Optimised Chiral Light-Matter Interactions at Polarisation Singularities for Quantum Photonics

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ABSTRACT
Photonic crystal waveguides support chiral-point polarisation singularities which give rise to local chirality even in the absence of a global chiral symmetry. Placing a quantum dot at such a C-point gives rise to a uni-directional emission dependent on the electron spin – ideal for applications in quantum information as it entangles the spin direction of electrons on the quantum dot (static qubits) to the path in the waveguide (flying qubits). Here we discuss the optimisation of this chiral light-matter interaction using slow-light waveguides, and show designs with 8.6 times enhancement of the local density of optical states at a C-point.
Keywords: photonic crystal waveguide, slow-light, chiral photonics.

1. INTRODUCTION

A polarisation singularity [1] is a position in a vector field where one of the parameters describing the local polarisation ellipse is undefined. For example, at linear lines of polarisation (L-lines) the handedness of the polarisation is undefined, and at circular points (C-points) the orientation of the ellipse is undefined. Thanks to the complicated spatial dependence of the polarisation of the electric field, the modes of photonic crystal waveguides are known to support polarisation singularities [2]. C-points allow photonic crystal waveguides to display local chirality, even though the global chirality is zero. Chirality in the modes of photonic crystal waveguides are a valuable resource for quantum optics [3].

Photonic crystal waveguides are a flexible, integrated and miniaturised technology platform that can be tailored and adapted to many applications such as filtering [4], optical switching [5,6], optical delay [7,8], frequency conversion [9,10,11] and fibre-coupling [12]. Photonic crystal waveguides are attractive for the exploitation of polarisation singularities for quantum photonics because they simultaneously support: (i) the in-plane integration of quantum dots; (ii) C-points, which allows for spin-dependent unidirectional emission of photons from these quantum dots, entangling static spin-qubits to flying path-qubits; and (iii) slow-light, which enhances the density of optical states and thereby the strength of the light-matter interaction.

(i) Quantum dots can be integrated in-plane and coupled to the optical modes of the photonic crystal waveguide. Usually epitaxial quantum dots are grown in random positions in the waveguide, but site-control to better than 50nm has recently been demonstrated [13]. Such site control has the potential to greatly increase yields, especially when the alignment to the optical modes of nanophotonic structures such as photonic crystal waveguide is desired.

(ii) Chirality in waveguides couples the spin direction of electrons to the direction of light [14,15]. This chiral light-matter interaction is at its most useful when the chirality reaches 100% at a C-point, where it has particular consequences for local dipole-like quantum emitters such as quantum dots. The angular momentum imparted by quantum dot transitions causes the emitted photons to also possess their own chirality. Placing the quantum dot at a C-point singularity results in a spin-dependent unidirectional emission, whereby all spin-up electrons emit photons in one direction, and spin-down electrons emit photons in the opposite direction. The propagation direction of the light is what gives rise to the symmetry breaking needed. Such spin-dependent unidirectional emission is an attractive property for quantum information applications as it entangles the electron spin with the photon direction and allows spin-encoded static qubits to be converted to path-encoded flying qubits. Many waveguides display highly chiral positions in their modes, but photonic crystal waveguides are perhaps unique in that they possess many C-points – singular positions of 100% chirality. Thanks to the photonic band-gap guiding, photonic crystal waveguide modes are highly confined. They possess a large longitudinal component of the electric field, as well as the transverse component. It is the complicated spatial profile of these two components and the phase difference between them that allows for such a rich and unique polarisation landscape.

(iii) By altering the design details of a photonic crystal waveguides, the dispersion relation can be flexibly tailored to new applications, such as the supporting of slow-light modes [16,17]. Slow-light is an interference effect that occurs in periodic optical media. When $\lambda/2$ matches the periodicity of the photonic crystal (at the bandedge), the forward and backward travelling components of the wave are equal, and interfere to give a standing wave. Just away from the bandedge, these same components are almost equal, and interfere to give an envelope that moves slowly forward. Slow-light modes in photonic crystal waveguides feature an enhanced density of optical states and very tight mode confinement, leading to greatly enhanced light-matter coupling. It is the local density of states at the position of an emitter that determines the efficiency and brightness of the light-matter interaction, and near-deterministic coupling of $\beta > 98\%$ between a slow-light mode of a photonic crystal waveguide and an embedded quantum dot has been demonstrated [18].

