Citation for final published version:

Publishers page: https://doi.org/10.1016/j.ejor.2017.05.014

Please note:
Changes made as a result of publishing processes such as copy-editing, formatting and page numbers may not be reflected in this version. For the definitive version of this publication, please refer to the published source. You are advised to consult the publisher’s version if you wish to cite this paper.

This version is being made available in accordance with publisher policies. See http://orca.cf.ac.uk/policies.html for usage policies. Copyright and moral rights for publications made available in ORCA are retained by the copyright holders.
Identifying the causes of the bullwhip effect by exploiting control block diagram manipulation with analogical reasoning

Mohamed M. Naim ¹, Virginia L. Spiegler ², Joakim Wikner ³, Denis R. Towill ¹#

¹ Cardiff Business School, Cardiff University, United Kingdom (UK)
Corresponding author: NaimMM@cf.ac.uk

² Kent Business School, Kent University, UK

³ Department of Management and Engineering, Linköping University, Sweden

# Posthumously: Prof. Denis R. Towill, 1933-2015 – a colleague and friend who was instrumental in motivating the research described in this paper

Highlights

- Develops an interdisciplinary method to identify causes of the ‘bullwhip’ effect
- A complex, mathematically intractable model is designated as the ‘target model’
- The complex model is shown to have ‘behavioural similarities’ with a simple model
- The simpler model has known solutions to be exploited for the complex model
- Our approach exploits control engineering and analogical reasoning
Identifying the causes of the bullwhip effect by exploiting control block diagram manipulation with analogical reasoning

Abstract

Senior managers when solving problems commonly use analogical reasoning, allowing a current ‘target problem’ situation to be compared to a valid previous experienced ‘source problem’ from which a potential set of ‘candidate solutions’ may be identified. We use a single-echelon of the often-quoted Forrester (1961) production-distribution system as a case ‘target model’ of a complex production and inventory control system that exhibits bullwhip. Initial analogical reasoning based on ‘surface similarity’ would presuppose a classic control engineering ‘source model’ consisting of a phase-lag feedback system for which it is difficult to derive the transfer function. Simulation alone would have to be relied on to mitigate the bullwhip effect. By using z-transform block diagram manipulation, the model for a single-echelon, consisting of 17 difference equations with five feedback loops is shown to have exact analogy to Burns and Sivazlian’s (1978) second order system that has no feedback. Therefore, this more appropriate ‘source model’ is based on a deeper understanding of the ‘behavioural similarities’ which indicates that the bullwhip effect is not in the case of the ‘target model’ due to feedback control but due to a first-order derivative, ‘phase advance’, term in the feed forward numerator path. Hence a more appropriate ‘candidate solution’ can be found via the use of a ‘recovery’ filter. An interdisciplinary framework for exploiting control engineering block diagram manipulation, utilising analogical reasoning, in a practical setting is presented, as is an example in a contemporary supply chain setting.

Keywords: (P) Systems dynamics, Forrester effect, system simplification, z-transform, simulation.

1. Introduction

Forrester’s (1958, 1961) seminal work on Industrial Dynamics is still cited to this day as an explanation for, or used synonymously with, the ‘bullwhip effect’ (e.g. in EJOR, Zhang & Burke, 2011, Ma et al., 2015, Wang and Disney, 2015). The ‘bullwhip effect’ is the phenomenon by which variance in the order flow increases upstream from one business to the next in the supply chain (Croson and Donohue, 2006). Lee et al. (1997a, b) first coined the term and suggested a number of categories for the causes of bullwhip including demand signal processing, order batching, inventory rationing, and price fluctuations. The former is also termed the Forrester Effect (Towill, 1997) and is attributed to the structure of an ordering system, the combination of decision rules, material and information delays, feedback loops and nonlinearities present in the system. The original Forrester paper (1958) and the subsequent text book (1961) formed the foundation for Industrial Dynamics, or what is now termed System Dynamics, the school of thought that relates system structures to dynamic behaviour in organisations. A fundamental principle of System Dynamics is that “feedback theory explains how decisions, delays, and predictions can produce either good control or dramatically unstable operation” (Forrester, 1958).

Gary et al. (2008) note that the use of system archetypes to understand problems and find solutions relates to the use of analogical reasoning (AR) (Gavetti and Rivkin, 2005). AR has been studied in the System Dynamics arena by Gonzalez
and Wong (2011). They undertook experiments into how decision makers draw analogies between different but apparently similar stock and flow problems and how they differentiate between surface and behavioural similarity:

“surface similarity is based on the mere appearance between two objects, whereas behavioural similarity is based on the function, matching relations, and final goal of the problems even when they do not appear to be similar.” (Gonzalez and Wong, 2011)

As Gavetti and Rivkin (2005) point out more generally - “Dangers arise when strategists draw an analogy on the basis of superficial similarity, not deep causal traits”, that is, there is reliance on what is termed ‘surface similarity’. But as Forrester himself noted in an interview - “The trouble with systems thinking, is it allows you to misjudge a system. You have this high-order, nonlinear, dynamic system in front of you as a diagram on the page. You presume you can understand its behaviour by looking at it, and there’s simply nobody who can do that” (Fisher, 2005). This reinforces Richardson’s (1991) argument that simple visual inspection of causal loop diagrams to determine system stability is insufficient and deeper understanding of the underlying control mechanisms is required.

Our research therefore covers the interdisciplinary space that brings together three disciplines, namely, General Management, as per Gavetti and Rivkin (2005), System Dynamics, (e.g. Gary et al., 2008) and Control Engineering, as typified by Wikner et al. (1992). While, from an Operational Research perspective, System Dynamics was originally considered to lack methodological rigour, as discussed by Sharp and Price (1984), it is now a commonly utilised method (e.g. Saleh et al., 2010). The latter has strong foundational contributions to Operational Research studies of inventory control systems (e.g. Vassion, 1955) and is still of value to the present (e.g. Dejonckheere et al., 2004, Spiegler et al., 2016). Our approach to methodological unification is commensurate with modern day management challenges that brings together “a wide variety of disciplines such as OM [operations management], OR [operational research] and systems dynamics” and may be branded as many different names including “supply chain, OM, management science, industrial and production engineering and OR” (MacCarthy et al., 2013).

