Who Provides the Capital for Chinese Growth: The Public or the Private Sector?

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Abstract

We focus on the role of the government in the provision of investment in China, through the medium of a DSGE model of the economy in which the form of the production function reflects this governmental role. Using indirect inference, we estimate and test for the elasticity of substitution between government and nongovernment capital in both CES and Cobb-Douglas technologies. The results underscore the strong substitution relationship between government and nongovernment capital from 1949, supporting CES rather than the Cobb-Douglas technology. They also show that the orientation of public investment changed after the start of the 'Socialist Market Economy' in 1992: government capital became more complementary to nongovernment capital as it focused more on infrastructure and withdrew from industrial production, intervening only in times of crisis, for stabilisation purposes, indirectly via the state banks.

Key Words

China, Government Investment, Indirect Inference, Economic Growth

JEL Classification

E22, E62, O47

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1 Introduction

Public infrastructure is one of the major determinants of economic growth, especially in developing economies. Public infrastructure can be a bottleneck for sustainable growth and poverty reduction. The pioneering work of Barro (1990) which incorporates the flow of public services into private production is the first endogenous growth model in which long-run economic growth is driven by fiscal policy. Futagami, Morita and Shibata (1993), Turnovsky (1997) and Fisher, and Turnovsky (1998) then introduced a stock of public capital as an input in the production along the lines of the early work of Arrow and Kurz (1970). Other studies by Baxter and King (1993), Glomm and Ravikumar (1994) and Cassou and Lasing (1998) also suggest that the accumulated stock of public capital rather than the flow of government expenditure is more relevant to the production process.

The limit of the early endogenous growth models with fiscal policy in the literature is that they use a Cobb-Douglas technology to specify the relationship between private and public capital in production and then restrict the factor substitutability between them to be unity. More recently, Chatterjee and Ghosh (2011) and Bucci and Del Bo (2012) allow for a flexible degree of complementarity/substitutability between them by a more general Constant Elasticity of Substitution (CES) aggregate technology. The lack of public capital stock data for a large number of countries has forced most relevant studies to focus on theoretical analysis only. Few empirical studies have investigated the effects of public capital on growth, not to mention the relationship between public and private capital[2]. The main goal of this paper is to investigate production technology using Chinese data and to analyse how government productive expenditure affects China's economic growth[3]. However, many producers in China are state owned enterprises (SOEs) and township collective enterprises. Hence, for purposes of accuracy, we use 'nongovernment capital' and 'government capital', rather than private capital and public capital, to define the two categories of physical capital in production, even if today nongovernment capital in China is mainly held by private enterprises because of the economic reform programme.

We find that since 1992 the 'Socialist Market Economy' diverted the orientation of public investment from direct input to industrial production to infrastructure, so that government capital became more complementary to nongovernment capital.

[2] Singh (2012) suggests the long-run crowding-in effects of public capital on private capital in India.
[3] Throughout this paper, 'China' refers to the mainland China, excluding Taiwan, Hong Kong and Macau.

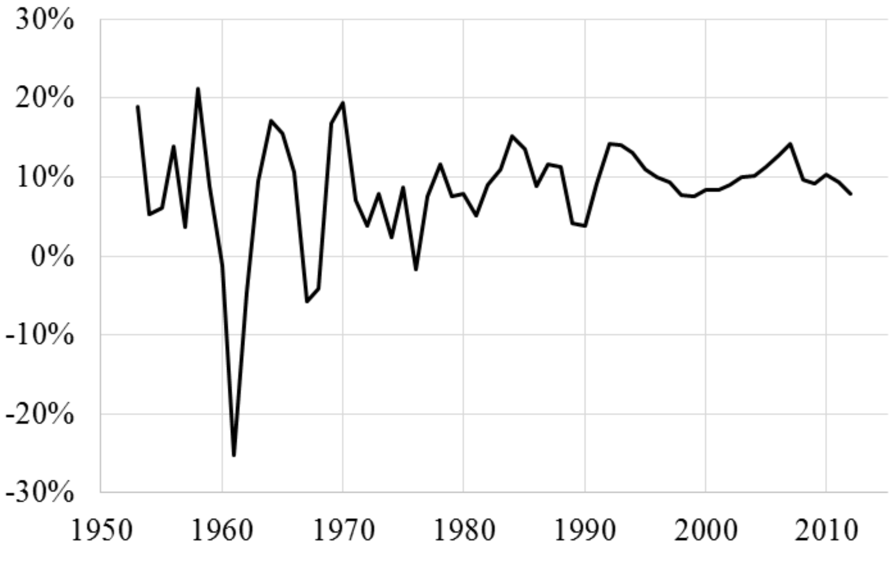
The rest of the paper is organised as follows. The next section introduces some aspects of China's economy as the background of our model; section 3 discusses the data; section 4 sets out the DSGE model to be estimated and tested; sections 5-7 in turn discuss the indirect inference method and the results under two different production functions; section 8 looks at structural change in 1992. In section 9 we consider the policy implications and section 10 concludes.

2 Stylised Facts of the Chinese Economy: An Overview

China is the fastest growing major economy in the world, with remarkable growth rates, averaging about 8% after 1949 and around 10% over the three decades before 2013. According to the World Bank (2015), China overtook the United States as the world's largest economy in 2014, accounting for 17% of the world economy.

Figure 1 demonstrates the six economic troughs after 1949: four of them happened before 1990 and were all because of political disarrays; the other two were caused by external factors, the 1997 Asian financial crisis and the 2008 global financial and economic crisis. There were large economic fluctuations before 1978. Since 1978, the economy has achieved rapid and stable growth.

Figure 1: The Economic Growth Rate in China



Data Source: National Data, National Bureau of Statistics of China.

The Six Economic Troughs since 1949 can be seen in Figure 1 and accounted for as follows: (i) 1959-1961: 'Three Years of Natural Disasters' along with policy failures of the Great Leap Forward; (ii) 1966: the Cultural Revolution; (iii) 1976: the Tangshan earthquake; the deaths of Zhou Enlai, Zhu De and Mao Zedong; the ousting of the Gang of Four; (iv) 1989:

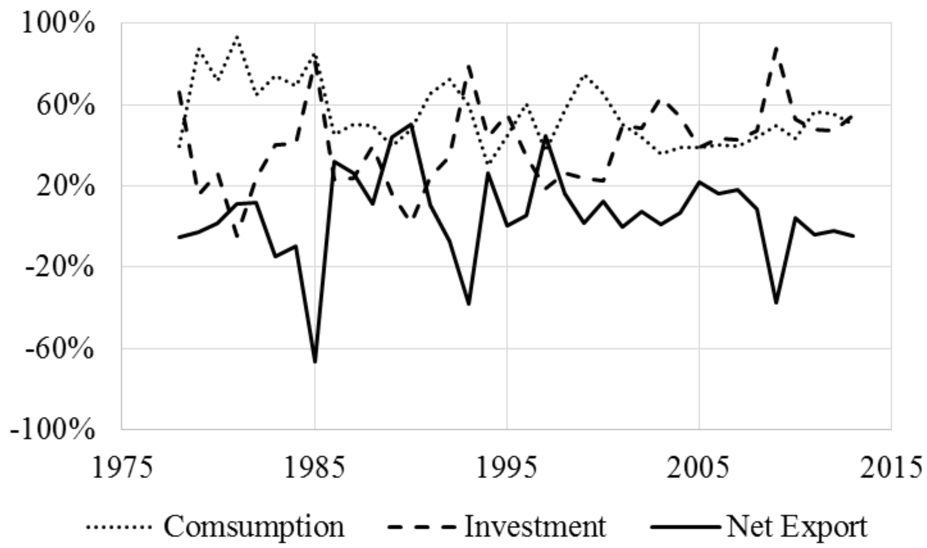
student riots and economic sanctions from Western countries; (v) 1997: the Asian financial crisis; (vi) 2008: the global financial crisis.

