1   **Title:** A Decoupled Advection-Dispersion Method for Determining Wall Thickness of

2   Slurry Trench Cut-off Walls

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24

25   **Abstract:** Low-permeability slurry trench cut-off walls are commonly constructed as

26   barriers for containment of subsurface point source pollution or as part of seepage control

27   systems on contaminated sites. A method to estimate wall thickness in slurry wall design

28   is proposed based on decoupling the advective and dispersive components of contaminant

29   fluxes through the wall. The relative error of the result obtained by the proposed method

30   to that by an analytical solution increases as the ratio of the specified breakthrough exit

31   concentration (c^*) to the source concentration (c_0) increases. For c^*/c_0 of less than 0.1,

32   which covers common practical situations, the relative error is not greater than 4% and is

33   always conservative indicating that the proposed method provides sufficient accuracy for

34   design. For a given breakthrough criterion (that is, c^*/c_0), the relative error is low for the

35   scenarios having either a low or high column Peclet number, where either dispersion and

36   advection dominate the contaminant migration, respectively; and the relative error is high

37   for the scenario having an intermediate column Peclet number, in which case the

38   coupling effect of advective and dispersive migrations is relatively high.

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Introduction

Slurry trench cut-off walls (termed as slurry walls hereafter) utilizing low-permeability backfill materials are commonly constructed as barriers for containment of subsurface point source pollution or as part of seepage control systems on contaminated sites. In slurry wall design, one key requirement is that the breakthrough time t_b should be not less than the designed service life. Transient contaminant transport through slurry walls can be regarded as a one-dimensional advective-dispersive process (see Fig. 1). Analytical solutions (Lapidus and Amundson 1952; Ogata and Banks 1961; Brenner 1962; Lindstrom et al. 1967) are available for varied boundary conditions to calculate the contaminant transport in slurry walls. However, these analytical solutions contain non-elementary functions (such as complementary error function) or require the solution of eigen equations. The evaluation of these analytical solutions is nontrivial and generally requires the use of a computer (Rowe et al. 2004). Accordingly the wall thickness corresponding to a designed service life has to be searched in a trial and error manner. This often leads to determination of the wall thickness by practical experience instead of contaminant transport analysis in design. An alternative simplification of the analytical solution of Ogata and Banks (1961) was presented by Cavalcante and de Farias (2013) however a numerical computation was required to iteratively obtain a solution.

A new method for determining the wall thickness of slurry walls is proposed in this paper. Representation of contaminant migration through the slurry wall is simplified by superposition of decoupled solutions for advective and dispersive fluxes. The error of the

66   proposed method is investigated by comparing the results with those obtained by an

67   analytical solution commonly used for slurry wall design. Finally, an example is

68   presented to illustrate the procedure of implementing the proposed method for slurry wall

69   design.

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71

72   **Method**

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74   As illustrated in Fig. 1, the slurry wall keys into the impervious soil layer. The backfill is

75   assumed to be homogenous, fully saturated and non-deformable. The pore water flow in

76   the backfill is assumed to be in a steady state condition. Contaminant migration in the

77   slurry wall can be regarded as a one-dimensional advective-dispersive process (Freeze

78   and Cherry 1979), that is,

79    $$R\frac{\partial c}{\partial t} = D_h \frac{\partial^2 c}{\partial x^2} - v_s \frac{\partial c}{\partial x}$$ (1)

80   where c is the volume-average concentration of contaminant in the pore water of backfill;

81   t is time; R is the retardation factor of contaminant for the backfill; and D_h is the

82   hydrodynamic dispersion coefficient of contaminant in the backfill. v_s is the seepage

83   velocity of the pore water flow and can be written as follows,

84    $$v_s = \frac{v_d}{n} = \frac{kh}{nL}$$ (2)

85   where v_d is the discharge velocity of pore water flow given by Darcy's law (see Eq. (2));

86   n and k are the porosity and hydraulic conductivity of the backfill, respectively; L is the

87   thickness of the slurry wall; and h is the hydraulic head difference between the entrance

88   and exit boundaries of the slurry wall and is assumed to be independent of L.

