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# Beyond LIFO and FIFO: Exploring an Allocation-In-Fraction-Out (AIFO) policy in a two-warehouse inventory model

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## Abstract

The classical formulation of a two-warehouse inventory model is often based on the Last-In-First-Out (LIFO) or First-In-First-Out (FIFO) dispatching policy. The LIFO policy relies upon inventory stored in a rented warehouse (RW), with an ample capacity, being consumed first, before depleting inventory of an owned warehouse (OW) that has a limited capacity. Consumption works the other way around for the FIFO policy. In this paper, a new policy entitled “*Allocation-In-Fraction-Out (AIFO)*” is proposed. Unlike LIFO and FIFO, AIFO implies simultaneous consumption fractions associated with RW and OW. That said, the goods at both warehouses are depleted by the end of the same cycle. This necessitates the introduction of a key performance indicator to trade-off the costs associated with AIFO, LIFO and FIFO. Consequently, three general two-warehouse inventory models for items that are subject to inspection for imperfect quality are developed and compared – each underlying one of the dispatching policies considered. Each sub-replenishment that is delivered to OW and RW incurs a distinct transportation cost and undertakes a 100 per cent screening. The mathematical formulation reflects a diverse range of time-varying forms. The paper provides illustrative examples that analyse the behaviour of deterioration, value of information and perishability in different settings. For perishable products, we demonstrate that LIFO and FIFO may not be the right dispatching policies. Further, relaxing the inherent determinism of the maximum capacity associated with OW, not only produces better results and implies comprehensive learning, but may also suggest outsourcing the inventory holding through vendor managed inventory.

**Keywords:** Two-warehouse inventory; Imperfect quality; Deterioration; Perishable items.

## 1. Introduction and research motivation

The classical Economic Order Quantity (EOQ) model is based on the assumption that a single owned warehouse (OW) has unlimited capacity, which is often unrealistic. However, there are many factors that may lead to purchasing an amount of units that may exceed the limited capacity of OW, resulting in the excessive units being stored in another, rented, warehouse (RW), which is assumed to be of an ample capacity (Hartley, 1976). Such factors may include a discounted price of goods offered by the supplier, revenue (acquisition price) being higher than the holding cost in RW, and evading high inflation rates (Chung et al., 2009; Hsieh et al., 2008; Lee and Hsu, 2009; Liang and Zhou, 2011; Yang, 2004; 2006; 2012; Zhong and Zhou, 2013; Zhou and Yang, 2005).

The classical formulation of a two-warehouse inventory model assumes that the lot size entering the system first fulfils the maximum storage capacity of the OW with the remaining quantity, over and above that maximum capacity, being kept at the RW. Subsequently, this entails two types of dispatching policies. The first one is to consume the goods of the RW at the earliest, which is termed Last-In-First-Out (LIFO) dispatching policy. Researchers advocating such a policy assume a higher (lower) holding cost (deterioration rate) in RW due to the availability of better preserving environmental conditions (e.g. Jaggi et al., 2015). Conversely, when the First-In-First-Out (FIFO) dispatching policy is employed then the goods of the OW are consumed first before considering the RW inventory. This case is usually justified by holding cost reduction, especially when the holding cost in RW is lower than that in OW due to competition, i.e. various offers are available in the market (e.g. Lee, 2006; Niu and Xie, 2008).

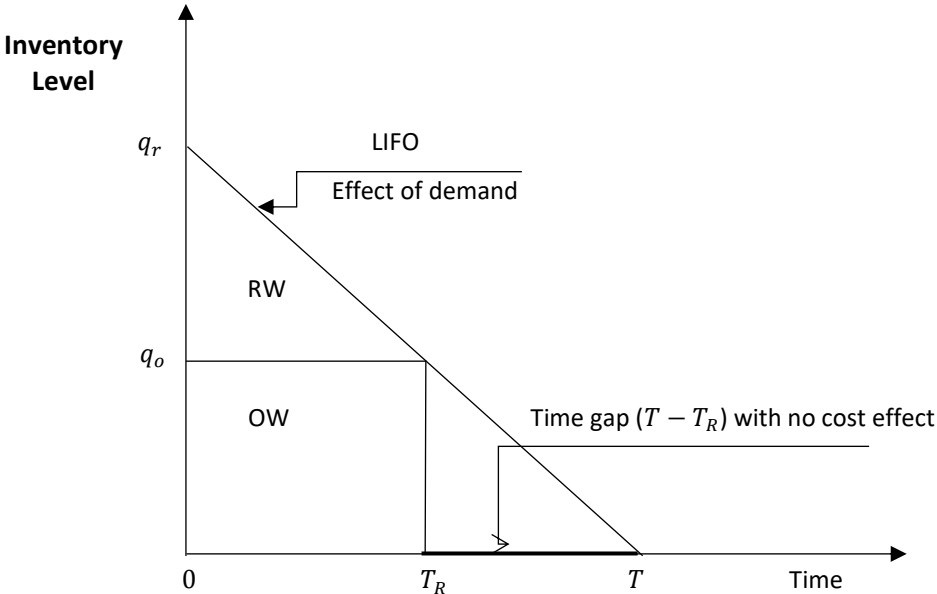
At this point it is important to note that the terms LIFO and FIFO are often associated with cost accounting, and indeed there is a considerable amount of research conducted in this area. However, these terms are solely used, for the purposes of this work, to indicate which warehouse is being used first.

Although, the literature related to the formulation of two-warehouse inventory models is quite mature, the inventory formulation is based on a number of explicitly or implicitly made unrealistic mathematical assumptions that may never reflect reality. In more detail, the assumption that the lot size is delivered to the inventory system in one batch ignores the cost effects of transporting items to distinct warehouses, and whether those items are transported to OW first and then to RW, or vice versa. It is worth noting that if no penalty charges are payable to the supplier when a replenishment (bulk quantity) is divided into two sub-replenishments, then there is no reason why the second sub-

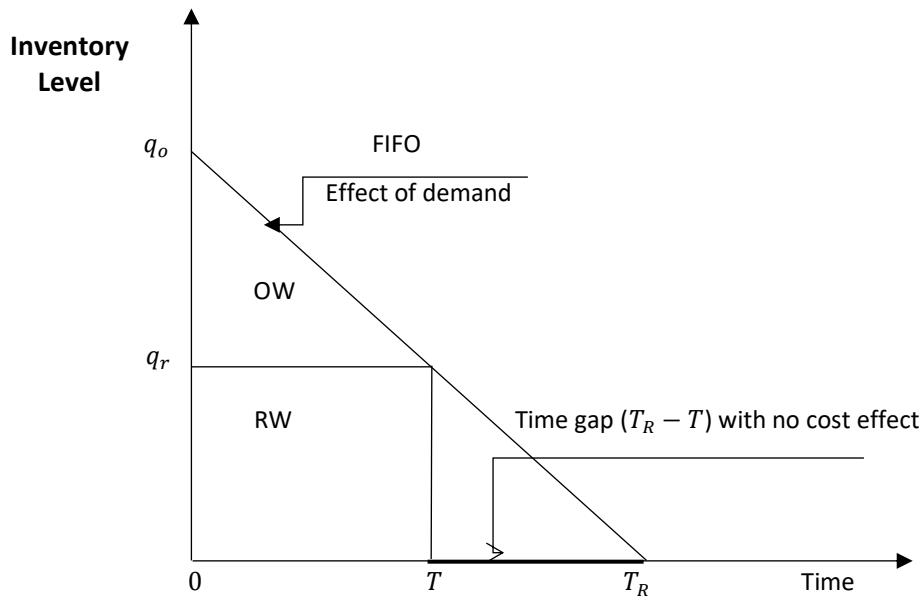
replenishment is not delivered at or just before the stored items in either warehouse are completely consumed. That is, the mathematical formulation of a two-level storage has no meaning. Therefore, considering differing unit transportation costs among supply chain levels may have a considerable effect on the optimal order quantity. This can be justified by the distinct location of each warehouse, i.e. there exists at least a marginal difference in distance that incurs an additional transportation cost payable for inventory movements.

From a managerial point of view, there is indeed a time gap between consecutive sub-replenishments that are delivered to OW and RW. The LIFO policy may influence the warehouse rental contract, i.e. the time gap may affect the availability of RW (Fig.1). On the other hand, the FIFO policy renders the OW unusable during the consumption period of RW (Fig.2). Because of this, both LIFO and FIFO assume no cost effect while the initially used warehouse is idle. Finally, in the case of managing perishable products, LIFO and FIFO may not be the right choices, given that the order quantity needs to be consumed based on a First-Expired-First-Out (FEFO) policy.

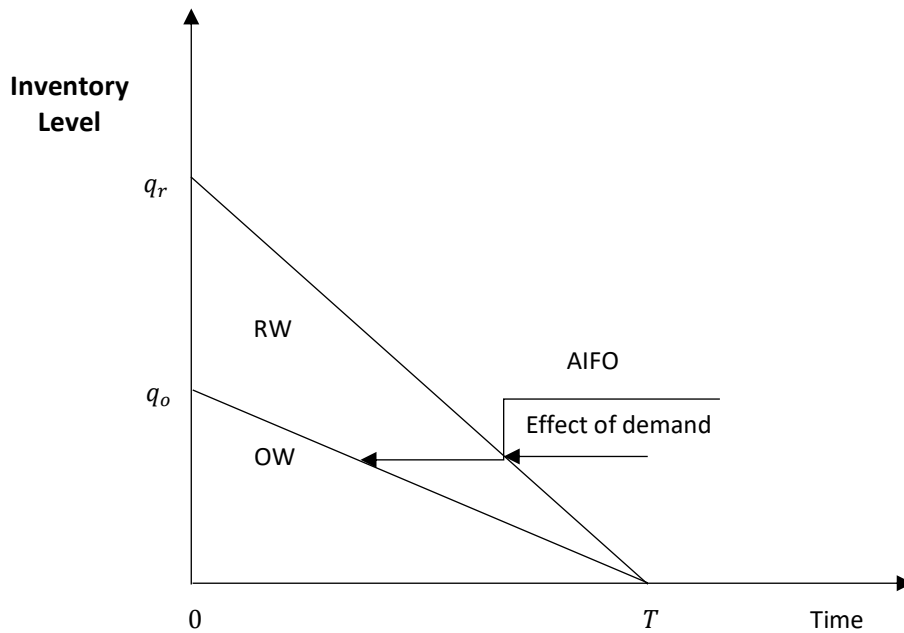
In this paper, a new policy entitled “Allocation-In-Fraction-Out (AIFO)” is developed. Under an AIFO dispatching policy the goods at RW and OW experience simultaneous consumption fractions, which implies that the inventories at both warehouses are depleted by the end of the same cycle (Fig.3). Note that under the LIFO (FIFO) policy, the sub-replenishment  $q_r(q_o)$  that is delivered to RW (OW) is consumed first by time  $T_R(T)$ , then the sub-replenishment  $q_o(q_r)$  that is delivered to OW (RW) is consumed by time  $T(T_R)$ .



**Fig.1.** Inventory variation of the two-warehouse model during one cycle (LIFO).



**Fig.2.** Inventory variation of the two-warehouse model during one cycle (FIFO).



**Fig.3.** Inventory variation of the two-warehouse model during one cycle (AIFO).

## 2. Research background and contribution

In this Section, we first address some product quality related issues that are associated with the formulation of a two-warehouse inventory model, followed by some discussion on the value of information (VOI) and inspection processes in supply chains. This provides the necessary background to position our study in the current body of literature and elaborate on its research contributions.

