# Taxes, Tariffs and Trade Costs under Oligopoly

By

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### Abstract

This study compares *ad valorem* and per-unit taxes in public finance and international trade and examines the welfare effects of trade cost in general oligopolistic equilibrium (GOLE). In chapter two, following Grazzini (2006), the welfare comparison of *ad valorem* and per-unit taxation is conducted in an exchange economy under Cournot competition. It is shown that the exceptional result in Grazzini (2006) that a per-unit tax can be welfare superior to an *ad valorem* tax, entirely depends on the form of social welfare function. Furthermore, the possibility of the dominance of per-unit taxation is due to the effect of taxation on the redistribution of income rather than from any efficiency gain.

In chapter three, assuming that the home government maximises the tariff revenue, the welfare with *ad valorem* tariff is higher than that with per-unit tariff given the constraint of FDI cost. The maximum revenue collected by the home government is always higher with *ad valorem* tariff under Cournot competition. However, under Bertrand competition with differentiated products, the maximum revenue with per-unit tariff is higher than that with *ad valorem* tariff if the FDI cost is sufficiently low. This is because the introduction of product differentiation and nature of Bertrand competition both intensify the competition and lower the prices.

In chapter four, by using the general oligopolistic equilibrium (GOLE) model developed by J. Peter Neary, it is shown that social welfare is also U-shaped in the trade cost under Cournot competition. The result is in line with Brander (1981) and Brander and Krugman (1983). In particular, when the trade cost is sufficiently high, a reduction in trade cost will increase the competitive wage due to the redistribution of labour, and the equilibrium prices as a function of trade costs follows a hump-shaped pattern if the products are homogeneous.

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### **Chapter 1 : Introduction**

#### 1.1 Modelling oligopoly

Models based on perfect competition dominated mainstream thinking about both positive and normative aspects of the trade from its inception in the work of Ricardo (1817). Ricardo (1817) outlined the principle of comparative, making an international trade the first applied domain of political economy. Traditional trade theory has been developed using models of perfect competition in which each firm is short of market power and does not act strategically.

The so-called new trade theory attracts a great number of efforts on the implications for international trade of imperfectly competitive models. Since 1979, there are two distinct directions have been developed to incorporate imperfect competition into international theory, which are monopolistically competitive models and oligopolistic models. It is worth noting that the former one fascinates a great amount of literature and immediately turns to be the more popular one. It is so-called "two and a half theories of trade" in the words of Paul Krugman (Peter Neary (2010)). There is also a "new" new trade theory starting with the pioneering work by Melitz (2003), assuming that firms within an industry differ in productivity (i.e., firms are heterogeneous) and using monopolistic competition .

In monopolistic competition models, it is natural to make the following three assumptions. To begin with, it is assumed that there are no strategic interactions. In other words, firms assume that their price will not affect the marginal utility of income of consumers. Due to the great number of firms, each firm ignores the effect of other firms' prices on their demand. Meanwhile, with respect to monopoly pricing, each firm confronts a downward-sloping demand curve. In terms of the issue of free entry, firms enter the industry until profits are driven to zero for all participants. Models of monopolistic competition permit each firm to have a finite level of market power but rule out strategic interaction by supposing that firms recognise themselves too tiny to influence the prices of others.

Oligopoly is a market structure in which a market is shared by a small number of large firms. These firms have significant influence over the industry. The followings are some reallife examples of oligopolies in the UK: Six utilities (EDF Energy, Centrica, RWE npower, E.on, Scottish Power and Scottish and Southern Energy) share the majority of the UK retail electricity market; Four core signal providers (EE, Vodafone, O2 and 3 Mobile) dominate the mobile phone networks; The detergent market is dominated by two companies (Unilever and Procter & Gamble). The basic difference between monopolistic competition models and oligopolistic competition models is that under oligopoly firms perceive that their action impacts the action of their rivals and they perceive that this is known by their rivals and they perceive that their rivals know this situation and so on and so forth (i.e. what is the so-called common knowledge).

It is widely acknowledged that embedding oligopoly model in general equilibrium suffers a number of severe technical problems<sup>1</sup>. To begin with, if firms are large in their own sector, then the firms can directly affect both economy-wide income and factor prices. Consequently, assuming the firms act rationally, they should take this into account in making their decisions. Such behaviour is of extreme difficulty to model. In addition, large firms have impacts on the cost of living, and rational shareholder should take account of their effect on the marginal utility of income when making their decisions. This issue was put forward by Gabszewicz and Vial (1972), who claim that modelling oligopoly in general equilibrium is sensitive to the choice of numeraire. Lastly, according to Roberts and Sonnenschein (1977), it is difficult to obtain the resulting reaction functions so that there may be no equilibrium. For instance, discontinuous and concave reaction functions may exist in general equilibrium.

There have been some attempts to embed oligopoly models in general equilibrium. For example, Cordella and J Gabszewicz (1997) assume that firms are owned by worker-producers, who maximise utility rather than profits. Dierker and Grodal (1999) assume that firms aim at maximising shareholders' wealth, taking account of how their choices influence the deflator for nominal wealth. Ruffin (2003a, 2003b) attempts to model oligopoly in general equilibrium by assuming a finite number of sectors and firms can set prices in good markets but not in factor markets. In other words, the typical agent is assumed to behave schizophrenically, i.e., affecting prices as a producer and accepting prices as given as a consumer. Chapter two and chapter four model oligopoly in general equilibrium. The analysis of chapter two is cast into a particular oligopoly model, in which oligopolists are simultaneously consumers and workers, while chapter four adopts the general oligopolistic equilibrium (GOLE) model developed by Neary (2002b, 2003a, 2003b).

<sup>&</sup>lt;sup>1</sup> Bonanno (1990) provides a comprehensive survey on equilibrium theory with imperfect competition.

#### **1.2 Taxes and tariffs under oligopoly**

Taxes are involuntary charged by a government entity in order to finance government activities. In general, taxes can be divided into two main categories: direct taxes and indirect taxes. Direct taxes such as income tax and wealth tax, are directly paid by the person on whom they are imposed. The tax burden is borne by the same person on whom they are levied. Direct taxes affect decisions about labour supply, savings, investments, and so forth. In contrast to direct taxes, indirect taxes are levied by a government on goods and services. A typical example of indirect tax is the consumption tax. The burden can be shifted onto the ultimate consumer of the product. An indirect tax may lead to an increase in the price of a good.

The effect of commodity taxes can differ according to whether the proposed tax is based upon the quantity (per-unit tax) or upon the value (*ad valorem*). An *ad valorem* tax is a fixed percentage of the value of the transaction on which it is imposed. Value added tax (VAT) is a typical example of *ad valorem* taxes. In the UK, VAT is charged at the standard rate of 20% on most goods offered for sale<sup>2</sup>. A reduced rate of 5% is payable on items such as children's car seats and gas and electricity. By contrast, a per-unit tax is a tax of a fixed amount on each unit of the commodity. Along with the standard rate VAT at 20%, petrol, tobacco, and alcohol also have per-unit taxes in the UK: Fuel duty is charged at a fixed rate of 57.95 pence per litre; Tobacco duty on cigarettes is charged at 16.5% of the retail price plus £4.34 on a packet of twenty; For a pint of 5.0% strength lager, the beer duty is charged at a standard rate of 19.08 pence litre.

Chapter two compares two forms of tax in public finance. The welfare comparison between per-unit and *ad valorem* taxation has been a popular subject in the study of public finance. In order to study the welfare properties of different forms of taxation, it is of help to understand the meaning of over-shifting of a tax and the measurement of social welfare. A consumption tax will generally increase the consumer price. Over-shifting occurs when price increases by more than the amount of the consumption tax and under-shifting when it increases by less. Over-shifting can only happen in the case of imperfect competition. This is because firms realise that an increase in the tax will reduce demand for their product and due to the existence of market power and strategic behaviour among firms in imperfectly competitive

<sup>&</sup>lt;sup>2</sup> Some things are exempt from VAT, e.g., postage stamps, financial and property.

markets, they are able to increase the price more than the increase in tax in order to compensate for the potential loss owing to the tax policy.

With a change in a government policy, some individuals might be better off while others might be worse off. In welfare economics, one may consider the following issues: whether the allocation of recourses is regarded to be economically efficient or whether the distribution of income is considered to be equitable. However, one problem of the evaluation of welfare is that the normative judgements cannot be avoided. Social welfare functions are often employed to address the problem by assigning weights to different individuals. To be more specific, social welfare function is an aggregation mechanism, which determines a social ordering as a function of individual ordering. A social welfare function aggregates the level of utility received by members of society, and it is a summary of society's attitudes toward different distributions of income and welfare. As individual welfare is increasing in the level of income, social welfare is also positively related to each individual's income.

It is widely known that, under perfect competition, ad valorem taxation and per-unit taxation are equivalent. In imperfect competition, the conventional wisdom suggests that an ad valorem tax is welfare superior to a per-unit tax, as the tax revenue received by the government is higher with an *ad valorem* tax if the price is the same with both taxes. Under monopoly, the pioneer to propose that these two types of taxation required separate analysis was Cournot (1971), writing in the 1830s. Wicksell (1896) put forward that ad valorem taxes dominate perunit taxes in the case of constant marginal cost. He argues that an *ad valorem* tax causes a lower consumer price and therefore more production when tax revenue is the same with both taxes. This dominance was completely demonstrated, with general cost functions, by Suits and Musgrave (1953). In particular, they argued that the government receives lower revenue with per-unit taxes than under a method of *ad valorem* taxes, given that the consumer price and the quantity of the monopoly good stayed unchanged. Under oligopoly, Delipalla and Keen (1992) confirm the welfare dominance of ad valorem taxation both in the short run with a fixed number of firms and the long run with free entry and exit of firms. Skeath and Trandel (1994b) show that, under monopoly, ad valorem taxation Pareto dominates (i.e., higher consumer surplus, profit and tax revenue) per-unit taxation. In addition, they show that the Pareto dominance of ad valorem taxation applies to symmetric Cournot-Nash oligopoly setting when the tax rate exceeds a critical level. Grazzini (2006) embeds oligopoly in general equilibrium to compare ad valorem and per-unit taxation. In particular, Grazzini (2006) considers an exchange

economy with two groups of agents (i.e., consumers and oligopolists) and two types of goods (i.e., good 1 and good 2). Consumers behave competitively on the exchange market, and initially own good 1. Good 2 can only be produced by oligopolists, using good 1 as input. The oligopolists do not own any good initially and behave strategically on the exchange market. The aggregate welfare is measured by the sum of welfare of both groups of agents. Grazzini (2006) claims that per-unit taxation can be welfare superior to *ad valorem* taxation when the number of oligopolists is sufficiently low compared to the number of consumers. In the following analysis in chapter two, I will show that the results Grazzini (2006) are not convincing as the welfare ranking entirely depends on the form of social welfare function. Besides, if the government imposes a lump-sum transfer in the setting of Grazzini (2006), the aggregate welfare is ambiguously higher with *ad valorem* taxation. It can be concluded that the dominance of per-unit tax in Grazzini (2006) is due to the effect of taxation on the redistribution of income rather than from any efficiency gain.

Chapter three examines the welfare comparison between per-unit and *ad valorem* tariffs in international trade. *Ad valorem* and per-unit tariffs were firstly compared by Hillman and Templeman (1985). They show that, when the home country is supplied with imports by a foreign monopoly, an *ad valorem* tariff is welfare superior to a per-unit tariff. The comparison of *ad valorem* and per-unit tariffs under oligopoly starts with Helpman and Krugman (1989). They show that an *ad valorem* tariff welfare dominates a per-unit tariff. Skeath and Trandel (1994a) demonstrate that any per-unit tariff can be replaced by a Pareto-dominating *ad valorem* tariff if the home country is supplied by a foreign monopolist. Moreover, they study the case when the home country faces a foreign oligopoly and find that the Pareto dominance of an *ad valorem* over a per-unit tariff holds if tariffs are sufficiently large. Unlike most of the relevant literature that directly compares the two tariffs, this study introduces a constraint from the potential FDI strategy by foreign firms.

Foreign direct investment (FDI) is an investment (e.g., building new facility) made by a multinational corporation to acquire lasting interest in enterprises operating outside of the economy. For example, Japanese car firms (e.g., Nissan, Toyota and Honda) had chosen the UK as a European base to access the EU market and avoid tariffs. Another example, Apple Inc. has FDI in China due to the fact that it has cheap labour, strong economy, and the biggest population in the world. According to World Investment Report 2017, the global FDI flows reduced by 2 per cent to \$1.75 trillion after a strong rise in 2015. The volume is over four times as large as those in 1995 (\$0.4 trillion). The report also reveals that global flows are expected to increase by about 5 percent in 2017 to almost \$1.8 trillion, continuing to \$1.85 trillion in 2018. In particular, in 2016, flows to developing economies decreased by 14 per cent. However, FDI still remains the largest and most constant external source of finance for developing economies. Flows to developed economies increased by 5 per cent in 2016. As a result, developed economies' share in global FDI inflows rose to 59 per cent.

FDI can be either greenfield (i.e., a brand new plant is built in the host country) or merger and acquisition (i.e., the existing facility was purchased by the multinational corporation). Traditional FDI can be divided into two categories: vertical and horizontal FDI. Vertical foreign direct investment refers to that the situation when the production process is geographically fragmented by stages of production. There are generally two forms of vertical FDI: forward and backward. Forward vertical FDI is an investment in a plant aboard that sells output for a firm's domestic production processes, whereas backward vertical FD is an investment in a plant aboard that offers input for a firm's domestic production processes. Multinational corporations benefit from vertical FDI by moving different stages of the production process to countries with lower costs. Horizontal FDI refers to investment in the same industry abroad that the firm operates in at home. It is a strategy to increase market share in a global economy and is a more attractive way when the trade costs and government intervention are high. In general, FDI is horizontal rather than vertical. This suggests that market access is more important than decreasing production costs as a motive for FDI. There has been a substantial growth in export-platform FDI globally over the past few decades. This type of investment occurs when the output of a foreign affiliate is largely sold in third markets rather than in the host markets.

It is assumed that the foreign firms can supply the domestic market by two alternative ways: by exporting or by undertaking FDI. Importantly, the foreign firms will only choose to export if it is more profitable than undertake FDI. The tariff revenue collected by the home government will be zero if the rate of import tariff is set too high. Therefore, the rate of import tariff imposed by the home government will depend on the value of FDI cost. This study will compare the maximum tariff revenue and welfare between the two regimes of import tariff with the constraint of FDI cost. It is shown that the welfare with *ad valorem* tariff is higher with *ad valorem* tariff than that with per-unit tariff under both modes of competition. The maximum revenue collected by the home government is always higher with *ad valorem* tariff

under Cournot competition. However, the tariff revenue with per-unit tariff can be higher than that with *ad valorem* tariff under Bertrand competition with differentiated products. This is because the introduction of product differentiation and nature of Bertrand competition both intensify the competition and lower the prices, the tariff revenue with *ad valorem* tariff will be very small when the FDI cost is close to zero.

#### **1.3 International trade under oligopoly**

Chapter four focuses on how trade costs affect the economy under oligopoly. Brander (1981) first presents a reciprocal-markets model to study the welfare effects of trade costs under Cournot duopoly. The basic structure of the reciprocal-markets model can be seen as follows. Assume that there are two identical countries. Within each country, two (or more) goods are consumed. The domestic and foreign firms compete in both markets. The reciprocal-markets model is of great help to analyse each country's market in isolation. A crucial assumption of the reciprocal-markets model which makes this possible is that markets are assumed to be segmented, in the sense that oligopolistic firms make separate strategic choices concerning domestic and foreign markets. This assumption indicates that there are no arbitrary opportunities so that prices in the two countries are considered as independent variables. Another common assumption to study one market in isolation is to assume that firms produce under constant marginal costs. This ensures that prices or output decisions in one market do not affect the costs at which other markets can be served. Market segmentation and the assumption of constant marginal costs indicate that changes in the exogenous variables in one market have no implication on the other market.

Brander (1981) considers a single oligopolistic industry and there is only one firm in each country that competes in this industry. The model is symmetric, where both home and foreign firms have the same marginal cost of production and face the same trade costs. Brander and Krugman (1983) extend the analysis to allow general demand functions. Both of them demonstrate that intra-industry trade can happen in equilibrium even when products are identical. In addition, there is a U-shaped relationship between welfare and trade costs. Following by Leahy and Neary (2013), the results in a more general setup that allows for product differentiation will be presented as follows.

#### **1.3.1 Quantity competition**

By considering multilateral free trade between two identical countries under quantity competition with differentiated products in the presence of transport costs, Leahy and Neary (2013) show that oligopolistic competition is an independent determinant of trade. This conclusion can also be applied to the case of two-way trade in Brander (1981) when the products are identical. When products become more differentiated, the volume of trade increases further due to the love of variety property. Another output effect of trade liberalisation is reflected by the dumping margin (i.e., the gap between the prices received by each firm in the domestic and foreign market). Each firm is selling more in its home market than abroad as the marginal cost is higher in its foreign market with the trade cost. As a consequence, the equilibrium price of each firm yields a lower mark-up cost on its foreign sales than on its home sales. The result is called "reciprocal dumping" by Brander and Krugman (1983). It is shown by Leahy and Neary (2013) that the dumping is positively related to the level of trade costs and the substitutability between goods.

Second, consider the effects of trade costs on the profits. By looking at the home firm, its total profits are calculated as the sum of its profits from the domestic and foreign markets. Leahy and Neary (2013) illustrate that profits are decreasing in trade costs in the neighbourhood of zero trade costs, while it is increasing in trade costs in the neighbourhood of autarky. Therefore, with linear demand, profits as a function of the transportation cost are U-shaped in the region where trade occurs. To begin with, from zero trade costs, an increase in the trade costs has a negative effect on the export due to the higher costs in the foreign market. Meanwhile, the home sales are enhanced by an equal increase in its rival's costs. It is shown that the former effect dominates and total profits and sales decrease for a tiny rise in trade costs when the initial trade cost is zero. From autarky, there is no export initially, a small decrease in trade costs have no effect on profits in the export market. However, a fall in the trade costs of the foreign firm will reduce the sales and the profits of the home sales, as they were initially at the monopoly level. Therefore, overall profits are falling in trade cost at autarky.

Lastly, the effect of trade costs on welfare will next be examined. Focusing on the home firm, the welfare equals to the sum of home consumer surplus and the profits of domestic firms in both domestic and foreign markets. It is shown that trade costs are positively related to the prices, as trade liberalisation leads to an increase in competition, in turn, reduces the price. Therefore, consumer surplus increases monotonically as trade costs decrease. Starting from

zero trade costs, both profits and consumer surplus are harmed by an increase in the firm's own costs. Hence, the overall welfare initially falls for a small increase in trade cost. Second, starting from autarky, if there is a reduction in trade costs, consumer surplus will increase since the price decreases as a result of more intensive competition. Meanwhile, profits reduce due to the negative effects on the prices and sales. Therefore, price effects cancel, leaving the negative effect of sales on the overall welfare. In conclusion, welfare as a function of the transport cost is also U-shaped, reaching its maximum at zero trade cost but its minimum below the prohibitive level of trade costs. Brander and Krugman (1983) provide an alternative explanation, where they believe that trade imposes waste because of the transport costs while at the same time decreasing monopoly distortion. First, if the transport costs are very small, cross-hauling is costless and the procompetitive effect outweighs the increased waste of resources. Second, if the transport costs are reduced marginally from prohibitive levels, the procompetitive effect is dominated by the increased waste of resources.

#### **1.3.2 Price competition**

Most of the literature on the welfare effects of international trade under imperfect competition has focused on the Cournot duopoly model. Clarke and Collie (2003) are the pioneers to investigate the welfare effects of international trade in the Bertrand duopoly model with product differentiation. Assuming a two-country model with linear demands and constant marginal costs, they illustrate that welfare under both unilateral and multilateral free trade is always greater than welfare in autarky for any trade cost.

The effects of trade cost on profits and welfare under price competition are very similar to quantity competition for trade costs between zero and the threshold level at which no trade occurs: they are U-shaped in trade costs. However, because the procompetitive effect under Bertrand duopoly is stronger than under Cournot duopoly, there may be a potential threat of exports even when trade costs exceed the threshold level which affects home firm's behaviour. Leahy and Neary (2013) derive that the home firm's outputs at the prohibitive level of trade costs under Bertrand competition are higher than the unconstrained monopoly output level. It follows that the home firm has no incentive to increase its price, as the foreign firm would make positive sales and lower the home firm's profits. The potential competition from the foreign firm will exist until the trade costs reach the prohibitive level under Cournot competition, then the home firm can behave as an unconstrained monopolist. That is to say, a

change in the trade cost still has an impact on the profits and welfare where trade does not actually occur under Bertrand competition because of the stronger procompetitive effect.

The intuition of the U-shape relationship between trade costs and welfare can be seen as follows. Starting from autarky, trade liberalisation increases welfare. This is the region where the home firm is faced with the potential threat of exports even though no actual trade occurs between two countries, so that there are no wasteful transport costs. According to the home firm's profit-maximising strategy, a reduction in trade costs from autarky lead to a lower price. As a result, welfare will be higher. As trade costs fall further to be lower than the prohibitive level in the case of Bertrand competition, imports become profitable and actual trade occurs, leading to a U-shaped relationship between welfare and trade costs as in Cournot competition. Remarkably, Clarke and Collie (2003) have found that there are always gains from trade whatever the level of trade costs, i.e., the minimum level of welfare under trade is higher than the autarky welfare.

This study will analyse the effects of trade costs on social welfare in general equilibrium. Following Neary (2002b, 2003a, 2003b), it is assumed that oligopolistic firms are large in their own sectors but small in the whole economy. Therefore, oligopolistic firms have market power in deciding their output, while they have ignorable effects on aggregate variables. Unlike partial equilibrium, the wage is endogenously determined by the full employment condition in the general equilibrium, and it provides the solution of the aggregate profits and social welfare. when the trade cost is relatively high, a fall in trade cost will increase the equilibrium wage because of a redistribution of labour. The production costs will increase for firms across all sectors as the cost function is assumed to be the Ricardian cost structure in GOLE approach. The equilibrium prices will also increase in response to the increased cost. On the other hand, a reduction in trade costs leads to a fall in the prices due to the competition effect. It is shown that the cost effect dominates when the trade cost is relatively high while the competition effect dominates when the trade cost is low enough. Therefore, the equilibrium prices as a function of trade costs follows a hump-shaped pattern when the products are identical. In terms of social welfare, it is U-shaped in the trade cost, reaching its minimum level below the prohibitive trade level. In addition, social welfare when the trade cost is zero is higher than the autarky welfare.

#### **1.4 Outline of this study**

The organisation of this study is as follows.

Chapter 2 compares welfare under two forms of taxation (per-unit versus *ad valorem*) in general equilibrium. Following Grazzini (2006), it is assumed that oligopolists are simultaneously producers and consumers. In an exchange economy, the government implements a commodity tax, either an ad valorem tax or a per-unit tax, on the goods produced by the oligopolists. This chapter shows the limitations in Grazzini (2006), who claims that perunit taxation is welfare superior to ad valorem taxation when the number of oligopolists is sufficiently low. I will argue that the result of Grazzini's model entirely depends on the form of social welfare function. In addition, there may be no efficiency gain during the shift from one taxation regime to the other in Grazzini's model and instead, redistribution of income plays a critical role in increasing welfare when the number of oligopolists is sufficiently low. Two different approaches will be presented. Section 2.2 provides a social welfare function that differs from Grazzini (2006), and it is shown that the superiority of which taxation over the other depends on the form of social welfare function. Section 2.3 introduces the lump-sum transfers from the group of gainers to the group of losers to the model of Grazzini (2006). The result shows that welfare with ad valorem tax is ambiguously greater than that with per-unit tax in the present of the lump-sum transfers.

Chapter 3 compares welfare under two forms of tariff (per-unit versus *ad valorem*) in partial equilibrium given a constraint of FDI cost under both Cournot and Bertrand competition. It is assumed that a home firm competes with its foreign rival in the domestic market and both firms have the same marginal cost of production. The home government implements an import tariff, either an *ad valorem* tariff or a per-unit tariff, on the foreign firm. The foreign firm needs to pay the import tariff if it chooses to export, and incurs a sunk cost if it undertakes FDI. An increase in import tariffs will reduce the profitability of exporting relative to the profitability of undertaking FDI. Thus, a rise in import tariffs would cause foreign firms to switch from exporting to undertaking FDI. It is assumed that the home government maximises the tariff revenue and the revenue is increasing in import tariff up to the critical level where the foreign firm is just willing to supply the home country by exporting. The results suggest that *ad*  *valorem* tariff is welfare superior to per-unit tariff under both Cournot and Bertrand competition.

Chapter 4 analyses the welfare effects of trade cost in general oligopolistic equilibrium (GOLE). It is assumed that there are two perfectly symmetric countries and a continuum of industries. In each industry, there is one home firm that competes with its foreign rival in a Cournot fashion. The linear demand function is linear and the cost function is the Ricardian cost structure. In section 4.2, it allows for product differentiation in the so-called "featureless" economy, i.e., all sectors have identical costs in two identical countries. It is shown that social welfare under zero trade cost is always greater than that under prohibitive trade cost if the products are differentiated. In section 4.3, products are assumed to be homogeneous, but there exists comparative advantage for both countries. Social welfare is U-shaped in trade costs under both cases in section 4.2 and section 4.3.

Chapter 5 is the conclusion. It summaries the main findings and limitations of this study.

# Chapter 2 : *Ad Valorem* Versus Per-unit (Specific) Taxation in an Oligopoly Model

### **2.1 Introduction**

In order to study the welfare properties of different forms of taxation in imperfectly competitive economies, the public finance literature has paid attention to the comparison between per-unit (specific) and *ad valorem* taxes. It is widely known that, in the context of perfect competition, *ad valorem* taxation and per-unit taxation are equivalent. This is because firms do not have control over prices, and they will treat the *ad valorem* tax as a fixed amount which equals to a unit tax of that amount. If the taxes are set so that the consumer prices are the same, the tax revenue will be the same with both taxes. Therefore, the balance between these two taxations is a matter of no significance. To see this issue in more details, the figure 2-1 is drawn.



Figure 2–1: Ad valorem and per-unit taxes in competitive markets

 $D_0D_1$  is the before-tax demand curve. The *ad valorem* tax can be treated as a rotating down the demand curve, and there is no tax at a zero price (where the demand curve intersects the x-axis). Therefore, the effect of *ad valorem* tax is to rotate the demand curve. The price that the firm receives is a fixed percentage of the price paid by the consumer.  $D_1D_1$  is *ad valorem* demand curve and  $E_1$  is the after-tax equilibrium. In the case of per-unit taxation, the price that the firm received will be less than the original price as the firm has to pay the per-unit tax on each unit to the government. The per-unit-tax demand curve  $D_2D_2$  is also drawn in figure 2-1 which has the identical magnitude at the equilibrium  $E_1$ . The demand curve  $D_2D_2$  is moved down by the same amount at that level of output. As a result, if the taxes are set so that the consumer price is the same with both taxes, the equilibrium output and tax revenues will be the same. In summary, a per-unit tax and an *ad valorem* tax which raise the same revenue have the identical impact on equilibrium output.

Due to the growing interest in the comparison of per-unit taxation and *ad valorem* taxation in the context of imperfect competition, a fairly large body of literature has been published<sup>3</sup>. Suits and Musgrave (1953) argued that the government receives lower revenue with per-unit taxes than under a method of *ad valorem* taxes, given that the consumer price and the quantity of the monopoly good stayed unchanged. This can be seen from figure 2-2 with a case of linear demand.

<sup>&</sup>lt;sup>3</sup> Keen (1998) offers a comprehensive review of the welfare comparison under two types of taxation.



Figure 2–2: Ad valorem and per-unit taxes in monopoly

OS, FD and FA stand for the original marginal cost, demand and marginal revenue schedules before the introduction of tax in Figure 2-2, respectively. Suppose that the government introduced a per-unit tax equals to HK on the monopoly, EB and EL are the new demand curve and marginal revenue curve faced by the supplier once the tax is taken into account. As a result, the equilibrium price becomes OG and the equilibrium output decreases to OX. Therefore, tax revenue is represented by the region a. In order to compare the effect of two types of taxation on tax yield, assume that the level of final output and price are the same with both taxes. Hence, the new marginal revenue curve, RA, must pass through point T. Combined with the new demand curve, the tax revenue under *ad valorem* tax is obtained as the combination of the region a and b in Figure 2-2. As can be clearly seen, tax revenue is higher with an *ad valorem* tax than with a per-unit tax. The reason is that the per-unit tax decreases marginal revenue by exactly the same amount as the tax, while the *ad valorem* tax lowers marginal revenues by less than

the amount of the tax<sup>4</sup>. As a monopolist produces the quantity at which marginal revenue equals marginal cost, output decreases by less if marginal cost is decreased by less. Therefore, for any given level of equilibrium output, the ad valorem tax leads to more tax revenue. Skeath and Trandel (1994b) further strengthened this claim. They point out that ad valorem taxation Pareto dominates (i.e., higher consumer surplus, profit and tax revenue) per-unit taxation. To be more specific, by considering a shift from per-unit tax to ad valorem tax, a monopoly tends to have an increasing incentive to expand its output. This is because the expansion of output reduces the tax wedge<sup>5</sup> in the case of ad valorem tax, while has no impact on the tax wedge in the case of per-unit tax. The expansion of output directly demonstrates that consumer surplus is higher with ad valorem tax. In addition, as a profit-maximising monopoly continues producing with ad valorem tax, the profit exceeds what it would have earned with per-unit tax. Moreover, Skeath and Trandel (1994b) show that an increase in tax revenue follows with an increase in the total revenue(PQ). The switch from per-unit tax to ad valorem tax causes the total revenue to increase, so that raises the tax revenue. It is worth noting that as a profit-maximising monopoly always sets the price on the elastic part of the demand curve. Therefore, an expansion of output gives rise to an increase in total revenue. Blackorby and Murty (2007) study a general equilibrium model with a monopoly sector. They show that the set of per-unit-tax Pareto optima is identical to the set of ad valorem-tax Pareto optima when profits are taxed at 100% and tax revenues are returned to consumers. This result contradicts with the literature mentioned above relating the comparison of per-unit and ad valorem taxation. They argue that the

the same, we have  $\tau q = tqp(q) \Rightarrow \tau = tp(q)$ . As price is greater than marginal revenue,  $\tau > tMR(q)$ .

<sup>&</sup>lt;sup>4</sup> If the monopoly pays a per-unit tax  $\tau$ , then marginal revenue is reduced by  $\tau$ . By contrast, an *ad* valorem tax at rate t reduces marginal revenue by tMR(q). Assuming that the equilibrium production is

<sup>&</sup>lt;sup>5</sup> Tax wedge is the deviation between what consumers pay and what producers receive from a transaction due to the taxation of a good. In an *ad valorem* regime, the tax wedge is  $P - p_t = tp_t$ , while in a per-unit tax regime, the tax wedge is  $P - p_t = \tau$ .

previous literature ignores the fact that the monopoly profits must be returned to consumers in a way such as government taxation and a lump-sum transfer.

The systematic comparison of *ad valorem* and per-unit taxation under oligopoly starts with Delipalla and Keen (1992). Delipalla and Keen (1992) study a model of homogeneous-product Cournot oligopoly with and without free entry. They show that per-unit taxes are more likely to be over-shifted than ad valorem taxes. By considering a tax reform (denoted as P-shift) that raises the ad valorem tax and decreases the perunit tax whilst leaving the tax revenues at the initial equilibrium price unchanged, they confirm the welfare dominance of *ad valorem* taxation both in the short run with a fixed number of firms and the long run with free entry and exit of firms. In addition to the Pareto comparison of ad valorem and per-unit taxes in a monopoly setting, Skeath and Trandel (1994b) also show that the Pareto dominance of ad valorem taxation applies to symmetric Cournot-Nash oligopoly setting when the tax rate exceeds a critical level. Intuitively, higher values of initial tax cause a reduction in the equilibrium output, and thus an increase in the elasticity of demand at the equilibrium. Therefore, Pareto dominance tends to hold if the per-unit tax is sufficiently high. The welfare dominance of ad valorem taxation over per-unit taxation is established by Denicolò and Matteuzzi (2000) for the case of asymmetric Cournot oligopoly. Asymmetries in costs lead to an additional consideration that tends to favour ad valorem taxation: switching from perunit to ad valorem taxes advantages the more efficient firms. Anderson et al. (2001a, 2001b) extend the welfare comparison of the two taxes under imperfect competition to settings with Bertrand competition and differentiated products. According to Anderson et al. (2001a), under Cournot competition with homogeneous and differentiated products, ad valorem taxation dominates per-unit taxation as more tax revenue is obtained under an *ad valorem* tax if the consumer prices are the same with both taxes, both in the short and long run. However, per-unit taxation can be more efficient under Bertrand competition with product differentiation. Anderson et al. (2001a) put forward a question whether this inverse ranking is due to the mode of competition or the existence of product differentiation in the short run and they provide weak evidence in

favour of the former one (i.e., the Bertrand competition)<sup>6</sup>. When there is free entry, the superior ability of *ad valorem* tax to extract firm profits tends to reduce their incentives to enter the market. This will decrease the number of product varieties as well as welfare. By contrast, a higher degree of over-shifting implies that firm profits are higher with a per-unit tax and this will advantage the number of varieties and welfare. Anderson et al. (2001a, 2001b) focus on tax incidence and the conditions under which taxes are overshifted/undershifted. They demonstrate that the high level of demand convexity that is necessary to cause the over-shifting of taxes is ruled out by the standard oligopoly assumptions. The degree of the over-shifting of excise taxes depends on the ratio of curvature of the firm's demand to the elasticity of the market demand. As Anderson et al. (2001b) state, the particular regime of excise tax can have distinct impacts for tax incidence, over-shifting of taxes can happen, and firm profits can increase under either taxes. Using quadratic preferences, Wang and Zhao (2009) model a single-differentiated product oligopoly model in the presence of substantial firm heterogeneity. Their result shows that, with sufficiently differentiated goods and a high enough cost variance, perunit taxation can be superior to ad valorem taxation under either Cournot or Bertrand competition<sup>7</sup>. Hamilton (2009) extends the analysis to multi-product transactions where firms are selling a wide variety of product. According to Hamilton (2009), over-shifting is more likely to happen in settings with multi-product firms than in the single-product case. The intuitive explanation is that if there is an increase in the excise tax, the multiproduct firms will reduce product variety, leading to less competition among firms for the remaining products. In addition, Hamilton (2009) shows that when the marginal production cost is fixed, ad valorem taxation is superior to per-unit taxation in the

<sup>&</sup>lt;sup>6</sup> Anderson, De Palma and Kreider (2001a) show that the product differentiation tends to play no role in this issue. Either tax regime may be preferred depending on revenue requirement even with a small amount of production differentiation.

<sup>&</sup>lt;sup>7</sup> According to Cheung (1998), failures to extend the ordinary ranking to some oligopoly studies can be explained. In the original framework where the dominance consequence is acquired, a homogeneous product is assumed to be sold by a single firm. As a result, the price distortions are simply exaggerated by these two types of taxation. However, in the extended framework, due to the more complex setting, the additional distortions may happen. The original price distortion may have opposite effects under per-unit and *ad valorem* tax which destroys the initial dominance.

