1   **Effects of sand-shale anisotropy on amplitude variation with angle (AVA)**

2   **modelling: The Sawan Gas Field (Pakistan) as a key case-study for South**

3   **Asia's sedimentary basins**

4   Hafiz Mubbasher Anwer[1*], Tiago M. Alves[2], Aamir Ali[1], Zubair [3]

5   [1]Department of Earth Sciences, Quaid-i-Azam University, Islamabad, 45320, Pakistan.

6   [2]3D Seismic Lab, School of Earth and Ocean Sciences, Cardiff University, Main Building-Park

7   Place, Cardiff CF10 1JZ, United Kingdom

8   [3]Software Integrated Solutions, Schlumberger, Pakistan.

9   [*]Corresponding author

10

11   **Abstract**

12   Amplitude variation with angle (AVA) is a technique widely used in the characterisation of

13   hydrocarbon reservoirs and assumes the Earth's crust to be an isotropic medium. Yet, seismic

14   anisotropy is known to have first-order effects on seismic AVA responses when investigating

15   subsurface prospects. This work analyses the effects of anisotropic strata on AVA responses

16   using the Lower Goru Formation, middle Indus basin (Pakistan) as a case-study. In the study

17   area, shale intervals are interbedded with reservoir sands of the Sawan gas field. Shales in this

18   gas field form laminae or are dispersed within reservoir sands, making the Lower Goru

19   Formation an example of a vertically transversely isotropic (VTI) medium. In this work, we

20   calculate the effective (saturated) mechanical properties of the Lower Goru Formation based on

21   rock physics templates; the Backus (1962) average typically designed for layered media,

22   combined with the empirical relations of Brown and Korringa (1975) and Wood (1955). The

23   input data used in our rock physics modelling is based on a detailed petrophysical analysis of

24   well data. Using the saturated effective mechanical properties of the Lower Goru Formation, we

generate angle-dependent reflection coefficient curves (and seismic AVA responses) based on exact and approximate solutions, for both isotropic and anisotropic reservoir scenarios. Our results suggest that the effects of lithological anisotropy are more pronounced in places with thick shale beds within reservoir sands. Conversely, angle-dependent reflection curves, and seismic AVA responses based on isotropic or anisotropic cases, give similar solutions in the presence of thin shale beds. As a corollary of this work, we present a Bayesian inversion method for the estimation of porosity in VTI media.

Keywords: South Asia; VTI medium; Rüger's approximation; PP reflection coefficients; Anisotropic AVA Modelling; Bayesian inversion.

1. Introduction

Modern techniques based on the systematic analysis of amplitude variations of seismic waves with changing distance between source and receiver (AVO), or with angle of incidence (AVA), are widely used in the petroleum industry to: a) detect subsurface gas, b) identify lithological variations, and c) analyse subsurface fluid volumes and compositions (Floridia and Teles, 1998; Grechka, 1998; Margesson and Sondergeld, 1999; Feng and Bancroft, 2006; Almutlaq and Margrave, 2010; Chao et al., 2012). Seismic AVA is of particular interest as its use is based on the realisation that variations in amplitude, at varying incident angles, result from contrasts in lithology and fluid content in rocks above and below a layer boundary (Zhang and Brown, 2001). Seismic AVA modelling is systematically carried out in reservoir studies by considering the Earth as an isotropic medium. However, all sedimentary rocks exhibit anisotropic behaviour at different scales (Xu et al., 2005). It has been known for a long time that anisotropy resulting from shaley beds and lenses within sandy successions can affect seismic AVA responses when investigating sand-shale reservoirs (Banik, 1987; Wright, 1987; Kim et al., 1993;

Thomsen, 1995; Blangy, 1994; Ball, 1995; Grechka, 1998; Besheli et al., 2005; Brajanovski et al., 2009; Chao et al., 2012; Nourollah et al., 2015). By ignoring the effects of anisotropy during AVA modelling, interpreters may complete erroneous characterisations of productive reservoir intervals (Besheli et al., 2005). Importantly, erroneous quantification of volumes and reservoir distribution(s) should arise when analysing the petroleum potential of sedimentary basins in South Asia, and other parts of the world, where stratified reservoir sequences of shale and silt contain significant amounts of 'tight oil' and 'tight gas' (McGlade et al., 2012; Katz and Lin, 2014; English et al., 2015; Rodriguez et al., 2016).

In recent decades, stratigraphic sequences of thin-layered sands and shales have become key exploration targets for which seismic anisotropy methods (also referred to as long-wavelength anisotropy) are useful and broadly successful in their characterisation (Crampin et al., 1984; Sayers, 2013; Sone and Zoback, 2013; Das and Zoback, 2013). Shale layers within a horizontally-bedded sand matrix behave elastically as transversely isotropic (TI) media, i.e. they have similar wave properties in two perpendicular directions, but are significantly different in a third orthogonal direction (Sone and Zoback, 2013). In these same sequences, vertical axes of symmetry are often described as vertically transversely isotropic (VTI) media (da Silva et al., 2016; Li et al., 2016).

The anisotropic behaviour of VTI media is usually quantified by using Thomsen's anisotropic parameters ε, δ, γ (Thomsen, 1986), in which ε denotes the fractional difference between the horizontal and vertical P-wave velocities, δ describes the variation in P-wave velocity with phase angle for near-vertical propagation, and γ denotes the fractional difference between the horizontal and vertical SH-wave velocity (Rüger, 2002). The empirical forms of ε, δ, γ are provided by Rüger (2002) as:

$$\gamma = \frac{C_{66} - C_{55}}{2C_{55}}, \tag{1}$$

$$\varepsilon = \frac{C_{11} - C_{33}}{2C_{33}}, \qquad (2)$$

and

$$\delta = \frac{(C_{13} + C_{55})^2 - (C_{33} - C_{55})^2}{2C_{33}(C_{33} - C_{55})}. \qquad (3)$$

Here, C represents the stiffness and its indices denote standard constants for VTI media. In essence, elastic anisotropy in VTI media can be characterised by five independent elastic constants, which can be expressed in two notations: a) in terms of stiffness constants ($C_{11}, C_{13}, C_{33}, C_{55}$ & C_{66}), or b) in terms of vertical velocities (V_P and V_s) and Thomsen's anisotropic parameters ($\varepsilon, \delta, \gamma$) (Tsvankin, 1997a, 1997b). These five independent elastic constants are very difficult to apply to AVA analyses, and most of these latter are developed by assuming a non-VTI medium.

Several researchers have attempted to estimate seismic anisotropy and incorporate it into AVA-based methods using seismic, VSP, well logs and core data (Blangy, 1992, Leaney, 1993; Blangy, 1994, Margesson and Sondergeld, 1999; Besheli et al., 2005, Luo et al., 2005, Wang et al., 2006; Brajanovski et al., 2009; Goodway et al., 2010; Wang, 2011), but seldom for hydrocarbon-rich basins in South Asia. However, in the Indian subcontinent (including Pakistan and bordering countries), important tight oil and gas reservoirs have been found in the hinterland basins that surround the Himalayan and Baluchistan mountain ranges. The Indus basin is currently the main focus of oil and gas exploration and production in Pakistan (Alam et al., 2014; Asim et al., 2015) (Figure 1a). In this basin, as in other hydrocarbon-productive regions of South and Southeast Asia, the successful use of AVA techniques in reservoir characterisation is critically dependent upon the careful estimation of the anisotropic parameters of sedimentary rocks (Zaigham and Mallick, 2000; Chengzao et al., 2012; Zhu et al., 2012; Jinhu et al., 2014).

99   In this work, we use well log data from the Sawan gas field (Figure 1b), middle Indus basin

100   (Pakistan), to focus on the Cretaceous Lower Goru C-sand reservoir, which is interbedded with

101   shales. We use reservoir properties estimated from borehole data as key data inputs to our rock

102   physics models, and to later carry out anisotropic and isotropic AVA analyses. Rock physics

103   models are often used to link seismic data to reservoir properties (Avseth et al., 2005). Seismic-

104   based estimates of reservoir properties can be uncertain, but the inclusion of rock physics models

105   in exploration workflows is, nevertheless, capable of reducing the level of uncertainty by

106   decreasing the number of unknown parameters in AVA analyses. In other words, rock physics

107   modelling represents a type of regularization within the context of seismic inversion (Ali and

108   Jakobsen, 2011a, 2011b).

109   The aim of this work is to analyse the effects of anisotropy generated by interbedded shales

110   within the reservoir sands, and its implications to AVA-based reservoir studies. In parallel, we

111   investigate how accurate are seismic AVA methods for both exact (Zoeppritz, 1919; Daley and

112   Horn, 1977) and approximate (Rüger, 1998; 2002) solutions for PP-reflection coefficients within

113   the context of isotropic and anisotropic cases.

114

115   **2. Geological setting**

116

117   The Sawan gas field lies in the middle Indus basin, in the eastern border of Pakistan (Zaigham

118   and Mallick, 2000). The study area is bounded by the Sargodha High to the north, and by the

119   Jacobabad and Mari-Kandkot Highs to the south (Figure 1a). The Indian Shield bounds the

120   eastern side of the study area, whereas the Kirthar and Suleiman fold-and-thrust belts mark its

121   western boundary (Kadri, 1995; Afzal et al., 2009). Regional geological data indicate that the

122   structural evolution of the Sawan gas field was closely controlled by three post-rift tectonic

123   events: a) Late Cretaceous uplift and erosion, b) NW–trending thick-skinned wrench faulting and

124   c) Late Tertiary to present-day tectonic uplift of the Jacobabad and Khairpur Highs (Ahmad et

al., 2004; Afzal et al., 2009; Azeem et al., 2016). These latter structural highs played an important role in the formation of structural and stratigraphic traps, not only in the Sawan area, but also in multiple oil and gas fields in the region (Ahmad et al., 2004; Fink et al., 2004; Berger et al., 2009).

Cretaceous black shales in the Sembar Formation are the proven source rock in the middle and lower Indus basins, which were mainly filled with shale and minor amounts of black siltstone, sandstone and nodular argillaceous limestone (Quadri and Shuaib, 1986; Kadri, 1995). The thickness of the Sembar Formation ranges from 0 m to more than 260 m (Iqbal and Shah, 1980). The Sembar Formation is deeply buried and matures thermally towards the western edge of the Indus basin. It becomes shallower and less mature towards its eastern edge (Wandrey et al., 2004). In the study area, the tectonic uplift recorded by the Khairpur High controlled the depositional patterns of the main reservoir unit (Goru Formation); reservoir sands in proximal depositional systems are positioned in structurally deep areas, whereas non-reservoir distal shales are positioned up-dip to form major structural traps (Azeem et al., 2016, Berger et al., 2009). The depositional environment of the Lower Goru Member, which comprises the main reservoir interval above the Sembar Formation, was marine and reflects deposition at the western (passive) margin of the Indian plate.

The Sembar Formation is, therefore, overlain by the Lower Goru Member of the Goru Formation, i.e. the proven reservoir interval in the Sawan gas field (Fig. 2). The upper part of the Lower Goru Member is chiefly composed of shales, whereas its lower part comprises medium to coarse-grained sandstones with thin layers of shale and limestone. The lower portion of the reservoir interval can be subdivided into distinct sand intervals: A, B, C and D (Krois et al., 1998) (Figure 2). The deposition of these sand intervals took place in deltaic, shallow-marine environments during sea-level lowstands, when medium to coarse-grained sediment was deposited on top of the distal (shale and siltstone) strata of previous highstand systems tracts (HSTs) (Berger et al., 2009, Azeem et al., 2016).

151   Petrographically, intervals A and B can be classified as quartz-rich arenites (Berger et al.,

152   2009). The C interval includes significant amounts of partially altered volcanic rock fragments

153   and pore lining iron chlorite cement. Thus, the sands of interval C can be classified as sublithic

154   to lithic arenites (McPhee and Enzendorfer, 2004; Berger et al., 2009; Azeem et al., 2016). Sands

155   in the B and C intervals comprise the main gas reservoirs in the study area (Munir et al., 2011),

156   showing high porosity (i.e. around 16 %) and permeability at depths between 3000–3500 m

157   (Azeem et al., 2016). These two reservoir intervals of the Lower Goru Member are draped by

158   transgressive shales and siltstones of the Upper Goru Member, which acts as a regional seal in

159   the middle and lower Indus basins (Berger et al., 2009).

160

161   **3. Dataset and methods**

162

163   Data from three exploration wells (Sawan-01, Sawan-3B and Sawan-06) are used in this

164   study to analyse the acoustic effects of the lithologic anisotropy created by interbedded shales in

165   the C-sand reservoir. The rationale behind selecting the Sawan gas field as a case study results

166   from the fact that it is located in the tectonically stable part of the Thar Platform, in the middle

167   Indus basin. It also shows marked alternations of sands and shales, being a typical example of a

168   VTI medium.

169   The workflow adopted in this work is shown in Figure 3. In a first instance, we analysed

170   sand-shale distribution patterns in the main reservoir intervals using the methodology proposed

171   by Thomas and Stieber (1975). In a second stage, rock physics modelling was undertaken using

172   the Backus (1962) approach for dry composite porous media (sand-shale), used together with the

173   relationships of Brown and Korringa (1975) and Wood (1955). These latter relationships allowed

174   us to incorporate fluid effects on the mechanical properties of strata. The aim of the second stage

175   of our analysis was to obtain (saturated) effective elastic properties for the Lower Goru reservoir

176   intervals (see also Jakobsen et al., 2003a, 2003b; Ali and Jakobsen, 2011a, 2011b; Ali et al.,

177   2016).