In deriving our interdisciplinary method, we use the Forrester (1961) model as a case example of what at first sight seems a highly complicated production and inventory control system. As the Forrester model is often quoted synonymously with the ‘bullwhip effect’ then it seems reasonable to use it as a classic reference, as done by Wikner et al. (1992) and more recently Spiegler et al. (2016), by which to test new innovations in mitigating the ‘bullwhip effect’. Also, given the fact that Forrester himself criticised the superficial visual inspections of feedback systems, it seems highly appropriate to use his seminal model as a reference.

The original Forrester (1961) model, was documented as series of simulation equations which we retain for easy cross-referencing and as given in Appendix 1. We do not show all the equations for all echelons here but rather, in exemplifying the control engineering approach, we utilise the equations for the factory-warehouse echelon to develop a z-transform representation as in Figure 1 a). It would be extremely difficult to relate the original simulation equations to Figure 1 b), and even with a cursory glance the model of Figure 1 a), looks complicated and, from a surface similarity visual comparison, still totally different from Figure 1 b). If we now try to use control engineering criteria to have a more analytical comparison we then have Table 1. Hence, surface similarity suggests two very different systems with no analogy. Using a system simplification approach originating in hardware control engineering (Biernson, 1988) and subsequently exploited by Wikner et al. (1992), using the Laplace s-domain, to developed an equivalent linear, time
invariant representation of the Forrester (1958) decision ordering rule, we will show the analogy of Figure 1 a) with the Burns and Sivazlian (1978) of Figure 1 b).

In this way our aim is to develop an interdisciplinary approach, exploiting control engineering in an AR context, in production and inventory control system design so as to understanding the causes of the ‘bullwhip effect’, a symptom of the system's dynamics, and a precursor to its reduction / elimination. Hence we provide the basis for future research in Operational Research in providing robust and structured approaches to AR (Knott, 2006). Also, by using control engineering within an AR context we then seek to avoid the inherent dangers that a purely quantitative approach will not be usable by decision makers (Akkermans and Bertrand, 1997). We will further show the potential of our integrated approach for other general supply-chain modelling problems by applying it in a contemporary setting.

2. Control engineering design of a complex production and inventory control system using analogical reasoning

We use Gavetti and Rivkin’s (2005) suggested three steps for the development of AR in management decision making. These are;

1. Target model – the observed or current situation / problem to be addressed is identified, documented and modelled.
2. Source model(s) – through direct / indirect experience considers other settings and, through a process of similarity mapping, identifies a setting that displays similar attributes, such as archetypes and benchmarks.
3. Candidate solution(s) – from the source model an actual, or potential, benchmark solution is identified.

2.1 Target model. This is the Forrester (1961) model of the factory-warehouse echelon as given by the equations and associated notation of Appendix 1. A fuller description of the meaning of the notation can be found in Forrester (1961) and their relationship with control engineering notation in Wikner et al. (1992). The latter translate the simulation equations into causal loop diagrams before deriving the Laplace block diagram representation. Here we go directly to a block diagram representation as given in Figure 1 a), using z-transform notation to be commensurate with the modelling approach utilised by Burns and Sivazlian (1978) and others (e.g. Popplewell and Bonney, 1987). z notation has more recently been utilised in operational research, analysing the bullwhip effect induced by ordering replenishment rules whether at the unit of analysis of a single-echelon (e.g. Disney et al., 2006) or multi-stage supply chains (e.g. Agrawal et al., 2008). A fuller description of the formulation and use of block diagrams and the z-transform may be found in Nise (2011). Appendix 1 explains how the z-transform notation relates to the original simulation equations.

Simply looking at the block diagram ‘as is’ would suggest the following;

- There exist the basic building blocks for a generic system archetype; feedback, stocks and flows, policies or decision rules, and lags or delays.
- There are a number of feedback loops and delays.
- The feedback loops are monitoring systems states or the stocks in the system.
- The feedback loops influence the ordering decision, MD, that is, the manufacturing rate.
- The feedback loops are balanced, suggesting a homeostatic system, which are also suggested by running the three-echelon simulation.

If the above ‘surface similarity’ deductions are to be believed then intuitively a manager would be looking to solve the problem traditionally associated with a phase-lag, or delayed response, system and that the bullwhip solution lies with proportional control / phase-lead compensation. The relative complexity of the block diagram suggests that it will be
difficult to derive the transfer function and any quantitative analysis would have to rely on simulation alone. Also, the complexity seems quite unique, again posing difficulties to identifying analogous production and inventory control systems with potential candidate solutions.

To better grasp ‘behavioural similarity’ the next step is to undertake a simplification procedure in order to understand the underlying mechanisms. We follow a similar procedure as given by Wikner et al. (1992), which ensures replication of their work in the Laplace s domain using the alternate z transform method, and as given in Appendix 2. The simplification yields Figure 2.

![Figure 1: a) Forrester (1958, 1961) model in block diagram z notation form and b) Burns and Sivazlian (1978) model - all parameters and variables will be explained later in the paper](image)

> Figure 1: a) Forrester (1958, 1961) model in block diagram z notation form and b) Burns and Sivazlian (1978) model - all parameters and variables will be explained later in the paper
Criteria for comparative purposes | Forrester (Figure 1a) | Burns and Sivazlian (Figure 1b) |
--- | --- | --- |
Number of variables | 17 | 5 |
Number of feedback loops | 8 | 0 |
Number of parameters (total) | 9 | 3 |
Number of first order lags / delays | 0 | 2 |
Number of second order lags / delays | 0 | 0 |
Number of third order lags / delays | 2 | 0 |
Number of integrators / stocks | 5 | 1 |
Number of time varying parameters | 1 | 0 |
Number of continuous non-linearities | 2 | 0 |
Number of discontinuous non-linearities | 2 | 0 |
Ease of transfer function formulation | Low | High |

Table 1: Control engineering comparison of the two systems shown in Figure 1.

If required, we can reinstate other variables of interest, such as IA or SS, but for the purposes of identifying the target model herein and the subsequently identified source model then Figure 2 highlights the relationship of interest, $\frac{MD}{RR}$. It can be clearly seen from Figure 2 that the system contains no linear state feedback in the ordering rules which consists of two components; the actual orders received, RR and a safety component, $RR \cdot \frac{Kz(z-1)}{(DR(z-1) + z)(DI(z-1) + z)}$. That is, the system states, given by IA, inventory actual levels, and LA, pipeline orders actual in transit, do not affect the ordering rule and hence have no impact on the ‘bullwhip effect’. Without this insight, considerable time and effort may be wasted by decision makers on exploring, say through protracted System Dynamics simulation studies, the impact of reducing pipeline lead-times and/or adjusting inventory feedback rules on the ‘bullwhip effect’.