"normal" case, that is consumer preferences are increasing in variety. However, if the consumers have decreasing preferences for variety, the opposite result will occur. Lapan and Hennessy (2011) consider a two-good multi-product oligopoly where marginal costs can differ across firms and products, and preferences of a representative consumer are convex. They demonstrate that *ad valorem* taxation dominates per-unit taxation only in a qualified way when *ad valorem* tax rates are identical across products. When *ad* valorem taxes differ across products, ad valorem taxes cannot successfully replace perunit taxes. Besides, Lapan and Hennessy (2011) show that the welfare ranking of the two taxes depends on unit cost covariance across multi-product firms and complementarity in demand. Colombo and Labrecciosa (2013) extend the comparison of the two forms of taxation to allow the possibility of collusion in imperfectly competitive markets using the P-shift employed by Delipalla and Keen (1992). By considering an infinitely-repeated game with discounting where collusion is supported by either a permanent reversion to the Nash equilibrium strategy or an optimal punishment strategy, they show that a switch from per-unit to ad valorem taxation makes it easier for firms to sustain a collusive agreement. Therefore, they claim that the conventional wisdom (i.e., ad valorem taxation is preferred on welfare grounds) may not hold when collusion is sustainable with an *ad valorem* but not with a per-unit tax. Azacis and Collie (2014) argue that the use of P-shift in Colombo and Labrecciosa (2013) is flawed because the necessary tax reforms will not be valid when prices differ in the different phases of the game. Instead of using P-shifts, Azacis and Collie (2014) compare the effects of ad valorem and per-unit taxes that yield the same consumer price in all phases of the game and they demonstrate that *ad valorem* taxation will always yield higher revenue than per-unit taxation if partial collusion is considered. Vetter (2014) studies the effect of taxes on pass-through rates<sup>8</sup> and competition in a model of homogeneous-good duopoly. It is shown that the taxes have different competitive effects due to the equilibrium is a consistent-conjectures equilibrium which is affected

<sup>&</sup>lt;sup>8</sup> The pass-through rate is defined as the rate at which prices to consumers rise when a tax is imposed on producers.

by the taxes. According to Vetter (2014), when taxes co-determine market conduct, ad valorem taxation may have a stronger anti-competitive effect than per-unit taxes. Therefore, per-unit taxation becomes more attractive when market conduct is endogenous. Häckner and Herzing (2016) systematically examine the welfare effects of taxation in oligopolistic markets. They focus on how pass-through rates and the marginal costs of public funds of *ad valorem* and per-unit taxes respond to the changes in variables such as the number of varieties, the degree of product differentiation, the substitutability of goods and the mode of competition. They find that pass-through rate is negatively related to the marginal costs of public funds and this relationship is of great help from a policy point of view in situations where the marginal cost of public fund is difficult to obtain. According to this, Häckner and Herzing (2016) find that the marginal cost of public fund is lower for ad valorem taxes as ad valorem taxes have a pro-competitive element. Therefore, it is always preferred to increase revenue by adjusting the ad valorem taxes. Recently, Vetter (2017) argues that the dominance of ad valorem taxation in Häckner and Herzing (2016) holds only if market conduct is unaffected by the mix of the taxes. i.e., the mode of competition is exogenous. If market conduct switches from the Bertrand type to the Cournot type, the conclusion will be reversed. In particular, Vetter (2017) compares per-unit and ad valorem taxes in a twostage differentiated-product duopoly: firms pre-commit to capacity in the first stage and compete in prices in the second stage. The Bertrand-Edgeworth duopoly model with soft capacity constraints enables to relate taxes to market conduct through the relationship between the long-term decisions of firms and the taxes. According to Vetter (2017), a change of taxes away from an *ad valorem* and towards a per-unit tax enhances the competitive pressure, which suggests that the per-unit tax is superior to the ad *valorem* tax. In addition, it is shown that a combination of two taxes or a pure per-unit tax is preferred to a pure *ad valorem* tax as shifting from pure *ad valorem* taxation to pure per-unit taxation may change market conduct and bring lower consumer price and higher tax revenue.

In a general equilibrium setting where the firms are large relative to the size of the economy and firms maximise the utility of shareholders, Collie (2015) analyses the incidence of different types of taxes (i.e., lump-sum transfers, profits taxes, per-unit taxes and *ad valorem* taxes) and compares the revenue with per-unit and *ad valorem* tax that yields the same price (therefore, the same aggregate output). Collie (2015) shows that an *ad valorem* tax generates higher tax revenue than a per-unit tax with both homothetic and quasi-linear preferences. Besides, Collie (2015) also demonstrates that when profits are taxed at 100%, as in Blackorby and Murty (2007), there is no difference in total tax revenue between the two taxes.

Grazzini (2006) considers the case of a Cournot oligopoly that involves two groups of players as well as two goods. The first group of players (consumers) are supposed to behave competitively on the exchange market. Furthermore, at the beginning, they are equally endowed with only good 1. By contrast, the second type of players (oligopolists) are not initially endowed with any good. Instead, each oligopolist owns a firm which can produce good 2 by using good 1 as input. Unlike consumers, they behave strategically on the exchange market. Grazzini (2006) separately analyses the preferences of these two groups with two regimes of taxation. She concludes that consumers would be more willing to accept *ad valorem* taxation while oligopolists prefer per-unit taxation. By focusing on the social welfare point of view, she shows that per-unit taxation is superior to *ad valorem* taxation if the number of oligopolists is sufficiently low compared to the number of consumers.

Grazzini (2006) assumes that all agents have identical utility functions given by the product of the consumption of two goods ( $u = x_1x_2$ ). This utility function is homogeneous of degree two. As a utilitarian social welfare function ( $W = \sum_{i=1}^{n} u_i$ ) is applied in Grazzini's model, it can be shown that if goods are transferred from a poorer player to a richer player then social welfare will increase. To see this, simply consider an economy with two agents and the social welfare is calculated as  $W = \sum_{i=1}^{2} u_i = u_A + u_B = x_1^A x_2^A + x_1^B x_2^B$ . Further suppose that agent A has strictly more

allocation of both goods than agent B, i.e.,  $x_1^A > x_1^B$  and  $x_2^A > x_2^B$ . If a redistribution of allocation occurs between them that transfers a quantity  $\Delta x$  of good two from agent B A, the social welfare be obtained to agent new can as  $W' = u'_A + u'_B = x_1^A \left( x_2^A + \Delta x \right) + x_1^B \left( x_2^B - \Delta x \right)$ . The new social welfare is greater than the original one before the redistribution of allocation:  $W' - W = \Delta x \left(x_1^A - x_1^B\right) > 0$ . This conclusion also extends to an economy with n agents. Hence, it is ambiguous whether an improvement of social welfare is the result of an efficiency gain or not as the redistribution of income may also increase the social welfare. Therefore, the conclusion of Grazzini's article seems not to be persuasive.

Though the comparison of welfare between per-unit and *ad valorem* taxation has been one of the earliest issues in the study of public finance, the analysis under general equilibrium remains limited. By using a strategic market game<sup>9</sup>, Grazzini (2006) argues that per-unit taxation welfare dominates *ad valorem* taxation if the number of oligopolists is sufficiently low in general equilibrium. However, this result tends to be driven by the effect on income distribution rather than the effect of general equilibrium. The aim of this chapter is to compare social welfare under two forms of taxation (perunit versus *ad valorem* taxation) in an exchange economy and to argue that *ad valorem* taxation is welfare superior to per-unit taxation under general equilibrium. The results show that the conventional wisdom can also be extended to the case under general equilibrium. Two different approaches will be applied. Section 2.2 provides the first method by changing the utility function from  $x_1x_2$  to  $x_1^{1/2}x_2^{1/2}$  and comparing the magnitude of welfare with per-unit and *ad valorem* taxation. Grazzini (2006) measures the welfare by a utilitarian social welfare function. As mentioned above, the social

<sup>&</sup>lt;sup>9</sup> The strategic market game (SMG) is defined as the general equilibrium mechanism of strategic allocation of resources. It is of great help to study the interaction among players in a game and the influence of their decisions on essential elements (e.g., prices, income distribution).

welfare put more weight on the wealthy consumers if the utility function is homogeneous of degree two. The updated utility function which is homogeneous of degree one would avoid this issue. Section 2.3 offers the second approach by introducing the lump-sum transfers from the group of gainers to the group of losers. The introduction of the lump-sum transfers can be of help to distinguish whether the welfare domination of one type of taxation is due to the efficiency gain or the redistribution of income. The same result will be derived in sections 2.2 and 2.3: *ad valorem* tax is welfare superior to per-unit tax.

#### 2.2 A new social welfare function

Consider an exchange economy with two goods (i.e., good 1 and good 2) and two types of agents (i.e., consumers and oligopolists). Suppose the number of consumers is n and they behave competitively in the market. Each consumer is equally endowed with good 1. The number of oligopolists is assumed to be m and each oligopolist is characterised by no initial endowment of good 1. Instead, he can produce good 2. More importantly, good 1 is needed as input in the process of producing good 2. In addition, he keeps some good 2 for private consumption and sends the rest to the exchange market for trade. Assume that all players have the following utility function:

$$U(x_1, x_2) = x_1^{1/2} x_2^{1/2}$$
(2.2.1)

This utility function is a monotonic transformation of the one used in Grazzini's model  $(x_1x_2)$  so that the two functions represent the same preferences of consumers. However, the measurement of social welfare will be different. The application of a new utility function is aimed at examining whether the conclusion of Grazzini (2006) could be applied to a game with a different measurement of social welfare.

The initial endowments for consumers are defined by

$$e_{i}^{c} = \left(\frac{1}{n}, 0\right), i = 1, ..., n$$
 (2.2.2)

And for oligopolists

$$e^{o}_{j} = (0,0), j = 1,...,m$$
 (2.2.3)

The consumers equally share one unit endowment of good 1 while the oligopolists do not own any good.

In addition, the oligopolists can produce good 2. An amount  $z_j$  of good 1 is employed by oligopolist *j* in order to produce an amount  $y_j$  of good 2. To capture the relationship between the amount of input and output, we consider the following technology function

$$y_j = \frac{1}{\alpha} z_j, \alpha > 0 \tag{2.2.4}$$

where  $\alpha$  is a positive constant. It follows that one unit of good 2 can be produced out of  $\alpha$  units of good 1. As an exchange market exists, the oligopolists can exchange good 2 for good 1 from consumers, after deciding how much of good 2 they are going to produce and consume. The production level is determined by the amount of good 1 they buy from consumers. Suppose that they send  $q_j$  out of the amount  $y_j$  produced of the second good to the exchange market for trade. Naturally, they will keep the rest amount  $y_j - q_j$  for their private consumption. It is clear that the amount  $q_j$  of good 2 determines the equilibrium exchange rate between two goods. Assume that each oligopolist can individually choose the share  $q_j$  and the total supply of good 2 is then

described as  $\sum_{k=1}^{m} q_k$ . For the agents of type 2, they behave strategically on the exchange market with strategies defined by pairs  $(q_j, y_j)$ .

The government implements a commodity tax on good 2. In this section, we consider two forms of tax: an *ad valorem* tax and a per-unit tax. Denote  $p_2$  as the consumer price. Under the regime of *ad valorem* tax, the producer price of second good becomes  $P_2 = p_2(1-t)$  with a tax rate t (0 < t < 1). Clearly, the total *ad valorem* tax

can be calculated as  $R_t = tp_2 \sum_{k=1}^{m} q_m$ . Regarding the per-unit tax, the producer price of good

2 is defined as  $P_2 = p_2 - \tau$  with a per-unit tax  $\tau (0 < \tau < 1/\alpha)$ . Accordingly, the total per-unit tax is formulated as  $R_{\tau} = \tau \sum_{k=1}^{m} q_m$ .

Let  $(p_1, p_2)$  be a price vector with  $p_h > 0$  (h = 1, 2), a competitive agent i, i = 1, ..., n, solves the following problem

$$\max_{x_1, x_2} x_1^{1/2} x_2^{1/2} \quad s.t. \ x_1 + p \, x_2 \le \frac{1}{n}$$

where  $p = p_2 / p_1$ . Let  $\lambda$  be the Lagrange multiplier and consider the following Lagrangian function

$$L(x_1, x_2) = x_1^{1/2} x_2^{1/2} + \lambda \left(\frac{1}{n} - x_1 - p x_2\right)$$

The first-order necessary conditions for the problem are

$$\frac{\partial L}{\partial x_1} = \frac{1}{2} x_1^{-\frac{1}{2}} x_2^{\frac{1}{2}} - \lambda = 0$$
(2.2.5)

$$\frac{\partial L}{\partial x_2} = \frac{1}{2} x_1^{\frac{1}{2}} x_2^{-\frac{1}{2}} - \lambda p = 0$$
(2.2.6)

$$\frac{\partial L}{\partial \lambda} = x_1 + p x_2 - \frac{1}{n} = 0 \tag{2.2.7}$$

Solve the system above; it gives rise to the equilibrium allocation:

$$\mathbf{x}_{i}(p) = \left(\frac{1}{2n}, \frac{1}{2np}\right), \quad i = 1, \dots n$$
 (2.2.8)

The total demand for good 2 is  $D^T = \frac{1}{2np} \cdot n = \frac{1}{2p}$ .

The indirect utility function v of the representative consumer can be obtained by substituting the equilibrium allocation (2.2.8) into the utility function (2.2.1):

$$\mathbf{v}(p) = \sqrt{\left(\frac{1}{2n}\right)\left(\frac{1}{2np}\right)} \tag{2.2.9}$$

Regarding the group of oligopolists that we have discussed above, they strategically choose  $(q_j, y_j)$ , j = 1, ..., m. The profit of oligopolist j in the case of *ad valorem* taxation can be defined as

$$\pi_i^{\ t}(q_i, y_i) = p(1-t)q_i - z_i \tag{2.2.10}$$

In contrast, the payoff under a per-unit taxation can be calculated as

$$\pi_{j}^{\tau}(q_{j}, y_{j}) = (p - \tau)q_{j} - z_{j}$$
(2.2.11)

Suppose that each oligopolist will spend the entire profit to purchase good 1. As a result, he can purchase  $p(1-t)q_j - \alpha y_j$  and  $(p-\tau)q_j - \alpha y_j$  amount of good 1 under the regime of *ad valorem* tax and per-unit tax, respectively.

The utility payoffs in the case of an ad valorem tax is given as

$$\sqrt{\left(p(1-t)q_j - \alpha y_j\right)\left(y_j - q_j\right)}$$
(2.2.12)

The utility payoffs with a per-unit tax for a representative oligopolist are:

$$\sqrt{\left(\left(p-\tau\right)q_{j}-\alpha y_{j}\right)\left(y_{j}-q_{j}\right)}$$
(2.2.13)

In addition, market clearing condition implies that total demand equals total supply

of good 2: 
$$\sum_{k=1}^{m} q_k = \frac{1}{2p}$$
(2.2.14)

Rearranging the formula above yields the equilibrium exchange rate

$$p = \frac{1}{2\sum_{k=1}^{m} q_k}$$
(2.2.15)

The utility functions of oligopolists with both taxes can be obtained as

$$W(q_{j}, y_{j}) = \sqrt{\left(\left(\frac{1-t}{2\sum_{k=1}^{m} q_{k}} - \tau\right)q_{j} - \alpha y_{j}\right)(y_{j} - q_{j})}; j = 1,...,m$$
(2.2.16)

By setting t = 0, the payoffs of the game in the case of per-unit *ad valorem* taxation can be expressed as follows,

$$W^{\tau}(q_{j}, y_{j}) = \sqrt{\left(\frac{1}{2\sum_{k=1}^{m} q_{k}} - \tau)q_{j} - \alpha y_{j}\right)(y_{j} - q_{j})}; j = 1,...,m$$
(2.2.17)

Similarly, by setting  $\tau = 0$ , the payoffs of the game in the case of *ad valorem* taxation can be expressed as follow

$$W^{t}(q_{j}, y_{j}) = \sqrt{\left(\frac{1-t}{2\sum_{k=1}^{m} q_{k}} q_{j} - \alpha y_{j}\right)(y_{j} - q_{j})}; j = 1, ..., m$$
(2.2.18)

Each oligopolist maximises the utility level by choosing his own strategy  $(q_j, y_j)$ .

Let Q be the total supply of good 2 (i.e.,  $Q = \sum_{k=1}^{m} q_k$ ). The optimality condition with respect to  $q_j$  gives

 $\frac{\partial W}{\partial q_i} = \frac{1}{2} \frac{\left( \left( y_j - q_j \right) \left( \Phi_1 - \tau - q_j \left( \frac{\Phi_1}{Q} \right) \right) - \left( \left( \Phi_1 - \tau \right) q_j - \alpha y_j \right) \right)^{-\frac{1}{2}}}{\sqrt{\left( \Phi_1 q_j - \alpha y_j \right) \left( y_j - q_j \right)}} = 0$   $\Rightarrow \qquad \left( y_j - q_j \right) \left( \Phi_1 - \tau - q_j \left( \frac{\Phi_1}{Q} \right) \right) - \left( \left( \Phi_1 - \tau \right) q_j - \alpha y_j \right) = 0 \qquad (2.2.19)$ 

where  $\Phi_1 = \frac{1-t}{2Q}$ 

It is clear that

$$\frac{1}{y_j - q_j} = \frac{\Phi_1 - \tau - q_j \left(\frac{\Phi_1}{Q}\right)}{(\Phi_1 - \tau)q_j - \alpha y_j}$$

The optimality condition with respect to  $y_j$  yields

$$\frac{\partial W}{\partial y_i} = \frac{1}{2} \frac{\left( \left( \Phi_1 - \tau \right) q_j - \alpha y_j - \left( y_j - q_j \right) \left( -\alpha \right) \right)^{-\frac{1}{2}}}{\sqrt{\left( \Phi_1 q_j - \alpha y_j \right) \left( y_j - q_j \right)}} = 0$$

$$\Rightarrow \qquad \left( \Phi_1 - \tau \right) q_j - \alpha y_j - \left( y_j - q_j \right) \left( -\alpha \right) = 0 \qquad (2.2.20)$$

It follows that

$$\frac{1}{y_j - q_j} = \frac{\alpha}{(\Phi_1 - \tau)q_j - \alpha y_j}$$

The first-order conditions can be rewritten as

$$\frac{1}{y_j - q_j} = \frac{\left(\Phi_1 - \tau\right) - q_j\left(\frac{\Phi_1}{Q}\right)}{\left(\Phi_1 - \tau\right)q_j - \alpha y_j} = \frac{\alpha}{\left(\Phi_1 - \tau\right)q_j - \alpha y_j}$$
(2.2.21)

From the second equality of the equation (2.2.21),

$$\left(\Phi_1 - \tau\right) - q_j \left(\frac{\Phi_1}{Q}\right) = \alpha$$
 (2.2.22)

Rearranging the expression above gives

$$1 - \frac{q_j}{Q} = 2\frac{\alpha + \tau}{1 - t}Q, \quad j = 1, ..., m$$
(2.2.23)

Summing up equation (2.2.23) yields the equilibrium supply of good 2 to consumers

$$Q_h^* = \frac{(m-1)(1-t)}{2m(\alpha+\tau)}, \quad h = t, \tau$$
 (2.2.24)

Where the subscript h,  $h = t, \tau$ , represents a variable in the case of *ad valorem* or per-unit taxation. It follows that the total supply of good 2 is negatively related to whichever tax regime introduced by the government (i.e.,  $\partial Q_h^* / \partial t < 0$ ,  $\partial Q_h^* / \partial \tau < 0$ ). Intuitively, in order to enhance the utility, oligopolists will trade less with consumers if the government raises the tax rate and increase their consumption on good 2.
From (2.2.15) and (2.2.24), the equilibrium price can be derived as

$$p_h^* = \frac{m(\alpha + \tau)}{(m-1)(1-t)}, \quad h = t, \tau$$
 (2.2.25)

It implies that a rise in either an *ad valorem* tax or a per-unit tax would increase the equilibrium price. As the producer price is defined as  $P_h = p_h(1-t) - \tau$ ;  $h = t, \tau$ , the effect of taxes on prices is given by:

$$\frac{\partial P_{\tau}}{d\tau} = \frac{\partial p_{\tau}}{d\tau} - 1 = \frac{1}{m-1} > 0$$

in the case of per-unit taxation, and

$$\frac{\partial P_t}{p_t dt} = \frac{\partial p_t}{p_t dt} (1-t) - 1 = 0$$

in the case of *ad valorem* taxation.

Since  $\partial p_h^* / \partial \tau > 1$ , per-unit taxes are over-shifted. It indicates that consumer price increases by more than the amount of the tax. In contrast, the *ad valorem* taxes are fully shifted<sup>10</sup>, that is, the proportional change in the consumer price is the same as the proportional change in the tax.

The total tax revenue in the case of ad valorem taxation equals to

$$R_t^* = t p_t^* Q_t^* = \frac{1}{2}t$$
 (2.2.26)

<sup>&</sup>lt;sup>10</sup> Grazzini (2006) claims that *ad valorem* taxes may be under shifted, but the result does not seem to be correct.

Similarly, total tax revenue in the case of per-unit taxation can be calculated as

$$R_{\tau}^{*} = \tau Q_{\tau}^{*} = \frac{(m-1)\tau}{2m(\alpha+\tau)}$$
(2.2.27)

In addition, using (2.2.23) and (2.2.24), we obtain

$$q_{jh}^{*} = \frac{(m-1)(1-t)}{2m^{2}(\alpha+\tau)}, \quad j = 1,...,m; \quad h = t,\tau$$
 (2.2.28)

and

$$y_{jh}^{*} = \frac{(2m\alpha + \tau - \alpha)(1 - t)}{4m^{2}\alpha(\alpha + \tau)}, \quad j = 1, ..., m; \quad h = t, \tau$$
(2.2.29)

Following Grazzini (2006), we analyse a transformation from an *ad valorem* tax t to a per-unit tax which leads to the same tax revenue (i.e., a revenue-neutral change). The tax revenue with per-unit tax and *ad valorem* tax are given in equations (2.2.26) and (2.2.27). Equating these two formulas yields

$$\frac{1}{2}t = \frac{(m-1)\tau}{2m(\alpha+\tau)}$$
(2.2.30)

It is easy to find the value of  $\tau$  which is employed as a basis of comparison as:

$$\tau = \frac{m\alpha t}{m(1-t)-1} \tag{2.2.31}$$

It implies that the tax revenue in the case of per-unit tax with an amount of  $\frac{m\alpha t}{m(1-t)-1}$  is the same as the tax revenue in the case of *ad valorem* tax with a rate of *t*.

By substituting (2.2.31) into (2.2.25), the price at equilibrium in the case of perunit taxation obtains as

$$p_{\tau}^{*} = \frac{m\alpha}{m(1-t)-1}$$
(2.2.32)

By substituting (2.2.28) and (2.2.29) into (2.2.18), the utility level for each oligopolist under *ad valorem* tax obtains as

$$W_t(q^*_{jt}, y^*_{jt}) = \frac{1-t}{4m^2\alpha^{\frac{1}{2}}}$$
(2.2.33)

Similarly, by substituting (2.2.28) and (2.2.29) into (2.2.17) and using (2.2.31), the utility level for each oligopolist under per-unit tax obtains as

$$W_{\tau}\left(q_{j\tau}^{*}, y_{j\tau}^{*}\right) = \frac{1}{4m^{2}\alpha^{\frac{1}{2}}}$$
(2.2.34)

In order to understand how much each oligopolist would gain (lose) from the change in the taxes, calculate the gap in the utility level under two taxation regimes

$$\Delta W = W_t - W_\tau = \frac{-t}{4m^2 \alpha^{\frac{1}{2}}} < 0 \tag{2.2.35}$$

It can be clearly seen from the above equation that oligopolists are always better off with the regime of per-unit taxation based on the comparison of revenue-neutral tax changes.

With respect to consumers, by substituting (2.2.25) into (2.2.9), the utility level for each consumer in the case of *ad valorem* tax obtains as

$$V(p_{t}^{*}) = \frac{\sqrt{(m-1)(1-t)}}{2nm^{\frac{1}{2}}\alpha^{\frac{1}{2}}}$$
(2.2.36)

Similarly, the utility level for each consumer in the case of per-unit tax is obtained by substituting (2.2.32) into (2.2.9):

$$V(p_{\tau}^{*}) = \frac{\sqrt{m(1-t)-1}}{2nm^{\frac{1}{2}}\alpha^{\frac{1}{2}}}$$
(2.2.37)

The change of utility level with these two types of taxation obtains as:

$$\Delta V = V(p_{t}^{*}) - V(p_{\tau}^{*}) = \frac{\sqrt{m(1-t)-1+t} - \sqrt{m(1-t)-1}}{2nm^{\frac{1}{2}}\alpha^{\frac{1}{2}}} > 0 \qquad (2.2.38)$$

Thus, unlike oligopolists, consumers are willing to accept the regime of *ad valorem* taxation in the context of the revenue-neutral tax change.

**Proposition 1:** *Ad valorem* taxation is welfare superior to per-unit taxation regardless the number of consumers and oligopolists in general equilibrium.

## Proof.

Define social welfare as the arithmetic sum of each agent's utility level.

$$S = nV + mW$$

From (2.2.35) and (2.2.38), the difference in the social welfare with *ad valorem* and per-unit taxes obtains as

$$\Delta S = n\Delta V + m\Delta W = \frac{\sqrt{(m-1)(1-t)} - \sqrt{m(1-t)-1}}{2m^{\frac{1}{2}}\alpha^{\frac{1}{2}}} - \frac{t}{4m\alpha^{\frac{1}{2}}} = \frac{2\sqrt{m}\left(\sqrt{m(1-t)-1+t} - \sqrt{m(1-t)-1}\right) - t}{4m\alpha^{\frac{1}{2}}}$$
(2.2.39)

Suppose that the social welfare is positive, and then the following inequality must hold

$$2\sqrt{m}\left(\sqrt{m(1-t)-1+t}-\sqrt{m(1-t)-1}\right)-t>0$$

Rearranging the inequality gives

$$\sqrt{m(1-t)-1+t} - \sqrt{m(1-t)-1} > \frac{t}{2\sqrt{m}}$$

Squaring the both sides and the gap obtains as

$$\left(\sqrt{m(1-t)-1+t} - \sqrt{m(1-t)-1}\right)^2 - \left(\frac{t}{2\sqrt{m}}\right)^2 > 0$$

$$\Rightarrow \frac{t^2(t^2 + 8m(2m-1)t + 16m)}{4m} > 0 \qquad (2.2.40)$$

Let *k* denote the following equality

$$k = t^2 + 8m(2m-1)t + 16m$$

$$\frac{\partial \kappa}{\partial t} = 2t + 8m(2m-1)$$

Since *m* represents the number of oligopolists, and *t* is positive. Therefore, *k* is monotonously increasing in *t* (i.e.,  $\partial k / \partial t > 0$ ). It can be shown that

$$k > \overline{k} \Big|_{t=0} = 16m > 0$$

It confirms that  $\Delta S > 0$ , i.e., welfare with *ad valorem* taxation is always greater than that with per-unit taxation.

Q.E.D.

Grazzini (2006) argued that a revenue-neutral transformation from an *ad valorem* tax to a per-unit tax enhances social welfare if the number of consumers is sufficiently high with respect to that of oligopolists. However, the conclusion seems to contradict a large amount of literature with models under partial equilibrium. Grazzini (2006) then attributes this contradiction to the use of general equilibrium.

However, according to the result of this paper, *ad valorem* dominates per-unit taxation in the view of social welfare and the result is regardless of the number of consumers and oligopolists. Notice that Grazzini (2006) considers the following utility function:  $u = x_1x_2$ , and as an arithmetic social welfare function is used, social welfare put more weight on the rich consumers. In contrast, the utility function which is homogeneous of degree one has been applied in this paper, i.e.,  $u = x_1^{1/2}x_2^{1/2}$ . It is a monotonic transformation of the utility function used by Grazzini and represents the same consumer preferences. The only difference is the social welfare function. As a consequence, the results in the context of general equilibrium are in consonance with the standard results under partial equilibrium.

# 2.3 An introduction of lump-sum transfers

This section introduces another comparison between per-unit and *ad valorem* taxation by allowing lump-sum transfers in the model. To be more specific, the utility function will be the same as the one used by Grazzini (2006), i.e.,  $u = x_1x_2$ . Including the lump-sum transfers into the game permits the redistribution of income. In the context of this, we focus on the comparison of welfare between *ad valorem* and per-unit taxation and verify if both consumers and oligopolists can be made better off under *ad valorem* taxation with the lump-sum transfers.

As can be seen from equation (2.2.35) and (2.2.38), a revenue-neutral shift from one type of taxation to the other leads to a different level of utility for both consumers and oligopolists. Thus, a lump-sum transfer plays a key role in shrinking the gap between gainers and losers. To be more specific, gainers would have to pay an amount of tax in order to get them back to the same level of utility that they had before taxation changed. In contrast, losers would be willing to be offered an amount of subsidy that would make them as good as before the change in taxation. Regarding the government, it receives the tax from the gainers and compensates to the losers.

Suppose that both consumers and oligopolists have identical utility function U defined as in Grazzini (2006)

$$U(x_1, x_2) = x_1 x_2 \tag{2.3.1}$$

Where  $x_1$  and  $x_2$  are quantity consumed of good 1 and good 2. Initial endowments of consumers are defined by

$$e_i = \left(\frac{1}{n}, 0\right), \quad i = 1, ..., n$$
 (2.3.2)

Each consumer is equally endowed with  $\frac{1}{n}$  unit of good 1.

Each oligopolist, by contrast, is not endowed with anything at the beginning:

$$e_j = (0,0), \quad j = 1,...,m$$
 (2.3.3)

In order to survive, agents of type 2 are assumed to have a linear technology:

$$y_j = \frac{1}{\alpha} z_j, \quad \alpha > 0 \tag{2.3.4}$$

Each oligopolist produces good 2 using the technology above and they will decide how much of good 2 to keep as a private consumption and will send the rest  $q_j$  to exchange market for the consumption of good 1.

Now suppose that the government imposes a lump-sum transfer which affects agents' behaviour. As a result, the lump-sum transfer enters consumers' budget constraint. More precisely, a competitive agent i, i = 1, ..., n, solves the following problem

$$\max_{x_1, x_2} x_1 x_2 \quad s.t. \quad x_1 + p x_2 \le \frac{1}{n} + T_n$$

where  $p = p_2 / p_1$  and  $T_n$  denotes a lump-sum transfer. If  $T_n$  is negative, then each consumer pays  $T_n$  to the government as he receives higher utility level with the new form of taxation. However, if he is worse off with the change of taxation, he will obtain a lump-sum transfer  $T_n$  from the government.  $T_n$  is positive in the latter example. The government's tax revenue (expenditure) from all consumers could be easily formulated as  $nT_n$ . The Lagrangian function obtains as

$$L(x_1, x_2) = x_1 x_2 + \lambda \left(\frac{1}{n} + T_n - x_1 - p x_2\right)$$

First-order conditions yield:

$$\frac{\partial L}{\partial x_1} = x_2 - \lambda = 0 \tag{2.3.5}$$

$$\frac{\partial L}{\partial x_2} = x_1 - \lambda p = 0 \tag{2.3.6}$$

$$\frac{\partial L}{\partial \lambda} = x_1 + p x_2 - \frac{1}{n} + T_n = 0$$
(2.3.7)

Solve the system above; it gives rise to individual demand as:

$$\boldsymbol{x}_{i}(\boldsymbol{p}) = \left(\frac{1+nT_{n}}{2n}, \frac{1+nT_{n}}{2np}\right), \quad i = 1, \dots n$$
(2.3.8)

The total demand for good 2 is  $\frac{1+nT_n}{2p}$ .

To calculate the indirect utility function v of a competitive consumer, substitute the individual demand (2.3.8) into the utility function (2.3.1),

$$v(p) = \left(\frac{1+nT_n}{2n}\right) \left(\frac{1+nT_n}{2np}\right)$$
(2.3.9)

Regarding the oligopolists, each of them will receive a lump-sum transfer  $T_m$  from the government if he is worse off with the fresh type of taxation. Alternatively, he will have to pay a lump-sum transfer  $T_m$  to the government if he is better off. Consequently, a lump-sum transfer accordingly has an impact on the profits of oligopolists. The profit of oligopolist *j* in the case of *ad valorem* taxation obtains as

$$\pi_{j}^{t}(q_{j}, y_{j}) = p(1-t)q_{j} - z_{j} + T_{m}$$
(2.3.10)

And in the case of per-unit taxation as

$$\pi_{j}^{\tau}(q_{j}, y_{j}) = (p - \tau)q_{j} - z_{j} + T_{m}$$
(2.3.11)

where  $T_m$  denotes the lump-sum transfers to oligopolists. Thus, captures the total government tax revenue (expenditure).

Furthermore, the utility payoffs in the case of an *ad valorem* tax are:

$$\left(p(1-t)q_j - \alpha y_j + T_m\right)\left(y_j - q_j\right)$$
(2.3.12)

The utility payoffs in the case of a per-unit tax are:

$$\left(\left(p-\tau\right)q_{j}-\alpha y_{j}+T_{m}\right)\left(y_{j}-q_{j}\right)$$

$$(2.3.13)$$

Given these strategies  $(q_j, y_j)$ , j = 1, ..., m, the value of p which satisfies the market clearing condition is given by

$$\sum_{k=1}^{m} q_k = \frac{1 + nT_n}{2p} \tag{2.3.14}$$

Rearranging the formula above yields

$$p = \frac{1 + nT_n}{2\sum_{k=1}^{m} q_k}$$
(2.3.15)

By substituting the equilibrium price in the utility payoffs (2.3.12) and (2.3.13), the payoffs of the oligopolists can be expressed as

$$W(q_{j}, y_{j}) = \left( \left( \frac{(1-t)(1+nT_{n})}{2\sum_{k=1}^{m} q_{k}} - \tau \right) q_{j} - \alpha y_{j} + T_{m} \right) (y_{j} - q_{j}); \quad j = 1, ..., m \quad (2.3.16)$$

The payoffs of the oligopolists in the case of per-unit can be expressed as

$$W^{\tau}(q_{j}, y_{j}) = \left( \left( \frac{1 + nTn}{2\sum_{k=1}^{m} q_{k}} - \tau \right) q_{j} - \alpha y_{j} + T_{m} \right) (y_{j} - q_{j}); \quad j = 1, ..., m \quad (2.3.17)$$

Similarly, the payoffs of the oligopolists in the case of *ad valorem* taxation can be expressed as

$$W^{t}(q_{j}, y_{j}) = \left(\frac{(1-t)(1+nT_{n})}{2\sum_{k=1}^{m}q_{k}}q_{j} - \alpha y_{j} + T_{m}\right)(y_{j} - q_{j}); \quad j = 1,...,m \quad (2.3.18)$$

The optimality condition with respect to gives

$$\frac{\partial W}{\partial q_i} = -\left(\left(\Phi_2 - \tau\right)q_j - \alpha y_j + T_m\right) + \left(y_j - q_j\right)\left(\left(\Phi_2 - \tau\right) - q_j\left(\frac{\Phi_2}{Q}\right)\right) = 0$$
(2.3.19)

where Q represents the total supply of good 2 and  $\Phi_2 = \frac{(1-t)(1+nT_n)}{2Q}$ .

It is clear that  $\frac{1}{(y_j - q_j)} = \frac{(\Phi_2 - \tau) - q_j \left(\frac{\Phi_2}{Q}\right)}{(\Phi_2 - \tau)q_j - \alpha y_j + T_m}$ 

The optimality condition with respect to  $y_j$  yields

$$\frac{\partial W}{\partial y_i} = \left( \left( \Phi_2 - \tau \right) q_j - \alpha y_j + T_m \right) + \left( y_j - q_j \right) \left( -\alpha \right) = 0$$
(2.3.20)

It follows that 
$$\frac{\partial W}{\partial y_i} = ((\Phi_2 - \tau)q_j - \alpha y_j + T_m) + (y_j - q_j)(-\alpha) = 0$$

The first order conditions could be rewritten as

$$\frac{1}{y_j - q_j} = \frac{\left(\Phi_2 - \tau\right) - q_j\left(\frac{\Phi_2}{Q}\right)}{\left(\Phi_2 - \tau\right)q_j - \alpha y_j + T_m} = \frac{\alpha}{\left(\Phi_2 - \tau\right)q_j - \alpha y_j + T_m}$$
(2.3.21)

The second equality of the above equation shows that

$$\left(\Phi_2 - \tau\right) - q_j \left(\frac{\Phi_2}{Q}\right) = \alpha$$
 (2.3.22)

Rearranging the expression above gives

$$\left(1+nT_n\right)\left(1-\frac{q_j}{Q}\right) = 2\frac{\alpha+\tau}{1-t}Q \qquad (2.3.23)$$

Summing up equation (2.3.23) yields the equilibrium supply of good 2

$$Q_h^* = \frac{(1+nT_n)(m-1)(1-t)}{2m(\alpha+\tau)}, \quad h = t, \tau$$
 (2.3.24)

It shows that oligopolists are more (less) willing to exchange goods with consumers if there is an increase in lump-sum subsidy (tax) to consumers.

