A Rankine Theory Based Approach for Stability Analysis of Slurry Trenches

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Abstract: Stability of slurry trenches during excavation is a major concern in design and construction. This paper presents an approach for stability analysis of slurry trenches based on Rankine's theory of earth pressure. The proposed approach is verified via analysis of previously reported problems and is applied to a scenario with a nearby slope. The proposed approach gives the same factor of safety as found by Coulomb force equilibrium methods. Scenarios with soil stratification and variably distributed surcharges can be analyzed by hand calculation when the proposed approach is used. The investigation also indicates that slurry trench stability is sensitive to surcharges from nearby slopes and that they require particular attention in design.

Keywords: factor of safety; Rankine's theory; slurry trench; stability

Introduction

Slurry trenches are long narrow vertical excavations, typically used in the construction of diaphragm walls in civil engineering or cut-off walls (vertical barriers) in geoenvironmental engineering. During excavation, the trenches are filled with slurry, which exerts pressure on the trench walls to balance the earth pressure and hydraulic pressure from the surrounding soils, to prevent trench collapse (Li and Cleall 2017).

Stability of the slurry trench during excavation is a major concern in design and construction. Coulomb-type force equilibrium methods considering a two-dimensional wedge between the trench and a trial failure plane (Nash and Jones 1963; Morgenstern and Amir-tahmasseb 1965; Elson 1968; Filz et al. 2004) is typically used to analyze slurry trench stability (Tsai and Chang 1996). The contribution of shear forces at the sides of the planar wedge was taken into account by Piaskowski and Kowalewski (1965); Prater (1973); Washbourne (1984); Tsai and Chang (1996) and Fox (2004). Such Coulomb-type force equilibrium methods consider the whole failure mass as a wedge and so cannot consider scenarios of trench excavation in heterogeneous layered soils since the shear strength parameters on the trial slip surface are varied between the layers. The horizontal slice method was applied in stability analysis of slurry trenches by Li et al. (2013) for the scenario of layered soils; however, this method requires a computational analysis to perform the force equilibrium analysis of the slices. In practice, variably distributed surcharges from landfill berms, placed solid wastes, excavation machines or nearby buildings may induce large deformation in the surrounding soils and even trench collapses during excavation. For example, shallow trench failure and cracks in a nearby landfill berm that occurred during and after trench excavation were observed at two sites in China in 2017 (see Fig. 1). It is likely that the presence of additional surface surcharges contributed to the failure and cracks.

In this paper, an approach for stability analysis of slurry trenches is proposed based on Rankine’s theory of active earth pressure. This approach is able to consider soil stratification and varied distributed surcharges for slurry trench design. The proposed approach is verified by analysis of a previously reported problem and is applied to a scenario with a nearby slope.

Theory

A typical slurry trench in layered soils is illustrated in Fig. 2a. During trench excavation, the trench is filled with slurry to avoid trench collapse. The pressure on the trench side walls exerted by the slurry is typically less than the sum of the earth pressure at-rest and hydrostatic pressure before excavation. So the trench side walls move inwards towards the trench centerline (Filz 1996), which indicates the soils surrounding the trench tends towards a state of active earth pressure from the at-rest state. During the process the slurry in the trench penetrates into the surrounding soils and forms a low-permeability "filter cake" on the trench side walls. In this paper, the forces on the filter cake from the two sides (that is, the slurry side and the surrounding soil side) are considered (see Fig. 2b) and the factor of safety for the slurry trench, F_s, is defined by

$$F_s = \frac{P_s - P_w}{P_a} \tag{1}$$

where P_s is the total thrust exerted by the slurry on to the filter cake; P_w is the total hydrostatic force of the groundwater in the surrounding soils on to the filter cake; and P_a is the total active thrust exerted on to the filter cake and is the sum of thrust by surrounding soil and that by uniformly distributed surcharge. In Eq. (1), $P_s - P_w$, the seepage force on solid particles of the filter cake, is the resistance to sliding. An alternative definition of F_s, which is given in the following expression, was proposed by Xanthakos (1994),

$$F_s = \frac{P_s}{P_a + P_w} \tag{2}$$

A comparison between the two definitions of F_s is made in the next section.