### **2.1. Inventory quality issues**

One of the unrealistic assumptions underlying the EOQ model is that all items are of good quality. In practice, this assumption is technologically unattainable in most supply chain applications as defective items may affect the operational and financial performance of an inventory system (Chan et al., 2003; Cheng, 1991; Khan et al., 2011; Pal et al., 2013; Salameh and Jaber, 2000).

Another implicit assumption embedded in the EOQ model is that stored items preserve their physical characteristics indefinitely. However, in real-life settings, items are subject to 'perishability', 'deterioration' and 'obsolescence' that affect the physical state/fitness and behaviour of an item as it moves through the supply chain (Bakker et al., 2012; Dave, 1986; Elmaghraby and Keskinocak, 2003; Ferguson and Ketzenberg, 2005; Ferguson and Koenigsberg, 2007; Goyal and Giri, 2001; Jain and Silver, 1994; Joglekar and Lee, 1993; Ketzenberg and Ferguson, 2008; Kim et al., 2014; Liao et al., 2013; Olsson, 2009; Song and Zipkin, 1996; Teunter and Flapper, 2003). Factors such as changes in temperature and controlled atmosphere storage as well as increases in the storage time may result in a decrease (or an increase) of the deterioration rate of certain items.

Pahl and Voß (2014) provided a comprehensive literature review that addresses deterioration and lifetime constraints of items. Common examples are packaged foods, seafood, fruit, cheese, processed meat, pharmaceutical, agricultural or chemical products that are transported over long distances in refrigerated containers, where temperature variability has a significant impact on product shelf lifetime (Doyle, 1995; Koutsoumanis et al., 2005; Taoukis et al., 1999). Moreover, various conditions such as transportation, handling, the product's temperature history and humidity have a direct impact on product shelf lifetime (Alamri et al., 2016; Ketzenberg et al., 2015).

### **2.2. Value of information (VOI) and inspection process**

Value of information (VOI) in supply chains has become increasingly important and may relate to sharing data over and above demand and inventory information (Dong et al., 2014; Kahn 1987; Metters 1997). For example, modern technologies such as radio-frequency identification (RFID) systems, data loggers and time-temperature integrators and sensors are capable of recording, tracking and transmitting information regarding an item as it moves through the supply chain (Jedermann et al., 2008). The deployment of such technologies increases supply chain visibility, which in turn increases efficiency, lowers safety stocks and improves customer service level (Gaukler et al., 2007; Kim and Glock 2014). Ketzenberg et al. (2007) conducted an extensive literature review of papers considering VOI in the context of inventory control. The researchers indicated that the dominant research stream in this area focuses on the value of demand information to enhance supply chain performance.

The above discussion relates very much to knowledge acquired from an inspection process conducted at RW and OW. This means that the quality issues that render an item defective can be communicated to the supplier in order to reduce the potential risks affecting such defectiveness. These risks can be attributed to production, handling or transportation errors. Although the buyer is often credited so that no costs apply for defective items, the potential interest remains to eliminate the presence of defects in subsequent replenishments. Therefore, coordination may be pursued between supply chain members implying that any information gained through previous replenishments can be used to enhance subsequent deliveries. In many situations, products entail inspection to ensure an appropriate service to the customers (White and Cheong, 2012). Inspection may also be presumed essential for updating the Information System records with good items that are actually available in stock so as to avoid shortages (Rekik et al., 2015). Moreover, inspection may eliminate the return service cost associated with product recalls (Klassen and Vereecke 2012).

The combination of the quality related issues raised in Section 2.1 is important in many industries and may significantly influence the optimal order quantity. This is an important issue especially in the case of managing perishable products where inspection would imply that products may be classified according to quality, size, appearance, freshness, etc., and where a distinct selling price may be linked to its corresponding quantity. Moreover, a 100 per cent inspection will render a potential random lifetime of a product deterministic, i.e. it intersects the areas of fixed and random lifetimes of perishable products. Finally, inspection not only isolates defective and/or already perished items, but also leads to the consumption of the order quantity based on a FEFO policy. For example, isolation, i.e. dis-location of good and defective items, allows for an immediate disposal of defective and/or already perished items in case of any potential safety issues. It may also reduce holding costs due to the deployment of less preserving environmental conditions, i.e. the defective items are not usually stored in the same warehouse where the good items are stored (e.g. Wahab and Jaber, 2010).

### ***2.3. Contribution and organisation of the paper***

The contribution of this work goes beyond addressing the issues raised in Sections 1, 2.1 and 2.2 when formulating a two-warehouse inventory model for items that require 100 per cent screening. In particular, a new policy entitled "Allocation-In-Fraction-Out (AIFO)" is developed. Under an AIFO dispatching policy the goods at RW and OW experience simultaneous consumption fractions, which implies that the inventories at both warehouses are depleted by the end of the same cycle. On the other hand, the LIFO and FIFO policies assume no cost effect while the initially used warehouse is idle, which is unrealistic and a rare scenario to encounter in practice. Subsequently, this necessitates introducing costs associated with the OW or RW being idle when formulating a two-warehouse

inventory model. Therefore, three general EOQ models for items with imperfect quality are presented and compared. The first model underlies the LIFO policy, the second model underlies the FIFO policy and the third model relates to the AIFO policy. It becomes apparent that the tradeoff between the three policies constitutes a key business objective in supply chain management. Under both the LIFO and FIFO dispatching policy, the cost associated with the OW or RW being idle is treated as an input parameter as well as a decision variable. If the cost is a decision variable, then it constitutes a key performance indicator (KPI), i.e. an upper-bound (cost associated with OW (RW) being idle) that renders AIFO the optimal dispatching policy.

To the best of our knowledge, the maximum capacity of the OW is invariably treated in the academic literature as an input parameter. Relaxing the inherent determinism related to the maximum fulfilment of the capacity of OW may lead to maximizing net revenue. In addition, if the system is subject to learning, then the lot size may reduce for each successive replenishment. However, such reduction affects the amount allocated to the RW only, and the amount allocated to the OW remains at the maximum capacity. Relaxing the inherent determinism of this assumption implies comprehensive learning that can be achieved simultaneously, i.e. the amounts that are allocated to both the OW and RW are affected.

The proposed models may be viewed as realistic in today's competitive markets and reflective of a number of practical concerns with regards to product quality related issues. These issues relate to imperfect items received from suppliers, goods' deterioration during storage, potential dis-location of good and defective items, tracking the quality of perishable products in a supply chain and transfer of knowledge from one inventory cycle to another. The percentage of defective items per lot reduces according to a learning curve and different warehouses for the good and defective items are considered in the mathematical models. We show that the solution to each underlying inventory model, if it exists, is unique and global optimal. Practical examples that are published in the literature for generalised models in this area are shown to be special cases of our FIFO, LIFO and AIFO models. We observe and test the behaviour of the theoretical models in different settings (e.g. different transportation costs associated with OW and RW, functions for varying demand, screening, defective and deterioration rates, VOI, perishable items that are subject to deterioration while in storage and by means of relaxing the inherent determinism related to the maximum fulfilment of the capacity of OW).

The remainder of the paper is organised as follows: In Section 3, we present our three EOQ models for items with imperfect quality and the solution procedures. Illustrative examples, a comparison between the three models and special cases are offered in Section 4, where we also present the key findings of



our work along with the managerial insights. Concluding remarks and opportunities for further research are provided in Sections 5. The proof of the optimality and uniqueness of our solutions is presented in an electronic companion as supplementary material to this paper.

### 3. Formulation of the general models

#### 3.1. Assumptions and notation

We will use throughout the paper the subscript “ $o$  ( $r$ )” to indicate the quantity related to the OW (RW). We will also employ the subscript “ $g$  ( $d$ )” to refer to good (defective) items. So, for example, and denoting the cycle index by  $j$ ,  $I_{rgj}(t)$  denotes the inventory level of good items at time  $t$  in RW, and  $I_{odj}(t)$  refers to the inventory level of defective items at time  $t$  in OW. We will also use the subscript  $i$  ( $i = A, L, F$ ) to refer to the AIFO, LIFO and FIFO dispatching policy, respectively.

Our models are developed under the following assumptions and notation:

1. A single item is held in stock.
2. The lead-time is negligible, i.e. any replenishment ordered at the beginning of a cycle arrives just prior to the end of that same cycle.
3. The demand, screening and deterioration rates are arbitrary functions of time denoted by  $D(t)$ ,  $x(t)$  and  $\delta_y(t)$  respectively.
4. The OW has a fixed limited capacity and the RW has unlimited capacity.
5. The percentage defective per lot reduces according to a learning curve denoted by  $p_j$ , where  $j$  is the cycle index.
6. Shortages are not allowed, i.e. we require that  $(1 - p_j)x(t) \geq D(t) \forall t \geq 0$ .
7. The cost parameters are as follows:

$c$  = Unit purchasing cost;

$d$  = Unit screening cost;

$c_L$  = Charge payable per unit time if RW remains idle for the LIFO model;

$c_F$  = Cost incurred per unit time if OW remains idle for the FIFO model;

$s_o$  = Unit transportation cost for OW;

$s_r$  = Unit transportation cost for RW;

$h_{rg}$  = Holding cost of good items per unit per unit time for RW;

$h_{rd}$  = Holding cost of defective items per unit per unit time for RW;

$h_{og}$  = Holding cost of good items per unit per unit time for OW;

$h_{od}$  = Holding cost of defective items per unit per unit time for OW;

$k$  = Cost of placing an order.

At the beginning of each cycle  $j$  ( $j = 1, 2, \dots$ ), a lot of size  $Q_{ij}$  is delivered such that a quantity of size  $q_{oij}$  is allocated to the OW and the remaining amount of size  $q_{rij} = Q_{ij} - q_{oij}$  is allocated to the RW. Each sub-replenishment that enters the OW (RW) undertakes a 100 per cent screening process at a rate of  $x(t)$  that starts at the beginning of the cycle and ceases by time  $T_{oij}$  ( $T_{rij}$ ), by which point in time  $q_{oij}$  ( $q_{rij}$ ) units have been screened and  $y_{oij}$  ( $y_{rij}$ ) units have been consumed. Each sub-replenishment covers the actual demand and deterioration during both the first phase (screening) and the second phase (non-screening). During the screening phase, items not conforming to certain quality standards (defective items) are stored in different warehouses.

### 3.2. Allocation-In-Fraction-Out (AIFO) dispatching policy

As an application of an AIFO dispatching policy, items are simultaneously depleted from the RW and OW at rates  $\phi_{oj} D(t)$  and  $\phi_{rj} D(t)$  respectively, where  $\phi_{rj} = 1 - \phi_{oj}$ . Unlike LIFO and FIFO, the analysis of AIFO is limited to one case, i.e. the cycle length for the RW and OW is the same. The behaviour of such a model is depicted in Fig. 4.

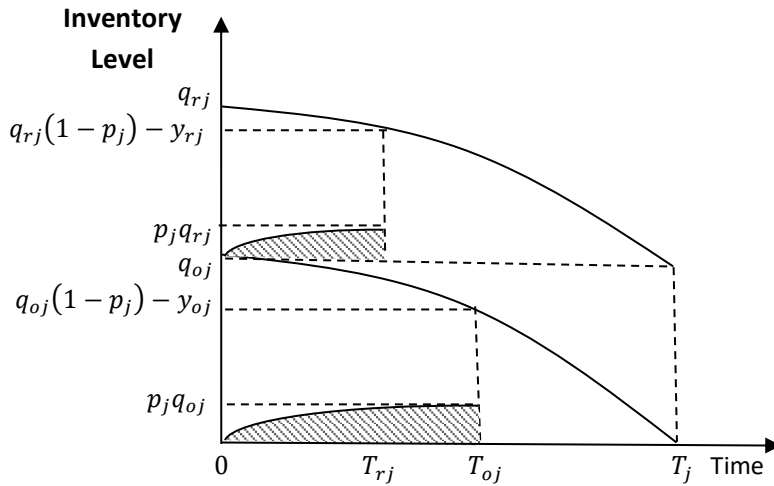


Fig. 4. Inventory variation of the two-warehouse model during one cycle (AIFO).