From (2.3.15) and (2.3.24), equilibrium price level can be then derived as

$$p_h^* = \frac{m(\alpha + \tau)}{(m-1)(1-t)}, \quad h = t, \tau$$
 (2.3.25)

It is worth noting that a lump-sum transfer plays no role in the equilibrium price.

Total tax revenue in the case of ad valorem taxation equals to

$$R_t^* = t p_t^* Q_t^* = \frac{1 + nT_n}{2} t$$
(2.3.26)

Similarly, total tax revenue in the case of per-unit taxation equals to

$$R_{\tau}^{*} = \tau Q_{\tau}^{*} = \frac{(1+nT_{n})(m-1)\tau}{2m(\alpha+\tau)}$$
(2.3.27)

Furthermore, using (2.3.23) and (2.3.24), we obtain

$$q_{jh}^{*} = \frac{(m-1)(1-t)(1+nT_{n})}{2m^{2}(\alpha+\tau)}, \quad j = 1,...,m; \quad h = t,\tau$$
(2.3.28)

and

$$y_{jh}^{*} = \frac{(2m\alpha + \tau - \alpha)(1 - t)(1 + nT_{n})}{4m^{2}\alpha(\alpha + \tau)} + \frac{T_{m}}{2\alpha}, \quad j = 1,...,m; \quad h = t,\tau \quad (2.3.29)$$

It can be seen from the equilibrium output that both lump-sum transfers from consumers and oligopolists will affect oligopolists' decision on the output level.

In order to examine if all players (i.e., consumers, oligopolists and the government), with the introduction of lump-sum transfers, can be better off under *ad valorem* tax, we then study a transformation from a per-unit tax  $\tau$  to an *ad valorem* tax *t* which raises the same direct tax revenue.

Equating (2.3.26) and (2.3.27)

$$\frac{1+nT_n}{2}t = \frac{(1+nT_n)(m-1)\tau}{2m(\alpha+\tau)}$$
(2.3.30)

yield the basis of comparison

$$\tau = \frac{m\alpha t}{m(1-t)-1} \tag{2.3.31}$$

It implies that by setting the per-unit tax  $\tau$  equal to  $\frac{m\alpha t}{m(1-t)-1}$ , the government

will receive an identical amount of indirect tax revenue as that of *ad valorem* taxation regime.

By substituting (2.3.31) into (2.3.25), the price at equilibrium in the case of perunit taxation obtains as

$$p_{\tau}^{*} = \frac{m\alpha}{m(1-t)-1}$$
(2.3.32)

By plugging (2.3.32) into (2.3.9), the utility level for each consumer in the case of per-unit tax obtains as

$$v_{\tau} = \frac{\left(m(1-t)-1\right)\left(1+nT_{n}\right)^{2}}{4\alpha mn^{2}}$$
(2.3.33)

The utility without lump-sum transfers could be derived by setting  $T_n = 0$ , it gives rise to

$$\overline{v}_{\tau} = \frac{m(1-t)-1}{4\alpha mn^2} \tag{2.3.34}$$

By plugging (2.3.25) into (2.3.9), the utility level for each consumer in the case of *ad valorem* tax obtains as

$$v_{t} = \frac{(m-1)(1-t)(1+nT_{n})^{2}}{4\alpha mn^{2}}$$
(2.3.35)

Suppose that the economy is absent from the lump-sum transfers, consumer's payoff could be expressed as

$$\overline{v}_{t} = \frac{(m-1)(1-t)}{4\alpha mn^{2}}$$
(2.3.36)

Comparing (2.3.34) and (2.3.36), it is clear that consumers receive unambiguously higher payoff under the regime of *ad valorem* taxation

$$\overline{v}_{\tau} - \overline{v}_{t} = -\frac{t}{4\alpha mn^{2}} < 0$$

Since consumers would obtain more payoffs if the government implement an *ad valorem* tax, this suggests government to collect the lump-sum taxes from consumers under *ad valorem* taxation.

In order to calculate the degree of lump-sum transfers that would make them indifferent between per-unit taxation and *ad valorem* taxation, we equate (2.3.34) and (2.3.35)

$$\overline{v}_{\tau} = v_t \tag{2.3.37}$$

$$\frac{m(1-t)-1}{4\alpha mn^2} = \frac{(m-1)(1-t)(1+nT_n)^2}{4\alpha mn^2}$$
(2.3.38)

Solving (2.3.38) gives rise to

$$\tilde{T}_n = \frac{1}{n} (\Phi_3 - 1) < 0 \tag{2.3.39}$$

Where  $\Phi_3 = \sqrt{\frac{m(1-t)-1}{(m-1)(1-t)}}$  and  $\tilde{T}_n$  represents the amount of lump-sum transfers

that consumers are indifferent between ad valorem and per-unit tax in terms of utility.

By substituting (2.3.28) and (2.3.29) into (2.3.16), the utility level for each oligopolist under per-unit tax obtains as

$$W_{\tau} = \frac{1}{\alpha} \left( \frac{1 + 2m^2 T_m + nT_n}{4m^2} \right)^2$$
(2.3.40)

In the absence of lump-sum transfers, each oligopolist obtains payoff which is equal to

$$\overline{W}_{\tau} = \frac{1}{\alpha} \left( \frac{1}{4m^2} \right)^2 \tag{2.3.41}$$

Similarly, by substituting (2.3.28) and (2.3.29) into (2.3.17), the utility level for each oligopolist under *ad valorem* tax obtains as

$$W_{t} = \frac{1}{\alpha} \left( \frac{1 - t + 2m^{2}T_{m} + nT_{n}(1 - t)}{4m^{2}} \right)^{2}$$
(2.3.42)

By setting both  $T_m$  and  $T_n$  equal to zero, each oligopolist receives the following payoff without lump-sum transfers

$$\overline{W}_t = \frac{1}{\alpha} \left( \frac{1-t}{4m^2} \right)^2 \tag{2.3.43}$$

Since  $\overline{W}_{\tau} - \overline{W}_{t} > 0$ , oligopolists acquire greater utility under the regime of per-unit taxation. They will be losers if government alters the taxation regime to *ad valorem* 

taxation. This suggests the government to offer a lump-sum subsidy to oligopolists, in order to compensate their loss.

In order to calculate the level of lump-sum transfers that would make indifferent between two taxations, we equate (2.3.41) and (2.3.42)

$$\bar{W}_{\tau} = W_t \tag{2.3.44}$$

$$\frac{1}{\alpha} \left(\frac{1}{4m^2}\right)^2 = \frac{1}{\alpha} \left(\frac{1-t+2m^2T_m+nT_n\left(1-t\right)}{4m^2}\right)^2$$

Solving (2.3.44) gives rise to

$$T_m = \frac{t - nT_n(1 - t)}{2m^2}$$
(2.3.45)

Substituting (2.3.39) into (2.3.45) implies

$$\tilde{T}_m = \frac{1 - (1 - t)\Phi_3}{2m^2} > 0 \tag{2.3.46}$$

By receiving a compensation equals to  $\tilde{T}_m$ , oligopolists are indifferent to the change of taxation regime.

During the shift from per-unit to *ad valorem* taxation, the government obtains a lump-sum transfer  $\tilde{T}_n$  from each consumer and pay a lump-sum transfer  $\tilde{T}_m$  to each oligopolist. As a result, all agents are indifferent between these two taxations. Meanwhile, the total government revenue from lump-sum can be calculated as

$$GR^{L} = -\left(n\tilde{T}_{n} + m\tilde{T}_{m}\right) \tag{2.3.47}$$

As can be seen from equation (2.3.27), taxes revenue from per-unit is obtained as  $R_{\tau}^{*}$ . Total tax revenues from *ad valorem* in the present of lump-sum transfers obtain as  $R_{t}^{*} + GR^{L}$ .

As has been discussed, the tax revenue from per-unit taxation is the same as that from *ad valorem* taxation. It follows that the difference of the tax revenue between per-unit and *ad valorem* taxation is the total lump-sum transfers that the government received.

**Proposition 2:** Both consumers and oligopolists can be made better off under *ad valorem* taxation than per-unit taxation with lump-sum transfers.

### Proof.

Substituting (2.3.39) and (2.3.46) into (2.3.47):

$$GR^{L} = -(n\tilde{T}_{n} + m\tilde{T}_{m}) = -\Phi_{3} + 1 - \frac{1 - (1 - t)\Phi_{3}}{2m}$$

The total government revenue from lump-sum transfers can be simplified as

$$GR^{L} = \frac{(1 - t - 2m)\Phi_{3} + 2m - 1}{2m}$$

Suppose that  $GR^L > 0$ 

$$GR^{L} = \frac{(1-t-2m)\Phi_{3} + 2m - 1}{2m} > 0$$

Therefore, the following inequality must hold:

$$(1-t-2m)\Phi_3+2m-1>0$$

Rearranging the inequality above yields:

$$\frac{2m-1}{(2m+t-1)} > \sqrt{\frac{m(1-t)-1}{(m-1)(1-t)}}$$

Squaring both sides of the inequality

$$\left(\frac{2m-1}{(2m+t-1)}\right)^2 > \frac{m(1-t)-1}{(m-1)(1-t)}$$

The inequality above can be expressed as

$$(2m-1)^{2}(m-1)(1-t) > (2m+t-1)^{2}(m(1-t)-1)$$

Therefore, the following inequality must hold

$$(2m-1)^{2}(m-1)(1-t)-(2m+t-1)^{2}(m(1-t)-1)>0$$

The inequality above can be simplified as

$$t\left(mt^{2} + \left(4m^{2} - 3m + 1\right)t + 2m - 1\right) > 0$$

Denote  $g = mt^2 + (4m^2 - 3m + 1)t + 2m - 1$ 

The first order condition implies that  $\partial g / \partial t = 2mt + 3m(m-1) + m^2 + 1$ . Since *m* is the number of oligopolists, *g* is monotonously increasing in *t* (i.e.,  $\partial g / \partial t > 0$ ) and

$$g > \overline{g}\Big|_{t=0} = 2m - 1 > 0.$$

It turns out that  $GR^L > 0$ , i.e., the government will receive positive revenue from the lump-sum transfers.

Q.E.D.

In conclusion, the government gains from the policy of lump-sum transfers. The money received from consumers outweighs government's expenditure. Thus, all players in this game can be made better off by altering the taxation policy.

According to Grazzini (2006), per-unit taxation can be welfare superior to *ad valorem* taxation in the context of general equilibrium. However, as can be seen from above, welfare under *ad valorem* taxation is always greater than that under per-unit taxation if a lump-sum transfer is introduced. The result of Grazzini's model is due to the effect of taxation on the distribution of income rather than from any efficiency gain.

In order to analyse why Grazzini derives a result that per-unit taxation is preferable in the view of social welfare, I calculate the marginal utility of transfer in the absence of taxes (i.e., t = 0 and  $\tau = 0$ ). From equation 2.3.33 (or equation 2.3.35), the marginal utility of transfer for each consumer obtains as:

$$MUT^{c} = \frac{\partial v}{\partial T_{n}} \bigg|_{T_{n}=0} = \frac{m-1}{2\alpha mn}$$
(2.3.48)

Similarly, from equation 2.3.40 (or equation 2.3.42), the marginal utility of transfer for each oligopolist obtains as:

$$MUT^{\circ} = \frac{\partial v}{\partial T_m} \bigg|_{T_m = 0, T_n = 0} = \frac{1}{4\alpha m^2}$$
(2.3.49)

The relation between the number of consumers and oligopolists when per-unit taxation welfare dominates *ad valorem* taxation in Grazzini's paper is

$$n > \frac{4m^2}{2-t}$$
 (2.3.50)

It can be shown that when the number of consumers exceeds the critical number above, the marginal utility transfer for each oligopolist is always greater than that of each consumer. The proof can be seen as follows. By substituting  $n = \frac{4m^2}{2-t}$  into equation (2.3.48) and considering zero tax (t = 0),

yields:

$$MUT_{c}^{c} = \frac{m-1}{2\alpha m \left(\frac{4m^{2}}{2}\right)} = \frac{m-1}{4\alpha m^{3}}$$

Calculate the difference in the marginal utility of transfer for each oligopoly and consumer when the number of consumers is set to be the critical value in Grazzini's model:

$$MUT_{c}^{o} - MUT_{c}^{c} = \frac{1}{4\alpha m^{2}} - \frac{m-1}{4\alpha m^{3}} = \frac{1}{4\alpha m^{3}} > 0$$

By setting  $\alpha = 1$  and n = 5, the following diagram shows the marginal utility of transfer for each oligopolist and consumer:



Figure 2–3: Marginal utility of income for consumers and oligopolists

The vertical line in figure 2-3 is the critical value for m in (2.3.50). It can be clearly seen that when the number of oligopolists is sufficiently low with respect to that of consumers, each oligopolist has a higher degree of marginal utility of transfer. As has been clarified above, oligopolists receive unambiguously higher payoff under the regime of per-unit taxation. During the shift from *ad valorem* taxation to per-unit taxation, transferring one unit of the consumption good from consumers to oligopolists will cause an increase in the social welfare.

# 2.4 Conclusion

Grazzini (2006) compares welfare under two types of taxation, i.e., per-unit and ad valorem taxation, in the case of general equilibrium. She shows that per-unit taxation may be superior to *ad valorem* taxation in terms of welfare. However, there are several limitations in Grazzini's model. First, as social welfare is measured by a utilitarian social welfare function and the utility function is formed as homogeneous of degree two  $(x_1x_2)$ . It implies that the social welfare put more weigh on rich consumers and the distribution of income may enter the determination of social welfare. One can argue that Grazzini's result may not be convincing because a different result would be derived with a new utility function. Section 2.2 shows that the model that involves a utility function that is homogeneous of degree one  $(x_1^{1/2}x_2^{1/2})$  will result in an opposite result, i.e., welfare under per-unit taxation is less than under *ad valorem* tax. This dominance is consistent with the majority of literature. The utility function is a monotonic transformation of  $x_1x_2$  and both functions represent the same preferences of consumers. The only difference is the measurement of social welfare. It can be summarised that the exceptional result from Grazzini's paper may be determined by the measurement of social welfare rather than the general equilibrium framework.

Second, the result of Gazzini's model is due to the effect of taxation on the redistribution of income rather than from any efficiency gain. In order to further compare welfare under per-unit and *ad valorem* taxation, a lump-sum transfer is introduced from one group to the other in section 2.3. We consider a shift from a per-unit tax to an *ad valorem* tax which gives the same tax revenues. Regarding the consumers, they receive higher utilities with *ad valorem* tax. As a result, they would pay up to an amount of  $\tilde{T}_n$  to enter an economy with *ad valorem* taxation. To be more specific, when they pay an amount of  $\tilde{T}_n$  to the government, they will be indifferent to these two taxation regulations and the government will receive extra revenue which

equals to  $n\tilde{T}_n$ . On the other hand, oligopolists obtain lower payoffs under *ad valorem* taxation. In order to encourage oligopolists to accept the new regulation, government promises to offer an amount  $\tilde{T}_m$  of lump-sum subsidies for each oligopolist. As a consequence, the government has to spend a total amount of  $m\tilde{T}_m$  and oligopolists will be indifferent under per-unit and *ad valorem* taxation. As can be seen from section 2.3, the government revenue from the lump-sum tax is greater than the government expenditure from the lump-sum subsidy. It turns out that by introducing the *ad valorem* taxation, the government will receive a positive revenue. It is clear that all players (i.e., consumers, oligopolists and government) are made better off under *ad valorem* taxation. Thus, welfare with *ad valorem* tax is ambiguously greater than that with per-unit tax.

Besides, the result of Grazzini's paper tends to be a consequence of the difference between the marginal utility of transfer for each oligopolist and consumer. Due to the fact that each oligopolist has a higher level of marginal utility of transfer when the number of oligopolists is relatively low compared to that of consumers, social welfare is higher in the case of per-unit tax since oligopolists obtain more payoffs with per-unit tax.

# Chapter 3 : *Ad Valorem* Versus Per-unit (Specific) Tariff with FDI Constraint under Cournot and Bertrand Duopoly

### 3.1 Introduction

The theoretical models have argued that strategic considerations affect the decision between exports and FDI, starting with Horstmann and Markusen (1987) and Smith (1987), where FDI is treated as a strategic investment in models of intra-industry trade under Cournot duopoly. Horstmann and Markusen (1992) and Rowthorn (1992) examine the decision between exporting and FDI in symmetric two-country models with endogenous market structure. It is assumed that there are fixed costs at the firm level, plant-specific fixed cost and increasing returns in production. They suggest that the decision of the entry depends on the trade-off between market size and trade costs (what is called proximity-concentration trade-off). FDI strategy is favoured when plantspecific fixed costs are sufficiently low compared to trade costs. By allowing for potential entry by domestic producers, Motta (1992) shows that the tariff may have opposite effects on the decision of the multinational firm. Specifically, the tariff may lead the multinational to decide not to invest as the local firm may enter the market with the introduction of the tariff. The tariff-jumping investment could improve the welfare of the host-country only when the local firm would not have participated in the competition under free trade. Motta and Norman (1996), Norman and Motta (1993) and Neary (2002a) analyse the effects of the trade liberalisation on the FDI pattern in twocountry and three-country models, and argue that a reduction in trade cost may encourage FDI. Norman and Motta (1993) assume asymmetric production costs and focus the analysis only on the FDI strategy of the outsider firm. In the following contribution in 1996, they consider the influence of market accessibility on both outsider and insider FDI with identical production costs. Neary (2002a) also studies the FDI strategy of the outsider firm and assumes that the potential multinational has a firstmove advantage. Unlike most literature on export and FDI under oligopoly that employs static game theory models, Leahy and Pavelin (2003) use an infinitelyrepeated game to present the follow-my-leader FDI put forward by Knickerbocker (1973). They show that there is a positive interdependence between FDI decisions of firms. In other words, foreign investment by one firm gives rise to the increased incentive for others to follow suit. Neary (2009) explains why trade liberalisation results in a massive increase in FDI in the real world. It happens for the following two reasons. To begin with, intra-bloc trade liberalisation has a positive effect on horizontal FDI in trading blocs, since foreign firms establish plants in one country as export platforms to serve the bloc as a whole. In addition, trade liberalisation encourages cross-border mergers and acquisitions (M&As) and this form of FDI (M&As) is quantitatively more important than greenfield FDI. Mukherjee and Suetrong (2008) consider the case of home-country export platform FDI, and show that both the positive and negative relationships between FDI and trade cost can happen. Besides, they present the implications of market size and competition between the asymmetric home and hostcountry firms. Collie (2009) uses an infinitely-repeated game with both Cournot duopoly and Bertrand duopoly models with differentiated goods. As in a static game, undertaking FDI by one firm will intensify competition and therefore, reduce its competitor's profits in its home market. The outcome is often a prisons' dilemma where all firms are made worse off if they all undertake FDI than if they all export. Collie (2009) shows that the prisons' dilemma can be avoided by implicitly colluding by choosing to export rather than undertake FDI. Then trade liberalisation may lead firms to switch from exporting to undertaking FDI when trade costs are sufficiently high. Recently, Collie (2011) explains the increases in both the amount of FDI and the volume of world trade by using a 'regional tariff jumping' model. In a setting with two regions and two countries in each of them, Collie (2011) shows that multilateral trade liberalisation may induce firms to shift from exporting to undertaking FDI when the inter-regional transport cost is sufficiently large.

Chapter 2 has outlined the existing literature regards the welfare comparison of per-unit and *ad valorem* taxation in public finance. The welfare comparison also applies to the study of international trade. Ad valorem and per-unit tariffs were firstly compared by Hillman and Templeman (1985). In an import monopoly market, they point out that, when the home country is supplied with imports by a foreign monopoly, any positive per-unit tariff could be replaced by an ad valorem tariff which gives a higher level of welfare. The intuition is that, since the tariff extracts revenue from the foreign monopolist, ad valorem tariff is necessarily superior to a per-unit tariff as the tariff revenue is higher with the ad valorem tariff. The Pareto comparison of ad valorem and per-unit tariff has been addressed by Kowalczyk and Skeath (1994). They have shown that, in a setting where a home country imports from a foreign monopolist, ad valorem tariffs are welfare superior to specific tariffs. However, there exists no general Pareto ranking of the two tariffs. When import demand is linear, it is shown that monopoly profits are higher with the optimal per-unit tariff than with the optimal ad valorem tariff. Skeath and Trandel (1994a) demonstrate that any per-unit tariff can be replaced by a Pareto-dominating ad valorem tariff if the home country is supplied by a foreign monopolist. In other words, the domestic consumer surplus, tariff revenue received by the home government and foreign profit are all larger with an ad valorem tariff. The result is driven by the fact that the foreign monopolist has an incentive to increase output when shifting from a per-unit to an ad valorem tariff due to the increased elasticity of import demand with an ad valorem tariff. In general equilibrium, Jørgensen and Schröder (2005) study welfare effect of two tariff regimes in a symmetric twocountry environment with homogeneous firms competing monopolistically and consumers who have the standard constant elasticity of substitution (CES) utility function. By considering that all tariff revenues will be redistributed to consumers, Jørgensen and Schröder (2005) demonstrate that per-unit tariff may welfare dominate ad valorem tariffs. The intuition has to do with the free entry and exit of firms. To be more specific, it is found that more domestic firms operate under a per-unit tariff regime, resulting in a larger number of varieties than under an *ad valorem* tariff. Therefore, consumer surplus under per-unit tariff is higher due to higher variety generated.

The comparison of *ad valorem* and per-unit tariffs under oligopoly starts with Helpman and Krugman (1989). They consider a foreign country that consists of nidentical firms who sell a quantity q of homogeneous product to an importing country and compete in a Cournot fashion. They find that an ad valorem tariff is preferred to a per-unit tariff. This is because the optimal per-unit tariff occurs when marginal revenue declines faster than price (i.e., p'(q) > MR'(q)) and the optimal *ad valorem* tariff occurs when the elasticity of the marginal revenue exceeds the elasticity of the demand (i.e.,  $\varepsilon^{MR(q)} > \varepsilon^{p(q)}$ ). It implies that whenever it is optimal to impose a per-unit tariff, it is also optimal to impose an *ad valorem* tariff because price exceeds marginal revenue. However, it does not apply to the converse. In addition to the Pareto comparison of ad valorem and per-unit tariff in a monopoly setting, Skeath and Trandel (1994a) also study the case when the home country faces a foreign oligopoly. They find that the Pareto dominance of an *ad valorem* over a per-unit tariff holds if tariffs are sufficiently large. The reason why the Pareto ranking does not generalise to all case under oligopoly is that, if one foreign oligopolist responds to the change from a per-unit tariff to an *ad* valorem tariff by increasing its exports to the home market, the price for imports will fall, which will harm the other foreign firms. As a result, the joint profits of all foreign oligopolists may be lower with ad valorem tariff. There is no guarantee that the market equilibrium must be at an elastic point on the import demand curve. Therefore, tariff revenues may decrease with an increase in imports. Collie (2006) analyses the welfare effects of per-unit and *ad valorem* trade policy instruments (import tariffs and production subsidies) in an asymmetric oligopoly market. He shows that these trade policy instruments have rationalisation effects (i.e., the effects relate to the equilibrium size of domestic and foreign firms) which depend on the curvature of demand and the variation of industry output. In particular, both per-unit and ad valorem tariffs move the industry up its demand curve thereby resulting in a flatter (steeper) demand curve if demand is concave (convex). There is an additional rationalisation effect by making the foreign firms' perceived demand curves flatter in the case of *ad valorem* tariff. By comparing welfare with the two tariffs that yield the same total consumption and price in the domestic market, Collie (2006) shows that the tariff revenue collected by the home government is higher with *ad valorem* tariff. Therefore, *ad valorem* tariff is welfare superior to a per-unit tariff. Shea and Shea (2006) examine the equivalence between per-unit and *ad valorem* tariffs in a quantity conjectural variation model. It is shown that if the tax revenue per unit of imports is the same for the two tariffs, an *ad valorem* tariff would generate more domestic output but less foreign output. In addition, they demonstrate that a per-unit tariff can be replaced by a Pareto-superior *ad valorem* tariff for all values of conjectural variation under duopoly.

In the literature of public finance (e.g. Anderson et al. (2001a) and Azacis and Collie (2014)), the firms' profits are proved to be higher with per-unit tax. The following analysis will confirm if the same result applies to tariffs in international trade for the foreign oligopolists. First, assume that there are two countries, a home and a foreign country and there are *n* firms in the home country and *m* firms in the foreign country that produce homogeneous goods and compete in the home market. Foreign firms can enter the home market by two alternative ways, namely, by exporting or by undertaking FDIs. In the first case, they bear a cost *t* for each unit exported if the home government imposes a per-unit or a proportion  $\tau$  of the value of the product if the method of *ad valorem* tariff is used<sup>11</sup>. In the second case, firms incur a sunk cost *G* of building a factory in the home market. All firms have the same marginal cost of production, c. Suppose that  $q_i^h$  and  $q_j^f$  are the quantities of the *i*th firm in the home country and *j*th firm in the foreign country, respectively. The inverse demand function is given by p = p(Q), where *P* is the consumer price and  $Q = \sum_{i=1}^n q_i^h + \sum_{j=1}^m q_j^f$  is total output. It is assumed that the inverse demand function is downward sloping (i.e.,

<sup>&</sup>lt;sup>11</sup> Please note that the letters used to denote two tariffs switched round from chapter two.

P'(Q) < 0) and there exists a Nash equilibrium in quantities denoted by  $q_1^{h^*}, \dots, q_n^{h^*}$  in the home country and  $q_1^{f^*}, \dots, q_m^{f^*}$  in the foreign country, with  $Q^* = \sum_{i=1}^n q_i^{h^*} + \sum_{j=1}^m q_j^{f^*}$ denoting the total equilibrium output. Following Anderson et al. (2001a), the comparison of the effects of the two regimes of import tariff will be achieved by seeking a per-unit and an *ad valorem* tariff that yields the same price.

The profits of the *i*th domestic firm are:

$$\pi_i^h = \left(p(Q) - c\right)q_i^h \tag{3.1.1}$$

The first-order condition of  $\pi_i^h$  in (3.1.1) with respect to  $q_i^h$  obtains as:

$$\frac{\partial \pi_i^h}{\partial q_i^h} = p(Q) - c + p'(Q)q_i^h = 0$$

$$\Rightarrow p(Q) + p'(Q)q_i^h = c, \quad i = 1,...n$$
(3.1.2)

The game is aggregative as in Bergstrom and Varian (1985) so that we can calculate the Cournot-Nash equilibrium without resorting to solving N first-order conditions.

Summing over all the first-order conditions from (3.1.2) yields:

$$np(Q^*) + p'(Q^*) \sum_{i=1}^{n} q_i^{h^*} = nc$$
 (3.1.3)

If the home government imposes a per-unit tariff, then the profits of the *j*th foreign firm can be calculated as:

$$\pi_{j}^{tf} = (p(Q) - c - t)q_{j}^{f}$$
(3.1.4)

The first-order condition of  $\pi_j^{tf}$  in (3.1.4) with respect to  $q_j^f$  obtains

$$\frac{\partial \pi_j^{tf}}{\partial q_j^f} = p(Q) - c - t + p'(Q)q_j^f = 0$$

$$\Rightarrow p(Q) + p'(Q)q_j^f = c + t, \quad j = 1,...m$$
(3.1.5)

Adding up all the first-order conditions from (3.1.5) yields:

$$mp(Q^{*}) + p'(Q^{*}) \sum_{j=1}^{m} q_{j}^{j*} = m(c+t)$$
(3.1.6)

Combining (3.1.3) and (3.1.6) yields:

$$(n+m)p(Q^*)+p'(Q^*)Q^*=nc+m(c+t)$$
 (3.1.7)

If the home government imposes an *ad valorem* tariff, then the profits of the *j*th foreign firm can be calculated as:

$$\pi_{j}^{\tau f} = ((1 - \tau) p(Q) - c) q_{j}^{f}$$
(3.1.8)

The first-order condition of  $\pi_j^{\tau f}$  in (3.1.8) with respect to  $q_j^f$  obtains as

$$\frac{\partial \pi_j^{tf}}{\partial q_j^f} = p(Q) - c - t + p'(Q)q_j^f = 0$$

$$\Rightarrow p(Q) + p'(Q)q_j^f = c + t, \quad j = 1,...m$$
(3.1.9)

Adding up all the first-order conditions from (3.1.9) yields:

$$mp(Q^{*}) + p'(Q^{*}) \sum_{j=1}^{m} q_{j}^{f^{*}} = \frac{mc}{(1-\tau)}$$
(3.1.10)

Combining (3.1.3) and (3.1.10) gives:

$$(n+m)p(Q^*)+p'(Q^*)Q^*=nc+\frac{mc}{(1-\tau)}$$
 (3.1.11)

Note that if  $t = \frac{c\tau}{1-\tau}$ , then the right-hand- sides of (3.1.7) and (3.1.11) are equal.

As  $Q^*$  is identical under the two tariffs, it follows that the equilibrium price is also the same. When  $t = \frac{c\tau}{1-\tau}$ , the profits of each foreign firm with per-unit and *ad valorem* tariff are, respectively:

$$\pi_{j}^{tf} = \left(p(Q) - c - t\right)q_{j}^{f} = \left(p(Q) - c - \frac{c\tau}{1 - \tau}\right)q_{j}^{f} = \left(p(Q) - \frac{c}{1 - \tau}\right)q_{j}^{f}$$

$$\pi_{j}^{\tau f} = \left((1 - \tau)p(Q) - c\right)q_{j}^{f} = (1 - \tau)\left(p(Q) - \frac{c}{1 - \tau}\right)q_{j}^{f} = (1 - \tau)\pi_{j}^{tf}$$
(3.1.12)

It can be concluded from (3.1.12) that, when the per-unit tariff and *ad valorem* tariff both cause the same price and output, the profits of the foreign firms with the *ad valorem* tariff are lower than that with the per-unit tariff. Intuition suggests that an increase in import tariffs will decrease the profitability of exporting relative to the profitability of undertaking FDI. Hence, a rise in import tariffs would cause foreign firms to switch from exporting to undertaking FDI. As the profits of the foreign firms are higher with per-unit tariff, this indicates that the foreign firms can supply the home market longer by exporting under per-unit tariff than under *ad valorem* tariff, if the policymaker continually increases the import tariff. If the government raises the tariff to a level that is above the critical level, the foreign firms will always prefer to undertake FDI and as a result, tariff revenue collected by the government will be zero.

There has been a lack of analysis of the comparison of per-unit and *ad valorem* tariffs given the constraint of FDI cost. The objective of this chapter is to check if the home government could receive higher tariff revenue with per-unit tariff than with *ad* 

*valorem* tariff given a constraint of FDI costs and to compare the welfare with two tariffs when the home government maximises the tariff revenue. For simplicity, it is assumed that each country has only one firm and the demand function is linear. Section 3.2 considers Cournot competition with homogeneous products and it is shown that government will always receive higher revenue with *ad valorem* tariff. Section 3.3 provides a Bertrand competition that allows for product differentiation, and the result reveals that the government may receive higher revenue with per-unit tariff when the FDI costs are sufficiently small. Regarding the welfare comparison, *ad valorem* tariff is superior to a per-unit tariff for both Cournot and Bertrand competition. The conclusions are in section 3.4.

## 3.2 Cournot competition with homogeneous products

In this section, it is assumed that a home firm competes with a foreign firm in a Cournot fashion, and they are producing homogeneous products as in the Brander (1981) and Brander and Krugman (1983) models of intra-industry trade.

In the home country, there is a representative agent who has the following quasilinear utility function:

$$U = a(q_1 + q_2) - \frac{b}{2}(q_1 + q_2)^2 + z$$
 (3.2.1)

where  $q_1$  and  $q_2$  are the consumption of the goods produced by the home firm and the foreign firm respectively, z is the consumption of the numeraire good. The parameter a is the consumers' maximum willingness to pay for the good and b is negatively related to the market size. The representative solves the utility maximisation problem by the first-order condition. Given the utility function (3.2.1), utility maximisation yields the inverse demand function:

$$p = a - b(q_1 + q_2) \tag{3.2.2}$$

### 3.2.1 The foreign firm chooses to export under per-unit tariff

First, assume that the foreign firm chooses to supply the home market by exporting and the home government imposes a per-unit tariff.

Denoting c as the marginal cost for both firms and assume that there is an interior solution where both firms make positive profits in the home market, the operating profits of the home firm and the foreign firm are:

$$\pi_1 = (p - c)q_1 \qquad \pi_2 = (p - c - t)q_2 \tag{3.2.3}$$

The fundamental assumption of Cournot competition is that firms realise that the quantity of output they supply to the market will affect their rival's optimal supply and vice versa. Therefore, each firm behaves optimally believing that its rival behaves optimally. As a result, each firm will choose its output to maximise its profits while treating its rival's output as given.

Taking the derivative of Cournot oligopoly ( $\partial \pi_1 / \partial q_1 = 0, \partial \pi_2 / \partial q_2 = 0$ ), the equilibrium outputs, prices and profits can be solved as:

$$q_{1E}^{t} = \frac{a - c + t}{3b} \qquad q_{2E}^{t} = \frac{a - c - 2t}{3b}$$

$$p_{E}^{t} = \frac{a + 2c + t}{3} \qquad (3.2.4)$$

$$\pi_{1E}^{t} = \frac{(a - c + t)^{2}}{9b} \qquad \pi_{2E}^{t} = \frac{(a - c - 2t)^{2}}{9b}$$

If the per-unit import tariff is prohibitive, i.e.  $q_{2E}^t = 0$ , when  $t \ge \overline{t} = \frac{a-c}{2}$ , the exports from the foreign firm to the home market and the profits of the foreign firm will be zero and the home firm will be a monopolist in the home market.

The revenue collected by the domestic government under per-unit tariff is:

$$R^{t} = t\left(q_{2E}^{t}\right) = t\frac{a-c-2t}{3b}$$
(3.2.5)

The maximum-revenue tariff is the tariff rate that maximises the tariff revenue.

The first-order condition for the maximisation of the tariff revenue obtains as:

$$\frac{\partial R^{t}}{\partial t} = \frac{a - c - 4t}{3b} = 0$$

The maximum-revenue tariff can then be solved as:

$$\tilde{t} = \frac{a-c}{4} \tag{3.2.6}$$

The welfare is measured by the sum of consumer surplus, the profits of the home firm and the tariff revenue as in Collie (1991).

Given the quasi-linear preferences (3.2.1), consumer surplus formulates as:

$$CS = U - p_E^t \left( q_{1E}^t + q_{2E}^t \right) - z$$
(3.2.7)

Using (3.2.7), the welfare of the domestic country obtains as

$$W_{E}^{t} = CS + \pi_{1E}^{t} + R^{t}$$
  
=  $\frac{2(a-c)^{2} + t(2a-2c-3t)}{6b}$  (3.2.8)

Collie (1991) compares the maximum-revenue tariff and the optimum-welfare tariff under oligopoly in a homogeneous product Cournot duopoly model with linear demand and constant marginal cost. He shows that, if domestic and foreign marginal costs are equal, the optimum-welfare tariff exceeds the maximum-revenue tariff due to the profit-shifting effects. The detailed proof can be seen in Appendix B5.

#### 3.2.2 The foreign firm undertakes FDI

Now consider the case that the foreign firm chooses to supply the home market by undertaking FDI.

