Word Embedding as Maximum A Posteriori Estimation[*]

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Abstract

The GloVe word embedding model relies on solving a global optimization problem, which can be reformulated as a maximum likelihood estimation problem. In this paper, we propose to generalize this approach to word embedding by considering parametrized variants of the GloVe model and incorporating priors on these parameters. To demonstrate the usefulness of this approach, we consider a word embedding model in which each context word is associated with a corresponding variance, intuitively encoding how informative it is. Using our framework, we can then learn these variances together with the resulting word vectors in a unified way. We experimentally show that the resulting word embedding models outperform GloVe, as well as many popular alternatives.

Introduction

Word embedding models learn low-dimensional vector representation of words based on co-occurrence information obtained from some large text corpus. The aim of such models is to learn representations which capture word similarities, analogies, and other lexical relationships. Various word embedding models have already been proposed in the literature, including approaches inspired by neural language models, such as Skipgram (SG) and the Continuous Bag-of-Word (CBOW) model (Mikolov et al. 2013), regression based models, such as the least squares regression model GloVe (Pennington, Socher, and Manning 2014) and the ordinal regression model from (Jameel and Schockaert 2017) and different kinds of probabilistic models, such as Gaussian embeddings (Vilnis and McCallum 2014), a Bayesian version of Skipgram (Barkan 2017) and an approach using Dirichlet-Multinomial language models (D-GloVe) (Jameel and Schockaert 2016).

Word embeddings play a crucial role in deep learning approaches to natural language processing tasks, as they allow for a natural way to represent textual input to neural networks. Word embeddings have similarly transformed related fields such as information retrieval (Zamani and Croft 2017), (Dehghani et al. 2017), (Ensan 2018), knowledge base completion (Yang, Tang, and Cohen 2016), (Wang et al. 2014), (Zhong et al. 2015) and recommender systems (Musto et al. 2015), (Zhao et al. 2017).

The approach we follow in this paper is based on GloVe (Pennington, Socher, and Manning 2014). This model aims to find two vector representations, denoted by $\mathbf{w_i}$ and $\tilde{\mathbf{w}}_\mathbf{i}$, and two bias terms b_i and $\tilde{b}_i$ for each word i such that $\mathbf{w_i} \cdot \tilde{\mathbf{w}}_\mathbf{j} + b_i + \tilde{b}_j$ approximates $\log x_{ij}$, with x_{ij} the number of times words i and j co-occur[1] that co-occurrence counts are weighted based on how closely together the words i and j appear. Given GloVe's formulation as a least squares regression problem, it can be naturally interpreted in terms of maximum likelihood estimation. Specifically, it is easy to see that the basic[2] GloVe objective is equivalent to maximizing the following expression:

$$\prod_{i,j} \mathcal{N}(\log x_{ij}; \mathbf{w_i} \cdot \tilde{\mathbf{w}}_\mathbf{j} + b_i + \tilde{b}_j, \sigma^2) \qquad (1)$$

where we write $\mathcal{N}(.; \mu, \sigma^2)$ for the Normal distribution with mean μ and variance σ^2. The variance σ^2 encodes how closely we expect $\mathbf{w_i} \cdot \tilde{\mathbf{w}}_\mathbf{j} + b_i + \tilde{b}_j$ to approximate $\log(x_{ij})$. In GloVe, this variance is assumed to be fixed for every word pair (i, j). In such cases, σ^2 can be chosen arbitrarily, as the choice of σ^2 does then not affect which word vectors and bias terms maximize (1). Intuitively, however, it seems desirable to use a variance σ_j^2 which depends on how informative the context word j is: if j is a highly informative context word, it should have a strong influence on the word vector representation of i and thus we intuitively want the associated variance σ_j^2 to be low. On the other hand, if j is a stop word, we may not want it to influence the word vector representation of i at all, and thus want σ_j^2 to be high.

The method we develop in this paper will allow us to learn a suitable variance σ_j^2 for each context word j. To this end, we will put priors on these variances, and maximize the resulting posterior distribution instead of (1), which will al-

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[1]As usual, we will assume throughout the paper
[2]The full GloVe model also includes a weight $f(x_{ij})$, which we do not consider here for simplicity.

low us to jointly learn the informativeness of each context word and the actual word embedding. More generally, by treating the problem of learning a word embedding as maximum a posteriori (MAP) estimation, rather than a maximum likelihood estimation problem, a wide range of parametrized variants of GloVe could be considered.

To the best of our knowledge, the idea of using priors to learn parametrized word embedding models has not yet been considered. However, the importance of priors has been extensively studied in the context of probabilistic topic models (Wallach, Mimno, and McCallum 2009). In particular, the widely used Latent Dirichlet Allocation (LDA) (Blei, Ng, and Jordan 2003) model can be seen as a variant of probabilistic Latent Semantic Analysis (pLSA) (Hofmann 1999) which includes priors. In this sense, the relationship between our proposed approach and the GloVe model is somewhat analogous to the difference between LDA and pLSA.