178   In a third stage exact solutions of P-wave reflection coefficients, derived from Zoeppritz

179   (1919) solutions for isotropic media and Daley and Horn's (1977) solutions for VTI media, were

180   used to generate angle dependent reflection coefficients and seismic AVA data from the top of

181   the reservoir, for both isotropic and anisotropic cases. In parallel, the Rüger's approximation

182   (Rüger, 1998, 2002) was used for angle dependent reflection coefficients and seismic AVA data

183   from the top of the reservoir, for both isotropic and anisotropic cases, over the Sawan reservoir

184   intervals. Finally, a Bayesian inversion scheme was utilised to investigate the implications of

185   isotropic and anisotropic AVA solutions on the estimation of porosity throughout the reservoir.

186

187   **3.1 Sand-Shale distribution analysis**

188

189   To understand shale distribution within reservoir sands is an integral part of forward

190   modelling using rock physics templates (Ali et al., 2016). Prior to developing the appropriate

191   rock physics model for AVA modelling, sand-shale distribution analyses are necessary to

192   identify the type of shale(s) distributed within reservoir sands (Ali et al., 2016). This step

193   improves confidence in the interpretation of sand-shale reservoirs because it helps interpreters to

194   decide which geophysical approach is appropriate for a particular reservoir (Kurniawan, 2005).

195   The distribution of shale within reservoir sands has a pronounced effect on reservoir production

196   performance due to decreasing porosity values and variable saturations that derive from the

197   presence of shales (Sames and Adrea, 2001; Ali et al., 2016). Hence, sand-shale distribution

198   analyses require robust stratigraphic correlations based on gamma-ray log data (Figure 4).

199   Gamma-ray curves are particularly helpful in the identification of shales within reservoir sand,

200   and for detailed petrophysical analyses and correlations. The petrophysical interpretation of wells

201 (Sawan-01, Sawan-3B and Sawan-06) was carried out to determine key reservoir properties such

202 as P-wave velocity (V_p), S-wave velocity (V_s), porosity (ϕ), density (ρ), volume of shale (V_{sh})

203 and water saturation (S_w) for the C-sand of the Lower Goru Member (Figures 5-7). The details

204 of estimated reservoir parameters are given in Tables 1-3.

205   The parameters commonly required in sand-shale distribution analyses are volume of shale,

206 porosity and water saturation, with the volume of shale being a critical parameter that controls

207 the two latter (Saxena et al., 2006). Once these parameters are estimated, the Thomas-Stieber

208 (1975) method in Figure 8 can be used to estimate shale distribution patterns (Saxena et al.,

209 2006). The Thomas-Stieber (1975) method estimates shale distribution utilizing a cross-plot with

210 volume of shale along the X-axis and total porosity along the Y-axis. The position of data points

211 on the cross-plot allows the identification of the type(s) of shale distribution within sands.

212   Usually, shales are distributed through four different ways within sands: a) as laminae, b)

213 through structures (faults, joints), c) dispersed in sands, and d) as any combinations of the three

214 latter ways (Clavaud et al., 2005; Sams and Andrea, 2001). The results from sand-shale analyses

215 undertaken for the three studied wells suggest that, in our study area, laminar and dispersed shales

216 are distributed within reservoir sands (Figures 9-11).

217

218 **3.2. Forward Modelling**

219

220   The nonlinear forward problem is defined as:

221

222 $$\mathbf{d} = G(\mathbf{m}). \tag{4}$$

223

224   In Equation (4), $\mathbf{d}$ is a vector of observable quantities (angle-dependent reflection coefficients

225 or seismic AVA data) and $\mathbf{m}$ is a vector of model parameters required to be estimated over the

226   model space **M** i.e.**m** $\in$ **M**. The operator G is a forward modelling operator used for generating

227   synthetic angle-dependent reflection coefficients, or seismic AVA data, via rock physics and

228   seismic modelling. In the subsequent section we will discuss the rock physics modelling for a

229   layered medium.

230

231   **3.2.1 Rock Physics modelling**

232

233   Rock physics models are important to correlate variations in reservoir properties (lithology,

234   porosity, permeability, pore fluid, etc.) with changes in the velocities (V_p, V_s) and density (ρ)

235   observed on well data (Uden et al., 2004). Rock physics models act as a bridge between

236   geological properties (reservoir parameters) and geophysical data by upscaling the reservoir

237   variables related to lithology, shale content and fluid parameters (Bachrach, 2006; Avseth et al.,

238   2005). Rock physics also provide a realistic and systematic basis for seismic-attribute generation

239   and interpretation (Avseth et al., 2005). In clastic reservoirs, shales are often found to behave

240   elastically as transversely isotropic media with a vertical axis of symmetry (Jakobsen and

241   Johansen, 1999, 2000). In forward models, rock physics models are used to calculate effective

242   elastic properties from petrophysical properties estimated from wireline-log data.

243   Results of shale distribution analyses within the Sawan gas field reservoir sands show that

244   laminar and dispersed shale types are distributed in the host medium. Such a medium can be

245   modelled using anisotropic rock physics (Ali et al., 2016; Ali et al., 2015; Jakobsen et al., 2003;

246   Sayers and Rickett 1997; Sayers, 1998; Backus, 1962). In this study, we follow the approach

247   given by Backus (1962), which was typically designed for a layered medium (see Appendix-A).

248   The fluid effects are incorporated via the Gassmann model for isotropic cases and via the Brown

249   and Korringa (1975) model for anisotropic cases, in conjunction with the Wood (1955) model

250   for homogenous saturations (Appendix-B).

251

3.2.2   Seismic amplitude variation with angle (AVA) modelling

253

254   Seismic AVA modelling is considered to be the most effective technique for reservoir

255   characterisation. In practice, AVA studies are the method most widely used for gas detection,

256   lithology identification and fluid parameter analyses (Feng and Bancroft, 2006). Seismic

257   response of variations in effective elastic properties obtained through rock physics tools for sand-

258   shale layers can be modelled using synthetic seismic AVA data. From the effective elastic

259   properties as calculated above, angle-dependent reflection coefficients and seismic AVA data

260   can be generated using the exact Zoeppritz (1919) formulation for isotropic media, and the Daley

261   and Horn's (1977) exact methods for anisotropic (VTI) media. We also tested the Rüger (2002)

262   approximation for the generation of reflection coefficients for both isotropic and VTI media. The

263   rationale behind only selecting Rüger's approximation for isotropic and VTI media is that the

264   Rüger's technique is only linearised in terms of small contrasts in medium parameters, without

265   having any additional assumption on Poisson's or V_p/V_s ratios (Rüger, 2002).

266

3.2.2.1 P-wave reflection coefficient for isotropic media

268

269   The exact Zoeppritz (1919) empirical relation (see Equation 5) allows us to calculate the PP-

270   wave reflection coefficients (R_{PP}) of a rock as a function of the incidence angle from the top of

271   an isotropic medium, which can be represented as a system of linear equations given as (Pujol,

272   2003):

273   $$A\boldsymbol{X} = B,$$ (5)

274   Where:

275

276
$$A = \begin{bmatrix} -Sine & Cosf & Sine' & Cosf' \\ Cose & \sin f & \cos e' & -Sinf' \\ Sin\,2e & -\frac{\alpha}{\beta}Cos\,2f & \frac{\rho'\alpha}{\rho\alpha}\left(\frac{\beta'}{\beta}\right)^2 Sin\,2e' & \frac{\rho'\alpha}{\rho\beta'}\left(\frac{\beta'}{\beta}\right)^2 Cos\,2f' \\ -Cos\,2f & -\frac{\beta}{\alpha}Sin\,2f & \frac{\rho'\alpha}{\rho\alpha}Cos\,2f' & -\frac{\rho'\beta'}{\rho\alpha}Sin\,2f' \end{bmatrix},$$ (6)

277
$$X = \begin{bmatrix} R_{PP} \\ T_{PP} \\ R_{PS} \\ T_{PS} \end{bmatrix},$$ (7)

278
$$\text{and } B = \begin{bmatrix} Sine \\ Cose \\ Sin\,2e \\ Cos\,2f \end{bmatrix}.$$ (8)

279

280 The unknown vector X can be obtained by:

281

282 $$X = A^{-1}B.$$ (9)

283

284 In these equations, α, β, ρ, e and f are the P-wave velocity, S-wave velocity, density, P-wave

285 transmission angle, and SV–wave transmission angle of the upper half space. Conversely, α', β',

286 ρ', e′ and f′ are the P-wave velocity, S-wave velocity, density and P-wave reflection angle, and

287 SV–wave reflection angle of the lower half space.

288

289 **3.2.2.2 Formulation of PP-wave reflection coefficients for VTI media**

290

291 The exact algebraic solution for reflection/transmission coefficients (R, T) of plane incident

292 P-waves in a VTI medium, developed by Daley and Horn (1977), is used in this work. The

293 concise form of the exact solution of P-wave reflection coefficients for a VTI medium (R_{PP}^{VTI}),

294 in matrix form, is given below (Graebner, 1992; Rüger, 2002):

295 $$MR = b,$$ (10)

296   where $\boldsymbol{b} = [-m_{11}, -m_{21}, m_{31}, m_{41}]^T$,   (11)

297   and $\boldsymbol{R} = \begin{bmatrix} R_{PP}^{VTI} \\ R_{PS}^{VTI} \\ T_{PP}^{VTI} \\ T_{PS}^{VTI} \end{bmatrix}$.   (12)

298   By using Cramer's rule, the solution for unknown vector $\boldsymbol{R}$ can be expressed in its analytic form

299   as:

300          $\boldsymbol{R} = \frac{1}{\det M} \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{bmatrix}^T \boldsymbol{b}$.   (13)

301   The values of matrices M and $\boldsymbol{b}$ are given by Rüger (2002), Graebner (1992) and Daley and Horn

302   (1977).

303

304   **3.2.2.3 Approximations for P-wave reflection coefficients**

305

306   We also investigate the accuracy of Rüger's (2002) approximation for both isotropic and VTI

307   media with the exact solutions given by Zeoppritz's (1919) and Daley and Horn's (1977).

308   Rüger's approximation for isotropic media is given below (Rüger, 2002):

309

310    $R_{PP}^{iso}(i) = \frac{1}{2}\frac{\Delta Z}{Z} + \frac{1}{2}\left\{\frac{\Delta V_{P0}}{\overline{V}_{P0}} - \left(\frac{2\overline{V}_{S0}}{\overline{V}_{P0}}\right)^2 \frac{\Delta G}{G}\right\} \sin^2 i + \frac{1}{2}\left\{\frac{\Delta V_{P0}}{\overline{V}_{P0}}\right\} \sin^2 i \tan^2 i.$ (14)

311

312   Where Z is the P-wave impedance, G is the shear wave modulus, V_{P0} is the vertical P-wave

313   velocity, and V_{S0} is the vertical shear-wave velocity. The character delta (Δ) stands for contrasts

314   across an interface ($\Delta Z = Z_2 - Z_1$), and the bar over a symbol represents its average ($\overline{Z} = \frac{Z_1 + Z_2}{2}$).

315   Subscript 1 corresponds to the upper layer and subscript 2 denotes the lower layer. Rüger's

316   approximation for the generation of reflection coefficients as a function of incidence angle (i),

317   in the case of VTI media (Rüger, 2002), is written as:

318

319    $$R_{PP}^{VTI}(i) = \frac{1}{2}\frac{\Delta Z}{\overline{Z}} + \frac{1}{2}\left\{\frac{\Delta V_{P0}}{\overline{V}_{P0}} - \left(\frac{2\overline{V}_{S0}}{\overline{V}_{P0}}\right)^2 \frac{\Delta G}{\overline{G}} + \Delta\delta\right\}\sin^2 i + \frac{1}{2}\left\{\frac{\Delta V_{P0}}{\overline{V}_{P0}} + \Delta\varepsilon\right\}\sin^2 i \tan^2 i. \qquad (15)$$

320

321   The additional terms ε and δ are Thomsen's (1985, 1996) anisotropy parameters for VTI

322   media.

323

324   **3.3 Inverse modelling**

325

326   In Bayesian settings, the solution of the inverse problem is given by the posterior probability

327   distribution $q(\mathbf{m}|\mathbf{d})$ applied over the model space M. In essence, $q(\mathbf{m}|\mathbf{d})$ carries all the

328   information about the model originating from the likelihood $L(\mathbf{m})$ and *a priori* probability

329   density function $p(\mathbf{m})$. Bayes' theorem allows to relate $q(\mathbf{m}|\mathbf{d})$ with $L(\mathbf{m})$ and $p(\mathbf{m})$ given

330   as (Aster et al., 2005):

331

332    $$q(\mathbf{m}|\mathbf{d}) \propto L(\mathbf{m})p(\mathbf{m}). \qquad (16)$$

333

334   Here $\propto$ is the sign of proportionality. The solution of the posterior distribution can be written

335   in a compact form as (Aster et al., 2005; Tarantola, 2005):

336

$$q(\mathbf{m} \mid \mathbf{d}) = \text{N} \cdot e^{-\text{J}(\mathbf{m})}, \tag{17}$$

338

339   where N is the normalization constant. The functional form of the objective function $\text{J}(\mathbf{m})$,

340   required to be minimised in the case of Gaussian statistics, and an uninformative prior

341   distribution, is given by Aster at al., (2005) as:

342

343     $$\text{J}(\mathbf{m}) = \min \sum_{i=1}^{n} \frac{\left((\text{G}(\mathbf{m}))_i - d_i\right)^2}{2\sigma^2}. \tag{18}$$

344

345   Here, σ is the standard deviation of the measured seismic data. The rationale behind assuming

346   an uninformative prior distribution is that we do not intend to constrain our inversion scheme by

347   incorporating *a priori* information (obtained mostly from log/core/laboratory data) about the

348   model parameters. There are different methods available for the evaluation of $q(\mathbf{m} \mid \mathbf{d})$, most of

349   which are described in Ali et al. (2011a; 2011b and 2015).