The variations in the inventory levels depicted in Fig. 4 for the OW and RW are given by the following differential equations:

$$\frac{dI_{ogj}(t)}{dt} = -\phi_{oj}D(t) - p_j x(t) - \delta_o I_{ogj}(t), \quad 0 \leq t < T_{oj} \quad (1)$$

$$\frac{dI_{ogj}(t)}{dt} = -\phi_{oj}D(t) - \delta_o I_{ogj}(t), \quad T_{oj} \leq t \leq T_j \quad (2)$$

$$\frac{dI_{rgj}(t)}{dt} = -\phi_{rj}D(t) - p_j x(t) - \delta_r I_{rgj}(t), \quad 0 \leq t < T_{rj} \quad (3)$$

$$\frac{dI_{rgj}(t)}{dt} = -\phi_{rj}D(t) - \delta_r I_{rgj}(t), \quad T_{rj} \leq t \leq T_j \quad (4)$$

with the boundary conditions  $I_{ogj}(0) = q_{oj}$ ,  $I_{ogj}(T_j) = 0$ ,  $I_{rgj}(0) = q_{rj}$  and  $I_{rgj}(T_j) = 0$

where

$$Q_{ij} = \int_0^{T_{oj}} x(u)du + \int_0^{T_{rj}} x(u)du. \quad (5)$$

Finally, the variations in the inventory levels for defective items (shaded area) depicted in Fig. 4 are given by the following differential equations:

$$\frac{dI_{rdj}(t)}{dt} = p_j x(t), \quad 0 \leq t \leq T_{rj} \quad (6)$$

$$\frac{dI_{odj}(t)}{dt} = p_j x(t), \quad 0 \leq t \leq T_{oj} \quad (7)$$

with the boundary conditions  $I_{rdj}(0) = 0$ ,  $I_{odj}(0) = 0$ ,  $I_{rdj}(T_{rj}) = p_j q_{rj}$  and  $I_{odj}(T_{oj}) = p_j q_{oj}$ .

Solving the above differential equations we get

$$I_{ogj}(t) = e^{-(g_o(t)-g_o(0))} \int_0^{T_{oj}} x(u)du - e^{-g_o(t)} \int_0^t [\phi_{oj}D(u) + p_j x(u)] e^{g_o(u)} du, \quad 0 \leq t < T_{oj} \quad (8)$$

$$I_{ogj}(t) = e^{-g_o(t)} \int_t^{T_j} \phi_{oj}D(u) e^{g_o(u)} du, \quad T_{oj} \leq t \leq T_j \quad (9)$$

$$I_{odj}(t) = \int_0^t p_j x(u)du, \quad 0 \leq t \leq T_{oj} \quad (10)$$

$$I_{rgj}(t) = e^{-(g_r(t)-g_r(0))} \int_0^{T_{rj}} x(u)du - e^{-g_r(t)} \int_0^t [\phi_{rj}D(u) + p_j x(u)] e^{g_r(u)} du, \quad 0 \leq t < T_{rj} \quad (11)$$

$$I_{rgj}(t) = e^{-g_r(t)} \int_t^{T_j} \phi_{rj}D(u) e^{g_r(u)} du, \quad T_{rj} \leq t \leq T_j \quad (12)$$

$$I_{rdj}(t) = \int_0^t p_j x(u)du, \quad 0 \leq t \leq T_{rj} \quad (13)$$

respectively, where

$$g_y(t) = \int \delta_y(t)dt. \quad (14)$$

Now, the per cycle cost components for the given inventory model are as follows:

Purchasing cost + Screening cost + Transportation cost =  $(c + d + s_o) \int_0^{T_{oj}} x(u)du + (c + d + s_r) \int_0^{T_{rj}} x(u)du$ . Note that the purchasing cost includes the defective and deteriorated items.

Holding cost for the RW =  $h_{rg}[I_{rgj}(0, T_{rj}) + I_{rgj}(T_{rj}, T_j)] + h_{rd}I_{rdj}(0, T_{rj})$ .

Holding cost for the OW =  $h_{og}[I_{ogj}(0, T_{oj}) + I_{ogj}(T_{oj}, T_j)] + h_{od}I_{odj}(0, T_{oj})$ .

Thus, the total cost per unit time of the underlying inventory model during the cycle  $[0, T_j]$ , as a function of  $T_{rj}, T_j$  and  $\phi_{oj}$  say  $Z_A(T_{rj}, T_j, \phi_{oj})$  is given by

$$\begin{aligned}
Z_A(T_{rj}, T_j, \Phi_{oj}) = & \frac{1}{T_j} \left\{ (c + d + s_o) \int_0^{T_{oj}} x(u) du + (c + d + s_r) \int_0^{T_{rj}} x(u) du + h_{og} \left[ [G_o(T_{oj}) - \right. \right. \\
& G_o(0)] e^{g_o(0)} \int_0^{T_{oj}} x(u) du - \int_0^{T_{oj}} [G_o(T_{oj}) - G_o(u)] [\Phi_{oj} D(u) + p_j x(u)] e^{g_o(u)} du + \int_{T_{oj}}^{T_j} [G_o(u) - \\
& G_o(T_{oj})] \Phi_{oj} D(u) e^{g_o(u)} du \left. \right] h_{od} \left[ \int_0^{T_{oj}} [T_{oj} - u] p_j x(u) du \right] + h_{rg} \left[ [G_r(T_{rj}) - \right. \\
& G_r(0)] e^{g_r(0)} \int_0^{T_{rj}} x(u) du - \int_0^{T_{rj}} [G_r(T_{rj}) - G_r(u)] [\Phi_{rj} D(u) + p_j x(u)] e^{g_r(u)} du + \int_{T_{rj}}^{T_j} [G_r(u) - \\
& G_r(T_{rj})] \Phi_{rj} D(u) e^{g_r(u)} du \left. \right] + h_{rd} \left[ \int_0^{T_{rj}} [T_{rj} - u] p_j x(u) du \right] + k \left. \right\}, \tag{15}
\end{aligned}$$

where

$$G_y(t) = \int e^{-g_y(t)} dt. \tag{16}$$

Our objective is to find  $T_{rj}, T_j$  and  $\Phi_{oj}$  that minimise  $Z_A(T_{rj}, T_j, \Phi_{oj})$ , where  $Z_A(T_{rj}, T_j, \Phi_{oj})$  is given by Eq. (15). But the variables  $T_{rj}, T_j$  and  $\Phi_{oj}$  are associated with each other through the following relations:

$$0 < T_{rj} < T_j, \tag{17}$$

$$e^{g_o(0)} \int_0^{T_{oj}} x(u) du = \int_0^{T_j} \Phi_{oj} D(u) e^{g_o(u)} du + \int_0^{T_{oj}} p_j x(u) e^{g_o(u)} du, \tag{18}$$

$$e^{g_r(0)} \int_0^{T_{rj}} x(u) du = \int_0^{T_j} \Phi_{rj} D(u) e^{g_r(u)} du + \int_0^{T_{rj}} p_j x(u) e^{g_r(u)} du. \tag{19}$$

Thus, our goal is to solve the following optimisation problem, which we shall call problem  $(m_A)$

$$(m_A) = \left\{ \begin{array}{l} \text{minimise } Z_A(T_{rj}, T_j, \Phi_{oj}) \text{ given by Eq. (15)} \\ \text{subject to Eqs. (17 – 19) and } 0 \leq \Phi_{oj} \leq 1 \end{array} \right\}.$$

From Eq. (19),  $T_{rj} = 0 \Rightarrow T_j = 0$  and  $T_{rj} > 0 \Rightarrow T_{rj} < T_j$ . Thus Eq. (19) implies constraint (17). Hence, if we temporarily ignore the monotony constraint (17) and call the resulting problem as  $(m_{A1})$  then constraint (17) does satisfy any solution of  $(m_{A1})$ . Therefore,  $(m_A)$  and  $(m_{A1})$  are equivalent. Moreover,  $T_{rj} > 0 \Rightarrow$  RHS of Eqs. (9) and (12)  $> 0$ , i.e. Eqs. (18) and (19) guarantee that the number of good items is at least equal to the demand and deterioration during screening.

### 3.2.1. Solution procedures

First, we note from Eqs. (18) and (19) that  $T_{rj}, T_j$  and  $\Phi_{oj}$  can be determined as functions of  $q_{rj}$ , say

$$T_{rj} = f_{rj}(q_{rj}), \tag{20}$$

$$T_j = f_j(q_{rj}), \tag{21}$$

$$\Phi_{oj} = \Phi_j(q_{rj}). \tag{22}$$

Thus, considering Eqs. (18)-(22) then the problem  $(m_A)$  is converted to the following unconstrained problem with the variable  $Q_{Aj}$  (which we shall call problem  $(m_{A2})$ ).

$$\begin{aligned}
W_A(Q_{Aj}) = & \frac{1}{f_j} \left\{ (c + d + s_o) \int_0^{f_{oj}} x(u) du + (c + d + s_r) \int_0^{f_{rj}} x(u) du + \right. \\
& h_{og} \left[ -G_o(0) e^{g_o(0)} \int_0^{f_{oj}} x(u) du + \int_0^{f_{oj}} p_j x(u) G_o(u) e^{g_o(u)} du + \phi_j \int_0^{f_j} D(u) G_o(u) e^{g_o(u)} du \right] + \\
& h_{od} \left[ \int_0^{f_{oj}} [f_{oj} - u] p_j x(u) du \right] + h_{rg} \left[ -G_r(0) e^{g_r(0)} \int_0^{f_{rj}} x(u) du + \int_0^{f_{rj}} p_j x(u) G_r(u) e^{g_r(u)} du + (1 - \right. \\
& \left. \phi_j) \int_0^{f_j} D(u) G_r(u) e^{g_r(u)} du \right] + h_{rd} \left[ \int_0^{f_{rj}} [f_{rj} - u] p_j x(u) du \right] + k \left. \right\}. \tag{23}
\end{aligned}$$

If we let  $W_A = \frac{w_A}{f_j}$ , then the necessary condition for having a minimum for problem  $(m_{A2})$  is

$$w'_{q_{rj}} f_j = f'_{j,q_{rj}} w_A, \tag{24}$$

where  $w'_{q_{rj}}$  and  $f'_{j,q_{rj}}$  are the derivatives of  $w_A$  and  $f_j$  with respect to  $q_{rj}$ , respectively.

Also, Eqs. (18) and (19) yield

$$f'_{j,q_{rj}} = \frac{(e^{g_r(0)} - p_j e^{g_r(f_{rj})}) \left( \int_0^{f_j} D(u) e^{g_o(u)} du \right)^2}{D(f_j) e^{g_r(f_j)} \left( \left( \int_0^{f_j} D(u) e^{g_o(u)} du \right)^2 - S \left( \int_0^{f_j} D(u) e^{g_o(u)} du - e^{(g_o(f_j) - g_r(f_j))} \int_0^{f_j} D(u) e^{g_r(u)} du \right) \right)}, \tag{25}$$

$$\phi'_{j,q_{rj}} = - \frac{S f'_{j,q_{rj}} D(f_j) e^{g_o(f_j)}}{\left( \int_0^{f_j} D(u) e^{g_o(u)} du \right)^2}, \tag{26}$$

where  $S = e^{g_o(0)} \int_0^{f_{oj}} x(u) du - \int_0^{f_{oj}} p_j x(u) e^{g_o(u)} du$ .

