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A Charging Pricing Strategy of Electric Vehicle Fast Charging Stations for the Voltage Control of Electricity Distribution Networks

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Abstract: With the increasing number of electric vehicles (EVs), the EV fast charging load will significantly affect the voltage quality of electricity distribution networks. On the other hand, EVs have potentials to change the choices of charging locations due to the incentives from the variations of charging prices, which can be considered as a flexible response resource for electricity distribution networks. In this paper, a charging pricing strategy of EV fast charging stations (FCSs) was developed to determine the pricing scheme for the voltage control of electricity distribution networks, which consisted of a simulation model of EV mobility and a double-layer optimization model. Considering the travel characteristics of users, the simulation model of EV mobility was developed to accurately determine the fast charging demand. Taking the total income of FCSs and the users’ response to the pricing scheme into account, the double-layer optimization model was developed to optimize the charging pricing scheme and minimize the total voltage magnitude deviation of distribution networks. A test case was used to verify the proposed strategy. The results show that the spatial distribution of EV fast charging loads was reallocated by the proposed charging pricing scheme. It can also be seen that the proposed strategy can make full use of the response capacity from EVs to improve the voltage profiles without decreasing the income of the FCSs.

The short version of the paper was presented at ICAE2017, Aug 21-24, Cardiff, UK. This paper is a substantial extension of the short version of the conference paper.
Keywords: Electric vehicle (EV); Electric vehicle mobility; Charging pricing strategy; Voltage control of electricity distribution networks

1. Introduction

With the growing concerns on the energy depletion and environmental issues around the world, the large-scale adoption of electric vehicles (EVs) is considered as an effective way in decarbonizing the transport sector. In recent years, the EV industry has made considerable progress with the great promotion from governments and automobile enterprises [1]. As the EV supply equipment, the charging infrastructure plays a crucial role in the EVs promotion [2]. With respect to the emergency charging of EVs, the fast charging station (FCS) is becoming the mainstream solution [3]. However, from the view point of electricity distribution networks, the fast charging load will cause the deterioration of voltage quality due to the short charging period and high power demand [4]. Thus, it is necessary to regulate the charging behaviors of EVs so as to improve the voltage quality of electricity distribution networks.

One way to support the operation of distribution networks is the direct control of EV charging load, due to the EVs’ flexibility in the charging time and the vehicle-to-grid (V2G) capability [5, 6]. In [7], a hierarchical coordinated charging framework was proposed to generate the charging curve for each aggregator of EVs in order to reduce the peak load of EV charging. In [8], the capacitor, the on-load tap changer and the EV chargers were coordinated to control the voltage of electricity distribution networks. In [9], the on-load tap changers and EVs were collaborated to mitigate the voltage fluctuations caused by generation variations of distributed solar panels. In [10], a high efficient valley-filling strategy was proposed to determine the charging priority of EVs at each time slot. In [11], EV charging loads were separately scheduled by changing the charging times and locations. In [12] , the operation of EV charging behavior was optimized by changing the charging time. In [13], the EV charging scheduling strategy of an aggregator was proposed by regulating the charging power in the charging process. In [14], the charging EV number in a certain period was calculated with the goals of peak-shaving and valley-filling. In [15], a double-layer smart charging strategy was developed. The first layer aims to determine the shortest path for EV users to reach a suitable charger. The second level controls the charging process in order to reduce the charging cost.
The above methods focus on adjusting the battery charging process of EVs. With the development of intelligent transportation systems [16], information and communication technology [17] and fast charging navigation system [18], the price mechanisms were applied to guide the EV charging behaviors.

In [19] and [20], the modeling of the EV driver’s response to the charging price was discussed and the EV charging loads were shifted to the valley time period. In [21], the effect of prices on the fast charging behavior of EV users was analyzed. In [22], a proper charging pricing mechanism was designed to guide the EVs’ charging behaviors. In [23], the load balancing of FCSs was achieved through a pricing mechanism, considering the quality-of-service targets and the spatial-temporal distribution of EVs. In [24], the fluctuation of renewable energy sources was balanced by adjusting the mobility behavior of EVs with the variations of price signals. The variable electricity prices are calculated based on marginal generation costs. In [25] and [26], it was assumed that the electricity was sold at the wholesale price to the EV users, ignoring FCS interests. And the electricity prices were optimized at the system level considering the operation of the power system and transportation system.

The existing researches have made good contributions to the optimization of EV fast charging load by the price incentives. The FCSs trend to privately-owned facilities [27, 28] and collaborate with distribution networks. Although the charging pricing scheme of FCSs can be applied to improve the voltage quality of distribution networks, the profit of FCSs should be guaranteed when the loads are redistributed through the charging pricing scheme. For this reason, a charging pricing strategy of EV FCSs was proposed to minimize the total voltage magnitude deviation of distribution networks. The charging pricing scheme can be determined to minimize the total voltage magnitude deviation without decreasing the income of FCSs.