350

351   **4. Numerical results and discussion**

352

353   In this work, we analyse the effects created by sand-shale anisotropy on AVA response with

354   the help of rock physics modelling. In parallel, we present a comparison of exact and approximate

355   AVA solutions for isotropic and anisotropic scenarios. Before applying rock physics modelling

356   to layered media, it is very important to identify the type of shale distribution within reservoir

357   sands, e.g. laminar, structural, dispersed, and so on. For this same purpose, we followed the

358   approach presented in Section 3.1 for the C-sand reservoir unit (Lower Goru Member) drilled by

359 wells Sawan-01, Sawan-3B and Sawan-06. Reservoir properties are estimated through

360 petrophysical analyses of well log data acquired in the Sawan field and results are presented in

361 Figures 5-7 and in Tables 1-2.

362   The analysis of shale distribution suggests that the C-sand comprises laminated and dispersed

363 shale types (Figures 8-11). Thus, the Lower Goru C-sand may be a potential candidate to be

364 characterised as a VTI medium, rather than isotropic. Laminar shales in the C-sand reservoir

365 comprise thin layers of allogenic clays and do not control effective porosity, water saturation, or

366 the horizontal permeability of rock. However, they significantly change vertical permeability

367 (Kurniawan, 2005). Each lamina differs in thickness, in a way that the amounts of sand, silt and

368 clay in the layer are repeated as depositional sequences (or cycles) under dual flow regimes that

369 denote contrasts in energy level.

370   The saturated effective elastic properties for isotropic and VTI media are obtained using the

371 methodology discussed in section 3.2. The input to our rock physics modelling, in the form of

372 elastic and reservoir properties of the Lower Goru C-sand, intra-reservoir shale layers and

373 overburden strata, is extracted from the petrophysical analyses summarised in Table 3. The

374 elastic properties of solid mineral (quartz) and fluid (water and gas), required to generate AVA

375 data, are given in Table 4.

376   For the generation of isotropic angle-dependent reflection coefficient curves are used the exact

377 solution of Zeoppritz (1919) for isotropic media, and Daley and Horn's (1977) solution for VTI

378 media. Rüger's (2002) approximation is also used for both isotropic and VTI media to investigate

379 the accuracy of exact and approximate solution of P-wave reflection coefficients as a function of

380 incidence angle.

381   The angle dependent reflection coefficient (RC) curves are key to explain the main results in

382 this work. We demonstrate in Figs. 12-14 that the intercept (normal incidence i.e. zero offset, P-

383 wave reflectivity) and slope of the curve (gradient) indicate how the amplitude of RC changes

384 with angle/offset. The gradient of each RC curve is almost the same, but there are small

16

385 differences in the magnitude of angle-dependent PP-reflection coefficients when comparing the

386 isotropic and anisotropic cases (Figures 12-14). If we examine variations in RC based upon

387 isotropy and anisotropy it is obvious that, in the Sawan-01 well, only small differences can be

388 observed between isotropic and anisotropic RC curves (Figure 12). This clearly demonstrates

389 that strata drilled in Sawan-01 have a weak anisotropy ($\varepsilon = 0.0214$, $\delta = 0.0306$, $\gamma = 0.0186$). In

390 Sawan-3B, there are insignificant variations between isotropic and anisotropic RC curves (Figure

391 13). Finally, for Sawan-06 there is a significant variation in RC curves when comparing the

392 isotropic and anisotropic cases (Figure 14). More importantly, if we relate these variations in RC

393 curves with varying shale content, we can conclude that the wells with comparatively thick shale

394 layers show substantial variations between their isotropic and anisotropic RC curves. The best

395 example is, naturally, the RC response documented in well Sawan-06 (Figure 14; see also thick

396 shale content marked in yellow in Figure 4).

397 We observe that AVA data is moderately sensitive to the anisotropy of the medium. In

398 essence, our results confirm that little or no effects created by anisotropy upon reflection curves

399 are diagnostic of very weakly anisotropic media. Significantly, as shales are strongly anisotropic

400 and heterogeneous (Kumar et al., 2012), exploration well(s) that encountered thick shale layers

401 in the study area show clear variations between the behaviour of isotropic and anisotropic angle

402 dependent PP-reflection coefficients (Figure 14). Based on data from the three interpreted wells

403 (Sawan-01, Sawan-3B and Sawan-06), shale anisotropy increases proportionally to shale content,

404 and the effect of this same shale anisotropy upon reflection curves becomes more pronounced.

405 As such, it is advisable to predict anisotropy in a medium during AVA studies, especially when

406 thick allogenic shale layers are present in reservoir successions.

407 The investigation of Rüger (2002) approximation's accuracy for both isotropic and VTI media

408 reveal that the predicted magnitude of reflection coefficients is significantly different in all the

409 three studied wells, at large offsets. Also, the gradient predicted by the Rüger (2002)

410 approximation is high when compared to the exact solutions.

411   For seismic AVA response in isotropic and VTI media, angle-dependent reflection

412   coefficients are convolved with the source (Ricker) wavelet (Figures 15-17). It can be observed

413   from our approach that the amplitude of synthetic AVA gathers show a decreasing trend with

414   increasing angle of incidence. The difference between exact and approximate solutions is more

415   pronounced in the magnitude of predicted seismic AVA amplitude values (Figures 15-17).

416   The AVA response of VTI media is sensitive to contrasts in Thomsen's anisotropic

417   parameters, ε and δ, across the interface (Blangy 1994; Margesson and Sondergeld, 1999). As

418   also identified by Daley and Horn (1977), generally P-P reflections indicate that the smaller the

419   contrast in isotropic properties (V_P, V_s, ρ), and the larger the contrast in δ (variation in P-wave

420   velocity with phase angle for near vertical propagation) across a reflection interface, the greater

421   are the effects of anisotropy on AVA signatures. Contrasts in δ are most important under small-

422   to-medium angles of incidence, as previously reported in Banik (1987), whereas contrasts in ε

423   (fractional difference between the horizontal and vertical P-wave velocities) can have a strong

424   influence on amplitudes for the larger angles of incidence (21º) commonly used in exploration

425   seismic data. The increasing trend in gradients is more pronounced at far offsets (21º to 40º),

426   revealing an increasing sensitivity in terms of anisotropy and isotropy with increasing offset

427   angles.

428   The AVA modelling also proves a strong relationship between porosity and reflection

429   coefficient for all offset ranges. Therefore, in order to check the accuracy of exact and

430   approximate solutions for P-wave reflection coefficients in AVA inversion, we tried to recover

431   true reservoir porosity distribution (with 20% uncertainty) under the Bayesian settings discussed

432   in Section 6. For this purpose, a correlated Gaussian field was generated representing the true

433   porosity (Buland and Omre, 2003) distribution throughout the reservoir within 100×100 grid

434   blocks (Figure 18). Later, a AVA modelling approach (exact/approximate VTI or

435   exact/approximate isotropic) was performed using a maximum-a-posteriori (MAP) solution. The

436 results suggest that the exact VTI solution recovers the porosity trends with much more accuracy

437 when compared to all other solutions, under significant noise conditions (Figure 18).

438

439 **5. Conclusions**

440   In industry, seismic AVA/AVO techniques are being increasingly used for amplitude-based

441 reservoir characterisation, but frequently assume that subsurface media are isotropic. Seismic

442 anisotropy is known to have a first order effect on AVA modelling but this effect is often ignored

443 during AVA studies, providing significant errors when describing reservoir intervals. In this

444 work we analyse the effects of anisotropy on AVA modelling and inversion for a sand-shale

445 reservoir in the Sawan gas field, Pakistan. The main conclusions of this work are as follows:

446

447   a) AVA modelling shows that anisotropy effects are more pronounced in stratigraphic

448 intervals where interbedded shales are relatively thick within reservoir sand (Sawan-06 well).

449 The exact/approximate isotropic or VTI solutions show smaller variations in the presence of thin

450 interbedded shale layers within reservoir sands.

451   b) The exact solution for VTI media provided by Daley and Horn's (1977) is one with the

452 highest potential for performing AVA inversion in sand-shale media with weak to strong

453 anisotropy. We have demonstrated this fact by completing a numerical synthetic experiment for

454 recovering porosity distributions through the Lower Goru reservoir.

455   c) The choice of approximate solution(s) for AVA modelling is crucial in any workflow since,

456 in most cases, there is a significant difference in the predictions of magnitude of reflection

457 coefficients, and gradient of the reflection curves, resulting from distinct approximate solutions.

458 As suggested by our own AVA modelling results, the Rüger's approximation is significantly

459 different to other techniques when considering the magnitude and gradient of reflection curves.

460 This fact can create additional uncertainty to the use of AVA inversion techniques in the

461 characterisation of shale-sand reservoirs.

462

471

References

473 Afzal, J., Kuffner, T., Rahman, A., Ibrahim, M., 2009. Seismic and well-log based sequence

474 stratigraphy of the early Cretaceous, Lower Goru "C" sand of the Sawan gas field, middle Indus

475 Platform, Pakistan. Annual Technical Conference, Islamabad.

476 Ahmad, N., Fink, P., Sturrock, S., Mahmood, T., Ibrahim, M., 2004. Sequence stratigraphy as

477 predictive tool in Lower Goru fairway, Lower Middle Indus Platform, Pakistan. PAPG-SPE

478 ATC, 85-105.

479 Alam, I., Azhar, A.M., Khan, M.W., 2014. Frontal structural style of the Khisor Range, northwest

480 of Bilot: Implications for hydrocarbon potential of the north-western Punjab foredeep, Pakistan.

481 Journal of Himalayan Earth Sciences, Volume 47, Issue No. 1, 87-98.

482 Ali, A., Anwer, H.M., Hussain, M., 2015. A comparisonal study in the context of seismic fracture

483 characterization based on effective stiffness and compliance methods, Arab. J. of Geosci., DOI

484 :10.1007/s12517-014-1478-8

485   Ali, A., Jakobsen, M., 2011a. On the accuracy of Rüger's approximation for reflection

486   coefficients in HTI media: implications for the determination of fracture density and orientation

487   from seismic AVAZ data. Journal of Geophysics and Engineering, issue No. 8, 372-393.

488   Ali, A., Jakobsen, M., 2011b. Seismic characterization of reservoirs with multiple fractures sets

489   using velocity and attenuation anisotropy data, Journal of Applied Geophysics, Issue No.75, 590-

490   602.

491   Ali, A., Shahraini, A., Jakobsen, M., 2011. Improved characterization of fault zones by

492   quantitative integration of seismic and production data, Journal of Geophysics& Eng. Issue 8,

493   259–274.

494   Ali, A., Zubair, Hussain, M., Rehman, K., Toqeer, M., 2016. Effect of Shale Distribution on

495   Hydrocarbon Sands Integrated with Anisotropic Rock Physics for AVA Modelling: A Case

496   Study, Acta Geophysica, vol. 64, issue No. 4, 1139-1163.

497   Almutlaq, M.H., Margrave, G.F., 2010. Tutorial: AVO inversion, CREWES Research Report,

498   Vol. 22, 1-23.

499   Asim, S., Zhu, P., Qureshi S.N., Naseer, M.T., 2015. A case study of Precambrian to Eocene

500   sediments' hydrocarbon potential assessment in Central Indus Basin of Pakistan, Arabian Journal

501   of Geosciences, ISSN 1866-7511, DOI 10.1007/s12517-015-1988-z.

502   Aster, R.C., Borchers, B., Thurber, C.H., 2005. Parameter Estimation and Inverse Problems.

503   ISBN: 0-12-065604-3, Elsevier Academic Press, Amsterdam.

504   Avseth, P., Mukerji, T., Mavko, G., 2005. Quantitative seismic interpretation: Applying rock

505   physics tools to reduce interpretation risk. ISBN: 0-13-978-0-521-81601-7, Cambridge

506   University Press, UK.

507 Azeem, T., Yanchun, W., Khalid, P., Xueqing, L., Yuan, F., Lifang, C., 2016. An application of

508 seismic attributes analysis for mapping of gas bearing sand zones in the Sawan gas field Pakistan.

509 Acta Geod. Geophys, DOI 10.1007/s40328-015-0155-z.

510 Bachrach, R., 2006. Joint estimation of porosity and saturation using stochastic rock-physics

511 modelling. Geophysics, 71(5), 53–63.

512 Backus, G.E.,1962. Long-wave elastic anisotropy produced by horizontal layering. J.

513 Geophysics, Res., issue No.67, 4427-4440.

514 Ball, G., 1995. Estimation of anisotropy and anisotropic 3-D Prestack depth migration, offshore

515 Zaire. Geophysics, issue No. 60, 1495-1513.

516 Banik, N. C., 1987. An effective anisotropy parameter in transversely isotropic media.

517 Geophysics, 52(12), 1654-1664.

518 Berger, A., Gier, S., Krois, P., 2009. Porosity-preserving chlorite cements in shallow-marine

519 volcaniclastic sandstones: evidence of the Sawan gas field Pakistan. AAPG Bull 93(5), 595-615.

520 Besheli, S. A., Urosevic, M., Li, R., 2005. The effect of seismic anisotropy on reservoir

521 characterization. SEG Annual Meeting Houston, USA.

522 Blangy, J.P.,1992. Anisotropy and AVO in TI media, an overview: Joint SEG. In EAEG summer

523 research workshop, Expanded Abstracts, Vol. 60, l-605.

524 Blangy, J.P.,1994. AVO in transversely isotropic media-An overview, Geophysics, Vol 59, No.5,

525 775-781.

526 Brajanovski, M., Nadri, D., Gurevich, B. Bona, A., 2009. Stress induced anisotropy in sandstone

527 reservoir and shale overburden - AVO modeling. International Geophysical Conference and

528   Exposition, Beijing, China, 24–27 April, 194-194. ISBN: 978-1-56080-284-6 doi:

529   10.1190/1.3603719.

530   Brown, R.J.S., Korringa, J., 1975. On the dependence of elastic properties of a porous rock on

531   the compressibility of the pore fluid. Geophysics 40, 608–616.

532   Buland, A., Omre, H., 2003. Joint AVO inversion-wavelet estimation and noise level estimation

533   using a spatially coupled and hierarchical Bayesian Model, Geophysical Prospecting, 51, 531-

534   550.