Related Work

Probabilistic Word Embedding Models. While, to the best our knowledge, our MAP based generalization of GloVe has not yet been considered, a number of probabilistic models for generating word embeddings have already been studied. One approach, called Gaussian embeddings, was introduced by Vilnis et al. (Vilnis and McCallum 2014) and further developed in (Chacón 2016). The main underlying idea is to represent words using Gaussian distributions, with the aim of capturing the diversity of word meaning, although this idea has also been applied in applications such as collaborative filtering (Dos Santos, Piwowarski, and Gallinari 2017) and knowledge graph embedding (He et al. 2015). Intuitively, the Gaussian representation of a given word w can be viewed as a soft region in the word embedding space, encoding the kinds of contexts in which the word may appear. One of the main underlying motivations is that such models should be better suited for modelling hypernymy. Note that Gaussians (with diagonal covariance matrices) are used in this approach to compactly represent regions, rather than for capturing uncertainty. Non-probabilistic representations for representing words as regions have been considered as well. For instance, (Jameel and Schockaert 2017) used ordinal SVM regression with a quadratic kernel to learn word regions. These approaches clearly differ from our work, as we still represent words using vectors, but use a probabilistic formulation to find these vectors.

Bayesian versions of Skipgram have been studied in (Barkan 2017) and (Bravzinskas, Havrylov, and Titov 2016), which respectively rely on standard variational techniques and on variational autoencoders for performing inference. The aim of such approaches is to replace point estimates by distributions of word vectors. While the representation of words as probability distributions is similar as in Gaussian embeddings, the motivation is slightly different: in the latter approach the Gaussian distributions are themselves viewed as representations of the words, whereas the Bayesian models assume that words are represented as vectors, but they model uncertainty about the true vector representations of the words. Still, experiments in (Bravzinskas, Havrylov, and Titov 2016) show that such Bayesian models can be used

in a similar way as Gaussian embeddings, with similar levels of performance. Our work is different in that we are using maximum a posteriori estimates, rather than aiming to characterize the full posterior distributions, which means that we can keep our models highly efficient. The motivation of our work is also different: whereas the aforementioned Bayesian Skipgram models aim at characterizing the uncertainty of word vector representations, our aim is to develop parametrized generalizations of the GloVe model.

Our model is also related to the work of (Jameel and Schockaert 2016), which introduced the idea that variance, in the probabilistic formulation of GloVe, can be related to the informativeness of context words. However, they estimate this informativeness in a heuristic way from GloVe embeddings that are initially trained in the standard way. In contrast, our work focuses on the role of priors for jointly learning the parameters of word embedding models and the resulting word vectors, and we use the idea of learning suitable variances to illustrate the potential of our setting. Another idea proposed in (Jameel and Schockaert 2016) is to use a Dirichlet-Multinomial language model to smooth co-occurrence counts and to estimate the randomness of these counts, to reduce the impact of infrequent words.

Probabilistic Topic Models. Probabilistic topic models (PTMs) are document representations based on latent variables, which intuitively represent topics. While conceptually different from word embeddings, they have served as an inspiration for this paper, in particular regarding the importance of priors in such models.

One of the key reasons for using priors in PTMs has been to avoid overfitting. This was motivated by the fact that the pLSA model, which is one of the first PTMs, was found to be prone to overfitting, even when using the tempered EM algorithm (Popescul, Pennock, and Lawrence 2001). In their seminal work, (Blei, Ng, and Jordan 2003) argue that the introduction of priors in their LDA model helps overcome this problem of overfitting. Similarly, in this paper, the introduction of priors will allow us to avoid overfitting when considering parametrized variants of the GloVe model.

The use of priors has also made it possible to consider various extensions of the LDA model. Some examples include the use of authorship (Rosen-Zvi et al. 2004), temporal information (Wang, McCallum, and Wei 2007) or class labels (Zhu, Ahmed, and Xing 2012), (Zhu et al. 2014). To the best of our knowledge, similar extensions to word embeddings have not yet been studied, although it is worth mentioning that several hybrid models have been proposed that combine topic and word embedding models (Das, Zaheer, and Dyer 2015), (Li et al. 2016), (Shi et al. 2017).

Model Description

In this section we describe our word embedding model, which is based on Maximum A Posteriori (MAP) estimation. From a conceptual point of view, one important difference with GloVe is that we associate a variance σ_j^2 with each word j, intuitively encoding how strongly the number of co-occurrences x_{ij} of words i and j should influence the word vector representation of i. These variances σ_j^2 are essentially

estimated from the data, i.e. from the co-occurrence statistics x_{ij} that are derived from some text corpus. However, the model will also incorporate a prior over these variances, which intuitively allows us to prevent overfitting.