2. Framework of the proposed charging pricing strategy

The framework of the proposed charging pricing strategy is shown in Fig. 1, which consists of a simulation model of EV mobility and a double-layer optimization model.

**The simulation model of EV mobility:** The travel chain method [29], graph theory [30] and the Monte Carlo Simulation (MCS) are used to determine the EV fast charging demand considering the travel characteristics of users. The demand is transferred to the lower layer.
The lower-layer optimization model: According to the fast charging demand supplied by the simulation model and a given charging pricing scheme supplied by the upper layer, the selected FCS of each user is optimized to minimize the corresponding cost. The loads of FCSs and the EV recharging capacity of each user are determined and then transferred to the upper layer.

The upper-layer optimization model: The charging pricing scheme of FCSs is generated and optimized based on the charging loads of FCSs and the EV recharging capacity of each user supplied by the lower layer. The scheme is then transferred to the lower layer.

The double-layer optimization model

The upper layer
Objective: minimize the total voltage deviation of distribution networks
Decision variables: charging prices of the FCSs

The lower layer
Objective: minimize the corresponding cost
Decision variables: the selected FCS
Determinations: the EV recharging capacity of each user
the charging loads of FCSs

The simulation model of EV mobility
(trip chain method, graph theory and MCS method)

3. The charging pricing strategy
3.1 The simulation model of EV mobility
The EV fast charging demand \( FC \) was predicted by the simulation model of EV mobility, considering the travel characteristics of users and the existing slow charging facilities in the urban area. \( FC \) was then transferred to the lower-layer optimization model.
1) Transportation network model

The extended graph is employed to describe the topology of the transportation network [30]. A graph $G$ is an ordered pair, which consists of a set of vertices $V$ connected by a set of edges $E$. The vertices represent the nodes of the transportation network, while the edges represent the arterial roads and their flow direction. It is assumed that FCSs are built on the arterial roads to avoid the traffic jams. The extended graph includes the virtual vertices representing FCSs and the corresponding edges.

The distance matrix $D$ is used to describe the distances between every two neighbor vertices of the extended graph. $D$ is a $N_v \times N_v$ symmetric matrix and all diagonal elements are zero, where $N_v$ is the total number of vertices in the extended graph and the element $D(v_i, v_j)$ represents the distance from vertex $v_i$ to vertex $v_j$.

The impedance matrix $IM$ is used to describe the driving time between every two neighbor vertices of the extended graph considering the traffic congestions. $IM$ is determined by $D$ and the average driving speed obtained from the history data of the traffic center. $IM$ is a $Q_1 \times N_v \times N_v$ matrix, where $Q_1$ is the number of time intervals. And the element $IM(t_i, v_i, v_j)$ of $IM$ represents the driving time from vertex $i$ to $j$ at the time interval $t_i$.

2) EV mobility model

The EV mobility is closely related with the travel characteristics of users, which is well described by the trip chain [29]. The concept of the trip chain has been widely applied in the travel demand forecast [31][32]. A trip chain is a time-ordered trip sequence which consists of locations and routes of daily trips. This chain can reflect the rules of user’s activities in space. In this paper, only the private EVs are considered to forecast their fast charging demand, because other kinds of EVs (such as buses, enterprise owned vehicles, etc.) generally have the proprietary charging stations. Also the charging choices of these EVs used for public services are not easy to be changed. The activities of the private EVs are a series of movements and stops describing by the trip chain theory.

The trip chain is composed of a spatial chain and a temporal chain.

It is supposed that the battery capacity consumption is linearly dependent on the real driving distance [33]. An energy chain is developed to describe the variations of available battery capacity of EVs with the moments as shown in Fig. 2, based on the trip chain theory. The dotted lines represent travel behaviors and the solid lines indicate parking behaviors. The variables of the spatial chain, the temporal chain and the energy chain for the EV $j$ are listed in Table I.
Table 1. The variables of the spatial chain, the temporal chain and the energy chain for the EV

<table>
<thead>
<tr>
<th>The variables of the spatial chain</th>
<th>The variables of the temporal chain</th>
<th>The variables of the energy chain</th>
</tr>
</thead>
<tbody>
<tr>
<td>s(0)</td>
<td>The type of the start location of a daily trip chain</td>
<td>ts/</td>
</tr>
<tr>
<td>s(k)</td>
<td>The type of the k\textsuperscript{th} stop</td>
<td>t/</td>
</tr>
<tr>
<td>d/</td>
<td>The driving distance of the k\textsuperscript{th} trip</td>
<td>tr/</td>
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<td></td>
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<td>ts/</td>
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<td>ΔC/</td>
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<tr>
<td></td>
<td></td>
<td>RC/</td>
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</tbody>
</table>

Fig. 2. Schematic diagram of the energy chain

The \( TY=\{ ty_m | m=1,2,\ldots,Q_2 \} \) represents the set of the stop types, where \( Q_2 \) is the total number of stop types. It is assumed that the driving distances on the minor roads are ignored and the stops are located at the nodes of the transportation network. The stop type and weight of each node in the transportation network are predefined.