Considering the above and also Eqs. (20)-(23) we have

$$\begin{aligned}
w'_{q_{rj}} = & (c + d + s_r) + h_{rg} \left[ \left( G_r(f_j) - G_r(0) \right) e^{g_r(0)} + \left( G_r(f_{rj}) - G_r(f_j) \right) p_j e^{g_r(f_{rj})} + \right. \\
& \left. \phi'_{j,q_{rj}} \left( G_r(f_j) \int_0^{f_j} D(u) e^{g_r(u)} du - \int_0^{f_j} D(u) G_r(u) e^{g_r(u)} du \right) \right] + \frac{h_{rd}}{x(f_{rj})} \int_0^{f_{rj}} p_j x(u) du + \\
& h_{og} \left[ \phi'_{j,q_{rj}} \int_0^{f_j} D(u) G_o(u) e^{g_o(u)} du + \phi_j f'_{j,q_{rj}} D(f_j) G_o(f_j) e^{g_o(f_j)} \right]. \tag{27}
\end{aligned}$$

$$\text{Also, Eq. (24)} \Leftrightarrow W_A = \frac{w_A}{f_j} = \frac{w'_{q_{rj}}}{f'_{j,q_{rj}}}. \tag{28}$$

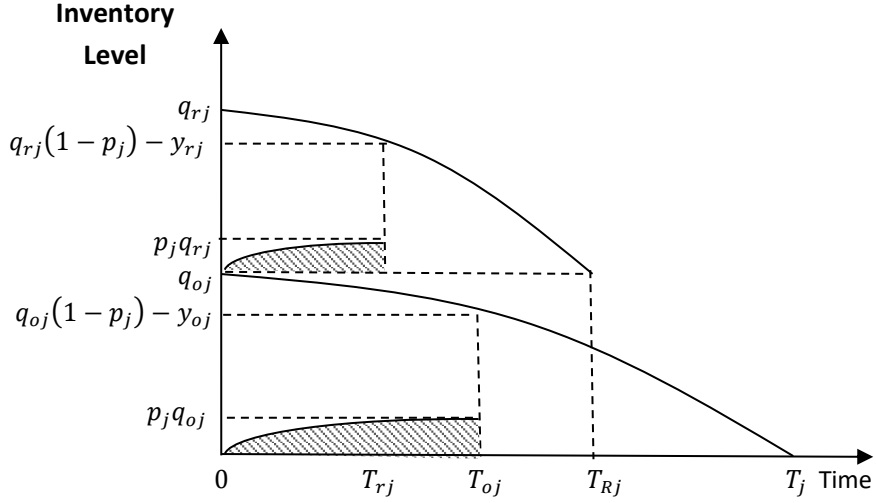
Eq. (28) can be used to determine the optimal value of  $Q_{Aj}$  and its corresponding total minimum cost.

Then the optimal values of  $T_{rj}$ ,  $T_j$  and  $\phi_j$  can be found from Eqs. (20), (21) and (22), respectively.

### 3.3. LIFO dispatching policy

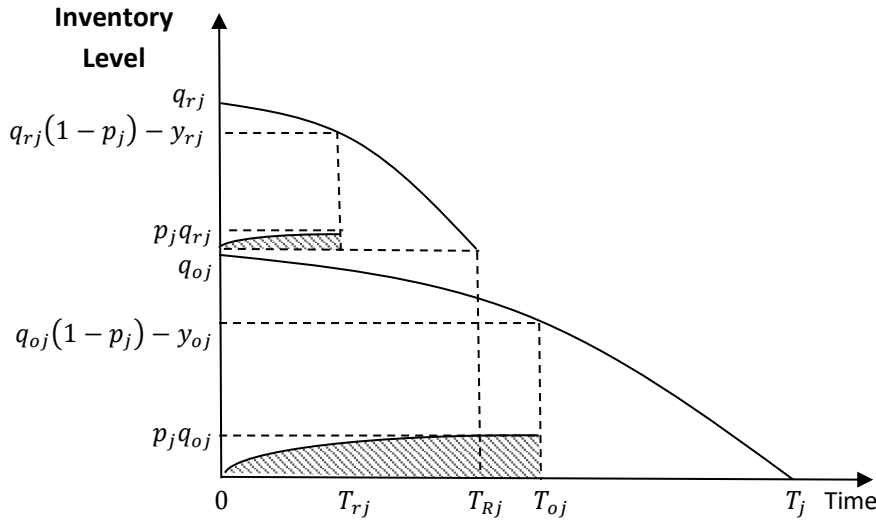
When applying a LIFO dispatching policy, items stored in the RW are depleted first by time  $T_{Rj}$ . In this model we distinguish two cases:

**Case 1.**  $T_{oj} \leq T_{Rj}$ . The behaviour of such a model is depicted in Fig. 5.



**Fig. 5.** Inventory variation of the two-warehouse model during one cycle when  $T_{oj} \leq T_{Rj}$  (LIFO).

**Case 2.**  $T_{oj} > T_{Rj}$ . The behaviour of such a model is depicted in Fig. 6.



**Fig. 6.** Inventory variation of the two-warehouse model during one cycle when  $T_{Rj} < T_{oj}$  (LIFO).

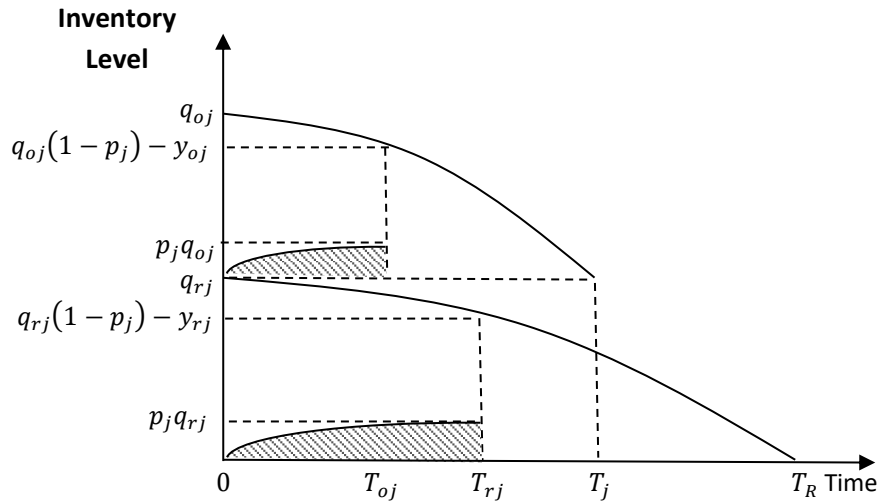
The mathematical formulation for cases 1 and 2 can be obtained in a similar way as that for AIFO (see Appendix A in the electronic companion), where the total cost per unit time of the underlying inventory model is identical for cases 1 and 2 and is given by

$$\begin{aligned}
 W_L(Q_{Lj}) = & \frac{1}{f_j} \left\{ (c + d + s_o) \int_0^{f_{oj}} x(u) du + (c + d + s_r) \int_0^{f_{rj}} x(u) du + \right. \\
 & h_{rg} \left[ -G_r(0) e^{g_r(0)} \int_0^{f_{rj}} x(u) du + \int_0^{f_{rj}} p_j x(u) G_r(u) e^{g_r(u)} du + \int_0^{f_{Rj}} D(u) G_r(u) e^{g_r(u)} du \right] + \\
 & h_{rd} \left[ \int_0^{f_{rj}} [f_{rj} - u] p_j x(u) du \right] + h_{og} \left[ -G_o(0) e^{g_o(0)} \int_0^{f_{oj}} x(u) du + \int_0^{f_{oj}} p_j x(u) G_o(u) e^{g_o(u)} du + \right. \\
 & \left. \int_{f_{Rj}}^{f_j} D(u) G_o(u) e^{g_o(u)} du \right] + h_{od} \left[ \int_0^{f_{oj}} [f_{oj} - u] p_j x(u) du \right] + k + c_{Lj} (f_j - f_{Rj}) \left. \right\}. \quad (29)
 \end{aligned}$$

### 3.4. FIFO dispatching policy

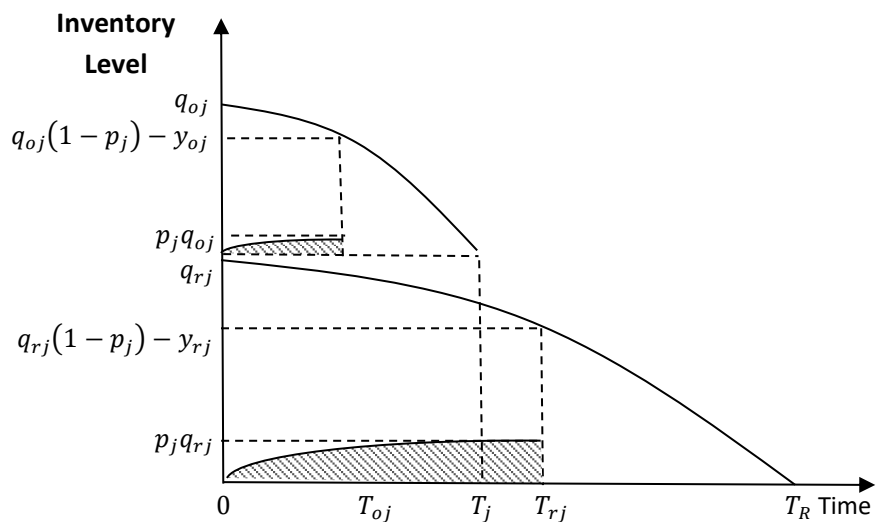
When applying a FIFO dispatching policy, the goods of the RW are consumed only after depleting the goods of OW, i.e.  $q_{oj}$  is consumed first, which implies that the cycle length for the OW is a predetermined value. The behaviour of such a model is depicted in Figs. 7 and 8.