89

90   The backfill is assumed to be initially free of contaminant. A constant concentration (that

91   is, $c=c_0$, where c_0 is the source concentration of contaminant at the upstream) at the

92   entrance boundary, as typically recommended for vertical barrier design and performance

93   assessment (Rabideau and Khandelwal 1998), is assumed in this paper.   The

94   breakthrough time is commonly defined to be the time when the exit concentration

95   reaches a specified value (c^*) which is often based on groundwater quality or other

96   standards.

97

98   As shown in Eq. (1), advective and dispersive migrations of the contaminant are coupled,

99   which leads to relatively complex analytical solutions.   In this paper, advection and

100   dispersion are assumed to be decoupled to allow development of a simplified method for

101   performing a suitable design of the wall thickness. The error caused by this assumption

102   is investigated in the next section.   The concentration of contaminant for the pure

103   advection segment is equal to the source concentration (that is, $c=c_0$) due to the effect of

104   dispersion being ignored. At the breakthrough time the distance between the advection

105   front and the entrance boundary due to pure advection, x_a, with consideration of

106   adsorption is

107   $$x_a = v_s \frac{t_b}{R}$$ (3)

108   The pure advection segment provides a moving constant concentration boundary, whose

109   velocity is equal to v_s/R, making the subsequent segment one of pure dispersion in a

110   semi-infinite medium, as illustrated in Fig. 2. Concentration continuity is assumed at the

111   interface of the two segments, in other words, the concentration at the inlet boundary of

112   the pure dispersion segment is equal to that of the advection front (which is c_0). The

113   analytical solution presented by Carslaw and Jaeger (1959) gives the following equation

114   for the pure dispersion segment at the breakthrough time,

115
$$\text{erfc}\left(\frac{x_d}{2\sqrt{D_h t_b / R}}\right) = \frac{c^*}{c_0} \tag{4}$$

116   where x_d is the dispersion distance between the advection front and the exit boundary, as

117   illustrated in Fig. 2. The ratio of the specified breakthrough exit concentration to the

118   source concentration, that is, c^*/c_0, represents the breakthrough criterion.

119

120   The complementary error function in Eq. (4) is a non-elementary function and a variable

121   m can be defined as the solution of the following equivalent equation,

122
$$\text{erfc}(m) = \frac{c^*}{c_0} \tag{5}$$

123   For c^*/c_0 in the range of 0.001 to 0.1, which covers the problems commonly considered,

124   the following approximating formula is proposed for the relationship between m and c^*/c_0

125   by fitting with least-square method

126
$$m = 3.56 - 3.33\left(c^*/c_0\right)^{0.142} \tag{6}$$

127   The relative error of Eq. (6) to Eq. (5) is less than 0.7% for the range of c^*/c_0 of 0.001 to

128   0.1. x_d can then be written as follows by substitution of Eq. (5) into Eq. (4),

129
$$x_d = 2m\sqrt{D_h \frac{t_b}{R}} \tag{7}$$

130   At the breakthrough time the wall thickness L is equal to the sum of x_a and x_d, and so can

131   be expressed as

132
$$L = v_s \frac{t_b}{R} + 2m\sqrt{D_h \frac{t_b}{R}} \tag{8}$$

133   The wall thickness corresponding to the designed service life of t_b for a breakthrough

134   criterion of c^*/c_0 can be obtained explicitly from Eq. (9) using Eq. (2), that is,

135
$$L = \left(m+\sqrt{m^2+P_L}\right)\sqrt{D_h \frac{t_b}{R}} \tag{9}$$

136   where P_L is the column Peclet number (van Genuchten and Parker 1984; Shackelford

137   1994; Shackelford 1995; Rabideau and Khandelwal 1998; Prince et al. 2000), which

138   represents the relative importance of advective migration to the dispersive migration and

