The Role of Energy Prices in the Great Recession — A Two-Sector Model with Unfiltered Data

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Abstract

We investigate the role of energy shocks during the Great Recession. We study the behaviour of the UK energy and non-energy intensive sectors firms in a real business cycle (RBC) model using unfiltered data. The model is econometrically estimated and tested by indirect inference. Output contraction during the Great Recession was largely caused by energy price and sector-specific productivity shocks, all of which are non-stationary and hence tend to dominate the sample variance decomposition. We also found that the channel by which the energy price shock reduces output in the model is via the terms of trade: these fall permanently when world energy prices increase and as substitutes for energy inputs are strictly limited there are few reactions via production channels. Therefore, there is no other way to balance the deteriorating current account than through lower domestic absorption.

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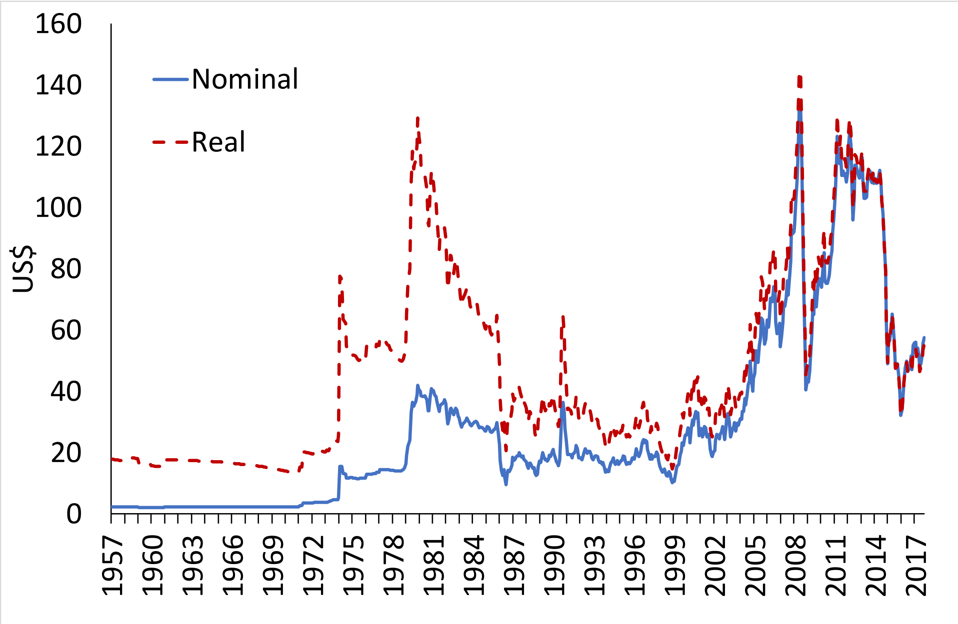
Introduction

Volatile world energy prices have affected economic activity and consumer prices across the world. Empirical evidence shows that energy prices are more volatile than prices of other commodities (Chaudhuri and Daniel, 1998; Hamilton, 2003; Sadorsky, 2012) whilst also being exogenous to a single economy, unlike prices of other goods (Kilian, 2008; Linn, 2008; Mork and Hall, 1980). The Great Recession is widely attributed to financial causes, however it was preceded by a massive spike in energy prices, with oil briefly reaching nearly $150 a barrel in July 2008 as can be seen in Figure 1.

Figure 1: Brent Oil Price (Real and Nominal)



Our aim in this paper is to examine the role of energy prices in the Great Recession. We explore the channels through which energy prices impact on the economy — specifically here the UK economy. One such channel could be the production structure of the economy between energy-intensive sectors and other sectors; it could be that as energy prices rise they cause strong demand substitution to these other sectors where supply is unavailable, so shifting the output gap downwards. Another could be the traditional terms of trade mechanism whereby to limit the current account deficit total demand must be reduced. To anticipate our results, we find that there is little demand substitution across sectors and that the terms of trade mechanism is the dominant channel through which energy prices determine output.

We propose an open economy two sector real business cycle (RBC) model for our purposes here. Previous work including Faccini et al. (2011), Kamber and Millard (2010), Kim and Loungani (1992) and Finn (1995) study the US and find that energy shocks provide little significance in explaining the real macroeconomic aggregate fluctuations in the economy. However, De Miguel et al. (2003) use a small open economy RBC model and find oil price shocks to be highly significant in explaining aggregate fluctuations. Their results show that oil shocks can explain a significant percentage of output fluctuations in many southern European countries. Their model also replicates the cyclical path of the periods of oil crisis in the European economies. The rise in the relative price of oil had a negative impact on welfare, mostly in the southern European countries, which in the historical data is also associated with a lax monetary policy in oil crisis periods. Millard (2011) found that the effects of energy price shocks (oil prices and gas prices) on the variability of output and relative prices is low. Aminu (2017) studied the behaviour of macroeconomic aggregates in the UK using stationary data and found the decline in output during the financial crisis was temporary; therefore, the UK authorities could have borrowed against such a fall.

All these studies used stationary data however, and world energy prices are nonstationary. The Hodrick-Prescott (HP) filter widely used to generate stationarity can create misleading cyclical features in the filtered data — see Christiano and Fitzgerald (2003).

Here we use unfiltered data and we find that both sectoral productivity and energy prices are nonstationary. This nonstationarity is an important contributor to our results.

The model[4]

We assume the UK is a two-sector small open economy that produces energy intensive goods and non-energy intensive goods/services. We also assume the UK is a net importer of energy[5], it imports at a world price, P_O. This model augments the models developed by Kim and Loungani (1992), and Finn (1995). It is similar to these models in the way that the UK's economic activities are carried out. The open economy is similar to Backus et al. (1993). The model maintains the assumption of perfectly competitive firms in the economy as well as real frictions. This is different from the previous authors' assumption in their models of the absence of real frictions since this is now a standard practice in the literature. There is a continuum of households of unit mass on the demand side while on the supply side there is a continuum of firms of unit mass. There are two sectors in the economy: energy intensive, denoted by *e*, and non-energy intensive, denoted by *n*. Households determine their total consumption and investment, with real rigidities that include habit formation in household's consumption and investment adjustment costs as well as capital utilisation. Total home spending, domestic absorption, consists of consumption C, investment I and government spending G; this spending is then allocated across imports and the home energy and non-energy sectors. The household can also choose to hold either domestic bonds or foreign bonds. Home production involves three combinations of inputs: labour, capital and primary energy (assumed to be crude oil). The firms produce the aggregate output of Y where each sector produces Y_e and Y_n of energy intensive and non-energy intensive output respectively. We assume immobility of labour and capital across borders while the accumulation of capital is subject to adjustment costs. The households supply differentiated labour, H, to each sector of the firms at a given

[4] A detailed explanation of the model is available in the appendix.
[5] This assumption follows reality since the production of crude oil in the United Kingdom is in decline (DUKES, 2017)

wage rate W. They also have the option of investing in two kinds of physical capital, K_e and K_n which are subject to adjustment costs.

Household

The household lifetime utility is:

$$\mathrm{E}_t \sum_{t=0}^{\infty} \beta^t \tau_t U((C_t - \psi_h C_{t-1}), \xi_{w,t} H_t) \quad (1)$$

where E_t represents the rational expectation, and $0 < \beta < 1$ denotes the discount factor. C represents aggregate demand by household (nominal consumption) and H denotes the work hours supplied by household with ψ_h representing the degree of habit formation by households. The household lifetime utility differs from Smets and Wouters (2003, 2007) as it includes habit formation as $(\psi_h > 0)$. τ denotes exogenous consumption preference shock and ξ_w denotes exogenous labour supply shock[6].

The Budget constraint closely follows Harrison et al. (2011) and shows how the end of period holdings of nominal government debt (B), nominal foreign bonds (B^f) and nominal capital (K) are given by their start of period holdings, plus net income. The net income includes earnings from labour supply (at wage) and capital services ($Z_t^j K_{t-1}^j$ rented at rate R_t^j), for $j = e, n$, to firms less expenditures on consumption (C) and lump-sum taxes (T). Adjustment costs will be discussed below, as well as the cost of servicing capital. Given that world import prices are the numeraire in the model, the values of the nominal variables are converted to US Dollars and deflated by world manufacturing prices.

$$\frac{B_t}{1+r_t} + \frac{B_t^f}{S_t\left(1+r_t^f\right)} + C_t + T_t + I_t = R_{t-1}B_{t-1} + R_{t-1}^f \frac{B_{t-1}^f}{S_t} + W_t H_t + \Pi_t + R_t^e Z_t^e K_{t-1}^e + R_t^n Z_t^n K_{t-1}^n \quad (2)$$

[6] The shock is assumed to follow a first order autoregressive process with an i.i.d. normal error term: $\varepsilon_t = \rho_\varepsilon \varepsilon_{t-1} + \eta_{\varepsilon,t}$. This is the same for all stationary shocks in the model.

This gives a clear picture of how household have the option to hold either domestic or foreign bonds. r denotes the domestic interest rate and r^f denotes the exogenous world interest rate, given that world prices are exogenous. Π denotes income profits from firm ownership.

Households decide on what capital stocks K_{t-1}^j to choose as new capital must take a one-quarter lag to become effective. The model assumes households have access to technology after the decision on which sector to install capital in the previous quarter.

$$K_t^e = \left(1 - \delta(Z_t^e)\right)K_{t-1}^e + \xi_{INV,t}^e I_t^e + \chi_e \left(\frac{K_t^e}{K_{t-1}^j}\right) \tag{3}$$

$$K_t^n = \left(1 - \delta(Z_t^n)\right)K_{t-1}^n + \xi_{INV,t}^n I_t^n + \chi_n \left(\frac{K_t^n}{K_{t-1}^n}\right) \tag{4}$$

where I^j denotes sector-specific gross nominal investment. $\delta(.)$ denotes sector-specific time varying depreciation rate of capital installed: for $0 \leq \delta(.) \leq 1, \delta'(.) > 0, and\ \delta''(.) > 0$. Z^j denotes the capital utilisation rate of each period's effective capital installation. ξ_{INV}^j denotes the sector-specific exogenous investment-specific technology shock. χ_j denotes adjustment costs which depends on the rate at which each sector adjusts its price, for $\chi_j > 1$. The assumption is consistent with standard RBC literature.

Subject to the budget constraint, the household maximise their expected lifetime utility value with the sequence $\left\{C_t, H_t, I_t, I_t^e, I_t^n, , B_{t+1}^f, Z_t^e, Z_t^n, K_t^e, K_t^n\right\}_{t=0}^{\infty}$. The first-order condition that solves the consumer's problem is derived.

Firms

The sectoral production functions are assumed to be homogeneous-of-degree-one, exhibiting constant returns to scale, like the work of Kim and Loungani (1992) that includes primary energy use E^e as input, which differs from the standard neo-classical practice. The representative firm's technology employs a production function that is characterized as a nested constant-elasticity of substitution (CES) specification of the form:

$$Y_t^e = A_t^e F^e(H_t^e, Z_t^e K_{t-1}^e, O_t^e E_t^e) = D_{d,t}^e + X_t^e \quad (5)$$

The equation above is the production function of the energy intensive sector with sector-specific endogenous variables and exogenous shocks. Y^e denotes the sector nominal output, measured in the nominal value of the *numeraire*, world imports price of US Dollars. $F^e(.)$ obeys the standard regularity conditions, A^e denotes the exogenous energy intensive sector productivity shock, H^edenotes sector's labour demand, $Z^e K_{t-1}^e$ denotes demand for capital services in the sector and O^e denotes the exogenous energy intensive sector energy input efficiency shock.[7] D_d^e denotes domestic absorption, stating that the sectoral output can either be consumed at home or to be exported X^e to satisfy the world demand. World prices are exogenous[8], hence, we assume energy prices, P_O, follow an exogenous process adjusting immediately to their world prices[9]. E^e denotes nominal energy use in the sector, which is measured in US Dollars given assumption of the numeraire of world imports.

The non-energy intensive sector output has a CES production function homogeneous-of-degree-one with properties similar to the energy intensive sector, denoted by superscript n:

$$Y_t^n = A_t^n F^n(H_t^n, Z_t^n K_{t-1}^n, O_t^n E_t^n) = D_{d,t}^n + X_t^n \quad (6)$$

where the exogenous shocks and endogenous variables are similar to the energy-intensive sector. Y^n denotes the sector nominal output, measured in the nominal value of the numeraire, world imports price of US Dollars. $F^n(.)$ obeys the standard regularity conditions, A^n denotes

[7] Sector-specific energy efficiency shock is a factor-augmenting technology. Energy efficiency in production captures a switch in the composition of capital towards machines with different energy intensities.

[8] Holtemoller and Mallick (2016) found oil prices do not respond contemporaneously to the shocks from other endogenous variables of the model.

