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A Monte Carlo procedure for checking identification in DSGE models[*]

Vo Phuong Mai Le (Cardiff University)[†]   David Meenagh (Cardiff University)[‡]

Patrick Minford (Cardiff University and CEPR)[§]

Michael Wickens (Cardiff University and CEPR)[¶]

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Abstract

We propose a numerical method, based on indirect inference, for checking the identification of a DSGE model. Monte Carlo samples are generated from the model's true structural parameters and a VAR approximation to the reduced form estimated for each sample. We then search for a different set of structural parameters that could potentially also generate these VAR parameters. If we can find such a set, the model is not identified. The test is both an alternative to using the rank condition and also can establish whether there is empirically weak identification.

Key words: identification, indirect inference, Monte Carlo, VAR, reduced form

1   Introduction

It has been usual for modellers to argue that DSGE models are over-identified because the rational expectations cross-equation restrictions supplement the normal identifying mechanisms of exclusion and sign. However in a recent paper Canova and Sala (2009) have questioned this view. They give examples of models in which the reduced form properties of DSGE models of different sorts are hard to distinguish and argue that a weak form of observational equivalence between DSGE models is widespread. They recommend careful exploration of these issues prior to estimation and testing of a particular DSGE model. In this paper we propose a numerical Monte Carlo method for checking the identification of a DSGE model which is simple to implement. We illustrate its application with two widely-used DSGE models, the Smets and Wouters (2007) model and the Gali et al. (2005) three-equation model.

To understand our approach consider the following straightforward definition of identification: a (structural DSGE) model is identified if and only if it has a reduced form representation which is not shared with any other such model. This simple definition makes it clear that for identification to fail there needs to be at least one other structural DSGE model that has the same reduced form; for it to exist there needs to be no other such model that has the same reduced form.

The indirect inference test we have discussed elsewhere (Le et al, 2016) compares the reduced form implied by a structural model possessing certain numerical parameters with the reduced form estimated on the sample data. If we generate the sample data by Monte Carlo resampling from the structural model, then plainly the reduced form estimated on this data will, because of sampling error, vary around the reduced form implied by the model. The indirect inference test of the model, at say 95% confidence, will have the generated data samples reject the model 5% of the time. The basis of our test of identification for this structural model is to check whether there is another model that can achieve the same 5% rate of rejection. If the model has the same reduced form, it will do so; if not, not. By increasing the size of

the sample and the number of Monte Carlo replicated samples we can make this check as precise as we like.

The standard test of identification is the rank condition. Iskrev (2010) and Komunjer and Ng (2011) follow this approach, finding the rank of matrices of reduced form coefficients of various types using mostly numerical methods. Another important theoretical contribution is by Qu and Tkachenko (2012) who establish their rank conditions using the spectral density matrix. Schorfheide (2011) notes that the extreme nonlinearity of DSGE model solutions in their structural parameters makes checking identification difficult except by numerical methods, as used by these authors. These papers consider only local identifiability of linearised DSGE models. These papers therefore consider identification in a 'narrow' sense that the answer is 'yes' or 'no'.

However the question raised by Canova and Sala (2009) could apply even when models obey narrow identification in this way: their point is that the likelihood surface for these models (and in particular the objective function surface based on impulse responses) is rather flat and that consequently different models, while having strictly different reduced forms, may in practice be hard to distinguish empirically even with very large samples because their reduced forms yield very similar data; a likelihood ratio test would not imply that one was any different from the other. This problem gets worse of course as the sample size falls — a problem they also stress as highly relevant given the small size of macro time series samples. They suggest that the problem may come about because the dynamics of these DSGE models can be generated in various different ways, including by error dynamics; these different dynamic forms may be difficult to distinguish in terms of their likelihood and specifically their impulse response functions. This 'observational equivalence' creates a problem of weak identification, which we distinguish from the issue of 'narrow identification' dealt with by checking the rank condition.

An echo of the findings of Canova and Sala (2009) can be found in Le et al. (2016) who examined the small sample power of tests based on likelihood. They noted that in likelihood tests model errors were routinely reestimated when different structural parameters were compared; this reestimation of error process parameters made it hard to distinguish the different parameter sets in terms of their predictive closeness to the data and so their likelihood. What seems to be going on is that the autoregressive parameters substitute closely for the structural parameters in terms of achieving predictive fit. This problem seems unlikely to be resolved by raising the sample size, which is essentially what Canova and Sala (2009) find.

