Testing a DSGE Endogenous Growth Model of R&D via Indirect Inference: Productivity Growth in a Panel of OECD

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Abstract

This thesis investigates the causal impact of research and development as the driver to the growth for a sample of 11 OECD countries over the period of 1980-2014. The R&D-driven growth hypothesis is embedded within a calibrated dynamic stochastic general equilibrium (DSGE) model to be tested via Indirect Inference simulation-based method of testing and evaluation; the method which relies on the comparison between the features of the model-generated and actual data through the auxiliary model. This method ensures the identification of the DSGE model hence there is no ambiguity in defining the direction of the causation in the model which comes from the R&D spending to productivity growth. The parameters of interest are also estimated using 'simulated annealing' algorithm and the parameter-modified model is tested by Indirect Inference Wald. The test results for the specified model satisfies the non-rejection condition where the relevant statistic lies within the 95% confidence interval. This thesis suggests an explicit empirical evidence that for the small open economies of OECD, the R&D spendings as a proxy for innovative activities causes a long-run growth episodes.

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I dedicate this thesis to my loved ones; to Reza and my beloved sister Mehrnoush.

Contents

Chapter 1

Introduction

A brief review of the literature on growth theories confirms significant developments in the number of studies focused on determining the causes and welfare effects of growth in nation's productivities. There are extensions to valid theories which depict the association of productivity growth with a variety of economic, social and political variables. Although there are extensive improvements in our understandings of the growth matters during past few decades, the question of what truly causes a sustainable growth is still a fresh progressive topic. Between all of the philosophical, societal and economic viewpoints about the growth, the new growth theories have a significant role in explaining the different aspects of the issue.

Among these highly accepted theories with the economic growth as their focal point, four direct extensions to Solow's (1956) model are the most influential ones: Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt's (1992) R&D-driven model of endogenous growth and Jones (1995b) semi-endogenous growth model with diminishing returns to knowledge of R&D are the examples of responses to the shortcomings of Solow's theory. Despite their differences, these theories believe that the steady growth can be generated endogenously in a sense that it can occur without any exogenous technical progress. Accordingly, the growth rates based on these models may depend on preference and technology parameters and even activist policies.

R&D-driven growth theories emphasise on the impact of innovative activities and research expenses as the growth-enhancing elements. Based on these models the innovative activities will ensure a sustainable long-run growth. Hence, the positive effect of R&D on TFP growth is highly approved analytically; however, the magnitude of the effect and the direction of the identified causal relation between TFP growth and R&D are still matters of empirical doubts. The controversy on the direction and magnitude of the effect may raise a question of how effective are the R&D enhancing policies as mostly these policies are very expensive. As accurate as the answer to this issue may be, the more efficient the implied policy decisions are.

One may ask if this hypothesis worth any further investigations. Despite the fact that most of the theoretical and empirical studies on the relation between R&D investment and TFP growth provide substantial evidence that these two factors are positively related (see Scherer, 1982; Griliches, 1982; Aghion, 1992; Zachariadis, 2003), these theoretical frameworks have never been tested and their ability to be empirically rejected has not been appropriately investigated. More importantly, not so much systematic causal analysis has been performed on the relation between TFP and R&D. Although in this case, one expects the causation to be bi-directional in nature, this certainly needs to be tested. On the other hand, access to improved data for research and development expenditures and more advanced computational methods, make it vital to call up empirical models and raise the accuracy of their results by inspecting them in more details.

The structure of this thesis is as following. In Chapter 2, a literature review on dominant endogenous growth models is given. The focus is on those endogenous models with research and development as the primary driver; accordingly, some pros and cons and improvements in the structure of these models are briefly noted. The summary of the analytical studies confirms a positive marginal impact of R&D. Following the literature of the analytical approaches, a brief overview of empirical studies with the aim to find the rate of return to R&D is given which shows controversial ideas for the magnitude and even the direction of the effect of innovative activities on TFP growth. This will raise questions about the empirical validation of the analytical theories and if they are following the correct path in analysing the specified determinants of growth. This is to motivate the growth model of Chapter 3 which is tested and estimated in Chapter 5.

In Chapter 3, a dynamic R&D-driven rational expectation model of endogenous growth is proposed. A representative agent model of the open economy is depicted, and the relations between aggregate macro variables are determined. This provides a simple but an identified model of endogenous growth which isolate the effect of innovation (R&D expenditure as a proxy for innovation) to evaluate the possible consequences of the relevant policies. Complications raised by augmenting a real business cycle model to endogenize the growth factor of productivity in an open economy make the simplification valid and defendable. Needless to say that this model wants to reevaluate the effectiveness of the highly accepted R&D on growth through a panel study, hence simplifying the model assists the rather complicated computational method described in Chapter 5. Subsequently, the model is calibrated using the stylised facts of the growth and RBC literature, and the impulse response functions of a one-off shock to R&D factor are illustrated.

The method of Indirect Inference as the methodology to evaluate and estimate the outlined RBC growth model is discussed in Chapter 4. The progressive method of Indirect Inference provides a classical statistical inferential framework for evaluating a partially estimated or a calibrated model, such as the one under my study, maintaining the spirit of the early methods of evaluation for RBC models. The main characteristic of this approach is using an auxiliary model to frame the actual data and model-generated data and compare these two frames to see if the model is "true" in the spirit of Friedman's positivism. Following Le and Meenagh (2013), the process of estimation via simulated annealing method is adopted, and the choice of panel VECM as the auxiliary model are described.

In the fifth chapter, the testing and estimation process described in Chapter 4 is used to evaluate the R&D-driven RBC model calibrated in Chapter 3. Firstly the data for the economic and science and technology indicators are introduced. The choice of gross domestic research and development expenditure share in GDP to proxy the innovation in the model is reasoned, and the details about the relation between R&D intensity and TFP are given. Secondly, the baseline calibrated model is tested, and the related Wald statistics are found and compared to the 5% critical value. Thirdly, the model is estimated using a simulated annealing algorithm discussed in Chapter 4. This method is based on a search about 35-45% above or below the initial calibration value for the parameters of interest and the model with a new set of parameters found in the specified range will be tested. These adjustments result in a model which certifies the positive effect of innovation hence confirm the necessity of a policy intervention to increase the growth-enhancing innovative activities.

Chapter 2

R&D as the Driving Force of Economic Growth : A Review of the Literature

"It is however always important to remember that the ability to see things in their correct perspective may be, and often is, divorced from the ability to reason correctly and vice versa. That is why a man may be a very good theorist and yet talk absolute nonsense...." -Joseph A. Schumpeter, 1943

2.1 Introduction

An overview of the literature reveals a notable growth in empirical and theoretical studies on the sources and causes of growth in nations' productivities. There are varieties of studies depicting the association of productivity growth with so many economic, social and political variables including many affected by government policies, while there are analytical models which tend to focus more narrowly on some specific sources of growth. Some of these efforts explain many of what Kaldor (1961) refers to the "stylised facts" of economic growth.

The first approach of these analytical models such as Jones and Manuelli (1990), Rebelo (1991) and King and Rebelo (1993) focuses on capital accumulation, broadly defined to include human capital as the driving force of economic growth. The second approach identifies the external economies as the driver for the growth in meaning that firm's investment in physical (inspired by Arrow's (1962) paper) and human capital (Lucas, 1988) contributes to the productivity of capital held by others. The third approach believes that the evolution and adaptation of new ideas are the channels to sustainable growth in productivity. The pioneers on the latter are Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992). Based on this approach, the profit-motivated innovations resulted from research and development lead to the accumulation of knowledge which becomes a primary source of growth. It must be considered that the main feature of R&D driven-growth theories is their explicit identification of a mechanism which ensures a long-term impact of policies on growth.

Alongside these analytical works there is a majority of empirical studies attempts to shed some light on the role of R&D as the proxy to innovative activities in enhancing TFP (proxy for growth), see the U.S. Bureau of Labor Statistics (1989) and Sveikauskas (2007), as well as past issues of the OECD STI Review; for the UK, see the report by Griffith et al. (2003), and for Canada, Longo (1984) and Mohnen (1992a). One cannot claim the existence of a consensus on the size of the R&D impact on growth or the direction of the effect in past empirical works as the results are mixed and not always confirming the theoretical proposition. Many studies find the marginal impact of R&D to be high and positive (see Hall and Mairesse, 1995; Adams and Jaffe, 1996) which is a confirmation for most of the theoretical works and several other studies which are indicating that although investment in R&D increases the knowledge accumulation of the society, the effects on the level of growth for representative firm can be insignificant or even negative (Link, 1981; Sassenou, 1989)- some firms may benefit from the increase in their R&D and some just suffer from the costs without experiencing the increase in their productivity.

The controversy on the effect may raise a question of how important are the R&D enhancing policies as mostly these policies are very expensive. In this thesis, I propose a R&D-driven DSGE model of endogenous growth which is outlined in Chapter 3. A panel data technique and data of 11-OECD countries for the period of 1980-2014 is used to investigate the following postulation of this endogenous growth model: R&D investment as the proxy to the innovation leads to permanent increases in TFP. This hypothesis is then tested and estimated using Indirect Inference method (see Chapter 4).

One may ask the question that if this hypothesis worth any further investigations. I shall answer it with a yes. Despite the fact that most of the theoretical and empirical studies on the relation between R&D investment and TFP growth provide substantial evidence that these two factors are positively related (see Scherer, 1982; Griliches and Lichtenberg, 1984; Aghion and Howitte, 1998; and Zachariadis, 2003), these theoretical frameworks have never been tested and their ability to be falsified has not been appropriately investigated. More importantly, not so many systematic causality analysis has been performed on the relation between TFP and R&D. Although one may, in this case, expect the causation to be bi-directional in nature, this certainly needs to be tested. On the other hand, access to improved data for research and development expenditures and patenting the innovations, make it vital to call up empirical models and raise the accuracy of their results by inspecting them in more details.

To motivate the R&D-driven growth model tested in Chapter 5, this chapter provides a review of some existing literature on the question of study as well as some historical facts about R&D intensity factor and the relevant policies for the 11-OECD countries of the study. Unlike most of the related micro-level studies which focusing on policies targeting particular firms, sectors or industries, in this thesis, I look at aggregate impacts of innovation channel (here R&D expenditures) at the macroeconomic level.

2.2 Theoretical Background of Economic Growth

"Economic growth, the process by which a nation's wealth increases over time...in the context of economic theory, it generally refers to an increase in wealth over an extended period." - Encyclopædia Britannica

2.2.1 Neoclassical vs. Endogenous Growth Analysis

Although the issue of economic growth received considerable attention directly after the second world war, there are more references to the economists' concerns about the growth-related issues even before that. Analysis of the process of growth was a central feature of the work of the English classical economists, as represented chiefly by Adam Smith, Thomas Malthus and David Ricardo. The interest of these economists on growth economy mostly rooted in the concrete conditions of their time. Live on the eve or in the full throes of the industrial revolution; they could not help it but be impressed by the social and economic changes of the time. The interest of these economists in economic growth was not just because of the time but also a philosophical concern with the possibilities of "progress" - the necessary conditions of developments of the material basis of society (Harris, 1975). It can be felt that the purpose of their analyses was to identify the societal forces ensuring these developments and consequently to provide a basis for policy and action to influence growth-amplifying forces. Smith's attack to monopolistic privileges associated with mercantilism (1776)[1], Malthus's concern with population growth (1798) [2] and Ricardo's campaign against the corn law in 1815 [3] are examples of activities condemning policies which prevent the nation's economic progress. Witnessing the growth in inputs such as capital resulted in the growth of output, made these economists, notably Smith, to determine the capital accumulation through deliberate saving or what Smith mentioned as *"parsimony"*, as a fundamental driving force behind economic growth. The question is,*How long this capital accumulation is going to last?*

Continuous capital accumulation is the heart of the classical theory which is systemised by John Stuart Mill (1854), and according to it any increase in capital investment increases the labour demand, and in the absence of growth in the numbers of workers, real wages rise which in return stimulate long-term population growth.

[1]Smith was not optimistic about the chances of Britain introducing free trade because of the outspoken opposition and political power of the vested interests. He freely compared the protected manufacturing interests to "an overgrown standing army" which would focus the "insolent outrage of furious and disappointed monopolists" which no politician would dare cross.

[2]"The superior power of population cannot be checked without producing misery or vice."-Essay on the Principle of Population(1798)

[3]In "Essay on the Influence of a Low Price of Corn on the Profits of Stock' which he published in 1815, he argued that raising the tariff on grain imports tended to increase the rents of the country gentlemen while decreasing the profits of manufacturers.

This then results in a growth in the number of the 'mouths' in the economy which means a higher demand for consumer's goods such as agricultural products; as them characterised by decreasing returns to scale, and it raises the issues related to the decline in marginal productivity of capital and fall of incentive to invest. Thus, in classic economic growth theory, it is highly believed that a non-declining marginal productivity of capital ends up to a sustained higher level of growth in the economy if it happens.

In this sense, Solow's (1956) neoclassical model of growth demonstrates that a sustainable positive per capita growth in long-run is possible. Based on this theory if labour is constant then technological progress can overcome the negative effect of diminishing rate of return on capital and hence deliver a sustained positive per capita growth in a long-run where the per capita output grows at the same rate as the rate of the technological progress. A contemporaneous study by Swan (1956) developed a similar analysis with a less explicit mathematical structure [4].

Before Solow and Swan, seminal papers by Harrod (1939) and Domar (1947) received extensive attention. They present that in a steady state of the economy it is necessary that product of the saving-output ratio and the output-capital ratio be equal to the rate of growth of potential output. In other words, if the output-potential output ratio is to remain constant, capital; K_t and the potential output; $\bar{Y}$ must grow at the same rate. They suggest that these three numbers; K_t, $\bar{Y}$ and $\frac{Y_t}{\bar{Y}}$ might be determined by different aspects of economic behaviour, hence it is unlikely for the market economy to be able to ensure the satisfaction of this condition and thus an active intervention of the government is required. On the other hand, Solow (1956) assumed that the ratio of output to potential output is constant which was the

[4] see discussion of the "golden rule" condition in papers by Phelps (1961) and Solow (1962).

matter of concern to Harrod and Domar. Solow observed that output-capital ratio could adjust endogenously, but as Hahn (1987) has noted, this observation does not actually speak to the Harrod-Domar problem.[5]

MacCallum (1996) defends Solow saying that although his contribution is not a complete optimising model, but he and Swan developed "something that might reasonably be called model", in the sense of a "falsifiable depiction of some economic phenomena", whereas Harrod and Domar had only derived a condition for a steady growth using a simple algebra which is required to be satisfied. He then continues with outlining the reason why the Solow-Swan's neoclassical approach fails to explain even some basic economic facts about actual growth behaviour. MacCallum (1996) believes that the failure is rooted in the scheme where the model predicts output per person to approach a steady-state path along with it grows at a rate which is exogenous to the model- the growth rate is independent of preferences, most aspects of the production function and also policy variables. As a consequence, the model suggests a constant growth rate for all economies or different values "about which it has nothing to say" for the real world in which the nations have different per capita growth rates, the rates that are systematically related to a variety of features of the economies. Attempts to response to these and other failings stressed by Romer (1986, 1987, 1989), Lucas (1988), King and Rebelo (1990) involved in the resurgence of growth theory which is known as endogenous growth models. The general feature of these 'new growth' theories is the presence of constant or increasing returns to

[5] "It will be noted straight away that [Solow's] argument has no bearing on Harrod's knife-edge claim. Harrod had not proposed that warranted paths diverge from the steady state, but that actual path did. The latter is neither characterised by a continual equality of ex-ante investment and savings nor by continual equilibrium in the market for labour. Thus although Solow thought he was controverting the knife-edge argument, he had only succeeded in establishing the convergence of warranted paths to the steady-state." (F.H. Hahn, 1987)

factors that can be accumulated such as capital (Barro, 1990).

In endogenous growth models, steady growth can be generated endogenously in the sense that it can occur without any exogenous technical progress. Accordingly, the growth rates based on these strand of models may depend upon preference and technology parameters and even activist policies. For the sake of clarity I relate the new growth theories to three categories: endogenous growth models with perfect competition (Romer, 1986; Lucas, 1988; Rebelo,1991), endogenous growth with imperfect competition which consists of two strands- models with horizontal product differentiation (Romer, 1990; Grossman and Helpman, 1991; Barro and Sala-i-Martin, 1995, ch.6) and models that assign economic growth to a vertical product differentiation (Segerstrom et al. 1990, Aghion and Howitt, 1992 and 1998; Barro and Sala-i-Martin, 1995, ch. 7) with both of them focused on the impact of R&D on economic growth and finally semi-endogenous growth models (Jones, 1995b) and Endogenous growth models without the scale effects (Young, 1998).

In the following sections, I briefly overview these approaches to endogenous growth.

2.2.2 Endogenous Growth Models with Perfect Competition

It is outlined in the previous section that the neoclassical theory of economic growth, Solow-Swan (1956) model of endogenous growth- attributes the long-run growth of output to the technological progress with a consideration that the level of technology is taken to be an exogenously growing factor *outside* of the system.

This then brings up a question: Why Solow and others (Cass, 1965; Koopmans, 1965) made this assumption? One answer to this question may be that models of perfect competition are the simplest existing models of firm behaviour with easy-to-understand implications, but these strand of models require constant returns to scale, in better words, the sum of the factor payments exhausts all the output as Euler equation says

$$Y_t = K.\frac{\partial Y_t}{\partial K_t} + N.\frac{\partial Y_t}{\partial N_t} \tag{2.1}$$

hence, the perfectly competitive firm has nothing left with which it can finance basic research, invent patentable technologies or do anything other than only paying capital rent and ensuring the payroll of production workers. According to this fact, the firm is not capable of financing the growth-enhancing activities; therefore, the only alternative is to assume the technological progress to occur exogenously. Paul Romer's (1986) with his famous paper, "Increasing Returns and Long-Run Growth", led the way to the formulation of the new generation of models with accumulated knowledge as the driver of the growth. Romer sketched his knowledge-based endogenous growth as following, firm j's production function, $j = \{1, ..., J\}$ is the number of representative firms, presented as

$$y_{j,t} = A_t F(k_{j,t}, n_{j,t}) \tag{2.2}$$

where $k_{j,t}$ and $n_{j,t}$ are capital and labour inputs for the firm j respectively. A_t is the aggregate output-augmenting technological progress. It is assumed that the capital of the firm accumulates without depreciation.

$$\dot{k_{j,t}} = i_{j,t} \tag{2.3}$$

with the assumption that there is no population growth and considering individuals are distributed along the unit interval, aggregate investment is

$$I_t = \int_0^1 i_{j,t} d_j \tag{2.4}$$

the most important assumption in Romer's (1986) model is that the aggregate stock of knowledge in the economy is proportional to the cumulative sum of past aggregate investment, which is identical to the size of the aggregate capital

$$\Xi_t = \int_{-\infty}^{t} I_v d_v = K_t \tag{2.5}$$

then he determines productivity via the effect of the stock of knowledge, $A_t = \Xi_t^{\psi}$, where $\psi < 1$. Accordingly the firm level Cobb-Douglas production function will be

$$y_{j,t} = k_{j,t}^{\alpha} n_{j,t}^{1-\alpha} \Xi_t^{\psi} \tag{2.6}$$

which is a constant return to scale (CRS) model of production in (k_t, n_t) at the firm level where the elasticity of substitution between capital and labour, α, is between zero and unity and aggregate knowledge, Ξ_t, is fixed. Therefore the aggregate level output is

$$Y_t = K_t^{\alpha} N_t^{1-\alpha} \Xi_t^{\psi} \tag{2.7}$$

Romer assumes the representative household maximises a typical CRRA[6] utility function and ignores the trivial effect her own investment decision has on aggregate

[6]The consumer maximises the discounted sum of Constant Relative Risk Aversion utility function

$$u(c) = Max \int_0^{\infty} [(\frac{1}{1-\rho})(\frac{c_t}{n_t})^{1-\rho}]e^{-\nu t} dt$$

where per capita consumption, $(\frac{c_t}{n_t})$ is assumed to be c_t, ρ measures the degree of relative risk aversion which is implicit in the utility function and ν is the time preference.

knowledge. Hence by using the marginal product of capital, $\alpha k_{j,t}^{\alpha-1} n_{i,t}^{1-\alpha} \Xi_t^{\psi}$, and normalising the model by assuming that the aggregate quantity of labour adds up to $N_t = 1$, one can set up and solve the Hamiltonian to obtain

$$\frac{\dot{c}_{j,t}}{c_{j,t}} = \frac{\alpha k_{j,t}^{\alpha-1} \Xi_t^{\psi} - \nu}{\rho} \tag{2.8}$$

if households are homogenous and the condition $\Xi_t = K_t$ is satisfied, the aggregate consumption per capita evolves according to

$$\frac{\dot{c}_t}{c_t} = \frac{\alpha k_t^{\alpha-1} \Xi_t^{\psi} - \nu}{\rho} \tag{2.9}$$

$$= \frac{\alpha k_t^{\alpha+\psi-1} - \nu}{\rho} \tag{2.10}$$

the economy can have a balanced growth path if $\alpha + \psi = 1$ and there will be a constant growth forever at a rate that depends on the degree of impatience and capital's share in output.

$$\frac{\dot{c}_t}{c_t} = \frac{\alpha - \nu}{\rho} \tag{2.11}$$

Romer's (1986) approach is consistent with both learning-by-doing, where learning is measured by cumulative investments and investment in research and development. Thus, this model can be related to Arrow's (1962) theory of technological change due to learning by doing, which assumes that the labor efficiency depends on capital stock but instead of assuming diminishing returns to learning, takes learning to be proportional to capital stock with the additional difference that here the capital-labour ratio measures the learning rather than the total capital. In Romer's analysis, the knowledge accumulation is an accidental byproduct of the investment decisions of representative firm. In a word, capital accumulation generates intra-firm knowledge accumulation through learning-by-doing, and it can spill over to other firms which provide an increasing return to scale which constitutes a type of positive externality. The spillover mechanism here ensures the perfect competition in endogenous growth, although the resulting equilibrium of spillover effect is sub-optimal. Hence, the social planner takes into account the fact that in the existence of externalities, there is a higher return to capital accumulation at the social level than at the individual level. The steady state chosen by the social planner is

$$\frac{\dot{c}_t}{c_t} = \frac{\alpha + \psi - \nu}{\rho} \tag{2.12}$$

Equation (2.12) implies that the steady-state growth rate depends upon α, ψ, ν and ρ. It can be concluded that the capital accumulation should be subsidised if the social planner wants to induce the private economy to move toward the social optimum which justifies the government intervention.

The second of the two basic endogenously driven growth mechanisms involves the accumulation of 'human capital'[7] in the sense that labour force's skills will be enhanced by the adequate investment of valuable resources on them. Lucas (1988) presents a human capital driven endogenous growth in which output is generated via a production function of the form

$$Y_t = A_t K_t^{\alpha} \big(\ell H_t N_t\big)^{1-\alpha} \tag{2.13}$$

[7]Human capital is defined in the Oxford English Dictionary as "the skills the labour force possesses and is regarded as a resource or asset." It encompasses the notion that there are investments in people (e.g. education, training, health, etc...) and that these investments increase an individual's productivity.

where ℓ is defined as the proportion of total labor time spent working [8] and H_t is the stock of human capital. Rewriting equation (2.13) in per capita terms gives a CRS production function as

$$y_t = A_t k_t^{\alpha} \big(\ell h_t\big)^{1-\alpha} \tag{2.14}$$

Capital accumulation proceeds via the usual differential equation

$$\dot{k}_t = y_t - c_t - (\kappa + \delta) k_t \tag{2.15}$$

while h_t evolves according to

$$\dot{h}_t = \phi h_t (1 - \ell) \tag{2.16}$$

hence

$$\frac{\dot{h}_t}{h_t} = \phi(1 - \ell) \tag{2.17}$$

where ϕ is the parameter that determines the efficiency of human capital accumulation. In Lucas's (1988) paper, he does not examine the Solow version of the model with a constant saving rate but instead he considers the version in which a social planner solves for the optimal perfect foresight paths of k_t and h_t. I will not go through the math here (as it is out of the scope of this literature review), I will just

[8] In Chapter 3, I use Lucas's approach on the time division between activities and I chose the notation ℓ for the time spend on leisurely activities and N for the time spend on normal wage-earning activities.

present the conclusion. The steady-state growth rate is

$$\frac{\dot{c}_t}{c_t} = \frac{\phi - \nu}{\rho} \tag{2.18}$$

Lucas also solves a version of the model with externality to human capital included. The idea is that each individual's productivity is enhanced if they are surrounded by those with a high level of human capital. Thus, the production function in this specific version is given as

$$y_{j,t} = A_t k_{j,t}^{\alpha} \big(\ell_j h_{j,t}\big)^{1-\alpha} \bar{h}^{\psi} \tag{2.19}$$

here $\bar{h}$ is the average human capital in the population and relative to it ψ determines externality in this model. Hence, the steady-state growth rate of human capital for the representative consumer will be

$$\gamma_h = \frac{\rho^{-1}(\phi - \nu)}{\left\{1 + \dfrac{\psi(1-\rho^{-1})}{1-\alpha}\right\}} \tag{2.20}$$

Following the consumer's homogeneity assumption, the derived growth rate of aggregate human capital (and the growth rate of all the rests) is the same for all individuals. If there were no externalities to affect the growth, $\psi = 0$, equation (2.18) would be the solution. Since saving is the source of growth in endogenous growth models, in the existence of the externality, the solution depends on the value for ρ^{-1} being equal, greater or less than unity. The $\bar{h}$ externality is like an increase in the interest rate; thus it's effect is determined by the trade-offs between income and substitution effects. If $\rho = 1$ the income effect exactly offsets the substitution effect and leaves the consumption unchanged in response to changes in $\bar{h}$ and (2.20) collapses to (2.18), but in a situation where consumers are very willing to cut current consumption in exchange for higher future consumption, $\rho^{-1} > 1$, the externality enhances saving and therefore growth, otherwise, the income effect outweighs the substitution effect, makes saving to fall and growth will be slower. Continuing his model, Lucas shows that this decentralised solution is sub-optimal since representative consumer does not obtain the full benefits to society of increasing their own stock of knowledge. Investment of more time on human capital-enhancing activities and accumulating $h_{j,t}$, individuals increase $\bar{h}$, which benefits the society in addition to themselves. Based on these features, Lucas shows that to obtain the socially optimal solution, the economy requires more of "the good thing" and increases the investment in human capital.

The difference between Romer and Lucas's theories is that in Romer (1986) the growth is caused by accumulating technology (or knowledge) and human capital is only seen as knowledge and ideas (in his later paper, Romer (1990) considers these 'ideas', non-rival and partly excludable) but in Lucas's approach it is the human capital formation itself which drives the endogenous growth via non-decreasing marginal returns. In both models, to get the benefits of positive externality and ensure the competitive optimal market solution, the role of a centralised social planner is emphasised. Thus, choosing any of the two human capital or accumulated knowledge as the channel of growth makes it inevitable for policy authorities to intervene in the market-based economy to push the society to the optimal level.

Rebelo (1990) proposes a similar class of economies with Romer (1986) sharing the property that growth is endogenous and increases in productivity attributed to technical progress in the neoclassical growth model. It is the simplest possible endogenous growth model, AK, where the production function is assumed to be linear in the only input, capital, and it rules out the labour income by assumption. Taking the taxation of income to account, it is shown that the public policy of increasing the income tax substantially reduces growth rates. The main difference between Rebelo (1990) and Romer (1986) is that Rebelo's model describes a constant returns to scale technologies which are compatible with the stylised facts of economic growth described in Kaldor (1961). Following Romer's paper, the competitive equilibrium in Rebelo's model is under the perfect foresight, and it can be computed as a solution to a planning problem considering the fact that in the absence of distortion the equilibrium is Pareto optimum. Although having much shared with Romer's model, Rebelo concludes that the increasing returns[9] and externalities are not necessary to generate endogenous growth. He then emphasises on the fact that as long as there is a "core" of capital goods whose production does not involve non-reproducible factors, endogenous growth will be consistent with the constant return to scale economies.

Three models described in section (2.2.1) allow public policies to affect the long-run growth rate through channels of human capital, externalities and direct intervention as taxation. This strand of endogenous growth model considers a typical Walrasian equilibrium wherein the absence of any shocks to the system the model generates a Pareto optimal solution. Section (2.2.2) outlines the literature on models with imperfect competition which are known as R&D-based endogenous growth.