Figure 2. The z-transform simplified representation of the Forrester (1958) model.

2.2 Source model. Here we note that the block diagram of Figure 2 and the principle of ‘real’ plus ‘safety’ orders has direct analogy with the model developed and analysed by Burns and Sivazlian (1978). While Burns and Sivazlian
(1978) used a flow graph and different notation \((Z=Z^{-1})\), as in Figure 1 b), Figure 3 a) shows the equivalent block diagram representation.

The terms used in Figure 3 a) are; \(h(n)\) = order placed in week \(n\), \(g(n)\) = order received in week \(n\), \(c\) = number of weeks of inventory ownership desired, \(f\) = hedging coefficient

Immediately it can be seen that Figure 3 a) resembles Figure 2 in that the order decision, \(h(n)\), consists of two components. The upper path is the order received, \(g(n)\), while the lower path is an additional component that aims to compensate for lags in the system and adjust inventory. The lower path consists of a number of functions that are, in order from left to right: exponential smoothing, with parameter \(\alpha\); a second exponential smoothing function, with parameter \(f\); differencing; and a constant, \(c\). Hence we can immediately deduce that there is ‘surface similarity’ between the Burns and Sivazlian (1978) model and simplified Forrester model of Figure 2.

**Figure 3. The Burns and Sivazlian (1978) model a) and its rationalisation, via b), to the model equivalent c)**

Figure 3 b) is the manipulation of the functions to get them in the form of \(z\) rather than \(z^{-1}\). Then, making the following correlations between terms used in Figures 2 and 3;

\[
\alpha = \frac{1}{1 + DR/\delta t} \quad \ldots(1)
\]

\[
f = \frac{1}{1 + DI/\delta t} \quad \ldots(2)
\]

\[
c = K \quad \ldots(3)
\]

\[
\delta t = 1 \quad \ldots(4)
\]

we derive Figure 3 c) which can be further reduced to be exactly equivalent to Figure 2.

This is an important result. We have now found ‘behavioural similarity’ between the Burns and Sivazlian (1978) model and the Forrester model (1961). The AR would not have been identified if the original Forrester model had been retained.
especially in the form of the simulation equations of Appendix 1 and even in the form of the original block diagram as shown in Figure 1 a).

Triangulating analytical approaches to verify the similarity between the Forrester (1961) and Burns and Sivazlian (1978), models, Figure 4 a) shows the MD unit step response comparison between the Wikner et al. (1992) and Forrester (1961) block diagram unit step responses using the MATLAB Simulink© software package, and the Wikner et al. (1992) / Burns and Sivazlian (1978) model inverse z-transform into the time domain using the Mathematica© software package.

The derived transfer function in z is

\[
\frac{MD}{RR} = \frac{z^2(DR \cdot DI + DR + DI + K)}{z^2(DR \cdot DI + DR + DI)} + z(-2DR \cdot DI - DR - DI - K) + DR \cdot DI \quad \cdots(5)
\]

For the parameter settings established in the original Forrester (1961) simulation tests, i.e. K = 9, DR = 8, DI = 4, the unit step responses are exact. This indicates that in the original Forrester model, for the value of AL used, the non-linearity established by the CLIP function never constraints MD. Figure 4 b) shows the IA deviation step response for the original Forrester model, which has DF as a time varying parameter, and compare it with the Wikner et al. (1992) model which can only be calculated by reinstating MO, SR, UO and ST. Also shown is the Forrester model with time varying parameters kept fixed and the non-linearity set by the CLIP function set at such a high level that it does not constrain SS. It can be seen that the latter directly mimics the Wikner et al. (1992) / Burns and Sivazlian (1978) model.

Figure 4 therefore suggests that even the time varying feedback that is present does not affect the ordering decision MD and hence does not influence the bullwhip effect. This is true also when the non-linearity also constrains SS.

2.3 Candidate solution. Our simplification in Section 2.1 and analysis in Section 2.2 now suggests the following properties associated with the Forrester model;

- There is no significant feedback into the ordering decision, MD
- There is a differencing term in the numerator of the transfer function which is the cause of the bullwhip effect and not any linear feedback loops.
- We should expect a phase-lead and not a phase-lag system
- It is easy to derive the transfer function. Hence, the model is mathematically tractable with simulation as support.
- The Forrester model has ‘surface similarity’ and ‘behavioural similarity’ with the Burns and Sivazlian (1978) model. Hence, a candidate solution will be found in Burns and Sivazlian (1978).

Burns and Sivazlian show selected unit step, random and sinusoidal responses to highlight the dynamic behaviour of one-, two- and six-echelon systems, which exhibit the ‘bullwhip effect’. Using numerical frequency response analysis they suggest a filtering approach so as to filter out unwanted ‘false orders’ in the lower path of Figure 3 c) while allowing ‘legitimate orders’ to pass through. The ‘false order’ is created by the differencing term, \( \frac{z-1}{z} \), a form of forecasting based on the rate of change. In hardware control engineering terms this generates the well-known “phase advance”, or predictive component (Truxal, 1955). While this has advantages when it comes to inventory replenishment, in essence ordering in advance to ensure stock availability, we now see that there must be some constraint (Porter 1952).

We do not replicate the analysis already undertaken by Burns and Sivazlian (1978). Instead we show the frequency response of the system graphically using discrete time bode plots given in Figure 5 which are based on the case when \( c = 9, \alpha = 0.111, f = 0.2 \), i.e. again for the original Forrester test condition when \( K = 9, DR = 8, DI = 4 \). Burns and Sivazlian,
using a continuous-time analogue of their model, calculated the natural frequency, $\omega_N$, corresponds to a period of

$$T_{\omega_N} = 2\pi \sqrt{\frac{1-\alpha}{\alpha f}} \text{ weeks.}$$

For the chosen parameter values, $T_{\omega_N} = 40$ weeks. It can be seen from the peak magnitude in Figure 5 that the damped natural frequency lies between 0.1-0.2 radians week$^{-1}$, so that $31.5 < T_{\omega_N} < 62.8$ weeks.