Le et al. (2016) also found that the indirect inference test, based on VAR coefficients as the auxiliary model, was not so vulnerable to this problem because being based on the correspondence between the (VAR) reduced form found in the data and the model's implied reduced form, a model that is narrowly identified will generate a different reduced form even in some quite apparently minor respects; such differences are picked up in the test.

Canova and Sala (2009) were particularly concerned with measures of model closeness to key impulse response functions (IRFs) found in the data. Here again they found that the closeness objective function surface was fairly flat near the peak. Minford, Wickens and Xu (2016) compared indirect inference tests based on IRFs with those based on VAR coefficients and found that they yielded highly similar power. But these tests are based on the full joint distribution of the IRFs (as of the VAR coefficients) which distinguishes them from the widespread 'informal' use of IRF closeness examined by Canova and Sala (2009).

Our identification criterion based on the Indirect Inference test thus acts as a 'narrow' identification test in the manner of the rank condition but it also confronts the empirical issue of observational equivalence or 'weak' identification: this is because we ask whether the candidate model for having the same reduced form as the model in question can be empirically distinguished via the test under ideal conditions of many samples of large size. We can also ask whether this ability to distinguish carries over to smaller samples, as in the concern of Canova and Sala (2009); though we have no scope here to pursue this issue in detail, we may note the findings of Le et al. (2016) on the high power of the Indirect Inference test in small samples which suggest that reducing sample size should not badly reduce model discrimination. Thus the key contribution of our proposed test is that it addresses both the narrow and weak identification issues. If either fail, our test will show this up, distinguishing between each type of failure.

We now note some particular features of our test that further contribute to establishing identification, over and above the existing literature on the rank condition. Being empirically based, it includes the error properties in the identifying information used; these may contribute power to distinguish model differences in indirect inference. Both the errors' ARMA parameters and all the error moments are included in the simulation process. While the error moments are not relevant for narrow identification,

they are highly relevant for weak identification as in Canova and Sala (2009). Another feature is that it can be applied to nonlinear DSGE models, such as those that occur with the Zero Bound when the model shifts regime: here our definition of identification asks whether the linear reduced form representation of the data behaviour produced by a nonlinear DSGE model is unique to that model. The test can also be extended to check for global identification, as it can look around a particular point in the parameter space as well as check for large numbers of points drawn randomly from everywhere in the possible space. Thus, if we wanted to, we could perform a global search for identification. The test's computational speed and convenience is also good for someone already using Indirect Inference since the identification test can be routinely carried out using the same methods required for the estimation and testing of the DSGE model. These methods are routinely available in programmes on the worldwide web and increasingly used by applied economists.

To illustrate our procedure we consider the two models already mentioned. The first is a standard three-equation New Keynesian model of the US, with a Phillips Curve, an IS curve (taken from the Euler equation and a market-clearing equation) and a Taylor Rule. We consider this model in a variety of forms: with and without autocorrelated disturbances and with and without persistence in the Phillips Curve (through indexation) and the Taylor Rule (through interest rate smoothing). The versions with autocorrelated disturbances, with or without persistence, we can show analytically are over-identified; the version with neither autocorrelated errors nor persistence we can show is under-identified. Our procedure can therefore be checked against these two analytical results.

The second model we consider is the Smets-Wouters model of the US (Smets and Wouters, 2007), based on Christiano et al. (2005). This model is too large for us to find its reduced form parameters analytically as above with the 3-equation New Keynesian model. The usual presumption that this model is over-identified is consistent with Iskrev's findings of narrow identification when he applies the rank condition to this model[1]. Here we use our numerical method to add information about this case.

2 Use of Indirect Inference to check identification

As just noted, the Indirect Inference testing method checks whether a certain assumed model could be the one generating the data description (the 'reduced form') we find. If we now allow for an infinite amount of data, we can exploit the asymptotic features of the procedure in large samples of which we have a large number in parallel; we may think of this as having access to a very large panel sample. These features imply that any reduced form will allow asymptotically exact estimates of the true structural parameters and that the true structural model if tested against this reduced form, will be rejected at exactly the correct size of the test — e.g. 5% for a 95% confidence test. It is this idea that we exploit to check for identification.