139   is defined by

140
$$P_L = \frac{v_s L}{D_h} = \frac{kh}{nD_h} \tag{10}$$

141   For many cases of slurry walls, the value of hydraulic conductivity of backfills, the range

142   of typical values of P_L is 0.01~1000. If the wall thickness L is given, the breakthrough

143   time for a breakthrough criterion of c^*/c_0 can be estimated as follows based on Eq. (9),

144
$$t_b = \frac{L^2 R}{\left(m+\sqrt{m^2+P_L}\right)^2 D_h} \tag{11}$$

145

146

147   **Error analysis**

148

149   The error associated with the assumption of the advective and dispersive fluxes being

150   decoupled is investigated in this section. The results found by the proposed method are

151   compared to those obtained from the analytical solution commonly used in slurry wall

152   design (Lapidus and Amundson 1952; Ogata and Banks 1961) that gives the following

153   equation at the breakthrough time t_b, when the exit concentration rises to c^* at $x=L$,

154
$$\frac{c^*}{c_0} = \frac{1}{2} \operatorname{erfc}\left(\frac{1}{2}A - \frac{1}{2}\frac{P_L}{A} \right) + \frac{1}{2}\exp(P_L)\operatorname{erfc}\left(\frac{1}{2}A + \frac{1}{2}\frac{P_L}{A} \right)$$
(12)

155   where A is a dimensionless parameter defined by

156
$$A = \sqrt{\frac{R}{D_h t_b}}L$$
(13)

157

158   The relative error, e_r, of the value of A obtained by the proposed method with respect to

159   that by the analytical solution (Eq. 12) is calculated as follows,

160
$$e_r = \frac{A_d - A_c}{A_c} \times 100\%$$
(14)

161   where A_d is the value of A obtained by the proposed method; and A_c is that found solving

162   the analytical solution with a Newton-Raphson method based search. The relationship

163   between e_r and P_L for varied breakthrough criteria (that is, c^*/c_0) is shown in Fig. 3. The

164   value of e_r is always positive, which indicates that the proposed method gives a

165   conservative result in terms of wall thickness for the designed service life (see Eq. (13)).

166

167   The relative error increases as c^*/c_0 increases and is not greater than 4% for $c^*/c_0 \leq 0.1$,

168   which is commonly used as a breakthrough criterion in slurry wall design. For $c^*/c_0 = 0.2$

169   and 0.3, the peak relative errors are 5.9% and 8.1%, respectively. So it is noted that the

170   wall thickness evaluated by the proposed method may be over 8.1 % higher than that

171   required according to contaminant transport analysis if $c^*/c_0 > 0.2$. Fig. 3 also shows that

172   the value of P_L corresponding to the peak value of e_r increases with decreasing c^*/c_0 due

173   to the higher impact of dispersion/diffusion on contaminant breakthrough.

174

175   For any given ratio of c^*/c_0 the relative error is low for relatively low or high values of P_L,

176   as in these scenarios dispersion and advection dominate contaminant migration,

177   respectively. In such cases, the coupling effects between advection and dispersion are

178   relatively low and subsequently the impact of assuming the two processes to be

179   decoupled becomes less significant. The relative error has a peak value for an

180   intermediate value of P_L, where both dispersion and advection are significant with a

181   relatively high degree of coupling occurring between the two migration processes.

182

183

184   **Example**

185

186   The procedure of implementing the proposed method to determine the wall thickness of

187   slurry walls is illustrated in this section. In the example considered, the porosity and

188   hydraulic conductivity of the backfill are taken as 0.4 and 1×10^{-9} m/s, respectively. The

189   contaminant is phenol, and its retardation factor is 30 based on Malusis et al. (2010) and

190   hydrodynamic dispersion coefficient is taken as 5×10^{-10} m^2/s (Rowe, et al. 2004). The

191   hydraulic head difference between the entrance and exit boundaries is assumed to be 0.8

192   m, and the entrance reservoir concentration of phenol is 1.0 mg/L according to the data of

193   typical landfill leachate (Rowe, et al. 2004). A concentration of 0.002 mg/L at the exit

194   boundary is used as the breakthrough criterion as per class III of Chinese Quality

195   Standard of Ground Water (GB/T 14848-1993). The designed service life of the slurry

196   wall is taken as 50 years.

