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When can advance notice be most beneficial for closed loop supply chains?

Takamichi Hosoda [1], Stephen M. Disney [2], Nagesh Gavirneni [3]

[1] Graduate School of International Management, Aoyama Gakuin University,
4-4-25 Shibuya, Shibuya-ku, Tokyo, JAPAN
hosoda@gsim.aoyama.ac.jp
[2] Logistics Dynamics Group, Cardiff Business School, Cardiff University,
Aberconway Building, Colum Drive, Cardiff, CF10 3EU, UK
disneysm@cardiff.ac.uk
[3] Johnson Graduate School of Management, Cornell University,
325 Sage Hall, Ithaca, NY, USA
nagesh@cornell.edu

Abstract

We investigate the impact of advance notice of product returns on the performance of a closed loop supply chain with lead-times. Our closed loop supply chain consists of a manufacturer and an external remanufacturer. The market demands and the product returns are stochastic and correlated with each other. After a lead-time the used products are converted into "as-good-as-new" products to be used, together with new products, to satisfy market demand. The remanufacturing process is subject to random yield. We investigate the benefit of the manufacturer obtaining advance notice of the product return quantities from the remanufacturer. We demonstrate that lead-times, random yields and parameters describing the return rate have a significant impact on the manufacturer's performance with this additional information. The interesting and counter-intuitive result is that increasing the lead-time at the remanufacturer can improve the benefit coming from the advance notice.

Keywords: Closed loop supply chain, information sharing, random yield, lead-time.

1. Introduction

Closed loop supply chains have attracted a lot of research attention recently, due to growing concern with environmental issues (Akçalı and Çetinkaya, 2011). At the same time, closed loop supply chains are generally acknowledged to be more complex than traditional supply chains. One reason is that both the demands and the product returns should be incorporated into a model, and both of them can be correlated each other. In addition, two different lead-times should be considered: the manufacturing lead-time and the remanufacturing lead-time. Furthermore, in most practical cases, due to the unpredictable quality of the returned products, the remanufacturing process yield is random. To achieve better performance in such a complicated closed loop supply chain, sharing information between the manufacturer and the remanufacturer is a natural course of action. The importance of the value of information sharing in multi-level supply chains is well recognized (see Gavirneni et al., 1999; Lee et al., 2000, for example), however, there is little literature that addresses this issue in the field of closed loop supply chains.

This research investigates the economic impact of the advance notice from the remanufacturer of the product return rate on the performance of the manufacturer in a closed loop supply chain. We focus on the question when to share such information. The lead-times, the degree of correlation between the demand and the product returns, and random yield of the remanufacturing process are all incorporated into the model to investigate the impact of those on the advance notice. After a constant lead-time the returned products are converted into "as-good-as-new" products that are used to meet the market demand alongside newly manufactured products. In order to cope with the uncertainty in its supply chain, the manufacturer must forecast both the market demand and the product returns. The product returns are already known to the external remanufacturer, and this information is shared in an advanced notice scheme and used to decide how many new units to produce. We demonstrate that the remanufacturing and the manufacturing lead-times and the remanufacturing yield as well as the parameters in the product return rate process can have a significant impact on the manufacturer's economic performance.

This paper is organized as follows. In the next section, the literature review is provided. Then the details of our closed loop supply chain model is introduced. After managerial properties are induced from the model, insights from a numerical analysis are shown. We conclude in the final section with a discussion of potential future research directions.

2. Literature Review

The complexity of closed loop supply chains provides a rich model to study. One of key issues in closed supply chains is how to reduce uncertainty with respect to demands, product returns and recovery yield. As in traditional supply chains without returns, to reduce such uncertainties, the value of information sharing is well recognized in closed loop supply chains (Ketzenberg et al., 2006; Ketzenberg, 2009; de Brito and van der Laan, 2009; Flapper et al., 2012; Ferrer and Ketzenbarg, 2004).

Using some approximations when it is necessary, Ketzenberg et al. (2006) present analytical models to quantify the value of information in a closed loop supply chain. In their model, the benefit comes from shared information of the market demand, the return rate, and the yield. Two analytical models are developed: a one period model and a multi-period model. Assuming a capacitated closed loop supply chain, Ketzenberg (2009) analyses the value of information of the demand, the return, the yield and the capacity utilization. The cost benefits are quantified by using heuristics and a simulation study. It is shown that information on capacity utilization can bring about the largest average benefit, though no type of information is dominant. de Brito and van der Laan (2009) investigate the impact of imperfect information on the forecast of lead-time demand under remanufacturing setting. Inventory cost is used to quantify the impact. Based on the result of analysis of four different forecasting methods, it is concluded that the most informed method does not always produce the lowest cost.

Flapper et al. (2012) consider the impact of having imperfect advance return information on inventory cost using a Markov decision formulation. A random return lead-time is assumed. They conclude that advance return information can reduce the inventory cost by 5% at most. It is also shown that the value of the advanced return information is affected by the expected length of return lead-time. Ferrer and Ketzenbarg (2004) also suggest that the value of information is dependent on the length of a constant lead-time. Guide (2000) reports that 60% of remanufacturing executives are under pressure to reduce remanufacturing lead-times.

