On the statistics of proto-cluster candidates detected in the Planck all-sky survey

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ABSTRACT
Observational investigations of the abundance of massive precursors of local galaxy clusters (“proto-clusters”) allow us to test the growth of density perturbations, to constrain cosmological parameters that control it, to test the theory of non-linear collapse and how the galaxy formation takes place in dense environments. The Planck collaboration has recently published a catalogue of > ∼ 2000 cold extra-galactic sub-millimeter sources, i.e. with colours indicative of z ∼ > 2, almost all of which appear to be over-densities of star-forming galaxies. They are thus considered as proto-cluster candidates. Their number densities (or their flux densities) are far in excess of expectations from the standard scenario for the evolution of large-scale structure. Simulations based on a physically motivated galaxy evolution model show that essentially all cold peaks brighter than S_{545GHz} = 500 mJy found in Planck maps after having removed the Galactic dust emission can be interpreted as positive Poisson fluctuations of the number of high-z dusty proto-clusters within the same Planck beam, rather than being individual clumps of physically bound galaxies. This conclusion does not change if an empirical fit to the luminosity function of dusty galaxies is used instead of the physical model. The simulations accurately reproduce the statistic of the Planck detections and yield distributions of sizes and ellipticities in qualitative agreement with observations. The redshift distribution of the brightest proto-clusters contributing to the cold peaks has a broad maximum at 1.5 ≤ z ≤ 3. Therefore follow-up of Planck proto-cluster candidates will provide key information on the high-z evolution of large scale structure.

Key words: galaxies: clusters: general – galaxies: evolution – large-scale structure of Universe – submillimetre: galaxies

1 INTRODUCTION
Observations of the early evolution of large scale structure are important tools for testing galaxy formation theories, investigating galaxy evolution in dense environments and getting independent constraints on fundamental cosmological parameters (e.g., Harrison & Coles 2012; Casey 2016). This has motivated several observational efforts to identify high-redshift proto-clusters of galaxies (for a review see Overzier 2016) defined as high-z galaxy over-densities that will evolve into the local galaxy clusters.

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The detection of these objects, however, proved to be
challenging. Above $z \simeq 1.5$ the searches via the X-ray emission or the Sunyaev-Zeldovich (SZ) effect as well as the optical detection of the galaxy red sequence are inefficient, not only because of the difficulty of high-$z$ observations but even more because these methods are appropriate to detect evolved clusters with mature galaxy populations or the signature of a hot inter-cluster medium (ICM). But in high-$z$ proto-clusters the ICM may not have yet reached the high temperatures necessary for X-ray or SZ detections, and member galaxies may be still in the active star-formation phase.

Indeed, (sub-)millimeter surveys with Herschel, SCUBA-2 or APEX LABOCA proved to be capable of detecting candidate high-$z$ proto-clusters of star-bursting galaxies (Valtchanov et al. 2013; Dannerbauer et al. 2014; Rigby et al. 2014; Ma et al. 2015; Casey et al. 2015; Clements et al. 2016). These objects are however rare and therefore very large area surveys are necessary to get a good statistics. As first pointed out by Negrello et al. (2005) low-resolution sub-mm surveys, such as those carried out by the Planck satellite, are very well suited for detecting proto-clusters because they integrate over the emission of all star-forming member galaxies, not only of those above some detection limit.

The Planck Collaboration XXXIX (2016, PCXXXIX hereafter) provided a catalog of 2151 cold sub-mm sources with spectral energy distributions peaking between 353 and 857 GHz at 5$^\text{th}$ resolution, with flux density at 545 GHz above 500 mJy. This is referred to as the Planck high-$z$, or PHz, sample. The vast majority of the sources in the sample appear as overdensities of dusty star-forming galaxies, having colours consistent with $z > 2$ and were considered as proto-cluster candidates.

Planck Collaboration XXVII (2015) used the Spectral and Photometric Imaging Receiver (SPIRE) on the Herschel satellite to perform follow-up observations of 234 cold Planck sources, satisfying the criterion of having their rest-frame far-IR peak redshifted to the frequency range 353–857 GHz, as expected for galaxies in the redshift range $z = 2–4$. About 94% of the SPIRE sources associated to the 228 Planck objects were found to be consistent with being galaxy overdensities; their (photometric) redshift distribution peaks at $z \simeq 2$ for a typical dust temperature of 35 K. Seven sources (about 3%) are candidate lensed systems; all of them have spectroscopic redshifts and are at $z > 2.2$ (Cañameras et al. 2015).