Figure 4. Triangulation methods in comparing the Wikner et al. (1992) / Burns and Sivazlian (1978) and Forrester (1961) dynamic responses ($K = 9$, $DR = 8$, $DI = 4$)

The bode plot also shows that the peak magnitude corresponds with little phase lead in the output. While Burns and Sivazlian, as with other authors who have utilised filter theory in supply chain design, focussed on the amplitude ratio or magnitude characteristics of such systems (e.g. Towill & del Vecchio, 1994, Dejonckheere et al., 2002, Towill et al., 2003), due consideration of the phase shift is also needed.
4. Conclusion

Now we may propose an interdisciplinary framework, with control engineering at its core and exploiting analogical reasoning, for identifying the causes of the bullwhip effect, and identifying potential solutions, as given in Table 2. The approach has been tested using the Forrester (1961) supply chain as a target model. The method does not assume that an initial complicated Forrester model will lead to the right AR. By undertaking block diagram formulation, manipulation and simplification, it is possible to establish the correct ‘target model’ (simplified Forrester model) and hence identify an appropriate ‘source model’ (Burns and Sivazlian, 1978 model) from which to establish a correct ‘candidate solution’ (using filter theory).

In identifying the causes of the bullwhip effect resulting from complicated production and inventory control systems the method establishes behavioural similarity and not just surface similarity i.e. understanding the underlying mechanisms that lead to a particular dynamic behaviour.

The method developed contributes to an interdisciplinary approach as it utilises control engineering, supported by AR, to gain insights into the underlying mechanisms to system dynamics problems and providing solutions. While the research has utilised an often-quoted model to highlight the utilisation of a block diagram simplification approach, and AR a second contemporary example, namely the Intel supply chain, to test the approach suggested in this paper is given in Appendix 3. Hence, our method can be potentially used to make a bridge between theoretical and practical modelling approaches. Further empirical testing of our approach given in Table 2 is suggested for future research, especially through empirical studies as suggested in Appendix 4, which would enhance its credibility in practical problem solving situations. Such future research need not be constrained to just the bullwhip effect but should be extended to solve other supply chain dynamics phenomena such as rogue seasonality, ripple effect, inventory drift and inventory variance,
among others. Also, the development of more formal rules to compare and contrast target and source models will be of interest to the Operational Research community.

<table>
<thead>
<tr>
<th>Generic phases</th>
<th>Forrester Model</th>
<th>Intel Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identify correct target model</td>
<td>Based on just a visual comparison between figures 1 a) and 1 b), that is merely ‘surface similarity’ comparison, there is no analogy between the Forrester and Burns-Sivazlian models. Figures 1a illustrates a complex model with several feedback loops, delays and nonlinearities, which can deceive designers into believing that the bullwhip problem is associated with a phase-lag, or delayed response ‘Behavioural similarity’, requiring block diagram manipulation and simplification, (comparing Figures 3 b and 3 c) subsequently reveals analogy between the simplified Forrester and Burns-Sivazlian models.</td>
<td>Based on just a visual comparison between Figures A3.1 and A3.3, that is merely ‘surface similarity’ comparison, there is no analogy between the Intel and IOBPCS family models. ‘Behavioural similarity’, requiring block diagram manipulation and simplification, (comparing Figures A3.3 with A3.5 and A3.6) subsequently reveals direct analogy between the Intel (pull mode) model with the VIOBPCS, but some similarity between the Intel (push mode) model with the APVIOBPCS.</td>
</tr>
<tr>
<td>Identify an appropriate source model</td>
<td>Without simplification it would not have been obvious that the Burns and Sivazlian’ model is analogous.</td>
<td>Without simplification it would not have been obvious that the IOBPCS family of models is analogous.</td>
</tr>
<tr>
<td>Establish a correct candidate solution</td>
<td>Surface similarity alone may have led to incorrect conclusion regarding the impact of feedback control. Behavioural similarity, revealed via simplification, gives new insights to potential solutions. Candidate solutions to bullwhip effect: Filter theory, as in Burns and Sivazlian (1978).</td>
<td>Surface similarity alone may have led to over reliance on simulation alone with a trial and error approach to finding solutions to the bullwhip effect. Behavioural similarity, revealed via simplification, gives new insights to known solutions from, as well as revealing an addition to, the IOBPCS family. Candidate solutions to the bullwhip effect: Conservative parameter settings from Edghill (1990) and adaptations of John et al. (1994).</td>
</tr>
</tbody>
</table>

*Table 2. An interdisciplinary framework, exploiting block diagram formulation and manipulation with analogical reasoning, for mitigating the bullwhip effect*
6. Acknowledgements

The authors would like to thank Cardiff Business School’s research committee for awarding an International Visitor Scheme grant allowing Prof. Wikner to co-locate with his co-authors. We would also like to thank Junyi Lin for spotting several typographical inconsistencies in our block diagrams, that are now corrected, and for his stock and flow representation of the Intel supply chain. In addition, we appreciate the considerable time and efforts of the editors and anonymous reviewers for the opportunity to develop and enhance the paper.

7. References


Appendix 1 – The Forrester model

Simulation equations used in the DYNAMO program (factory-warehouse echelon)

\[
\begin{align*}
UO.K &= UO.J + (DT)(RR.JK-SS.JK) \\
IA.K &= IA.J + (DT)(SR.JK-SS.JK) \\
ST.K &= UO.K/DF.K \\
NI.K &= IA.K/DT \\
SS.KL &= CLIP(ST.K,NI.K,NI.K,ST.K) \\
DF.K &= (ID.K/IA.K)(DU)+DH \\
ID.K &= (AI)(RS.K) \\
MD.KL &= CLIP(MW.K,AL,AL,MW.K) \\
LD.K &= (RS.K)(DC+DP) \\
LA.K &= CP.K+OP.K \\
UN.K &= (RS.K)(DH+DU) \\
CP.K &= CP.J+(DT)(MD.JK-MO.JK) \\
MO.KL &= DELAY3(MD.KL,DC) \\
OP.K &= OP.J+(DT)(MO.JK-SR.JK) \\
SR.KL &= DELAY3(MO.KL,DP)
\end{align*}
\]