Assuming that the outcome is an interior solution where both firms make positive profits in the home market, the operating profits of the home firm and the foreign firm are:

$$\pi_1 = (p - c)q_1 \quad \pi_2 = (p - c)q_2 - G \tag{3.2.9}$$
Where G is the fixed cost of FDI, i.e., a sunk cost of building a factory in the home market.

The equilibrium outputs, prices and profits of the two firms can be solved as:

$$q_{1F} = \frac{a-c}{3b} = q_{2F}$$

$$p_F = \frac{a+2c}{3}$$

$$\pi_{1F} = \frac{(a-c)^2}{9b} \qquad \pi_{2F} = \frac{(a-c)^2}{9b} - G$$
(3.2.10)

If the foreign firm undertakes FDI to supply the home market, the tariff revenue collected by the domestic government will be zero. The welfare is then measured by the combination of the consumer surplus and home firm's profit:

$$W_{F}^{t} = CS + \pi_{1F}^{t} = \frac{(a-c)^{2}}{3b}$$
 (3.2.11)

The foreign firm will undertake FDI if it is more profitable than exporting. Compare the profits of the foreign firm with FDI and per-unit tariff as below:

$$\pi_{2F} > \pi_{2E}^{t} \Longrightarrow \frac{\left(a-c\right)^{2}}{9b} - G > \frac{\left(a-c-2t\right)^{2}}{9b}$$

$$\Rightarrow G < G^{t} = \frac{4t\left(a-c-t\right)}{9b}$$
(3.2.12)

It follows that when the FDI cost is lower than the critical value  $G^{t}$ , the foreign firm will always prefer to supply the home market by undertaking FDI. By contrast, if the fixed cost of FDI exceeds  $G^{t}$ , the foreign firm will choose to export.

The first and second derivatives of the critical value  $G^t$  with respect to t obtain as:

$$\frac{\partial G^{t}}{\partial t} = \frac{4(a-c-2t)}{9b} > 0$$
$$\frac{\partial^{2}(G^{t})}{\partial t^{2}} = -\frac{8}{9b} < 0$$

The critical value of FDI cost  $G^t$  is a concave quadratic curve in import tariff and  $G^t$  reaches its peak when the import tariff is at the prohibitive level  $\overline{t}$ .

Figure 3-1 shows the relationship between the critical value of FDI cost and the level of per-unit tariff with the following parameter values: a = 10, b = 1, c = 2.



Figure 3–1: The profitability of FDI and exporting with per-unit tariff

As can be seen from figure 3-1, the foreign firm will choose to supply the domestic market by exporting in the region where  $G > G^t$ . The critical value  $G^t$  is increasing in import tariff t up to the prohibitive tariff  $\overline{t} = \frac{a-c}{2}$  and there is no export thereafter. The method of undertaking FDI is preferred for the foreign firm in the region where G < G', i.e., when the fixed cost of FDI is sufficiently low. It can be concluded from the diagram that an increase in import tariff will shift the foreign firm from exporting to undertaking FDI.

## 3.2.3 The foreign firm chooses to export under ad valorem tariff

Next, consider a case that the foreign firm chooses to supply the home market by exporting and the home government imposes an *ad valorem* tariff.

Assuming that there exists an interior solution where both firms make positive profits, the operating profits of the home firm and the foreign firm in the home market are:

$$\pi_1 = (p - c)q_1 \qquad \pi_2 = ((1 - \tau)p - c)q_2 \qquad (3.2.13)$$

The equilibrium outputs, prices and profits can be solved as:

$$q_{1E}^{\tau} = \frac{a - c - \tau (a - 2c)}{3b(1 - \tau)} \qquad q_{2E}^{\tau} = \frac{a - c - \tau (a + c)}{3b(1 - \tau)}$$

$$p_{E}^{\tau} = \frac{a + 2c - \tau (a + c)}{3(1 - \tau)} \qquad (3.2.14)$$

$$\pi_{1E}^{\tau} = \frac{(a - c - \tau (a - 2c))^{2}}{9b(1 - \tau)^{2}} \qquad \pi_{2E}^{\tau} = \frac{(a - c - \tau (a + c))^{2}}{25b(1 - \tau)}$$

If the *ad valorem* import tariff is prohibitive, i.e.  $q_{2E}^{\tau} = 0$ , when  $\tau \ge \overline{\tau} = \frac{a-c}{a+c}$ , the

exports from the foreign firm to the home market and the profits of the foreign firm will be zero.

The import tariff revenue under ad valorem tariff obtains as:

$$R^{\tau} = \tau p_{E}^{\tau} \left( q_{2E}^{\tau} \right) = \tau \left( \frac{a + 2c - \tau \left( a + c \right)}{3(1 - \tau)} \right) \left( \frac{a - c - \tau \left( a + c \right)}{3b(1 - \tau)} \right)$$
(3.2.15)

The maximum-revenue tariff  $\tilde{\tau}$  is calculated by solving the first-order condition for the maximisation of the import tariff revenue collected by the home government:

$$\frac{\partial R^{\tau}}{\partial \tau} = \frac{(1-\tau)^{3} (a+c)^{2} - (1-\tau)c(a-c) - 4c^{2}}{9b} = 0$$

The welfare of the home country is measured by the sum of consumer surplus, the profits of the home firm and the tariff revenue:

$$W_{E}^{\tau} = CS + \pi_{1E}^{t} + R^{\tau}$$

$$= \frac{2a^{2}(1-\tau)^{2}(3+\tau) + 2ac(2\tau^{3}-9\tau^{2}+13\tau-6) + c^{2}(2\tau^{3}+7\tau^{2}-16\tau+6)}{18b(1-\tau)^{2}}$$
(3.2.16)

The foreign firm will undertake FDI if the profits by undertaking FDI are greater than the profits by exporting:

$$\pi_{2F} > \pi_{2E}^{\tau} \Rightarrow \frac{(a-c)^{2}}{9b} - G > \frac{(a-c-\tau(a+c))^{2}}{25b(1-\tau)}$$

$$\Rightarrow G < G^{\tau} = \frac{\tau(a^{2}(1-\tau)+2ac(1-\tau)-c^{2}(3+\tau))}{9b(1-\tau)}$$
(3.2.17)

If the FDI cost is lower than the critical value  $G^{\tau}$ , undertaking FDI is preferred for the foreign firm, while if the FDI cost is sufficiently high, the foreign firm will choose to supply the home market by exporting.

The first and second derivatives of the critical value  $G^r$  with respect to  $\tau$  obtain as:

$$\frac{\partial G^{\tau}}{\partial \tau} = \frac{\left(a+c+\frac{2c}{1-\tau}\right)\left(a+c-\frac{2c}{1-\tau}\right)}{9b} > 0; \quad \frac{\partial^2\left(G^{\tau}\right)}{\partial \tau^2} = -\frac{8c^2}{9b(1-\tau)^3} < 0$$

The critical value of FDI cost  $G^r$  under *ad valorem* tariff is also a concave quadratic function in import tariff, reaching its peak when the import tariff is prohibitive, that is  $\tau = \overline{\tau}$ .

The relationship between the critical value of the fixed cost of FDI  $G^r$  and the level of *ad valorem* tariff is presented in figure 3-2, given the following parameter values a = 10, b = 1, c = 2.



*Figure 3–2: The profitability of FDI and exporting with ad valorem tariff* 

It can be seen from figure 3-2 that, as in the case of per-unit tariff, the critical value of the fixed cost  $G^{\tau}$  is increasing in the *ad valorem* tariff up to the prohibitive tariff  $\overline{\tau}$ .

## 3.2.4 The profitability of FDI and exporting with two import tariffs

To make these two regimes of import tariff comparable, the approach of Anderson et al. (2001a) will be used. The key idea is to assume the equilibrium price and total output are the same with the two import tariffs.

Equating the equilibrium price under per-unit tariff from (3.2.4) and under *ad valorem* tariff from (3.2.14):

$$p_E^t = p_E^\tau \Longrightarrow \frac{a + 2c + t}{3} = \frac{a + 2c - \tau (a + c)}{3(1 - \tau)}$$
$$\Longrightarrow \tau = \frac{t}{c + t}$$

This indicates that, when  $\tau = \frac{t}{c+t}$ , the equilibrium price, total consumption and consumer surplus are the same for the two regimes of the import tariff when the foreign firm chooses to supply the home market by exporting.

By setting  $\tau = \frac{t}{c+t}$ , the critical value of FDI cost under *ad valorem* tariff becomes  $t(a^2 + 2ac - c(3c + 4t))$ 

$$G^{r(t)} = \frac{t(a^2 + 2ac - c(3c + 4t))}{9b(c + t)}$$

Figure 3-3 shows the critical value of the fixed cost under per-unit tariff and the corresponding *ad valorem* tariff that results in the same equilibrium price, with the following parameter values a = 10, b = 1, c = 2.



Figure 3–3: The profitability of FDI and exporting with two import tariffs

Exporting is preferred for the foreign firm in the region *EE* under both per-unit and *ad valorem* tariff, while undertaking FDI is preferred in the region *FF* when the fixed cost of FDI is sufficiently low. In the region  $EF^{12}$ , the foreign firm will shift from exporting to undertaking FDI in the case of *ad valorem* tariff, while will continue supplying the home market by exporting in the case of per-unit tariff. It suggests that the foreign firm can supply the home market by exporting for a longer period with perunit tariff than with *ad valorem* tariff. This is because, as has been shown before, the foreign firm can make more profits with per-unit tariff. One important question is, if the import tariff revenue collected by the home government is increasing in import tariff up to the critical level where the foreign firm is just willing to supply the home country by exporting, can the home government receive higher revenue under per-unit tariff than under *ad valorem* tariff given the constraint of FDI cost?

<sup>&</sup>lt;sup>12</sup> Notice that in region DF, price will not be the same as there is no tariff revenue with the *ad valorem* tariff.

# 3.2.5 The maximum tariff revenue with two import tariffs

As has been shown above, the maximum-revenue tariff is  $\tilde{t} = \frac{a-c}{4}$ , substitute it into the critical level of the fixed cost of FDI yields:

$$G^{i} = \frac{(a-c)^{2}}{12b}$$
(3.2.18)

This implies that if the FDI cost exceeds  $G^i$ , the government has no incentive to increase the import tariff to any level above  $\tilde{t}$ .

The maximum tariff revenue is calculated by substituting the maximum-revenue tariff  $\tilde{t}$  into (3.2.5):

$$R^{\tilde{i}} = \frac{(a-c)^2}{24b}$$
(3.2.19)

In summary, for any level of FDI cost that is lower than  $G^i$ , the tariff revenue is increasing in t and the maximum revenue will be at the critical value of import tariff where  $G = G^i$ . For any level of FDI cost that exceeds  $G^i$ , the maximum import tariff revenue collected by the government will be constant at  $R^{\tilde{i}}$ .

Recall the critical value of the fixed cost of FDI in the case of per-unit tariff (3.2.12), the critical value of import tariff can be solved:

$$G = \frac{4t(a-c-t)}{9b} \Longrightarrow$$

$$\hat{t} = \frac{1}{2} \left( a - c - \sqrt{(a-c)^2 - 9bG} \right)$$
(3.2.20)

For any given level of FDI cost that is below  $G^{i}$ , if the rate of the per-unit import tariff is higher than  $\hat{t}$ , the foreign firm will choose to supply the home market by

undertaking FDI, and the tariff revenue collected by the home government will be zero. If the rate of import tariff is lower than  $\hat{t}$ , the foreign firm chooses to export so that the home government receives positive revenue. In order to obtain the maximum tariff revenue for a constraint of FDI cost, substitute  $\hat{t}$  into the government revenue in the case of per-unit tariff (3.2.5):

$$R^{\hat{i}} = \frac{\sqrt{(a-c)^2 - 9bG} \left(a - c - \sqrt{(a-c)^2 - 9bG}\right)}{6b}$$
(3.2.21)

As revenue is increasing in the trade costs up to  $\tilde{t}$  and  $\partial G^t / \partial t > 0$ , It follows that  $\partial R^{\hat{t}} / \partial G > 0$  up to the "unconstrained" maximum-revenue tariff<sup>13</sup>.

The relationship between the fixed cost of FDI and maximum revenue collected by the home government is shown from the following figure, given the following parameter values a = 10, b = 1, c = 2.



<sup>&</sup>lt;sup>13</sup> See Appendix B1 for the proof.

The maximum tariff revenue is increasing in the fixed cost of FDI up to  $G^{\overline{i}}$ . For  $G^{\overline{i}} < G < G^{\overline{i}}$ , the maximum tariff revenue remains at the "unconstrained" maximum level  $\frac{(a-c)^2}{12b}$  and it is independent of the FDI cost.

The "unconstrained" maximum tariff revenue  $R^{\tilde{t}}$  in the case of *ad valorem* tariff can be obtained by substituting  $\tilde{\tau}$  into (3.2.15). The corresponding critical value of FDI cost  $G^{\tilde{t}}$  is achieved by substituting  $\tilde{\tau}$  into (3.2.17). When the fixed cost exceeds  $G^{\tilde{t}}$ , the government has no incentive to increase the tariff rate as it has reached the "unconstrained" maximum level. If the fixed cost is lower than  $G^{\tilde{t}}$ , the maximum tariff revenue will be at the critical level of import tariff where  $G = G^{\tau}$ . As shown in Appendix B2,  $G^{\tilde{t}} > G^{\tilde{t}}$ . Therefore, if the FDI cost is increasing from  $G^{\tilde{t}}$  to  $G^{\tilde{t}}$ , the home government will impose a constant rate of import tariff equal to  $\tilde{t}$  in the case of per-unit tariff, while the import tariff is increasing in the fixed cost of FDI if the government imposes an *ad valorem* tariff.

According to the critical value of the fixed cost of FDI in the case of *ad valorem* tariff (3.2.17), the critical value of import tariff can be solved as:

$$G = \frac{\tau \left( (a-c)^{2} (1-\tau) - 2c^{2} (1+\tau) \right)}{9b(1-\tau)} \Longrightarrow$$

$$\hat{\tau} = \frac{(a-c)(a+3c) - \sqrt{\left( (a-c)^{2} - 9G \right) \left( (a+3c)^{2} - 9G \right)} + 9G}{2(a+c)^{2}}$$
(3.2.22)

With the constraint of FDI cost, the home government will receive zero tariff revenue if the import tariff exceeds  $\hat{\tau}$  since undertaking FDI is more profitable for the foreign firm. For any level of FDI cost that is lower than  $G^{\bar{\tau}}$ , the maximum tariff revenue given the constraint of FDI cost will be  $\hat{\tau}$ . Substituting  $\hat{\tau}$  into the tariff revenue  $R^{\tau}$ , the maximum tariff revenue when the fixed cost of FDI is lower than  $G^{\bar{\tau}}$ is:

$$R^{\hat{r}} = \frac{\left((a+c)^2 - 9bG + \Delta_1\right)\left((a+3c)(a-c) + 9bG + \Delta_1\right)\left(a^2 + 4ac + 7c^2 - 9bG + \Delta_1\right)}{18b\left(a^2 + 2ac + 5c^2 - 9bG + \Delta_1\right)}$$
(3.2.23)  
where  $\Delta_1 = \sqrt{\left((a+3c)(a-c) + 9bG\right)^2 - 36b(a+c)^2 G}$ 

The relationship between the fixed cost of FDI and the maximum tariff revenue in the case of *ad valorem* tariff is demonstrated in the following figure with the parameter values a = 10, b = 1, c = 2:



Figure 3–5: Import tariff revenue with ad valorem tariff

As in the case of per-unit tariff, the maximum tariff revenue with *ad valorem* tariff is increasing in the fixed cost of FDI up to  $G^{\tilde{\tau}}$ . For any FDI cost that is between  $G^{\tilde{\tau}}$ and  $G^{\tilde{\tau}}$ , the maximum tariff revenue will be at the constant at the "unconstrained" maximum level. The following figure combines figure 3-4 and 3-5 to compare the maximum tariff revenue given the constraint of FDI cost with the two import tariffs.



Figure 3–6: Import tariff revenue with two tariffs

It appears to show that, for any given fixed cost of FDI, the home government can receive higher revenue under *ad valorem* tariff than that under per-unit tariff.

**Proposition** 1: The maximum revenue with ad valorem tariff is always higher than that with per-unit tariff given the constraint of FDI cost under Cournot duopoly with homogeneous products.

# Proof.

Define  $\Delta R$  as the gap between the maximum tariff revenue with *ad valorem* tariff  $R^{t}$  and with per-unit tariff  $R^{t}$ .

The first-order derivative at zero FDI cost obtains as:

$$\left. \frac{\partial \Delta R}{\partial G} \right|_{G=0} = \frac{a-c}{4a+12c} > 0 \tag{3.2.24}$$

It indicates that the maximum revenue with *ad valorem* tariff dominates that with per-unit tariff in the neighbourhood of free FDI cost.

Solving  $\Delta R = 0$  gives the only one positive real root:

$$G^r = \frac{\left(a-c\right)^2}{9b}$$

It is easy to check that  $G^r = \frac{(a-c)^2}{9b} > \frac{(a-c)^2}{12b} = G^{\tilde{r}}$ . It follows that there is no

intersection between the curves of maximum revenue  $R^{t}$  and  $R^{r}$  if  $G \in (0, G^{i}]$ . Combine this with the fact that the revenue with two tariffs is identical with zero FDI cost (i.e.,  $\Delta R|_{G=0} = 0$ ) and the first-order derivative at the zero FDI cost is positive  $\frac{\partial \Delta R}{\partial G}\Big|_{G=0} > 0$ , it is sufficient to show that  $\Delta R$  is always positive when the fixed cost of

FDI is between 0 and  $G^i$ .

# Q.E.D.

Under per-unit tariff, for any level of FDI cost that is greater than  $G^i$ , the home government will stick with the maximum-revenue tariff  $\tilde{t}$  and receive a constant tariff revenue. In the case of *ad valorem* tariff, as the maximum tariff revenue is increasing in the fixed cost of FDI up to  $G^{\tilde{t}}$  which is higher than  $G^i$ , the revenue gap  $\Delta R$  will be even larger if FDI cost is between  $G^i$  and  $G^{\tilde{t}}$ . The relationship between  $\Delta R$  and fixed cost of FDI can be clearly seen from the following figure with the parameter values a = 10, b = 1, c = 2:



*Figure 3–7: The revenue gap between two tariffs* 

It can be concluded from Figure 3-7 that the maximum revenue with *ad valorem* tariff is always higher than that with per-unit tariff and the gap  $\Delta R$  is increasing in the level of FDI cost, reaching its peak at  $G^{\tilde{\tau}}$  and stabilising thereafter.

### 3.2.6 The profits of the home firm with two import tariffs

For any level of FDI that is lower than  $G^{i}$ , the profit of the home firm depends on the FDI cost if the policymaker aims at maximising the tariff revenue. The corresponding profits of the home firm are obtained by substituting  $\hat{t}$  from (3.2.20) into the profit function (3.2.4):

$$\pi^{\hat{i}} = \frac{\left(3(a-c) - \sqrt{(a-c)^2 - 9bG}\right)^2}{36b}$$
(3.2.25)

When the fixed cost of the FDI is greater than  $G^i$ , the government will impose the rate of per-unit tariff that equals to the "unconstrained" maximum-revenue tariff  $\tilde{t}$ . As

the profits of the home firm are increasing in the import tariff<sup>14</sup>, the home firm receives the highest profits when the rate of import tariff is  $\tilde{t} = \frac{a-c}{4}$ . Substituting  $\tilde{t}$  from (3.2.6) into the profits of the home firm  $\pi_{1E}^{t}$  from (3.2.4) yields:

$$\pi^{\tilde{i}} = \frac{25(a-c)^2}{144b}$$
(3.2.26)

Figure 3-8 shows the profits of the home firm given the constraint of the FDI cost with the following parameter values a = 10, b = 1, c = 2. The profit of the home firm is increasing in the fixed cost of FDI up to  $G^{i}$ . With an increased FDI cost, the home government can impose a higher import tariff on the foreign firm, and the home firm is benefiting from the rise in its rival's cost.



Figure 3–8: The profits of the home firm with per-unit tariff

<sup>&</sup>lt;sup>14</sup> To see this, taking the first order derivative of the home firm's profits from (3.2.4):  $\frac{\partial \pi_{1E}^{t}}{\partial t} = \frac{2(a-c+t)}{9b} > 0$ . Intuitively, the profits of the home firm are helped if there is an increase in the import tariff as the marginal cost of its foreign rival will increase.

In the case of *ad valorem* tariff, the profits of the home firm given the constraint of the FDI cost is obtained by substituting  $\hat{\tau}$  from (3.2.22) into  $\pi_{1E}^{\tau}$  from (3.2.14):

$$\pi^{\hat{t}} = \frac{a(a^2 + 2ac + 5c^2) - 8c^3 + (a - 2c)(\Delta_2 - 9bG)}{9b(a^2 + 2ac + 5c^2 - 9bG + \Delta_2)^2}$$
(3.2.27)

where 
$$\Delta_2 = \sqrt{((a-c)^2 - 9bG)((a+3c)^2 - 9bG)}$$
.

As the profits of the home firm are increasing in  $\tau^{15}$ , the maximum profits of the home firm  $\pi^{\tilde{\tau}}$  are achieved when the rate of *ad valorem* tariff equals to  $\tilde{\tau}$ . When the FDI cost exceeds  $G^{\tilde{\tau}}$ , the profits of the home firm will be identical at the level of  $\pi^{\tilde{\tau}}$ . Figure 3-9 illustrates the profits of home firm given the constraint of the FDI cost with the following parameter values a = 10, b = 1, c = 2. As in the case of per-unit tariff, the profits of the home firm given the constraint of FDI cost is increasing in G up to  $G^{\tilde{\tau}}$ .



Figure 3–9: The profits of the home firm with ad valorem tariff

<sup>&</sup>lt;sup>15</sup> See Appendix B3 for more detail.

Figure 3-10 compares the profits of the home firm with two tariffs. It suggests that the profits of the home firm with per-unit tariff always dominate the profits with *ad valorem* tariff.



Figure 3–10: The profits of the home firm with two import tariffs

**Proposition** 2: The profits of the home firm with per-unit tariff are always higher than that with ad valorem if the domestic government maximises the tariff revenue given the constraint of FDI cost.

# Proof.

Define  $\Delta \pi$  as the gap between the profits of the home firm with *ad valorem* and per-unit tariff.

It is easy to check that  $\Delta \pi |_{G=0} = 0$ . When the fixed cost of FDI is zero, undertaking FDI is always preferred to exporting whatever the way of import tariff is, thus the profits of the home firm will be identical.

The first-order derivative at zero FDI cost obtains as:

$$\left. \frac{\partial \Delta \pi}{\partial G} \right|_{G=0} = -\frac{a-c}{2(a+3c)} < 0 \tag{3.2.28}$$

 $\Delta \pi$  is decreasing in the FDI cost in the neighbourhood of zero FDI cost.

Solving  $\Delta \pi = 0$  yields the only one positive real root:

$$G^{r} = \frac{(a-c)^{2}}{9b}$$
(3.2.29)

Since 
$$G^{r} = \frac{(a-c)^{2}}{9b} > \frac{(a-c)^{2}}{12b} = G^{i}$$
,  $\pi^{t}$  and  $\pi^{\tau}$  are not equal if  $G \in (0, G^{i}]$ .

Notice that there is no difference between the profits of the home firm with per-unit and *ad valorem* tariff at zero FDI cost and  $\frac{\partial \Delta \pi}{\partial G}\Big|_{G=0} > 0$ , it is sufficient to say that  $\Delta \pi$  is

always negative when the fixed cost of FDI is between 0 and  $G^{i}$ .

# Q.E.D.

Since  $G^{\tilde{r}} > G^{\tilde{i}}$ , the profits of the home firm reach the peaks when the FDI cost is  $G^{\tilde{i}}$  in the case of per-unit tariff, while in the case of *ad valorem* tariff, the profits will continue increasing until the FDI cost rises to  $G^{\tilde{r}}$ . It is necessary to compare the maximum profits of the home firm with the two regimes of the tariffs. As shown in Appendix B4 that, given the constraint of FDI cost, the maximum profits with *ad valorem* tariff  $\pi^{\tilde{r}}$  are dominated by the maximum profit  $\pi^{\tilde{i}}$  with per-unit tariff. Therefore, the profits of the home firm are always higher with per-unit tariff given the constraint of FDI cost.



Figure 3–11: The gap in the profits of the home firm between two tariffs

Figure 3-11 shows the gap between the profits of the home firm with *ad valorem* and per-unit tariff with the parameter values a = 10, b = 1, c = 2. It can be clearly seen that the gap is decreasing in the FDI cost and reaching the minimum level when the FDI cost is  $G^{\tilde{i}}$ .

#### 3.2.7 The welfare comparison of the two import tariffs

As shown in Appendix B5, the optimal-welfare tariff exceeds the maximumrevenue tariff under both methods of the import tariffs, which is in line with Collie (1991). Therefore, the welfare is also increasing in import tariff up to the "unconstrained" maximum-revenue tariff.

In the case of per-unit tariff, the welfare when the FDI cost is lower than  $G^{i}$  can be calculated by substituting  $\hat{t}$  from (3.2.22) into the welfare function (3.2.8)

$$W^{i} = \frac{6(a^{2} + c^{2}) + 27bG + 2(a - c)\left(\sqrt{(a - c)^{2} - 9bG} - 6c\right)}{24b}$$
(3.2.30)

If the FDI cost exceeds  $G^i$ , the import tariff is independent of the fixed cost of FDI, and the level of welfare is constant at its peak. The highest welfare given the constraint of FDI cost when the tariff revenue is maximised is obtained by substituting the "unconstrained" maximum-revenue tariff  $\tilde{t}$  into the welfare function (3.2.8):

$$W^{\tilde{i}} = \frac{37(a-c)^2}{96b}$$

Figure 3-12 shows the welfare of the home country in the case of per-unit tariff with the following parameter values a = 10, b = 1, c = 2. As shown in Figure 3-12, the level of welfare is increasing the fixed cost of FDI.



Figure 3–12: The welfare with per-unit tariff

In the case of *ad valorem* tariff, the welfare of the home country given the constraint of the FDI cost is achieved by substituting  $\hat{\tau}$  from (3.2.22) into the welfare function (3.2.16):

$$W^{\hat{r}} = \frac{1}{576bc^2}$$

$$[(-a^4 - 12a^3c + 159c^4 + 522bc^2G - 81b^2G^2 + 6a^2(31c^2 + 3bG) \quad (3.2.31)$$

$$-4a(83c^3 - 27bcG) + (a^2 + 10ac - 11c^2 - 9bG)\Delta_3]$$
where  $\Delta_3 = \sqrt{((a-c)^2 - 9bG)((a+3c)^2 - 9bG)}$ 

If the home government aims at maximising the tariff revenue, the highest welfare given the constraint of FDI cost occurs when the rate of *ad valorem* tariff equals to  $\tilde{\tau}$ . For any level of FDI cost that is greater than  $G^{\tilde{\tau}}$ , the welfare of the home country will be constant at  $U^{\tilde{\tau}}$  as the government has no incentive to raise the import tariff to any level that is higher than  $\tilde{\tau}$ . Figure 3-13 demonstrates the welfare of the home country with the following parameter values a = 10, b = 1, c = 2.



Figure 3–13: The welfare with ad valorem tariff

Figure 3-14 compares the overall welfare with two regimes of the import tariffs. As can be observed from the figure, the welfare with *ad valorem* tariff dominates that with per-unit tariff.



Figure 3–14: The welfare with two import tariffs

**Proposition** 3: The welfare of the home country with ad valorem tariff is always higher than that with per-unit tariff if the domestic government maximises the tariff revenue given the constraint of FDI cost.

## Proof.

Define  $\Delta W$  as the gap between the welfare of the home country with *ad valorem* and per-unit tariff.

If the fixed cost of FDI is zero, the foreign firm will always choose to undertake FDI. Hence, imposing a per-unit tariff or an *ad valorem* tariff is a matter of no importance for the home government. As can be expected,  $\Delta W|_{G=0} = 0$ .

The first-order derivative at zero FDI cost obtains as:

$$\left. \frac{\partial \Delta W}{\partial G} \right|_{G=0} = \frac{a-c}{4a+12c} > 0$$

This shows that the  $\Delta W$  is increasing in the fixed cost of FDI cost at free FDI.

Solving  $\Delta W = 0$ , the only one positive real root is  $G^r = \frac{(a-c)^2}{9b}$ .

Because  $G^r$  exceeds the critical value  $G^i$ ,  $W^t$  and  $W^r$  are not equal if  $G \in (0, G^i]$ . Moreover, as the welfare gap is zero when the FDI cost is zero  $\Delta W|_{G=0} = 0$  and the first-order derivative at the zero FDI cost is positive  $\frac{\partial \Delta W}{\partial G}\Big|_{G=0} > 0$ , it can be concluded

that  $\Delta W$  is always positive when the fixed cost of FDI is between 0 and  $G^i$ . As has been shown above,  $G^{\tilde{i}} < G^{\tilde{r}}$ , the welfare gap  $\Delta W$  is even bigger when FDI cost is between  $G^i$  and  $G^{\tilde{r}}$ .

Q.E.D.

Figure 3-15 shows the gap of welfare  $\Delta W$  with the parameter values a = 10, b = 1, c = 2



Figure 3–15: The welfare gap between two tariffs

It can be seen from figure 3-15 that  $\Delta W$  is always positive and increasing in the fixed cost of FDI up to  $G^{\tilde{t}}$ . If the fixed cost of FDI increases from  $G^{\tilde{t}}$  to  $G^{\tilde{t}}$ , the welfare with per-unit tariff will remain unchanged, while the welfare with *ad valorem* tariff will continue growing.

Figure 3-16 shows the equilibrium price with the two import tariffs for different values of FDI cost.



Figure 3–16: The equilibrium prices with two import tariffs

It can be seen from Figure 3-16 that the equilibrium price is always lower with *ad valorem* tariff. As consumer surplus is defined as the difference between the maximum price a consumer is willing to pay and the actual price he pays, the consumers in the home country benefit from the lower price with *ad valorem* tariff and gain more consumer surplus.

# 3.3 Bertrand competition with differentiated products

In this section, it is assumed that the home firm competes with the foreign firm in a Bertrand fashion, and unlike section 3.2, the products are assumed to be differentiated as in Clarke and Collie (2003).

The utility function of a representative agent in the home country is given by:

$$U = a(q_1 + q_2) - \frac{b}{2}(q_1^2 + q_2^2 + 2rq_1q_2) + z$$
(3.3.1)

 $q_1$  and  $q_2$  are the consumption of the goods produced by the home firm and the foreign firm respectively, z is the consumption of the numeraire good.  $r \in [0,1]$  represents the degree of product differentiation and ranges from the case of independent demands (r = 0) to that of perfect substitutes (r = 1).

Given the quadratic utility function (3.3.1), utility maximisation yields the inverse demand functions:

$$p_{1} = a - b(q_{1} + rq_{2})$$
  

$$p_{2} = a - b(q_{2} + rq_{1})$$
(3.3.2)

By inverting the inverse demand functions above, the direct demand functions are obtained as:

$$q_{1} = \frac{a - p_{1} - ar + p_{2}r}{b - br^{2}}$$

$$q_{2} = \frac{a - p_{2} - ar + p_{1}r}{b - br^{2}}$$
(3.3.3)

# 3.3.1 The foreign firm chooses to export under per-unit tariff

To begin with, suppose that the foreign firm supplies the domestic market by exporting under per-unit tariff regime.

Assuming that there exists an interior solution where both firms make positive profits in the home market, the operating profits of the home and foreign firms are:

$$\pi_1 = (p_1 - c)q_1$$
  

$$\pi_2 = (p_2 - c - t)q_2$$
(3.3.4)

Under Bertrand competition, it is assumed that firms choose prices simultaneously and independently and sell the goods that are demanded at these prices as given by the inverse demand functions. Taking the derivatives of Bertrand oligopoly  $\partial \pi_1 / \partial p_1 = 0$ and  $\partial \pi_2 / \partial p_2 = 0$ , the equilibrium outputs, prices and profits can be solved as:

$$q_{1E}^{t} = \frac{(a-c)(2-r-r^{2})+rt}{b(2-r)(2+r)(1-r)(1+r)}$$

$$q_{2E}^{t} = \frac{(a-c)(1-r)(r+2)-(2-r^{2})t}{b(2-r)(2+r)(1-r)(1+r)}$$

$$p_{1E}^{t} = \frac{2(a+c)-ar(1+r)+r(c+t)}{4-r^{2}}$$

$$p_{2E}^{t} = \frac{2(a+c)-ar(1+r)+cr+2t}{4-r^{2}}$$

$$\pi_{1E}^{t} = \frac{\left((a-c)(2-r-r^{2})+rt\right)^{2}}{b(4-r^{2})^{2}(1-r^{2})}$$

$$\pi_{2E}^{t} = \frac{\left((a-c)(2-r-r^{2})-(2-r^{2})t\right)^{2}}{b(4-r^{2})^{2}(1-r^{2})}$$

(3.3.5)

If the per-unit import tariff is prohibitive, i.e.  $q_{2E}^t = 0$ , which happens when  $t \ge \overline{t} = \frac{(a-c)(2-r-r^2)}{2-r^2}$ , the foreign firm will not supply the home country as the profits of the foreign firm will be zero.

The tariff revenue collected by the home government when the import tariff is below  $\overline{t}$  is

$$R^{t} = t\left(q_{2E}^{t}\right) = t\frac{(a-c)(1-r)(r+2) - (2-r^{2})t}{b(2-r)(2+r)(1-r)(1+r)}$$
(3.3.6)

In order to obtain the maximum-revenue tariff, solving the first-order condition:

$$\frac{\partial R^{t}}{\partial t} = 0 \Longrightarrow$$

$$\tilde{t} = \frac{(a-c)(2-r-r^{2})}{2(2-r^{2})} \qquad (3.3.7)$$

When the rate of per-unit import tariff is  $\frac{(a-c)(2-r-r^2)}{2(2-r^2)}$ , the home government

receives the maximum tariff revenue.

The welfare is measured by the sum of consumer surplus, the profits of the home firm and the tariff revenue:

$$W_{E}^{t} = CS + \pi_{1E}^{t} + R^{t}$$
  
=  $\frac{2(a-c)^{2}(2-r-r^{2}) + 2(a-c)(1-r)(1+r)t - (3-2r^{2})t^{2}}{2b(4-5r^{2}+r^{4})}$  (3.3.8)

Clarke and Collie (2006) compare the maximum-revenue tariff and the optimumwelfare tariff under Bertrand duopoly with differentiated. It is shown that the optimalwelfare tariff may exceed the maximum-revenue tariff. In the symmetric case where the firms have the same cost, this will happen if the degree of product substitutability is sufficiently high. The detailed proof will be shown in Appendix B6.

# 3.3.2 The foreign firm undertakes FDI

Next, consider the case that the foreign firm undertakes FDI to supply the home market.

Assuming there is an interior solution where both firms make positive profits, the operating profits of the home and foreign firms are:

$$\pi_1 = (p - c)q_1 \qquad \pi_2 = (p - c)q_2 - G \tag{3.3.9}$$

The equilibrium outputs, prices and profits of the two firms can be calculated as:

$$q_{1F} = \frac{a-c}{2b+br-br^{2}} = q_{2F}$$

$$p_{1F}' = \frac{a+c-ar}{2-r} = p_{2F}'$$

$$\pi_{1F} = \frac{(a-c)^{2}(1-r)}{b(2-r)^{2}(1+r)} \qquad \pi_{2F} = \frac{(a-c)^{2}(1-r)}{b(2-r)^{2}(1+r)} - G$$
(3.3.10)

The tariff revenue is zero as the foreign firm does not export goods to the home country.