As pointed out by PCXXXIX the nature of the high-$z$ galaxy overdensities is still uncertain. Given the large uncertainties in the photometric redshift estimates, it is impossible, at present, to establish whether galaxies making up the overdensities are physically related or are just part of random fluctuations in the galaxy number density within the Planck resolution element.

PCXXXIX found that the distribution of the total (8–1000 μm) infrared (IR) luminosities of overdensities peaks around $2 \times 10^{14} L_\odot$ for their reference dust temperature of 35 K. Using the calibration by Kennicutt & Evans (2012), the corresponding star formation rate (SFR) is $\simeq 3 \times 10^4 M_\odot \text{yr}^{-1}$. The associated halo mass, $M_h$, can be estimated using the SFR–$M_h$ relation at $z = 2$ derived by Aversa et al. (2015) exploiting the abundance matching technique. For mean SFRs in the range 30–1000 $M_\odot$ yr$^{-1}$ (the typical SFR of $z \simeq 2$ star-forming galaxies is $3 \times 10^2 M_\odot$ yr$^{-1}$, see Fig. 9 of Cai et al. 2014) the sum of halo masses of star-forming galaxies is $M_{h,\text{st}} \simeq 5 \times 10^{14} M_\odot$, independently of the SFR. If the lifetime of the star-forming phase is $t_{\text{sf}}$, the total halo mass is $M_h \simeq M_{h,\text{st}}(t_h/t_{\text{sf}})$, where $t_h \simeq 3.3 \text{Gyr}$ is the age of the universe at $z = 2$. Since $t_{\text{sf}}$ is likely $< 1 \text{Gyr}$, $M_h$ is probably several times larger than $M_{h,\text{st}}$. The redshift-dependent halo mass function by Sheth & Tormen (1999) gives a surface density of halos with mass larger than $M_{h,\text{st}}$ at $z > 2$ of $\sim 5 \times 10^{-5} \text{deg}^{-2}$ (and much lower than that for $M_h$, since the density sinks down exponentially in this mass range) to be compared with the surface density of Planck overdensities of 0.21 deg$^{-2}$. This already highlights a problem with the interpretation of overdensities as proto-clusters.

To look for a plausible explanation of the results by PCXXXIX we have updated the study by Negrello et al. (2005) who worked out a formalism to derive the IR luminosity functions and number counts of proto-clusters of dusty galaxies. In the last decade the amount of data on the cosmological evolution of the IR luminosity function and on the clustering properties of dusty galaxies has grown enormously, allowing us to put our estimates on a much more solid basis than was possible in 2005. We will adopt as our reference the latest Cai et al. (2013) version of the self-regulated galaxy evolution model developed by Granato et al. (2004) and Lapi et al. (2006, 2011). However we checked that our results and conclusions do not significantly change if we adopt an empirical fit the luminosity function of dusty galaxies (e.g. Mancuso et al. 2016).

The paper is organized as follow. In Section 2 we review and update the formalism introduced by Negrello et al. (2005), while in Section 3 we briefly describe the model adopted for the luminosity function and the clustering properties of dusty galaxies. Predictions for the number counts of proto-clusters and comparison with data are presented in Section 4. In Section 5 we investigate the effect on the estimated number counts of Poisson fluctuations of the number of proto-clusters within the Planck beam. Section 6 summarizes and discusses our main conclusions.

Throughout the paper we assume a flat cold dark matter cosmology with $\Omega_{0,m} = 0.315$, $\Omega_{0,b} = 0.044$, $h = 0.67$ and $\sigma_8 = 0.81$, with an index $n = 1$ for the power spectrum of primordial density fluctuations, consistent with the Planck results (Planck Collaboration XIII 2016).