3. Model

In this research, the following set of notation is used:

t	time period
D_t	market demand rate at time t for the finished goods
R_t	product return rate realized by remanufacturer at t
μ_D	mean of the market demand rate
μ_R	mean of the return rate
τ	correlation time lag parameter $\tau \in \mathbb{N}_0$
ε_t	i.i.d. error term realized at t
σ_ζ	standard deviation of ζ_t and $\sigma_\zeta = k\sigma_\varepsilon$
T_r	remanufacturing lead-time plus delivery time $T_r \in \aleph_0$
T_p	manufacturing lead-time $T_p \in \mathbb{N}_0$
P_t	production order rate at t
ξ_t	i.i.d. yield rate realized at t, independent of R_t, $0 \le \xi_t \le 1$, and $E[\xi_t] = \bar{\xi}$
$\Xi(\cdot)$	yield of remanufacturing (i.e. $\Xi(R_t) = \xi_t R_t$), $0 \le \Xi(R_t) \le R_t$

ζ_t i.i.d. error term realized at t (ε_t and ζ_t are mutually independent)
θ correlation coefficient $|\theta| \leq 1$
k non-negative scale parameter
σ_ε standard deviation of ε_t
NS_t net stock level at the end of t (a negative value of NS_t represents a backlog at t

Figure 1 shows the schematic of our model. It is a periodic review system where both the manufacturer and the remanufacturer employ the same review period. The manufacturer uses an order-up-to policy (see Hosoda and Disney, 2006, for example) to determine its production order quantity. Both the remanufacturing and the manufacturing processes have unlimited capacities. It is assumed that there is no differences between the remanufactured products and brand-new products in terms of quality. As such customers cannot recognise the difference between the two products. Random yield is modelled by the stochastically proportional yield model (Hening and Gerchak, 1990). This model is appropriate when the system is subject to material variations (Yano and Lee, 1995), and used in many remanufacturing studies (see Tao et al., 2012, for example). The yield rate is identified at the beginning of the remanufacturing process in what is generally called the triage process. In our model, when R_t is realised, ξ_t is also identified as in Ketzenberg (2009).

3.1 Market Demand and Return Rate. It is assumed that both the market demand rate (D_t) and the product return rate (R_t) follow white noise processes. This white noise assumption is widely used in much of the closed loop supply chain literature (e.g. Ketzenberg et al., 2006; Ketzenberg, 2009). The demand and the product return rates models are given by

$$\begin{aligned} D_t &= \mu_D + \varepsilon_t, \\ R_t &= \mu_R + \theta k\, \varepsilon_{t-\tau} + \sqrt{1-\theta^2}\zeta_t, \end{aligned} \tag{1}$$

where the correlation between $D_{t-\tau}$ and R_t becomes θ, as shown in Appendix 1. It should be noted that in our model τ in $D_{t-\tau}$ can be any non-negative integer. This model is useful when the impact of the correlation is investigated since the value of θ can be any arbitrary value on the condition that $|\theta| \leq 1$.

As shown in Appendix 1 the standard deviations of D_t and R_t are σ_ε and $k\sigma_\varepsilon$, respectively. If k is greater than unity, for example, the standard deviation of R_t becomes larger than that of D_t. Without loss of generality, it is assumed that $\mu_D \gg \mu_R$ as in van der Laan et al. (1999b) since the product return is a portion of the demand in practical situations.

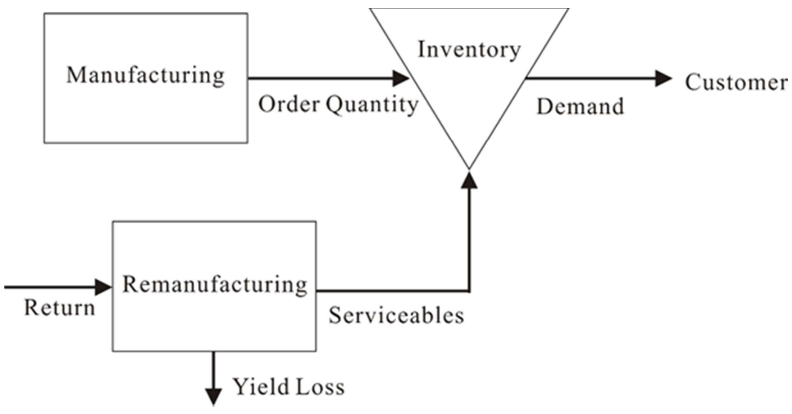

Figure 1. Schematic of material flow

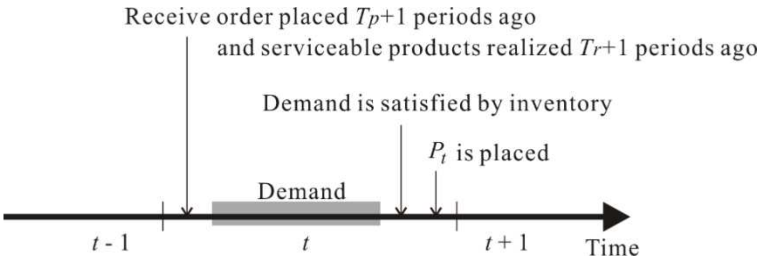

Figure 2. Sequence of events at manufacturer

3.2 *Sequence of Events*. The sequence of events in the model can be described as follows: At the beginning of t, the remanufacturer observes the total number of remanufacturable products R_t that have been returned from the market place. The remanufacturing process is not capacitated but subject to a random yield. If the remanufacturer receives R_t at t, the amount of serviceable goods the remanufacturer actually produces is $\Xi(R_t)$. It is assumed that the value of $\Xi(R_t)$ is recognised by the remanufacturer at t and the yield distribution does not depend on time or the quantity of R_t. The expected yield rate $\bar{\xi}$ $(= E[\xi_t])$, μ_R and T_r are known by the manufacturer. Remanufactured products are then delivered to the manufacturer's inventory at the beginning of $t + T_r + 1$, in order to partially satisfy market demand of D_{t+T_r+1}. At the beginning of t, the manufacturer receives brand-new goods from its production line, the order placed in period $t - (T_p + 1)$, in addition to remanufacturerd products from the remanufacturer. Next, the market demand D_t is observed and filled from the on-hand inventory. If the manufacturer does not have sufficient on-hand inventory to fill the all of the demand, unmet demand is backlogged. At the end of t, the manufacturer places a production order P_t to meet the future demand, taking account of future product return rate as well (see Figure 2). Therefore, NS_t, the net stock level of the manufacturer at the end of t, follows

$$NS_t = NS_{t-1} + \Xi\left(R_{t-(T_r+1)}\right) + P_{t-(T_p+1)} - D_t. \tag{2}$$