197

198   The wall thickness of the slurry wall can be determined by the following steps using the

199   proposed method:

200   Step 1: Calculate the ratio of the specified breakthrough exit concentration to the source

201   concentration,

202    $$\frac{c^*}{c_0} = \frac{0.002}{1.0} = 0.002 \tag{15}$$

203   Step 2: Approximate the value of m via Eq. (6),

204    $$m = 3.56 - 3.33 \left(\frac{c^*}{c_0} \right)^{0.142} = 3.56 - 3.33 \times 0.002^{0.142} = 2.18 \tag{16}$$

205   Step 3: Calculate the value of P_L using Eq. (10),

206    $$P_L = \frac{kh}{nD_h} = \frac{1 \times 10^{-9} \times 0.8}{0.4 \times 5 \times 10^{-10}} = 4.0 \tag{17}$$

207   Step 4: Calculate the wall thickness L from Eq. (9),

208    $$L = \left(m + \sqrt{m^2 + P_L} \right) \sqrt{D_h \frac{t_b}{R}} = \left(2.18 + \sqrt{2.18^2 + 4.0} \right) \sqrt{5 \times 10^{-10} \frac{50 \times 3.1536 \times 10^7}{30}} = 0.83 \text{ m}$$

209    $$\tag{18}$$

210   The calculated wall thickness of 0.83 m corresponds to a designed service life of 50 years

211   for the specified breakthrough criterion. As a result, L=0.9 m can be used as the designed

212   wall thickness of the slurry wall.

213

214   The concentration profiles in the slurry wall at the calculated breakthrough time are

215   shown in Fig. 4. For the proposed method, the advection front x_a is 0.14 m and the

216   dispersion distance x_d is 0.76 m, which indicates that dispersion/diffusion dominates the

217   contaminant migration for the scenario considered (that is, P_L=4.0). At the breakthrough

218   time the contaminant concentration profile obtained by the proposed method is close to

219   that produced by the analytical solution in which the advective and dispersive migrations

220   are coupled.

221

222   Concentration profiles, at the calculated breakthrough time, for the scenarios with P_L=0.4,

223   40 and 400, are also shown in Fig. 4. It can be observed that for cases having low and

224   high P_L the proposed method does not result in a significant error in the determination of

225   wall thickness or breakthrough time despite the assumption of a decoupled advection-

226   dispersion problem. For P_L=0.4, the concentration profiles obtained by these two

227   methods are close with a relative error of 0.2% in the calculated t_b due to this scenario

228   being dispersion dominated. For P_L=400, advection dominates contaminant migration,

229   and the concentration profiles obtained by the two methods are also close to each other.

230

231   For the scenario with P_L=40 the relative error of the predicted t_b is 2.5%, which is fully

232   acceptable in design, though the difference between the concentration profiles is relative

233   large compared to the other scenarios. The advection front x_a is 0.45 m and the

234   dispersion distance x_d is also 0.45 m (see Fig. 4). This indicates that for this scenario the

235   contaminant migration is controlled by both advection and dispersion.

236

237

238 **Conclusions**

239

240 A simplified method has been proposed to determine the thickness of slurry walls via an

241 assumption of decoupled advection-dispersion in the analysis of contaminant migration.

242 The relative error for the column Peclet number P_L in the range of 0.01 and 1000 is not

243 greater than 4% when the breakthrough criterion of c^*/c_0 is less than 0.1, which covers

244 common practical situations in slurry wall design. For a given breakthrough criterion, the

245 relative error is relatively low for a low or high P_L, when dispersion or advection

246 dominate the contaminant migration, respectively; but for intermediate values of P_L,

247 when the coupling effects between dispersion and advection migrations are more

248 significant the relative errors are higher. Finally, it should be fully recognised that such a

249 decoupling approach may not be extended to other contaminant transport problems

250 without careful calculation and comparison of the results to those obtained by suitable

251 analytical solutions.