In order to decide how to supply the home country, the foreign firm will compare its profits in the way of exporting and undertaking FDI. The foreign firm will only choose to undertake FDI if it is more profitable than export:

$$\pi_{2F} > \pi_{2E}^{t} \Rightarrow \frac{(a-c)^{2}(1-r)}{b(2-r)^{2}(1+r)} - G > \frac{((a-c)(2-r-r^{2})-(2-r^{2})t)^{2}}{b(4-r^{2})^{2}(1-r^{2})}$$

$$\Rightarrow G < G^{t} = \frac{(2-r^{2})t(2(a-c)(2-r-r^{2})-(2-r^{2})t)}{b(4-r^{2})^{2}(1-r^{2})}$$
(3.3.11)

Therefore, the foreign firm will only undertake FDI if the fixed cost of FDI is lower than the critical value  $G^{t}$ .

It is easy to check that the first-order derivative  

$$\frac{\partial G'}{\partial t} = \frac{2(2-r^2)((a-c)(1-r)(2+r)-(2-r^2)t)}{b(4-r^2)^2(1-r^2)}$$
is positive and the second derivative  

$$\frac{\partial^2 (G')}{\partial t^2} = -\frac{2(2-r^2)^2}{b(4-r^2)^2(1-r^2)}$$
is negative. Thus, the critical value of FDI cost is a concave

quadratic in the import tariff which can be seen from figure 3-17 with the parameter values a = 10, b = 1, c = 2, r = 0.25 / 0.5 / 0.75



Figure 3–17: The profitability of FDI and exporting with per-unit tariff

The critical value of FDI cost is increasing in t up to the prohibitive tariff  $\overline{t}$ . The foreign firm will choose to undertake FDI if the import tariff is sufficiently high. A reduction in import tariff at the critical value may shift the foreign firm undertaking FDI to exporting. In addition, it tends to show that the foreign firm is more willing to export rather than undertake FDI if the degree of product differentiation is low and vice versa.

#### 3.3.3 The foreign firm chooses to export under ad valorem tariff

Now suppose that the foreign firm chooses to supply the home market by exporting in the case of *ad valorem* tariff. Assuming the outcome is an interior solution where both firms make positive profits in the home market, the operating profits of the home and foreign firms in the home market are:

$$\pi_1 = (p-c)q_1 \qquad \pi_2 = ((1-\tau)p-c)q_2 \qquad (3.3.12)$$

The equilibrium outputs, prices and profits can be solved as:

$$q_{1E}^{\tau} = \frac{(a-c)(2-r-r^{2})(1-\tau)+cr\tau}{b(4-5r^{2}+r^{4})(1-\tau)}$$

$$q_{2E}^{\tau} = \frac{(a(1-\tau)-c)(2-r-r^{2})-cr\tau}{b(4-5r^{2}+r^{4})(1-\tau)}$$

$$p_{1E}^{\tau} = \frac{c(2+r-2\tau)+a(2-r-r^{2})(1-\tau)}{(4-r^{2})(1-\tau)}$$

$$p_{2E}^{\tau} = \frac{a(2-r-r^{2})(1-\tau)+c(2+r-r\tau)}{(4-r^{2})(1-\tau)}$$

$$\pi_{1E}^{\tau} = -\frac{(a(2-r-r^{2})(1-\tau)+c(2-r-r^{2}-2\tau+r^{2}\tau))^{2}}{b(4-r^{2})^{2}(1-r^{2})(1-\tau)^{2}}$$

$$\pi_{2E}^{\tau} = \frac{(a(2-r-r^{2})(1-\tau)-c(2-r-r^{2}+r\tau))^{2}}{b(4-r^{2})^{2}(1-r^{2})(1-\tau)}$$

(3.3.13)

If the *ad valorem* import tariff is prohibitive, i.e.  $q_{2E}^{\tau} = 0$ , which happens when  $\tau \ge \overline{\tau} = \frac{(a-c)(2-r-r^2)}{cr+a(2-r-r^2)}$ , the foreign firm will not supply the home market as the

profits will be zero.

The import tariff revenue collected by the home government  $R^{\tau}$  is obtained as:

$$R^{\tau} = \tau p_{2E}^{\tau} \left( q_{2E}^{\tau} \right)$$
  
=  $\tau \left( \frac{a \left( 2 - r - r^2 \right) \left( 1 - \tau \right) + c \left( 2 + r - r \tau \right)}{\left( 4 - r^2 \right) \left( 1 - \tau \right)} \right) \left( \frac{a \left( 2 - r - r^2 \right) \left( 1 - \tau \right) - c \left( 2 - r - r^2 + r \tau \right)}{b \left( 4 - 5r^2 + r^4 \right) \left( 1 - \tau \right)} \right)$   
(3.3.14)

The maximum-revenue tariff  $\tilde{\tau}$  is achieved when the first-order condition  $\partial R^{\tau} / \partial \tau$  equals to zero.

The welfare of the home country is measured by the sum of consumer surplus, the profits of the home firm and the tariff revenue collected by the home government:

$$W = CS + \pi_{1E}^{\tau} + R^{\tau}$$
(3.3.15)

Undertaking FDI is a preferred way to supply the home market if it is more profitable than exporting:

$$\pi_{2F} > \pi_{2E}^{\tau} \Rightarrow \frac{(a-c)^{2}(1-r)}{b(2-r)^{2}(1+r)} - G > \frac{((a(1-\tau)-c)(2-r-r^{2})-cr\tau)^{2}}{b(4-r^{2})^{2}(1-r^{2})(1-\tau)}$$
  
$$\Rightarrow G < G^{\tau} = \frac{(a(a(2-r-r^{2})+2cr)((2-r-r^{2})(1-\tau))-c^{2}(4+r^{4}-r^{2}(5-\tau)))\tau}{b(4-r^{2})^{2}(1-r^{2})(1-\tau)}$$
  
(3.3.16)

It follows that the foreign firm will only undertake FDI if the fixed cost is lower than the critical value  $G^r$ . If the FDI cost exceeds  $G^r$ , exporting is a preferred way for the foreign firm.

The relationship between the critical value of the fixed cost of FDI  $G^r$  and the level of *ad valorem* tariff is shown in figure 3-18, given the following parameter values a = 10, b = 1, c = 2, r = 0.25 / 0.5 / 0.75.



Figure 3–18: The profitability of FDI and exporting with ad valorem tariff

 $G^r$  is increasing in import tariff and for any given level of FDI cost, the critical value of import tariff imposed by the home government is lower with a higher degree of r (i.e., lower level of product differentiation).

## 3.3.4 The maximum tariff revenue with two import tariffs

Substituting the maximum-revenue tariff  $\tilde{t}$  from (3.3.7) into the critical value of FDI cost (3.3.11) yields:

$$G^{\tilde{i}} = \frac{3(a-c)^2(1-r)}{4b(2-r)^2(1+r)}$$
(3.3.17)

For any level of FDI cost that is greater than  $G^{\tilde{t}}$ , the government will always impose the maximum-revenue tariff  $\tilde{t}$ . The "unconstrained" maximum revenue is achieved by substituting  $\tilde{t}$  into (3.3.6):

$$R^{\tilde{t}} = \frac{\left(a-c\right)^{2} \left(2-r-r^{2}\right)}{4b\left(4+2r-4r^{2}-r^{3}+r^{4}\right)}$$
(3.3.18)

Rearranging (3.3.11), the critical value of import tariff in the case of per-unit tariff obtains as:

$$G^{t} = \frac{(2-r^{2})t(2(a-c)(2-r-r^{2})-(2-r^{2})t)}{b(4-r^{2})^{2}(1-r^{2})}$$

$$\Rightarrow \hat{t} = \frac{(2+r)((1-r)a-c-\Delta_{4})}{2-r^{2}}$$
(3.3.19)

where 
$$\Delta_4 = \sqrt{(1-r)((a-c)^2(1-r)-bG(2-r)^2(1+r))}$$

The foreign firm will choose to undertake FDI if the import tariff is greater than the critical value  $\hat{t}$  and the tariff revenue will be zero. As the tariff revenue is increasing in import tariff up to  $\tilde{t}$ , for any level of FDI cost that is lower than  $G^{\tilde{t}}$ , the maximumrevenue tariff will be at the critical value  $\hat{t}$ . Substituting  $\hat{t}$  into the revenue function (3.3.6) yields:

$$R^{\hat{t}} = \frac{\left(2 - r - r^{2}\right)\left(\left(a - c\right)\Delta_{4} - \left(\left(a - c\right)^{2}\left(1 - r\right) - bG\left(2 - r\right)^{2}\left(1 + r\right)\right)\right)}{b\left(4 - 2r - 6r^{2} + 3r^{3} + 2r^{4} - r^{5}\right)}$$
(3.3.20)

Figure 3-19 shows the maximum revenue collected by the home government given the constraint of the fixed cost of FDI with the following parameter values a = 10, b = 1, c = 2, r = 0.25/0.5/0.75:



Figure 3–19: The import tariff revenue with per-unit tariff

As can be seen from figure 3-19, the maximum revenue is increasing in FDI cost up to  $G^{\tilde{i}}$ , then it reaches the "unconstrained" maximum level when the FDI cost reaches  $G^{\tilde{i}}$ . In addition, it turns out that the government receives higher import tariff revenue with less differentiated products when the FDI cost is sufficiently low. However, the unconstrained maximum tariff revenue is always higher with more differentiated products.

In the case of *ad valorem* tariff, the unconstrained maximum revenue  $R^{\tilde{\tau}}$  is obtained by substituting  $\tilde{\tau}$  into the revenue function (3.3.14) and the critical value of FDI cost  $G^{\tilde{\tau}}$  when the import tariff reaches the maximum-revenue tariff is obtained by substituting  $\tilde{\tau}$  into (3.3.16). If the FDI cost is higher than  $G^{\tilde{\tau}}$ , the government will always set the rate of import tariff equals to  $\tilde{\tau}$ . While if the FDI cost is lower than  $G^{\tilde{\tau}}$ , the maximum-revenue tariff will be at the critical value where  $G = G^{\tau}$ .

Rearranging the critical value of FDI cost (3.3.16) gives:

$$G^{\tau} = \frac{\left(a\left(a\left(2-r-r^{2}\right)+2cr\right)\left(\left(2-r-r^{2}\right)(1-\tau)\right)-c^{2}\left(4+r^{4}-r^{2}\left(5-\tau\right)\right)\right)\tau}{b\left(4-r^{2}\right)^{2}\left(1-r^{2}\right)(1-\tau)}$$
  
$$\Rightarrow \tau = \hat{\tau}$$
(3.3.21)

If the import tariff is higher than  $\hat{\tau}$ , undertaking FDI is preferred and the tariff revenue collected by the home government will be zero. Therefore, the maximum tariff revenue will be at  $\hat{\tau}$ . Substituting  $\hat{\tau}$  into the revenue function in the case of *ad valorem* tariff (3.3.14), the maximum tariff  $R^{\hat{t}}$  given the constraint of FDI cost can be obtained.

The relationship between FDI cost and maximum revenue in the case of *ad valorem* tariff is illustrated in figure 3-20 with the parameter values a = 10, b = 1, c = 2, r = 0.25 / 0.5 / 0.75:



Figure 3–20: The import tariff revenue with ad valorem tariff
The maximum tariff revenue is increasing in the fixed cost of FDI up to  $G^{\tilde{\tau}}$ . As in the case of per-unit tariff, when the FDI cost is sufficiently low, the tariff revenue with less differentiated products is higher than with more differentiated products. Regarding the "unconstrained" maximum tariff revenue, the higher the degree of product differentiation is, the more import tariff revenue the government will collect.

Figure 3-21 shows the relationship between  $\Delta R$  and the fixed cost of FDI given the following parameter values a = 10, b = 1, c = 2, r = 0.25 / 0.5 / 0.75



Figure 3–21: The revenue gap between two tariffs

**Proposition** 4: The maximum revenue with per-unit tariff is higher than that with ad valorem tariff if the FDI cost is sufficiently low under Bertrand duopoly with differentiated products.

### Proof.

Define the gap between  $R^{\tau}$  and  $R^{t}$  as  $\Delta R$ .

If the FDI cost is sufficiently low, the government revenue with per-unit tariff outweighs that with *ad valorem* tariff, and the gap is higher with the less differentiated product. However, the home government is in favour of *ad valorem* tariff when the FDI cost is relatively high and the products are more differentiated.

If the fixed cost of FDI is zero, the foreign firm will always choose to undertake FDI regardless what the regime of import tariff is. The first-order derivative at zero FDI cost is calculated as:

$$\frac{\partial \Delta R}{\partial G}\Big|_{G=0} = -\frac{(a-c)r^2(2-r-r^2)}{2(2-r^2)(a(2-r-r^2)+c(2+r-r^2))}$$
(3.3.22)

As the degree of product differentiation r is between 0 and 1, it is easy to check that  $\frac{\partial \Delta R}{\partial G}\Big|_{G=0} < 0$ . It shows that  $\Delta R$  is decreasing in the FDI cost in the neighbourhood

of free FDI.

### Q.E.D.

The introduction of product differentiation and nature of Bertrand competition both intensify the competition and lower the prices, the tariff revenue with *ad valorem* tariff will be very small when the FDI cost is close to zero. As tariff revenue in the case of *ad valorem* tariff is the products of prices, the quantity of exports from the foreign firms and the level of *ad valorem* tariff, the tariff revenue with per-unit tariff is more likely to be higher than that with *ad valorem* tariff when the FDI cost is close to zero.

### 3.3.5 The profits of the home firm with two import tariffs

When the trade cost is lower than  $G^{\tilde{t}}$ , the government will set the import tariff to the critical value  $\hat{t}$ , and the corresponding profits of the home firm can be calculated by substituting  $\hat{t}$  into (3.3.5):

$$\pi^{\hat{t}} = \frac{r\Delta_4 - (a-c)(2-r)(1-r)(1+r)}{b(2-r)^2(2-r^2)^2(1-r^2)}$$
(3.3.23)

When the trade cost exceeds  $G^{\tilde{t}}$ , the rate of import tariff will be constant at  $\tilde{t}$ , and the profits of the home firm are obtained by substituting  $\tilde{t}$  into (3.3.5):

$$\pi^{\tilde{i}} = \frac{(a-c)^2 (1-r) (4+r-2r^2)^2}{4b (2-r)^2 (1+r) (2-r^2)^2}$$
(3.3.24)

Figure 3-22 shows the profits of the home firm given the constraint of the FDI cost with the following parameter values a = 10, b = 1, c = 2, r = 0.25/0.5/0.75:



Figure 3–22: The profits of the home firm with per-unit tariff

It shows that the profits of the home firm are increasing in the fixed cost of FDI, and is positively related to the degree of product differentiation, i.e., the more differentiated the goods are, the higher profits the home firm will receive.

In the case of *ad valorem* tariff, the profits of the home firm given the constraint of the FDI cost is calculated by substituting  $\hat{\tau}$  into  $\pi_{1E}^{\tau}$  from (3.3.13). The highest profits  $\pi_{1E}^{\tilde{\tau}}$  are achieved when the rate of import tariff equals to  $\tilde{\tau}$ . Figure 3-23 shows the profits of the home firm given the constraint of the FDI cost with the following parameter values a = 10, b = 1, c = 2, r = 0.25/0.5/0.75:



Figure 3–23: The profits of the home firm with ad valorem tariff

Define  $\Delta \pi$  as the gap between the profits of the home firm with *ad valorem* and per-unit tariff. When the FDI cost is zero, undertaking FDI is always preferred for the foreign firm, and the profits of the home firm will be the same under two regimes of the import tariffs. Hence,  $\Delta \pi |_{G=0} = 0$ .

The first-order derivative at zero FDI cost shows:

$$\frac{\partial \Delta \pi}{\partial G}\Big|_{G=0} = -\frac{(a-c)(1-r)r(2+r)}{(2-r^2)(c(2+r-r^2)+a(2-r-r^2))} < 0$$
(3.3.25)

 $\Delta \pi$  is decreasing in the fixed cost of FDI in the neighbourhood of free FDI.

Figure 3-24 illustrates with  $\Delta \pi$ the parameter values a = 10, b = 1, c = 2, r = 0.25 / 0.5 / 0.75. It shows that  $\Delta \pi$  is decreasing in FDI cost when the FDI is low enough and reaching its minimum when FDI cost equals to  $G^{\tilde{i}}$ . For any level of FDI cost that is between  $G^{\tilde{t}}$  and  $G^{\tilde{t}}$ , the profits of the home firm with per-unit tariff are constant as the home government will always impose the "unconstrained" maximum-revenue tariff. By contrast, in the case of ad valorem tariff, the import tariff will continue growing until it reaches  $\tilde{\tau}$ . The profits gap will become constant if the FDI cost is above  $G^{\tilde{i}}$ . The figure suggests that the profits of the home firm are higher with per-unit tariff, which is consistent with the case of Cournot duopoly.



Figure 3–24: The profits gap of the home firm between two tariffs

### 3.3.6 The welfare comparison of the two import tariffs

In the case of per-unit tariff, given the constraint of FDI cost, the welfare  $W^{\hat{t}}$  can be calculated by substituting  $\hat{t}$  into the welfare function (3.3.8):

$$W^{i} = \frac{bG(2-r)^{2}(3-2r^{2})(1+r)(2+r)+2(a-c)((a-c)(3r-2r^{3})+(4+r-3r^{2}-r^{3})\Delta_{4}}{2b(2-r^{2})^{2}(2+r-r^{2})}$$
(3.3.26)

If the fixed cost of FDI is greater than  $G^{\tilde{i}}$ , the import tariff will be constant at  $\tilde{t}$ , and the corresponding welfare  $W^{\tilde{i}}$  is obtained by substituting  $\tilde{t}$  into the welfare function (3.3.8):

$$W^{\tilde{i}} = \frac{\left(a-c\right)^{2} \left(34+3r-37r^{2}-2r^{3}+10r^{4}\right)}{8b\left(2-r^{2}\right)^{2} \left(2+r-r^{2}\right)}$$
(3.3.27)

In the case of *ad valorem* tariff, the welfare  $W^{\hat{\tau}}$  given the constraint of the FDI cost is calculated by substituting  $\hat{\tau}$  into the welfare function (3.3.15). When the FDI cost is higher than  $G^{\tilde{t}}$ , the welfare of the home country will be constant at  $W^{\hat{\tau}}$ .

Lastly, define  $\Delta W$  as the gap between the welfare of the home country with *ad* valorem and per-unit tariff. If the FDI cost is zero, then undertaking FDI is always preferred. Therefore, there is no difference between imposing a per-unit tariff and an *ad valorem* tariff so that  $\Delta W|_{G=0} = 0$ .

The first-order derivative at zero FDI cost obtains as:

$$\frac{\partial \Delta W}{\partial G}\Big|_{G=0} = \frac{(a-c)(2-r-r^2)^2}{2(2-r^2)(c(2+r-r^2)+a(2-r-r^2))} > 0$$

It implies that the welfare of the home country is higher with *ad valorem* tariff in the neighbourhood of free FDI. Figure 3-25 illustrates the welfare gap  $\Delta W$  with the parameter values a = 10, b = 1, c = 2, r = 0.25 / 0.5 / 0.75.



*Figure 3–25: The welfare gap between two import tariffs when* r = 0.25 / 0.5 / 0.75

Figure 3-25 shows that the welfare of the home country is higher with *ad valorem* tariff as  $\Delta W$  is always positive. In addition, it appears to show that the welfare gap  $\Delta W$  is increasing in the degree of product differentiation. Unlike the case of Cournot competition where  $\Delta W$  is increasing in FDI cost up to  $G^{\tilde{t}}$ ,  $\Delta W$  under Bertrand competition reaches its peak at  $G^{\tilde{t}}$  and then starts decreasing thereafter until  $G^{\tilde{t}}$ . This is because the maximum-revenue tariff may exceed the optimal-welfare tariff when the value of r is sufficiently low. As noted by Clarke and Collie (2006) the maximum-revenue tariff is higher than the optimal-welfare tariff if r is between 0 and 0.78 under Bertrand duopoly with per-unit tariff<sup>16</sup>. If the value of r is above 0.78, then the optimal-welfare tariff exceeds the maximum-welfare tariff. The following figure shows  $\Delta W$  when r equals to 0.9.

<sup>&</sup>lt;sup>16</sup> See Appendix B6 for the detailed proof.



*Figure 3–26: The welfare gap between two import tariffs when* r = 0.9

Figure 3-26 suggests that, when the products are sufficiently close substitutes, then the welfare gap  $\Delta W$  between two regimes of import tariff is always increasing in the FDI cost.

# **3.4 Conclusion**

In summary, assume that the home government maximises the tariff revenue and changes the method of import tariff from per-unit to *ad valorem* type, the tariff revenue and consumer surplus will increase while the profits of the home firm will fall, and the overall welfare of the home country is higher with *ad valorem* tariff. This is because, when changing from per-unit to *ad valorem* tariff, the increase in the consumer surplus due to a lower equilibrium price, and the extra gain of the government revenue overweight a fall in the profits of the home firm. In addition, as the critical value of FDI cost of the maximum-revenue tariff under *ad valorem* tariff  $G^{t}$  exceeds that under per-unit tariff  $G^{t}$ , the home government is in favour of *ad valorem* tariff when the fixed cost of FDI is sufficiently high. Due to the optimal-welfare exceeds maximum-welfare tariff in Cournot duopoly with constant marginal cost, welfare is always increasing in the level of tariff. As a result, there is an additional welfare gain with *ad valorem* tariff when the fixed cost of FDI is relatively high.

Under Bertrand duopoly with differentiated products, the following conclusions can be drawn. First, the home firm is in favour of per-unit tariff as the profits are always higher with per-unit tariff than with *ad valorem* tariff. Besides, if the FDI cost is sufficiently low, the home government will receive higher tariff revenue with per-unit tariff, while *ad valorem* tariff always leads to higher tariff revenue under Cournot competition. This is because the introduction of product differentiation and nature of Bertrand competition both intensify the competition and lower the equilibrium prices. Lastly, the overall welfare is always higher with *ad valorem* tariff. This result is consistent with the case of Cournot competition. Switching from per-unit tariff to *ad valorem* tariff, if the FDI cost is high enough, the profits of the home firm will decrease while the increase in consumer surplus and government revenue will exceed the loss of the home firm. If the FDI cost is sufficiently small, there are two negative effects: both profits of the home firm and tariff revenue collected by the home government will fall.

However, the consumer surplus will increase and the gain from consumer surplus will outweigh the loss from the home firm and the government. Therefore, welfare is higher with *ad valorem* tariff even when FDI cost is close to zero.

# **Appendix B**

1. To show that the maximum revenue with per-unit tariff is increasing in

FDI costs 
$$\frac{\partial R^{i}}{\partial G} = \frac{3\left(2\sqrt{(a-c)^{2}-9bG}-(a-c)\right)}{4\sqrt{(a-c)^{2}-9bG}} > 0$$
 when  $G < G^{i} = \frac{(a-c)^{2}}{12b}$ 

Assume that  $2\sqrt{(a-c)^2-9bG} > (a-c)$ , squaring both sides yields:

$$2\left(\left(a-c\right)^2-9bG\right)>\left(a-c\right)^2$$

Alternatively, we have  $\frac{2((a-c)^2 - 9bG) - (a-c)^2 > 0}{3((a-c)^2 - 12bG) > 0}$ 

As 
$$G < \frac{(a-c)^2}{12b}$$
, therefore,  $\frac{\partial R^i}{\partial G} > 0$  is proved.

2. To compare the critical value of FDI costs with the maximum import tariff in both regimes of tariffs and show that  $G^{\tilde{\tau}} > G^{\tilde{t}}$ .

$$G^{i} = \frac{(a-c)^{2}}{12b}$$
 is the critical value of FDI cost it reaches the "unconstrained"

maximum import tariff in the case of per-unit tariff. We can find out an equivalent *ad valorem* tariff that yields the same critical value of FDI by using (3.2.17):

$$\frac{(a-c)^2}{12b} = \frac{\tau \left(a^2 \left(1-\tau\right)+2ac \left(1-\tau\right)-c^2 \left(3+\tau\right)\right)}{9b \left(1-\tau\right)}$$
$$\Rightarrow \overline{\tau} = \frac{7a^2+2ac-9c^2-(a-c)\sqrt{a^2+30ac+33c^2}}{2\left(4a^2+8ac+4c^2\right)}$$

Next, confirm that if the first-order derivative  $\frac{\partial R^{\tau}}{\partial \tau}$  is increasing when  $\tau = \overline{\tau}$ 

$$\frac{\partial R^{r}}{\partial \tau}\Big|_{\tau=\bar{\tau}} = \frac{1}{9a^{2}b + 126abc + 153bc^{2} + 9b(a-c)\sqrt{a^{2} + 30ac + 33c^{2}}} + (a-c)(a+c)^{2}[a^{5} + 27a^{4}c - 278a^{3}c^{2} - 1706a^{2}c^{3} - 2779ac^{4} - 1409c^{5} + \sqrt{a^{2} + 30ac + 33c^{2}}(a^{4} + 12a^{3}c + 150a^{2}c^{2} + 364ac^{3} + 241c^{4})]$$

Define 
$$\frac{y = a^5 + 27a^4c - 278a^3c^2 - 1706a^2c^3 - 2779ac^4 - 1409c^5}{+\sqrt{a^2 + 30ac + 33c^2} \left(a^4 + 12a^3c + 150a^2c^2 + 364ac^3 + 241c^4\right)}$$

It remains to show that y > 0. Divided by  $a^5$  on both sides and define  $x = \frac{c}{a}$ ,

we have:

$$\frac{y}{a^5} = 1 + 27x - 278x^2 - 1706x^3 - 2779x^4 - 1409x^5 + \sqrt{1 + 30x + 33x^2} \left(1 + 12x + 150x^2 + 364x^3 + 241x^4\right)$$

Since a > c > 0, it can be shown that .1 > x > 0. The following figure shows the value of  $\frac{y}{a^5}$  when the value of x is between 0 and 1:



Therefore,  $\check{\tau}$  is lower than the "unconstrained" maximum-revenue tariff  $\tilde{\tau}$ . Thus, it is sufficient to show that  $G^{\tilde{\tau}} > G^{\tilde{t}}$ .

3. To show that the profits of the home firm are increasing in the level of

ad valorem tariff 
$$\frac{\partial \pi_{1E}^{\tau}}{\partial \tau} > 0$$

To see this, taking the first-order derivative of the home firm's profits from (3.2.14):

$$\frac{\partial \pi_{1E}^{\tau}}{\partial \tau} = \frac{2c(a(1-\tau)+2c\tau-c)}{9b(1-\tau)^3}$$
. It remains to show that  $a(1-\tau)+2c\tau-c>0$ 

Define  $f(\tau) = a(1-\tau) + 2c\tau - c$ , the first-order derivative shows that  $\frac{\partial f(\tau)}{\partial \tau} = a - 2c$ . There are two possibilities. First, if a > 2c, then it follows that

 $\frac{\partial f(\tau)}{\partial \tau} > 0. \text{ The minimum value of } f(\tau) \text{ is } f(\tau)\Big|_{\tau=0} = a - c > 0. \text{ Therefore } f(\tau) > 0.$ 

Second, if a < 2c and  $\frac{\partial f(\tau)}{\partial \tau} < 0$ . The minimum value of  $f(\tau)$  is  $f(\tau)|_{\tau=1} = c > 0$ .

Therefore,  $f(\tau) > 0$ .  $\frac{\partial \pi_{1E}^{\tau}}{\partial \tau} > 0$  is then proved.

4. To compare the profits with the maximum import tariff in both regimes of tariffs and show that  $\pi^{\tilde{i}} > \pi^{\tilde{r}}$ .

First, from the critical value of the fixed cost of FDI in the case of *ad valorem* tariff (3.2.17), find out an import tariff that is equal to the maximum profits of the home firm in the case of per-unit tariff (3.2.26):

$$\frac{\tau\left(a^{2}\left(1-\tau\right)+2ac\left(1-\tau\right)-c^{2}\left(3+\tau\right)\right)}{9b\left(1-\tau\right)} = \frac{25(a-c)^{2}}{144b}$$
$$\Rightarrow \hat{\tau} = \frac{a-c}{a+3c}$$

Next, it has been shown that, in the case of *ad valorem* tariff, the home government collects the maximum revenue when the rate of import tariff is  $\tilde{\tau}$  and the corresponding critical value of FDI cost is  $G^{\tilde{\tau}}$ . Therefore, any amount of *ad valorem* tariff that is higher than  $\tilde{\tau}$ , the first-order derivative  $\frac{\partial R^{\tau}}{\partial \tau}$  will be negative and thus, the critical

value of FDI cost will be higher than  $G^{\tilde{\tau}}$ . The first-order derivate  $\frac{\partial R^{\tau}}{\partial \tau}$  when the import

tariff is  $\hat{\tau} = \frac{a-c}{a+3c}$  obtains as:

$$\left. \frac{\partial R^{\tau}}{\partial \tau} \right|_{\tau=\hat{\tau}} = -\frac{\left(a-c\right)^2 \left(a+c\right)}{72bc} < 0$$

Therefore,  $\hat{\tau} > \tilde{\tau}$  and  $G^{\hat{\tau}} > G^{\tilde{\tau}}$ . It shows that the maximum profits of the home firm in the case of *ad valorem* tariff  $\pi^{\tilde{\tau}}$  are lower than that in the case of per-unit tariff  $\pi^{\tilde{\tau}}$  if the policymaker maximises the tariff revenue.

5. To show that the optimal-welfare tariff is higher than the maximumrevenue tariff in the case of a per-unit tariff.

From (3.2.8), the first-order condition of the welfare shows:

$$\frac{\partial W_E^{t}}{\partial t} = \frac{a - c - 3t}{3b} = 0$$
$$\implies \tilde{t}^w = \frac{a - c}{3}$$

Compare  $\tilde{t}^w$  to the maximum-revenue tariff, we have:  $\tilde{t}^w > \tilde{t} = \frac{a-c}{4}$ . This has been proved by Collie (1991).

In the case of *ad valorem* tariff, from (3.2.16), the first-order condition of the welfare shows:

$$\frac{\partial W_E^{\tau}}{\partial \tau} = 0$$
$$\Rightarrow \tilde{\tau}^w = \frac{2a + c - \sqrt{c}\sqrt{4a + 5c}}{2(a + c)}$$

Substituting  $\tilde{\tau}^w$  into the first-order derivative of the tariff revenue

$$\frac{\partial R^{\tau}}{\partial \tau}\bigg|_{\tau=\tilde{\tau}^{W}} = -\frac{4\sqrt{c}\left(a+c\right)^{2}\left(2a+c-\sqrt{c}\sqrt{4a+5c}\right)}{3b\left(\sqrt{c}+\sqrt{4a+5c}\right)^{3}}$$

Define  $y = 2a + c - \sqrt{c}\sqrt{4a + 5c}$ . Divided by *a* on both sides of *y* and define  $x = \frac{c}{a}$ , we have:

$$\frac{y}{a} = 2 + x - \sqrt{x}\sqrt{4 + 5x}$$

Since a > c > 0, it can be shown that 1 > x > 0. The following figure shows the value of  $\frac{y}{a}$  when the value of x is between 0 and 1:



The minimum value of  $\frac{y}{a}$  is calculated when  $x = 1: \frac{y}{a}\Big|_{x=1} = 0$ .

Therefore y > 0 and then  $\frac{\partial R^{\tau}}{\partial \tau}\Big|_{\tau = \tilde{\tau}^{w}} < 0$ . As the first-order derivative is negative,

one can conclude that  $\tilde{\tau}^w$  exceeds the maximum-revenue tariff  $\tilde{\tau}$ .

6. To show that the maximum-revenue tariff may exceed the optimalwelfare tariff when the value of r is sufficiently low in the case of *ad valorem* tariff.

Recall the welfare function (3.3.8) under Bertrand competition, the first-order condition gives:

$$\frac{\partial W^{t}}{\partial t} = \frac{2a(-1-r)(-1+r)+2c(-1+r^{2})+2(-3+2r^{2})t}{2b(4-5r^{2}+r^{4})} = 0$$
$$\Rightarrow \tilde{t}^{w} = \frac{(a-c)(1-r^{2})}{3-2r^{2}}$$

Define  $\Delta t$  as the gap between the maximum-revenue tariff and the optimal-welfare tariff.

$$\Delta t = \tilde{t} - \tilde{t}^{w} = \frac{(a-c)(2-3r-r^{2}+2r^{3})}{2(6-7r^{2}+2r^{4})}$$

The following figure shows  $\frac{\left(2-3r-r^2+2r^3\right)}{2\left(6-7r^2+2r^4\right)}$  when *r* changes from 0 to 1:



Solving  $\Delta t = 0$  gives the roots of *r*:

$$r_1 = 1, r_2 = 0.78, r_3 = -1.28$$

Therefore, when r is between 0 and  $r_2$ , the maximum-revenue tariff is higher than the optimal-welfare tariff. If r is greater than  $r_2$ , then  $\Delta t < 0$  and the optimal-welfare tariff exceeds the maximum-revenue tariff. The proof has been done in Clarke and Collie (2006).

# **Chapter 4 : Welfare Effects of Trade cost in General Oligopolistic Equilibrium (GOLE)**

## **4.1 Introduction**

Tracing back the evolution of the standard theory of international trade, models based on perfect competition dominated the mainstream thinking. The imperfectly competitive models became the kernel of international trade theory after the so-called "new trade theory" revolution from 1979<sup>17</sup>. Since then, two distinct directions have been advanced to embed imperfect competition into international theory, which are monopolistically competitive models and oligopolistic models. Monopolistically competitive models immediately became the more popular ones among the researchers, but the theory of oligopoly is currently an increasingly important field that has affected the view of international trade, especially over the last few decades<sup>18</sup>. Leahy and Neary (2013) provide a comprehensive review of the welfare effects of trade liberalisation under oligopoly.

Brander (1981) and Brander and Krugman (1983) use the reciprocal-market structure to study the welfare effects of trade liberalisation between two identical countries in a homogeneous product Cournot model. The model is symmetric, where both home and foreign firms have the same marginal cost of production and the markets are assumed to be segmented. The main result is that the relationship between welfare and trade costs is U-shaped. When the initial trade cost is sufficiently low, the procompetitive effect dominates the increased waste of resources, and then trade liberalisation improves welfare. However, the negative effects of expending wasteful

<sup>&</sup>lt;sup>17</sup> See Krugman (1979) for an early contribution.

<sup>&</sup>lt;sup>18</sup>Head and Spencer (2017) provide a comprehensive survey regards the recent reappearance of oligopolistic competition in international trade in both theory and empirical work.

resources outweigh the pro-competitive effect if the trade cost is too high. Therefore, trade liberalisation can lower welfare. Brander and Krugman (1983) extend the reciprocal-market model to allow general demand function. Brander and Krugman (1983) and Venables (1985) demonstrate that there will always be gains from multilateral trade when there is free entry and exit of firms. This is because the waste on the transport cost is dominated by the pro-competitive effect of having more firms and a larger overall market. Collie (1996) examines the welfare consequences of unilateral free trade under Cournot duopoly. By assuming homogeneous goods and linear demand functions, Collie (1996) demonstrates that entry of a foreign firm reduces the welfare of home country unless the cost of the foreign firm is sufficiently lower than that of the home firm. He also shows that a sufficient requirement for a country to have higher welfare from unilateral free trade is that its firm stops producing under free trade. Cordelia (1993) analyses the welfare effects of free trade under Cournot oligopoly with many firms in each country assuming linear demand functions and zero marginal costs. He shows that entry of foreign firms increases domestic welfare if the domestic industry is very concentrated, that is, the number of domestic firms is much lower than the number of foreign firms. Bernhofen (2001) extends the analysis to allow for exogenous product differentiation using the Bowley demand functions and assuming that there are no transportation costs. In particular, Bernhofen (2001) focuses on the effect of product differentiation on the volume of trade, and the effect of trade liberalisation on profits and consumer surplus. He shows that the amount of intra-industry trade is positively related to the level of product differentiation, and the gains from trade are driven by the pro-competitive effect and increased product variety. Marjit and Mukherjee (2015) assume that there is free entry of domestic firms that compete with a single foreign firm in the home market and the products are differentiated. It is shown that trade liberalisation has no impact on the prices set by domestic firms as they do not export, and trade liberalisation causes a fall in consumer surplus. Collie (2016) considers a Cournot oligopoly model that allows free entry and exit of firms and products are differentiated. He shows that, when the trade cost is close to the prohibitive level, there

may be losses from trade. The intuition is that, though there is an increase in the total number of varieties to consumers, the loss of domestically-produced varieties outweigh the gains of imported varieties, because the imported varieties are very small when the trade cost is close to the prohibitive level. The analysis also applies to models with integrated markets. Markusen (1981) first derives the welfare effects of trade costs under Cournot duopoly for the case of integrated markets. It is shown that multilateral trade will increase outputs of both firms because of a procompetitive effect and there are gains from trade even though no trade actually occurs. When countries differ in terms of market size, international trade will lead to a higher world real income, the small country will always gain from trade but the large country may suffer a welfare loss. A sufficient condition for the large country to gain from trade is that there is an expansion in the output of its firm due to the trade.