3.3 *Ordering Policy*. Let us use IP_t^+, the inventory position for the manufacturer at t the moment after P_t is determined. Thus, IP_t^+ is the net stock level at t (NS_t) plus the total of open orders, $\{P_{t-T_p}, \dots, P_t\}$. The value of IP_t^+ is known to the manufacturer since all such information is local. Hosoda and Disney (2012) show that in a traditional supply chain setting when there is no remanufacturing, whatever ordering policy is used, we always have $NS_{t+T_p+1} = IP_t^+ - \sum_{i=1}^{T_p+1} D_{t+i}$. In a closed loop supply chain, however, it is necessary to incorporate the pipeline inventory coming from the remanufacturer which will be available for the manufacturer during the time interval of $(t, t + T_p + 1]$. Consequently, we have the following relationships:

$$NS_{t+T_p+1} = IP_t^+ - \sum_{i=1}^{T_p+1} D_{t+i} + PIR_t + FPIR_t, \tag{3}$$

where

$$IP_t^+ = NS_t + P_{t-T_p} + \cdots + P_t,$$

$$PIR_t = \begin{cases} \sum_{i=T_r-T_p}^{T_r} \Xi(R_{t-i}) & T_r \geq T_p \\ \sum_{i=0}^{T_r} \Xi(R_{t-i}) & T_r < T_p, \end{cases}$$

and

$$FPIR_t = \begin{cases} 0 & T_r \geq T_p \\ \sum_{i=1}^{T_p - T_r} \Xi(R_{t+i}) & T_r < T_p \,. \end{cases}$$

PIR_t is the total pipeline inventory of successfully remanufactured products. $FPIR_t$ represents the future pipeline inventory of successfully remanufactured products at t and its value is not known yet by anybody when $T_r < T_p$. Note that PIR_t is not known by the manufacturer when there is no advance notice scheme in the supply chain. Without advance notice, the manufacturer needs to take the expected value of PIR_t, $\widehat{PIR}_t$, to determine P_t. Thus the advance notice of the product return information may have some impact on the performance of the manufacturer. Furthermore, when $T_r < T_p$, since the manufacturer must estimate the value of $FPIR_t$ as well, the magnitude of the relationship between T_r and T_p may have an impact on the performance of the manufacturer. With knowledge of (2), we have the following relationship between IP_t^+ and IP_{t-1}^+: $IP_t^+ = IP_{t-1}^+ - D_t + \Xi\big(R_{t-(T_r+1)}\big) + P_t$. Thus, P_t can be written as

$$P_t = D_t - \Xi\big(R_{t-(T_r+1)}\big) + IP_t^+ - IP_{t-1}^+. \tag{4}$$

From (3), we obtain another form of IP_t^+,

$$IP_t^+ = \sum_{i=1}^{T_p+1} D_{t+i} - PIR_t - FPIR_t + NS_{t+T_p+1}. \tag{5}$$

The RHS of (5) however, includes some unknown values for the manufacturer. Thus the manufacturer may want to take the expected value of IP_t^+, which yields

$$E[IP_t^+] = \widehat{D} - \widehat{PIR_t} - \widehat{FPIR}_t + TNS, \tag{6}$$

where

$$\widehat{D} = E\left[\sum_{i=1}^{T_p+1} D_{t+i}\right] = \big(T_p + 1\big)\mu_D,$$
$$\widehat{PIR_t} = E[PIR_t],\ \widehat{FPIR}_t = E[FPIR_t], TNS = \mathrm{E}[NS_{\mathrm{t+T_p+1}}],$$

and *TNS* stands for the target net stock level; a time invariant constant predetermined by the manufacturer to minimise its inventory cost. From (4) we have the order-up-to policy for a closed loop supply chain; $P_t = D_t - \Xi\big(R_{t-(T_r+1)}\big) + \big(E[IP_t^+] - E[IP_{t-1}^+]\big)$. We assume $P_t \geq 0$ in order to obtain closed-form expressions of variances. This non-negative assumption is not as strong as it appears to be when $\mu_D \gg \mu_R$ holds and can be seen in many other remanufacturing studies (e.g. Ketzenberg et al., 2006). Note that the values of $E[IP_t^+]$ and $E[IP_{t-1}^+]$ are dependent on the assumption about the availability of the advance notice. We have the following two cases in our setting: 1) the case that advance notice is not available (case N) and 2) the case that advance notice is available (case A).

3.4 Case N: No Advance Notice Case. This is the base case where the remanufacturer does not share the information of R_t, ξ_t and $V[\xi]$. In this circumstance, the manufacturer does not know $\Xi(R_t)$. The expected value $E[\Xi(R_t)] = \bar{\xi}\mu_R$ should be used instead. The estimated values of $\widehat{FPIR}_t$ and $\widehat{PIR}_t$ for the manufacturer in this case become

$$\widehat{FPIR}_t = E[FPIR_t] = \begin{cases} 0 & T_r \geq T_p \\ (T_p - T_r)\bar{\xi}\mu_R & T_r < T_p, \end{cases} \tag{7}$$

and

$$\widehat{PIR}_t = E[PIR_t] = \begin{cases} (T_p + 1)\bar{\xi}\mu_R & T_r \geq T_p \\ (T_r + 1)\bar{\xi}\mu_R & T_r < T_p. \end{cases} \tag{8}$$

From (6), (7) and (8) we can see that $E[IP_t^+] = E[IP_{t-1}^+]$ and P_t reduces to

$$P_t^N = D_t - \Xi(R_{t-(T_r+1)}) \tag{9}$$

Note that the manufacturer knows only the value of $\Xi(R_{t-(T_r+1)})$ and does not know the values of $R_{t-(T_r+1)}$ and ξ_t.