The conditional probability \( TP \) is used to describe the transition probability from the stop type \( ty_v \) to stop type \( ty_w \), which are determined by the National Household Trip Survey (NHTS) data. \( TP \) is a \( Q_3 \times Q_2 \times Q_2 \) matrix, where \( Q_3 \) is the number of time intervals. According to the given time interval \( t_2 \), \( TP \) is depicted in (1).

\[
TP(ty_v, ty_w | t_2) = \begin{bmatrix}
TP(ty_v, ty_v | t_2) & \cdots & TP(ty_v, ty_w | t_2) \\
\vdots & \ddots & \vdots \\
TP(ty_v, ty_v | t_2) & \cdots & TP(ty_v, ty_w | t_2)
\end{bmatrix}
\]

(1)

where \( TP(ty_v, ty_w | t_2) \) represents the probability that a user transfers from the type \( ty_v \) to type \( ty_w \) at the time interval \( t_2 \). And the sum of probabilities of each row in (1) is equal to 1.

3) Numerical implementation

The following assumptions are adopted in this paper:
1. The EV users, especially the risk-averse ones, reserve a safety margin $RC_i$, to hedge against running out the power capacity.

2. Only fast charging and slow charging with constant power are considered in this paper for the private EV users. It is assumed that all the stops are equipped with enough slow chargers.

3. The EV users will choose the fast charging mode only in emergency.

4. It is assumed that an EV needs at most one fast charging for a day. And the status “stay at home” indicates the travel is finished for the whole day.

The simulation flowchart for the EV fast charging demand is shown in Fig. 3.

Step 1: Set $n=1$;

Step 2: Set $j=1$;

Step 3: $N_{EV}$ is the EV number. If $j \leq N_{EV}$, go to Step 5. Otherwise, $n=n+1$, go to Step 4;

Step 4: $N_d$ is the number of typical days. If $n \leq N_d$, go back to Step 2. Otherwise, save and output the fast charging demand $FC$;

Step 5: Generate and determine the initial parameters of the EV $j$; set $k=1$;

1) Generate $t_{s0}^j$ based on the probability distributions of $t_{s0}$ determined by the NHTS data [34];

2) Generate $RC^j$ for the EV $j$;

3) Determine the battery rated capacity $Cap^j$ of EV $j$ and the power consumption $e$ under the urban dynamometer driving schedule;

4) Determine $RC^j_0$ based on the (2).

$$RC^j_0 = SOC^j_0 \times Cap^j$$ (2)

where $SOC^j$ is the initial state of charge for the EV $j$; it is assumed that the initial state of charge ($SOC$) varies in the range of [0.8, 0.9], considering the factors such as the battery safety and users’ psychology [35][36].

Step 6: Generate $s(k)$ based on $TP$, $s(k-1)$ and $ts_{k-1}^i$;

Step 7: Determine the stop, $d^j$, $t^j$ and $tr^j$;

1) Determine the stop based on $s(k)$ and weights of the transportation network nodes;

2) The travel paths are determined by the modified Floyd algorithm [37] to minimize the driving time of the trip based on $IM$ and $ts_{k-1}^i$. Thus, $d^j$ and $t^j$ are determined based on the travel paths and $D$. 

3) Determine $tr_{kj}$ based on the probability distribution of $tr_{ij}$ according to the NHTS data.

Step 8: Determine $\Delta CE_{kj}$ based on the (3);

$$\Delta CE_{kj} = e \times d_{ij}^j$$ (3)

Step 9: If $RC_{kj} < (\Delta CE_{kj} + RC_{kj})$, update the fast charging demand $FC$. That is $j$, the stop $k$-1, the stop $k$, $RC_{kj}$, and $RC_{kj}$ are recorded to the matrix $FC$. And set $j=j+1$ and go back to Step 5. Otherwise, go to Step 10;

Step 10: Determine $ta_{kj}$ and $ts_{kj}$ based on the (4) and (5).

$$ta_{kj} = ts_{kj} + tr_{kj}^j$$ (4)

$$ts_{kj} = ta_{kj} + tr_{kj}^j$$ (5)

Step 11: Determine $\Delta C_{kj}$, $RC_{kj}$, and $RC_{kj}$ based on the (6), (7) and (8).