For the problem considered, P_w and P_s can be written as follows,

$$P_s = \frac{1}{2}\gamma_s H_s^{\,2} \tag{3}$$

$$P_w = \frac{1}{2}\gamma_w H_w^{\,2} \tag{4}$$

where γ_s and γ_w are the unit weights of slurry and water, respectively; and H_s and H_w are the heights of the slurry surface and groundwater table, respectively, as illustrated in Fig. 2a.

P_a is calculated by Rankine's theory of earth pressure. The active earth pressure, which implies a lower bound plasticity solution, corresponds to the surrounding soils being at the state of plastic equilibrium, and can be written as follows,

$$p_{ai}^{T} = \sum_{k=1}^{k=i-1} \gamma_k h_k K_{ai} - 2c_i\sqrt{K_{ai}} + qK_{ai} \tag{5}$$

$$p_{ai}^{B} = \sum_{k=1}^{k=i} \gamma_k h_k K_{ai} - 2c_i\sqrt{K_{ai}} + qK_{ai} \tag{6}$$

$$K_{ai} = \tan^2\left(45^{\circ} - \phi_i / 2\right) \tag{7}$$

where p_{ai}^{T} and p_{ai}^{B} are the active earth pressures at the top and the bottom, respectively, of the ith soil layer; K_{ai} is the active earth pressure coefficient of the ith soil layer; q is a uniformly distributed surcharge; c_i and ϕ_i are the cohesion intercept and internal friction angle, respectively, of the ith soil layer; γ_k and h_k are the unit weight and thickness, respectively, of the kth soil layer. For the soil layers below the groundwater table the effective unit weight and the effective shear strength parameters are used. A soil layer intersected by the presence of the ground water table should be divided into two layers. It is noted that at the interface between soil layers the active earth pressure at the bottom of the upper soil layer may be not equal to that at the top of lower soil layer if the two soil layers have different properties parameters. Tension cracks are likely to develop near the surface and the part of the pressure distribution should be neglected if $qK_{a1} - 2c_1\sqrt{K_{a1}} < 0$ as illustrated in Fig. 2b. P_a can then be written as follows,

$$P_a = \sum_{i=1}^{i=n} \frac{\left(p_{ai}^{T} + p_{ai}^{B}\right)h_i}{2} \tag{8}$$

where n is the total number of the layered soils corresponding to the depth of the trench considered. For the scenario with layered soils, in contrast to the force equilibrium analysis of the horizontal slice method (Li, et al. 2013), P_a can be calculated by hand in the proposed method. It is noted that if the tension crack is filled with water the hydrostatic pressure must be considered (Barnes 2011).

As Rankine's theory is applied in this paper, the stress on the filter cake caused by varied distributed surcharges (Das 1998) can be taken into consideration in the stability analysis of slurry trenches. For the scenario where a slope is near the excavated slurry trench (see Fig. 3), the additional stress on the filter cake caused by the slope, Δp, can be calculated as follows (MOHURD 2012),

$$\Delta p=\begin{cases} 0 & \text{for} \quad z<a/\tan\theta \\ K_{ai}\dfrac{\gamma h}{b}(z-a)+K_{ai}\dfrac{E_a(a+b-z)}{b^2K_a} & \text{for} \quad a/\tan\theta\le z\le(a+b)/\tan\theta \\ K_{ai}\gamma h & \text{for} \quad z\ge(a+b)/\tan\theta \end{cases} \tag{9}$$

$$E_a=\frac{1}{2}\gamma h^2K_a-2ch\sqrt{K_a}+\frac{2c^2}{\gamma} \tag{10}$$

where z is the depth from the trench surface; a and b are the horizontal distance from the slope toe to the trench side wall and that of the slope face, respectively, as shown in Fig. 3; θ is the spread angle and $\pi/4$ is recommended; h is the height of the slope; γ and c are the unit weight and cohesion intercept, respectively, of slope soil; E_a is the active earth pressure caused by the weight of the sloping part soil; and K_a is the active earth pressure coefficient of the slope soil using Eq. (7) with the cohesion intercept c and the internal friction angle ϕ of slope soil. The weighted average values of γ, c and K_a are used if the slope consists of varied layers.