The four stylised facts of the Chinese economy below can be treated as the background for our model and the comparison between the two subsamples separated by the year of 1992.

2.1 Investment

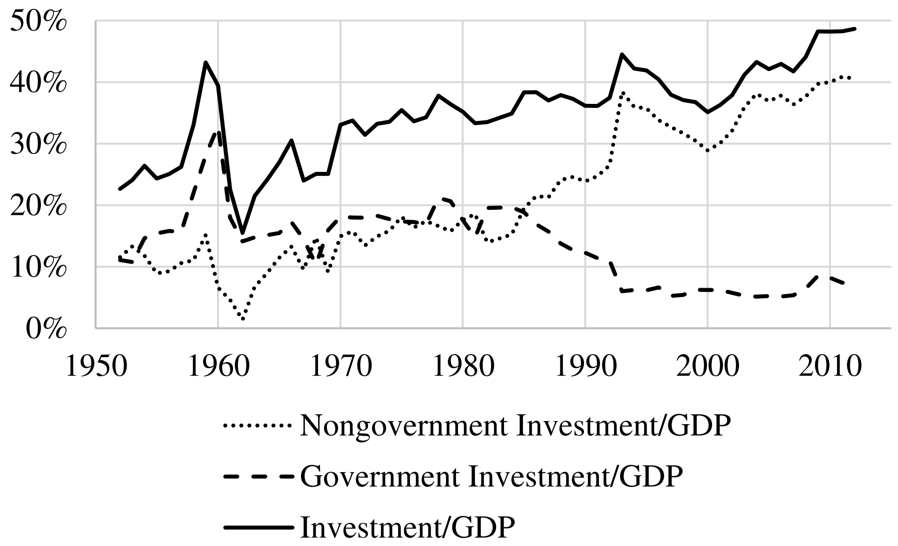
China's sustained and rapid economic growth as well as its increasing competitiveness in manufacturing and international trade have been underpinned by massive capital formation. Figure 2 reveals that investment contributes much more than net exports to China's economic growth. In 2009 when the world financial crisis hit exports and resulted in rising unemployment, investment contributed 87.6% of total growth, protecting China from the repercussions of the global economic crisis. As shown in Figure 3, the proportion of investment in GDP has been above 30% since 1970, reaching almost 50% in recent years. The majority of investment was carried out by the government directly or by state-owned entities before 1978. In the mid-1980s, nongovernment investment began to outpace government investment, as shown in Figure 3. In recent years, more than 80% of investment has been from the nongovernment sector. However, the share of public investment expenditure in the government spending has been always much higher than most of other countries, which is a distinguishing feature of China's economy and also the reason for incorporating the government capital in our model.

Figure 2: Contribution Shares of the Three Main Components in GDP Growth



Data Source: National Data, National Bureau of Statistics of China.

Figure 3: Investment as a Share of GDP



Data Source: The data is calculated on the base of National Data, National Bureau of Statistics of China.

2.2 Human Capital

Production grew substantially between 1949 and the beginning of economic reform in 1978. Without the political turmoil of that era, China would have had a higher growth rate then. Also, economic growth was accompanied by population growth. As a result, productive capacity was unable to outrun essential consumption needs by a significant margin, which offset the efforts of the government to invest more resources in capital goods. The relatively small size of the capital stock per capita led to low productivity per worker, which in turn perpetuated the economy's inability to generate a substantial surplus. In 1979, family planning policy was introduced to alleviate social, economic, and environmental problems in China, averting more about 300 million births. In the post reform period, productivity rather than population has accounted for China's economic growth (see Chow and Li, 2002). Since 1949, human capital has increased steadily, mainly from the growing population in the first three decades and from the increasing quality and capability of the workforce in the following three decades. It is believed that human capital has played an important role in China's economic miracle (See, for example, Fleisher and Chen, 1997, Démurger, 2001, and Fleisher, Li and Zhao, 2009). Hence, our model considers the contribution of human capital which is included in A_t of the production function.

2.3 Tax System

Before 1978, public expenditure was mainly financed from the revenues and profits of the State Owned Enterprises (SOEs). Since 1978, however, the profits claimed by the government have been replaced with taxes. In the beginning of this tax reform period, the tax system was adjusted for differences in capitalisation and pricing situations among various firms, but more uniform tax schedules were introduced in the early 1990s. After 1992, value added tax (proportional to the net value of production) has been the largest revenue resource among all tax categories. In 2012, tax revenue accounted for 85.8% of government revenue and most items of tax revenue are proportional rather than progressive. The two progressive tax categories are individual income tax and land appreciation tax, together accounting for 8.6% of the total tax revenue in 2012. Since most government tax revenue is proportional to output, we can simply use a proportional tax in our model.

2.4 The transformation of government functions

Before the early 1990s, central control over planning and the state ownership of the financial system and certain important industries had enabled the government to mobilise whatever surplus was available and greatly increase the proportion of GDP devoted to investment. In 1992, Deng Xiaoping made his famous 'southern tour speech', following which the 'socialist market economy' was legitimised and institutionalised. Since then, private investment[4] has been widely encouraged and it has made up a growing share in total investment. Meanwhile, the government has changed the structure of public investment by cutting the financial support to SOEs and inputting more resources in high-quality public infrastructures, which motivates the examination on the possibility of a structural break in 1992 for our model.

In the light of the four features identified above, this paper will examine the elasticity of substitution between government and nongovernment capital[5], investigate the differing orientation of public investment before and after 1992, and discuss the policy implications.

3 The Data

This paper employs annual data from 1952 to 2012 which is listed in **Appendix A**. The raw data are collected from China Statistical Yearbook (2001, 2007, 2008) and National Data, National Bureau of Statistics of China. The items of raw data are: (i) Gross Domestic Product (GDP), (ii) Household Consumption (private consumption or nongovernment consumption), (iii) Gross Capital Formation (total investment), (iv) Government Budgetary Revenue, (v) Government Extra-budgetary Revenue, (vi) Government Expenditure by Function (1952-2006), (vii) Main Items of National Government Expenditure of Central and Local Govern-

[4] In this paper, private investment is a part of nongovernment investment since nongovernment sector also includes state owned enterprises and township collective enterprises.
[5] The private sector has occupied a larger and larger share in the nongovernment capital since the 1978 reform.

ments (2007-2012), (viii) Extra-budgetary Revenue by Item (1952-2010), and (ix) Extra-budgetary Expenditure by Item[6].

The National Bureau of Statistics of China does not directly provide data on government investment. However, government investment can be approximately estimated by totalling the corresponding productive expenditure sub-items in the items (vi)-(ix). Thus, nongovernment investment can be calculated by the gap between total investment and government investment. Nominal data are transformed into real by dividing by the GDP deflator[7].

The tax rate is simply estimated as the share of budgetary plus extra-budgetary revenue in GDP. We calculate the capital stock values following the conventional 'Perpetual Inventory Method' (PIM)[8] expressed as equations (3)-(4) below. The value of the capital stock is estimated by using annual data for capital formation as investment.