Let us write τ_i for the tuple $(\mathbf{w_i}, \tilde{\mathbf{w}}_\mathbf{i}, b_i, \tilde{b}_i)$, D (the co-occurrence statistics from) for the given document collection and V for the vocabulary. Our word embedding model has the following general form:

$$P(\mathbf{m}, \mathbf{s}, \mathbf{t}, \psi | D) \propto P(D|\mathbf{m}, \mathbf{s})P(\mathbf{m}, \mathbf{s}|\mathbf{t}, \psi)P(\mathbf{t})P(\psi)$$

where the vector $\mathbf{m} = (\mu_1, ..., \mu_{|V|^2})$ associates a mean μ_{ij} with each word pair (i, j), $\mathbf{s} = (\sigma_1^2, ..., \sigma_{|V|}^2)$ associates a variance σ_i^2 with each word i, and $\mathbf{t} = (\tau_1, ..., \tau_{|V|})$ contains the usual word embedding parameters. The role of ψ will be explained below. For simplicity, we will assume a uniform prior on $\mathbf{t}$ and ψ, and thus not consider the probabilities $P(\mathbf{t})$ and $P(\psi)$ in the remainder. The probability $P(D|\mathbf{m}, \mathbf{s})$ is evaluated similarly to the maximum likelihood formulation of the GloVe model (1). In particular, we assume that the model defines a Normal distribution for each pair of words (i, j), which acts as a prediction for $\log x_{ij}$:

$$P(D|\mathbf{m}, \mathbf{s}) = \prod_{\substack{i,j \\ x_{ij} \neq 0}} \mathcal{N}(\log x_{ij}; \mu_{ij}, \sigma_j^2) \qquad (2)$$

where the product ranges over all pairs (i, j) for which $x_{ij} > 0$. Note that differently from the GloVe model, in our case the mean μ_{ij} is not directly determined from the word embedding parameters. Instead, we consider a probability distribution over possible values of μ_{ij}, which will be determined by the parameters of the word embedding model (see below). We refer to the model corresponding to (2) as WeMAP (for "Word Embedding as Maximum A Posteriori estimation").

We will also consider two variants to (2). First, the GloVe model uses a weighting function f to reduce the impact of rare words. It is defined as $f(x_{ij}) = (\frac{x_{ij}}{x_{max}})^\alpha$ if $x_{ij} \leq x_{max}$ and $f(x_{ij}) = 1$ otherwise, where usually $\alpha = 0.75$ and $x_{max} = 100$ are chosen. This weighting function can be taken into account in the probabilistic formulation (WeMAP1) as follows:

$$P(D|\mathbf{m}, \mathbf{s}) = \prod_{\substack{i,j \\ x_{ij} \neq 0}} \mathcal{N}(\log x_{ij}; \mu_{ij}, \sigma_j^2)^{f(x_{ij})} \qquad (3)$$

For the second variant (WeMAP2), $\log x_{ij}$ is replaced by the Pointwise Mutual Information (PMI) between occurrences of i and j. Let $\theta > 0$ be the smoothing parameter. To estimate this PMI, we use Bayesian smoothing as follows:

$$pmi(i, j) = \log\left(\frac{p_{ij}}{p_i \cdot p_j}\right)$$

$$p_{ij} = \frac{x_{ij} + \theta}{x_{**} + |V|^2\theta} \qquad p_i = \frac{x_{i*} + \theta}{x_{**} + |V|\theta}$$

$$x_{i*} = x_{*i} = \sum_{j \in V} x_{ij} \qquad x_{**} = \sum_{i,j \in V} x_{ij}$$

This leads to the following variant of (2):

$$P(D|\mathbf{m}, \mathbf{s}) = \prod_{i,j} \mathcal{N}(pmi(i, j); \mu_{ij}, \sigma_j^2) \qquad (4)$$

One advantage of using this smoothed version of PMI is that $pmi(i, j)$ is also defined if $x_{ij} = 0$, which allows us to consider negative examples. In particular, the product in (4) could, in principle, range over any word pair (i, j), regardless of whether $x_{ij} = 0$. In practice, the resulting quadratic time complexity would be prohibitive, hence we limit the number of pairs for which $x_{ij} = 0$ to a small sample, similar to how negative samples are used in e.g. the Skipgram model (Mikolov et al. 2013). In our experiments with this variant, for each j, we set the number N_j of negative examples (i.e. the number of randomly sampled words i for which $x_{ij} = 0$) as $N_j = 2 \cdot |\{i \mid x_{ij} \neq 0\}|$.

The probability $P(\mathbf{m}, \mathbf{s}|\mathbf{t}, \psi)$ acts as a prior on the means in $\mathbf{m}$ and variances in $\mathbf{s}$. It is defined using the Normal Inverse Gamma (NIG) distribution, which is the usual conjugate prior for the normal distribution (Murphy 2007). This is a probability distribution over pairs (μ, σ^2), which is defined as the product of a Normal distribution and an Inverse Gamma (IG) distribution. In particular, we evaluate $P(\mathbf{m}, \mathbf{s}|\mathbf{t}, \psi)$ as follows[3]:

$$P(\mathbf{m}, \mathbf{s}|\mathbf{t}, \psi) = \prod_{\substack{i,j \\ x_{ij} \neq 0}} NIG(\mu_{ij}, \sigma_j^2; \mu_{ij}^0, \lambda, \alpha, \beta) \qquad (5)$$

