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DEBONDING OF CELLULAR STRUCTURES UNDER SHEAR DEFORMATION

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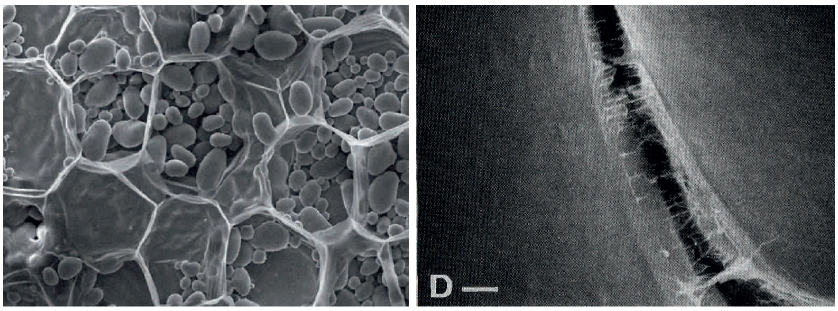
ABSTRACT

Many natural materials at millimetre scale are cellular structures, while at micrometre scale, the cell walls are fibrous elastic composites (*e.g.* plant stems, vegetables, fruit). Cell separation through debonding of the middle lamella in cell walls is key in explaining some important characteristics or behaviour. To model such phenomena, we consider cellular structures with nonlinear hyperelastic cell walls under large shear deformations and incorporate unilateral contact between neighbouring cells. Numerically, we show that, when finite element models of periodic structures with hexagonal cells are sheared, significant cell separation is captured diagonally across the structure. Our analysis further reveals that separation is less likely between cells with high internal cell pressure than between cells where the internal pressure is low.

Key Words: *cellular solids; unilateral contact; hyperelastic materials; finite element method; fruit softening.*

1. Introduction

Cellular tissues such as apples, pears and potatoes are a collection of fluid filled parenchyma cells (Figure 1a) bound together by inter-cellular cohesion. In a ripe and juicy apple, fluid is released from cells as the cell wall ruptures (cell bursting). In overripe or cold-stored fruit the strength of the inter-cellular cohesion decreases and the cell wall strength increases, such that it takes less energy to separate cells than to burst [1]. The texture of the fruit becomes dry and bitty (known as 'mealy') as the cells fall apart in small clumps and little fluid is released [1,2]. The phenomena of cell separation, or debonding, (Figure 1b) is key in explaining the behaviour of fruit and legumes during storage or cooking, and is decisive for the quality of food products [1].



(a) Potato parenchyma. (b) Cell separation in apple tissue.

Figure 1: Scanning electron microscopy images of potato and apple parenchyma.

Cell properties determine tissue behaviour and applied external forces change the cell responses as deformation progresses [4,5]. These relations lead to nonlinear mechanical behaviour and the requirement for a multi-scale approach. This study uses numerical models to provide evidence of how the cell wall, cell contents and inter-cellular cohesion contribute to cell debonding in soft fruits and tissues. Particular focus is given to shear deformation as this has been largely neglected in literature.

2. Contact Problems in Finite Elasticity

The finite (large strain) elastic regime is used to capture nonlinear behaviours in large deformations. The cell walls are modelled by a Mooney-Rivlin hyperelastic material, described by the strain energy function $\mathcal{W} = C_1 (I_1 - 3)/2 + C_2 (I_2 - 3)/2$ with material constants $C_1 > 0$ and $C_2 > 0$. The walls of neighbouring cells are assumed to be in unilateral (non-penetrative) contact. The problem is to find the displacement field $\mathbf{u}(\mathbf{X}) \in \mathbb{R}^3$ satisfying [6]:

• The Lagrangian equation of non-linear elastostatic equilibrium in the body Ω (no body forces):

$$\mathrm{Div}\mathbf{P}(\mathbf{X}) = 0, \tag{1}$$

where $\mathbf{P}(\mathbf{X})$ is the 1st Piola-Kirchhoff stress, representing the force per unit area in the reference configuration.

• The Dirichlet (prescribed displacement) conditions on the boundary Γ_D:

$$\mathbf{u}(\mathbf{X}) = \mathbf{u}_D. \tag{2}$$

• The Neumann (prescribed surface pressure) conditions on the boundary Γ_N:

$$\mathbf{P}(\mathbf{X})\mathbf{N} = \mathbf{g}_N, \tag{3}$$

where $\mathbf{N}$ is the outward unit normal vector to Γ_N.