### 2 FORMALISM

The mean luminosity of a clump of clustered galaxies, or of a proto-cluster, at redshift $z$ is (Peebles 1980)

$$L_{\text{cl}}(z) = \int_{V} \frac{dN(L', z)}{dL' dV} L' dL' \cdot \int_{V_{\text{cl}}} \xi(r, z) dV,$$

$^1$ At variance with Negrello et al. (2005) we have now dropped from the equations all the Poisson terms, as we are specifically interested in physically related galaxies.
where \( dN(L, z)/dLdV \) is the comoving luminosity function of galaxies and \( \xi(r, z) \) is their spatial correlation function; \( V_{cl} \) is the volume of the proto-cluster (the choice of the proto-cluster radius will be discussed in Section 3), while \( r \) is the comoving radial distance to the volume element \( dV \). The variance of the clump luminosity is

\[
\sigma_{\text{cl}}^2(z) = \int_{\mathcal{L}} \frac{dN(L', z)}{dL'dV} L'^2 dL' \cdot \int_{V_{cl}} \xi(r, z) dV \\
+ \left[ \int_{\mathcal{L}} \frac{dN(L', z)}{dL'dV} L'dL' \right]^2 \cdot \int_{V_{cl}} \int_{V_{cl}} [\xi(r_1, r_2, z) \\
+ \xi(r_{12}, z) - \xi(r_1, z)\xi(r_2, z)] dV_1 dV_2 \tag{2}
\]

where \( r_{12} \) is the comoving distance between the volume elements \( dV_1 \) and \( dV_2 \), and \( \zeta \) is the reduced part of the three-point spatial correlation function for which we assume the standard hierarchical formula:

\[
\zeta(r_1, r_2) = Q(z) [\xi(r_1)\xi(r_2) + \xi(r_1)\xi(r_{12}) + \xi(r_{12})\xi(r_2)] \cdot \tag{3}
\]

The variance of clump luminosities [eq. (2)] is the sum of the contributions from fluctuations in the total luminosity and in the number of member sources.

Using Eq. (3) we rewrite Eq. (2) as

\[
\sigma_{\text{cl}}^2(z) = \int_{\mathcal{L}} \frac{dN(L', z)}{dL'dV} L'^2 dL' \cdot \int_V \xi(r, z) dV \\
+ \left[ \int_{\mathcal{L}} \frac{dN(L', z)}{dL'dV} L'dL' \right]^2 \cdot \tag{4}
\]

\[
[Q(z) - 1] \cdot \left( \int_V \xi(r, z) dV \right)^2 \\
+ \left[ \int_{\mathcal{L}} \frac{dN(L', z)}{dL'dV} L'dL' \right]^2 \\
2Q(z) \cdot \int_V \int_V \xi(r_{12}, z)\xi(r_1) dV_1 dV_2 \\
+ \left[ \int_{\mathcal{L}} \frac{dN(L', z)}{dL'dV} L'dL' \right]^2 \cdot \int_V \xi(r_{12}, z) dV_1 dV_2.
\]

The statistics of the matter-density distribution found in N-body simulations is successfully reproduced by a log-normal function (Coles & Jones 1991; Kofman et al. 1994; Taylor & Watts 2000; Kayo, Taruya, & Suto 2001; Taruya, Hamana, & Kayo 2003), not only in weakly non-linear regimes, but also up to density contrasts \( \delta \approx 100 \). If light is a (biased) tracer of mass, fluctuations in the luminosity density should reflect the statistics of the matter-density field. Therefore we adopt, following NO5, a log-normal shape for the distribution of \( L_{cl} \).

This function is completely determined by its first (mean) and second (variance) moments:

\[
dN(L_{cl}, z)/dL_{cl}dV = \exp\left[-\frac{1}{2} \ln L_{cl} - \mu(z)/\sigma^2(z) \right]/\sqrt{2\pi\sigma^2(z)L_{cl}} \tag{5}
\]

where

\[
\mu(z) = \ln\left[ \frac{L_{cl}^2(z)}{\sigma_{L_{cl}}^2(z) + L_{cl}^2(z)} \right], \tag{6}
\]

\[
\sigma^2(z) = \ln\left[ \frac{\sigma_{L_{cl}}^2(z)}{L_{cl}^2(z)} + 1 \right]. \tag{7}
\]

The function is normalized by requiring the conservation of the luminosity density

\[
\int_{L_{cl}} \frac{dN(L_{cl}, z)}{dL_{cl}dV} L_{cl}' dL_{cl}' = \int_{\mathcal{L}} \frac{dN(L', z)}{dL'dV} L'dL'. \tag{8}
\]