3.5 Case A: Advance Notice of R_t and ξ_t Case. In this case the information about R_t and ξ_t is shared with the manufacturer. It is also assumed that the manufacturer is capable of time series data analysis and the manufacturer can obtain the values of $\{\varepsilon_t, \varepsilon_{t-1}, \ldots\}$, θ, k and τ as well as μ_D, μ_R and $\bar{\xi}$ from historical data of D_t and R_t. In this setting, $\widehat{FPIR}_t$ and $\widehat{PIR}_t$ for the manufacturer become

$$\widehat{FPIR}_t = E[FPIR_t] = \begin{cases} 0 & T_r \geq T_p \\ (T_p - T_r)\bar{\xi}\mu_R & T_p > T_r \wedge \tau = 0 \\ (T_p - T_r)\bar{\xi}\mu_R + \bar{\xi}\theta k \sum_{i=1}^{\tau} \varepsilon_{t+1-i} & T_p - T_r \geq \tau \geq 1 \\ (T_p - T_r)\bar{\xi}\mu_R + \bar{\xi}\theta k \sum_{i=1}^{T_p - T_r} \varepsilon_{t-\tau+i} & \tau > T_p - T_r > 0, \end{cases}$$

and

$$\widehat{PIR}_t = \sum_{i=(T_r - T_p)^+}^{T_r} \Xi(R_{t-i}).$$

The formula for P_t^A thus depends on the values of T_p, T_r and τ,

$$P_t^A = \begin{cases} D_t - \Xi\left(R_{t-(T_r - T_p)}\right) & T_r \geq T_p \\ D_t - \Xi(R_t) & T_p > T_r \wedge \tau = 0 \\ D_t - \Xi(R_t) + \bar{\xi}\theta k(\varepsilon_{t-\tau} - \varepsilon_t) & T_p - T_r \geq \tau \geq 1 \\ D_t - \Xi(R_t) + \bar{\xi}\theta k\left(\varepsilon_{t-\tau} - \varepsilon_{t-(\tau - T_p + T_r)}\right) & \tau > T_p - T_r > 0. \end{cases} \tag{10}$$

In the next section, analytical expressions of the variances of production and net stock levels are described.

4. Variance Analysis

To conduct a full economic study for our problem, complete PDFs for the production and the net stock levels are required. Due to the methodological challenges introduced by the random yield rates, however, we have not pursued this line of study. Instead, to measure the benefit of the advance notice, the variances of production order and the net stock levels are used in this research. The use of variances as a measure of a supply chain performance is quite popular (Disney and Towill, 2003). It should be noted that the variance expressions shown in this section are obtained without specific assumptions on the distribution of D_t, R_t or ξ_t.

4.1 Case N: The Closed Loop Supply Chain with No Information Sharing. When no advance notice is given, P_t is given by (9) and its variance is

$$V[P^N] = \sigma_\varepsilon^2 + V[\Xi(R)], \tag{11}$$

where $V[\Xi(R)] = \bar{\xi}^2 k^2 \sigma_\varepsilon^2 + (\mu_R^2 + k^2\sigma_\varepsilon^2)V[\xi]$ as shown in Appendix 2. In the RHS of (3), the manufacturer knows only the locally available information, IP_t^+. Thus $V[NS^N]$ can be written as

$$V[NS^N] = \mathrm{E}\left[\left(PIR_t + FPIR_t - \sum_{i=1}^{T_p+1} D_{t+i} - E\left(PIR_t + FPIR_t - \sum_{i=1}^{T_p+1} D_{t+i}\right)\right)^2\right]$$
$$= \begin{cases} (T_p+1)(\sigma_\varepsilon^2 + V[\Xi(R)]) - 2\bar{\xi}\theta k(T_p - T_r - \tau)\sigma_\varepsilon^2, & T_p - T_r \geq \tau \\ (T_p+1)(\sigma_\varepsilon^2 + V[\Xi(R)]), & \text{otherwise.} \end{cases} \tag{12}$$

4.2 Case A: The Closed Loop Supply Chain with Advance Notice. The ordering policy in this case has multiple formulae as shown in (10). Fortunately, the variance expression of P_t^A reduces to the following two expressions.

$$V[P^A] = \begin{cases} V[P^N] - 2\bar{\xi}\theta k\sigma_\varepsilon^2, & T_p - T_r \geq \tau \\ V[P^N], & \text{otherwise.} \end{cases} \tag{13}$$

By following the similar method for the case N, we can have $V[NS^A]$;

$$V[NS^A] = \mathrm{E}\left[\left(PIR_t + FPIR_t - \sum_{i=1}^{T_p+1} D_{t+i} - E\left(PIR_t + FPIR_t - \sum_{i=1}^{T_p+1} D_{t+i}\right)\right)^2\right]$$
$$= \begin{cases} (T_p+1)\sigma_\varepsilon^2, & T_r \geq T_p \\ (T_p+1)\sigma_\varepsilon^2 + (T_p - T_r)V[\Xi(R)] - \tau\bar{\xi}^2\theta^2k^2\sigma_\varepsilon^2 - 2\bar{\xi}\theta k(T_p - T_r - \tau)\sigma_\varepsilon^2, & T_p - T_r \geq \tau \\ (T_p+1)\sigma_\varepsilon^2 + (T_p - T_r)\left(V[\Xi(R)] - \bar{\xi}^2\theta^2k^2\sigma_\varepsilon^2\right), & \tau > T_p - T_r > 0. \end{cases}$$

Our analytical expressions of the variances for each case provide the following insights.

Property 1. When information about the return rate and the yield rate is shared, the variance of net stock levels always reduces (*i.e.*$V[NS^A] < V[NS^N]$).

An intuitively understandable explanation on this property is that the shared information about the incoming returns is exploited to absorb the impact from the demands, in order to keep the net stock levels constant as much as possible. This property means the manufacturer can reduce its inventory related costs as a result of the advance notice scheme.

Property 2. When information about the return rate and the yield rate is shared, the variance of the production order will become smaller (*i.e.* $V[P^A] < V[P^N]$), *if and only if* $T_p - T_r \geq \tau$ *and* θ *is positive.*

This suggests that the advance notice of the product return rate information enables the manufacturer to mitigate the well-known Bullwhip effect; however, this preferable outcome occurs only in a limited set of circumstances. For example, if we use the well-accepted assumption where D_t and R_t are mutually independent (i.e. $\theta = 0$), we always have $V[P^A] = V[P^N]$. This may lead us to the conclusion that the advance notice scheme does not influence Bullwhip. When the condition $T_p - T_r \geq \tau$ is met and θ takes a negative value, the variance of the production order can even increase in the case A. Therefore, if the reduction of Bulllwhip effect is a major concern, we should be careful about the outcome of the advance notice scheme. Managers should pay attention to the value of θ, in addition to the relationship between T_p, T_r and τ.