**Case 1.**  $T_{rj} \leq T_j$ .



**Fig. 7.** Inventory variation of the two-warehouse model during one cycle when  $T_{rj} \leq T_j$  (FIFO).

**Case 2.**  $T_{rj} > T_j$ .



**Fig. 8.** Inventory variation of the two-warehouse model during one cycle when  $T_j < T_{rj}$  (FIFO).

The mathematical formulation for cases 1 and 2 can be obtained in similar way as that for AIFO (see Appendix B in the electronic companion), where the total cost per unit time of the underlying inventory model is identical for cases 1 and 2 and is given by

$$\begin{aligned}
W_F(Q_{Fj}) = & \frac{1}{f_{Rj}} \left\{ (c + d + s_o) \int_0^{f_{oj}} x(u) du + (c + d + s_r) \int_0^{f_{rj}} x(u) du + \right. \\
& h_{og} \left[ -G_o(0) e^{g_o(0)} \int_0^{f_{oj}} x(u) du + \int_0^{f_{oj}} p_j x(u) G_o(u) e^{g_o(u)} du + \int_0^{f_{oj}} D(u) G_o(u) e^{g_o(u)} du \right] + \\
& h_{od} \left[ \int_0^{f_{oj}} [f_{oj} - u] p_j x(u) du \right] + h_{rg} \left[ -G_r(0) e^{g_r(0)} \int_0^{f_{rj}} x(u) du + \int_0^{f_{rj}} p_j x(u) G_r(u) e^{g_r(u)} du + \right. \\
& \left. \int_{f_j}^{f_{Rj}} D(u) G_r(u) e^{g_r(u)} du \right] + h_{rd} \left[ \int_0^{f_{rj}} [f_{rj} - u] p_j x(u) du \right] + k + c_{Fj} (f_{Rj} - f_j) \left. \right\}. \quad (30)
\end{aligned}$$

#### 4. Numerical analysis and special cases

In this Section, we present illustrative examples and special cases to support the application of our mathematical models and solution procedures in different realistic situations. In practice, the demand function may increase (decrease) over time with linear, exponential, quadratic, and stock-dependent trends. For example, exponentially increasing demand fits well products such as new spare parts, new electronic chips and seasonal goods in which the demand rate is likely to increase very fast with time (Hariga and Benkherouf, 1994). On the other hand, essential commodities and seasonal products may follow steadily increasing quadratic or linear demand functions over time (Sana, 2010). As such, the mathematical formulation presented in this paper considers arbitrary functions of time, which allows the decision maker to assess the consequences of a diverse range of strategies by employing a single inventory model. For example, the variation of demand, screening, deterioration and defective rates with time (or due to any other factors) is a quite natural phenomenon (Alamri, 2011; Benkherouf et al., 2014; Datta et al. 1998; Grosse et al., 2013; Jaber et al., 2008; Karmarkar and Pitbladdo, 1997; Murdeshwar, 1988). There is almost unanimous agreement among researchers and practitioners that the preponderant form of a learning curve is either an S-shaped (Jordan, 1958; Carlson, 1973) or a power one as suggested by Wright (1936) (Alamri and Balkhi, 2007; Dar-El, 2000; Jaber, 2006).

##### 4.1. Varying rates

In this example we consider the following functions for varying demand, screening, defective and deterioration rates:

$$x(t) = at + b, D(t) = at + r, p_j = \frac{\tau}{\pi + e^{\gamma j}}, \delta_o(t) = \frac{l_o}{z_o - \beta_o t} \text{ and } \delta_r(t) = \frac{l_r}{z_r - \beta_r t},$$

where  $b, r, \pi, z_y > 0$ ;  $a, \alpha, l_y, \tau, \gamma, \beta_y, t \geq 0$  and  $\beta_y t < z_y$ .

The parameter “ $\alpha$ ”, represents the rate of change in the demand. The case of  $\alpha = 0$  corresponds to a constant demand rate, when then  $D(t) = r \forall t \geq 0$ . A similar behaviour holds true for the effect of



“ $a$ ”, the rate of change in the screening rate. Note that  $\delta_y(t)$  is an increasing function of time. The case of  $\beta_y = 0$  corresponds to a constant deterioration rate and  $l_y = 0$  reflects the case associated with no deterioration. The percentage defective per lot reduces according to an S-shaped logistic learning curve (Jordan, 1958; Carlson, 1973), where  $\tau$  and  $\pi$  are model parameters,  $\gamma$  is the learning exponent and  $j$  is the cycle index. The case  $\gamma = 0$  applies to a constant percentage of defective items per lot.

Each problem ( $m_{i2}$ ) has been coded in *MATLAB* for the above functions and solutions were obtained for a wide range of the control parameter values. Here, and for comparison purposes, we thematically consider situations with parameters that are presented in Table 1 below.

**Table 1.** Input parameters for varying rates.

$h_{og}$	$h_{od}$	$h_{rg}$	$h_{rd}$	$q_o$	$k$
20	5	25	5	2000	3000
Dollars/unit/year	Dollars/unit/year	Dollars/unit/year	Dollars/unit/year	Unit	Dollars/cycle
$a$	$b$	$\alpha$	$r$	$c$	$d$
1000	100200	500	50000	100	0.50
Unit/year	Unit/year	Unit/year	Unit/year	Dollars/unit	Dollars/unit
$l_o$	$l_r$	$z_o$	$z_r$	$\beta_o$	$\beta_r$
1	1	20	33.33	25	20
Unit/year	Unit/year	Unit/year	Unit/year	Unit/year	Unit/year
$s_o$	$s_r$	$\tau$	$\pi$	$\gamma$	
0.50	0.75	70.067	819.76	0.7932	
Dollars/unit	Dollars/unit	Unit/year	Unit/year	Unit/year	

Despite the fact that  $c_{ij}$  is formulated for the LIFO and FIFO models, its associated value is set to be equal to zero. That is, rather than assigning  $c_{ij}$  a specific value that would render the AIFO policy performing better than LIFO and FIFO, the ignorance of such a value implies that AIFO is optimal unless  $c_{ij} \leq \Delta_{ij}$ .

Now, let  $\Delta_{Lj} = \varepsilon h_{rg} + \frac{T_{Lj}^*(W_{Aj}^* - W_{Lj}^*)}{T_{Lj}^* - T_{LRj}^*}$  and  $\Delta_{Fj} = \varepsilon h_{og} + \frac{T_{FRj}^*(W_{Aj}^* - W_{Fj}^*)}{T_{FRj}^* - T_{Fj}^*}$ , where  $\varepsilon = 0.5$  denotes the minimum average inventory of one unit that can be stored in either unused warehouse. This appears realistic since we either store at least one unit, in the unused warehouse, or do not keep any (e.g. EOQ). On the other hand, an AIFO policy implies simultaneous consumption fractions associated with RW and OW, where the goods at both warehouses are depleted by the end of the same cycle, i.e.  $\varepsilon = 0$ . Thus,  $\Delta_{ij}$  constitutes KPI, i.e. an upper-bound (cost applied if OW (RW) is idle) that renders AIFO the

optimal dispatching policy. Note that as  $T_{ij}^* \rightarrow T_{iRj}^*$ , then  $\varepsilon \rightarrow 0 \Rightarrow W_{Aj}^* = W_{Lj}^* = W_{Fj}^* \Rightarrow \text{EOQ} \Rightarrow c_{ij} = 0 = \Delta_{ij}$  (recall  $m_{i2}$ ). Therefore, if  $W_{Lj}^* > W_{Fj}^*$  then  $c_{Fj}$  represents the cost incurred per year when the OW remains unusable (empty). Conversely, if  $W_{Lj}^* < W_{Fj}^*$  then  $c_{Lj}$  denotes the charge payable per year if the RW remains idle or the charge incurred per year in order to guarantee that the RW is available. As can be seen below, this cost is typically very small with respect to the minimum average holding cost per year incurred to store items in either warehouse.

**Table 2.** Optimal results for varying demand, screening, defective and deterioration rates.

$j$	$p_j$	$f_{rj}^*$	$f_j^*$	$f_{Rj}^*$	$Q_j^*$	$\omega_{oj}^*$	$\omega_{rj}^*$	$W_j^*$	$w_j^*$	Policy	$c_{ij}$ vs. $\phi_j^*$
1	0.08524	0.0121	0.0586	0.0221	3209	3.88	0.39	5618002	329280	<b>LIFO</b>	$c_{Lj} \leq \mathbf{1446}$
1	0.08524	0.0110	0.0365	0.0568	3107	1.78	1.45	5619757	318980	FIFO	$c_{Fj} = 0$
1	0.08524	0.0116	0.0577	-	3158	2.79	0.95	5618896	324160	AIFO	$\phi_j^* = 0.63$
2	0.08497	0.0121	0.0586	0.0221	3209	3.88	0.39	5616361	329210	<b>LIFO</b>	$c_{Lj} \leq \mathbf{1446}$
2	0.08497	0.0110	0.0366	0.0568	3106	1.78	1.45	5618116	318910	FIFO	$c_{Fj} = 0$
2	0.08497	0.0116	0.0577	-	3158	2.79	0.95	5617256	324120	AIFO	$\phi_j^* = 0.63$
3	0.08436	0.0120	0.0586	0.0221	3207	3.88	0.39	5612658	329050	<b>LIFO</b>	$c_{Lj} \leq \mathbf{1446}$
3	0.08436	0.0110	0.0366	0.0568	3104	1.79	1.45	5614413	318720	FIFO	$c_{Fj} = 0$
3	0.08436	0.0115	0.0577	-	3156	2.79	0.94	5613552	323920	AIFO	$\phi_j^* = 0.63$
4	0.08305	0.0120	0.0586	0.0221	3204	3.88	0.39	5604720	328700	<b>LIFO</b>	$c_{Lj} \leq \mathbf{1446}$
4	0.08305	0.0110	0.0366	0.0568	3100	1.79	1.45	5606477	318330	FIFO	$c_{Fj} = 0$
4	0.08305	0.0115	0.0577	-	3152	2.80	0.94	5605616	323540	AIFO	$\phi_j^* = 0.63$
5	0.08030	0.0119	0.0587	0.0220	3196	3.89	0.38	5588132	327970	<b>LIFO</b>	$c_{Lj} \leq \mathbf{1445}$
5	0.08030	0.0109	0.0367	0.0568	3092	1.80	1.44	5589892	317510	FIFO	$c_{Fj} = 0$
5	0.08030	0.0114	0.0577	-	3144	2.80	0.94	5589030	322760	AIFO	$\phi_j^* = 0.64$

Table 2 shows the effect of learning on the optimal values of  $Q_{ij}^*$ ,  $T_{rij}^*$ ,  $T_{Rij}^*$ ,  $T_{ij}^*$ ,  $\omega_{ij}^*$ ,  $\phi_j^*$  and the corresponding total minimum costs for 5 successive cycles. In the first cycle, the optimal order quantities for the three models are  $Q_{L1}^* = 3209$  units,  $Q_{F1}^* = 3107$  units and  $Q_{A1}^* = 3158$  units (and  $\phi_1^* = 0.63$ ), respectively. The corresponding total minimum costs per year are  $W_{L1}^* = 5618002$  dollars,  $W_{F1}^* = 5619757$  dollars and  $W_{A1}^* = 5618896$  dollars and the total minimum costs per cycle are  $w_{L1}^* = 329280$  dollars,  $w_{F1}^* = 318980$  dollars and  $w_{A1}^* = 324160$  dollars, respectively. The amount of deteriorated items is  $\omega_{i1}^*$ , which signifies the difference between the actual demand and the amount held in either warehouse at the beginning of the cycle. The amount of defective items is  $p_1 Q_{i1}^*$ , which can be sold at a salvage price at times  $T_{oi1}^*$  and  $T_{ri1}^*$  or incur a disposal penalty charge. As learning increases, i.e. the percentage of defective items per lot decreases, all

optimal quantities for the three models decrease except the amount of deteriorated items in the OW that experiences a minor increase due to the slight increase in the cycle length (Table 2).