Nomenclature

At time \( K \) the previous level is said to be \( \text{LEVEL.J} \) and the rate of change in the previous time interval, \( DT \) is \( \text{RATE.JK} \). By assuming a linear relationship the level at time \( K=J+DT \) may be calculated as \( \text{LEVEL.K} = \text{LEVEL.J} + DT \cdot \text{RATE.JK} \) for \( DT \to 0 \). NB For the purposes of the exercise it is assumed \( DT = 1 \), i.e. 1 week or time unit. Hence \( NI.K = IA.K \) and therefore NI is not shown in the block diagram representation of Figure 3.
Variables and constants used in the DYNAMO program (factory-warehouse echelon)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AI</td>
<td>constant for inventory</td>
<td>LD</td>
<td>pipeline orders desired in transit</td>
</tr>
<tr>
<td>AL</td>
<td>constant specifying capacity limit</td>
<td>MD</td>
<td>manufacturing rate decision</td>
</tr>
<tr>
<td>CP</td>
<td>clerical in-process orders</td>
<td>MO</td>
<td>manufacturing orders</td>
</tr>
<tr>
<td>DC</td>
<td>delay clerical</td>
<td>MW</td>
<td>manufacturing rate wanted</td>
</tr>
<tr>
<td>DF</td>
<td>delay (variable) in filling orders</td>
<td>NI</td>
<td>negative inventory limit rate</td>
</tr>
<tr>
<td>DH</td>
<td>delay due to minimum handling time</td>
<td>OP</td>
<td>orders in production</td>
</tr>
<tr>
<td>DI</td>
<td>delay in inventory/pipeline adjustment</td>
<td>RR</td>
<td>requisition (orders) received</td>
</tr>
<tr>
<td>DP</td>
<td>delay in production lead time</td>
<td>RS</td>
<td>requisition (orders) smoothed</td>
</tr>
<tr>
<td>DR</td>
<td>delay in smoothing requisitions</td>
<td>SR</td>
<td>shipment received inventory</td>
</tr>
<tr>
<td>DU</td>
<td>delay, average, in unfilled orders</td>
<td>SS</td>
<td>shipment sent</td>
</tr>
<tr>
<td>IA</td>
<td>inventory actual</td>
<td>ST</td>
<td>shipping rate tried</td>
</tr>
<tr>
<td>ID</td>
<td>inventory desired</td>
<td>UN</td>
<td>unfilled orders normal</td>
</tr>
<tr>
<td>LA</td>
<td>pipeline orders actual in transit</td>
<td>UO</td>
<td>unfilled orders</td>
</tr>
</tbody>
</table>

The equations are expressed as difference equations with two predefined DYNAMO functions included. DELAY3 represents a third order delay and CLIP introduces a non-linearity equivalent to setting a capacity constraint.

For example,

\[ MO.KL = \text{DELAY3}(MD.KL, DC) \]  
\[ MD.KL = \text{CLIP}(MW.KL, AL, AL, MW.KL) \]

represents a third-order delay with a lag of \( \frac{DC}{3} \) whose input is MD and output is MO.

\[ MD.KL = \text{CLIP}(MW.KL, AL, AL, MW.KL) \]

means that the maximum possible value of MD is AL.

As an example of the use of the z-transform notation, Equation A1.1 becomes

\[ \frac{MO(z)}{MD(z)} = \left( \frac{z}{\frac{DC}{3}(z-1)+z} \right)^3 \]

and is represented visually in block diagram form as shown in the top right hand corner of Figure of 1 a), with Equation A1.2 shown visually by the preceding block. The term \( \frac{z}{z-1} \) is an accumulator, or stock and circles are comparator or dividers. For example,

\[ LA = OP + CT \]

and

\[ ST = \frac{DF}{UO} \]
Appendix 2 – Simplifying the Forrester (1961) model using block diagram manipulation

The four stage approach we follow is;

1. Convert time varying parameters to a typical fixed value within the range thus creating a time invariant estimate of the system – in Figure A2.1 the parameter DF varies with time but we will assume it is a fixed so that $ST = \frac{DF}{UO}$ now only varies due to changes in $UO$ and not DF.

2. Convert the non-linear relationships to a linear approximation – Figure A2.2 shows the two CLIP functions that are eliminated.

3. Collect terms – Equation A2.1 collected all the parameters, AI, DH, DU, DC and DP, into a single term, K.

4. Eliminate redundancies (Figures A2.4 and A2.5)

$$K = AI - DH - DU + DC + DP \quad \text{…(A2.1)}$$

Figure A2.1. Element of the model converted from time varying to time invariant

Figure A2.2. Eliminating the CLIP functions

Figure A2.3 shows the revised block diagram based on the simplifications elements 1-3.
Figures A2.4 and A2.5 show two stages of identifying and eliminating mathematical redundancy in the system. Figure A2.4 gives the mathematical manipulation of terms in equations.

Figure A2.5 indicates that the manufacturing ordering, $MD$, is not influenced by $UO$. Noting that the input-output relationship, $\frac{RS}{RR}$, represents a first order lag, then Figure A2.5 b) can be redrawn as Figure A2.6 a) and rationalised as Figure A2.6 b).
Figure A2.4. a) First elements identified that contain redundancy, b) First set of redundancies removed
Figure A2.5. a) Second elements identified that contain redundancy, b) Second set of redundancies removed
Figure A2.6. The Wikner et al. (1992) representation of the simplified Forrester model in $z$. 
Appendix 3 – Example of the exploitation of AR in a contemporary empirical supply chain setting: the case of Intel

While this paper has exploited the widely quoted Forrester (1958, 1961) model to highlight the potential applicability of control theoretic block diagram formulation and manipulation to identify ‘behavioural similarities’ with seemingly the seemingly ‘surface dissimilar’ Burns and Sivazlian (1978) model, it may be argued that both these models are dated and no longer represent what happens in the contemporary world. In addition, the Forrester model was developed in an archaic simulation language, DYNAMO, that is no longer commercially available.

A3.1 Target model. We draw on the Intel supply chain model for semi-conductor manufacture and distribution developed by Gonçalves et al. (2005). They developed a Vensim® simulation model, using stock and flow representations as shown in Figure A3.1 that, according to Gonçalves et al. (2005), yields “a ninth-order non-linear differential equation system. Since the equations are highly non-linear it is not possible to obtain closed form solutions”.