535   Chao G., Cadore, T., Deplante, C., Gratzer, O., 2012. Detailed AVA analysis of a class IIp

536   reservoir in presence of VTI shale anisotropy. A case study from offshore West Africa. SEG

537   Annual Meeting, Las Vegas, DOI:10.1190/segam2012-1280.1.

538   Chengzao, J., Min, Z., Yongfeng, Z., 2012. Unconventional Hydrocarbon Resources in China

539   and the Prospect of Exploration and Development. Petroleum Exploration and Development, v.

540   39/2, 129-136.

541   Clavaud, J.B., Nelson, R., Guru, U.K., Wang, H., 2005. Field example of enhanced hydrocarbon

542   estimation in thinly laminated formation with a triaxial array induction tool: A laminated sand-

543   shale analysis with anisotropic shale. SPWLA 46th Annual Logging Symposium, 26-29 June,

544   New Orleans, USA.

545   Crampin, S., Chesnokov, E.M., Hipkin, R.G., 1984. Seismic Anisotropy- the state of the art: II.

546   Geophysics. J. Int. issue 76(1), 1-16.

547   Da Silva, N.V., Ratcliffe, A., Vinje, V., Conroy, G., 2016. A new parameter set for anisotropic

548   multiparameter full-waveform inversion and application to a North Sea dataset. Geophysics, Vol.

549   81, issue No. 4, U25–U38.

550   Daley, P.F., Horn, F., 1977. Reflection and transmission coefficients for transversely isotropic

551   media. Bull. Seis. Soc. Am., issue No. 67, 661-675.

552   Das, I., Zoback, M.D., 2013. Long-period, long-duration seismic events during hydraulic

553   stimulation of shale and tight-gas reservoirs Part 1: Waveform characteristics. Geophysics, Vol.

554   78, No. 6, KS97-KS108.

555   English, J.M., Lunn, G.A., Ferreira, L., Yacu, G., 2015. Geologic evolution of the Iraqi Zagros

556   and its influence on the distribution of hydrocarbons in the Kurdistan region. AAPG Bulletin,

557   V.99, No.2, 231-272, DOI: 10.1306/06271413205.

558   Feng, H., Bancroft, J.C., 2006. AVO principles, processing and inversion. CREWES Research

559   Report, Vol. 18, 1-19.

560   Fink, P., Schuh, M., Ahmad, N., Koehazy, R., 2004. Can the American Securities and Exchange

561   Commission and the central limit theorem ever become friends? A practical proposal for

562   reconciliation from OMV's Sawan Reserves Estimation exercise. PAPG/SPE Annual Technical

563   Conference, 8-9 October, Islamabad, Pakistan.

564   Floridia, C., Teles, N.T., 1998. Effects of anisotropy in the AVO analysis. SEG Technical

565   Program, Expanded Abstracts, 212-215. doi:10.1190/1.1820378.

566   Gassmann, F., 1951. Über die Elastizität poröser Medien.Viertel.Naturforsch.Ges. Zürich, 96, 1-

567   23.

568   Goodway, B., Perez, M., Varsek, J., Abaco, C., 2010. Seismic petrophysics and isotropic-

569   anisotropic AVO methods for unconventional gas exploration. The Leading Edge, 29(12), 1500-

570   1508. Doi: 10.1190/1.3525367.

571   Graebner, M., 1992. Plane-wave reflection and transmission coefficients for a transversely

572   isotropic solid. Geophysics, 57(11), 1512-1519, doi:10.1190/1.1443219.

573   Grechka, V.,1998. AVO analysis in finely layered azimuthally anisotropic media. SEG Technical

574   Program, Expanded Abstracts. 1649-1652. doi: 10.1190/1.1820238.

575   Iqbal, M.W.A., Shah, S.M.I., 1980. A guide to the stratigraphy of Pakistan, Geological Survey

576   of Pakistan, Records. Geol. Surv. Pak. Quetta, 53-34.

577   Jakobsen, M., Johansen, T.A., 1999. A test of ANNIE based on ultrasonic measurements on a

578   shale. J. Seis. Explor.issue No. 8, 77-89.

579   Jakobsen, M., Hudson, J.A., Johansen, T.A., 2003a. T-matrix approach to shale acoustics.

580   Geophysical Journal International, Issue No.154, 533-558.

581   Jakobsen, M., Johansen, T.A., 2000. Anisotropic approximations for mud rocks: A seismic

582   laboratory study. Geophys. **65,** 1711-1725.

583   Jakobsen, M., Johansen, T.A., McCann, C., 2003b. The acoustic signature of fluid flow in

584   complex porous media. Journal of Applied Geophysics, 54, 219-246.

585   Jinhu, D., He, L., Desheng. M., Jinhua, F., Yuhua, W., Zhou, T., 2014.Discussion on effective

586   development techniques for continental tight oil in China. Petroleum Exploration and

587   Development, Volume 41, Issue 2, 217-224.

588   Kadri, I.B., 1995. Petroleum geology of Pakistan: Pakistan Petroleum Limited. Graphic

589   Publishers Karachi.

590   Katz, B., Lin, F., 2014. Lacustrine Basin unconventional Resource Plays: Key differences.

591   Marine and Petroleum Geology, Volume 56, 255-265.

592   Kim, K.Y., Wrolstad, K.H., Aminzadeh, F., 1993. Effects of Transverse Isotropy on P-wave

593   AVO for gas sands. Geophysics, 58, 883-888.

594   Krois P, Mahmood T, Milan, G., 1998. Miano field, Pakistan a case history of model driven

595   exploration. Proceedings of the Pakistan Petroleum Convention, Pakistan Association of

596   Petroleum Geologists Islamabad, 111-131.

597   Kumar, D., Hoversten, G.M., 2012. Geophysical model response in a shale gas. 9th Biennial

598   international conference & Exposition on Petroleum Geophysics, Hyderabad, India.

599   Kurniawan, F., 2005. Shaly Sand Interpretation Using CEC-Dependent Petrophysical

600   Parameters, MS Dissertation, Louisiana State University and Agricultural and Mechanical

601   College.

602   Leaney, W.S., 1993. AVO and anisotropy from logs and walkaways. Exploration Geophysics,

603   24, 623-630.

604   Li, V., Tsvankin, I., Alkhalifah, T., 2016. Analysis of RTM extended images for VTI media

605   Geophysics, Vol. 81, NO.3, S139–S150, DOI: 10.1190/GEO2015-0384.1.

606   Luo, X., Lin, Y., Wu, L., Yao, F., 2005. Estimation of anisotropy parameters in VTI Media from

607   surface P-wave seismic data and VSP in Tarim Basin, China. SEG Annual Meeting, Houston.

608   178-181.

609   Margesson, R.W., Sondergeld, C.H., 1999. Anisotropy and amplitude versus offset: a case

610   history from the West of Shetlands. Proceedings of the 5th Conference, 635-643. Petroleum

611   Geology '86 Ltd. Published by the Geological Society, London.

612   Mavko, G., Mukerji, T., Dvorkin, J. 2009. The Rock Physics Handbook: Tools for Seismic

613   Analysis in Porous Media. Cambridge University Press, UK.

614   McGlade, C., Speirs, J., Sorrell, S., 2012. A review of regional and global estimates of

615   unconventional gas resources. A report to the Energy Security Unit of the Joint Research Centre

616   of the European Commission.

617   McPhee, C.A., Enzendorfer, C.K., 2004. Sand management solutions for high-rate gas wells,

618   Sawan field. SPE, Sindh.

619   Munir, K., Iqbal, M.A., Farid, A., Shabih, S.M. 2011. Mapping the productive sands of Lower

620   Goru Formation by using seismic stratigraphy and rock physical studies in Sawan area, southern

621   Pakistan: a case study. Journal of Petroleum Exploration & Production Technology, issue No.1,

622   33-42, doi:10.1007/s13202-011-0003-9.

623   Nourollah, H., Urosevic M., Keetley, J., 2015. Seal potential of shale sequences through seismic

624   anisotropy: Case study from Exmouth Sub-basin, Australia. Society of Exploration

625   Geophysicists Vol. 3, Issue No. 4, T257-T267.

626   Pujol, J., 2003. Elastic Wave Propagation and Generation in Seismology. ISBN 0-521-52046-0,

627   Cambridge University Press, UK, 160-161.

628   Quadri, V.N., Shuaib, M.,1986. Hydrocarbon prospects of Southern Indus Basin, AAPG

629   Bulletin, v. 70, pp.730–747.

630   Rodriguez, K., Hodgson, N., Hewitt, A., 2016. The future of oil exploration. First Break, Vol 34,

631   issue No 2, 95-101.

632   Rüger, A., 2002. Reflection Coefficients and Azimuthal AVO Analysis in Anisotropic Media:

633   Geophysical Monograph Series, No. 10, Society of Exploration Geophysics.

634   Rüger, A.,1998. Variation of P-wave reflectivity with offset and azimuth in anisotropic media.

635   Geophysics, issue Nno.63, 935-947.

636   Sams, M.S., Andrea, M., 2001. The effect of clay distribution on the elastic properties of

637   sandstones. Geophysical Prospecting, 49,128-150.

638   Saxena, K., Tyagi, A., Klimentos, T., Morriss, C., Mathew, A. 2006. Evaluating Deepwater Thin-

639   Bedded Reservoirs with the RT Scanner. Petromin, Kaula Lumpur.

640   Sayers, C.M. 2013. Rock Physics for Reservoir Exploration, Characterisation and Monitoring.

641   Geophysical Prospecting, 61, 251–253. doi: 10.1111/1365-2478.12034.

642   Sayers, C.M., 1998. Long-wave seismic anisotropy of heterogeneous reservoirs, Geophysical J.

643   Int. 132, 667-673.

644   Sayers, C.M., Rickett, J.E., 1997. Azimuthal variation in AVO response for fractured gas sands,

645   Geophysical Prospecting, 45, 165-182.

646   Shahraini, A., Ali, A., Jakobsen, M. 2011. Seismic history matching in fractured reservoirs using

647   a consistent stiffness-permeability model: Focus on the effects of fracture aperture, Geophysical

648   Prospecting, 59, 492-505.

649   Sone, H., Zoback, M.D., 2013. Mechanical properties of shale-gas reservoir rocks Part 2: Ductile

650   creep, brittle strength, and their relation to the elastic modulus. Geophysics , Vol. 78, No. 5,

651   D393-D402.

652   Tarantola, A., 2005. Inverse Problem Theory and Methods for Model Parameter Estimation.

653   Society for Industrial and Applied Mathematics, USA.

654   Thomas, E.C., Stieber S.J., 1975. The distribution of shale in sandstones and its effect on

655   porosity. SPWLA 16th Annual Logging Symp., 4-7 June1975.

656   Thomsen, L., 1995. Elasticity anisotropy due to aligned cracks in porous rock, Geophysical

657   Prospecting, 43, 805-829.

658   Thomsen, L.,1986. Weak elastic anisotropy. Geophysics,51(10), 1954-1966.

659   Tsvankin, I., 1997a. Anisotropic parameters and P-wave velocity for orthorhombic media

660   Geophysics, 62, 1292–1309.

661   Tsvankin, I., 1997b. Reflection move out and parameter estimation for horizontal transverse

662   isotropy. Geophysics, issue No.62, 614-629.

663   Tyagi, A.K., Guha, R., Voleti, D., Saxena, K., 2009. Challenges in The Reservoir

664   Characterization of a Laminated Sand Shale Sequence. 2nd SPWLA-India Symposium,

665   November 19-20.

666   Uden, R., Dvorkin, J., Walls, J., Carr, M., 2004. Lithology substitution in a sand/shale sequence.

667   ASEG 17th Geophysical Conference and Exhibition, Sydney.

668   Wandrey, C.J., Law, B.E., Shah, H.A., 2004. Sembar Goru/Ghazij composite total petroleum

669   system Indus and Sulaiman-Kirthar geologic Provinces Pakistan and India. US Geological

670   Survey, USA; Geological Survey Bulletin.

671   Wang, L., Dai, H., Li, X.Y., 2006. Estimating anisotropic parameters from PS converted-wave

672   data: a case study. British Geological Survey, Xianyi Sun, Petro China, Daqing Oil Field.

673   Wang, Y., 2011. Seismic anisotropy estimated from P-wave arrival times in cross hole

674   measurements. Geophysical Journal International, 184, 1311-1316.

675   Wood, A.W., 1955. A Textbook of Sounds, New York, McMillan Co. USA.

676 Wright, J., 1987. The effects of transverse isotropy on reflection amplitude versus offset.
677 Geophysics, 52, 564-567.

