differences between experimental and theoretical intensities near 0, 100, 200 ms, etc. are due to the inability of the domain wall to move instantaneously from one limit position to another. The differences at the starting and ending stages of the transition region (near 75, 175 ms, etc.) are most likely due to the shape of the laser beam spot being of a Gaussian distribution instead of a circular one, which, however, does not affect the linearity of the sensor. The theoretically computed intensity was used to find the dependence of ΔT on the angular displacement $\Delta\alpha$ based on the similar procedure used for the experiment. It was found that theoretical values agreed very well with experimental values, as shown in Fig. 3.

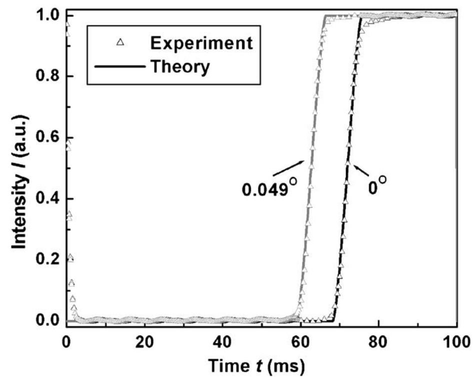

FIG. 5. Theoretical calculations (solid line) of the intensity compared with the experimental data (symbols) for a remote MOAD sensor at a reference position ($\alpha=26.7°$, $\Delta\alpha=0°$) and under a small increased angle ($\Delta\alpha=0.049°$). One period is shown only. The discrepancies between experimental and theoretical intensities near 0 ms are due to the inability of the domain wall to instantaneously move from one limit position to another. The differences at the starting and ending stages of the transition region are most likely due to the shape of the laser beam spot being of a Gaussian distribution instead of a circular one.

The current device operates in a quasistatic operation; i.e., the speed of the displacement is much less than the speed of the domain wall motion. We have tested the current device successfully at frequencies up to 200 Hz and observed the similar results without losing resolution. We do not expect the change in the behavior in the domain wall movement at high frequencies such as around 1 kHz. However, the difficulties could arise due to the electronic equipment (Data Acquisition Card and LabView) and due to the proposed way to determine the temporal changes ΔT. We plan to study the operation of our sensor at high frequencies in the nearest future.

To conclude, the truly remote magneto-optic angular displacement (MOAD) sensor for measuring small angular displacements using the magnetic domain wall motion was investigated. A linear relationship between the change of the time characteristics ΔT and the angular rotation of the sample was observed. The theoretical modeling provided an excellent support for the experimental results. The angular resolution of the sensor was found to be 1×10^{-3} deg, which is ten times higher than the previous MOAD sensor. Newly developed remote magneto-optic angular displacement sensors can be applied to the calibration and the alignment of the machines and tools with high precision.

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