The idea behind our numerical procedure starts from the proposition that as data samples get larger and more numerous, the VARs estimated on them have mean VAR coefficients that tend to the true values — thus the true values are the probability limit of the actual values. This also implies that in the limit the distribution of the VAR coefficients generated by the DSGE model will converge on their true VAR distribution.

Thus with many samples of large size (we choose N=500 throughout) we can test by indirect inference whether a DSGE model is generating these VARs. At 5% significance the VAR distribution generated by the true DSGE model will be rejected 5% of the time; if there is another DSGE model that can produce the same VAR coefficients then it too will be rejected only 5% of the time (the choice of 5% is arbitrary; any significance level could be chosen). However, if it cannot produce these VAR coefficients, it will be rejected more, possibly much more, of the time.

What is found with Indirect Inference is that the power of the Wald test at any significance level can be raised as high as one wishes by a) increasing the order of the VAR description and b) increasing the amount of data used by raising the sample size while c) increasing the number of samples increases its accuracy. Thus the test of identification can be made as precise as one wishes.

Thus we follow the following numerical procedure:

a) we generate by Monte Carlo sampling a large number of samples of large size from the true DSGE model being checked.

b) we compute the VAR coefficient distribution implied by these samples for a high order VAR on the maximum number of variables.

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[1] Here we refer to Iskrev's findings under full information when he examines the identification of the estimateable structural parameters. As noted below, some deep parameters cannot be recovered because combined with other deep parameters to form the estimateable parameters.

c) we create an alternative DSGE models by varying the structural parameters in the region of the true model parameters in various systematic ways, searching for one that could generate the same VAR behaviour as the true model.

d) we use the information provided by the true model samples (the 'data') to estimate for this alternative DSGE model the implied error time-series processes and also their innovation moments. We do this by extracting the implied residuals on each true data sample, and reestimating the AR coefficient on each until this converges; we then average across all the different sample AR coefficients to find the implied best AR coefficient for the false model.

e) we carry out a Wald test on this alternative DSGE model with the full set of true samples, to find the rejection rate of the model with these samples at some nominal test frequency (we use 5%).

f) if we can find no alternative model that is rejected at only the same nominal frequency we regard the DSGE model as identified. In the spirit of Canova and Sala (2009) we mean by this that alternative models show a well defined rise in rejection as we move further away from the true model in the direction defined by the alternative.

Thus in this procedure we are combining testing with an estimation search for a DSGE model that can compete closely with the true model on the basis of its rejection rate from the Wald test. This search should find other models if the model is not identified, since clearly changing the unidentified parameters away from their true values will not change the model's reduced form and so the VAR distribution it implies should be the same as that of the true model.

We now carry out this procedure first on the small New Keynesian model. We look in turn at:

a) the 3-equation model with autocorrelated disturbances which we can show analytically to be over-identified

b) the same model but where the errors are all *i.i.d.* and persistence parameters are set to zero. In this case, we know the model is under-identified because ω (the parameter that determines how backward looking the Phillips Curve is) cannot be retrieved.

These two cases are a check on how well the procedure can deal with the variety of identification possibilities including those where we know there is lack of identification. The fact that we can check this model's identification analytically allows us to use it as a check on our numerical method.

3 The full New Keynesian 3-equation model

3.1 The New Keynesian model with persistence

For our Monte Carlo experiment we use the following model, which is similar to that used originally by Clarida, Gali and Gertler (1999) as their prototype New Keynesian model.

$$\pi_t = \omega E_t \pi_{t+1} + (1 - \omega)\pi_{t-1} + \lambda y_t + \varepsilon_t^\pi \tag{1}$$

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}\left(r_t - E_t \pi_{t+1}\right) + \varepsilon_t^y \tag{2}$$

$$r_t = \rho r_{t-1} + (1 - \rho)\left(\gamma \pi_t + \eta y_t + \psi\left(y_t - y_{t-1}\right)\right) + \varepsilon_t^r \tag{3}$$

where the shocks follow the univariate AR(1) processes

$$\begin{aligned}
\varepsilon_t^\pi &= \rho_\pi \varepsilon_{t-1}^\pi + u_t^\pi \\
\varepsilon_t^y &= \rho_y \varepsilon_{t-1}^y + u_t^y \\
\varepsilon_t^r &= \rho_r \varepsilon_{t-1}^r + u_t^r
\end{aligned}$$

The first equation is the New Keynesian Phillips curve. If $\omega = 0$ it is a backward-looking Phillips Curve and, if $\omega = 1$, it is a forward-looking Phillips Curve. The second equation is the aggregate demand equation and the last equation is an interest rate rule where the interest rate is 'smoothed' by the parameter ρ.