The corresponding results of trade liberalisation under price competition in a reciprocal-market setting were first derived by Clarke and Collie (2003). They demonstrate that trade imposes a stronger competition effect under price competition. Even if there is no actual trade between two countries, the home firm behaves as a constrained monopolist. As a result, trade liberalisation lowers the prices and leads to an increase in welfare. As trade costs decrease further, it generates a U-shaped relationship between welfare and the trade costs. Moreover, the level of welfare is always greater than that under autarky, which is not in the case under Cournot competition. Friberg and Ganslandt (2008) analyse if the conclusion in Clarke and Collie (2003) can generalise to other market structures. By assuming two identical countries, each with *n* identical local firms, they find that international trade may reduce welfare compared to autarky in a Bertrand oligopoly if the local markets are sufficiently competitive and goods are sufficiently close substitutes. This is because, in a more competitive market with relatively similar goods, the negative effect of costly trade may dominate the benefits of having more competition and variety of products. The U-shape relationship between welfare and trade costs holds in this more competitive market structure. Collie and Le (2015) extends the analysis of Bernhofen (2001) to allow a positive trade cost under both Cournot and Bertrand duopoly. In contrast to Bernhofen (2001) who uses the Bowley demand functions, Collie and Le (2015) employ Shubik-Levitan demand functions as the size of the market will not be affected by a change of product differentiation with the Shubik-Levitan demand functions. It is shown that the volume of intra-industry trade is positively related to the level of product differentiation when the trade cost is relatively high, but is negatively related to the level of product differentiation when the trade cost is relatively high, but is negatively related to the level of product differentiation when the trade cost is relatively low. Recently, Brander and Spencer (2015) show how the type of market conduct affects the incentive to undertake endogenous horizontal product differentiation and investigate the gains from trade under Cournot and Bertrand oligopoly. It is demonstrated that product differentiation plays a key role in determining the pattern of trade, and can increase the gains from trade under both Cournot and Bertrand oligopoly.

As mentioned in chapter one that embedding oligopoly model in general equilibrium suffers a number of difficulties. Neary has suggested a new approach to avoid these problems, which he refers to as general oligopolistic equilibrium (GOLE) approach, successfully deriving the equilibrium by assuming simple demand and cost functions. According to Neary (2002b, 2003a, 2003b), the GOLE approach relies on a simple assumption that firms are large in their own sectors but small in the economy as a whole: the economy involves a continuum of sectors and there are just a few oligopolists in each sector. Neary (2002b, 2003a, 2003b) argues that, with this setting, firms are small in all markets other than their own, they treat factor prices and national income as given when choosing the profit-maximising level of output or price. Moreover, firms are not able to affect factor rewards while they strategically influence the good prices since there is no input-output linkage among sectors that is considered. The sensitive issue with the choice of numeraire is addressed by normalizing the marginal utility of national income to unity, because the firms are not able to influence factor markets and nation-wide income. It is assumed that utility is additively separable over goods and each sub-utility function is quadratic. It follows that demands only depend on own price and the marginal utility of income which is a "sufficient statistic" for the rest of the economy.

The GOLE approach has been widely adopted and applied to the recent and growing economic research in multiple fields<sup>19</sup>. Neary (2016) employs the GOLE approach to analyse the existence of gains from trade, the effects on income distribution and trade patterns. To be more specific, Neary (2016) considers a two-country model with a continuum of sectors, each of which has a small number of firms that compete in Cournot fashion. Production of home firms and foreign firms are respectively decreasing and increasing in the order of sector, and there is no trade cost or other barriers to international trade. When all sectors have identical costs in two identical countries (what is called the "featureless" economy in Neary (2002b), there exists no gain from trade. If sectors are heterogeneous, there are gains from trade, which is driven by the competition effect and the extent of gains is increasing in the comparative advantage. Neary (2016) also sheds light on the explanation of the "missing trade mystery" stated by Trefler (1995): the volume of real-world trade is far less than predicted by the Heckscher-Ohlin Model. He shows that the volume of imports and the average share of net imports in consumption across all sectors are both increasing in the number of firms. Thus oligopoly tends to reduce trade volumes. Bastos and Kreickemeier (2009) build a framework that allows for the interaction between unionised and non-unionised sectors in general equilibrium, and Kreickemeier and Meland (2013) extend the analysis of Bastos and Kreickemeier (2009) to allow for nontraded goods and the presence of labour unions in some of shielded sectors. Both articles show that moving from autarky to free trade will increase the aggregate welfare. Bastos and Straume (2012) introduce the degree of horizontal product differentiation into a general oligopolistic equilibrium model and study the effect of freer trade on the distribution of income, resources allocation and aggregate welfare. It is shown that the

<sup>&</sup>lt;sup>19</sup> See the following articles: Neary (2007) for cross-border mergers; Eckel and Neary (2010), Egger and Koch (2012) for multiproduct firms, Basile and De Benedictis (2008) and Neary and Tharakan (2012) for unions and unemployment, and Colacicco (2012) for strategic trade policy. Colacicco (2015) provide a comprehensive survey on the GOLD model and some of its implications.

welfare effect is ambiguous if parts of the economy are shielded from international competition. Fujiwara (2017) studies the welfare effects of FDI liberalisation and trade liberalisation in a general oligopolistic equilibrium model. It is shown that, when the productivity difference between exporting and FDI industries is small enough, trade liberalisation improves the aggregate welfare.

The motivation of this chapter is to analyse the effect of trade costs and product differentiation on the economy, including the aggregate profits and social welfare in GOLE model. All contributions above in GOLE model simply compare the free trade welfare to the autarky welfare. In this analysis, I will show how the trade cost continually affects the social welfare from zero trade cost to the prohibitive trade cost. More importantly, I will compare the results to that under partial equilibrium. In particular, unlike the assumption that wage plays no role in Brander (1981) and Brander and Krugman (1983), wage is endogenously determined in GOLE model. I will examine how the equilibrium wage affects the whole economy. In addition, the effect on price are different from that under partial equilibrium and I will demonstrate the relationship between the variability of price and the social welfare in this analysis. This chapter is organised in the following way. In section 4.2, products are assumed to be differentiated, and the level of technology is identical across all sectors in both countries. In section 4.3, products are homogeneous, but there exists comparative advantage for both countries. Two cases will then be presented. For the first case, when the trade cost is sufficiently low, all firms can supply goods in both markets. In each industry, there exists one home firm that competes with its foreign rival. For the second case, when the trade cost is sufficiently high, some firms will not export goods. In the absence of the foreign firms in the home market, some domestic firms will become the only suppliers in their sectors and behave as monopolists. The conclusions are in section 4.4.

## 4.2 Featureless economy with differentiated products

### 4.2.1 The model

Suppose that there are two perfectly symmetric countries, the home country and the foreign country, and a continuum of industries,  $z \in [0,1]$ . In each industry, there is one home firm that competes with its foreign rival in a Cournot fashion. Therefore, firms are considered to be large in their own sectors and have market power in deciding their output. However, they are modelled as very small in the economy as a whole, so that have ignorable effects on aggregate variables. Labour is the only factor of production and the labour market is competitive in both countries. The wage rate is endogenously determined and plays a key role in giving the solution of the aggregate profits and social welfare in general equilibrium.

Assume that each country is inhabited by a representative consumer, who owns L units of labour. Preferences in the home country are represented by an additively separable utility function over a continuum of products,  $z \in [0,1]$ :

$$U[\{\mathbf{x}(z)\}] = \int_0^1 u[\mathbf{x}(z)] dz \qquad (4.2.1)$$

Each sub-utility function is quadratic and is given by:

$$u[x(z)] = a[x_1(z) + x_2(z)] - \frac{b}{2}[x_1(z)^2 + x_2(z)^2 + 2rx_1(z)x_2(z)];$$
  

$$u' > 0, \ u'' < 0, a > 0, b > 0$$
(4.2.2)

where  $x_1(z)$  and  $x_2(z)$  are the consumption of the good produced in sector z by the domestic firm (denoted as good 1) and the foreign firm (denoted as good 2), supplying in the home market.  $r \in [0,1]$  represents the degree of product differentiation and ranges from the case of independent demands (r = 0) to that of perfect substitutes (r = 1). The budget constraint of the representative consumer is given by:

$$\int_{0}^{1} \left[ p_{1}(z) x_{1}(z) + p_{2}(z) x_{2}(z) \right] \leq I$$
(4.2.3)

where I is the aggregate income and  $p_1(z)$  and  $p_2(z)$  represent the price of good 1 and good 2 in sector z.

It is further assumed that wage income and profits will be costlessly distributed to the representative consumer, who can use them for the consumption. Therefore, the national income is given by the sum of aggregate wage and the aggregate profits:  $I = wL + \Pi$  where w is the competitive wage level and  $\Pi$  stands for the aggregate profits.

The Lagrangian function can be written as:

$$\mathscr{Z} = \int_{0}^{1} u \left[ x(z) \right] dz - \lambda \left( \int_{0}^{1} \left[ p_{1}(z) x_{1}(z) + p_{2}(z) x_{2}(z) \right] - I \right)$$
(4.2.4)

where  $\lambda$  is the Lagrange multiplier of the budget constraint.

The first-order conditions are as follows:

$$\frac{d\mathscr{X}}{dx_1(z)}: \quad a - b \Big[ x_1(z) + r x_2(z) \Big] = \lambda p_1 \tag{4.2.5}$$

$$\frac{d\mathscr{X}}{dx_2(z)}: \quad a - b \Big[ rx_1(z) + x_2(z) \Big] = \lambda p_2 \tag{4.2.6}$$

$$\frac{d\mathscr{X}}{d\lambda}: \quad \int_0^1 \left[ p_1(z) x_1(z) + p_2(z) x_2(z) \right] = I \tag{4.2.7}$$

Combining (4.2.5) and (4.2.6), the linear inverse demand function formulates as:

$$\lambda p_{i} = \left( a - b \left[ x_{i}(z) + r x_{j}(z) \right] \right); \quad i, j = 1, 2$$
(4.2.8)

By rearranging (4.2.8), the direct demand function displays as:

$$x_{i} = \frac{(1-r)a + \lambda(rp_{j} - p_{i})}{(1-r^{2})\beta}; \quad i, j = 1,2$$
(4.2.9)

The Lagrange multiplier of the budget constraint, as usual, measures the marginal utility of national income. It can be expressed by substituting (4.2.9) into (4.2.3) and integrating over all sectors:

$$\lambda = \frac{a(1-r)\mu_1^p - b(1-r^2)I}{\mu_2^p - rv^p}$$
(4.2.10)

Where  $\mu_1^p \equiv \int_0^1 [p_1(z) + p_2(z)] dz$ ,  $\mu_2^p \equiv \int_0^1 [p_1(z)^2 + p_2(z)^2] dz$  and  $v^p \equiv 2 \int_0^1 [p_1(z) p_2(z)] dz$ .

 $\mu_1^p$  is the measure of the first moment of the price distribution across all sectors,  $\mu_2^p$  stands for the second moment of the price distribution,  $\nu^p$  measures the covariance between the price of two goods. It is worth mentioning that, as can be seen below, the value of  $\lambda$  is not independent in general equilibrium. All nominal variables are interpreted as relative to the inverse of the marginal utility of national income ( $\lambda^{-1}$ ).

Firms have complete information and compete under Cournot competition, selecting their own profit-maximising production and treating the outputs of their rival as given. The "large in the small but small in the large" assumption suggests that firms take  $\lambda$  as given when deciding output as each firm is unlikely to impact the price distribution, factor reward as well as national income.

The wage rate w can be pinned down since the labours can move across all sectors freely. The cost function is linear in the output owing to the assumption of the constant return to the scale of technology. Each domestic firm maximises its profits subject to the inverse demand function (4.2.8) given that the outputs of rivals and wage rate are known:

$$\pi_1(z) = [p_1(z) - c_1(z)]y_1(z) + [p_1^*(z) - c_1(z) - \tau_1(z)]y_1^*(z)$$
(4.2.11)

where  $y_1(z)$  represents the home sales of the domestic firm and  $c_1(z)$  is the marginal cost of the home firm in sector z. Asterisk measures variables in the foreign market.  $y_1^*(z)$  is the level of exports for the home firms in sector z. The linearity in the inverse demand and cost functions ensures that there exists unique Cournot-Nash equilibrium in pure strategies and no firm is willing to move away from this equilibrium. The marginal cost in sector z is endogenously determined which depends on the competitive wage rate and sector-specific labour input requirement. Therefore, the marginal cost of the domestic firm in sector z is written as:

$$c_1(z) = w\beta(z) \tag{4.2.12}$$

where  $\beta(z)$  is the unit labour requirement of production in sector z.

Similarly, the profit of the foreign firm in sector z obtains as:

$$\pi_2(z) = [p_2(z) - c_2(z)]y_2(z) + [p_2^*(z) - c_2(z) - \tau_2(z)]y_2^*(z)$$
(4.2.13)

where  $y_2(z)$  and  $y_2^*(z)$  are the sales of the foreign firm supplying the domestic market and foreign market,  $c_2(z)$  is the marginal cost of the foreign firms and  $\tau(z)$  is the trade cost per-unit of exports.

The marginal cost of the foreign firm in sector z formulates as

$$c_2(z) = w^* \beta(z)$$
 (4.2.14)

The trade cost per unit of exports in sector z is expressed as

$$\tau(z) = wt(z) \tag{4.2.15}$$

where t(z) is the trade costs in terms of labour. For simplify, t(z) is assumed to be constant across all sectors, i.e. t(z) = t.

Market clearing condition implies that consumption equals to production:

$$x_i(z) = y_i(z)$$
 (4.2.16)

Substituting the inverse demand function (4.2.8) into (4.2.11) and (4.2.13) and solving the system of the first-order conditions gives the best response function. Thus, the Cournot-Nash (*CN*) equilibrium productions of the domestic firm and foreign firm in the home market are represented as:

$$y_1(z)^{CN} = \frac{(2-r)\left[a - \lambda w\beta(z)\right] + \lambda rtw}{b(4-r^2)}$$
(4.2.17)

$$y_{2}(z)^{CN} = \frac{(2-r)[a - \lambda w\beta(z)] - 2\lambda tw}{b(4-r^{2})}$$
(4.2.18)

Due to the full symmetry, the Cournot-Nash equilibrium productions of the domestic firm and foreign firm in the foreign market can be represented as:

$$y_{1}^{*}(z)^{CN} = \frac{(2-r)[a - \lambda w\beta(z)] - 2\lambda rtw}{b(4-r^{2})}$$
(4.2.19)

$$y_{2}^{*}(z)^{CN} = \frac{(2-r)[a - \lambda w\beta(z)] + \lambda tw}{b(4-r^{2})}$$
(4.2.20)

The profit of domestic firm in sector z can be written as:

$$\Pi(z)^{CN} = b \left[ y_1(z)^{CN} \right]^2 + b \left[ y_1^*(z)^{CN} \right]^2$$
(4.2.21)

The profits of the home firm in sector z stem from the profits from the domestic market  $b \left[ y_1(z)^{CN} \right]^2$  and the foreign market  $b \left[ y_1^*(z)^{CN} \right]^2$ .

The aggregate profits of the home firms are given by:

$$\Pi = \int_{0}^{1} b \left[ y_{1}(z)^{CN} \right]^{2} dz + \int_{0}^{1} b \left[ y_{1}^{*}(z)^{CN} \right]^{2} dz \qquad (4.2.22)$$

The full employment condition suggests that the exogenous total labour supply L equals to the total labour demand:

$$L = \int_{0}^{1} \beta(z) y_{1}(z)^{CN} dz + \int_{0}^{1} \left[ \beta(z) + t \right] y_{2}(z)^{CN} dz \qquad (4.2.23)$$

In this section, it is assumed that there is no comparative advantage and all firms have identical labour input requirement:

$$\beta(z) = \beta \tag{4.2.24}$$

The first and second moment of unit labour requirement obtain as

$$\beta = \int_0^1 \beta(z) dz \tag{4.2.25}$$

$$\beta^{2} = \int_{0}^{1} \left[ \beta(z) \right]^{2} dz \qquad (4.2.26)$$

Now the equilibrium wage rate can be solved by substituting (4.2.17) and (4.2.18) into (4.2.23), integrating over all sectors and combining with (4.2.25) and (4.2.26):

$$w = \frac{(2-r)\left[2a\beta - (2+r)bL + at\right]}{2\lambda\left(t^{2} + (2-r)\beta t + (2-r)\beta^{2}\right)}$$
(4.2.27)

Alternatively, we can express the consumer's real wage as:

$$\overline{w} = w\lambda = \frac{(2-r)[2a\beta - (2+r)bL + at]}{2(t^2 + (2-r)\beta t + (2-r)\beta^2)}$$
(4.2.28)

By symmetry, the equilibrium wage rate in the foreign country is also  $\overline{w}$ . It is worth noting that each domestic and foreign firms have the same marginal cost in sector z with the same competitive wage rate and unit labour requirement of production. This is what Neary called the "featureless economy".

Substituting the competitive wage rate into the Cournot-Nash equilibrium productions (4.2.17) - (4.2.20):

$$y_1^{CN} = \frac{at^2 + (a\beta - bLr)t + (2 - r)bL\beta}{2b(t^2 + (2 - r)\beta t + (2 - r)\beta^2)}$$
(4.2.29)

$$y_2^{CN} = \frac{(2bL - a\beta)t + (2 - r)bL\beta}{2b(t^2 + (2 - r)\beta t + (2 - r)\beta^2)}$$
(4.2.30)

$$y_{1}^{*CN} = \frac{(2bL - a\beta)t + (2 - r)bL\beta}{2b(t^{2} + (2 - r)\beta t + (2 - r)\beta^{2})}$$
(4.2.31)

$$y_{2}^{*CN} = \frac{at^{2} + (a\beta - bLr)t + (2 - r)bL\beta}{2b(t^{2} + (2 - r)\beta t + (2 - r)\beta^{2})}$$
(4.2.32)

As shown in Appendix C1 and C2 that  $\partial y_1^{CN} / \partial t > 0$  and  $\partial y_2^{CN} / \partial t < 0$ , which means that an increase in the trade costs will lead to an increase in home sales, but a reduction in exports.

In the featureless economy, as z does not enter the solution for the production, the above four expressions also display the aggregate production level. In particular, the total consumption in the home country can be calculated as:

$$X = \int_{0}^{1} y_{1}^{CN} dz + \int_{0}^{1} y_{1}^{*CN} dz = \frac{at^{2} + bL(2-r)(t+2\beta)}{2b(t^{2} + (2-r)t\beta + (2-r)\beta^{2})}$$
(4.2.33)

Notice that if the trade cost is too high, it will expel the foreign firms from the domestic market, i.e., the foreign firms will not export, and the home firm in each sector will be the sole supplier in the home market. This will occur when the level of export is zero:

$$y_2^{CN} = 0 \tag{4.2.34}$$

Solving (4.2.34) yields the prohibitive trade level:

$$\tilde{t} = \frac{(2-r)bL\beta}{a\beta - 2bL}; \quad a\beta > 2bL \tag{4.2.35}$$

If the trade cost is below  $\tilde{t}$ , the home firms compete with their foreign rivals. However, if the trade cost exceeds  $\tilde{t}$ , all foreign firms will not export goods to the home market and all domestic firms will become monopolists in their industries.

### 4.2.2 The equilibrium wage

One distinct feature of GOLE approach is that, wage is endogenously determined and it plays a key role in solving the aggregative variables. Unlike the conventional assumption that marginal cost is constant under partial equilibrium, in GOLE approach, marginal cost is measured by the product of wage rate and unit labour requirement. Therefore, a change in trade costs will affect the competitive wage, resulting in a change in the production costs. Therefore, it is necessary to examine the effect of trade costs on the equilibrium wage. When the trade cost is prohibitive, i.e.,  $t = \tilde{t}$ , the equilibrium wage rate is obtained as:

$$\overline{w}\Big|_{t=\tilde{t}} = \frac{a\beta - 2bL}{\beta^2} \tag{4.2.36}$$

The first-order derivative of wage with respect to t when the trade cost is prohibitive obtains as:

$$\frac{\partial \overline{w}}{\partial t}\Big|_{t=\overline{t}} = -\frac{a(a\beta - 2bL)^2}{2\beta^2(\Delta_1)}$$
(4.2.37)

where  $\Delta_1 = b^2 L^2 (2+r) - abL (2+r) \beta + a^2 \beta^2$ .

It is shown in Appendix C1 that  $\Delta_1 > 0$ . It follows that  $\frac{\partial \overline{w}}{\partial t}\Big|_{t=\tilde{t}}$  is negative. Hence, a

reduction in trade costs leads to an increase in the competitive wage when t is prohibitive.

In the case of zero trade cost, the equilibrium wage calculates as:

$$\overline{w}\Big|_{t=0} = \frac{-bL(2+r) + 2a\beta}{2\beta^2}$$
(4.2.38)

It is easy to check that  $\overline{w}|_{t=0} > \overline{w}|_{t=\tilde{t}}$ , that is, the equilibrium wage at zero trade cost is higher than that at prohibitive trade cost.

The first-order derivative of wage with respect to t at zero trade cost obtains as:

$$\left. \frac{\partial \overline{w}}{\partial t} \right|_{t=0} = \frac{bL(2+r) - a\beta L}{2\beta^3}$$
(4.2.39)

The effect is ambiguous when t = 0. The equilibrium wage is increasing in t at zero trade cost only when  $L > \frac{a\beta}{b(2+r)}$ .

Figure 4-1 and figure 4-2 show two possible shapes of the equilibrium wage on the trade costs:

1. The equilibrium wage is monotonically decreasing in trade costs given the following parameter values  $a = 10, b = 1, L = 10, \beta = 3, r = 0.5$ . This can be seen from figure 4-1.

2. The wage curve is increasing in trade costs when t is close to zero, but starts falling after the wage reaches its maximum, given the following parameter values  $a = 10, b = 1, L = 10, \beta = 2.2, r = 0.5$ . This can be seen from figure 4-2.



Figure 4–1: Wage rate with trade cost I



Figure 4–2: Wage rate with trade cost II

Labour supply is exogenous and fixed, and labour demand is determined by the labour input requirement, the level of output and the trade costs as shown in (4.2.23). The full employment condition implies that an increase in wage is driven by a higher labour demand across all sectors.

As the home sales and exports are identical for the firms across all sectors, the function of total labour demand can be simplified as:

$$L = \beta y_1^{CN} + [\beta + t] y_2^{CN}$$
(4.2.40)

Taking the derivative of total labour demand with respect to t gives:

$$\frac{\partial L}{\partial t} = \beta \frac{\partial y_1^{CN}}{\partial t} + [\beta + t] \frac{\partial y_2^{CN}}{\partial t} + y_2^{CN}$$
(4.2.41)

If  $\left[\beta + t\right] \left| \frac{\partial y_2^{CN}}{\partial t} \right| > \beta \frac{\partial y_1^{CN}}{\partial t} + y_2^{CN}$ , a reduction in trade cost will lead to a higher

demand for labour. Therefore, a reduction in trade cost will raise the equilibrium wage. This is more likely to happen when the trade cost is relatively high: For one thing, t in the bracket on the left-hand-side of the inequality is large. For another,  $y_2^{CN}$  on the right-hand-side of the inequality is small. As can be seen from figure 4-1 and figure 4-2, the equilibrium rate is decreasing in trade cost when trade cost is high.

Starting from the prohibitive trade cost, if there is a reduction in t, the home sales  $y_1^{CN}$  decrease while the exports  $y_2^{CN}$  increase, labour will be redistributed. As the labours demanded for producing good 2 outweigh the labours sacked from the sectors that produce good 1, the overall demand for labour increases. As a result, wage rate will increase. When t is close to zero, the effect of trade costs on wage is ambiguous. This is because the inequality  $\left[\beta + t\right] \left| \frac{\partial y_2^{CN}}{\partial t} \right| > \beta \frac{\partial y_1^{CN}}{\partial t} + y_2^{CN}$  may be reversed: t is small

while the exports are large when the trade cost is low. To be concluded, in the GOLE approach, a change in trade cost has an impact on the competitive wage due to the change in labour demand.

Recall the function of the trade costs per-unit of exports in (4.2.15), t is measured in terms of labour, the competitive wage responds to the change of t and also has an impact on the trade costs per-unit of exports  $\tau$ . Therefore, it is worth seeing the effect of t on  $\tau$ .

As shown in Appendix C3 that  $\partial \tau / \partial t > 0$  for all possible values of t. Hence, trade costs per-unit of exports  $\tau$  and unit labour requirement of exports t are positively related. With a reduction in t, though wage rate may increase especially when t is high, the trade cost per-unit of exports will always fall.

#### 4.2.3 The aggregate profits and social welfare

The domestic aggregate profits are comprised of two parts: the profits from domestic sales and the profits from exporting. The first component of profit can be obtained as:
$$\Pi_{1} = b \left( y_{1}^{CN} \right)^{2} = \frac{\left[ at^{2} + \left( a\beta - bLr \right)t + \left( 2 - r \right)bL\beta \right]^{2}}{4b \left( t^{2} + \left( 2 - r \right)\beta t + \left( 2 - r \right)\beta^{2} \right)^{2}}$$
(4.2.42)

The domestic firms make the following aggregate profits from exporting:

$$\Pi_{1}^{*} = b \left( y_{2}^{CN} \right)^{2} = \frac{\left[ (2bL - a\beta)t + (2 - r)bL\beta \right]^{2}}{4b \left( t^{2} + (2 - r)\beta t + (2 - r)\beta^{2} \right)^{2}}$$
(4.2.43)

The aggregate profits can then be calculated by summing (4.2.42) and (4.2.43):

$$\Pi = \frac{\left[at^{2} + (a\beta - bLr)t + (2 - r)bL\beta\right]^{2} + \left[(2bL - a\beta)t + (2 - r)bL\beta\right]^{2}}{4b(t^{2} + (2 - r)\beta t + (2 - r)\beta^{2})^{2}}$$
(4.2.44)

Finally, in order to obtain the social welfare function, substituting (4.2.29) and (4.2.31) into (4.2.2) and integrating over all sectors yields:

$$U = \frac{At^{4} + Bt^{3} + Ct^{2} + Dt + E}{8b(t^{2} + (2 - r)\beta t + (2 - r)\beta^{2})^{2}}$$
(4.2.45)

where 
$$A = 3a^2$$
;  $B = 2a(bL(4-3r) + a(3-r)\beta)$ ;

$$C = -b^{2}L^{2}(4-3r^{2}) + 4abL(8-7r+r^{2})\beta + 2a^{2}(3-r)\beta^{2} \qquad ;$$

$$D = 2bL\beta \left( bL \left( 4 + 3r^2 - r^3 \right) + 6a \left( 2 - r \right)^2 \beta \right)$$
 and  
$$E = -2bL \left( 2 - r \right)^2 \beta^2 \left( bL \left( 1 + r \right) - 4a\beta \right)$$

According to the aggregate profits function (4.2.44), calculate the first-order derivative at zero trade cost:

$$\left. \frac{\partial \Pi}{\partial t} \right|_{t=0} = -\frac{bL^2}{2\beta^3} < 0 \tag{4.2.46}$$

It reveals that an increase in the trade cost will lead to a drop in the aggregate profits in the neighbourhood of zero trade cost. In particular, the measurement of product r differentiation does not enter the first-order derivative above.

As the aggregate profits are comprised of the profits from the domestic and foreign market, the effect of trade cost on profits can be divided into the following two parts: one on the profits from home sales and the other on the profits from exporting. Recall (4.2.22), and separately analyse the impact of trade cost on profits from two markets:

$$\frac{\partial \Pi}{\partial t}\Big|_{t=0} = \frac{\partial \Pi_1}{\partial t}\Big|_{t=0} + \frac{\partial \Pi_1^*}{\partial t}\Big|_{t=0} = \frac{L(a\beta - 2bL)}{2(2-r)\beta^3} + \left(-\frac{L(a\beta - bLr)}{2(2-r)\beta^3}\right) < 0 \quad (4.2.47)$$

As can be seen from above, trade cost is positively related to the profits from domestic market while negatively related to the profits from exporting. The intuition is as follows. An increase in t leads to a higher marginal cost for export goods (tw), resulting in a fall in the level of exports as well as the profit from international trade. Therefore, trade cost has a negative effect. On the other hand, the foreign firms confront the same problem so that they will reduce the supply in the home market and home firms benefit. As the overall effect on the aggregate profits is negative, one can conclude that, in the case of zero trade cost, exports are harmed by a rise in the firm's own cost more than home sales are helped by the same increase in the other firm's marginal costs. Accordingly, consumers will consume more goods produced by the home firms but fewer goods supplied by the foreign firm, and the overall consumption will decrease. Therefore, a tiny increase in t results in a decrease in total consumption and profits. This conclusion is in line with Brander and Krugman (1983).

In the neighbourhood of the prohibitive trade cost, the first-order effect shows:

$$\frac{\partial \Pi}{\partial t}\Big|_{t=\tilde{t}} = \frac{L(a\beta - bLr)(a\beta - 2bL)^2}{(2-r)\beta^3\Delta_1}$$
(4.2.48)

As it is shown in Appendix C1 that  $\Delta_1 > 0$ ,  $\frac{\partial \Pi}{\partial t}\Big|_{t=\overline{t}}$  is positive.

When the trade cost is evaluated at the prohibitive trade cost, exports are zero at the beginning. Hence, a small reduction in trade cost has zero effect on profits from international trade. However, profits from home market reduce due to a small fall in trade cost as they were at the monopoly level before the change of trade costs. In addition, a reduction in t at the prohibitive trade cost will unambiguously increase the competitive wage rate w, and in turn, increase the marginal cost of production. Therefore, a reduction in the trade cost evaluated at the prohibitive trade cost will lead the aggregate profits to fall.

Figure 4-3 shows the relationship between the aggregate profits and trade costs given the following parameter values  $a = 10, b = 1, L = 10, \beta = 3, r = 0.25 / 0.5 / 0.75$ .



Figure 4–3: Aggregate profits with trade cost in the featureless economy

It shows that the aggregate profits are decreasing in t when the trade cost is sufficiently low. If the trade cost is close to the prohibitive trade cost  $\tilde{t}$ , the aggregate profits are positively related to the trade costs. If the trade cost exceeds  $\tilde{t}$ , there is no export from the foreign firms and the domestic firms will be monopolists in their own sectors, and the aggregate profits are constant at the monopoly level.

In order to examine the effect of trade cost on social welfare at zero trade cost, derive the first-order derivative of social welfare with respect to t and evaluate t to zero:

$$\left. \frac{\partial U}{\partial t} \right|_{t=0} = \frac{\left[ (1+r)bL - 2a\beta \right]L}{4\beta^3} < 0 \tag{4.2.49}$$

It is clear that social welfare is harmed by an increase in trade cost when trade cost is zero.

On the other hand, when the trade cost is prohibitive, the first-order derivative shows

$$\left. \frac{\partial U}{\partial t} \right|_{t=\tilde{t}} = \frac{\left(2bL - a\beta\right)\left(brL - a\beta\right)L}{2\left(2 - r\right)\Delta_1\beta^3} > 0 \tag{4.2.50}$$

It indicates that a decrease in the trade cost evaluated at the prohibitive trade cost will lower the social welfare:

To see how trade costs affect the social welfare, figure 4-4 is drawn with the parameter values:  $a = 10, b = 1, L = 10, \beta = 3, r = 0.25/0.5/0.75$ . It is clearly shown that social welfare is U-shaped in the trade cost.



Figure 4-4: Social welfare with trade cost in the featureless economy

The results mentioned above lead to the following proposition:

**Proposition 1:** In the featureless economy, the aggregate profits and social welfare are decreasing in t in the neighbourhood of zero trade cost, and increasing in t in the neighbourhood of the prohibitive trade cost.

The social welfare in the case of zero trade cost can be calculated as

$$U\big|_{t=0} = \frac{L\big[4a\beta - (1+r)bL\big]}{4\beta^2}$$
(4.2.51)

The first-order derivative with respect to r show that  $\partial U|_{t=0} / \partial r < 0$ . This suggests that the social welfare is higher if the goods are more differentiated when the trade cost is zero. This can also be seen from figure 4-4.

When the trade cost is prohibitive, the social welfare obtains as:

$$U|_{t=\tilde{t}} = \frac{L[2a\beta - bL]}{2\beta^2}$$
(4.2.52)

Since there is no trade between two countries, the representative consumer in home country can only consume goods produced by the home firms. Therefore, the degree of product differentiation does not enter function (4.2.52).

**Proposition 2**: There are losses from trade when the trade cost is close to the prohibitive level in the featureless economy, and social welfare under zero trade cost is always greater than that under prohibitive trade cost if the products are differentiated.

#### Proof.

Calculate the gap between the social welfare under zero trade cost (4.2.51) and the prohibitive trade cost (4.2.52):

$$U\big|_{t=0} - U\big|_{t=\tilde{t}} = \frac{(1-r)bL^2}{4\beta^2} \ge 0$$
(4.2.53)

#### Q.E.D.

It implies that when the goods are identical (i.e., r = 1), the social welfare under zero trade cost is the same as that of prohibitive trade cost. If the products are differentiated, i.e.,  $1 > r \ge 0$ , welfare under zero trade cost is always greater than that under prohibitive trade cost.

In summary, in the featureless economy, it suggests that there is a U-shape relationship between social welfare and trade cost. Besides, there are gains from trade unless the products are perfect substitutes. However, the result is subject to the simplest assumption with two identical countries and the same technology level across all sectors (i.e., the featureless economy). In the next section, it is assumed that the level of technology differs across sectors so that it allows for comparative advantage between two countries.

# 4.3 The economy with comparative advantage and homogeneous products

# 4.3.1 The model

Assume the country is populated by a representative consumer with L units of labour, inelastically supplied to a perfectly competitive labour market. It is assumed that preferences are additively separable over a continuum of goods and the products are homogeneous.