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[9]Romer(1990) worked on a consistent increasing rate of return.

2.2.3 Endogenous Growth Models with Imperfect Competition

According to Hornung (2002), R&D-based endogenous growth models consist of two major types: models comprise horizontal product differentiation (i.e. expanding product's variety) such as Romer (1990), Grossman and Helpman (1991a, ch. 3), Barro and Sala-i-Martin (1995, ch. 6) and also the type which encompasses models that assign economic growth to vertical product differentiation (i.e. improving product quality) studied by Segerstrom et al. (1990), Grossman and Helpman (1991a, 1991b), Aghion and Howitt (1992), Barro and Sala-i-Martin (1995, ch. 7) and Aghion and Howitt (1998, ch. 3). Throughout both, the growth models are based on R&D-capturing the imperfect competition effect through the R&D marginal effect on endogenously driven growth in which the pace of the long-run sustainable growth is determined by the number of the researchers and scientists and in general case the research and development expenditures, hence subsidising the research and development activities will unambiguously and positively affects total factor productivity.

Romer's (1990) seminal paper provides a general idea about the first-generation of R&D-based endogenous growth structure. This model has four variables; output, Y_t, capital, K_t, labour, N_t and knowledge (the terms technology or ideas can also be used), A_t. The simplest model of R&D-based endogenous growth has two sectors (later developments in Romer-type models consist an intermediate goods sector too); a final goods sector and R&D sector which produce final goods and knowledge respectively. Now the allocation of the labour should be considered; here it is assumed that part of labour allocate their time on producing final goods and the rest are producing knowledge (scientists and innovators); hence, the economy as a total has the labour endowment of $N_{Y,t} + N_{A,t} = N_t$. Now a Cobb-Douglas Production function for output is assumed

$$Y_t = K_t^{\alpha}(A_t N_t)^{1-\alpha} \tag{2.21}$$

with $0 < \alpha < 1$ as the marginal elasticity of substitution of inputs. The important part of these models is the "new ideas" production function which happens in the R&D sector by assumption.

$$\dot{A}_t = \bar{\delta}_A N_{A,t} \tag{2.22}$$

where $\dot{A}_t$ represents the flow of new knowledge or the number of new ideas generated by the scientists in R&D sector, $\bar{\delta}_A$ is the average research productivity having following structure

$$\bar{\delta}_A = \delta_A A_t^{\psi_A} N_{A,t}^{\lambda-1} \tag{2.23}$$

where $\delta_A > 0$, ψ_A, and λ_A are constant parameters, A_{t-1} is the existing stock of knowledge at time t, and $A_t^{\psi_A}$ intended to capture the effect of stock of knowledge [10] on the current research productivity. Equation (2.23) presents that any increase in the stock of knowledge may increase the present current research productivity, hence $\psi_A > 0$ will be the positive "spillover of knowledge" to future researchers, and it is referred to "standing on the shoulder" effect in literature. It is also possible for effect to be negative, $\psi_A < 0$, which means discovering new ideas becomes more difficult during the time as most of the original distinct ideas are already found- it is called "fishing out effect". The situation where $\psi_A = 0$ is referred to a time when standing on the shoulder completely offsets the negative effects of fishing out; thus

[10] The knowledge stock can simply be identified as the accumulation of all ideas already been discovered by scientists or developed by inventors.

current research productivity becomes independent of the knowledge stock. One may question the presence of $N_{A,t}^{\lambda-1}$ in equation (2.23). This captures the effect of the number of labour contributing in discovering or developing new ideas on the research productivity at time t. It is probable that by increasing the number of workers allocating their time on research and development, increases in the frequency of duplication or overlapping of discoveries happen. In that case, by doubling the number of researchers, $N_{A,t}$, the number of unique ideas, $\dot{A}_t$ is less than double of "stepping on toes" effect which can be captured by allowing $0 < \lambda < 1$. Combining (2.22) and (2.23) gives

$$\dot{A}_t = \delta_A N_{A,t}^{\lambda} A_t^{\psi_A} \tag{2.24}$$

Equation (2.24) suggests that any growth in the number of new ideas at any given point in time depends on the number of researchers and the existing stock of knowledge. Given the above set up the balanced growth path will be

$$g_Y = g_K = g_A \tag{2.25}$$

where g_Y, g_K and g_A are the steady state growth of output, capital and technology respectively. Hence, R&D-based endogenous growth shares the same prediction as the neoclassical Solow model. Romer (1990) imposes restrictions, $\psi_A = 1$ and $\lambda = 1$ to equation (2.24), where the first restriction makes $\dot{A}_t$ linear in A_t hence generates growth in the stock of knowledge as

$$\frac{\dot{A}_t}{A_t} = \delta_A N_{A,t} \tag{2.26}$$

and the steady state of the growth rate of the stock of knowledge , g_A will be

$$g_A = \delta_A N_A \tag{2.27}$$

(2.27) shows the steady state growth rate of the knowledge stock and according to (2.25) the steady state of per capita output depends positively on the number of researchers work in R&D sector. This proportionality of the size of the population and the growth rate is called “scale-effects” property which characterises most of the first-generation R&D-based endogenous growth models. Being based on imperfect competition in R&D, Romer’s analysis has important policy implications: policies which encourage more researchers to devote their time to innovative activities (i.e. subsidising R&D sector) have a permanent long-run growth-enhancing impact on the economy. Romer’s (1990) original paper uses the Dixit-Stiglitz (1977) model of product variety assuming new products are no better than existing ones, and there is no uncertainty involved in the structure. This is the hallmark of Romer (1990) in a nutshell and many Romer’s style R&D-based growth models, including Grossman and Helpman (1991a, 1991b) and Aghion and Howitt (1992), follow similar principles in explaining the growth. Grossman and Helpman (1991a, 1991b) construct a model of vertical innovation which implicitly integrates the analysis of Segerstrom, Anant, and Dinopoulos (1990) and Aghion and Howitt (1992) establish a simple Schumpeterian-style[11] growth, modelling the innovation process similar to patent-race literature surveyed by Tirol (1988, ch. 10) and Reinganum (1989). I briefly discuss Aghion and Howitt’s (1992) model as it is more significant of all the similar

[11] Aghion and Howitt (1992 and 1998, ch. 3) embody Schumpeter’s idea of ”creative destruction.” Each innovative activity is the act of creation whose goal is to capture the monopoly rents, but it also destroys the monopoly rents which motivated the previous creation.

models recalled in this section. The structure of Aghion and Howitt's (1992) model is as following. There assumed a three-section economy with a perfectly competitive final good's (manufacturing) sector, a monopolistic sector in charge of producing a single intermediate good and finally a R&D sector comprised of many identical researchers. Similar to Romer (1990) the allocation of labour across the sectors is the matter of interest here; labour divided into those who allocate their time producing a single monopolistic intermediate good, x, and those who devotes their time (labours) to research and development, n. It is assumed that the final good's sector produces goods using an intermediate input purchased from a monopolistic supplier.

$$y = Ax^{\alpha} \tag{2.28}$$

where $0 < \alpha < 1$ and A measures the stock of technological progress. In this model, an innovation raises A by a constant factor like γ. Hence, consider t indexes innovation (and not time in Aghion and Howitt, 1992), $\frac{A_{t+1}}{A_t} = \gamma$. It must be noted that innovations consist of creating new varieties of intermediate goods that replace the old ones, and whose use increases the productivity of the intermediate goods in the production of final goods. Considering the price of intermediate input to be p, the firm will maximise its own profit

$$\Pi = Ax^{\alpha} - px \tag{2.29}$$

from (2.29), the price of the intermediate input can be derived as $p = \alpha Ax^{\alpha-1}$ which implies that final goods producers employ x until its marginal product equals its price. Similar to Romer's model, Aghion and Howitt's has two positive spillovers such as the monopoly rent of the firm is lower than the consumer surplus generated by intermediate output and also because the result of the innovative activities are accessible, the researchers can “stand on the shoulder's of inventors by using their innovative ideas and move on to the next invention (partially non-excludable good). There is also the negative externality of “business stealing effect” which indicates the effect of successful monopolist destroying the previous by making its invention 'obsolete'. Note that the incentive for innovators in this model is the opportunity to get the monopolistic rent, in that case let V_{t+1} denotes the expected discounted payoff to the innovation, hence λV_{t+1} will be the gross income of any innovators in the absence of the new rivals, where λ is the productivity of the research technology. Therefore the net expected profit of this new innovation is

$$\pi = \lambda n V_{t+1} - w_t n \tag{2.30}$$

with n indicates the number of hours or labour devoting themselves to the research and w_t is the wage rate after innovation t. Free entry into the research sector ensures that (2.30) is equal to zero giving; $w_t = \lambda V_{t+1}$ which states that the expected value of one unit of devoted research equals to its costs (no-arbitrage condition). V_{t+1} is determined by

$$V_{t+1} = \frac{\pi_{t+1}}{r + \lambda n_{t+1}} \tag{2.31}$$

Here π_{t+1} is the profit flow attainable by the $(t + 1)$ intermediate goods and the denominator can be interpreted as “obsolescence-adjusted interest rate” presenting the creative destruction effect in the sense that the more research is expected to happen after the current innovation, and the more productive the research is, the shorter will be the duration of the life of monopoly profits enjoyed by the creator of the next innovation, hence the smaller payoff to the innovation. Recalling Romer model, innovations become more valuable over time (complementarities) whereas here they become less valuable (obsolescence). To go further, the profit maximisation problem of the successful innovator is solved. It is out of the scope of this literature work to go through all algebraic steps hence some steps are ignored. To have more information on the details see Aghion and Howitt (1998, ch. 3).

First order conditions are

$$x = \Big(\frac{\alpha^2 A_t}{w_t}\Big)^{\frac{1}{1-\alpha}} = \tilde{x}\Big(\frac{w_t}{A_t}\Big) \tag{2.32}$$

$\tilde{x}$ expresses employment in manufacturing as a function of productivity adjusted wage, $\frac{w_t}{A_t}$ which will be denoted as w_t from now on. Equation (2.32) shows that when the productivity adjusted wage rises, employment in manufacturing decreases.

$$\tilde{\pi} = \Big(\frac{1-\alpha}{\alpha}\Big) w_t \tilde{x} \tag{2.33}$$

profit is a function of employment in manufacturing and productivity adjusted wage where it is decreasing in the latter. Combining no-arbitrage condition with equation (2.31), it can be written

$$w = \frac{\lambda\gamma\tilde{\pi}(w_{t+1})}{r + \lambda n_{t+1}} \tag{2.34}$$

equation (2.34) combined with the constraint of $\{x_t + n_t = L_t\}$, give multiple solutions to this system of equations where these different solutions are associated with different equilibria in the model and average growth rate will be a step-function in which actual growth is a random function of time. The system can be solved for the unique $\hat{w}$ and $\hat{n}$ where the system is at the steady state hence w_t and n_t are constant.

Aghion and Howitt (1992) presents that the average growth rate along the balanced growth path is

$$\hat{g} = \lambda(\hat{n})\, ln\, \gamma \tag{2.35}$$

hence, increases in the arrival parameter, the size of the skilled labor endowment, the size of the innovations all raise the average growth rate in this model. Notice that this model is also characterised by scale-effects property as rises in population will increase $\hat{n}$ and therefore increases $\hat{g}$. The model outlined above can gain richness and realism if there were introduced some sort of capital (physical, human or R&D capital) that affects the arrival rate of innovations, and as Aghion and Howitt note in their seminal paper one of the advantages of this model is it's simplicity which makes it feasible to extend it further.

The scale-effect prediction of the first-generation growth models is at odds with the empirical outcomes of Jones (1995b). Hence a new literature has been developing around the objective of eliminating this scale-effect from the R&D-based growth models. In next section Jones'(1995a, 1995b) criticism to the scale-effects property of the R&D-based model is discussed, the alternative approach of a semi-endogenous growth is outlined, and finally, a fully-endogenous Schumpeterian model of growth is introduced.

2.2.4 Endogenous vs. Semi-endogenous Growth

Jones (1995a) argues that the "scale effects" prediction of the R&D-based endogenous models developed by Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992) are inconsistent with the time series evidence from industrialised economies. He uses the number of the scientists and engineers to proxy the labour population in R&D sector and total factor productivity (TFP) for growth in productivity. The data is for the United States over the post-war period. He presents that the prediction of the R&D-based model about the growth rate of the economy being proportional to the size of its labour force is "easily falsifies". Historical data over 25 years show that the size of the labour force has grown dramatically, but the growth rate is constant or even declining over the period. Figure 2.1 presents that the amount of labour engaged in R&D increases by more than five times from about 160,000 in 1950 to nearly a million by 1988- same pattern for Japan, France and West Germany. As it is depicted in Jones (1995a) the average growth of TFP for the post-war period is relatively constant or even declining at some points.

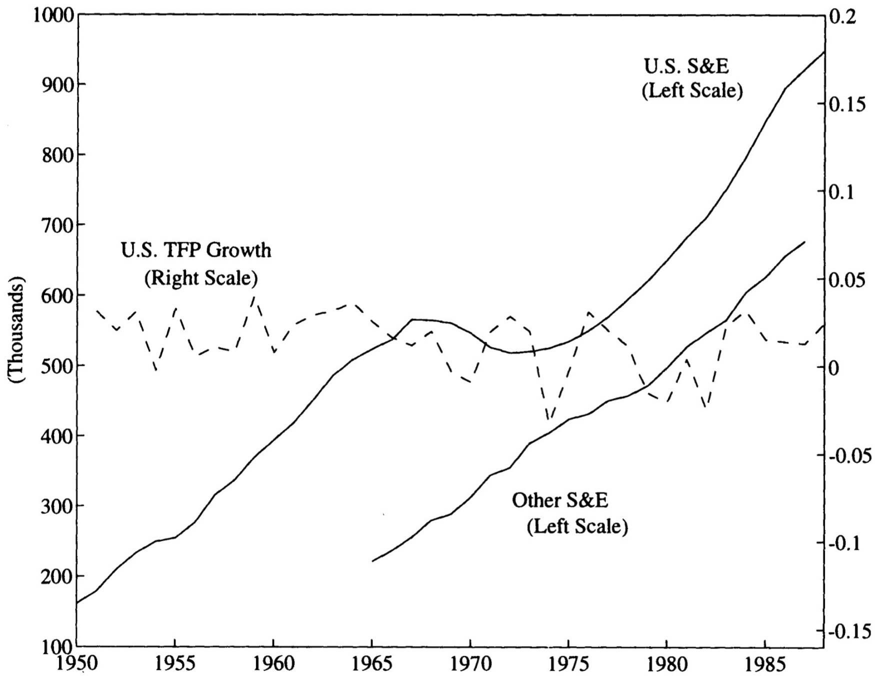

Figure 2.1: Scientists and engineers engaged in R&D. Other S&E is the sum of scientists and engineers in Japan, West Germany and France. Source: Jones (1995a).

Hence, he concluded that the assumption embedded in R&D-based growth model of first-generation that is the growth rate of the economy is proportional to the level of resources allocated to research and development is falsified. Jones (1995b) is an attempt to modify Romer (1990) specification to get a structure more consistent with the time series evidence. To establish a brief review of Jones (1995b) adjusted model, review Romer's (1990) model discussed in section (2.23). Jones relaxes Romer's assumption of the knowledge spillovers being greater than unity. He believes that Romer's value for ψ_A is 'arbitrary'. By rearranging equation (2.24) and dividing it by A_t, he derives

$$\frac{\dot{A}_t}{A_t} = \delta_A \Big\{ \frac{N_{A,t}^{\lambda}}{A_t^{1-\psi_A}} \Big\} \tag{2.36}$$

meaning in the steady state, the growth rate of A_t is constant by definition; therefore, the RHS of equation (2.36) is constant which indicates that $N_{A,t}$ and $A_t^{1-\psi_A}$ must grow at the same rate.

$$\lambda(\frac{\dot{N}_{A,t}}{N_{A,t}}) = (1-\psi_A)\frac{\dot{A}_t}{A_t} \tag{2.37}$$

hence a constant steady state growth rate of A_t is consistent with a rising number of scientists and researchers, $N_{A,t}$ if the condition of $\psi_A < 1$ is satisfied. Imposing this constraint eliminates the scale effects assumed in first-generation R&D-based growth models, thus

$$g_A = \frac{\lambda}{1-\psi_A}(\frac{\dot{N}_{A,t}}{N_{A,t}}) \tag{2.38}$$

that is the long-run growth rate of the stock of knowledge (according to equation (2.25), this equals the long-run growth rate of per capita output) depends on the growth rate of the labour force engaged in R&D rather than its level. Note that the positive knowledge spillovers are not ruled out. The parameter capturing the spillover effects, ψ_A is still strictly positive. Jones argues that the degree of positive knowledge spillovers assumed by Romer (1990) is arbitrary and to achieve the consistency with the historical evidence a weaker magnitude for this parameter is required. Along the balanced growth path, the rate of growth in the number of scientists and engineers equals the rate of growth in the labour force, $\frac{\dot{N}_{A,t}}{N_{A,t}} = \frac{\dot{N}_t}{N_t} = n$, hence

$$g_A = \frac{\lambda}{1-\psi_A}n \tag{2.39}$$

this implies that a long-run growth depends upon the value of parameters ψ_A, λ and n which are assumed to be exogenous to the model, hence, on the contrary to Romer (1990), Helpman and Grossman (1991) and Aghion and Howitt (1992), in

Jones' (1995b) model, the long-run growth is independent of policy variables and the channel of subsidy to R&D changes the long-run level of stock of knowledge but it does not alter the long-run growth rate, this growth is invariant to policy variable. This type of models in which the technological change is endogenous but the long-run growth is pinned down by an exogenous population growth is known as semi-endogenous growth. Following Jones' (1995b) semi-endogenous growth structure, Eaton and Kortum (1998), Dinopoulos and Thompson (1998), Segerstrom (1998), Giordani and Luca (2008) assume similar characteristics for the growth model.

The second-generation of endogenous growth theories consist of another class of models called 'fully endogenous' growth models following Aghion and Howitt (1998, ch. 12), Howitt (2000), Peretto (1998) and Young (1993). These models retain the assumption of constant returns to knowledge stock from the first-generation of growth models before Jones (1995a) critique. Thus, on the contrast to semi-endogenous growth theory, long-run policy implications of the first-generation models are applicable. One of the major assumptions here is that the effectiveness of R&D is diluted due to the products proliferation (Ha and Howitt, 2007) as the economy expands. In other words, it is required to increase R&D over time to offset the negative effect of increasing range and complexity of products which weaken the positive productivity effects of research and development activities.

Using notation in Ha and Howitt (2007), and Madsen (2008) a simple model of Schumpeterian scale-free endogenous growth model is structured as following

$$\frac{\dot{A}_t}{A_t} = \lambda\big(\frac{x}{Q}\big)^{\sigma} A_t^{\psi-1} \tag{2.40}$$

where x is the innovative activities divided by the product variety, Q. The practical result of Q in this models is that it eliminates the scale-effects property of the first-generation models. σ is the duplication parameter- it will be 0 if the innovation is duplication, 1 otherwise- which is assumed to equal unity in Ha and Howitt (2007).

$$Q \propto L^{\beta} \tag{2.41}$$

Aghion and Howitt (1998, ch. 12) and Ha and Howitt (2007) define β as product proliferation, and ψ_A is the return to scale to knowledge parameter. The values for these two parameters will distinguish the endogenous growth models. In the absence of product proliferation effects, $\beta = 0$, the first-generation models assume a constant returns to scale to the stock of knowledge, $\psi_A = 1$, where semi-endogenous growth models assume diminishing returns to knowledge, $\psi_A < 1$. Fully-endogenous models maintain the constant returns to scale property of the first-generation models but assuming there are product proliferation effects in the model, $\beta = 1$.

An interpretation of this trend of models is that as the population grows during the time, there will be an increase in the number of innovators entering the market with a new variety of product hence more horizontal innovations. This dilutes resource spendings on R&D over a larger number of separate projects. The restriction $\beta = 1$ indicates the idea that in the long-run innovative activities and product variety grow at the same rate; hence the growth-enhancing effect of R&D is counterbalanced by the negative effect of the increasing in product variety (Dinopoulos and Thompson, 1998). Jones (1999) criticises the assumption $\beta = 1$ in fully-endogenous models as being a "knife-edge" assumption and claims that by relaxing this assumption the growth model is not fully endogenous anymore since the scale effects are not eliminated. He then proposes a "hybrid" semi-endogenous model with partial product proliferation, $\beta = 1$, and diminishing returns to knowledge, $\psi_A < 1$ which predicts the scale effect.

Peretto & Smulders (2002) is a response to Jones (1999) critique. They establish a model in which the scale effects may be positive or negative, but they always are eliminated asymptotically. The claim that the value $\beta = 1$ is not just an assumed parameter restriction but is a result of very specific microeconomic mechanisms in the knowledge externalities.

There is still debate about which of these two styles of growth models are more empirically correct, hence there is not enough for consensus on the matter. The are scores of empirical economists who test and compare the outcomes of these two strands of models to capture the real effect of R&D channel on growth and test the validity of policy implications, i.e. testing if subsidising R&D is growth-enhancing or just a significant cost imposed on the system. Section (2.3) discusses an overview of the empirical literature focused on testing and estimating the impact of R&D on growth using different approaches.

2.3 Empirical Literature on the Rate of Return to R&D

The empirical studies concerned with the effect of research and development on growth generally involve testing or estimating the effect of R&D variables on total factor productivity (TFP) growth. In section (2.2.4), I briefly discussed Jones' (1995b) approach to the effect of R&D on growth. By using the data for TFP growth and R&D which is proxied by the number of scientists and engineers f Germany, France, Japan and United States; Jones tests the validity of R&D-based endogenous growth models and he finds no evidence to support the positive impact of R&D on TFP growth which has been emphasized by these models. Aghion and Howitt (1998) provide explanations for this contradiction which is known as "Jones' Paradox". One of the reasons they discuss is the need for continuity in raising the R&D over time to keep the innovation rate constant for each product, and the second cause of this paradoxical result of Jones might be that by increasing the number of outputs, the proportional spillover-effect on the aggregate stock of knowledge will be smaller. They also argue that instead of using the number of scientists and engineers to proxy R&D, the GDP share of R&D investment should be used. Using these facts, Aghion and Howitt (1998), and Zachariadis (2003) provide strong evidence that in the U.S. economy R&D investment and TFP growth are positively related.

The literature on the estimation of the rate of return to R&D is divided into two major groups: 1) the literature on the estimation of the private returns and 2) the social returns to R&D where it received substantial attention with the recommendations of Edwin Mansfield (1971) to the National Science Foundation on topics in R&D which needs further investments. Within each of these two groups, cross-section analysis and panel data studies are recognisable. Frantzen (2000), Griffith (2000), and Griffith, Redding and Reenen (2004) use international panel data and confirm a positive relationship between countries' own R&D and productivity growth. Coe, Helpman and Hoffmaister (1995); Griffith, Redding and Reenen (2002) also present substantial evidence that R&D spillovers from industrialised countries to developing countries have positive effects on the TFP growth of the latter. About R&D investments, Savvides and Zachariadis (2003) discuss the effect of both domestic and foreign direct R&D investment increase the domestic total factor productivity growth. And Zachariadis in his paper in 2003, compares the R&D effect on aggregate and manufacturing output and concludes that the R&D effect is much higher for the aggregate economy than the manufacturing sector.

These are just a few examples of the empirical studies attempt to estimate and evaluate the impact of R&D on the growth of total factor productivity. Have it in mind that there is an extensive literature which analyses this matter from different angles using various methods, therefore in this literature review, I go through only those few relevant studies which might help to clarify some aspects of the model outlined in Chapter 3.

2.3.1 Private vs. Social Rate of Return to R&D

The empirical literature on the rate of return to research and developments shows a constant attempt of economists in developing the method of estimating the impact of R&D in the knowledge-based economy. Most of these studies augment the conventional growth accounting framework with measures of R&D investment or capital at firm, sector, industry or all the way up at aggregate-macro level. By regressing TFP growth on the common production factors, i.e. capital, labour or intermediate inputs, the residuals growth factor is assumed to be the product of R&D that may be the growth-enhancing element. R&D can increase the productivity in various ways such as increasing the productivity by improving the quality, reducing the cost of producing it or it widens the spectrum of final goods or intermediate inputs. Hence, R&D affects the system by increasing the profit, reducing the prices or/and reallocating factors across the appropriate level. One more important channel with which R&D spendings may increase the growth is via knowledge spillovers; R&D carried out in one firm, sector, industry or country may increase the productivity in other firms, sectors, industries, or countries.

Hence, if one must summarise the findings of the related literature on the impact of R&D investment on growth, she may find it appropriate to divide them into two groups: first group concentrates on estimating private rate of return to research and development which received lots of attention since Mansfield (1971) and Griliches (1973) and the second group estimates the social rate of return believing that the social rate of return to R&D is much higher than the private rate i.e. Sveikauses (1981) estimates a social rate of return to R&D of 50%, while Griliches and Lichtenberg (1984) estimate a social rate of return to R&D of 41-62% where the private rate is about 7-10%.

The private rate of return can just be estimated by looking at the effect of the firm's own R&D on its own output. The most common way according to Griffith (2000) to obtain the estimates of the rate of return to R&D is from the parameters of the production function, hence with a production function of the general form $Y_{i,t} = A_{i,t}F(K_{i,t}, N_{i,t})$, total factor productivity will be affected by many elements which the stock of knowledge, G, is considered to be one of them

$$ln\, A_{i,t} = \eta_A\, ln\, G_{i,t} + \beta\, ln\, X_{i,t} \tag{2.42}$$

where $X_{i,t}$ denotes all other factors affecting the TFP.

$$\eta_A = \Big(\frac{\partial Y_{i,t}}{\partial G_{i,t}}\Big)\Big(\frac{G_{i,t}}{Y_{i,t}}\Big) \tag{2.43}$$

The parameter η_A is the elasticity of output with respect to knowledge stock and $\frac{G_{i,t}}{Y_{i,t}}$ can be defined as R&D intensity[12]. $r_A = \dfrac{\partial Y_{i,t}}{G_{i,t}}$ on the other hand is the rate of

[12] "R&D intensity (R&D expenditure as a percentage of GDP) is used as an indicator of an economy's relative degree of investment in generating new knowledge".

return to the accumulation of the knowledge which can proxy by R&D expenditure, the flow of investment in knowledge. In principle, the choice between estimating an elasticity or a rate of return depends on which one of the two is more likely to be constant. Hall (1996) reports estimates to private rates of return to R&D which cluster around 10-15%, however, it can be as high as 30% in some studies. Wieser (2005) surveys 50 studies of the private rate of return to R&D and finds that about 50% of the studies so far report statistically significant estimate such as annual rates of return in a range between 7% to 69% with an average value of 28.8%. It should be noted that there are differences between the estimated returns extracted from cross section data, time series and panel data as Wieser (2005) reports the highest rates belong to cross-section studies and the lowest to time series analysis with panel data-based studies have an estimated value in between these two values.

The social rate of return to R&D is obtained by estimating the impact on growth in one firm of R&D done in other firms. These other firms can be in a same industry/country or in a different but related industry/country (Griffith et al. 2004). The dominant belief is the higher social rate of return to R&D relative to the private rates. The reason behind this is the idea that knowledge spills over from the innovator to other firms. Hence, when a new idea is discovered, it can be imitated by others (the properties of non-rivalry and only partial-excludability of knowledge make it happens), although the innovator can have a share of the rent for her new idea with the patent protection and the delay in distribution of the new idea between others. To interpret the social rate of returns one must consider that estimation results at the firm level capture the social return to that firm; hence those estimations at the industry level captures the social rates of return to that industry but not spillovers

Source: OECD Science, Technology and Industry Scoreboard (2011) to the other industries. Similarly, estimates conducted at national level presents the social rate of return to R&D within the country but not those between the countries. Jones and Williams (1998) present social rates of return which are integrated into a macroeconomic model of endogenous innovation and growth. In their survey they show that the social rates of return to research and development in literature are under-evaluated, and the results of most studies in the literature provide only the lower bound to the true social rate of return, once we take into account the dynamic general equilibrium effects emphasised in the endogenous growth literature, see table (2.1).