Although the LIFO dispatching policy performs better than the AIFO policy, fact remains that the former policy ignores cost effects during the time elapsed between consuming the goods of the RW and the time by which the next sub-replenishment is delivered. On the other hand, the latter operates in a simultaneous consumption fashion at the OW and RW, i.e. the goods at both warehouses are depleted by the end of the same cycle. It should be noted that when a LIFO policy is considered, the idle time has been found to be significant for a wide range of the control parameter values. In this example,  $T_{L1}^* - T_{LR1}^* = 0.0586 - 0.0221 = 0.0365 \cong 13$  days, which constitutes more than 62% of the cycle length. That said, the RW remains idle for more than 227 days per year and free of charge, which is unrealistic and rare to encounter in practice. Thus, a LIFO dispatching policy is optimal if and only if the charge payable to keep the RW available is less than or equals the upper-bound, i.e.  $c_{L1} \leq \Delta_{L1} = 0.5h_{rg} + \frac{T_{L1}^*(W_{A1}^* - W_{L1}^*)}{T_{L1}^* - T_{LR1}^*} = 10 + 1436 = 1446$  dollars per year. Note that this cost is typically very small with respect to the average holding cost per year incurred to store items in the RW, which is given by  $\frac{T_{LR1}^* h_{rg}}{2T_{L1}^*} \times 1209 = 5702$  dollars per year, assuming also that  $h_{rd} = 0$ . This is so, since  $T_{L1}^* > T_{LR1}^*$ , i.e. there is a time gap (free of charge) between consecutive sub-replenishments that are delivered to the RW. If for instance  $T_{L1}^* = T_{LR1}^* \Rightarrow \text{EOQ}$ , then this cost increases to 15113 dollars per year. Therefore,  $c_{L1}$  denotes the cost per year incurred if no items are stored in the RW. Considering  $m_{L2}$  and Table 2, this cost is less than 53 dollars per cycle or less than 900 dollars per year. This can be further justified, if for instance this cost (e.g. 53 dollars) is included in the ordering cost applied for LIFO and setting  $c_{L1} = 0$ , then  $W_{A1}^* < W_{L1}^*$ . For a FIFO dispatching policy, the time elapsed for the OW to remain unusable is more than 7 days, which constitutes more than 36% of the cycle length, i.e. 130 days per year of an empty space. In many industrial situations, substantial portion of holding cost applies for an empty space as well. It should be noted that the AIFO dispatching policy not only overcomes this issue, but may also lead to a discounted holding cost that can be gained if a continuous and long-term rental contract is beneficial and hence further reduction in the total minimum cost per year can be achieved.

As illustrated in Table 2, other forms of varying demand, screening, defective and deterioration rates may be incorporated in each model in order to allow managers to assess the consequences of a diverse range of strategies.

In the next Section, we analyse the behaviour of the theoretical models in different settings, taking into account that the associated value of  $c_i$  is set to be equal to zero for every single case. Tables 3, 4, 5 and 6 depict the effect of each model parameter on the optimal values. Table 7 tests and compares the effect of learning when the maximum fulfilment of the capacity of OW is relaxed. Fig. 9 compares LIFO and AIFO for consecutive cycles in order to observe the effect of different learning curves on the optimal order quantities. Finally, Fig. 10 indicates the effect of different learning curves on the maximum rental cost associated with the RW, i.e.  $c_{Lj}$  (upper-bound).

#### 4.2. Sensitivity analysis

First, we note that for any  $s_o = s_r \geq 0$ , the optimal order quantity is identical for each model, which signifies the importance of considering differing transportation costs in the mathematical formulation of two-warehouse inventory models (Table 3). With this consideration in mind, further interesting insights can be obtained. For example, the dis-location of good and defective items significantly influences the optimal order quantity (Tables 3 and 4).

**Table 3.** Sensitivity analysis for transportation costs.

Parameter	$f_o^*$	$f_r^*$	$f^*$	$f_R^*$	$q_o^*$	$q_r^*$	$\omega_o^*$	$\omega_r^*$	$W^*$	Policy	$c_i$ vs. $\emptyset^*$
$s_o = 0.50$ $s_r = 0.75$	0.020	0.0121	0.0586	0.0221	2000	1209	3.88	0.39	5618002	<b>LIFO</b>	$c_L \leq 1446$
	0.020	0.0110	0.0365	0.0568	2000	1107	1.78	1.45	5619757	FIFO	$c_F = 0$
	0.020	0.0116	0.0577	-	2000	1158	2.79	0.95	5618896	AIFO	$\emptyset^* = 0.63$
$s_o = 0.50$ $s_r = 0.50$	0.020	0.0149	0.0638	0.0273	2000	1494	4.38	0.59	5612482	<b>LIFO</b>	$c_L \leq 1781$
	0.020	0.0140	0.0365	0.0621	2000	1402	1.78	1.95	5614482	FIFO	$c_F = 0$
	0.020	0.0145	0.0630	-	2000	1449	3.04	1.30	5613495	AIFO	$\emptyset^* = 0.58$
$s_o = s_r = 0$	0.020	0.0149	0.0638	0.0273	2000	1495	4.38	0.59	5585101	<b>LIFO</b>	$c_L \leq 1781$
	0.020	0.0140	0.0365	0.0621	2000	1402	1.78	1.95	5587111	FIFO	$c_F = 0$
	0.020	0.0145	0.0630	-	2000	1450	3.04	1.30	5586119	AIFO	$\emptyset^* = 0.58$
$s_o = 0.50$ $s_r = 0.40$	0.020	0.0152	0.0644	0.0279	2000	1527	4.44	0.62	5610115	<b>LIFO</b>	$c_L \leq 1828$
	0.020	0.0151	0.0365	0.0642	2000	1513	1.78	2.15	5612175	FIFO	$c_F = 0$
	0.020	0.0155	0.0650	-	2000	1558	3.14	1.44	5611145	AIFO	$\emptyset^* = 0.56$
$q_o^*$	0.016	0.0193	0.0647	0.0354	1609	1935	3.86	0.99	5609991	<b>LIFO</b>	$c_L \leq 2271$
$s_o = 0.50$	0	0.0338	0.0620	0.0620	0	3391	0	3.07	5611828	FIFO	$\Rightarrow$ EOQ
$s_r = 0.40$	0.014	0.0270	0.0639	-	1420	2078	2.17	1.90	5611016	AIFO	$\emptyset^* = 0.41$

Note that the assumption that the OW is fulfilled with the maximum capacity is indeed not the optimal choice for specific input parameters (Tables 3, 4, 5 and 7). Although such observation may appear to be counterintuitive, it is indeed an important observation for practitioners since the objective is to minimise the total system cost. Note that under FIFO and LIFO policies, it may become optimal that no

items are stored in the OW, i.e. the problem reduces to the EOQ (Tables 3, 4 and 7), which is consistent with the behaviour of outsourcing inventory holding through a vendor managed inventory (VMI) or other inventory intermediary arrangement (Table 6). Moreover, relaxing the inherent determinism of this assumption, not only produces better results, but may also reduce the value of the upper-bound significantly (Tables 4, 5 and 7).

**Table 4.** Sensitivity analysis for holding costs with  $s_o = s_r = 0$ .

Parameter	$f_o^*$	$f_r^*$	$f^*$	$f_R^*$	$q_o^*$	$q_r^*$	$\omega_o^*$	$\omega_r^*$	$W^*$	Policy	$c_i$ vs. $\emptyset^*$
$h_{og} = h_{od} = 20$ $h_{rg} = h_{rd} = 25$	0.020	0.0147	0.0634	0.0269	2000	1472	4.34	0.57	5585796	<b>LIFO</b>	$c_L \leq 1764$
	0.020	0.0138	0.0365	0.0618	2000	1381	1.78	1.91	5587789	FIFO	$c_F = 0$
	0.020	0.0142	0.0626	-	2000	1428	3.03	1.27	5586805	AIFO	$\emptyset^* = 0.58$
$h_{og} = h_{rg} = 20$	0.020	0.0172	0.0680	0.0315	2000	1724	4.79	0.79	5583382	<b>LIFO</b>	$c_L \leq 2198$
	0.020	0.0182	0.0365	0.0699	2000	1827	1.78	2.76	5581324	FIFO	$c_F = 0$
	0.020	0.0177	0.0690	-	2000	1777	3.33	1.75	5582364	AIFO	$\emptyset^* = 0.58$
$q_o^*$ $h_{og} = h_{rg} = 20$	0	0.0371	0.0679	0.0679	0	3719	0	3.69	5581411	LIFO	$\Rightarrow$ EOQ
	0.0103	0.0271	0.0188	0.0685	1030	2718	0.47	3.40	5580732	<b>FIFO</b>	$c_F \leq 886$
	0.0035	0.0337	0.0680	-	348	3373	0.56	3.34	5581367	AIFO	$\emptyset^* = 0.09$

**Table 5.** Sensitivity analysis for deterioration rates with  $s_o = s_r = 0$ .

Parameter	$f_o^*$	$f_r^*$	$f^*$	$f_R^*$	$q_o^*$	$q_r^*$	$\omega_o^*$	$\omega_r^*$	$W^*$	Policy	$c_i$ vs. $\emptyset^*$
$z_o = 10$ $z_r = 20$	0.020	0.0128	0.0599	0.0235	2000	1285	8.24	0.73	5593672	LIFO	$c_L = 0$
	0.020	0.0135	0.0365	0.0611	2000	1348	3.62	3.12	5592737	<b>FIFO</b>	$c_F \leq 1236$
	0.020	0.0132	0.0605	-	2000	1318	6.01	1.90	5593230	AIFO	$\emptyset^* = 0.60$
$q_o^*$ $z_o = 10$ $z_r = 20$	0.0097	0.0228	0.0595	0.0418	971	2288	4.84	2.33	5593342	LIFO	$c_L = 0$
	0.0134	0.0196	0.0245	0.0603	1343	1959	1.62	3.97	5592454	<b>FIFO</b>	$c_F \leq 741$
	0.0117	0.0210	0.0599	-	1173	2108	3.44	3.04	5592887	AIFO	$\emptyset^* = 0.36$
$\beta_o = \beta_r = 0$	0.020	0.0151	0.0641	0.0276	2000	1510	4.28	0.60	5584866	<b>LIFO</b>	$c_L \leq 1914$
	0.020	0.0140	0.0365	0.0622	2000	1406	1.76	1.93	5587011	FIFO	$c_F = 0$
	0.020	0.0146	0.0632	-	2000	1458	2.97	1.29	5585950	AIFO	$\emptyset^* = 0.58$
$l_o = l_r = 0$	0.020	0.0175	0.0687	0.0321	2000	1756	0	0	5576235	<b>LIFO</b>	$c_L \leq 3965$
	0.020	0.0155	0.0366	0.0651	2000	1558	0	0	5580380	FIFO	$c_F = 0$
	0.020	0.0165	0.0669	-	2000	1658	0	0	5578342	AIFO	$\emptyset^* = 0.55$

For equal holding costs and deterioration rates, the optimal order quantity for the three models is identical, i.e.  $c_i = 0$  (Table 6). This result is fundamental since not only shows the validity and robustness of the proposed models, but also underpins and portrays the value added for integrating the upper-bound in the mathematical formulations. The same behaviour is observed in Table 6 when  $f_o = 0$ , i.e. all models are reduced to that of Alamri et al. (2016).