If your situation allows you to change the values of T_p and T_r, the following three properties (property 3 - 5) might be quite useful.

Property 3. When the information about the return rate and the yield rate is shared, the variance of the net stock levels ($V[NS^A]$) is decreasing in T_r, if and only if $T_p - T_r \geq \tau \wedge \theta < V[\Xi(R)]/(2\bar{\xi}k\sigma_\varepsilon^2)$.

Property 4. When the information about the return rate and the yield rate is shared and $\tau > T_p - T_r > 0$, the variance of the net stock levels ($V[NS^A]$) is decreasing in T_r. Increasing the value of T_r (until $T_r = T_p$), reduces the value of $V[NS^A]$.

Properties 3 and 4 produce counter-intuitive insights: under certain conditions, a shorter remanufacturing lead-time (T_r) can increase the net stock variance at the manufacturer. For example, if demand and return rate are independent each other (i.e. $\theta = 0$, which is always less than $V[\Xi(R)]/(2\bar{\xi}k\sigma_\varepsilon^2)$), $T_p > T_r$ and $\tau = 0$, shorter remanufacturing lead-time increases the net stock variance. In a serially linked supply chain it is known that shorter lead-time always reduces the net stock variance (see Lee et al., 2000; Chen et al., 2000; Hosoda and Disney, 2006, for example). This property indicates that such an insight obtained from a serially linked supply chain setting without returns may not be true in a closed loop supply chain. A similar finding is shown in van der Laan et al. (1999a). They show that this counter intuitive phenomenon could be observable when $T_p > T_r$ by using numerical analysis. Property 4 is proved by the fact that $(T_p - T_r)\left(V[\Xi(R)] - \bar{\xi}^2\theta^2k^2\sigma_\varepsilon^2\right) \geq 0$, when $\tau > T_p - T_r > 0$.

Property 5. When $T_r > T_p$, the value of T_r does not have any impact upon $V[NS^A]$.

Property 5 suggests that shorter remanufacturing lead-time T_r does not decrease the net stock variance as long as the relation $T_r > T_p$ holds in a closed loop supply chain.

Finally, our variance expressions reveal that irrespective of the availability of the advance notice, the following two fundamental trade-off issues exist in closed loop supply chains.

Property 6. When the mean of the returns μ_R is higher, the production and the net stock variances become higher. A single exception is $V[NS^A]$ when $T_r \geq T_p$.

It is obvious from the variance expressions that when μ_R increases, $V[\Xi(R)]$ will increase, which could result in lower supply chain performance in the end. Therefore, companies should be very careful about employing a strategy to increase μ_R. van der Laan et al. (1999b) identify a similar phenomenon. They conclude that it may be unwise to remanufacture all returned products. Those findings suggest that for the sake of the better environment, for example, larger values of μ_R are preferable, however, for better performance of the supply chain, lower values of μ_R are required. $V[NS^A]$ is independent of μ_R only when $T_r \geq T_p$.

Property 7. When the mean of the random yield rate $\bar{\xi}$ and/or the variance of the random yield rate $V[\xi]$ increases, the production and the net stock variances increases. A single exception is $V[NS^A]$ when $T_r \geq T_p$.

$V[\Xi(R)]$ increases in $\bar{\xi}$ and $V[\xi]$. If the mean of yield rate $\bar{\xi}$ is improved (thanks to a Kaizen activity, for example) but $V[\xi]$ remains constant, the production and the net stock variances become higher, which may lead to lower supply chain performance.

5. Uniform Distribution Example

In this section, we assume that the remanufacturing yield rate ξ_t is uniformly distributed between $0 \leq a \leq b \leq 1$, in order to complete a numerical analysis. The PDF is given by

$$f(x) = \begin{cases} \dfrac{1}{b-a}, & a \leq x \leq b \\ 0, & x < a \vee x > b \end{cases}$$

giving an average yield of $\bar{\xi} = (a+b)/2$ with a variance of $\mathrm{V}[\xi] = (b-a)^2/12$.

5.1 Value of Advance Notice. It is assumed that the following expression is a good indicator of the value of advance notice on the inventory related cost in the closed loop supply chain;

$$\widehat{\Delta} = \frac{\sqrt{V[NS^N]} - \sqrt{V[NS^A]}}{\sqrt{V[NS^N]}} \times 100.$$

Unless otherwise stated, the following values are assumed; $\mu_D = 100$, $\mu_R = 50$, $\sigma_\varepsilon = 1$, $k = 1$, $T_p = 5$, $T_r = 1$, $\tau = 2$, $\theta = 0.7$, $a = 0$ and $b = 1$.

Figure 3 illustrates the condition that $\theta < V[\Xi(R)]/(2\bar{\xi}k\sigma_\varepsilon^2)$ is met, when $0 < k \leq 4$, $0 \leq a \leq 1$ and $b = 1$. Most of the cases $V[\Xi(R)]/(2\bar{\xi}k\sigma_\varepsilon^2)$ is greater than unity. Since $|\theta| \leq 1$, we can conclude that under our setting, if $T_p - T_r \geq \tau$, $V[NS^A]$ is decreasing in T_r, as predicted by property 3.

Figure 4 illustrates the impact of T_r and μ_R on $\widehat{\Delta}$: longer T_r or larger μ_R results in higher benefits from advance notice. The left-hand side graph shows that when $T_r \geq T_p$ (=5, in this case), the value of $\widehat{\Delta}$ is maximised. The right-hand side of Figure 4 explains the impact of μ_R on $\widehat{\Delta}$. The value of μ_R is varied from 10 to 90. Generally, $\widehat{\Delta}$ is increasing in μ_R and is affected by the length of T_r, but $\widehat{\Delta}$ becomes less sensitive as μ_R increases. It should be noted that when μ_R increases, the cost could increase, according to property 6, but the value of $\widehat{\Delta}$ improves. Figure 4 suggests that the advance notice is most valuable when $T_r \geq T_p$ and μ_R is large.