Unfortunately, there are not many good quality studies which estimate the private rate of return to R&D at the national level. But there are some studies which calculate the social rate of return at the national level. Griffith (2000) and Griffith, Redding and Van Reenen (2000) are examples of those studies. They use the value of average relative TFP to calculate the implied total rate of return to R&D and consider a non-zero imitation cost which is ignored by Jones and William (1998). Hence Griffith et al. (2000) assume two channels which affect productivity at the national level, one is the innovation, and the other is the channel of increased potential for imitation. The imitation factor is more crucial for those countries which are behind the technological frontier. Accordingly, they use the country's distance from the technological frontier to measure the potential for imitation in their study where the frontier is defined as the country with the highest level of total factor productivity (TFP). Based on their assumption the further a country is behind the frontier the more impact the R&D may have on increasing TFP through the transfer from more advanced countries, see Table (2.2).

Study	(1) Own R&D	(2) Used R&D	(1)+ (2)	Period	No. observations
Terleckyj, 1980	0.25 (0.08)	0.82 (0.21)	1.07	1948-66	20
Sveikauskas, 1981	0.17 (0.06)	-	-	1959-69	144
Scherer, 1982	0.29 (0.14)	0.74 (0.39)	1.03	1973-78	87
Griliches and Lichtenberg (1984a)	0.34 (0.08)	-	-	1969-73	27
Griliches and Lichtenberg(1984b)	0.30 (0.09)	0.41 (0.20)	0.71	1969-78	193
Griliches, 1994	0.30 (0.07)	-	-	1978-89	143

Table 2.1: Estimates of the social rate of return to R&D in manufacturing-industry level
Source: Jones and Williams (1998)

Column 1 in Table (2.1) presents the social rate of returns to the industry from R&D conducted by firms within the same industry which range from 17% to 34%. Column 2 shows the social rate of return to R&D which is conducted in one industry but is used in another industry. These estimates of the rates are relatively high. Jones and Williams show the total rate of return to R&D in column 3 which is the product of (1) and (2) added together. The value for the total rate of return is as high as 100% for Terleckyj (1980) and Scherer (1982). Note that the estimates are based on the data from the manufacturing sector.

	(1) Mean relative TFP Frontier=1.000	(2) National return to R&D including innovation & imitation
Canada	0.826	57.2%
Denmark	0.728	67.9%
Finland	0.525	95.2%
France	0.849	54.9%
Germany	0.901	49.9%
Japan	0.703	70.8%
Netherlands	0.905	49.6%
Norway	0.663	75.6%
Sweden	0.726	68.0%
UK	0.626	80.5%
US	0.994	41.7%

Table 2.2: Estimates of the Social Rate of Return to R&D in Manufacturing National level for 1974-90
Source: Griffith (2000)

Griffith et al. (2000) assumed a constant rate of return to innovation of about 41.2%. As it is presented in table (2.2), the social rate of return to R&D in U.S. is due to almost entirely to innovation where the average relative TFP in Finland is just over 50% of the frontier's, and less than half of R&D's social rate of return (95.2%) is due to innovation and imitation where it shows the imitation potential is more important than innovation itself. They conclude that in non-frontier countries, there is potential to increase TFP via the channel of technology transfers. This is one of the critical studies which targeted OECD countries and estimated their social rate of return to R&D at national levels.

Although there are studies which question the evidence of the large social rate of return to R&D (i.e. Comin (2005) argues that econometric R&D spillover studies potentially suffer from omitted variable bias) due to some econometrics difficulties, the majority of empirical studies confirm the large and significant effect of R&D

spillover. Hence, an overview of the literature shows that the estimated private rate of return to R&D is high and its social rate of return is even higher. If one accepts the fact that there is a large significant return to research and development and that the market is incapable of allocating the optimal amount of resources to R&D factor in order to enhance the level of the economy, then she may consider the impact of policy decisions and evaluate the effect of these decisions on increasing R&D, therefore growth in TFP. Subsidising research and development can be an example of these policy decisions. According to Lisbon Strategy (2010) leveraging R&D is the key element in the aim of European Union to become "the most competitive and dynamic knowledge-based economy in the world" and to achieve this goal, countries are required to increase R&D investment to 3% of their GDP. Due to uncertainty and complexity of the models and cause and effects relations involved in these analyses, the consequences of these policies are still ambiguous. R&D is expensive hence it should be well evaluated beforehand. More questions may arise, for example, how much governments' involvement is required? What instruments policy makers must choose to get to the optimal value of the social investment on R&D to get the high return? And questions regarding of testing the effects of the related policies.

2.3.2 Effectiveness of the Public Spending on R&D

The existence of positive externalities for R&D investment and the fact that the governments may need to invest in reallocating sources to more specific growth-enhancing elements such as R&D make it necessary to study the effects of publicly funded R&D on growth. Despite the fact that majority of the studies in empirical literature confirm a positive rate of return to publicly funded R&D, they conclude that estimated returns are smaller compared to the privately funded R&D (Mansfield, 1980; Griliches and Lichtenberg, 1984; Griliches, 1986; Lichtenberg and Siegel, 1991; Nadiri and Mamuneas, 1994; and Di Cagno et al. 2014). Griliches (1992) paper even suggests that the rate of return to privately funded and publicly funded R&D is not significantly different at the company level. Bassanini et al. (2000) and Rodriguez-Pose et al. (2004) show inconclusive results on the capacity of publicly funded R&D to promote innovative activities hence the economic growth.

A comprehensive OECD survey on the sources of economic growth in 2003, shows that in the developed countries during the 80s and the 90s only privately funded research contributed to economic growth and publicly funded research based on this survey not only has no significant impact on increasing TFP but also it is suspected to have a negative effect on the amount of privately funded research by displacing it (companies using public funds to substitute the private funds for researching and produce the same level of R&D output as before). Hence the possibility of the publicly funded R&D crowd out privately funded R&D is noted in many studies. Goolsbee (1998) and David and Hall (2000) claim that any increases in the public fund to research and development only enhance the wage of the personnel working in the R&D sector and it does not increase the growth (at least in the short run). David et al. (2000) mainly come up with the evidence of crowd-out effect using U.S. data. There are also some studies which suggest that public and private R&D are complements, in a better word, some particular type of public R&D can have a distinctive positive effect on the funding to research and development provided by private sector. Cohen and Levinthal (1989), Geroski (1995), and Branstetter and Sakakibara (1998) provide empirical evidence in support of the impact of publicly funded R&D which can increase the absorbance of the privately funded research while Griffith, Redding and Van Reenen (2004) provide the theoretical foundations underlying the hypothesis of absorptive capability. Adams (1990) and Mansfield(1991, 1998) emphasise the fact that public funds to universities and research institutes which focus mostly on discovery of the basic knowledge have strong positive spillover effects on commercial R&D, although the knowledge spills over slowly from universities to commercial sector (the delay of response makes detecting the effect difficult).

Guellec and van Pottelsberghe de la Potterie (2003) in their dynamic panel of 16 OECD member countries conclude a positive effect of publicly funded R&D on private R&D, however they continue that for the public R&D to have the related positive impact of TFP growth, governments are required to carry out a broad and coherent innovation policy approach due to interactions between variety of elements involved in the growth-enhancing process. This finding is supported by Afonso et al. (2005). Herrera and Pang (2005) and Jaumotte and Pain (2005a, 2005b) note that a well-functioning national innovation system which facilitates innovative activities and the process of discovering new ideas and transferring it to the commercial sector is required to increase the welfare of the nations. This system can be financed by policy authorities and the optimal spot-on policy channels. Hence, due to the complications and confusion about the effect of the policy channel (i.e. direct subsidy to R&D) on TFP growth, as Griliches (1995) stresses, the relationship between publicly funded innovative activities and the economic growth remains a mystery. Publicly funded research may or may not be effective in increasing the private research hence inducing an economic growth. Theories noted in section (2.2) with their complicated aspects are still too simplified to take into account all the effective elements of growth, for example, R&D institutions such as universities (Nelson, 1998) can be a source of an increase in R&D stock thus growth in TFP at the end.

2.3.3 Applied DSGE models of R&D-driven Growth

Based on Hall et al. (2009), the impact of research and development of productivity is usually studied econometrically. Although there is no consensus on the size of the effect, almost all microeconometric and macroeconometric empirical results present positive, significant and even large rate of returns to research. Believing that the social returns may be at least 3-4 times higher than the private returns to this important element of sustainable growth (Jones and Williams, 1998), makes the policy intervention inevitable. While being very useful tools for the ex-post evaluation of R&D effects, econometric models cannot be employed for ex-ante impact assessment of innovation policies (Di Comite and Kancs, 2015). Instead, macroeconomic models need to be used for simulation of R&D and innovation policies and comparing the results to the baseline (without policy) to see how policy should be structured if one wants to ensure the positive or a large magnitude of the effect.

Hence, evaluating policy strategies requires tools such as a dynamic stochastic general equilibrium structure which can be used to make a simulated world to help to predict the impact of the policy shock to the economy. Most of the earlier DSGE models lack the long-run growth structure. But there are more recent studies which are focused on introducing the long-run growth to DSGE models. Wang and Wen (2008) and Annicchiarico et al. (2010) growth-augmented a DSGE model through a simple AK approach. Examples of R&D driven growth DSGE are Comin and Gertler (2006), Comin et al. (2009a) and Holden (2011).

One of the latest versions of the class of DSGE models which considers R&D as the driver to the growth is QUEST III developed by the European Commission. The QUEST model is a simulation-based model to analyse the effects of any structural reforms and the response of the economy to changes i.e. subsidy to research and development to achieve higher growth. Hence this class of models are well-structured tools for evaluating and testing the R&D-related policies. For example one version of QUEST III used in the Department of Treasury which is an extension of the DSGE model for quantitative policy analysis developed by the Directorate-General for Economic and Financial Affairs at the European Commission (see Ratto et al. 2008) and it is augmented to consider the R&D driven endogenous growth (Roeger, 2010). Roeger (2010) uses the framework introduced by Jones (1995b, 2005) to adapt the Romer's (1990) model including the research and development effects on growth (horizontal innovations). Di Comite and Kancs (2015) suggest that QUEST R&D appears to be the most suitable simulation-based DSGE model for assessing the impact of R&D and innovation policies over time (given the fact that R&D investment decisions are inherently dynamic) as QUEST models are the only class of models with inter-temporal optimisation of economic agents.

Notes on DSGE model of growth of Chapter 3: The empirical model of growth discussed in Chapter 3 which is used in Chapter 5 to test and estimate the effect of research and development subsidy on growth is a dynamic stochastic general equilibrium augmented to include the endogenous growth element. The relationship between growth and innovative activities derived through the representative agent's decision on spending time on these activities, hence the difference between this type of model with QUEST or the rest of theoretical and empirical studies that attempt to find the impact of knowledge and innovation on growth is the fact that here the time spent on innovation is directly 'incentivised' by spendings/subsidies (R&D expenditures of any kind). In better words, an example of government's subsidy to research and development (which increases the R&D intensity) encourages the high-skilled representative agent to spend more time on growth enhancing knowledge-producing activities. The time dedicated to innovative activities and not wage-earning activities is the choice of the agent which is notionally conducted outside of the firm but in the world of the model. The model I outline in next chapter is not a firm-based R&D endogenous growth model (unlike the most R&D-based endogenous models). This assumption is required to ensure the perfect competition in this model (similar to Boldrin and Levine, 2002). The assumption of innovation being perfectly competitive may come against the Schumpeterian 'creative destruction' approach which relates the increase in innovative activities to monopoly rights. The model in this study assumes no monopoly right given to firms by subsidy. The policy variable is here just to encourage the household to spend more time on innovative activities to increase the total productivity of the firm, and this is reflected in their share of dividends in their budget constraint. Hence, this model does not emphasise the non-rival, partially non-excludability condition, standing on the shoulder or business stealing effect which is discussed in this literature overview. It attempts to present the difference between returns to household coming from the higher firm productivity encouraged by R&D variable in a frictionless market and the returns in the absence of it, thus testing the effect and probably estimate the magnitude of the policy effect. This is what I am interested in by isolating the policy determinants of growth.

These simplifications will save troubles regarding the complications coming from the firm side; however, the logic is empirically valid and testable. Although the model may not be the best microeconometric representation of the relation between R&D and growth, it is a simulation-based model that can be used as a tool for policy evaluation, and as it is a dynamic stochastic general equilibrium model, the criticisms to DSGE models are valid for this one too.

2.4 Summary

In Chapter 2, I reviewed some theoretical and empirical literature on the endogenous growth models. Most of the recent growth studies confirm the role of knowledge and innovation in sustainable growth, and theoretical frameworks acknowledge the positive impact of innovation on growth in nation's wealth. The journey of the theoretical growth literature from the neoclassical approach to Jones' semi-endogenous and R&D-based Schumpeterian growth model is filled with different and even paradoxical viewpoints on what really guarantee a long-run growth of production which gives various policy recommendations. Hence it is required to have reliable empirical results to check policies (i.e. research subsidies) overtaken. A brief review of the empirical literature shows flaws and uncertainties in the results and measurement of the impact of R&D on growth. Wieser (2005) and Mairesse et al., (2010) in their metastudies survey the literature on the rate of return to R&D. Wieser reports micro level studies measuring the private rate of return to research and development and Mairesse et al. reports both macro and micro-level empirical results where microeconometrics attempts are doing better as related macro-level studies suffer the lack of identification. This thesis provides a simple but identified model of endogenous growth which isolate the effect of policy to evaluate the possible consequences of it. A micro-founded model of growth outlined in Chapter 3 is a legitimate framework for the empirical analysis of Chapter 5. Complications raised by augmenting a real business cycle model to endogenize the growth factor of productivity in an open economy make the simplification valid and defendable. Ultimately, this panel study of the 11-OECD country wants to test and reevaluate the effectiveness of the highly accepted R&D factor on growth.

Chapter 3

A Structural Small Open Economy Model of Endogenous Growth

"Models are to be used, not believed." -Henri Theil, 1971

Here I present an open economy RBC model following Meenagh et al. (2006). The economy is open as it exchanges goods and services with the world and it can borrow on global markets. Although it is an actor in the international trade, it is 'small' in the strictest sense; that is, it cannot influence the key macroeconomic variables such as prices or interest rates. The addition to this set up is an endogenous growth mechanism which has been extensively used in the literature: innovative activities. This endogenous growth process is similar to Lucas (1988), in that growth in productivity depends on investment in time spending on productivity enhancing activities; such as training to accumulate human capital with the difference that in the model I adapted the endogenous growth is characterised by investment in intellectual capital as research and development; more similar to Grossman and Helpman

(1991) and Aghion and Howitt (1992).[1]

In this chapter, an innovation-based endogenous growth model that can be tested and estimated using unfiltered data[2] is introduced. Section (3.2) undertakes the workhorse model, and in section (3.3) a baseline calibration and the solution method is presented, followed by a discussion of the model simulation and Impulse Response Functions generated from a productivity shock. Finally, section (3.4) summarises.

3.1 The Model

The model of the economy is composed of an identical infinitely living representative agents, which maximises the discounted sum of instantaneous utility. A single good as output is produced in this economy that is used both for the consumption and investment. The home economy coexists with 'the rest of the world' as the foreign economy. Assuming the foreign economy to be large relative to the home country, its income is unaffected by the developments in the home economy. There is a perfectly competitive final goods market.

At the beginning of each period, the representative consumer chooses to consume, to hold savings for investment and to divide the time between the production led activities and leisure. The representative consumer has the choice to withdraw from its own normal work activities and invest time on innovative activities which increase the productivity. This can be done through the channel of human capital (Lucas, 1988), learning by doing or knowledge spillover (Romer, 1986), or inten-

[1]They developed the 'Schumpeterian' theory of endogenous growth- early models were produced by Segerstrom, Anant and Dinopoulos (1990), and Corriveau (1991)- focusing on quality-improving innovations that render old products obsolete, through 'creative destruction'. The model presented here does not have the monopolistic specifications emphasised in Schumpeterian-style of R&D-based growth models.

[2]Following Meenagh, Minford and Wickens (2012).

tional knowledge creation and technological progress, i.e. R&D activities (Romer, 1990; Grossman and Helpman; 1991; Aghion and Howitt, 1992; and Jones, 1995b). The consumer choices are restrained by the budget constraint and the time availability as mentioned above. Although as an open economy, goods can be traded in this model, for the simplicity it is assumed that these do not enter the production process and are only exchanged as final goods. Setting up the model; initially, the productivity is considered to be exogenous, later the model is extended to endogenize the productivity considering R&D intensity as the driver.

3.1.1 Representative Consumer

The representative consumer chooses her consumption as a composite of home goods C_t^d and foreign goods C_t^f i.e. $C_t = f\{C_t^d, C_t^f\}$. The consumption bundle is treated as the numeraire, hence all the prices are expresses relative to the general price level, P_t. Taking in to account that the economy is very 'small', prices in home country is equal to the foreign prices. A classical Armington (1969) aggregator for the composite consumption utility index in the assumed two country-single industry model can be of the form as below

$$C_t = [\nu(C_t^d)^{-\varrho} + (1-\nu)\zeta_t({C_t}^f)^{-\varrho}]^{\frac{-1}{\varrho}} \tag{3.1}$$

where ν is preference bias associated with the share of home goods in the aggregate consumption. If ν equals to 0.5, consumers are indifferent between home and foreign goods thus $0.5 < \nu < 1$ reflects the assumption that the representative consumer has some fixed preference bias towards home goods. The demand for foreign goods is subject to a stochastic shock, ζ_t.

The representative consumer maximises her expected lifetime utility represented as

$$U = MaxE_0\left\{\sum_{t=0}^{\infty}\beta^t u(C_t, \ell_t)\right\} \tag{3.2}$$

where E_0 is the mathematical expectations operator, $0 < \beta < 1$ is the discount factor, C_t and ℓ_t is the consumption and the amount of leisure time consumed in period t respectively. The utility function $u(.)$ follows a specific time-separable[3] form as McCallum and Nelson (1999a).

$$u(C_t, \ell_t) = \theta(1-\rho_1)^{-1}\gamma_t C_t^{(1-\rho_1)} + (1-\theta)(1-\rho_2)^{-1}\xi_t \ell_t^{(1-\rho_2)} \tag{3.3}$$

$0 < \theta < 1$ is a preference weighting on consumption, ρ_1, $\rho_2 > 0$ are Arrow-Pratt coefficients of relative risk aversion for consumption and leisure[4] and γ_t and ξ_t are the preference errors.

The representative agent splits her time between leisure activities and supplying labour, N_t to the firm to earn the real wage, w_t to consume and unpaid activities, z_t which known to have future returns. The total time endowment for her is normalised to unity

$$N_t + \ell_t + z_t = 1 \tag{3.4}$$

I left aside the choice of z_t in (3.4) for now- it is discussed in 3.1.5 on endogenizing

[3]Time-separability of utility means that past work and consumption do not influence current and future tastes according to Barro and King (1982). This form of utility functions does not restrict the size of intertemporal substitution effects, but it limits the relative responses of leisure and consumption to changes in relative prices and in permanent income. These limits are important for evaluating the effect of shifts in expectations about future.

[4]The inverse of $\rho_1(\rho_2)$ can be interpreted as the intertemporal substitution elasticity between consumption(leisure) in period t and period t+1.

the productivity growth (also see Appendix I for some detailed information). Hence, $N_t + \ell_t = 1$[5]. Here the representative agent choices are restricted to the leisure activities and some non-leisure activities such as consumption and saving in the form of domestic and foreign bonds. The budget constraint is

$$C_t + \frac{b_{t+1}}{1+r_t} + \frac{b^f_{t+1}}{1+r^f_t} + q_t S^p_t = w_t N_t + b_t + b^f_t + (q_t + d_t) S^p_{t-1} \tag{3.5}$$

where b_t and b^f_t are home and foreign bonds respectively. The consumer's income includes the wage received from the firm for the hours of work, $w_t N_t$, bonds, and the firm's profit share in the form of dividends d_t. Here the assumption is that in every period, the representative consumer's share holdings are equivalent to a single share, $S^p_t = S^p_{t-1} = \bar{S} = 1$ hence the value of the firm as total which is considered in the budget constraint is equal to $q_t S^p_t - (q_t + d_t) S^p_{t-1} = d_t$. Consumer maximises the expected discounted stream of utility (equation 3.6) subject to her budget constraint. The Lagrangian is

$$\begin{aligned} L_0 = E_0 \sum_{t=0}^{\infty} \beta^t E_t \Big\{ & \theta(1-\rho_1)^{-1} \gamma_t C_t^{(1-\rho_1)} + (1-\theta)(1-\rho_2)^{-1} \xi_t \ell_t^{(1-\rho_2)} - \\ & \lambda_t [C_t + \frac{b_{t+1}}{1+r_t} + \frac{b^f_{t+1}}{1+r^f_t} - w_t N_t - b_t - b^f_t - d_t] \Big\} \end{aligned} \tag{3.6}$$

first order conditions respect to C_t, ℓ_t, b_{t+1} and b^f_{t+1} are as below

$$\frac{\partial L}{\partial C_t} = \gamma_t \theta C_t^{-\rho_1} - \lambda_t \tag{3.7}$$

[5]Furthermore for convenience in the logarithmic transformations we assume that approximately $L = N$ on average as the consumer spends 50% of her time on leisure activity and the rest on providing labour for the firm.

$$\frac{\partial L}{\partial \ell_t} = \xi_t(1-\theta)\ell_t^{-\rho_2} - w_t\lambda_t \tag{3.8}$$

$$\frac{\partial L}{\partial b_{t+1}} = \frac{-\lambda_t}{1+r_t} + \beta E_t\lambda_{t+1} \tag{3.9}$$

$$\frac{\partial L}{\partial b^f_{t+1}} = \frac{-\lambda_t}{1+r^f_t} + \beta E_t\lambda_{t+1} \tag{3.10}$$

substituting (3.13) in (3.11) yields Euler Equation, describes intertemporal substitution in consumption as,

$$(1+r_t) = \frac{1}{\beta}E_t\Big[\frac{\gamma_t}{\gamma_{t+1}}\Big]\Big[\frac{C_t}{C_{t+1}}\Big]^{-\rho_1} \tag{3.11}$$

as the cost of one extra unit of utility at time t is $\frac{1}{(1+r_t)}$ in terms of the discounted expected consumption utility at time $t+1$. Combining equations (3.11) and (3.12) gives

$$w_t = \frac{(1-\theta)\xi_t\ell_t^{-\rho_2}}{\theta\gamma_t C_t^{-\rho_1}} \tag{3.12}$$

Intratemporal condition equates the marginal rate of substitution between consumption and leisure to their price ratio. This will be w_t as the price for consumption is the numeraire. To derive the uncovered interest parity condition in real terms (RUIP), equation (3.13) is substituted into (3.14)

$$\frac{(1+r_t)}{(1+r^f_t)} = 1 \tag{3.13}$$

The economy is small and open, hence $r_t = r^f_t$. Contrary to the situation in the closed economy, in this economy, the interest rate is independent of savings and investments. At the time of investment surplus, the agent borrows from the foreign countries and lends to them when she has the saving surplus.

3.1.2 Representative Firm

The representative firm hires labour and buys capital to produce a homogenous consumption good using a constant-returns-to-scale production technology with diminishing marginal products to labour and capital. Then it sells the good to representative consumer or government. Choosing a Cobb-Douglas production function, the firm's technology is described as:

$$Y_t = A_t K_t^{1-\alpha} N_t^{\alpha} \tag{3.14}$$

where Y_t is aggregate output per capita, A_t represents the state of the technology at time t, K_t is the capital carried over from the last period $(t-1)$, N_t is the labour demanded by the firm and $0 \leq \alpha \leq 1$ is the elasticity of substitution between inputs. It is assumed that production technology $f(N_t, K_t)$ is strictly concave and satisfied Inada conditions[6]. The law of motion for capital, $K_t = I_t + (1-\delta)K_{t-1}$, shows how capital per unit of effective workers evolves during the time where I_t is the gross investment and δ is the depreciation rate.

[6]Based on Inada assumptions (1963), a per capita production function $f : R_+ \rightarrow R_+$ should satisfy

$$f(0) = 0 \qquad f'(\infty) = 0$$
$$f'(0) = \infty \qquad f(\infty) = \infty$$

and it also needs to be strictly increasing; $f'(h) > 0$ and strictly concave; $f''(h) < 0$ for all $h \in R_+$. Inada type of conditions is widely used in the applied economic literature. Usawa used these conditions in his series of two-sector growth models realising that these conditions are sufficient to ensure the existence of equilibria. These conditions are intuitively very plausible and easily justified i.e. in the Cobb-Douglas production function presented here as Inada conditions guarantee that the marginal product of capital (or labour) approaches infinity as capital (or labour) goes to zero and approaches zero as capital (or labour) goes to infinity.

Hence, the firm's profit function includes the cost of capital and labour and the cost of capital covers the return to debt holders[7], capital depreciation δ, adjustment costs a_t. Firm's profit function is:

$$\Pi_t = Y_t - K_t(r_t + \delta + a_t + \kappa_t) - N_t(w_t + \chi_t) \tag{3.15}$$

where κ_t and χ_t are shocks that capture the effect of excluded imposts or regulations on the firm's use of capital and labour respectively. For analytical and econometric convenience, the adjustment costs are assumed to be strictly convex[8] and quadratic[9], $a_t = \frac{\varphi}{2}\frac{(\Delta K_t)^2}{K_t}$ with φ as the adjustment cost parameter. Subject to these constraint, the representative firm maximises its present discounted stream of cash flows, V, as following:

$$V = E_0 \sum_{t=0}^{\infty} d_k^t E_t \Big\{ Y_t - K_t(r_t + \delta + \kappa_t) - \frac{\varphi}{2}(\Delta K_t)^2 - N_t(w_t + \chi_t) \Big\} \tag{3.16}$$

d_k is the firm's discount factor. The first order conditions are

$$\frac{\partial V}{\partial K_t} = (1-\alpha)(\frac{Y_t}{K_t}) - (r_t + \delta + \kappa_t) - \varphi(\Delta K_t) + d_k \varphi(E_t \Delta K_{t+1}) \tag{3.17}$$

$$\frac{\partial V}{\partial N_t} = \alpha(\frac{Y_t}{K_t}) - (w_t + \chi_t) \tag{3.18}$$

[7]The firm undertakes capital investment by issuing debt at time t and pays the cost, r_t at time $t+1$.

[8]Strictly convex adjustment costs imply that it is always optimal to make a continuous, non-zero adjustment as the costs increase with the size of adjustments.

[9]As Sargent (1978, 1987) identified, quadratic objective functions have the advantage of generating linear decision rules which allows the optimisation problem to remain tractable.

a non-linear demand for capital is derived from equation (3.17)

$$\begin{aligned} K_t = & \frac{1}{(1+d_k)} K_{t-1} + \frac{d_k}{(1+d_k)} E_t K_{t+1} + \frac{(1-\alpha)}{\varphi(1+d_k)} (\frac{Y_t}{K_t}) \\ & - \frac{1}{\varphi(1+d_k)} (r_t + \delta) - \frac{1}{\varphi(1+d_k)} \kappa_t \end{aligned} \tag{3.19}$$

and rearranging (3.18) gives the representative firm's demand for labour

$$N_t = \frac{\alpha Y_t}{w_t + \chi_t} \tag{3.20}$$

3.1.3 The Government

It is assumed that the markets are perfect and there are no market failures. According to this fact, the government does not directly affect the agents' decision making, and the stabilisation policy is neutral; thus all the fluctuations and cycles are only the optimal responses of the representative agents to the exogenous shocks. Hence, in this framework, the government consumption, G_t, is assumed to be non-productive and strictly for the welfare transfers. As it is a no-tax economy, the government compensates its consumption only by borrowing (issuing bonds at time t, maturing in the following period, $t+1$).

$$G_t + b_t = \frac{b_{t+1}}{1 + r_t} \tag{3.21}$$

Another assumption is that the government's consumption of current output follows a non-negative stochastic process such that $G_t \leq Y_t$ for all t.