## 3 ADOPTED MODEL FOR DUSTY GALAXIES

We compute the clump total-infrared (8 - 1000 \( \mu \)m) luminosity function using the redshift–dependent IR luminosity function of dusty galaxies provided by the Cai et al. (2013, C13 hereafter) model which accurately reproduces the observational determinations, available up to \( z \approx 4 \) (see Fig. 1 of Bonato et al. 2014), as well as a broad variety of other relevant data\(^2\). In order to assess the robustness of our

\(^2\) A tabulation of the IR luminosity function at several redshifts as well as figures showing fits to other relevant data sets.
results we also work out predictions based on a parametric fit to the infrared luminosity function of dusty galaxies. Specifically we use the results by Mancuso et al. (2016) who preformed a fit to the global, redshift-dependent, SFR function of galaxies measured at UV and far-IR wavelengths by the Hubble and the Herschel space telescopes. As we are interested in the dust-obscured phase of the formation of galaxy clusters at $z \geq 1$, we just need the part of the global SFR function above $SFR \gtrsim 50 \, M_{\odot} \, yr^{-1}$ [corresponding to luminosities $L_{SFR} \gtrsim few \times 10^{11} \, L_{\odot}$ (Kennicutt & Evans 2012)], where, generally, the star formation is heavily obscured by dust. In practise we identify the value of the SFR at which the global SFR function is a factor of 10 higher than the SFR function derived by Mancuso et al. from UV (dust-corrected) data alone and apply an exponential cut-off below that value. The cut-off corresponds to $SFR \approx 60 \, M_{\odot} \, yr^{-1}$ to $120 \, M_{\odot} \, yr^{-1}$ from $z = 2$ to $z = 5$, or to infrared luminosities $L_{IR} \approx 4 \times 10^{11} \, L_{\odot}$ to $\approx 8 \times 10^{11} \, L_{\odot}$, respectively.

The two-point correlation function of dusty galaxies was computed using the Halo Occupation Distribution (HOD) formalism. The power spectrum of the galaxy distribution is parameterized as the sum of two terms: a 1-halo term, dominating on small scales, that accounts for the distribution of galaxies within the same halo, and a 2-halo term, that describes the correlations among galaxies residing on different halos and, therefore, dominates on larger scales. The HOD provides a statistical description of how galaxies populate dark matter halos and we model it using the central-satellite formalism (see, e.g., Zheng et al. 2005). This assumes that the first galaxy to be hosted by a halo lies at its center, while the remaining galaxies are distributed in proportion to the halo mass profile and are classified as satellites. The mean halo occupation function of satellite galaxies is parameterized as: $(N_{\text{sat}}) \propto (M_{\text{vir}}/M_{\text{sat}})^{\alpha_{\text{sat}}}$, where $M_{\text{vir}}$ is the halo mass and the power-law index $\alpha_{\text{sat}}$ is a free parameter. As for the 2-halo term, the key parameter, determining the amplitude of the effective bias function $b_{\text{eff}}(z)$, is the minimum halo mass, $M_{\text{vir,min}}$.

We have adopted the HOD parameter values derived by C13 by fitting the power spectra of the Cosmic Infrared Background (CIB) measured by Planck and Herschel. The spatial correlation function, $\xi(r, z)$, was then computed as the Fourier anti-transform of the 3D power spectrum. We refer the reader to Xia et al. (2012) for all the relevant details on the formalism used. The clustering radius, or correlation length, $r_0$, defined by $\xi(r_0, z) = 1$, is found to be, in comoving units, $r_0 \approx 10 - 20 \, \text{Mpc}$ in the redshift range $z \sim 1 - 7$. It is shown by the solid line in the upper panel of Fig. 1.

Recently, Chen et al. (2016b) have measured the correlation length of a sample of $\approx 3000$ sub-millimetre galaxies, with redshifts $z \sim 1 - 5$ and star formation rates $\gtrsim 60 - 100 M_{\odot} \, yr^{-1}$, identified using a new colour selection technique, which combines three optical-near-infrared colours (Chen et al. 2016a). Their estimates of $r_0$ in 3 red-
Statistics of proto-clusters