The NIG distributions in (5) have four parameters. The parameter μ_{ij}^0 defines our prior belief about the mean. This key parameter is where the parameters of the actual word embedding come into play:

$$\mu_{ij}^0 = \mathbf{w_i} \cdot \tilde{\mathbf{w}}_\mathbf{j} + b_i + \tilde{b}_j$$

The parameters α and β are respectively the shape and scale parameters of the Inverse Gamma distribution, and λ is a precision parameter which controls how much μ_{ij} can deviate from our prior belief μ_{ij}^0. Rather than treating these parameters as encoding our prior beliefs, as in typical probabilistic models, we will assume that these parameters α, β and λ are encoded in the vector $\psi = (\alpha, \beta, \lambda)$. In particular, in our MAP formulation, these parameters will then be chosen such that they maximize the posterior probability $P(\mathbf{m}, \mathbf{s}, \mathbf{t}, \psi | D)$.

Putting everything together, we learn a word embedding model by maximizing the following:

$$\prod_{\substack{i,j \\ x_{ij} \neq 0}} \mathcal{N}(\log x_{ij}; \mu_{ij}, \sigma_j^2) NIG(\mu_{ij}, \sigma_j^2; \mu_{ij}^0, \lambda, \alpha, \beta)$$

Because the NIG distribution is a conjugate prior of the Normal distribution, this expression simplifies to a product of NIG distributions, which will make the computations easier.

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[3] For the ease of presentation, we will assume that variant (2) or (3) of the likelihood function is used; for variant (4), $P(\mathbf{m}, \mathbf{s}|\mathbf{t}, \theta)$ is defined in the same way, except that the range of the product then also includes the negative examples.

Note that we do not need to perform any probabilistic inference, as we end up with an optimization problem, which we can simply solve using SGD. Compared to Bayesian inference methods, this MAP approach has the advantage of being computationally much easier. Interestingly, it was recently found in (Mandt, Hoffman, and Blei 2017) that there are also relationships between approximate Bayesian inference and gradient methods.

Experiments

In this section, we present a series of experiments in which we compare our model with popular and recent state-of-the-art word embedding and topic models, all of which can be classified as dimension reduction methods. Topic models such as LDA are an interesting baseline to additionally consider in our experiments given the important role they have played in our motivation. Apart from some common intrinsic evaluation tasks for word embeddings, we also consider three extrinsic evaluation tasks: text classification, document retrieval and named entity recognition (NER).

Methodology

Corpora: We have considered the May 2018 dump of the English Wikipedia. We pre-processed the dump using a publicly available script[4], which yielded 4.3 billion tokens. After removing tokens that appear fewer than 500 times in the collection, we ended up with a vocabulary of 117835 distinct tokens. We have lower-cased and removed punctuations. We have considered a symmetric context window size of 10 words, which is a common choice that has been shown to give good results (Pennington, Socher, and Manning 2014).

Intrinsic evaluation tasks: We have considered three standard evaluation tasks for word embedding models. First, we considered three analogy datasets: the Google Word Analogy dataset[5], the Microsoft Research Syntactic Analogies Dataset (MSR)[6], and the BATS 3.0 dataset[7]. Second, we considered 14 word similarity datasets[8], and finally two outlier detection datasets[9].

We formatted the BATS 3.0 dataset to be similar to the Google and MSR word analogy datasets, so that we can use the same evaluation script for all word analogy experiments[10]. The BATS 3.0 dataset has 4 superclasses: "Inflectional Morphology (DI)", "Derivational Morphology (DM)", "Encyclopedia Semantics (ES)", and "Lexicographic Semantics (LS)". Within each superclass there are

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[4] https://github.com/facebookresearch/fastText
[5] https://aclweb.org/aclwiki/Analogy_(State_of_the_art)
[6] https://aclweb.org/aclwiki/Syntactic_Analogies_(State_of_the_art)
[7] http://vsm.blackbird.pw/bats
[8] https://github.com/mfaruqui/eval-word-vectors
[9] http://lcl.uniroma1.it/outlier-detection/ and https://github.com/belph/wiki-sem-500
[10] We have made this formatted dataset along with codes which created them available here: https://bit.ly/2J5MtXj

10 subclasses consisting of 50 unique word pairs. We compute average accuracy for each of these 10 subclasses and report results for the superclass.

Evaluation results were obtained using publicly available implementations. For instance, word analogy results for the Google dataset were obtained using the code from the GloVe project[11], word similarity results were obtained using the VSMLib tool[12], and outlier detection results were also obtained from available code[13]. We share our code, pre-processing scripts and datasets online[14]. We have indexed the word similarity datasets as follows: EN-MC-30 (E1), EN-MEN-TR-3k (E2), EN-MTURK-287 (E3), EN-MTURK-771 (E4), EN-RG-65 (E5), EN-RW-Stanford (E6), EN-SIMLEX-999 (E8), EN-Verb-143 (S8), EN-WS-353-ALL (E9), EN-WS-353-REL (E10), EN-WS-353-SIM (E11), EN-YP-130 (E12), SimVerb-3500 (E13), and RareWords[15] (E14). For outlier detection, we use O1 to refer to the dataset by (Camacho-Collados and Navigli 2016) and O2 for the WikiSem-500 dataset (Blair, Merhav, and Barry 2016).