4 The Model

Assume that the representative agent maximises lifetime utility:

$$\max E_0\left[\sum_{t=0}^{\infty}\beta^t u\left(c_t,h_t\right)\right], \text{ where } u\left(c_t,h_t\right)\equiv\frac{c_t^{1-\sigma}-1}{1-\sigma}+\varphi\frac{h_t^{1-\sigma}-1}{1-\sigma} \quad (1)$$

The model adopts a transformed 'constant inter-temporal elasticity of substitution' (CIES) utility function, where c_t is private consumption (or nongovernment consumption), h_t is public consumption (or government consumption), $\sigma > 0$ and σ is the elasticity of marginal utility with respect to private and public consumption, φ is the weight given to public consumption relative to private consumption. For simplicity, we assume that government capital k_{Gt}, tax rate τ_t and h_t are exogenous variables, so for the representative, h_t is determined outside

[6] Since 2011, extra-budgetary revenue and expenditure were included in intra-budgetary revenue and expenditure.

[7] The base year for calculating the GDP deflator is 2000. GDP deflator from 1960 to 2012 is from World Development Indicators (WDI) & Global Development Finance (GDF), The World Bank (issued in July, 2013); GDP deflator from 1952 to 1959 is calculated from the nominal GDP and indices of GDP (1952-1960). There are no price indices for government investment and nongovernment investment, therefore we use GDP deflator to calculate their real values.

[8] PIM: The net capital stock at period t, Capital_t, can be written as a function of its previous value, Capital_{t-1}, gross investment in period t, Investment_t, and the depreciation of capital in previous period, $\text{Depreciation}_{t-1}$: $\text{Capital}_t = \text{Capital}_{t-1} + \text{Investment}_t - \text{Depreciation}_{t-1}$. Assuming depreciation at a constant rate δ, then we can rewrite the capital stock as: $\text{Capital}_t = (1-\delta)\text{Capital}_{t-1} + \text{Investment}_t$. The method is called "perpetual" because all assets are forever part of the inventory of capital stocks. The quantity of services provided by an asset declines as it ages, but it never reaches zero. PIM is a popular method of estimating capital stock, for example, Chow and Li (2002) and Zhang and Zhang (2003) use this method in calculating China's capital stock, and Kamps (2006) adopts it to provide internationally comparable capital stock estimates for 22 OECD countries. The estimation results in sections 7 and 8 also show the depreciation rates in different periods, which is another contribution to the empirical literature on China.

the model. The utility function in the Xie (1997) is a special case of Equation (1) by setting $\varphi = 1$ and letting σ approach zero so that the utility follows the logarithm form.

The optimization problem is subject to the budget constraint (2), the laws of motion for the two capital stocks (3)-(4), and the production function (5) are:

$$c_t + i_t = (1-\tau_t) y_t \tag{2}$$

$$i_t = k_{t+1} - (1-\delta) k_t \tag{3}$$

$$i_{Gt} = k_{Gt+1} - (1-\delta_G) k_{Gt} \tag{4}$$

$$y_t = f(k_t) = A_t^{1-\theta} \left[\alpha k_t^r + (1-\alpha) k_{Gt}^r \right]^{\frac{\theta}{r}} \exp(v_t^y) \tag{5}$$

where A_t is a deterministic trend which tracks the long-run growth of human capital, including both quantity and quality of the labour force[9], v_t^y is the logarithm of the productivity shock, k_t is nongovernment capital and k_{Gt} is government capital, i_t is nongovernment investment and i_{Gt} is government investment, δ_t and δ_{Gt} are the rates of depreciation in non-government and government sectors.

The homogenous Constant Elasticity of Substitution (CES) production function was introduced by Arrow, Chenery, Minhas and Solow (1961). The elasticity of substitution measures the percent change in factor proportions due to a one percent change in the marginal rate of technical substitution. Equation (5) is an extended CES production function which uses a CES structure to describe the relationship between government and nongovernment capital, but specifies the relationship between broad physical capital and A_t in the Cobb-Douglas form. α and $(1-\alpha)$ are the share parameters for nongovernment and government capital respectively, $(1-r)^{-1}$ is the elasticity of substitution between the two types of capital, θ and $(1-\theta)$ are the output elasticities of total capital and A_t, respectively. If r approaches zero, in the limit we get the Cobb-Douglas function: $y_t = f(k_t) = A_t^{1-\theta} k_t^{\theta\alpha} k_{Gt}^{\theta(1-\alpha)} \exp(v_t^y)$.

The equilibrium conditions of the optimisation problem can be derived and stationarised by A_t following the standard procedure detailed in **Appendix B**.

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[9] The production can be write as $y_t = A_{TFP}(l_t h_t)^{1-\theta}[\alpha k_t^r + (1-\alpha)k_{Gt}^r]^{\frac{\theta}{r}} \exp(v_t^y)$, here A_{TFP} total factor productivity, l_t and h_t are the quantity and quality of the labour force, hence, $A_t = A_{TFP}^{1/(1-\theta)} l_t h_t$. Until now, there has been almost no comprehensive method that directly measures the quality of the labour force in China. Moreover, there is no official data on total factor productivity. Nevertheless, we can evaluate A_t from Equation (5) since we already have the data of output and the two different physical capital stocks.

5 Methodology

A DSGE model as such is usually taken to the data using either classical inference (e.g. maximum likelihood used by Ireland, 2004; GMM used by Christiano and Eichenbaum, 1992) or Bayesian inference (e.g. Smets and Wouters, 2003, 2007). With the development of computer power, there is a trend in macroeconomic literature to use simulation-based inference, such as indirect inference (Le et al, 2011) which can be used for testing and estimating a DSGE models. One advantage of indirect inference test is that it provides an absolute verification (or falsification) of the model against the data, while the classical and Bayesian tests (using likelihood ratio and posterior odds) are relative tests between models. A more comprehensive comparison between these inferences in the scenario of Chinese economy is done in Dai, Minford and Zhou (2015).

Another advantage of indirect inference is its use of auxiliary model to summarise the data features. It provides a flexible choice of which aspects you want the model to match. All models are wrong, but some are useful. It is unlikely for any reasonably complicated model to match everything. Our model, for example, has a focus on the contribution of capital to economic growth, so we would like to place higher weights on the capital related dynamics. VAR is a commonly chosen auxiliary model to summarise the dynamic and volatility features of the data.

The process of indirect inference testing is to bootstrap the structural residuals and generate a large number of sample replications, based on which a distribution of the VAR parameters is obtained; finally to test whether the VAR parameters from actual data lie within this distribution at some level of confidence. Estimation using indirect inference involves choosing a set of parameters for the structural model, so that, when this model is simulated, it generates estimates of the auxiliary model as close as possible to the estimates of the auxiliary model using actual data. In other words, indirect estimation chooses the structural parameters that can minimise the distance between the two sets of estimated parameters. Smith (1993) was the first to propose this method for estimating nonlinear macro models—see also Gourieroux, Monfort and Renault (1993) and Canova (2005). The Indirect Inference testing and estimation procedure used here is set out by Dai, Minford and Zhou (2015) in an earlier application to China and more generally by Le et al (2016) who explain its considerable power in small samples and the literature in which it was developed.

5.1 The Indirect Inference Test Procedure

The null hypothesis H_0 is that the model is true in the sense that the data behavior as summarized by the auxiliary model, usually a VAR, is close to the simulated data behavior based on the model. In practice, the test is carried out in the following steps.