Figure 3. Cumulative infrared luminosity function of proto-clusters with redshifts in the range $2 < z < 3.5$ (upper panel) and with $z > 4$ (lower panel). The predictions, based on the Cai et al. model are shown by the thick dashed red curve while, for comparison, the abundance of unlensed protospheroids is shown by the thick solid black curve. The excess density of proto-clusters compared to unlensed protospheroids is compensated by a deficit at faint luminosities (not shown): the luminosity density is conserved [eq. (8)]. The lighter curves are predictions based on the infrared luminosity function of Mancuso et al. (2016) for dusty galaxies (solid back curve) and proto-clusters (dashed red curve). The data points mark the constraints on the abundance of dusty proto-clusters set by the discoveries of such objects reported in Capak et al. (2011), Ivison et al. (2013), Clements et al. (2014) and Wang et al. (2016). The luminosities of detected proto-clusters are the sum of those of member galaxies above the detection limits. Hence, they do not include the contribution of fainter member galaxies, whose summed emission may be dominant (see text).

We adopt the transition scale between the 2- and the 1-halo terms as the radius of the proto-cluster, $r_{cl}$. As illustrated by Fig. 2, the corresponding angular scale ranges from $\approx 1'$ to $\approx 0.4'$ for $z$ varying from $\approx 1$ to $\approx 5$. Hence, the Planck resolution is not ideal for detecting individual proto-clusters. Higher resolution surveys, such as those planned with the Cosmic Origins Explorer (CORE), will be far more efficient for this purpose (De Zotti et al. 2016).

The additional power-law contribution to $w(\theta)$ dominates on scales $\lesssim 0.1’-0.2’$ corresponding to physical linear scales of 50–100 kpc. Interestingly, this is the scale of the proto-cluster cores at $z \approx 2.4-2.5$ detected by Ivison et al. (2013) and Wang et al. (2016).

The integrals over $r$ in eqs. (1) and (4) are carried out up to $r_{sup} = r_{cl}$, while the lowest value of $r$ is set by the minimum angular scale probed by the Chen et al. measurements, i.e., $\sim 0.025’$ (or 1.5’), which corresponds to a comoving length of $\sim 25-64$ kpc for $z = 1-7$. In our calculations we set $r_{inf} = 40$ kpc (comoving), corresponding to $\approx 13$ kpc (physical) at $z \approx 2$ where the distribution of Planck proto-cluster candidates peaks. The values of $r_{inf}$ and $r_{sup}$, as a function of redshift, are shown by the upper and lower limits of the shaded region in lower panel of Fig. 1. In the same figure, the dotted curve represents the scale sampled by the beam of Planck at sub-millimeter wavelengths (i.e. 5’). It is clear that such proto-clusters would appear as point-like sources in the Planck maps.

For the amplitude $Q$ of the three point correlation function $\zeta$ we assumed $Q(z) = 1/b^2 (M_{eff}, z)$. This formula was derived by Szapudi et al. (2001) from N-body simulations and holds on scales $\lesssim 10$ Mpc $h^{-1}$, i.e. on the scales of interest here. We neglect any dependence of $Q$ on the linear scale. At variance with N05 we do not consider a model with $Q = 1/b$ suggested by perturbation theory (Fry & Gaztanaga 1993) as this holds on larger scales, i.e. in the limit of small density fluctuations. We also discard the unrealistic model corresponding to $Q(z) = 1$, as this holds for a pure dark-matter distribution, i.e. not for visible galaxies (e.g. Szapudi et al. 1996).

We note that since $Q(z)$ decreases with increasing redshift, the contribution to $\sigma_8^2$ from the second term in the right-hand side of eq. (2) may become negative. When this happens we set such contribution to zero.
due to unresolved/undetected cluster members. Therefore they do not account for the emission in Fig. 3. The quoted infrared luminosities are those derived by studying the CO line properties of the sub-mm bright-galaxy members. By exploiting the wealth of data covering the entire accessible electromagnetic spectrum in the 2 deg² Cosmological Evolution Survey (COSMOS; Scoville et al. 2007) field, Capak et al. (2011) searched for z > 2 proto-clusters finding one at z = 5.30. This primordial cluster is characterized by an overdensity of galaxies within a region of ∼ 2 Mpc (comoving) radius around the extreme starburst galaxy COSMOS AzTEC-3 and has an estimated total (baryonic + dark) mass of ∼ 4 × 10¹⁴ M⊙.