$$\Delta C_{kj} = \begin{cases} \delta_k (RC_{kj} - RC_{kj}) & (RC_{kj} - RC_{kj}) \leq \eta tr_{kj}^j p_{rate}^{slow} \\ \delta_k tr_{kj}^j p_{rate}^{slow} & (RC_{kj} - RC_{kj}) > \eta tr_{kj}^j p_{rate}^{slow} \end{cases}$$ (6)

$$RC_{kj} = RC_{kj} + \Delta C_{kj}$$ (7)

Step 12: If $s(k)$ is “stay at home”, set $j=j+1$ and go back to Step 5. Otherwise, set $k=k+1$ and go back to Step 6.
3.2 The lower-layer optimization model

The lower-layer optimization model optimizes the selections of users and minimizes the corresponding cost considering the users’ response in a given charging pricing scheme from the upper-layer optimization model. The EV recharging capacity for each user and the charging loads of FCSs are determined and then transferred to the upper-layer optimization model.

1) The detour to charge the EV battery

An EV user is assumed to make a detour for a charge when the EV battery runs out of power before reaching the next stop or the destination. For example, when an EV \( j \) needs the fast charging from the \((k-1)\)th stop to the \(k\)th stop, it will make a detour to the FCS \( i \), as shown in Fig. 4. The stop \( k-1 \), the stop \( k \), \( RC_{k-1} \), and \( RC_k \) were obtained from \( FC \).
In Fig. 4, $t_{k-1}$ is the driving time from the $(k-1)^{th}$ stop to the FCS $i$; $t_{k}$ is the driving time from the FCS $i$ to the $k^{th}$ stop; $d_{k-1}$ is the driving distance from the $(k-1)^{th}$ stop to the FCS $i$; $d_{k}$ is the driving distance from the FCS $i$ to the $k^{th}$ stop; $t_{k-1}$, $t_{k}$, $d_{k-1}$ and $d_{k}$ are determined by $D$, $IM$ and the modified Floyd algorithm.

**Spatial chain**

![Spatial Chain Diagram](image)

**Temporal chain**

![Temporal Chain Diagram](image)

**Energy chain**

![Energy Chain Diagram](image)

Fig. 4. The chains’ diagram of the trip when an EV $j$ needs the fast charging between the $(k-1)^{th}$ stop and the $k^{th}$ stop

$tr_{i}$ is the dwell time at the FCS $i$, as depicted in (9). $ta_{i}$ is the time arriving at the FCS $i$, as depicted in (10). $ts_{i}$ is the time leaving the FCS $i$, as depicted in (11). $t_{k}$ is the total travel time of the trip from the $(k-1)^{th}$ stop to the $k^{th}$ stop, as depicted in (12).

\[
tr_{i} = tw_{i} + tc_{i} \quad (9)
\]

\[
ta_{i} = ts_{i-1} + t_{i-1} \quad (10)
\]

\[
rs_{i} = ta_{i} + tc_{i} + tw_{i} \quad (11)
\]

\[
t_{k} = t_{i-1} + t_{i} + tr_{i} \quad (12)
\]

For the EV $j$, $tc_{i}$ is the charging time at the FCS $i$, as depicted in (13). It is assumed that the available capacity after fast charging is $RC_{i}$. And $\Delta FC_{i}$ is the recharging capacity at the FCS $i$, as depicted in (14). $\eta_{2}$ is the fast charging efficiency. $P_{rate^{fcs}}$ is the rated power of the fast charger.

\[
tc_{i} = \Delta FC_{i} / (\eta_{2} \times P_{rate^{fcs}}) \quad (13)
\]

\[
\Delta FC_{i} = RC_{i} - \left( RC_{i-1} \times d_{i-1} \times \epsilon \right) \quad (14)
\]

2) **Determine the waiting time**

The activities of the EV $j$ at the FCS are shown in Fig. 5. It is assumed that the EVs are served based on a first-
come first-served rule. The arrival and departure of the EVs at the FCS are shown in Fig. 6 which is taken as an example. Before the time $m_1$, the accumulated amounts of EV arrival and departure are 4 and 1, respectively. If there are only 2 chargers available at the FCS, the EV $j_1$ is queuing to wait for charging. At the time $l_1$, the accumulated amount of EV departure is 3. So no EVs are queuing before the EV $j_1$ and there is an idle charger at the time $l_1$. The EV $j_1$ will start to charge at the time $l_1$ and its waiting time is $l_1 - m_1$.