3.1.4 The Foreign Sector

A further extension to RBC model following Kydland, Backus, Kehoe (1991) to have the open economy elements in the model makes it possible to assume that the country's consumption and investment decisions are not restricted to its own production anymore. The representative agent can smooth its consumption during the time by considering import and export in its demand for consumption. By using Armington (1996) assumption in section (3.1.1), it is presented that the homogenous consumption goods produced in this framework 'can' be distinguished by their origin of production, thus given the demand for foreign goods in equation (3.5), the import and export equations are

$$IM = C_t^f = C_t[(1-\nu)\zeta_t]^{\sigma} \tag{3.22}$$

$$EX = (C_t^d)^* = (C_t)^*[(1-\nu^F)\zeta_t^*]^{\sigma^F} \tag{3.23}$$

where the demand for export is given in equation 3.27, where $(C_t^d)^*$ is foreign demand for home goods, $(C_t)^*$ is the general consumption in the foreign country, ν^F and σ^F are the foreign equivalents to home bias and the elasticity of marginal substitution between home and imported goods. Equation (3.27) describes that export is positively related to the total consumption in foreign country. Foreign bonds evolve during the time to satisfy the balance of payments constraint.

$$\Delta b_{t+1}^f = r_t^f b_t^f + (EX_t - IM_t) \tag{3.24}$$

as equation (3.28) presents the current account surplus (sum of the trade balance plus net income flows from foreign assets) is equal to capital account deficit (decrease in the country's net foreign assets).

3.1.5 Endogenizing Productivity Growth

In this section, I extend the model presented in section (3.1.1) and (3.1.2) to include a new determination for productivity growth, z_t. In endogenous growth models, one key channel of growth is via labour being withdrawn from 'normal activities' and being used for an activity that raises productivity. Here I think of it as 'innovation' similar to Klette and Kortum (2004). In Lucas' model (1988), this productivity-enhancing activity is education, in Aghion and Howitt (1998) following a Schumpeterian method of 'creative destruction', the channel to increase total factor productivity is firm-specific research and developments. Notice that the maximisation problem is the same for all of these models; the agent diverts appropriate time of her into productivity-enhancing activities. She decides the optimal time to devote to z_t by maximising its expected welfare. The productivity growth is

$$\frac{A_{t+1}}{A_t} = \alpha_0 + \alpha_1 z_t + u_t \tag{3.25}$$

where z_t is the time spent on innovative activities, α_1 is the marginal impact of the innovations on the total factor productivity growth with $\alpha_1 > 0$, and u_t is the error process. Here in this model by considering innovative activities as the channel to derive growth, all the other factors that might systematically affect the growth are included in the error term.

The consumer's optimising decision is developed for z_t as following :

$$L_0 = E_0 \sum_{t=0}^{\infty} \beta^t E_t \Big\{ \theta(1-\rho_1)^{-1}\gamma_t C_t^{(1-\rho_1)} + (1-\theta)(1-\rho_2)^{-1}\xi_t \ell_t^{(1-\rho_2)} - \lambda_t[C_t + \frac{b_{t+1}}{1+r_t} + \frac{b_{t+1}^f}{1+r_t^f} + \pi_t z_t - w_t(1-\ell_t-z_t) - b_t - b_t^f - d_t] \Big\} \quad (3.26)$$

note that the dividend income, d_t in (3.30) is the output reduced by the capital and labour costs, i.e. profit. This is important since the agent is the sole shareholder of the firm. Given the relationship in (3.29), any increase in z_t will permanently raise the productivity from time t+1. By this increase, the agent's income expected to increase through the dividends as the firm gets higher profit. F.O.C respect to z_t is

$$\frac{\partial L}{\partial z_t} = -\beta^t \lambda_t(w_t + \pi_t) + E_t \sum_{i=1}^{\infty} \beta^{t+i}\lambda_{t+i}\frac{\partial d_{t+i}}{\partial z_t} \quad (3.27)$$

to capture the effect of productivity enhancing activity in $\frac{\partial d_{t+i}}{\partial z_t}$ first we need to find $\frac{\partial A_{t+i}}{\partial z_t}$.

$$\frac{\partial A_{t+i}}{\partial z_t} = (\frac{\partial A_{t+i}}{\partial A_{t+i-1}})(\frac{\partial A_{t+i-1}}{\partial A_{t+i-2}})...(\frac{\partial A_{t+2}}{\partial A_{t+1}})(\frac{\partial A_{t+1}}{\partial z_t}) \quad (3.28)$$

given equation (3.29), $(\frac{\partial A_{t+i}}{\partial A_{t+i-1}}) = (\frac{A_{t+i}}{A_{t+i-1}})$ [10] , for all $i \geq 1$, there will be

$$\frac{\partial A_{t+i}}{\partial z_t} = \alpha_1 A_t (\frac{A_{t+i}}{A_{t+1}}) \quad (3.29)$$

[10] $\frac{A_{t+1}}{A_t} = \alpha_0 + \alpha_1 z_t + u_t$ gives $\Delta ln\, A_{t+1} + 1 = \alpha_0 + \alpha_1 z_t + u_t$.

$$\frac{\partial d_{t+i}}{\partial z_t} = (\frac{Y_{t+i}}{A_{t+i}})(\frac{A_{t+i}}{A_{t+1}})\alpha_1 A_t \tag{3.30}$$

substituting $\frac{\partial d_{t+i}}{\partial z_t} = (\alpha_1 Y_{t+i})(\frac{A_t}{A_{t+1}})$ in equation (3.31), gives

$$\beta^t \lambda_t (w_t + \pi_t) = \{E_t \sum_{i=1}^{\infty} \beta^{t+i} \lambda_{t+i} Y_{t+i}\}\{\frac{\alpha_1}{\alpha_0 + \alpha_1 z_t + u_t}\} \tag{3.31}$$

rearranging equation (3.35) for z_t and substituting for the multiplier λ_t and λ_{t+i} derived in equation (3.11) will give

$$z_t = \frac{E_t \sum_{i=1}^{\infty} \beta^{t+i}(\gamma_{t+i}\theta C_{t+i}^{-\rho_1})Y_{t+i}}{\beta^t(\gamma_t \theta C_t^{-\rho_1})(w_t + \pi_t)} - \frac{\alpha_0 + u_t}{\alpha_1} \tag{3.32}$$

as $\alpha_1 z_t = \frac{A_{t+1}}{A_t} - (\alpha_0 + u_t)$, equation (3.36) will be

$$\frac{A_{t+1}}{A_t} = \frac{\alpha_1 E_t \sum_{i=1}^{\infty} \beta^i \gamma_{t+i} C_{t+i}^{-\rho_1} Y_{t+i}}{\gamma_t C_t^{-\rho_1}(w_t + \pi_t)} \tag{3.33}$$

I assume an AR(1) process for γ_t, such that $\gamma_t = \mu_\gamma \gamma_{t-1} + \epsilon_{\gamma,t}$, thus $E_t \sum_i^{\infty} \gamma_{t+i} = \gamma_t \sum_i^{\infty} \mu_t^i$. If we assume ρ_1 to be approximately equal to unity[11] and approximating $\frac{C_t}{Y_t}$ as a random walk[12], equation (3.37) will be as following

[11]Section (3.2.1) presents that $\rho_1 = 1$ gives the elasticity of intertemporal substitution (EIS) for the consumption, $\frac{1}{\rho_1}$ equal to one which is consistent with the literature.

[12]$E_t(\frac{Y_{t+i}}{C_{t+i}}) = \frac{Y_t}{C_t}$(see Appendix I).

$$\frac{A_{t+1}}{A_t} = \alpha_1 . \frac{(\frac{\beta\mu_\gamma}{1-\beta\mu_\gamma})(\frac{Y_t}{C_t})}{(\frac{w_t}{C_t})(1+\pi'_t)} \tag{3.34}$$

where $\pi'_t = \frac{\pi_t}{w_t}$, determines the subsidy to innovative activities which is presented as the opportunity cost of withdrawal from 'standard' wage-earning activities. Using Taylor approximation on equation (3.38) [13] and linearising it will give

$$\frac{A_{t+1}}{A_t} = \psi_0 + \psi_1 \pi'_t + error_t \tag{3.35}$$

as $\frac{A_{t+1}}{A_t} = \Delta A_{t+1} + 1$, after collecting the constant the expression is

$$\Delta A_{t+1} = \psi'_0 + \psi_1 \pi'_t + u'_t \tag{3.36}$$

where ψ_1 is the rate of return to innovative activities[14]. By taking z_t choice, the agent takes all other sources of productivity growth such as human capital accumulation as exogenous therefore affecting the constant ψ'_0 and the disturbance in the productivity time series. To summarise the process of endogenizing the productivity, I used the agent's optimal choice for z_t to derive productivity growth as a linear function of π'_t; the subsidy rate on innovative activities. The choice of calibration for ψ_1 is discussed in section (3.2.1).

[13] Approximation around the point where $\frac{Y_t}{w_t} = \frac{Y}{w}$ and $\pi'_t = \pi'$ gives

$$\frac{A_{t+1}}{A_t} = \alpha_1 . \frac{(\frac{\beta\mu_\gamma}{1-\beta\mu_\gamma})}{(1+\pi'_t)}(\frac{Y}{w}) + \alpha_1 . \frac{(\frac{\beta\mu_\gamma}{1-\beta\mu_\gamma})}{(1+\pi'_t)}(\frac{\partial Y_t}{\partial w_t}) + \alpha_1 . \frac{(\frac{\beta\mu_\gamma}{1-\beta\mu_\gamma})(\frac{Y}{C})}{(\frac{w}{C})(1+\pi'_t)} \partial \pi'_t$$

[14] derived from the first order approximation; $\psi_1 = \alpha_1 . \frac{(\frac{\beta\mu_\gamma}{1-\beta\mu_\gamma})(\frac{Y}{C})}{(\frac{w}{C})(1+\pi'_t)}$.

3.1.6 Completing the Model

To complete the model we need a balanced government budget and goods market clearing condition to guarantee the market clearing conditions in the model discussed above.

$$b_{t+1} = (G_t - \pi_t z_t) + (1 + r_t)b_t \tag{3.37}$$

$$Y_t = C_t + I_t + G_t + NX_t \tag{3.38}$$

Equation (3.41) is the government budget constraint which brings together the revenues it raises from households and the transfer it; one channel of this transfer is via subsidising the innovative activities, z_t. Equation (3.42) is goods market clearing condition in which households buy consumption and investment goods from firms who supply them domestically or from the net output they purchase on the world market at the exogenous world prices which are set to unity. Taking the representative agent's budget constraint as following

$$C_t + b_{t+1} + b^f_{t+1} + \pi_t z_t = w_t N_t + (1 + r_t)b_t + (1 + r^f_t)b^f_t + d_t \tag{3.39}$$

where $d_t = \Pi_t = Y_t - [K_t - (1 - \delta)K_{t-1}] - w_t N_t$ is the profit of the firm which is transferred to the agent in the form of dividends. Substituting (3.41) and (3.42) into (3.43) gives the balance of payments where the capital account deficit (net lending abroad) is equal to the net foreign interest plus the net revenue of the trading goods.

$$b^f_{t+1} - b^f_t = r^f_t b^f_t + NX_t \tag{3.40}$$

It is necessary to have a transversality condition to ensure a balanced growth equilibrium for this small open economy meaning that the trade surplus (deficit) cannot run forever by lending (borrowing) from abroad. Transversality condition imposes a restriction on the balance of payments in the sense that trade surplus should clear the current level of debt at a terminal time T.

$$r_T^f b_T^f = -NX_T \tag{3.41}$$

since changes in net foreign asset; Δb_T^f is equal to zero.

3.1.7 Behavioural Equations and Stochastic Processes

Before I discuss the calibration and solution method in section (3.2), log-linearised equations for the RBC models argued above is presented as below,

$$ln\, C_t = E_t\, ln\, C_{t+1} + (\frac{1}{\rho_1})\{\, ln\, (\frac{1}{\beta}) - r_t\} + \epsilon_{c,t} \tag{3.42}$$

$$ln\, Y_t = ln\, A_t + (1-\alpha)\, ln\, K_t + \alpha\, ln\, N_t \tag{3.43}$$

$$ln\, N_t = ln\, \alpha + ln\, Y_t + ln\, w_t + \epsilon_{N,t} \tag{3.44}$$

$$ln\, K_t = \eta_1\, ln\, K_{t-1} + \eta_2\, ln\, K_{t+1} + \eta_3\, ln\, Y_t + \eta_4\, r_t + \epsilon_{k,t} \tag{3.45}$$

$$ln\,NX_t = (\frac{\bar{Y}}{\bar{NX}})\,ln\,Y_t + (\frac{\bar{C}}{\bar{NX}})ln\,C_t + (\frac{\bar{G}}{\bar{NX}})ln\,G_t + (\frac{\bar{K}}{\bar{NX}})[ln\,K_t - (1-\delta-\gamma_k)ln\,K_{t-1}] \quad (3.46)$$

$$\Delta b_{t+1} = r_t^f b_t^f + NX_t \quad (3.47)$$

$$ln\,G_t = ln\,\alpha_g + ln\,Y_t \quad (3.48)$$

$$ln\,w_t = ln\,(\frac{1-\theta}{\theta}) + \rho_1\,ln\,C_t + \rho_2\,ln\,N_t + \epsilon_{w,t} \quad (3.49)$$

$$ln\,A_t = ln\,A_{t-1} + \psi_1\,\pi'_{t-1} + \epsilon_{A,t} \quad (3.50)$$

$$\pi'_t = \rho_{\pi'}\,\pi'_{t-1} + \epsilon_{\pi',t} \quad (3.51)$$

Equations (3.50) and (3.51) hold as identities; the former is the market clearing constraint and the latter is foreign bonds evolution equation respectively. Some of the equations are log-linearised and some such as capital equation and market clearing constraint which are not straightforwardly linear in logs, are linearised around the sample mean values in equation (3.23)[15] and (3.50). This linearisation is solely for the sake of simplicity because the written program[16] for these structural equations

[15]The capital demand equation consists of intertemporal dynamics, thus is linearised around a point at which $K_t = \bar{K}$ and K_{t-1} and K_{t+1} are related to $\bar{K}$ by a fixed balanced growth rate.

[16]This program is RATEXP which is developed by Matthews et al. (1979) and further extensions are added by Minford (1984). This program uses a complex solution method inspired by Taylor (1983).

can easily handle non-linear and complex models.

There are six autoregressive shocks in the model denoted by $\epsilon_{i,t}$ where i stands for the dependent variable that the disturbance is allocated to. Five of these six shocks are residuals from the structural equation and only one is an exogenous variable which we consider as a shock to output productivity (R&D shock). Productivity shock is the only non-stationary process in this model and the rest of them are straightforwardly stationary or trend stationary.

$$\epsilon_{i,t} = c_i + \rho_{i,t}\epsilon_{i,t-1} + \alpha_i T + \eta_{i,t} \tag{3.52}$$

Innovations are identically and independently distributed disturbances for the error processes in equation (3.56); $\eta_{i,t}$. To extract these innovations, the structural errors $\epsilon_{i,t}$ are derived using McCallum (1976) and Wickens (1982) IV method for the expectational variables of the model. Innovations are used to bootstrap the model. The process in brief is as following

- $\epsilon_{i,t}$ is estimated to find $\hat{\epsilon}_{i,t} = \rho_i\hat{\epsilon}_{i,t-1} + \eta_{i,t}$ where $\hat{\epsilon}_{i,t} = \epsilon_{i,t} - \hat{c}_i - \hat{\alpha}_i T$
- $\eta_{i,t}$ are approximated using the fitted residuals
- estimated innovations; $\hat{\eta}_{i,t}$, are used for bootstrapping

Consumption, labour demand, capital, the real wage are endogenous variables with stationary, I(0), shocks, policy variable (R&D expenditures) is a stochastic exogenous variable that is treated as stationary AR(1) process. Only Solow residual, A_t, is considered non-stationary and modelled as a unit root process,

$$\Delta A_t = \psi_1\pi_{t-1} + \epsilon_{A,t} \tag{3.53}$$

$$\epsilon_{A,t} = \rho_A \epsilon_{A,t-1} + \eta_{A,t} \tag{3.54}$$

Non-stationarity of Solow residual implies that a temporary shock to research and development expenditure, the only policy variable in this model, can induce a long run increase in productivity. The aim of this empirical work is to examine the marginal impact and the duration of the effect of the policy variable on the growth of productivity. Chapter 5 presents the behaviour of the productivity around the transitional path when it is shocked out by R&D.

3.2 Calibration and Solution Algorithm

As a DSGE model, the small open economy RBC model I discussed above is considered a standard tool of quantitative macroeconomics; thus it is used to measure the importance of different economic phenomena and perhaps to provide a policy prescription. Following Kydland and Prescott's immensely influential paper in 1982, most of the related empirical literature faces three major issues: 1) determining the values of the parameters that describe preferences and technology (structural parameters), 2) measuring the fit of the model, and more importantly 3) choosing the existing theory which explains the data to the best. Kydland and Prescott propose to 'calibrate' their model; to select parameter values by matching some moments of the data and by taking them from different studies which have estimated these parameters on a microeconometric basis. In section (3.2.1) the calibration method is discussed and the structural parameters consistent with the logic of the model are chosen.

Subsequent to the calibration of the structural parameters, a solution algorithm for the rational expectation model is to be selected. Following Minford et al. (1984, 1986) a solution (projection method) to the model is outlined in section (3.3.2).

3.2.1 Calibration

Similar to *natural* science, economics uses theory to understand and explain the observed features of the world, and one interpretation of the word *theory* is considering a set of idealised or hypothetical facts about those features (Cooley, 1997). Looking at the literature on the economic theories, it is not difficult to realise that these principles are evolved during past five decades; from being just verbal arguments, these ideas progressed to use mathematical logic in order to provide more precision and clarity and due to the limitations to mathematical arguments; economists turned to computation as a way of theorising. With all the developments in economic theories and technological advances in computing, it becomes feasible to answer some important economic questions by studying the behaviour of *calibrated* models.

What is calibration? Using the process known as calibration has a long tradition in economics. Shoven and Whaley(1984) and Auerbach and Kotlikoff (1988) present a process of adapting calibration method to CGE models of public finance and international economics. As Cooley (1997) noted "Calibration is a strategy for finding numerical values for the parameters of artificial economic worlds", calibration uses economic theory as the basis to make a restricted general framework and then maps the created framework into the data. Koopmans (1950, 1953) and the Cowles Commission[17] followers emphasize on a point that measurement without theory is a

[17] Cowles Commission in the 1940s and early 1950s outgrow Haavelmo's seminal monograph. The Cowles Commission economists' primary concern was the identification problem-how to map the economic theory to data. They developed the theory of identification to a high degree of completeness.

very limited enterprise[18].

Calibration encompasses the idea of the identification- to use economic theory to be able to extract more information from the data, furthermore, it shows that the relationship between theory and measurement is not necessarily unidirectional, as in the calibration approach someone can use measurements to give a content to a theory and also the theory gives path of thoughts to concentrate on *'what to measure'* and *'how to measure'* matters. The noted relationship between measurements and theory distinguishes the quantitative economics from the conventional econometrics approach.

What is the difference between calibration and estimation? Further discussion on calibrated models by Kydland and Prescott (1996), Sims (1996), and Hansen and Heckman (1996) reinforce a view that quantitative theory represented by calibration approach and conventional statistical estimation and inference method are competing methodologies. There are economists who believe that statistical econometrics is less 'casual' [19] and it is more accurate than the calibrated models, as it relies on well-known parametric forms. Their emphasis is on a point that conventional statistical econometrics produces not only point estimates of parameters but also it measures the uncertainty about these statistics. The conventional econometric approach which dominated the economic research since the 1940s consider the observed data as given and use them to estimate the structural parameters; thus they value econometrics as a search for the 'data generation process'. In other words,

[18]In the "measurement-without-theory" debate with Vining, Koopmans firmly maintained that theory must be prior to data (Hendry and Morgan, 1995) and data could not be interpreted without theoretical presuppositions.

[19]Hoover (1995) describes calibration as 'A model is calibrated when its parameters are quantified from casual empiricism or unrelated econometric studies or are chosen to guarantee that the model mimics some particular feature of the historical data'. He also describes Kydland and Prescott's (1982) choices of structural parameters in their study as 'casual' and their robustness check as 'profanatory'.

this approach takes the data and searches for a model which has generated this data. On the other hand, calibrationists view the appropriate data or measurements to be determined partially by the features of the theory since some of the parameters are chosen based on the traditional econometrics method while the rests are based on micro-founded theories. Concluded from the discussion above, calibration and estimation are complements, not substitutes and both are instruments to capture a better picture of the economic world.

I chose to calibrate the structural parameters and use them to generate impulse response functions analysed in section (3.3.2). The selected parameters capturing the specification of the RBC model outlined in section (3.1) are consistent with both the logic of the model and the actual data. The calibrated parameters are explained as below.

To ensure that total utility is well defined, the discount factor is bounded in an intertemporal choice problem. There is a substantial set of microeconometric studies that estimate the time discount factor of utility, β, for individuals. They found a range of values between 0.9 and 0.99. Lower discount factor means a greater preference of individuals for immediate rewards over delayed rewards and vice versa. Following Meenagh et al. (2010), the quarterly discount factor is set as 0.97 which is relatively lower than the standard value of 0.99 in the literature (Backus, Kehoe, and Kydland, 1992). By looking at the table of different empirically estimated discount factors presented by Frederick et al. (2002), it is clear that there is no strong consensus on time preference discount factor but it can be concluded that the 0.97 is a value within the range.

Following Meenagh et al. (2010), the fixed consumption and leisure preferences, θ and $(1-\theta)$, is calibrated at 0.5 presenting an equal weight. The relative risk aversion coefficients are chosen as 1.2 and 1.0 for ρ_1 and ρ_2 respectively. $\rho_2 = 1.0$ gives the elasticity of intertemporal substitution (EIS) for the leisurely activities, $\frac{1}{\rho_2}$, equal to one which is consistent with the literature. For example in Kydland and Prescott (1982) and Jones et al. (2000) set up of equilibrium business cycle models, they argue that an EIS between 0.8 and 1 gives the best fit to the data. Following Meenagh et al. (2010), ρ_1 is chosen comparatively larger implying a less than one intertemporal elasticity of substitution for consumption. Lucas (1990) uses the consumption Euler equation and US average consumption and interest rates and rules out an EIS less than 0.5; hence the chosen value of 1.2 is defendable.

According to McDaniel and Balistreri (2002), Schuerenberg-Frosch (2012), Siddig and Grethe (2012) and others, the choice of Armington elasticities in the import demand function has a strong influence on the simulation results in GE models; hence calibration values for these elasticities should be chosen in a sensible way. The determined value for σ and the foreign equivalent for this elasticity σ^F, is unity and it is consistent with the literature. According to Feenstra et al. (2014) any value "in the neighbourhood of unity regardless of sector" would be defendable presenting the fact that the countries competitiveness exists but it is not sensitive to the foreign alternatives. Following this choice, the Armington weight associated with the proportion of domestic consumption to the total consumption assumed to be 0.5 and likewise, the foreign equivalent of it set to 0.5 by symmetry. Since the foreign and domestic goods considered to be identical in both markets, the only driver for the difference between the domestic good's consumption and the foreign good's consumption ratio is the preference shock, ζ_t designated in the utility function.

The quarterly capital depreciation rate is set at 0.025 corresponding to about 10% depreciation per annum consistent with the US estimates reported by Prescott(1986).

Parameters	Role	Value
α	labour share in input	0.7
β	quarterly discount factor	0.97
ν, ν^f	home(foreign) bias in consumption	0.5
δ	quarterly depreciation rate	0.025
θ	preference weight on C_t	0.50
ρ_1	CRRA coefficient for C_t	1.20
ρ_2	CRRA coefficient for L_t	1.00
$\eta_1, \eta_2, \eta_3, \eta_4$	Capital equation coefficients	0.5, 0.475, 0.025, 0.25

Table 3.1: Calibrated values for RBC model's coefficients

Following Minford et al. (2010), the labour's share of national income is set at 0.7 and the capital equation coefficients after the linear approximation, $\eta_1, \eta_2, \eta_3, \eta_4$, are set at values as following

$$ln\, K_t = 0.5\, ln\, K_{t-1} + 0.475\, ln\, K_{t+1} + 0.025\, ln\, Y_t + 0.25\, r_t \tag{3.55}$$

where $\eta_3 = 1 - \eta_1 - \eta_2$ is a crucial constraint set to guarantee a long-run consistency since the capital equation must not contradict the terminal conditions imposed to the solution, see section (3.2.2).[20]

Recollecting equation (3.50), the coefficients for the log-linearised market clearing constraint are derived from the data averages for 11-OECD countries from the period of 1981-2014. Initially, the average values are calculated for each country, as they are developed European countries they share similar characteristics; hence the averages are very close. To provide consistency, same values are chosen for all 11 countries. It is assumed that the long-run quarterly growth of capital, γ_k, is equal to 0.005.

$$ln\, NX_t = ln\, Y_t + 0.76\, ln\, C_t + 0.29\, ln\, G_t + 6.15\, [ln\, K_t - ln\, K_{t-1}] \tag{3.56}$$

[20] Capital and output should converge in the long run, (K=Y).

Study	Sample	Period	Type of Estimation	R&D rate of return
Griliches-Mairesse (1990a)	U.S. 525 mfg firms	1973-80	growth rates on R&D	41%
	Japan 406 mfg firms	1973-80	Intensity	56%
Bartelsman et al. (1996)	Netherlands ~200 mfg firms	1985,89,93	4 yrs growth rate	30%
			VA,4 yrs growth rate	173%
Harhoff(1998)	Germany 443 mfg firms	1979-89	long diff growth rate	74% (net)
Rogers (2009)	UK 719 firms	1989-2000	VA Prod function with R&D flow on input	40% to 58% (mfg) 53% to 108% (non-mfg)

Table 3.2: Estimated rate of return to R&D - Firm Data

There is not much of a consensus about calibrating the rate of return to innovation since the macroeconometric literature does not offer a strong prior for the relationship between total productivity growth, ΔA_{t+1} and subsidies to innovative activities, π'_t. In dominant literature for research and innovation marginal impact on TFP, R&D intensity (total R&D expenditure's ratio to GDP) is considered to be a good proxy for the time spent on innovative activities. Thus, I chose it to represent the innovation in my model. Now the question is: 'how much is the marginal impact of this proxy on TFP?' Literature presents different even sometimes contradicting results regarding the private and social (considering spill-over effects) rate of return to R&D. Among those studies concentrated on the rate of return, some important works are summarised in Tables (3.2),(3.3) and (3.4) which are extracted from Wieser (2005) and Hall et al. (2009).

As it is explained in section (2.3.1) the social rate of return is obtained by adding

Study	Sample	Period	Own R&D	External R&D
Griliches-Lichtenberg (1984a)	US 193 mfg industries	1959-78	11% to 13%	50% to 90%
Odagiri (1985)	Japan 15 mfg industries	1960-77	157% to 315%	-606% to 734%
Sterlacchini (1989)	UK 15 mfg industries	1945-83	12% to 20%	19% to 35%
Goto-Suzuki (1989)	Japan 50 mfg industries	1978-83	26%	80%
Mohnen-Lepine (1991)	Canada 12 mfg industries	1975, 77 79, 81-83	56%	30%
Griffith-Redding van Reenen (2004)	OECD 12 industries 12 countries	1974-90	47% to 67%	57% to 105%

Table 3.3: Estimated rate of return to R&D - Industry Data

the private rate of return which is defined as the benefit to the firm, industry or particular country that performs the R&D to the sum of the returns outside R&D for all recipients of spillovers from that firm, industry or country[21]. The magnitude of the social rate of return depends upon the number of spillover receivers. For example the social rate of return of Sweden's R&D will be greater if all countries of the world are included as potential recipients of the Swedish R&D spillover than if only the G-7 countries[22]are involved (Coe and Helpman, 1995; Coe, Helpman and Hoffmaister 1997). According to Jones and Williams (1998), the magnitude of the effect of R&D consists of the spillover effects and based on it the social rate of return

[21]external R&D in Table (3.2) and (3.3)

[22]G-7=France, Germany, UK, Italy, Japan, Canada, US

to R&D is much more than just a return estimated using the input and output of the firms. They argue that the actual rate of return is 3 to 4 times greater than the initial private rate of return to R&D.

