Business Cycles, Endogenous Growth, and Monetary Cycles

By

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Abstract

This dissertation sets out to introduce a new calibration procedure building on Jermann (1998) and the iterative shock identification scheme of Benk et al. (2005) in Chapter 1. It incorporates the use of *Simulated Annealing*, a global optimization algorithm, into the Jermann (1998) calibration methodology that is applied to search for the combination of structural parameters within a bounded parameter space that yields the lowest distance between a vector of US data moments and its simulated moments counterpart in the frequency domain. It also extends the methodology of Jermann (1998) with the identification scheme of Benk et al. (2005) to obtain convergent estimates for shock parameters.

After illustrating the workings of this new calibration methodology on the two sector business cycle model of Dang et al. (2011) with endogenous growth and human capital in Chapter 2 this dissertation sets out in Chapter 3 to introduce an extended version of the model of Dang et al. (2011) and to explain a number of real business cycle (RBC) problems that include the Gali (1999) labor response, the basic consumption-output and labor-output relationship, and the lack of an internal propagation mechanism as pointed out by Cogley and Nason (1995) and Rotemberg and Woodford (1996).

This extension follows the suggestions of King and Rebelo (2000) to incorporate an external labor margin through a human capital investment sector and a physical capital utilization margin in the form of physical capital utilization rate to improve the performance of the standard RBC model. In the model introduced in Chapter 3 the physical capital utilization rate is further amended by the introduction of entrepreneurial capacity as in Friedman (1976) and Lucas (1988). The added margin of physical capital utilization is intra-temporal in nature, which enables the new calibration scheme to improve on the ability of the model significantly to explain the underlying real business cycle problems and US data moments in the frequency domain.

Lastly, in Chapter 4 a simple monetary extension of the model in Chapter 3 is presented. In this chapter it is shown that the added physical capital utilization in a monetary model combined with the proposed calibration scheme is successful in explaining the empirical negative long term relationship between inflation and output.

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May the Force be with all!

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Introduction

Since the seminal paper by Kydland and Prescott (1982) a number of problems have emerged with the real business cycle (RBC) model leading to a lesser role in policy or research over time.

One of these, as pointed out by Summers (1986) and Mankiw (1989), is concerned with the driving force of RBC models: the goods sector technology shock. More specifically, they pointed out that early estimates of the Solow residual had standard deviations of close to 1 percent per quarter, which would require these shocks to be very much apparent to the public and it may not be a proper measure of technological progress. The second main criticism concerns the internal propagation mechanism of the standard RBC model. Cogley and Nason (1995) and Rotemberg and Woodford (1996) show that output dynamics closely follow TFP innovations and this suggests inadequate internal dynamics, which translate into less predictable output movements in the RBC model.

Third, the real business cycle model is widely criticized for not being able to match movements in labor hours with output. This criticism concerned with the standard model's mechanism of inter-temporal labor supply driven by the variations in incentives to work over time. More specifically, the incentive to supply labor due to changes in wages [i.e. changes in labor demand] is inadequate to generate fluctuations found in U.S. data. Furthermore, Gali (1999) found that labor hours in the United States initially fall upon the impact of a positive neutral TFP shock unlike what standard RBC models suggest.

In response to these criticisms Einarsson and Marquis (1998) add a Lucas (1988) type human capital investment sector to an otherwise standard business cycle model with the only source of fluctuations being a good sector TFP shock. With this approach they have been able to further improve the ability of the business cycle model to match labor market movements. This has been due to the fact that in their model the agents have an ability to reallocate their time endowment toward acquiring human capital in formal training, and hence they further outline the importance of the external labor margin in altering the standard model's mechanism of inter-temporal labor supply.

King and Rebelo (2000) point out that physical capital capacity utilization is key to improve the amplification mechanism of the business cycle model. DeJong et al. (1996) included not only a human capital sector, but added physical capital capacity utilization rate to their model that works through the intra-temporal margin. Their source of fluctuation was still the goods sector productivity shock. Their inclusion of the utilization rate allowed for the representative agent to treat both human and physical capital in a symmetric fashion by permitting to use both stocks of capital at less than full capacity. They found that this can further explain the co-movement of labor hours with output and can very well match business cycle volatilities. They also showed that in such a model learning time [i.e. time in education / human capital investment production] is countercyclical, which is a direct result of the symmetric treatment of both capital stocks. Also, Maffezzoli (2000) extended this literature by explaining international business cycle facts.

Also, Benhabib et al. (1997) focused on the lack of a robust internal propagation mechanism of the standard RBC model in order to explain output growth persistence, which they attempted to resolve by explaining the composition of output within a multi-sector business cycle model.[1]

Most recently Dang et al. (2011) used a more general production function in the human sector and used a human sector productivity shock to capture output growth persistence and labor movements including the Gali (1999) labor response and labor volatility. Dang et al. (2011) have been able to capture output growth persistence to a standard neutral TFP shock at the first lag, however, they have been unable to capture the quick drop in persistence after the impact of a shock similarly to Benhabib et al. (1997).

This dissertation presents a two sector business cycle model with endogenous growth in the Lucas (1988) tradition and with physical capital utilization rate extended with the idea of entrepreneurial capacity as explained by Friedman (1976) and Lucas (1978). This benchmark model is a direct extension of the model of Dang et al. (2011). Furthermore, a monetary extension of the benchmark model of this dissertation is also presented to explain the empirical output-inflation relationship without losing the model's ability to capture RBC co-movements similarly to Gomme (1993), even though he used an exogenous growth model. The models' are evaluated in the full frequency domain that includes the Comin and Gertler (2006) type "medium term" cycle, which extracts a trend of the length of the full frequency domain by using a Christiano and Fitzgerald (2003) type asymmetric band-pass filter. Lastly, in order maximize the explanatory power of the baseline model and its monetary extension a new calibration procedure is introduced that builds upon the iterative calibration procedure of Jermann (1998).

[1] Evidence by Cogley and Nason (1995) shows that output growth is positively autocorrelated over short horizons and has a weak / negative autocorrelation over higher lags in U.S. data.

The first contribution of this dissertation is a new calibration procedure. This proposed calibration procedure generalizes and extends the iterative calibration methodology of Jermann (1998) and extends it with the shock identification and estimation scheme introduced by Benk et al. (2005), which was built on the original procedure of Ingram et al. (1997). In Chapter 1 this new procedure is introduced and discussed.

In Chapter 2 this new calibration methodology is illustrated on the model of Dang et al. (2011) and the performance of the new calibration is compared to the original one.

The second contribution of this dissertation is that it adds physical capital capacity utilization with entrepreneurial capacity into the model of Dang et al. (2011). Thirdly, it shows that through a second intra-temporal margin the additional features can provide a robust enough propagation mechanism to spread the shocks in order to match output growth persistence beyond the first degree and to narrow the gap between labor market movements in U.S. data and business cycle models. The description of this benchmark model is in Chapter 3, where it's performance is also compared to that of the newly calibrated model of Dang et al. (2011).

The fourth contribution of the dissertation is that it shows when the benchmark model of Chapter 3 is extended with exchange money in Chapter 4 by applying the proposed new calibration scheme the model can capture the empirical output-inflation relationship in the frequency domain with specific focus on the long-run output-inflation relationship.

Chapter 1

Methodology for Calibration and Simulation of Business Cycle Models with Endogenous Growth, and Human Capital

1.1 Introduction

Chapter 1 describes in detail the methodology for evaluating the performance of endogenous growth models with a non-market human capital investment sector such as the models in the later chapters of this dissertation. The methodology described and proposed here is specifically concerned with the calibration, solution, and simulation of these models to be able to match simulated moments to data across the frequency domain. Such models are non-stationary in nature and therefore pose a challenge to evaluate them using standard methods, including finding their recursive solution, for example as described in Uhlig (1998).

This chapter proposes a new calibration procedure, which generalizes the iterative calibration method of Jermann (1998) and combines it with the iterative shock extraction method of Ingram et al. (1997) and Benk et al. (2005). In this new procedure the actual search for parameters is done by *Matlab's Simulated Annealing* global optimization algorithm in a similar fashion to Bayesian estimation, in which we bound our parameter search with prior information available to us from the literature. Lastly, we present in detail the U.S. data used for calibration purposes and for shock estimation, in which we closely followed Gomme and Rupert (2007).

In Section 1.2 we outline the solution, calibration, and simulation methodology. In Section 1.3 we present a concise description of methods in the literature that are required to solve such models. These methods include stochastic discounting, the solution methodology, and the simulation methodology. In Section 1.4 we describe the proposed calibration procedure. Section 1.5 describes the constructed U.S. data; and lastly, Section 1.6 concludes Chapter 1.

1.2 Methodology Outline

Two sector business cycle models with a human capital investment sector in the Lucas (1988) tradition exhibit non-stationary features. More specifically, in the models' described in later chapters the state variables including the human capital stock denoted by h_{t+1} and the physical capital stock denoted by k_{t+1} along with other variables including output, y_t, consumption, c_t, and physical capital investment, i_{kt}, are growing at a common gross rate of $(1 + g)$ along the balanced growth path (BGP). This feature of the standard set of equilibrium conditions as given in the Appendices of the relevant chapters for the respective models make it unfeasible to solve these models directly by using standard techniques such as the solution method of undetermined coefficient as described in Uhlig (1998).

In these models we normalize each growing variable with the stock of human capital in time period t as suggested by Maffezzoli (2000) and Dang et al. (2011), which is a standard technique in the growth literature to stationarize such models. We denote the normalized stationary variables with '$\tilde{.}$' throughout this dissertation.

After the underlying set of equilibrium conditions are transformed we can use standard techniques to find the solutions of these models in a straightforward fashion in terms of the normalized variables. For this one has to solve for the stationary balanced growth path equilibrium as is standard in the DSGE literature.

Given the steady state solution and the stationary system of equations with normalized variables one can log-linearize these set of equations around their respective steady states, after which by applying the solution methodology of Uhlig (1998) one can obtain the recursive solutions for the respective stochastic models in terms of the normalized variables.

In order to evaluate these models one may obtain the impulse response function for the normalized variables. As in Maffezzoli (2000) these impulse responses coincide with those of the non-stationary variables. Impulse responses are obtained by performing a short-run deterministic simulation [i.e. comparative static exercise]. In the deterministic case, as along the BGP, the stock of human capital is represented by a straight line, which has a slope of $(1 + g)$. The non-stationary variables grow at the same rate and therefore by normalizing with the human capital stock one extracts the growth trend in the deterministic case. Hence, the impulse responses based on the solution for normalized variables coincide with the impulse responses of non-stationary variables relative to the BGP growth trend.

In order to evaluate the performance of the model relative to U.S. data there are two main approaches presented here in Section 1.3. One approach is to construct stationary growth rates using the model solution for the normalized variables as described in Dang et al. (2011) and King et al. (1988). An alternative approach is to follow Restrepo-Ochoa and Vazquez (2004) that entails extracting synthetic non-stationary log-level series for the original non-stationary variables by using the model solutions of the normalized variables. After applying any of these methods the evaluation of the models' performances become a straightforward exercise.

Lastly, after we established the model solution and evaluation methodology we propose a calibration procedure that generalizes the iterative method of Jermann (1998) and extends it with the shock estimation procedure of Ingram et al. (1997) and Benk et al. (2005). In short, the outline of the solution and calibration methodology is as follows:

1. Obtain the model's non-stationary equilibrium conditions;

2. Transform the system of equilibrium conditions into a stationary system by normalizing growing variables with the stock of human capital;

3. Solve for the balanced growth path solution of the stationary system obtained in the previous step;

4. Log-linearize the stationary system of the model equilibrium conditions around its steady state;

5. Apply the method of undetermined coefficients method in Uhlig (1998) to obtain the recursive solutions for the normalized and non-normalized stationary variables;

6. Obtain synthetic log-level non-stationary series for the growing variables and/or their respective growth rates;

7. Filter the simulated data using a Christiano and Fitzgerald (2003) band-pass filter at pre-defined frequencies;

8. Calibrate the model using band-pass filtered data moments.

1.3 Normalization, Solution Methodology, Simulation Methodology

1.3.1 Normalization

As pointed out in the previous section a standard procedure in the growth and RBC literature to transform a non-stationary endogenous growth model with human capital is to normalize all growing variables by the human capital stock as in Maffezzoli (2000) and Restrepo-Ochoa and Vazquez (2004). We denote normalized variables by '$\tilde{.}$' .

For Model 1 and Model 2 in Chapters 2 and 3 the variables that grow at a common rate along the BGP include the physical capital stock, k_{t+1}, the human capital stock, h_{t+1}, output, y_t, consumption, c_t, physical capital investment, i_{kt}, and human capital investment, i_{ht}. Then to transform the model we define the physical capital to human capital ratio as $\tilde{k}_{t+1} \equiv \frac{k_{t+1}}{h_{t+1}}$, the gross growth rate of the human capital stock, $g_{ht+1} \equiv \frac{h_{t+1}}{h_t}$, the output to human capital ratio, $\tilde{y}_t \equiv \frac{y_t}{h_t}$, the consumption to human capital ratio, $\tilde{c}_t \equiv \frac{c_t}{h_t}$, the physical investment to human capital ratio, $\tilde{i}_{kt} \equiv \frac{i_{kt}}{h_t}$, and the human investment to human capital stock ratio, $\tilde{i}_{ht} \equiv \frac{i_{ht}}{h_t}$. In addition, in the simple monetary extension of Model 1 and 2 in Chapter 2 and 3 the additional growing variable is the money stock, denoted by m_t for which models we define the money stock to human capital ratio as $\tilde{m}_t \equiv \frac{m_t}{h_t}$ in Chapter 4.

1.3.2 Solution Methodology

After obtaining the log-linearized systems of the respective stationary models one can directly apply the well established solution methodology of Uhlig (1998). The method of Uhlig (1998) is based on the method of undetermined coefficients following King et al. (1988). We chose this method as it is relatively simple to implement. In order to be able to apply this solution method one has to rewrite the log-linear versions of the first order conditions in the following matrix form:

$$Ax_t + Bx_{t-1} + Cy_t + Dz_t = 0, \tag{1.1}$$

$$E_t[Fx_{t+1} + Gx_t + Hx_{t-1} + Jy_{t+1} + Ky_t Lz_{t+1} + Mz_t] = 0, \tag{1.2}$$

$$z_{t+1} = Nz_t + \epsilon_{t+1}, \tag{1.3}$$

where $E_t(\epsilon_{t+1}) = 0$; the vector x_t (size $mx1$) contains the endogenous state variables; y_t (size $nx1$) is the vector of all other endogenous variables; meanwhile, z_t (size $kx1$) is the vector of exogenous stochastic variables. It is assumed that the coefficient matrix C is of size lxn, where $l \geq n$ and of rank n. l is the number of deterministic equations, F is a coefficient matrix of size $(m + n - l)xm$, and N has only stable eigenvalues.

In our models x_t contains the log-linear versions of $\hat{\tilde{k}}_t$ and $\hat{g}_{ht}$. There are two exogenous variables in z_t, namely, $\hat{z}_t^g$ and $\hat{z}_t^h$. All other endogenous variables in both models, including other normalized variables, are in y_t.[1] Then the log-linear solution method by Uhlig (1998) is seeking to find a recursive equilibrium law of motion of the following form:

$$x_t = Px_{t-1} + Qz_t; \tag{1.4}$$

$$y_t = Rx_{t-1} + Sz_t. \tag{1.5}$$

Where P,Q, R, and S are the coefficient matrices of the recursive solution.

1.3.3 Simulation Methodology

In order to assess the performance of the models in Chapters 2, 3, and 4 after obtaining the recursive solution of the models, two methods are used. One is based on King et al. (1988) in which instead of obtaining log-level series the focus of analysis shifts to analyzing the stationary growth rates of the actual and simulated data. The other approach as in Restrepo-Ochoa and Vazquez (2004) utilizes the recursive model solution to extract model implied non-stationary log-level series for growing variables in endogenous growth models.

Methodology for Model Simulated Growth Rates

The first approach, used partially for the purposes of evaluating model performances follows King et al. (1988) by which instead of using variables in log-levels to cal-

[1]The growth rate of human capital $g_{ht} \equiv h_{t+1}/h_t$ is defined as a state variable in order to satisfy the requirement that the $l \geq n$ condition is imposed by the log-linear approximation. Since it is not a state variable in the proper sense it will vanish from the recursive policy functions.

culate key moments of the simulated data, the growth rates of output, investment, and consumption are used. The underlying reason for this that both the actual and simulated data series are stationary for growth rates. Then the asymmetric Christiano and Fitzgerald (2003) type band-pass filter is applied to the data after which we obtain moments at different frequencies.

The methodology that enables to obtain simulated data for the growth rates of growing variables [i.e. y_t, i_{kt}, c_t] is illustrated through the example of consumption. Recall that $\tilde{c}_t \equiv \frac{c_t}{h_t}$, then consumption growth in logarithmic form can be calculated as,

$$\begin{aligned} g_{c,t+1} &= \log c_{t+1} - \log c_t \\ &= \log \tilde{c}_{t+1} - \log \tilde{c}_t + \log h_{t+1} - \log h_t \\ &= (\log \tilde{c}_{t+1} - \log \tilde{c}) - (\log \tilde{c}_t - \log \tilde{c}) + \log \frac{h_{t+1}}{h_t} \\ &= \hat{\tilde{c}}_{t+1} - \hat{\tilde{c}}_t + (\log g_{h,t+1} - \log g_h) + \log g_h \\ &= \hat{\tilde{c}}_{t+1} - \hat{\tilde{c}}_t + \hat{g}_{ht+1} + \log g_h, \end{aligned} \tag{1.6}$$

where $\tilde{c}$ and g_h are steady state values of $\tilde{c}_t$ and g_{ht+1}, $\hat{\tilde{c}}_t$ is normalized consumption's log-deviation from its steady state, and $\hat{g}_{ht+1}$ is the log-deviation of the human capital growth rate from its steady state. Growth rates of output, physical investment and other growing variables' can be derived in an identical fashion.

Methodology for Model Simulated Non-Stationary Data Series

As an alternative method, the extraction of non-stationary synthetic data for the variables of interest [i.e. y_t, i_{kt}, and c_t] implied by the model solution can be used to calculate simulated moments. It is relatively straightforward to obtain the log-levels of output, consumption, and investment as in Restrepo-Ochoa and Vazquez (2004) which we illustrate for the models described in Chapter 2 and 3. Consider the definitions of stationary normalized series as defined in Subsection 1.3.1, where n_t denotes a non-stationary variable of the model of choice:

$$\log n_t = \log \tilde{n}_t + \log h_t, \tag{1.7}$$

where $\tilde{n}_t \equiv n_t/h_t$. Then one may observe about the growth rate of human capital, g_{ht}, where g_{ht} we define as the gross growth rate of human capital. Then one can write that

$$\frac{h_{t+1}}{h_t} = g_h e^{g_{ht}} \approx g_h g_{ht}, \tag{1.8}$$

where $g_h = 1 + g$ is the steady state value of h_{t+1}/h_t. From the log-linear solution of the model in (1.4) the laws of motion of the log-deviations from the steady state values of $\tilde{k}_{t+1}$ and g_{ht+1} are

$$\hat{g}_{ht} = P_{21}\hat{k}_t + Q_{21}z_t^g + Q_{22}z_t^h, \tag{1.9}$$

$$\hat{k}_{t+1} = P_{11}\hat{k}_t + Q_{11}z_t^g + Q_{12}z_t^h, \tag{1.10}$$

where P_{ij} and Q_{ij} denote the generic elements of matrices P and Q respectively. Next take natural logarithm of equation (1.8) to obtain,

$$\log h_{t+1} = \log h_t + \log g_h + \hat{g}_{ht}. \tag{1.11}$$

Then combining (1.9), (1.10), and (1.11) one can obtain an expression for the solution of the model implied synthetic human capital stock in log levels as,

$$\begin{aligned} \log h_{t+1} = \log h_t + \log g_h + P_{21}P_{11}\hat{k}_{t-1} + [P_{21}Q_{11} + Q_{21}]z_t^g \\ + [P_{21}Q_{12} + Q_{22}]z_t^h. \end{aligned} \tag{1.12}$$

Then using the expression for the human capital stock in equation (1.12) one can easily obtain the log-level synthetic series for other variables of interest by using equation (1.7). Given the non-stationary time series obtained from the model for the log-levels of output, consumption, and investment, a band-pass filter can be applied at different frequencies as defined earlier following Baxter and King (1999),Benk et al. (2005, 2008, 2010), Basu et al. (2012), and Comin and Gertler (2006).

1.4 The Calibration and Shock Estimation Methodology

Given that one obtains a solution for a model such as in Chapters 2 and 3 we may turn our attention to a new proposed calibration procedure. The balanced growth path of the models are calibrated quarterly on the basis of the data of Gomme and Rupert (2007) updated from 1959:Q2 until 2014:Q2. Gomme and Rupert (2007) refine the methodology of calibration, in particular, for a two-sector market and non-market household economy. In our models the human capital investment sector is the equivalent of their non-market sector. Next, we propose a generalized calibration procedure and summarize the methodology for estimating shocks.

1.4.1 The Calibration Procedure

In order to further refine the calibration procedure and to make it more efficient we have implemented a similar method to that of Jermann (1998) using our data updated U.S. data. In order to make the process more efficient instead of the iterative setup of Jermann (1998) to find structural and initial shock parameters to match our targets, we have implemented an automated search by using one of *Matlab's* global optimization functions called *Simulated Annealing.*

This allowed our calibration method to exhibit similar features to a *Bayesian* estimation. More specifically, we set the *Simulated Annealing* algorithm to search for the best parameter values including structural and shock parameters within a pre-defined parameter space set by prior information in order to minimize the distance between the vectors of selected model generated moments and our U.S. data based targets. The first vector is based on US data $\theta_{1,i}$, and the second is based on the model simulated data $\theta_{1,i}$, where $i = 1, 2$ represent the model.

For Model 1 $\theta_{1,2}$ denotes the vector of seven model calculated moments: $\theta_{1,2} = [\psi_y, \psi_c, \psi_{i_k}, \rho^{bc}_{c,y}, \rho^{lf}_{c,y}, \rho^{bc}_{i_k,y}, (c/y)]'$, where ψ_y stands for the autocorrelation of output growth at one lag; ψ_c is the autocorrelation of consumption growth at one lag; and ψ_{i_k} is the autocorrelation of physical investment at one lag. $\rho^{bc}_{c,y}$ and $\rho^{bc}_{i_k,y}$ are the correlations of consumption and physical investment with output at the business cycle frequency; $\rho^{lf}_{c,y}$ denotes the correlation of consumption with output at the low frequency, and lastly, (c/y) is the long-run value of the ratio of consumption and output. The vector $\theta_{1,2}$ is effectively a function of the model parameters: $\Theta_1 = [\beta, A, \sigma, \phi_1, \phi_2, A_g, A_h, \delta_k, \delta_h, \rho_g, \rho_h, \rho_{g,h}]$. $\theta_{1,1}$ is the vector of corresponding target moments.

For Model 2 $\theta_{2,1}$ denotes the vector of 8 model calculated moments: $\theta_{2,1} = [\psi_y, \psi_c, \psi_{i_k}, \rho^{bc}_{c,y}, \rho^{bc}_{u,y}, \rho^{lf}_{c,y}, \rho^{bc}_{i_k,y}, (c/y)]'$, where ψ_y stands for the autocorrelation of output growth at one lag; ψ_c is the autocorrelation of consumption growth at one lag; and ψ_{i_k} is the autocorrelation of physical investment at one lag. $\rho^{bc}_{c,y}, \rho^{bc}_{u,y}$, and $\rho^{bc}_{i_k,y}$ are the correlations of consumption, the physical capital utilization rate and physical investment with output at the business cycle frequency; $\rho^{lf}_{c,y}$ denotes the correlation of consumption with output at the low frequency, and lastly, (c/y) is the long-run value of the ratio of consumption and output. The vector $\theta_{2,1}$ is a function of the model parameters: $\Theta_2 = [\beta, \sigma, A, B, \phi_1, \phi_2, A_g, A_h, \delta_k, \delta_h, \psi, \rho_g, \rho_h, \rho_{g,h}]$. $\theta_{2,2}$ is the vector of corresponding target moments.

Then the *Simulated Annealing* global optimization algorithm of *Matlab's* Global Optimization Toolbox is set up to minimize the distance between $\theta_{1,1}$ and $\theta_{2,1}$, $\theta_{1,2}$ and $\theta_{2,2}$ respectively.

Given that Ω is a weighting matrix, the objective function for the algorithm to minimize was defined in three different ways, which also served as a robustness check and a way to determine the most efficient method. These definitions are the following:

1. The norm of the target and model based moment vectors: $\parallel \theta_{1,2} - \theta_{1,1} \parallel$, and $\parallel \theta_{2,1} - \theta_{2,2} \parallel$.

2. Following Jermann (1998), the distance is defined as: $[\theta_{1,2} - \theta_{1,1}]'\Omega[\theta_{1,2} - \theta_{1,1}]$, and $[\theta_{2,1} - \theta_{2,2}]'\Omega[\theta_{2,1} - \theta_{2,2}]$, where Ω is the weighting matrix, which is in this case an identity matrix.

3. The distance is defined as $[\theta_{1,2} - \theta_{1,1}]'\Omega[\theta_{1,2} - \theta_{1,1}]$, and $[\theta_{2,1} - \theta_{2,2}]'\Omega[\theta_{2,1} - \theta_{2,2}]$where Ω is the weighting matrix, which in this case has the average of all the data based moments in $\theta_{1,1}$ and $\theta_{2,2}$ on the diagonal, and zeros elsewhere. The reason for this is to transform each moment in the vector space to the same units.

After numerous runs of the algorithm we found that definition three is the most efficient to find the best set of parameters for the model.

In order to resolve any issues of the algorithm picking an economically unfeasible equilibrium we have implemented for each search iteration of the *Simulated Annealing* algorithm the iterative method described in Benk et al. (2005, 2008, 2010) and Nolan and Thoenissen (2009), based on the shock estimation procedure of Ingram et al. (1997). This entails that at each iteration of the search, the recursive model solution and U.S. data is used to extract the TFP and human productivity shock series, whose auto-correlation parameters are then estimated with the method of Seemingly Unrelated Regressions. The estimation is performed until the shock parameters converge. This way any set of parameters that yield an unfeasible equilibrium that the search algorithm gives can be ruled out.

Simulated Annealing is a global optimization algorithm that given the initial conditions for each parameter value searches in the pre-defined parameter space for a combination of parameters that yields the lowest possible value for the metric function that one may define. The search by the algorithm can yield different results at each run due to the fact that the search is driven by calculating the temperature metric of the optimization algorithm from the previous guess the algorithm makes, which is a relative distance measure compared to the best calibration case up to that point. Therefore, from the initial parameter values that one may provide it may lead to multiple directions each time given the algorithms first few guesses around the initial values provided. By narrowing the parameter bounds we experimented with refining our calibration of the models in this dissertation. Furthermore, to neglect non-convergent shocks from the estimation procedure we implemented a rule to neglect any guesses that yield non-convergent shock parameters. Each run on average took 2 hours, however, narrowing parameter bounds can reduce the time required for the calibration procedure to run as size of the parameter space becomes smaller.

1.4.2 Construction of Shocks

To construct the shocks of the models in later chapters we assume that they follow an AR(1) process as defined in equations (1.15) and (1.16). After the model is linearized around its non stochastic steady state (BGP), denoting any variable $\hat{n}_t$ the variable's deviation from its own steady state. Then the log-deviations of the model variables can be written as a linear function of the the state $s = [\hat{\tilde{k}}, z_t^g, z_t^h]$, where variables with a tilde represent any variable normalized by the human capital stock and variables with a hat represent variables log-deviation from their respective steady states. The solution of the model variables in terms of the endogenous and exogenous state variables can be expressed in a more general form as,

$$X_t = AA[\hat{\tilde{k}}_t] + BB[z_t^g z_t^h]. \tag{1.13}$$

Then by knowing the solution to AA, and BB the shock series $[z_t^g z_t^h]$ can be constructed using data on at least two model variables and the state variable. It is the case because identification requires at least as many series in X_t as shock series and the state variable.[2] Then the series can be estimated as,

$$[z_t^g z_t^h]' = (BB'BB)^{-1}BB'(X_t - AA[\hat{\tilde{k}}_t]). \tag{1.14}$$

[2] Here more variables are used with the goal of ensuring robust solutions.

Then we take the estimated series for z_t^g and z_t^h and estimate the following equations with the method of *Seemingly Unrelated Regressions*:[3]

$$z_t^g = \rho_g z_{t-1}^g + \epsilon_t^g; \tag{1.15}$$

$$z_t^h = \rho_h z_{t-1}^h + \epsilon_t^h; \tag{1.16}$$

This way we obtain estimates of the first order auto-correlation coefficients of both the goods and human sector TFP and their correlation coefficient. Then we proceed by implementing an iterative procedure. We start at an initial guess for the auto-correlation parameters and use those to estimate new shock auto-correlation coefficients. We use these again to repeat the procedure until the process ends when the three parameters $\{\rho_g, \rho_h, \rho_{g,h}\}$ converge. Then these are used to obtain the simulated moments and impulse responses for both models.[4]

1.5 U.S. Business Cycle Data

The US business cycle data used for calibration and shock estimation purposes in *Chapters 1* and *2* is from 1959:Q1 until 2014:Q2 except for that of the physical capital utilization rate, which is only available from 1971:Q4. In constructing real data series for US macroeconomic variables we have closely followed Gomme and Rupert (2007). Analogously to their methodology the following aggregate series are constructed:

- *Nominal Market Investment* = Nonresidential Fixed Investment + Change in Private Inventories
- *Nominal Home Investment* = Residential Fixed Investment + PCE on Durables
- *Nominal Investment* = Nominal Home Investment + Nominal Market Investment

[3] We apply the Seemingly Unrelated Regressions (SUR) method as presented in Greene (2003).

[4] As noted in Benk et al. (2005) the estimated shock processes are sensitive to the combinations of the data series used in the process. However, since the focus of this exercise is to obtain goods sector TFP series that can be constructed using conventional methods [i.e. $\hat{z}_t^g = \hat{y}_t - \phi_1 \hat{k}_t - (1-\alpha)\hat{l}_{gt}$] we used the combination that is most highly correlated with that of the conventional TFP. Given our data and performing a great number of iterations with different combinations we find that defining $X_t = [\widehat{c_t/y_t} \widehat{i_{kt}/y_t} \widehat{i_{kt}/h_t} \hat{u}_t \hat{l}_{gt}]$ for Model 2 and the same less the series for the physical capital utilization rate for the Model 1, we obtained the most highly correlated shock series with the traditionally obtained ones.

- *Real Investment*= Nominal Investment / (Average Price Deflator / 100)
- *Nominal Market Output* = Gross Domestic Product - PCE: Housing Services
- *Nominal Private Market Output* = Nominal Market Output - Employee Compensation: Government
- *Real Market Output* = Nominal Market Output / (Average Price Deflator / 100)
- *Real Private Market Output* = Nominal Private Market Output / (Average Price Deflator / 100)
- *Physical Capital Utilization Rate* = Total Capacity Utilization: Manufacturing
- *Labor Hours* = Non-farm Business Sector: Average Weekly Hours
- *Nominal Market Consumption* = PCE on Nondurable Goods + PCE on Services - PCE on Housing Services
- *Real Market Consumption*= Nominal Market Consumption / (Average Price Deflator/100)
- *Average Price Deflator* = (Implicit Price Deflator:Nondurables + Implicit Price Deflator: Services)/2

According to Gomme and Rupert (2007), output (y) is measured by real per capita GDP less real per capita Gross Housing Product as defined above. It is due to the argument that home sector production should be removed when calculating market output using the *National Income and Product Accounts*. The price deflator is constructed by taking the average of the implicit price deflators on nondurables and services. Population is measured by the non-institutionalized persons aged over 16 years. Consumption (c) is measured by real personal expenditures on nondurables and service less Gross Housing Product. Investment is measured by the sum of real nonresidential fixed investment, the change in private inventories, residential fixed investment, and Personal Consumption Expenditures on durables. Lastly, working hours is measured by the average weekly labor hours.

The annual index of human capital per person data series we used in this paper is based on years of schooling [Barro and Lee (2013)], and returns to education [Psacharopoulos (1994)]. The series have been constructed by Feenstra et al. (2013) using the perpetual inventory method. To obtain quarterly human capital data we have interpolated the annual data of Feenstra et al. (2013) by following Baier et al. (2004) where they define the depreciation rate to human capital as the average of death rates in different age groups for which the data we have obtained from the Center for Disease Control database.

The quarterly physical capital data is constructed from BEA annual US capital stock estimates and quarterly data on investment expenditures. Due to the availability of human capital data until 2012:Q2 we have constructed physical capital from 1954:Q1 until 2012:Q2.

Lastly, the Civilian Labor Force Participation Rate (Series ID: LNS11300000) is from Employment Situation release of the U.S. Bureau of Labor Statistics.

Raw Data Sources					
Description	Units	Seasonally Adjusted	BEA / BLS Code / Source	Frequency	Time Range
PCE on Nondurable Goods	Billions of Dollars	SAAR	DNDGRC1	Quarterly	1959Q1 - 2014Q2
PCE on Services	Billions of Dollars	SAAR	DSERRC1	Quarterly	1959Q1 - 2014Q2
PCE on Housing Services	Billions of Dollars	SAAR	DHUTRC1	Quarterly	1959Q1 - 2014Q2
Implicit Price Deflator (Nondurables)	Index 2009=100	SA	DNDGRD3	Quarterly	1959Q1 - 2014Q2
Implicit Price Deflator (Services)	Index 2009=100	SA	DSERRD3	Quarterly	1959Q1 - 2014Q2
Gross Domestic Product	Billions of Dollars	SAAR	A191RC1	Quarterly	1959Q1 - 2014Q2
Employee Compensation: Government	Billions of Dollars	SAAR	B202RC1	Quarterly	1959Q1 - 2014Q2
Civ. Noninst. Pop. 16 and over	1000s of Persons	NSA	CNP160V	Quarterly	1959Q1 - 2014Q2
Capacity Utilization: Manufacturing	Percentage	SA	TCU:MAN.	Quarterly	1971Q4 - 2014Q2
Nonfarm Bus. Sector: Avg. Weekly Hours	Index 2009=100	SA	PRS585006023	Quarterly	1959Q1 - 2014Q2
Nonres. Fixed Investment	Billions of Dollars	SAAR	A008RC1	Quarterly	1959Q1 - 2014Q2
Residential Fixed Investment	Billions of Dollars	SAAR	A011RC1	Quarterly	1959Q1 - 2014Q2
Change in Private Inventories	Billions of Dollars	SAAR	A014RC1	Quarterly	1959Q1 - 2014Q2
PCE on Durables	Billions of Dollars	SAAR	DDURRC1	Quarterly	1959Q1 - 2014Q2
Index of Human Capital per Person	Index	NSA	Penn World Tables 8.0	Annual	1950 - 2011

Table 1.1: Raw Data Sources.

1.6 Conclusion

In Chapter 1 we have described a 'blueprint' for solving and evaluating business cycle models with endogenous growth in the Lucas (1988) fashion. More specifically, we have described the step-by-step procedure we have used and implemented to solve the models and to obtain simulated results from Model 1 and 2 in Chapters 2 and 3.