Based on the chronological order, the two sets ($A$ and $B$) are used to record the arrival and departure time of the EVs at the FCS, respectively. $\text{size}(A(t))$ and $\text{size}(B(t))$ are the accumulated amounts of EV arrival and departure at the FCS before the time $t$. $\text{nev}(i,t)$ is the existing EV number at the FCS $i$ at the time $t$, which is depicted in (15). When $\text{nev}(i,ta_i) < C_i$ of the FCS $i$, the waiting time $tw_i$ for the EV $j$ at the FCS $i$ is equal to 0. Otherwise, $tw_i$ is the minimum time $t$ satisfying the constraint, as depicted in (16).

$$\text{nev}(i,t) = \text{Size}(A(t)) - \text{Size}(B(t))$$ (15)

$$\text{Size}(B(ta_i) + t) + C_i > \text{Size}(A(ta_i)) \quad (ta_i + t) \in B$$ (16)

3) Determine the selections for electric vehicle users

$\Omega_c$ is the set of optional FCSs for the EV $j$, as depicted in (17), based on the fast charging demand obtained by the simulation model of EV mobility. The EV $j$ can reach the optional FCS $i$ supported by $RC_{i,k}$. And $tw_i$ is less than the threshold $thr_i$ at the optional FCS $i$.

$$\Omega_c = \{i | \Delta CE_{i,k}^{l-1} + RC_{i,k}^{l-1} \leq RC_{i,k}^{l} \& \& tw_i < thr_i, i \in \Omega\}$$ (17)

where $\Omega$ is the set of the FCS serial numbers, namely $\Omega = \{1,2,3...,N_{fcs}\}$; $N_{fcs}$ is the total number of FCSs.
When the EV needs a fast charging, the corresponding user selects the FCS due to different cost priorities. To highlight these priorities, the following three types for the EV are introduced.

**Type I:** Selection with the minimum charging cost \((f_c)\), as depicted in (18).

**Type II:** Selection with the minimum total travel time cost \((f_t)\), as depicted in (19).

**Type III:** Selection with the minimum total cost \((f)\), as depicted in (20).

\[
\min f_c^i = \Delta FC_j \times cp_i, \quad i \in \Omega
\]  

(18)

\[
\min f_t^i = f_t^i \times dc, \quad i \in \Omega
\]  

(19)

\[
\min f_j^i = f_c^i + f_t^i, \quad i \in \Omega
\]  

(20)

where \(cp_i\) is the fast charging price of the FCS \(i\) obtained by the upper-layer optimization model.

The selection is optimized to minimize the corresponding cost solved by the traversal method in the \(\Omega\). According to the (14) and (15), \(\Delta FC\) and \(nev(i,t)\) are determined and transferred to the upper-layer optimization model. \(P_{i,fcs}\) is the average fast charging load of the FCS \(i\) at the time interval \(t\) as depicted in (21), which is transferred to the upper-layer optimization model.

\[
P_{i,fcs} = \begin{cases} 
\frac{1}{N_d} nev(i,t) \times P_{fcs}^{rate} & nev(i,t) \leq C_i \times N_d \\
\frac{1}{N_d} C_i \times P_{fcs}^{rate} & nev(i,t) > C_i \times N_d
\end{cases}
\]  

(21)

### 3.3 The upper-layer optimization model

The voltage magnitude deviation index developed in [38] was utilized and depicted in (22). \(NVD_{n,t}\) represents the voltage magnitude deviation of the node \(n\) at the time interval \(t\). \(U_{n,t}\) is the voltage magnitude of the node \(n\) at the time interval \(t\) due to fast charging load. \(U_{n,p}\) is the voltage standard value of the node \(n\).

\[
NVD_{n,t} = \|U_{n,t} - U_{n,p}\|
\]  

(22)

It is supposed that FCSs collaborate with distribution networks and the charging prices of FCSs are fixed for a day. The initial charging prices of FCSs are \(cp_0\) and the same. Thus, the fast charging prices of FCSs are optimized to minimize the total voltage magnitude deviation of the distribution networks \((TNVD)\), as depicted in...
Meanwhile, the total income of the FCSs remains unchanged before and after the optimization as depicted in (24).

\[
\min \; TNVD = \sum_{n=1}^{N} \sum_{t=1}^{T} NVD_{n,t} \tag{23}
\]

\[
= \sum_{i=1}^{N} c_{p_i} \times \sum_{j \in \Omega(i,c_{p_j})} \Delta FC_i = \sum_{i=1}^{N} c_{p_0} \times \sum_{j \in \Omega(i,c_{p_0})} \Delta FC_i \tag{24}
\]

where \( N_D \) is the node number in the distribution networks; \( T \) is the number of time intervals for a day; \( \Omega(i,c_{p_j}) \) and \( \Omega(i,c_{p_0}) \) are the sets of the users selecting the FCS \( i \) with the charging price \( c_{p_j} \) and the \( c_{p_0} \), respectively.