[9] We use world crude oil prices for observed energy prices. Initially, we assumed $P_t^o = \frac{P_t^{of} P_t^f}{S_t}$ as the energy price shock, like in Harrison, et al., (2011). $P_t^{of} P_t^f$ is the assumed world exogenous price but after linearisation, the data residual is equal to observed price.

the exogenous non-energy intensive sector productivity shock, H^n denotes sector's labour demand, $Z^n K_{t-1}^n$ denotes demand for capital services in the sector and O^n denotes the exogenous non-energy intensive sector energy input efficiency shock. D_d^n denotes domestic absorption, stating that the non-energy intensive sector output can either be consumed at home or to be exported X^n to satisfy the world demand. Given the above assumption, and firms in the non-energy intensive sector are also perfectly competitive, the typical firm maximises the following profit function subject to the budget constraint in equations (5) and (6) $max\,\Pi_t = P_t^j Y_t^j - \left(W_t H_t^j + R_t^j Z_t^j K_{t-1}^j + P_{O,t} E_t^j\right)$ where P^j and R^jdenotes the relative price of non-energy intensive goods and rental rate of capital services for the sector, respectively. P_O represents the world price of energy. E^j denotes nominal energy use, in each sector, the value is measured in US Dollars given the assumption of the numeraire of world imports.

Government

Following An and Schorfheide (2007) and Justiniano et al. (2009), we assume the fiscal authorities to be fully Ricardian, and the budget constraint does not change over time. We assume the UK government will employ the HM Treasury and the Bank of England to continue to adjust taxes and adjusts interest rates to achieve its policy objective:

$$G_t + B_t = T_t + E_t R_{t+1} B_{t+1} \qquad (7)$$

where G represents the exogenous government spending shock following a univariate autoregressive form. The budget deficit of the government is financed by issuing short-term bonds to households. Therefore, households can access the domestic bond market where nominal government bonds, that pay a gross interest rate R_t, are traded.

International Trade

Given we assume the UK is an open economy, we also assume that consumption, investment and government are composites of the UK's and world's sectoral goods. We note that by definition:

$$C_t = \Phi_c\left(C_t^{d,e} C_t^{d,n} C_t^{f,e} C_t^{f,n}\right) \quad (8)$$

$$I_t = \Phi_I\left(I_t^{d,e} I_t^{d,n} I_t^{f,e} I_t^{f,n}\right) \quad (9)$$

and $$G_t = \Phi_G\left(G_t^{d,e} G_t^{d,n} G_t^{f,e} G_t^{f,n}\right) \quad (10)$$

where Φ is the *Armington aggregator*, CES utility function with homothetic preferences assumed to be homogenous-of-degree-one and increasing. For all variables, superscripts d, e denotes demand for domestically produced goods in the energy intensive sector, while superscripts d, n denotes demand for domestically produced goods in the non-energy intensive sector. Superscripts f, e denotes demand for foreign produced goods in the energy intensive sector, while superscripts f, n denotes demand for foreign produced goods in the non-energy intensive sector. In order to maintain focus on the macro-variables, we choose to use aggregate expenditures of these variables, and in that way, the total sum of these variables yields the domestic absorption:

$$D_t = C_t + I_t + G_t \quad (11)$$

where $$D_t = \kappa\left(D_t^d, M_t\right) \quad (12)$$

This means D is a composite for the four outputs. The *Armington aggregator* function here, κ, is assumed to be homogeneous-of-degree-one and increasing in both arguments. D^d represents the households' demand for goods produced in the UK and M denotes the total spending on imported goods. Unlike Backus et al. (1993) where they assumed two goods in an open economy, we assume four produced goods in the world which require some more disaggregation.

Here, D and M are assumed to be a function of both sectoral outputs, we can note that by definition:

$$D_t = \Sigma(D_t^e, D_t^n) \quad (13)$$

and $$M_t = \Xi(M_t^e, M_t^n) \quad (14)$$

where the *Armington aggregator* functions of Σ *and* Ξ are homogeneous-of-degree-one and increasing in both arguments. D^e and D^n represents the nominal expenditure on domestic output from the energy intensive and non-energy intensive sectors by domestic agents respectively. Similarly, M^e and M^n represents the nominal expenditure on imports from the energy intensive and non-energy intensive sectors by households respectively.

The domestic agents will $min\{P_t^d D_t^d + P_{M,t}M_t - S_t D_t\}$ subject to equation (12) where P^d is the price index of composite goods produced in the UK while S is the consumer price index of the UK as well as also the nominal exchange rate. P_M is the world's price index of composite goods, assumed to be the *numeraire* in the model. The agents have another problem of $min\{P_t^e D_t^e + P_t^n D_t^n - S_t D_t\}$ subject to equation (13) with the assumption of Walras' law that "all markets clear", the non-energy intensive sector goods market is silent, here, as the law implies the market will clear. The domestic agents will also solve the problem of the share of imported goods expenditure in the respective sectors by using the budget constraint of equation (14) to solve $min\{P_{M,t}^e M_t^e + P_{M,t}^n M_t^n - M_t\}$. P_M^e and P_M^n are imports prices in the energy intensive and non-energy intensive sectors, respectively. Like energy prices, world prices are exogenous and therefore we treat imports price as an exogenous process. The derived first order condition[10] shows that the representative agents' problem of the world is similar to the agent's problem in the domestic economy. This is why the imports function will be used to set-up the world's demand (exports) function:

$$D_t^w = \kappa^w\left(D_t^f, M_t^f\right) \tag{15}$$

Similarly, we assume $D_t^w = C_t^w + I_t^w + G_t^w$ as the aggregate world demand, D^f denoted as world's demand for home goods and M^f denotes the total imports in the world's economy

[10] See full model in the appendix

which signifies the UK's nominal exports (X). Where κ^w is homogeneous-of-degree-one and increasing in both its arguments.

Aggregation, Market clearing and the Resource Constraint

The assumption of this two-sector model is to have a total nominal output that is produced in the domestic country. The nominal sectoral outputs are measured in US dollars then added to give total output, measured in US Dollars because of the assumption of the world imports prices as the *numeraire*[11]. The total output is simply given as:

$$Y_t = Y_t^e + Y_t^n \tag{16}$$

where Y is denoted as total nominal output and Y^e and Y^n are the sectoral output of the energy intensive firm and the non-energy intensive firm respectively. The aggregate for labour supply and total energy use are:

$$H_t = H_t^e + H_t^n \tag{17}$$

$$E_t = E_t^e + E_t^n \tag{18}$$

Aggregate investment is defined as:

$$I_t = I_t^e + I_t^n \tag{19}$$

where $I^e \ and \ I^n$ are sector-specific investment.

Energy intensive sector market clears

$$Y_t^e = D_t^e + X_t^e - M_t^e \tag{20}$$

Final production satisfies demand as:

$$Y_t = C_t + I_t + G_t + X_t - M_t - E_t \tag{21}$$

It means the aggregate resource constraint is describing how the output is absorbed by consumption, investment, governments exogenous spending, net exports and energy use.

The dynamics of the current account equation is given as:

[11] We provide a detailed explanation of data collection and construction in the appendix.

$$B_t^f - \left(1 + R_{t-1}^f\right)B_{t-1}^f = S_t X_t - M_t - P_{O,t}E_t \tag{22}$$

The equation above denotes the aggregate resource constraint describing how net foreign assets of the economy evolve. The left-hand side shows the changes made in foreign asset holdings within one period lag while the right-hand side states the expenditures of net exports, with imports price assumed to be the *numeraire* in the model, and primary energy use yielding adjustment of bond wealth.

Calibration

Parameters

As we prepare to evaluate the log-linearized model, we set values for the parameters. We first split the parameters into two groups. The first group of parameters are the set that are important in deriving the model's steady state[12]. Derived by taking average ratios of the data used in the study covering the period 1990-2014, with little influence on the dynamic properties. These parameters are set to match steady-state values. When we estimate the model, these set of parameters remain unchanged, hence the name fixed parameters. We set the discount factor β at 0.96, this means that the model will generate a steady-state annual real interest rate of 4%. The cost shares between labour and capital services, α_e and α_n are set to 0.35 and 0.28 for energy intensive sector and non-energy intensive sector, respectively. This means that steady-state labour share is 65% and 72% in energy intensive sector and non-energy intensive sector, respectively.

The depreciation rate is set at 0.0125 per quarter, which implies 5% annual depreciation on capital. Nonetheless, we have the opportunity to estimate this using the model's structural parameters in steady-state as follows: we divided the depreciation rate of capital into two sectors for $j = e, n$. $\delta U^j = \delta_{j0} + \delta_{j1}\left(\mu_j\right)^{-1}\left(U^j\right)^{\mu_j}$. In setting $\delta_{j1} = 1$ and assuming

[12] See Fixed parameters in the appendix

households optimality conditions with regards to capital utilisation rates conditioned on the values for the respective sectors' steady-state real capital rental rate, $\delta_{j1}\left(U^{j}\right)^{\mu_j} = R^{j} = \frac{1}{\beta} - (1-\delta U^{j})$. Having calibrated δU^{j} using the data, $\delta_{j1}\left(U^{j}\right)^{\mu_j} = 0.0544$ and 0.0606 for energy intensive sector and non-energy intensive sector respectively. To calibrate the elasticity in capital utilisation rate μ_j, we augmented the previous result which we assumed the conditioned values of the discount factor and rental rate as $\mu_j = \frac{\delta_{j1}\left(U^{j}\right)^{\mu_j}}{1+\delta_{j1}\left(U^{j}\right)^{\mu_j}-\frac{1}{\beta}}$ =1.404 and 1.1 for energy intensive sector and non-energy intensive sector respectively. The cost share parameter between capital services and energy is calibrated using the capital-energy ratio from the sample period and the structural parameter that results in $\theta_j = \frac{1}{1+\frac{Po}{\delta_{j1}\left(U^{j}\right)^{\mu_j}}\left(\frac{e^{j}}{k^{j}}\right)^{1+v_e}}$ where $\frac{e^{j}}{k^{j}}$ is the steady-state ratio of energy-capital and Po is the steady-state value of energy prices.

We set the parameter for the degree of habit formation parameter at 0.7 to be consistent with standard RBC models, intertemporal elasticity of substitution to 2 and the Frisch inverse elasticity of labour supply parameter at 3. We have the choice of either to assume the UK has a balanced current account by setting the foreign bonds' adjustment cost to 0 or as a creditor greater than 0, we chose the latter and set the parameter at 0.25. The elasticity of substitution between capital services and energy use in the respective sectors, $v_e\ and\ v_n$, are set to 0.7. The value of the capital adjustment cost which is set at 5. This means that cost of capital costs gives incentives for households to change the capital stock. That means, ceteris paribus, a higher capital adjustment cost parameter will decrease the elasticity of the change in capital stock relating to real interest rate. The parameters governing foreign trade are assumed to follow the standard RBC literature.

The elasticity of substitution between consumption of the sectoral goods, ς, is set to unity, the elasticity of demand for imports, η, is set at 1.5 which is also the case for the rest of the world equation η_w as we assume the world has the same agent's problem as the UK. The elasticity of demand for imports of energy intensive goods is set at 0.4. All values of shares and ratio are consistent with the RBC model of the UK literature.

Error Processes

We use the data and the model to back out the model errors. The sectoral productivity shocks can be directly estimated while the world energy price shock is measured with the observed data. The sectoral productivity shocks together with the energy prices shock are tested and treated as nonstationary shocks, we model the shocks as ARIMA(1,1,0) processes. Other shocks are tested to be stationary or trend stationary, hence, treated as stationary or trend-stationary ARMA(1,0) processes with a deterministic trend (if needed). The properties of the errors are also shown in Table 1, we report the persistence estimated from the AR(1) process and the standard deviation estimated from the errors' innovations. One can see the volatility of energy price is quite high. In sectoral comparison, one can see productivity and energy efficiency of the energy intensive firms are more volatile, while the investment is lower than non-energy firms. Foreign shocks have high persistence while investment specific-technology shocks possess high persistence and volatility.

Table 1: Error Properties

Shock	**Persistence ρ_j**	**Volatility σ_j**
Productivity (energy intensive sector)- non-stationary, differenced	0.3394	0.0259
Productivity (non-energy intensive sector)- non-stationary, differenced	0.1896	0.0241
Consumption preference	0.4367	0.0807
Government spending	0.9894	0.0235
Investment Specific-Technology (energy intensive sector)	0.9209	0.1689
Investment Specific-Technology (non-energy intensive sector)	0.8696	0.2335
Energy efficiency (energy intensive sector)	0.9039	0.3071
Energy efficiency shock (non-energy intensive sector)	0.8954	0.3053
World exports price	0.9741	0.0181
Energy price- non-stationary, differenced	0.2257	0.1388
World interest rate	0.9227	0.0031
Labour supply	0.8568	0.1158
World demand	0.9250	0.0430

Methodology: Model Evaluation by Indirect Inference Testing

Evaluating a model using the method of Indirect Inference offers a classical econometrics inferential structure for assessing models. Meenagh et al. (2009) first proposed the method then Le et al. (2011) augmented the methodology to what is now widely used. This method is used to judge partially or fully estimated models while maintaining the fundamental ideas utilised in the evaluation of early RBC models of comparing data generated moments from the model simulation with those from the actual data. Instead of using moments for comparison with no distributions, this method provides a simple model (auxiliary model) that includes the conditional mean of the distribution with which one can compare the features of the model estimated from actual and simulated data. The method has similar features in the widely used indirect estimation method. The primary feature of this similarity is utilisation of the auxiliary model in addition to the structural macroeconomic model. Estimation by indirect inference chooses the parameters of the RBC model in a way that the simulated model generates estimates of the auxiliary model that are similar to those obtained from the data.