Study	Sample	Period	Own R&D	External R&D
Mohnen (1992ba)	OECD 5 countries	1964-85	6% to 9%	4% to 18%
Coe-Helpman (1995)	22 countries	1971-90	123% (G-7) 85% (other)	32% (G-7 to RoW)
Kao et al (1999)	22 countries	1971-90	120% (G-7) 79%(other)	29% (G-7 to RoW)
van Pottelsberghe -Lichtenberg(2001)	13 countries	1971-90	68% (G-7) 15% (other)	-

Table 3.4: Estimated rate of return to R&D - Country Data

Considering Table (2.1), (3.2) and (3.3) and using Jones and Williams (1998, 2000) the intertemporal spillover effects will be added to the selected private rate of return; hence the actual rate is going to be approximately four times of the value for the private rate of return. It should be noted that the starting value for ψ_1 should not be chosen too low as it becomes difficult to distinguish the model from an exogenous growth model. The selected value of 0.29 is consistent with both the empirical literature of rate of return to R&D (Wieser, 2005; Hall et al., 2009) and endogenously driven growth theories. This initial value will be adjusted by estimation as the other coefficients are altered correspondingly so initial value for ψ_1 would not affect the model

The initial calibration values are used to derive Impulse Response Functions in section

(3.3.2). The R&D shock chosen to be large (%100 increase in subsidy at time $t = 0$) in order to have reasonably clear images for the after-shock behaviours of the system.

3.2.2 Solution Algorithm

The model introduced in Chapter (3) is a rational expectational model with forward-looking variables such as consumption and foreign debt. The general problem with this strand of models in which there are current or lagged expectations of future variables is that there is infinity of solution path (Shiller, 1978) and as it is noted in Minford et al. (1979), some of these paths are also divergent. This problem can be solved typically by imposing the additional condition on the model to make the solution convergent (Sargent and Wallace, 1973; Minford, 1978). The condition sets the coefficient on divergent roots within the general solution to zero to 'rule out speculative bubbles' (Sargent and Wallace, 1973). Minford and Matthews (1978) proposal is that imposing a terminal equilibrium condition will ensure unique solutions for this kind of models whether linear or non-linear. It is useful to mention that these solutions are identical with the solutions obtained when the convergence condition is imposed to those linear rational expectation forward-looking models.

Thus, to solve the model discussed in this chapter, a terminal date, T, which needs to be relatively large to deal with the sensitivity of the algorithm, is chosen to ensure a solution for the model. In brief, the idea of the terminal condition is that beyond T, all the expectational variables are set to their long-run equilibrium value.

There are several iterative methods to solve a rational expectation model. In this study, a first-order solution which is built in a computer program called RATEXP, developed by Matthews (1979) and extended further by Minford et al., in 1984 using Fair and Taylor (1983); is chosen. The algorithm uses a backward-solving method similar to dynamic programming with a difference that the solution vector is approached simultaneously for all t=1,2,..,T but the convergences happen following a backwards process. Thus the process begins with the initial 'guesses'[23] for the forward-looking variables, then the equality between the expectations and the forecasted values from the model is checked, the initial values for these variables are gradually changing till the convergence is obtained.

To discuss further, a first order solution method for a non-linear dynamic model with consistent expectations similar to Fair and Taylor (1983) is assumed.

$$f(Y_t, \Gamma_t, Y_{t-1}, x_t, u_t; \theta) = 0 \tag{3.57}$$

with

$$\Gamma_t = \{Y_{t+1}, Y_{t+2}, ..., Y_{t+k}\} \tag{3.58}$$

rewriting the system of the equation in a normalised form will give

$$Y_t = g\{Y_t, \Gamma_t, Y_{t-1}, x_t, u_t; \theta\} \tag{3.59}$$

where Y_t is a vector of endogenous variables, Γ_t is a vector of expected future endogenous variables, Y_{t-1} is a vector of predetermined endogenous variables, x_t is a vector of endogenous variables, u_t is a vector of exogenous shock processes and θ is a vector of calibrated parameters. First an initial guess for the expectation variable i.e., Y°_{t+1}, $t = \{1, ..., T\}$ is made[24]. Then the system of equation is solved for the

[23] i.e., Jacobi Method.

[24] The model includes the lagged variable for Y_{t-1} which is predetermined in each period of the time so it is considered to be fixed. Note that only in the first period the value for Y° is outside of the model solution period which is known as the initial conditions and usually set as the actual historical data hence

iteration of υ, $\upsilon = \{1, ..., V\}$.

$$Y_t^{\upsilon} = g(Y_t^{\upsilon}, \Gamma_t^{\upsilon-1}, Y_{t-1}, x_t, u_t; \theta)$$
$$t = 1, ..., T$$

By treating $\Gamma_t^{\upsilon-1}$ as fixed, the system of equations can be solved using first order solution method such as Gauss-Seidel or Newton's method [25]. The next step is to update the value for the expectational variables in the model and repeat the iterations till the convergence happens. By convergence we mean

$$\max_t \mid \Gamma_t^{\upsilon} - \Gamma_t^{\upsilon-1} \mid < \varepsilon \tag{3.60}$$

where ε is a pre-assigned tolerance level. It is often computationally efficient to choose the loose tolerance to pace up the convergence procedure. These iterations are called Type II iterations by Fair and Taylor (1983).

It must be noted that for simplicity the REFV[26] model of this chapter is log-linearised as it is more convenient to have all variables in the log or fractional form (i.e. interest rate). The reason to use RATEXP is solely because of the non-stationarity characteristic of some of the variables such as net foreign assets. It is most crucial to impose a terminal condition on foreign debt to guarantee the unique solution for the model. Although it is not necessary to log-linearise the REFV model

treated as exogenous in the model

[25] In RATEXP, Gauss-Seidel method of iterations is used because of its simplicity and less computational time per each iteration. However, the slow rate of convergence may result in a much larger number of iterations. Using Newton's method needs less number of iterations to reach convergence than Gauss-Seidel, but it needed more complex programming techniques

[26] Rational Expectation Forward-looking Variables

of the study, to achieve straightforward intuition we have the behavioural equations in (3.1.7) linearised. As this does not lead to rejection, hence there won't be any inadequate approximation problem.

3.3 Model Simulations and the Impulse Response Functions

Once the model is calibrated, and initial values are assigned to the parameters of the preference the model can be solved numerically and the characteristics of the model in transition to the steady state be studied. The reasons for this attempt is: 1) economy is not at the steady state in the first place or 2) there are policy interventions altering the steady state. As the second reason seems to be of greater concern, there is always demands to evaluate the impact of the policy rules; hence it is crucial to simulate the dynamic model to study the characteristics of these pivotal systematic movements and even shed lights on the causation of these movements.

3.3.1 Simulations

By looking at the literature, one can conclude that experimentalism finally reached economic methodology (Reiss, 2008, chap. 5). In spite of that every day there are more studies focusing on performing economic experiments, it is still valid to believe that one cannot subject the whole economy to experimental control mostly because some economic phenomena are inaccessible due to ethical, technical or practical reasons. Most importantly we have to accept the fact that observational studies in economics suffer from 'problem of co-founders' (Steel, 2004) meaning that in occasions researchers are incapable of distinguishing the causal effects of phenomena by empirical means. These difficulties do not make the experimental economic studies inscrutable to the rational evidence-based investigation, thanks to computer simulations which increase the ability of economists to produce more developed empirical analyses of the facts.

There are three primary dominant definitions of (computer) simulation in methodological and philosophical literature.

- using computer to solve equations that are not or cannot be solved analytically (Pritzker, 1979; Troitzsch, 1997; Frigg & Reiss, 2009).
- using computer to mimic a process by another (computer) process (Zeigler, 1976; Pritzker, 1984; Humphreys, 2004).
- using computer simulations to explore the properties of the model aimed at drawing inferences (Reiss,2006)

The advantage of using the simulation-based methodology is that although it requires the analytical relations in the background, it reduces the complexity. In a better word, the high computational power of simulations decreases the degree of 'idealisations' in the underlying model[27]. Some may argue that the simulation does not generate optimal solutions to the model, but it has the advantage of being flexible; hence changes in the system variables can be made, and the best solution among various alternatives can be selected.

The complexity of the dynamic stochastic model discussed in this chapter makes it inevitable to use simulation method. In brief, here assumed a system of simultaneous equations representing the body and a chosen exogenous variable which its

[27] Paul Humphreys (2004) writes "Because of their increased computational power, simulations usually allow a reduction in the degree of idealisation needed in the underlying model, idealisations that often must be made in analytical modelling to achieve tractability. Because they are free from many of the practical limitations of real experiments, simulations and numerical experiments allow more flexibility and scope for changing boundary and initial conditions than do empirical experiments."

co-movements with the state variables are going to be studied. To achieve this goal the model is presented in a possible simple format for the sake of tractability. To evaluate the policy variable, the standard simulation methods compare the solution of the model where the exogenous variable is perturbed with the base[28] where no alterations occurred. Specifying an exogenous variable to be the policy instrument of the model, the idea of evaluating the policy effect will be the comparison between the perturbed solutions and the base. Simulated results are obtained as following. Firstly, the model is numerically solved using the calibration and solution method outlined in the previous section, secondly, the residuals remains after solving the model construct the structural errors to be used to generate sets of innovations (assuming ARIMA(1,0,0) process for the errors to capture the effect of the dynamic), thirdly, the information in these innovations define the shocks in order to simulate the model (using the bootstrapping method to withdraw these shocks in the process randomly) and finally, the difference between the simulation and base is generated, and the impact of the policy changes will be tested or estimated.

There is also the matter of length of simulations. Following Lewis et al. (1999) a large simulation period has been chosen. One of the main reason is that in solving a non-linear rational expectation model similar to one we have here, it is necessary to select a sufficiently far terminal date for the simulation in order to ensure that the simulation results are not affected by this choice.

In Chapter 5, the simulation results are presented then the effect of specified policy variable is analysed and the direction of causation has been studied.

[28] In this study the solution to the system of equation using Fair and Taylor (1983) Type II fix in the absence of any shocks is called Base.

3.3.2 The IRFs

The impulse response to the policy variable, z_t, at time $t+i$ to an impulse in period t is defined as

$$IR(z_{t+i}) = E_t z_{t+i} - E_{t-1} z_{t+i} \tag{3.61}$$

The impulse response functions trace the expected behaviour of the system at time t given the available information at that period and compare it to what was expected at time $t-1$. Here I consider a difference between the base-run of the model -the solution to the model (go to 3.2.2) in the absence of any shocks which replicate the data- and the simulation run after a one-off shock to the policy variable in the first period of simulation as IRFs for the model.

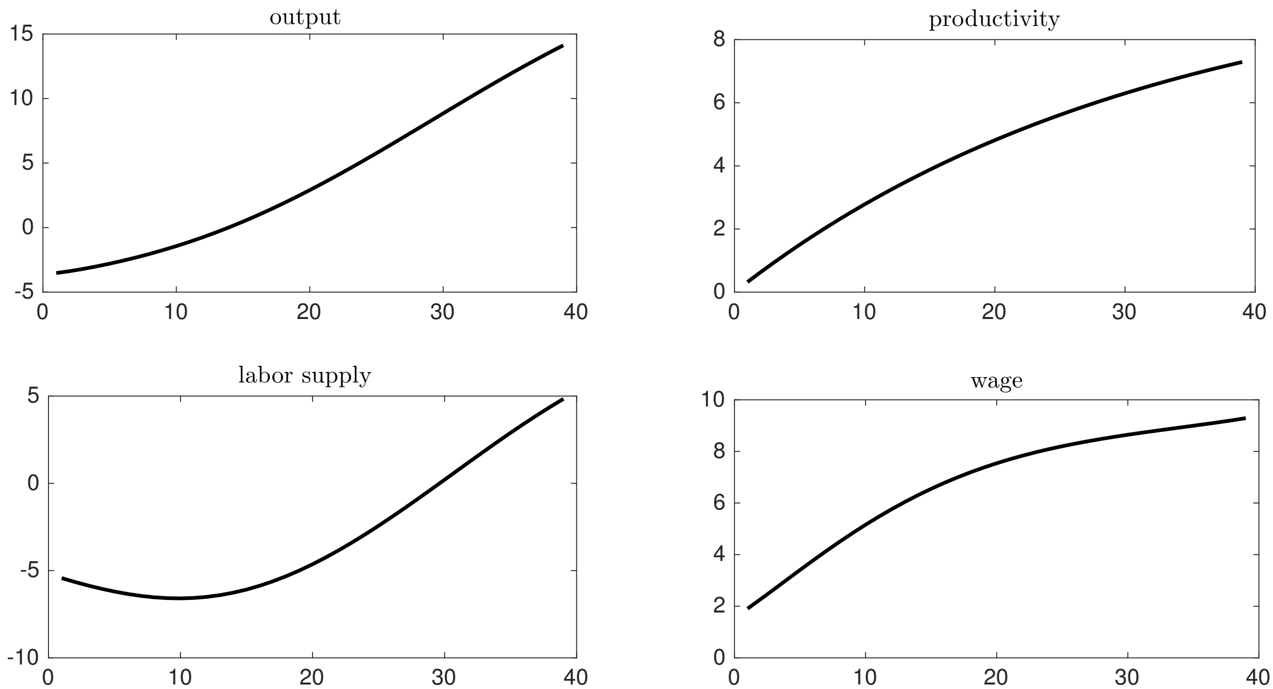

Figure 3.1: Growth Episodes for a positive R&D shock (40 quarters)

To provide some intuition for the results, Figures 3.1 and 3.2 plot the behaviour of the model after a controlled shock is imposed to it. Since the focus of this thesis is testing the dynamic effect of the specified subsidy, I restrict the IRF analysis to responses of the variables to one-time increase in the policy instrument, R&D intensity which is measured as the ratio of GERD (total intramural expenditure on research and development performed on the national territory) to total GDP. The shock based on the assumption is very consistent with the AR(1) coefficient of 0.97; hence it is expected that the imposed one-off shock has a long-run positive effect on productivity which lasts over 40 quarters (Figure 3.1). It is illustrated that the output and labour supply are still growing after 40 quarters, while real wages are smoothening out although converging to higher levels. In early quarters labour supply tends to decrease (smaller than the base-run) after the one-off shock to productivity. It can be interpreted as the lower opportunity cost of z makes labour a relatively less attractive way to earn, hence output falls in initial periods in response to this 'off-the-wage-earning' activity decision of agent but as productivity increases more, the output rises steeply from period 10. The real consumer wage rises to offset the income effect on labour supply from the productivity increase, but it is not a dominant effect. Eventually, Y and w will converge to higher levels implying the permanent effect of the positive exogenous growth.

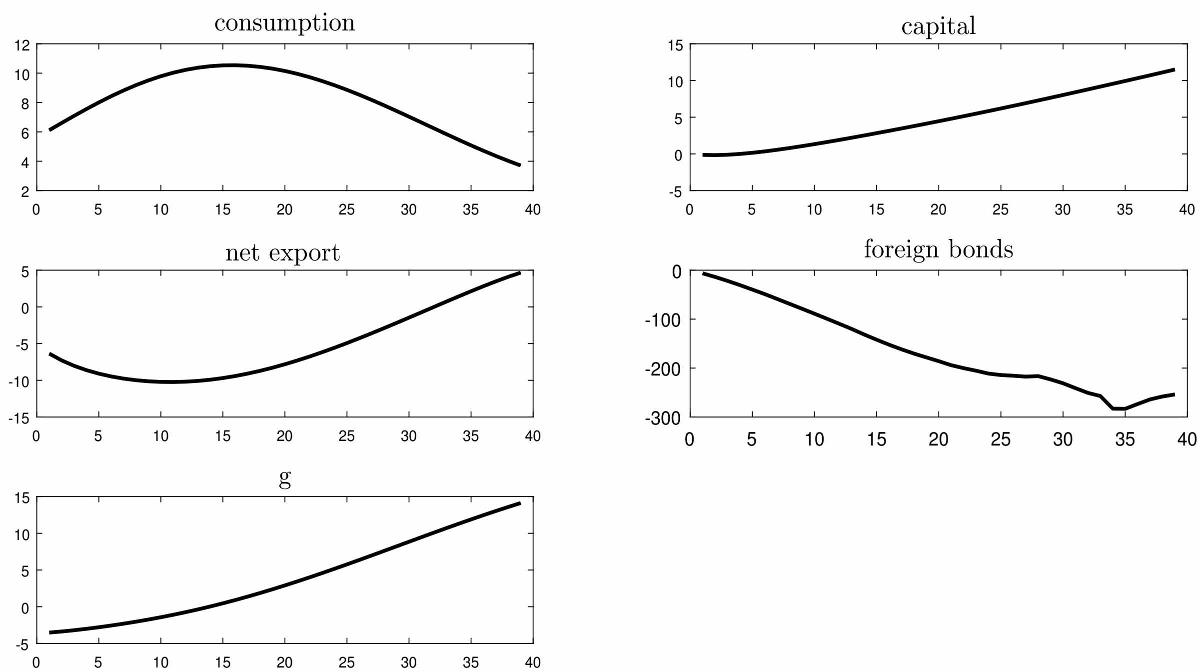

Figure 3.2: Business Cycles for a positive R&D shock (40 quarters)

Figure (3.2) depicts the response of capital to the shock where adjustment costs prevent the 'investment overshooting' and results in a higher level of capital. Government spending has the similar attitude toward the shock as the output, first decreases at some levels then converges to a higher level permanently. Net foreign assets accumulate throughout the simulations and the transversality condition ensures that net foreign assets stabilise by the end of the simulation. Consumption increases due to increases in output and real consumer wages and finally stabilises at the end of the simulation period. The convergence will happen for all the variables however it takes longer for some to stabilise after the shock.

3.4 Summary

In this chapter, a testable micro-founded macroeconomic model for a small open economy is constructed. It is proposed to be calibrated using priors extracted from literature to ensure the theoretical consistency. A standard rational expectation model adequately simplified by excluding money, taxes and unemployment benefits is prepared to be used for a panel study consisting of 11 countries in Chapter 5. IRF's are provided to sketch the model's reactions to a one-time shock of the policy variable. This step confirms that the model is ready to be tested and the reasons beneath the growth movements in stated elements can be studied in more details.

Chapter 4

Methodology: Testing and Estimation via Indirect Inference

One of the major unresolved issue in macroeconomics over past three decades is how best to test an already or partially estimated macroeconomic models, particularly DSGE models. In this chapter, I present the Indirect Inference Wald testing methodology which has been applied to an endogenous growth DSGE model presented in Chapter 3. This chapter is set out as following. In section (4.1) we consider how in recent studies the DSGE models have been evaluated empirically. In section (4.2) a brief comparison between indirect and direct inference testing method is presented. And section (4.3) summarises the chapter.

4.1 The Empirical Evaluation of DSGE Model

The previous formulation of macroeconometric models failed to fulfil some crucial specifications; hence DSGE models appeared as a response to these shortcomings.

One of the most significant of these flaws was that the conventional Klein and Goldberger's macroeconometric approach- which was dominant in the 1960s- lacked being structural- despite being referred to as structural macroeconometric models, thus, they were vulnerable to Lucas critique that they must not be used for policy evaluation (Lucas, 1976), and that they are not general equilibrium models of the economy.

Despite all the theoretical advantages of DSGE models as being micro-founded, its failure in providing empirical results which fit the economic facts caused sets of questions about its superiority toward the conventional macroeconometric models. Sims (1980) provided a complete answer to Lucas (1976), arguing that micro-foundations are not a necessary condition for policy evaluation. He believed that DSGE models are not empirically corroborated and even that the conventional econometric models are performing well for economic policy evaluation. There have been subsequent developments in response to the empirical failure of DSGE models. Smets-Wouters (2007) is an example of an empirical success which addressed most of the empirical limitations and some specification issues, although this achievement has been questioned.

One of the major criticisms of DSGE models is about why these models have been calibrated rather than been estimated and tested. Estimating using Bayesian rather than the classical macroeconometric method is an attempt to answer this question. Rare attempts to test them via the traditional method of testing such as Likelihood Ratio ended in rejecting 'too many good models'[1]. Estimating models using Bayesian

[1]Sargent remarked of the early days of testing DSGE models: "...my recollection is that Bob Lucas and Ed Prescott were initially very enthusiastic about rational expectations econometrics. After all, it simply involved imposing on ourselves the same high standards we had criticised the Keynesian for failing to live up to. But after about five years of doing likelihood ratio tests on rational expectations models, I recall Bob Lucas and Ed Prescott both telling me that those tests were rejecting too many good models."Tom Sargent, interviewed by Evans and Honkapoja (2005).

methods of estimation is an attempt to response to these questions, as estimating via Bayesian method gives more freedom to data to influence the final estimation and it is more flexible in choosing the prior beliefs about the structural parameters compare to the calibration method, though, as Hensen and Heckman (1996) pointed out, if the priors are uninformative or they cannot be justified properly, then there are not many differences between the Bayesian estimation and classical maximum likelihood estimation as still "too many good models" are going to be rejected.

In response to the empirical failure of DSGE models, there is a more radical approach as it believes that all models are misspecified-in another word "wrong"- but the question is "how wrong do they have to be not to be useful"[2]. Wickens (2014) mentions all models are deliberate simplifications, but nonetheless, they may be useful. By accepting the fact that all models are misspecified what will be the point in testing them in a Popperian manner under the null hypothesis that they are true? Canova (1994), argues that one should ask "how true is your false model?" and evaluate this using a closeness measure rather than concentrating on them being false thus useless.

Following Minford (1984), I adapt a different approach considering the role of formal statistical tests of DSGE models which reflect the effect of Friedman's[3] widely accepted economic methodology. As it has been briefly explained, no DSGE model, or any model, is capable of being completely 'true' as the 'true world' is more complicated to be fully explained in a model. Thus according to Friedman, a model should be tested 'as if it is true' and be judged on its ability to explain the facts by

[2]Box, G. E. P., and Draper, N. R., (1987).

[3]Friedman in his famous essay 'The Methodology of Positive Economics (1953)' argued that economics as a science should not be judged on its objectivity; instead it should be judged by its simplicity and fruitfulness as an engine of prediction.

measuring the distribution of the data it can generate. In this sense, I evaluate a DSGE growth model described in Chapter 3 using a formal misspecification tests. In this spirit, the probability of rejecting this model gives a measure of 'closeness' of it to the facts and the reality.

The progressive method of Indirect Inference provides a classical statistical inferential framework for evaluating a partially estimated or a calibrated model, such as the one under my study, maintaining the spirit of the early methods of evaluation for RBC models. This approach also compares the moments generated by the data simulated from the model with the actual data (see; Kydland and Prescott, 1982; Kydland and Prescott, 1991; Chari et al., 2002). The difference between the classical method of evaluation for RBC model and what I adapted following Le et al. (2011), is that Indirect Inference considers the joint behaviour of the variables as well. Here an auxiliary model is in charge of describing the joint behaviour of the variables both simulated and actual. In the next section, I discuss the method of Indirect Inference, the testing and estimation procedure and the choice of the auxiliary.

4.2 Model Evaluation by Indirect Inference

Recent developments in computer science and increases in the computational power of the machines made it possible to use simulation-based methods of estimation for highly complex dynamic models. The basic idea behind all of these methods is to compare the closeness of some specific properties of model-generated data with those features of empirical evidence. These methods are known by various names, including Simulated Method of Moments (SMM) (McFadden, 1989; Pakes and Pollard, 1989; Duffie and Singleton, 1993; Lee and Ingram, 1991; and Jalali et al., 2013), Efficient

Method of Moments (EMM) (Gallant and Tauchen, 1996; Durlauf and Blume, 2008) and Indirect Inference (II) (Smith, 1993; Gourieroux, Monfort, and Renault, 1993; Gourieroux and Monfort, 1996; Gourieroux, Phillips, and Yu, 2010). These methods are mostly useful for models with intractable likelihood function such as nonlinear dynamic models and models with missing or incomplete data.

Indirect Inference as a simulation-based method of testing and estimation for the DSGE models uses an auxiliary model to mimic the behaviour of the structural model. In this approach, the auxiliary is a window to give the view to the actual and simulated data. Indirect Estimation may be distinguished from Indirect Inference testing. In Indirect Estimation, the parameters of the structural model are chosen in a way that the simulated model generates similar estimates of the auxiliary model to those obtained from the data. To use Indirect Inference to test the model, the parameters of the structural model are considered given (calibrated or estimated before the test). I use indirect Inference test as the baseline for the Indirect Inference estimation carried out in Chapter 5. For more information about the Indirect Inference as a method of testing see, Le et al. (2011). They present Indirect Inference Wald test that concentrates on some specific features of RBC models. In the next sections, the indirect inference as the method of evaluation and estimation is discussed.

4.2.1 Definition for Indirect Inference

One of the hallmarks of the method of Indirect Inference is that it compares the performance of the auxiliary model- general but simple formal model- estimated from the actual data with the performance of the simulated data derived from the model. As a window to view both actual and simulated data, auxiliary models do not require to be very accurate descriptions of the data generating process. It can represent particular aspects of the model on which the study is focused. Le et al. (2013, 2015) present a VAR as the auxiliary model, but they point out that IRFs or moments could also be used as that window to look at the actual and model-generated data and to check their closeness as a measurement of model's performance. It is not necessary to estimate the parameters of the structural model when we test it via Indirect Inference. The parameters of the structural model might be already calibrated or partially estimated using different methods of estimation such as the Bayesian method. The null hypothesis is that "the model is true". If the DSGE model under the study is true, then the selected moment of the VAR estimates based on these simulation results will not be significantly different from those derived from the actual data. The main idea is to bootstrap the estimated DSGE model, generating 1000 pseudo-samples which represent what the model and its "true errors" could have generated for the historical sample data. The aim of the test is to compare the coefficients of the estimated VAR using the actual data with the mean of the distributions of the estimated coefficients for the simulated data.

Le et al. (2011, 2015) use the notation of Canova (2005), design a formal statement of inferential problem as following. They define a $m \times 1$ vector of observed data, y_t; t=1,...,T, a $m \times 1$ vector of simulated time series of S observations generated from the structural model, $x_t(\theta)$ where θ is a $k \times 1$ vector of the parameters of the macroeconomic model under our study. It is assumed that $x_t(\theta)$ and y_t are stationary and ergodic. Then they define the auxiliary model as $f[y_t, \alpha_t]$[4]. The null hypothesis is $H_0 : \theta = \theta_0$ where θ_0 can be the calibrated or estimated value of the coefficients for the structural model, based on this the auxiliary model will be $f[x(\theta_0), \alpha(\theta_0)]$. They

[4]An example of the auxiliary model is the $VAR(p) y_t = \sum_{i=1}^{p} A_i y_{t-i} + \eta_t$, where α is a vector consists of elements of the A_i and the covariance matrix of y_t.

consider α_T as the estimator of α using actual data and $a_T(\theta_0)$ as the estimator of α for simulated data using θ_0[5]. After bootstrapping and deriving N independent sets of simulated data, it is required to find the mean of $a_T(\theta_0)$ as $\frac{1}{N}\sum_{k=1}^{N} a_T(\theta_0)$. The Wald statistic is as presented in (4.1).

$$WS(\theta_0) = (a_T - \overline{a_T(\theta_0)}\ )'W(\theta_0)(a_T - \overline{a_T(\theta_0)}\ ) \tag{4.1}$$

as $\overline{a_T(\theta_0)}$ is the arithmetic mean of the estimated parameters of VAR and $W(\theta_0) = \omega(\theta_0)^{-1}$ is the inverse of the estimated variance-covariance matrix. The variance-covariance matrix, $W(\theta_0)$ can be obtained from the asymptotic distribution of $(a_T - \overline{a_T(\theta_0)}\ )$.

Section (4.2.2) summarises the implementation of the Wald test and process to estimate the model via Indirect Inference.

4.2.2 Indirect Inference Testing Process

Step 1: Calculate the structural errors using the actual data and θ_0

Solving the structural model and finding the related errors ϵ_t, is the first step. The number of the independent structural errors is less than or equal to the number of endogenous variables in the DSGE model, and the errors are *not* assumed to be normally distributed. If there is no expectation in the model the errors could just be withdrawn from the equations and the data but considering the expectations makes it necessary to use the robust instrumental variables methods of McCallum (1976) and Wickens (1982), with the lagged endogenous data as instruments. This gives

[5]The number of simulations S can be considered equal to the number of actual data T, since it is required for the actual data to be regarded as potential replication from the population of the bootstrapped simulation samples, thus it can be chosen as the same size as the actual data or a coefficient of it e.g $cT; c \geqslant 1$.

the auxiliary model as e.g. $VAR(p)$[6].