Step 1: Calculate the Innovations

To recover the innovations ($e_t^{\tau}, e_t^{KG}, e_t^{y}$) from the stationarised equation system (**Appendix B**) which is derived from the model in Section 4, the parameters need to be calibrated. In particular, the deterministic growth rate γ is estimated on the basis of the average growth rates of GDP and private consumption. For the value of A_{1952}, set the shocks in the stationarised equation system as zero, substitute the definition $A_t = A_{1952}\gamma^{(t-1952)}$ into the production function, then obtain the value of 'A_{1952}' in each year, and obtain A_{1952} so that the right and left hand sides are equal at their means. τ_E and $\hat{k}_{GE}$ (the steady states of τ_t and $\hat{k}_{Gt}$) in the shock processes are estimated based on the mean values of τ_t and $\hat{k}_{Gt}$. Finally, $\rho_y, \rho_\tau, \rho_{KG}$ are estimated by AR(1) models.

Step 2: Simulate Data

Solve the equation system numerically using Dynare, and then obtain N sets of simulated data based on N bootstraps of the innovations.

Step 3: Estimate the Auxiliary Model

Use a VAR(1) as the auxiliary model, then estimate it with both actual data and the N samples of the simulated data. β^a is obtained from the actual data and includes the VAR parameters and the volatilities of the three variables in the VAR model; β^i ($i = 1,2, \ldots N$) is obtained from one sample of simulated data; $\bar{\beta}$ is the average of N sets of β^i. The auxiliary VAR(1) model is:

$$\begin{bmatrix} \hat{y}_t - \hat{y}_{ss} \\ \hat{k}_t - \hat{k}_{ss} \\ \hat{c}_t - \hat{c}_{ss} \end{bmatrix} = \begin{bmatrix} \beta_{11} & \beta_{21} & \beta_{31} \\ \beta_{12} & \beta_{22} & \beta_{32} \\ \beta_{13} & \beta_{23} & \beta_{33} \end{bmatrix} \begin{bmatrix} \hat{y}_{t-1} - \hat{y}_{ss} \\ \hat{k}_{t-1} - \hat{k}_{ss} \\ \hat{c}_{t-1} - \hat{c}_{ss} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^y \\ \varepsilon_t^k \\ \varepsilon_t^c \end{bmatrix} \tag{6}$$

where $\hat{y}_{ss}, \hat{k}_{ss}, \hat{c}_{ss}$ are the steady state values of the three endogenous variables: output, nongovernment capital and nongovernment consumption in the model.

Step 4: Compute Wald Statistics

$$W^x = \left(\beta^x - \bar{\beta}\right)' \Omega^{-1} \left(\beta^x - \bar{\beta}\right), \text{ where } \Omega = \operatorname{cov}\left(\beta^i - \bar{\beta}\right);\ x = a \text{ or } i = 1, \ldots, N \tag{7}$$

Compare W^a (the Wald statistics from the actual data) with W^{95th} (the 95th percentile of the Wald statistics from the simulated data), if W^a is outside the 95% confidence interval, the model with the selected parameters does not pass the test. In other words, the actual data is very unlikely to be generated by the model and H_0 should be rejected. Therefore, the model can be accepted if:

$$WR = \frac{W^a}{W^{95th}} \le 1 \tag{8}$$

5.2 The Indirect Inference Estimation Procedure

As described above, indirect inference can be employed to test whether the model with a given set of parameters can generate the actual data and the Wald statistic measures the distance between the observed and the simulated data behaviour. However, if this set of structure parameters cannot explain the data then another set of parameters may be able to. If no combination of parameters can be found under which the model passes, then the model should be rejected. Even if the model has already passed the test, it is still necessary to seek an alternative set of parameters that could reduce the value of WR. The main idea of indirect inference estimation is to search for the optimal set of parameters which minimises the value of WR.

6 Indirect Inference Test Results

This section looks at the indirect inference test results of the calibrated model under both CES production function and Cobb-Douglas as a special case. Based on the assumed parameters, our test results accept the CES production function and reject the more restrictive Cobb-Douglas specification.

The key aspects of PIM are to set the base year capital stock and to estimate the asset depreciation rates. Table 1 and Table 2 show that different values of initial capital stock and depreciation rate have been used in the literature and the choices are controversial. Any initial value of capital stock (the capital stock in 1952) with moderate depreciation rates makes the ratio of total capital to GDP around 2 during the period after 1960, which suggests the capital-GDP ratio is about 2 (See Figure 4), similar to those in other countries. The GDP in 1952 was about 274 billion, therefore, the indirect inference test sets the initial capital stock as 600 billion Yuan with the price level in 2000, which is close to Wang and Fan (2000). For the estimations in the next section, the initial capital stock is allowed to range from 500 to 800 billion Yuan. Since government and nongovernment investment from 1952 to 1954 are rather close, it is reasonable to assume the initial capital was almost equally divided into government and nongovernment sectors. Hence, the initial values of the two types of physical capital are both set as 300 billion in the test[10].

[10] The initial value of capital does not have a large effect on the future capital after several years due to a large depreciation rate. Our experiments show that different sets of initial values of nongovernment and government capital only lead to rather small gaps between their corresponding empirical results, especially in the period from 1993 to 2012.

Table 1: Literature of Initial Capital Stock in China (in 2000 prices, billion Yuan)

Literature	Capital Stock in 1952
Zhang (1991), cited in Zhang and Zhang (2003)	808.1
He (1992), cited in Zhang and Zhang (2003)	186.5
Chow (1993)	707.1
Perkins (1998), cited in Wu (2009)	832.3
Chow and Li (2002)	894.1
Wang and Fan (2000)	646.5
Zhang and Zhang (2003)	323.2

Table 2 provides the literature of depreciation rate in China. The selection of depreciation rates of both types of physical capital in Equation (3) and Equation (4) follows Zhang (2008) with 9.6% in the test, and the rates are in an interval from 7% to 12% for estimations.

Table 2: Literature of Depreciation Rate in China (%)

Literature	Depreciation Rate
World Bank (1997), cited in Wu (2009)	4.0
Perkins (1998), cited in Wu (2009)	5.0
Chow and Li (2002)	5.4
Wu (2004)	7.0
Maddison (2007)	17.0
Zhang (2008)	9.6

The study of Luo and Zhang (2009) shows that the labour income share in GDP drops from 52% in 1987 to 40% in 2006; while Bai and Qian (2010) have it that the labour income share in the industry sector increased from 34.52% to 42.21% in the period between 1978 and 2004. The labour income share is much lower in China than in the US and most of other countries, in other words, the capital income share in China is higher than that in most of other countries.

Since there is government capital in the production function, the elasticity of output with respect to total capital should be higher than the capital income share. The average ratio of nongovernment capital to government capital is about 5:4 in the period from 1952 to 2012. Hence, the value of θ in Equation (5) is possibly in the interval from 0.55 to 0.75. The value in testing can be set as 2/3=0.667. Many studies adopt a 3 percent or a little larger discount rate according to the real rate of interest on Treasury bonds or other economic figures, therefore, the rate of time preference in Equation (1) is fixed as $\beta = 0.97$ for tests with the data in China, a future-oriented society, and between the lower bound 0.96 and the upper bound 0.99 in estimation.

Table 3 gives the test results with the CES production function, showing that the model passes the test since $WR < 1$. Moreover, all VAR parameters and variances from the actual data are within the 95% lower and upper bounds, in other words, based on the assumed parameters, the model with the CES production function captures the data behaviour well.