We explain the inferential problem using Canova (2005) notations designed for indirect inference estimation, where y_t is defined as $m \times 1$ vector observed data $(t = 1, \dots, T)$ and $x_t(\theta)$ is a $m \times 1$ vector of simulated (time series) data with the number of observations S which is generated from the structural model, θ is a $k \times 1$ vector of the model's structural parameters. Then set $S = T$ with the requirement of the actual data sample being regarded as the expected imitation from the population of the samples that have been bootstrapped by the data. The auxiliary model is assumed as $f(y_t, \alpha)$, with α as the vector of descriptors. From the given null hypothesis $H_0: \theta = \theta_0$, the auxiliary model then becomes $f[x_t\,(\theta), (\theta_0\,)]$ as $f[y_t, \alpha]$. The test of the null hypothesis is by a $q \times 1$ vector of a continuous function $g(\alpha)$. Therefore, under the null hypothesis, one is going to have $g(\alpha) = g(\alpha(\theta_0))$. The estimator for α using the actual data is a_T while the estimator for (θ_0) based on simulated data is $a_S(\theta_0)$.

This gives us $g(\alpha_T)$ and $g(\alpha_S(\theta_0))$. We then get the mean of the bootstraps as: $\overline{g(\alpha(\theta_0))} = \frac{1}{N}\sum_{k=1}^{N} g_k(\alpha(\theta_0))$. From here, we get the Wald statistic (*WS*) by using the bootstrapped distribution of $g(\alpha_S)$- $\overline{g(\alpha_S(\theta_0))}$. This is then defined as:

$$WS = \left(g(\alpha_T) - \overline{g\big(\alpha_S(\theta_0)\big)}\right)' W^{-1}(\theta_0)\big(g(\alpha_T) - \overline{g(\alpha_S(\theta_0))}\big) \tag{23}$$

where $W(\theta_0)$ is the variance-covariance of the bootstrapped distribution of $g(\alpha_S)$- $\overline{g(\alpha_S(\theta_0))}$. Furthermore, $W(\theta_0)$ is obtained from the asymptotic distribution of $g(\alpha_S)$- $\overline{g(\alpha_S(\theta_0))}$ and then the asymptotic distribution of the Wald statistic would then be chi-squared. We choose a VECM (or a VAR in the case of stationary data) as the auxiliary model, and the coefficients from this supplemented by the variance of the auxiliary model disturbances as the data descriptors we are trying to match. Therefore we are trying to match the dynamic properties of the data as well as the size. The Wald test by bootstrap is conducted in the following three steps:

Step 1: *Estimating the errors of the structural model based on observed data and* θ_0.

We assume the errors will follow an AR(1) process. However, given the non-stationarity of three structural shocks, these shocks follow ARIMA(1,1,0). We verify that the number of exogenous shocks in the model are less than the endogenous variables in our model, however, the assumption allows the number of shocks to be at least equal. We then estimated the 13 structural residuals ε_t from the model $x_t(\theta_0)$, we give it stated values of θ_0 and the actual observed data. Figure 2 shows the estimated structural residuals using observed data. The structural equations that contains no expectation have their residuals backed out of the equation using the observed data of each variable in the structural equation. For equations that include expectations on some variables we use the McCallum (1976) and Wickens (1982) robust instrumental variables method with lagged endogenous observed data as the instruments to determine the expectations.

Figure 1: Estimated Structural Residuals

Step 2: Deriving the simulated data

In this model, like many RBC models, the structural shocks are assumed to be autoregressive processes rather than being serially independent. OLS is used to estimate the innovations from the residuals[13]. We produce 1000 bootstrap simulations by randomly drawing the innovations, drawn in time vector to preserve any simultaneity between the shocks.

Step 3: Compute the Wald Statistic

The auxiliary model is then estimated, a VAR(1)[14], on the bootstrap sample and the actual data to obtain the estimates[15], of the distribution of the observed data and the VAR coefficients, a_T and a_S of the vector α. We are able to obtain the covariance matrix $W(\theta_0)$ of the distribution (a_S)- $\overline{(a_S(\theta_0))}$ through estimating the auxiliary VAR on the 1000 bootstrapped simulations of $a_S(\theta_0)$ while the covariance of the simulated variables from the bootstrap samples were

[13] The coefficients of the lagged residuals from the OLS estimation are the shock's persistence.
[14] We employed VECM in our testing instead of VAR due to non-stationary data.
[15] Actual and simulated data variances have been included in the estimates to determine the model's dynamics and volatility.

obtained. This shows the variations in the data sampling as implied by the model from the resulting set of a_k vectors ($k = 1, \dots . N$), thus the estimate of $W(\theta_0)$ will be:

$$\frac{1}{N}\sum_{k=1}^{N}(a_k - \overline{a_k})'(a_k - \overline{a_k}) \tag{24}$$

where $\overline{a_k} = \frac{1}{N}\sum_{k=1}^{N} a_k$. From here, the Wald statistic is calculated for the data sample and then the bootstrap distribution of the Wald from the 1000 samples of the bootstrap is estimated.

A combination of output (Y) and consumption (C) were chosen as the variables of interest in the auxiliary model for the evaluation of fit to the data. This auxiliary model allows for joint distribution testing, with the null hypothesis as the structural macroeconomic model is the data generating mechanism.

Using a VAR (or VECM) as the auxiliary model supplemented by the variances of the data means that our II estimation procedure will find a set of coefficients that produce simulations that have dynamic properties and volatility that are closest to those seen in the data. We feel this is a better descriptor that just looking at correlations.

Using Nonstationary Data

Filtering observed data distorts the dynamic properties of the model in several ways that one cannot tell. It also changes the forward-looking properties of the structural model as the filtering method is two-sided. Since the RBC model is supposed to mimic the activities of the economy a distortion of world oil prices will show huge imperfections of the model. In a model like this, where the expectation structure and IRFs are critical, using filtered data will be a flaw in the study. It is common knowledge that the data generated by an RBC model on most occasions proved to be nonstationary as generated by the model structure or due to incorporation of nonstationary exogenous variables, which are unobservable, such as the productivity shocks or world oil prices function which is an observed variable. Therefore, the linearized model's solution will be denoted by a vector error correction model (VECM) to allow the model to have higher number of endogenous variables than cointegrating vectors if there are unobservable nonstationary variables. With this, we have nonstationary errors in the long-run structural model. The auxiliary model is now a VECM with nonstationary processes represented as observable variables. This method includes the nonstationary errors estimated from the structural model in the auxiliary model as the auxiliary model is required to have key variables for cointegration that will allow the VECM to be stationary. One should also remember that the auxiliary model is partly conditioned by the structural model that is also the null hypothesis. Therefore, the construction of the VECM came through the null hypothesis. A non-rejection is far from certain under this condition of data generated VECM because the RBC structural model picks a range of parameters which could be inconsistent with the RBC structural model. Rather, the objective of the null hypothesis constraint is to make sure the VECM obtains cointegration under the null hypothesis which is also the assumption of the errors.

We do not carry out any test for cointegration because we treat all nonstationary errors as valid cointegrating variables. Without cointegration an RBC model will not have a solution which means there will be no simulation and that will be impossible to have the Wald test. Therefore, the indirect inference testing method we carry out imposes cointegration and tests the simulation performance of the RBC model at the later stage.

Assessing the estimated model fit

We estimate the model parameters by employing the powerful simulated annealing[16] algorithm to find the best set of estimated model parameters that fits the real economic data. We then use conventional tools like the impulse response functions (IRFs) to understand how a simulated RBC model works and variance decomposition to study the structural shock effects on each

[16] Simulated Annealing algorithm due to Ingber (1996). It is a method of getting a solution for large number of parameters by unconstrained and bound-constrained optimization technique.

macroeconomic variable of the model. We also examine what the model says about energy prices shock by accounting for the crisis period with the model's shock decomposition.

We used a wide ranging set of variable combinations in the model testing with the aggregate output (*Y*) remaining a constant in each of these sets. We finally used output (*Y*) and consumption (*C*), since the model is a study of a UK open economy with world's prices and foreign bonds included in the model.

Table 2 shows the values of the estimated parameters alongside the initial calibration for comparison. The value of the habit persistence parameter is similar to the assumed value reported by Smets and Wouters (2003) to be between 0.5 and less than 1 and also close to Boldrin, Christiano and Fisher (2001) where they argued the ability of a standardized RBC model to account for the equity premium among other points. The elasticity of labour supply is consistent with Chadha et al. (2001), as we have a similar utility function. The shock persistence and volatility follow an AR(1) process for the stationary shocks and ARIMA(1,1,0) process for the nonstationary shocks[17]. The energy efficiency shock in the energy intensive sector has a high persistence and volatile rate. However, government spending has the highest persistence and low volatility while the volatility of energy price shock is high.

[17] The energy price shock and both productivity shocks as are treated as non-stationary.

Table 2: Estimates of model parameters

Parameter	Description	Calibrated	Estimated
ω	Frisch elasticity of labour supply	0.33	0.2078
ψ_h	Habit formation in consumption	0.7	0.8318
σ_c	Intertemporal elasticity of substitution	2	1.1688
η	Elasticity of demand for imports	1.5	3.2899
η_w	Elasticity of demand for exports	1.5	2.1813
μ_e	Elasticity in capital utilisation rate; energy intensive sector	1.404	1.6856
μ_n	Elasticity in capital utilisation rate; non-energy intensive sector	1.1	1.0858
v_e	Elasticity of substitution between energy and capital in energy intensive production	0.7	1.8880
v_n	Elasticity of substitution between energy and capital in non-energy intensive production	0.7	2.873
ς	Elasticity of substitution between consumption of energy intensive and non-energy intensive goods	1	0.595
χ_e	Cost parameter: capital stock in energy intensive sector	5	78.1
χ_n	Cost parameter: capital stock in non-energy intensive sector	5	49.5
ϕ	Elasticity of demand for imports of energy intensive goods	0.6145	0.4506
ϕ_w	Elasticity of demand for exports of energy intensive goods	0.5	0.5310
ϵ	Share of energy intensive goods	0.5	0.4750
$\delta_{e1}z^{\mu_e}$	Cost of capital utilisation in energy intensive sector	0.0544	0.0171
$\delta_{n1}z^{\mu_n}$	Cost of capital utilisation in non-energy intensive sector	0.0606	0.0022
χ_{BF}	Cost of adjusting portfolio of foreign bonds	0.25	0.7548

Shock (*j*)	**Persistence ρ_j**		**Volatility σ_j**	
	Calibrated	**Estimated**	**Calibrated**	**Estimated**
Productivity (energy intensive sector)	0.3394	0.3297	0.0259	0.0280
Productivity (non-energy intensive sector)	0.1896	0.2628	0.0241	0.0370
Consumption preference	0.4367	0.4362	0.0807	0.1001
Government spending	0.9894	0.9894	0.0235	0.0235
Investment Specific-Technology (energy intensive sector)	0.9209	0.9008	0.1689	0.1104
Investment Specific-Technology (non-energy intensive sector)	0.8696	0.8639	0.2335	0.1031
Energy efficiency (energy intensive sector)	0.9039	0.9059	0.3071	0.1769
Energy efficiency shock (non-energy intensive sector)	0.8954	0.8917	0.3053	0.1007
World exports price	0.9741	0.9741	0.0181	0.0181
Energy price	0.2257	0.2257	0.1388	0.1388
World interest rate	0.9227	0.9227	0.0031	0.0031
Labour supply	0.8568	0.7741	0.1158	0.1299
World demand	0.9250	0.9250	0.0430	0.0430

From Table 3 one can see how, following estimation, the simulated behaviour of the model matches the simulated behaviour of the data. The table reports the auxiliary model (VECM) coefficients (the dynamics) and data variances (the volatility) from the actual data alongside the upper and lower bound from the model simulations. We also show the Wald statistic transformed to a t-statistic individually for the dynamics and volatility as well as the overall Wald, which includes both. The results show that, as well as each coefficient being within the bounds individually, the joint test is also passed, as shown by the p-values of the Wald Statistics being greater than 0.05. Generally, one can say that the tests imply that this model performs very well in its context, as a RBC model, as it can explain output and consumption among the real variables.

Table 3: VECM results and summary

	95% lower	95% upper	Actual	IN/OUT
A_y^y	0.267471	0.879874	0.684021	IN
A_y^c	-0.142090	0.117258	0.040889	IN
A_c^y	-0.400267	0.208570	-0.062125	IN
A_c^c	0.642383	0.926467	0.820774	IN
σ_y^2	0.000166	0.000426	0.000218	IN
σ_c^2	0.000174	0.000458	0.000237	IN
Summary of results	Normalised t-statistic	p-value		
Dynamic	0.023	0.448		
Volatility	0.014	0.590		
Overall	0.482	0.267		

Impulse response functions (IRFs)[18]

The impulse responses of the model result from the 13 exogenous shocks, given each shock's persistence and volatility of the estimated model. Given that the sectoral productivity shocks and energy prices shock are treated as nonstationary, the shock responses will show a permanent effect on some of the variables instead of the conventional temporary effects shocks have on variables. The two sectors have the same production function that follows a similar linearized equation. The responses follow the same pattern but have different values.