678 Xu, S., Wu, X., Huang, X., Yin, H., 2005. Evaluation of Anisotropic Rock Properties in
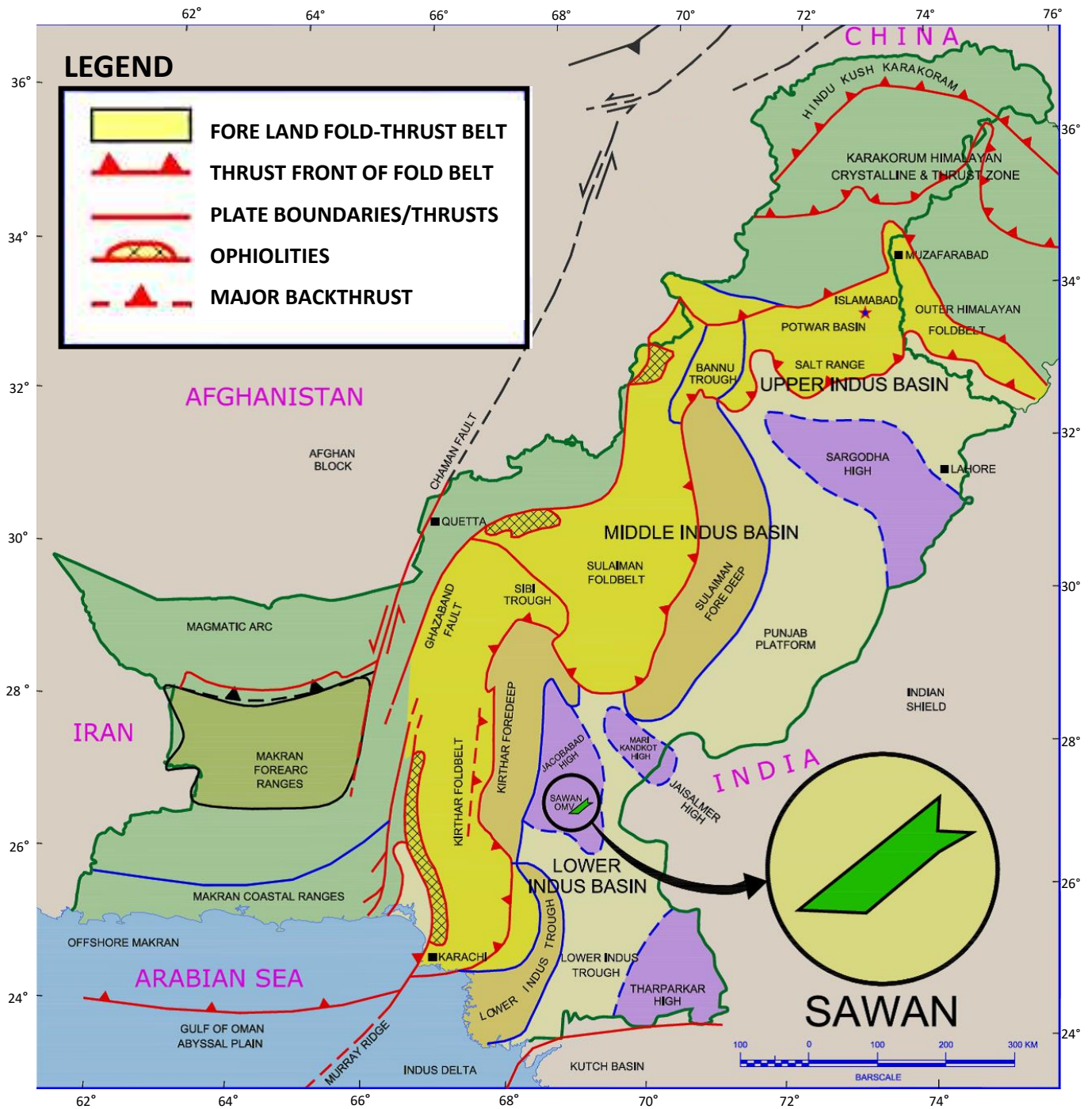679 Sedimentary Rocks from well log. Offshore Technology Conference Houston, TX, USA.

680 Zaigham, N.A., Mallick, K.A., 2000. Prospect of hydrocarbon associated with fossil-rift
681 structures of the southern Indus basin, Pakistan. AAPG, Bull 84(11), 1833-1848.

682 Zhang, H., Brown, R.J., 2001. A review of AVO analysis, CREWES Research Report, Volume
683 13, 337-378

684 Zhu, Y., Xu, S., Payne, M., Martinez, A., Liu, E., Harris, C., Bandyopadhyay, K., 2012.
685 Improved Rock-Physics Model for Shale Gas Reservoirs. SEG Technical Program Expanded
686 Abstracts, 1-5.doi: 10.1190/segam2012-0927.1

687 Zoeppritz, K.E., 1919. On the reflection and propagation of seismic waves. Göttinger
688 Nachrichten, 1(5), 66-84.

689

690

Figure 1a: Regional map showing the regional structural setting of Pakistan and the location of

multiple sedimentary basins in this country. The location of Sawan gas field is highlighted by

the black circle.

694

695   Figure-1b: Local map showing the boundaries of the Sawan gas field and the relative location

696   of exploration wells in the study area.

AGE	STRATIGRAPHY			LITHOLOGY	RESERVOIR POTENTIAL			OIL / GAS SHOWS	FIELDS
					SOURCE	CAPROCK	RESERVOIR		
RECENT / PLIOCENE	ALLUVIUM / SIWALIKS								
EOCENE	KIRTHAR FM.	DRAZINDA MB.				C			
		PIRKOH MB.					R		
		SIRKI MB.				C			
		HABIB RAHI MB.					R	☀	Mari
	LAKI FM.	GHAZIJ MB.				C			
		SUI MAIN LST. MB.					R	☀	Kandhkot, Sui Qadirpur, Pirkoh
PALEOCENE	DUNGHAN FM.					C	R	☀	Zarghun
	RANIKOT FM.					C	R	☀	Pirkoh
	PARH FM.								
UPPER CRETACEOUS	GORU FM.	LOWER GORU MB.	UPPER GORU MB.						
			SHALE INTERVAL		S	C			
			"D" INTERVAL			C			
			"C" INTERVAL		S	C	R	☀	Sawan, Mari Latif
LOWER CRETACEOUS			"B" INTERVAL		S	C	R	☀	Miano, Rehmat, Kadanwari
			"A" INTERVAL				R	☀	
	SEMBAR				S				
JURASSIC	CHILTAN								

Figure 2: Generalised stratigraphy of the Sawan gas field highlighting the presence of multiple lithological units, including the Lower Goru C-sand interval and shaley intervals within and adjacent to these reservoir sands. This C-sand interval comprises the principal gas-producing reservoir of the Sawan field (Azeem et al., 2015).
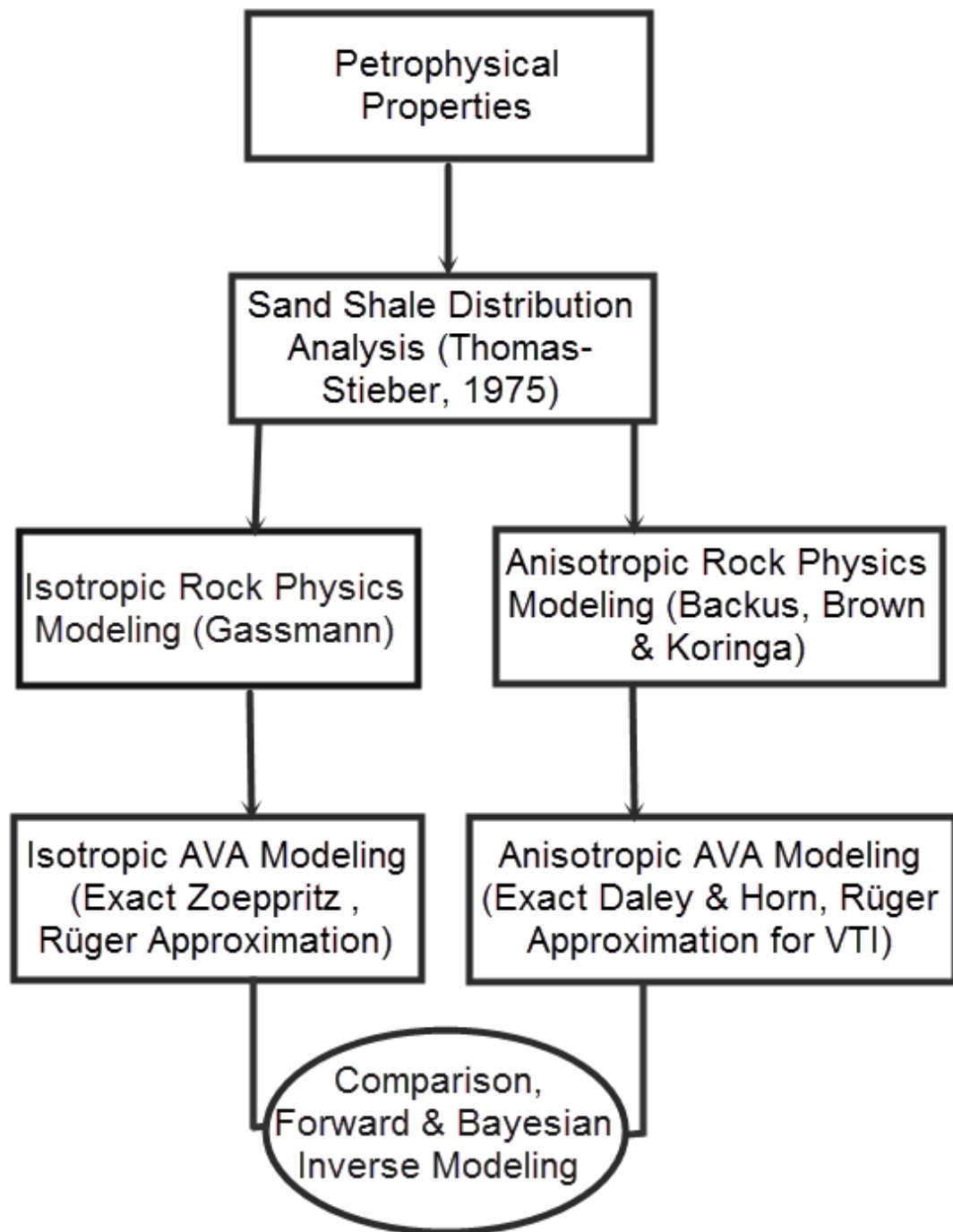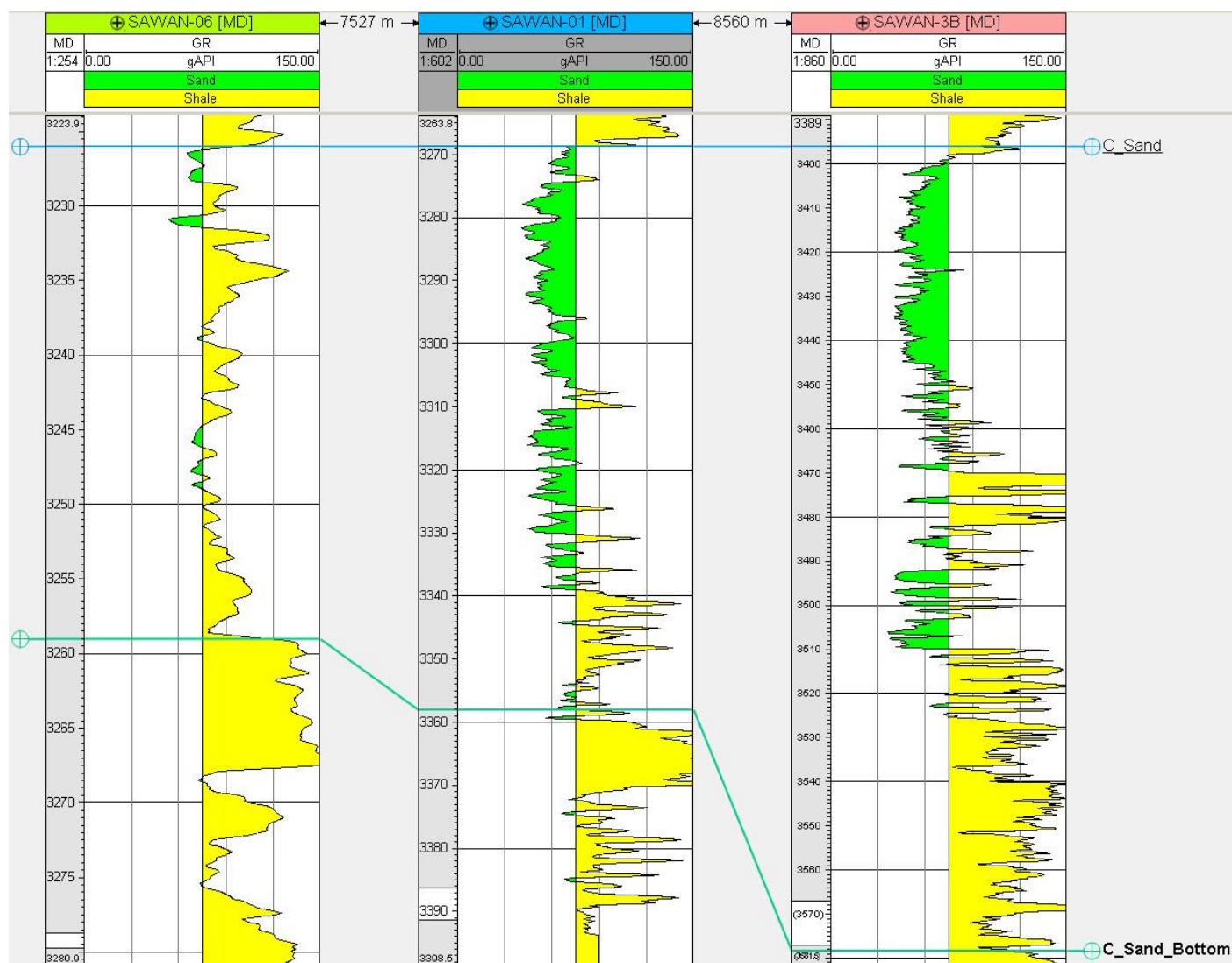
Figure 3: Work Flow diagram showing the methodology adopted to compare isotropic and anisotropic AVA modelling.