Table 3: Test Results of the Model with Different Production Functions

Auxiliary Parameters	CES	IN/OUT[11]	Cobb-Douglas	IN/OUT
β_{11}	0.8206	IN	0.8452	IN
β_{21}	0.0573	IN	0.0533	IN
β_{31}	0.1703	IN	0.1856	IN
β_{12}	-0.0978	IN	-0.0144	IN
β_{22}	1.0737	IN	1.0554	OUT
β_{32}	-0.1834	IN	-0.2703	OUT
β_{13}	0.0102	IN	0.0195	IN
β_{23}	0.0053	IN	0.0037	IN
β_{33}	0.9446	IN	0.9489	IN
$var(\hat{y})$	0.0268	IN	0.0182	IN
$var(\hat{k})$	0.2771	IN	0.1885	OUT
$var(\hat{c})$	0.0095	IN	0.0065	IN
W^a	10.3766	**IN**	1383.2690	**OUT**
WR	0.3367 < 1		46.3973 > 1	

The calibrated values of the parameters are: $k_{1951} = 3000$, $k_{G1952} = 3000$, $\delta = 0.096$, $\delta_G = 0.096$, $\beta = 0.97, \theta = 0.6667$, $\alpha = 0.55$, $r = 1$, $\sigma = 1.5$, $\gamma = 1$. $A_{1952}, \tau_E, \hat{k}_{GE}, \rho_y, \rho_\tau, \rho_{KG}$ are estimated by the indirect inference method.

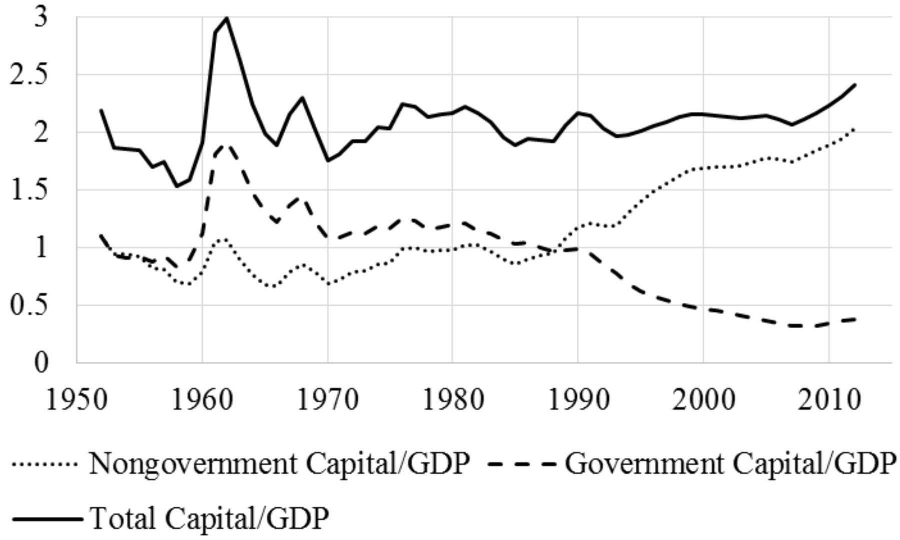
The test of the model with Cobb-Douglas production function adopts the same values of $k_{1951}, k_{G1952}, \delta, \delta_G, \beta, \theta, \alpha, r, \sigma$ as in the test with CES production function above, while the values of $A_{1952}, \tau_E, \hat{k}_{GE}, \rho_y, \rho_\tau, \rho_{KG}$ are re-estimated. Table 3 shows that, with the Cobb-Douglas production function, the Wald statistic from the test is very high. Hence, we cannot accept the Cobb-Douglas production function under the assumed parameters.

According to the testing results, the CES production function is much better than the Cobb-Douglas production function, which is not surprising since the CES production function is a more general case. The results imply the elasticity of substitution between the two types of capital $1/(1-r)$ is extremely high since the test result becomes better as r gets close to 1.

[11] IN means ‘inside the 95% confidence interval’; OUT means ‘outside the 95% confidence interval’.

Figure 4 may justify these apparently odd results. The ratio of total capital to GDP is rather stable, while the ratio of nongovernment capital to GDP doubled in the six decades and the ratio of government capital had decreased to less than half of the initial ratio, suggesting there may be a strong substitution relation between the two types of capital.

Figure 4: Ratios of Nongovernment, Government and Total Capital to GDP



Note: The data is estimated by setting $k_{1951} = 3000$, $k_{G1952} = 3000$, $\delta = 0.096$, $\delta_G = 0.096$.

Table 4 shows that the two types of capital are gross substitutes ($0 < r \leq 1$). The model passes the test when $r = 0.8$ but gets closest to the data behaviour when the two types of capital are perfect substitutes ($r = 1$). WE show the details in the following section.

Table 4: Test Results with Different Values of r

r value	0	0.2	0.4	0.6	0.8	1
W^a	1383.2690	373.5062	114.4094	39.0750	16.4111	10.3766
WR	46.3973	13.0236	3.9217	1.3285	0.5401	0.3367

7 Indirect Inference Estimation Results

Table 5 shows that the set of estimated parameters (i.e. the parameters that get closest to the data behaviour) under the CES production function captures the reality well with a low Wald statistic and a low value of WR. The impulse response functions for this model are listed in **Appendix C**.

Table 5: Estimation Results of the Model with Different Production Functions

Auxiliary Parameters	CES	IN/OUT	Cobb-Douglas	IN/OUT
β_{11}	0.7923	IN	0.5924	OUT
β_{21}	0.0775	IN	0.1278	IN
β_{31}	0.1818	IN	0.3969	IN
β_{12}	-0.0791	IN	0.1180	IN
β_{22}	1.1108	IN	1.0340	OUT
β_{32}	-0.3322	IN	-0.4426	OUT
β_{13}	-0.0060	IN	-0.0685	OUT
β_{23}	0.0112	IN	0.0299	IN
β_{33}	0.9519	IN	1.0409	IN
$var(\hat{y})$	0.0437	IN	0.0122	OUT
$var(\hat{k})$	0.3445	IN	0.1163	IN
$var(\hat{c})$	0.0036	IN	0.0032	IN
W^a	4.8892	**IN**	50.9957	**OUT**
WR	$0.1502 < 1$		$1.7445 > 1$	

The estimated parameter values can be found in Table 6. The estimated elasticity of substitution r is close to 1, implying a strong substitution between government and nongovernment capital. Most estimated parameters under CES technology are close to the calibrated values, while the estimated initial government capital in 1952 is much higher than the calibrated one. However, even if we restrict the initial government capital to the calibrated value, other CES estimated parameters still yield a small value of WR and the CES model passes the test easily.

In contrast, the model with the Cobb-Douglas production function does not pass the test (Table 5): the Wald statistic is very large and the value of WR is larger than 1, which means that no set of estimated parameter under the Cobb-Douglas production function can capture actual date behaviour well. Hence, the Cobb-Douglas production function is not suitable to describe the relationship between government and nongovernment capital. The estimated parameter values can be found in Table 6.

8 Structural Break

As argued in the introduction and Subsection 2.4, we examine the possibility of a structural break in 1992, the time of Deng Xiaoping's important south tour speech.