We choose the parameter values shown in Table 1. These values are assumed to be those for the True model.

Using the parameter values set out in Table 1 (denoted by θ), we generate 10,000 Monte Carlo samples of 500 observations each — the 'true data samples' from this DSGE model. We estimate a VAR(3) for the three variables, output, inflation and interest rates, on all 10,000 samples. We use the resulting 10,000 coefficient vectors, α_T.

Next we examine the number of times a DSGE model with parameter vector θ_i is rejected by the true samples at the 5% level. Plainly if $\theta_i = \theta$ the rejection rate is 5% by construction. For each other θ_i

Parameter	Value	Parameter	Value
ω	0.7640	η	0.8830
σ	3.4550	ψ	0.0727
λ	0.0997	ρ_y	0.8654
ρ	0.4029	ρ_π	0.7999
γ	1.1624	ρ_r	0.7829

Table 1: Parameters of True model used in Monte Carlo simulations

we take 10,000 samples by Monte Carlo simulation and construct the Wald statistic distribution implied by that DSGE model as:

$$WS(\theta_i) = (a_S(\theta_i) - \overline{a_S(\theta_i)})'W(\theta_i)(a_S(\theta_i) - \overline{a_S(\theta_i)})$$

where $\overline{a_S(\theta_i)}$ is the mean of the 10,000 vectors $a_S(\theta_i)$ and $W(\theta_i) = \Omega(\theta_i)^{-1}$ is the inverse of the variance-covariance matrix.

Now we calculate how often this DSGE model with vector θ_i is rejected on the 10,000 true samples, whose VAR coefficient vectors are given by α_T. In effect this is done by counting the percent of values of

$$WS_T = (a_T - \overline{a_S(\theta_i)})'W(\theta_i)(a_T - \overline{a_S(\theta_i)})$$

that are greater than the 5% critical value of $WS(\theta_i)$.

We are looking for a vector θ_i that could generate the same reduced form coefficient vector, α_T, as the true vector θ; if so we could say that this θ_i could be 'confused with' the true vector θ. Numerically, we require that its rejection rate be at or arbitrarily close to 5%; in this case this model cannot be rejected as the true model any more than the true model itself. Since it is not the true model, this can only occur if it behaves just like the true model — i.e. the true model is not identified.

Our method is straightforward. First, we look for any arbitrary set of parameters different from those of the true model and assess the effect on the rejection rate, remembering that we are seeking to find a 5% rate as evidence of a model with the same reduced form. In seeking this variation we change all the parameters by a very small amount. Furthermore we do this variation for each parameter alone, to see whether any one parameter fails to be identified (i.e. variations in this alone in the model can generate the same reduced form). What we see is that any such variation generates a higher rejection rate than 5%; though for four parameters $(\sigma, \eta, \psi, \rho_\pi)$ this does not rise markedly with increasing falsity, the lowest rejection rate at 5.4% is safely above the 5% level[2]. What this reveals is that there is weak identification of these parameters, with flat ridges fanning out from just below the peak.

The results for falsifying all parameters together are shown in Table 2, and the results when falsifying individual parameters are in Table 3. When falsifying all parameters together we have shown the results for 3 different methods. Firstly we increased all parameters by $x\%$; secondly we alternately increased or decreased the parameters by $x\%$; thirdly we increased or decreased the parameters by $x\%$ in a random way (the table shows the range of rejection rates from several sets of randomly false parameters). For the individual parameters we increased each parameter by $x\%$.

	Rejection Rate			
Degree of falseness	0.1%	0.2%	0.5%	1.0%
Falsify by + only	6.00	6.40	15.49	80.05
Falsify by alternately +/-	5.81	7.28	23.19	84.47
Falsify by randomly +/- (min,max)	$(5.68, 6.12)$	$(6.12, 8.34)$	$(13.48, 32.91)$	$(38.67, 99.4)$

Table 2: Rejection Rates for NK 3 equation model, all parameters falsified

We can also look at varying combinations of parameters if we suspect these could jointly produce the same reduced form. We do this by an estimation algorithm that searches for such combinations, θ_i, that can get closest to replicating the reduced form found in the 10,000 data samples.