Ivison et al. (2013) identified at least four intrinsically luminous galaxies at z = 2.41 across an ∼ 100 kpc region by studying the CO line properties of the sub-mm brightest sources in the Herschel Astrophysical Terahertz Large Area Survey (H-ATLAS; Eales et al. 2010). Specifically, they focused on a sample of candidate lensed galaxies with 2.1 ∼ z ∼ 3.5, extracted from ∼ 330 deg² of the H-ATLAS (i.e. the Equatorial + North Galactic pole fields), for which measurements of the CO(1-0) emission line were previously obtained with the Green Bank Telescope (Harris et al. 2012). The clump of luminous galaxies discovered by Ivison et al. was interpreted as the core of an overdensity that will evolve into a massive (∼ 10¹⁴.6 M⊙) cluster.

Wang et al. (2016) looked instead for concentrations of Ks-band selected red galaxies at z > 2 over a 1.62 deg² region of the COSMOS field, finding a significant overdensity of massive galaxies at z = 2.506 associated with extended X-ray emission. The high star formation rate (∼ 3400 M⊙ yr⁻¹) measured in the central 80 kpc region of the overdensity suggests that also in this case we are witnessing the rapid build-up of a cluster core.

Clements et al. (2014) examined the Herschel/ SPIRE images of the 16 Planck Early Release Compact Source Catalog sources lying within the 90 deg² of the Herschel Multitiered Extragalactic Survey (HerMES; Oliver et al. 2012), finding 4 overdensities of sub-mm sources. Using existing mult wavelength photometric data they estimated redshifts in the range z ∼ 0.8 – 2.3 and typical star formation rates SFR > 1000 M⊙ yr⁻¹ for the protoclusters.

In Fig. 3 we compare these observations with the predictions based on our model. The measured infrared luminosities of the proto-clusters, listed in Table 1, are presented as lower limits. In fact they only account for cluster members detected above the flux limit of the observations, while a significant, or even dominant, contribution is expected from fainter members. For example, using the Cai et al. (2013) model we find that sources brighter than 50 mJy at 500 μm, approximately the 90% completeness level of Herschel obser-

Table 1. Information about the proto-clusters at redshift z > 2 in Fig. 3. The quoted infrared luminosities are those derived by summing the contributions from all the detected star forming galaxy members. Therefore they do not account for the emission due to unresolved/undetected cluster members.

<table>
<thead>
<tr>
<th>z</th>
<th>LIR (×10¹² L⊙)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.05±0.09</td>
<td>31±6</td>
<td>Clements et al. (2014)</td>
</tr>
<tr>
<td>2.27±0.12</td>
<td>73±11</td>
<td>Clements et al. (2014)</td>
</tr>
<tr>
<td>2.41</td>
<td>57±6</td>
<td>Ivison et al. (2013)</td>
</tr>
<tr>
<td>2.506</td>
<td>15.8</td>
<td>Wang et al. (2016)</td>
</tr>
<tr>
<td>5.30</td>
<td>17±8</td>
<td>Capak et al. (2011)</td>
</tr>
</tbody>
</table>
with observations, that refer to the redshift range probed by the sample of PCXXXIX, we compare, in Fig. 4, our predictions (dashed red curve; thicker for the Cai et al. model and lighter for the Mancuso et al. empirical luminosity function) with the counts at 545 GHz (550 μm) of the PHz sources, reported in the latter paper (filled blue circles). The number counts at a given wavelength are derived from the IR luminosity function of the clumps [eq. (5)] by adopting the spectral energy distribution (SED) of the $z = 2.3$ star forming galaxy SMM J2135-0102 (Swinbank et al. 2010) for the proto-spheroids, as done by Cai et al. (2013).

Apart from the small fraction of strongly lensed galaxies (around 3%; Planck Collaboration XXVII 2015), these candidate high-$z$ sources may be either over-densities of bright star-forming galaxies (i.e. proto-clusters) or high peaks of the cosmic infrared background (CIB) fluctuations with red colours because are contributed by physically unrelated high-$z$ galaxies.

Figure 4 also shows, for comparison, the differential number counts of strongly lensed galaxies. Such counts were computed using the formalism of Lapi et al. (2012, their SISSA model) with a maximum amplification $\mu_{\text{max}} = 15$ that is found by Negrello et al. (2017) to reproduce the counts of strongly lensed galaxies in the H-ATLAS. The number densities of the PHz sample are well in excess of those expected from our model, also when using the empirical luminosity function of Mancuso et al., and, more importantly, of the number densities of high-$z$ halos capable of hosting them, as pointed out in Sect. 1. In the next Section we discuss a plausible explanation of the discrepancy.