The model consists of 26 variables and 15 constants as given in Table A3.1. We develop the block diagram, again using z notation, as given in Figure A3.2. Looking at the block diagram ‘as is’ suggests:

- There exist the basic building blocks for a generic system archetype; feedback, stocks and flows, policies or decision rules, and lags or delays.
- There are a number of feedback loops and delays.
- The linear feedback loops are monitoring systems states or the stocks in the system.
- The linear feedback loops influence the production, $A_G$, that is, the gross assembly completion rate.
- The linear feedback loops are balanced, suggesting a homeostatic system, which are also suggested by the running the Vensim® simulation (see Gonçalves et al., 2005).
- There exists a ‘switch’, defined by the CLIP function, so that $A_G$ is either governed by the lower value of $PullA_G$ or $PushA_G$ but not both.
If the above ‘surface similarity’ deductions are to be believed then intuitively a manager would be looking to solve the problem traditionally associated with a phase-lag, or delayed response, system and that the bullwhip solution lies with proportional control / phase-lead compensation. The relative complexity of the block diagram suggests that it will be difficult to derive the transfer function and any quantitative analysis would have to rely on simulation alone. Also, the complexity seems quite unique, again posing difficulties to identifying analogous production and inventory control systems with potential candidate solutions.

A3.2 Source model.

As we did with the Forrester simulation representation, the control theoretic block diagram representation, even without any further simplification, helps us draw analogue with a well-known benchmark model suite, namely the inventory and order based production control system (IOBPCS) family (Sarimveis et al., 2008, Wikner et al., 2007, Lalwani et al., 2006) as represented in Figure A3.3 and Table A3.2.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_G$</td>
<td>Gross assembly completion rate</td>
</tr>
<tr>
<td>$A^*_G$</td>
<td>Desired AG</td>
</tr>
<tr>
<td>$A_N$</td>
<td>Net assembly completion rate</td>
</tr>
<tr>
<td>$A^*_N$</td>
<td>Desired $A_N$</td>
</tr>
<tr>
<td>AWIP</td>
<td>Assembly work in process</td>
</tr>
<tr>
<td>AWIP*</td>
<td>Desired AWIP</td>
</tr>
<tr>
<td>AWIPAdj</td>
<td>AWIP adjustment</td>
</tr>
<tr>
<td>B</td>
<td>Backlog</td>
</tr>
<tr>
<td>B*</td>
<td>Target backlog</td>
</tr>
<tr>
<td>BAdj</td>
<td>B adjustment</td>
</tr>
<tr>
<td>D</td>
<td>Actual order</td>
</tr>
<tr>
<td>$D^*_i$</td>
<td>Desired die inflow</td>
</tr>
<tr>
<td>DD*</td>
<td>Desired delivery delay</td>
</tr>
<tr>
<td>$D_i$</td>
<td>Die completion rate</td>
</tr>
<tr>
<td>DPW</td>
<td>Dies per wafer</td>
</tr>
<tr>
<td>ED</td>
<td>Long term demand forecast</td>
</tr>
<tr>
<td>ES</td>
<td>Expected shipments</td>
</tr>
<tr>
<td>$F_G$</td>
<td>Gross fabrication rate</td>
</tr>
<tr>
<td>$F_G^*$</td>
<td>Desired $F_G$</td>
</tr>
<tr>
<td>FGI</td>
<td>Finished goods inventory stock</td>
</tr>
<tr>
<td>FGI*</td>
<td>Target FGI</td>
</tr>
<tr>
<td>FGIAdj</td>
<td>FGI adjustment</td>
</tr>
<tr>
<td>FWIP</td>
<td>Fabrication work in process</td>
</tr>
<tr>
<td>FWIP*</td>
<td>Desired FWIP</td>
</tr>
<tr>
<td>FWIPAdj</td>
<td>FWIP adjustment</td>
</tr>
<tr>
<td>Pull$A_G$</td>
<td>Governs $A_G$ in ‘pull’ mode</td>
</tr>
<tr>
<td>Push$A_G$</td>
<td>Governs $A_G$ in ‘push’ mode</td>
</tr>
<tr>
<td>$S$</td>
<td>Actual shipments</td>
</tr>
<tr>
<td>$S^*$</td>
<td>Desired shipments</td>
</tr>
<tr>
<td>$S_{MAX}$</td>
<td>Feasible shipments</td>
</tr>
<tr>
<td>WOI*</td>
<td>Desired weeks of inventory $= \tau_{OP} + \tau_{SS}$</td>
</tr>
<tr>
<td>WS</td>
<td>Wafer starts $= WS^*$</td>
</tr>
<tr>
<td>WS*</td>
<td>Desired wafer starts</td>
</tr>
<tr>
<td>$W_{SN^*}$</td>
<td>Desired net WS</td>
</tr>
<tr>
<td>$Y_D$</td>
<td>Die yield</td>
</tr>
<tr>
<td>$Y_L$</td>
<td>Line yield</td>
</tr>
<tr>
<td>$Y_{U}$</td>
<td>Unit yield</td>
</tr>
<tr>
<td>$\tau_A$</td>
<td>Assembly time</td>
</tr>
<tr>
<td>$\tau_B$</td>
<td>Time to adjust backlog</td>
</tr>
<tr>
<td>$\tau_{AWIP}$</td>
<td>Time to correct AWIP discrepancy</td>
</tr>
<tr>
<td>$\tau_{DAdj}$</td>
<td>Demand smoothing constant</td>
</tr>
<tr>
<td>$\tau_F$</td>
<td>Fabrication time</td>
</tr>
<tr>
<td>$\tau_{FGI}$</td>
<td>Time to adjust FGI</td>
</tr>
<tr>
<td>$\tau_{FWIP}$</td>
<td>FWIP correction time</td>
</tr>
<tr>
<td>$\tau_{OP}$</td>
<td>Minimum, or sum of, order processing time</td>
</tr>
<tr>
<td>$\tau_{SAdj}$</td>
<td>Shipping smoothing constant</td>
</tr>
<tr>
<td>$\tau_{SS}$</td>
<td>Safety stock coverage</td>
</tr>
</tbody>
</table>

*Table A3.1 – The Intel supply chain model variables and constants*
Figure A3.2: Block diagram representation of the Intel supply chain (based on Gonçalves et al., 2005)