We have described a stochastic normalization of the models with the stock of human capital to transform the system of non-stationary equilibrium conditions of the respective models to a stationary form that enabled us to implement standard solution techniques such as Uhlig (1998).

We also described the two methods of model evaluation. One was concerned with the methodology of obtaining synthetic non-stationary log-level data by using the model solution as in Restrepo-Ochoa and Vazquez (2004); meanwhile, the other method was based on King et al. (1988), in which case for the study of growth rates we have shown how one can obtain simulated growth rate series from the recursive solutions of the models.

Lastly, we have described the methodology for constructing shocks as in Ingram et al. (1997), Benk et al. (2005, 2008, 2010), and Nolan and Thoenissen (2009), which we have also incorporated in a new generalized calibration procedure. In that we have used *Matlab's* optimization algorithm called *Simulated Annealing* to generalize the iterative calibration procedure of Jermann (1998). This enabled us to automate the search for parameter values within a pre-defined interval based on prior information in the literature. The algorithm's did it so that it minimized the distance between a vector of US and simulated data moments. In order to force the algorithm to pick parameters that yield an economically feasible long-run equilibrium, we have implemented the iterative shock construction and estimation of Benk et al. (2005, 2008, 2010), and Nolan and Thoenissen (2009) for each guess of the optimization algorithm.

Given the methodology that we have covered in this chapter in Chapter 2 we illustrate this methodology in action for the endogenous growth model of Dang et al. (2011). In Chapter 3 we extend the model of Dang et al. (2011) with physical capital utilization as suggested by King and Rebelo (2000) with an amended concept of entrepreneurial capacity as in Friedman (1976) and Lucas (1978). We show that this through an additional margin allows to improve the model's ability to explain U.S. data moments in the frequency domain. Lastly, in Chapter 4 we extend the model in Chapter 2 with exchange money. In this we show that by the added margin through the physical capital utilization in a simple monetary economy setting with an endogenous growth mechanism can capture the empirical inflation-output relationship in the frequency domain.

Chapter 2

Human Capital in Business Cycles: A Calibration Exercise

2.1 Introduction

Real business cycle models have been criticized for their lack of an internal propagation mechanism to spread the effect of shocks over time, as pointed out by Cogley and Nason (1995) and Rotemberg and Woodford (1996) Further criticisms point to the inability of the standard RBC model to match movements of labor hours with output and their inability to capture the Gali (1999) negative labor response to the goods sector productivity shock.[1]

The success of endogenous growth models in explaining long term growth generated a new literature in the last decade of the 20th century beginning with Benhabib et al. (1991) and Greenwood and Hercowitz (1991). This literature tried to respond to the main criticisms of the RBC model by adding growth features to it and an external labor margin.

Building upon the endogenous growth models Benhabib et al. (1991) and Greenwood and Hercowitz (1991) added a household sector with the production of non-maket goods. Einarsson and Marquis (1998) add a Lucas (1988) type human capital sector . DeJong et al. (1996) included not only a human capital sector, but added physical capital utilization rate to their model. Maffezzoli (2000) extended the literature to international business cycle theory. Lastly, Dang et al. (2011) used a human capital investment sector and correlated sectoral shocks to answer some of the criticisms of the business cycle model.

In Dang et al. (2011) human capital is produced according to a constant-returns-

[1] This evidence is still somewhat controversial. Chari et al. (2008) and Christiano and Davis (2006) find evidence against, meanwhile; Canova (2009) finds evidence in support of it. More specifically, Canova (2009) shows that by removing any low frequency cycles at the business cycle frequency "hours robustly fall to neutral shocks".

to-scale Cobb-Douglas production function with inputs being effective labor and physical capital unlike in Lucas (1988), where human capital investment is produced with effective labor in a linear fashion. With this model setup Dang et al. (2011) have been able to capture the first degree output growth persistence and the Gali (1999) labor effect, when an aggregate shock hits the economy.

In this chapter it is shown that the distance between U.S. data and such models as in Dang et al. (2011) can be reduced with a new calibration technique at the business cycle frequency. We show this by applying a Christiano and Fitzgerald (2003) type asymmetric band-pass filter to the actual and simulated data at the high, business cycle, and low frequencies along with the Comin and Gertler (2006) "medium term cycle" to match moments.[2] At the same time, it is also shown that this can be done by not losing earlier results pointing to the fact that trade-offs between a traditional and the proposed new calibration technique are minimal.

This chapter proceeds with the description of the model environment as in Dang et al. (2011) in Section 2.2. In Section 2.3, the model calibration is given and the TFP shock series obtained from the solution of the two different calibrations of the model as in Benk et al. (2005, 2008, 2010) and Ingram et al. (1997) are compared to a traditionally obtained Solow residual based TFP series for the U.S. In Section 2.4, the simulation results and impulse responses are presented that include key correlations, volatilities, and the persistence of output growth being matched to U.S. data. Section 2.5 concludes Chapter 2.

2.2 The Model Environment

In this section a model is presented with endogenous growth, which includes a separate human capital investment sector as in Lucas (1988). It includes a more general human investment production function and correlated sectoral productivity shocks as in Maffezzoli (2000) and Dang et al. (2011).

2.2.1 The Model

The representative agent maximizes its expected sum of discounted utility. The agent derives utility from consumption, c_t, and leisure, x_t, at each time period t. With $A > 0$, and $\sigma > 0$, the time t utility is given by

$$U(c_t, x_t) = \frac{[c_t x_t^A]^{1-\sigma} - 1}{1-\sigma}, \tag{2.1}$$

[2]The detailed calibration procedure is described in Chapter 1.

which satisfies the necessary conditions for the existence of a balanced growth path equilibrium.[3] The representative agent is confined by a normalized time endowment for each period t, where l_{gt} is the fraction of time spent in goods production, and l_{ht} in human capital investment production,

$$1 = x_t + l_{gt} + l_{ht}. \tag{2.2}$$

The representative agent invests in physical capital denoted by i_{kt}, according to the following standard physical capital accumulation constraint,

$$k_{t+1} = (1 - \delta_k) + i_{kt}, \tag{2.3}$$

where k_t is the physical capital stock at the beginning of time period t; and δ_k is the constant depreciation rate of physical capital.

Denote by y_t the real goods output; A_g is a positive factor productivity parameter; z_t^g is a productivity shock; k_t is the physical capital stock that has been accumulated by the beginning of period t; v_{gt} is the share of physical capital stock being used in the goods sector; and $v_{gt}k_t$ is the amount of physical capital in the goods sector. h_t is the stock of human capital at the beginning of period t; l_{gt} denotes the share of time used in goods production [i.e. the share of human capital stock that is utilized in the goods sector] ; and $l_{gt}h_t$ represents the effective labor input. With $\phi_1 \in [0, 1]$ being the share of physical capital in goods production, the output is produced according to the following constant returns to scale production function,

$$y_t = F(v_{gt}k_t, l_{gt}h_t) = A_g e^{z_t^g} (v_{gt}k_t)^{\phi_1} (l_{gt}h_t)^{1-\phi_1}, \tag{2.4}$$

where z_t^g is assumed to evolve according to a stationary first order autoregressive process in logarithmic form:

$$z_t^g = \rho_g z_{t-1}^g + \epsilon_t^g. \tag{2.5}$$

The innovations ϵ_t^g is a sequence of independently and identically distributed normal random variables with zero mean and constant variance σ_g^2.

The representative agent also accumulates human capital in a separate home

[3] For more details, see King et al. (1988).

sector, h_t. The human capital stock is then accumulated over time according to the following standard law of motion,

$$h_{t+1} = (1 - \delta_h)h_t + i_{ht}, \tag{2.6}$$

where δ_h is the assumed constant depreciation rate of human capital, and i_{ht} is the per period investment in human capital. Here human capital is reproducible in a separate sector as in Lucas (1988). The human capital investment technology is given by equation (2.7), where A_h is a positive factor productivity parameter in the human sector; z_t^h represents the productivity shock to the human sector; $v_{ht} = 1 - v_{gt}$ is the remaining fraction of physical capital used in human capital production; and $v_{ht}k_t$ is the amount of physical capital in the human sector used for human investment production. Also, l_{ht} denotes the share of human capital utilized in human capital investment production; and with $\phi_2 \in [0, 1]$ being the share of physical capital in human capital investment production, it is produced according to the following constant returns to scale production function,

$$i_{ht} = G(v_{ht}k_t, l_{ht}h_t) = A_h e^{z_t^h}(v_{ht}k_t)^{\phi_2}(l_{ht}h_t)^{1-\phi_2}. \tag{2.7}$$

z_t^h is the human sector productivity shock process that is assumed to evolve according to a stationary first order autoregressive process in logarithmic form:

$$z_t^h = \rho_h z_{t-1}^h + \epsilon_t^h, \tag{2.8}$$

where the innovations ϵ_t^h is a sequence of independently and identically distributed normal random variables with zero mean and constant variance σ_h^2.

With no externalities, the optimal allocations of the economy coincides with the result of the social planner's problem, which can be stated as[4]

$$\max_{\{c_t, l_{gt}, l_{ht}, v_{gt}, v_{ht}, k_{t+1}, h_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \frac{[c_t x_t^A]^{1-\sigma} - 1}{1 - \sigma} \tag{2.9}$$

subject to (2.2), (2.3), (2.4), (2.6), and (2.7).

[4]The first-order and the equilibrium conditions of the model can be found in *Appendix A.1.*

2.2.2 Definition of the Optimal Allocations

Definition 1 *The optimal allocations of this model is a set of contingent plans* $\{c_t, k_{t+1}\}$ $\{h_{t+1}, v_{gt}, v_{ht}, x_t, l_{gt}, l_{ht}\}$ *that solve the central planner's maximization problem in (2.9) for some initial endowment* $\{k_0, h_0\}$ *and exogenous stochastic technology processes* $\{z_t^g, z_t^h\}$*, with initial conditions* $\{z_0^g, z_0^h\}$*.*

Definition 2 *A deterministic balanced growth path allocations of this model is a set of paths* $\{\bar{c}_t, \bar{k}_{t+1}, \bar{h}_{t+1}, \bar{v}_{gt}, \bar{v}_{ht}, \bar{x}_t, \bar{l}_{gt}, \bar{l}_{ht}\}$ *that solve the central planner's maximization problem in (2.9) for some initial endowment* $\{k_0, h_0\}$ *and exogenous technology parameters* $\{\bar{z}_t^g = 0, \bar{z}_t^h = 0\}$*, such that* $\{\bar{c}_t, \bar{k}_{t+1}, \bar{h}_{t+1}\}$ *grow at a common trend, and* $\{\bar{v}_{gt}, \bar{v}_{ht}, \bar{x}_t, \}$ $\{\bar{l}_{gt}, \bar{l}_{ht}\}$ *are constant.*

2.3 Model Simulation

By normalizing the variables that grow along the *balanced growth path* (BGP), and then log-linearizing all of the equilibrium conditions of the model around its normalized growth path, one gets a stochastic system of linear equations. Here we normalize by the human capital stock ,h_t, as in Dang et al. (2011), Benk et al. (2010) and Maffezzoli (2000). For both models the respective systems are solved for in terms of k_t/h_t and the two shock processes, z_t^g and z_t^h. The calibrated models are solved with the method of undetermined coefficients as described in Uhlig (1998).

2.3.1 Data

The data series used for calibration purposes are for the United States from 1959:Q1 to 2014:Q2.[5]. The data series have been filtered by a Christiano and Fitzgerald (2003) type asymmetric band-pass filter following Baxter and King (1999), Benk et al. (2010), and Basu et al. (2012). For matching key moments and calculating simulated moments the low frequency component of the data has a periodicity of 32 to 200 quarters; the business cycle component has a periodicity of 6 to 32 quarters; and the high frequency is defined as 2 to 6 quarters in periodicity . The simulated data is also filtered by the underlying band-pass filter at the medium term frequency, which includes all of the components and has a defined periodicity of 2 up to 200 quarters in line with the definition of Comin and Gertler (2006). Altogether there are 55 years of data.

[5] All data series are of quarterly frequency and from the period noted above except where noted otherwise in Chapter 1.

2.3.2 The Calibration

Table 2.1 contains 9 different pieces of information about the data from different sources. These pieces of information then are used as the calibration targets for structural parameters when implementing our calibration procedure.

Description	Calibration Target	Target Value/Range	Achieved Value	Target Source
Output Growth Rate	g	0.0035	0.0035	Gomme and Rupert (2007); NIPA
Consumption Output Ratio	c/y	0.70	0.81	Gomme and Rupert (2007)
Phy. Investment Output Ratio	i_k/y	0.30	0.19	Gomme and Rupert (2007)
Leisure	x	0.50	0.50	Gomme and Rupert (2007)
Labor Time	l_g	0.30	0.28	Jones et al. (2005)
Learning Time	l_h	0.24	0.22	Gomme and Rupert (2007)
Real Interest Rate	r^k	$0.050 - 0.019$	0.051	Greenwood et al. (1993); ?
Phy. Capital Depreciation Rate	δ_k	0.025	0.016	Gomme and Rupert (2007)
Hum. Capital Depreciation Rate	δ_h	$0.008 - 0.0025$	0.029	Jorgenson and Fraumeni (1991)

Table 2.1: Target values of the baseline calibrations for the U.S. - Model 1.

Following Gomme and Rupert (2007) for the calibration of the model in Section 1.2 we set the balanced growth rate, g, and leisure, x, equal to their long-run target values. Their values are 0.0035 and 0.50. Given these strict targets the leisure preference weight A, and the human capital sector scale parameter A_h can be directly pinned down from the long-run version of the intra-temporal margin and the inter-temporal margin with respect to human capital combined with the other 7 structural parameters.

After setting the strict primary targets for the calibration the rest of the structural parameters are searched for by the so-called *Simualated Annealing* algorithm of the Global Optimization Toolbox for *Matlab*, in such a way that the distance between a vector of selected U.S. data moments and the model simulation based moments is minimized. This methodology further generalizes the iterative calibration procedure of Jermann (1998). Furthermore, in order to calibrate the shock parameters within each guess of the structural parameter search convergence is created between the computer-chosen calibration parameters and the estimated parameters of the shocks that are identified using the Ingram et al. (1997), Benk et al. (2005), and Nolan and Thoenissen (2009) methodology of shock construction for the designated time period.

For this model the set of structural parameters that we directly search for with *Simulated Annealing* within our procedure is $\{\beta, \sigma, \phi_1, \phi_2, A_g, \delta_k, \delta_h\}$. Given this information the baseline parameterization shall imply the following for the real interest rate r^k:

$$\frac{(1+g)^{\sigma}}{\beta} = [1 + r^k - \delta_k]. \tag{2.10}$$

In Gomme and Rupert (2007) the pre-tax interest rate implied is 13.2 percent per annum, which corresponds to 3.3 percent per quarter. Poterba (1998) suggests an annual return of 8.6 percent; Siegel (1992) finds it to be 7.7 percent similarly to Mehra and Prescott (1985)) for overlapping long periods in U.S. data. In Greenwood et al. (1993) on the other hand the pre-tax real interest rate, which coincides with our measure of interest rate in the absence of taxes, is implied to be 20.5 percent per annum, which corresponds to a 5 percent interest rate per quarter. Consequently, we set our target range for the real interest rate between 5 and 2 percent per quarter. Given the long-run value of g the real interest rate further depends on the depreciation rate of physical capital δ_k, the time preference parameter β, the CES parameter σ.

Estimates for the depreciation rate of physical capital vary in the literature. In this we target the physical capital depreciation rate at 2.5 percent per quarter, which is standard in the literature and supported by the estimate of 2.7 percents by Gomme and Rupert (2007) based on data from the Bureau of Economic Analysis (BEA).[6] Then allowing our calibration algorithm to search within the range of 1.5 and 3 percents undershoots our target and yields a quarterly depreciation rate of 1.6 percent.

For the CES parameter σ we set our target range between 0.34 to 2, which corresponds with a lower bound based on the estimates of Hall (1988) and an upper bound based on Mehra and Prescott (1985). Within this range our procedure pins down σ as 0.84. Then by limiting the discount factor search range in our procedure to 0.95 to 0.99 we are able to pin down β so that the BGP relationship in equations (2.10) holds. This yields the value of 0.9692 for β.

For the human sector parameters considering the depreciation rate of human capital Jorgenson and Fraumeni (1991) suggest an annual depreciation rate between 1 and 3 percent annually; Jones et al. (2005) estimate a lower bound for human capital depreciation at 1.5 percent yearly; whereas DeJong and Ingram (2001) estimate a 0.5 percent rate per quarter. In our calibration process we allow for a lower bound of 0.1 percent per quarter remaining close to DeJong and Ingram (2001) and an upper bound of 3 percent as in DeJong et al. (1996). The calibration process yielded a quarterly depreciation rate of 2.8 percent.

The share of capital in goods production, ϕ_1, and in human production ϕ_2, is chosen by our procedure to target 0.3 time share in goods labor as in Jones et al. (2005), and a time share of 0.24 in the human investment sector. We bounded our algorithm to search for the capital share in goods production within the range of 0.25 and 0.42; and to search within the range of 0.08 and 0.25 for the capital share in human investment production. For the share in human production the lower bound corresponds to the estimate of Jorgenson and Fraumeni (1991) for educational

[6]For other estimates, see Musgrave (1992), Epstein and Denny (1980), Kollintzas and Choi (1985), Bischoff and Kokkelenberg (1987), and Nadiri and Prucha (1993) among many other.

output; whereas they also estimate 17 percent share of physical capital in on the job training. The capital share in goods production is then 0.41. The capital share in human investment production is 0.146. Given the values the calibration procedure yielded the labor time obtained in goods production is 0.28, whereas the learning time is 0.22.

The capital shares, the human and physical capital depreciation rates, and the goods sector scale parameter combined with our strict targets then pin down the scale parameter for the human sector so that the inter-temporal margins hold. Finally, through the intra-temporal margins in the respective models we can pin down the value of the preference weight of leisure. The complete list of calibrated structural parameters can be found in Table 2.2 below. Table 2.3 lists the parameters of the exogenous shock processes, which are obtained through the iterative step of the calibration procedure.

Parameter	Description	New Value	Dang et al. (2011)
β	Subjective Discount Factor	0.969	0.986
σ	CES Parameter	0.844	1
A	Weight of Leisure in Preference	1.27	1.55
A_g	Scale Parameter of Goods Sector	1.86	1
A_h	Scale Parameter of Human Sector	0.09	0.0461
ϕ_1	Physical Capital Share in Goods Production	0.41	0.36
ϕ_2	Physical Capital Share in Human Investment	0.14	0.11
δ_k	Depreciation Rate of Physical Capital	0.016	0.02
δ_h	Depreciation Rate of Human Capital	0.028	0.005

Table 2.2: Structural parameter values of the baseline calibrations for the U.S. - Model 1 and 1a

Parameter	Description	New Value	Dang et al. (2011)
ρ_g	Auto-correlation of TFP	0.979	0.95
ρ_h	Auto-correlation of Human Shock	0.982	0.95
σ_g^2	Variance of TFP	0.00015	0.0007
σ_h^2	Variance of Human Productivity Shock	0.00015	0.0007
$\sigma_{g,h}$	Correlation of Shock Innovations	0.996	1.00

Table 2.3: Shock process parameter values of the baseline calibration - Model 1 and 1a.

2.3.3 The Estimated Goods Sector TFP: 1971:Q4 to 2012:Q2

Using the methodology in Ingram et al. (1997), Benk et al. (2005) and Nolan and Thoenissen (2009) a traditionally obtained Solow residual series for the goods sector TFP shock is compared to estimated series obtained from the model solutions of the two variants of the model. As noted in Benk et al. (2005) the estimated shock processes are sensitive to the combinations of the data series used in the process. However, since the focus of this exercise is to obtain only the goods sector TFP series that can be constructed using conventional growth accounting based methods [i.e. $\hat{z}_t^g = \hat{y}_t - \phi_1 \hat{k}_t - (1-\alpha)\hat{l}_{gt}$] we used the combination that yielded the most highly correlated series with that of the conventional TFP. For obtaining the traditional TFP series the capital share parameter specified by the new calibration is used [i.e. $\phi = 0.41$].

In Figure 2.1 we present the unfiltered TFP series obtained by using the new calibration, denoted by Model 1, and the original calibration as inDang et al. (2011), denoted by Model 1a, compared to a traditionally obtained goods sector TFP using the new calibration described above. The series presented here are the ones with the highest correlations to the traditional one with values of 0.50 cross-correlation with Model 1 and 0.60 for Model 1a. Here we compare the two DSGE derived TFP series to a traditionally obtained one and decompose them at different frequencies using a Christiano and Fitzgerald (2003) band-pass filter.[7]

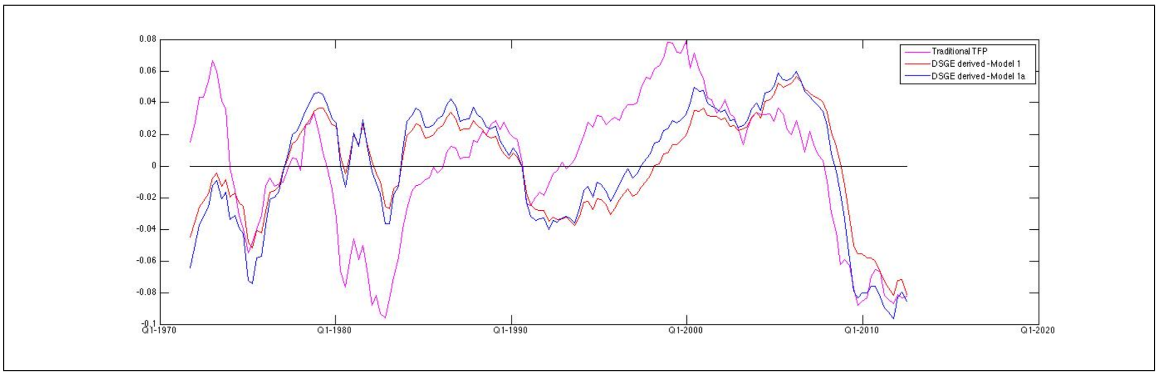

Figure 2.1: Total Factor Productivity - DSGE derived (Model 1 and 1a) versus traditionally estimated shocks.

Despite the different cross-correlations of the DSGE derived TFP series one can observe in Figure 2.1 that the two DSGE based series closely follow each other. In case of both series relative to the traditionally obtained TFP series it is notable that in terms of shape both DSGE series closely follow the traditionally obtained one, even though, both fail to capture the recessions in the early 1980s and the high growth periods in the early 1970s and 1990s in terms of magnitude. On the other hand, the model does extremely well in capturing the fall in productivity over the Great Recession for both calibrations.

Figure 2.2 and 2.3 show the DSGE derived TFP series and the traditional one decomposed at different frequencies. Figure 2.2 shows the traditional one relative to the Model 1 based series, where it is notable that during the early 1970s, early 1980s, and mid 1990s the two series follow each other at a larger distance in the low frequency; otherwise, they closely follow each other at the business cycle frequency. This explains why in the unfiltered series the estimated series for Model 1 fails to capture the magnitude during the periods in question.

[7]The windows for the band-pass filter frequencies are defined in detail in Section 2.4. Furthermore, the detailed U.S. data description and the shock estimation methodology can be found in

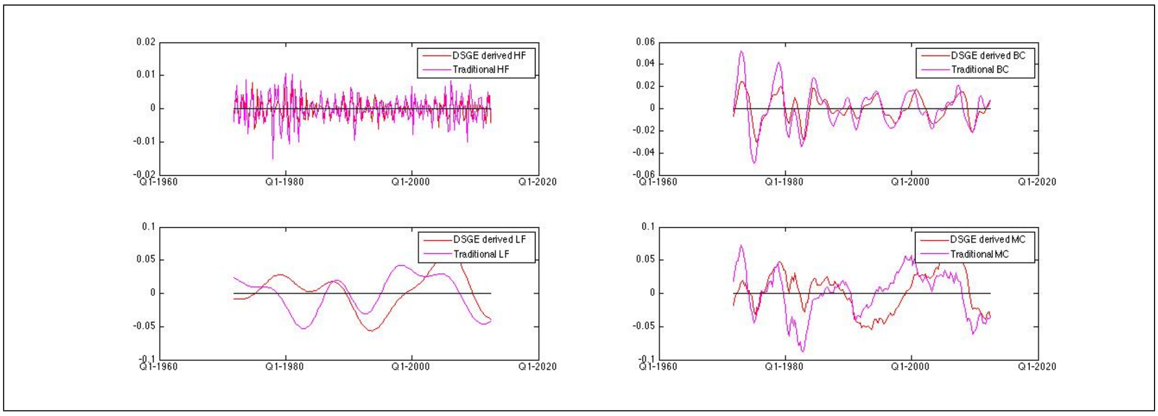

Figure 2.2: Total Factor Productivity - Model 1 DSGE derived versus traditionally estimated shocks frequency components.

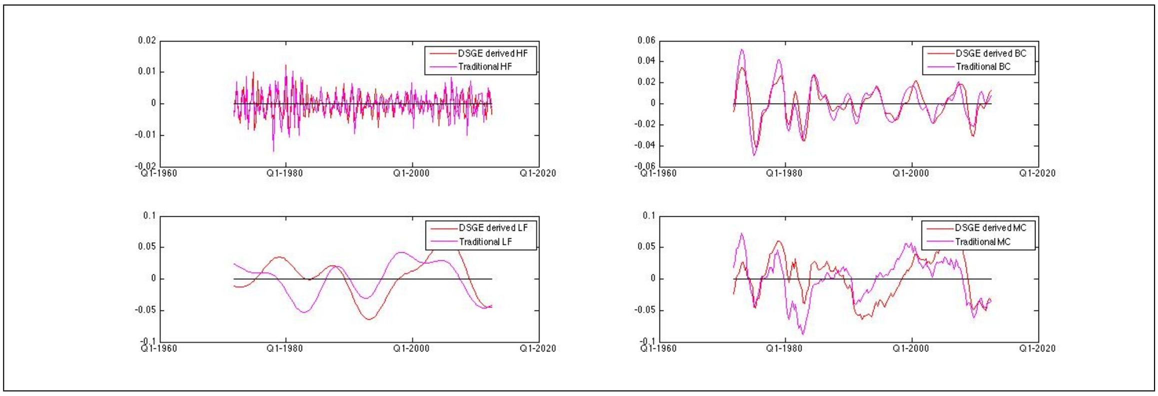

Figure 2.3: Total Factor Productivity - Model 1a DSGE derived versus traditionally estimated shocks frequency components.

In Figure 2.3 we present the co-movement of the Model 1a based TFP series with that of the Solow residual based one. Here it is evident that the Model 1a based series relative to the traditional TFP follows an identical trend as for Model 1 with minimal differences in magnitude. More specifically, the estimated series fail to closely follow the traditional one at the low and medium term cycle frequencies, which the business cycle component cannot fully offset.

2.4 Simulation Results

In this section the model simulation results are presented that include the impulse responses to three different type of shocks; matching business cycle correlations, volatilities and the output growth persistence to U.S. data and comparing it to the calibration of Dang et al. (2011). The three different shocks are the goods sector TFP shock; the human sector productivity shock; and an aggregate economy wide shock, which is defined as a shock that hits both the human sector and the goods

Chapter 1.

sector at the same time similarly to Benhabib et al. (1997). In comparing the simulation results to the calibration of Dang et al. (2011) Model 1 represents the model with the calibration presented in the previous section; whereas, Model 1a is the model with the original calibration of Dang et al. (2011).

2.4.1 Impulse Responses

In this subsection the impulses of selected key variables are shown in Figures 2.4 to 2.7. Impulse responses for normalized [i.e. by human capital] variables coincide with responses of growing variables as in Maffezzoli (2000).

The reason for this lies in the fact that impulse responses can be classified as a short-run deterministic simulation of the model subject to a change in one or both of the exogenous processes. In the deterministic case the human capital stock is a linear trend. By normalizing (i.e. dividing) with it translates in a log-linear environment to linear detrending. Hence the impulse responses for normalized variables are equivalent to the impulse responses of the non-stationary growing original variables. The interpretation of the impulse responses in this case must be made relative to the BGP trend, which is equivalent to the trend defined by the human capital stock. Therefore, in the impulse responses that follow the zero line represents the balanced growth path of the underlying model.

Impulse Response Functions

Here the impulse responses are shown to selected variables for Model 1. Figure 2.4 shows the responses of selected variables to a goods sector TFP shock. It shows that a goods productivity shock makes the agent substitute away from human capital investment time and leisure towards labor. This effort increases the physical capital investment rate (i_k/y), with a consequent gradual increase in the physical capital to human capital ratio. More physical investment in turn increases output growth temporarily above the balanced growth rate.
Figure 2.5 shows the responses of selected variables to a positive human sector productivity shock. This makes the agent allocate more time to the human capital investment sector instead of the the physical sector or leisure. This is due to the relatively cheaper cost of human capital to goods output. This results in a lower rate of investment in the physical capital sector and lower rate of output growth temporarily.
Figure 2.6 and 2.7 show the responses of selected variables to a positive aggregate shock. Similarly to Benhabib et al. (1997) we define the aggregate shock as highly correlated shocks that occur at the same time in both sectors. This on impact will decrease the physical investment rate relative to output, which gradually increases

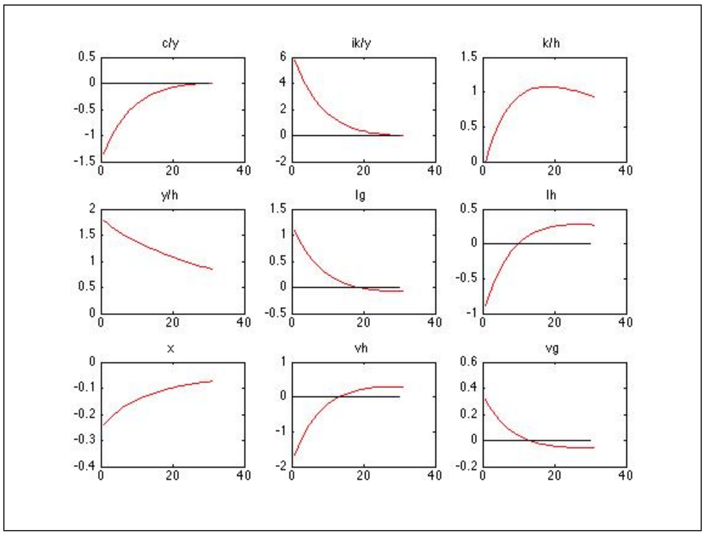

Figure 2.4: Impulse responses in percent deviation from the steady state - Goods TFP Shock in Model 1.

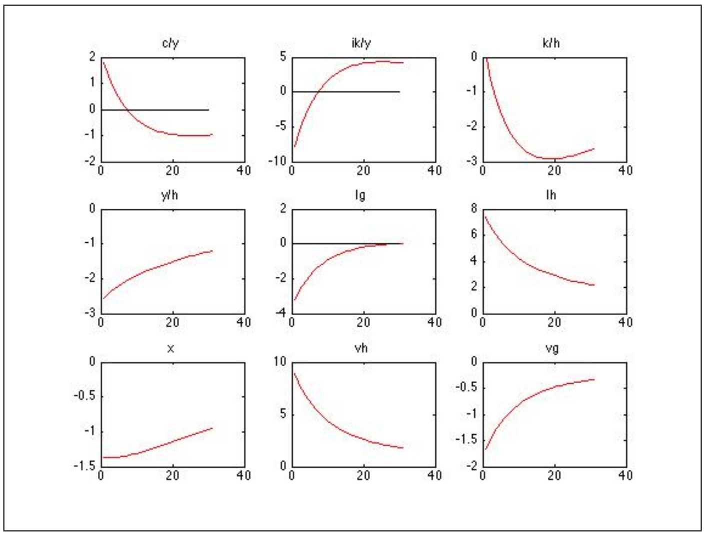

Figure 2.5: Impulse responses in percent deviation from the steady state - Human Productivity Shock in Model 1.

and overshoots the balanced growth rate. Leisure and labor decrease on impact as a result of a small decrease in the rate of physical investment. The decline in working hours on impact is consistent with the empirical finding of Gali (1999), who identifies a negative correlation between productivity and working hours using VAR evidence. Therefore, the observed decline in working hours in the presence of higher labor productivity is consistent with Model 1. The aggregate shock's effect in the human sector outweighs that of the goods sector's effect. Labor and physical investment increase and the latter overshoots its BGP level as the result of a more persistent effect of the aggregate shock in the goods sector.

Figure 2.7 then shows that this model is in line with the findings of Bond et al. (1996) regarding the fact that the allocation of factor inputs are solely determined

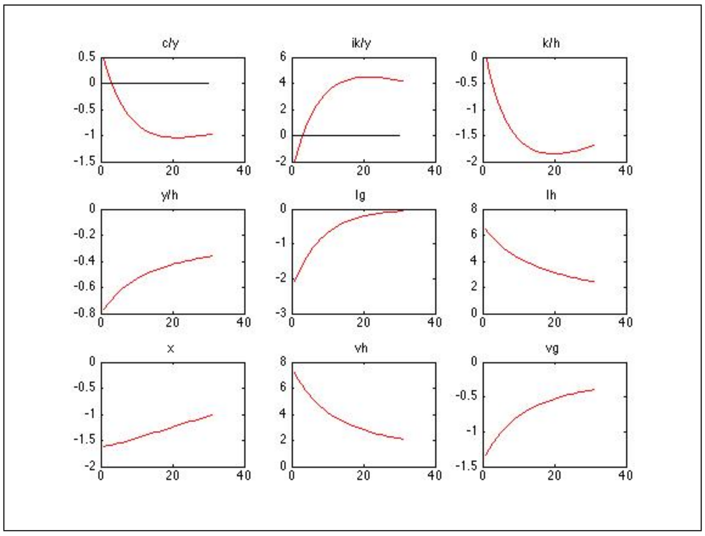

Figure 2.6: Impulse responses in percent deviation from the steady state (1) - Aggregate Shock in Model 1.

by factor prices and the Stolper and Samuelson (1941) and the Rybczynski (1955) effects. More specifically, the relative price of human capital, p_{ht} jumps on impact above its steady state value. This occurs when wages, w_t, are higher than the real return on physical capital, r_t^k. This can be clearly observed on Figure 2.7 on impact of the aggregate shock. As a result, physical capital becomes relatively cheaper and is allocated more towards goods production, which in turn raises the return on it.

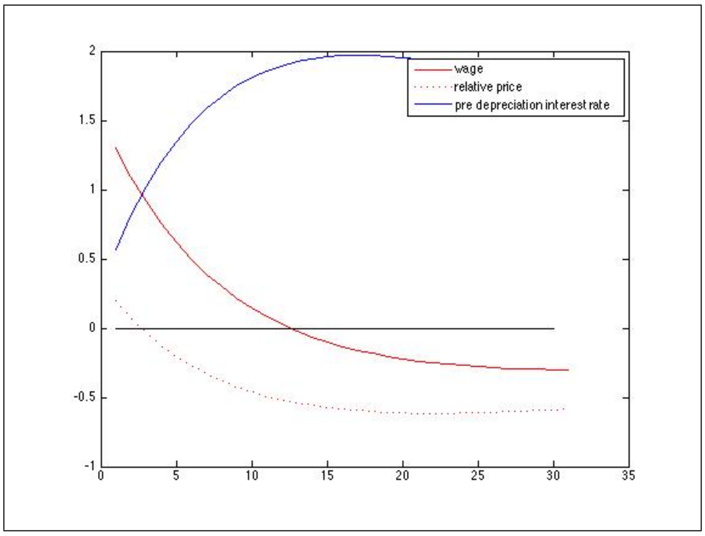

Figure 2.7: Impulse responses in percent deviation from the steady state (2) - Aggregate Shock in Model 1.