[18] The IRFs for the remaining shocks are available on request, including the VAR-IRFs.

Figure 2: Productivity shock (Energy intensive sector)

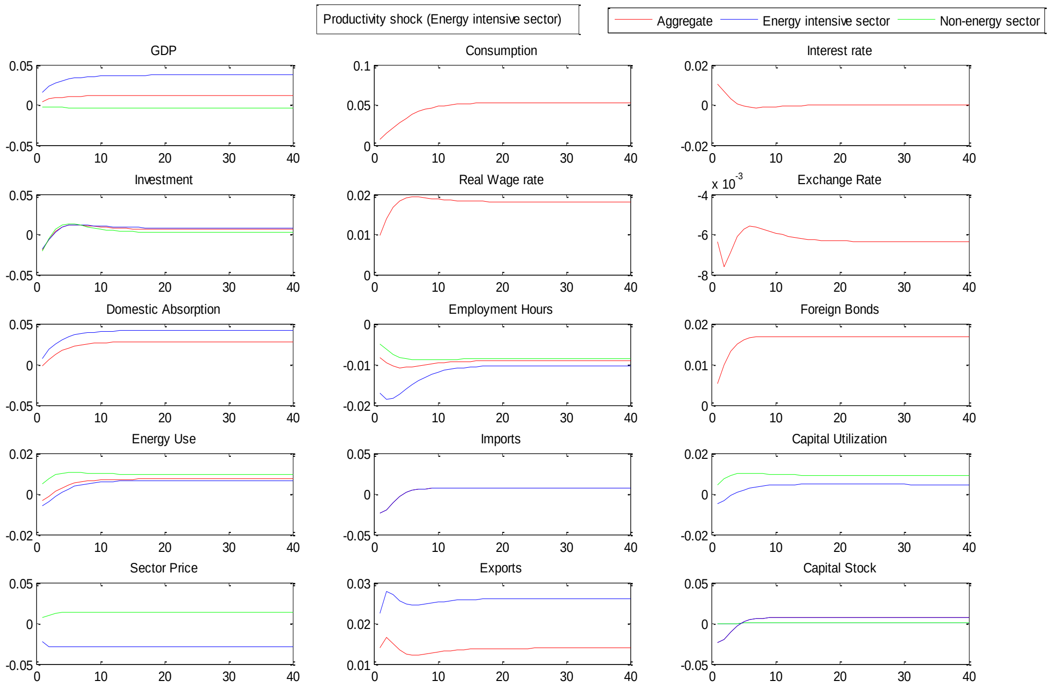


Figure 2 shows the effects of a one standard deviation shock to productivity of the energy intensive sector with an increase in sector output of almost 5% for given labour and capital which leads to higher output supplied by about 5%. Due to the slow adjustment of prices, firms' demand for labour and capital utilisation falls which then reduces the marginal cost of production for firms. The rise in output increases welfare in the economy. The real wages will increase which will have a 'knock-on' effect on employment. As households become richer (through the wealth effect of lower prices of output), they consume more and have more leisure since income increases and domestic absorption increases. The rise in productivity allows households to set their wages higher due to increasing productivity. This increase also comes at the expense of high-energy use, decreasing capital stock in the short-term and its utilisation rate. Monetary policy reacts to productivity by raising domestic interest rates in the short-run, to remain around the potential output. Thereby driving down investment in a similar fashion, but the effects of the latter will be permanent. The impact of high output will push down domestic prices since goods in the UK have a lower cost of production, which makes the exchange rate appreciate relative to domestic prices. The latter, being foreign asset prices, causes foreign bond investment to rise. The demand for UK goods rises due to lower prices and the UK households' demand for foreign goods rises. One should note that our model is not restricted to developed economies, Mallick and Sousa (2011) find developing economies respond to shocks in similar pattern.

Figure 3 shows that the productivity shock to the non-energy intensive sector responses are qualitatively similar to the energy-intensive sector. The slight difference is how a decline in energy use in the non-energy intensive sector raises energy use in the energy intensive sector to increase productivity.

Figure 3: Productivity shock (Non-energy intensive sector)

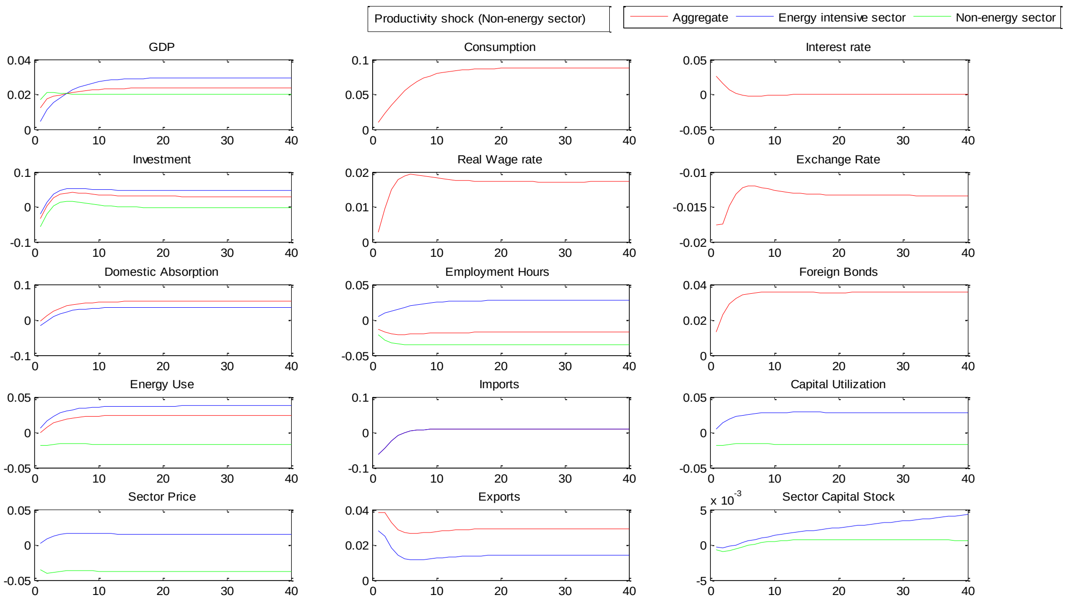
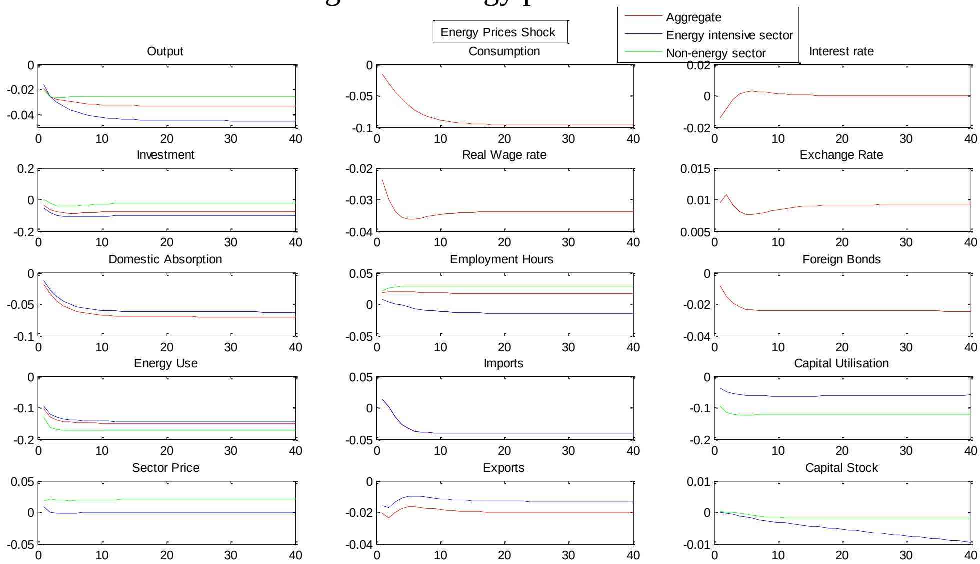


Figure 4: Energy price shock



From Figure 4, a one standard deviation shock to world energy price will have permanent effects on the real macroeconomic aggregates. The impact of this shock is mostly in the energy intensive sector due to the energy intensity in that sector. There is a fall in output of over 4% while aggregate output falls by about 3%. As output declines, the economy faces a welfare loss thereby causing aggregate demand to fall as income is reduced. Firms' demand for inputs will decline as energy use, and capital utilisation falls, this mean lower price elasticity of demand. As revenue declines, households choose to work more. Therefore, the employment is skewed to the non-energy sector as households reduce their real wages to gain employment. The energy prices shock drives up prices in both sectors as expected. The firms' marginal cost will decline in this case. As positive energy price shock is likely to send the economy into a recession as empirical evidence suggests, monetary policy will react to this shock by decreasing interest rates to finance borrowing and investment in the economy as the exchange rate appreciates[19]. The effect of the nonstationary energy price shock on declining output means that the terms of trade decline permanently when energy price changes as it is nonstationary and there is no other

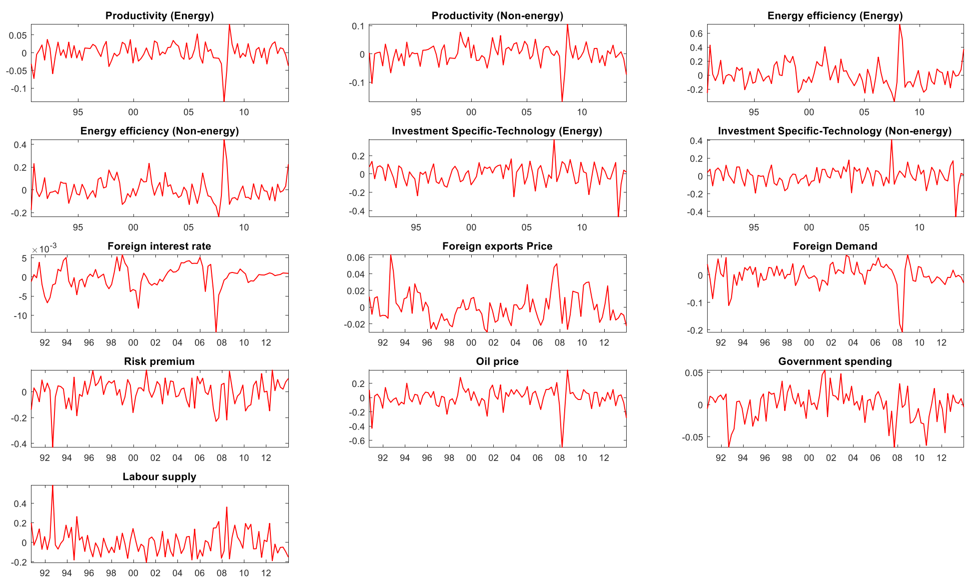
[19] Holtemöller and Mallick (2016) show that inflationary supply shocks demand an aggressive behaviour from central banks towards inflation stabilisation following shocks to global energy prices.

way to balance the current account than to reduce absorption — because there is no way to substitute away from energy by enough to eliminate the problem. A rise in energy prices can make the current account of countries that are net importers of oil worse, like the UK, by increasing their current account deficits and depreciating their currencies. The situation would be the same if an economy were a net exporter of oil faced with low oil prices.

What does the model say about energy price shocks?

Given the high volatility of the energy prices shock, the world economies are hugely affected and this has affected world demand. From Figure 6 one can see that the shocks to sector investment-specific technology, sector energy efficiency and labour supply have been highly volatile over the sample period, 1990Q2-2014Q4. Conversely, foreign export prices, foreign interest rates, sector productivity and government spending shocks have low volatility. These shocks reflect what happened to world trade during the 2008 and 2009. Observing the consumer preference shock (risk premium shock), it is easy to see the loss of consumer confidence due to credit rationing in that period. The government spending shock reflects the quantitative easing during the same period followed by the austerity measures of the 2010 political regime.

Figure 5: Shock's innovations



A Stochastic Variance Decomposition

The analysis of variance decomposition is one of the outstanding features of this model compared to other studies. This is because of the dominant role that the energy price shock plays in explaining the variance of real macroeconomic aggregates of the model. Table 4 shows the variance decomposition of the aggregate variables from the benchmark model with respect to the contribution of the 13 shocks.[20]

The energy price shock, with its high volatility rate, explains 56% of the output variance in the model. This shock also explains 45% of consumption variation, 25% of foreign assets variance, and 20% of asset prices (real exchange rate). Also, 10% of domestic interest rates and 57% of wage rate and about 62% of total investment in the economy. It also explains 36% of the variability of total employment in the economy, 55% of total domestic absorption of the economy. The shock has effects on the variance of total exports with 18% contribution while it dominates as it explains 43% of total imports. Comparing with the related literature, authors like Bjornland (2000) as well as Jiménez-Rodriguez and Sanchez (2004) find the oil price shock explains 9% of the variability in the output in the UK. Allowing the energy price shock to be non-stationary vastly increases the importance of this shock.