In the model, the following constraints are considered:

1) Upper and lower boundary constraints of the \( c_{p_i} \)

To ensure the profit of the FCS, the lower boundary of the charging price should be larger than the electricity price \( c_{p_{min}} \) of the distribution network and the upper boundary of the charging price should be less than the fuel cost converted to the same mileage \( c_{p_{max}} \). The constraints are depicted in (25).

\[
c_{p_{min}} \leq c_{p_i} \leq c_{p_{max}} \tag{25}
\]

2) Voltage constraints

\[
U_{n,\text{min}} \leq U_{n,t} \leq U_{n,\text{max}} \tag{26}
\]

where \( U_{n,\text{min}} \) and \( U_{n,\text{max}} \) are the minimum and maximum voltage magnitudes at node \( n \), respectively.

3) Current constraints of lines

\[
I_{l,\text{min}} \leq I_{l,t} \leq I_{l,\text{max}} \quad l \in \Omega^k \tag{27}
\]

where \( I_{l,t} \) is the current of the line \( l \) at the time interval \( t \); \( I_{l,\text{min}} \) and \( I_{l,\text{max}} \) are the minimum and maximum current values of the line \( l \), respectively; \( \Omega^k \) is the set of lines.

4) Power flow constraints

\[
P_{n,t} = U_{n,t} \sum_{g=1}^{N} U_{g,t} (G_{n,g} \cos \theta_{n,g} + B_{n,g} \sin \theta_{n,g}) \tag{28}
\]

\[
Q_{n,t} = U_{n,t} \sum_{g=1}^{N} U_{g,t} (G_{n,g} \sin \theta_{n,g} - B_{n,g} \cos \theta_{n,g}) \tag{29}
\]
\[ P_{n,t} = P_{n,t}^D + \sum_{k=0}^{D_{fs}} P_{n,t}^{fs} \] (30)

where \( P_{n,t}^D \) and \( Q_{n,t}^D \) are the values of active and reactive power of the node \( n \) in the distribution network at the time interval \( t \) without fast charging loads from FCSs, respectively; \( \Omega_n \) is the set of FCSs connected to the node \( n \) of the distribution network.

4. Test case

4.1 Test system and simulation parameters

**EVs:** four types of private EVs are considered in this case based on the top proportions in the Chinese market on the 2016 as listed in Table II [39]. Based on the NHTS data from US Department of Transportation [34], 6 types of the stops are considered, which are “home”, “work” (\( ty_2 \)), “shopping” (\( ty_3 \)), “recreation” (\( ty_4 \)), “pick up somebody” (\( ty_5 \)) and “meal” (\( ty_6 \)), respectively. The “home” is further classified into two statuses: “temporary stay at home” (\( ty_1 \)) and “stay at home” (\( ty_7 \)) for the stop. \( TP \) is obtained in [40], which is 24×6×7 matrix. \( TP \) at 7:00-8:00 and 17:00-18:00 are shown in Fig. 7. The probability distribution of \( ts_0/j \) and the dwell time for different types of stops can be found in [40].

**Transportation network:** A real transportation network in the urban core of Hangzhou, China is selected as the test system as shown in Fig. 8, which consists of 116 edges and 42 vertices. The 42 vertices include 31 nodes of the transportation network and 11 FCSs corresponding to 32-41 vertices. In each FCS, there are 8 chargers. The distribution of different stop types is shown in Fig. 8. \( D \) of the 42 vertices is listed in Table III. \( IM \) is obtained from the traffic center [41]. Weights of various stop types for nodes in the transportation network are listed in Table IV.

**Electricity distribution network:** In China, most FCSs are connected to 10-kV feeder [42]. Each FCS is taken as a centralized load of the 10-kV distribution network. The structures of four 10-kV distribution networks are configured based on the IEEE 33 standard distribution network [43] as shown in Fig. 9. The peak load of each node in the distribution networks is listed in Table V. The load profile is obtained from [44].

Other simulation parameters are listed in Table VI.
Table II: The main EVs in the Chinese market [45]

<table>
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<tr>
<th>EV Manufacturer</th>
<th>EV type</th>
<th>Proportion</th>
<th>The rated capacity of the battery (kWh)</th>
<th>e (kWh/km)</th>
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Fig. 7. TP of the stops

Fig. 8. The transportation network

Table III: The distance of neighbor vertices

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<th>vj</th>
<th>d(vi, vj)</th>
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<th>vj</th>
<th>d(vi, vj)</th>
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Table IV Weights for each node in the transportation network

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Fig. 9. The corresponding distribution networks

Table V The peak load for each node in the corresponding distribution networks

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Table VI The other simulation parameters