2.4.2 Key Correlations

Table 2.4 reports the contemporaneous correlations between output and other endogenous variables at different frequencies including the Comin and Gertler (2006) "medium term" frequency, at which domain a single cycle matches a "medium tern" cycle equivalent in length to all the other three frequencies combined. The different frequencies are the high frequency [2 - 6 quarters]; business cycle frequency [6 - 32 quarters]; low frequency [32 - 200 quarters]; and the medium cycle [2 - 200 quarters]. The main focus of this paper in terms of matching correlations is to see if such endogenous growth models are able to explain the standard business cycle facts as well as the low frequency, long-term correlations.

The co-movement of consumption and investment with output is closely matched by Model 1 and 1a at the business cycle frequency with Model 1a slightly closer to the data. On the other hand, both models fail to fully capture the high frequency co-movement between consumption and output. At the low frequency and the medium cycle both models are able to match the data reasonably well, with once again Model 1a doing slightly better in this respect.

Variable	High freq. 2 -6 qrs.	Bus. cyc. 6 - 32 qrs.	Low freq. 32 - 200 qrs.	Med. term 2 - 200 qrs.
$corr(c_t, y_t)$				
Data	0.469	0.894	0.979	0.962
Model 1	0.968	0.928	0.903	0.898
Model 1a	0.873	0.903	0.968	0.899
$corr(i_{kt}, y_t)$				
Data	0.811	0.943	0.809	0.815
Model 1	0.990	0.987	0.829	0.839
Model 1a	0.993	0.980	0.896	0.901
$corr(l_{gt}, y_t)$				
Data	0.404	0.731	0.614	0.614
Model 1	-0.154	0.512	0549	0.454
Model 1a	0.159	-0.918	-0.459	-0.493
$corr(l_{ht}, y_t)$				
Data	-	-	-	-
Model 1	0.133	-0.336	-0.119	-0.128
Model 1a	-0.166	0.936	0.532	0.556
$corr(c_t, l_{gt})$				
Data	0.215	0.764	0.621	0.624
Model 1	0.010	0.592	0.647	0.507
Model 1a	0.158	-0.719	-0.053	-0.106

Table 2.4: Matching Key Correlations at Different Frequencies - Model 1 and 1a.

Labor hours' correlation with output is surprisingly well-matched by Model 1 with Model 1a being unable to capture the co-movement of labor hours with output predicting it to be counter-cyclical. As in DeJong et al. (1996) Model 1 is also able to generate a negative correlation between learning time and output as suggested by theory; whereas, Model 1a with the calibration of Dang et al. (2011) fails to capture this.[8]

Lastly, one may observe as it can be seen in Table 2.4 that the endogenous growth mechanism is able to break the negative relationship between labor hours and consumption given the new calibration that is one of the main reasons why the

[8]For more on data evidence, see Dellas and Sakellaris (2003).

standard RBC model cannot capture output growth persistence as pointed out by Benhabib et al. (1997).

Both calibrations perform reasonably well at explaining the correlations of consumption and physical investment with output; meanwhile, the new calibration applied to the model enables it to better capture the movement of labor and learning hours.

2.4.3 Volatilities

Table 2.5 reports the simulated and actual data standard deviations of the underlying key variables at all of the defined four frequencies. Output, investment, and consumption growth volatilities are well matched at the business cycle frequency by Model 1; however, using the calibration of Dang et al. (2011) the model performs better at the high and low frequencies as well as the medium cycle. Model 1 overshoots the volatility at the high and low frequencies, whereas Model 1a undershoots the volatilities at the business cycle frequency.

Variable	High freq. 2 -6 qrs.	Bus. cyc. 6 - 32 qrs.	Low freq. 32 - 200 qrs.	Med. term 2 - 200 qrs.
$vol(g_{y,t})$				
Data	0.0069	0.0065	0.0035	0.0100
Model 1	0.094	0.0064	0.0065	0.0130
Model 1a	0.0062	0.0043	0.0035	0.0083
$vol(g_{c,t})$				
Data	0.0038	0.0037	0.0024	0.0058
Model 1	0.0034	0.0027	0.0063	0.0075
Model 1a	0.0016	0.0015	0.0023	0.0032
$vol(g_{i_k,t})$				
Data	0.0203	0.0210	0.0101	0.0305
Model 1	0.0360	0.0240	0.1700	0.0460
Model 1a	0.0200	0.0140	0.0089	0.0260
$vol(y_t)$				
Data	0.0044	0.0169	0.0464	0.0498
Model 1	0.0070	0.0140	0.1000	0.1000
Model 1a	0.0038	0.0130	0.0430	0.0450
$vol(c_t)$				
Data	0.0024	0.0098	0.0376	0.0390
Model 1	0.0032	0.0053	0.0890	0.0890
Model 1a	0.0010	0.0056	0.0330	0.0330
$vol(i_{kt})$				
Data	0.0131	0.0547	0.0891	0.1075
Model 1	0.0240	0.0530	0.2300	0.2400
Model 1a	0.0130	0.0370	0.0940	0.100
$vol(l_{gt})$				
Data	0.0017	0.0050	0.0224	0.0230
Model 1	0.0140	0.0260	0.0340	0.0450
Model 1a	0.0110	0.0250	0.0380	0.0470
$vol(l_{ht})$				
Data	-	-	-	-
Model 1	0.0430	0.0790	0.1500	0.1800
Model 1a	0.0630	0.1500	0.250	0.3000

Table 2.5: Matching Volatilities at Different Frequencies - Model 1 and 1a.

The volatilities of the model generated log-level series for output, consumption, and physical investment Model 1 perform extremely well at the business cycle frequency. On the other hand, the same trend can be observed at the low frequencies as in the case of the growth rates, meaning, that Model 1 tends to overshoot U.S. data volatilities; meanwhile, Model 1a is able to match them relatively well at the high and low frequencies. The only clear shortcoming of both models is that they overpredict the volatility of labor hours at all frequencies.

Overall, it can be stated that the underlying model with the calibration presented in the previous section can match volatilities better at the business cycle frequency; meanwhile, the calibration of Dang et al. (2011) performs better at the low frequency and the medium cycle.

2.4.4 Persistence

Table 2.6 reports the autocorrelations of key variables up to three lags. Normally, as in Benhabib et al. (1997) the focus is usually on explaining output growth and consumption persistence to a traditional goods sector TFP shock. Traditional RBC models fail to reproduce the output, consumption, and physical investment growth persistence beyond the first degree or at all.

Variable	Lag 1	Lag 2	Lag 3
$\rho(g_{y,t})$			
Data	0.265	0.216	0.157
Model 1	0.328	0.280	0.283
Model 1a	0.208	0.212	0.196
$\rho(g_{c,t})$			
Data	0.365	0.280	0.305
Model 1	0.712	0.689	0.687
Model 1a	0.717	0.696	0.667
$\rho(g_{ik,t})$			
Data	0.262	0.75	0.082
Model 1	0.149	0.084	0.08
Model 1a	0.090	0.102	0.089
$\rho(l_{gt})$			
Data	0.987	0.974	0.961
Model 1	0.851	0.720	0.619
Model 1a	0.932	0.866	0.801

Table 2.6: Matching Auto-correlation Functions of Key Variables - Model 1 and 1a.

Model 1 is able to capture the first degree autocorrelation of output growth even though it somewhat overshoots it; while Model 1a undershoots it. Model 1a may be perceived with a clear advantage over Model 1 in terms of explaining higher degree autocorrelation of output growth, however; as it can be seen in Figure 2.8 it only appears so because it undershoots it at the first degree. Neither calibrations is able to capture to drop in higher degree autocorrelations of output growth persistence.

Consumption growth persistence is significantly overestimated by both calibra-

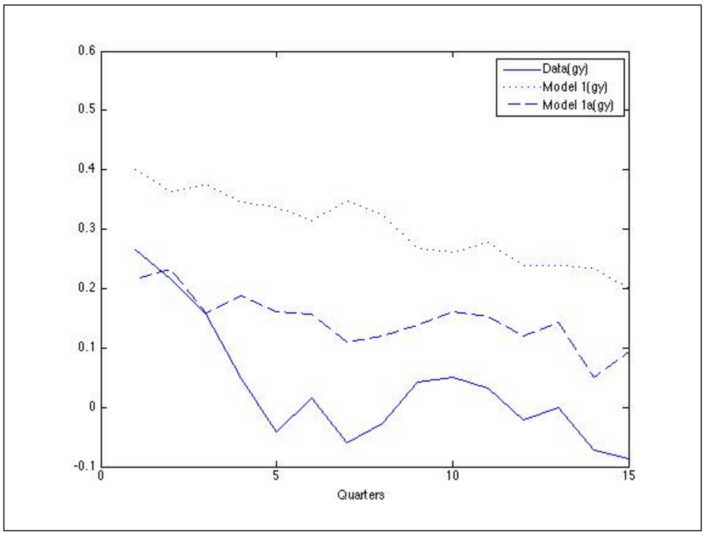

Figure 2.8: Persistence of output growth (US Data, Model 1, Model 1a).

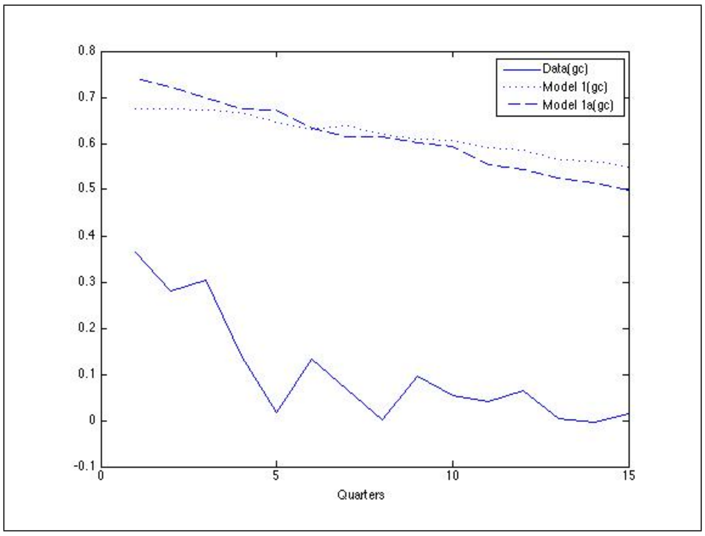

Figure 2.9: Persistence of consumption growth (US Data, Model 1, Model 1a).

tions of the model, with almost identical results at higher degrees as well. Investment growth persistence is underestimated by both model calibrations with Model 1a performing worse at the first degree, whereas it must be noted that in both cases the consumption and investment growth persistence have a similar slope with the starting point [i.e. first degree auto-correlation] being different as it can be observed in Figures 2.9 and 2.10. Lastly, in Table 2.6 one can see that Model 1a is clearly performing better in capturing the persistence of labor hours.

Overall, even though output growth persistence is captured by both models relatively well at the first degree, none of the two calibrations can reproduce higher degree auto-correlations of the growth rates of output, consumption and investment.

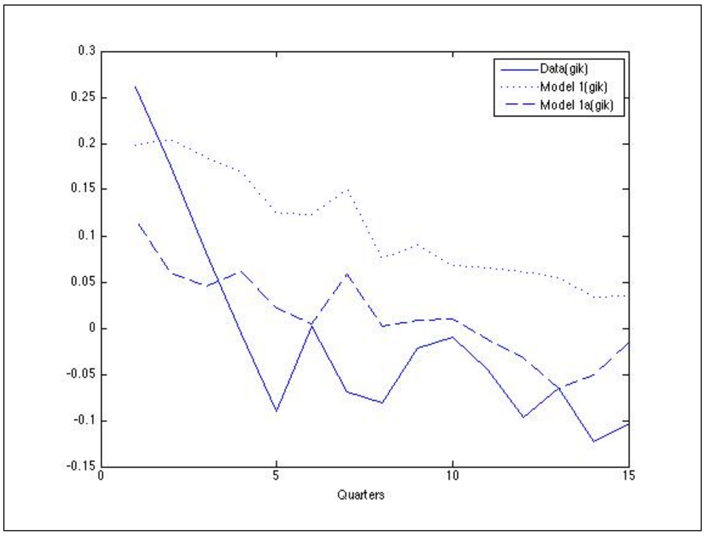

Figure 2.10: Persistence of physical investment growth (US Data, Model 1, Model 1a).

2.5 Conclusion

In conclusion, in this chapter it has been shown that by applying a new calibration to an otherwise standard business cycle model with endogenous growth one can obtain somewhat improved results. Applying a new calibration procedure to the model of Dang et al. (2011) we could clearly improve upon the model's performance in terms of standard business cycle frequency correlations and the co-movement of labor and learning hours with output. Furthermore, we could show that with different calibrations there could be a trade-off between explaining volatilities at the business cycle and the lower frequencies.

Overall, both calibrations perform similarly. The common shortcomings of both calibrations of the model are that they overshoot labor volatility; and that they are unable to match the drop in output, consumption, and investment growth persistence at higher degrees. In the meantime, the clear improvement due to a new calibration is the ability to capture the strong positive correlation between labor hours and output at the business cycle and lower frequencies as well; while being able to generate counter-cyclical learning hours.

A good sign in terms of capturing output growth persistence more accurately is that with the new calibration the positive correlation between consumption and labor hours can be captured relatively well, in which standard RBC models and the model with the original calibration of Dang et al. (2011) failed.

In view of the improvements and the shortcomings of the underlying endogenous growth model with a new calibration there is room for further improvements.

A.1 Equilibrium Conditions

Define the Lagrange multiplier of the representative agent's budget constraint as λ_t, and that of the human capital accumulation's as χ_t. Then the agent's first order conditions are the following,

$$c_t: \quad c_t^{-\sigma} x_t^{A(1-\sigma)} = \lambda_t; \tag{A.I-1}$$

$$l_{gt}: \quad A c_t^{1-\sigma} x_t^{A(1-\sigma)-1} = \lambda_t w_t h_t; \tag{A.I-2}$$

$$l_{ht}: \quad A c_t^{1-\sigma} x_t^{A(1-\sigma)-1} = \chi_t (1-\phi_2) A_h e^{z_t^h} \left[\frac{v_{ht} k_t}{l_{ht} h_t}\right]^{\phi_2} h_t; \tag{A.I-3}$$

$$v_{gt}: \quad \lambda_t \phi_1 A_g e^{z_t^g} \left[\frac{v_{gt} k_t}{l_{gt} h_t}\right]^{\phi_1 - 1} k_t = \chi_t \phi_2 A_h e^{z_t^h} \left[\frac{(1-v_{gt}) k_t}{l_{ht} h_t}\right]^{\phi_2 - 1} k_t; \tag{A.I-4}$$

$$\begin{aligned} k_{t+1}: \quad \lambda_t = {} & \beta E_t \lambda_{t+1} \left[1 + r_{t+1}^k v_{gt+1} - \delta_k\right] \\ & + \beta E_t \chi_{t+1} \phi_2 A_h e^{z_{t+1}^h} \left[\frac{v_{ht+1} k_{t+1}}{l_{ht+1} h_{t+1}}\right]^{\phi_2 - 1} v_{ht+1}; \end{aligned} \tag{A.I-5}$$

$$\begin{aligned} h_{t+1}: \quad \chi_t = {} & \beta E_t \chi_{t+1} \left[1 + (1-\phi_2) A_h e^{z_{t+1}^h} \left[\frac{v_{ht+1} k_{t+1}}{l_{ht+1} h_{t+1}}\right]^{\phi_2} l_{ht+1} - \delta_h\right] \\ & + \beta E_t \lambda_{t+1} w_{t+1} l_{gt+1}; \end{aligned} \tag{A.I-6}$$

where r_t^k and w_t denote the own marginal productivity conditions of physical and human capital such that $r_t^k \equiv F_{1t} = \phi_1 A_g e^{z_t^g} (v_{gt} k_t)^{\phi_1 - 1} (l_{gt} h_t)^{1-\phi_1}$ and $w_t \equiv F_{2t} = (1-\phi_1) A_g e^{z_t^g} (v_{gt} k_t)^{\phi_1} (l_{gt} h_t)^{-\phi_1}$. Also, define $p_{ht} \equiv \frac{\chi_t}{\lambda_t}$ as the relative price of human capital in terms of physical capital. Note that since physical capital and goods output are perfect substitutes in the absence of adjustment costs then p_{ht} also denotes the relative price of human capital investment good to the consumption good. Then the representative agent's equilibrium conditions can be stated as:

$$A_g e^{z_t^g}(v_{gt}k_t)^{\phi_1}(l_{gt}h_t)^{1-\phi_1} = c_t + i_{kt}; \tag{A.I-7}$$

$$A_h e^{z_t^h}((1-v_{gt})k_t)^{\phi_2}(l_{ht}h_t)^{1-\phi_2} = h_{t+1} - (1-\delta_h)h_t; \tag{A.I-8}$$

$$p_{ht} = \left[\frac{A_g}{A_h}\right]\left[\frac{e^{z_t^g}}{e^{z_t^h}}\right]\left[\frac{1-\phi_1}{1-\phi_2}\right]^{1-\phi_2}\left[\frac{\phi_1}{\phi_2}\right]^{\phi_2}\left[\frac{v_{gt}k_t}{l_{gt}h_t}\right]^{\phi_1-\phi_2}; \tag{A.I-9}$$

$$\frac{A}{x_t}\frac{c_t}{h_t} = (1-\phi_1)A_g e^{z_t^g}\left[\frac{v_{gt}k_t}{l_{gt}h_t}\right]^{\phi_1}; \tag{A.I-10}$$

$$x_t = 1 - l_{gt} - l_{ht}; \tag{A.I-11}$$

$$\frac{1-\phi_1}{\phi_1}\frac{v_{gt}k_t}{l_{gt}h_t} = \frac{1-\phi_2}{\phi_2}\frac{(1-v_{gt})k_t}{l_{ht}h_t}; \tag{A.I-12}$$

$$i_{kt} = k_{t+1} - (1-\delta_k)k_t; \tag{A.I-13}$$

$$1 = \beta E_t\left[\left(\frac{c_t}{c_{t+1}}\right)^{\sigma}\left(\frac{x_{t+1}}{x_t}\right)^{A(1-\sigma)}\left[1 + r_{t+1}^k - \delta_k\right]\right]; \tag{A.I-14}$$

$$\begin{aligned} 1 = \beta E_t &\left(\frac{c_t}{c_{t+1}}\right)^{\sigma}\left(\frac{x_{t+1}}{x_t}\right)^{A(1-\sigma)}\frac{p_{ht+1}}{p_{ht}} \\ &\left[1 + (l_{gt+1}+l_{ht+1})(1-\phi_2)A_h e^{z_{t+1}^h}\left[\frac{(1-v_{gt+1})k_{t+1}}{l_{ht+1}h_{t+1}}\right]^{\phi_2} - \delta_h\right]. \end{aligned} \tag{A.I-15}$$

Equation (A.I-7) is the goods market clearing condition; equation (A.I-8) is the human capital law of motion; equation (A.I-9) defines the relative price of human capital in units of consumption goods; equation (A.I-10) is the intra-temporal condition that governs the substitution between leisure and consumption. Equation (A.I-11) is the time constraint; equation (A.I-12) equates the weighted factor intensities across sectors; and (A.I-13) is the physical capital law of motion. Equations (A.I-14) and (A.I-15) are the inter-temporal capital efficiency conditions with respect to physical and human capital, where the capacity utilization of human capital is equivalent to total working time, $(1-x_t)$.

The set of 9 equations in (A.I-7) - (A.I-15) and the marginal efficiency conditions fully describes Model 1. All together, there are 11 equations in 11 unknowns $\{k_{t+1}, h_{t+1}, c_t, i_{kt}, l_{gt}, l_{ht}, x_t, v_{gt}, p_{ht}, r_t^k, w_t\}$. Furthermore, the exogenous variables $\{z_t^g, z_t^h\}$ are governed by the AR(1) processes defined in equations (2.5) and (2.8).

A.2 Stochastic Discounting: Model 1

After normalizing the growing endogenous variables with the human capital stock, h_t, the equilibrium conditions of Model 1 in equations (A.I-7) to (A.I-15) become the following stationary system:

$$A_g e^{z_t^g}(v_{gt}\tilde{k}_t)^{\phi_1} l_{gt}^{1-\phi_1} = \tilde{c}_t + \tilde{i}_{kt}; \tag{A.II-1}$$

$$A_h e^{z_t^h}((1-v_{gt})\tilde{k}_t)^{\phi_2} l_{ht}^{1-\phi_2} = g_{ht+1} + (1-\delta_h); \tag{A.II-2}$$

$$p_{ht} = \left[\frac{A_g}{A_h}\right]\left[\frac{e^{z_t^g}}{e^{z_t^h}}\right]\left[\frac{1-\phi_1}{1-\phi_2}\right]^{1-\phi_2}\left[\frac{\phi_1}{\phi_2}\right]^{\phi_2}\left[\frac{v_{gt}\tilde{k}_t}{l_{gt}}\right]^{\phi_1-\phi_2}; \tag{A.II-3}$$

$$\frac{A}{x_t}\tilde{c}_t = (1-\phi_1)A_g e^{z_t^g}\left[\frac{v_{gt}\tilde{k}_t}{l_{gt}}\right]^{\phi_1}; \tag{A.II-4}$$

$$x_t = 1 - l_{gt} - l_{ht}; \tag{A.II-5}$$

$$\frac{1-\phi_1}{\phi_1}\frac{v_{gt}\tilde{k}_t}{l_{gt}} = \frac{1-\phi_2}{\phi_2}\frac{(1-v_{gt})\tilde{k}_t}{l_{ht}}; \tag{A.II-6}$$

$$\tilde{i}_{kt} = g_{ht+1}\tilde{k}_{t+1} - (1-\delta_k)\tilde{k}_t; \tag{A.II-7}$$

$$1 = \beta E_t \left[\left(\frac{\tilde{c}_t}{\tilde{c}_{t+1}}\right)^{\sigma} \left(\frac{1}{g_{ht+1}}\right)^{\sigma} \left(\frac{x_{t+1}}{x_t}\right)^{A(1-\sigma)} \left[1 + r^k_{t+1} - \delta_k\right] \right]; \tag{A.II-8}$$

$$\begin{aligned} 1 = \beta E_t & \left(\frac{\tilde{c}_t}{\tilde{c}_{t+1}}\right)^{\sigma} \left(\frac{1}{g_{ht+1}}\right)^{\sigma} \left(\frac{x_{t+1}}{x_t}\right)^{A(1-\sigma)} \frac{p_{ht+1}}{p_{ht}} \\ & \left[1 + (l_{gt+1} + l_{ht+1})(1-\phi_2) A_h e^{z^h_{t+1}} \left[\frac{(1-v_{gt+1})\tilde{k}_{t+1}}{l_{ht+1}}\right]^{\phi_2} - \delta_h \right]; \end{aligned} \tag{A.II-9}$$

where the factor prices become $r^k_t = \phi_1 A_g e^{z^g_t} (v_{gt}\tilde{k}_t)^{\phi_1 - 1} l_{gt}^{1-\phi_1}$ and $w_t \equiv F_{2t} = (1 - \phi_1) A_g e^{z^g_t} (v_{gt}\tilde{k}_t)^{\phi_1} l_{gt}^{-\phi_1}$.

Therefore, the stationary system in equations (A.II-1) to (A.II-9) fully describe Model 1 in terms of the normalized variables $g_{ht+1} \equiv \frac{h_{t+1}}{h_t}$; $\tilde{k}_t \equiv \frac{k_t}{h_t}$; $\tilde{y}_t \equiv \frac{y_t}{h_t}$ $\tilde{i}_{kt} \equiv \frac{i_{kt}}{h_t}$; $\tilde{i}_{ht} \equiv \frac{i_{ht}}{h_t}$ and $\tilde{c}_t \equiv \frac{c_t}{h_t}$.

A.3 The BGP Solution: Model 1

First, express the stationary model equilibrium conditions for Model 1 in equations (A.II-1) - (A.II-9) in terms of the variables' long-run values. Then, given that g_{ht+1} becomes $(1+g)$ along the BGP, where g is the net BGP growth rate of the economy, the equilibrium conditions become,

$$A_g (v_g \tilde{k})^{\phi_1} l_g^{1-\phi_1} = \tilde{c} + \tilde{i}_k; \tag{A.III-1}$$

$$A_h ((1-v_g)\tilde{k})^{\phi_2} l_h^{1-\phi_2} = g + \delta_h; \tag{A.III-2}$$

$$p_{ht} = \left[\frac{A_g}{A_h}\right] \left[\frac{1-\phi_1}{1-\phi_2}\right]^{1-\phi_2} \left[\frac{\phi_1}{\phi_2}\right]^{\phi_2} \left[\frac{v_g \tilde{k}}{l_g}\right]^{\phi_1 - \phi_2}; \tag{A.III-3}$$

$$\frac{A}{x}\tilde{c} = (1-\phi_1)A_g\left[\frac{v_g\tilde{k}}{l_g}\right]^{\phi_1}; \tag{A.III-4}$$

$$x = 1 - l_g - l_h; \tag{A.III-5}$$

$$\frac{1-\phi_1}{\phi_1}\frac{v_g\tilde{k}}{l_g} = \frac{1-\phi_2}{\phi_2}\frac{(1-v_g)\tilde{k}}{l_h}; \tag{A.III-6}$$

$$\tilde{i}_k = [g+\delta_k]\tilde{k}; \tag{A.III-7}$$

$$1 = \beta\left[\left(\frac{1}{1+g}\right)^{\sigma}\left[1 + r^k - \delta_k\right]\right]; \tag{A.III-8}$$

$$1 = \beta\left(\frac{1}{1+g}\right)^{\sigma}\left[1 + (1-x)(1-\phi_2)A_h\left[\frac{(1-v_g)\tilde{k}}{l_h}\right]^{\phi_2} - \delta_h\right]; \tag{A.III-9}$$

where factor prices become $r^k = \phi_1 A_g(v_g\tilde{k})^{\phi_1-1}l_g^{1-\phi_1}$ and $w = (1-\phi_1)A_g(v_g\tilde{k})^{\phi_1}l_g^{-\phi_1}$.

Now, define the auxiliary variables $f_g \equiv \frac{v_g k}{l_g h}$ and $f_h \equiv \frac{(1-v_g)k}{l_h h}$. Then the above system can be narrowed down to 11 equations in 11 unknowns, $\{f_g, f_h, g, \tilde{k}, \tilde{i}_k, \tilde{c}, v_g, l_g\}$ $\{l_h, w, r^k, p_h\}$ as,

$$A_g f_g^{\phi_1} l_g = \tilde{c} + \tilde{i}_k; \tag{A.III-10}$$

$$A_h f_h^{\phi_2} l_h = g + \delta_h; \tag{A.III-11}$$

$$p_{ht} = \left[\frac{A_g}{A_h}\right]\left[\frac{1-\phi_1}{1-\phi_2}\right]^{1-\phi_2}\left[\frac{\phi_1}{\phi_2}\right]^{\phi_2} f_g^{\phi_1-\phi_2}; \tag{A.III-12}$$

$$\frac{A}{x}\tilde{c} = (1-\phi_1)A_g f_g^{\phi_1}; \tag{A.III-13}$$

$$x = 1 - l_g - l_h; \tag{A.III-14}$$

$$\frac{1-\phi_1}{\phi_1} f_g = \frac{1-\phi_2}{\phi_2} f_h; \tag{A.III-15}$$

$$\tilde{i}_k = [g + \delta_k]\tilde{k}; \tag{A.III-16}$$

$$1 = \beta \left[\left(\frac{1}{1+g} \right)^{\sigma} \left[1 + r^k - \delta_k \right] \right]; \tag{A.III-17}$$

$$1 = \beta \left(\frac{1}{1+g} \right)^{\sigma} \left[1 + (1-x)(1-\phi_2) A_h \left[\frac{(1-v_g)\tilde{k}}{l_h} \right]^{\phi_2} - \delta_h \right]; \tag{A.III-18}$$

Given the exogenous information set of parameters $(\phi_1, \phi_2, A_g, A_h, \delta_k, \delta_h, \beta, \sigma, A)$, the uniqueness of the solution to the system in (A.III-10) - (A.III-18) can be narrowed down to the uniqueness of the variable g. In order to show this, one can solve for $f_g, f_h, \tilde{k}, \tilde{i}_k, \tilde{c}, v_g, l_g, l_h, w, r^k, p_h$ in terms of g, which leaves a single equation, (A.III-13) in one unknown g. First, let's solve for f_g using (A.III-17) and the factor price of physical capital. This yields,

$$f_g = \left[\frac{\frac{(1+g)^{\sigma}}{\beta} - 1 + \delta_k}{\phi_1 A_g} \right]^{\frac{1}{\phi_1 - 1}}. \tag{A.III-19}$$

Then f_h directly follows from (A.III-15),

$$f_h = \frac{1-\phi_1}{1-\phi_2}\frac{\phi_2}{\phi_1} f_g. \tag{A.III-20}$$

Next one can express total labor time $(l_g + l_h) = (1 - x) \equiv D$ from equation

(A.III-18):

$$D = (l_g + l_h) = \left[\frac{\frac{(1+g)^\sigma}{\beta} - 1 + \delta_h}{(1-\phi_2) A_h f_h^{\phi_2}} \right]. \tag{A.III-21}$$

To express the time shares one can express l_h in terms of g from equation (A.III-11) and then use the solution for total labor time for l_g,

$$l_h = \left[\frac{g + \delta_h}{A_h} \right] f_h^{-\phi_2}; \tag{A.III-22}$$

$$l_g = D - l_h. \tag{A.III-23}$$

Next by using equation (A.III-12) and the obtained expression for f_g in (A.III-19) it follows that the relative price of human capital in terms of g is,

$$p_h = \left[\frac{A_g}{A_h} \right] \left[\frac{1-\phi_1}{1-\phi_2} \right]^{1-\phi_2} \left[\frac{\phi_1}{\phi_2} \right]^{\phi_2} f_g^{\phi_1 - \phi_2}. \tag{A.III-24}$$

Now one can obtain an expression in g for c/k from equation (A.III-10), after dividing both sides with $\tilde{k}$ and noticing that $v_g = \left[\frac{l_g f_g}{l_g f_g + l_h f_h} \right]$, as

$$\frac{\tilde{c}}{\tilde{k}} = A_g f_g^{\phi_1 - 1} \left[\frac{l_g f_g}{l_g f_g + l_h f_h} \right] - g - \delta_k. \tag{A.III-25}$$

From the equation the (A.III-15) and the definition of f_g and f_h it follows that,

$$\tilde{k} = l_g f_g + l_h + f_h. \tag{A.III-26}$$

Lastly, the solution for $\tilde{c}$ and $\tilde{i}_k$ directly follows from combining equations (A.III-25) and (A.III-26), and (A.III-16) and (A.III-26) respectively.

Then after substituting (A.III-19) - (A.III-26) into equation (A.III-13) one can obtain a highly nonlinear equation in g such that: $\Phi(g) = 0$. This equation then can be solved numerically for the baseline calibration of parameters.

Chapter 3

Business Cycles, Endogenous Growth, and Physical Capital Capacity Utilization

3.1 Introduction

In Chapter 2 based on the model by Dang et al. (2011) it has been shown that by applying different calibrations there is a small trade-off between the results obtainable at different frequencies. It has been concluded that improvements could be made in matching business cycle facts by performing an alternative calibration exercise, most notably in terms of labor market dynamics. It has also been pointed out in the previous chapter that output, consumption, and investment growth autocorrelation functions to a standard TFP shock cannot fully be explained beyond the first degree.

In Chapter 3 we show that introducing entrepreneurial capacity, defined in Friedman (1976) and applied in Lucas (1978) and Gillman (2011), to the model in Chapter 2 one can significantly improve the explanatory power of the model in terms of output growth persistence, labor market movements, and volatilities.

Entrepreneurial capacity here represents the ability to run a firm, or in other words the ability to make decisions over at what capacity physical capital shall be used in production. This enables that from the perspective of production, used entrepreneurial capacity proxies physical capital utilization rate. Entrepreneurial capacity by assumption is an ability of the representative agent with which it has been endowed and can derive disutility from if not used. This concept is based on the on the job satisfaction. Using a simple analogy, if one is not using his or her skills and intellectual ability at the workplace, or it is not challenging, then the person can become bored and would not be happy with his or her job.

In terms of economic theory it provides the agent with an additional margin where a trade-off between consumption [and thus labor] and entrepreneurial capacity must be made. This combined with an endogenous convex physical capital depreciation rate as a function of the utilization rate and the external labor margin through a human capital investment sector provide a more robust internal propagation mechanism to spread the effects of shocks,.

The addition of entrepreneurial capacity and an endogenous depreciation rate of physical capital after calibrating the model enable us to capture output, consumption, and investment growth persistence beyond the first degree. The model is also able to match standard business cycle correlations and volatilities very closely at the business cycle and low frequencies; and able to do so with estimated shock variances that are many magnitudes smaller than the standard in the business cycle literature. More specifically, the model presented in this chapter also does a better job at explaining labor market movements while still capturing the Gali (1999) labor effect.

This chapter continues in Section 3.1 with the description of the model environment. Section 3.3 gives the calibration of the underlying model and compares the estimated shocks of the newly calibrated model in Chapter 2 and the model in this chapter to a traditionally obtained TFP series. Section 3.4 reports the simulation results and the impulse responses. Section 3.5 summarizes a sensitivity analysis of shock parameters; and Section 3.6 concludes the chapter.

3.2 The Model Environment

In this section the model is presented, which includes entrepreneurial capacity applied as a proxy of the physical capital capacity utilization rate. A convex endogenous physical capital depreciation rate is included as a function of the utilization rate as in DeJong et al. (1996) and Benhabib and Wen (2004). It further includes a more general human investment production function and correlated sectoral productivity shocks as in Chapter 2, Dang et al. (2011), andMaffezzoli (2000).