The utility function for the representative consumer is given by:

$$U[\{\mathbf{x}(z)\}] = \int_0^1 u[x(z)] dz$$
 (4.3.1)

Any quadratic sub-utility function is given by:

$$u[x(z)] = a[x_1(z) + x_2(z)] - \frac{b}{2}[x_1(z) + x_2(z)]^2;$$
  

$$u' > 0, \ u'' < 0, a > 0, b > 0$$
(4.3.2)

The utility function is maximised subject to the budget constraint:

$$\int_{0}^{1} \left[ p_{1}(z) x_{1}(z) + p_{2}(z) x_{2}(z) \right] \leq I$$
(4.3.3)

The Lagrangian function obtains as:

$$\mathscr{Z} = \int_{0}^{1} u \left[ x(z) \right] dz + \lambda \left( \int_{0}^{1} \left[ p_{1}(z) x_{1}(z) + p_{2}(z) x_{2}(z) \right] - I \right)$$
(4.3.4)

For a representative sector, the first-order conditions imply:

$$\frac{d\mathscr{X}}{dx_1(z)}: \quad a - b \Big[ x_1(z) + x_2(z) \Big] = \lambda p_1 \tag{4.3.5}$$

$$\frac{d\mathscr{X}}{dx_2(z)}: \quad a - b \Big[ x_1(z) + x_2(z) \Big] = \lambda p_2 \tag{4.3.6}$$

$$\frac{d\mathscr{X}}{d\lambda}: \quad \int_{0}^{1} \left[ p_{1}(z) x_{1}(z) + p_{2}(z) x_{2}(z) \right] = I$$
(4.3.7)

As can be seen from expression (4.3.5) and (4.3.6), the linear inverse demand function formulates as:

$$p_{i}(z) = \frac{1}{\lambda} \left( a - b \left[ x_{i}(z) + x_{j}(z) \right] \right); \quad i, j = 1, 2$$
(4.3.8)

Solving for  $\lambda$  by multiplying demand function in (4.3.8) by x(z) and substituting into the budget constraint gives  $\lambda [\{p(z)\}, I] = \frac{a\mu_1^p - bI}{\mu_2^p}$ 

The impacts of prices on  $\lambda$  can be summarised by two price terms  $\mu_1^p$  and  $\mu_2^p$ , which are the first and the second moments of the distribution of prices:  $\mu_1^p = \int_0^1 p(z) dz$  and  $\mu_2^p = \int_0^1 p(z)^2 dz$ .

Under Cournot competition, each domestic firm maximises its profits subject to the inverse demand function (4.3.8) given that the outputs of rivals and wage rate are known. The profits of the domestic firm in industry z are given by the sum of its profits obtained from the domestic market and foreign market:

$$\pi_1(z) = [p_1(z) - c_1(z)]y_1(z) + [p_1^*(z) - c_1(z) - \tau_1(z)]y_1^*(z)$$
(4.3.9)

The marginal cost of the home firm in sector z formulates as:

$$c_1(z) = w\beta_1(z) \tag{4.3.10}$$

Similarly, the profit of the foreign firm in sector *z* obtains as:

$$\pi_2(z) = [p_2(z) - c_2(z)]y_2(z) + [p_2^*(z) - c_2(z) - \tau_2(z)]y_2^*(z)$$
(4.3.11)

The marginal cost of the foreign firm in sector z formulates as

$$c_2(z) = w^* \beta_2(z) \tag{4.3.12}$$

Assume that all firms have the same trade cost in terms of labour, the trade costs per-unit of exports can be written as

$$\tau = wt \tag{4.3.13}$$

Market clearing condition implies that all goods produced by domestic and foreign firms will be exactly consumed:

$$x_i(z) = y_i(z)$$
 (4.3.14)

Substituting the inverse demand function into the problem of both the home and foreign firm in sector z and solving the system of first-order conditions yields the best response function. Thus, the Cournot-Nash equilibrium productions of the domestic firm and foreign firm in the home market in sector z can be expressed as:

$$y_1(z)^{CN} = \frac{a - \lambda w \left(2\beta_1(z) - \beta_2(z) - t\right)}{3b}$$
(4.3.15)

$$y_{2}(z)^{CN} = \frac{a + \lambda w (\beta_{1}(z) - 2\beta_{2}(z) - 2t)}{3b}$$
(4.3.16)

The superscript CN is short for Cournot-Nash equilibrium outputs.

Similarly, the Cournot-Nash equilibrium productions of the home firm and foreign firm in foreign market in sector z can be expressed as:

$$y_{1}^{*}(z)^{CN} = \frac{a - \lambda w (2\beta_{1}(z) - \beta_{2}(z) + 2t)}{3b}$$
(4.3.17)

$$y_{2}^{*}(z)^{CN} = \frac{a + \lambda w (\beta_{1}(z) - 2\beta_{2}(z) + t)}{3b}$$
(4.3.18)

Regarding the level of technology, it is assumed that the unit labour requirement of the home firms increases in z while the unit labour requirement of the foreign firms decreases in z. The function of unit labour requirement for the home and foreign country can be seen as following

$$\beta_1(z) = d + z \tag{4.3.19}$$

$$\beta_2(z) = 1 + d - z \tag{4.3.20}$$

where d is assumed to be positive. With a higher value of d, firms need more units of labour in the production, which follows a reduction in the technology level.

According to (4.3.19) and (4.3.20), that home country has an advantage from sector 0 to 0.5, while the foreign country is more productive from sector 0.5 to 1. This can be clearly seen from the following diagram.



Figure 4–5: Comparison of the productivity for the two countries

The domestic aggregate profits are given by the sum of profits from domestic sales and total exports. Suppose that all home firms can make positive profits from exporting so that they supply both markets. The aggregate profits can be expressed as:

$$\Pi = \int_{0}^{1} b \left[ y_{1}(z)^{CN} \right]^{2} dz + \int_{0}^{1} b \left[ y_{1}^{*}(z)^{CN} \right]^{2} dz \qquad (4.3.21)$$

It is possible that some firms do not export in equilibrium due to a high trade cost. As the technology level of the home country decreases in *z*, the aggregate profits are represented as:

$$\Pi = \int_{0}^{1} b \left[ y_{1}(z)^{CN} \right]^{2} dz + \int_{0}^{n} b \left[ y_{1}^{*}(z)^{CN} \right]^{2} dz$$
(4.3.22)

where *n* is the last domestic industry that still exports goods to the foreign country.

It is of convenience to use what is called "Frisch indirect utility function" to express the levels of utility with quadratic preferences. An important property of this form of utility function is that utility can be written as a function of prices and the marginal utility of income. To do this, substitute the demand function (4.3.8) into the direct utility function (4.3.2) and integrate over all sectors:

$$V[\lambda, \{p(z)\}] = \int_0^1 [x_i(z) + x_j(z)] \left(a - \frac{1}{2}b[x_i(z) + x_j(z)]\right) dz$$
  
$$= \frac{1}{2b} \int_0^1 [a - \lambda p(z)] [a + \lambda p(z)] dz \qquad (4.3.23)$$
  
$$= \frac{1}{2b} \int_0^1 [a^2 - \lambda^2 p(z)^2] dz$$
  
$$\Rightarrow V[\lambda, \{p(z)\}] = \frac{1}{2b} \left(a^2 - \lambda^2 \mu_2^p\right) \qquad (4.3.24)$$

Where  $\mu_2^p = \int_0^1 p(z)^2 dz$ .

Therefore, the social welfare depends on the variability of price: reducing price variability across sectors will raise the social welfare.

Ignoring the constant term, utility equals to minus the product of the squared marginal utility of income and the second moment of prices. Define U as  $-\lambda^2 \mu_2^p$ . Then social welfare V and U are related as:

$$U = -\lambda^2 \mu_2^p \quad \text{where } U \equiv 2bV - a^2 \tag{4.3.25}$$

#### 4.3.2 Case 1: All domestic and foreign firms are active in the markets.

The condition of full employment suggests that the total labour demand equals to the exogenous labour supply:

$$L = \int_{0}^{1} \beta_{1}(z) y_{1}(z)^{CN} dz + \int_{0}^{1} \left[ \beta_{1}(z) + t \right] y_{1}^{*}(z)^{CN} dz \qquad (4.3.26)$$

L is the total labour supply in the home country and the RHS of (4.3.26) is the total labour demand, summing up the labour demand from supplying the domestic and foreign markets.

The competitive wage rate in home country can be solved by substituting (4.3.15), (4.3.17) and (4.3.19) into (4.3.26).

$$w = \frac{a + 2ad - 3bL + at}{\lambda \left(2t^2 + 2d^2 + (1 + 2d)(1 + t)\right)}$$
(4.3.27)

Labour market clearing condition in the foreign country shows that:

$$L = \int_{0}^{1} \beta_{2}(z) y_{2}(z)^{CN} dz + \int_{0}^{1} \left[ \beta_{2}(z) + t \right] y_{2}^{*}(z)^{CN} dz \qquad (4.3.28)$$

## 4.3.2.1 The equilibrium wage

The competitive wage rate in the foreign country can be solved by substituting (4.3.16), (4.3.18) and (4.3.20) into (4.3.28):

$$w^* = \frac{a + 2ad - 3bL + at}{\lambda \left(2t^2 + 2d^2 + (1 + 2d)(1 + t)\right)}$$
(4.3.29)

The equilibrium wage rate in the foreign country is the same as in the home country.

One can display the consumer's real wage as:

$$\overline{w} = w\lambda = \frac{a + 2ad - 3bL + at}{\left(2t^2 + 2d^2 + (1 + 2d)(1 + t)\right)}$$
(4.3.30)

Under zero trade cost, the equilibrium wage rate becomes

$$\overline{w}\Big|_{t=0} = \frac{a+2ad-3bL}{2d^2+1+2d}$$
(4.3.31)

It is natural to assume a positive wage rate. Thus, it follows that  $a > \frac{3}{1+2d}bL$ .

Taking the derivative of  $\overline{w}$  with respect to t and evaluating it at t = 0 yields:

$$\frac{\partial \overline{w}}{\partial t}\Big|_{t=0} = \frac{3b(1+2d)L - 2ad(1+d)}{(1+2d+2d^2)^2}$$

The effect of trade costs on the equilibrium wage rate is ambiguous when t = 0. Only if the given labour supply is sufficiently large  $\left(L > \frac{2ad(1+d)}{3b(1+2d)}\right)$ , an increase in t will cause a rise in the equilibrium wage in the neighbourhood of zero trade cost.

The trade cost per-unit of exports is unambiguously increasing in t at zero trade  $\operatorname{cost:} \frac{\partial \tau}{\partial t}\Big|_{t=0} = \frac{a + 2ad - 3bL}{1 + 2d + 2d^2} > 0$ . It implies that a rise in t will increase the marginal

cost of the export goods at zero trade cost.

Substitute the equilibrium wage rate into the equilibrium outputs (4.3.15) - (4.3.18):

$$y_1(z)^{CN} = \frac{3at^2 - 3bLt + 2ad^2 + (a + 2ad)(1 + 2t) + (at + a + 2ad - 3bL)(1 - d - 3z)}{3b(2t^2 + 2d^2 + (1 + 2d)(1 + t))}$$

$$y_{2}(z)^{CN} = \frac{6bLt + 2ad^{2} + (a + 2ad)(1 - t) + (at + a + 2ad - 3bL)(d + 3z - 1)}{3b(2t^{2} + 2d^{2} + (1 + 2d)(1 + t))}$$

(4.3.33)

$$y_{1}^{*}(z)^{CN} = \frac{6bLt + 2ad^{2} + (a + 2ad)(1 - t) + (at + a + 2ad - 3bL)(2 + d - 3z)}{3b(2t^{2} + 2d^{2} + (1 + 2d)(1 + t))}$$
(4.3.34)

$$y_{2}^{*}(z)^{CN} = \frac{3at^{2} - 3bLt + 2ad^{2} + (a + 2ad)(1 + 2t) + (at + a + 2ad - 3bL)(3z - d - 2)}{3b(2t^{2} + 2d^{2} + (1 + 2d)(1 + t))}$$
(4.3.35)

As mentioned above, the home country is more efficient and has a comparative advantage with lower values of z. It can be seen from (4.3.32) and (4.3.34) that the outputs of the home firm are decreasing in z. By contrast, the outputs of the foreign firm are positively related to z.

Due to the fact that an increase in trade cost will lessen the profit as well as the level of exports, those firms with relatively lower technology do not export if the trade cost is set to be too high for them (i.e., marginal cost is too expensive). There must exist a critical trade cost that the firm with the lowest technology still survives in the opposite market. In order to find out this critical value, we focus on the foreign firm with highest labour input requirement that does not export. As the technology level of the foreign country is increasing in *z*, the critical trade level  $\overline{t}$  can be calculated by setting z = 0:  $\beta_1(z=0) = d$  and  $\beta_2(z=0) = 1+d$ .

Substituting the two labour input requirements into (4.3.35) and solving the trade cost when the output is zero. The critical trade cost can be solved:

$$\overline{t} = \frac{6bL + 3bdL - a - 3ad}{3(a + ad - 2bL)} \quad ; \frac{6 + 3d}{1 + 3d}bL > a \tag{4.3.36}$$

It is clear that the when the trade cost is below  $\frac{6bL+3bdL-a-3ad}{3(a+ad-2bL)}$ , all the

domestic and foreign firms supply both markets. While, if trade cost exceeds this critical value, the low-technology firms will be efficiently dropped out of the market.

## 4.3.2.2 The aggregate profits and social welfare

Note that  $\beta_1(z) + \beta_2(z) = d + z + d + 1 - z = 2d + 1$  for all z. It can be clearly seen from the above equation that sector z disappears when summing up labour input requirements for home and foreign market in each sector. As shown below, the aggregate profits, consumption level and equilibrium price will also be the same across all sectors.

Summing (4.3.32) and (4.3.33), the consumption level for the representative consumer in each sector can be calculated as:

$$\overline{x} = y_1(z)^{CN} + y_2(z)^{CN} = \frac{a + 3at^2 + 3bL(1 + t + 2d)}{3b(2t^2 + 2d^2 + (1 + 2d)(1 + t))}$$
(4.3.37)

As z does not enter (4.3.37), the aggregate consumption is the same as  $\overline{x}$ ( $X = \int_0^1 \overline{x} dz = \overline{x}$ ). Obtain the first-order derivative at zero trade cost:

$$\frac{\partial \overline{x}}{\partial t}\Big|_{t=0} = \frac{a(-1-2d)-6b(d+d^2)L}{3b(1+2d+2d^2)^2} < 0$$
(4.3.38)

This follows that the representative agent's consumption is harmed by the introduction of trade cost.

The aggregate profits can be measured by the sum of profits from domestic and foreign sales:

$$\Pi = \int_{0}^{1} b \left[ y_{1}(z)^{CN} \right]^{2} dz + \int_{0}^{1} b \left[ y_{1}^{*}(z)^{CN} \right]^{2} dz \qquad (4.3.39)$$

Substituting (4.3.32) and (4.3.34) into the aggregate profits function and integrating over all sectors:

$$\Pi = \frac{1}{9b(1+2d^{2}+t+2t^{2}+2d(1+t))^{2}}$$

$$[-6abL(1+2d+t)(1+3t^{2})+9b^{2}L^{2}(2+2d^{2}+t+5t^{2}+2d(1+t))$$

$$+a^{2}(1+3t^{2})(2+6d^{2}+3t+3t^{2}+6d(1+t))]$$
(4.3.40)

To see how trade cost affects the profit at zero trade cost, partially differentiating the aggregate profits with respect to t and evaluating it at  $t = 0^{20}$ :

$$\frac{\partial \Pi}{\partial t}\Big|_{t=0} = -\frac{a^2 \left(1+8d+18d^2+12d^3\right)-6ab \left(1+6d+6d^2\right)L+9b^2 \left(3+8d+6d^2+4d^3\right)L^2}{9b \left(1+2d+2d^2\right)^3} < 0$$
(4.3.41)

It is clear that the aggregate home profits decrease for a tiny increase in t at zero trade cost. This is because the combination of home sales and exports fall with an increase in t when the initial trade cost is zero.

Substituting (4.3.37) into (4.3.8) yields the equilibrium prices:

$$p = \frac{a(3t^{2} + 6d^{2} + 3t + 2 + 6d(1+t)) - 3(1+2d+t)bL}{3\lambda(2t^{2} + 2d^{2} + (1+2d)(1+t))}$$
(4.3.42)

Alternatively, the real price can be written as:

$$\overline{p} = \frac{a(3t^2 + 6d^2 + 3t + 2 + 6d(1+t)) - 3(1+2d+t)bL}{3(2t^2 + 2d^2 + (1+2d)(1+t))}$$
(4.3.43)

<sup>&</sup>lt;sup>20</sup> See Appendix C4 for the proof.

Taking the derivative of  $\overline{p}$  with respect to t and evaluating it at t = 0 gives:

$$\frac{\partial \overline{p}}{\partial t} = \frac{a + 2ad + 6bd(1+d)L}{3(1+2d+2d^2)^2} > 0$$
(4.3.44)

It follows that an increase in t at zero trade cost will increase the equilibrium price.

Lastly, substituting the second moment of prices into (4.3.25) yields the social welfare:

$$U = -\lambda^{2} \mu_{2}^{p} = -\left(\frac{a\left(3t^{2} + 6d^{2} + 3t + 2 + 6d\left(1 + t\right)\right) - 3\left(1 + 2d + t\right)bL}{3\left(2t^{2} + 2d^{2} + \left(1 + 2d\right)\left(1 + t\right)\right)}\right)^{2} (4.3.45)$$

At zero trade cost, the first-order derivative shows that<sup>21</sup>

$$\frac{\partial U}{\partial t}\Big|_{t=0} = \frac{2((3+6d)bL - (6d^2 + 6d + 2)a)((6d + 6d^2)bL + (1+2d)a)}{9b(2d^2 + 2d + 1)^3} < 0 \quad (4.3.46)$$

Starting from zero trade cost, a small increase in t will lead to a fall in social welfare. This is because the equilibrium price will become more variable if there is an increase in t at zero trade cost.

The results mentioned above lead to the following proposition:

**Proposition 3:** An increase in trade cost will lower the aggregate profits and social welfare in general equilibrium when the trade cost is zero.

In order to examine how the trade cost affects the social welfare at  $\overline{t}$ , obtain the following partial derivative:

<sup>&</sup>lt;sup>21</sup> See the Appendix C5 for the proof.

$$\frac{\partial U}{\partial t}\Big|_{t=\bar{t}} = -\frac{18(a+ad-2bL)^2 \left(a\left(2+3d+3d^2\right)-3bdL\right) \left(a\left(2+6d+6d^2\right)-\left(4+12d+3d^2\right)bL\right)}{b\left(3d^2+3d+4\right)^3 \left(\left(6d^2+6d+2\right)a^2-9\left(1+2d\right)abL+18b^2L^2\right)}$$
(4.3.47)

There are two possibilities<sup>22</sup>:

1. If 
$$a < \frac{4+12d+3d^2}{2+6d+6d^2}bL$$
, then  $\frac{\partial U}{\partial t}\Big|_{t=\bar{t}} > 0$ 

2. If 
$$a > \frac{4+12d+3d^2}{2+6d+6d^2}bL$$
, then  $\frac{\partial U}{\partial t}\Big|_{t=\overline{t}} < 0$ 

Therefore, the effect is ambiguous at the critical trade level  $\overline{t}$ . Social welfare can be decreasing or increasing in the trade cost at  $\overline{t}$ . However, one may ask what would happen to the economy if the trade costs continue growing and exceed  $\overline{t}$  so that the low-technology firms start to drop out of the market? And more importantly, are there gains from trade from the viewpoint of social welfare? The following analysis will offer the answers.

<sup>&</sup>lt;sup>22</sup> See Appendix C6 for more details

#### 4.3.3 Case 2: some firms do not export due to a high trade cost

Suppose that trade cost exceeds prohibitive trade level  $\overline{t} = \frac{6bL + 3bdL - a - 3ad}{3(a + ad - 2bL)}$ ,

then some foreign firms with low technology do not supply the home market and similarly, due to the full symmetry, the less efficient domestic firms do not export.



Figure 4–6: Labour input requirements of the two countries

The industry with the lowest technology will be firstly dropped out when there is a tiny increase in trade cost at  $\overline{t}$ . With the increasing trade cost, more foreign firms with relatively low technology do not supply the home market. The diagram above shows the case that the domestic firm in sector 1-h is producing zero output for exports. In other words, in the home country, only the firms in sector  $z \in [0, 1-h]$  will supply the foreign market. By symmetry, only the foreign firms in sector  $z^* \in [h, 1]$  will export goods to the home country. Notice that the domestic firm in any sector z between [0,h] is a monopoly since a high trade costs prevent the foreign firms in the relevant industries from entering the home market. If there is an increase in the trade costs, more foreign firms will not export and therefore, h will rise.

Now consider the case that the domestic firms behave as monopolists in the sector  $z \in [0, h]$ .

The quadratic sub-utility function now becomes

$$u[x(z)] = a[x_1(z)] - \frac{b}{2}[x_1(z)]$$
(4.3.48)

The linear inverse demand function formulates as:

$$p_1(z) = \frac{1}{\lambda} (a - bx_1(z))$$
(4.3.49)

As a monopoly, each domestic firm maximises the following profits:

$$\pi_1(\tilde{z}) = [p_1(\tilde{z}) - c_1(\tilde{z})]y_1(\tilde{z})$$
(4.3.50)

Market clearing condition implies that all goods produced by the domestic firm will be exactly consumed by the representative consumer, i.e.  $x_1(\tilde{z}) = y_1(\tilde{z})$ 

Substituting the inverse demand function (4.3.49) into (4.3.50), the monopoly output in sector  $\tilde{z} \in [0, h]$  can be expressed as:

$$y_{1}^{m}(\tilde{z}) = \frac{a - \lambda \beta_{1}(\tilde{z})w}{2b}$$
 (4.3.51)

The superscript *m* is short for monopoly.  $y_{1}^{m}(z)$  represents the sales of the home firm in sector  $z \in [0, h]$ .

Now consider the whole economy: firms in sector  $z \in [0, h]$  behave as monopolists while firms in  $z \in [h, 1]$  compete with the foreign firms. The condition of full employment suggests that the total labour demand equals to the exogenous labour supply L:

$$L = \int_{0}^{h} \beta_{1}(\tilde{z}) y^{m}_{1}(\tilde{z}) d\tilde{z} + \int_{h}^{1} \beta_{1}(\hat{z}) y_{1}(\hat{z}) d\hat{z} + \int_{0}^{1-h} \left[ \beta_{1}(\hat{z}) + t \right] y^{*}_{1}(\hat{z}) d\hat{z}$$
(4.3.52)

The total labour demand is comprised of the following three parts. This first component  $\int_{0}^{h} \beta_{1}(\tilde{z}) y_{1}^{m}(\tilde{z}) d\tilde{z}$  shows how many units of labour are needed when domestic firms are monopolies in  $\tilde{z} \in [0, h]$ . For  $z \in [h, 1]$ , domestic and foreign firms compete with each other in the home market.  $\int_{h}^{1} \beta_{1}(\hat{z}) y_{1}(\hat{z}) d\hat{z}$  represents the total labour demand when the foreign firms supply the home market. For any sector between 0 and 1-h, domestic firms export goods to the foreign market, and due to a high trade cost, domestic firms in sectors between 1-h and 1 will be efficiently dropped out of the foreign market. Therefore,  $\int_{0}^{1-h} [\beta_{1}(z) + t] y_{1}^{*}(z) dz$  indicates the total labour demand for supplying the foreign market.

## 4.3.3.2 The equilibrium wage

Substituting (4.3.51), (4.3.15) and (4.3.17) into (4.3.52), integrating across the relevant sectors, the equilibrium wage rate can be solved as:

$$\overline{w} = w\lambda$$

$$= \frac{12bL - a(3h^{2} + 2d(4-h) + 4(1+t)(1-h))}{2((4h-4)t^{2} + (4dh+8h-6h^{2}-4d-2)t+3h^{3} + (d^{2}-6-3d)h^{2} + (4d+1)h-4d^{2}-4d-2)}$$
(4.3.53)

*h* is endogenously determined by the trade cost. The prohibitive trade cost prevents the foreign firms from exporting. Denote  $\tilde{t}(h)$  as the prohibitive trade level for the foreign firms in sector *h*. The labour unit requirement of the domestic and foreign firm in sector *h* are  $\beta_1(h) = d + h$  and  $\beta_2(h) = 1 + d - h$  respectively. By substituting (4.3.53) into (4.3.16), the exports by the foreign firm in sector *h* can be calculated as:

$$y_{2}(h) = \frac{a(4-3h^{3}+12t-12h(1+t)+6h^{2}(1+t)+3d(4-8h+h^{2}+4t))-12(2+d-3h+2t)bL}{6b((4h-4)t^{2}+(4dh+8h-6h^{2}-4d-2)t+3h^{3}+(d^{2}-6-3d)h^{2}+(4d+1)h-4d^{2}-4d-2)}$$
(4.3.54)

In sector *h*, the exports of the firm are zero, i.e.,  $\tilde{y}_2(h) = 0$ . The corresponding prohibitive trade cost in sector *h* can be solved as:

$$\tilde{t}(h) = \frac{12ah - 12ad - 4a + 24adh - 6ah^2 - 3adh^2 + 3ah^3 + 24bL + 12bdL - 36bhL}{6(ah^2 - 2ah - 4bL + 2a + 2ad)}$$
(4.3.55)

The first-order derivative shows:

$$\frac{\partial \tilde{t}(h)}{\partial h} = \frac{a^2 \left(16 + 12d \left(4 - h\right)(1 + d) - 16h + 18h^2 - 12h^3 + 3h^4\right) - 72ab \left(1 + 2d\right)L + 144b^2 L^2}{6 \left(a \left(2 + 2d - 2h + h^2\right) - 4bL\right)^2}$$
(4.3.56)

It is shown in Appendix C7 that  $\partial \tilde{t}(h) / \partial h$  is positive. It implies that an increase in trade costs will lead more foreign firms to leave the domestic market and accordingly, *h* will be higher.

Since the unit labour requirement is decreasing in z in the foreign country, the foreign firm in sector 0 is the least efficient. By setting h = 0 and substituting into

(4.3.55), the prohibitive trade level for the foreign firm in sector 0 is calculated as  $\tilde{t}\Big|_{h=0} = \frac{6bL + 3bdL - a - 3ad}{3(a + ad - 2bL)}$ . Unsurprisingly, the result is consistent with (4.3.36).

All domestic and foreign firms supply goods in both markets if the trade cost is below  $\tilde{t}|_{h=0}$ . However, if trade cost exceeds  $\tilde{t}|_{h=0}$ , the low efficient firms will be dropped out of the markets.

If the trade cost increases to an extraordinarily high level that expels all foreign firm from entering the home market. It is easy to obtain the prohibitive trade cost at autarky by setting h = 1:

$$\tilde{t}\big|_{h=1} = \frac{a(5+9d)-12(d-1)bL}{6(a+2ad-4bL)}; a(5+9d) > 12(d-1)bL$$
(4.3.57)

If trade cost exceeds  $\tilde{t}|_{h=1}$ , the economy will become an autarky, i.e., there is no trade between two countries

Substituting (4.3.55) into (4.3.53), the reduced form of equilibrium wage rate can be obtained as<sup>23</sup>:

$$\bar{w} = w\lambda = \frac{3\left(a\left(2+2d-2h(\tilde{t})+h(\tilde{t})^{2}\right)-4bL\right)}{2\left(4+3d+3d^{2}-9h(\tilde{t})+9h(\tilde{t})^{2}-3h(\tilde{t})^{3}\right)}$$
(4.3.58)

The competitive wage rate at the prohibitive trade cost  $\tilde{t}|_{h=1}$  obtains as:

$$\overline{w}\Big|_{h(\overline{i})=1} = \frac{3(a(1+2d)-4bL)}{2(1+3d+3d^2)}$$
(4.3.59)

<sup>&</sup>lt;sup>23</sup> It is more convenient to express the variables as a function of  $\tilde{t}(h)$  so that the effect of trade cost on the economy is straightforward. However, the formulas containing  $\tilde{t}(h)$  are too complicated. Indeed, because *h* is endogenous and determined by the trade cost, expressing in term of  $\tilde{t}(h)$  or  $h(\tilde{t})$  is a matter of no importance.

It is shown in Appendix C8 that  $\overline{w}|_{t=0} > \overline{w}|_{h(\tilde{t})=1}$ . The wage rate at zero trade cost is unambiguous larger than that at the prohibitive trade cost.

The effect of trade cost on the equilibrium wage rate at the prohibitive trade cost can obtain as:

$$\frac{\partial \overline{w}}{\partial h(\tilde{t})}\Big|_{h(\tilde{t})=1} = 0$$
(4.3.60)

It shows that a small change of trade cost at autarky has no effect on the wage rate, and as can be seen below, it also has no effect on aggregate profits and social welfare.

Suppose that the trade cost is prohibitive, i.e.,  $h(\tilde{i}) = 1$ , the most efficient foreign firm in sector 0 is now making zero profits in the home market and the economy is of autarky. Unlike the case that all foreign firms have the same cost, the foreign industries are ordered in terms of technology, i.e., from the most efficient (z = 0) to the lowest efficient (z = 1). Also, the model is assumed to have a continuum of sectors. Therefore, a small reduction in t may have very little impact on the number of foreign firms that will enter the home market and compete with the home firms. Neary (2016) has provided a similar intuition: "labor demand is unaffected by small changes in the threshold sectors. Changes in either of these thresholds implies entry or exit of extra firms which are just at the margin of profitability". If there is a small reduction in t at the prohibitive trade cost, the labour demand remains constant. As a result, the effect on aggregate labour demand can be ignored.

In order to have a better view of how trade cost affects the equilibrium wage, we assign specific values on the parameters.

The effect of trade costs on the equilibrium wage can be seen from figure 4-7 with the parameter values: a = 10, b = 1, L = 10, d = 2.



*Figure 4–7: Trade cost and wage rate* 

Figure 4-7 suggests that a reduction in trade costs will lead to an increase in the equilibrium wage rate when trade cost is relatively high. As has been shown in section 4.2, the effect is because of a redistribution of labour: at the extensive margin, some firms with high technology level will be able to compete and export in response to a reduction in t. As a result, the equilibrium wage  $\overline{w}$  will increase due to an increase in overall demand for labour. The effect of t on the wage at zero trade cost has been shown in the analysis of case 1: the effect is ambiguous and depends on the specific values of parameters.

Next, consider the effect of trade costs on the equilibrium prices.

The consumption level is identical for sector  $z \in [h, 1]$  in the home country, using  $\beta_1(z) + \beta_2(z) = 2d + 1$ , we have:

$$\overline{x}(\hat{z}) = y_1(z) + y_2(z) = \frac{a(8 - 24h + 3(8 - d)h^2 - 9h^3) + 12b(d + h)L}{4b(4 + 3d + 3d^2 - 9h + 9h^2 - 3h^3)};$$
(4.3.61)  
$$z \in [h, 1]$$

The first moment of the distribution of prices obtains as:

$$\mu_{1}^{p} = \int_{h}^{1} \left[ \frac{1}{\lambda} \left( a - b\overline{x} \left( \hat{z} \right) \right) \right] d\hat{z} + \int_{0}^{h} \left[ \frac{1}{\lambda} \left( a - bx^{m} \left( \tilde{z} \right) \right) \right] d\tilde{z}$$
  
$$= \frac{a \left( 16 + 24d + 24d^{2} - 24h + 18h^{2} - 3h^{4} \right) - 12b \left( 2d + (2-h)h \right) L}{8 \left( 4 + 3d + 3d^{2} - 9h + 9h^{2} - 3h^{3} \right)}$$
(4.3.62)

As  $\frac{\partial \mu_1^p}{\partial h(\tilde{t})}\Big|_{h(\tilde{t})=1} = 0$ , a reduction in the trade cost evaluated at the prohibitive trade

cost has zero effect on the first moment of the distribution of prices.

Figure 4-8 shows the effect of trade cost on the price with the parameter values: a = 10, b = 1, L = 10, d = 2.



Figure 4-8: Trade cost and the first moment of prices distribution

It can be clearly seen from figure 4-8 that, the first moment of prices is a humpshaped function in the trade costs. The intuitions are given as follows. Starting from the prohibitive trade cost, if there is a reduction in trade cost at the extensive margin, the wage will lead to an increase in the labour demand. This raises production costs for firms across all sectors. Consequently, the price will increase in response to an increase in production costs. On the other hand, the competition effect tends to reduce the price if there is a reduction in trade costs. When the trade cost is sufficiently high, the competition effect is dominated by the high production costs and a reduction in the trade cost leads to higher consumer prices. When the trade cost is relatively small, the competition effect outweighs the effect of the production cost, and a reduction in the trade cost leads the price to fall.

Substituting (4.3.58) into (4.3.51) and (4.3.15) -(4.3.18) yield the equilibrium productions:

$$y_{1}^{m}(z) = \frac{a}{2b} + \frac{\left(6ah(\tilde{t}) + 12bL - 6a - 6ad - 2ah(\tilde{t})^{2}\right)\beta_{1}(z)}{4b\left(4 + 3d + 3d^{2} - 9h(\tilde{t}) + 9h(\tilde{t})^{2} - 3h(\tilde{t})^{3}\right)}; \qquad (4.3.63)$$
$$z \in \left[0, h(\tilde{t})\right]$$

$$y_{1}(z) = \frac{1}{4b(4+3d+3d^{2}-9h(\tilde{t})+9h(\tilde{t})^{2}-3h(\tilde{t})^{3})} [4a(1+d^{2})+8ah(\tilde{t})(d-1)+ah(\tilde{t})^{2}(10-d)-3ah(\tilde{t})^{3} +4bL(2+d-3h(\tilde{t}))+(4a+4ad-4ah(\tilde{t})+2ah(\tilde{t})^{2}-8bL)(\beta_{2}(z)-2\beta_{1}(z))];$$
  
$$z \in [h(\tilde{t}),1]$$

$$y_{2}(z) = \frac{\left(a\left(h\left(\tilde{t}\right)^{2} - 2h\left(\tilde{t}\right) + 2d + 2\right) - 4bL\right)\left(2 + d - 3h\left(\tilde{t}\right) - 2\beta_{2}(z) + \beta_{1}(z)\right)}{2b\left(4 + 3d + 3d^{2} - 9h\left(\tilde{t}\right) + 9h\left(\tilde{t}\right)^{2} - 3h\left(\tilde{t}\right)^{3}\right)};$$
  
$$z \in \left[h\left(\tilde{t}\right), 1\right]$$

$$y_{1}^{*}(z) = \frac{\left(a\left(h\left(\tilde{t}\right)^{2} - 2h\left(\tilde{t}\right) + 2d + 2\right) - 4bL\right)\left(2 + d - 3h\left(\tilde{t}\right) - 2\beta_{1}(z) + \beta_{2}(z)\right)}{2b\left(4 + 3d + 3d^{2} - 9h\left(\tilde{t}\right) + 9h\left(\tilde{t}\right)^{2} - 3h\left(\tilde{t}\right)^{3}\right)};$$
  
$$z \in \left[0, 1 - h\left(\tilde{t}\right)\right]$$

(4.3.66)

$$y_{2}^{*}(z) = \frac{1}{4b(4+3d+3d^{2}-9h(\tilde{t})+9h(\tilde{t})^{2}-3h(\tilde{t})^{3})} [4a(1+d^{2})+8ah(\tilde{t})(d-1)+ah(\tilde{t})^{2}(10-d)-3ah(\tilde{t})^{3} +4bL(2+d-3h(\tilde{t}))+(4a+4ad-4ah(\tilde{t})+2ah(\tilde{t})^{2}-8bL)(\beta_{2}(z)-2\beta_{1}(z))];$$
  
$$z \in [1-h,1]$$

# 4.3.3.3 The aggregate profits and welfare

The aggregate profits can be expressed as<sup>24</sup>:

$$\Pi = \int_{0}^{h} b\left(\left[y_{1}^{*}(z)\right]^{2} + \left[y_{1}^{m}(z)\right]^{2}\right) dz + \int_{h}^{1-h} b\left(\left[y_{1}(z)\right]^{2} + \left[y_{1}^{*}(z)\right]^{2}\right) dz + \int_{1-h}^{1} b\left(\left[y_{1}(z)\right]^{2}\right) dz$$

$$(4.3.68)$$

The first term measures the aggregate profits from sector  $z \in [0,h]$ , where the home firms behave as monopolists in the home market and compete with the foreign firms in the foreign market. The second term represents the total profits from sector  $z \in [h, 1-h]$ , where both domestic and foreign firms compete with each other in both markets. The last term captures the aggregate profits from the sector  $z \in [1-h, 1]$ , where domestic firms do not export due to the prohibitive trade cost.