For a typical case, the trench is in the most critical condition and has the minimum F_s when the trench is fully excavated to the designed depth. However, following the work of Li et al. (2013) it is recommended the values of F_s be calculated for the scenarios that the trench is excavated to the bottoms of each soil layer, especially for layers with low shear strength parameters in design.

Verification and Investigation

In this section, the proposed method is verified via analysis of a slurry trench stability problem defined by Filz et al. (2004) and Fox (2006). The trench is 20 m deep and the groundwater table is 3 m below the ground surface (H_w=17 m). The property parameters for the soil layers above and below the groundwater table are: γ_1=19 kN/m^3, c_1=0, ϕ_1=37°; and γ_2=20 kN/m^3 c_2'=0, and ϕ_2'=37°. The slurry surface is maintained at the ground surface (H_s=20 m) and the unit weight of slurry is 11.8 kN/m^3. No surcharge pressure is applied on the ground surface (q=0). The unit weight of groundwater used in the calculation is 10 kN/m^3. The following calculations can be made using Eqs. (1), (3)~(8),

$$P_s = \frac{1}{2} \times 11.8 \times 20^2 = 2360.0 \text{ kPa/m} \quad (11)$$

$$P_w = \frac{1}{2} \times 10 \times 17^2 = 1445.0 \text{ kPa/m} \quad (12)$$

$$K_{a1} = K_{a2} = \tan^2\left(45^\circ - 37^\circ/2\right) = 0.249 \quad (13)$$

$$p_{a1}^T = 0.0 \text{ kPa/m}^2 \quad (14)$$

$$p_{a1}^B = p_{a2}^T = \left(19 \times 3\right) \times 0.249 = 14.2 \text{ kPa/m}^2 \quad (15)$$

$$p_{a2}^B = \left(19 \times 3 + 10 \times 17\right) \times 0.249 = 56.5 \text{ kPa/m}^2 \quad (16)$$

$$P_a = \frac{\left(0.0 + 14.2\right) \times 3}{2} + \frac{\left(14.2 + 56.5\right) \times 17}{2} = 622.2 \text{ kPa/m} \quad (17)$$

$$F_s = \frac{2360.0 - 1445.0}{622.2} = 1.47 \quad (18)$$

As shown above, hand calculation can be performed for the stability analysis of slurry trenches when the proposed approach is used. The F_s obtained by the proposed method is 1.47, which is the same as reported by Filz et al. (2004) and Fox (2006) using Coulomb force equilibrium methods. Additional calculations on other published examples, that is, the example with full tension cracks of Fox (2004) and the example with β=0° of Li et al. (2013), were also performed and the results confirm that the proposed approach gives identical values of F_s to those obtained by Coulomb force equilibrium methods. The sliding resistance term, $P_s - P_w$, is implicitly considered in the equations of the Coulomb force equilibrium methods (see Eq. (2a) in Filz, et al. (2004) and Appendix) and is regarded as the sliding resistance in the proposed definition of F_s (see Eq. (1)) The same expression in terms of P_s and P_w used in these two methods results in an identical value of F_s. However, the definition of F_s presented by Xanthakos (1994) in Eq. (2) gives

$$F_s = \frac{2360.0}{622.2 + 1445.0} = 1.14 \tag{19}$$

which is much lower than that reported by Filz et al. (2004) and Fox (2006). In this definition, the filter cake is in effect regarded as a fully impermeable layer, which causes a hydraulic discontinuity between the two sides of the filter cake. This definition is similar to the factor of safety against sliding of gravity retaining walls with the sliding resistance between the wall base and the soil beneath replaced by the thrust exerted by the slurry on to the filter cake.