To consider the impact of $\bar{\xi}$, we increase the value of a from zero to unity and hold $b = 1.0$. Note that in this setting, as a is large, $\bar{\xi}$ becomes larger (since $\bar{\xi} = (a+b)/2$) but $V[\xi]$

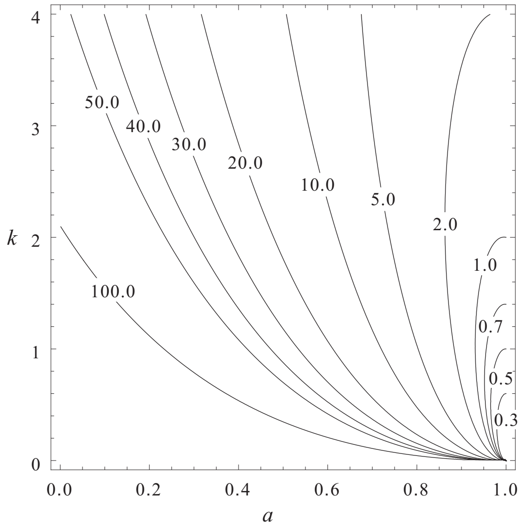

Figure 3. Value of $\frac{V[\Xi(R)]}{2\bar{\xi}k\sigma_\varepsilon^2}$ *when* $0 < k \leq 4$, $0 \leq a \leq 1$ *and* $b = 1$

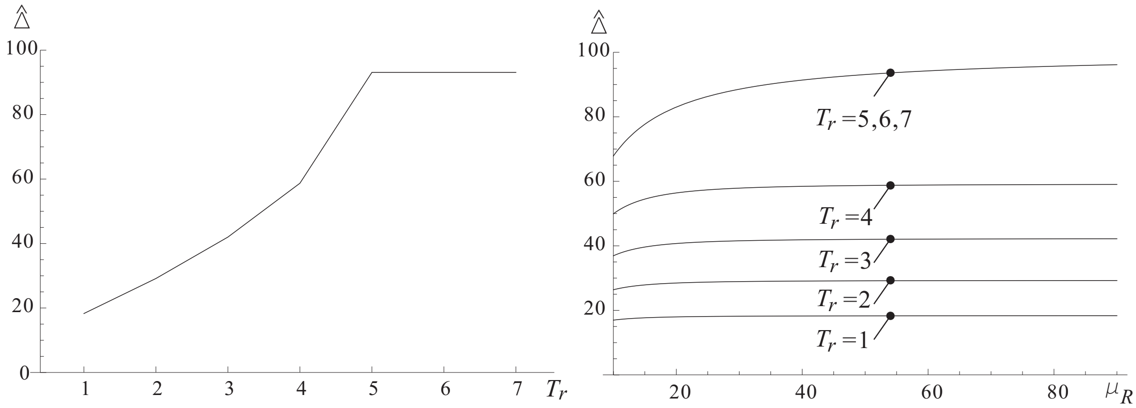

Figure 4. Impact of T_r *and* μ_R *when* $T_p = 5$

becomes smaller (since $V[\xi] = (b - a)^2/12$). The result is summarised in Figure 5. It is shown that the impact of a or $\bar{\xi}$ is largely dependent on the value of k and the correlation factor θ, especially when a or $\bar{\xi}$ is large. High values of a imply higher values of $\bar{\xi}$ and smaller values of $V[\xi]$, will result in high value of $\widehat{\Delta}$ if and only if the demand and the return are highly correlated (e.g. $\theta \geq 0.8$). It is also concluded that the values of a (or $\bar{\xi}$), k and θ have almost no impact on $\widehat{\Delta}$, when a (or $\bar{\xi}$) is small (e.g. a < 0.4 or $\bar{\xi}$ < 0.7). Figure 5 also indicates that there is a benefit of the advanced notice scheme, even when the value of a is small (which also means that $\bar{\xi}$ is small and $V[\xi]$ is high). This implies that a high yield rate is not necessary to enjoy the benefit coming from the advance notice scheme. When θ is small, yield rate improvement can deteriorate the value of the advance notice scheme.

We consider the situation where $T_p = T_r = 5$. Figure 6 illustrates the results. Note that under the condition $T_r \geq T_p$, $\widehat{\Delta}$ is independent of θ. The value of $\widehat{\Delta}$ in Figure 6 is almost always better than in Figure 5. Only when $a = 1$, $\theta = 1$ and $k = 1$ will those two values become equal. Figure 6 shows that $\widehat{\Delta}$ is decreasing in a and $\bar{\xi}$.

The results of our numerical analysis shown indicate that a large value of T_r is a key factor to have better value of $\widehat{\Delta}$. Figure 7 illustrates when to exploit the advance notice scheme from

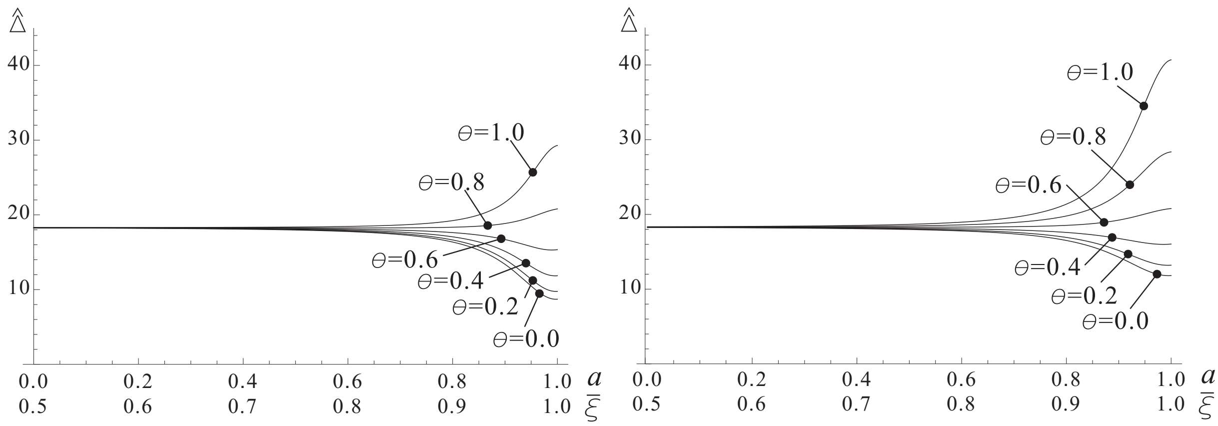

Figure 5. Impact of a, $\bar{\xi}$ and θ on $\hat{\Delta}$ when $k = 1$ (left) and $k = \sqrt{2}$ (right)