• The non-penetrative frictionless contact conditions on the boundary Γ_C:

$$\eta(\mathbf{X} + \mathbf{u}(\mathbf{X})) \leq 0, \qquad \mathbf{P}(\mathbf{X})\mathbf{N} \cdot \mathbf{N} \leq 0, \qquad (\eta(\mathbf{X} + \mathbf{u}(\mathbf{X})))(\mathbf{P}(\mathbf{X})\mathbf{N} \cdot \mathbf{N}) = 0, \tag{4}$$

where η is the relative distance between contacting cell walls and $\mathbf{N}$ is the unit normal vector to the contact interface.

3. Successive Deformation Decomposition Procedure

To improve computational efficiency, we implement the *successive deformation decomposition procedure* (SDDP) proposed in [6]: (i) first, a continuous deformation is computed for the entire the structure, as in a compact elastic solid, where only the external boundary conditions are imposed while the cells remain in mutual contact; (ii) then, from the pre-deformed structure, the unilateral contact between cell walls are taken into account and cells are able to separate (Figure 2). For our computer simulations, the two-step procedure proved significantly faster and more robust than when the external boundary conditions and contact constraints were imposed simultaneously in a single step.

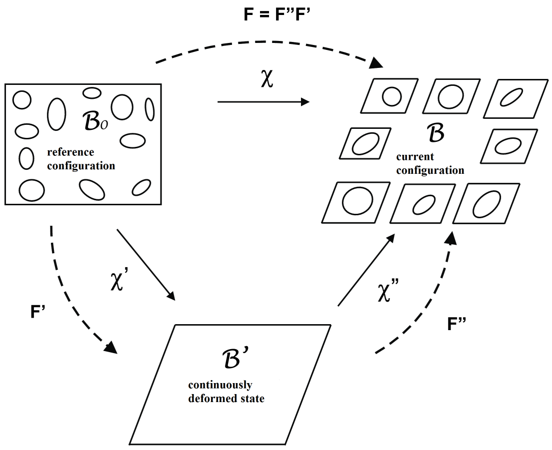
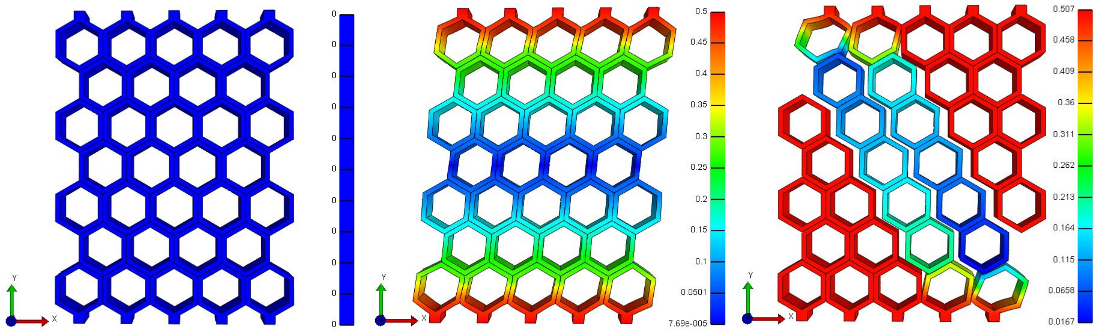


Figure 2: Diagrammatic representation of the successive deformation decomposition procedure.

4. Empty Cells with Unilateral Contact

The SDDP is applied to periodic structures with hexagonal prismatic cells, subject to horizontal shear, modelled using FEBio (Finite Element for Biomechanics) [3]. Results show that, when structures are subject to shear deformation, gaps appear between adjacent cells, causing extensive cell separation diagonally across the structure (Figure 3).

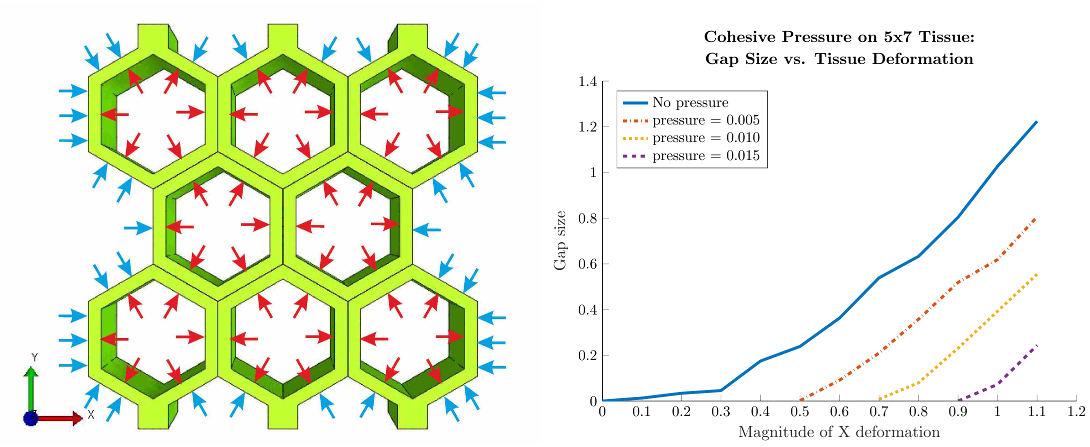


(a) Reference configuration. (b) Continuously deformed state. (c) Current configuration.