707

708   Figure 4 Well-log correlations for the Lower Goru C-sand and shale intervals in the three exploration wells used in AVA modelling.
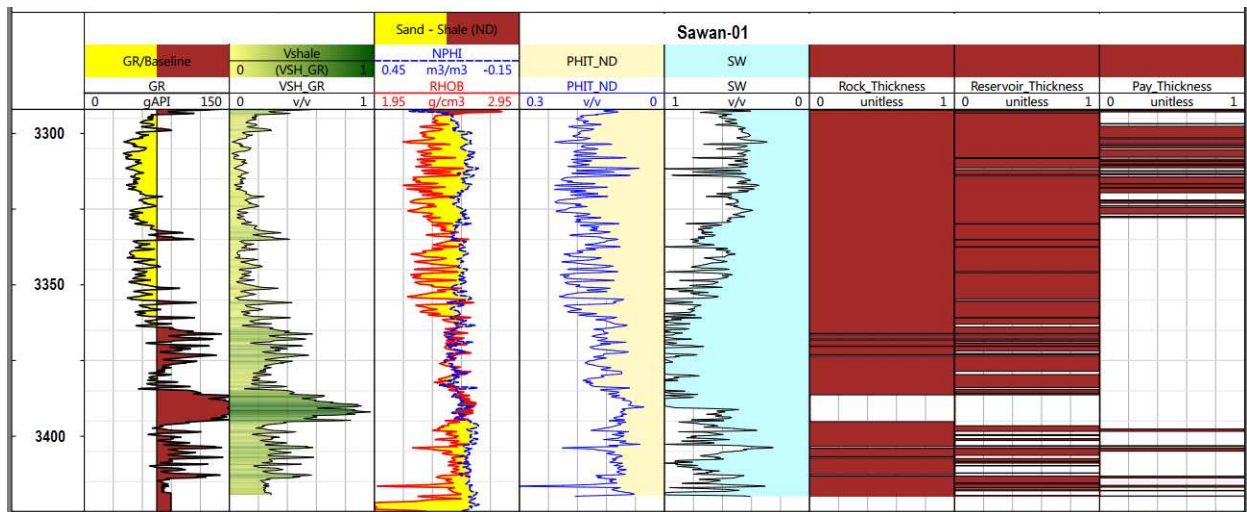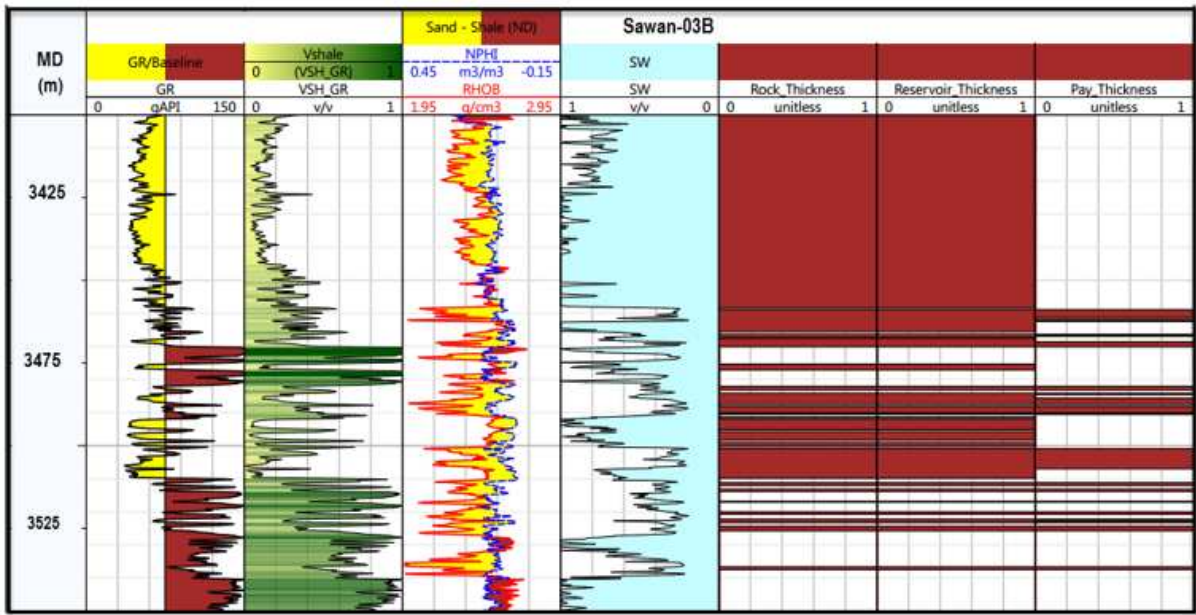
709

710   Figure 5: Results of petrophysical analyses for the estimation of reservoir properties of sand-
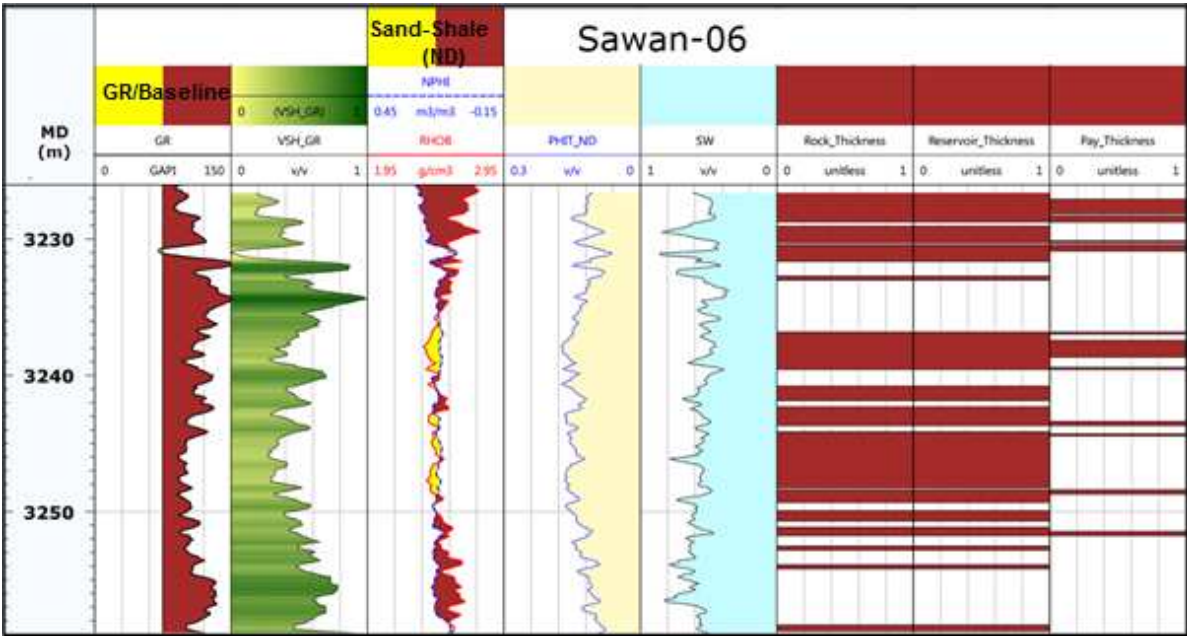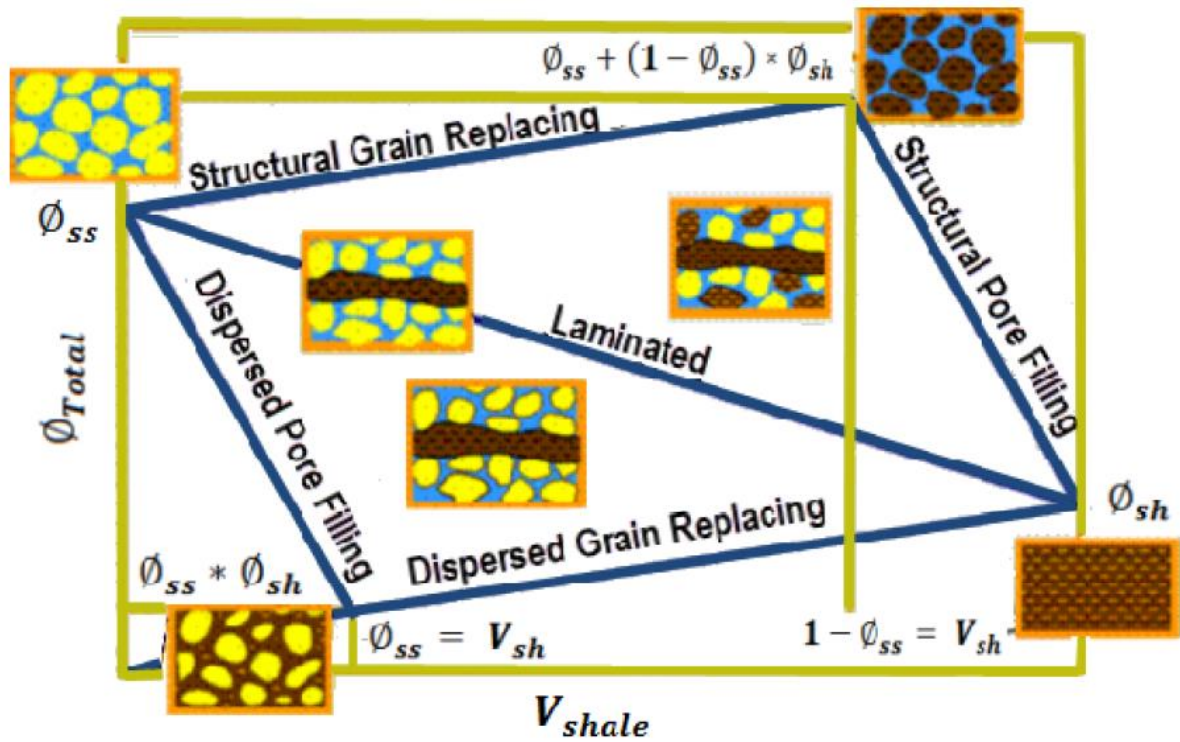
711   shale intervals in the Sawan-01 well.

712

713

Figure 6: Results of petrophysical analyses for the estimation of reservoir properties of sand-

shale intervals in the Sawan-3B well.

716

717 Figure 7. Results of petrophysical analyses for the estimation of reservoir properties of sand-

718 shale intervals in the Sawan-06 well.

$$\emptyset_{ss} + (1 - \emptyset_{ss}) \times \emptyset_{sh}$$

Structural Grain Replacing

Structural Pore Filling

$\emptyset_{ss}$

$\emptyset_{Total}$

Dispersed Pore Filling

Laminated

$\emptyset_{sh}$

$\emptyset_{ss} * \emptyset_{sh}$

Dispersed Grain Replacing

$\emptyset_{ss} = V_{sh}$

$1 - \emptyset_{ss} = V_{sh}$

V_{shale}

719

720   Figure 8: Shale distribution model proposed by Thomas and Strieber (1975) (figure modified

721   from Tyagi et al., 2009). In this diagram, V_{shale} is the volume of shale, ϕ_{total} is the total

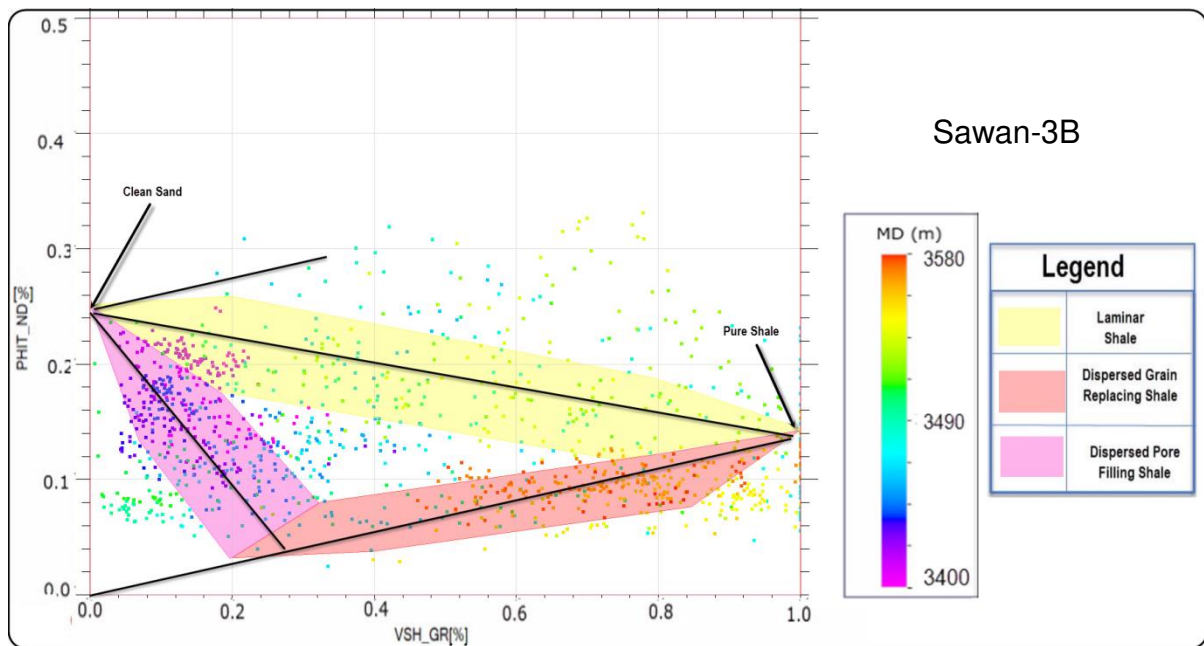722   porosity, ϕ_{max} is the maximum porosity, and ϕ_{sh} is the porosity in shales.
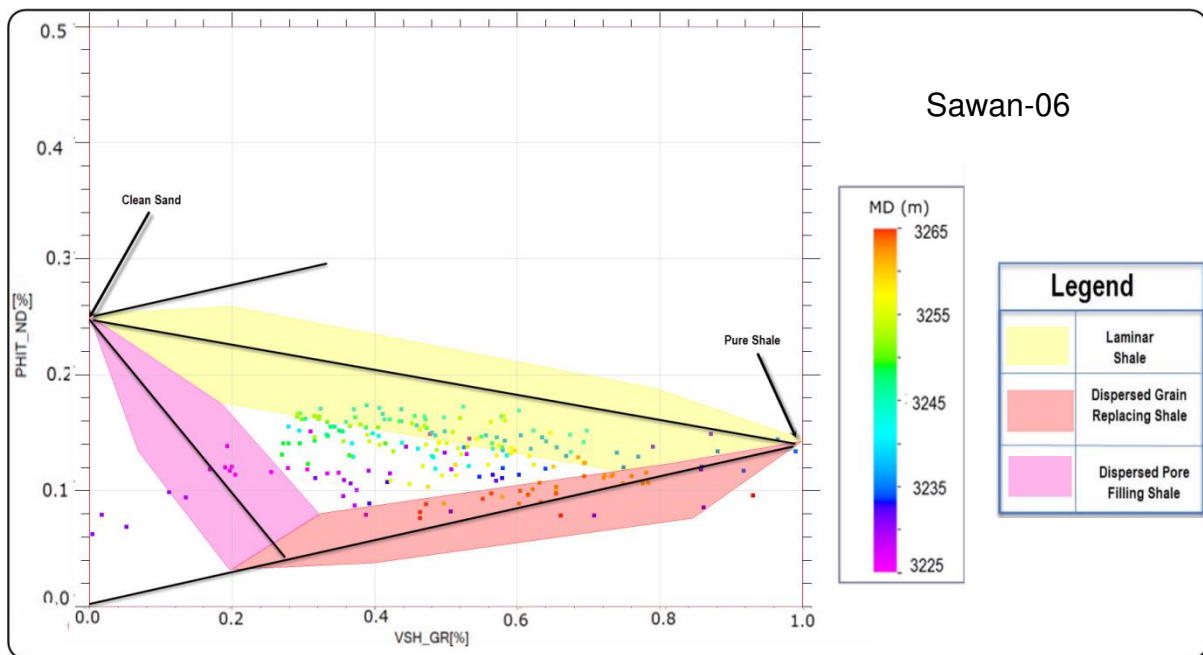
723

726 Figure 9: Cross-plots between volume of shale (X-axis) and total porosity (Y-axis). Based

727 upon the position of data points, shale distribution was characterised within the C-sand

728 reservoir in the Sawan-01 well.