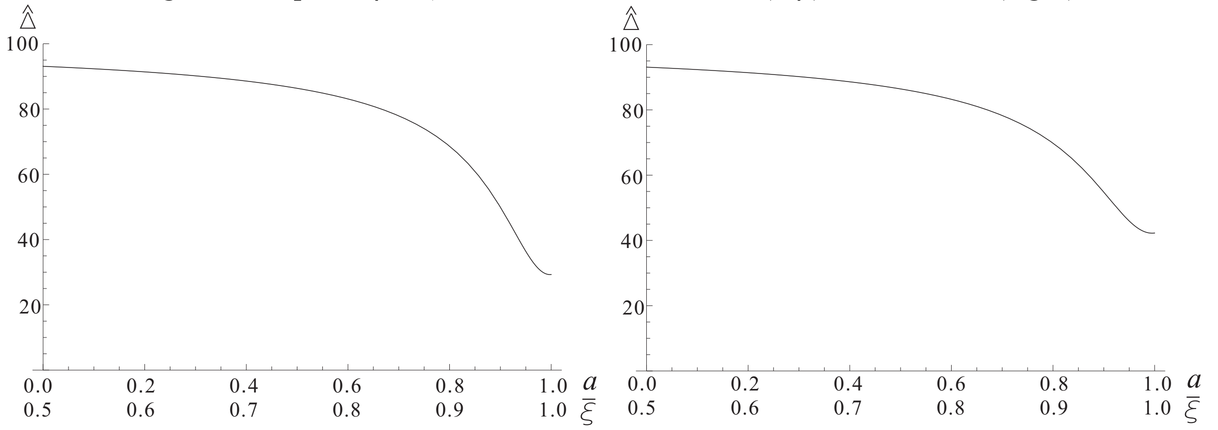

Figure 6. Impact of a, $\bar{\xi}$ and θ on $\hat{\Delta}$ when $T_p = T_r = 5$, $k = 1$ (left) and $k = \sqrt{2}$ (right)

the point of τ (x-axis) and $T_p - T_r$ (y-axis). Each number in Figure 7 represents the value of $\widehat{\Delta}$. As you see, the advance notice scheme provides the largest value when $T_r \geq T_p$,irrespective of τ.

6. Conclusions

Using an analytical model and numerical analysis, we have investigated the economic benefit of an advance notice scheme and the impact of the lead-times, random yield and correlation in a closed loop supply chain. To analyse the impact on performance, production and inventory variances were used. It is shown that the sharing of the product return and yield rate information could bring a benefit to the manufacturer. In certain scenarios, however, Bullwhip could increase, even though the net stock variance decreases as the result of the advance notice scheme. It is also shown that random yields and parameters in the product return rate process have an impact upon the magnitude of the benefit, in addition to the manufacturing and the remanufacturing lead-times. A counterintuitive finding is that shorter remanufacturing lead-time T_r could result in the net stock variance increases in certain situations. The value of advance notice can be improved by setting $T_r \geq T_p$.

Our findings yield the following general guideline for managers. The advance notice of the return product rate information will always reduce the net stock variance. On the other hand, to reduce Bullwhip, we should ensure $T_p - T_r \geq \tau$ and a positive value of θ exists. In addition, when $T_p \geq T_r$, reducing T_r could result in a higher net stock variance. To avoid this, the values of $\{k, \theta, \tau, \bar{\xi}, V[\xi], \mu_R\}$ should be carefully investigated. Numerical analysis shows that the advance notice scheme can provide the largest benefit when $T_r \geq T_p$.

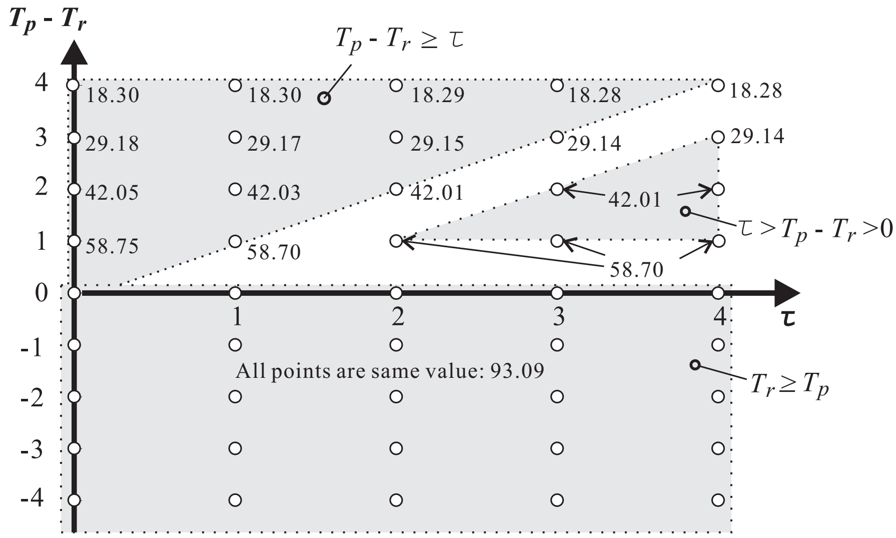

Figure 7. Impact of T_r and τ on $\hat{\Delta}$ when $T_p = 5$, $1 \leq T_r \leq 9$, and $0 \leq \tau \leq 4$

Finally, increasing the product return rates and the mean of yield rates could result in lower supply chain performance, since larger value of μ_R and/or $\bar{\xi}$ can have a negative impact. This finding indicates the existence of a fundamental underlying trade-off problem in a closed loop supply chain.