Based on the problem above, the scenario with a nearby slope is considered. The horizontal distance from the slope toe to the trench side wall is 2 m (a=2 m). The inclination of the slope is 45° (that is, b=h). The cohesion intercept and internal friction angle of the slope soil are 5 kPa and 30°, respectively (c=5 kPa and ϕ=30°). The unit weight of slope soil is 18 kN/m^3 (γ=18 kN/m^3). The additional stress on the filter cake caused by the slope with a height from 0.0 m to 5.0 m is calculated with θ=π/4 used. For example, when h=2.0 m,

$$\Delta p = \begin{cases} 0 \ \text{kPa} & \text{for} \quad z < 2\text{m} \\ 3.88z - 6.55 \ \text{kPa} & \text{for} \quad 2\text{m} \le z \le 4\text{m} \\ 8.96 \ \text{kPa} & \text{for} \quad \text{for} \quad z \ge 4\text{m} \end{cases} \tag{20}$$

The total additional force on the filter cake exerted by the slope, ΔP, is

$$\Delta P = \frac{1.21+8.96}{2} \times 2 + 8.96 \times 16 = 153.5 \ \text{kPa} \tag{21}$$

With a consideration of the additional force exerted by the slope, the factor of safety of the slurry trench defined by Eq. (1) is

$$F_s = \frac{2360.0-1445.0}{622.2+153.5} = 1.18 \tag{22}$$

Fig. 4 shows the relationship between the factor of safety (F_s) and the height of slope (h) for h=0.0 m to 5.0 m. The value of F_s defined by Eq. (1) is reduced from 1.47 to 1.1, which is typically the criterion for geotechnical structures in short-term condition, as h increases from 0.0 m to 2.75 m and becomes less than 1.0 when h>4.0 m. From the calculation of the scenarios considered, it can be seen that the stability of slurry trench is sensitive to the surcharge from a nearby slope. Neglect of the additional stress from the nearby slope results in non-conservative F_s in stability analysis of slurry trenches. Fig. 4 also gives the relationship between F_s defined by Eq. (2) and h. The value of F_s is less than that defined by Eq. (1), which indicates it is relatively conservative, when $F_s>1.0$; however, it turns to be greater than that defined by Eq. (1) when $F_s<1.0$; the values of F_s defined by Eqs. (1) and (2) are equal when $F_s=1.0$, which corresponds to $P_s=P_a+P_w$.

Conclusions

This paper presents an approach for stability analysis of slurry trenches based on the Rankine's theory of active earth pressure. The scenarios with soil stratification and varied distributed surcharges can be analyzed via hand calculation when the presented approach is used. The verification example shows the proposed definition of the factor of safety gives identical value of F_s with the Coulomb force equilibrium methods. Further investigation indicates the stability of slurry trench is sensitive to the surcharge from a nearby slope, which requires attention in design. It is noted that in the presented approach a two-dimensional analysis is performed and so the side force on the sliding mass is not included, and so yielding a conservative result.

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Appendix

The Coulomb's force equilibrium method is recast herein to demonstrate the term $P_s - P_w$ in the calculation of the factor of safety for slurry trenches.