One can also see how sector-specific productivity shocks play important roles in explaining the variance of key variables in the model. The non-energy productivity shock is more important than the energy productivity shock, explaining 30% of output variation compared to 7.7%. Overall, one can say the sectoral productivity shocks have played a vital role in explaining the key variable’s variations in the model.

These findings, especially in energy prices, are significant to this study as it opposes Kim and Loungani (1992); Finn (1995); Hamilton (2003); Kilian and Vigfusson (2014). They argued that energy prices shocks are of little significance in RBC models.

[20] We show the variance decomposition for aggregate variables please see the appendix for sectoral variables.

Table 4: Variance decomposition (Benchmark model)

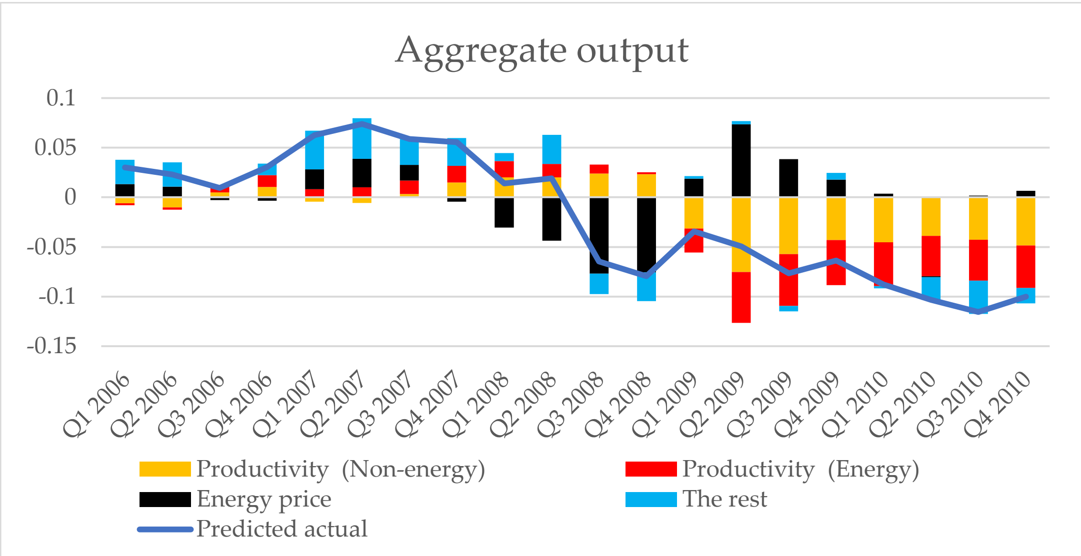
	Productivity (Energy)	**Productivity (Non-energy)**	**Energy price**	**Invest. specific-Technology (Energy)**	**Invest. specific-Technology (Non-energy)**	**Government spending**	**Consumption preference**	**Energy efficiency (Energy)**	**Energy efficiency (Non-energy)**	**Foreign interest rate**	**Foreign demand**	**Foreign exports price**	**Labour supply**
Output	7.7	30.0	56.0	0.7	0.1	0.4	0.3	2.7	0.2	0.0	0.2	0.0	1.6
Consumption	14.0	39.1	45.1	0.0	0.0	0.8	0.1	0.4	0.1	0.0	0.1	0.0	0.4
Foreign Bonds	12.1	54.9	24.5	1.7	0.2	0.0	0.3	0.0	1.9	0.2	2.3	0.0	1.9
Interest rate	5.7	31.8	10.3	8.0	0.4	0.1	5.5	3.5	9.9	0.6	12.6	0.0	11.4
Exchange rate	10.2	48.4	20.2	3.9	0.3	0.0	1.5	0.7	4.6	0.0	5.8	0.0	4.3
Wage rate	16.6	15.6	56.5	0.2	0.0	0.0	0.7	5.8	0.5	0.0	2.2	0.0	2.0
Investment	1.0	12.6	61.8	2.7	0.9	0.3	1.8	12.1	0.8	0.0	2.6	0.0	3.5
Employment	11.3	43.3	36.4	0.1	0.0	1.3	1.4	0.1	0.2	0.0	0.1	0.0	5.8
Energy use	0.2	2.4	91.9	0.8	0.0	0.0	0.0	3.1	1.3	0.0	0.2	0.0	0.1
Domestic absorption	9.0	32.1	54.5	0.4	0.1	0.0	0.0	2.3	0.2	0.0	0.5	0.0	1.0
Exports	9.1	43.1	18.0	3.5	0.2	0.0	1.3	0.6	4.1	0.0	16.1	0.0	3.9
Imports	3.7	14.6	43.0	3.0	0.1	0.0	1.8	10.0	5.7	0.1	13.7	0.0	4.4

Shock Decomposition for the Crisis Period

We analyse the crisis period by looking at the shock decomposition for the 2006–2010 period. Our focus is on aggregate as well as sectoral output and the real exchange rate. We report the contribution of the three dominant shocks for each variable (energy price and the two sectoral productivity shocks) and combine the other then shocks as 'the rest'.

Figure 7 shows the timeline for aggregate output. Energy prices can be seen to be a significant determinant of movements of aggregate output. High world energy prices during the crisis period, 2008 especially, significantly reduced aggregate output as can be seen with the energy price shock. A reduction in productivity in both sectors dragged aggregate output down further. Furthermore, a reduction in productivity occurred due to low demand for inputs in the non-energy intensive sector which helped in increasing similar demand in the energy intensive sector as firms substituted towards energy input. The rest, representing other shocks, shows how labour supply, consumption preference (risk premium) and world demand were significant shocks that contributed to pulling down UK output as observed in the effects of the 2008 financial crisis.

Figure 6: Shock decomposition of aggregate Output



The downward movement in the output of the energy intensive sector is driven by the energy prices shock during the crisis period despite some initial effort for the productivity shocks to push up the output as shown in Figure 7. As the crisis deepened in 2008Q4, one can see how other shocks contributed to the loss of output as well as the energy-intensive sector productivity shock that dominates the sample period. The low demand for inputs contributed to negative productivity.

Figure 7: Shock decomposition of energy intensive sector output

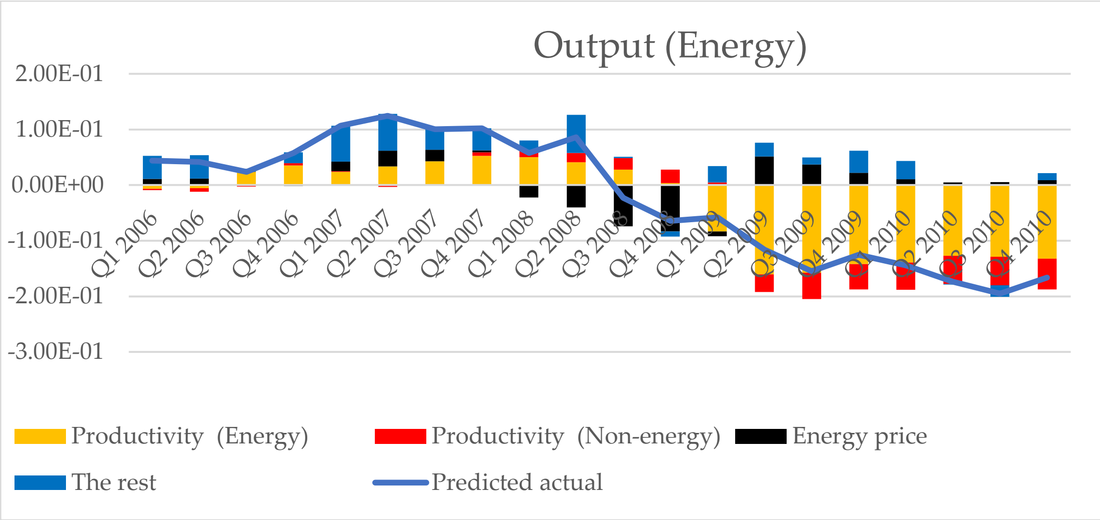
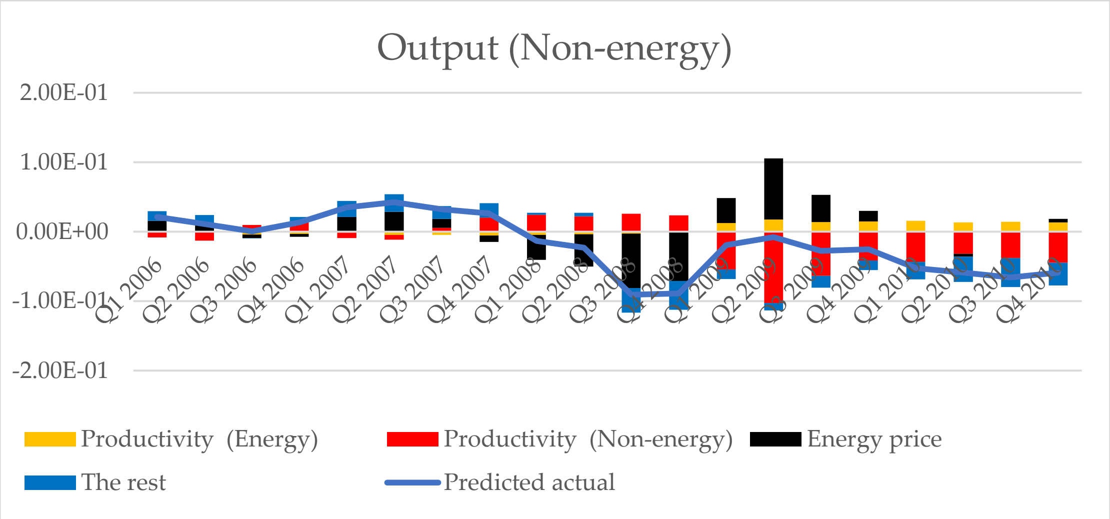


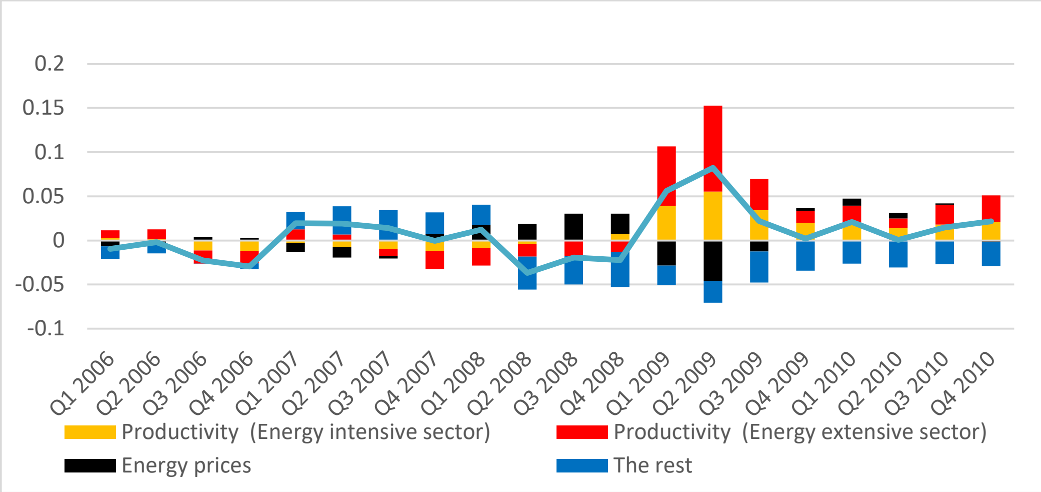
Figure 8**Error! Reference source not found.** shows that the decomposition of movements in the output of the non-energy intensive sector is strongly attributed to the energy prices shock. The energy intensive sector productivity shock contributed to the downward movement with lack of substitution of input from energy as world energy prices rise, therefore, reducing energy use. Also, other shocks played important roles in pushing down output. These factors included: credit rationing to firms and households, corporation closures, labour supply shock and the depreciation of the exchange rate in the previous quarter. This made UK exports less attractive. However, as one can see from the data, the decline of output in the non-energy-intensive sector did not last as long as the energy-intensive sector. One explanation is that a fall in world energy prices increased the sector's output. The appreciation of the exchange rate in the sector contributed to higher demand for UK services, since the data shows that the services sector contributed to over 70% of UK exports.

Figure 8: Shock decomposition of non-energy intensive sector output



From the timeline of the real exchange rate, as shown in Figure 9**Error! Reference source not found.**, one can observe that the energy price shock was pushing down the pound substantially from between 2008:Q1-2008Q4. The emprical evidence that oil prices and real exchange rates have an inverse relationship is evident here. Historical data show energy prices peaked during that quarter and the real exchange rate appreciated. The non-energy intensive sector productivity shock contributed in moving the real exchange as corporate firms in the sector traded with foreign currency as well as the UK exports that are dominated by the output of this sector. The energy intensive sector played a significant role in the movement of the exchange rate as can be seen in 2009Q1 when world energy prices dropped. Other shocks that contributed to the real exchange rate movement in the crisis period include world demand as UK products were competitive in the world market since domestic prices fell in the UK.