Step 2: Bootstrap, simulate data, and construct the empirical distribution conditional on the null hypothesis

To simulate the data, the structural errors $\epsilon_t; \{t = 1, ..., T\}$- some are stationary and some are non-stationary- are modelled as autoregressive processes. The residuals of the estimated AR processes are called "innovations" which are identically independently distributed. The bootstrapping procedure then involves drawing randomly with replacement from this set of i.i.d innovations to preserve any simultaneity between them, then the projection method due to Minford et al. (1983, 1986) is used to solve the "*true model*" and generate the simulated data. To obtain $N = 1000$ bootstrapped simulations, this randomly drawing with replacement is repeated independently for each sample.

Step 3: Establish the Wald Statistic

To reject or not reject the null of the model being true, estimation of an auxiliary model using the actual and simulated data is required. In this study- which is a Panel analysis of an endogenous growth DSGE model- a PVAR(1) is chosen as the auxiliary model- see 4.2.4 for details on choosing the auxiliary model. The aim is to estimate the selected auxiliary model using both actual and N samples of simulated data to obtain estimates of a_T and $a_T(\theta_0)$ of the vector α. By estimating the auxiliary on each of the bootstrapped simulations from *Step 2*, the distribution of $a_T - \overline{a_T(\theta_0)}$ and its covariance matrix $W(\theta_0)$ are found.

After computing the Wald statistic in 4.1, the inference can proceed by comparing the percentile of the Wald distribution at which the critical Wald statistic falls within

[6]They used the limited information maximum likelihood (LIML) procedure to estimate the expectations on the right-hand side of the model.

the chosen size of the test; for a 5% significance level, a percentile above 95% would fall into the rejection area. As an alternative, Mahalanobis distance can be used which is the square root of the Wald value. As it is the square root of a chi-squared distribution, it is possible to convert it to a t-statistic by adjusting the mean and the size and normalising it to ensure that the resulting t-statistic is 1.645 at the 95% point of distribution. Thus the null hypothesis can be rejected if this t-value takes a number greater than the t-statistic at 5% [7].

The VAR covariance parameter is important as assuming it to be equal to zero or non-zero will change the value of the Wald statistics. In their experiment, Minford et al. (2008) present non-zero VAR covariances generated by the bootstrap sample variation make a Wald statistic of 100 (outright rejection) whereas zero covariances will push the test to the non-rejection area (below 100 Wald statistics). Hence considering a high covariance creates a 'ridge' out of the density function. Figure (4.1) sheds more light on how this method is working. Assume a VAR with two parameters which their variables are regressed on the own lagged value, and that the model distribution is centred around 0.5 and 0.5, and the data-based VAR produced values for their partial autocorrelations of 0.1 and 0.9 respectively. As we can see when the correlation is 0, the model seems to explain a great deal of the reality (data) where the filled square represents the estimated VAR parameters using the actual data which is in a lesser distance to the mean of the estimated VAR parameters of the simulation-generated data. By considering a correlation of 0.9, the mountain

[7] Following Wilson and Hilferty (1931) the normalised Mahalanobis distance is given as

$$t = \left\{ \frac{\sqrt{2MD} - \sqrt{2n}}{\sqrt{2MD_{95\%}} - \sqrt{2n}} \right\} \times 1.64 \tag{4.2}$$

where MD is the value of the Mahalanobis distance derived using the actual data in Wald and $MD_{95\%}$ is the value for it corresponding to 95% tail of simulated distribution and n is the degree of freedom of the variant.

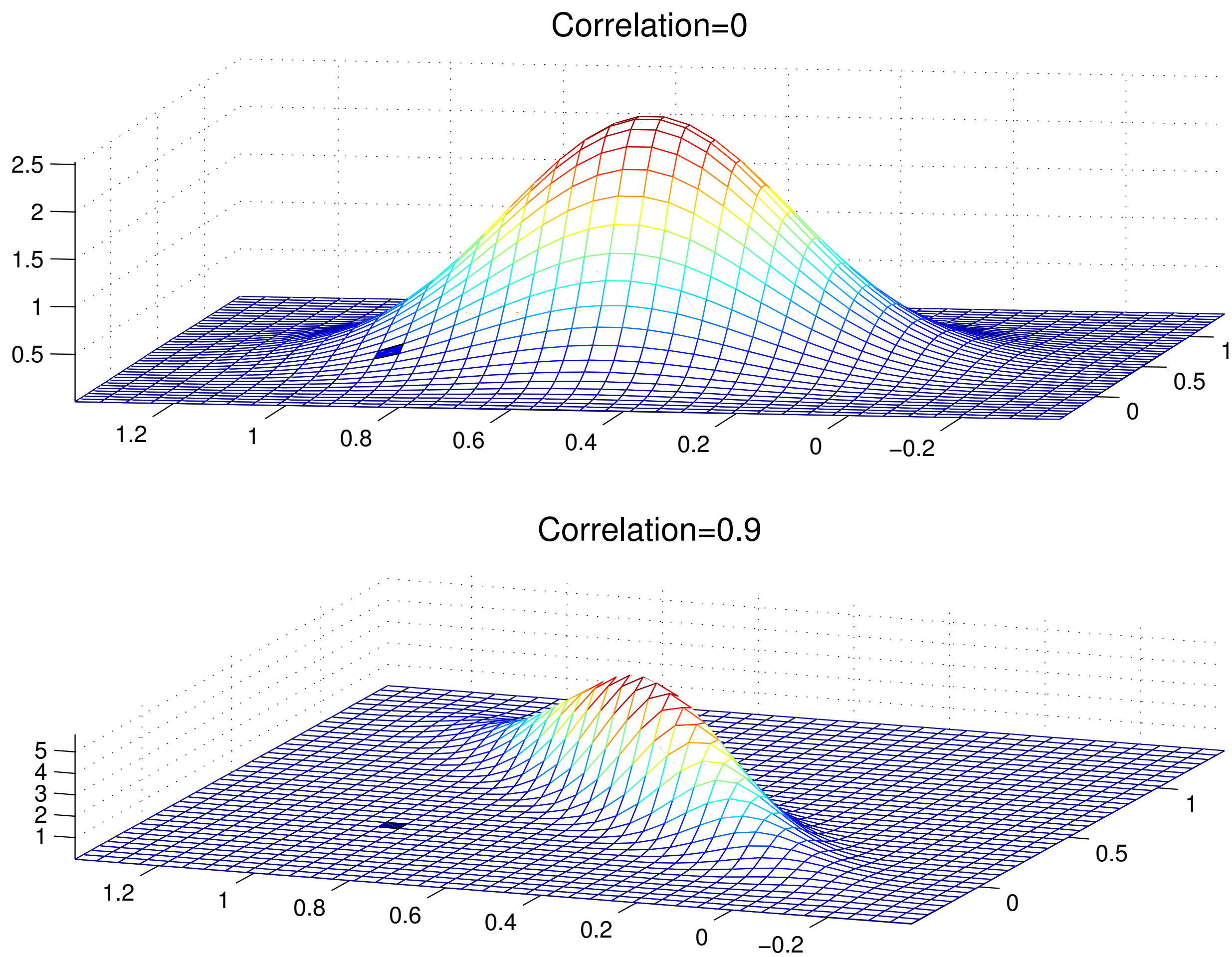

Figure 4.1: Bivariate Normal Distributions (0.1, 0.9 shaded)

rotates in such a way that the estimation of the VAR parameter of the actual data falls far from the mean of the estimated VAR parameters for the model generated data; hence the test rejects the model in explaining the properties of the data.

4.2.3 Indirect Inference Estimation Process

As mentioned earlier, Indirect Inference test is the basis for the Indirect Inference estimation. The estimation process involves choosing the parameters of the macroeconomic model so that when the model under the null hypothesis is simulated, it generates estimates of the auxiliary model which are similar to those from the actual data. Here the question is how can we find 'the best choice' of parameters. The estimation procedure involves searching over parameter space, within certain bounds, to find that vector of parameters. The Wald statistic measures the distance between the data and the model thus to find 'the best calibrations', any minimising algorithm to minimise the Wald can be used. Here the Wald minimising process refers to the systematic process of minimising the distance between the coefficients of the VAR of the data with the average of those of the model-generated data. The process, in brief, is as following: 1) the function to minimise takes the coefficients as an input, 2) it does steps 1-3 for testing the model and then 3) gives the Wald as the output. In Chapter 5, due to Le et al. (2013), a 'simulated annealing algorithm' is chosen as the minimising algorithm to perform the Indirect Inference Wald test for 1000 points in parameter space. I searched for the possible best parameters within the 45% interval of the initial calibration value. The parameters chosen to be estimated, are mostly those preference-related parameters or parameters for which no strong priors exists in the literature.

According to Le et al. (2015), for small samples, Indirect Inference estimates have a low small sample bias compared with the FIML systems estimator. Bayesian estimators would bias the estimator towards the priors which when we do not have information about the model parameters that will give a bias. Hence, using indirect inference estimation method is valid and efficient.

There is this question about the power of the Indirect Inference evaluation method after doing the estimation. As Le et at. (2015) present Indirect Inference is a very robust test in rejecting the false models. The algorithm is as following, the new sets of calibrated values for parameters will set up a new model which we consider as the 'true' model. If the model is wrong, then the probability that Indirect Inference rejects the model will be very high. Le et al. (2015) experiment of SW's model checked the validity of the power of Indirect Inference test. In evaluating the power of the test, they use their Monte Carlo procedure and generate 10,000 samples from 'True' model and find the distribution of the Wald for these true samples. Then they generate 10,000 sample from the 'False' model with parameters and find the Wald distribution for it and finally they calculate how many of the actual samples from the 'true' model would reject the 'false' model on this calculated distribution with 95% confidence level. This will give the rejection rate for a given percentage degree of misspecification. Their experiment shows that with 5% misspecification, the Wald statistic rejects 99% of the time while Likelihood Ratio test rejects only 15% of the time. At a sufficiently high degree of falseness both of them reject at 100% of the time.

Le et al. (2015) ensure that the if the estimated parameters are far from establishing a 'true' model, the powerful Indirect Inference evaluation method will reject it with a very high probability. For a high degree of falseness Indirect Inference and LR both are as good and as powerful in rejecting 100% of the time, and for the lower degrees, it seems Indirect Inference will do better than LR. However, one must consider that these two tests are measuring different things, as LR measures the forecasting ability of the model while the Indirect Inference Wald test measures the model's ability to explain the sample data behaviour.

4.2.4 The Choice of the Auxiliary Model

After log-linearisation, almost all macroeconomic and financial econometric models - even an endogenous growth DSGE such as the one in this study- can be represented as VAR (or a VARMA) with restrictions (Wickens, 2014).

The data generated by a DSGE model are often non-stationary. The reasons for the non-stationarity of the model generated data can be either because 1) the model structure causes non-stationarity for example by making state variables functions of variables that depend on accumulated shocks, such as net foreign assets, in this study, or 2) because a non-stationary variables such as technology shock in the production function is included in the model. Thus after log-linearising, the solution of the model can be represented by Vector Error Correction model- an auxiliary model with stationary errors. Following Meenagh et al. (2012)[8] and Le et al. (2015a), I show that the chosen auxiliary model is an approximation of the reduced form of the DSGE model under the null of the cointegration[9] and it can be represented as a

[8]They assume that the solution of the model can be represented by a VECM. Then they consider that if there are unobservable non-stationary variables, such as a technology shock, the number of the cointegrating vectors will be less than the number of the endogenous variables. This means that one or more of the structural equations will have a non-stationary residual. To solve this matter they estimate all of the coefficients of the model, construct the residuals, and they treat all these estimated residuals as the observable variables. Now there will be as many cointegrating relations as the number of endogenous variables.

[9]Constraint of the null guarantees that the VECM achieves cointegration under the null and the residual assumption ensures that the DSGE model achieves cointegration.

cointegrated VARX.

After log linearisation the DSGE model can be presented in the form of

$$A(L)y_t = BE_t y_{t+1} + C(L)x_t + D(L)e_t \tag{4.3}$$

where y_t is a $p \times 1$ vector of endogenous variables, $E_t y_{t+1}$ is a $r \times 1$ vector of expected future endogenous variables, x_t is a $q \times 1$ of non-stationary variables which are assumed to be driven by

$$\Delta x_t = a(L)\Delta x_{t-1} + d + b(L)z_{t-1} + c(L)\epsilon_t \tag{4.4}$$

x_t is a vector of unit root processes, elements of which may have a systematic dependency on the lag of z_t which is a stationary exogenous variable. Both e_t and ϵ_t are i.i.d with zero means. As y_t is linearly dependent on x_t and x_t is non-stationary, as a result it will be non-stationary. The general solution of y_t is

$$y_t = G(L)y_{t-1} + H(L)x_t + f + M(L)e_t + N(L)\epsilon_t \tag{4.5}$$

where f is a vector of constants and polynomial functions in the lag operator have roots outside of the unit circle. The solution to the model has p cointegrated relations[10]

[10]The matrix Π is $p \times p$ and we found it when we solve for the terminal conditions on the model. Terminal conditions restricts the expectations in the structural model and make them consistent with the model's long run equilibrium.

$$
\begin{aligned}
y_t &= [I - G(1)]^{-1}[H(1)x_t + f] \\
&= \Pi x_t + g
\end{aligned}
\tag{4.6}
$$

in the short run y_t is a function of deviation from the equilibrium; $y_t - [\Pi x_t + g] = \eta_t$ where η_t is the error correction term. In the long run solution to the model is

$$\bar{y}_t = \Pi \bar{x}_t + g \tag{4.7}$$

$$\bar{x}_t = [1 - a(1)]^{-1}[dt + c(1)\xi] \tag{4.8}$$

$$\xi_t = \sum_{s=0}^{t-1} \epsilon_{t-s} \tag{4.9}$$

as we see the long run solution to x_t can be decomposed to two components, the deterministic trend; $[1 - a(1)]^{-1}dt$ and a stochastic trend; $[1 - a(1)]^{-1}c(1)\xi$. The endogenous variable consists of this trend and the deviations from it, therefore the solution for y_t can be written as this trend plus a VARMA in deviations from it. As an alternative Meenagh et al. (2012) formulates this as a cointegrated VECM with a mixed moving average error term, ω_t.

$$\Delta y_t = -[I - G(1)](y_{t-1} - \Pi x_{t-1}) + P(L)\Delta y_{t-1} + Q(L)\Delta x_t + f + M(L)e_t + N(L)\epsilon_t \tag{4.10}$$

$$= -[I - G(1)](y_{t-1} - \Pi x_{t-1}) + P(L)\Delta y_{t-1} + Q(L)\Delta x_t + f + \omega_t$$

$$\omega_t = M(L)e_t + N(L)\epsilon_t \tag{4.11}$$

This can be approximated by the VARX with ζ_t as an i.i.d zero mean process.

$$\Delta y_t = -K(y_{t-1} - \Pi x_{t-1}) + R(L)\Delta y_{t-1} + S(L)\Delta x_t + g + \zeta_t \tag{4.12}$$

since $\bar{y}_t - \Pi\bar{x}_{t-1} - g = 0$, the VECM can be written as

$$\Delta y_t = -K[(y_{t-1} - \bar{y}_{t-1}) - \Pi(x_{t-1} - \bar{x}_{t-1})] + R(L)\Delta y_{t-1} + S(L)\Delta x_t + h + \zeta_t \tag{4.13}$$

Either equations (4.12) or (4.13) can be used as the auxiliary model. Rewriting (4.12) as below

$$y_t = [I - K]y_{t-1} + K\Pi x_{t-1} + n + \phi t + \sigma_t \tag{4.14}$$

we will have a VARX(1) approximation to the reduced form of the model where the errors σ_t now includes the lagged difference regressors, and by considering the time trend the deterministic trend in $\bar{x}_t$ which influence both endogenous and exogenous variables is captured.

I use PVARX as the auxiliary to assist the process of evaluation in Chapter 5. Panel VARs have the same structure as VAR models in the sense that all variables are assumed to be endogenous and interdependent with the difference that PVARs are applied to panel data hence a cross sectional dimension is added to the VAR representation. Following Canova (2013) notation, we consider Y_t as the stacked version of y_{it}, the vector of G variables where $\{i = 1, ..., N\}$ as i in this study indicates the number of countries. Hence, Y_t can be written as $Y_t = \{y'_{1t}, y'_{2t}, ..., y'_{Nt}\}'$. Then the panel VAR is

$$\begin{aligned} y_{it} &= A_{0i}(t) + A_i(\ell)Y_{t-1} + u_{it} \\ i &= \{1, ..., N\} \\ t &= \{1, ..., T\} \end{aligned} \tag{4.15}$$

here u_{it} is a $G \times 1$ vector of random errors, A_{0i}(t)[11] and $A_i(\ell)$ which is a polynomial in the lag operator may depend on the unit. A panel VARX is assumed in this study. The representation is

$$y_{it} = A_{0i}(t) + A_i(\ell)Y_{t-1} + F_i(\ell)W_t + u_{it} \tag{4.16}$$

where $u_{it} = [u_{1t}, u_{2t}, ..., u_{Nt}]'$ is i.i.d with mean zero and variance of Σ_u, $F_{i,j}$ are $G \times M$ matrices for each lag $j = \{1, ..., q\}$, and W_t is a $M \times 1$ vector of predetermined or exogenous variables, common to all units i.

One of the disadvantages in using PVARs is that one needs to impose cross-section homogeneity on the relationship between the endogenous and exogenous variables. Most of the literature related to using PVAR models are implicitly assumed cross-

[11] All the deterministic components of data is compacted in $A_{0i}(t)$, thus it should be understood that equation (4.14) may include seasonal dummies, constants or even deterministic polynomial in time (Canova and Ciccarelli, 2013).

section homogeneity for simplicity in the process of estimation.[12]

Hence, using panel VAR (pooling observations for a number of countries over a given sample period), one needs to impose cross-country homogeneity on the relationships among the variables. Some econometric adjustments are made to deal with this objection. In particular, country-specific constant terms and country-specific linear time trends are included in the regression to eliminate any cross-country contemporaneous residual correlation. These features reduce the amount of heterogeneity. Also, the fact that 11 OECD countries studied in Chapter 5, share many similarities is conducive to reducing heterogeneity.

4.3 Summary

This chapter introduced the concept of the Indirect Inference testing and estimation methodology applied in the empirical work presented in this thesis. I provided a definition for the Indirect Inference and a step by step procedure to the testing and estimation method, following Le et al. (2013). Then, VAR, VECM, and PVAR as the relevant auxiliary models are introduced. In Chapter 5, I first present the outcome of the Indirect Inference test of the model given the initial calibration values for the parameters, and then I proceed to estimate the model parameters and find the optimal calibrations value using the *simulated annealing* search algorithm.

[12] Beetsma and Giuliadori (2011) and Lane and Benetrix (2011) look at the transmission of government spending shocks and Boubtbane et al. (2010) examine how immigration shocks are transmitted in a variety of countries. Love and Zicchino (2006) measure the effect of shocks to financial factors on a cross-section of U.S. firms, and they all assumed a cross-section homogeneity.

Chapter 5

Testing and Estimating a Multi-Country R&D-driven Model of Growth

"An indispensable hypothesis, even though still far from being a guarantee of success, is however the pursuit of a specific aim, whose lighted beacon, even by initial failures, is not betrayed." -Max Planck, Nobel Lecture (1920)

This chapter presents the process of testing a dynamic model of growth outlined in Chapter 3 via the method of Indirect Inference. The aim is to verify the impact of the knowledge-based policy variable (R&D intensity) on total factor productivity growth. I firstly introduce the data- economic and Science & Technology indicators in section 5.1, where they are illustrated and analysed through suitable analysis method. Section (5.2) explains the details of the theory under the study, the testing procedure and finally, the results of the Wald test on the baseline calibrated model is provided, then the parameters of interest are estimated using a 'simulated-annealing algorithm' discussed in Chapter 4.

5.1 Description of Data and Methodology

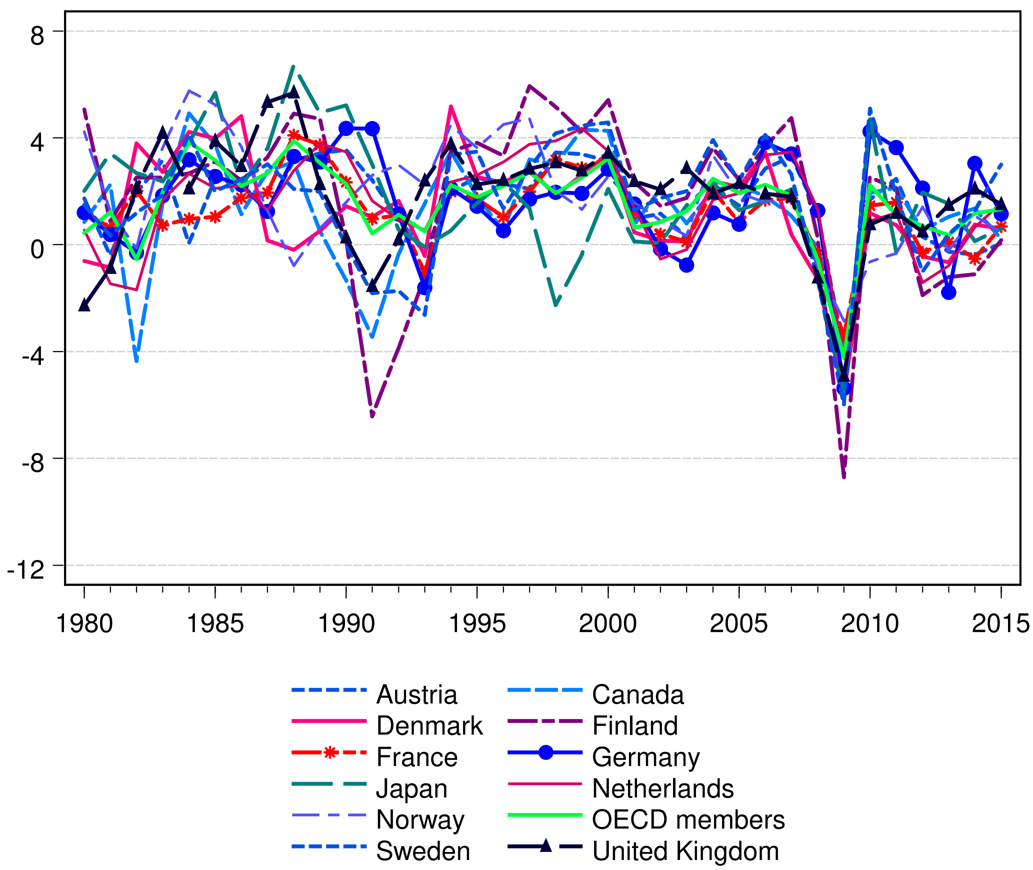

Figure 5.1: GDP growth (annual %)

Source: World Bank and OECD National Accounts data files.

Figure (5.1) illustrates the annual GDP growth rate for 11-OECD countries under the study for the period of 1980-2015. The data on gross domestic products of the countries is extracted from OECD Main Economic Indicator (MEI)- OECD National Accounts and the annual data on GDP growth from the World Bank database. As it is depicted the OECD countries in the 1990s are characterised by widening disparities in growth rates of GDP per capita (see Aiginger and Landesmann, 2002) with Finland having the lowest growth rate of -6.42% in 1991 and the highest rate of 5.9% in 1997. Japan experienced the lowest of -2.26 % in 1998 while it benefited from the highest rate of 5.2% of the annual growth in 1990. It is evident from Figure(5.1) that all 11 economies collapse in 2009 with their lowest rate of growth (decrease in growth of 4.22%) for OECD members with a recovery rate of approximately 2.3% in the following year.

Country	Minimum Value	Maximum Value
Austria	-4.05	3.55
Canada	-4.36	4.91
Denmark	-5.59	5.17
Finland	-8.71	5.94
France	-3.44	4.09
Germany	-5.38	4.35
Japan	-5.52	6.69
Netherlands	-4.26	4.35
Norway	-2.86	5.75
Sweden	-5.99	5.09
United Kingdom	-4.91	5.69

Table 5.1: Minimum and Maximum level of Annual percentage growth rate of GDP per capita based on constant local currency.

Hence, it can be concluded that there has been a tremendous economic and technological catching up in past three decades. Despite the economic ups and downs, the economic growth is still the primary concern of all nations and upgrading the technological state of the country to achieve this growth requires effective policy decisions which suitably decompose the elements of the growth and allocate resources to effective factors. As discussed in previous chapters technological change needs relevant innovation-based policies; thus R&D is acknowledged as a principal mean to enhance economic growth and competitiveness. How to evaluate and test this effectiveness is the focus of this study.

The data of the leading economic indicators are extracted from OECD National Accounts data files, International Finance Statistics database (IFS), Office of National Statistics (ONS) for data on UK, Bank of England (BoE), Unesco Institute for Statistics, Statistics Canada, Federal Reserve Bank of St. Louis (FRED) and The World Bank's DataBank (WB). The collected data consists of quarterly data for the period of 1980Q1-2014Q4. In the cases where the quarterly data were not available (i.e. data on average manufacturing earning index as a proxy for wages), the quarterly data is derived from annual data using the extrapolation or interpolation. All the data are seasonally adjusted and in constant prices unless specified otherwise. Table (5.2) includes the definitions and descriptions of data used in this study, as well as symbol keys.

Notes on Table (5.2):

1. Seasonally Adjusted using U.S. Census Bureau's software package X12-ARIMA.

2. Hourly Average Earnings includes earnings series in manufacturing and for the private economic sector. Mostly the sources of the data are business surveys covering different economic sectors, but in some cases, administrative data are also used. The target series for hourly earnings correspond to seasonally adjusted average total earnings paid per employed person per hour, including

Symbol	Variables	Data Definition
Y	Output	Gross Domestic Product, Constant Prices, SA[1] USD, 2010 Prices
I	Investment	Fixed Investment, Constant Prices, SA, USD, 2010 Prices
w	Consumer's Real Wage	The Hourly Earnings Index: Year 2010 = 100[2] divided by GDP Deflator
N	Labour Demand	Seasonally Adjusted Employment divided by 16+ working population
K	Capital Stock	Capital Stock calculated using '*perpetual inventory equation*[3]'
A	Total Factor Productivity	Calculated as the Solow Residual
C	Consumption	Consumption, Private, Constant Prices, SA, USD, 2010 Prices
G	Government Spending	Government Consumption , Constant Prices, SA, USD, 2010 Prices
EX	Export	Exports, Goods and Services, Constant Prices, SA, USD, 2010 Prices
IM	Import	Imports, Goods and Services, Constant Prices, SA, USD, 2010 Prices
NX	Net Export	(Exports -Imports)
b_f	Net Foreign Assets	Ratio of Nominal Net Foreign Assets (NFA) to Nominal GDP[4]
C_f	Foreign Consumption Demand	World Exports, Goods and Services, Constant Prices, USD, 2010 Prices
r	Real Domestic Interest Rate	Nominal Interest Rate (Short Term) - next period's inflation
r_f	Real Foreign Interest Rate	Foreign Interest Rate (weighted average of EU(19%), US(60%), JP(21%), then made real using P_f
P	Domestic Price	Consumer Price Index, SA, Index, 2010=100
P_f	Foreign Price	World, Consumer Price Index, Index, 2010=100
	GDP Deflator	Price Deflator, Gross Domestic Product, Index, 2010=100
π	R&D Intensity	$GERD$[5] as a % of GDP

Table 5.2: Data Description

overtime pay and regularly recurring cash supplements.

3. Unobservable variable, the capital stock is created following Caselli (2004):

$$K_t = I_t + (1 - \delta)K_{t-1} \tag{5.1}$$

where initial level of capital K_0 is given by $K_0 = \dfrac{I_0}{(g+\delta)}$.

4. Nominal NFA is accumulated current account surpluses, taking the Balance of Payments international investment position as a starting point.

$$NFA(1981Q2) = NFA(1981Q1) + CAS(1981Q2) \tag{5.2}$$

5. Gross domestic expenditure on research and development (GERD) is total intramural expenditure on research and development performed on the national territory during a given period.
   Source Publication: OECD Frascati Manual, Sixth Edition.

Direct subsidy to research and development is proxied by the ratio of gross expenditure on research and development to total GDP. The detail of the chosen policy variable is discussed in the following section.