**Table 6.** Sensitivity analysis for special cases of the general models with  $s_o = s_r = 0$ .

Parameter	$f_o^*$	$f_r^*$	$f^*$	$f_R^*$	$q_o^*$	$q_r^*$	$\omega_o^*$	$\omega_r^*$	$W^*$	Policy	$c_i$ vs. $\emptyset^*$
$\alpha = -500$	0.020	0.0162	0.0662	0.0297	2000	1525	4.61	0.70	5581490	<b>LIFO</b>	$c_L \leq 1939$
	0.020	0.0153	0.0366	0.0645	2000	1530	1.78	2.19	5583594	FIFO	$c_F = 0$
	0.020	0.0158	0.0654	-	2000	1579	3.16	1.47	5586805	AIFO	$\emptyset^* = 0.56$
$l_o = l_r = 0$ $\alpha = 0$ $a = 0$	0.020	0.0183	0.0701	0.0335	2000	1833	0	0	5574310	<b>LIFO</b>	$c_L \leq 4142$
	0.020	0.0163	0.0365	0.0664	2000	1631	0	0	5578555	FIFO	$c_F = 0$
	0.020	0.0173	0.0683	-	2000	1733	0	0	5576466	AIFO	$\emptyset^* = 0.54$
$h_{og} = h_{rg} = 20$ $z_r = z_o = 20$ $\beta_o = \beta_r = 25$	0.020	0.0161	0.0659	0.0295	2000	1611	4.59	1.15	5584203	LIFO	$c_L = 0$
	0.020	0.0161	0.0365	0.0659	2000	1611	1.78	3.96	5584203	FIFO	$c_F = 0$
	0.020	0.0161	0.0659	-	2000	1611	3.19	2.55	5584203	AIFO	$\emptyset^* = 0.55$
$h_{og} = h_{rg} = 20$ $z_r = z_o = 20$ $\beta_o = \beta_r = 25$ $f_o = 0 \Rightarrow \text{EOQ}$	0	0.0354	0.0648	0.0648	0	3550 <sup>a</sup>	0	5.4	5585464	LIFO	$c_L = 0$
	0	0.0354	0.0648	0.0648	0	3550 <sup>a</sup>	0	5.4	5585464	FIFO	$c_F = 0$
	0	0.0354	0.0648	-	0	3550 <sup>a</sup>	0	5.4	5585464	AIFO	$\emptyset^* = 0$

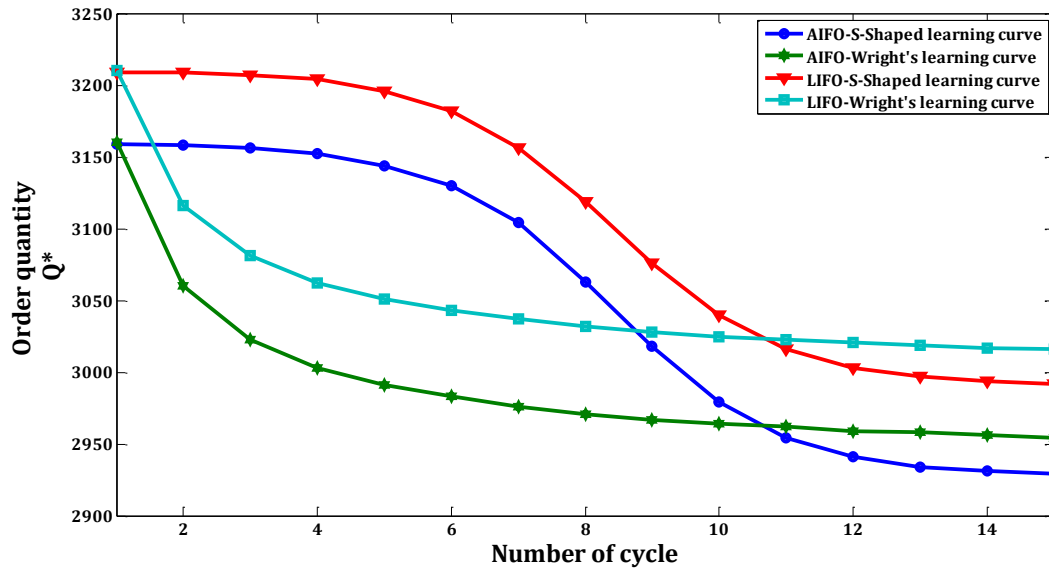
<sup>a</sup>The order quantity as in Alamri et al. (2016).

Table 7 replicates the first two rows of Table 5 for two consecutive cycles in order to observe the effect of Wright's learning curve, i.e.  $p_j = \frac{\tau}{\pi+1}j^{-\gamma}$  on the optimal order quantity when the capacity of the OW is a decision variable with that of fixed value. It is worth noting that the results presented in Table 2 reveal that the reduction in the optimal order quantity does not affect the OW. That said, although the OW may benefit from the VOI that reduces the defective items per lot, it still keeps the maximum capacity of goods, and consequently the effect of learning does not really apply here. On the other hand, this is not the case when relaxing the inherent determinism of the maximum capacity associated with OW. In particular, such relaxation implies comprehensive learning that can be achieved simultaneously, i.e. the amounts that are allocated to both the OW and RW are affected. (Table 7). However, and despite the fact that the lot size may reduce for each successive replenishment, the amount that is allocated to the OW either remains static (due to capacity restriction) or experiences further reduction, but the amount that is allocated to the RW may decrease (increase) subject to the input parameter. This is a key observation, which demonstrates the impact of learning on the operational and financial performance of an inventory system with a two-level storage.

Fig. 9 compares the optimal order quantity of AIFO with that of LIFO for 15 consecutive cycles with respect to  $p_j = \frac{\tau}{\pi+e^{\gamma j}}$  (Jordan, 1958; Carlson, 1973) and  $p_j = \frac{\tau}{\pi+1}j^{-\gamma}$  (Wright, 1936). The same behaviour observed in Fig. 9 holds true for the total minimum cost per year, which can be further justified by the reduction gained in the maximum rental cost per year (upper-bound) (Fig. 10). The S-shaped logistic learning curve generates greater quantities in the incipient phase, which is consistent with the behaviour of slow improvement that is observed in practice (Dar-El, 2000).

**Table 7.** The effect of Wright’s learning curve on variable capacity of the OW with  $s_o = s_r = 0$ .

Parameter	$j$	$f_{oj}^*$	$f_{rj}^*$	$f_j^*$	$f_{Rj}^*$	$q_{oj}^*$	$q_{rj}^*$	$\omega_{oj}^*$	$\omega_{rj}^*$	$W_j^*$	Policy	$c_{ij}$ vs. $\emptyset_j^*$
$z_o = 10$ $z_r = 20$	1	0.020	0.0128	0.0599	0.0235	2000	1285	8.24	0.73	5593672	LIFO	$c_{Lj} = 0$
	1	0.020	0.0135	0.0365	0.0611	2000	1348	3.62	3.12	5592737	FIFO	$c_{Fj} \leq 1236$
	1	0.020	0.0132	0.0605	-	2000	1318	6.01	1.90	5593230	AIFO	$\emptyset_j^* = 0.60$
$z_o = 10$ $z_r = 20$	2	0.020	0.0119	0.0604	0.0226	2000	1189	8.45	0.66	5384686	LIFO	$c_{Lj} = 0$
	2	0.020	0.0124	0.0379	0.0616	2000	1246	3.83	3.04	5383925	FIFO	$c_{Fj} \leq 1075$
	2	0.020	0.0122	0.0610	-	2000	1219	6.22	1.82	5384334	AIFO	$\emptyset_j^* = 0.62$
$q_o^*$ $z_o = 10$ $z_r = 20$	1	0.0097	0.0228	0.0595	0.0418	971	2288	4.84	2.33	5593342	LIFO	$c_{Lj} = 0$
	1	0.0134	0.0196	0.0245	0.0603	1343	1959	1.62	3.97	5592454	FIFO	$c_{Fj} \leq 741$
	1	0.0117	0.0210	0.0599	-	1173	2108	3.44	3.04	5592887	AIFO	$\emptyset_j^* = 0.36$
$q_o^*$ $z_o = 10$ $z_r = 20$	2	0	0.0319	0.0606	0.0606	0	3196	0	4.83	5384265	LIFO	$\Rightarrow$ EOQ
	2	0.0123	0.0197	0.0235	0.0610	1237	1977	1.45	4.11	5383677	FIFO	$c_{Fj} \leq 552$
	2	0.0093	0.0226	0.0607	-	936	2262	2.86	3.39	5384011	AIFO	$\emptyset_j^* = 0.29$



**Fig. 9.** A comparison of the optimal lot sizes of AIFO and LIFO for S-shaped and Power learning curves.

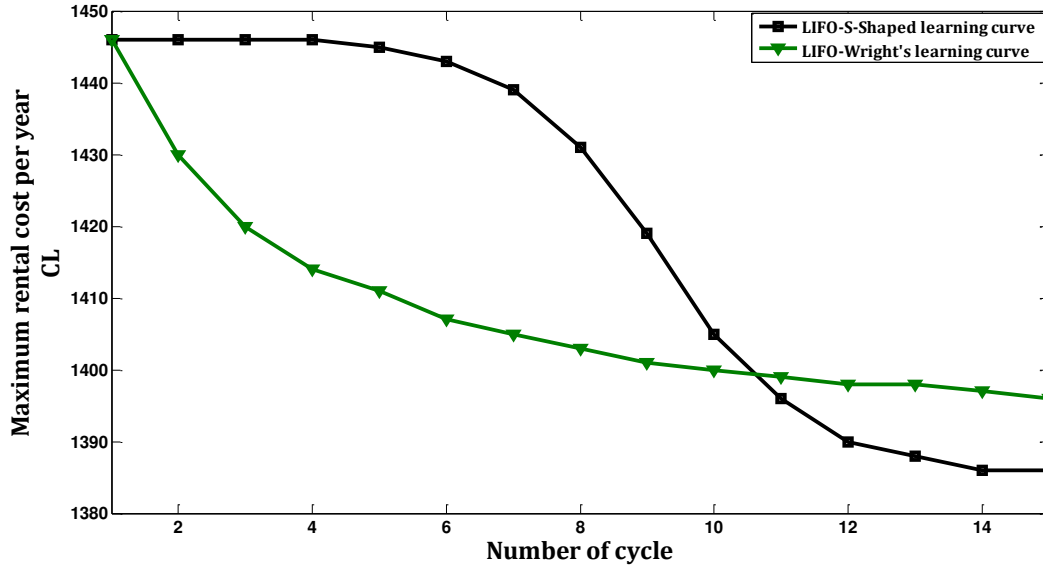


Fig. 10. A comparison of the maximum rental cost per year for S-shaped and Power learning curves.

The proposed models are not limited to the above contributions; they may trigger other applications that can be disseminated from the general formulation as shown in Sections 4.3. and 4.4 below.

### 4.3. Perishable products and lifetime constraints

The implications of the inspection process in inventory decision-making can be further explored to accommodate an inventory system with a two-level storage. Specifically, a lot of size  $Q_{ij} = q_{oij} + q_{rij}$  is delivered such that a quantity of size  $q_{oij}$  is allocated to the OW and the remaining amount of size  $q_{rij} = Q_{ij} - q_{oij}$  is allocated to the RW. The assumption that each sub-replenishment that enters the RW undertakes a 100 per cent screening would imply that  $\underline{q}_{rij} = (q_{rmj}, q_{rm-1j}, \dots, q_{r0j})$  where  $q_{rkj}$  is the number of units with  $k(k = 0, 1, \dots, m)$  useful periods of shelf lifetime. Here,  $q_{r0j}$  denotes newly replenished items that have arrived already perished or items not satisfying certain quality standards (defective items). A similar argument holds true for the quantity  $q_{oij}$  that is allocated to the OW. Although no buyer would pay for defective and already perished items, they would surely be interested in seeing a reduction in the presence of such quantities in subsequent replenishments. Our formulation allows for an immediate disposal of the amount  $q_{r0j}$  and  $q_{o0j}$  in case of any potential safety issues, i.e.  $h_{od} = h_{rd} = 0$ . Now, let  $\omega_{rkj}$  denote the quantity of the on-hand inventory of shelf lifetime  $k$  that perishes by the end of period  $k$  in RW.