C-points in photonic crystal waveguides have been demonstrated both theoretically and experimentally, but there is a trade-off when using them for coupling a quantum dot to their optical mode. C-points tend to occur at positions and frequencies where the local density of optical states (LDOS) is relatively low, limiting the efficiency of the chiral coupling. As it is the LDOS that determines the strength and efficiency of the coupling between a quantum dot and the optical mode, we have investigated ways to optimise this density at a C-point. One obvious contender is to use the slow-light regime. There, as the bandedge is approached and the group velocity $v_g \rightarrow 0$, the density of optical states diverges in what is known as the van Hove singularity. Theoretically there is no limit to the enhancement of the density of optical states, but the disorder found in real-world fabricated waveguides limits practical designs to $v_g < c/100$.

2. W1 PHOTONIC CRYSTAL WAVEGUIDE

Our search for photonic crystal waveguide designs with enhanced LDOS at the position of a C-point begins with the archetypical W1 waveguide [19] of one row of missing holes from an hexagonal lattice of holes with radius $r = 0.3a$ in a GaAs dielectric membrane (see Fig. 1 (a)). We have calculated the eigenmodes of the photonic crystal waveguides using MPB, an open source mode-solver. Fig. 1 (b-d) shows one such mode, broken down into the longitudinal component and transverse component of the electric field, as well as the phase difference between them. From these calculations the polarisation properties of the waveguide mode can be neatly summarised by the Stokes parameters [20]. The important point here is the third Stokes parameter S_3, (Fig. 1 (e)) which indicates the degree of local chirality u/v. Positions with $S_3 = \pm 1$ are the singular C-points, marked with white crosses. Fig. 1 (f) shows the orientation of the local polarisation ellipse, with the contours marking lines of equal orientation. Notice that at C-points, this orientation is undefined and can take any value – in Fig. 1 (f) this can be recognised as the positions where the contours cross.

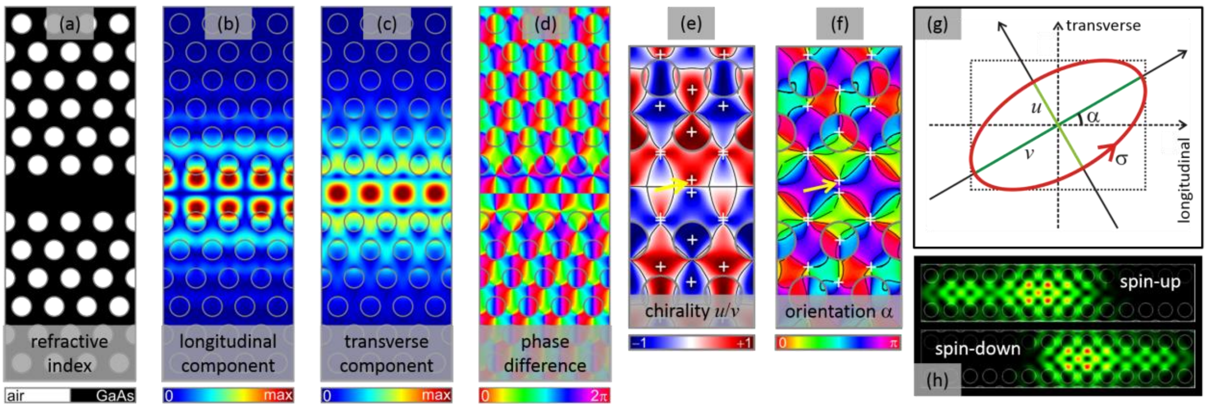

Figure 1. Finding polarisation singularities in the modes of a W1 photonic crystal waveguide. (a-d) Photonic crystal waveguide eigenmodes. (a) Refractive index profile; (b) the longitudinal component of the electric field; (c) the transverse component; and (d) the phase difference between the two. (e) The chirality and (f) orientation of the local polarisation ellipse. White crosses: C-points in the waveguide. (g) Schematic showing the polarisation ellipse and definitions of its parameters. (h) FDTD simulation of emission from a quantum dot at a C-point into a photonic crystal waveguide. Yellow arrow in (e-f): position of quantum dot in simulations.