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[2]To judge the necessary distance above 5% required to establish identification, we may use the value found when we test the true model by Monte Carlo simulation: this is slightly inaccurate, at 5.16% in this 3-equation model. Hence 5.4% cannot be accounted for by this inaccuracy.

Degree of falseness	Rejection Rate					
	0.1%	0.2%	0.5%	1.0%	3.0%	5.0%
ω	5.41	5.48	5.85	6.34	15.06	38.60
σ	5.71	5.76	5.66	5.47	5.71	5.68
λ	5.44	5.80	5.80	5.62	6.27	7.77
ρ	5.83	5.28	5.76	5.72	6.14	6.96
γ	5.29	5.52	5.61	5.37	9.75	19.10
η	5.45	5.39	5.52	5.42	5.26	6.14
ψ	5.43	5.72	5.55	5.67	5.41	5.44
ρ_π	5.29	5.24	5.41	5.61	5.82	5.65
ρ_r	5.70	5.38	5.46	5.67	11.02	27.26
ρ_y	5.61	6.54	20.91	94.20	100.00	100.0

Table 3: Rejection Rates for NK 3 equation model, individual parameters falsified

There is an important detail to be clarified about θ_i. This vector includes the error moments. Now if we choose the other parameters in θ_i, which are of course false, then the errors implied by this model are given by the true data samples interacting with these parameters. Thus we can extract the moments of these errors from the true data and the false other parameters; of course if we did not, then the θ_i would be rejected by the true sample data directly, as the error moments would be incompatible with both the data and the other structural parameters. Thus the error moments cannot be chosen freely given the true data and these other structural parameters.

To deal with this important detail we tackle the search for another θ_i with rejection rate at 5% as follows: as the structural parameters are falsified we use the true data samples to generate the implied errors, recompute the ρ on these (iterate until the expectations generate a converged ρ) and extract the innovation moments. It is this falsified model, whose errors are extracted from the true data samples, that is examined for rejection by the true data.

Our results for the 3-equation model were as follows.

We searched within a 2% region around the true θ, assuming that the error moments remained the true ones. We found the set of parameters in Table 4 that gave the next best rejection rate. We found that the next best rejection rate was 11.88%, which is far from the 5% of the true model, and therefore, according to our test, the model is identified.

Parameter	True	Varied
ω	0.7640	0.7677
σ	3.4550	3.4297
λ	0.0997	0.1002
ρ	0.4029	0.3979
γ	1.1624	1.1621
η	0.8830	0.8688
ψ	0.0727	0.0725
ρ_y	0.8654	0.9102
ρ_π	0.7999	0.7675
ρ_r	0.7829	0.8269
Rejection Rate	5.00	11.88

Table 4: Parameter Combinations

If we summarise these results, we could say that there is clearly narrow identification; but that there is also weak identification, with four parameters showing ridges along which the rejection rate barely moves. Two of these parameters are in the Phillips Curve: the inflation response to output and the autoregressive coefficient in the error process. One is the intertemporal consumption response, and one is the output response in the Taylor Rule. With realistic sample sizes this would badly undermine discrimination between model versions.

3.2 The New Keynesian model without persistence

We now turn to the version of the New Keynesian model where all persistence is removed from both the model and its error processes, as follows:

$$\pi_t = \omega E_t \pi_{t+1} + \lambda y_t + \varepsilon_t^\pi \tag{4}$$

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}(r_t - E_t \pi_{t+1}) + \varepsilon_t^y \tag{5}$$

$$r_t = \gamma \pi_t + \eta y_t + \varepsilon_t^r \tag{6}$$

where the shocks are all white noise.

This leads to lack of identification of the parameter ω: this is obvious from the lack of persistence since $E_t \pi_{t+1} = 0$. We should find that our test discovers this under-identification. We vary each parameter by a small amount and see whether this variation can be rejected.

In Table 5 we show for a small variation in each parameter from the true value how the rejection rate alters.