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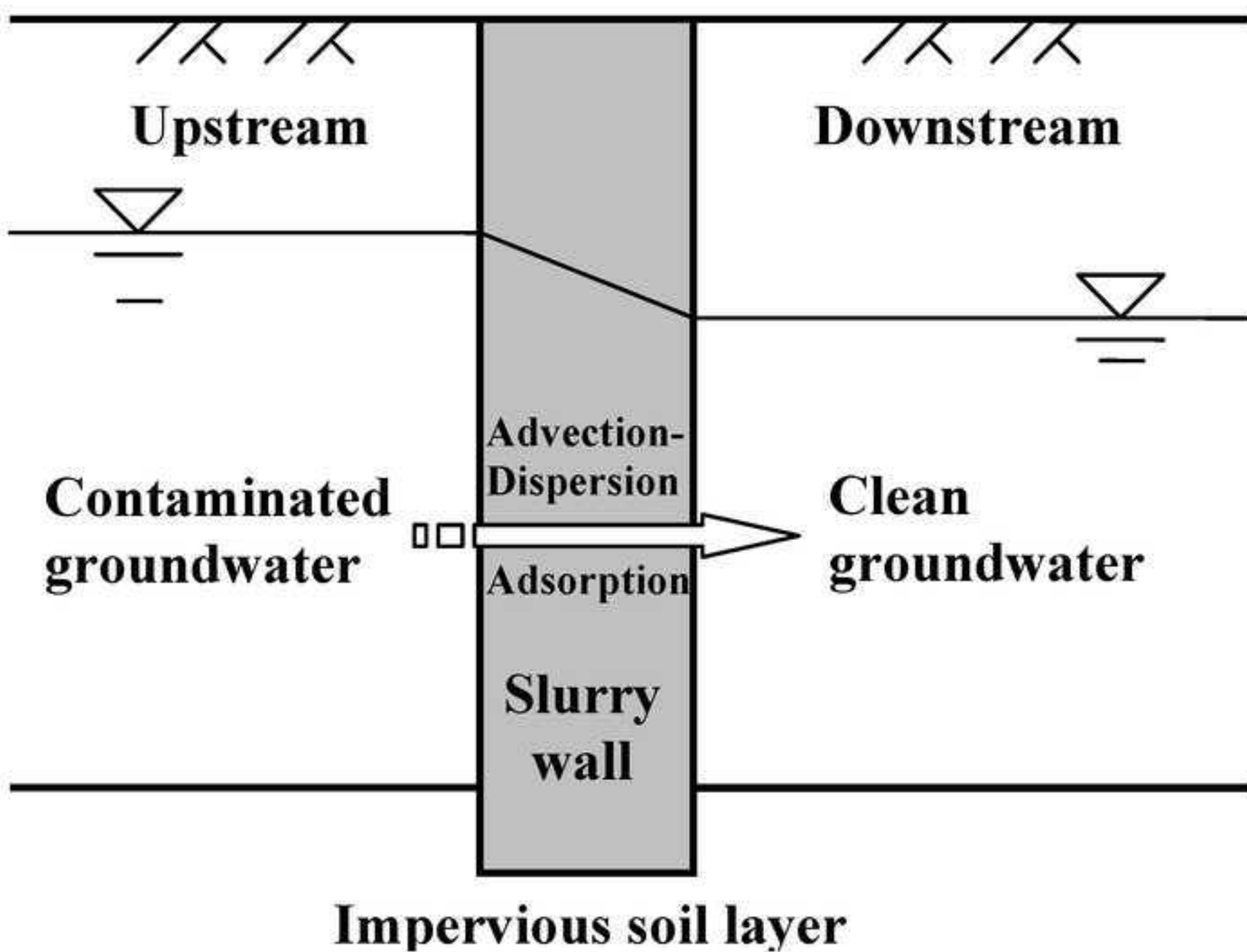
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