Figure A3.3: The IOBPCS family (from Wikner et al., 2007 and Sarimveis et al. 2008)
<table>
<thead>
<tr>
<th>Variables</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>CONS</td>
<td>Customer consumption, or demand</td>
</tr>
<tr>
<td>AVCON</td>
<td>Average consumption, or smoothed / forecast demand</td>
</tr>
<tr>
<td>DINV</td>
<td>Desired or target inventory – either fixed or AVCON multiplied by a constant $k_1$</td>
</tr>
<tr>
<td>AINV</td>
<td>Actual inventory</td>
</tr>
<tr>
<td>EINV</td>
<td>Error in inventory – the difference between DINV and AINV</td>
</tr>
<tr>
<td>ORATE</td>
<td>Order rate – equal to AVCON plus a function $G_i(s/z)$ of EINV plus a function $G_w(s/z)$ of EWIP</td>
</tr>
<tr>
<td>COMRATE</td>
<td>Completion rate – a function of ORATE dependent on $G_p(s/z)$, which represents a delay</td>
</tr>
<tr>
<td>DWIP</td>
<td>Desired work-in-progress – AVCON multiplied by a constant $k_2$</td>
</tr>
<tr>
<td>AWIP</td>
<td>Actual work-in-progress</td>
</tr>
<tr>
<td>EWIP</td>
<td>Error in work-in-progress – the difference between DWIP and AWIP</td>
</tr>
</tbody>
</table>

Table A3.2. Variables in the IOBPCS family – functions $G(s/z)$ may be either in the $s$ or $z$ domains.

We can now manipulate the block diagram to draw direct analogues with a subset of the IOBPCS suite. Following a similar process as we undertook with the Forrester model, we

1. convert non-linear relationships to a linear approximation –
   a. assuming that variables are never negative, we can eliminate the three CLIP functions that govern $D^*$, $A^*$, and $WS$
   b. we assume that $S^* < S_{MAX}$ and hence $S = S^*$, hence the CLIP function governing $S$ is eliminated.
2. collect terms – we set
   a. $K_1 = \frac{1}{DPW \cdot Y \cdot D \cdot Y_L}$
   b. $K_2 = K_1 \cdot T_F$
   c. $K_3 = \frac{1}{K_1}$
3. eliminate redundancies – given the assumption regarding $S$ in 1 above, then we ship whatever is demanded, that is $S = D$. Thus
   a. $B = DD \cdot D$ and $B^* = DD^* \cdot D$ so that $B_{Adj} = 0$
   b. $ED = ES$
   c. $S_{MAX}$ is redundant given 1 b above
4. convert time varying parameters to a typical fixed value within the range thus creating a time invariant estimate of the system – parameter $S_{MAX}$ varies with time but, given the assumption regarding $S^*$ and $S_{MAX}$ in 1 above, $S_{MAX}$ is redundant as in 3 above.

We now have the block diagram in the form given in Figure A3.4. We can inspect the block diagram under the two scenarios –

1. $A_G = PullA_G$

This yields Figure A3.5. Given that $A_N = Y_U \cdot A_G$ and $A_G = A^* N / Y_U$ then $A_G = A^* N$. This in itself is an interesting result, giving insight that the model suggests instantaneous assembly and what is required to be assembled is actually achieved without, in fact, any yield loss. In terms of the model structure, and by comparing with Figure A3.3, the ordering policy for $A^* N$ (≡ ORATE) is a function of average demand (≡ AVCON) and FGIAdj (≡ EINV·$G_i(s/z)$), where the target inventory is a function, WOI of forecast demand, $ED$ (≡ $k_2 \cdot AVCON$). Given that $AG = A^* N$ the
equivalent in Figure A3.3 is that \( G_p(s/z) = 1 \) and hence COMRATE = ORATE. This scenario is therefore a direct analogue with the VIOBPCS variant (Edghill, 1990) of the IOBPCS family.

Figure A3.4: Simplified block diagram representation of the Intel supply chain

Figure A3.5: Simplified block diagram representation of the Intel supply chain in ‘pull’ scenario

2. \( A_G = \text{Push}A_G \)

Figures A3.6 gives the model form for this scenario. There does not seem to be a direct analogue with the IOBPCS family, although we do again have the ordering rule for \( A*_{N} = \text{ORATE} \) as in Scenario 1 but, with a product hierarchy.
defined as opposed to an aggregate product as in Figure A3.3. There is also a lower level ordering rule for $D^*$. This suggests a ‘type’ of APIOBPCS form where there are multiple ordering rules for the product hierarchy that includes a pipeline policy, where there is a total pipeline of FWIP and AWIP ($= DWIP$). Hence, although there is no direct analogy, there is similarity between Figures A3.6 and A3.3.

Figure A3.6: Simplified block diagram representation of the Intel supply chain in ‘push’ scenario

**A3.2 Candidate.**

For scenario 1 we can turn to existing know-how regarding the VIOBPCS model, which may be most comprehensively found in Edghill (1990). She recommended alternative forms of the IOBPCS family in order to avoid bullwhip and stock-out costs. But if the VIOBPCS form is used then her recommendation for a ‘conservative design’ when choosing model parameters. For scenario 2 we could utilise existing knowledge of the APVIOBPCS model, say through studies by John et al. (1994) and Georgiadis and Michaloudis (2012), when designing the system. Perhaps more importantly, we can claim a new form to add to the IOBPCS family, which can be brought under control engineering scrutiny for future research.

**Additional references used in Appendix 3**


Appendix 4 – Proposed empirical exploitation of AR in alleviating the ‘bullwhip effect’

Naim and Towill (1994) developed a framework that utilises multiple methods to aid supply chain managers in ensuring smooth material flow in supply chains with a particular focus on mitigating the ‘bullwhip effect’. Bechtel and Jayaram (1997) note that Naim and Towill’s (1994) “methodology is a direct offshoot of the pioneering works of Jay Forrester”, which has subsequently been advocated, utilised and adapted by many authors (Tibben-Lembke, 1998; Hong-Minh et al., 2000; Kumar and Yamaoka, 2007; Raj and Lakshminarayanan, 2008; Kumar and Nigmatullin, 2011) to design efficient supply chains, re-engineer processes and analyse supply chains’ dynamic behaviours.