	Parameter		Rejection rate (at 5%)
	True	Varied	MC replications = 3000 Sample = 500
λ	0.0997	0.0969	9.2000
σ	3.4553	3.3670	8.5667
γ	1.1624	1.1738	9.8333
η	0.8830	0.8728	10.033
ω	0.7640	0.6195	5.0000

Table 5: Checking on the non-identification of individual parameters in the NK Model without persistence

What this reveals is that indeed each parameter is identified except ω, as is obvious analytically since effectively it does not enter the model. Column 3 shows that the rejection rate roughly doubles the nominal rate with the false parameter with a sample size of 500, and 3000 Monte Carlo replications.

Thus we have established here that not only does our method work with extreme precision to find that identified models are indeed so; but it also works exactly to find unidentified parameters. We have found that large perturbations of these parameters, either singly or in combination, do not shift the rejection rate from 5%. Thus plainly here we have found combinations of parameter values for the model that yield identical reduced forms.

3.3 Conclusions from the New Keynesian 3-equation models

What we have shown in these two exercises is that our numerical method accurately captures the identification of model parameters where we already know their status. If we were not sure about their status, we would perform the identification procedure in two stages. First, we determine the identification status of each coefficient individually, searching over different values while keeping other coefficients unchanged. Second, we go on to investigate combinations of all coefficients other than those that are not identified individually.

In the full model version, where we know there is over-identification of all the parameters, the method indeed finds that no other local parameter values can be accepted; but identification is weak. In the model version with no persistence again the method rightly finds that the ω parameter is not identified, as values for them other than the true values are not rejected at more than the 5% level chosen but that alternative values of all the other parameters are clearly rejected at higher rates and so are identified, both narrowly and strongly.

These results suggest that the method is reliable. It also clearly discriminate between narrow and strong identification. We now go on to apply it to a case where we are unable to establish identification analytically.

4 Identification of the Smets-Wouters model

We now discuss an application of these procedures to the Smets-Wouters model, which we expect to be over-identified by the cross-equation restrictions imposed on the model by the RE assumption; hence it

should satisfy the rank condition and so narrow identification. As noted above, Iskrev (2010) indeed finds that the rank condition is satisfied. In this section we use our suggested method to check its identification both in the narrow and the empirical senses. This can be considered the first application where we try to bring fresh information about the identification of a major model in current use.

The Smets-Wouters model (2007) marks a major development in macroeconometric modelling based on DSGE models. Its main aim is to construct and estimate a DSGE model for the United States in which prices and wages, and hence real wages, are sticky due to nominal and real frictions arising from Calvo pricing in both the goods and labour markets, and to examine the consequent effects of monetary policy which is set through a Taylor rule. It is, therefore, a New Keynesian model. Smets and Wouters combine both calibration and Bayesian estimation methods and use data for the period 1966Q1–2004Q4. Wright (2015) notes that the implied reduced form VAR for the Smets-Wouters model is a VAR of order 4 with seven observable endogenous variables. In what follows we use this VAR.

Again, first we look for arbitrary changes in all the parameters to see whether in total or individually there is any sign that the reduced form is the same. Notice that we follow the normal practice of defining as 'parameters' the operative parameters of the model, which in some cases combine several 'deeper' parameters that cannot each be retrieved (including for example the elasticities ε_p and ε_w); clearly these by construction cannot be identified but we, as also previously Iskrev (2010), are not here concerned with them. Table 6 shows that when all the parameters are varied away from the true model values, whether alternately positively and negatively, or randomly, there is a rising rejection rate: hence the average criterion distance of the data samples from the models rises the further they are from the true model. Thus there is both a clear gap between the other models and the true model and also this gap gets larger the further away the model, indicating a clear peak. There is no flatness in the criterion function around the true model.

In Table 7, we do the same for each parameter individually, simply raising each by $x\%$. Notice that as we raise each parameter we also adjust the ARMA coefficients (simple ARs in practice) of the implied residuals so that the structural model gets as close as possible to the data given the wrong parameters. What we said above for the generally falsified models applies also to the individually falsified parameters except of course that the criterion peak is less pronounced as each parameter on its own naturally worsens the criterion less than all together do.