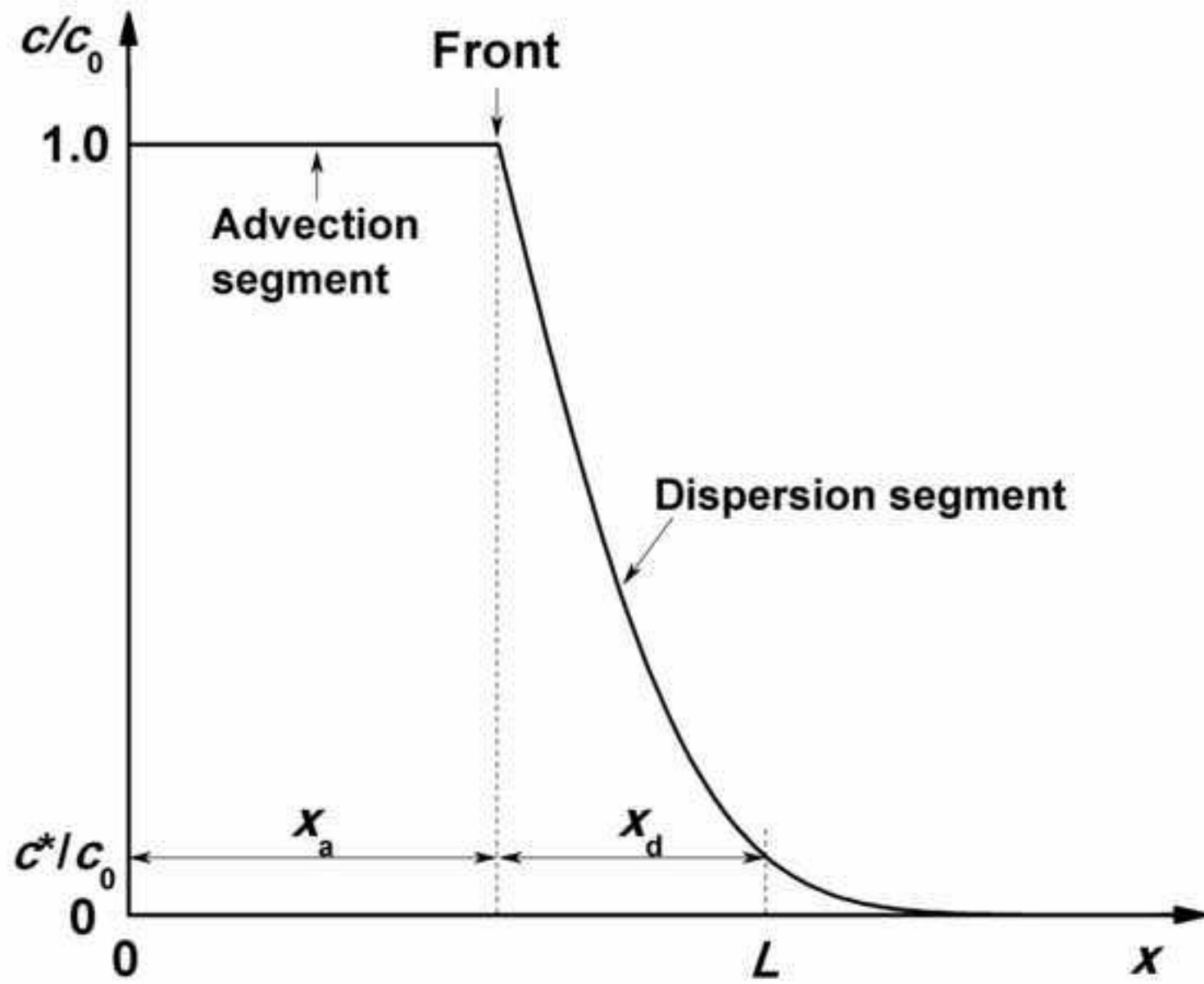291   *Geotechnical Engineering-ASCE*, 120(10), 1804-1828.