To confirm the spin-dependent unidirectional emission, we have conducted FDTD simulations of a quantum dot placed at the position of a C-point (indicated by the yellow arrow in Fig. 1 (e-f)) in the photonic crystal waveguide [14]. We find a correlation of greater than 99% between the electron spin direction and the emission direction of a photon into the waveguide, with the small deviation from 100% due to the finite gridding of the FDTD simulation. A snap-shot of the simulation result is shown in Fig. 1 (h).

3. TIME REVERSAL SYMMETRY CONSTRAINS UNIDIRECTIONAL PHOTON EMISSION

Maxwell's equations are reciprocal in time-invariant, linear media, which imposes the condition $\mathbf{E}_k(\mathbf{r}) = \mathbf{E}_{-k}^*(\mathbf{r})$ for two counter-propagating modes with wavevectors k and $-k$. Both these mode exist simultaneously, one as the time-reverse of the other. If we could watch the evolution of the electric field backwards, then it would be indistinguishable from the counter-propagating mode. For the polarisation, the Stokes parameters of two counter-propagating modes have the relations $S_1 \rightarrow S_1$, $S_2 \rightarrow S_2$, $S_3 \rightarrow -S_3$, when $k \rightarrow -k$. In other words, left-handed chirality becomes right-handed in the counter-propagating mode (and vice versa), whereas transverse/longitudinal components remain transverse/longitudinal. At the bandedge, there exists a standing wave with group velocity $v_g = 0$, and so there is no difference between the forwards and backwards travelling modes (both are the same standing wave). Therefore, no chiral components can exist at the bandedge, as the standing wave is its own time-reverse. Time-reversal symmetry requires that the counter-propagating mode be equal to the forward one, $\mathbf{E}_k(\mathbf{r}) = \mathbf{E}_{-k}{}^*(\mathbf{r}) = \mathbf{E}_{-k}(\mathbf{r})$, and so $S_3 = -S_3 = 0$. All chiral components of the polarisation vanish. Just away from the bandedge, in the slow-light regime, we therefore have a situation where the value of S_3 approaches zero everywhere.

Fig. 2 shows the chirality of the waveguide mode as the bandedge is approached and the group velocity drops. As anticipated, there is no chirality found in the standing wave mode with $v_g = 0$ at the bandedge, but how we approach this point is remarkable. As one may expect, there is a general "washing-out" of the chirality as the group velocity is reduced. However, C-points are still found deep into the slow-light regime [21].

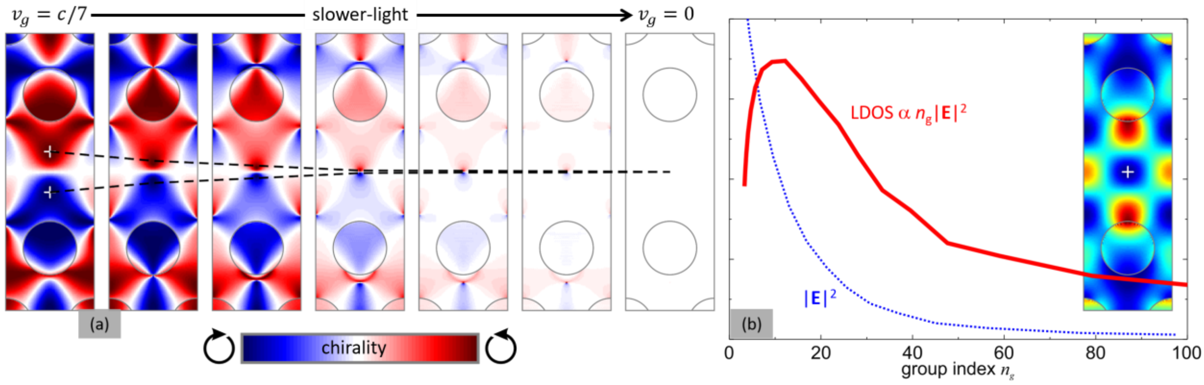

Figure 2. (a) Chirality in the waveguide mode near the central C-points as a function of the group velocity. The left-most panel has the positions of the C-points marked with white crosses, and the dashed lines track the C-points as they approach and annihilate. (b) LDOS and $|E|^2$ as a function of $n_g = c/v_g$. Inset: The electric field strength in the mode with $v_g = 0$. White cross: position of the C-points at the node.