Figure 9: Shock decomposition of real exchange rate



Robustness

As a robustness check to emphasise the importance of the energy price shock we also show the variance decomposition analysis after excluding energy from the model. The results in Table 5 show that now the productivity shock dominates in the contribution to the variance of output, consumption, employment, real wages and domestic absorption. The variation of investment is now spread more equally among all shocks except for foreign interest rates and exports price. This is in line with the results of Le et al. (2011) who find that, when allowing it to be non-stationary, the productivity shock dominates the variance decomposition. This shows that excluding energy from an RBC model means the model is missing a vital element explaining macroeconomic variability.

Table 5: Variance Decomposition (model ex-energy)

	Productivity	Investment specific-technology	Government spending	Consumption preference	Foreign interest rates	Foreign demand	Foreign exports price	Labour supply
Output	79.9	2.2	3.8	2.4	0.0	0.5	0.0	11.2
Consumption	85.0	0.2	12.1	0.7	0.0	0.3	0.0	1.7
Foreign Bonds	61.9	0.5	4.6	1.3	0.8	24.4	0.0	6.6
Interest rate	17.7	1.0	0.9	11.0	1.5	48.8	0.0	19.1
Exchange rate	36.9	0.8	2.3	5.2	0.2	42.6	0.0	11.9
Wage rate	72.8	0.9	0.2	3.2	0.1	14.1	0.0	8.8
Investment	16.9	14.2	10.8	14.1	0.4	18.0	0.1	25.6
Total Hours	59.9	0.2	15.0	4.7	0.0	0.8	0.0	19.3
Domestic Absorption	84.7	1.1	5.1	0.1	0.1	2.5	0.0	6.3
Total Exports	35.7	0.8	2.2	5.0	0.2	44.5	0.0	11.5
Total Imports	20.9	0.3	1.4	4.9	0.5	61.0	0.0	11.0

Conclusion

Our aim in this paper was to examine the extent to which energy and other real shocks could account for the Great Recession in the UK, given that these factors have generally been ignored in favour of a financial explanation. We have found that these factors did indeed play a substantial role in explaining the UK's experience in this episode. Our findings show the effect of the nonstationary energy price shock on declining output means that the terms of trade decline permanently when the oil price falls as it is nonstationary and there is no other way to balance the current account than to reduce absorption — because there is no way to substitute away from energy by enough to eliminate the problem. From the viewpoint of policy this makes any stabilisation policy problematic because policy could not alter this fundamental determinant of long-run equilibrium. No doubt policy could apply some smoothing of adjustment for the output gap; but we do not discuss this here, largely because this function would naturally be taken by monetary policy which is excluded in our RBC set-up. Fiscal policy in most countries during the crisis was dominated by the need to restore the economy to long-run equilibrium through the reduction of absorption whose necessity is expalined by this model.

The study also demonstrates that a linear model, here a standard RBC model, can show the real effects of energy prices, contrary to Hamilton (2003) and Kilian and Vigfusson (2014). The treatment of the energy price shock, as well as unobserved productivity shocks in the energy intensive sector and non-energy intensive sector provided the key focus for this study. Further work could usefully incorporate financial factors and examine whether they were themselves caused by these real shocks or were a major independent source of recession. The model can also be applied to study the growing demand for renewable energy use.

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Appendix
Model listing

Household

The household lifetime utility is prescribed as:

$$\mathrm{E}_t \sum_{t=0}^{\infty} \beta^t \tau_t U((C_t - \psi_h C_{t-1}), \xi_{w,t} H_t) \tag{25}$$

where E_t represents rational expectation of household, and $0 < \beta < 1$ denotes discount factor. C represents aggregate demand by household (nominal consumption) and H denotes the work hours supplied by household with ψ_h representing the degree of habit formation by households. The household lifetime utility differs from Smets and Wouters (2003, 2007) as it includes habit formation as ($\psi_h > 0$). τ denotes exogenous consumption preference shock and ξ_w denotes exogenous labour supply shock[21].

The Budget constraint, closely follows Harrison, et al., (2011), shows how the end of period holdings of nominal government debt(B), nominal foreign bonds (B^f) and nominal capital (K) are given by their start of period holdings, plus net income. The net income includes earnings from labour supply (at wage) and capital services ($Z_t^j K_{t-1}^j$ rented at rateR_t^j), for $j = e, n$, to firms less expenditures on consumption (C), lump-sum taxes (T), adjustment costs will be discussed below as well as the cost of servicing capital. Given that we use world imports prices as the numeraire in the model, the values of the nominal variables are converted to US Dollars and deflated by world manufacturing prices.

$$\frac{B_t}{1+r_t} + \frac{B_t^f}{S_t\left(1+r_t^f\right)} + C_t + T_t + I_t = R_{t-1}B_{t-1} + R_{t-1}^f \frac{B_{t-1}^f}{S_t} + W_t H_t + \Pi_t + R_t^e Z_t^e K_{t-1}^e + R_t^n Z_t^n K_{t-1}^n \tag{26}$$

This gives a clear picture of how household have the option to hold either domestic or Foreign bonds. r denotes domestic interest rate and r^f[22] denotes exogenous world interest rates, given that world prices are exogenous. Π denotes income profits from firm ownership.

Households decide on what capital stocks K_{t-1}^j to choose as new capital must take one-quarter lag to become effective. The model assumes households have access to technology after decision on which sector to install capital in the previous quarter.

[21] The shock is assumed to follow a first order autoregressive process with an i.i.d. normal error term: $\varepsilon_t = \rho_\varepsilon \varepsilon_{t-1} + \eta_{\varepsilon,t}$

[22] See footnote 1.

$$K_t^e = \left(1 - \delta(Z_t^e)\right)K_{t-1}^e + \xi_{INV,t}^e I_t^e + \chi_e \left(\frac{K_t^e}{K_{t-1}^j}\right) \tag{27}$$

$$K_t^n = \left(1 - \delta(Z_t^n)\right)K_{t-1}^n + \xi_{INV,t}^n I_t^n + \chi_n \left(\frac{K_t^n}{K_{t-1}^n}\right) \tag{28}$$

where I^j denotes sector-specific gross nominal investment. $\delta(.)$ denotes sector-specific time varying depreciation rate of capital installed: for $0 \leq \delta(.) \leq 1,\ \delta'(.) > 0, and\ \delta''(.) > 0$. Z^j denotes capital utilization rate of each period's effective capital installation. ξ_{INV}^j denotes sector-specific exogenous investment-specific technology shock[23]. χ_j denotes adjustment costs which depends on the rate at which each sector adjusts its price, for $\chi_j > 1$. The assumption is consistent with standard DSGE literature.

Subject to the budget constraint, the household maximise[24] their expected lifetime utility value with the sequence $\left\{C_t, H_t, I_t, I_t^e, I_t^n, , B_{t+1}^f, Z_t^e, Z_t^n, K_t^e, K_t^n\right\}_{t=0}^{\infty}$.

2 Firms

The sectoral outputs' production functions are assumed to be homogeneous-of-degree-one, following Kim and Loungani (1992) that includes primary energy use E^e as input, which differs from the standard neo-classical practice. The representative firm's technology employs a production function which can be characterized as a nested constant-elasticity of substitution (CES) specification of the form:

$$Y_t^e = A_t^e F^e(H_t^e, Z_t^e K_{t-1}^e, O_t^e E_t^e) = D_{d,t}^e + X_t^e \tag{29}$$

The equation above is the production function of the energy intensive sector with sector-specific endogenous variables and exogenous shocks. Y^e denotes the sector nominal output, measured in the nominal value of the numeraire, world imports price of US Dollars. $F^e(.)$ obeys the standard regularity conditions, A^e denotes the exogenous energy intensive sector productivity shock, H^edenotes sector's labour demand, $Z^e K_{-1}^e$ denotes demand for capital

[23] See footnote 1.

[24] A consolidated budget constraint of the model is shown in the sub-chapter of log-linearized version of the model.

services in the sector and O^e denotes the exogenous energy intensive sector energy input efficiency shock.[25]

D_d^e, denotes domestic absorption, states that the sectoral output can either be consumed at home or to be exported X^e to satisfy the world demand.

Given the above assumption, and firms in the energy intensive sector are also perfectly competitive, the typical firms maximises the following profit function subject to the budget constraint in equation (29): $max\,\Pi_t = P_t^e Y_t^e - \left(W_t H_t^e + R_t^e Z_t^e K_{t-1}^e + P_{O,t} E_t^e\right)$ where P^e and R^e denotes the relative price of energy intensive goods and rental rate of capital services for the sector, respectively. World prices are exogenous, hence, we assume energy prices[26], P_O, to follow an exogenous process adjusts immediately to their world prices. E^e denotes nominal energy use, in the sector, the value is measured in US Dollars given assumption of the numeraire of world imports.

United Kingdom is a net importer of energy (crude oil, in this study). The energy (non-energy) extensive sector output has a CES production function of homogeneous-of-degree-one with properties similar to the energy intensive sector, denoted by superscript n, is

$$Y_t^n = A_t^n F^n(H_t^n, Z_t^n K_{t-1}^n, O_t^n E_t^n) = D_{d,t}^n + X_t^n \tag{30}$$

where the exogenous shocks and endogenous variables are similar to the energy-intensive sector. Y^n denotes the sector nominal output, measured in the nominal value of the numeraire, world imports price of US Dollars. $F^n(.)$ obeys the standard regularity conditions, A^n denotes the exogenous energy extensive sector productivity shock, H^n denotes sector's labour demand, $Z^n K_{-1}^n$ denotes demand for capital services in the sector and O^n denotes the exogenous energy extensive sector energy input efficiency shock. D_d^n, denotes domestic absorption, states that the energy extensive sector output can either be consumed at home or to be exported X^n to satisfy the world demand. The UK has a very high of its exports, services, from this sector.

Given the above assumption, and firms in the energy extensive sector are also perfectly competitive, the typical firms maximises the following profit function subject to the budget

[25] Sector-specific energy efficiency shock is a factor-augmenting technology. This energy efficiency in production, which might capture a switch in the composition of capital towards machines with different energy intensities. Also, see footnote 1.

[26] Initially, we assumed $P_t^o = \frac{P_t^{of} P_t^f}{S_t}$ as the energy price shock, like in Harrison, et al., (2011). $P_t^{of} P_t^f$ is the assumed world exogenous price but after linearization, the data residual is equal to observed price. See residual plots in previous chapter. We simply assumed world energy price shock to avoid complications in the model and reduce the number of equations. Again, see footnote 1.

constraint in equation (6): $max\,\Pi_t = P_t^n Y_t^n - \left(W_t H_t^n + R_t^n Z_t^n K_{t-1}^n + P_{O,t} E_t^n\right)$ where P^n and R^ndenotes the relative price of energy extensive goods and rental rate of capital services for the sector, respectively. P_O represents the world price of energy. E^n denotes nominal energy use, in the sector, the value is measured in US Dollars given assumption of the numeraire of world imports.

Given that, the respective demand for labour, capital and energy use in the energy intensive sector is

$$W_t = P_t^e A_t^e F_1^e (H_t^e, Z_t^e K_{t-1}^e, O_t^e E_t^e) \tag{31}$$

$$R_t^e = P_t^e A_t^e F_2^e (H_t^e, Z_t^e K_{t-1}^e, O_t^e E_t^e) \tag{32}$$

and

$$Q_t = P_t^e A_t^e O_t^e F_3^e (H_t^e, Z_t^e K_{t-1}^e, O_t^e E_t^e) \tag{33}$$

and the respective demand for labour, capital and energy use in the energy extensive sector is

$$W_t = P_t^n A_t^n F_1^n (H_t^n, Z_t^n K_{t-1}^n, O_t^n E_t^n) \tag{34}$$

$$R_t^n = P_t^n A_t^n F_2^n (H_t^n, Z_t^n K_{t-1}^n, O_t^n E_t^n) \tag{35}$$

and

$$Q_t = P_t^n A_t^n O_t^n F_3^n (H_t^n, Z_t^n K_{t-1}^n, O_t^n E_t^n) \tag{36}$$

The first-order condition, of the above two sectors, gives the marginal productivity of each input relative to its marginal cost given the assumption of perfect competitive firms.

3 Government

Following An and Schorfheide (2007) and Justiniano, et al., (2009), the fiscal authorities are assumed to be fully Ricardian, and the following budget constraint does not change over time. This is based on the assumption that the government will continue to adjust taxes and through the monetary authority adjusts interest rates to achieve its policy objective.

$$G_t + B_t = T_t + E_t R_{t+1} B_{t+1} \tag{37}$$

where G represents the exogenous government spending shock following a univariate autoregressive form[27]. The budget deficit of the government is financed by issuing short term bonds to households. Therefore, households can access the domestic bond market where nominal government bonds, that pay a gross interest rate R_t, are traded.

[27] See footnote 1.