The estimated parameter values of the two subsamples are compared with those of the whole sample in Table 6. The small values of Wald statistics (3.9566) and $\mathrm{WR} = 0.1232 < 1$ suggest that, based on the data from 1952 to 1992, the model with CES production function can be accepted. All actual VAR coefficients and volatilities are within the 95% lower and upper bounds. Again, r is close to 1, which still implies strong substitution between government capital and nongovernment capital from 1952 to 1992. Similarly, Wald statistics (10.1558) and $\mathrm{WR} = 0.3419 < 1$ are small, so that, based on the data from 1993 to 2012, the model with CES production function can be accepted. All actual VAR coefficients and volatilities in this period are within the 95% lower and upper bounds.

Table 6: The Summary of Calibrated and Estimated Parameters

Parameter	**Calibrated**	**CES Estimation**	**Cobb-Douglas Estimation**	**1952-1992 Estimation**	**1993-2012 Estimation**
k_{1951}	3000	2896	3445	3500	3000
k_{G1952}	3000	3999	3999	3500	3000
A_{1952}	–	4475	4821	2945	–
A_{1993}	–	–	–	–	57130
δ	0.096	0.1192	0.1138	0.1009	0.12
δ_G	0.096	0.0928	0.1138	0.1048	0.07
β	0.97	0.9720	0.9895	0.9700	0.9760
θ	0.6667	0.7469	0.7498	0.6063	0.7400
α	0.55	0.5012	0.7703	0.5500	0.5536
r	–	0.9998	–	0.9988	0.8816
σ	1.5	1.6518	2.2445	1.7406	1.3114
γ	1	1.0702	1.0780	1.0776	1.0744
τ_E	–	0.3475	0.2621	0.4060	0.1743
$\hat{k}_{GE}$	–	0.7048	0.4775	0.9672	0.7051
ρ_y	–	0.7880	0.8266	0.7615	0.9155
ρ_τ	–	0.9674	0.9440	0.9147	0.9950
ρ_{KG}	–	0.9900	0.9900	0.9900	0.9904
W^a	–	4.8892	50.9957	3.9566	10.1558
WR	–	0.1502	1.7445	0.1232	0.3419

The estimated elasticity of substitution r=0.8816, which is less than those in the full sample period and the subsample period between 1952 and 1992. The result is consistent with the transformation of government functions since 1992. The depreciation rate of government capital after 1992 is only 7%, much lower than that for nongovernment capital in the same period (12%), and also lower than that before 1992 (10.48%) and the calibrated value for the full sample. The possible reason is that since 1992 the government investment has been dominated by public infrastructures rather than the direct finance support to SOEs, while public infrastructures usually have longer useful lives and lower depreciation rates than business assets and seldom depreciate intangibly. The depreciation rate in the nongovernment sector in the second subsample period is 2% higher than that in the first subsample, which may be because the nongovernment assets, such as machinery and vehicles, have experienced higher intangible depreciation due to the dramatic technology advances since the begging of 'socialist market economy' in 1992.

The lower value of elasticity of substitution between government capital and nongovernment capital after 1992 suggests that the government has become more complementary to the nongovernment sector. The estimation result with CES production function ties in with the fact that China has deepened the reforms of SOEs and fiscal policies since 1992.

9 Discussion and Policy Implications

As illustrated by both test and estimation results, there is a strong substitution relationship between government and nongovernment capital. The estimation for CES technology is generally close to the calibration, which enforces reinforce our conclusion. Hence, the Cobb-Douglas specification cannot be applicable to China's macroeconomy, especially when the economy was based on the dominance of the state-owned sector before 1992. Our empirical findings identify the potential problems in previous studies, such as Barro (1990) and Turnovsky (1997): they do not address the issue of different degrees of complementarity/substitutability between the two types of capital.

The substitution relationship between government and nongovernment sectors was in line with the reality in China before 1992. Influenced by the Soviet Union, the Chinese government nationalised virtually most private industrial enterprises during the 1950s. A large proportion of government productive expenditure was poured into SOEs. Under a centrally planned economy, the government 'invaded' the private sector, making the difference between government and nongovernment investment ambiguous. In other words, sometimes the government did what the nongovernment sector should do. After 1978, the government initiated intensive reforms and private investment was encouraged. In 1990s, some SOEs were privatised. Nongovernment capital, especially private capital, then 'fought back' into the field once occupied by the nongovernment capital.

In the 1980s and 1990s, the Chinese government intentionally reduced its share of GDP in order to allow rural and urban households and firms to have more resources and better incentives. Furthermore, the SOEs tended to be more market orientated and could not receive public investment freely. Meanwhile, public productive investment flowed to infrastructure construction rather than SOEs. The government became a service supplier. Government and nongovernment capital stocks became more complementary, which is verified by the estimated r being reduced from close to 1 in the period between 1952 and 1992 to 0.8816 after 1992.

The merits and drawbacks of large-scale public investment were thoroughly exhibited in the Maoist era. In that period, consumption, especially in rural areas, was contained by taxes and compulsory delivery quotas, which were imposed in order to finance public investment and feed the urban population at low prices. Absolute egalitarianism totally destroyed market forces. Enthusiasm for work was mainly from discipline and political beliefs rather than economic benefits. Nevertheless, economic performance was greatly improved, as argued by Maddison (2007). Real GDP had expanded more than fivefold from 1949 to 1976; human capital and labour productivity were also enhanced due to better education and doubled average life span; the economic structure was transformed with industry's share of GDP increasing from one tenth to almost one half. China achieved this despite the self-inflicted wounds that hampered economic growth during the Great Leap Forward (1958-1960) and the Cultural Revolution (1966-1976), China's political and economic isolation, and its hostile diplomatic relations with both the United States and the Soviet Unions. Obviously, without the political instability and the economic sanctions imposed by some Western countries, China would have enjoyed a better economic performance from its development effort and large-scale public investment. We cannot jump rashly to the conclusion that the macroeconomy before the 1978 reform was a complete failure. However, it is safe to draw the conclusion that Deng Xiaoping's path of opening up and economic reform was superior to the development strategies before the reform, as evidenced by higher and more stable economic growth after 1978. Committing to continuing Deng Xiaoping's path, China has adopted fiscal policies to deal with the trade-off in the public investment, as simply summarised in three aspects below.

Firstly, the government is decentralising the economy, partially privatising the SOEs and transforming most of them into joint-stock companies. The state retains ownership of some large SOEs but has little direct control over their operations. Moreover, the government does not pour public investment into SOEs freely any longer, which lessens the financial burden of the government, making it possible to reduce the tax rate so that households and firms have more resources and better incentives.

Secondly, the government has changed the structure of public investment. By stopping the free injection of funds into SOEs and cutting the financial support to them, the government has more resources to invest in high-quality and large-quantity infrastructures and improve the investment environment. The government directly or indirectly (through SOEs) builds more expressways, international airports, high-speed railways and telecommunication facilities. The government has transformed itself from being a competitor of the private sector to being a service provider to private investors.

Lastly, the government is using public investment, together with other policies, as a tool for stabilising the economy when faced with slipping business confidence domestically or externally. Beijing's response to the 2008 economic crisis was swift: development of polysilicon supplies and manufacturing technology of clean cars were declared as national priorities with financial support and tax advantages. Money poured into manufacturers and overseas acquisition of cheap assets, resources and technology from state-owned companies and banks, local governments expedited approvals for new plants. The leveraged investment from the government and SOEs offset the reduction of private investment, reduced unemployment, and finally restored confidence and liquidity, achieving rather high annul growth rates: 9.6%, 9.2% and 10.4% from 2008 to 2010. In 2009, 7.6% of growth was contributed by investment.