729

730

Figure 10: Cross-plots between volume of shale (X-axis) and total porosity (Y-axis). Based

upon the position of data points, shale distribution was characterised within the C-sand

reservoir in the Sawan-3B well.

734

Figure 11: Cross-plots between volume of shale (X-axis) and total porosity (Y-axis). Based

upon the position of data points, shale distribution was characterised within the C-sand

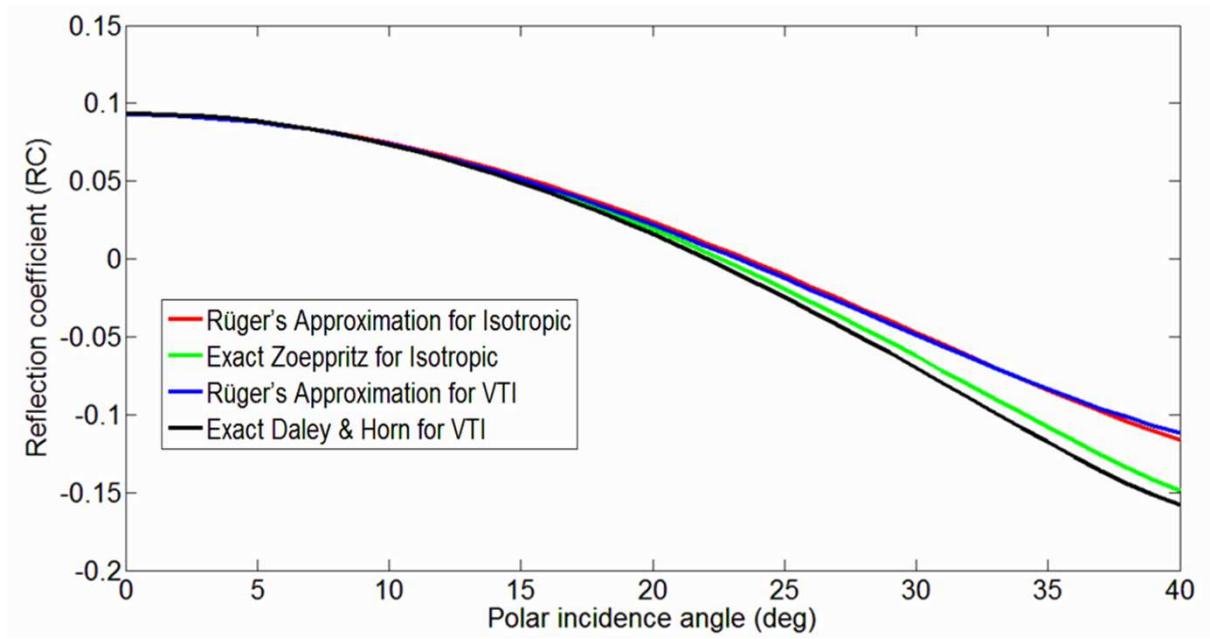reservoir in the Sawan-06 well.

738

739 Figure 12: Angle-dependent reflection coefficient data generated through the exact and

740 approximate solutions of PP-wave for isotropic and anisotropic (VTI) media at the top of C-

741 sand reservoir (Sawan-01 well).

742

743

Figure 13: Angle-dependent reflection coefficient data generated through the exact and approximate solutions of PP-wave for isotropic and anisotropic (VTI) media at the top of C-sand reservoir (Sawan-3B well).
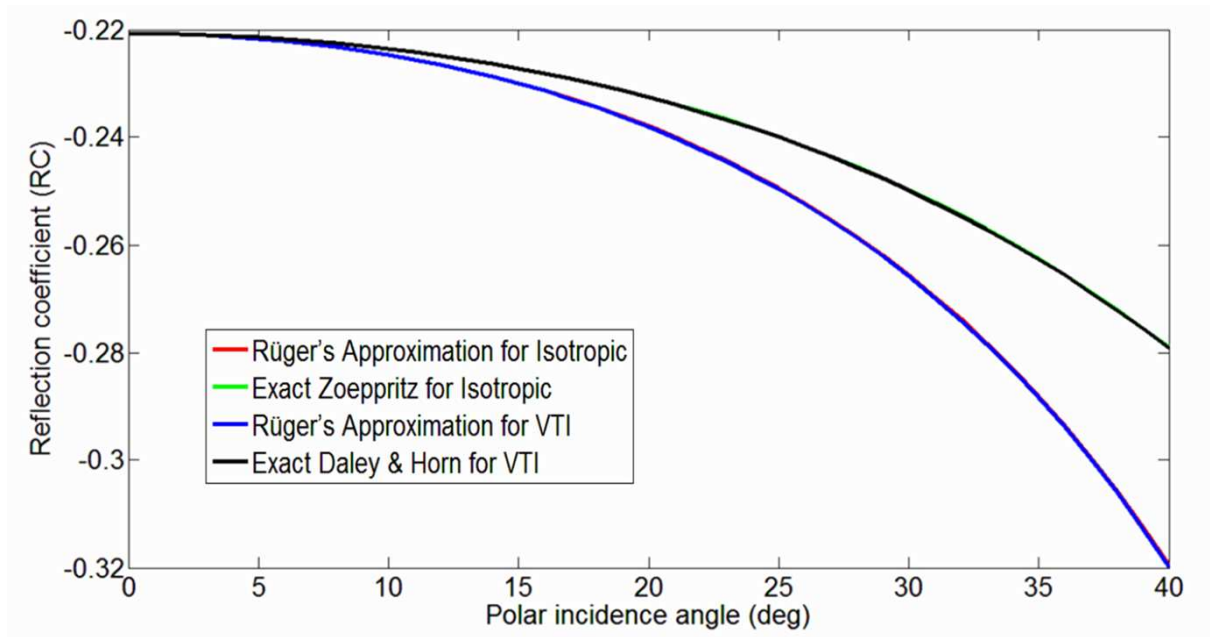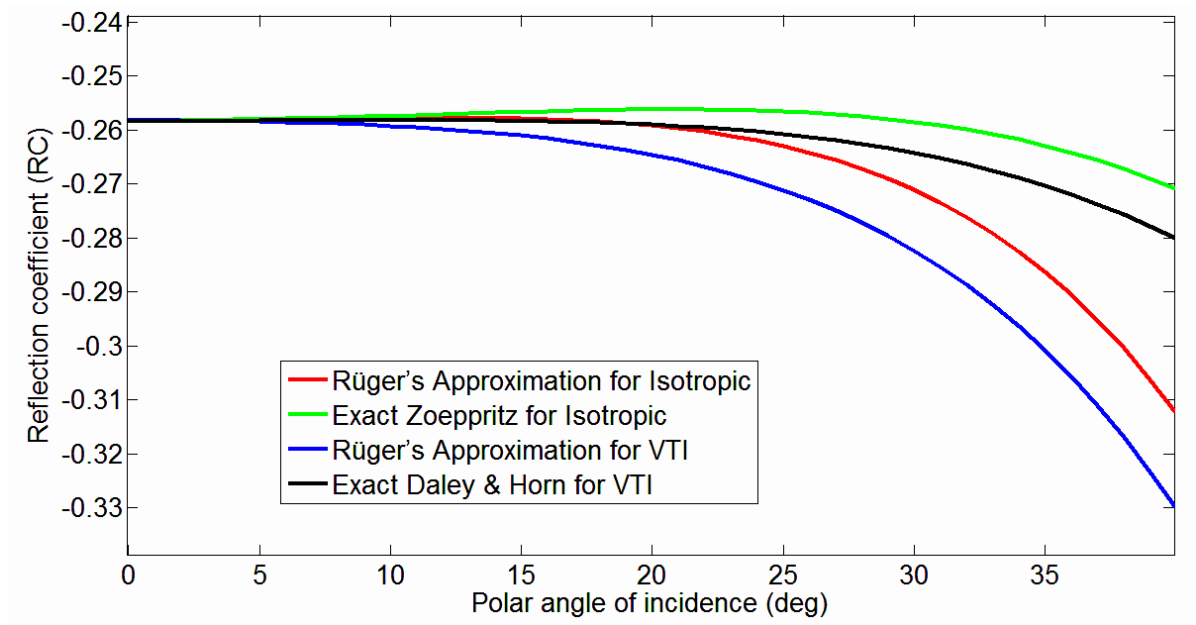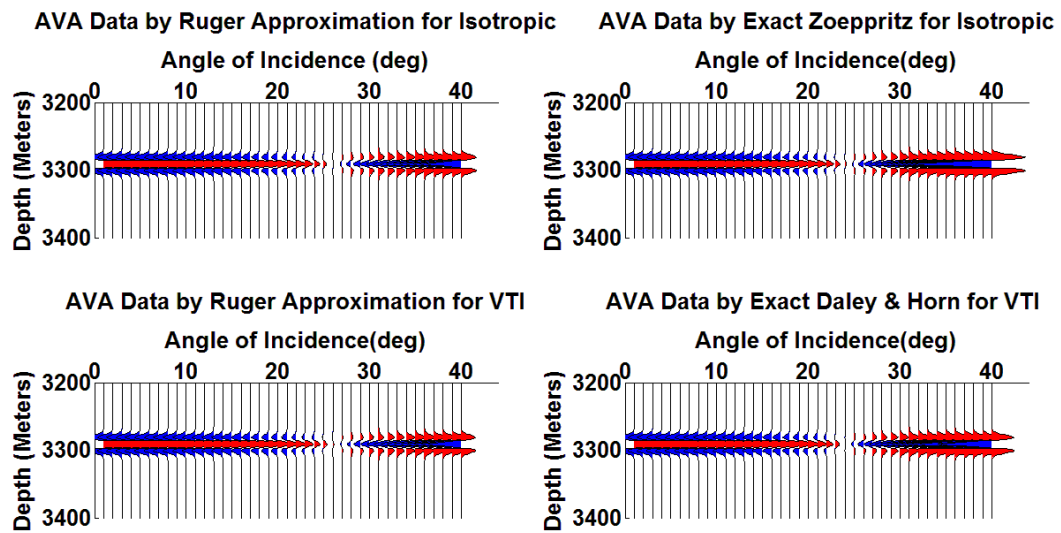
744

745

746

747

748

749   Figure 14: Angle-dependent reflection coefficient data generated through the exact and

750   approximate solutions of PP-wave for isotropic and anisotropic (VTI) media at the top of C-

751   sand reservoir (Sawan-06 well).

752

753

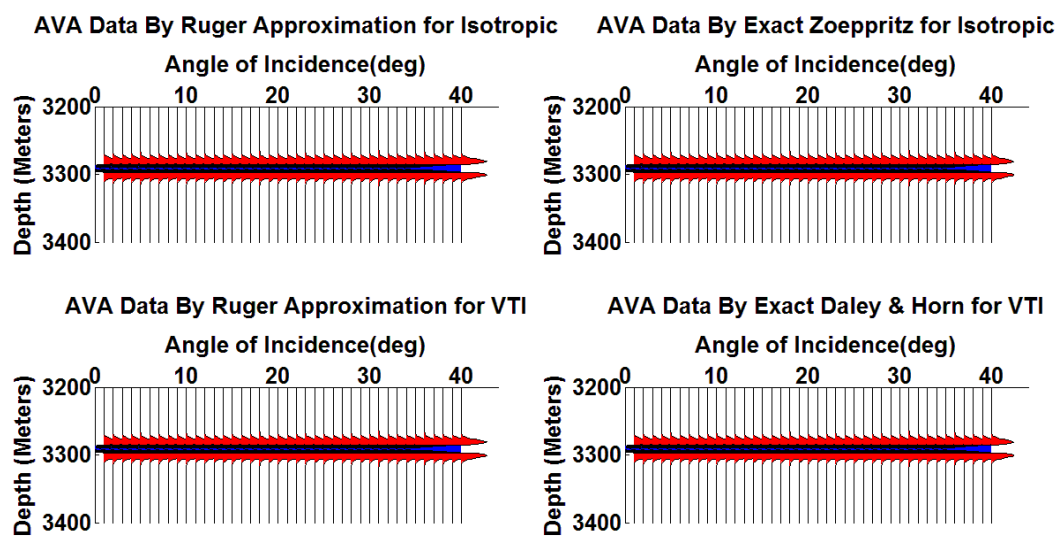754   Figure 15: Seismic AVA response for the Lower Goru C-sand, Sawan-01 well.