Based on Naim and Towill’s (1994) approach, an empirical method to exploit AR is presented in Figure A4.1. The main difference between Naim and Towill's (1994) framework and the one presented Figure A4.1 is the introduction of a ‘simplification’ stage and the incorporation of existing known models and solutions in the supply chain design decision making. The new elements of the process in Figure A4.1 are given by the shaded block and dashes arrows.

In this method, there are two distinct, but overlapping, phases of analyses. In the qualitative phase, both the objective of the study and the key drivers are identified through an intuitive and conceptual modelling process. Then, the relationships among key drivers are represented in a block diagram. The second phase is the quantitative analysis, which is associated with the development of mathematical and simulation models.

This phase starts by exploring a particular supply chain system and defining its boundaries and interfaces. For that, knowing the business objectives is very important. Forrester (1961) also indicated that in designing a model of an organisation the elements that must be included arise directly from the questions that are to be answered or objectives that are to be achieved. Moreover, since there is no all-inclusive model, different models should be created to address different questions about the same system and models can be extended or altered so that new objectives are achieved. Naim and Towill (1994) suggested that four main business objectives can be evaluated using their framework; inventory reduction target, controlled service levels, minimum variance in material flow and minimum total cost of operations and procurement.

The next step is to describe how the material and information flows occur and how the ordering rules are defined. This input-output analysis (Parnaby, 1979, Bonney and Jaber, 2013) will identify material and information delays, production and logistics constraints, how information is processed and how planning and scheduling operations are undertaken. The information obtained from this step supports the development of a suitable conceptual model, which can be illustrated in the form of causal loop diagrams, cognitive maps or other appropriate soft system methodology tools (Wilson, 2001). These illustrative diagrams are reported to help in communicating with the relevant people in the supply chain and extracting more information to refine the model (Naim and Towill, 1994; Hicks, 2004). The data for such visual models may come from a variety of opinion, archival, analytical and observations sources.

Finally, as the operations and control procedures become known, the soft system diagrams can be converted into block diagram form. The latter contains mathematical descriptions of the relationships between the various interacting variables in the conceptual model. Each element in the block diagram establishes a relationship by including a mathematical expression that, for example, may represent delays or stock levels.

In the case of the analysis presented in this paper the former steps in the empirical study are assumed to have been undertaken and that they have yielded the block diagram of Figure 1 a). In the Forrester (1961) approach his conceptual
model was created by establishing the business objectives and by generating a flow diagram for each supply chain echelon. However, Forrester (1961) has never translated his model into block diagram form since the only method advocated by him was Industrial Dynamics computer simulation. Hence, he advanced from a flow diagram to the description of the system equations, as in Appendix 1, that need to be defined in the simulation programme, which currently may be defined by such software as Vensim® or iThink® rather than DYNAMO which is no longer available.

The first step in the adapted approach given in Figure A4.1 for the quantitative analysis of complicated models is to undertake simplification for that is when the underlying control mechanisms are revealed (Wikner et al., 1992). The technique we give here is block diagram manipulation, which is the rearrangement of the block diagram obtained from the conceptual model into a reduced form by identifying and eliminating redundancies, collecting constants and moving blocks to create familiar forms in the model. Although this technique may reduce the number of variables and equations, it ensures that no causal relationships between variables are lost.

Moreover, as we have shown in Section 2, there is the opportunity to also exploit AR. The simplification will ensure that we have correctly identified the target model and hence also the source model and solution. In the example given in this paper the target model is defined by Figure 1 a) and the source model is given by Figure 3 c). A source model is given by the findings of Burns and Sivazlian (1978), which is the use of filtering to reduce the ‘bullwhip effect’. The source model is one input into the dynamic analysis to be undertaken in determining a range of cost-benefits of different options to improve the performance of the supply chain.

After having a better understanding of the system's behaviour and its underlying structures, single or repeated simulations can be carried out to confirm the insights acquired in the previous step and to obtain a more exact result of the system responses. The advantage of simulations is that the original conceptual model can be studied without simplification, but from previous researcher, it is noted that simulation on its own can be `dangerous' (Atherton, 1975; Towill, 1981; Rugh, 2002) as a "guess and check" approach may overlook underlying mechanisms and dynamic behaviour as well as being very time consuming and inefficient.

For brevity we do not dwell on the other elements of Figure A4.1, which are documented in the original paper (Naim and Towill, 1994). The quantitative stage brings together control theory, simulation and statistical techniques, each having its own strengths and weaknesses but when combined provides comprehensive synthesis. The first method is a key element of the AR approach advocated which is supplemented by computer simulation. The third method has not been addressed in this paper but may involve detrending, smoothing, range analysis, auto- and cross-correlations to identify features in the data, such as degree of scatter, short/long term trends, cyclical variation and exogenous events.

Comparison and validation of the model normally involves consultation with the interested parties in the supply chain to ensure correctness of the model. Then, real data is inputted from the supply chain system into the model and validation is obtained by comparing the outputs from the model with the output of the real system. In this research, this type of validation was not carried out since the author used pre-existing and well-established supply chain models. Hence, it is assumed that both the models of Forrester (1961) and Burns and Sivazlian (1978) have been formerly validated with real data.

Following the validation process, the model can be subjected to extensive dynamic analysis. The objective of this stage is to determine the dynamic performance of the supply chain by subjecting the model to test inputs, such as steps, pulses, cycles and stochastic time series. Finally, the supply chain models can be further inspected by changing the control
procedure, creating various scenarios and undertaking sensitivity analysis to reveal how vulnerable the supply chain is. For this type of analysis, computer simulation methods can be used for generating results relatively easily and quickly. Naim and Towill (1994) suggest a structured approach to exploit supply chain models:

- Tuning existing parameters: supply chains can be redesigned by maintaining the original supply chain structure but varying the control parameters to improve performance.
- Structural redesign: this involves altering the model's structure, such as removing an echelon or including a feedback information into the control system.
- `What if?' business scenarios: this involves testing how the supply chain performs for alternative business propositions or unexpected changes in the business scenario, e.g. the impact of changes in physical parameters, such as lead-times.

The final major feedback loop in Figure A4.1 suggests a substantive change management programme, whereby the outcomes of the supply chain redesign are implemented in the ‘real-world’.

**Additional references used in Appendix 4**


Figure A4.1: Adaptation of the Naim and Towill (1994) supply chain design method to incorporate AR.