4 International Trade

We assume in this model, logically, that the United Kingdom is an open economy. We also assume that consumption, investment and government are composites of United Kingdom's and world's sectoral goods. We can note that by definition:

$$C_t = \Phi_c\left(C_t^{d,e} C_t^{f,n} C_t^{d,e} C_t^{f,n}\right) \tag{38}$$

$$I_t = \Phi_I\left(I_t^{d,e} I_t^{d,n} I_t^{d,e} I_t^{f,n}\right) \tag{39}$$

and

$$G_t = \Phi_G\left(G_t^{d,e} G_t^{d,n} G_t^{f,e} G_t^{f,n}\right) \tag{40}$$

for $J = C, I, G$, where Φ_j is the Armington aggregator, CES utility function with homothetic preferences assumed to be homogenous-of-degree-one and increasing. For all variables J, superscripts d, e denotes demand for domestically produced goods in the energy intensive sector, while superscripts d, n denotes demand for domestically produced goods in the energy extensive sector (non-energy). Superscripts f, e denotes demand for foreign produced goods in the energy intensive sector, while superscripts f, n denotes demand for foreign produced goods in the energy extensive sector (non-energy). In order to maintain focus on the macro-variables, we choose to use aggregate expenditures of variables j, and in that way, the total sum of these variables yields the domestic absorption:

$$D_t = C_t + I_t + G_t \tag{41}$$

where

$$D_t = \kappa\left(D_t^d, M_t\right) \tag{42}$$

This means D is a composite for the four outputs. The Armington aggregator function here, κ, is assumed to be homogeneous-of-degree-one and increasing in both arguments. D^drepresents the households' demand of goods produced in the United Kingdom and M denotes the total spending on imported goods. Unlike Bakus et al., (1993) where they assumed two goods in an open economy, this model assumes four produced goods in the world which require some more disaggregation.

Here, D and M are assumed to be a function both sectoral outputs, we can note that by definition:

$$D_t = \Sigma(D_t^e, D_t^n) \tag{43}$$

and $\quad M_t = \Xi(M_t^e, M_t^n)$ (44)

where the Armington aggregator functions of $\Sigma\ and\ \Xi$ are homogeneous-of-degree-one and increasing in both arguments. $D^e\ and\ D^n$ represents the nominal expenditure on domestic output from the energy intensive and energy extensive (non-energy) sectors by domestic agents, respectively. Similarly, $M^e\ and\ M^n$ represents the nominal expenditure on imports from the energy intensive and energy extensive (non-energy) sectors by households, respectively.

The domestic agents will $min\{P_t^d D_t^d + P_{M,t}M_t - S_t D_t\}$ subject to equations (12) where P^d is the price index of composite goods produced in the United Kingdom while S is the consumer price index of the United Kingdom. P_M is world's price index of composite goods, assumed to be the numeraire in the model. Given that, therefore, S is also the nominal exchange rate variable. The agents have another problem of $min\{P_t^e D_t^e + P_t^n D_t^n - S_t D_t\}$ subject to equation (13) with the assumption of Walras' law that "all markets clear", the energy extensive sector goods market is silent, here, as the law implies the market will clear. The domestic agents will, also, solve the problem of share of imported goods expenditure in the respective sectors by using the budget constraint of equation (14) to solve $min\{P_{M,t}^e M_t^e + P_{M,t}^n M_t^n - M_t\}$. P_M^e and P_M^n are imports prices in the energy intensive and extensive sectors, respectively. Like energy prices, world prices are exogenous as they adjust to their world prices, therefore, imports prices are treated as exogenous shocks in this model[28]. The imports function will be used to set-up the world's demand (exports) function:

$$D_t^w = \kappa^w\left(D_t^f, M_t^f\right) \tag{45}$$

Similarly, where the model assumes $D_t^w = C_t^w + I_t^w + G_t^w$ as the aggregate world demand, D^f denoted as world's demand for home goods and M^f denotes the total imports in the world's economy which signifies the United Kingdom's nominal exports (X). where κ^w is homogeneous-of-degree- one and increasing in both its arguments.

5 Aggregation, Market clearing and the resource constraint

The assumption of this two-sector model is to have a total nominal output that produce in the domestic country, simply given as:

$$Y_t = Y_t^e + Y_t^n \tag{46}$$

[28] See footnote 1.

where Y is denoted as total nominal output and Y^e and Y^n are the sectoral output of the energy intensive firm and the energy extensive firm, respectively. The aggregate for labour supply and total energy use are:

$$H_t = H_t^e + H_t^n \tag{47}$$

$$E_t = E_t^e + E_t^n \tag{48}$$

Aggregate investment is defined as:

$$I_t = I_t^e + I_t^n \tag{49}$$

where I^e and I^n are sector-specific investment.

Energy intensive sector market clears

$$Y_t^e = D_t^e + X_t^e - M_t^e \tag{50}$$

Final production satisfies demand as:

$$Y_t = C_t + I_t + G_t + X_t - M_t - E_t \tag{51}$$

It means the aggregate resource constraint is describing how the output is absorbed by consumption, investment, governments exogenous spending, net exports and energy use.

The dynamic of the current account equation is given as:

$$B_t^f - \left(1 + R_{t-1}^f\right)B_{t-1}^f = S_t X_t - M_t - P_{O,t} E_t \tag{52}$$

Above denotes aggregate resource constraint describing how the net foreign assets of the economy evolve. The left-hand side shows the changes made in foreign asset holdings within one period lag while the right-hand side states the expenditures of net exports, with imports price assumed to be the numeraire in the model, and primary energy use yielding adjustment of bond wealth.

Table 6: Fixed Parameters

Parameter	Value	Description
β	0.99	Discount factor
δ^e	0.0125	Depreciation rate energy intensive sector
δ^n	0.0125	Depreciation rate non-energy intensive sector
$1-\alpha_e$	0.65	Labour share in energy intensive sector
$1-\alpha_n$	0.72	Labour share in non-energy intensive sector
θ_e	0.9998	Capital services weight in energy intensive sector
θ_n	0.9999	Capital services weight in non-energy intensive sector
$\frac{c}{y}$	0.1773	Share of private consumption in total output
$\frac{i}{y}$	0.2019	Ratio of investment to total output
$\frac{x}{y}$	0.2933	Share of exports in total output
$\frac{m}{y}$	0.3126	Ratio of imports to total output
$\frac{m^e}{y^e}$	0.0990	Ratio of imports to output in energy intensive sector
$\frac{e}{y}$	0.2355	Share of energy use in total output
$\frac{g}{y}$	0.1773	Share of government consumption in total output
$\frac{y^e}{y}$	0.6145	Ratio of energy intensive output to total output
$\frac{y^n}{y}$	0.3855	Ratio of non-energy intensive output to total output
$\frac{i^e}{i}$	0.3320	Ratio of investment in energy intensive sector to total investment
$\frac{i^n}{i}$	0.6680	Ratio of investment in non-energy intensive sector to total investment
$\frac{e^e}{e}$	0.710	Ratio of energy usage in energy intensive sector to total energy usage
$\frac{i^e}{k^e}$	0.0420	Ratio of investment to capital in energy intensive sector
$\frac{i^n}{k^n}$	0.0362	Ratio of investment to capital in non-energy intensive sector
$\frac{c}{d}$	0.6514	Share consumption in domestic absorption
$\frac{i}{d}$	0.1869	Ratio of investment in domestic absorption
$\frac{g}{d}$	0.1617	Share of government consumption in domestic absorption
$\frac{p^e}{s}$	0.7753	Ratio of price to exchange rate in energy intensive sector
$\frac{p^n}{s}$	0.9448	Ratio of price to exchange rate in non-energy intensive sector

$\frac{e^e}{k^e}$	0.0827	Energy-capital ratio in energy intensive sector
$\frac{e^n}{k^n}$	0.0289	Energy-capital ratio in non-energy intensive sector
$\frac{h^e}{h}$	0.1710	Ratio of employment in energy intensive sector to total employment
$\frac{h^n}{h}$	0.8290	Ratio of employment in non-energy intensive sector to total employment
$s\frac{x}{b^f}$	0.2057	Ratio of demand for exports to foreign bonds
$\frac{m}{b^f}$	0.2134	Ratio of demand for imports to foreign bonds
$\frac{e}{b^f}$	0.1584	Ratio of energy demand to foreign bonds

Data

The data for endogenous variables and exogenous forcing processes cover the period 1990Q1 to 2014Q4. We aim at going further back, but the data availability of some structural variables such as sectoral output and energy use only starts from 1990Q1. Due to this constraint, we are able cover the crisis periods during of the 2008 financial crisis. The definition of energy intensive sector as regards to data collection is the combination of industries in the UK that spends over 3% of their production cost on crude oil products. This definition is similar to the definition of EU 2000 Regulation on Pollution Prevention and Control that define energy intensive sector in terms of energy use. These industries include Agriculture, Production Sector, Construction sector, and finally Transport & Storage from the Services sector. The non-energy intensive sector is the sector of the economy that use less than 3% of their cost on crude oil products. These include: Services industry that includes Accommodation & Food Service Activities, Information & Communication, Financial and Insurance Activities, Real Estate Activities, Professional, Scientific & Technical Activities, Administrative and Support Service Activities, Public Administration, Education, Health and Social Work, Arts, Entertainment and Recreation, and Other Service Activities.

We use the three-month Treasury bill rate series, for the interest rate, from Bank of England database (IUQAAJNB). For exchange rate, we use Quarterly Average Effective exchange rate index XUQABK67 from Bank of England. The data for output (GDP), capital stock, energy use, are collected from the ONS. We use final consumption expenditure of households and NPISHs (ABJQ.Q + HAYE.Q). For total hours of employment, We use ONS series of (YBUS.Q). Real wages We divided UK wages (XPEWF.B) from ONS series by total hours worked (YBUS.Q) and then divided through by consumption deflator where the consumption deflator is (ABJQ.Q + HAYE.Q)/(ABJM.Q + HAYO.Q). The foreign bonds are the UK investment abroad which net acquisition of financial assets are by Monetary financial institutions, Central government Local government, Public corporations and other sectors (UKHBNR). Capital utilisation rate is represented by Manufacturing sector utilisation rate and the corporate sector utilisation rate for the energy intensive sector and non-energy intensive sector, UKCBICAPE and UKXCAPU.R, respectively.

For world data, we use the series of the world import prices (Q76.X.F), for energy (crude oil, as proxy) prices we collected the world prices of crude oil (WDXWPOB). We deflate the variables by, the numeraire, world's manufacturing price index by using the weighted average of some OECD countries: Canada, Germany, France, Japan, Italy, South Africa and the United States. We seasonally adjust energy use, world prices and world demand. Likewise, the foreign interest rate is a weighted average of the stated OECD countries. All variables are in per capita basis, this is done by dividing through by a UK working-age population before taking natural logs and all were detrended using Hodrick-Prescott (HP) filter setting the smoothing parameter λ=1600 we use the ONS quarterly series (UKMGSL.Q) for population.

Table 7 List of industries that make up the two-sector firms

Industries	
Energy intensive firms	Non-energy intensive firms
Agriculture, Forestry & Fishing	Wholesale Trade
Mining & Quarrying	Retail Trade & Repairs
Food Products, Beverage and Tobacco	Accommodation & Food Service Activities
Textiles, Leather & Clothing	Information & Communication

Chemicals & Man-made Fibres	Financial & Insurance Activities
Basic Metals & Metal Products	Real Estate Activities
Engineering & Allied Industries	Professional, Scientific & Technical Activities
Other Manufacturing	Administrative & Support Service Activities
Electricity, Gas & Water Supply	Public Administration
Transport & Storage	Education
Construction	Health & Social Work

Some observations of the sectoral data

Observed output of the UK during the 2008 financial crisis show output declining during that period. However, sector contribution of output show that the non-energy intensive sector, see Figure 11, did not suffer as much as the energy intensive sectors. This is despite the financial sector taking a big hit during the financial crisis. Oil, a feedstock and transportation fuel, is the most significant commodity in terms of economic effects to the transportation sector among many other sectors that are energy intensive in their production. One can see the contribution of output in the United Kingdom from transportation and storage sector is only recovering due to energy prices shock since the financial crisis, see Figure 12, due to the sector's dependence on transport fuels. The study of energy prices in a macroeconomic model will likely make the improvement by giving households, firms and policymakers a window to plan for alternatives. Energy prices have also directly affected other macroeconomics variables, such as exchange rates, foreign demand of goods and foreign exports prices as found in this study.