5 THE EFFECT OF SOURCE CONFUSION

Thanks to the very low instrumental noise of the Planck High Frequency Instrument (HFI), the fluctuation field measured in the HFI maps is signal-dominated. In regions with low Galactic dust content, clustering of high-$z$ sources making up the CIB is the dominant source of intensity fluctuations, at sub-mm wavelengths, on the several arcmin scales of interest here. The probability distribution function (PDF) of such intensity fluctuations is highly skewed, with an extended tail towards high signal-to-noise ratios (cf. Fig. 10 of De Zotti et al. 2015).

To investigate the nature of high signal-to-noise intensity peaks we have resorted to Monte Carlo simulations. Proto-clusters have been randomly distributed in infrared luminosity and redshift over an area of 5000 deg$^2$, according to the modelled luminosity functions. The area was divided into $1.7' \times 1.7'$ pixels$^3$ and the flux densities of proto-clusters within each pixel were summed up. The map was then convolved with the Planck beam and background subtracted. The source extraction was performed by looking for connected pixels with signal-to-noise ratio (SNR)$^3$ SNR$> 5$ as in PCXXXIX. A couple of examples of the content of simulated intensity peaks with SNR $> 5$ are shown in Fig. 6. One or two very luminous proto-clusters are generally present; however, most of the flux density within the Planck beam

- $^3$ 1.7’ corresponds to 1/3 of the Planck Full Width at Half Maximum (FWHM) at 545 GHz.
- $^4$ A detection requires at least one pixel above the SNR threshold.
is accounted for by much fainter objects, as implied by the steepness of the bright portion of the proto-cluster counts (Figs. 4 and 5).

The source flux density is obtained by first fitting a Gaussian to a 20′ × 20′ postage stamp centered on the detected object and then integrating the best-fit model. The differential number counts measured in the simulated map are shown in the right-hand panel of Fig. 4 by the yellow stars. The effect of source confusion is dramatic and brings the number counts in line with the measured ones (blue dots). The same conclusion is reached when we use the Mancuso et al. empirical luminosity function for dusty galaxies, as illustrated by the orange pentagons in the same figure.

The model integral number counts of individual high-z (z ≥ 2) proto-clusters, displayed in Fig. 5, show that those brighter than the flux limit of the PHz sample (500 mJy at 545 GHz), hence detectable by Planck, are expected to be very rare. The CORE project can do much better. Its estimated 4σ detection limits, at the nearby frequency of 520 GHz, range from 141 mJy for the 1 m telescope option to 82.5 mJy for a 1.5 m telescope (De Zotti et al. 2016). From Fig. 5 the corresponding surface densities range from 230 sr⁻¹ to 3300 sr⁻¹.

The yellow histograms in Fig. 7 show the distributions of the FWHMs and of the ellipticities, ι, of sources detected in simulated maps with SNR ≥ 5 and F₅₄₅GHz ≥ 500 mJy (very similar results are obtained using the empirical luminosity function of Mancuso et al.). The distributions are qualitatively consistent with those of the PHz sample (solid histograms, taken from Fig. 14 of PCXXXIX), suggesting that the elongated shape of the detected sources is an effect of source confusion rather than being an intrinsic property of the proto-clusters. Note that a close agreement between the results of our simulations and the observed distributions is not to be expected because simulations cannot reproduce exactly the selection criteria adopted by PCXXXIX (simultaneous detection within a 5′ radius in the 545 GHz excess map, with SNR > 5, and in the 857, 545, and 353 GHz cleaned maps with SNR > 3; absence, at 100 GHz, of any local maximum with SNR > 3 within a radius of 5′ of the 545 GHz position; colour-colour selection) and their procedures for determining sizes and ellipticities.

Finally, in Fig. 8 we show the redshift distribution of the brightest proto-clusters associated with the sources detected in the simulated map with SNR ≥ 5 and F₅₄₅GHz ≥ 500 mJy. For comparison, we also show the photometric redshift distribution (obtained assuming a modified black body spectrum with dust temperature T = 35 K and dust emissivity index β = 1.5) of the Herschel/SPire detected sources associated with the Planck candidate proto-clusters (Planck Collaboration XXVII 2015, black histogram in the same figure). Both histograms peak in the redshift range z = 1.5 − 3 showing the great potential offered by the Planck catalogue for studying the early phases of clusters formation.