5.1.1 Policy Variable

Over the past decades, most of EU countries have the objectives encouraging the increase in the level of investment to provide stimulus to the competitiveness. Lisbon strategy was an attempt to set the EU a target of devoting at leat 3% of the member's GDP to research and development activities by 2010 which the target was not reached thus the 3%-target remained one of the five key objectives of the Europe's 2020 strategy adopted in 2010.

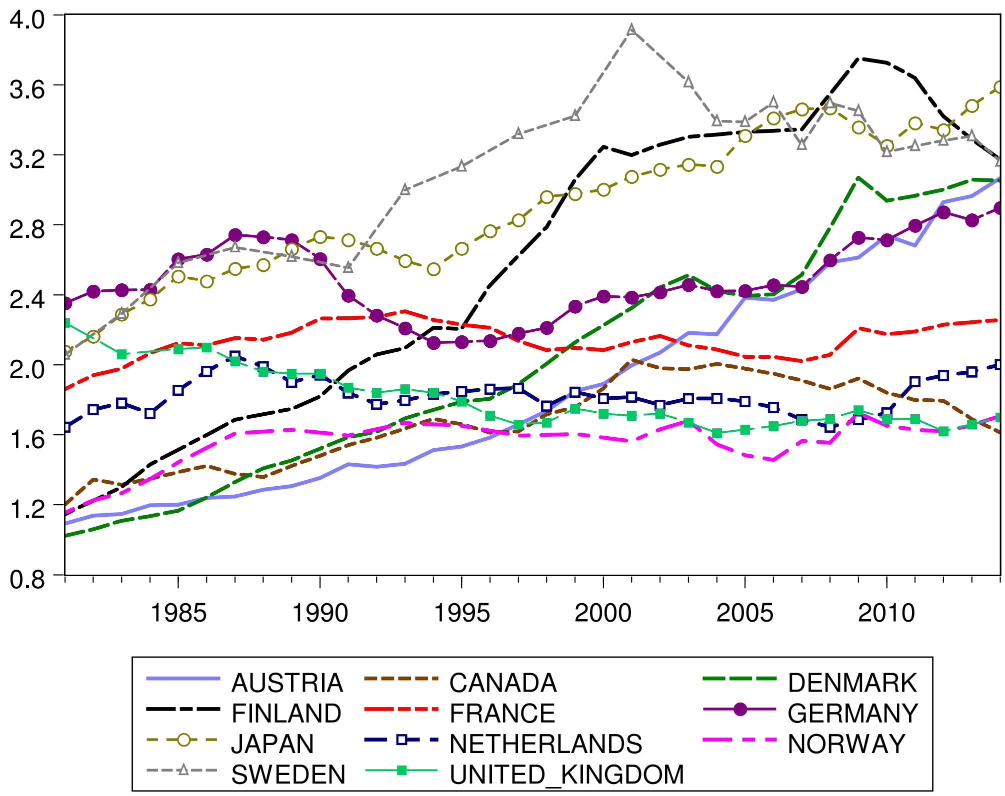

Figure 5.2: R&D intensity of 11-OECD countries (1981-2014)

Source: OECD Stats (MSTI)

Gross domestic expenditure on R&D (GERD) for total OECD zone increased from 161,688 million USD in 1981 to 1,181,495 million in 2014. Some countries experienced faster growth in their GERD, and some grew slower during this period. To make figures more comparable, GERD is often expressed relative to GDP which is also known as R&D intensity. See Figure (5.2) which illustrates the gross domestic expenditure ratio to GDP for countries of the study. The ratio declined modestly during 1990-1994 from the average of 2.18% to 1.9%. This follows by an increase from 2000 to 2010.

Between 2004 and 2008 there was an increase in the relative importance of GERD in the Japanese economy, as its ratio to GDP rose by 0.34 percentage points during this period; note that Japanese economic growth was relatively subdued during this period. However, in the period between 2009 and 2010, the ratio of GERD to GDP in the Japanese economy fell by 0.22 percentage points, before bouncing back somewhat (a gain of 0.13 percentage points) in 2011. Norway and Canada are two countries with the lowest average R&D intensity of less than 1.7% for this period. UK spendings on research and development declined enormously since 1981 (from 2.24% to 1.6% in 2014). The information in Figure (5.2) may imply that different countries adopt different policies regarding their spendings on R&D or as this rate is relative to the GDP, any decline or improvements in the value of it makes significant changes in the spending rate. It is possible that not much of increase happened in the research sector and it is only due to declining in GDP from 2009, some improvements in the value of the intensity are observed. There are three main sectors performing research and development: Government, Higher Education and Business Enterprise. The OECD National Account Statistics and Eurostat use the information of these four sectors to calculate GERD for each country. Overviewing the data for different sectors which are financing GERD, one can easily see that most of the R&Ds are performed by the private sector (the business enterprise). The lowest budget spent on research and development belong to higher education sector as predicted. The average ratio of GERD financed by higher education for 11 OECD countries in the period under the study, decreased 27.7% where the percentage of GERD funded by governments decreased even more by the approximate rate of 33%. The percentage of GERD financed by the private sector for all countries present an increasing trend for the period of 1984-2009. It is increased by the rate of 24%. To have a better view on the improvement of R&D intensity relative to GDP growth and compare the ratio of R&D funded by different sectors to total R&D, see Figures (1)-(3) in Appendix V.

To test the effect of R&D on the growth of total factor productivity the gross domestic expenditure on R&D is chosen to include the total effect of any spendings on research and developments on growth. If R&D shows an empirically identified effect on TFP growth based on the model, one may conclude that the policy must support any methods which generate more innovations i.e. government can directly subsidise research activities, or it can be increased by motivating private and higher education sectors to invest more on R&D. In this thesis, the focus is on the causal relation between growth in TFP and R&D expenditures which is supposed to increase innovations. R&D intensity as the ratio of GERD to GDP is not the product of innovations but is an input to the machine of innovations. It is discussed in Chapter 2 that many empirical studies use the patent data as the output for the innovative activities which seems a better proxy for knowledge productivity. The model outlined in Chapter 3 does not have a representative firm with different sectors to take the R&D input and generate the output. It is a simple model concentrated on the household's decision on choosing to do creative productivity-enhancing activities or not; thus it is not necessary to use the patent proxy. As long as incentivising the representative household to do innovations is the matter of concern, R&D intensity can correctly proxy these activities.

5.1.2 On Choice of not Filtering the Data

It is a common practice to filter the data before estimating a DSGE model. Since Whittaker (1923), methods of graduating data which now is commonly called data smoothing or data filtering are designed to remove the effects of measurement error and reveal the underlying trend in the data. Hodrick-Prescott (HP), Band-Pass and Baxter-King filters are the standard techniques in macroeconomics for separating the long-run trend in a data series from short-run fluctuations. Two well-cited papers in which HP filter is applied are Kydland and Prescott (1990) and Backus and Kehoe (1992). Following these two articles, HP filter used universally to stationarise data despite the fact that there are criticisms to its properties (Cogley and Nason, 1995; Ravn and Uhlig, 2002; Canova, 2008; and Phillips and Jin, 2015). It is widely known that filtering the data, in general, may distort the dynamic properties of the model in some way which are not easy to uncover. For example, Cogly and Nason (1995) in their paper demonstrate that HP filter is likely to generate 'spurious' cyclical structure at business cycle frequencies in a case where it applied to difference stationary series. The similar defects are considered for Band-Pass and Baxter-King filters which make them targets for major modifications if one wants to apply them appropriately.

Following Meenagh et al. (2012), I decided to use raw data which are not filtered using the conventional and frequently used HP, Band-Pass and Baxter-King filtering method mostly for the reasons such as these methods (particularly HP-filter) alter the lag dynamic structure of the data and make non-existent cycles or transform the forward-looking properties of the DSGE model. Instead, all the quarterly data are only seasonally adjusted using X12-ARIMA method and to preserve their real business cycle fluctuation no filter applied. Hence, unfiltered data is used for all the endogenous variables when solving, testing and estimating the model of Chapter 2 and trend-stationary exogenous variables are detrended.

There still exists the issue of non-stationary data generated by the model. This could be either because the structure of the model generates non-stationarity i.e., by making the state variables to be related to some pre-determined variables that are dependent upon some accumulated shock, such as net foreign assets or the model consider a non-stationary variable such as technology shock in the production function (for the model outlined in Chapter 2 both of these two reasons are valid). To handle these model generated non-stationary data, a VECM model as the auxiliary is chosen where unobservable non-stationary variables such as technology shock are assumed observable using the residuals of the data which can make it possible to have as many cointegrating relations as endogenous variables hence any non-stationary residual is treated as a legitimate cointegrating variable. More details on the solution method with non-stationary data are discussed in (4.2.4).

5.2 Empirical Work

Recollecting equation 3.4-3.58, the theory represented by the model of Chapter 3 is structured as following

$$ln\, A_t = ln\, A_{t-1} + \psi_1\, \pi'_{t-1} + \epsilon_{A,t} \tag{5.3}$$

$$\pi'_t = \rho_{\pi'}\, \pi'_{t-1} + \epsilon_{\pi',t} \tag{5.4}$$

Thus the first approach is to test whether the null hypothesis of the model that research activities drive the growth of TFP. In the case that the theory is supported empirically the policy implications may be necessary to motivate the research activities to enhance the growth. As it implicitly mentioned the policy decisions alters based on the characteristics of the economy including the direct government's subsidy, tax incentive policies or any other policies which encourage the sectors responsible for research are required to act more on the matter.

$$d\,ln\,A_t = \psi_1\,\pi'_{t-1} + \epsilon_{A,t} \tag{5.5}$$

The main assumption is $\psi_1 > 0$, and it is supported by theories discussed in Chapter 2; hence it is assumed that the marginal impact of research activities is positive. The choice of the value for ψ_1 in this R&D driven growth theory is already discussed in Chapter 2 and calibration section of Chapter 3. The conclusion was that there is still debate about the actual value of the rate of return to research and development in the empirical literature which allows a considerable freedom around this choice. The chosen value for $\psi_1 = 0.29$, which is an average estimated rate of return to R&D for 11 countries under the study and it is consistent with the literature (see, Wieser, 2005; Hall et al. 2009). In the following sections, I investigate whether the % of GDP allocated to GERD (R&D intensity) play a role in causing productivity growth, in addition to the other productivity growth determinants wrapped in errors.

5.2.1 Indirect Inference Baseline Calibration Test Result

The summary of calibrated parameters and the allocated symbols to them is given in Table (5.3). Those parameters with * in front of them are going to be estimated in section (5.2.2).

Parameters	Role in the model outlined in Chapter 3
α	labour share in input
β	quarterly discount factor
ν, ν^f	home(foreign) bias in consumption
δ *	quarterly depreciation rate
θ	preference weight on C_t
ρ_1 *	CRRA coefficient for C_t
ρ_2	CRRA coefficient for L_t
η_1 *	Marginal effect of lagged capital stock on current capital demand (natural logs)
η_2 *	Marginal effect of expected capital on current capital
η_3 *	Marginal effect of output on current capital
η_4 *	Marginal effect of the current real interest rate on current capital
$\frac{\bar{Y}}{\bar{NX}}$ **	Average output- NX ratio
$\frac{\bar{C}}{\bar{NX}}$ **	Average consumption- NX ratio
$\frac{\bar{G}}{\bar{NX}}$ **	Average government spendings-NX ratio
$\frac{\bar{K}}{\bar{NX}}$ **	Average capital-NX ratio
ψ_1**	Rate of Return to R&D

Table 5.3: Calibrated Parameters for RBC Model
* The parameters which are estimated in section (5.2.2). ** These values are calibrated using the sample averages of 11 OECD countries (1981-2014)

Table (5.4) shows parameters, and long-run ratios held fixed throughout the investigations and Table (5.5) provides the starting calibration values for other parameters of interest.

α	β	ν, ν^f	ρ_2	$\frac{\bar{C}}{\bar{NX}}$	$\frac{\bar{G}}{\bar{NX}}$	$\frac{\bar{K}}{\bar{NX}}$	ψ_1
0.7	0.97	0.5	1.0	6.15	.0.29	0.76	0.29

Table 5.4: Parameters and ratios held fixed

The stationary residuals of the structural equations take the AR(1) form, and the related coefficients are estimated using the IV method discussed in (3.1.7). Consump-

δ	ρ_1	θ	η_1	η_2	η_3	η_4
0.025	1.20	0.5	0.5	0.475	0.025	0.25

Table 5.5: Starting calibration for other parameters

tion, labour demand, capital, the real wage are endogenous variables with stationary; I(0), shocks, policy variable (R&D expenditures) is a stochastic exogenous variable that is treated as stationary AR(1) process and TFP (Solow residual); A_t is the only one with non-stationary shocks hence modelled as a unit root process. The estimated AR(1) coefficients are presented in Table (5.6).

Country	ϵ_c	ϵ_n	ϵ_k	ϵ_w	$\epsilon_{\pi'}$
Austria	0.499	0.907	0.615	0.919	0.951
Canada	0.581	0.921	0.628	0.964	0.997
Denmark	0.402	0.924	0.632	0.933	0.9975
Finland	0.335	0.958	0.950	0.961	0.996
France	0.546	0.985	0.612	0.988	0.957
Germany	0.436	0.941	0.606	0.961	0.998
Japan	0.419	0.969	0.701	0.967	0.963
Netherlands	0.494	0.957	0.688	0.999	0.961
Norway	0.429	0.927	0.639	0.971	0.950
Sweden	0.069	0.929	0.073	0.978	0.984
UK	0.594	0.913	0.620	0.971	0.969

Table 5.6: AR(1) coefficients of the structural shocks to variables indicated by subscript given the starting calibration. π' modelled as exogenous stationary AR(1) process.

The test results for this calibration are based on the auxiliary Panel VECM with output and productivity as endogenous variables for each country. The choice of VECM is due to the non-stationarity of the technology shock which may cause one or more of the long-run structural equations to generate non-stationary shocks. Assuming the cointegration according to the fact that it is not possible to test all equations for the cointegration (it would be useful if one could test each equation for the existence of the cointegration), I chose PVARX approximating the auxiliary model (see Chapter 4). PVARX in this study aims at identifying the R&D-driven growth while allowing for the fixed effects (i.e. country-specific unobserved heterogeneity in the levels of the variables). If country-specific time dummies are considered in PVARX, the aggregate country-specific shocks to macro variables that may affect all countries at the same time can be captured.

To evaluate the R&D-driven RBC growth model of the study, a Panel VARX(1) with a limited group of variables is preferred as a higher order of VARs or increased number of variables will increase the 'stringency' of the overall test. However, if the model is rejected by a PVARX(1), any attempt to proceed with the higher order of PVAR will be unnecessary. Although it is theoretically accepted that a 'true' DSGE model must survive the infinite order of PVAR, raising the lag order of it worsens the fit to the data as it may increase the complexity of the captured behaviour of the model (Minford, Meenagh, and Theodoridis; 2008). The general form of a PVARX(1) is given as

$$Z_{it} = f_i + \Phi_1 Z_{it-1} + \Phi_2 x_{it} + \delta_i T + u_{it} \tag{5.6}$$

where f_i is the individual-specific effect (only fixed-effect specification is considered); that captures the unobserved ability of each country. δ_i is the country specific effect over time and control the non-stationarity effect of the endogenous variables[1] denoted by Z_{it}. Φ_1 is the country-common coefficients for the stacked endogenous variables of each country. Hence, no "dynamic-interdependencies" is assumed which means that slope coefficients are the same for all cross-sections (Love and Zicchino,

[1]The time trend is the balanced growth path BGP for each country.

2006). It is conceptually (not mathematically identical) closer to taking an average of the slopes (cross-country homogeneity condition). This is the classical approach to deal with the cross-section heterogeneity as the dynamic interdependency increases the complexity of a VAR with more endogenous variables and time series which would be an empirical issue.

Φ_2 captures the effect of exogenous macro variable (GERD intensity) and u_{it} are i.i.d shocks. Z_{it} may be a VAR of 3,4 or 5 macro variables.

To illustrate what this approach implies, an example of PVARX(1) with two endogenous variable, $Y_{i,t}$, A_{it} and an exogenous variable π'_{it} is given as following:

$$\begin{pmatrix} A_{it} \\ y_{it} \end{pmatrix} = \begin{pmatrix} f_{1i} \\ f_{2i} \end{pmatrix} + \begin{pmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \delta_{1i} \\ \phi_{21} & \phi_{22} & \phi_{23} & \delta_{2i} \end{pmatrix} \begin{pmatrix} A_{it-1} \\ y_{it-1} \\ \pi'_{it} \\ T \end{pmatrix} + \begin{pmatrix} u_{1it} \\ u_{2it} \end{pmatrix} \tag{5.7}$$

where $t = 1, ..., T$ and $i = 1, ..., N$ identify the time and country specifications respectively. Note that the orthogonality condition is satisfied and $E[Z_{it}u_{it}] = E[f_i u_{it}] = 0$. The idea is to take the data and use ordinary least square to estimate the coefficients on the balanced panel [2]. Given the dimension of my panel with a 134-period time se-

[2] The stacking procedure for a chosen endogenous variable such as A_t will be as

$$\begin{pmatrix} A_{1t} \\ A_{1t+1} \\ \vdots \\ A_{1T} \\ A_{2t} \\ \vdots \\ A_{NT} \end{pmatrix} = \begin{pmatrix} f1 & f2 & ... & f_N \end{pmatrix} \begin{pmatrix} 1 & 0 & ... & 0 \\ 1 & 0 & ... & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 1 & ... & 0 \\ 0 & 1 & ... & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & ... & 1 \end{pmatrix} + \phi_{11} \begin{pmatrix} A_{1t-1} \\ A_{1t} \\ \vdots \\ A_{1T-1} \\ A_{21t-1} \\ \vdots \\ A_{NT-1} \end{pmatrix} + \phi_{13} \begin{pmatrix} \pi'_{1t} \\ \pi'_{1t+1} \\ \vdots \\ \pi'_{1T} \\ \pi'_{2t} \\ \vdots \\ \pi'_{NT} \end{pmatrix} + \delta_{11} \begin{pmatrix} 1 & 0 & ... & 0 \\ 2 & 0 & ... & 0 \\ \vdots & \vdots & \ddots & \vdots \\ T & 0 & ... & 0 \\ 0 & 1 & ... & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & ... & T \end{pmatrix} + \begin{pmatrix} u_{1t} \\ u_{1t+1} \\ \vdots \\ u_{1T} \\ u_{2t} \\ \vdots \\ u_{NT} \end{pmatrix}$$

ries and short cross section (11 countries), OLS panel VAR will be a consistent and defendable estimator. By capturing the country-specific fixed effect and the time trends in the PVAR, only the dynamic remains (which was the purpose of including constant and time trend). This is applied to both actual and model-generated simulation for different combinations of VAR to get the Wald statistics discussed in 4.2.1 and 4.2.2. Table 5.7 shows the summary of the baseline calibration test results.

	(1)*	(2)	(3)	(4)
Included Endog.	Y, A	Y, A, N	Y, A, K	Y, A, w
Included Exog.	π'_{t-1}	π'_{t-1}	π'_{t-1}	π'_{t-1}
Normalised MD(t-stat)	2.622*	5.087	4.418	4.171
	(5)	(6)	(7)	(8)
Included Endog.	Y, A, C	Y, A	Y, A, w	Y, A, C
Included Exog.	π'_{t-1}	π'_{t-1}, b^f_{t-1}	π'_{t-1},b^f_{t-1}	π'_{t-1},b^f_{t-1}
Normalised MD(t-stat)	3.825	4.389	4.912	6.352

Table 5.7: Wald Test Results (Baseline Calibration): Alternative VECMs.
*Trend and constant are included in all alternative auxiliaries.

Table (5.7) implies massively rejections of the directed Wald test at 5% of significance level for different combinations of panel VARs. The Mahalanobis Distance measure shows test statistics higher than 1.64 which implies a rejection of the model, in other words, the test results show the Wald percentile of 100. As it is discussed earlier increasing the number of variables in the VECM's combinations or raising the order worsens the test results. The best MD statistic belongs to the case of the auxiliary with only output, productivity and exogenous policy variable confirms the fact that increasing the number of parameters of the VAR depresses the Wald statistics and increases its distance with the critical value of 1.64. A variance decomposition established for the productivity growth generated by the R&D model. To conduct the variance-decomposition, the shocks to the R&D intensity and AR(1) error of productivity growth are simulated to see how much of the variation in productivity growth is because of the shock to the R&D intensity and how much is due to the independent productivity shock, e_A. The idea is to see if the rate of return to R&D which is based on the empirical facts has a significant effect on the growth rate of output or it has a negligible effect where the latter implies the policy effect is exogenous to the model.

Variation in Productivity Growth	Variance	Percentage of the Total
Total Variance	0.01324	-
Due to R&D shock π'	0.00151	11.4%
Due to other shocks e_A	0.0117	88.6%

Table 5.8: Variance decomposition for the initial calibration values

Table (5.8) presents the shares of the variation in growth due to the shock to R&D intensity and innovation to the productivity growth. This is generated using the mean of the variances for 11 countries of the study. The 11.4% value for the proportion of variance due to the R&D shock to the system confirms the distinction of the model of Chapter 3 from an exogenous model.

5.2.2 Indirect Inference Estimation Results

Due to the rejection of the baseline calibration model, those parameters of interest whose changes will not contradict the micro-founded structure of the model is estimated via 'simulated annealing' search algorithm. To ensure the alteration without getting too far from the theory, the search is limited to the 45% either side of the initial values of the coefficients. Recall that the primary objective of the simulated annealing algorithm is not to find the magnitude of the effects, but to see whether a set of parameters in a controlled range can be found in which the size of the effect of

R&D is large enough, such that the R&D-driven growth model of Chapter 3 is not rejected.

More than 200 sets of coefficients in a range of 45% either side of the initial calibration value are found to plug in the model. When the process of bootstrapping the model for each set of coefficients for each country is finished, the model-generated data is compared to the actual data through the pane of the panel VAR and the related Wald statistics are calculated to see which set of discovered coefficients is the best fit for the model (generates the smallest Wald statistic). A panel VAR consists of only productivity and R&D intensity as the initial attempt is used to see if it passes the test or not. In a case of no-rejection to the model, more variables will be included.

δ	ρ_1	θ	η_1	η_2	η_3	η_4
0.0143	1.156	0.492	0.507	0.492	0.0143	0.1904

Table 5.9: Wald Minimising Coefficients Values

Table (5.9) reports the discovered Wald minimising coefficients which survive the Indirect Inference test. Also, the average implied AR(1) coefficients for the stochastic processes are reported in Table (5.10). Assessed on output and productivity, the normalised transformed Mahalanobis Distance statistic is 0.862 which means the statistic falls within the 91st percentile of the Wald distribution. It is presented that on the auxiliary model (1) and (3), the Wald statistic is within the non-rejection area of the bootstrap distribution.

Some of the estimated coefficients moved significantly from the initial values; for example, the capital depreciation decreased by 42.8%, the marginal effect of the output and current real interest rate on current capital declined by 42% and 24%

	ϵ_c	ϵ_n	ϵ_k	ϵ_w	$\epsilon_{\pi'}$
Average 11-OECD	0.4373	0.9396	0.6675	0.9643	0.9755

Table 5.10: AR(1) coefficients for the structural shocks after estimation
Average of the coefficients for 11-OECD Countries

	(1)**	(2)	(3)**	(4)
Included Endog.	Y, A	Y, A, N	Y, A, K	Y, A, w
Included Exog.	π'_{t-1}	π'_{t-1}	π'_{t-1}	π'_{t-1}
Wald percentile	91.1**	98.33	94.81**	100
	(5)	(6)	(7)	(8)
Included Endog.	Y, A, C	Y, A	Y, A, w	Y, A, C
Included Exog.	π'_{t-1}	π'_{t-1}, b^f_{t-1}	π'_{t-1}, b^f_{t-1}	π'_{t-1}, b^f_{t-1}
Wald percentile	100	100	100	100

Table 5.11: Wald Test Results (Estimation): Alternative VECMs.
Trend and constant are included in all alternative auxiliaries.

respectively. The changes in CRRA coefficient in the utility function for consumption decreased only by less than 4% and the preference weight on consumption, θ, decreased just by 1.6%.

	C	Y	N	K	NX	G	w	b^f	A
e_c	0.22653	0.10147	0.06572	0.13146	0.15748	0.07248	0.04934	0.02948	-
e_n	0.05837	0.04281	0.05535	0.07526	0.16088	0.07157	0.04277	0.04928	-
e_k	0.03713	0.02892	0.05514	0.07243	0.15804	0.06685	0.02187	0.04160	-
e_w	0.06956	0.04583	0.06128	0.07801	0.16117	0.07287	0.04566	0.04963	-
$e_{\pi'}$	0.06226	0.04396	0.05748	0.07638	0.16094	0.07225	0.04609	0.04927	0.12281
e_A	0.54616	0.73701	0.70502	0.56646	0.20149	0.64398	0.79426	0.78075	0.87719

Table 5.12: Variance Decomposition, R&D-driven Model
*Average across 11-OECD Countries.

A variance decomposition is calculated for the logarithm of A for which only the R&D innovation and the independent productivity innovation are relevant and also for the rest of other endogenous variables. Calculating the var-decomposition values for stationary shocks are different from the non-stationary one, hence to get the results presented in Table (5.12), different approaches are applied. For stationary consumption, labour, capital, wages shocks and also for exogenous AR(1) process of policy variable, the average standard deviations of the shocks are determined and the model is bootstrapped one by one for each of these shocks (holding the rest of the shocks zero) separately for each country. For non-stationary productivity shock, a column of shocks are considered, the model is simulated and a 1000 pseudo-sample for each country is discovered, then the variances of the difference between the model-simulated data and the base-run for each variable due to the productivity shock are calculated. Finally, an average across the 11-OECD countries is taken to have more analytical understandings of the effect of this shock across these assumed small economies (see Appendix IV for the variance decomposition tables for each country).

As it is expected on a non-stationary set up the non-stationary shock to dominate the sample variance, the variance decomposition depicts that within-sample variations in endogenous variables are due to the productivity shock components and that the other shocks are more or less equally spread in effect. We may still believe in the model to be distinct from the exogenous growth model as in column 10 of Table (5.12) the 12.3% of the variation in total factor productivity is because of the R&D intensity factor. I have to mention that the table of the variance decomposition illustrates the average of the variations for the countries, by looking at the variance decomposition tables for each country, one may conclude that each country data set is responding differently to some of the shocks and R&D's role on changes of variables differs from one country to another.

Looking back at the results of Table (5.11) and (5.12), the role of R&D in making effects in the transition of productivity can be concluded, although the magnitude of the effect is still the matter of concern. The inferential test used as a tool to evaluate a simple open RBC model with R&D as the driver of productivity suggests the model to be 'true' although restrictions are applied in a sense that the model's micro foundation may not be the most appropriate structure when one considers the effect of research and development on productivity. Even with all the assumptions and restrictions due to the nature of the impact, the model shows acceptable outcomes.

5.3 Summary

In this chapter, the open economy RBC model of growth outlined in Chapter 3 is tested and estimated using the Indirect Inference method when the productivity is driven endogenously by the expenditure on research and development across 11 developed countries in Europe. Before starting the engine of the test process, the data descriptions and the main sources of them are introduced. To proxy the representative household's innovative activities, the expenditure approach is chosen which is a valid proxy for the time spending on innovation if these activities are compensated by the subsidy, hence the share of gross expenditure on research and development (GERD) in total GDP known as R&D intensity defined by Frascati Manual is chosen to represent the household's innovative activities. Both science and technology indicators and aggregate economic indicators of the countries are for the period of 1981-2014.

Using the initial calibrated parameters, the model is tested via Indirect Inference and the normalised Mahalanobis distance statistics for different choices of panel VAR are given where all indicate that the model is strongly rejected as these statistics fall in the 100th percentile of the Wald distribution. Following the extensive rejection of the model some calibrated parameters are estimated using 'simulated annealing algorithm" which searches within about 45% either side of the initial values to find the best fit with minimum Wald statistics. The result of the chapter shows changes in capital parameters and slightly changes in consumer preferences will drive the model to present more appropriate Wald statistics and pass indirect inference Wald test at 5% significant level implying that the R&D spendings lead to the productivity growth during this period and for this sample of 11 countries. Although the effect of productivity may be different for each country but as average the magnitude of over 12% is predicted for the impact of research and development on productivity factor.