Extrinsic evaluation tasks: We have used the benchmark TREC WT2G[16] dataset for document retrieval which comes with associated TREC topics (queries) and document annotations. We represent each document and each query as the average of all the words via their corresponding vectors. Then, given a query, the similarity can be modeled as cosine similarity between the document vector and the query vector. For document retrieval tasks, the method of only using embeddings is a weak ranker (Mitra et al. 2016), (Nalisnick et al. 2016). Following Mitra et al. (2016), we use a weighted average of the similarities between embeddings and the Okapi BM25 model (Robertson and Walker 1994). We have used Elasticsearch[17] to construct the index and to retrieve the related documents using the default BM25 model (Robertson and Walker 1994).

For document classification, we used the standard Reuters (Reu52 and Reu8), 20Newsgroups (20NG), TechTC300[18] (TTC), and WebKB (WKB) collections, which are available online[19] in preprocessed form. We also considered two sentence classification datasets from the Text-Top-Model[19] project: the Movie Review Polarity dataset (MRP) and the Subjectivity dataset (SUB). For the named entity recognition (NER) task, we have used the CoNLL-2003 English benchmark dataset[20]. To generate the document classifi-

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[11] https://github.com/stanfordnlp/GloVe
[12] http://vsm.blackbird.pw/tools
[13] http://lcl.uniroma1.it/outlier-detection/
[14] https://bit.ly/2J5MtXj
[15] https://nlp.stanford.edu/~lmthang/morphoNLM/
[16] http://ir.dcs.gla.ac.uk/wiki/Terrier/WT2G
[17] https://github.com/elastic/elasticsearch
[18] http://techtc.cs.technion.ac.il/techtc300/techtc300.html
[19] https://github.com/nadbordrozd/text-top-model
[20] https://gist.github.com/JackNhat/

Table 1: Word analogy results in Accuracy

	GSem	GSyn	Avg.	MSR	DI	DM	ES	LS
SG	71.58	60.50	65.45	51.71	55.45	13.48	08.78	67.11
CBOW	64.81	47.39	55.17	45.33	50.58	10.11	07.02	76.43
Gauss	66.33	51.67	58.22	41.38	45.17	12.87	07.90	**90.32**
GloVe	78.85	62.81	69.97	53.04	55.21	14.82	10.56	88.13
D-GloVe	82.34	62.84	71.55	54.47	55.82	13.39	10.22	77.45
MM (L)	78.89	62.86	70.02	53.54	53.79	14.48	09.09	76.91
MM (Q)	80.80	**63.49**	71.22	54.47	52.20	12.07	10.24	74.73
NWE	78.84	62.76	71.55	54.26	55.04	13.93	09.18	68.02
SV	57.11	39.25	47.22	29.16	47.01	12.01	08.43	60.21
SVD	73.19	50.29	60.51	42.47	54.97	12.07	10.32	67.12
NMF	62.09	31.50	45.16	27.11	43.91	11.69	09.05	52.55
LDA	59.28	38.16	47.59	29.21	43.69	11.45	08.82	51.54
HDP	63.72	36.97	48.91	28.67	45.02	11.56	08.66	55.76
WeMAP	83.50	63.01	72.16	**55.08**	56.02	**14.95**	10.61	**90.32**
WeMAP1	83.50	63.02	72.18	**55.08**	56.02	**14.95**	10.62	**90.32**
WeMAP2	**83.52**	63.08	**72.19**	**55.08**	**56.03**	**14.95**	10.62	**90.32**

cation results we have used the Text-Top-Model[19] project, which has implementations for many text classification algorithms, as well as code for automatic tuning and model selection. In particular, we used the Convolutional Neural Network (CNN) model from Keras for document classification, and the CNN-based model described in (Kim 2014) for sentence classification. For NER we have used the model described in (Chiu and Nichols 2016) with its online implementation[21]. This model takes word embeddings as input. We used the default settings and trained the model for 50 epochs.

Baseline Models: We compare our method with several popular and strong baseline models: GloVe (Pennington, Socher, and Manning 2014), Skipgram (SG) (Mikolov et al. 2013), Continuous Bag-of-Words (CBOW) (Mikolov et al. 2013), Gaussian word embeddings (Gauss) (Vilnis and McCallum 2014), D-Glove (Jameel and Schockaert 2016), maximum-margin embeddings (MM) (Jameel and Schockaert 2017), both in their linear (denoted by L) and quadratic (denoted by Q) versions, SemanticVectors (SV)[22] (Widdows and Ferraro 2008), Singular Value Decomposition (SVD) (Golub and Reinsch 1970), Non-Negative matrix factorization (NMF) (Lee and Seung 2001), LDA topic models (Blei, Ng, and Jordan 2003), and their non-parametric counterpart called Hierarchical Dirichlet Processes (HDP) (Teh et al. 2005), and finally the Bayesian extension to the SG model proposed in (Barkan 2017) (NWE). Note that SVD and NMF are applied to a PMI-weighted word-word co-occurrence matrix. The topic models LDA and HDP represent topics as probability distributions over words. We then represent a word as a vector containing the probability of that word in each of the topics.