In sum, the Chinese government began post-war by dominating the economy with public investment, substituting actively for investment in areas that would have been provided for by the private sector in a non-communist economy. However, after 1978 and particularly after the reforms of 1992, it increasingly withdrew from these areas and concentrated on the provision of infrastructure, only reentering them in times of crisis for stabilisation reasons and even in those times indirectly via the active direction of credit from the state-controlled banks.

10 Conclusion

This paper focuses on the role of the government in the provision of investment in China, through a DSGE model of the economy in which the form of the production function reflects this governmental role. Specifically, using the method of indirect inference, we estimate and test for the elasticity of substitution between government and nongovernment capital in both CES and Cobb-Douglas technologies. Both test and estimation results underscore the strong substitution relationship between government and nongovernment capital after the founding of People's Republic of China in 1949, and thus the general CES technology rather than the Cobb-Douglas technology is the suitable structure for modelling the two types of capital in the production function.

Furthermore, the estimation results corroborate the way in which the orientation of public investment changed after the start of the 'Socialist Market Economy' in 1992. From this point onwards government capital became more complementary to nongovernment capital as it focused more on infrastructure and withdrew from industrial production. It only deviated from this focus in times of crisis, especially the recent world financial crisis, for reasons of macro stabilisation; even then it did so indirectly via the state banks.

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Appendix A: Real Data (1952-2012)

100 million Yuan

	GDP	Nongovernment Consumption	Nongovernment Investment	Government Investment	Tax Rates
1952	2743.95	1830.64	316.79	304.33	0.28
1953	3265.02	2096.90	433.92	351.82	0.27
1954	3437.42	2200.91	404.76	503.21	0.30
1955	3647.27	2415.21	325.78	561.99	0.29
1956	4155.76	2614.73	384.82	656.55	0.29
1957	4307.32	2769.11	455.51	673.75	0.31
1958	5222.65	2893.04	577.80	1148.43	0.33
1959	5683.32	2729.89	859.56	1595.84	0.41
1960	5614.64	2858.19	371.18	1844.62	0.47
1961	4199.66	2811.36	192.19	753.07	0.34
1962	4004.53	2922.30	54.88	565.67	0.33
1963	4382.73	3000.00	295.91	646.88	0.32
1964	5134.18	3141.24	459.96	776.97	0.32
1965	5927.81	3286.70	678.62	917.58	0.32
1966	6563.60	3587.84	871.37	1130.74	0.34
1967	6189.46	3773.55	589.37	895.98	0.28
1968	5935.58	3708.58	876.18	612.63	0.25
1969	6938.42	4037.59	633.88	1105.83	0.32
1970	8285.03	4438.40	1239.02	1500.59	0.34
1971	8865.18	4610.89	1397.92	1594.41	0.36
1972	9200.22	4874.68	1237.34	1653.06	0.36
1973	9926.67	5226.19	1481.37	1814.87	0.37
1974	10156.17	5340.37	1607.74	1799.98	0.36
1975	11039.78	5629.83	1997.31	1915.39	0.36
1976	10862.36	5861.62	1778.13	1875.37	0.36
1977	11690.03	6016.06	2033.12	1976.00	0.37
1978	13055.95	6300.50	2170.35	2764.83	0.41
1979	14047.65	6955.39	2212.69	2901.07	0.39
1980	15147.02	7768.08	2663.10	2667.46	0.38
1981	15933.42	8559.93	2956.79	2353.31	0.36
1982	17379.53	9477.31	2425.11	3399.90	0.38
1983	19277.89	10446.49	2812.19	3780.12	0.39
1984	22205.95	11528.03	3381.21	4367.10	0.39
1985	25205.59	13104.28	4916.61	4749.32	0.39
1986	27422.42	14150.25	5888.66	4631.49	0.38
1987	30605.63	15548.48	6518.02	4806.85	0.35
1988	34064.36	17817.26	8212.25	4695.81	0.31
1989	35459.77	18390.23	8715.88	4499.27	0.31
1990	36805.64	18633.48	8779.73	4522.71	0.30
1991	40194.69	19801.81	9937.37	4581.92	0.29
1992	45897.51	22161.78	12154.53	5039.98	0.27
1993	52323.29	24303.42	20128.59	3146.54	0.16
1994	59182.05	26822.45	21295.33	3681.46	0.15
1995	65630.71	30626.90	23446.85	4049.75	0.14
1996	72194.53	34441.53	24405.00	4791.57	0.16
1997	78909.90	36891.99	25798.00	4146.04	0.15
1998	85065.79	39537.69	26950.93	4609.44	0.15
1999	91525.87	42784.65	27914.08	5716.76	0.17
2000	99214.55	45854.60	28668.24	6174.56	0.17
2001	107452.40	48442.78	32285.00	6685.51	0.19
2002	117226.20	51686.87	37634.00	6754.70	0.19
2003	128949.74	54732.56	46312.63	6818.48	0.19
2004	141975.26	57915.35	54122.97	7300.00	0.19
2005	158012.11	62336.56	58264.80	8256.75	0.20
2006	178080.54	67980.12	67335.95	9188.36	0.21
2007	203374.38	73705.05	73950.23	10933.67	0.22
2008	222901.15	79260.70	83867.19	14312.45	0.22
2009	243397.69	88236.91	96675.93	20747.47	0.22
2010	268714.23	94203.35	107513.19	22057.09	0.22
2011	293707.51	104889.89	120040.23	21718.07	0.22
2012	316711.44	116195.95	128529.96	25581.27	0.23

Appendix B: The Derivation of the Equilibrium Conditions

Rewrite the optimisation problem defined by (1)-(5) in the form of Bellman equation:

$$v\left(k_t, k_{Gt}, \tau_t\right) = \max_{c_t, k_{t+1}} \left\{u\left(c_t\right) + \beta E_t\left[v\left(k_{t+1}, k_{Gt+1}, \tau_{t+1}\right)\right]\right\},$$

subject to:

$$c_t + k_{t+1} - \left(1-\delta\right)k_t = \left(1-\tau_t\right)A_t^{1-\theta}\left[\alpha k_t^r + \left(1-\alpha\right)k_{Gt}^r\right]^{\frac{\theta}{r}} \exp\left(v_t^y\right)$$

Derive the Euler equation:

$$\frac{\partial v\left(k_t, k_{Gt}, \tau_t\right)}{\partial k_{t+1}} = -c_t^{-\sigma} + \beta E_t\left[\frac{\partial v\left(k_{t+1}, k_{Gt+1}, \tau_{t+1}\right)}{\partial k_{t+1}}\right] = 0$$

Apply envelope theorem to the first derivative of value function:

$$\frac{\partial v\left(k_t, k_{Gt}, \tau_t\right)}{\partial k_t} = c_t^{-\sigma}\left\{\theta\alpha\left(1-\tau_t\right)A_t^{1-\theta}\left[\alpha k_t^r + \left(1-\alpha\right)k_{Gt}^r\right]^{\frac{\theta}{r}-1} k_t^{r-1}\exp\left(v_t^y\right) + \left(1-\delta\right)\right\}$$

Hence,

$$\frac{\partial v\left(k_{t+1}, k_{Gt+1}, \tau_{t+1}\right)}{\partial k_{t+1}}$$
$$= c_{t+1}^{-\sigma}\left\{\theta\alpha\left(1-\tau_{t+1}\right)A_{t+1}^{1-\theta}\left[\alpha k_{t+1}^r + \left(1-\alpha\right)k_{Gt}^r\right]^{\frac{\theta}{r}-1} k_{t+1}^{r-1}\exp\left(v_{t+1}^y\right) + \left(1-\delta\right)\right\}$$