Substituting (4.3.63), (4.3.64) and (4.3.66) into the equation of aggregate profit yields:

$$\Pi = \frac{1}{16b\left(4 + 3d + 3d^2 - 9h\left(\tilde{t}\right) + 9h\left(\tilde{t}\right)^2 - 3h\left(\tilde{t}\right)^3\right)^2} \left(Aa^2 - BabL^2\right) \quad (4.3.69)$$

<sup>&</sup>lt;sup>24</sup> Notice that the equation suits for any *h* less than 1/2. If *h* is greater than 1/2, then the aggregate profits should be express as:  $\Pi = \int_0^{1-h} b\left(\left[y_1^*(z)\right]^2 + \left[y_1^{m_1}(z)\right]^2\right) dz + \int_{1-h}^{h} b\left(\left[y_1^{m_1}(z)\right]^2\right) dz + \int_{h}^{1} b\left(\left[y_1(z)\right]^2\right) dz$ These two equations are exactly the same by replacing *h* by *1-h*.

with

•

$$A = 64 + 3d^{2} (-2 + h(\tilde{t}))^{2} (8 + h(\tilde{t})(-16 + 9h(\tilde{t}))) +3(-2 + h(\tilde{t}))h(\tilde{t}) \{48 + h(\tilde{t})(-72 + h(\tilde{t})[72 + h(\tilde{t})(-34 + h(\tilde{t})(5 + 2h(\tilde{t})))])\} +3d \{32 + h(\tilde{t})(-96 + h(\tilde{t})(108 + h(\tilde{t})[-56 + h(\tilde{t})(20 + h(\tilde{t})(-10 + 3h(\tilde{t})))]))\} \}$$

and  
$$B = 24b(4+3d-h(\tilde{t}))(-1+h(\tilde{t}))^{2}(2+2d+(-2+h(\tilde{t}))h(\tilde{t}))L +48b^{2}(8+6d+3d^{2}-6(3+d)h(\tilde{t})+3(5+d)h(\tilde{t})^{2}-4h(\tilde{t})^{3}).$$

In order to analyse the effect of trade cost on the aggregate profits at autarky, partially differentiating with respect to h and evaluating it at h = 1 gives:

$$\frac{\partial \Pi}{\partial h(\tilde{t})}\Big|_{h(\tilde{t})=1} = 0$$
(4.3.70)

It shows that a small change of t has no effect on the aggregate profits since there are no entry or exit of extra firms.

The effect of trade cost on aggregate profits can be seen from figure 4-9 with the parameter values: a = 10, b = 1, L = 10, d = 2.



Figure 4–9: Trade cost and the aggregate profits

Figure 4-9 shows that the aggregate profits are decreasing in t when the trade cost is initially zero. The intuition has been demonstrated in the case 1. If the trade cost is relatively high, the aggregate profits are positively related to the trade costs. If the trade cost is prohibitive, a reduction in t leads the wage to increase, the production costs will be higher, and as a result, the aggregate profits will fall.

Now consider the effect of trade costs on the social welfare.

The second moment of the distribution of prices obtains as:

$$\mu_2^p = \int_h^1 \left[ \frac{1}{\lambda} \left( a - b\overline{x} \left( \hat{z} \right) \right) \right]^2 d\hat{z} + \int_0^h \left[ \frac{1}{\lambda} \left( a - bx^m \left( \tilde{z} \right) \right) \right]^2 d\tilde{z}$$
(4.3.71)

Substituting (4.3.61) and (4.3.51) into (4.3.71) and substituting the second moment of prices into (4.3.25) yields the aggregate social welfare:

$$\tilde{U} = -\lambda^2 \mu_2^p = \frac{Aa^2 + BabL - Cb^2 L^2}{16(4 + 3d + 3d^2 - 9h + 9h^2 - 3h^3)^2}$$
(4.3.72)

with

$$A = -16(2+3d+3d^{2})^{2} + 96(2+3d+3d^{2})h - 72(4+3d+3d^{2})h^{2} + 12(17-2d-d^{2})h^{3} + 3(2+d)(15d-4)h^{4} - 6(13+5d)h^{5} + 3(19+3d)h^{6} - 12h^{7}$$
$$B = 24[12d^{3} + 6d^{2}(2+2h-h^{2}) + d(8+h^{2}(3+2h-3h^{2})) + h(8-h(16+h(8-h)h-17))]$$

 $C = 48(3d^{2} + 3d(2-h)h + (3-2h)h^{2})$ 

Examine the effect of trade cost on social welfare when *h* equals to 0:

$$\frac{\partial \tilde{U}}{\partial h(\tilde{t})}\Big|_{h(\tilde{t})=0} = -\frac{6\left(a\left(2+3d+3d^{2}\right)-3bdL\right)\left(a\left(2+6d+6d^{2}\right)-b\left(4+12d+3d^{2}\right)L\right)}{\left(4+3d+3d^{2}\right)^{3}}$$
(4.3.73)

There are two possibilities:

1. If 
$$a < \frac{4+12d+3d^2}{2+6d+6d^2}bL$$
, then  $\left.\frac{\partial \tilde{U}}{\partial h}\right|_{h=0} > 0$   
2. If  $a > \frac{4+12d+3d^2}{2+6d+6d^2}bL$ , then  $\left.\frac{\partial \tilde{U}}{\partial h}\right|_{h=0} < 0$ 

Again, these results are consistent with section 4.2 above.

Regarding the case of autarky when h = 1, the first-order derivative obtains as

$$\frac{\partial \tilde{U}}{\partial h(\tilde{t})}\Big|_{h(\tilde{t})=1} = 0$$
(4.3.74)

Therefore, social welfare remains constant with a small change of trade cost at autarky.

As has been explained above, the effect of trade costs on the aggregate variables in the neighbourhood of autarky can be ignored. **Proposition 4:** Social welfare when the trade cost is zero is higher than the social welfare when the trade cost is at the prohibitive level in general equilibrium.

## Proof.

By substituting t = 0 into the social welfare function (4.3.45), the social welfare under zero trade cost obtains as:

$$U\big|_{t=0} = -\left(\frac{a\left(6d^2 + 2 + 6d\right) - 3(1 + 2d)bL}{3\left(2d^2 + 2d + 1\right)}\right)^2$$
(4.3.75)

Substituting h = 1 into (4.3.72), the social welfare under autarky obtains as:

$$U^{a} = -\frac{a^{2}(13 + 48d + 18d^{2}) - 48ab(1 + 2d)L + 48b^{2}L^{2}}{16(1 + 3d + 3d^{2})}$$
(4.3.76)

Alternatively, recall the analysis of the monopoly sector from (4.3.48) -(4.3.51). If the economy is an autarky, then h = 1. The condition of full employment implies:

$$L = \int_{0}^{1} \beta(z) y_{1}^{m}(\tilde{z}) dz \qquad (4.3.77)$$

Substitute the monopoly output into the condition above, the equilibrium wage under autarky obtains as:

$$w^{a} = \frac{3(a+2ad-4bL)}{2(1+3d+3d^{2})r}$$
(4.3.78)

The monopoly price is obtained as:

$$p^{a} = \frac{-12bL\beta_{1}(z) + a\left(2 + 6d^{2} + 3\beta_{1}(z) + 6d\left(1 + \beta_{1}(z)\right)\right)}{4\left(1 + 3d + 3d^{2}\right)}$$
(4.3.79)

Lastly, substitute the second moment of the monopoly prices into (4.3.25) yields the aggregate social welfare:

$$U^{a} = -\frac{a^{2} \left(13 + 48d + 18d^{2}\right) - 48ab \left(1 + 2d\right)L + 48b^{2}L^{2}}{16 \left(1 + 3d + 3d^{2}\right)}$$
(4.3.80)

Define  $\Delta U$  as the gap between social welfare under zero trade cost and under prohibitive trade cost:

$$\begin{split} \Delta U &= U \Big|_{t=0} - U^a \\ &= - \left( \frac{a \left( 6d^2 + 2 + 6d \right) - 3 \left( 1 + 2d \right) bL}{3 \left( 2d^2 + 2d + 1 \right)} \right)^2 + \frac{a^2 \left( 13 + 48d + 18d^2 \right) - 48ab \left( 1 + 2d \right) L + 48b^2 L^2}{16 \left( 1 + 3d + 3d^2 \right)} \\ &= \frac{a^2 \left( 53 + 324d + 792d^2 + 936d^3 + 468d^4 \right) - 48ab \left( 5 + 22d + 36d^2 + 24d^3 \right) L + 144b^2 \left( 2 + 5d + 5d^2 \right) L^2}{144 \left( 1 + 2d + 2d^2 \right)^2 \left( 1 + 3d + 3d^2 \right)} \end{split}$$

Denote x = bL, the numerator can be expressed as the following:

$$f(x) = a^{2} (53 + 324d + 792d^{2} + 936d^{3} + 468d^{4})$$
  
-48a (5 + 22d + 36d^{2} + 24d^{3})x + 144 (2 + 5d + 5d^{2})x^{2}

The discriminant of the quadratic function is calculated as:

$$\left(-48a\left(5+22d+36d^{2}+24d^{3}\right)\right)^{2}$$
  
-4\left(144\left(2+5d+5d^{2}\right)\left(a^{2}\left(53+324d+792d^{2}+936d^{3}+468d^{4}\right)\right)  
=-1728a^{2}\left(1+2d+2d^{2}\right)^{2}\left(2+3d+3d^{2}\right)<0

Therefore, f(x) > 0 and  $U|_{t=0} - U^a > 0$ .

# Q.E.D.

As in the partial equilibrium, there are gains from trade under general equilibrium. As has shown under the "featureless economy" without comparative advantage, social welfare is identical under zero trade cost and autarky if the products are identical. However, though the products are homogeneous, social welfare at zero trade costs is still higher than the autarky welfare if there exists comparative advantage across industries. This confirms that one source of the gains from trade is the existence of the comparative advantage.

The "real" social welfare can be obtained from equation (4.3.25): 
$$V = \frac{U + a^2}{2b}$$

As mentioned above, the effect of trade costs on social welfare at  $\overline{t}$  depends on the values of parameters. Therefore, two groups of parameter values will be considered.

The first group of parameter values follows that 
$$\frac{\partial U}{\partial t}\Big|_{t=\bar{t}} > 0$$

The effect of trade cost on social welfare can be seen from figure 4-10 with the parameter values: a = 10, b = 1, L = 10, d = 2.



Figure 4–10: Social welfare with trade cost I

The second group of parameter values ensures that  $\frac{\partial U}{\partial t}\Big|_{t=\overline{t}} < 0$ .

The effect of trade cost on social welfare can be seen from figure 4-11 with the parameter values: a = 10, b = 1, L = 10, d = 2.5.



Figure 4–11: Social welfare with trade cost II

The diagrams above suggest that an increase in d will lower both aggregate profits and social welfare. This is because, with a higher value of d, the unit labour requirement will increase, and as a result, the firms will become less efficient.

Figure 4-10 and figure 4-11 also suggest that social welfare is a U-shape function in the trade costs. Following Neary (2016), the relationship between social welfare and trade costs can be explained by the change of the variability of prices. Starting from zero trade cost, prices tend to increase, raising their variability and so reducing the social welfare. On the other hand, if the trade cost is initially at the prohibitive level, prices tend to increase in response to an increase in t. Again the variability of prices will increase so that the social welfare will fall.
## 4.4 Conclusion

Unlike the usual analysis under partial equilibrium, the wage is endogenously determined by the full employment condition in the general equilibrium. A reduction in trade cost will increase the competitive wage particularly when the trade cost is high. This is because the labour will be redistributed and the total labour demand exceeds the total labour supply. Since wage is not fixed under general equilibrium, the trend of prices is totally different from that in partial equilibrium. In the analysis of partial equilibrium, a reduction in trade costs always lowers the prices to the home consumers. However, as the Ricardian cost structure is used in the GOLE approach: the costs are the products of the wage and unit labour requirement. Therefore, prices are also affected by the change of costs. It is shown that, when the products are homogeneous, prices are hump-shaped in the trade cost is sufficiently high.

In the featureless economy where the labour input requirement is identical for all firms in both countries, social welfare has a U-shape relation with the trade costs, reaching its minimum level below the prohibitive trade level at autarky. Also, there are gains from trade if the products are differentiated. This suggests that love-of-variety is one source of the gains from trade.

If the unit labour requirement is increasing in the sector z for the home country and decreasing in the sector z for the foreign country, that is, the home country has a comparative advantage in sector  $z \in [0,0.5]$  and the foreign country has a comparative advantage in sector  $z \in [0.5,1]$ , social welfare is also U-shaped in the trade cost. As in the featureless economy, there are gains from trade even if the products are assumed to be homogeneous. Therefore, another source of the gains from trade is confimed to be the comparative advantage.

## Appendix C

1. To show that the home sales are increasing in the trade costs:  $\frac{\partial y_1^{CN}}{\partial t} > 0$ .

Evaluating the first order derivative at t = 0 gives:

$$\frac{\partial y_1^{CN}}{\partial t}\Big|_{t=0} = \frac{a\beta - 2bL}{2b(2-r)\beta^2} > 0$$

Evaluating the first order derivative at  $t = \tilde{t}$  gives:

$$\frac{\partial y_1^{CN}}{\partial \overline{t}}\Big|_{t=\tilde{t}} = \frac{(a\beta - bLr)(a\beta - 2bL)^2}{2b(2-r)\beta^2(b^2L^2(2+r) - abL(2+r)\beta + a^2\beta^2)}$$

Denote x as bL, then we have the following quadratic function:

$$f(x) = (2+r)x^{2} - a(2+r)\beta x + a^{2}\beta^{2}$$

To check if there exist real roots for this function:

$$(-a(2+r)\beta)^{2} - 4(a^{2}\beta^{2})((2+r)x^{2}) = a^{2}(-4+r^{2})\beta^{2} < 0$$

Therefore, f(x) > 0 and  $\frac{\partial y_1^{(CN)}}{\partial \overline{t}}\Big|_{t=\overline{t}} > 0$ .

Solving the first-order condition  $\frac{\partial y_1^{CN}}{\partial t} = 0$ , the two roots are:

$$r_{1} = \frac{-(2-r)\beta(a\beta - bL) + \sqrt{(2-r)\beta^{2}(b^{2}L^{2}(2+r) - abL(2+r)\beta + a^{2}\beta^{2})}}{a(1-r)\beta + bLr}$$

$$r_{2} = \frac{-(2-r)\beta(a\beta - bL) - \sqrt{(2-r)\beta^{2}(b^{2}L^{2}(2+r) - abL(2+r)\beta + a^{2}\beta^{2})}}{a(1-r)\beta + bLr}$$

As the denominator is positive,  $r_1 > r_2$ .

Suppose that  $r_1 < 0$ , then we must have:

$$(2-r)\beta^{2}(b^{2}L^{2}(2+r)-abL(2+r)\beta+a^{2}\beta^{2})-((2-r)\beta(a\beta-bL))^{2}>0$$

Simplify the left-hand-side of inequality above:  $(2-r)\beta^2(a\beta-bL)(a(1-r)\beta+bLr)$ , which is positive. Therefore, it can be shown that  $0 > r_1 > r_2$ .

 $\frac{\partial y_1^{CN}}{\partial t}$  is monotonically increasing in t when the trade costs are positive.

$$\frac{\partial y_1^{CN}}{\partial t} > 0 \text{ is proved.}$$

2. To show that the exports are decreasing in the trade costs:  $\frac{\partial y_2^{CN}}{\partial t} < 0$ .

Evaluating the first order derivative at t = 0 gives:

$$\left. \frac{\partial y_2^{CN}}{\partial t} \right|_{t=0} = \frac{rbL - a\beta}{2b(2-r)\beta^2} < 0$$

Evaluating the first order derivative at  $t = \tilde{t}$  gives:

$$\frac{\partial y_2^{CN}}{\partial \overline{t}}\Big|_{t=\tilde{t}} = -\frac{\left(a\beta - 2bL\right)^3}{2b\left(2-r\right)\beta^2\left(b^2L^2\left(2+r\right) - abL\left(2+r\right)\beta + a^2\beta^2\right)} < 0$$

Solving the first-order condition  $\frac{\partial y_2^{CN}}{\partial t} = 0$ , the two roots are:

$$r_{1} = \frac{(2-r)bL\beta + \sqrt{((2-r)bL\beta)^{2} + 4\beta^{2}(a\beta - 2bL)(2-r)(a\beta - bL)}}{2(a\beta - 2bL)}$$
$$> \frac{(2-r)bL\beta}{2(a\beta - 2bL)} = \tilde{t}$$

$$r_{2} = \frac{(2-r)bL\beta - \sqrt{((2-r)bL\beta)^{2} + 4\beta^{2}(a\beta - 2bL)(2-r)(a\beta - bL)}}{2(a\beta - 2bL)} < 0$$

It follows that  $r_1 > \tilde{t} > 0 > r_2$ .  $\frac{\partial y_2^{CN}}{\partial t}$  is monotonically decreasing in t when

the trade costs are positive.  $\frac{\partial y_2^{CN}}{\partial t} < 0$  is proved.

3. To show that the trade costs per-unit of exports are positively related to unit labour requirement of exports:  $\frac{\partial \tau}{\partial t} > 0$ .

Evaluating the first order derivative at t = 0 gives:

$$\left. \frac{\partial \tau}{\partial t} \right|_{t=0} = \frac{2a\beta - (2+r)bL}{2\beta^2} > 0$$

Evaluating the first order derivative at  $t = \tilde{t}$  gives:

$$\frac{\partial \tau}{\partial t}\Big|_{t=\tilde{t}} = \frac{\left(2a\beta - (2+r)bL\right)\left(a\beta - 2bL\right)^2}{2\beta^2\left(b^2L^2\left(2+r\right) - abL\left(2+r\right)\beta + a^2\beta^2\right)} > 0$$

Solving the first-order condition  $\frac{\partial \tau}{\partial t} = 0$ , the two roots are:

$$r_{1} = \frac{-a(2-r)\beta^{2} + \sqrt{(4-r^{2})\beta^{2}(b^{2}L^{2}(2+r) - abL(2+r)\beta + a^{2}\beta^{2})}}{bL(2+r) - ar\beta}$$
$$r_{2} = \frac{-a(2-r)\beta^{2} - \sqrt{(4-r^{2})\beta^{2}(b^{2}L^{2}(2+r) - abL(2+r)\beta + a^{2}\beta^{2})}}{bL(2+r) - ar\beta}$$

There are two possibilities.

First, assume that  $bL(2+r) - ar\beta > 0$ .

We want to show that  $0 > r_1 > r_2$ . Therefore, the following inequality must hold:

$$\left(-a(2-r)\beta^{2}\right)^{2}-\left(4-r^{2}\right)\beta^{2}\left(b^{2}L^{2}(2+r)-abL(2+r)\beta+a^{2}\beta^{2}\right)>0$$

Simplify the inequality above gives:

$$(-2+r)\beta^{2}\left(b^{2}L^{2}(2+r)^{2}-abL(2+r)^{2}\beta+2a^{2}r\beta^{2}\right)>0$$

It remains to show that  $b^2 L^2 (2+r)^2 - abL(2+r)^2 \beta + 2a^2 r \beta^2 < 0$ .

Define x = bL, then we have the following quadratic function:

$$f(x) = (2+r)^{2} x^{2} - (2+r)^{2} a\beta x + 2ra^{2}\beta^{2}$$

As it is assumed that  $bL(2+r) - ar\beta > 0$  and according to (4.2.36), we have the

range of  $x : x \in \left(\frac{ar\beta}{2+r}, \frac{a\beta}{2}\right)$ 

Obtain the axis of symmetry of the parabola:  $-\frac{-(2+r)^2 a\beta}{2(2+r)^2} = \frac{a\beta}{2}$ 

Therefore, f(x) is decreasing in x. The last step is to show that  $f(x)|_{x=\frac{\alpha r\beta}{2+r}}$  is non-positive.

It can be shown that  $f(x)\Big|_{x=\frac{ar\beta}{2+r}} = 0$ . Therefore, f(x) < 0 and  $0 > r_1 > r_2$ . It turns out that  $\tau$  is monotonically increasing in t if  $bL(2+r) - ar\beta > 0$ .

Second, assume that  $bL(2+r) - ar\beta < 0$ . We want to show that  $r_1 < 0$  and  $r_2 > \tilde{t}$ .

To show that  $r_1 < 0$ , the following inequality must hold:

$$f(x) = (2+r)^{2} x^{2} - (2+r)^{2} a\beta x + 2ra^{2}\beta^{2} > 0$$

and

$$x > \frac{ar\beta}{2+r} > \frac{a\beta}{2}.$$

It is shown that the axis of symmetry of the parabola is  $\frac{a\beta}{2}$  and  $f(x)\Big|_{x=\frac{ar\beta}{2+r}} = 0$ . Therefore, f(x) > 0 is proved, and  $r_1 < 0$ .

To show that  $r_2 > \tilde{t}$ , we have:

$$\frac{a(2-r)\beta^{2} + \sqrt{(4-r^{2})\beta^{2}(b^{2}L^{2}(2+r) - abL(2+r)\beta + a^{2}\beta^{2})}}{ar\beta - bL(2+r)} > \frac{r(2-r)bL\beta}{r(a\beta - 2bL)} = \frac{(2-r)bL\beta}{a\beta - 2bL}$$

compare the denominator,  $ar\beta - bL(2+r) < r(a\beta - 2bL)$  as r is between 0 and 1. Calculate the gap between  $a(2-r)\beta^2$  and  $r(2-r)bL\beta$ , we have:

$$a(2-r)\beta^2 - r(2-r)bL\beta = (2-r)\beta(a\beta - rbL) > 0$$
. The numerator of the left-

hand-side exceeds that of the right-hand-side.  $r_2 > \tilde{t}$  is then proved.

Therefore,  $\tau$  is also monotonically increasing in t if  $bL(2+r) - ar\beta < 0$ .

- $\frac{\partial \tau}{\partial t} > 0 \text{ must hold for all possible value of } t.$
- 4. To show that the aggregate profits are decreasing in *t* in the neighbourhood of zero trade cost:

$$\frac{\partial \Pi}{\partial t}\Big|_{t=0} = -\frac{a^2 \left(1+8d+18d^2+12d^3\right)-6ab \left(1+6d+6d^2\right)L+9b^2 \left(3+8d+6d^2+4d^3\right)L^2}{9b \left(1+2d+2d^2\right)^3} < 0$$

First, let x = bL, then  $a^{2}(1+8d+18d^{2}+12d^{3})-6ab(1+6d+6d^{2})L+9b^{2}(3+8d+6d^{2}+4d^{3})L^{2}$  can be can be rewritten as a function of:

$$f(x) = 9(3+8d+6d^{2}+4d^{3})x^{2} - 6a(1+6d+6d^{2})x + a^{2}(1+8d+18d^{2}+12d^{3})$$

Calculate the discriminant of the quadratic function:  $-72a^{2}(1+2d+2d^{2})^{2}(1+6d+6d^{2}) < 0$ 

Therefore, f(x) > 0 and the first order derivative  $\frac{\partial \Pi}{\partial t}\Big|_{t=0} < 0$ 

5. To show that social welfare is decreasing in t in the neighbourhood of zero trade  $\operatorname{cost:} \left. \frac{\partial U}{\partial t} \right|_{t=0} < 0$  We just need to show  $(3+6d)bL - (6d^2+6d+2)a > 0$  or equivalently  $a > \frac{3+6d}{6d^2+6d+2}bL$ 

Since 
$$a > \frac{2}{1+d}bL$$
, we then show if  $\frac{2}{1+d} > \frac{3+6d}{6d^2+6d+2}$ . Calculate the gap as:

$$\frac{2}{1+d} - \frac{3+6d}{6d^2+6d+2} = \frac{2(6d^2+6d+2)-(1+d)(3+6d)}{(1+d)(6d^2+6d+2)}$$
$$= \frac{6d^2+3d+1}{(1+d)(6d^2+6d+2)} > 0$$

Therefore, 
$$\frac{\partial U}{\partial t}\Big|_{t=0} < 0$$

5. To check the first-order derivate of social welfare with respect to *t* when the trade cost is prohibitive:

$$\frac{\partial U}{\partial t}\Big|_{t=\overline{t}} = -\frac{18(a+ad-2bL)^2 \left(a\left(2+3d+3d^2\right)-3bdL\right) \left(a\left(2+6d+6d^2\right)-\left(4+12d+3d^2\right)bL\right)}{b\left(3d^2+3d+4\right)^3 \left(\left(6d^2+6d+2\right)a^2-9\left(1+2d\right)abL+18b^2L^2\right)}$$

First, prove that  $a(2+3d+3d^2)-3bdL > 0$  or equivalently that  $a > \frac{3bdL}{2+3d+3d^2}$ .

Since  $a > \frac{2}{1+d}bL$ , we then show if  $\frac{2}{1+d} > \frac{3d}{2+3d+3d^2}bL$ . Calculate the gap as:

$$\frac{2}{1+d} - \frac{3d}{2+3d+3d^2} = \frac{2(2+3d+3d^2) - 3d(1+d)}{(1+d)(2+3d+3d^2)}$$
$$= \frac{3d^2 + 3d + 4}{(1+d)(6d^2 + 6d + 2)} > 0$$

Then we show that  $(6d^2 + 6d + 2)a^2 - 9(1+2d)abL + 18b^2L^2 > 0$ 

Denote x as bL, then  $(6d^2 + 6d + 2)a^2 - 9(1+2d)abL + 18b^2L^2$  can be rewritten as :

$$f(x) = (6d^{2} + 6d + 2)a^{2} - 9(1 + 2d)ax + 18x^{2}$$

Obtain the discriminant of the quadratic function:

$$\left(-9(1+2d)a\right)^{2}-4(18)\left(6d^{2}+6d+2\right)a^{2}=-9a^{2}\left(7+12d+12d^{2}\right)<0$$

It follows that f(x) > 0

Therefore, 
$$\frac{\partial U}{\partial t}\Big|_{t=\bar{t}}$$
 is positive only if when  $a(2+6d+6d^2)-(4+12d+3d^2)bL$  is

positive.

6. To show that 
$$h$$
 and  $\tilde{t}$  are positively related  

$$\frac{\partial \tilde{t}}{\partial h} = \frac{a^2 \left(16 - 12d \left(-4 + h\right) - 12d^2 \left(-4 + h\right) - 16h + 18h^2 - 12h^3 + 3h^4\right) - 72ab \left(1 + 2d\right)L + 144b^2L^2}{6\left(a \left(2 + 2d - 2h + h^2\right) - 4bL\right)^2} > 0$$

Denote x = bL, then the numerator can be expressed as the following function:

$$f(x) = a^{2} \left( 16 - 12d \left( -4 + h \right) - 12d^{2} \left( -4 + h \right) - 16h + 18h^{2} - 12h^{3} + 3h^{4} \right)$$
  
-72a (1+2d) x + 144x<sup>2</sup>

Obtain the discriminant of the quadratic function:

$$(-72a(1+2d))^{2} - 4(144) \Big[ a^{2} (16 - 12d(-4+h) - 12d^{2}(-4+h) - 16h + 18h^{2} - 12h^{3} + 3h^{4}) \Big]$$
  
= 576a<sup>2</sup> (-1+h)(7+12d+12d^{2} - 9h + 9h^{2} - 3h^{3})

Define  $y(h) = 7 + 12d + 12d^2 - 9h + 9h^2 - 3h^3$ 

The first order derivative shows:

$$y'(h) = -9(h^2 - 2h + 1) = -9(h - 1)^2 \le 0$$

Therefore, y(h) is decreasing in *h* and *h* ranges from 0 to 1. Therefore, we have the following inequality:

$$y(h) \ge y(1) = 4 + 12d + 12d^2 > 0$$

 $\frac{\partial \tilde{t}}{\partial h} > 0$  is then proved.

7. To show that the equilibrium wage at zero trade cost is higher than that at the prohibitive trade cost:  $\overline{w}|_{t=0} > \overline{w}|_{h(\tilde{t})=1}$ 

Calculate the gap between  $\overline{w}|_{t=0}$  and  $\overline{w}|_{h(\tilde{t})=1}$ :

$$\frac{a+2ad-3bL}{2d^2+1+2d} - \frac{3(a(1+2d)-4bL)}{2(1+3d+3d^2)} = \frac{-a(1+2d)+6b(1+d+d^2)L}{2(1+5d+11d^2+12d^3+6d^4)}$$

It remains to show that  $L > \frac{a(1+2d)}{6b(1+d+d^2)}$ 

According to (4.3.36), we have  $L > \frac{a(1+3d)}{b(6+3d)}$ 

As 
$$\frac{a(1+3d)}{b(6+3d)} - \frac{a(1+2d)}{6b(1+d+d^2)} = \frac{ad(1+2d+2d^2)}{2b(2+d)(1+d+d^2)} > 0$$

Therefore,  $\overline{w}|_{t=0} > \overline{w}|_{h(\tilde{t})=1}$  is proved.

## **Chapter 5 : Conclusion**

The welfare comparison between ad valorem and per-unit taxation has been one of the oldest issues in public finance. Under Cournot competition, conventional wisdom has it that an ad valorem is welfare superior to a per-unit tax. Grazzini (2006) compares ad valorem and per-unit taxation in an exchange economy in which oligopolists are simultaneously producers and consumers. She claims that, in a general equilibrium setting, per-unit taxation can dominate ad-valorem taxation when the number of oligopolists is sufficiently low compared to the number of consumers. In this study, it is shown that the dominance of which taxation over the other depends on how the social welfare is measured. In Grazzini (2006), social welfare is measured by a utilitarian social welfare function and the utility function is homogeneous of degree two  $(x_1x_2)$ . In section 2.2, the utility function which is homogeneous of degree one is used  $(x_1^{1/2}x_2^{1/2})$ , which is a monotonic transformation of the utility function used in Grazzini (2006). Therefore, it yields the same demands as in Grazzini (2006). It is shown that ad valorem taxation is preferred on welfare grounds. To further analyse why the exceptional result occurs in Grazzini (2006), a lump-sum transfer is introduced from the gainers to the losers in section 2.3, by using some social welfare function in Grazzini (2006). It is shown that welfare with ad valorem tax is unambiguously higher than that with per-unit tax. It can be concluded that the result in Grazzini (2006) is due to the effect of taxation on the redistribution of income rather than from any efficiency gain.

The implication of this chapter, from a policy perspective, is that an *ad valorem* tax is superior to an equal-yield per-unit tax. This reinforces the conventional wisdom and the usual argument. In particular, assuming that the government has the same revenue under two forms of taxation, it is shown that the consumers are in favour of *ad valorem* taxation while the oligopolists are better off with per-unit taxation. From the viewpoint of the aggregate welfare (i.e., the arithmetic sum of each agent's utility level), *ad*  *valorem* dominates per-unit taxation regardless the number of consumers and oligopolists. Moreover, if the government can regulate the economy by transferring goods from gainers to losers, all players are made better off under *ad valorem* taxation. This suggests that the government should adopt an *ad valorem* tax. However, the analysis of chapter 2 does not provide the necessary condition of the social welfare function in which the opposite result in Grazzini (2006) can hold.

The welfare comparison also applies to the study of international trade and conventional wisdom suggests that any positive per-unit tariff can be replaced by an ad valorem tariff that yields a higher level of welfare. However, very few studies consider the welfare with the two tariffs in the presence of the constraint of FDI cost. The proximity-concentration trade-off suggests that the FDI is discouraged when trade costs fall and the foreign firms will only choose to undertake FDI if it is more profitable than export. It is assumed in chapter 3 that the home government maximises the tariff revenue rather than the welfare. As the import tariff revenue is increasing in import tariff up to the critical level where the foreign firm is just willing to supply the home country by exporting, the maximum-revenue tariff will be at the critical value in which the foreign firms can just make positive profits. Given the constraint of FDI cost, the maximum revenue collected by the home government is always higher with ad valorem tariff under Cournot competition. However, under Bertrand competition with differentiated products, the maximum revenue with per-unit tariff is higher than that with ad valorem tariff if the FDI cost is sufficiently low. This is because when the FDI cost is small enough, the critical value of maximum-revenue ad valorem tariff implemented by the home government will be very small. The introduction of product differentiation and nature of Bertrand competition both intensify the competition and lower the prices. Note the tariff revenue in the case of *ad valorem* tariff is calculated as the products of prices, the quantity of exports from the foreign firms and the level of ad valorem tariff. As a result, for a small FDI cost, the import revenue will be very small in the case of *ad valorem* tariff. From the viewpoint of welfare, an *ad valorem* tariff is superior to a per-unit tariff for both Cournot and Bertrand oligopoly.

This welfare dominance of ad valorem tariff given the constraint of FDI cost can also be extended to the case that the government maximises the welfare. To begin with, encouraging FDI is always not the optimal policy. This is because welfare is increasing in the level of import tariff up to the optimal-welfare tariff and as has been shown in chapter 3 that the critical value of FDI cost and import tariff are positively related. This implies that welfare is also increasing in the level of FDI cost. The welfare with a policy of encouraging FDI is equivalent to the welfare with zero FDI cost. Therefore, the welfare with any positive value of FDI outweighs the welfare with encouraging FDI. In addition, the optimal-welfare tariff may exceed the maximum-revenue tariff under both modes of competition. Under per-unit tariff, the tariff revenue rises as the level of exports and import tariff increases. Under ad valorem tariff, an increase in the tariff revenue is due to a rise in the level of exports and import tariff as well as prices. It is shown in chapter 3 that the tariff revenue with an *ad valorem* tariff is unambiguously higher than that with a per-unit tariff in the neighbourhood of the "unconstrained" maximum-revenue tariff. One could expect that that, the tariff and welfare gaps between the two tariffs are even higher if the FDI cost exceeds the critical value with the maximum-revenue tariff. Also, it is noted that the results in chapter 3 could be extended to models that allow for different marginal costs for the two firms and have more than one firm in each country. However, it is unclear that if the results also apply to a model with non-linear demand function.

The sign and magnitude of the gains from trade and from a reduction in trade costs continue to be among the central issues in international trade. In partial equilibrium, welfare as a function of trade costs follows a U-shaped pattern. However, it is of great difficulty to embed oligopoly model in general equilibrium. Thus, there is a very limited number of studies that analyse the relationship between trade costs and welfare in general equilibrium. Chapter 4 adopts the GOLE approach developed by Neary, by assuming that firms are large in their own sectors but small in the economy as a whole. Therefore, the oligopolists are not able to influence factor rewards because they are many in demanding scarce inputs, and they take other good prices and national income

as given. This simple assumption permits to have a theory of oligopoly in general equilibrium, by also addressing the factors markets.

It is shown in chapter 4 that social welfare is also U-shaped in the trade cost in general equilibrium. In particular, in the featureless economy where the level of technology is identical for all firms in two identical countries, there are gains from trade if the products are differentiated. With identical products and comparative advantage, social welfare with zero trade cost is higher than that with the prohibitive trade cost. Therefore, from these two examples, it can be said that gains from trade can be driven by love-of-variety and comparative advantage.

Unlike partial equilibrium, the wage rate is endogenously determined in general equilibrium. An increase in wage is driven by a higher labour demand across all sectors. It is shown that, when the trade cost is sufficiently high, a reduction in trade cost will increase the competitive wage due to the redistribution of labour. Therefore, the production costs will increase for firms across all sectors as the cost function is the Ricardian cost structure. This will also increase the level of prices. On the other hand, the competition effect tends to reduce the price if there is a reduction in trade costs. It is shown that the cost effect dominates when the trade cost is sufficiently high while the competition effect dominates when the trade cost is low enough. As a result, the equilibrium prices as a function of trade costs follows a hump-shaped pattern if the products are identical.

One limitation of this analysis is that the measurement of labour input requirement for the two firms is too "specific", and the comparative advantage is "constant" and will not be affected by changing the values of exogenous parameter values. By contrast, Neary (2016) assumes that the home sales and the exports are respectively decreasing and increasing in sector z and proves a measure of the technological dissimilarity between the two countries (i.e., a measure of comparative advantage)<sup>25</sup>. However,

<sup>&</sup>lt;sup>25</sup> See equation 26 in Neary (2016).

though the model involves the comparative advantage in chapter 4.3, it fails to capture the relationship between welfare and the degree of comparative advantage, which has been done in Neary (2016).

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