As illustrated in Fig. 5, the limit equilibrium of the failure wedge is considered. The angle of the trial failure plane from the horizontal is α. The force on the hydrostatic pressure on the trial failure plane U can be decomposed into U_x and U_y, which are respectively,

$$U_x = \frac{1}{2}\gamma_w H_w \frac{H_w}{\sin\alpha}\sin\alpha = \frac{1}{2}\gamma_w {H_w}^2 = P_w \tag{23}$$

$$U_y = \frac{1}{2}\gamma_w H_w \frac{H_w}{\sin\alpha}\cos\alpha = \frac{1}{2}\gamma_w {H_w}^2 \cot\alpha \tag{24}$$

It is noted that U_x=P_w in Eq. (23). Summing force components for the failure wedge in the horizontal and vertical directions,

$$P_s + S\cos\alpha = U_x + N'\sin\alpha \tag{25}$$

$$W = U_y + N'\cos\alpha + S\sin\alpha \tag{26}$$

where S is the shear force on the failure plane; N' is the effective normal force on the failure plane and W is the weight of the failure wedge. It can be observed that U_y is equal to the buoyant force on the failure wedge, so we define W' as follows,

$$W' = W - U_y \tag{27}$$

where W' is calculated with the buoyant weight used for the portion of the failure wedge below the groundwater table. The shear force on the failure plane S is calculated as

$$S = \frac{C + N'\tan\phi'}{F_s} \tag{28}$$

where C is the total cohesion force on the failure plane.

The following equation can be obtained using Eqs. (23), (25)~(28),

$$\frac{\cos\alpha+\frac{\tan\phi'}{F_\mathrm{s}}\sin\alpha}{\sin\alpha-\frac{\tan\phi'}{F_\mathrm{s}}\cos\alpha}\left(P_\mathrm{s}-P_\mathrm{w}\right)+\left(\frac{\cos\alpha+\frac{\tan\phi'}{F_\mathrm{s}}\sin\alpha}{\sin\alpha-\frac{\tan\phi'}{F_\mathrm{s}}\cos\alpha}\frac{\cos\alpha}{F_\mathrm{s}}+\frac{\sin\alpha}{F_\mathrm{s}}\right)C-W'=0 \quad (29)$$

The factor of safety F_s is implicitly included in Eq. (29) and can be calculated using mathematic or numerical methods. The minimum F_s should be found by searching for the critical inclination angle of α. It can be observed that P_s and P_w in Eq. (29) are only shown by the term $P_\mathrm{s}-P_\mathrm{w}$. It is noted that the search of the critical inclination angle of failure wedge is not required in the proposed approach.

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List of Figure Captions

Fig. 1 Cases of shallow trench failure and large deformation in nearby landfill berm induced by slurry trench excavation: (a) Case 1, Shallow trench failure; (b) Case 2, Cracks in nearby landfill berm.

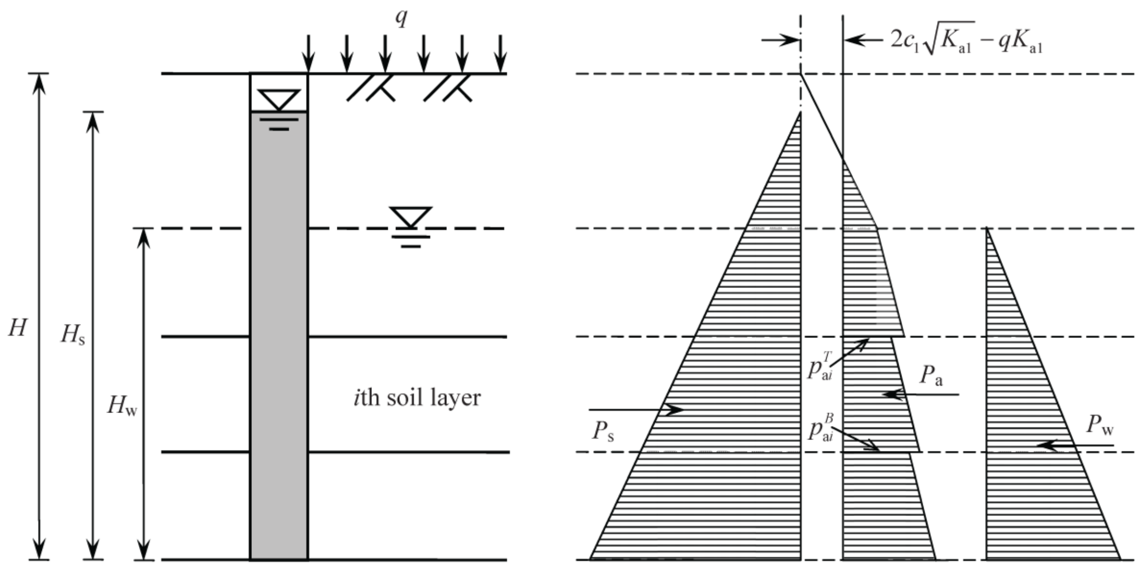
Fig. 2 Configuration of a slurry trench and pressures on ‘filter cake’.