Thus, we have



$$\omega_{rkj} = \begin{cases} q_{rkj} - [D_{kj} - (\sum_{n=1}^{k-1} q_{rnj} - \sum_{n=1}^{k-1} \omega_{rnj} - \sum_{n=1}^k d_{rnj})]^+ \\ 0 & \text{otherwise,} \end{cases}$$

where  $D_{kj} < (\sum_{n=1}^k q_{rnj} - \sum_{n=1}^{k-1} \omega_{rnj} - \sum_{n=1}^k d_{rnj})$  is the actual demand observed up to the periodic review  $k$ , and  $d_{rkj}$  is the number of items of shelf lifetime  $k$  that deteriorate in RW while on storage. Hence,  $\sum_{y=k}^m d_{ryj}$  denotes the total sum of deteriorated items in RW in period  $k$ , i.e. an item may not retain the same utility throughout its shelf lifetime, and consequently  $\sum_{k=1}^m \omega_{rkj}$  refers to the total sum of inventory in RW that perishes in cycle  $j$ , excluding any replenished items that have arrived already perished. A similar argument holds true for the quantity  $\omega_{okj}$  that perishes in OW. It is important to note that if LIFO or FIFO are considered, then  $\sum_{k=1}^m \omega_{rkj} + \sum_{k=1}^m \omega_{okj}$  is likely greater than that experienced under the AIFO policy. This can be justified by the marginal difference in cycle length (Sections 4.1 and 4.2) and the fact that under the LIFO and FIFO policies, only one warehouse is utilised at a time. This is an important issue, especially in the case when a distinct selling price  $v_k$  may be linked to its corresponding quantity  $q_{kj}$ , i.e.  $\underline{V} = (v_m, v_{m-1}, \dots, v_0)$  is applied for the set  $\underline{Q}_{ij} = (q_{mj}, q_{m-1j}, \dots, q_{0j})$ . Therefore, our formulation is viable if, for instance, an item partially loses its value based on its perceived actuality (obsolescence).

The above discussion underpins and demonstrates how the terms ‘deterioration’, ‘perishability’ and ‘obsolescence’ may collectively apply to an item. Note that  $m_{i2}$  can still be used to drive the optimal quantity that needs to be added to the on-hand inventory for the next replenishment, i.e.  $q_{ikj} = Q_{ij+1} - I_{ogij}(t_{kj}) - I_{rgij}(t_{kj})$ , where  $t_{kj}$  denotes the time up to the periodic review. This relation holds true for Sections 4.1 and 4.2, i.e.  $q_j = Q_j - I_{ogj-1}(T_{j-1}) - I_{rgj-1}(T_{j-1})$  for AIFO and LIFO and  $q_j = Q_j - I_{ogj-1}(T_{Rj-1}) - I_{rgj-1}(T_{Rj-1})$  for FIFO; please refer to Alamri et al. (2016) for a discussion on this issue.

As an example of lifetime constraint, we can assume that  $T$  denotes the remaining shelf lifetime of an item and  $^{\circ}C_y$  and  $t_y$  represent, respectively, the temperature and time elapsed of an item in a supply chain entity  $y$ . Then we have  $T = M - \zeta(^{\circ}C_a)t_a - \zeta(^{\circ}C_b)t_b$ , where  $\zeta(^{\circ}C_y) = (0.1^{\circ}C_y + 1)^2$  and  $M = m + t_a + t_b$  (Bremner, 1984; Ronsivalli and Charm, 1975). In this case,  $f_j \leq T$  for AIFO and LIFO and  $f_{Rj} \leq T$  for FIFO, and consequently, the VOI can be quite valuable in reducing the cost per cycle (Alamri et al., 2016; Ketzenberg et al., 2015). Note that  $z_y = 1 + T$  can fit here as well.

#### 4.4. Stochastic parameters

Let  $\underline{X}_j$  refers to a set of random variables that are predetermined according to the VOI gained by the system due to its coordination as an output of the  $j^{th}$  inspection process. Suppose that  $\underline{X}_j \sim U[\underline{Y}_j - \sqrt{3}\underline{Z}_j, \underline{Y}_j + \sqrt{3}\underline{Z}_j]$ . Note that  $E(\underline{X}_j) = \underline{Y}_j$ , i.e. if  $D_j \sim U[\mu_j - \sqrt{3}\sigma_j, \mu_j + \sqrt{3}\sigma_j]$ , then  $E(D_j) = \mu_j = D(t) = r$ . A similar argument holds true for other input parameters. Note that  $\underline{X}_j$  and hence the actual yield of may vary from one cycle to another (e.g. the parameters are nonstationary).

Hence, we have  $E(W_i) = \frac{E(w_i)}{E(f_{ij})} = \frac{E(w'_{q_{rj}})}{E(f'_{j,q_{rj}})}$ , for AIFO and LIFO, and  $E(W_F) = \frac{E(w_F)}{E(f_{Rj})} = \frac{E(w'_{q_{rj}})}{E(f'_{Rj,q_{rj}})}$ , for FIFO, where  $(1 - E[p_j])x(t) > D(t)$ .

In the example provided by Jaggie et al. (2015) they have assumed  $E[p] = 0.02, q_o = 800, h_{og} = h_{od} = 5, h_{rg} = h_{rd} = 7, \beta_o = \beta_r = 0, c = 25, k = 100, r = 50000, l_o = l_r = 1, z_o = 3.33, z_r = 5, \alpha = 0$  and  $b = 175200$ , resulting in  $Q^{J et al} = 915$  units. This quantity is greater than our optimal  $Q_L^* = 909$  units and  $w_L^* = 1312381$  dollars. However,  $Q_F^* = 943$  units and  $w_F^* = 1312315$ , and consequently FIFO performs better than LIFO. Moreover, if  $q_o$  is taken as a decision variable, then  $q_o^* = 0, Q_L^* = 920$  units and  $w_L^* = 1312126$  dollars  $\Rightarrow$  the solution of FIFO does not exist. This result is consistent with the results obtained in Tables 3, 4, 5 and 7 and seems realistic given that the objective is to minimise the total system cost. Setting  $l_o = l_r = 0$ , the result is identical with that of Chung et al. (2009) and Jaggie et al. (2015), where  $Q_L^* = Q^{C et al} = Q^{J et al} = 1290$  units.

#### 4.5. Key findings

In this Section, we emphasise the key findings of our work and relate the results of the study to the general body of knowledge in the discipline.

- For any  $s_o = s_r \geq 0$ , the optimal order quantity is identical for each model, which signifies the importance of considering differing transportation costs in the mathematical formulation of two-warehouse inventory models.
- The dis-location of good and defective items significantly influences the optimal order quantity.
- The assumption that the OW is fulfilled with the maximum capacity may not be the optimal choice for specific input parameters.
- Under FIFO and LIFO policies, it may become optimal that no items are stored in the OW, i.e. the problem reduces to the EOQ, which is consistent with the behaviour of outsourcing inventory holding through a VMI or other inventory intermediary arrangement.
- For equal holding costs and deterioration rates, the optimal order quantity for the three models

is identical, i.e.  $c_i = 0$ , which implies that LIFO (AIFO) is optimal if and only if  $c_i = 0$ . This finding is fundamental, since it not only shows the validity and robustness of the proposed models, but also underpins and portrays the value added for integrating the upper-bound in the mathematical formulations.

- When  $f_o = 0$ , all models are reduced to a single-level warehouse base model presented by Alamri et al. (2016). In this case,  $c_i = 0$ , which also shows the validity and robustness of the proposed models.
- Relaxing the inherent determinism of the maximum capacity associated with OW implies comprehensive learning that can be achieved simultaneously.
- The optimal order quantity and the total minimum costs per year follow the same fashion as that of  $p_j$ .
- Previously published models in this area are shown to be special cases of our models (Table 6 and Appendices A, B and C in the electronic companion).

#### **4.6. Summary of implications and managerial insights**

In this Section, we highlight a summary of the implications and managerial insights related to our research contributions.

- The versatile nature of our models allows the incorporation of the desired functions that are suitable to a system. Consequently, the list of implications and managerial insights outlined in Alamri et al. (2016) fit our models as well.
- Each model emerges as a viable solution that manages and controls the efficient and cost-effective flow of perishable and non-perishable products.
- General solution procedures for LIFO, FIFO and AIFO to determine the optimal dispatching policy for continuous intra-cycle periodic review applications for two-level storage EOQ models are presented.
- A detailed method to illustrate how the terms deterioration, perishability and obsolescence may collectively affect inventories in a two-level storage is explored.
- The accuracy of RFID temperature tags that capture the TTH, and the use of that TTH data are adopted to model the shelf lifetime of an item under LIFO, FIFO and AIFO dispatching policies.
- The mathematical formulations are linked to the renewal theory, which have led to further interesting insights.
- The trade-off between the three policies constitutes a key business objective in supply chain management.
- In the case of managing perishable products, LIFO and FIFO may not be the right dispatching

policies, given that the order quantity needs to be consumed based on a FEFO policy. This is so, since under the LIFO or FIFO dispatching policies, the total sum of inventory that perishes in each cycle is likely greater than that experienced under the AIFO policy.

- The dimension of risk influencing the management of perishable products may increase if, for instance, a distinct selling price is linked to its corresponding quantity with a distinct useful period of shelf lifetime.
- Under an AIFO dispatching policy, a discounted holding cost can be gained if a continuous and long-term rental contract is beneficial and hence further reduction in the total minimum cost per year can be achieved.

## **5. Conclusion and further research**

In this paper, we have been concerned with the implications of dispatching policies associated with a two-level storage, where each lot is subjected to a 100 per cent screening. Three general EOQ models for items with imperfect quality were presented and compared, and it has been shown that the solution to each inventory model, if it exists, is unique and global optimal. The first model underlies LIFO, the second model underlies FIFO and the third model relates to simultaneous consumption fractions associated with OW and RW and is entitled Allocation-In-Fraction-Out (AIFO). Items not conforming to certain quality standards are isolated in a separate facility with different holding costs of the good and defective items being considered.

Under an AIFO dispatching policy, the cycle length is the same for both OW and RW, and consequently the upper-bound (cost applied if OW (RW) is idle) that renders AIFO the optimal dispatching policy has also been provided. When a LIFO (FIFO) policy is considered, the idle time has been found to be significant for a wide range of the control parameter values and free of charge, which is unrealistic and rare to encounter in practice. An AIFO dispatching policy not only overcomes this issue, but may also lead to a discounted holding cost. We have shown that the upper-bound is typically very small with respect to the minimum average holding cost per year incurred to store items in either warehouse.

The analytical results illustrate the impact of considering different transportation costs associated with OW and RW as well as the incorporation of varying demand, deterioration, defective and screening rates on the optimal order quantity.

This study is viable for fixed and random lifetimes of perishable products, where VOI is adopted to model a shelf lifetime of an item. The versatile nature of our models and the fact that they may reflect

a diverse range of strategies has been emphasised, where the validity of the general models are ascertained, i.e. the solution is the same as in published sources or in some cases produces better results. To the best of our knowledge, this seems to be the first time that such an Allocation-In-Fraction-Out (AIFO) policy is presented, which necessitates a general formulation of LIFO and FIFO EOQ models for investigation and numerical comparison purposes.

Further research can be addressed for finite or infinite planning horizons that may include extensions such as allowing for shortages, considering that the screening rate follows learning and forgetting curves and the risk of failure during screening (Type I and Type II errors). In addition, it seems plausible to assess the formulation of EOQ models for a two-level storage to consider multiple items or to study the effect of different supplier trade credit practices.

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