3.2.1 The Model

The representative agent maximizes its expected sum of discounted utility. The agent derives utility from consumption,c_t, leisure, x_t, and disutility form unused managerial capacity, e_t, at each time period t. With $A > 0$, $B < 0$, and $\sigma > 0$, the time t utility is given by

$$U(c_t, x_t, e_t) = \frac{[c_t x_t^A e_t^B]^{1-\sigma} - 1}{1-\sigma}, \tag{3.1}$$

which satisfies the necessary conditions for the existence of a balanced growth path equilibrium. The representative agent is confined by a normalized time endowment for each period t, where l_{gt} is the fraction of time spent in goods production, and l_{ht} in human capital investment production,

$$1 = x_t + l_{gt} + l_{ht}. \tag{3.2}$$

The representative agent is also confined by an endowment of entrepreneurial capacity, which is normalized to unity, where u_t is the used entrepreneurial capacity,

$$1 = e_t + u_t. \tag{3.3}$$

The representative agent invests in physical capital, i_{kt}, according to the following physical capital accumulation constraint following DeJong et al. (1996),

$$k_{t+1} = k_t - \delta(u_t)k_t + i_{kt}, \tag{3.4}$$

where k_t is the physical capital stock at the beginning of time period t; $\delta(u_t)$ is the endogenous depreciation rate of physical capital; and u_t is the physical capital capacity utilization rate [i.e. used entrepreneurial capacity].[1] The endogenous depreciation rate is as such so that a faster rate of utilization results in a higher rate of depreciation. More specifically,

$$\delta(u_t) = \frac{\delta_k}{\psi} u_t^{\psi}, \tag{3.5}$$

with $\psi > 1$ and $\delta_k > 0$. Note that it directly follows that $\delta'(u) > 0$ and $\delta''(u) > 0$ so that the marginal cost of utilization of the physical capital stock is increasing in the utilization rate.

y_t corresponds to the notion of GDP; A_g is a positive factor productivity parameter; z_t^g is a productivity shock; k_t is the physical capital stock that has been

[1] Others with similar endogenous depreciation rate as a function of the utilization rate include Greenwood et al. (1988), DeJong et al. (1996), and Benhabib and Wen (2004).

accumulated by the beginning of period t; v_{gt} is the share of physical capital stock being used in the goods sector; u_t is the economy wide physical capital utilization rate; $v_{gt}u_tk_t$ is the amount of physical capital in the goods sector that is utilized for production. h_t is the stock of human capital at the beginning of period t; l_{gt} denotes the share of time used in goods production [i.e. the share of human capital stock that is utilized in the goods sector] ; and $l_{gt}h_t$ represents the effective labor input. With $\phi_1 \in [0,1]$ being the share of physical capital in goods production, the output is produced according to the following constant returns to scale production function,

$$y_t = F(v_{gt}u_tk_t, l_{gt}h_t) = A_g e^{z_t^g}(v_{gt}u_tk_t)^{\phi_1}(l_{gt}h_t)^{1-\phi_1}, \tag{3.6}$$

where z_t^g is assumed to evolve according to a stationary first order autoregressive process in logarithmic form:

$$log z_t^g = \rho_g log z_{t-1}^g + \epsilon_t^g. \tag{3.7}$$

The innovations ϵ_t^g is a sequence of independently and identically distributed normal random variables with zero mean and constant variance σ_g^2.

The representative agent also accumulates human capital in a separate home sector, h_t. The human capital stock is then accumulated over time according to the following standard law of motion,

$$h_{t+1} = (1-\delta_h)h_t + i_{ht}, \tag{3.8}$$

where δ_h is the assumed constant depreciation rate of human capital, and i_{ht} is the per period investment in human capital. For the human investment production technology, A_h is a positive factor productivity parameter in the human sector; z_t^h represents the productivity shock to the human sector; $v_{ht} = 1 - v_{gt}$ is the remaining fraction of physical capital used in human capital production; u_t is the economy wide utilization rate of physical capital; and $v_{ht}u_tk_t$ is the amount of physical capital in the human sector that is utilized for human investment production. l_{ht} denotes the share of human capital utilized in its production; and with $\phi_2 \in [0,1]$ being the share of physical capital in human capital investment production, it is produced according to the following constant returns to scale production function,

$$i_{ht} = G(v_{ht}u_tk_t, l_{ht}h_t) = A_h e^{z_t^h}(v_{ht}u_tk_t)^{\phi_2}(l_{ht}h_t)^{1-\phi_2}. \tag{3.9}$$

Where z_t^h is the human sector productivity shock process that is assumed to evolve according to a stationary first order autoregressive process in logarithmic form:

$$log z_t^h = \rho_h log z_{t-1}^h + \epsilon_t^h, \tag{3.10}$$

where the innovations ϵ_t^h is a sequence of independently and identically distributed normal random variables with zero mean and constant variance σ_h^2.

With no externalities, the optimal allocations of the economy coincide with the result of the social planner's problem, which can be stated as[2]

$$\max_{\{c_t,l_{gt},l_{ht},v_{gt},v_{ht},u_t,e_t,k_{t+1},h_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \frac{[c_t x_t^A e_t^B]^{1-\sigma} - 1}{1-\sigma} \tag{3.11}$$

subject to (3.2), (3.3), (3.4) (3.5), (3.6), (3.8), and (3.9).

3.2.2 Definition of the Optimal Allocations

Definition 3 *An optimal allocation of this model is a set of contingent plans* $\{c_t, k_{t+1}\}$ $\{h_{t+1}, v_{gt}, v_{ht}, u_t, x_t, l_{gt}, l_{ht}\}$ *that solve the central planner's maximization problem in (3.1) for some initial endowment* $\{k_0, h_0\}$ *and exogenous stochastic technology processes* $\{z_t^g, z_t^h\}$*, with initial conditions* $\{z_0^g, z_0^h\}$*.*

Definition 4 *A deterministic balanced growth path equilibrium of this model is a set of paths* $\{\bar{c}_t, \bar{k}_{t+1}, \bar{h}_{t+1}, \bar{v}_{gt}, \bar{v}_{ht}, \bar{u}_t,\}$ $\{\bar{x}_t, \bar{l}_{gt}, \bar{l}_{ht}\}$ *that solve the central planner's maximization problem in (3.1) for some initial endowment* $\{k_0, h_0\}$ *and exogenous technology parameters* $\{\bar{z}_t^g = 0, \bar{z}_t^h = 0\}$*, such that* $\{\bar{c}_t, \bar{k}_{t+1}, \bar{h}_{t+1}\}$ *grow at a common trend, and* $\{\bar{v}_{gt}, \bar{v}_{ht}, \bar{u}_t, \bar{x}_t,\}$ $\{\bar{l}_{gt}, \bar{l}_{ht}\}$ *are constant.*

[2]The first - order conditions of the representative agent for Model 2 can be found in *Appendix B.1*.

3.2.3 Inter-sectoral Dynamics of Factor Inputs

For the model here consider the equilibrium condition in (B.I-14), which after some simplification becomes

$$\frac{v_{gt}(1-x_t-l_{gt})}{l_{gt}(1-v_{gt})}=\frac{\phi_1(1-\phi_2)}{\phi_2(1-\phi_1)}. \tag{3.12}$$

If a shock causes a decrease in leisure, x_t, and/or labor in the goods sector, l_{gt}, then in order for the above relationship to hold in each time period, t, the sectoral allocation of physical capital, v_{gt}, must adjust. This is in line with the impulse responses in Section 3.4 This demonstrates the sense in which leisure creates a type of asymmetry that drives factor input allocations between sectors as described by the Rybczynski (1955) theorem.[3]

In order to show that apart from the Rybczynski (1955) theorem a Stolper and Samuelson (1941) effect is responsible for driving factor input allocation across sectors. For this consider the capital factor rewards w_t and r_t^k in terms of the relative price of human capital, p_{ht}, as,

$$r_t^k=\phi_1(z_t^h)^{\frac{\phi_1-1}{\phi_1-\phi_2}}(z_t^g)^{\frac{1-\phi_2}{\phi_1-\phi_2}}A_h^{\frac{\phi_1-1}{\phi_1-\phi_2}}A_g^{\frac{1-\phi_2}{\phi_1-\phi_2}}\left[\frac{\phi_2}{\phi_1}\right]^{\frac{\phi_2(\phi_1-1)}{\phi_1-\phi_2}}\left[\frac{1-\phi_2}{1-\phi_1}\right]^{\frac{(1-\phi_2)(\phi_1-1)}{\phi_1-\phi_2}}p_{ht}^{\frac{\phi_1-1}{\phi_1-\phi_2}} \tag{3.13}$$

$$w_t=(1-\phi_1)(z_t^h)^{\frac{\phi_1}{\phi_1-\phi_2}}(z_t^g)^{\frac{-\phi_2}{\phi_1-\phi_2}}A_h^{\frac{\phi_1}{\phi_1-\phi_2}}A_g^{\frac{-\phi_2}{\phi_1-\phi_2}}\left[\frac{\phi_2}{\phi_1}\right]^{\frac{\phi_2\phi_1}{\phi_1-\phi_2}}\left[\frac{1-\phi_2}{1-\phi_1}\right]^{\frac{(1-\phi_2)\phi_1}{\phi_1-\phi_2}}p_{ht}^{\frac{\phi_1}{\phi_1-\phi_2}} \tag{3.14}$$

Proposition 5 *The sign of the derivative of r_t^k and w_t with respect to p_{ht} depends only on the factor intensity ranking.*

Proof. Given the assumption that human capital investment is relatively more intensive in human capital than goods production, so that $\phi_1>\phi_2$, then by equations (3.13) and (3.14), $\frac{\partial r_t^k}{\partial p_{ht}}<0$ and $\frac{\partial w_t}{\partial p_{ht}}>0$. ■

Lastly, let us combine equations (3.13) and (3.14) and log-linearize it along the

[3] The same relationship is identical in the model in Chapter 2.

model's BGP denoted by variables with a $\hat{}$.

$$\hat{w}_t - \hat{r}_t^k = \frac{\hat{p}_{ht} - \hat{z}_t^g + \hat{z}_t^h}{\phi_1 - \phi_2} \tag{3.15}$$

With $\phi_1 > \phi_2$, one may observe that the relative price decreases on impact relative to its steady state value when the TFP and human productivity shocks [i.e. aggregate shock] hits the economy. As the relative price of human capital decreases the reward to producing physical capital, r_t^k, rises more than the reward on labor. This is consistent with the Stolper and Samuelson (1941) theorem.[4]

The reason behind this behavior of the Stolper and Samuelson (1941) effect is due to the effect of shocks on the relative price and factor rewards. As described in the next section, the variance of the TFP shock is significantly higher than that of the human productivity shock's.[5] The correlation between the shocks are very high, close but not equal to unity.

On impact of a positive aggregate shock (with different magnitudes in sectors), the relative price of human capital decreases initially. This is due to the fact that now with adjusting the utilization rate the effect of an aggregate shock can be reduced, which also means that the deprecation rate of physical capital slows down and decreases the level of physical investment relative to the steady state. This induces the reallocation of resources to the human capital sector. However, the reduction in leisure keeps pressuring the price of human capital investment goods upwards and in subsequent periods this price increases and the direction of the inter-sectoral resource transfer reverses. As the effect of the aggregate shock dies out labor flows back to the goods sector due to an optimal spreading of the inter-sectoral adjustment cost across periods.

3.2.4 Duality

Factor input movements between sectors are determined by the Rybczynski (1955) effect and its dual the Stolper and Samuelson (1941) effect. The sectors are the goods sector and the human investment sector.[6] One factor input in the normalized economy is the ratio of physical to human capital stock, k_t/h_t, and the other is time allocated into production in the respective sectors, i.e. l_{gt} and/or l_{ht}. The following analysis is done for a normalized model as described in Appendix B.2 because in this

[4]For an illustration, see Section 3.4.

[5]It is approximately eight times larger.

[6]In this analysis to quantify and to show the dynamic Rybczynski (1955) and Stolper and Samuelson (1941) effects and their dual relationship, our two sectors are analogous to good producing countries in an international setting.

case all factor inputs are stationary along the BGP, which makes the illustration of the role of the underlying effects a straightforward exercise.

Now one may describe the Rybczynski (1955) theorem in a dynamic setting, following Van Long (1992), as

Proposition 6 *(A dynamic Rybczynski Theorem): An increase in the initial allocation of a factor input in a sector will expand the output of that sector if it is more intensive in the underlying input, whereas, the output of the other sector more intensive in the other factor input will increase with a relatively lower quantity.*

One sector produces goods, y_t [or alternatively physical capital, k_t], whose price is normalized to unity, meanwhile, the second sector produces human investment good, i_{ht}, at a relative price , p_{ht}, in terms of the consumption good . This relative price is unique and well defined along the BGP and is given by the ratio of the marginal products of the goods and human investment technologies with respect to a given factor input.

$$p_{ht} = \frac{\phi_1 A_g e^{z_t^g}\left[\frac{v_{gt}u_t(k_t/h_t)}{l_{gt}}\right]^{\phi_1-1}}{\phi_2 A_h e^{z_t^h}\left[\frac{v_{ht}u_t(k_t/h_t)}{l_{ht}}\right]^{\phi_2-1}} = \frac{(1-\phi_1)A_g e^{z_t^g}\left[\frac{v_{gt}u_t(k_t/h_t)}{l_{gt}}\right]^{\phi_1}}{(1-\phi_2)A_h e^{z_t^h}\left[\frac{v_{ht}u_t(k_t/h_t)}{l_{ht}}\right]^{\phi_2}}. \tag{3.16}$$

Then one can define and quantify the Rybczynski (1955) effect as the change in the output of of a sector given a change in either of the sectoral allocations of a factor input. Then the Rybczynski (1955) effect in the human investment sector for the different scenarios can be written as[7]

$$R_1^h = \frac{\partial i_{ht}}{\partial v_{ht}u_t(k_t/h_t)} = \phi_2 A_h e^{z_t^h}\left[\frac{v_{ht}u_t(k_t/h_t)}{l_{ht}}\right]^{\phi_2-1} = \frac{r_t^k}{p_{ht}} \tag{3.17}$$

$$R_2^h = \frac{\partial i_{ht}}{\partial l_{ht}} = (1-\phi_2)A_h e^{z_t^h}\left[\frac{v_{ht}u_t(k_t/h_t)}{l_{ht}}\right]^{\phi_2} = \frac{w_t}{p_{ht}} \tag{3.18}$$

Then one may ask the question what is the effect on factor inputs when the relative price changes, p_{ht}. The Stolper and Samuelson (1941) theorem applied to a two sector two inputs and two goods dynamic closed economy setting can give a good explanation of the factor movements.

[7]We choose to show the dual relationship by using the human sector because the price of the physical investment and consumption goods is normalized to unity.

Proposition 7 *(A dynamic Stolper and Samuelson Theorem): An increase in the relative price of a good more intensive in one of the factor inputs, will increase relatively more the rent earned from a unit of that input in that sector.*

In the case of the human sector the rent earned by the representative agent from the factor inputs can be defined as,

$$q_{ht}^{k} = r_t^k, \tag{3.19}$$

$$q_{ht}^{l} = w_t, \tag{3.20}$$

where one can note that,

$$w_t = p_{ht}(1-\phi_2)A_h e^{z_t^h}\left[\frac{v_{ht}u_t(k_t/h_t)}{l_{ht}}\right]^{\phi_2}; \tag{3.21}$$

and

$$r_t^k = \phi_2 A_h e^{z_t^h}\left[\frac{v_{ht}u_t(k_t/h_t)}{l_{ht}}\right]^{\phi_2-1}. \tag{3.22}$$

It is clear that factor prices are linear in the relative price of the human investment good, which follows from the first order conditions of the model in *Appendx B.1.* After the rent from factor inputs in the human sector are defined, one can quantify the Stolper and Samuelson (1941) effect as the change in the rentals to a change in the relative price p_{ht}. Then,

$$S_1^h = \frac{\partial q_{ht}^k}{\partial p_{ht}} = \phi_2 A_h e^{z_t^h}\left[\frac{v_{ht}u_t(k_t/h_t)}{l_{ht}}\right]^{\phi_2-1} = \frac{r_t^k}{p_{ht}}, \tag{3.23}$$

$$S_2^h = \frac{\partial q_{ht}^l}{\partial p_{ht}} = (1-\phi_2)A_h e^{z_t^h}\left[\frac{v_{ht}u_t(k_t/h_t)}{l_{ht}}\right]^{\phi_2} = \frac{w_t}{p_{ht}}. \tag{3.24}$$

One can observe that $R_1^h = S_1^h$ and $R_2^h = S_2^h$, which shows that there is a strong duality in this model between the Rybczynski (1955) and Stolper and Samuelson (1941) effects between the two sectors.

3.3 Model Simulation

Here we present the calibration of the model with physical capital capacity utilization, denoted by Model 2. Also, the model based TFP shock series from Model 2 and Model 1 in Chapter 2 are compared to a traditionally obtained TFP series.

3.3.1 Data

The data series are for the United States from 1959:Q1 to 2014:Q2.[8]. The data series have been once again filtered by a Christiano and Fitzgerald (2003) type asymmetric band-pass filter following Baxter and King (1999), Benk et al. (2010), and Basu et al. (2012).

3.3.2 The Calibration

Table 3.1 contains 11 different pieces of information about the data from different sources. These pieces of information then are used as the calibration targets for structural parameters.

Description	Target	Achieved Value	Target Source
Output Growth Rate	0.0035	0.0035	Gomme and Rupert (2007); NIPA
Phys. Capital Utilisation Rate	0.7852	0.7852	US Census Bureau; NAICS
Leisure	0.50	0.50	Gomme and Rupert (2007)
Consumption Output Ratio	0.70	0.71	Gomme and Rupert (2007)
$Corr(y_t, u_t)^{bc}$	0.809	0.935	NIPA, NAICS
$Corr(y_t, c_t)^{bc}$	0.894	0.917	NIPA
$Corr(y_t, c_t)^{lc}$	0.979	0.853	NIPA
$Corr(y_t, i_{kt})^{bc}$	0.943	0.986	NIPA
$\rho(g_y)$	0.265	0.284	NIPA
$\rho(g_c)$	0.365	0.327	NIPA
$\rho(g_{ik})$	0.262	0.313	NIPA

Table 3.1: Target values of the baseline calibrations for the U.S. - Model 2.

Once again following Gomme and Rupert (2007) for the calibration of the model we set the balanced growth rate, g, the physical capital utilization rate, u, the endogenous depreciation rate $\delta(u)$; and leisure, x, equal to their long-run target values, which are 0.0035, 0.7852, 0.025, and 0.50. These strict targets allow us to directly pin down the leisure preference weight A, the human capital sector scale parameter A_h, the entrepreneurial capacity weight in preference, B, and the constant depreciation parameter δ_k, from the long-run versions of the intra-temporal margins, the inter-temporal margin with respect to human capital, and the definition of the endogenous depreciation rate combined with the other 7 structural parameters that our calibration algorithm picks. Then the set of structural parameters that we directly search for with *Simulated Annealing* within our calibration procedure is $\{\beta, \sigma, \phi_1, \phi_2, A_g, \psi, \delta_h\}$, meanwhile, the shock parameters are directly estimated. Simulated Annealing is set up to minimize the vector distance between the U.S.

[8]All data series are quarterly and from the period noted above except where noted otherwise in Chapter 1.

data moments in Table 3.1apart from the strict targets and their simulated moment counterparts as described in Chapter 1. These include the correlation between output and consumption at the business cycle and low frequencies; the correlation between output and investment at the business cycle frequency; the correlation between output and the utilization rate; and the first degree auto-correlation coefficients of output, consumption, and investment growth. Given the algorithm results the baseline parameterization shall imply the following for the real interest rate r^k:

$$\frac{(1+g)^{\sigma}}{\beta} = [1 + r^k u - \frac{\delta_k}{\psi} u^{\psi}]. \tag{3.25}$$

Given the long-run values of g and u the real interest rate further depends on the depreciation rate of physical capital δ_k, the time preference parameter β, the CES parameter σ, and the endogenous depreciation rate's convexity parameter ψ.

Estimates for the depreciation rate of physical capital varies in the literature. In this we target the physical capital depreciation rate at 2.5 percent per quarter, which is standard in the literature and supported by the estimate of 2.7 percents by Gomme and Rupert (2007) based on BEA estimates. This as pointed out above we set as a strict target and keep it fixed.

Then our algorithm with allowing for a variation in the convexity parameter of the endogenous depreciation rate ψ between the values of 1 and 4 yields a parameter value of 3.34. Given our target, it pins down the depreciation parameter δ_k at 0.19. The convexity parameter in our calibration implies that the depreciation rate of physical capital is highly elastic with the utilization rate when our target is matched.

For the CES parameter σ we set our target range between 0.34 to 2 as in Chapter 2 that coincides with the quarterly estimate of Hall (1988) and the estimate of Mehra and Prescott (1985) respectively. Within this range our procedure pins down σ as 0.41. Then by limiting the discount factor search range in our procedure once again to 0.95 to 0.99 we are able to pin down β in the BGP relationship in equation (3.25). This yields the value of 0.9862 for β.

For the human sector parameters as in Chapter 2 we set our search range based on the estimates of Jones et al. (2005), DeJong and Ingram (2001) and Jorgenson and Fraumeni (1991) of the depreciation rate of human capital. In our calibration process we allow for a lower bound of 0.1 percent per quarter remaining close to DeJong and Ingram (2001) and an upper bound of 3 percent as in DeJong et al. (1996). Our calibration process yielded a quarterly depreciation rate of 0.1 percent.

The share of physical capital in goods production, ϕ_1, and in human production ϕ_2, are chosen by our procedure. We bounded our algorithm to search for the capital share in goods production within the range of 0.3 and 0.42; and to search within the range of 0.08 and 0.25 for the capital share in human investment production. For the share in human production the lower bound corresponds with the estimate of Jorgenson and Fraumeni (1991) for educational output. In the model the calibration algorithm picks out a physical capital share of 0.36 in the goods sect; whereas, it picks a capital share in human investment production of 0.198. Given the values, our calibration procedure yielded a labor time of 0.40 in the goods sector and a learning time of 0.1.

The capital shares, the human and physical capital depreciation rates, and the goods sector scale parameter combined with our strict targets then pin down the scale parameter for the human sector so that the inter-temporal margins hold. Finally, through the intra-temporal margins we can pin down the values of preference weights. The complete list of calibrated structural parameters for our model can be found in Table 3.2 below. Table 3.3 lists the parameters of the exogenous shock processes, which are obtained through the iterative step of the calibration procedure described more in detail in Chapter 1.

Parameter	Description	Value
β	Subjective Discount Factor	0.986
σ	CES Parameter	0.412
A	Weight of Leisure in Preference	1.11
B	Weight of Man. Capacity in Preference	-0.16
A_g	Scale Parameter of Goods Sector	0.80
A_h	Scale Parameter of Human Sector	0.032
ϕ_1	Physical Capital Share in Goods Production	0.36
ϕ_2	Physical Capital Share in Human Investment	0.20
δ_k	Depreciation Parameter (Physical Capital)	0.19
ψ	Convexity of Endog. Depr. Rate	3.34
δ_h	Depreciation Rate of Human Capital	0.001

Table 3.2: Structural parameter values of the baseline calibrations for the U.S. - Model 2.

Parameter	Description	Value
ρ_g	Auto-correlation of TFP	0.979
ρ_h	Auto-correlation of Human Shock	0.978
σ_g^2	Variance of TFP	$8.8 * 10^{-8}$
σ_h^2	Variance of Human Productivity Shock	$1.2 * 10^{-8}$
$\sigma_{g,h}$	Correlation of Shock Innovations	0.999

Table 3.3: Shock process parameter values of the baseline calibrations for the U.S. - Model 2.

3.3.3 The TFP Shock Process: 1971:Q4 to 2012:Q2

In this section, similarly to Chapter 2, the shocks of Model 2 are constructed using U.S. data following the procedure in Benk et al. (2005, 2008, 2010) and Nolan and Thoenissen (2009). Then the DSGE based TFP series are compared to a traditional Solow residual based TFP and that of the newly calibrated Model 1 based TFP series in the previous chapter. We call the model in this chapter 'Model 2', and the newly calibrated model based on Dang et al. (2011) from Chapter 2 we call 'Model 1'. The comparison is made relative to the Solow residual based TFP series obtained by using the calibration presented in above [i.e. $\phi_1 = 0.36$].

In Figure 3.1 the unfiltered TFP series obtained from Model 1 and 2 are compared to the traditionally obtained goods sector TFP. The series presented here are the ones with the highest correlations to the traditional one with values of 0.84 cross-correlation with the Model 1 based series and 0.72 for the Model 2 based series. We also decompose them at different frequencies using a Christiano and Fitzgerald (2003) band-pass filter at the high, business, cycle, low, and the Comin and Gertler (2006) medium cycle frequencies.[9]

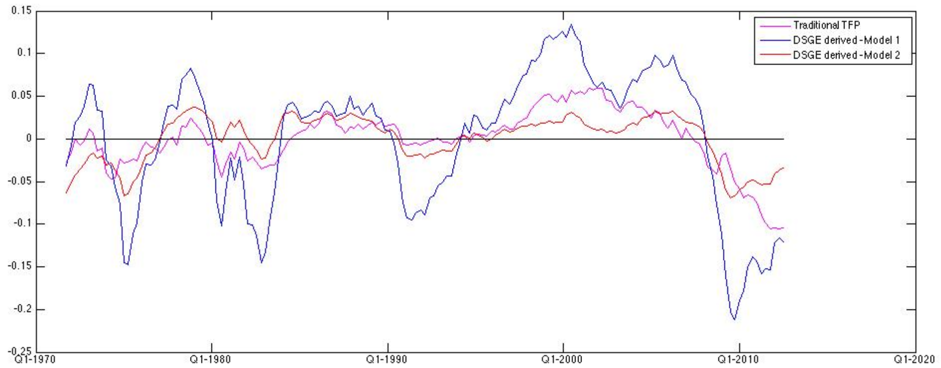

Figure 3.1: Total Factor Productivity - DSGE derived versus traditionally estimated shocks.

Despite a lower correlation with the traditional one the TFP series obtained from the solution of Model 2 follows the traditional TFP series more closely in terms of magnitude. Both are able to capture well the recession periods in the early and mid 1980s with the Model 1 based TFP overstating and Model 2 understating the magnitude of the TFP relative to the traditional one. The same is the case during the recession in 1991, meanwhile, the relationship of the DSGE derived series to the traditional one reverses during the growth period during the mid and late 1990s. The difference between the TFP series obtained from Model 1 and 2 lies in the behavior of the series during and after the Great Recession, which explains the lower correlation of Model 2 with the traditional TFP series. Both series capture the drop in productivity during the financial crisis of 2008, however, only the TFP series obtained from Model 1 follows the traditional series, when it starts increasing in 2010 just after the recession in the U.S. ended, meanwhile, the Model 2 based series keeps decreasing.

Figure 3.2 and 3.3 shows the DSGE derived TFP series and the traditional one decomposed at different frequencies. Figure 3.2 shows the traditional one relative

[9]The windows for the band-pass filter frequencies are defined in detail in Section 3.4.

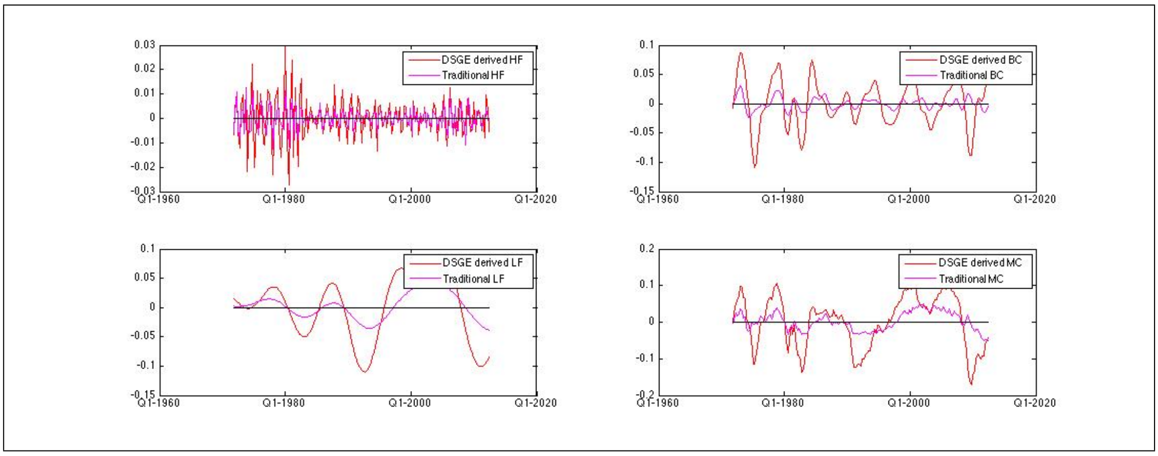

Figure 3.2: Total Factor Productivity - Model 1 DSGE derived versus traditionally estimated shocks frequency components.

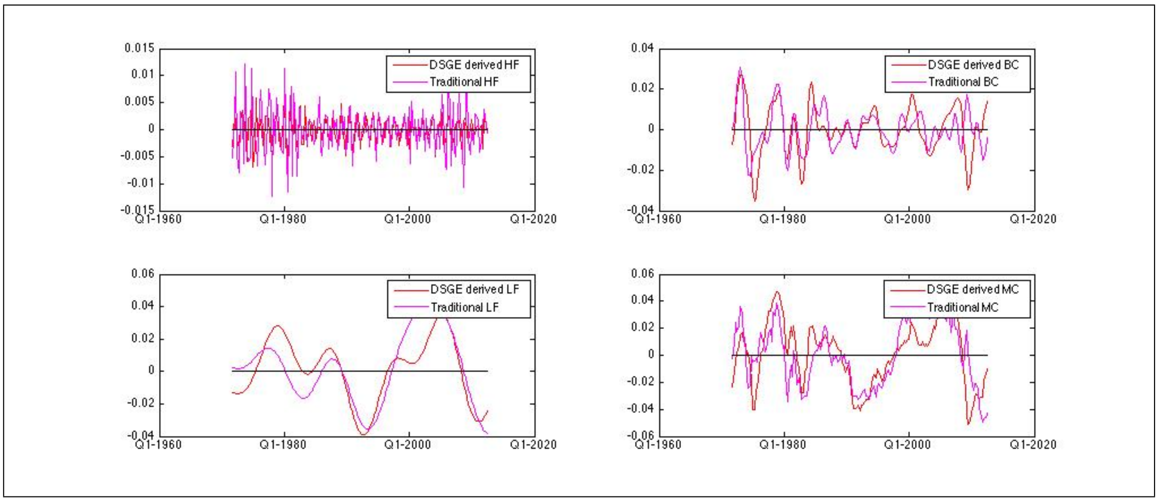

Figure 3.3: Total Factor Productivity - Model 2 DSGE derived versus traditionally estimated shocks frequency components.

to the Model 1 based series, where it is notable that during the crisis period the two series stop following one another at the business cycle frequency. In the meantime, at the low frequency the co-movement of the two series remain strongly correlated, which offsets the business cycle frequency movement of the Model 1 based TFP series with the traditional one.

In Figure 3.3 we present the co-movement of the Model 2 based TFP series with that of the Solow residual based one. Here it is evident that the Model 2 based series moves in the opposite direction similarly to the Model 1 based series as in Figure 3.2, but in a much more robust way, which is not fully offset by the low frequency component.

An explanation for these trends at the end of the Great Recession lies in the relatively slow labor market movements. Figure 3.4 shows the traditional and the two DSGE derived TFP series relative to an amplified US labor participation rate

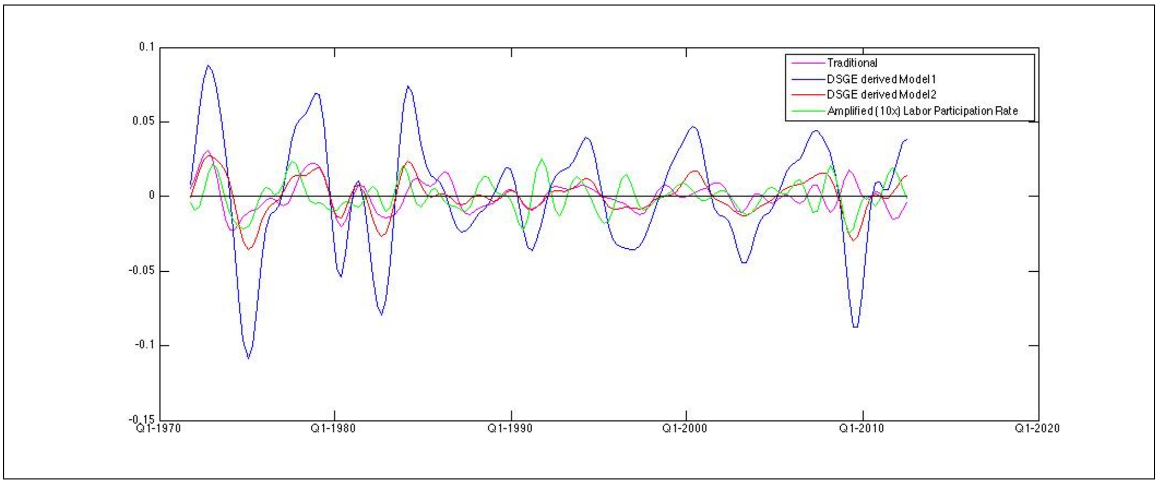

Figure 3.4: Traditional and DSGE Derived Total Factor Productivities (Model 1 and 2) vs. Amplified Labor Participation Rate (x10).

at the business cycle frequency. At the end of the Great Recession the labor participation rate fails to pick up quickly and shows a similar movement as suggested by the DSGE derived TFP series. In these models the goods labor time movements are similar in nature to what the labor participation rate data captures in the U.S. economy. Given the DSGE solution based shocks it provides support for the idea that the added endogenous growth mechanism through the workings of the Stolper and Samuelson (1941) and the Rybczynski (1955) in factor inputs is able to capture within the estimated shocks the trend in the U.S. labor participation rate during the Great Recession as suggested by the estimated shock series unlike in the case of the standard Solow residual based series.

3.4 Simulation Results

In this section we are showing three sets of results based on the model simulations and the impulse response functions to the different shocks. The three sets of key results are those of the persistence of key variables; correlations of key variables and volatilities at different frequencies using a Christiano and Fitzgerald (2003) band-pass filter, where the frequencies include the high frequency [2 - 6 quarters], the business cycle frequency [6 - 32 quarters], the low frequency [32 - 200 quarters]; and the Comin and Gertler (2006) type medium term frequency [2 - 200 quarters]. In each case we are comparing the results of Model 2 as defined in this chapter to the results of Model 1 [i.e. calibrated model of Chapter 2 based on Dang et al. (2011)], in order to show how physical capital utilization and entrepreneurial capacity improves the results relative to Model 1.

3.4.1 Impulse Responses

The impulse responses are shown to selected variables for the extended model, Model 2. Figure 3.5 shows the responses of selected variables to a goods sector TFP shock. It shows that a goods productivity shock makes the agent substitute away from leisure towards labor. This effort increases the physical capital investment rate (i_k/y), with a consequent gradual increase in the physical capital to human capital stock ratio. Also, reallocating and utilizing more physical capital in goods production initially, and a gradual increase in physical investment in turn increases output growth temporarily above the balanced growth rate.

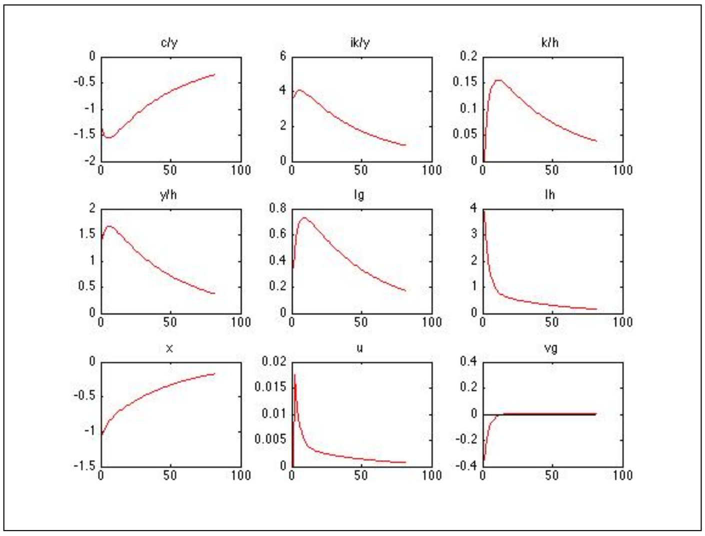

Figure 3.5: Model 2 - Impulse responses in percent deviation from the steady state - Goods TFP Shock.