C-points in photonic crystal waveguides are always found in pairs of left- and right-handed points – this is required as there is no global chiral symmetry in the waveguides. The chirality disappears at the bandedge when these pairs approach each other, colliding and annihilating at the bandedge. We find that C-points still exist deep into the slow-light regime, but the separation between them reduces (our calculation grid means that we cannot resolve the two C-points for $v_g < c/300$). The dashed lines in Fig. 2 mark the approach of these two oppositely-handed C-points.

At first glance, this result appears encouraging – although C-points disappear at $v_g = 0$ due to time-reversal symmetry, they can none-the-less be found at arbitrary small group velocities. It appears we can use the slow-light regime to enhance the coupling between the optical mode and the quantum dot placed at a C-point. However, this is not practical for two reasons. Firstly, as the oppositely handed C-points approach each other, it becomes difficult to place a quantum dot over just one of them. Secondly, and perhaps more fundamentally, as the two C-points collide, there is a point that is simultaneously a right-handed and a left-handed point. This configuration can only be satisfied at a node where the electric field strength $|\mathbf{E}|^2 = 0$. Therefore, although the density of states rises due the reduced group velocity in the slow-light regime, this is counteracted by the fall in the local density of states at the C-point as the electric field strength $|\mathbf{E}|^2$ drops. We find that this drop is faster than the corresponding rise, and so there exists a finite group velocity for which the light-matter coupling enhancement is optimal. For our W1 waveguide, this occurs for modes with $v_g = c/10$ [21].

4. SEARCHING FOR OPTIMISED DESIGNS

We form new designs by displacing the holes in a W1 waveguide. Each hole in the first row of holes closest to the waveguide is displaced a distance D_1 towards the waveguide core, and each hole in the second row a distance D_2 (see Fig. 3). Fig. 3 presents the main results, showing the enhancement of the LDOS at C-points where they occur in a single-mode waveguide. Most designs show no enhancement no enhancement over the standard W1 (i.e. the ratio of LDOS at the C-points is below one). There is, however, a small region of the design space where we find significant enhancements of the LDOS near $(D_1, D_2) = (-0.11, 0.15)a$. In this design there exists a C-point at frequency $0.2791c/a$ and position $(x, y) = (0.50, 1.17)a$ from the origin that we calculate has an LDOS 8.6 times higher than any C-point in a standard W1 waveguide [22].

5. CONCLUSIONS

We have found designs for photonic crystal waveguide that display an 8.6 times enhancement of the LDOS at a C-point over their standard W1 counterparts. These designs will allow the efficient coupling between the optical mode of the waveguide and a quantum dot positioned at a C-point.

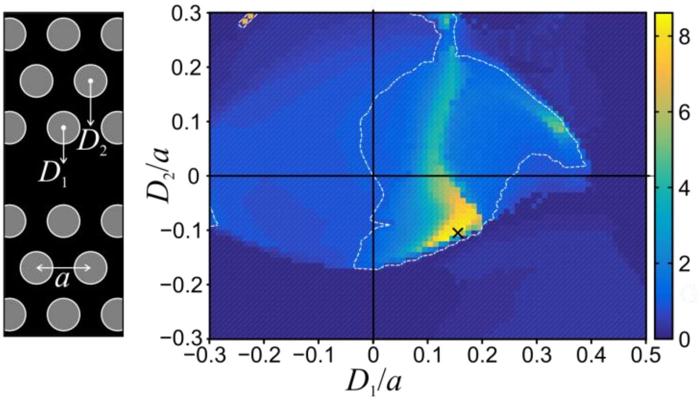

Figure 3 Left: New designs of photonic crystal waveguide are formed from the W1 waveguide by shifting the first row of hole by a distance D_1 and the second row D_2. Right: The calculated LDOS enhancement at a C-point for waveguides as a function of (D_1, D_2). The white contour: designs equal to a W1; black cross: optimised design.

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