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<td>17</td>
<td>[47]</td>
</tr>
<tr>
<td>Q1</td>
<td>24</td>
<td></td>
<td>U_{min}</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>RC/</td>
<td>0.12</td>
<td>kWh</td>
<td>c_{max}</td>
<td>1.08</td>
<td>RMB/kWh</td>
</tr>
<tr>
<td>theta</td>
<td>120</td>
<td>min</td>
<td>c_{min}</td>
<td>3.3</td>
<td>RMB/kWh</td>
</tr>
<tr>
<td>\eta_1</td>
<td>90</td>
<td>%</td>
<td>\eta_1</td>
<td>1.6</td>
<td>[45]</td>
</tr>
<tr>
<td>\eta_2</td>
<td>3.3</td>
<td>kW</td>
<td>N_0</td>
<td>100</td>
<td>day</td>
</tr>
<tr>
<td>\eta_3</td>
<td>99</td>
<td>%</td>
<td>\eta_3</td>
<td>30000</td>
<td>-</td>
</tr>
<tr>
<td>\eta_4</td>
<td>120</td>
<td>kW</td>
<td>T</td>
<td>96</td>
<td>-</td>
</tr>
<tr>
<td>\eta_5</td>
<td>20</td>
<td>min</td>
<td>U_{max}</td>
<td>1</td>
<td>-</td>
</tr>
</tbody>
</table>

The distribution network with 25 nodes (MVA)
The distribution network with 27 nodes (MVA)
4.2 The Results and Analysis

1) Optimal results and analysis

The EV users of Type I, Type II and Type III account for 40%, 30% and 30%, respectively. The optimal charging pricing scheme of the FCSs is shown in Fig. 10. The results provide a pricing scheme for the FCSs to remain the FCS total income while the voltage profiles of the distribution networks are improved. Compared with the \(c_{p_0}\) pricing scheme, the fast charging prices of the FCS 4, 7 and 10 are lower, while the fast charging prices of the remaining FCSs are higher.

![Fig. 10. The optimal charging pricing scheme of the FCSs](image)

The load difference between the optimal charging pricing scheme and the \(c_{p_0}\) pricing scheme is shown in Fig. 11. The fast charging loads are reallocated among the spatial adjacent FCSs in response to the pricing scheme. Because the FCS 6 price is larger than the FCS 4 price, the FCS 6 load is partially transferred to the FCS 4. Compared with the corresponding load under the \(c_{p_0}\) scheme, the FCS 6 load under the optimal pricing scheme is decreasing, while the FCS 4 load is increasing. That is, the fast charging load at the node 2 of the distribution network \(b\) is partially transferred to the node 15 of the distribution network \(c\). Similarly, because the FCS 11 price is larger than the prices of the FCS 4 and 8, the FCS 11 load is partially transferred to the FCS 4 and 8. Thus, the FCS 11 load under the optimal pricing scheme is decreasing compared with the load under the \(c_{p_0}\) scheme. Because the loads are partially transferred to other nodes of other distribution networks, the voltage profiles of distribution network \(b\) under the optimal pricing scheme are improved as shown in Fig. 12.
2) Comparison and analysis

In order to verify the effectiveness of the proposed strategy in this paper, three scenarios are considered for $N_{EV}$ EVs and the corresponding results are compared and analyzed.

**Scenario I:** The EV users of Type I, Type II and Type III account for 20%, 50% and 30%, respectively.

**Scenario II:** The EV users of Type I, Type II and Type III account for 40%, 30% and 30%, respectively.

**Scenario III:** The EV users of Type I, Type II and Type III account for 50%, 20% and 30%, respectively.

The optimal pricing schemes under different scenarios are shown in Fig. 13. Compared with the total voltage magnitude deviation $TNVD_0$ of the distribution networks with the $cp_0$ scheme, $TNVD$ is shown in Fig. 14 under different scenarios. The total voltage magnitude deviations for different distribution networks under different scenarios are listed in Table VII. The differences between $TNVD_0$ and $TNVD$ under these scenarios are also listed in Table VII.
The optimal charging pricing schemes compared with the \( cp_0 \) pricing scheme under different scenarios are shown in Fig. 13. Because some EV users will select relatively far FCS due to the optimal pricing scheme, \( TD \) is larger than \( TD_0 \) under each scenario. Thus, EV users’ convenience for the fast charging is reduced, which indicates the voltage profiles of the distribution networks are improved at the expense of the EV users’ convenience on the whole.