Figure 3.6 shows the responses of variables to a positive human productivity shock. As in the case of the Model 1 the agent allocates more time to human capital production instead of leisure and labor. This in turn reduces the physical investment rate and the utilization rate of physical capital, meanwhile, it reallocates physical capital to the human sector as the relative price of human capital to goods becomes lower.

Figure 3.7 and 3.8 show the responses of selected variables to a positive aggregate shock. This on impact will decrease the physical investment rate relative to output, which gradually reaches the balanced growth rate. Leisure and labor decrease on impact as a result of the decrease in the rate of physical investment. The decline in labor once again is consistent with the empirical finding of Gali (1999). Unlike in Model 1, in Model 2 the aggregate shock's effect in the human sector does not outweigh that of the goods sector's effect.

Figure 3.8 then shows that Model 2 is also in line with the findings of Bond et al. (1996) regarding the fact that the allocation of factor inputs are solely determined

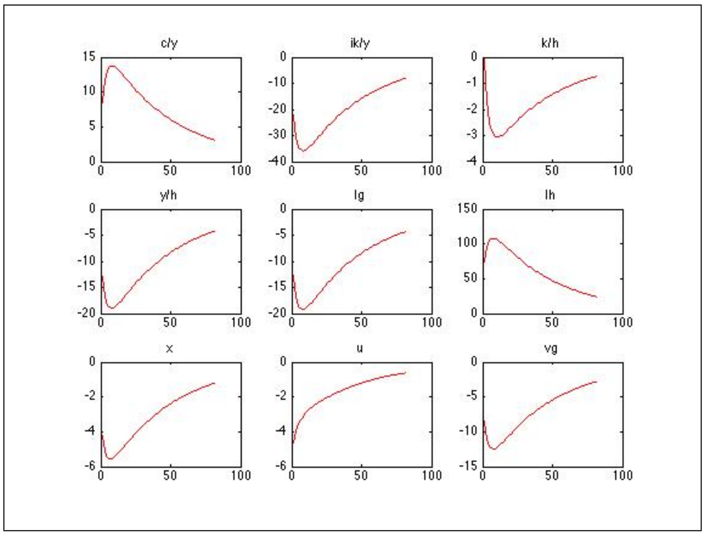

Figure 3.6: Model 2 - Impulse responses in percent deviation from the steady state - Human Prod. Shock.

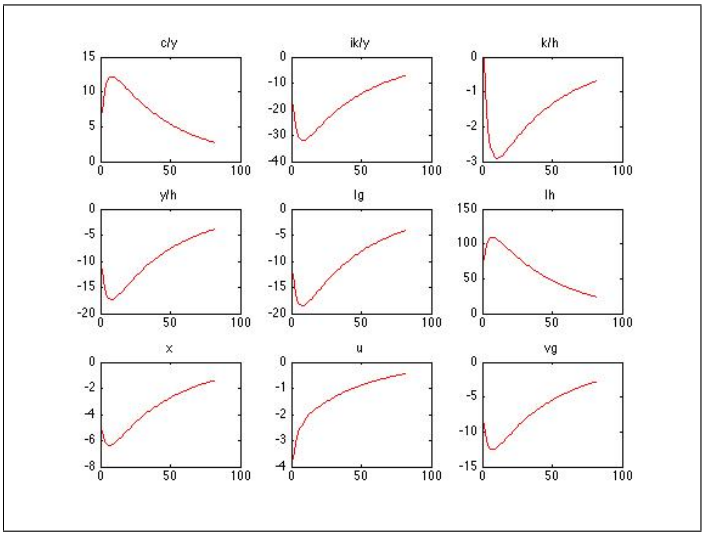

Figure 3.7: Model 2 - Impulse responses in percent deviation from the steady state (1) - Aggregate Shock.

by factor prices and the Stolper and Samuelson (1941) effect and the Rybczynski (1955) theorem. More specifically, the relative price of human capital, p_{ht} jumps down on impact below its steady state value. This occurs when wages, w_t, are lower then the real return on physical capital, r_t^k. This can be clearly observed on Figure 3.8 on impact of the aggregate shock. As a result physical capital becomes relatively more expensive and allocated more towards human capital production, which in turn reduces the return on it. The relative changes in prices and that of the larger effect of a human sector when an aggregate shock occurs is due to the physical capital utilization rate. When a shock hits reallocating and increasing the utilization rate reduces the magnitude of the effect of an aggregate shock through the goods sector.

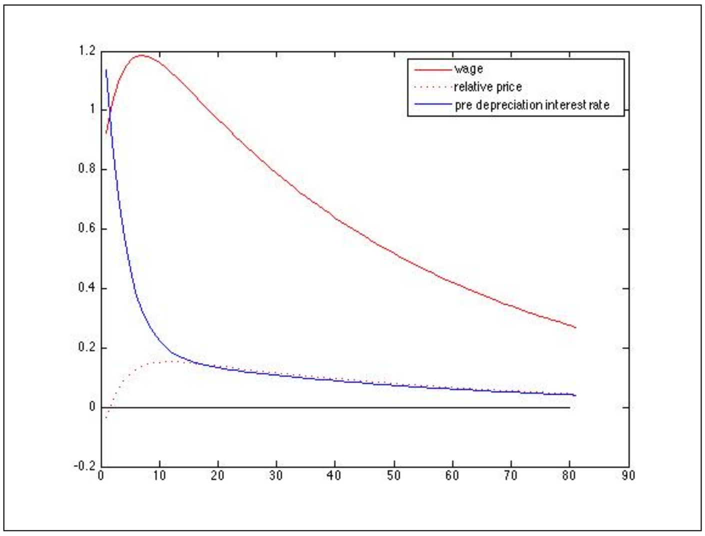

Figure 3.8: Model 2 - Impulse responses in percent deviation from the steady state (2) - Aggregate Shock.

3.4.2 Persistence

Table 3.4 reports the auto-correlations of key variables up to three lags. We have shown in the previous chapter that by including human capital and endogenous growth in an otherwise standard RBC model one can match output growth persistence relatively closely at the first degree.

Variable	Lag 1	Lag 2	Lag 3
$\rho(g_{y,t})$			
Data	0.265	0.216	0.157
Model 1	0.328	0.280	0.283
Model 2	0.284	0.250	0.176
$\rho(g_{c,t})$			
Data	0.365	0.280	0.305
Model 1	0.712	0.689	0.687
Model 2	0.327	0.331	0.279
$\rho(g_{ik,t})$			
Data	0.262	0.75	0.082
Model 1	0.149	0.084	0.08
Model 2	0.313	0.255	0.169
$\rho(l_{gt})$			
Data	0.987	0.974	0.961
Model 1	0.851	0.720	0.619
Model 2	0.989	0.973	0.958
$\rho(l_{ht})$			
Data	-	-	-
Model 1	0.944	0.894	0.854
Model 2	0.987	0.973	0.954
$\rho(u_t)$			
Data	0.956	0.862	0.749
Model 1	-	-	-
Model 2	0.930	0.875	0.821

Table 3.4: Matching Auto-correlation Functions of Key Variables.

Model 1 and Model 2 are both able to capture the first degree autocorrelation of output growth with Model 2 doing slightly better. Model 2 shows a clear advantage over Model 1 in terms of explaining higher degree autocorrelations, where it is able to match the steep drop in persistence beyond the first lag as seen in Table 3.4 and Figure 3.9.

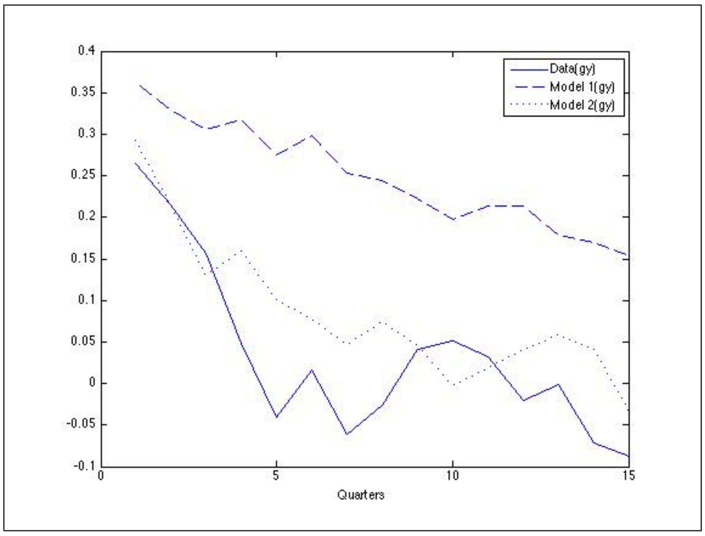

Figure 3.9: Persistence of output growth (US Data, Model 1, Model 2).

Consumption growth persistence is only closely matched by Model 2. Model 1 shows some weakness in matching investment and consumption growth persistence. On the other hand, Model 2 is able to explain both consumption and investment growth persistence strikingly well not just at the first degree but beyond that as it can be seen in Table 3.4 and Figures 3.10 and 3.11.

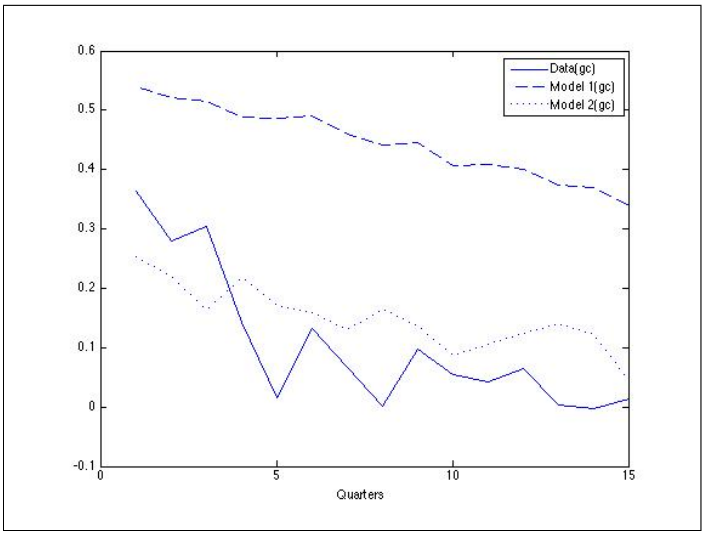

Figure 3.10: Persistence of consumption growth (US Data, Model 1, Model 2).

Overall, it is clear from the results in Figures 3.9 - 3.11 and Table 3.4 that by adding entrepreneurial capacity and physical capital utilization with a convex endogenous depreciation rate one can capture the persistence of output growth,

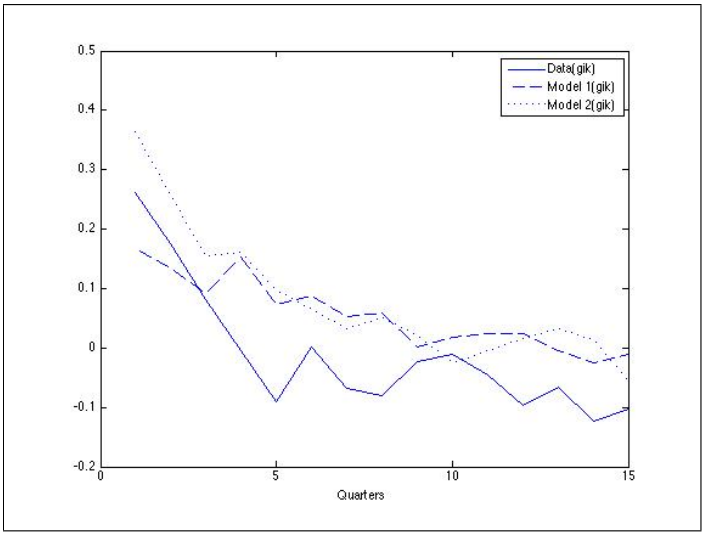

Figure 3.11: Persistence of phy. investment growth (US Data, Model 1, Model 2).

consumption, and investment growth not just after impact but to higher degrees as well. Now the question remains how this model performs relative to Model 1 in terms of key correlations and volatilities.

3.4.3 Key Correlations

Table 3.5 reports the contemporaneous correlations between output and other variables at different frequencies including the Comin and Gertler (2006) medium term cycle or frequency. Both models perform relatively well in matching U.S. data moments.

The co-movement of consumption and investment with output is closely matched by Model 1 and 2 at the business cycle frequency. On the other hand both models fail to fully capture the low frequency co-movement between consumption and output.

Labor hours' correlation with output is surprisingly well-matched by both models with Model 1 slightly performing better. As in DeJong et al. (1996) both models are able to generate negative correlation between learning time and output as suggested by theory. Model 2 in this respect gives a more robust negative correlation both at the business cycle and low frequencies. Model 2 is also able to capture well the physical capital utilization rate and output correlation at the business cycle frequency.

Lastly, one may observe as it can be seen in Table 3.5 that the endogenous growth mechanism is able to break the negative relationship between labor hours and consumption that is one of the main reasons why the standard RBC model cannot capture output growth persistence as pointed out by Benhabib et al. (1997). Model 2 in this respect performs extremely well at the business cycle frequency and is reasonable at the lower frequencies.

Variable	High freq. 2 -6 qrs.	Bus. cyc. 6 - 32 qrs.	Low freq. 32 - 200 qrs.	Med. term 2 - 200 qrs.
$corr(c_t, y_t)$				
Data	0.469	0.894	0.979	0.962
Model 1	0.968	0.928	0.903	0.898
Model 2	0.990	0.917	0.853	0.853
$corr(i_{kt}, y_t)$				
Data	0.811	0.943	0.809	0.815
Model 1	0.990	0.987	0.829	0.839
Model 2	0.996	0.986	0.933	0.937
$corr(l_{gt}, y_t)$				
Data	0.404	0.731	0.614	0.614
Model 1	-0.154	0.512	0549	0.454
Model 2	-0.142	0.894	0.801	0.799
$corr(l_{ht}, y_t)$				
Data	-	-	-	-
Model 1	0.133	-0.336	-0.119	-0.128
Model 2	0.115	-0.917	-0.809	-0.8138
$corr(u_t, y_t)$				
Data	0.083	0.809	0.335	0.416
Model 1	-	-	-	-
Model 2	0.006	0.935	0.912	0.848
$corr(c_t, l_{gt})$				
Data	0.215	0.764	0.621	0.624
Model 1	0.010	0.592	0.647	0.507
Model 2	-0.255	0.656	0.379	0.387

Table 3.5: Matching Key Correlations at Different Frequencies - Model 1 and 2.

In summary, it is clear that both models are able to capture the business cycle frequency correlations well. They both do a reasonable job at explaining lower frequency correlations, where the different calibration of Model 1 by Dang et al. (2011) performs better.

3.4.4 Volatilities

Table 3.6 reports the simulated and actual data standard deviations of the underlying key variables at all of the defined four frequencies. Output, investment, and consumption growth volatilities are well matched at the business cycle frequency despite both models' inability to closely capture it at lower frequencies. Model 2, however, performs better in the respect.

The volatilities of the three growth rates generated by the Model 1 and Model 2 are almost identical at the business cycle frequency. At the low frequency both models tend to overshoot the volatilities of growth rates, meanwhile, at the medium cycle the low and high frequencies balance each other out and both models are able to match the medium cycle growth rate volatilities.

With regards to the volatilities of the model generated log-level series for output, consumption, and physical investment both model performs extremely well in the cases of output and investment and reasonably in the case of consumption at the business cycle frequency. On the hand the same trend can be observed at the low

Variable	High freq. 2 -6 qrs.	Bus. cyc. 6 - 32 qrs.	Low freq. 32 - 200 qrs.	Med. term 2 - 200 qrs.
$vol(g_{y,t})$				
Data	0.0069	0.0065	0.0035	0.0100
Model 1	0.094	0.0064	0.0065	0.0130
Model 2	0.0072	0.0065	0.0100	0.0140
$vol(g_{c,t})$				
Data	0.0038	0.0037	0.0024	0.0058
Model 1	0.0034	0.0027	0.0063	0.0075
Model 2	0.0038	0.0027	0.0042	0.0061
$vol(g_{i_k,t})$				
Data	0.0203	0.0210	0.0101	0.0305
Model 1	0.0360	0.0240	0.1700	0.0460
Model 2	0.0160	0.0180	0.0310	0.0370
$vol(y_t)$				
Data	0.0044	0.0169	0.0464	0.0498
Model 1	0.0070	0.0140	0.1000	0.1000
Model 2	0.0047	0.0180	0.0790	0.0810
$vol(c_t)$				
Data	0.0024	0.0098	0.0376	0.0390
Model 1	0.0032	0.0053	0.0890	0.0890
Model 2	0.0025	0.0074	0.0520	0.0510
$vol(i_{kt})$				
Data	0.0131	0.0547	0.0891	0.1075
Model 1	0.0240	0.0530	0.2300	0.2400
Model 2	0.0100	0.0470	0.1800	0.1900
$vol(l_{gt})$				
Data	0.0017	0.0050	0.0224	0.0230
Model 1	0.0140	0.0260	0.0340	0.0450
Model 2	0.0060	0.0220	0.0780	0.0820
$vol(l_{ht})$				
Data	-	-	-	-
Model 1	0.0430	0.0790	0.1500	0.1800
Model 2	0.0440	0.1500	0.500	0.5300
$vol(u_t)$				
Data	0.0057	0.0262	0.0322	0.0429
Model 1	-	-	-	-
Model 2	0.0015	0.0042	0.0120	0.013

Table 3.6: Matching Volatilities at Different Frequencies - Model 1 and 2.

frequencies as in the case of the growth rates, meaning, that both models tend to overshoot volatilities, even though Model 2 clearly performs better.

The only clear shortcoming of both model is that they over predicts the volatility of labor hours at all frequencies as pointed in the case of Model 1 in Chapter 2, whereas, in the case of Model 2 the physical capital utilization rate has a significantly lower volatility than in the data.

3.5 Sensitivity Analysis

In this section a standard sensitivity analysis for both of the shock persistence parameters and their correlation similarly to Maffezzoli (2000).

Looking at the role of the correlation between the TFP and the human productivity one can observe in Table 3.7 that the Benchmark model's ability to match output growth persistence is highly sensitive to the correlation of the two shocks. The calibrated model's assumed correlation is close to unity but the two shocks are not fully correlated (0.996). It is clear that as the correlation is reduced the model can capture less and less of the output growth persistence. In the meantime, output growth and output level volatilities keep increasing to the point of overshooting the actual volatilities at the business cycle frequencies.

In contrast, Model 2, shown in Table 3.10, is less sensitive to changes in the shock correlation parameter. Volatilities at the business cycle frequency remains near the baseline value. Output growth persistence's response to a lower correlation is marginal compared to the Benchmark model, even though as it becomes smaller a clear trend of decreasing persistence can be observed.

Furthermore, for both Model 1 and 2 a sensitivity analysis is performed to assess the role of the shock persistence parameters in Tables 3.8 - 3.9 and 3.11 - 3.11 respectively. The underlying question is that how much of the output growth persistence is generated by the shock persistences themselves or the models' internal propagation mechanisms? In the case of Model 1 it is clear that the output growth persistence is highly dependent not just on the correlation between the shocks and their resonance through the economy but also the shock persistences. Also, it seems to be more dependent on the human productivity shock persistence. For Model 1 even for a persistence parameter of the human shock $\rho_h = 0.90$ the output growth persistence drops to close to zero. Model 2 exhibits similar trends, however, here with a decrease in any of the persistence parameters of the shocks output persistence gradually decreases unlike in the case of Model 1 thus suggesting a significantly more robust internal propagation mechanism for the two shocks of the model.

	Persistence of x					Volatility of x						
	$x =$	g_y	g_c	g_{ik}	l_g	g_y	g_c	g_{ik}	l_g	y	c	i_k
US data		0.265	0.365	0.262	0.987	0.0065	0.0037	0.0210	0.0050	0.0169	0.0098	0.0547
Baseline ($\sigma_{gh} = 0.996$)		0.328	0.712	0.149	0.851	0.0064	0.0027	0.0240	0.0260	0.0140	0.0053	0.0530
$\sigma_{gh} = 0.995$		0.428	0.802	0.197	0.878	0.0068	0.0027	0.0260	0.0280	0.0160	0.0060	0.0600
$\sigma_{gh} = 0.99$		0.407	0.718	0.241	0.901	0.0073	0.0032	0.0270	0.0320	0.0190	0.0071	0.0710
$\sigma_{gh} = 0.95$		0.269	0.692	0.052	0.864	0.0084	0.0033	0.0330	0.0330	0.0200	0.0072	0.0770
$\sigma_{gh} = 0.9$		0.297	0.649	0.054	0.895	0.0094	0.0037	0.0380	0.0300	0.0200	0.0076	0.0820
$\sigma_{gh} = 0.8$		0.124	0.666	-0.009	0.899	0.0100	0.0034	0.0420	0.0330	0.0230	0.0075	0.0930
$\sigma_{gh} = 0.7$		0.049	0.483	-0.034	0.844	0.0130	0.0042	0.0520	0.0330	0.0260	0.0086	0.1100
$\sigma_{gh} = 0.5$		0.074	0.369	-0.008	0.867	0.0160	0.0059	0.0650	0.0390	0.0320	0.0100	0.1300

Table 3.7: Model 1 - Sensitivity Analysis of the Shock Correlation Parameter.

	Persistence of x					Volatility of x						
	$x =$	g_y	g_c	g_{ik}	l_g	g_y	g_c	g_{ik}	l_g	y	c	i_k
US data		0.265	0.365	0.262	0.987	0.0065	0.0037	0.0210	0.0050	0.0169	0.0098	0.0547
Baseline ($\rho_g = 0.979$)		0.328	0.712	0.149	0.851	0.0064	0.0027	0.0240	0.0260	0.0140	0.0053	0.0530
$\rho_g = 0.95$		0.689	0.581	0.489	0.930	0.0035	0.0032	0.0110	0.0200	0.0092	0.0059	0.0300
$\rho_g = 0.90$		0.741	0.658	-0.007	0.976	0.0022	0.0028	0.0170	0.0180	0.0043	0.0055	0.0310
$\rho_g = 0.85$		0.625	0.749	-0.036	0.988	0.0047	0.0029	0.0300	0.0200	0.0095	0.0047	0.0600
$\rho_g = 0.80$		0.201	0.639	-0.111	0.982	0.0071	0.0025	0.0410	0.0250	0.0140	0.0044	0.0800
$\rho_g = 0.70$		0.010	0.631	-0.170	0.955	0.0110	0.0023	0.0590	0.0290	0.0210	0.0041	0.1100
$\rho_g = 0.60$		-0.010	0.646	-0.160	0.934	0.0140	0.0027	0.0730	0.0370	0.0270	0.0051	0.1400
$\rho_g = 0.50$		0.066	0.815	-0.192	0.936	0.0150	0.0032	0.0770	0.0370	0.0280	0.005	0.1400

Table 3.8: Model 1 - Sensitivity Analysis of the TFP Persistence Parameter.

	Persistence of x				Volatility of x						
$x =$	g_y	g_c	g_{ik}	l_g	g_y	g_c	g_{ik}	l_g	y	c	i_k
US data	0.265	0.365	0.262	0.987	0.0065	0.0037	0.0210	0.0050	0.0169	0.0098	0.0547
Baseline ($\rho_h = 0.982$)	0.328	0.712	0.149	0.851	0.0064	0.0027	0.0240	0.0260	0.0140	0.0053	0.0530
$\rho_h = 0.95$	0.223	0.774	0.072	0.811	0.0085	0.0025	0.0360	0.0330	0.0200	0.0059	0.0830
$\rho_h = 0.90$	0.069	0.587	-0.022	0.736	0.0100	0.0025	0.0460	0.0340	0.0230	0.0063	0.1000
$\rho_h = 0.85$	0.060	0.578	-0.009	0.733	0.0130	0.0029	0.0570	0.0410	0.0310	0.0081	0.1400
$\rho_h = 0.80$	-0.043	0.715	-0.104	0.667	0.0130	0.0024	0.0600	0.0390	0.0300	0.0070	0.1300
$\rho_h = 0.70$	-0.030	0.665	-0.081	0.619	0.0150	0.0027	0.0720	0.0410	0.0350	0.0080	0.1600
$\rho_h = 0.60$	-0.069	0.674	-0.117	0.562	0.0170	0.0031	0.0810	0.0430	0.0400	0.0100	0.1800
$\rho_h = 0.50$	-0.201	0.579	-0.240	0.414	0.0160	0.0027	0.0770	0.0360	0.0340	0.0084	0.1500

Table 3.9: Model 1 - Sensitivity Analysis of Human Shock Persistence Parameter.

	Persistence of x					Volatility of x							
$x =$	g_y	g_c	g_{ik}	l_g	u	g_y	g_c	g_{ik}	l_g	u	y	c	i_k
US data	0.265	0.365	0.262	0.987	0.956	0.0065	0.0037	0.0210	0.0050	0.0262	0.0169	0.0098	0.0547
Baseline ($\sigma_{gh} = 0.999$)	0.284	0.327	0.313	0.989	0.930	0.0065	0.0027	0.0180	0.0220	0.0042	0.0180	0.0074	0.0470
$\sigma_{gh} = 0.995$	0.314	0.319	0.366	0.989	0.907	0.0069	0.0031	0.0170	0.0190	0.0025	0.0160	0.0065	0.0410
$\sigma_{gh} = 0.99$	0.308	0.337	0.354	0.991	0.937	0.0061	0.0027	0.0150	0.0190	0.0023	0.0150	0.0062	0.0400
$\sigma_{gh} = 0.95$	0.306	0.312	0.361	0.990	0.935	0.0064	0.0028	0.0170	0.0200	0.0028	0.0160	0.0066	0.0440
$\sigma_{gh} = 0.9$	0.291	0.300	0.330	0.988	0.925	0.0068	0.0029	0.0180	0.0200	0.0031	0.0170	0.0066	0.0450
$\sigma_{gh} = 0.8$	0.262	0.343	0.273	0.993	0.968	0.0068	0.0027	0.0180	0.0190	0.0035	0.0170	0.0064	0.0430
$\sigma_{gh} = 0.7$	0.191	0.267	0.196	0.989	0.937	0.0076	0.0030	0.0200	0.0210	0.0039	0.0180	0.0069	0.0490
$\sigma_{gh} = 0.5$	0.213	0.214	0.241	0.987	0.966	0.0076	0.0027	0.0210	0.0210	0.0050	0.0200	0.0067	0.0560

Table 3.10: Model 2 - Sensitivity Analysis of the Shock Correlation Parameter.

	Persistence of x					Volatility of x							
$x =$	g_y	g_c	g_{ik}	l_g	u	g_y	g_c	g_{ik}	l_g	u	y	c	i_k
US data	0.265	0.365	0.262	0.987	0.956	0.0065	0.0037	0.0210	0.0050	0.0262	0.0169	0.0098	0.0547
Baseline ($\rho_g = 0.9794$)	0.284	0.327	0.313	0.989	0.930	0.0065	0.0027	0.0180	0.0220	0.0042	0.0180	0.0074	0.0470
$\rho_g = 0.95$	0.161	0.279	0.164	0.984	0.914	0.0770	0.0210	0.2300	0.1900	0.0390	0.1800	0.0480	0.5500
$\rho_g = 0.90$	0.280	0.530	0.263	0.978	0.883	0.0400	0.0075	0.1300	0.1100	0.0160	0.1000	0.0190	0.3300
$\rho_g = 0.85$	0.219	0.809	0.179	0.956	0.759	0.0600	0.0067	0.2000	0.0490	0.0180	0.1500	0.0170	0.5000
$\rho_g = 0.80$	0.133	0.390	0.074	0.941	0.618	0.1200	0.0082	0.4100	0.3100	0.0280	0.2800	0.0190	0.9800
$\rho_g = 0.70$	0.124	-0.272	0.039	0.884	0.679	0.1400	0.0190	0.5500	0.4000	0.0280	0.3600	0.0280	1.3000
$\rho_g = 0.60$	0.037	-0.270	-0.051	0.851	0.920	0.0470	0.0110	0.1900	0.1100	0.0100	0.1000	0.0170	0.4000
$\rho_g = 0.50$	-0.068	-0.329	-0.158	0.840	0.944	0.0290	0.0089	0.1300	0.0690	0.0081	0.0610	0.0140	0.2500

Table 3.11: Model 2 - Sensitivity Analysis of the TFP Persistence Parameter.

	Persistence of x					Volatility of x							
$x =$	g_y	g_c	g_{ik}	l_g	u	g_y	g_c	g_{ik}	l_g	u	y	c	i_k
US data	0.265	0.365	0.262	0.987	0.956	0.0065	0.0037	0.0210	0.0050	0.0262	0.0169	0.0098	0.0547
Baseline ($\rho_h = 0.9788$)	0.284	0.327	0.313	0.989	0.930	0.0065	0.0027	0.0180	0.0220	0.0042	0.0180	0.0074	0.0470
$\rho_h = 0.95$	0.164	0.240	0.169	0.976	0.892	0.0740	0.0190	0.2200	0.1800	0.0410	0.1700	0.0420	0.5200
$\rho_h = 0.90$	0.204	0.443	0.189	0.954	0.839	0.0240	0.0035	0.0770	0.0580	0.0120	0.0590	0.0089	0.1900
$\rho_h = 0.85$	0.159	0.889	0.123	0.949	0.816	0.0270	0.0023	0.0920	0.0680	0.0100	0.0670	0.0064	0.2300
$\rho_h = 0.80$	0.178	0.237	0.123	0.937	0.718	0.0500	0.0032	0.1800	0.1200	0.0140	0.1200	0.0069	0.4100
$\rho_h = 0.70$	0.148	-0.236	0.060	0.888	0.859	0.0580	0.0082	0.2200	0.1500	0.0100	0.1300	0.0110	0.4800
$\rho_h = 0.60$	0.027	-0.313	-0.081	0.815	0.977	0.0150	0.0037	0.0620	0.0400	0.0045	0.0330	0.0052	0.1300
$\rho_h = 0.50$	-0.003	-0.347	-0.136	0.781	0.986	0.0088	0.0029	0.0380	0.0240	0.0041	0.0180	0.0041	0.0730

Table 3.12: Model 2 - Sensitivity Analysis of the Human Shock Persistence Parameter.

3.6 Conclusion

In Chapter 3 we have shown that Model 2 described in this chapter can clearly match output, consumption, and investment growth persistence beyond the first degree and also shows a strong internal propagation mechanism to spread shocks, in which the more symmetric treatment of both capital stocks along the inter-temporal margins play a key role as noted by Benhabib et al. (1997) and DeJong et al. (1996). Furthermore, the estimated very small TFP and human sector productivity shock variances imply that the amplification through two intra-temporal margins is more powerful, thus explaining how small technological advances that are not so apparent to the public can drive real fluctuations in the U.S. economy.

In the process of adding entrepreneurial capacity and an endogenous physical capital depreciation rate to the initial model in Chapter 2 there have been very little trade-offs to obtain the above mentioned results while being able to still match the movements of consumption, investment, and labor and learning hours with output at the business cycle, the low frequency, and the medium term frequency of Comin and Gertler (2006). Lastly, the estimated TFP series based on the model solution of the calibrated Model 2 of Chapter 3 is able to match the Solow residual obtained by traditional methods better over the period of 1971:Q4 to 2012:Q2. In this respect Model 2 only performs worse than Model 1 during the Great Recession period.

Despite all the results that have been shown in Chapter 3 the major shortcoming of both models presented in the previous two chapters is that they over estimate the volatility of labor relative to output, which has been a reversal of the problem in standard RBC models, where labor tends to be less volatile as pointed out by King and Rebelo (2000).

Overall, however, with Model 2 and partially with Model 1 the RBC issues of a lack of strong propagation mechanism, the lack of an amplification mechanism, the Gali (1999) labor response and the consumption-output puzzle have all been explained making a strong case for including the endogenous growth mechanism of Lucas (1988) and entrepreneurial capacity.