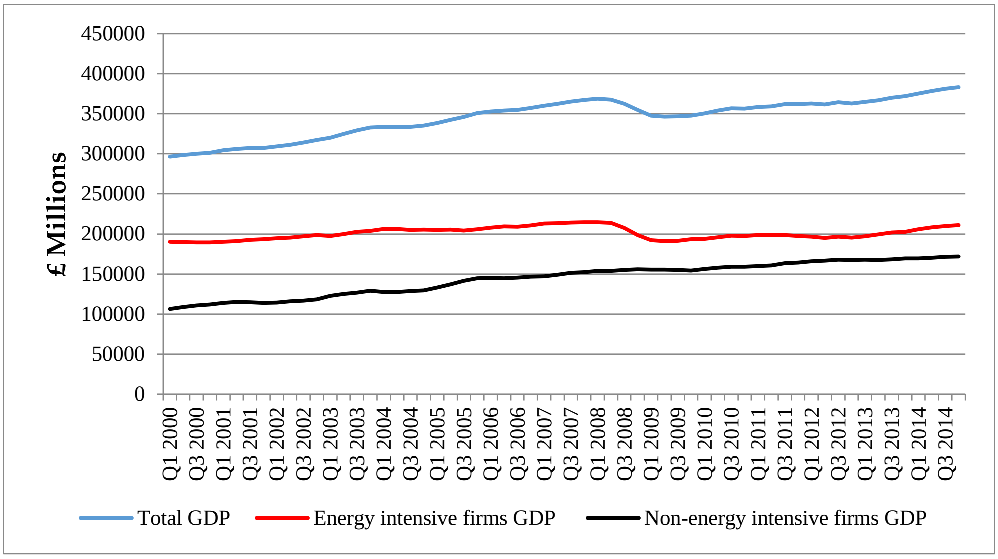

Figure 10 Energy intensive sector and Non-energy intensive sector contribution of output in the United Kingdom Source: Authors from ONS data

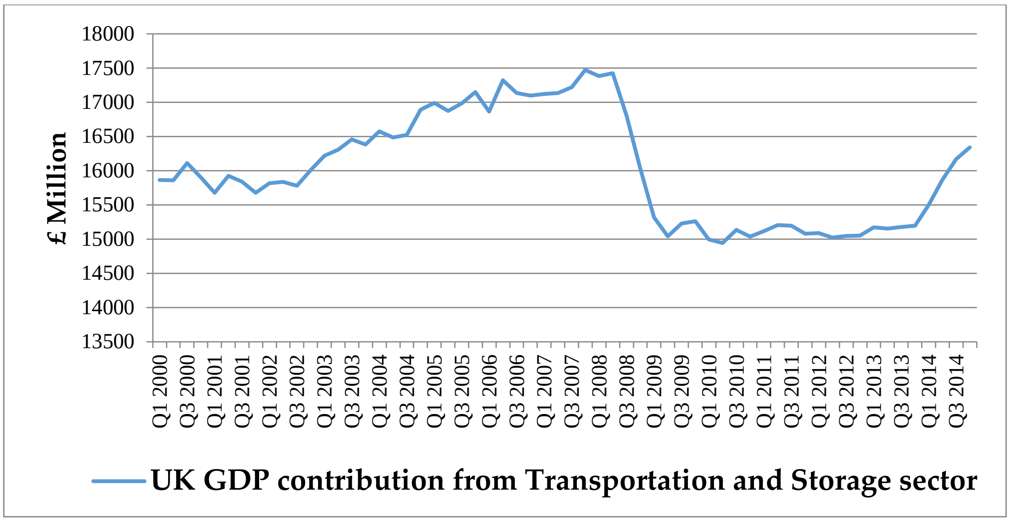

Figure 11 output contribution of the Transportation sector in the United Kingdom. Source: Authors from ONS data

Figure 12 Unfiltered data of the UK

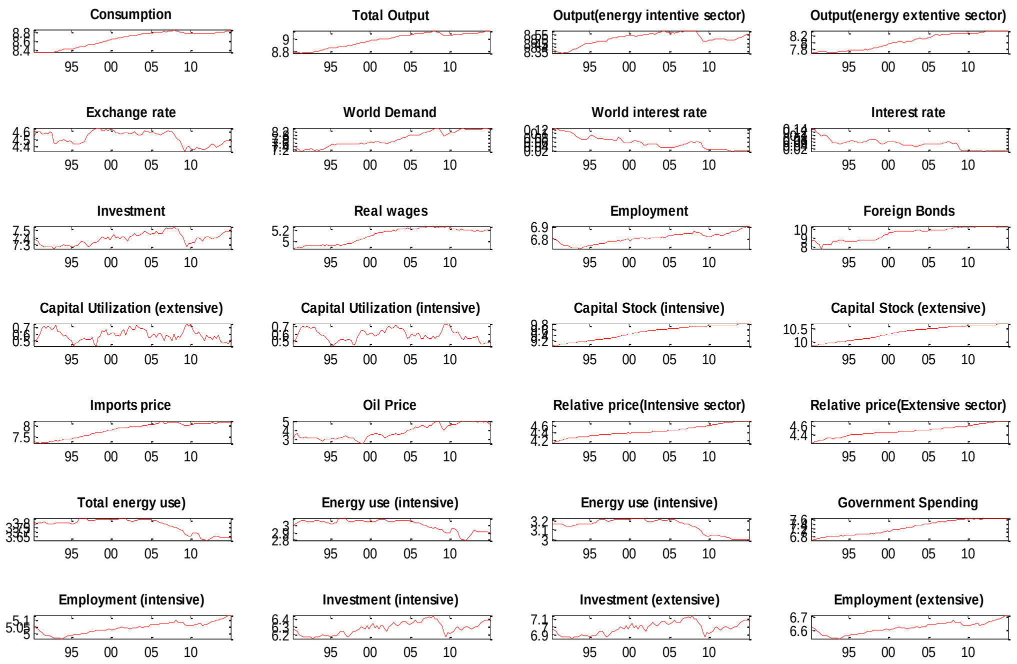

Table 8: Stationarity Test

Shock	**Process**	***c***	**trend**	**AR(1)**
Productivity (energy-intensive sector)[29]	Non-stationary		-2.3387	0.3394
Productivity shock (non-energy intensive sector)	Non-stationary		-1.0939	0.1896
Consumption preference	Stationary	0.1966		0.4367
Government spending *[30]	Stationary	0.2082		0.9894
Investment Specific-Tech. shock (non-energy)	Stationary	0.1082		0.9209
Investment Specific-Tech. shock (non-energy)	Stationary	0.1045		0.8696
Energy efficiency (energy intensive sector)	T-stationary		0.0589	0.9039
Energy efficiency shock (non-energy)	T-stationary		0.0599*	0.8954
World exports price	T-stationary		0.1013	0.9741
Energy price	Non-stationary		-3.6603	0.2257
World interest rate	T-stationary		0.0904*	0.9227
Labour supply	T-stationary		0.2108*	0.8568
World demand	T-stationary		0.1587*	0.9250

[29] Negative numbers come from ADF test while others show result from KPSS test.
[30] * 1% level of significance

Following the result above showed the sector-specific productivity shocks and energy price shocks are tested to be nonstationary[31]. The results are concluded following a stationarity test of KPSS and ADF.

The auxiliary equation

A linearized RBC model can be written as:

$$A(L)y_t = BE_t y_{t+1} + C(L)x_t + D(L)e_t \tag{53}$$

where y_t are the number, p, of endogenous variables and x_t are the number, q, of exogenous variables that are driven by the assumed equation:

$$\Delta x_t = a(L)x_{t-1} + d + c(L)\epsilon_t \tag{54}$$

As stated earlier, based on using non-stationary data, the exogenous variables can have observed and unobserved variables such as the world oil prices shock and productivity shocks. The errors $e_t \, and \, \epsilon_t$ are iid variables each with a zero mean. L symbolises the lag operator where $\pi_{t-s} = L^s \pi_t$ and $A(L), B(L)$… are polynomial functions each with its root outside the unit circle. Therefore, the solution for y_t, where it follows $y_t \, and \, x_t$ are non-stationary, will be:

$$y_t = G(L)y_{t-1} + H(L)x_t + f + M(L)e_t + M(L)\epsilon_t \tag{55}$$

where polynomial functions each with its root outside the unit circle. As $y_t \, and \, x_t$ are non-stationary, a p cointegration relation will have the solution as:

$$y_t = [I - G(1)]^{-1}[H(1)x_t + f] \tag{56}$$

$$= \Pi x_t + g \tag{57}$$

and a long-run solution of the model will be:

$$\bar{y}_t = \Pi \bar{x}_t + g \tag{58}$$

$$\bar{y}_t = [1 - a(1)]^{-1}[dt + c(1)\xi_t] \tag{59}$$

[31] Thus, we use first-difference in the shock estimation: $\varepsilon_t = \varepsilon_{t-1} + \rho(\varepsilon_{t-1} - \varepsilon_{t-1}) + \eta_t$.

$$\xi_t = \Sigma_{i=0}^{t-1} \epsilon_{t-s} \tag{60}$$

In the long-run solution, x_t, defined as $\bar{x}_t = \bar{x}_t^D + \bar{x}_t^S$ will have a deterministic trend represented as $\bar{x}_t^D = [1 - a(1)]^{-1} dt$ and a stochastic trend represented as $\bar{x}_t^S = [1 - a(1)]^{-1} c(1) \xi_t$.

One can now re-write the solution for y_t as the VECM

$$\Delta y_t = -[I - G(1)](y_{t-1} - \Pi \bar{x}_{t-1}) + P(L)\Delta y_{t-1} + Q(L)\Delta x_t + f + M(L)e_t + N(L)\epsilon_t \tag{61}$$

$$= -[I - G(1)](y_{t-1} - \Pi \bar{x}_{t-1}) + P(L)\Delta y_{t-1} + Q(L)\Delta x_t + f + \omega_t \tag{62}$$

$$\omega_t = M(L)e_t + N(L)\epsilon_t \tag{63}$$

The disturbance of ω_t is assumed to be a mixed moving average process which means that the VECM may be estimated by the VARX

$$\Delta y_t = K(y_{t-1} - \Pi x_{t-1}) + R(L)\Delta y_{t-1} + S(L)\Delta x_t + g + \zeta_t \tag{64}$$

where ζ_t is an iid process with a zero mean as $\bar{x}_t = \bar{x}_{t-1} + [1 - a(1)]^{-1}[d + \epsilon_t]$ and Finally, the VECM can be written as

$$\Delta y_t = K(y_{t-1} - \bar{y}_{t-1}) + \Pi(x_{t-1} - \bar{x}_{t-1}) + R(L)\Delta y_{t-1} + S(L)\Delta x_t + h + \zeta_t \tag{65}$$

The latter two equation can be used as the auxiliary model, but equation (65) shows the difference between the effects of the trend elements in *x* and temporary deviations it has from the trend. The estimation of (65) is done by OLS because it is straight forward and efficient.

Table 9: Variance decomposition of sectoral variables

	Productivity (Energy)	Productivity (Non-energy)	Energy price	Invest. specific-Technology (Energy)	Invest. specific-Technology (Non-energy)	Government spending	Consumption preference	Energy efficiency (Energy)	Energy efficiency (Non-energy)	Foreign interest rate	Foreign demand	Foreign exports price	Labour supply
Energy intensive sector													
GDP	30.8	19.2	42.4	2.2	0.1	0.0	0.2	2.7	0.4	0.0	0.4	0.1	1.6
Investment	0.7	15.1	65.3	3.4	0.0	0.1	0.8	10.7	0.4	0.0	1.6	0.0	1.8
Employment	12.6	55.0	16.0	0.1	0.3	0.1	1.3	2.7	2.0	0.0	0.6	0.4	9.1
Energy use	0.2	5.8	85.6	1.4	0.0	0.0	0.0	6.7	0.0	0.0	0.2	0.0	0.1
Capital stock	0.4	6.5	29.9	55.8	0.0	0.5	1.2	2.8	0.2	0.0	0.6	0.0	2.1
Capital utilisation	0.4	13.1	62.7	14.4	0.0	0.0	0.0	8.5	0.1	0.0	0.5	0.0	0.2
Price of goods	68.1	18.3	1.2	6.0	0.0	0.0	0.4	2.4	0.3	0.0	2.0	0.0	1.2
Domestic Absorption	23.6	18.0	51.0	0.9	0.0	0.0	0.0	4.6	0.2	0.0	0.6	0.0	1.1
Exports	39.4	14.0	9.8	7.1	0.2	0.0	1.6	0.1	3.7	0.0	19.5	0.0	4.6
Imports	3.6	14.3	42.1	3.0	0.1	0.0	1.7	9.7	5.6	0.1	13.4	2.1	4.3
Energy extensive sector													
GDP	1.1	36.4	56.5	0.0	0.2	1.9	0.4	1.9	0.1	0.0	0.1	0.1	1.3
Investment	2.3	9.2	23.8	2.3	17.3	1.5	8.9	11.8	3.1	0.2	7.1	0.0	12.6
Employment	3.4	57.2	35.7	0.1	0.0	1.0	0.5	0.1	0.6	0.0	0.1	0.0	1.4
Energy use	0.3	1.0	84.5	0.0	0.1	0.1	0.0	0.5	13.5	0.0	0.1	0.0	0.0
Capital stock	0.2	0.2	1.0	0.0	82.3	1.2	3.0	5.0	0.4	0.0	1.4	0.0	5.2
Capital utilisation	0.6	2.1	94.2	0.0	1.5	0.2	0.0	0.9	0.3	0.0	0.2	0.0	0.0
Price of goods	7.7	67.2	17.9	0.1	0.2	0.0	0.3	2.6	2.0	0.0	1.0	0.0	0.9