**Table VII** The total voltage magnitude deviations for different distribution networks under different scenarios

<table>
<thead>
<tr>
<th>Scenario</th>
<th>The voltage magnitude deviations</th>
<th>Distribution network a</th>
<th>Distribution network b</th>
<th>Distribution network c</th>
<th>Distribution network d</th>
<th>The difference between ( TNVD ) and ( TNVD_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Under the optimal pricing scheme</td>
<td>50.2684</td>
<td>62.8003</td>
<td>49.9785</td>
<td>69.8459</td>
<td>1.1951</td>
</tr>
<tr>
<td></td>
<td>Under the ( cp_0 ) scheme</td>
<td>50.7840</td>
<td>63.4273</td>
<td>50.2758</td>
<td>69.6011</td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>Under the optimal pricing scheme</td>
<td>49.9201</td>
<td>62.792</td>
<td>50.4169</td>
<td>68.8859</td>
<td>2.1247</td>
</tr>
<tr>
<td></td>
<td>Under the ( cp_0 ) scheme</td>
<td>50.977</td>
<td>63.5656</td>
<td>50.4763</td>
<td>69.1208</td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>Under the optimal pricing scheme</td>
<td>50.9933</td>
<td>63.2646</td>
<td>49.9685</td>
<td>68.3356</td>
<td>2.3737</td>
</tr>
<tr>
<td></td>
<td>Under the ( cp_0 ) scheme</td>
<td>51.4274</td>
<td>63.411</td>
<td>50.6303</td>
<td>69.4671</td>
<td></td>
</tr>
</tbody>
</table>

\( TD \) and \( TD_0 \) are the sums of \( d_{e,j} \) for all the EVs before and after the optimization, respectively. \( TD \) and \( TD_0 \) under different scenarios are shown in Fig. 15.
The difference of the sum of $d_k$ before and after the optimization for different users is listed in Table VIII.

Type I and Type III users respond to the prices, so the sum of $d_k$ for these users under each scenario is different before and after the optimization. Type II users do not respond to the prices, so $TD_2$ is equal to $TD_{20}$. The distribution of EV fast charging loads is changed and the voltage profiles of distribution networks are improved.

More users respond to the optimal pricing scheme and the difference between $TNVD$ and $TNVD_0$ is larger, as shown in Table VII.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$TD_1$ (km)</th>
<th>$TD_{10}$ (km)</th>
<th>$TD_2$ (km)</th>
<th>$TD_{20}$ (km)</th>
<th>$TD_3$ (km)</th>
<th>$TD_{30}$ (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>688.5</td>
<td>0</td>
<td>-185.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>1399.5</td>
<td>0</td>
<td>-160</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>1764</td>
<td>0</td>
<td>-224</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table VIII** The difference of the sum of $d_k$ before and after the optimization for different users

The cost difference before and after the optimization for different users is listed in Table IX. If the difference is positive, the corresponding cost is increased after the optimization. $FC_1$ and $F_3$ are decreasing under Scenario I and II, and $FT_2$ is not changed after the price optimization. However, $F_3$ is increased after the optimization under Scenario III. It indicates that the optimal pricing scheme will increase the total cost of Type III users with the proportion increase of Type I users, to some extent.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$FC_1$ (RMB)</th>
<th>$FC_{10}$ (RMB)</th>
<th>$FT_2$ (RMB)</th>
<th>$FT_{20}$ (RMB)</th>
<th>$F_3$ (RMB)</th>
<th>$F_{30}$ (RMB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>-232.37</td>
<td>0</td>
<td>-190.66</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>-231.17</td>
<td>0</td>
<td>-65.22</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>-392.26</td>
<td>0</td>
<td>175.64</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table IX** The cost difference before and after the optimization for different users
3) The sensitivity analysis of the EV number

The sensitivity of the EV number on the TNVD/TNVD₀ is analyzed and shown in Fig. 16. TNVD and TNVD₀ grow with the EV number increasing from 10000 to 50000 at a fixed step. However, the difference between TNVD₀ and TNVD keeps increasing. This is because more EVs will respond to the optimal pricing scheme as the EV number increases.

![Fig. 16. TNVD₀ and TNVD under different EV numbers](image)

5. Conclusion and future work

This paper develops a charging pricing strategy of EV FCSs for the voltage control of electricity distribution networks. Considering the travel characteristics of EV users, the fast charging demand is determined using an energy chain. Through the coordination between the upper and lower layers, the fast charging prices are optimized to minimize the total voltage magnitude deviation of distribution networks without decreasing the total income of the FCSs. A real urban transportation network with 11 FCSs is used to validate the proposed strategy. The results show that the voltage profiles of the test system can be significantly improved due to the reallocated fast charging load by the proposed strategy. This is because the users respond to the optimal charging pricing scheme. The strategy is to fully explore the characteristics of different type EV users, and guide these users to participate in the voltage control of distribution networks. Future research will enhance the EV users’ willingness to participate in the voltage control of distribution networks. The game theory will be also introduced to coordinate the benefits of distribution networks, FCSs and EV users.
Acknowledgements

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Reference