12. References

Akçalı, E., Çetinkaya, S., 2011. Quantitative models for inventory and production planning in closed-loop supply chains. *International Journal of Production Research*, 49, 2373–2407.

Chen, Y.F., Drezner, Z., Ryan, J.K., Simchi-Levi, D., 2000. Quantifying the bullwhip effect in a simple supply chain: The impact of forecasting, lead times, and information. *Management Science*, 46(3), 436–443.

de Brito, M.P., van der Laan, E.A., 2009. Inventory control with product returns: The impact of imperfect information. *European Journal of Operational Research*, 194, 85–101.

Disney, S.M., Towill, D.R., 2003. On the bullwhip and inventory variance produced by an ordering policy. *OMEGA, The International Journal of Management Science*, 31, 157–167.

Ferrer, G., Ketzenberg, M.E., 2004. Value of information in remanufacturing complex products. *IIE Transactions*, 36, 265–277.

Flapper, S., Gayon, J., Vercraene, S., 2012. Control of a production-inventory system with returns under imperfect advance return information. *European Journal of Operational Research*, 218, 392–400.

Gavirneni, S., Kapuscinski, R., Tayur, S., 1999. Value of information in capacitated supply chain. *Management Science*, 45(1), 16–24.

Guide, V.D.R. Jr., 2000. Production planning and control for remanufacturing: industry practice and research needs. *Journal of Operations Management*, 18, 467–483.

Henig, M., Gerchak, Y., 1990. The structure of periodic review policies in the presence of random yield. *Operations Research*, 38, 634–643.

Hosoda, T. Disney, S.M., 2012. On the replenishment policy when the market demand information is lagged. *International Journal of Production Economics*, 135, 458–467.

Hosoda, T., Disney, S.M., 2006. On variance amplification in a three-echelon supply chain with minimum mean square error forecasting. *OMEGA, The International Journal of Management Science*, 34(4), 344–358.

Ketzenberg, M.E., Laan, E.A. van der, Teunter, R.H., 2006. Value of information in closed loop supply chains. *Production and Operations Management*, 15, 393–406.

Ketzenberg, M.E., 2009. The value of information in a capacitated closed loop supply chain. *European Journal of Operational Research*, 198, 491–503.
Laan, E.A. van der, Salomon, M., Dekker, R., 1999a. An investigation of lead-time effects in manufacturing/remanufacturing systems under simple push and pull control strategies. *European Journal of Operational Research*, 115, 195–214.
Laan, E.A. van der, Salomon, M., Dekker, R., Van Wassenhove, L., 1999b. Inventory control in hybrid systems with remanufacturing. *Management Science*, 45, 733–747.
Lee, H.L., So, K.C., Tang, C.S., 2000. The value of information sharing in a two-level supply chain. *Management Science*, 46(5), 626–643.
Tao, Z., Zhou, S. X., Tang, C.S., 2012. Managing a remanufacturing system with random yield: Properties, observations, and heuristics. *Production and Operations Management*, 21, 797–813.
Yano, C.A., Lee, H.L., 1995. Lot sizing with random yields: A review. *Operations Research*, 43, 311–334.

Appendix 1: Variance of the Returns and Correlation between Demand and Returns

The variance of R_t and the correlation coefficient of $D_{t-\tau}$ and R_t are addressed herein. Generally, since the variance of a random variable X is the expected value of its squared deviations from the mean, $E[(X-\mu)^2]$ where $\mu = E[X]$ and $\sigma_\zeta = k\sigma_\varepsilon$, the variance of R_t is

$$V[R] = \mathrm{E}[(\mathrm{R_t} - \mu_R)^2] = E\left[\left(\theta k \varepsilon_{t-\tau} + \sqrt{1-\theta^2}\zeta_t\right)^2\right] = \theta^2 k^2 \sigma_\varepsilon^2 + (1-\theta^2)k^2\sigma_\varepsilon^2 = k^2\sigma_\varepsilon^2.$$

Using the covariance of $D_{t-\tau}$ and R_t, $Cov(D_{t-\tau}, R_t)$, the correlation coefficient is given by

$$\frac{Cov(D_{t-\tau}, R_t)}{\sigma_\zeta \sigma_\varepsilon} = \frac{E\left[\varepsilon_{t-\tau}\left(\theta k \varepsilon_{t-\tau} + \sqrt{1-\theta^2}\zeta_t\right)\right]}{\sigma_\zeta \sigma_\varepsilon} = \frac{\theta k \sigma_\varepsilon^2}{k\sigma_\varepsilon^2} = \theta.$$

Appendix 2: Variance of the Remanufacturing Yield

The process to obtain the variance of $\Xi(R)$ is shown herein. Generally the variance of a random variable X is equal to the difference between the expected value of its squared value and the square of its expected value: $V[X] = E[X^2] - E[X]^2$. Thus, we can have

$$V[\Xi(R)] = E[\Xi(R_t)^2] - E[\Xi(R_t)]^2 = E[\mu_R^2\xi_t^2 + \theta^2k^2\xi_t^2\varepsilon_{t-\tau}^2 + (1-\theta^2)\xi_t^2\zeta_t^2] - \bar{\xi}^2\mu_R^2.$$

Since we already know $E[X^2] = V[X] + E[X]^2$, $\mathrm{E}[\xi_t^2]$ can be written as $V[\xi] + \bar{\xi}^2$, which yields the final expression of $V[\Xi(R)]$,

$$V[\Xi(R)] = \bar{\xi}^2k^2\sigma_\varepsilon^2 + (\mu_R^2 + k^2\sigma_\varepsilon^2)V[\xi].$$

This result suggests that $V[\Xi(R)]$ is increasing in $\bar{\xi}$, k^2, σ_ε^2, μ_R and $V[\xi]$. It is interesting that the levels $\bar{\xi}$ and μ_R are in the variance expression. This does not happen in linear systems, but is clearly present in this non-linear system.