		Rejection Rate		
Degree of falseness	1.0%	1.5%	2.0%	3.0%
Falsify by alternately +/-	13.82	13.89	15.43	20.11
Falsify by randomly +/- (min,max)	(17.92, 25.95)	(15.63, 33.93)	(15.73, 33.68)	(19.89, 61.36)

Table 6: Rejection Rates for SW model, all parameters falsified

	Rejection Rate								
Degree of falseness	1.0%	1.5%	2%	3%	5%	10%	20%	50%	75%
Steady-state elasticity of capital adjustment φ	7.28	7.97	7.25	7.31	7.47	7.86	8.35	18.41	36.20
Elasticity of consumption σ_c	7.12	8.08	6.92	7.65	8.12	8.14	9.71	18.81	36.37
External habit formation λ	7.63	7.32	7.65	7.22	8.54	8.15	9.63	18.25	36.16
Probability of not changing wages ξ_w	7.72	7.32	7.65	7.34	8.10	8.45	9.22	19.32	35.43
Elasticity of labour supply σ_L	7.83	7.25	7.71	8.07	7.86	8.61	9.27	18.83	34.05
Probability of not changing prices ξ_p	7.77	7.25	7.11	7.35	7.15	7.62	9.43	17.48	37.02
Wage indexation ι_w	7.61	7.95	7.70	7.71	7.80	8.54	9.41	18.80	34.55
Price indexation ι_p	7.27	7.43	7.50	7.55	7.55	7.82	8.97	18.11	36.72
Elasticity of capital utilization ψ	7.47	7.35	7.64	7.55	7.38	7.97	9.85	18.23	36.55
Share of fixed costs in production (+1) Φ	7.20	7.72	7.30	7.35	8.11	8.38	9.51	18.76	34.90
Taylor Rule response to inflation r_p	7.57	7.48	7.32	6.77	7.45	7.83	9.65	17.32	35.37
Interest rate smoothing ρ	8.23	7.82	7.47	7.89	7.50	7.97	9.52	18.37	37.08
Taylor Rule response to output r_y	7.76	7.43	7.13	8.07	7.83	8.35	9.72	18.89	33.74
Taylor Rule response to change in output $r_{\Delta y}$	7.93	7.50	7.09	7.94	7.41	8.93	9.72	19.37	35.71
Quarterly steady-state inflation $\bar{\pi}$	7.21	7.73	7.31	7.40	7.80	8.38	8.72	18.27	36.45
Discount factor β	7.25	7.15	7.49	7.87	7.73	7.74	10.17	17.81	34.00
Steady-state hours worked $\bar{L}$	7.14	7.97	6.92	7.57	7.57	7.86	8.84	18.41	35.59
Quarterly steady-state output growth $\bar{\gamma}$	7.30	6.85	7.79	7.78	7.85	7.89	9.54	16.58	35.89
Share of capital in production α	7.74	7.59	7.22	7.30	7.58	8.00	8.97	18.66	35.57

Table 7: Rejection Rates for SW model, individual parameters falsified

These results suggest that when we search among all the parameters individually or in combination

without any systematic search we cannot find any models that obtain the necessary 5% rejection rate of the true model[3]. In this model every parameter individually shows strong identification, with the rejection rate rising quite strongly as falsity increases.

Then again we go on to investigate actively whether there is any combination of parameters that could generate the same reduced form, using our global search procedure within a moderate distance of the true parameters. Here we look for a set of structural parameters that, after adjusting for the AR coefficient that best fits the data on average, can minimise the rejection rate on the Wald test.

The best nearby set of parameters found by our search algorithm is shown in the Table 8.

Parameter	True	Varied	Parameter	True	Varied
φ	5.7400	5.7060	r_p	2.0400	2.0259
σ_c	1.3800	1.3714	ρ	0.8100	0.8072
λ	0.7100	0.7086	r_y	0.0800	0.0802
ξ_w	0.7000	0.7052	$r_{\Delta y}$	0.2200	0.2262
σ_L	1.8300	1.7861	$\bar{\pi}$	0.7800	0.7643
ξ_p	0.6600	0.6634	β	0.1600	0.1620
ι_w	0.5800	0.5726	$\bar{L}$	0.5300	0.5296
ι_p	0.2400	0.2377	$\bar{\gamma}$	0.4300	0.4298
ψ	0.5400	0.5471	α	0.1900	0.1906
Φ	1.5000	1.4843	Rejection Rate	5.00	9.96

Table 8: Best Rejection Rate from search algorithm

It can be seen that the closest set of parameters we can find is rejected by 10% of the true samples and so clearly is different in reduced form from the true model.