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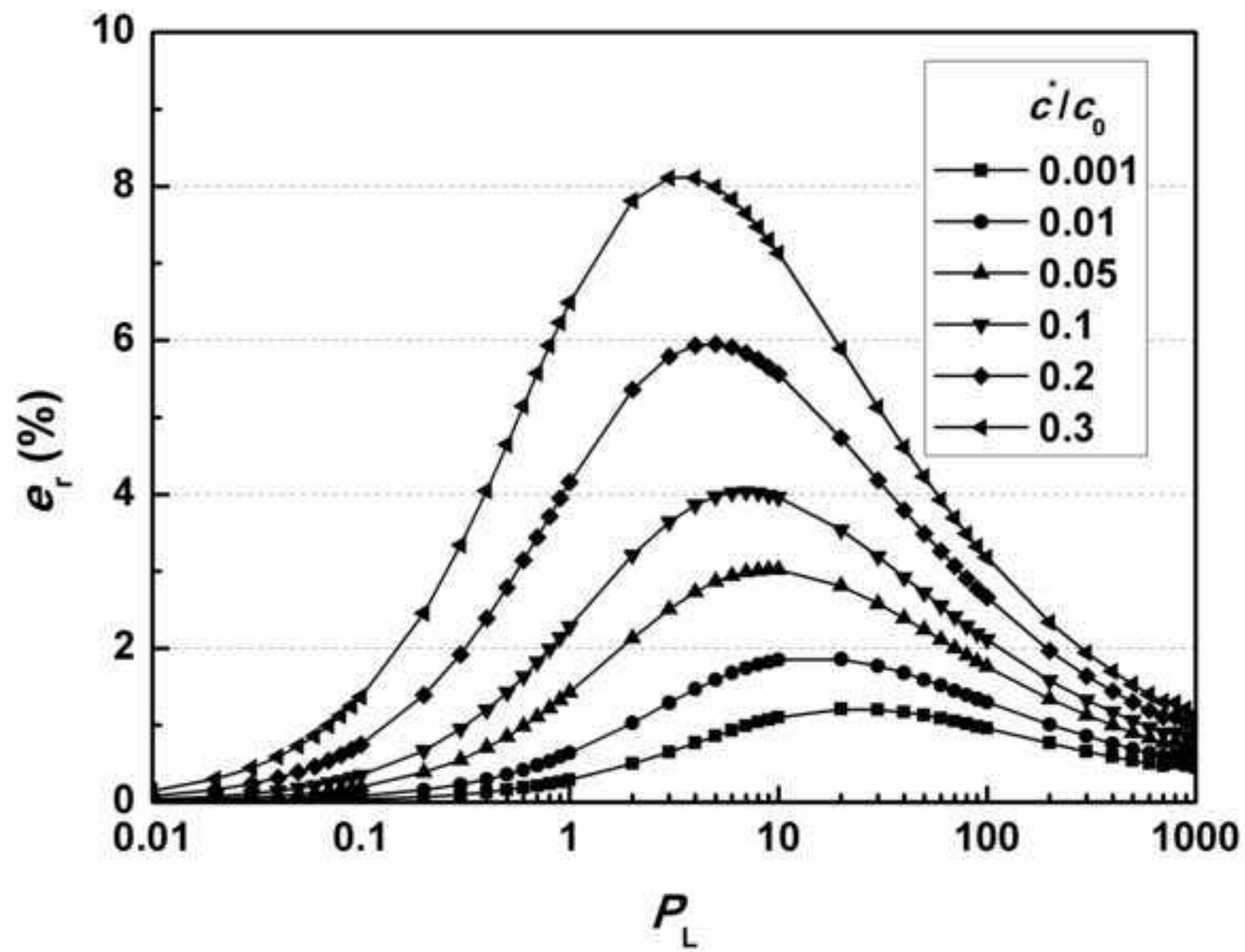294   York, 355-372.