It can be noted that the hypothesis of R&D intensity driving productivity growth survives the Indirect Inference test. However as the power of the test rises by the increase in the number of endogenous variables to take part in the panel VAR, the test fails. It would be interesting to do a power analysis for the test and see how the range of possible falseness of the model and how any R&D policy effects are altered which will be a motive for the future work. The results of the chapter may also suggest a more detailed and complex structured model to explain the 'real' effect of R&D channel on growth which can also be the material for future research. This chapter undoubtedly presented a hard effort to explain different aspects of a model empirically with all related shortcomings and strengths.

Chapter 6

Conclusions

The results of the empirical study achieved in Chapter 5 confirms the positive effect of research and development as the driver of TFP growth for the sample period. Consistent with the argument of Griffith et al. (2004) and Guellec and van Pottelsberghe de la Potterie (2001), the growth-enhancing influence of research and development indicates the need for a sustainable activist policy for the OECD countries of the sample.

By reviewing the existing empirical work on R&D-based growth models in Chapter 2, it was concluded that the literature suffers from the defect of "unidentified" regression defining the relationship between R&D and TFP growth hence there is not enough research concentrated on the direction of the causality or a well-inferenced structural model with identified reduced form relationships. The ambiguity in determining the accurate correlation between growth and R&D was the main reason motivated this research to adopt the method of Indirect Inference outlined in Chapter 4 to re-evaluate and estimate an identified R&D-driven dynamic stochastic general equilibrium model of growth depicted in Chapter 3 in which policy channel of R&D

(if subsidy is to be considered) causes a permanent upward shift in productivity followed by long-run growth episodes.

The "true" structural model is simulated using a bootstrapping technique, and 1000 pseudo samples are generated for 11-OECD countries. The simulated features of the bootstrapped model are then summarised by the chosen auxiliary model of panel VAR and compared to the features of the 11-OECD sample data. These captured features discovered to be close in the statistical sense as the Wald statistics are small enough for the impact not to be rejected. That is to say that the model-generated data shows specifications which are close to the actual data thus the model has not failed to explain the part of reality which is captured by the auxiliary model. For the sample of chosen individuals, the research and development expenditure which is decided to proxy the innovation in the workhorse DSGE model of Chapter 3, shows a considerable effect on the productivity growth episodes over the period for the sample as through the pane of panel VAR, the difference between the features of the simulated data is close enough to reckon the model as the true one.

The variance decompositions are provided in Chapter 5 to show that the value assigned to the marginal impact of innovation on productivity growth for the sample period is large enough. They implied that the value for the effect of R&D spendings on TFP growth is acceptable in the sense that the model is far from being exogenous, and it can comfortably be considered endogenously driven by R&D factor, but it should be noted that the study is not focused on the accuracy of the magnitude of the effect due to the fact that this thesis is driven by the motivation to find the sign of the effect rather than the precise magnitude. The significant achievement of this empirical work is that the strict and powerful method of Indirect Inference does not reject the hypothesis of the positive growth-enhancing effect of R&D (the ratio of expenditure on research and development GERD to GDP) and this result is conclusive and defendable.

It should be noted that considering R&D intensity to represent innovation in the model shows that this thesis did not try to specifically concentrate on evaluating the policy implication which in that case instead of GERD proportion to GDP, the extensively used BERD to government spendings ratio may have been chosen but by some minimal changes in the model and selected proxy to characterise R&D, the policy impact can be directly tested and evaluated. It is not difficult to conclude from the results that R&D spendings being responsible for the positive changes in productivity growth can drive policy makers to allocate resources to research or to establish a system of policies to incentivise the innovative activities.

Believing in empiricism and the Popperian falsifiability feature of science, the main purpose of this thesis was to empirically test a highly accepted theoretical specification that R&D is positively related to productivity growth of nations. A rather simplified R&D-based endogenous growth model is chosen to signify the theoretical aspect of the research, hence the further step for this research may be to extend the model to explain more details of the actual world and to apply the procedure discussed in Chapter 4 to alternative micro-founded macro models.

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Appendix I. The Choice of z_t

To have a better understanding of representative agent's behaviour in choosing the amount of time spent on innovative activities, consider the firm's problem in section (3.1.2). The expected impact of z_t on the firm's production function at time $(t+1)$ is

$$\begin{aligned}\frac{\partial \pi_{t+1}}{\partial z_t} = & \frac{\partial Y_{t+1}}{\partial A_{t+1}}.\frac{\partial A_{t+1}}{\partial z_t} + \frac{\partial Y_{t+1}}{\partial K_{t+1}}.\frac{\partial K_{t+1}}{\partial z_t} + \frac{\partial Y_{t+1}}{\partial N_{t+1}}.\frac{\partial N_{t+1}}{\partial z_t} \\ & - \frac{\partial K_{t+1}}{\partial z_t}(\delta + r_{t+1} + \kappa_{t+1} + a_{t+1}) - \frac{\partial N_{t+1}}{\partial z_t}(w_{t+1} + \chi_{t+1})\end{aligned} \tag{1}$$

This equation presents that the choice of z_t will change profit to the firm via affecting productivity, capital and also labour. Decomposing the relation further will give

$$\begin{aligned}\frac{\partial \pi_{t+1}}{\partial z_t} = & \frac{\partial Y_{t+1}}{\partial A_{t+1}}.\frac{\partial A_{t+1}}{\partial z_t} + (MPK + \Delta MPK).\frac{\partial K_{t+1}}{\partial z_t} + (MPN + \Delta MPN).\frac{\partial N_{t+1}}{\partial z_t} \\ & - \frac{\partial K_{t+1}}{\partial z_t}(\delta + r_{t+1} + \kappa_{t+1} + a_{t+1}) - \frac{\partial N_{t+1}}{\partial z_t}(w_{t+1} + \chi_{t+1})\end{aligned} \tag{2}$$

where MPK is the expected marginal product of capital at time $t+1$ assuming no effect coming from z_t and ΔMPK is the expected increase in the marginal product due to the marginal impact of z_t and similar approach for MPN which is the expected marginal product of labour with no marginal impact from z_t and ΔMPN as the difference which choice of z_t will make in labour. In the absence of any changes in z_t

$$MPK = (\delta + r_{t+1} + \kappa_{t+1} + a_{t+1}) \tag{3}$$

$$MPN = w_{t+1} + \chi_{t+1} \tag{4}$$

since the choice of z_t has no effect on the rents of capital and labour the equation for the marginal effect of z_t on profit of the firm can be reduced to

$$\frac{\partial \pi_{t+1}}{\partial z_t} = \frac{\partial Y_{t+1}}{\partial A_{t+1}}.\frac{\partial A_{t+1}}{\partial z_t} + \Delta MPK.\frac{\partial K_{t+1}}{\partial z_t} + \Delta MPN.\frac{\partial N_{t+1}}{\partial z_t} \tag{5}$$

On the RHS of equation (5), the second order terms can be ignored, hence it can be assumed that the increase representative firm's profit is only due to the higher productivity. The world of the model outlined in Chapter 3, is the perfect competition hence the profit of the firm will be zero after small changes in z_t, meaning after a small increase in the time of innovative activities, the dividend income ,$\pi_{t+1} = d_{t+1}$ will be zero. One may question how the extra income enters the budget constraint. This only happens through the increase in wages due to increase in productivity. To summarise

$$\begin{aligned}\frac{\partial \pi_{t+1}}{\partial z_t} &= \frac{\partial Y_{t+1}}{\partial A_{t+1}}.\frac{\partial A_{t+1}}{\partial z_t} \\ &= \frac{\partial d_{t+1}}{\partial z_t}\end{aligned} \tag{6}$$

as it is assumed the second order effects on firms demand for labour and capital can be ignore, hence $\frac{\partial Y_{t+1}}{\partial A_{t+1}} = \frac{Y_{t+1}}{A_{t+1}}$ and based on this assumption the marginal impact of z_t on the the firm's profit (or the dividends) is reduced to

$$\frac{\partial \pi_{t+1}}{\partial z_t} = \frac{Y_{t+1}}{A_{t+1}} . \frac{\partial A_{t+1}}{\partial z_t} \tag{7}$$

Appendix II. On Non-stationarity of $\frac{C_t}{Y_t}$

Recalling the representative consumer's budget constraint from equation (3.9):

$$C_t + \frac{b_{t+1}}{1+r_t} + \frac{b^f_{t+1}}{1+r^f_t} = w_t N_t + b_t + b^f_t + d_t \tag{8}$$

where where $d_t = \Pi_t = Y_t - [K_t - (1-\delta)K_{t-1}] - w_t N_t$ is the profit of the firm which is transferred to the consumer in the form of dividends. By substituting out wages and dividends, budget constraint will be

$$C_t + \frac{b_{t+1}}{1+r_t} + \frac{b^f_{t+1}}{1+r^f_t} = Y_t - [K_t - (1-\delta)K_{t-1}] + b_t + b^f_t \tag{9}$$

based on the first order conditions for consumer problem, $r_t = r^f_t$, hence it can easily be assumed that $\left\{b_{t+1} + b^f_{t+1} \equiv b'_{t+1}\right\}$ and $\left\{b_t + b^f_t \equiv b'_t\right\}$

$$C_t + \frac{b'_{t+1}}{1+r_t} = Y'_t + b'_t \tag{10}$$

where $Y'_t = Y_t - [K_t - (1-\delta)K_{t-1}]$. In expectational form the representative consumer plan must satisfy the following constraint where there is an infinite forward recursion in the value of the future bonds.

$$b'_t = C_t - Y'_t + E_t \sum_{i=0}^{\infty} \Big[\Big\{ \prod_{j=0}^{i} (1 + r_{t+j}) \Big\}^{-1} \big(C_{t+i} - Y'_{t+i} \big) \Big] \tag{11}$$

From the representative first order condition, one can derive

$$C_t = \frac{1}{\beta} E_t \frac{C_{t+1}}{(1 + r_{t+1})} = \frac{1}{\beta^2} E_t \frac{C_{t+2}}{(1 + r_{t+1})(1 + r_{t+2})} = ... \tag{12}$$

hence,

$$E_t \Big\{ \prod_{j=0}^{i} (1 + r_{t+j} \Big\}^{-1} C_{t+i} = \beta^i C_t \tag{13}$$

thus

$$C_t = (1 - \beta) \Big\{ b'_t + Y'_t + E_t \sum_{i=0}^{\infty} \Big[\Big\{ \prod_{j=0}^{i} (1 + r_{t+j}) \Big\}^{-1} Y'_{t+i} \Big] \Big\} \tag{14}$$

The term inside the bracket is the representative consumer's disposable income hence the whole RHS expression is permanent income

$$C_t = (1 - \beta) b'_t + Y_t^P \tag{15}$$

at a terminal date like T, consumption, consumer's permanent income and bonds are growing at a constant rate of g.

$$C_T = (1 - \beta) \Big\{ b'_T + \sum_{i=0}^{\infty} \Big\{ \frac{1 + g}{1 + r*} \Big\}^i [1 + \frac{\delta \gamma}{(r * + c)}] Y_T \Big\} \tag{16}$$

$$= (1 - \beta) b'_T + Y_T^P \tag{17}$$

dividing both sides of equation (17) by Y_t will give

$$\frac{C_t}{Y_t} = (1-\beta)\frac{b'_t}{Y_t} + \frac{Y_t^P}{Y_t} \tag{18}$$

by using linear approximation equation (18) will be

$$ln(\frac{C_t}{Y_t}) = \text{(share of household's income from abroad)}.ln\left(\frac{b'_t}{Y_t}\right) + ln\left(\frac{Y_t^P}{Y_t}\right) \tag{19}$$

in steady state $(\frac{b'_t}{Y_t})$ will tend to some steady level because of representative consumer's behaviour and expectations. In a meantime $(\frac{b'_t}{Y_t})$ is driven by a difference equation of the form

$$x_{t+1} = (1+q_t) + x_t + \varepsilon_t \tag{20}$$

where q_t considers the growth rate in debts and can have positive or negative sign. With ε_t moving randomly between steady states, $x_t = \frac{b'_t}{Y_t}$ will have an explosive randomly distributed behaviour (unit root). Therefore with $\frac{b'_t}{Y_t}$ being non-stationary one can conclude a non-stationary property for $ln(\frac{C_t}{Y_t})$.

I also tested the unit-root test for $ln(\frac{C_t}{Y_t})$ using Augmented Dicky Fuller, and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) for all 11-OECD countries under the study. Without any exception, the non-stationarity is confirmed. Simulation generated series also present a random walk behaviour, thus, the random walk assumption for $ln(\frac{C_t}{Y_t})$ is defendable.

Appendix III. Steady State and Terminal Conditions

System of equations after log-linearisation and substitution of calibrated parameters is as

$$C_t = E_t C_{t+1} + 0.83[ln\,(\frac{1}{0.97}) - r_t] \tag{21}$$

$$Y_t = 0.7N_t + 0.3K_t \tag{22}$$

$$N_t = -0.3566749 + Y_t - w_t \tag{23}$$

$$K_t = 0.5K_{t-1} + 0.475K_{t+1} + 0.025[-1.6094 + Y_t - 10r_t] \tag{24}$$

$$w_t = 1.2C_t - 0.69314718 + N_t \tag{25}$$

$$b_t = (1 + r_t^f)b_{t-1} + NX_t \tag{26}$$

$$NX_t = Y_t - \frac{8}{1.3}[K_t - K_{t-1}] - \frac{0.381}{1.3} \tag{27}$$

$$G_t - \frac{1}{1.3}C_t \tag{28}$$

$$G_t = -1.203972804 + Y_t \tag{29}$$

at T, $K_T = Y_T$(plus constant which drops out). To find the steady state's parameters I tried:

$$K = Y$$

$$C = 1.3B + 1.3Y - 0.381G$$

$$G = Y$$

$$w = 1.2C + N$$

$$Y = 0.7N + 0.3K + A$$

$$N = Y - w$$

and the solution for the steady state is

$$C = 1.2487A + 0.61822B \tag{30}$$

$$G = 1.3587A - 0.74187B \tag{31}$$

$$K = 1.3587A - 0.74187B \tag{32}$$

$$N = -6.9839 \times 10^{-2}A - 0.74187B \tag{33}$$

$$Y = 1.3587A - 0.74187B \tag{34}$$

$$w = 1.4286A \tag{35}$$

B is equal to $r_t^f b_t$, and all the constant has been excluded. Hence, by using steady state calculated above,the terminal conditions for the Fortran program are provided; equation (30), equation (34) and the condition where $(K = Y)$.

Appendix IV. Variance Decomposition Table for Countries

	C	Y	N	K	NX	G	w	b^f	A
e_c	0.74000	0.65050	0.16954	0.59265	0.19806	0.16825	0.00976	0.01260	-
e_n	0.00000	0.00095	0.00017	0.00387	0.19302	0.16068	0.00020	0.00001	-
e_k	0.00000	0.00076	0.00004	0.00390	0.19303	0.16068	0.00005	0.00001	-
e_w	0.00000	0.00181	0.00032	0.00487	0.19304	0.16069	0.00006	0.00006	-
$e_{\pi'}$	0.00000	0.00077	0.00004	0.00390	0.19301	0.16068	0.00005	0.00001	0.13959
e_A	0.26000	0.34520	0.82989	0.39082	0.02985	0.18903	0.98988	0.98729	0.86041

Table 1: Variance Decomposition, Austria
one standard deviation shock

	C	Y	N	K	NX	G	w	b^f	A
e_c	0.00191	0.00685	0.16968	0.01587	0.16144	0.03501	0.00496	0.00975	-
e_n	0.00150	0.00716	0.170	0.01635	0.16369	0.03528	0.00482	0.01290	-
e_k	0.00140	0.00727	0.17010	0.01763	0.16268	0.03537	0.00483	0.01301	-
e_w	0.00156	0.00757	0.17029	0.01662	0.16427	0.03562	0.00482	0.01213	-
$e_{\pi'}$	0.00151	0.00707	0.16995	0.01616	0.16359	0.03520	0.00483	0.01266	0.20824
e_A	0.99213	0.96408	0.14997	0.91736	0.18434	0.82353	0.97573	0.93955	0.79176

Table 2: Variance Decomposition, Canada
one standard deviation shock

	C	Y	N	K	NX	G	w	b^f	A
e_c	0.03349	0.00081	0.00155	0.07690	0.10805	0.01613	0.00117	0.00062	-
e_n	0.03354	0.00082	0.00155	0.07692	0.10806	0.01614	0.00118	0.00063	-
e_k	0.03346	0.00082	0.00159	0.07685	0.10790	0.01614	0.00119	0.00063	-
e_w	0.03369	0.00085	0.00157	0.07697	0.10807	0.01617	0.00125	0.00062	-
$e_{\pi'}$	0.03361	0.00083	0.00155	0.07694	0.10807	0.01616	0.00125	0.00063	0.02500
e_A	0.83220	0.99588	0.99219	0.61541	0.45985	0.91925	0.99397	0.99686	0.97500

Table 3: Variance Decomposition, Denmark
one standard deviation shock

	C	Y	N	K	NX	G	w	b^f	A
e_c	0.98662	0.02428	0.03617	0.11773	0.13194	0.03041	0.02525	0.01291	-
e_n	0.00237	0.02330	0.02338	0.12586	0.14790	0.02947	0.02239	0.04037	-
e_k	0.00851	0.02269	0.02391	0.13379	0.14154	0.02889	0.02127	0.04166	-
e_w	0.00038	0.02380	0.02474	0.12495	0.14786	0.02996	0.02259	0.03913	-
$e_{\pi'}$	0.00211	0.02322	0.02351	0.12501	0.14784	0.02939	0.02264	0.03999	0.12412
e_A	0.00000	0.88271	0.86828	0.37265	0.28292	0.85187	0.88585	0.82594	0.87588

Table 4: Variance Decomposition, Finland
one standard deviation shock

	C	Y	N	K	NX	G	w	b^f	A
e_c	0.00987	0.00577	0.00254	0.01824	0.18399	0.02573	0.00725	0.00805	-
e_n	0.00854	0.00601	0.00403	0.01856	0.18476	0.02594	0.00650	0.00769	-
e_k	0.00833	0.00601	0.00457	0.01788	0.18361	0.02594	0.00678	0.00770	-
e_w	0.01101	0.00666	0.00505	0.02002	0.18514	0.02653	0.00721	0.00926	-
$e_{\pi'}$	0.00937	0.00609	0.00390	0.01895	0.18468	0.02602	0.00720	0.00780	0.20374
e_A	0.95287	0.96946	0.97991	0.90635	0.07783	0.86985	0.96506	0.95951	0.79626

Table 5: Variance Decomposition, France
one standard deviation shock

	C	Y	N	K	NX	G	w	b^f	A
e_c	0.23825	0.00510	0.00521	0.19907	0.12319	0.05645	0.01198	0.00032	-
e_n	0.18359	0.00526	0.00541	0.17320	0.12790	0.05657	0.01033	0.00472	-
e_k	0.18249	0.00538	0.00553	0.18026	0.12759	0.05666	0.01055	0.00474	-
e_w	0.20929	0.00587	0.00597	0.18708	0.12834	0.05702	0.01068	0.00475	-
$e_{\pi'}$	0.18637	0.00526	0.00516	0.17378	0.12791	0.05657	0.01079	0.00463	0.12305
e_A	0.00000	0.97314	0.97273	0.08661	0.36508	0.71674	0.94567	0.98084	0.87695

Table 6: Variance Decomposition, Germany
one standard deviation shock

	C	Y	N	K	NX	G	w	b^f	A
e_c	0.27899	0.20282	0.16768	0.19299	0.19941	0.20044	0.23448	0.18741	-
e_n	0.19682	0.08701	0.20363	0.17898	0.20264	0.10675	0.15254	0.10430	-
e_k	0.05928	0.14004	0.18150	0.19892	0.09814	0.09488	0.09527	0.17157	-
e_w	0.02041	0.09650	0.22113	0.20502	0.10953	0.20391	0.16306	0.10494	-
$e_{\pi'}$	0.21089	0.21704	0.20807	0.20179	0.20279	0.10702	0.16137	0.10307	0.11467
e_A	0.23361	0.25660	0.01800	0.02230	0.18749	0.28700	0.19328	0.32871	0.88533

Table 7: Variance Decomposition, Japan
one standard deviation shock

	C	Y	N	K	NX	G	w	b^f	A
e_c	0.19975	0.21659	0.16416	0.22723	0.19344	0.20859	0.24568	0.07420	-
e_n	0.21293	0.20854	0.19446	0.20389	0.20441	0.20442	0.20573	0.24117	-
e_k	0.11135	0.13092	0.19427	0.14790	0.19125	0.16422	0.09750	0.19802	-
e_w	0.25158	0.22886	0.23745	0.21399	0.20599	0.21495	0.22566	0.24545	-
$e_{\pi'}$	0.22439	0.21509	0.20966	0.20699	0.20492	0.20781	0.22542	0.24116	0.037
e_A	0.00000	0.0011	0.00023	0.00014	0.0007	0.000254	0.000101	0.00136	0.962

Table 8: Variance Decomposition, Netherlands
one standard deviation shock

	C	Y	N	K	NX	G	w	b^f	A
e_c	0.00032	0.00001	0.00008	0.00004	0.09673	0.01732	0.00001	0.000071	-
e_n	0.00113	0.00018	0.00019	0.00267	0.09960	0.01747	0.00020	0.00013	-
e_k	0.00184	0.00108	0.00089	0.01413	0.10273	0.01830	0.00124	0.00031	-
e_w	0.00216	0.00022	0.00021	0.00323	0.09932	0.01751	0.00038	0.00009	-
$e_{\pi'}$	0.00156	0.00025	0.00030	0.00308	0.09948	0.01754	0.00033	0.00011	0.19992
e_A	0.99299	0.99825	0.99833	0.97686	0.50214	0.91186	0.99785	0.99930	0.80008

Table 9: Variance Decomposition, Norway
one standard deviation shock

	C	Y	N	K	NX	G	w	b^f	A
e_c	0.00122 0	.00340	0.00544	0.00526	0.15404	0.00577	0.00221	0.01823	-
e_n	0.00065	0.00330	0.00497	0.00530	0.15564	0.00567	0.00207	0.02053	-
e_k	0.00062	0.00296	0.00465	0.00524	0.15413	0.00533	0.00193	0.01988	-
e_w	0.00065	0.00358	0.00613	0.00530	0.15555	0.00595	0.00211	0.01988	-
$e_{\pi'}$	0.00066	0.00335	0.00522	0.00529	0.15557	0.00571	0.00219	0.02029	0.08060
e_A	0.99620	0.98341	0.97358	0.97362	0.22508	0.97158	0.98948	0.90119	0.91940

Table 10: Variance Decomposition, Sweden
one standard deviation shock

	C	Y	N	K	NX	G	w	b^f	A
e_c	0.00141	0.00006	0.00089	0.00002	0.18207	0.03321	0.00001	0.00006	-
e_n	0.00097	0.00007	0.00109	0.00003	0.18205	0.03322	0.00006	0.00003	-
e_k	0.00110	0.00013	0.00154	0.00018	0.18209	0.03327	0.00001	0.00006	-
e_w	0.00084	0.00009	0.00122	0.00006	0.18207	0.03323	0.00001	0.00003	-
$e_{\pi'}$	0.01433	0.00464	0.00493	0.00832	0.18243	0.03703	0.00651	0.00208	0.09395
e_A	0.98135	0.99502	0.99033	0.99139	0.08929	0.83005	0.99339	0.99774	0.90605

Table 11: Variance Decomposition, UK
one standard deviation shock

Appendix V. Graphs for GERD

A. Indicators on R&D expenditures, budgets and personnel are derived from the OECD's Research and Development Statistics (RDS) database, which is based on the data reported to OECD and Eurostat in the framework of the joint OECD/Eurostat international data collection on resources devoted to R&D where Eurostat?s statistics on R&D expenditure are compiled using guidelines laid out in the Frascati manual, published in 2002 by the OECD.

B. The main analysis of R&D is by four sectors of performance: 1) Government sector, 2) Higher Education sector, 3) Business Enterprise Sector, and 4) Private non-Profit Sector. Based on Eurostat, Gross domestic expenditure on R&D (GERD) is composed of expenditure from each of these four sectors.

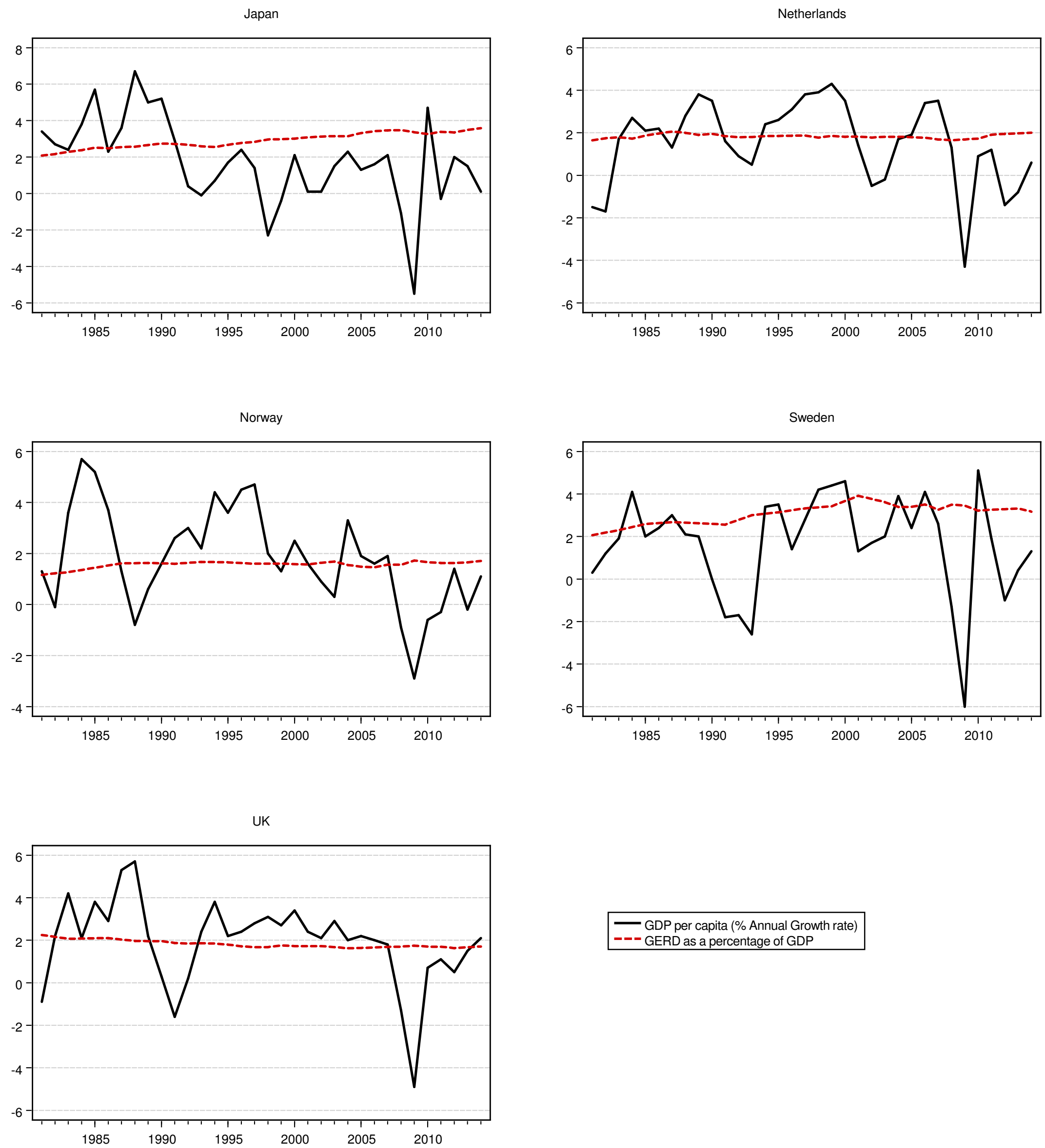

Figure 1: Annual GDP per capita growth and gross domestic expenditure on R&D as the percentage of GDP (1981-2014)

Source of the data: OECD iLibrary.

Figure 2: Number of researchers in government, higher education and business sectors as the ratio of the total number of researchers in each country (1981-2014). Source: OECD Science, Technology and R&D Statistics.

Figure 3: percentage of gross domestic expenditure R&D financed by different sectors
Source: OECD Science, Technology and R&D Statistics.

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