Fig. 3 Lateral earth pressure caused by a nearby slope on the surface.

Fig. 4 Relationship between the factor of safety and the height of nearby slope.

Fig. 5 Configuration of a slurry trench for Coulomb’s force equilibrium method.


Longitudinal crack on landfill berm surface
Transverse cracks on landfill berm surface at turning cover
Slurry trench
Landfill berm



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$2c_1\sqrt{K_{a1}} - qK_{a1}$
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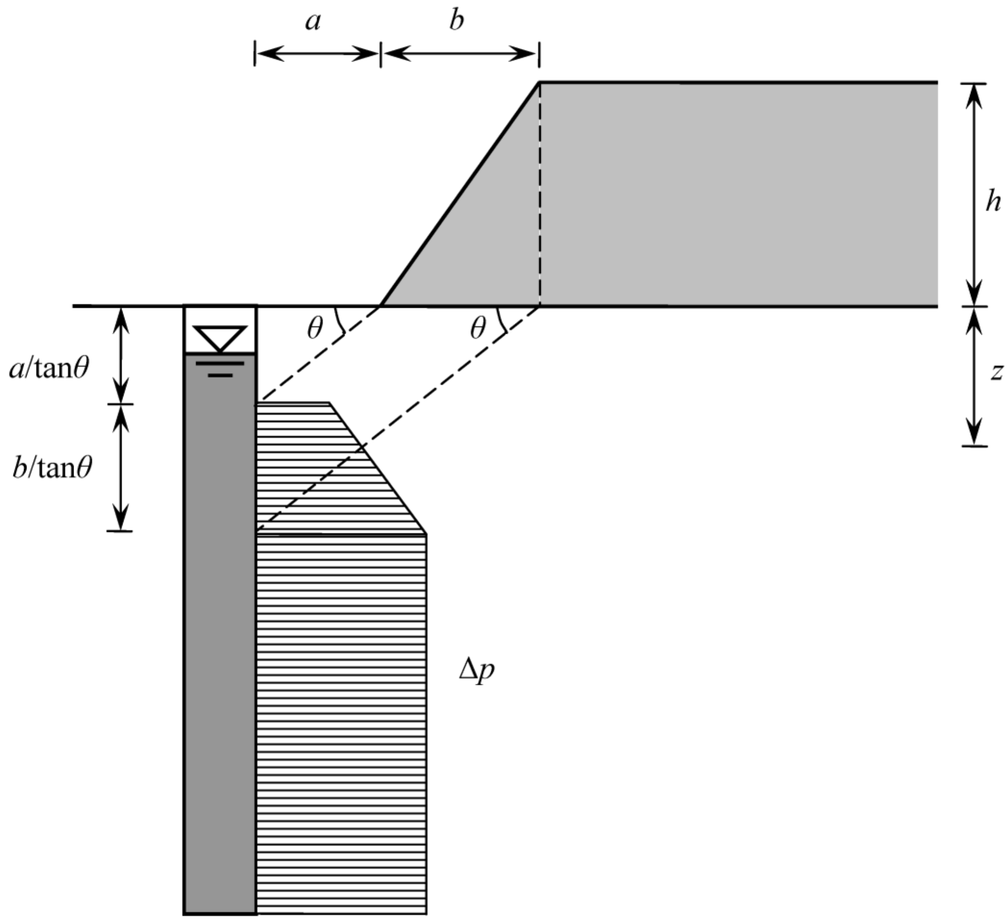
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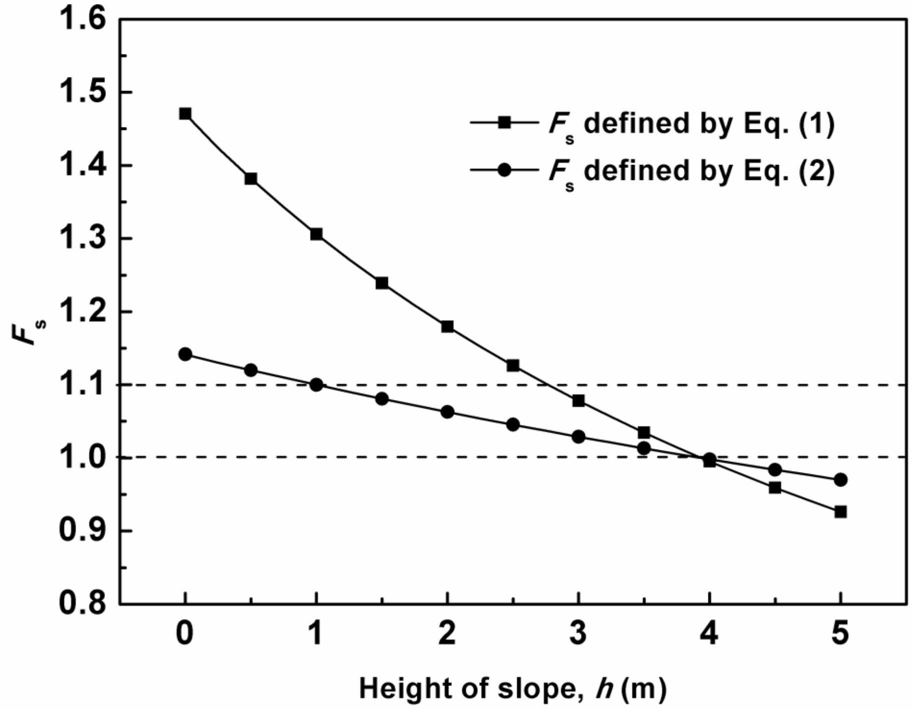