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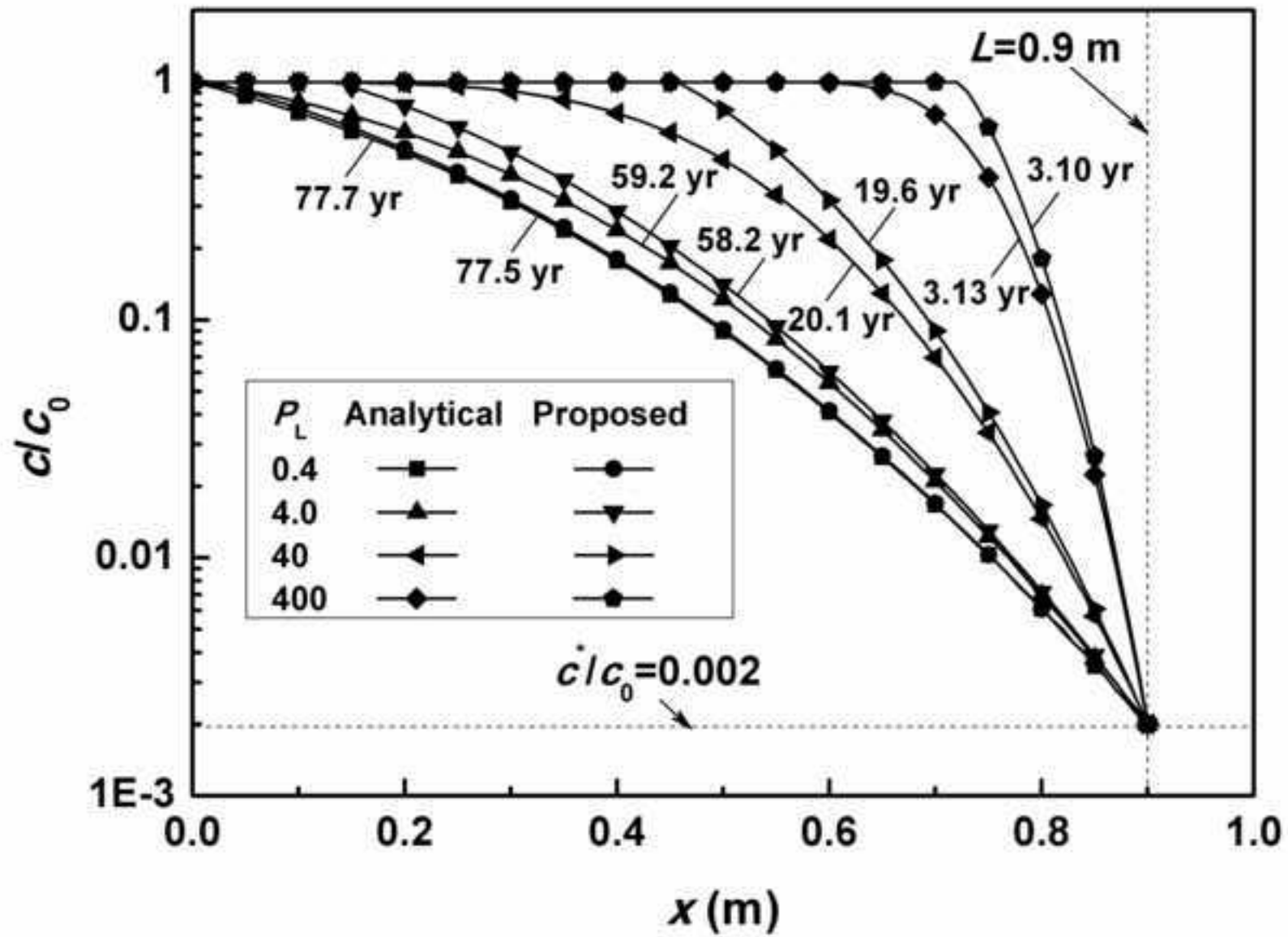
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Upstream

Downstream

Advection-
Dispersion

Contaminated
groundwater

Clean
groundwater

Adsorption

Slurry
wall

Impervious soil layer

Figure 3   Figure 3.tiff

Figure 4   Click here to download Figure Fig. 4.tiff ⤓

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