Parameter selection: All models have some free parame-

ters that need to be tuned. For datasets that have pre-defined tuning and testing splits, we used these standard splits. For the other datasets, we randomly selected 20% as tuning data, and we report results on the remaining 80%. The number of dimensions for each model was selected from {50, 100, 300, 400}. For CBOW and SG, we chose the number of negative samples from a pool of {1, 5, 10, 15}. For GloVe, we selected the $x_{\max}$ value from {10, 50, 100} and α from {0.1, 0.25, 0.5, 0.75, 1}. For the Gaussian word embedding approach, we used the spherical Gaussians with KL-divergence, which gave better results than the diagonal model in our experiments. For D-GloVe, we selected the Dirichlet prior constant from {0.0001, 0.001, 0.01, 0.1, 1000, 2000, 5000, 8000}. For WeMAP2, we selected θ from {0.1, 0.01, 0.001, 0.0001, 0.00001, 0.000001}. The number of iterations for all word embedding models was fixed to 20 and the number of posterior inference iterations for all topic models was fixed to 1000. We have also experimented with the different number of iterations for different models, and in each case we found that no major changes occurred after 20 iterations for the word embedding models and 1000 iterations for the topic models. We also experimented with different learning rate parameters, namely {0.01, 0.001, 0.0001, 0.00001}. For the variational inference-based LDA model, we began with the default hyper-prior values hard-coded in the implementation[23], which were later updated after each iteration by the sampler, and we did same for the HDP[24] model. Note that parameter selection was done for all tasks, both for the baselines and for the proposed models. The only hyperparameters of WeMAP that need to be tuned are the number of dimensions and the learning rate for SGD. In particular, WeMAP requires less tuning than Skip-Gram (where the parameters of the negative sampling strategy need to be tuned) and GloVe (where the parameters of the $f(x_{ij})$ weighting function need to be tuned). WeMAP1 uses the $f(x_{ij})$ function from GloVe and thus shares the same hyperparameters as GloVe. We have found that 0.75 for alpha

[21]https://github.com/kamalkraj/
Named-Entity-Recognition-with-\
Bidirectional-LSTM-CNNs

[22]https://github.com/semanticvectors/
semanticvectors/wiki

[23]http://www.cs.columbia.edu/~blei/lda-c/

[24]https://github.com/blei-lab/hdp

Table 2: Word similarity results in Spearman's ρ

	E1	E2	E3	E4	E5	E6	E7	E8	E9	E10	E11	E12	E13	E14	Avg.
SG	0.741	0.742	0.651	0.653	0.757	0.470	0.356	0.289	0.662	0.643	0.726	0.565	0.195	0.470	0.566
CBOW	0.727	0.615	0.637	0.555	0.639	0.419	0.279	0.307	0.618	0.563	0.682	0.227	0.168	0.419	0.490
Gauss	0.632	0.710	0.650	0.620	0.695	0.436	0.283	0.273	0.624	0.601	0.690	0.510	0.147	0.436	0.522
GloVe	0.739	0.746	0.648	0.651	0.752	0.473	0.347	0.308	0.675	0.659	0.732	0.582	0.184	0.422	0.566
D-GloVe	0.750	0.746	0.652	**0.659**	0.779	0.474	0.347	0.315	0.677	0.659	0.735	0.579	0.188	0.473	0.574
MM (L)	0.740	0.737	0.656	0.651	0.764	0.465	0.339	0.296	0.663	0.649	0.726	0.554	0.186	0.465	0.564
MM (Q)	**0.770**	0.742	0.649	0.658	**0.779**	**0.480**	0.356	0.289	**0.688**	**0.672**	**0.751**	0.565	0.196	0.470	0.576
NWE	0.745	0.677	0.627	0.585	0.737	0.450	0.360	**0.334**	0.681	0.638	0.749	0.487	**0.212**	0.450	0.552
SV	0.653	0.671	0.632	0.599	0.591	0.393	0.245	0.276	0.582	0.555	0.672	0.421	0.127	0.393	0.486
SVD	0.606	0.697	0.644	0.620	0.674	0.427	0.279	0.246	0.605	0.596	0.676	0.502	0.141	0.427	0.510
NMF	0.589	0.617	0.618	0.568	0.537	0.324	0.228	0.290	0.537	0.525	0.615	0.372	0.106	0.324	0.446
LDA	0.643	0.657	0.630	0.595	0.592	0.376	0.242	0.261	0.569	0.558	0.655	0.413	0.122	0.376	0.478
HDP	0.632	0.646	0.627	0.584	0.571	0.358	0.234	0.274	0.563	0.548	0.648	0.421	0.113	0.358	0.470
WeMAP	0.764	0.751	0.651	0.657	0.777	0.470	**0.361**	0.303	0.682	0.663	0.746	0.592	0.196	**0.480**	0.578
WeMAP1	0.766	0.751	0.651	0.657	0.777	0.470	**0.361**	0.303	0.683	0.663	0.746	0.592	0.196	**0.480**	0.578
WeMAP2	0.769	**0.752**	**0.657**	**0.659**	**0.779**	0.472	**0.361**	0.303	0.684	0.663	0.748	**0.593**	0.196	**0.480**	**0.580**

Table 3: Outlier detection.