B.1 Equilibrium Conditions

Define the Lagrange multiplier of the representative agent's budget constraint as λ_t, and that of the human capital accumulation's as χ_t. Then the agent's first order conditions are the following,

$$c_t : \quad c_t^{-\sigma} x_t^{A(1-\sigma)} e_t^{B(1-\sigma)} = \lambda_t; \tag{B.I-1}$$

$$l_{gt} : \quad A c_t^{1-\sigma} x_t^{A(1-\sigma)-1} e_t^{B(1-\sigma)} = \lambda_t w_t h_t; \tag{B.I-2}$$

$$l_{ht} : \quad A c_t^{1-\sigma} x_t^{A(1-\sigma)-1} e_t^{B(1-\sigma)} = \chi_t (1-\phi_2) A_h e^{z_t^h} \left[\frac{v_{ht} u_t k_t}{l_{ht} h_t}\right]^{\phi_2} h_t; \tag{B.I-3}$$

$$\begin{aligned} u_t : B c_t^{1-\sigma} x_t^{A(1-\sigma)} e_t^{B(1-\sigma)-1} = {} & \lambda_t \phi_1 A_g e^{z_t^g} \left[\frac{v_{gt} u_t k_t}{l_{gt} h_t}\right]^{\phi_1 - 1} (v_{gt} k_t) + \\ & + \chi_t \phi_2 A_h e^{z_t^h} \left[\frac{v_{ht} u_t k_t}{l_{ht} h_t}\right]^{\phi_2 - 1} (v_{ht} k_t) - \lambda_t \delta_k u_t^{\psi - 1} k_t; \end{aligned} \tag{B.I-4}$$

$$v_{gt} : \quad \lambda_t \phi_1 A_g e^{z_t^g} \left[\frac{v_{gt} u_t k_t}{l_{gt} h_t}\right]^{\phi_1 - 1} (u_t k_t) = \chi_t \phi_2 A_h e^{z_t^h} \left[\frac{(1 - v_{gt}) u_t k_t}{l_{ht} h_t}\right]^{\phi_2 - 1} (u_t k_t); \tag{B.I-5}$$

$$\begin{aligned} k_{t+1} : \quad \lambda_t = {} & \beta E_t \lambda_{t+1} \left[1 + r_{t+1}^k u_{t+1} v_{gt+1} - \frac{\delta_k}{\psi} u_{t+1}^{\psi}\right] + \\ & + \beta E_t \chi_{t+1} \phi_2 A_h e^{z_{t+1}^h} \left[\frac{v_{ht+1} u_{t+1} k_{t+1}}{l_{ht+1} h_{t+1}}\right]^{\phi_2 - 1} (u_{t+1} v_{ht+1}); \end{aligned} \tag{B.I-6}$$

$$\begin{aligned} h_{t+1} : \quad \chi_t = {} & \beta E_t \chi_{t+1} \left[1 + (1-\phi_2) A_h e^{z_{t+1}^h} \left[\frac{v_{ht+1} u_{t+1} k_{t+1}}{l_{ht+1} h_{t+1}}\right]^{\phi_2} l_{ht+1} - \delta_h\right] \\ & + \beta E_t \lambda_{t+1} w_{t+1} l_{gt+1}; \end{aligned} \tag{B.I-7}$$

where r_t^k and w_t denote the own marginal productivity conditions of physical and human capital such that $r_t^k \equiv F_{1t} = \phi_1 A_g e^{z_t^g} (v_{gt} u_t k_t)^{\phi_1 - 1} (l_{gt} h_t)^{1-\phi_1}$ and $w_t \equiv F_{2t} =$

$(1-\phi_1)A_g e^{z_t^g}(v_{gt}u_t k_t)^{\phi_1}(l_{gt}h_t)^{-\phi_1}$. Also, $p_{ht} \equiv \frac{\chi_t}{\lambda_t}$ denotes the relative price of human capital in terms of consumption goods. Then the representative agent's equilibrium conditions can be stated as,

$$A_g e^{z_t^g}(v_{gt}u_t k_t)^{\phi_1}(l_{gt}h_t)^{1-\phi_1} = c_t + i_{kt}; \tag{B.I-8}$$

$$A_h e^{z_t^h}((1-v_{gt})u_t k_t)^{\phi_2}(l_{ht}h_t)^{1-\phi_2} = h_{t+1} - (1-\delta_h)h_t; \tag{B.I-9}$$

$$p_{ht} = \left[\frac{A_g}{A_h}\right]\left[\frac{e^{z_t^g}}{e^{z_t^h}}\right]\left[\frac{1-\phi_1}{1-\phi_2}\right]^{1-\phi_2}\left[\frac{\phi_1}{\phi_2}\right]^{\phi_2}\left[\frac{v_{gt}u_t k_t}{l_{gt}h_t}\right]^{\phi_1-\phi_2}; \tag{B.I-10}$$

$$\frac{A}{x_t}\frac{c_t}{h_t} = w_t; \tag{B.I-11}$$

$$\frac{B}{(1-u_t)}\frac{c_t}{k_t} = r_t - \delta_k u_t^{\psi-1}; \tag{B.I-12}$$

$$x_t = 1 - l_{gt} - l_{ht}; \tag{B.I-13}$$

$$\frac{1-\phi_1}{\phi_1}\frac{v_{gt}u_t k_t}{l_{gt}h_t} = \frac{1-\phi_2}{\phi_2}\frac{(1-v_{gt})u_t k_t}{l_{ht}h_t}; \tag{B.I-14}$$

$$i_{kt} = k_{t+1} - k_t + \frac{\delta_k}{\psi}u_t^{\psi}k_t; \tag{B.I-15}$$

$$1 = \beta E_t\left[\left(\frac{c_t}{c_{t+1}}\right)^{\sigma}\left(\frac{x_{t+1}}{x_t}\right)^{A(1-\sigma)}\left(\frac{e_{t+1}}{e_t}\right)^{B(1-\sigma)}\left[1 + r_{t+1}^k u_{t+1} - \frac{\delta_k}{\psi}u_{t+1}^{\psi}\right]\right]; \tag{B.I-16}$$

$$1 = \beta E_t \left(\frac{c_t}{c_{t+1}}\right)\left(\frac{x_{t+1}}{x_t}\right)^{A(1-\sigma)}\left(\frac{e_{t+1}}{e_t}\right)^{B(1-\sigma)} \frac{p_{ht+1}}{p_{ht}}$$
$$\left[1 + (l_{gt+1} + l_{ht+1})(1-\phi_2)A_h e^{z^h_{t+1}} \left[\frac{(1-v_{gt+1})u_{t+1}k_{t+1}}{l_{ht+1}h_{t+1}}\right]^{\phi_2} - \delta_h\right]. \tag{B.I-17}$$

Equation (B.I-8) is the goods market clearing condition; equation (B.I-9) is the human capital law of motion; equation (B.I-10) defines the relative price of human capital in units of consumption goods; equation (B.I-11) is the intra-temporal condition that governs the substitution between leisure and consumption; meanwhile, equation (B.I-12) is the second intra-temporal condition governing the substitution between managerial capacity and consumption. Equation (B.I-13) is the time constraint; equation (B.I-14) equates weighted factor intensities across sectors; and (B.I-15) is the physical capital law of motion. Equations (B.I-16) and (B.I-17) are the inter-temporal capital efficiency conditions with respect to physical and human capital, where the capacity utilization of physical capital is the equivalent of used entrepreneurial capacity, u_t, and the capacity utilization of human capital is equivalent to total working time, $(1 - x_t)$.

The set of 10 equations in (B.I-8) - (B.I-17) and the marginal efficiency conditions fully describe the model. Altogether, there are 12 equations in 12 unknowns $\{k_{t+1}, h_{t+1}, i_{kt}, c_t, u_t, l_{gt}, l_{ht}, x_t, v_{gt}, p_{ht}, r_t, w_t\}$. Furthermore, the exogenous variables $\{z^g_t, z^h_t\}$ are governed by the AR(1) processes defined in equations (3.7), and (3.10).

B.2 Stochastic Discounting: Model 2

Once again normalizing the growing endogenous variables with the human capital stock, h_t, the equilibrium conditions of Model 2 in equations (B.I-8) to (B.I-17) become the following stationary system:

$$A_g e^{z^g_t}(v_{gt}u_t\tilde{k}_t)^{\phi_1} l_{gt}^{1-\phi_1} = \tilde{c}_t + \tilde{i}_{kt}; \tag{B.II-1}$$

$$A_h e^{z^h_t}((1-v_{gt})u_t\tilde{k}_t)^{\phi_2} l_{ht}^{1-\phi_2} = g_{ht+1} - (1-\delta_h); \tag{B.II-2}$$

$$p_{ht} = \left[\frac{A_g}{A_h}\right]\left[\frac{e^{z_t^g}}{e^{z_t^h}}\right]\left[\frac{1-\phi_1}{1-\phi_2}\right]^{1-\phi_2}\left[\frac{\phi_1}{\phi_2}\right]^{\phi_2}\left[\frac{v_{gt}u_t\tilde{k}_t}{l_{gt}}\right]^{\phi_1-\phi_2}; \qquad \text{(B.II-3)}$$

$$\frac{A}{x_t}\tilde{c}_t = w_t; \qquad \text{(B.II-4)}$$

$$\frac{B}{(1-u_t)}\frac{\tilde{c}_t}{\tilde{k}_t} = r_t^k - \delta_k u_t^{\psi-1}; \qquad \text{(B.II-5)}$$

$$x_t = 1 - l_{gt} - l_{ht}; \qquad \text{(B.II-6)}$$

$$\frac{1-\phi_1}{\phi_1}\frac{v_{gt}u_t\tilde{k}_t}{l_{gt}} = \frac{1-\phi_2}{\phi_2}\frac{(1-v_{gt})u_t\tilde{k}_t}{l_{ht}}; \qquad \text{(B.II-7)}$$

$$\tilde{i}_{kt} = g_{ht+1}\tilde{k}_{t+1} - \tilde{k}_t + \frac{\delta_k}{\psi}u_t^{\psi}\tilde{k}_t; \qquad \text{(B.II-8)}$$

$$1 = \beta E_t\left[\left(\frac{\tilde{c}_t}{\tilde{c}_{t+1}}\right)^{\sigma}\left(\frac{1}{g_{ht+1}}\right)^{\sigma}\left(\frac{x_{t+1}}{x_t}\right)^{A(1-\sigma)}\left(\frac{e_{t+1}}{e_t}\right)^{B(1-\sigma)}\left[1 + r_{t+1}^k u_{t+1} - \frac{\delta_k}{\psi}u_{t+1}^{\psi}\right]\right]; \qquad \text{(B.II-9)}$$

$$1 = \beta E_t\left(\frac{\tilde{c}_t}{\tilde{c}_{t+1}}\right)^{\sigma}\left(\frac{1}{g_{ht+1}}\right)^{\sigma}\left(\frac{x_{t+1}}{x_t}\right)^{A(1-\sigma)}\left(\frac{e_{t+1}}{e_t}\right)^{B(1-\sigma)}\frac{p_{ht+1}}{p_{ht}}$$
$$\left[1 + (l_{gt+1} + l_{ht+1})(1-\phi_2)A_h e^{z_{t+1}^h}\left[\frac{(1-v_{gt+1})u_{t+1}\tilde{k}_{t+1}}{l_{ht+1}}\right]^{\phi_2} - \delta_h\right]; \qquad \text{(B.II-10)}$$

where the factor prices become $r_t^k = \phi_1 A_g e^{z_t^g}(v_{gt}u_t\tilde{k}_t)^{\phi_1-1}l_{gt}^{1-\phi_1}$ and $w_t = (1-\phi_1)A_g e^{z_t^g}(v_{gt}u_t\tilde{k}_t)^{\phi_1}l_{gt}^{-\phi_1}$.

Therefore, as before the stationary system in equations (B.II-1) to (B.II-10) fully describe Model 2 in terms of the normalized variables $g_{ht+1} \equiv \frac{h_{t+1}}{h_t}$; $\tilde{k}_t \equiv \frac{k_t}{h_t}$; $\tilde{y}_t \equiv \frac{y_t}{h_t}$

$\tilde{i}_{kt} \equiv \frac{i_{kt}}{h_t}$; $\tilde{i}_{ht} \equiv \frac{i_{ht}}{h_t}$ and $\tilde{c}_t \equiv \frac{c_t}{h_t}$. Now we may proceed in the solution methodology of the model by solving for the BGP.

B.3 The BGP Solution: Model 2

Once again, express the stationary model equilibrium conditions for Model 2 in equations (B.II-1) - (B.II-10) in terms of the variables' long-run values. Then given that g_{ht+1} becomes $(1 + g)$ along the BGP, where g is the net BGP growth rate of the economy, the equilibrium conditions become,

$$A_g(v_g u\tilde{k})^{\phi_1} l_g^{1-\phi_1} = \tilde{c} + \tilde{i}_k; \tag{B.III-1}$$

$$A_h((1 - v_g)u\tilde{k})^{\phi_2} l_h^{1-\phi_2} = g + \delta_h; \tag{B.III-2}$$

$$p_h = \left[\frac{A_g}{A_h}\right] \left[\frac{1-\phi_1}{1-\phi_2}\right]^{1-\phi_2} \left[\frac{\phi_1}{\phi_2}\right]^{\phi_2} \left[\frac{v_g u\tilde{k}}{l_g}\right]^{\phi_1-\phi_2}; \tag{B.III-3}$$

$$\frac{A}{x}\tilde{c} = w; \tag{B.III-4}$$

$$\frac{B}{(1-u)}\frac{\tilde{c}}{\tilde{k}} = r^k - \delta_k u^{\psi-1}; \tag{B.III-5}$$

$$x = 1 - l_g - l_h; \tag{B.III-6}$$

$$\frac{1-\phi_1}{\phi_1}\frac{v_g\tilde{k}}{l_g} = \frac{1-\phi_2}{\phi_2}\frac{(1-v_g)u\tilde{k}}{l_h}; \tag{B.III-7}$$

$$\tilde{i}_k = \left[g + \frac{\delta_k}{\psi}u^{\psi}\right]\tilde{k}; \tag{B.III-8}$$

$$1=\beta\left(\frac{1}{1+g}\right)^{\sigma}\left[1+r^{k}u-\frac{\delta_k}{\psi}u^{\psi}\right]; \tag{B.III-9}$$

$$1=\beta\left(\frac{1}{1+g}\right)^{\sigma}\left[1+(1-x)(1-\phi_2)A_h\left[\frac{(1-v_g)u\tilde{k}}{l_h}\right]^{\phi_2}-\delta_h\right]; \tag{B.III-10}$$

where factor prices become $r^k=\phi_1 A_g(v_g u\tilde{k})^{\phi_1-1}l_g^{1-\phi_1}$ and $w=(1-\phi_1)A_g(v_g u\tilde{k})^{\phi_1}l_g^{-\phi_1}$.

Once again, define the auxiliary variables $f_g\equiv\frac{v_g uk}{l_g h}$ and $f_h\equiv\frac{(1-v_g)uk}{l_h h}$. Then the above system can be narrowed down to 12 equations in 12 unknowns, $\{f_g,f_h,g,\tilde{k},\tilde{i}_k,\tilde{c},v_g,l_g\}$ $\{l_h,u,w,r^k,p_h\}$ as,

$$A_g f_g^{\phi_1}l_g=\tilde{c}+\tilde{i}_k; \tag{B.III-11}$$

$$A_h f_h^{\phi_2}l_h=g+\delta_h; \tag{B.III-12}$$

$$p_h=\left[\frac{A_g}{A_h}\right]\left[\frac{1-\phi_1}{1-\phi_2}\right]^{1-\phi_2}\left[\frac{\phi_1}{\phi_2}\right]^{\phi_2}f_g^{\phi_1-\phi_2}; \tag{B.III-13}$$

$$\frac{A}{x}\tilde{c}=w; \tag{B.III-14}$$

$$\frac{B}{(1-u)}\frac{\tilde{c}}{\tilde{k}}=r^k-\delta_k u^{\psi-1}; \tag{B.III-15}$$

$$x=1-l_g-l_h; \tag{B.III-16}$$

$$\frac{1-\phi_1}{\phi_1}f_g=\frac{1-\phi_2}{\phi_2}f_h; \tag{B.III-17}$$

$$\tilde{i}_k = \left[g + \frac{\delta_k}{\psi} u^{\psi}\right] \tilde{k}; \tag{B.III-18}$$

$$1 = \beta \left(\frac{1}{1+g}\right)^{\sigma} \left[1 + r^k u - \frac{\delta_k}{\psi} u^{\psi}\right]; \tag{B.III-19}$$

$$1 = \beta \left(\frac{1}{1+g}\right)^{\sigma} \left[1 + (1-x)(1-\phi_2) A_h f_h^{\phi_2} - \delta_h\right]; \tag{B.III-20}$$

where the factor prices become $r^k = \phi_1 A_g f_g^{\phi_1 - 1}$ and $w = (1-\phi_1) A_g f_h^{\phi_1}$.
Given the exogenous information set of parameters $(\phi_1, \phi_2, A_g, A_h, \delta_k, \psi, \delta_h, \beta, \sigma, A, B)$, the uniqueness of the solution to the system in (B.III-11) - (B.III-20) can be narrowed down to the uniqueness of the variables g and u. In order to show this, one can solve for $f_g, f_h, \tilde{c}, \tilde{k}, \tilde{i}_k, x, l_g, l_h, v_g, p_h, w, r^k$ in terms of g and u, which leaves a system of two equations, (B.III-14) and (B.III-15), in two unknowns g and u. First, one may solve for f_g using (B.III-19) and the the expression for r^k, which yields,

$$f_g = \left[\frac{\frac{(1+g)^{\sigma}}{\beta} - 1 + \frac{\delta_k}{\psi} u^{\psi}}{\phi_1 A_g}\right]^{\frac{1}{\phi_1 - 1} u}. \tag{B.III-21}$$

Then f_h directly follows from (B.III-17) as,

$$f_h = \frac{1-\phi_1}{1-\phi_2} \frac{\phi_2}{\phi_1} f_g. \tag{B.III-22}$$

Next one can express total labor time $(l_g + l_h) \equiv D$ from equation (B.III-20):

$$D = (l_g + l_h) = \left[\frac{\frac{(1+g)^{\sigma}}{\beta} - 1 + \delta_h}{(1-\phi_2) A_h f_h^{\phi_2}}\right]. \tag{B.III-23}$$

To express the time shares one can express l_h in terms of g and u from equation (B.III-12) and then use the solution for total labor time,

$$l_h = \left[\frac{g + \delta_h}{A_h}\right] f_h^{-\phi_2}; \tag{B.III-24}$$

$$l_g = D - l_h. \tag{B.III-25}$$

Next by using equation (B.III-13) and the obtained expression for f_g it follows that the relative price of human capital in terms of g is,

$$p_h = \left[\frac{A_g}{A_h}\right]\left[\frac{1-\phi_1}{1-\phi_2}\right]^{1-\phi_2}\left[\frac{\phi_1}{\phi_2}\right]^{\phi_2} f_g^{\phi_1-\phi_2}. \tag{B.III-26}$$

Now one can obtain an expression in g and u for c/k from equation (B.III-11), after dividing both sides by $\tilde{k}$ and noticing that $v_g = \left[\frac{l_g f_g}{l_g f_g + l_h f_h}\right]$, as,

$$\frac{c}{k} = A_g f_g^{\phi_1-1}\left[\frac{l_g f_g}{l_g f_g + l_h f_h}\right] - g - \frac{\delta_k}{\psi}u^{\psi}. \tag{B.III-27}$$

After this from equation (B.III-17) and the definition of f_g and f_h it follows that,

$$\tilde{k} = l_g f_g + l_h + f_h. \tag{B.III-28}$$

Lastly, the solution for $\tilde{c}$ and $\tilde{i}_k$ directly follows from combining equations (B.III-27) and (B.III-28), and (B.III-18) and (B.III-28) respectively.

Then after substituting (B.III-21) - (B.III-28) into equations (B.III-14) and (B.III-15) one obtains a system of two highly nonlinear equations in g and u such that: $\Phi(g,u) = 0$. This system of two equations then can be solved numerically for the baseline calibration of parameters defined in Chapter 3.

Chapter 4

Monetary Business Cycles: The Inflation - Growth Relationship and the Role of Capacity Utilization

4.1 Introduction

The additional physical capital utilization margin combined with the application of the proposed calibration scheme of Chapter 1 has been shown to be effective in explaining a number of RBC issues in a two sector business cycle model with endogenous growth. In this chapter by adding exchange money to Model 2 we show that the empirical long-run negative relationship between output and inflation and output growth and inflation can be explained. An additional target to see that by applying the proposed calibration scheme we are able to explain the long run negative relationship between employment and inflation in the frequency domain.

There is a vast literature on the effect of inflation on output growth. Evaluation of this relationship dates back to early models of incorporating money such as Bailey (1956), Sidrauski (1967), Tobin (1969) Clower (1967), Lucas (1980) and Stockman (1981).

Cooley and Hansen (1989) extends the standard RBC model of Kydland and Prescott (1982) and Long and Plosser (1983) to include money in a Lucas (1980) fashion while concluding that inflation reduces the effective return to labor as money earned cannot be spent in the current time period but must be carried over to next period, which result is also supported by Gomme (1993), Aschauer and Greenwood (1983), and Carmichael (1989). Meanwhile, King and Rebelo (1990) suggest that a traditional exogenous growth models may be inappropriate to fully capture the long run inflation-output and employment-inflation relationships.

In U.S. data there is a clear postwar evidence of a negative correlation between output-growth and inflation in the frequency domain as it can be seen in Figures 4.1 and 4.2 as well as in Table 4.1. As it is highlighted in Table 4.1 there is a strong negative relationship across all frequencies between growth and inflation.

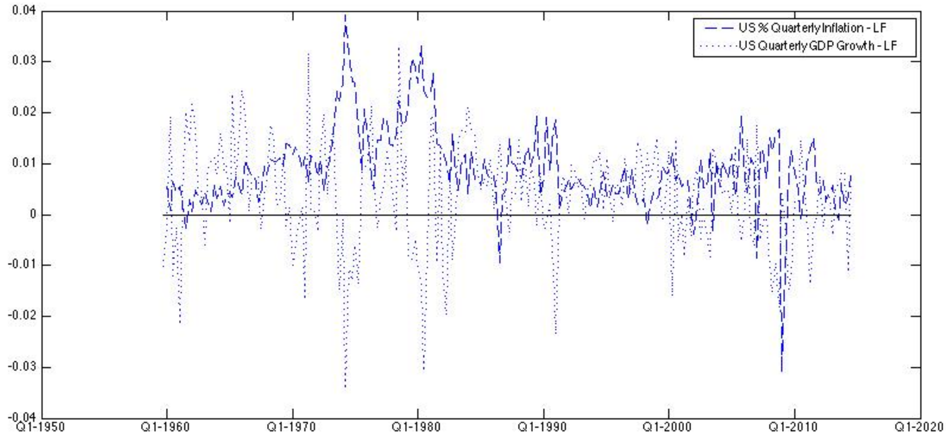

Figure 4.1: US Output Growth and Inflation: 1959:Q1 - 2014:Q2

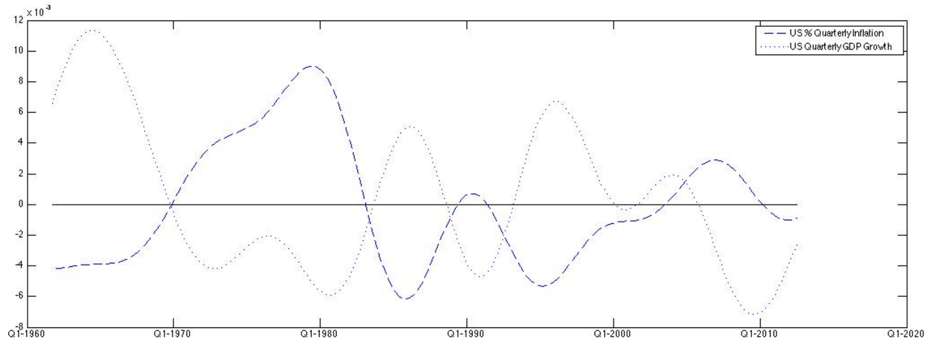

Figure 4.2: Low Frequency U.S. Output Growth - Inflation Relationship: 1961:Q4 - 2012:Q2

Band Pass	1967Q1 - 2004Q2
2 - 6 qrs	-0.397***
6 - 32 qrs	-0.498***
32 - 200 qrs	-0.735***
2 - 200 qrs	-0.522***

Table 4.1: Robustness Check for Low Frequency US GDP Growth - Inflation Correlation

In endogenous growth models without externalities such as Model 1 and 2 extended by exchange money inflation acts like a tax on capital, which in turn lowers growth as pointed out by Gillman et al. (2004); Gillman and Kejak (2005, 2011). It simply translates to more leisure time and less time spent in productive activity that requires both physical and human capital. Then given that the return to human capital falls, the return to physical capital must fall, which results in a Tobin type effect on the physical capital to human capital ratio as it increases in each productive sector and a negative effect on the growth rate of the economy as in Gillman and Nakov (2004).

The models presented in this chapter are direct extensions of Model 1 in Chapter 2 and Model 2 in Chapter 3 with exchange money and constant personal and corporate income taxes. The extension of Model 1 is denoted by Model 1m and the extension of Model 2 is denoted by Model 2m.

In Section 4.2 the model environment is described. In Section 4.3 the calibration is given with the extracted model based TFP and money supply shock processes. In Section 4.4 we present the models' impulse responses and the key real and monetary business cycle moments. Finally, Section 4.5 concludes the results.

4.2 The Model Environment

In this section the model is presented, which includes a convex endogenous physical capital depreciation rate as a function of the utilization rate to physical capital as in DeJong et al. (1996); and the addition of money balances with an exchange technology in line with Stockman (1981). In what we call Model 1m, we set the capacity utilization rate to one, and use the standard cash-only Clower (1967) constraint for the model's exchange constraint. Then it is the same as Gomme (1993) except that we also have income taxes as well as a human capital sectoral productivity shock. In the following general model specification this Model 1m case is achieved by setting parameters $B = \Omega = 0$ and $\psi = u = 1$; these parameters are specified below. We call this Model 1m. The full model, presented below, we then call Model 2m.

4.2.1 The Household's Problem

The representative household maximizes the present value of its infinite sum of period utilities. The agent derives utility from consumption, c_t, leisure, x_t, and disutility from unused entrepreneurial capacity, e_t, at each time period t. With

$A > 0$, $B < 0$, and $\sigma > 0$, the time t utility is given by

$$U(c_t, x_t, e_t) = \frac{[c_t x_t^A e_t^B]^{1-\sigma} - 1}{1-\sigma}, \tag{4.1}$$

which satisfies the necessary conditions for the existence of a balanced growth path equilibrium.[1] The representative consumer is confined by a normalized time endowment for each period t, where l_{gt} is the fraction of time spent in goods production, and l_{ht} in human capital investment production,

$$1 = x_t + l_{gt} + l_{ht}. \tag{4.2}$$

The representative household is also confined by an endowment of entrepreneurial capacity, which is normalized to unity, where u_t is the used entrepreneurial capacity,

$$1 = e_t + u_t. \tag{4.3}$$

The representative agent invests in physical capital, i_{kt}, according to the following physical capital accumulation constraint following DeJong et al. (1996),

$$k_{t+1} = k_t - \delta(u_t)k_t + i_{kt}, \tag{4.4}$$

where k_t is the physical capital stock at the beginning of time period t; and $\delta(u_t)$ is the endogenous depreciation rate of physical capital; and u_t is the physical capital utilization rate [i.e. used managerial capacity].[2] The endogenous depreciation rate is as such so that a faster rate of utilization results in a higher rate of depreciation. More specifically,

$$\delta(u_t) = \frac{\delta_k}{\psi} u_t^{\psi}, \tag{4.5}$$

with $\psi > 1$ and $\delta_k > 0$; note that it directly follows that $\delta'(u) > 0$ and $\delta''(u) > 0$ so that the marginal cost of utilization of the physical capital stock is increasing in the utilization rate.

The representative consumer also accumulates non-tradable human capital in a

[1] For more see King et al. (1988).

[2] Others with similar endogenous depreciation rate as a function of the utilization rate include Greenwood et al. (1988), DeJong et al. (1996), and Benhabib and Wen (2004).

separate home sector, h_t. The human capital stock is then accumulated over time according to the following standard law of motion,

$$h_{t+1} = (1-\delta_h)h_t + i_{ht}, \tag{4.6}$$

where δ_h is the assumed constant depreciation rate of human capital, and i_{ht} is the per period investment in human capital. Human capital is reproducible in a separate sector as in Lucas (1988). A_h is a positive factor productivity parameter in the human sector; z_t^h represents the productivity shock to the human sector in logarithmic form; $v_{ht} = 1 - v_{gt}$ is the remaining fraction of physical capital used in human capital production; u_t is the economy wide utilization rate of physical capital; $v_{ht}u_tk_t$ is the amount of physical capital in the human sector that is utilized for human investment production. l_{ht} denotes the share of human capital utilized in its production; and with $\phi_2 \in [0,1]$ meaning the share of physical capital in human capital investment production, it is produced according to the following constant returns to scale production function,

$$i_{ht} = H(v_{ht}u_tk_t, l_{ht}h_t) = A_h e^{z_t^h}(v_{ht}u_tk_t)^{\phi_2}(l_{ht}h_t)^{1-\phi_2}. \tag{4.7}$$

Where z_t^h is the human sector productivity shock process that is assumed to evolve according to a stationary first order autoregressive process in logarithmic form:

$$z_t^h = \rho_h z_{t-1}^h + \epsilon_t^h, \tag{4.8}$$

where the innovations ϵ_t^h is a sequence of independently and identically distributed normal random variables with zero mean and constant variance σ_h^2.

The agent then uses nominal money balances in the beginning of time period t denoted by M_t to buy consumption goods and to finance investment, which is governed by the following exchange constraintL

$$M_t + N_t \geq P_tc_t + \Omega P_t i_{kt}, \tag{4.9}$$

where N_t is nominal government transfers of money in time period t; and P_t is the price of goods [i.e. price level]. Normalizing with the price level then yields the following exchange constraint in real terms:

$$m_t + n_t \geq c_t + \Omega i_{kt}, \tag{4.10}$$

where $m_t \equiv \frac{M_t}{P_t}$ is real money balances in the beginning of time period t; $n_t \equiv \frac{N_t}{P_t}$ is the real government transfer. One may note, when the parameter $\Omega = 0$ the exchange constraint in (4.10) implies that real money balances are used to purchase consumption only. When $\Omega = 1$ the exchange technology constraint implies that both physical investment goods and consumption goods can be purchased. For the underlying model Ω is assumed to be one, which scenario has been introduced by Stockman (1981).

The household earns nominal labor income, $P_t w_t l_{gt} h_t$, where w_t is the real wage rate; nominal rents from capital, $P_t r_t^k v_{gt} u_t k_t$, where r_t^k is the real return on physical capital; and income on nominal risk-free government bonds held over the previous period, $(1 + R_t)B_t$, where $(1 + R_t)$ is the gross risk-free rate of return on government bonds. The household also receives a nominal lump-sum government transfer, N_t, to augment nominal money balances in the beginning of the time period carried over from the previous period, M_t. The household then uses these balances to purchase goods and/or investment good purchases. It also carries over to the next time period the remaining money balances after purchases denoted by M_{t+1}. The household's nominal expenditures include consumption purchases, $P_t c_t$, physical investment purchases, $P_t i_{kt}$, and nominal government bond purchases to be held over the given period, B_{t+1}. With constant tax rates on personal and corporate income taxes being denoted by τ_l and τ_k, and using the law of motion of physical capital in equation (4.4) the nominal budget constraint that the agent faces in each time period can be written as:

$$\begin{aligned} P_t c_t = (1-\tau_l) P_t w_t l_{gt} h_t + (1-\tau_k) P_t r_t^k v_{gt} u_t k_t - P_t \left[k_{t+1} - k_t + \frac{\delta_k}{\psi} u_t^{\psi} k_t \right] \\ - B_{t+1} + (1+R_t) B_t - M_{t+1} + M_t + N_t. \end{aligned} \tag{4.11}$$

Similarly to the exchange constraint the budget constraint can also be written in real terms after normalizing it with P_t. In order to be able to write it in real terms let's define gross inflation as $(1 + \pi_{t+1}) \equiv \frac{P_{t+1}}{P_t}$; and real government bonds as $b_t \equiv \frac{B_t}{P_t}$.

Then the representative consumer's budget constraint in real terms is the following:

$$\begin{aligned} c_t = (1-\tau_l)w_t l_{gt}h_t + &(1-\tau_k)r_t^k v_{gt}u_t k_t - k_{t+1} - k_t + \frac{\delta_k}{\psi}u_t^{\psi}k_t - (1+\pi_{t+1})b_{t+1} \\ &+ (1+R_t)b_t - (1+\pi_{t+1})m_{t+1} + m_t + n_t. \end{aligned} \tag{4.12}$$

Then the household maximises the expected present value of its lifetime stream of utility,[3]

$$\max_{\{c_t,x_t,l_{gt},l_{ht},v_{gt},v_{ht},u_t,e_t,k_{t+1},h_{t+1},m_{t+1},b_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \frac{[c_t x_t^A e_t^B]^{1-\sigma} - 1}{1-\sigma} \tag{4.13}$$

subject to (4.2), (4.3), (4.4) (4.5), (4.6), (4.7),(4.10), and (4.12).

4.2.2 The Goods Producer's Problem

The goods producer firm maximizes its real profits, Π_{gt}, in each time period t, given by:

$$\max_{\{(v_{gt}u_t k_t),(l_{gt}h_t)\}} \Pi_{gt} = y_t - w_t l_{gt}h_t - r_t^k v_{gt}u_t k_t, \tag{4.14}$$

subject to a constant returns to scale *Cobb-Douglas* production technology for goods, where the inputs are effective labor, $l_{gt}h_t$, and sectoral utilized physical capital, $v_{gt}u_t k_t$, similar to King and Rebelo (2000):

$$y_t = G(v_{gt}u_t k_t, l_{ht}h_t) = A_g e^{z_t^g}(v_{gt}u_t k_t)^{\phi_1}(l_{gt}h_t)^{1-\phi_1}, \tag{4.15}$$

where z_t^g is the goods sector productivity shock process (TFP) that is assumed to evolve according to a stationary first order autoregressive process in logarithmic form:

$$z_t^g = \rho_g z_{t-1}^g + \epsilon_t^g, \tag{4.16}$$

where the innovations ϵ_t^g is a sequence of independently and identically distributed normal random variables with zero mean and constant variance σ_g^2. The goods producer firm's first order conditions can be found in *Appendix C.1*.

[3] The household's first-order and equilibrium conditions can be found in *Appendix A.4*

4.2.3 The Government

The government spends a lump-sum nominal transfer of cash, N_t, that is given to the household each time period. The government finances this through a personal income, τ_{lt}, and a corporate income, τ_{kt}, tax, and by printing money, $M_{t+1} - M_t$. By assumption for asset markets to clear we assume that in each time period t the real bonds are zero, $b_t = 0$. Then the government's budget constraint can be written as,

$$N_t = M_{t+1} - M_t + P_t[(1-\tau_{lt})w_t l_{gt} h_t + (1-\tau_{kt}) r_t^k v_{gt} u_t k_t]. \tag{4.17}$$

Assuming that the underlying money supply is such that there is a constant rate of money supply growth along the *balanced growth path*, defined by

$$\theta_t \equiv \frac{M_{t+1} - M_t}{M_t} \equiv \frac{N_t - P_t[(1-\tau_{lt})w_t l_{gt} h_t + (1-\tau_{kt}) r_t^k v_{gt} u_t k_t]}{M_t}. \tag{4.18}$$

Given the definition of the money supply in equation (4.18) it can be written in a more compact form as:

$$M_{t+1} = M_t(1+\theta_t). \tag{4.19}$$

In order to be consistent with previous notation the money supply rule in real terms becomes,

$$(1+\pi_{t+1})m_{t+1} = m_t(1+e^{\theta_t}); \tag{4.20}$$

where θ_t is a stochastic money supply growth process, which is assumed to take the following AR(1) form in natural logarithms,

$$\theta_t = \bar{\theta} + \rho_m \theta_{t-1} + \epsilon_t^m, \tag{4.21}$$

where ϵ_t^m is a white-noise process with constant variance σ_m^2; and $\bar{\theta}$ is the constant *BGP* rate of money supply growth.

4.2.4 Characterization of the Equilibrium

(E.1): Given the processes $\{\pi_{t+1}\}$, $\{w_t\}$, $\{r_t^k\}$, $\{z_t^h\}$, $\{R_t\}$, and $\{n_t\}$ the household solves its utility maximization problem in (13).

(E.2): Given the processes $\{w_t\}$, $\{r_t^k\}$, $\{z_t^g\}$, and the production technology in equation (4.15), the goods producer maximizes (4.14)).

(E.3): Asset, goods, and money markets clear so that $b_t = 0$, $y_t = c_t + i_{kt}$, and $n_t = \theta_t m_{t-1}$.

4.3 Model Simulation

4.3.1 Data

The data series for calibration purposes and to obtain moments have been filtered by a Christiano and Fitzgerald (2003) type asymmetric band-pass filter following Baxter and King (1999), Comin and Gertler (2006), and Basu et al. (2012). As the US data, using the period of 1959:Q1 to 2014:Q2, is of quarterly frequency. The data is filtered by the underlying band-pass filter at the medium term frequency, which includes all components and has a defined periodicity of 2 up to 200 quarters in line with the definition of Comin and Gertler (2006). A detailed description of the real business cycle data can be found in Chapter 1. The monetary data series are described in detail in *Appendix C.2*.

4.3.2 Calibration

Our calibration uses the *Simulated Annealing* algorithm of *Matlab's* Global Optimization Toolbox that generalizes the methodology of Jermann (1998), whereby we also create convergence between the computer-chosen calibration parameters and the estimated parameters of the shocks that are identified using the Ingram et al. (1997) and Benk et al. (2005, 2008, 2010) methodology of shock construction for the designated time period as described in Chapter 1.

4.3.3 The Calibration

Table 4.2 contains 13 and 11 different pieces of information about the data from different sources that include business cycle and monetary information. These pieces of information then are used as our calibration targets for structural parameters in Models 1m and 2m and they determine how we set up the calibration algorithm described in Chapter 1.