So we find that the Smets-Wouters model is identified according to our test. This confirms the finding of Iskrev (2010) according to the rank condition. It also reveals that there is a clear observational difference between the true model reduced form and any models lying close to this; there is strong identification in the sense of Canova and Sala.

5 Conclusion

Whether a DSGE model is identified is a matter of ongoing concern, and recently it has been suggested by Canova and Sala (2009) that identification may be a problem in the sense that in the region of the true model parameters a variety of parameter sets can produce similar empirical closeness to the data. As noted by Schorfheide (2011), theoretical checks for identification are difficult to apply in practice; nevertheless both Iskrev (2010) and Komunjer and Ng (2011) have suggested practical methods using the rank condition. Here we suggest a numerical method based on estimation and testing by indirect inference for checking identification not merely in the narrow sense of satisfying the rank condition but also in the empirical sense defined by Canova and Sala. The idea is to set up a Monte Carlo experiment in which a true DSGE model generates data samples; each of these has a VAR estimated on it and indirect estimation and testing is used to search for a set of false DSGE parameters that could fail to be rejected as generating these estimates. If the DSGE model is identified then no such false parameters can be found and as the parameters move away from the true model set there should be a clear deterioration in closeness. If so, then we will have established that the reduced form (VAR) parameters implied by the DSGE model uniquely imply the True model parameters generating them: the condition for identification that the two parameter sets map exclusively into each other.

We have used the three equation model focused on by Canova and Sala as one example of this method in application. This is a good example since there are reasons to be concerned about identification in this model: the forward and lagged inflation terms and the autoregressive error term in the Phillips Curve can all substitute for each other in generating similar inflation behaviour. The cross-equation restrictions created by rational expectations ensure over-identification, as we show analytically for this model; and empirically as very large amounts of data are used the variant parameter sets can be clearly distinguished; however for a number of individual parameters we do find clear weakness of identification. Our numerical example thus confirms narrow but not strong identification.

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[3] With this model the Monte Carlo experiment yields a highly accurate rejection rate of 5.03% for the true model.

We also looked at the case where all dynamics in the model are suppressed, in which case the forward-looking parameter, ω, in the Phillips curve fails to be identified, whereas all the others are. Again our numerical procedure confirmed this, with the rejection rates rising sharply for small deviations from the true parameter values for all parameters other than ω.

We also looked at another example where identification seems assured by over-identification and where indeed narrow identification has been found according to the rank condition: the Smets-Wouters model of the US economy. Here our method again showed that as parameters leave their true values rejection rates rise rapidly. We could not find another parameter set that came close to mimicing the true model. On our test the true model also showed a clear peak in the criterion function and no flat ridges beyond, so that there was also strong identification.

We found that we could reconcile our findings with those of Canova and Sala, by appealing to three considerations: first, 'disappearing' parameters are possible but may be rare in DSGE models because of lag coefficients both in the model and in the error processes; second, informal matching of impulse response functions, as they note, use quite limited information, as compared with their joint distribution via our Wald test (which can be applied to IRFs as to VAR coefficients with similar power); third, the likelihood surface they review is less well-determined than the criterion of closeness to the VAR coefficients used in the Wald test. Effectively the Wald test brings to bear enough extra information to pin down parameters rather closely in empirical application. Nevertheless we do find weak identification in a popular small New Keynesian model as they do.

Plainly there are potential limitations to our results. Our numerical results could be vulnerable to holes in our search algorithms, an endemic issue with such methods; however so far we have not found evidence that it is affecting our results. Also, we cannot deal in any general way with the possibility that a given DSGE model could be confused with an entirely different model; nevertheless any suggested alternative can be tested ad hoc and, as reported in Meenagh et al. (2016), we have found in other Monte Carlo experiments that entirely different models are rejected at almost 100% rates even in small samples.

We therefore suggest that this numerical procedure could usefully be applied in empirical work to DSGE models when identification is in doubt either in a narrow sense or in the empirical sense noted by Canova and Sala. When we applied it here to two DSGE models widely used in applied macroeconomics, we found that both were narrowly but one only weakly identified.

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