	O1		O2	
	Acc	OPP	Acc	OPP
SG	59.375	90.430	30.466	70.794
CBOW	57.813	89.258	30.645	70.399
Gauss	60.938	90.625	37.957	74.354
GloVe	62.500	91.016	39.677	75.541
D-GloVe	**67.188**	91.797	**39.749**	75.555
MM (L)	59.375	89.844	38.530	74.645
MM (Q)	65.625	91.211	38.280	74.881
NWE	62.500	91.406	33.441	71.409
SV	57.813	87.695	29.677	69.951
SVD	57.813	88.281	35.197	72.938
NMF	57.813	87.500	27.814	68.490
LDA	56.250	86.719	28.244	68.242
HDP	59.375	88.281	27.993	68.759
WeMAP	**67.188**	**92.578**	39.642	**75.720**
WeMAP1	64.063	92.188	**39.749**	75.169
WeMAP2	**67.188**	**92.578**	39.677	75.541

Figure 1: Run time comparisons with competitive comparative methods (CPU hours) for 5 iterations and 300 dimensional vectors.

and $x_{max} = 100$ gave us good results most of the time for WeMAP1 which is also true for GloVe. For WeMAP2 we also need to tune a smoothing parameter, and most of the time $\theta = 0.00001$ gave good results. In our three models, a learning rate of 0.001 consistently gave good results. We have also observed that 300 dimensions almost always led to the best results.

Results

Intrinsic Evaluation We first present the results for traditional word embedding tasks. They are summarized in Table 1 for the analogy datasets, Table 2 for the word similarity datasets, and Table 3 for the outlier detection datasets. For the analogy datasets, our model clearly and consistently outperforms all of the baselines, with the only exception being the syntactic fragment of the Google dataset, where MM(Q) is marginally better. "Avg." measures the overall performance on the Google dataset in which WeMAP2 performs the best. For outli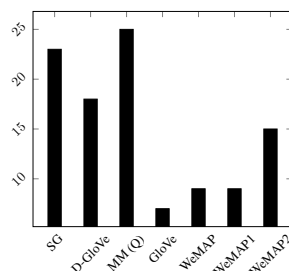er detection, our model also achieves the best results, where only D-GloVe is marginally better than some variants of our model on O2 in terms of accuracy. Note that we measure outlier detection performance using Accuracy and Outlier Position Percentage (OPP). The goal of OPP is to reflect the position of the outlier word w.r.t. to other similar words, where the outlier detection problem is thus evaluated as a ranking problem. These metrics are defined in more detail in (Blair, Merhav, and Barry 2016) and (Camacho-Collados and Navigli 2016).

For word similarity, on average our models outperform the baselines, although the improvement is not consistent across all datasets. Note that our model consistently outperforms the popular SG model. On most datasets, our model also outperforms GloVe and D-GloVe, which are the most closely related models.

We also conducted a running time analysis in CPU hours in Figure 1, where we can see that our model runs faster than most comparative models. This experiment was performed on 3.20 GHz machine with 25 threads.

Extrinsic Evaluation We summarize the document retrieval results in Figure 2. While the differences between the models are small, due to the fact that they all rely on BM25, our model achieved the best performance. In Table 4, we summarize the text classification results. For this ex-
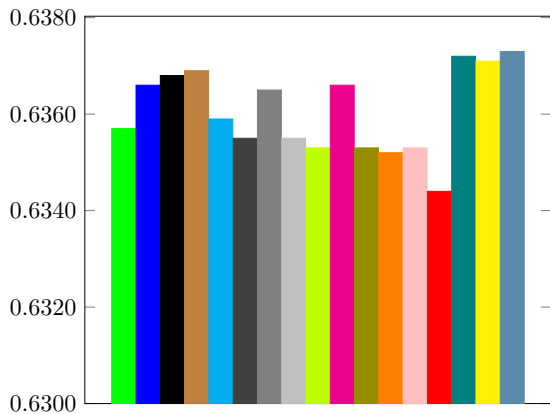
Figure 2: Mean Average Precision (MAP) scores on WT2G. In the figure, starting from left, colour - - - refers to SG, - - - CBOW, - - - Gauss, - - - GloVe, - - - D-GloVe, - - - MM (L), - - - MM (Q), - - - NWE, - - - SV, - - - SVD, - - - NMF, - - - LDA, - - - HDP, - - - Okapi BM25, - - - WeMAP, - - - WeMAP1, and - - - WeMAP2.