1.6
1.5
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1.1
1.0
0.9
0.8
F_s
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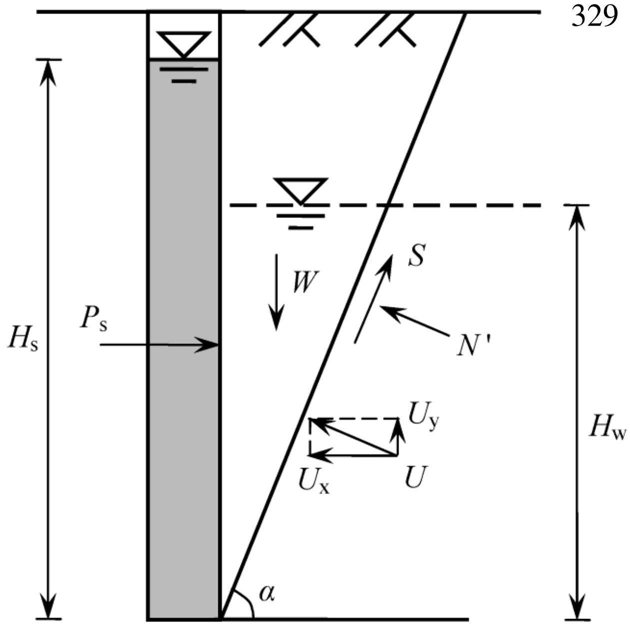
3
4
5
Height of slope, h (m)
F_s defined by Eq. (1)
F_s defined by Eq. (2)



H_s
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