Description	Calibration Target	Target Value/Range	Achieved Value Model 1m	Achieved Value Model 2m	Target Source
Output Growth Rate	g	0.0035	0.0035	0.0035	Gomme and Rupert (2007); NIPA
Phy. Capital Utilisation Rate	u	0.7852	-	0.7852	US Census Bureau; NAICS
Consumption Output Ratio	c/y	0.70	0.81	0.80	Gomme and Rupert (2007)
Phy. Investment Output Ratio	i_k/y	0.30	0.19	0.20	Gomme and Rupert (2007)
Leisure	x	0.50	0.50	0.50	Gomme and Rupert (2007)
Labor Time	l_g	0.30	0.36	0.28	Jones et al. (2005)
Learning Time	l_h	0.24	0.16	0.20	Gomme and Rupert (2007)
Real Interest Rate	r^k	$0.050 - 0.019$	0.072	0.088	Greenwood et al. (1993); Siegel (1992)
Phy. Capital Depreciation Rate	δ_k / $\delta(u)$	0.025	0.025	0.025	Gomme and Rupert (2007)
Hum. Capital Depreciation Rate	δ_h	$0.008 - 0.03$	0.025	0.029	Jorgenson and Fraumeni (1991); DeJong and Ingram (2001)
CES (CRRA) Coefficient	σ	$0.34 - 2.0$	0.42	0.43	Hall (1988); Mehra and Prescott (1985)
Nominal Interest Rate	R	0.048	0.054	0.038	3 Months T-bill Rate
Inflation Rate	π	0.98%	0.67%	0.67%	US CPI

Table 4.2: Target values of the baseline calibrations for Model 1m and 2m for the U.S.

Partially following Gomme and Rupert (2007) for the calibration of both models we set the balanced growth rate, g, the money supply growth rate, θ, and leisure, x, equal to their long run target values in Model 1m. In addition for Model 2m we once again set the endogenous depreciation rate $\delta(u)$ and we set the physical capital utilization rate, u, equal to their long run target values. Their values are 0.0035, 0.01, 0.50, 0.025, and 0.7852, respectively as it is shown in Table 4.2. Given these strict targets the leisure preference weight A, and the human capital sector scale parameter A_h can be directly pinned down from the long run versions of the intra-temporal margin and the inter-temporal margin with respect to human capital combined with the other 8 structural parameters in Model 1m. For Model 2m this allows us to directly pin down the leisure preference weight A, the human capital sector scale parameter A_h, the entrepreneurial capacity weight in preference B, and the constant depreciation parameter δ_k, from the long run versions of the intra-temporal margins, the inter-temporal margin with respect to human capital, and the definition of the endogenous depreciation rate combined with the other 8 structural parameters in Model 2m. For Model 1m then the set of structural parameters that we directly search for with *Simulated Annealing* within our procedure is $\{\beta, \sigma, \phi_1, \phi_2, A_g, \delta_k, \delta_h, \Omega\}$, and for Model 2m it is $\{\beta, \sigma, \phi_1, \phi_2, A_g, \psi, \delta_h\Omega\}$.

As in Chapter 2 and 3 we are targeting the real interest rate at 5 percent per quarter following Gomme and Rupert (2007). The physical capital depreciation rate along the BGP is once again targeted at 2.7 percents per quarter in line with Gomme and Rupert (2007). In Model 2m instead of the depreciation parameter we target the long run endogenous rate of depreciation similarly at 2.5 percent. Our calibration algorithm with allowing for a variation in the convexity parameter of the endogenous depreciation rate ψ between the values of 1 and 4 yields a parameter value of 3.49. Given our target it pins down the depreciation parameter δ_k at 0.202.

For the CES parameter σ we set our target range between 0.34 to 2 following Hall (1988) and Mehra and Prescott (1985). Within this range we calibrate σ as 0.418 in Model 1m and 0.433 in Model 2m. Then by limiting the discount factor search range in our procedure to 0.95 to 0.99 we find β to be of the value of 0.959 in Model 1m, and a value of 0.973 in Model 2m for β so that the inter-temporal margin in each model holds while it pins down the real rate of interest.

For the human sector parameters our algorithm finds that for Model 1m the quarterly depreciation rate of human capital is 2.5 percent, whereas, for Model 2m it gave us a depreciation rate of 2.9 percent per quarter. The share of physical capital in goods production, ϕ_1, and in human production ϕ_2, is chosen by our procedure to target 0.3 time share in goods labor and a time share of 0.24 in the human investment sector as in Gomme and Rupert (2007). We bounded our algorithm to search for the capital share in goods production within the range of 0.3 and 0.42;

Parameter	Description	Model 1m	Model 2m
β	Subjective Discount Factor	0.959	0.973
σ	CES Parameter	0.418	0.433
A	Weight of Leisure in Preference	1.11	1.21
B	Weight of Man. Capacity in Preference	-	-0.07
A_g	Scale Parameter of Goods Sector	0.737	1.25
A_h	Scale Parameter of Human Sector	0.117	0.104
ϕ_1	Physical Capital Share in Goods Production	0.29	0.32
ϕ_2	Physical Capital Share in Human Investment	0.22	0.25
δ_k	Depreciation Parameter (Physical Capital)	0.025	0.202
ψ	Convexity of Endog. Depr. Rate	-	3.49
δ_h	Depreciation Rate of Human Capital	0.025	0.029
$\bar{\theta}$	BGP Money Supply Growth Rate	0.01	0.01
Ω	Exchange Constraint Parameter	0.028	0.06
τ_l	Constant Personal Income Tax Rate	-	0.189
τ_k	Constant Corporate Income Tax Rate	-	0.189

Table 4.3: Structural parameter values of the baseline calibrations for Model 1m and 2m for the U.S.

and to search within the range of 0.08 and 0.25 for the capital share in human investment production. For Model 1m the capital share in goods production is 0.29 and in Model 2m it is 0.32. The capital share in human investment production for Model 1m is 0.22, and for Model 2m it is 0.25. Given the values our calibration procedure yielded the labor time obtained in Model 1m is 0.36 and 0.28 in Model 2m, whereas the learning time is 0.16 in Model 1m and 0.20 in Model 2m.

The capital shares, the human and physical capital depreciation rates, and the goods sector scale parameter combined with our strict targets then pin down the scale parameter for the human sector so that the inter-temporal margins hold.

By setting the money supply growth rate equal to its long-run value we are able to pin down the BGP inflation rate of the model, meanwhile, it also pins down the nominal interest through the Fisher relationship.

Parameter	Description	Model 1m	Model 2m
ρ_g	Auto-correlation of TFP	0.968	0.957
ρ_h	Auto-correlation of Human Shock	0.977	0.974
ρ_m	Auto-correlation of Human Shock	0.975	0.979
σ_g^2	Variance of TFP	1.1x10ˆ(-6)	9.8x10ˆ(-4)
σ_h^2	Variance of Human Productivity Shock	8.9x10ˆ(-7)	3.6x10ˆ(-4)
σ_m^2	Variance of Human Productivity Shock	0.0054	0.084
$\sigma_{g,h}$	Correlation of Shock Innovations	-0.849	-0.468
$\sigma_{m,h}$	Correlation of Shock Innovations	0.716	0.305
$\sigma_{g,m}$	Correlation of Shock Innovations	-0.976	0.611

Table 4.4: Shock process parameter values of the baseline calibrations for Model 1m and 2m for the U.S.

Finally, through the inter-temporal margins in the respective models we can pin down the values of preference weights. In Model 2m we used annual effective personal and corporate income tax rates from the Congressional Budget Office to calculate their long-run values. Then we calculated the average of the two tax rates, which we used as the calibrated The complete list of calibrated structural parameters for Model 1m and 2m can be found in Table 4.3 below.

Table 4.4 lists the parameters of the exogenous shock process, which are obtained through the iterative step of the calibration procedure described more in detail in the next section. Here we somewhat deviated from the procedure described in Chapter 1 in the sense that we used the iterative estimation procedure for shock parameters for their respective persistence parameters and the shock variances. For the shock correlations we used the correlations picked by our optimization algorithm.

4.3.4 The Shock Processes: 1971:Q4 to 2012:Q2

In this section, similarly to Chapter 2 and 3, the shocks of Model 1m and 2m are constructed using US data following the procedure in Benk et al. (2005, 2008, 2010) and Nolan and Thoenissen (2009). The data used that yielded the most highly correlated shock series are $[\widehat{c_t/y_t}\widehat{i_{kt}/y_t}\widehat{i_{kt}/h_t}\hat{u}_t\hat{l}_{gt}, \hat{\pi}_t, \hat{R}_t]$ for Model 2m and the same less the series for the physical capital utilization rate for the Model1m. Then the DSGE based TFP series are compared to a traditional Solow residual based TFP using the best calibration we obtained [i.e. Model 2m calibration in the previous section]. The same exercise is done for the money supply shock.

In Figure 4.3 the unfiltered TFP series obtained from Model 1m and 2m are compared to the traditionally obtained goods sector TFP. Figures 4.4 and 4.5 show the components of the DSGE shock series relative to the Solow residual based one after using a Christiano and Fitzgerald (2003) band-pass filter, where once again the high frequency component has a periodicity of 2 to 6 quarters; the business cycle component has a periodicity of 6 up to 32 quarters; the low frequency component is from 32 up to 200 quarters and the Comin and Gertler (2006) medium cycle containing all three components with a periodicity of 2 up to 200 quarters. It must be noted that despite most of the series being available from 1959:Q1 we are only extracting the shock series from 1971:Q4 due to the lack of data availability of the physical capital utilization rate. Also, the human capital index data is only available until 2012:Q2. Therefore as before, our shock series are obtained for the period of 1971:Q4 until 2012:Q2.

One can observe that the DSGE based TFP shock series have a larger volatility as the traditional series. On the other hand the DSGE based series do a very good job at capturing recessions over the defined period in U.S. economic history. More specifically, during the 1973 oil price shock despite suggesting a much larger drop in TFP it captures the recessionary period well. For both model based TFP series in terms of capturing the shape of the technology process they are able to match the traditional TFP series starting from 1984 throughout the 1990s. Similarly to the scenario in Chapter 3 the DSGE model based series fail to capture the pick-up in technology suggested by the the traditional seres from 2009.

Figure 4.4 and 4.5 show the four different components of the TFP series obtained

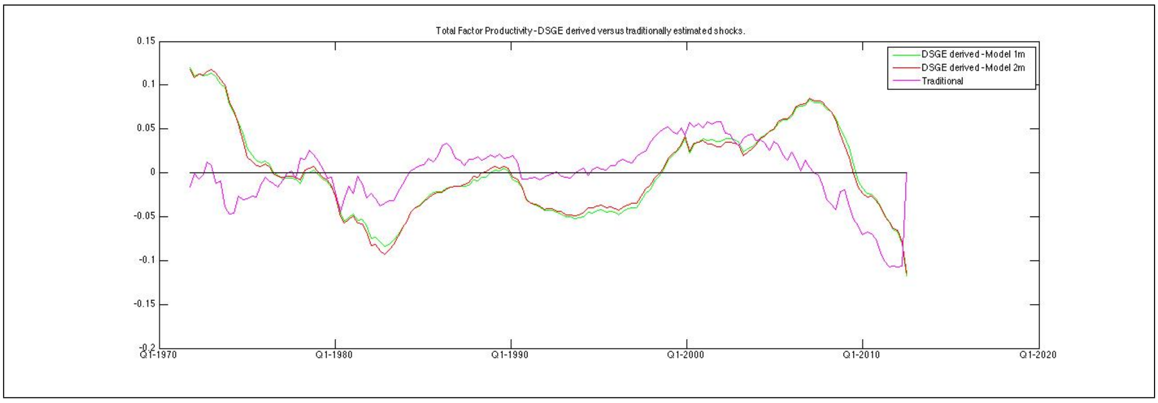

Figure 4.3: Total Factor Productivity - DSGE derived in Model 1m and Model 2m versus traditionally estimated shocks.

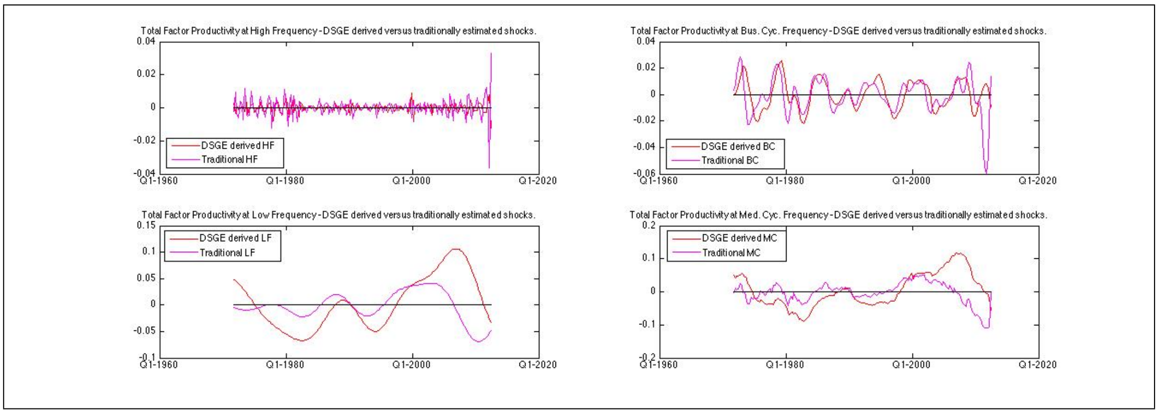

Figure 4.4: Total Factor Productivity - Model 1m DSGE derived versus traditionally estimated shock frequency components.

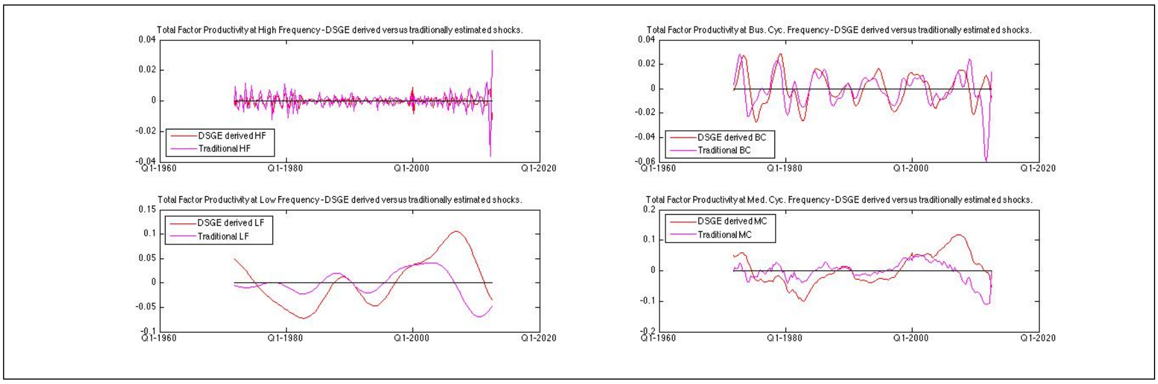

Figure 4.5: Total Factor Productivity - Model 2m DSGE derived versus traditionally estimated shock frequency components.

from Model 1m and Model 2m relative to the different frequency components of the Solow residual based series. In Figure 4.4 for the series based on the solution of Model 1m we can observe that at the business cycle frequency the DSGE based series closely matches the traditional TFP except for the recent crisis period. Unlike in Chapter 3 we note that at the low frequency and medium cycle the DSGE based series is still unable to match the pick up in technology after the Great Recession period. In Figure 4.5 we can see an identical trend in the decomposition of the TFP series based on Model 2m.

In Figure 4.6 one can see the traditionally obtained money supply shock series [i.e. based on the models' money supply rule] relative to the two DSGE based series. We must note that the two models after using the estimated shock variances overstate the magnitude of the money supply shock by a scale of 100 relative to the traditionally obtained series. Therefore, in order to be able to compare the cyclical features of the different series at different frequencies we have scaled the traditionally obtained series by 100.

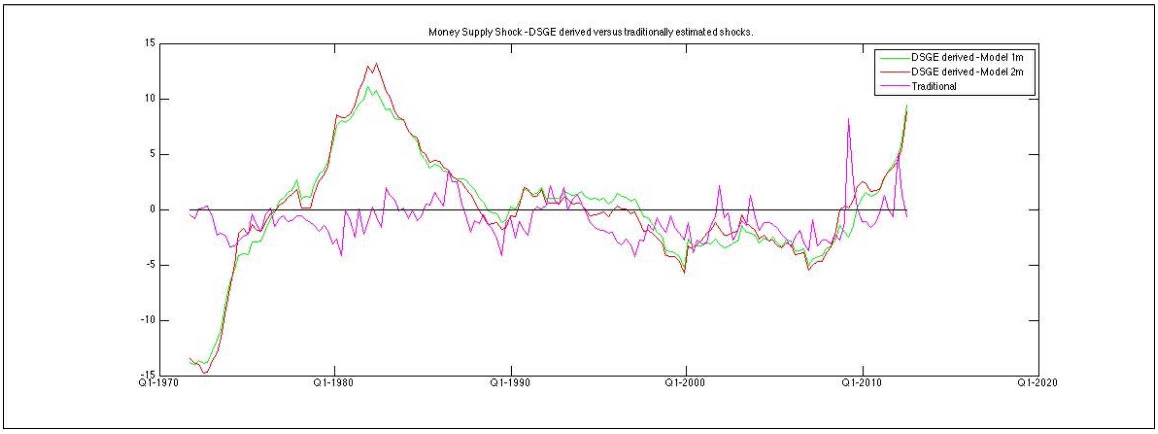

Figure 4.6: Money Supply Shock - DSGE derived in Model 1m and Model 2m versus amplified (x100) traditionally estimated shocks.

We observe in Figure 4.6 that the two DSGE based series follow each other very closely. Also, starting from the mid and late 1980s they are able to match the traditionally obtained money supply shock series very closely until the Great Recession period. During the 1970s we observe a prolonged increase in the money supply until the early 1980s. Despite the fact that the DSGE based series do not follow the traditionally obtained one this hike in the model based series during this period suggest that the model based series are able to capture the high inflationary period in U.S. postwar economic history. The model based series then converge toward the traditionally obtained series, which coincide with the beginning of the Great Moderation era. Lastly, we observe that the DSGE based series from the beginning of the recent crisis show a prolonged hike in the money supply that coincides with the quantitative easing implemented by the Federal Reserve. We must note though that during this last period the traditionally obtained series show more fluctuation in the money supply, which is not captured by the model based series.

Figure 4.7 presents the components of the traditional and Model 1m based money supply shock series components. We can observe that at the business cycle frequency the two series matches each other relatively well apart from the early 1970s and the recent crisis period. On the other hand, at the low and medium cycle the Model 1m

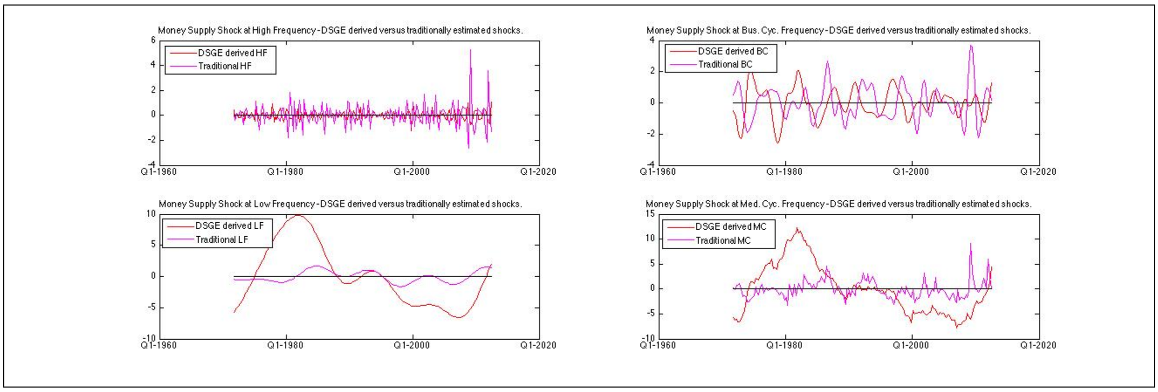

Figure 4.7: Money Supply Shock - Model 1m DSGE derived versus amplified (x100) traditionally estimated shocks frequency components.

based shock series until the mid 1980s and from around 2000 exhibits a much larger magnitude than the traditional and scaled money supply shock series. This trend at the low frequency is not offset by the business cycle component.

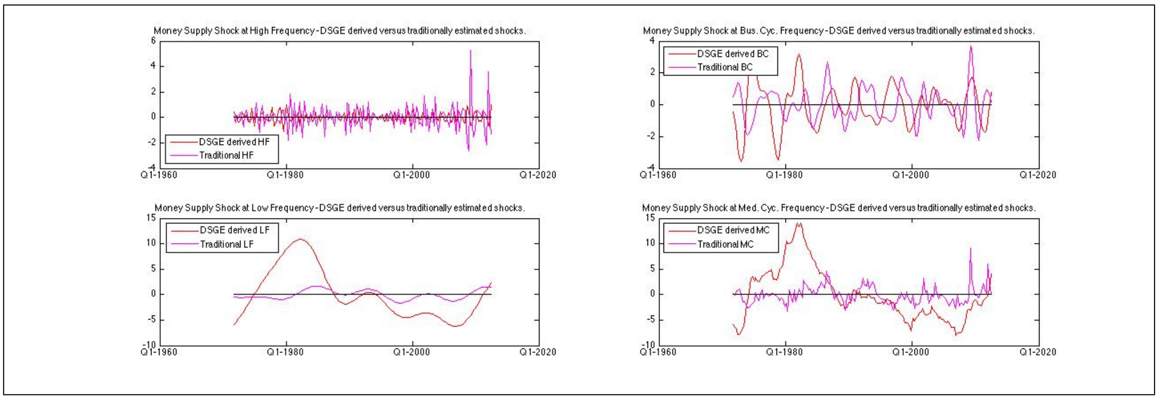

Figure 4.8: Money Supply Shock - Model 2 DSGE derived versus amplified (x100) traditionally estimated shocks frequency components.

Figure 4.8 presents the components of the Model 2m based series relative to the traditionally obtained one. The DSGE based series exhibits the same features and trends as the Model 1m based series at different frequencies with marginally different magnitudes.

Overall, we can note that the DSGE based TFP series are able to match the traditionally obtained goods sector TFP series well in the case of both model. On the other hand, the money supply shock estimated using the model solutions tend to overestimate the magnitude of the money supply shock relative to the traditionally obtained one.

4.4 Simulation Results

4.4.1 Impulse Responses

The short term dynamics of Model 1m can be analyzed via the impulse responses of orthogonalized shocks to to the goods sector TFP, z_t^g; the human sector productivity, z_t^h; and the money supply, θ_t. We also show the responses to an aggregate productivity shock, which occurs when the goods sector and the human sector is hit by their respective shocks simultaneously.

Model 1m: Impulse Response Analysis

Figures 4.9 through 4.16 plot the impulse responses to the respective shocks for Model 1m.

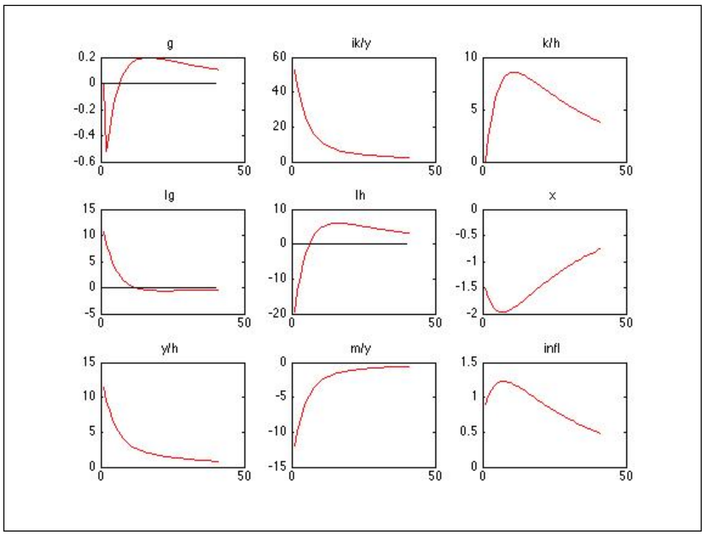

Figure 4.9: Impulse responses in percent deviation from the steady state (1) - Goods TFP Shock in Model 1m.

A positive 1 percent productivity shock in the goods sector makes the agent substitute time away from producing human capital investment goods and leisure towards labor in goods production. This lowers human investment and increases goods output. As a result of the substitution away from learning time the growth rate initially decreases. The TFP shock causes an initial jump in the rental rate of capital that drives the capital investment ratio upwards first. The surge in goods output and physical investment as a result of the exchange technology, in which capital investment is partially financed from money holdings, causes an upward jump in the inflation rate. The increase in savings as a result of the decreased leisure and higher investment generates inter-temporal substitution in leisure, which explains the subsequent increase in x.

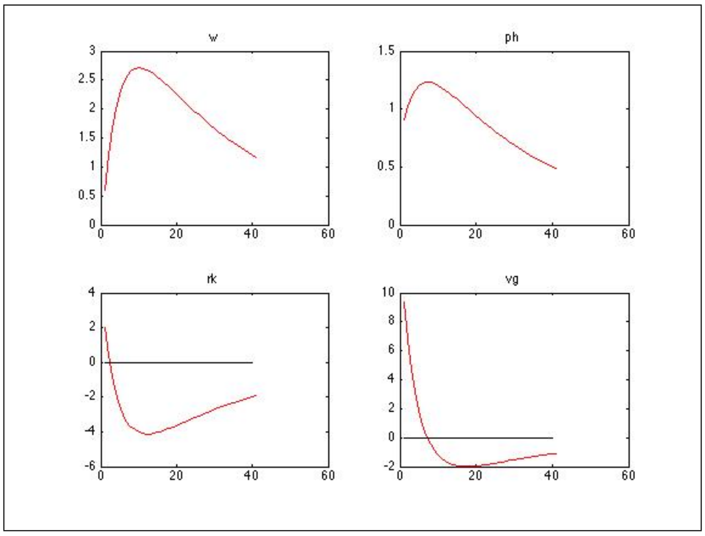

Figure 4.10: Impulse responses in percent deviation from the steady state (2)- Goods TFP Shock in Model 1m.

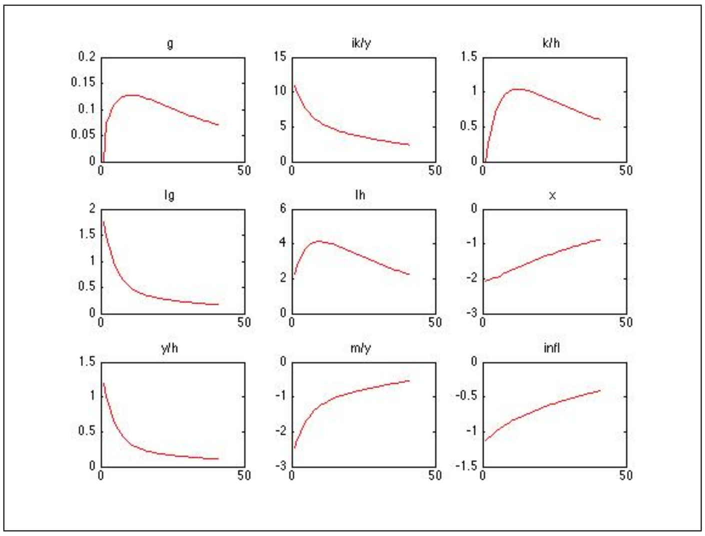

Figure 4.11: Impulse responses in percent deviation from the steady state (1) - Human Productivity Shock in Model 1m.

In response to a 1 percent positive human sector productivity shock in z_t^h, the agent substitutes away time from goods production and leisure towards time in human investment production. Physical investment declines as the agent allocates more resources to human capital production. The growth, however, increases as a result of the long-run positive effect of human capital on output. Inflation due to lower production and physical investment drops and as resources are reallocated back to goods production through the adjustment in the price of human capital it gradually returns to its BGP level.

In Figures 4.13 and 4.14 a positive money supply shock raises the inflation rate, which triggers the substitution of time from labor in the goods sector and leisure towards time in human investment production as the relative price of human capital

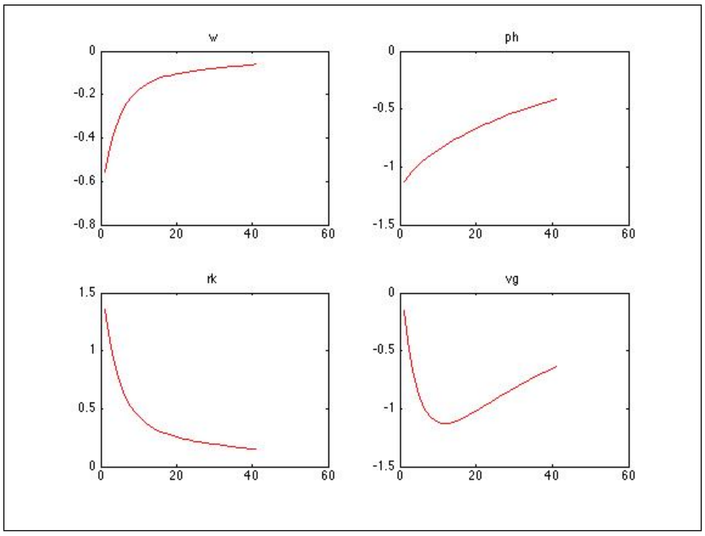

Figure 4.12: Impulse responses in percent deviation from the steady state (2) - Human Productivity Shock in Model 1m.

increases and thus resources are reallocated to the human sector. Parallel to this, the initial increase in the relative physical investment rate happens together with the rise of the physical to human capital ratio. This in turn results in an increase in the growth rate of the economy.

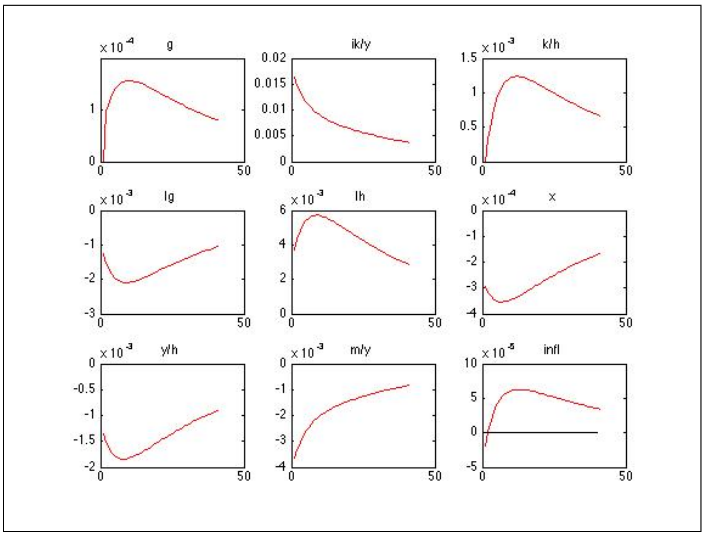

Figure 4.13: Impulse responses in percent deviation from the steady state (1) - Money Supply Shock in Model 1m.

Lastly, we turn our attention to the analysis of the responses to a positive aggregate shock. Upon impact due to the larger estimated variance of the goods sector TFP shock the input prices for goods production rise and trigger a substitution of time from human investment and leisure towards labor in goods production. This raises the output level, which in turn raises the inflation rate. The substitution towards goods labor shocks the relative physical investment rate upwards along with

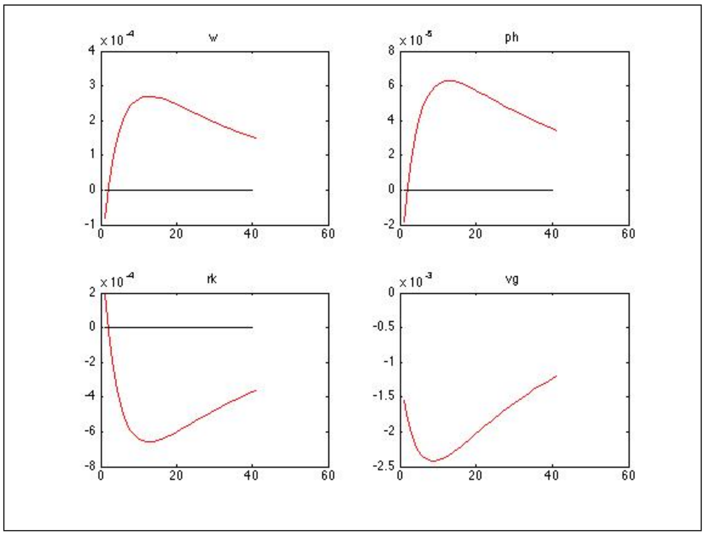

Figure 4.14: Impulse responses in percent deviation from the steady state (2)- Money Supply Shock in Model 1m.

the physical to human capital ratio. The rise in the physical capital investment rate pushes the growth rate down as a result of diminishing marginal returns to capital. As the relative price of human capital increases over time resources are reallocated towards the human sector and the growth rate increases again.

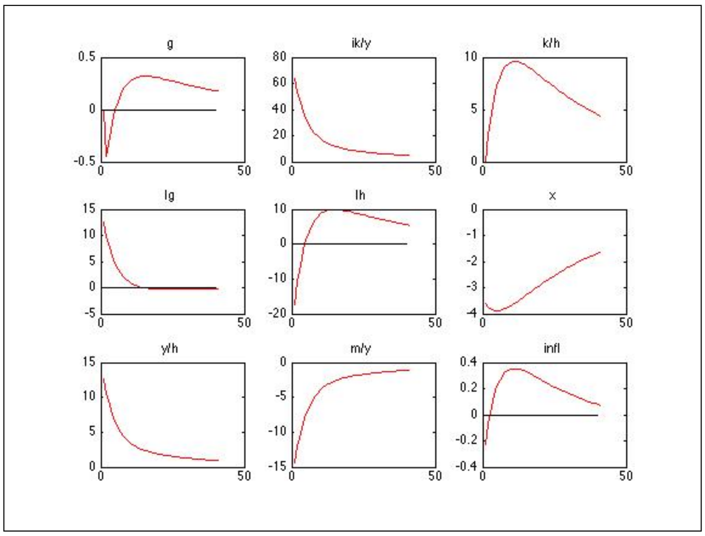

Figure 4.15: Impulse responses in percent deviation from the steady state (1) - Aggregate Productivity Shock in Model 1m.

Model 2m: Impulse Response Analysis

Figures 4.17 through 4.24 plot the impulse responses to the respective shocks for Model 2m.

A positive productivity shock in the goods sector makes the agent substitute time away from producing human capital investment goods and leisure towards labor in

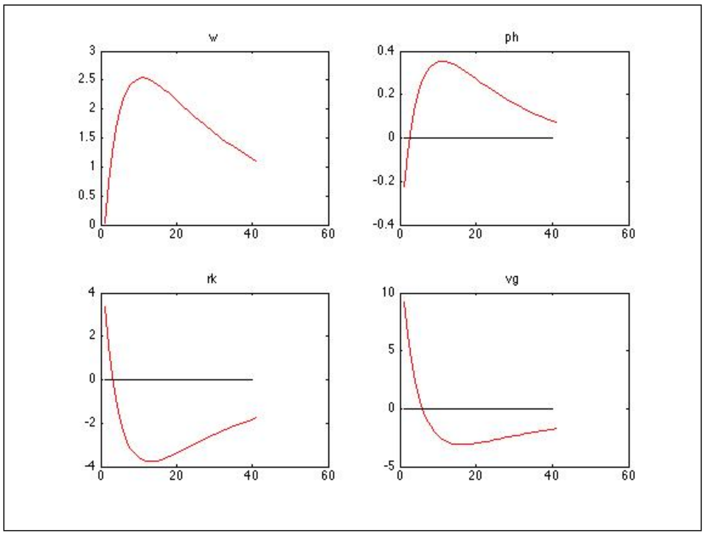

Figure 4.16: Impulse responses in percent deviation from the steady state (2) - Aggregate Productivity Shock in Model 1m.