Table 4: Text classification results (F1 score).

	TTC	20NG	Reu52	Reu8	WKB	MRP	SUB
SG	0.336	0.469	0.396	**0.720**	0.846	0.763	**0.879**
CBOW	0.313	0.382	0.385	0.483	0.800	0.716	0.813
Gauss	0.243	0.366	0.312	0.566	0.796	0.729	0.868
GloVe	0.297	0.480	0.426	**0.720**	0.846	0.771	**0.879**
D-GloVe	0.331	0.459	0.312	0.671	0.737	0.740	0.810
MM (L)	0.294	0.417	0.428	0.485	0.796	0.761	**0.879**
MM (Q)	0.331	0.322	0.312	0.485	0.786	0.753	0.811
NWE	0.333	0.440	0.394	0.485	0.784	0.762	0.808
SV	0.332	0.397	0.420	0.573	0.702	0.700	0.803
SVD	0.335	0.377	0.413	0.628	0.771	0.701	0.809
NMF	0.334	0.397	0.412	0.484	0.750	0.698	0.802
LDA	0.334	0.417	0.412	0.485	0.746	0.655	0.808
HDP	0.333	0.446	0.412	0.587	0.740	0.638	0.801
SVM (L)	0.309	0.480	0.444	0.700	0.846	0.765	0.874
SVM (R)	0.334	0.426	0.444	0.684	0.825	0.766	**0.879**
SVM (Q)	0.335	0.365	**0.460**	**0.720**	0.846	0.767	**0.879**
MLP	0.334	0.435	0.396	0.673	0.737	0.769	0.815
MultNB	0.335	0.447	0.429	0.688	0.737	0.770	0.861
BernNB	0.330	0.377	0.392	0.484	0.737	0.770	0.814
WeMAP	0.338	0.468	0.434	**0.720**	0.847	0.774	**0.879**
WeMAP1	0.334	0.469	0.446	0.713	**0.847**	**0.776**	**0.879**
WeMAP2	**0.355**	**0.481**	0.444	**0.720**	0.842	0.773	**0.879**

Table 5: NER results showing Precision (Prec.), Recall (Rec.) and F-measure (F1).

	Prec.	Rec.	F1
SG	0.871	**0.883**	**0.877**
CBOW	0.853	0.871	0.862
Gauss	0.864	0.880	0.872
GloVe	0.868	0.879	0.874
D-GloVe	0.865	0.877	0.871
MM (L)	0.869	0.879	0.874
MM (Q)	0.862	0.874	0.868
NWE	0.862	0.880	0.874
SV	0.859	0.876	0.867
SVD	0.859	0.873	0.866
NMF	0.844	0.861	0.853
LDA	0.832	0.849	0.840
HDP	0.839	0.856	0.847
WeMAP	0.874	0.878	0.876
WeMAP1	**0.876**	0.878	**0.877**
WeMAP2	0.871	0.879	0.875

periment, we also include some text classification baselines which use standard bag-of-words representations with tf-idf weighting. In particular, SVM (L), SVM (R) and SVM (Q) are SVM classifiers which respectively use a linear, RBF and quadratic kernel, MLP is a Multi-layer Perceptron classifier, MultiNB is a Naive Bayes classifier for Multinomial models, and BernNB is a Naive Bayes classifier for multivariate Bernoulli models. In document classification task, our model outperforms all baselines, except for the Reuters-52 (Reu52) dataset, where an SVM classifier using the bag-of-words representations achieved the best performance. Note, however, that for this dataset our model is still the best performing word embedding model. On the sentence classification datasets, our model again achieves the best performance (although many models achieve the same performance on SUB). In the NER task, as shown in Table 5, our model performs on par with SG in terms of F1, and it outperforms the other models.

Conclusions

In this paper, we have introduced a probabilistic word embedding model, in which word vectors are learned by finding the parameters that maximize a posterior probability. Despite its probabilistic formulation, our model learns standard word vectors, in contrast to e.g. Bayesian versions of Skipgram, which aim to learn word representations that take the form of probability distributions. The most immediate practical benefit of our model is the fact that it allows us to associate a separate variance with each word. In this way, we are able to implicitly take into account the fact that some words are more informative than others. We presented experimental results that showed our model to consistently perform well across a wide range of tasks, while remaining computationally efficient.

The setting we have developed in this paper opens up several promising avenues for future work. For example, we could straightforwardly extend our model with informed priors on the word vectors, e.g. to encode prior knowledge from a lexicon. Moreover, the use of priors may allow us to introduce additional parameters without overfitting. For example, in (Jameel and Schockaert 2017) a quadratic regression based word embedding model was found to outperform its linear counterpart. Similarly, we may replace the linear regression based formulation of GloVe with a model in which each context word is represented as a quadratic mapping, where priors could be used to encourage the model to stay close to being linear to avoid overfitting in the case of rare context words.

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