From the above equation and the Euler equation, obtain:

$$c_t^{-\sigma} = \beta E_t\left\{c_{t+1}^{-\sigma}\left\{\begin{array}{l}\theta\alpha\left(1-\tau_{t+1}\right)A_{t+1}^{1-\theta}\left[\alpha k_{t+1}^r + \left(1-\alpha\right)k_{Gt+1}^r\right]^{\frac{\theta}{r}-1} k_{t+1}^{r-1}\exp\left(v_{t+1}^y\right)\\ +\left(1-\delta\right)\end{array}\right\}\right\}$$

Thus, we have a system with three endogenous variables:

$$y_t = A_t^{1-\theta}\left[\alpha k_t^r + \left(1-\alpha\right)k_{Gt}^r\right]^{\frac{\theta}{r}}\exp\left(v_t^y\right)$$
$$c_t^{-\sigma} = \beta E_t\left\{c_{t+1}^{-\sigma}\left\{\begin{array}{l}\theta\alpha\left(1-\tau_{t+1}\right)A_{t+1}^{1-\theta}\left[\alpha k_{t+1}^r + \left(1-\alpha\right)k_{Gt+1}^r\right]^{\frac{\theta}{r}-1} k_{t+1}^{r-1}\exp\left(v_{t+1}^y\right)\\ +\left(1-\delta\right)\end{array}\right\}\right\}$$
$$c_t = \left(1-\tau_t\right)y_t - k_{t+1} + \left(1-\delta\right)k_t$$

Translate the system into the form with Dynare timing convention:

$$y_t = A_t^{1-\theta}\left[\alpha k_{t-1}^r + (1-\alpha)k_{Gt}^r\right]^{\frac{\theta}{r}} \exp\left(v_t^y\right)$$

$$c_t^{-\sigma} = \beta E_t\left[c_{t+1}^{-\sigma}\left\{\theta\alpha(1-\tau_{t+1})A_{t+1}^{1-\theta}\left[\alpha k_t^r + (1-\alpha)k_{Gt+1}^r\right]^{\frac{\theta}{r}-1} k_t^{r-1}\exp\left(v_{t+1}^y\right) + (1-\delta)\right\}\right]$$

$$c_t = (1-\tau_t)y_t - k_t + (1-\delta)k_{t-1}$$

Set the year 1952 as the initial period, and define the stationarised variable of x_t as $\hat{x}_t$:

$$A_t = A_{1952}\gamma^{(t-1952)};\ \hat{x}_t = \frac{x_t}{A_t}.$$

Hence, we have a stationary system:

$$\hat{y}_t = \left[\alpha \hat{k}_{t-1}^r\gamma^{-r} + (1-\alpha)\hat{k}_{Gt}^r\right]^{\frac{\theta}{r}} \exp\left(v_t^y\right)$$

$$\hat{c}_t^{-\sigma} = \beta E_t\left[\hat{c}_{t+1}^{-\sigma}\gamma^{-\sigma}\left\{\theta\alpha(1-\tau_{t+1})\left[\alpha \hat{k}_t^r\gamma^{-r} + (1-\alpha)\hat{k}_{Gt+1}^r\right]^{\frac{\theta}{r}-1} \hat{k}_t^{r-1}\gamma^{1-r}\exp\left(v_{t+1}^y\right) + (1-\delta)\right\}\right]$$

$$\hat{c}_t = (1-\tau_t)\hat{y}_t - \hat{k}_t + (1-\delta)\hat{k}_{t-1}\gamma^{-1}$$

The exogenous shock processes are defined as:

$$\tau_t = \tau_E \exp\left(v_t^\tau\right)$$

$$\hat{k}_{Gt} = \hat{k}_{GE}\exp\left(v_t^{KG}\right)$$

$$v_t^\tau = \rho_y v_{t-1}^\tau + e_t^\tau$$

$$v_t^{KG} = \rho_{KG} v_{t-1}^{KG} + e_t^{KG}$$

$$v_t^y = \rho_y v_{t-1}^y + e_t^y$$

As a special case, with the Cobb-Douglas production function, the stationary system is:

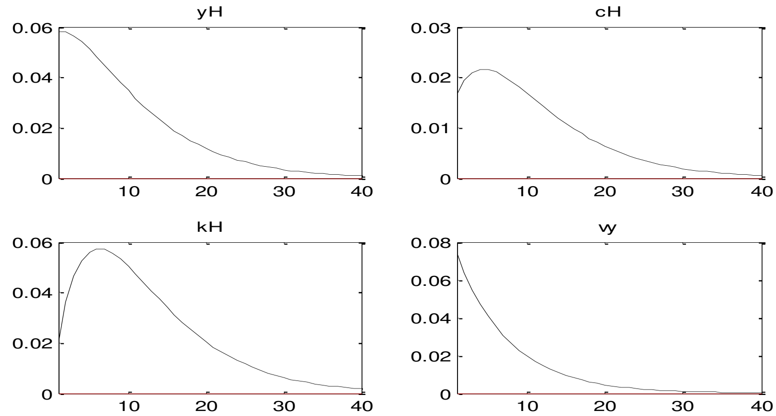
$$\hat{y}_t = \left(\hat{k}_{t-1}^\alpha\gamma^{-\alpha}\hat{k}_{Gt}^{1-\alpha}\right)^\theta \exp\left(v_t^y\right)$$

$$\hat{c}_t^{-\sigma} = \beta E_t\left[\hat{c}_{t+1}^{-\sigma}\gamma^{-\sigma}\left\{\theta\alpha(1-\tau_{t+1})\hat{k}_t^{\theta\alpha-1}\hat{k}_{Gt+1}^{\theta(1-\alpha)}\gamma^{1-\theta\alpha}\exp\left(v_{t+1}^y\right) + (1-\delta)\right\}\right]$$

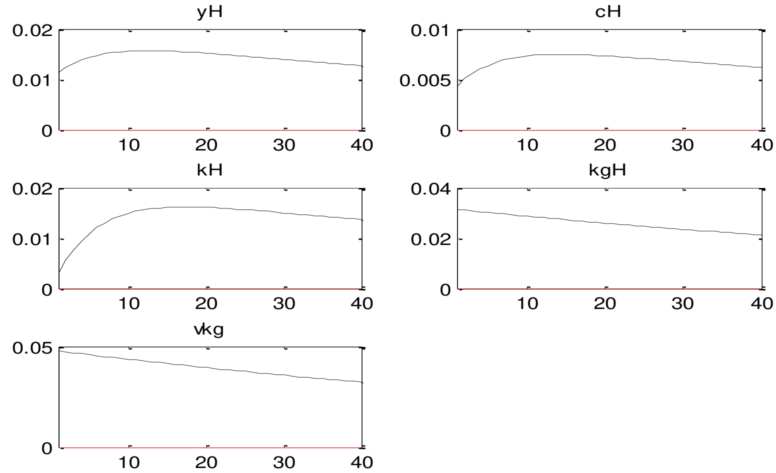
$$\hat{c}_t = (1-\tau_t)\hat{y}_t - \hat{k}_t + (1-\delta)\hat{k}_{t-1}\gamma^{-1}$$

Appendix C: IRFs (Estimation, 1952-2012)

e^{y}



e^{KG}



e^{τ}