Figure 3: SDDP of a 5x7 cellular tissue, subject to shear. Colour indicates X displacement.

5. Intercellular Cohesion

Cohesion on a contact interface is usually modelled by the condition $\mathbf{P}(\mathbf{X})\mathbf{N} \cdot \mathbf{N} \leq g$ on Γ_C, where $g > 0$ indicates that a tensile force of magnitude g is permissible whilst two bodies are in contact. Computationally, this leads to highly unstable systems, so we consider instead an internal cell pressure which is normal to the cell walls and has the same magnitude for each cell (Figure 4a). This creates a compressive normal force which must be overcome to separate cell walls, analogous to normal contact cohesion. Results show that a higher surface pressure delays the initiation of inter-cellular gap opening (Figure 4b).

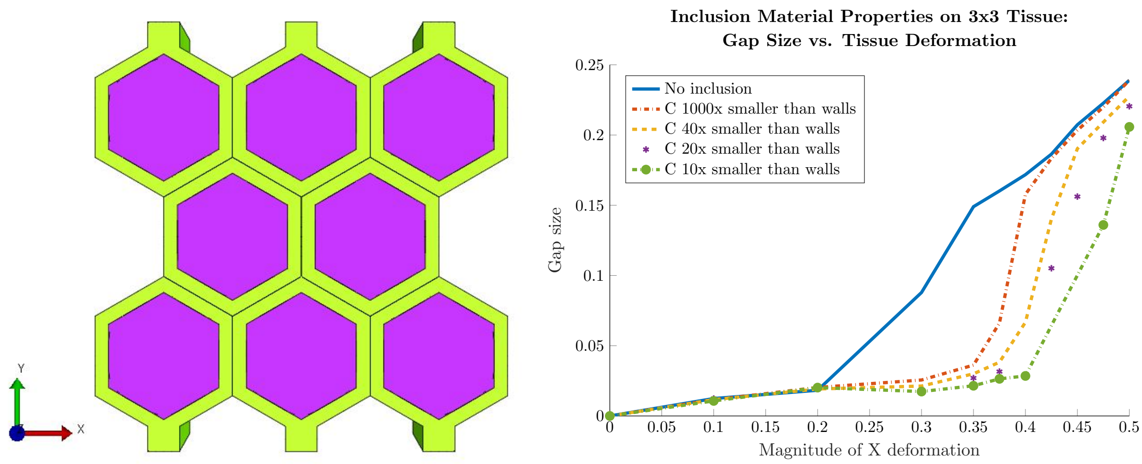


(a) Representative model with surface pressures. (b) Relationship between separation and pressure.

Figure 4: Cohesive pressure within cellular tissues.

6. Filled Cells

In our study, the influence of the cell inclusions on the inter-cellular contact is addressed by modelling the inclusions as a nearly-incompressible, softer Mooney-Rivlin material (Figure 5a). A primary effect of this is the cell volume constraint. As shown by our results, the rate of increase in the gap-size, occurring at X-displacement of ≈ 0.2 for empty cells, is delayed by the presence of cellular inclusion, and delayed further by increasing the stiffness of the inclusions (Figure 5b).



(a) Representative model with cellular inclusions.
(b) Relationship between separation and inclusion softness.

Figure 5: Soft cellular inclusions within cellular tissue.

Alternatively, the presence of cell inclusions could be modelled by imposing uniform normal pressure on the internal cell walls. Formally, this is similar to our model for inter-cellular cohesion, suggesting that higher cell pressure results in an increased inter-cellular cohesion.

7. Conclusion

We model computationally cellular bodies with nonlinear hyperelastic cell walls in mutual non-penetrative contact under large shear deformations, and propose a two-step strategy which we employ to solve the multi-body contact problems more efficiently. Our numerical results are in agreement with physical observations that tissue from overly mature fruit (apple, pear), where cell pressure is low and intercellular cohesion is weak, breaks down into small clumps of undamaged cells, whereas fruit of a lower maturity, with high cell pressure and intercellular cohesion will not debond easily.

Acknowledgements

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