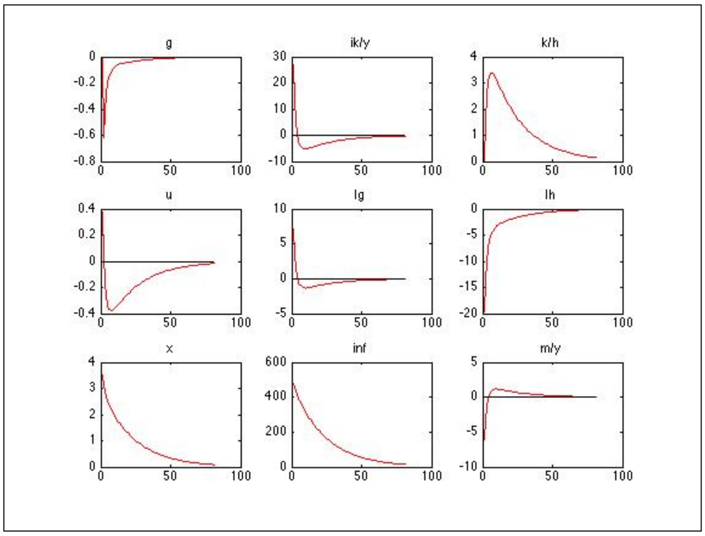

Figure 4.17: Impulse responses in percent deviation from the steady state (1) - Goods TFP Shock in Model 2m.

goods production. This lowers human investment and increases goods output. As a result of the substitution from learning time the growth rate decreases despite the rise in output, the relative physical investment rate, and the physical to human capital ratio. The agent as a result of increased labor supply lowers its capacity after increasing it first due to the lower rental rate of physical capital. As a result of a higher output level after impact the inflation rate rises.

After a positive temporary productivity shock in the human sector the relative price of human capital drops initially and quickly rises above its steady state value. Due to the negative correlation between the goods sector and human sector productivity shocks there is negative effect in goods TFP, which induces the relative physical investment rate to drop below its steady state level after an initial hike.

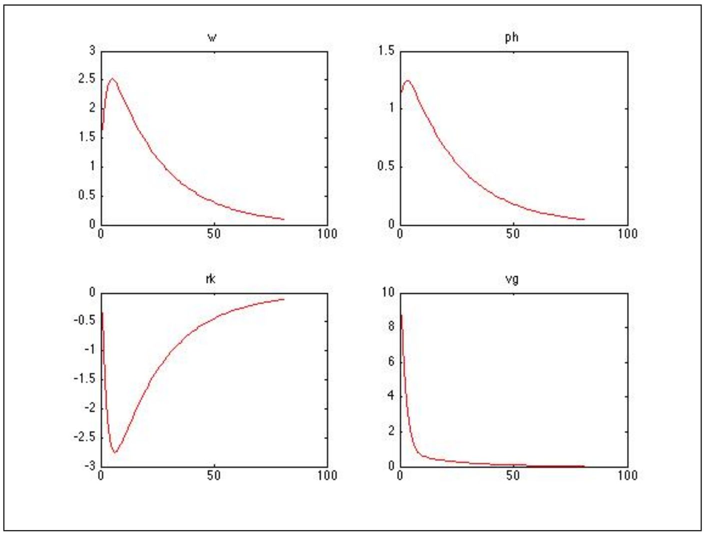

Figure 4.18: Impulse responses in percent deviation from the steady state (2)- Goods TFP Shock in Model 2m.

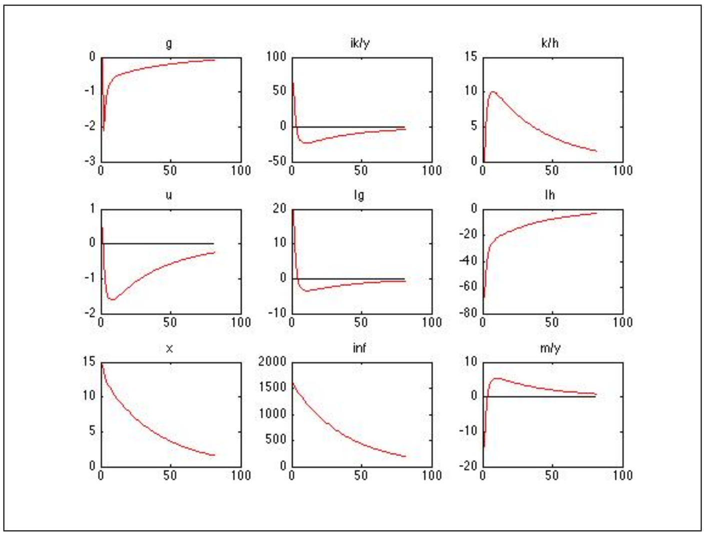

Figure 4.19: Impulse responses in percent deviation from the steady state (1) - Human Productivity Shock in Model 2m.

The agent at the same time substitutes away from human investment and leisure towards labor production and reallocates physical capital to the goods sector as a result of a higher wage rate and a lower rental rate of capital. Until the reallocation of resources physical capital utilization initially rises and after more capital is allocated to goods production it drops below its steady state level in conjunction with a rise in the level of the physical to human capital ratio. As a result real output rises initially which reduces the real balances to output ratio and increases the level of inflation temporarily. Lastly, despite an overall growth in output due to the substitution of inputs away from the human sector the growth rate drops but then quickly increases back to its steady state level as the price of human capital rises above its steady state level and resources are reallocated towards the human sector.

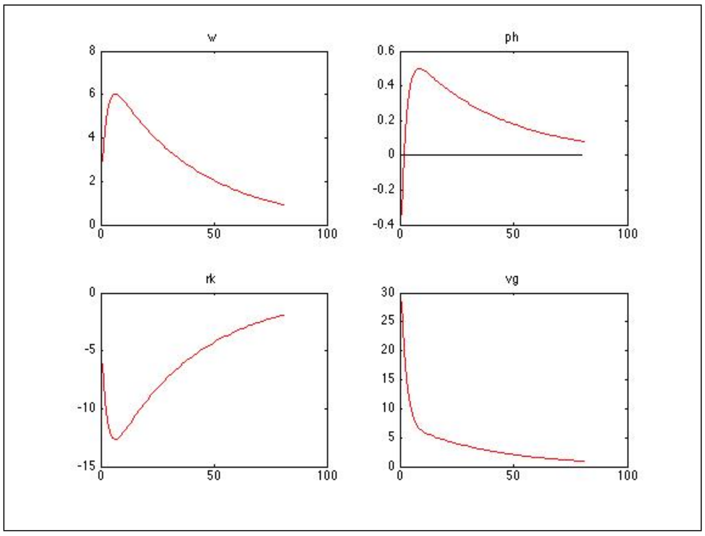

Figure 4.20: Impulse responses in percent deviation from the steady state (2) - Human Productivity Shock in Model 2m.

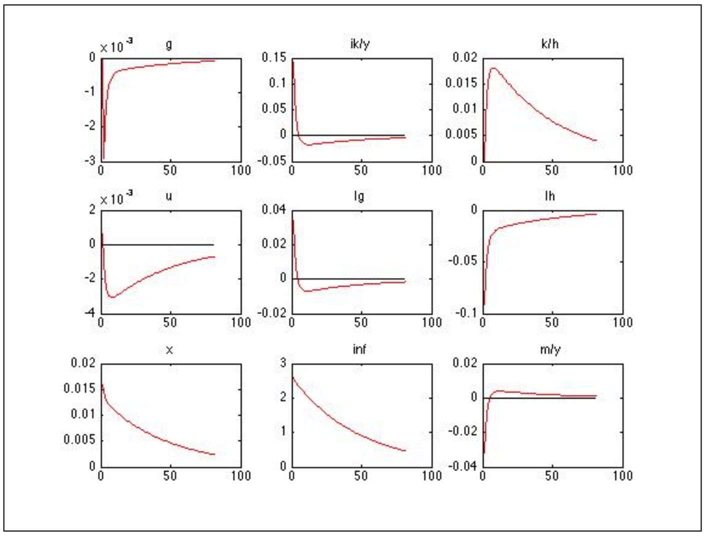

Figure 4.21: Impulse responses in percent deviation from the steady state (1) - Money Supply Shock in Model 2m.

After a positive temporary money supply shock the responses of Model 2m economy can be seen in Figure 4.21 and 4.23. Due to the strong positive correlation between the money supply and the TFP shock and the much smaller positive correlation between the money supply shock resources are substituted towards the goods sector initially as in the case of the human shock. Ultimately, the short run dynamics of Model 2m due to the structure of the calibrated shock covariances are going to be identical to the case of the goods sector shock except in terms of magnitude.

In Figure 4.24 and **??** one can observe the impulse responses of Model 2m due to a positive aggregate shock. It is evident once again that with the underlying calibration due to the shock covariances the dynamics are identical to the case of the goods sector shock upon impact after which the economy quickly adjusts. One

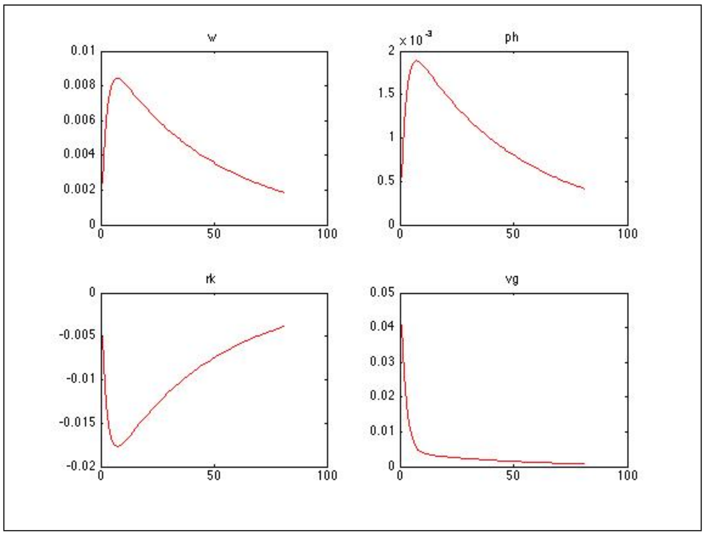

Figure 4.22: Impulse responses in percent deviation from the steady state (2)- Money Supply Shock in Model 2m.

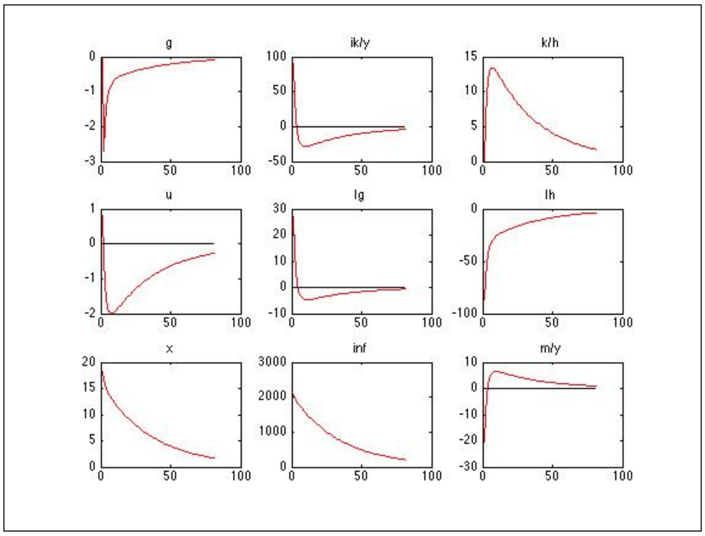

Figure 4.23: Impulse responses in percent deviation from the steady state (1) - Aggregate Productivity Shock in Model 2m.

interesting aspect is the extremely large response of the inflation rate. In response to the previous section, where the estimated money supply shock based on the model solutions was many magnitudes larger than the traditionally obtained one, we can see that the shock covariance structure of our calibrated model is responsible for the very high responsiveness of inflation to all of the shocks.

4.4.2 Key Correlations

In this section we are matching key correlations of simulated and actual data to evaluate the performance of Model 1m and 2m in terms of capturing the business cycle and long-run negative inflation-output growth relationship, and in terms of

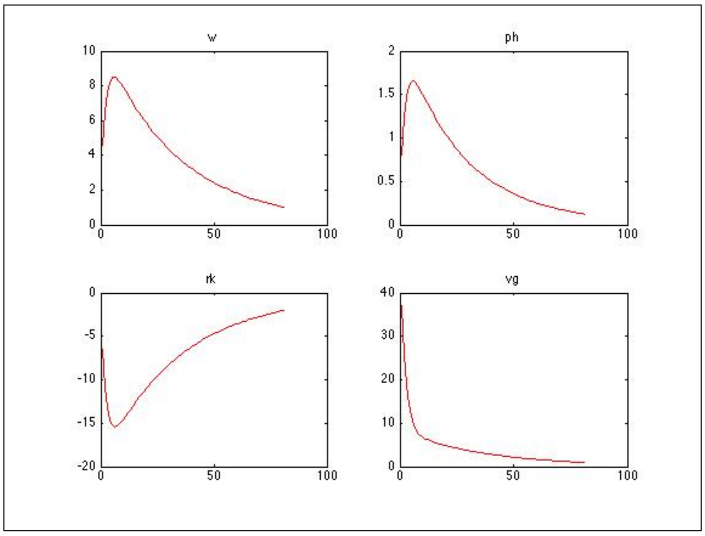

Figure 4.24: Impulse responses in percent deviation from the steady state (2) - Aggregate Productivity Shock in Model 2m.

the models' ability to explain standard business cycle facts. The latter is important as it gives us an idea how robust are the results of Chapter 3. More specifically, we want to see if we lose the key results by adding exchange money to the original Model 1 and 2. For this we are once again using a Christiano and Fitzgerald (2003) type band-pass filter to obtain the simulated data's low frequency component with a periodicity of 32 up to 200 quarters, the business cycle component with a periodicity of 6 to 32 quarters, and the medium term cycle with a defined periodicity of 2 up to 200 quarters as defined by Comin and Gertler (2006). This way we are able to pick up long-run relationships at the low and medium cycle frequencies.

In Table 4.5 we summarize three sets of relationships. The first is the inflation-output growth relationship; the second is the inflation-output relationship; and the third is the labor and inflation relationship. In terms of the inflation-output growth correlation both Model 1m and Model 2m capture the strong business cycle frequency negative correlation. At the low frequency and the medium cycle both models generate a negative relationship but Model 2m has a clear advantage in terms of magnitude.

The inflation-output and employment-inflation relationships are important not just for the sake of matching moments, but they represent the Phillips curve relationship that plays a significant role in even today's New Keynesian literature. U.S. data suggests that in the shorter term, at the business cycle frequency, generating inflation can stimulate output, however, it suggests in the case of the level of output that this positive effect fades out and converges to zero in the long-run. Regarding employment the data suggests that after the short-run positive effect on employment in the long term inflation can have a strong negative relationship with employment. Therefore, the data suggests the policy induced inflation in the short-run can have

Variable	High freq. 2-6 qrs.	Business cyc. 6 - 32 qrs.	Low freq., 32 - 200 qrs.	Medium term, 2 - 200 qrs.
$corr(g_{yt}, \pi_t)$				
US data	-0.397	-0.498	-0.735	-0.522
Model 1m	-0.675	-0.497	-0.547	-0.272
Model 2m	-0.868	-0.685	-0.759	-0.382
$corr(y_t, \pi_t)$				
US data	-0.275	0.164	0.029	0.035
Model 1m	-0.261	0.371	0.181	0.180
Model 2m	-0.356	0.204	0.004	0.021
$corr(l_{gt}, \pi_t)$				
US data	-0.002	0.262	-0.633	-0.299
Model 1m	0.746	0.524	0.054	0.217
Model 2m	0.962	0.546	-0.334	0.114

Table 4.5: Matching Key Correlations at Different Frequencies - Monetary Relationships Model 1m and 2m.

a positive effect and in the long-run in the case of output levels simply converges to zero, meanwhile, in the case of employment prolonged inflation has a strong negative impact.

Model 1m without physical capital utilization generates a positive correlation between inflation and output at the business cycle frequency, but fails to pick up the drop in correlation at the low frequency and the medium cycle. On the other hand, Model 2m is able to generate not just the positive business cycle correlation between inflation and output but also able to capture the fading out of this effect in the long-run.

Regarding the employment-inflation relationship, both models can capture and somewhat overshoot the positive correlation at the business cycle frequency. At the low frequency and the medium cycle Model 1m fails completely in capturing the long-run negative effect of inflation. Model 2m on the other hand can successfully capture a negative correlation at the low frequency. It can be observed in Table 4.5 that at the medium cycle Model 2m fails as the positive relationship in the short-run still outweighs the negative one in the long-run.

Now we may turn our attention to matching key business cycle correlations and long-run relationships in U.S. data similarly to Chapter 2 and 3. In Table 4.6 the first result that comes to our attention is that Model 1m with the addition of exchange money loses its ability to capture the strong positive pro cyclical movement of consumption in the short and the long-run as well. Model 2m on the other hand matches closely the consumption-output correlation at the business cycle frequency and does reasonably well at the low frequency and the medium cycle.

Regarding the correlation between output and physical investment one may note that both models are able to closely match it at all frequencies similarly to the results in Chapter 3 with Model 2m being marginally closer to what the data tells us. The co-movement of labor hours is also captured reasonably well by both models at

Variable	High freq. 2-6 qrs.	Business cyc. 6 - 32 qrs.	Low freq., 32 - 200 qrs.	Medium term, 2 - 200 qrs.
$corr(c_t, y_t)$				
US data	0.469	0.895	0.978	0.962
Model 1m	0.188	-0.241	-0.259	-0.185
Model 2m	-0.102	0.889	0.881	0.806
$corr(i_{kt}, y_t)$				
US data	0.811	0.943	0.810	0.815
Model 1m	0.999	0.995	0.920	0.956
Model 2m	0.997	0.992	0.868	0.925
$corr(l_{gt}, y_t)$				
US data	0.404	0.730	0.597	0.602
Model 1m	-0.169	0.868	0.917	0.735
Model 2m	-0.155	0.850	0.836	0.617
$corr(l_{ht}, y_t)$				
US data	-	-	-	-
Model 1m	0.162	-0.879	-0.807	-0.735
Model 2m	0.240	-0.585	-0.298	-0.311
$corr(u_t, y_t)$				
US data	0083	0.791	0.363	0.440
Model 1m	-	-	-	-
Model 2m	0.092	0.741	0.291	0.362
$corr(c_t, i_{kt})$				
US data	0.079	0.846	0.721	0.693
Model 1m	0.143	-0.334	-0.616	-0.453
Model 2m	0.915	0.824	0.531	0.520
$corr(c_t, l_{gt})$				
US data	0.215	0.764	0.616	0.619
Model 1m	0.174	-0.562	-0.506	-0.405
Model 2m	0.110	0.866	0.532	0.444

Table 4.6: Matching Key Correlations at Different Frequencies - Business Cycle Relationships Model 1m and 2m.

the business cycle and low frequencies but they generate a somewhat stronger relationship than what is in U.S. data. In the absence of data the theoretically implied countercyclical relationship between learning time and output is well captured by both models.

Model 1m similarly to the case of the correlation between consumption and output fails at capturing the strong positive relationship between consumption and investment, as well as, the relationship between consumption and employment. In both respects Model 2m does a very good job and it is able to match the correlation between consumption-investment, and consumption-employment very well at all three frequencies reported in Table 4.6. Lastly, as in Chapter 3 Model 2m captures the co-movement of the physical capital utilization rate and output in the data extremely well at the business cycle and low frequencies as well.

Overall, we may say that by adding exchange money to Model 2 in Chapter 3 we are able to capture the business cycle and long-run inflation-output growth relationship very accurately without losing much explanatory power over key business cycle and long-run movements of variables that we have already established. The extension of Model 1 with money; however, despite capturing some of the inflation-output growth relationship, due to the strong effect of the inflation tax it generated counterfactual relationships between key variables.

4.5 Conclusion

In Chapter 4 we extended Model 1 and Model 2 of Chapters 2 and 3 with exchange money. Such models of inflation and Lucas (1988) style endogenous growth have been used successfully to explain the output growth and inflation rate relationship. Here we presented our first extension in Model 1m, which excludes capacity utilization but a more general human investment production function as in Gomme (1993). Model 2m on the other hand includes a more general exchange constraint similar to Stockman (1981), and it also includes physical capital utilization rate and taxes to labor and capital income.

We evaluated the models' ability to capture the inflation-output growth; inflation-output; and inflation-employment relationship at different spectra along with standard business cycle correlations. We found that Model 2m performed better in all aspects. It is not only able to capture the negative inflation-output growth relationship at the business cycle and lower frequencies but can capture the long-run negative relationship between employment and inflation. In terms of key business cycle correlations we found that the general inclusion of money and constant taxes does not change the internal mechanism of Model 2 in Chapter 3 and all previous results have been reproduced. In the case of the extension of Model 1 in Chapter 2 and 3 we found that due to the addition of money we lose the positive consumption-output and consumption-labor relationships, which is a standard problem in monetary business cycle models.

Furthermore, after calibrating the models and extracting the U.S. data and DSGE model solution based goods TFP and money supply shock series following Ingram et al. (1997) and Benk et al. (2005, 2008, 2010)and comparing them to traditionally obtained series we found that both of our models are able to match the goods sector TFP series well, but they both tend to overestimate the magnitude of the money supply shock relative to the traditionally obtained one. This, as the short term dynamics have shown are due to the calibrated shock covariance structure.This points to the high sensitivity of these models to the shock covariance structure when correlated shocks are allowed.

In conclusion, the results in this chapter show that including capacity utilization and a more general Stockman (1981) style exchange technology, an endogenous growth model as our Model 2m can explain the U.S. inflation-output growth relationship, meanwhile, capturing the key business cycle correlations. This shows that Model 2 in Chapter 3 has a more robust internal propagation mechanism than

Model 1 in Chapter 2, and it draws our attention to the importance of physical capital utilization in explaining real and monetary business cycles.

C.1 Equilibrium Conditions

After substituting the time constraint in equation (4.2), and the entrepreneurial capacity constraint in equation (4.3) into the utility function in equation (4.1); and substituting the physical capital law of motion into the exchange constraint in equation (4.10); and denoting the Lagrange multiplier of the household as λ_t, the exchange constraint's as μ_t, and that of the human capital accumulation's as χ_t, the household's first order conditions are the following:

$$c_t : \quad c_t^{-\sigma} x_t^{A(1-\sigma)} e_t^{B(1-\sigma)} = \lambda_t + \mu_t; \tag{C.I-1}$$

$$l_{gt} : \quad A c_t^{1-\sigma} x_t^{A(1-\sigma)-1} e_t^{B(1-\sigma})} = \lambda_t (1-\tau_l) w_t l_{gt} h_t; \tag{C.I-2}$$

$$l_{ht} : \quad A c_t^{1-\sigma} x_t^{A(1-\sigma)-1} e_t^{B(1-\sigma)} = \chi_t (1-\phi_2) A_h e^{z_t^h} \left[\frac{v_{ht} u_t k_t}{l_{ht} h_t} \right]^{\phi_2} h_t; \tag{C.I-3}$$

$$\begin{aligned} u_t : \quad B c_t^{1-\sigma} x_t^{A(1-\sigma)} e_t^{B(1-\sigma)-1} &= \lambda_t (1-\tau_k) r_t^k v_{gt} k_t - \lambda_t \delta_k u_t^{\psi-1} k_t \\ &- \mu_t \Omega \delta_k u_t^{\psi-1} k_t \\ &+ \chi_t \phi_2 A_h e^{z_t^h} \left[\frac{v_{ht} u_t k_t}{l_{ht} h_t} \right]^{\phi_2 - 1} (v_{ht} k_t); \end{aligned} \tag{C.I-4}$$

$$v_{gt} : \quad \lambda_t (1-\tau_k) r_t^k u_t k_t = \chi_t \phi_2 A_h e^{z_t^h} \left[\frac{v_{ht} u_t k_t}{l_{ht} h_t} \right]^{\phi_2 - 1} (u_t k_t); \tag{C.I-5}$$

$$m_{t+1} : \lambda_t (1+\pi_{t+1}) = \beta E_t (\lambda_{t+1} + \mu_{t+1}); \tag{C.I-6}$$

$$b_{t+1} : \lambda_t (1+\pi_{t+1}) = \beta E_t \lambda_{t+1} (1+R_{t+1}); \tag{C.I-7}$$

$$
\begin{aligned}
k_{t+1}: \lambda_t + \mu_t = {} & \beta E_t \lambda_{t+1} \left[1 + (1-\tau_k) r^k_{t+1} (v_{gt+1} u_{t+1}) - \frac{\delta_k}{\psi} u^{\psi}_{t+1} \right] \\
& + \beta E_t \mu_{t+1} \Omega \left[1 - \frac{\delta_k}{\psi} u^{\psi}_{t+1} \right] \\
& + \beta E_t \chi_{t+1} \phi_2 A_h e^{z^h_{t+1}} \left[\frac{v_{ht+1} u_{t+1} k_{t+1}}{l_{ht+1} h_{t+1}} \right]^{\phi_2 - 1} (u_{t+1} v_{ht+1});
\end{aligned}
\tag{C.I-8}
$$

$$
\begin{aligned}
h_{t+1}: \chi_t = {} & \beta E_t \chi_{t+1} \left[1 + (1-\phi_2) A_h e^{z^h_{t+1}} \left[\frac{v_{ht+1} u_{t+1} k_{t+1}}{l_{ht+1} h_{t+1}} \right]^{\phi_2} l_{ht+1} - \delta_h \right] \\
& + \beta E_t \lambda_{t+1} (1-\tau_l) w_{t+1} l_{gt+1}.
\end{aligned}
\tag{C.I-9}
$$

For the goods producer the associated standard first order conditions are

$$
(l_{gt} h_t): \quad w_t = (1-\phi_1) A_g e^{z^g_t} \left[\frac{v_{gt} u_t k_t}{l_{gt} h_t} \right]^{\phi_1}; \tag{C.I-10}
$$

$$
(v_{gt} u_t k_t): \quad r^k_t = \phi_1 A_g e^{z^g_t} \left[\frac{v_{gt} u_t k_t}{l_{gt} h_t} \right]^{\phi_1 - 1}. \tag{C.I-11}
$$

After some straightforward algebra and considering the government's money supply rule and budget constraint one can summarize the model by the following set of equations:

$$
y_t = c_t + i_{kt}; \tag{C.I-12}
$$

$$
i_{kt} = k_{t+1} - k_t + \frac{\delta_k}{\psi} u^{\psi}_t k_t; \tag{C.I-13}
$$

$$
i_{ht} = h_{t+1} - (1-\delta_h) h_t; \tag{C.I-14}
$$

$$
y_t = A_g e^{z^g_t} (v_{gt} u_t k_t)^{\phi_1} (l_{gt} h_t)^{1-\phi_1}; \tag{C.I-15}
$$

$$i_{ht} = A_h e^{z_t^h} (v_{ht} u_t k_t)^{\phi_2} (l_{ht} h_t)^{1-\phi_2}; \tag{C.I-16}$$

$$\frac{A}{1 - l_{gt} - l_{ht}} \frac{c_t}{h_t} = \frac{(1-\tau_l) w_t}{1 + R_t}; \tag{C.I-17}$$

$$\frac{B}{1 - u_t} \frac{c_t}{k_t} = \frac{(1-\tau_k) r_t^k - (1 + \Omega R_t) \delta_k u_t^{\psi - 1}}{1 + R_t}; \tag{C.I-18}$$

$$(1 + \Omega R_t) = E_t \left[\left(1 + \frac{(1-\tau_k) r_{t+1}^k u_{t+1}}{1 + \Omega R_{t+1}} - \frac{\delta_k}{\psi} u_{t+1}^{\psi} \right) (1 + \pi_{t+1}) \right]; \tag{C.I-19}$$

$$\begin{aligned} 1 = \beta E_t \left(\frac{c_t}{c_{t+1}} \right)^{\sigma} \left(\frac{x_{t+1}}{x_t} \right)^{A(1-\sigma)} \left(\frac{e_{t+1}}{e_t} \right)^{B(1-\sigma)} \left(\frac{1 + R_t}{1 + R_{t+1}} \right) \\ \left(1 + \frac{(1-\tau_k) r_{t+1}^k u_{t+1}}{1 + \Omega R_{t+1}} - \frac{\delta_k}{\psi} u_{t+1}^{\psi} \right) \left[\frac{1 + \Omega R_{t+1}}{1 + \Omega R_t} \right]; \end{aligned} \tag{C.I-20}$$

$$\begin{aligned} 1 = \beta E_t \left(\frac{c_t}{c_{t+1}} \right)^{\sigma} \left(\frac{x_{t+1}}{x_t} \right)^{A(1-\sigma)} \left(\frac{e_{t+1}}{e_t} \right)^{B(1-\sigma)} \left(\frac{1 + R_t}{1 + R_{t+1}} \right) \\ \left(\frac{p_{ht+1}}{p_{ht}} \right) [1 + (1 - x_{t+1}) r_{t+1}^h - \delta_h]; \end{aligned} \tag{C.I-21}$$

$$w_t = (1 - \phi_1) A_g e^{z_t^g} \left[\frac{v_{gt} u_t k_t}{l_{gt} h_t} \right]^{\phi_1}; \tag{C.I-22}$$

$$r_t^k = \phi_1 A_g e^{z_t^g} \left[\frac{v_{gt} u_t k_t}{l_{gt} h_t} \right]^{\phi_1 - 1}; \tag{C.I-23}$$

$$r_t^h = (1 - \phi_2) A_h e^{z_t^h} \left[\frac{v_{ht} u_t k_t}{l_{ht} h_t} \right]^{\phi_2}; \tag{C.I-24}$$

$$p_{ht} = \frac{(1-\tau_l)w_t}{(1-\phi_2)A_h e^{z_t^h}\left[\frac{v_{ht}u_t k_t}{l_{ht}h_t}\right]^{\phi_2}}; \tag{C.I-25}$$

$$m_t = c_t + \Omega i_{kt}; \tag{C.I-26}$$

$$1 = e_t + u_t; \tag{C.I-27}$$

$$1 = x_t + l_{gt} + l_{ht}; \tag{C.I-28}$$

$$1 = v_{gt} + v_{ht}; \tag{C.I-29}$$

$$\left[\frac{1-\phi_1}{\phi_1}\right]\frac{v_{gt}u_t k_t}{l_{gt}h_t} = \left[\frac{1-\phi_2}{\phi_2}\right]\frac{v_{ht}u_t k_t}{l_{ht}h_t}; \tag{C.I-30}$$

where r_t^h is the marginal product of effective labor in the human sector.

In the above system equation (C.I-12) is the standard goods market clearing condition; equations (C.I-13) and (C.I-14) are the physical capital and human capital law of motion respectively. Equation (C.I-15) is the goods production technology; equation (C.I-16) is the human investment technology; and (C.I-17) is the marginal rate of substitution between consumption and leisure. Equation (C.I-18) is the second marginal rate of substitution between entrepreneurial capacity and consumption; equation (C.I-19) is the Fisher equation. Equation (C.I-20) and (C.I-21) are the inter-temporal margins with respect to physical and human capital. Equation (C.I-22) defines the wage rate as the marginal product of effective labor in the goods sector; equation (C.I-23) is gives the rental rate of physical capital; and equation (C.I-24) is the marginal product of effective learning time in the human sector. Equation (C.I-25) defines the relative price of human capital in terms of units of goods; equation (C.I-26) is the exchange technology; and equation (C.I-27) is the constraint of entrepreneurial capacity. Equation (C.I-28) is the time constraint; (C.I-29) defines the sectoral capital shares; and lastly, equation (C.I-30) is the equates the weighted sectoral intensities.

The set of equilibrium conditions in equations (C.I-12) - (C.I-30) together with the money supply rule in equation (4.20) and the forcing processes in equations (4.8), (4.16), and (4.21) fully describe this economy. After obtaining the equilibrium conditions one can apply the procedure outlined in Chapter 1 to solve and evaluate the underlying model.

C.2 Monetary Data Sources

For calibration purposes we have used real business cycle and key monetary U.S. data that covers the period of 1959:Q1 until 2014:Q2. The business cycle data used in this chapter is identical to the one described in Section 3.8 in *Chapter 3*. The monetary data description and sources are as follows:

1. U.S. M1 Money Stock (Seasonally Adjusted) [Code: M1SL] - Source: Board of Governors of the Federal Reserve System (H.6 Release);

2. U.S. 3-Months Treasury Bill Rate (Not Seasonally Adjusted) [TB3MS] - Source: Board of Governors of the Federal Reserve System (H.15 Release);

3. U.S CPI (Seasonally Adjusted) [CPIAUCSL] - Source: Bureau of Economic Analysis.

The 3-Months Treasury Bill rate is of monthly frequency, for which we have calculated 3-months averages as a quarterly measure. The inflation rate used for the calibration has been calculated from the CPI index. The M1 money stock data is nominal in nature, which we have transformed into real money balances by normalizing it with the price deflator series described in Chapter 1.

Summary and Conclusion

In summary, this dissertation in Chapter 1 proposed a new calibration scheme that extends the iterative calibration methodology of Jermann (1998) with the Simulated Annealing global optimization algorithm of Matlab and the shock identification scheme of Ingram et al. (1997) and Benk et al. (2005). The automation of the calibration methodology of Jermann (1998) allows this scheme to widen the parameter search for optimal calibrations constrained by U.S. data in a bounded parameter space in an efficient manner.

Chapter 2 and Chapter 3 demonstrated the power of this new calibration scheme on the two sector business cycle model of Dang et al. (2011) with endogenous growth following Lucas (1988) and its extension to find economically feasible calibrations that can explain a number of business cycle problems.

In Chapter 3 more specifically this dissertation contributed by extending the model of Dang et al. (2011) with a physical capital utilization margin [King and Rebelo (2000)] through adding entrepreneurial activity in the sense of Friedman (1976), Lucas (1978), and Gillman (2011). The combination of these two additions adds a new intra-temporal margin that allows for the symmetric treatment of human and physical capital and by using the calibration scheme could explain a number of RBC issues, which include the Gali (1999) negative labor response, RBC comovements of consumption and labor with output in the frequency domain, and the profile of output growth persistence to a goods sector TFP shock.

Lastly, in Chapter 4 by extending both the model of Dang et al. (2011) and the model in Chapter 3 with exchange money and constant taxes and applying the new calibration methodology, we could capture the basic and well-established relationship of inflation-output, inflation-output growth, and inflation-labor hours in the frequency domain with a specific focus on the long run.

Given the promising results of the new calibration scheme and the explanatory power of the extended business cycle model of Chapter 3 there is ample space for further research through further extensions or through empirical work. In terms of empirical work the estimation of the preference parameter to entrepreneurial capacity using time series data could further justify the introduction of entrepreneurial capacity into modeling economies. Furthermore, the results in Chapter 4 well motivate empirical work in estimating coefficients of the determinants of output growth. Such empirical research could be extended to single country and multiple country analysis depending on using time series or panel data.

From a modeling and theoretical perspective one extension could be to include labor market features to further improve upon the model's labor market dynamics. Such extension could be the introduction of indivisible labor as in Rogerson (1988). Also, the introduction of saving-investment intermediation approach as in Gillman (2011) could be a direct extension to capture the equity premium in both the real and monetary versions of our model.

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