6 CONCLUSIONS

The PHz catalogue appears to be dominated by overdensities of star-forming galaxies plus a small fraction of strongly gravitationally lensed galaxies (around 3%; Planck Collaboration XXVII 2015). Motivated by these results, we have updated the N05 predictions for the number counts of proto-clusters of star-forming galaxies and compared them with observational estimates.

We stress that our results are largely model-independent. The basic ingredients of our calculations are the sub-mm luminosity functions of high-z galaxies and their spatial correlation function. The luminosity functions have been observationally determined up to z ≃ 4, based on Herschel survey data (Gruppioni et al. 2013). The model we have used reproduces very well the observational determinations at each redshift (Cai et al. 2013; Bonato et al. 2014). Furthermore, we have shown that the results do not change.

Figure 6. Examples of sources detected in the 545 GHz simulated map (black and white image in the background). The red ellipse is the Gaussian fit to the detection, while the spots mark the five brightest proto-clusters within the ellipse, scaled in size according to their flux density (i.e. the brighter the clump, the bigger the spot).
not reproduce accurately the selection criteria of the PHz samples. Hence a precise match between the model and the observed distributions cannot be expected (see text).

Figure 7. Distribution of FWHMs and ellipticities, ε, derived from the simulations for detections with SNR ≥ 5 and $F_{545GHz} ≥ 500\,\text{mJy}$ (yellow histograms), compared to those measured for the PHz sample, normalized to the number of sources in the simulation (solid line). The FWHM is defined as $(\text{FWHM}_{\text{major}} \times \text{FWHM}_{\text{minor}})^{1/2}$ where FWHM_{major} and FWHM_{minor} are the major and the minor FWHM derived from the Gaussian fit, while the ellipticity is defined as $\epsilon = [1 - (\text{FWHM}_{\text{minor}}/\text{FWHM}_{\text{major}})^2]^{1/2}$. Note that the simulations cannot reproduce accurately the selection criteria of the PHz sample, nor the procedures for measuring the FWHM and the ellipticity. Hence a precise match between the model and the observed distributions cannot be expected (see text).

Figure 8. Redshift distribution of the brightest proto-clusters associated with the sources detected in the simulation with SNR ≥ 5 and $F_{545GHz} ≥ 500\,\text{mJy}$ (yellow histogram). For comparison we show the photometric redshift distribution of the Planck candidate proto-clusters (Planck Collaboration XXVII 2015) derived from the associated Herschel/SPire sources (black line) assuming a modified black body spectrum with dust temperature $T = 35\,\text{K}$ and dust emissivity index $\beta = 1.5$.

significantly if the model is replaced by an empirical fit of the observed luminosity function of dusty galaxies (Mancuso et al. 2016). As shown by Fig. 2, our model also provides an accurate representation of the observed redshift dependent correlation functions.

We find that the counts of sub-mm bright proto-clusters, obtained in the framework of the standard ΛCDM cosmology, are well below the observational estimates by PCXXXIX. A similar conclusion can be inferred from the results of Granato et al. (2015) who also predict, by using a completely different approach, a number of proto-clusters well below the one suggested by the statistics of Planck detections.

We have shown that this basically follows from the fact that, at high z, there are not enough sufficiently massive halos. On the other hand we have shown, by means of simulations, that the Planck cold intensity peaks are fully consistent with being mostly random fluctuations in the number of unrelated proto-clusters at $z \geq 2$ within the Planck beam. This is highlighted in Fig. 6 where examples of sources detected in the simulation are shown together with the brightest proto-clusters they are comprised of. The counts of these fluctuations match very well those of cold peaks and there is qualitative agreement also with the distributions of their FWHMs and of their ellipticities. Interestingly, the only Planck over-density for which spectroscopic or photometric redshifts of member galaxies have been obtained, was found to consist of 2 physically unrelated structures at $z \sim 1.7$ and $z \sim 2$ (Flores-Cacho et al. 2016).

The redshift distribution of the brightest proto-clusters contributing to the cold peaks, given by the model, has a broad maximum between $z = 1.5$ and $z = 3$. Therefore, follow-up observations of galaxies within the Planck overdensities providing redshift estimates would be a powerful tool to investigate the early phases of cluster formation, inaccessible by other means.

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