755

756

757 Figure 16: Seismic AVA response for the Lower Goru C-sand, Sawan-3B well.
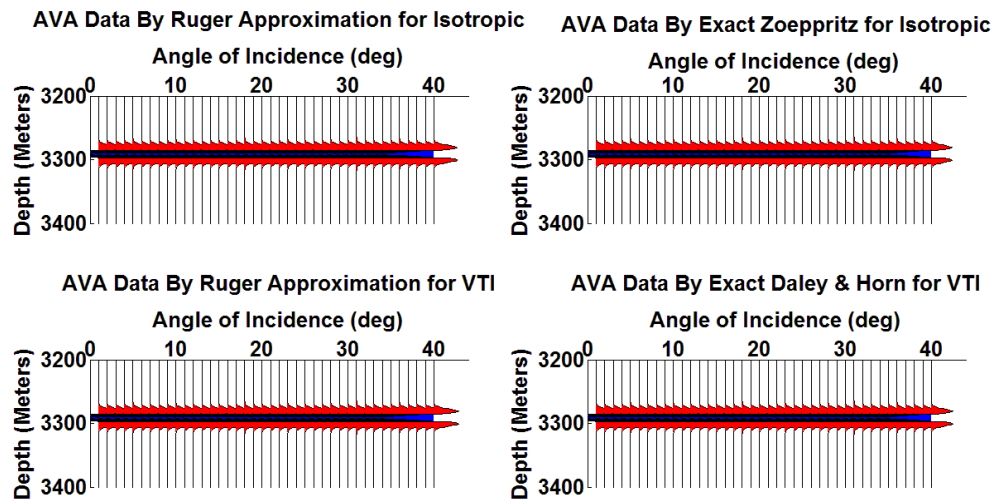
758

759

760   Figure 17: Seismic AVA response for the Lower Goru C-sand, Sawan-06 well.
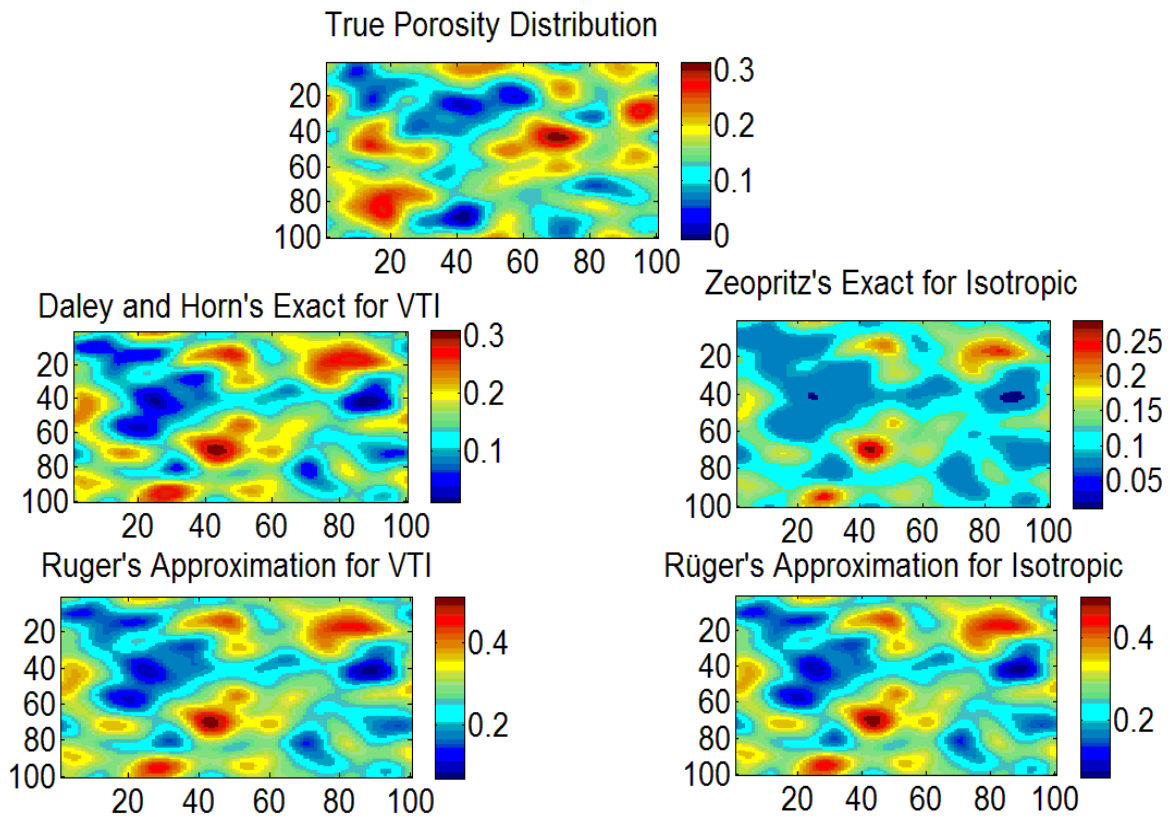
761

Figure 18: Maps highlighting the inversion with 20% uncertainty for true porosity distribution (top) generated via a correlated Gaussian field distribution throughout the reservoir interval at 100×100 grid blocks. These porosity plots were compiled via maximum a posteriori solution under Bayesian settings utilising the rock physical properties of C-sand reservoir interval (3268-3432 m) with exact and approximate solutions of PP-reflection coefficients for the isotropic and anisotropic (VTI) cases. Inversion results show that Daley & Horn's (1977) exact solution for VTI (left in 2nd row) returns porosity trends that are accurately aligned with the true porosity distribution of the reservoir.

Reservoir Properties	Sawan-01	Sawan-3B	Sawan-06
Depth Range (m)	3268-3432	3398-3592	3227-3312
Gross Thickness (m)	164	184	85
Net Reservoir Thickness (m)	80	126	36
Net Pay Thickness (m)	24	58	20
Avg Porosity (%)	15	14	12
Avg Volume of Shale (%)	28	22	34
Average Water Saturation (%)	46	40	45

772   **Table 1**: Reservoir properties of the Sawan wells used in this study.

773

Cut-off	GR<=75 API	0.1<=NPHI<=0.45	Sw<=50%
Rock Thickness (m)	Yes	No	No
Net Reservoir Thickness (m)	Yes	Yes	No
Net Pay Thickness (m)	Yes	Yes	Yes

774   **Table 2**: Details of cut-off values used in the calculation of reservoir properties in Table 1.

775

Well	Formation	$V_p(\alpha)$	$V_s(\beta)$	Density(ρ)	Porosity(ϕ)
Sawan-01	Sand	4165	4112	2.32	0.15
	Shale	2668	2170	2.67	0.08
	Overburden	2874.5	2367.5	2.54	
Sawan-3B	Sand	4251	2562	2.44	0.13
	Shale	4487	2628	2.60	0.10
	Overburden	4258	2547	2.62	
Sawan-06	Sand	4148.628	2579.0	2.51	0.13%
	Shale	4462.847	1741.0	2.61	0.085%
	Overburden	4322.960	2520.0	2.57	

776 **Table 3**: Mechanical properties of reservoir sand, intra-reservoir shale layers and overburden

777 used in the calculation of reflection coefficients.

778

Material	Shear modulus (GPa)	Bulk modulus (GPa)	Density (g/cm^3)
Solid Mineral (Quartz)	44	37	2.65
Gas	0.0	0.025	0.065
Fluid (water/brine)	0.0	2.2	1.035

Table 4: Values of elastic moduli and densities for solid mineral and fluid used in our rock physics modelling.

779

Appendix-A

Backus Averaging Approach

The effective stiffness is anisotropic for a stratified medium composed of transversely

isotropic layers in the limit of long-wavelength, and represented by the Backus (1962) matrix

below (Mavko et al., 2009):

$$
\begin{bmatrix}
A & B & F & 0 & 0 & 0 \\
B & A & F & 0 & 0 & 0 \\
F & F & C & 0 & 0 & 0 \\
0 & 0 & 0 & D & 0 & 0 \\
0 & 0 & 0 & 0 & D & 0 \\
0 & 0 & 0 & 0 & 0 & M
\end{bmatrix}, \qquad M = \frac{1}{2}(A - B), \tag{A-I}
$$

where A, B, C, D and F are the five independent elastic constants

[i.e., $C_{11}, C_{13}, C_{33}, C_{55} \& C_{66}$]. In terms of P- and S-wave velocities (V_P and V_S) and densities

(ρ), the five independent elastic constants can be written as:

$$
A = \left\langle 4\rho V_S^{\,2}\left[1 - \frac{V_S^{\,2}}{V_P^{\,2}}\right]\right\rangle + \left\langle 1 - 2\frac{V_S^{\,2}}{V_P^{\,2}}\right\rangle^2 \left\langle \left(\rho V_P^{\,2}\right)^{-1}\right\rangle^{-1}, \tag{A-II}
$$

$$
B = \left\langle 2\rho V_S^{\,2}\left[1 - \frac{2V_S^{\,2}}{V_P^{\,2}}\right]\right\rangle + \left\langle 1 - 2\frac{V_S^{\,2}}{V_P^{\,2}}\right\rangle^2 \left\langle \left(\rho V_P^{\,2}\right)^{-1}\right\rangle^{-1}, \tag{A-III}
$$

$$
C = \left\langle \left(\rho V_P^{\,2}\right)^{-1}\right\rangle^{-1}, \tag{A-IV}
$$

$$
D = \left\langle \left(\rho V_S^{\,2}\right)^{-1}\right\rangle^{-1}, \tag{A-V}
$$

$$
F = \left\langle 1 - 2\frac{V_S^{\,2}}{V_P^{\,2}}\right\rangle^2 \left\langle \left(\rho V_P^{\,2}\right)^{-1}\right\rangle^{-1}, \tag{A-VI}
$$

794 $\quad M = \left\langle \rho {V_S}^2 \right\rangle.$ (A-VII)

795 The brackets $\langle \cdot \rangle$ indicate averages of the enclosed properties weighted by their volumetric

796 proportions. Once the five independent constants are obtained, the Thomsen anisotropy

797 parameters for VTI can be obtained using the relationships given by Thomsen (1986, 1995):

798 $\quad \gamma = \frac{M-D}{2D},$ (A-VII)

799 $\quad \varepsilon = \frac{A-C}{2C},$ (A-VIII)

800 $\quad \delta = \frac{(F+D)^2 - (C-D)^2}{2C(C-D)}.$ (A-IX)

801 In order to calculate the effect of fluid saturation on the effective properties of a sand-shale

802 layered medium, we have used the Gassmann (1951) equation for isotropic media, and the

803 relationships of Brown and Korringa (1975) for anisotropic (VTI) media (Ali et al., 2011;

804 Shahraini et al., 2011).

805

Appendix-B

807   **1.   Gassmann (1951) fluid substitution model for isotropic media**

808   As the dry composite sand-shale medium was assumed as isotropic in this work, we used the

809   low frequency Gassmann (1951) model (Equation B-I) applicable to well-connected porous

810   media under isobaric conditions. We used this latter model in order to incorporate fluid

811   effects into the effective mechanical properties calculated in our own models. Equation (B-I)

812   defines a relationship between saturated bulk modulus, bulk modulus of the skeleton of the

813   rock, bulk modulus of mineral comprising rock matrix, fluid bulk modulus and porosity. A

814   typical form of the Gassmann (1951) equation is as follows:

815

816
$$K_{sat} = K_{dry} + \left\{ \frac{\left(1 - \left(K_{dry}/K_{grian}\right)\right)}{\left(\frac{\phi}{K_{fluid}} + \frac{1-\phi}{K_{grian}} - \frac{K_{dry}}{K_{grian}^2}\right)} \right\}. \qquad \text{(B-I)}$$

817   Here 'K_{sat}' is the saturated rock bulk modulus, 'K_{dry}' is the frame or dry bulk modulus,

818   'K_{grian}' is the grain bulk modulus, 'K_{fluid}' corresponds to the fluid bulk moduli, and 'ϕ' is

819   the porosity.

820

821   **2.   Brown and Korringa (1975) relations for fluid effects of an anisotropic medium**

822   **(VTI)**

823

824   In order to calculate the effect of fluid saturation on the effective properties of a sand-shale

825   interval, assuming it as VTI medium, we have used the anisotropic relations of Brown and

826   Korringa (1975), which can be written in the symbolic or matrix notation as (Ali et al., 2011):

827

828
$$S^* = S_d^* + \frac{\left((S_d^*-S_m): (I_2 \otimes I_2): (S_d^*-S_m)\right)}{\varphi^o\left(I_2 \cdot S_m \cdot I_2 - 1/_{k_f}\right) - I_2 \cdot (S_d^*-S_m) \cdot I_2}.$$
(B-II)

829 Here, $\otimes$ denotes the dyadic product, S_m is the compliance tensor of the solid mineral

830 component (properties of mineral quartz were used in the case of sand-shale model), S_d^* is the

831 effective compliance tensor for the dry sand-shale medium, and S^* is the effective compliance

832 tensor for the saturated sand-shale medium. In Equation (B-II), φ^o is the total porosity and I_2

833 is the (symmetric) identity matrix for second-rank tensors.

834 In the case of a composite porous medium that is partially saturated with oil, gas and water,

835 K_f may be regarded as the bulk modulus of an effective fluid given by Wood - also known as

836 the Reuss average (Mavko et al., 2009):

837

838
$$\frac{1}{K_f} = \frac{S_w}{K_w} + \frac{S_o}{K_o} + \frac{S_g}{K_g},$$
(B-III)

839 where

840

841 $S_w + S_o + S_g = 1.$
(B-IV)

842 Here, S_w, S_o and S_g represent the saturation for water, oil and gas, and K_w, K_o and K_g

843 represent the bulk modulus for water, oil and gas.