Essays on Trend Inflation, Nominal Rigidity, and Optimal Monetary Policy

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ABSTRACT

To some extent, the common assumption of zero-inflation steady state in modern macro models is theoretically flawed, empirically unfounded, and practically inconsistent. I build a medium-scale Generalised New Keynesian (GNK) model based on non-zero inflation steady state in order to study inflation persistence and the effect of trend inflation. This GNK model exhibits much more dynamics than a standard NK model in several ways: a) NKPC becomes flatter and more forward looking as trend inflation increases; b) price dispersion introduces huge inertia into the model as trend rises; c) backward looking feature is also present in the market wage equation, even though optimal reset wage is more forward looking. This model is then estimated using a Bayesian technique with quarterly US data from 1970 to 2017. The estimation results show the model is capable of capturing macro evidence in the postwar US, and annual trend inflation is estimated to be around 3 percent. Simulations show trend inflation does not generate significant alterations to macroeconomic dynamics under a moderately high degree of indexation, and this is consistent with the literature. However, once indexation is switched off, trend inflation alters the model dynamics in a very significant way: 1) in general, output and inflation fluctuate much more heavily with higher trend inflation after most shocks; 2) inflation exhibits a hump-shaped response with four percent trend inflation or above after a transitory monetary policy shock; 3) inflation reacts less on impact but becomes much more persistent with higher trend inflation after a monetary shock. The welfare loss, timing of the maximum effect and inflation persistence after a monetary policy shock provide very important implications to both economists and policy-makers.

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Introduction

The Background Story

Since the 1990s, there has been a large volume of macroeconomic literature that focuses on using New Keynesian dynamic stochastic general equilibrium (DSGE, henceforth) models to study business cycle and monetary policy (Clarida, Galí and Gertler (1999); Goodfriend and King (1997); Rotemberg and Woodford (1997)), these also include central banks in the advanced economies. In the majority of these researches, the economy is modelled based on the assumption that inflation is zero in the long run steady state, and equations are therefore approximated linearly around a zero inflation steady state. This has become a standard practice among New Keynesian macroeconomists.

Apparently, there are motivations why macroeconomists tend to and have been using models that are log-linearised around a zero inflation steady state for monetary policy research, the primary reasons can be summarised into three categories. First, there is a great temptation in terms of practicality, in other words, analytical convenience, as suggested by Ascari and Ropele (2007). Second, the zero inflation or nearly zero inflation status is optimal in a cashless economy, and some literatures have indeed shown this (see Goodfriend and King (2001); Woodford (2003); Schmitt-Grohé and Uribe (2007a), Schmitt-Grohé and Uribe (2010)). Third, the above zero inflation targeting only emerged in the late 1990s and did not become so popular among leading central banks in industrialised countries until the European Central Bank (ECB) formally adopted it in early 2000s, hence, the issue only started to attract practical attention since then. However, the zero or nearly zero inflation optimal stabilisation requires some very special conditions both in terms of microeconomic assumptions and the interaction between monetary and fiscal policies (see Schmitt-Grohé and Uribe (2007a), Schmitt-Grohé and Uribe (2007b)). Any deviation from these assumptions could lead to the optimal inflation rate significantly departing from zero.

Given all the above motivations of modelling macroeconomic DSGE models with zero inflation steady state, I believe there are plenty of appealing reasons to look at the roles of positive and moderate trend inflation in these contemporary New Keynesian models, which are designed for stabilisation policy studies. By positive I mean any steady state inflation above zero; by moderate I mean the net annual rate of trend inflation is around 2 to 6 percent. First of all, the assumption of zero long run inflation is unrealistic for the period that macroeconomists and policy makers have been studying for stabilisation policy: the post World World Two time in the United States. Cogley and Sbordone (2008) uses a VAR approach and their estimation results show the trend inflation level in the post-war US is considerably above zero, and it was well above 4 percent during the *Great Inflation* time in the 1970s. Second, even though studies show near zero inflation is optimal under some special conditions, leading central banks in the advanced economies (including central banks from some emerging economies) suggest zero inflation is not their long term target (the Federal Reserve, the ECB, the Bank of England, the Bank of Japan). Table 1 gives a brief look at the targets of inflation targeting central banks around the world.

Starting from the industrialised countries, the Federal Reserve, the ECB, the Bank of England and the Bank of Japan that are usually regarded as four most important monetary authorities in the world, all set their long term inflation target at 2 percent. The same target is also shared by the central banks of Sweden, Norway, and Switzerland [1]. There are three banks in the advanced economies that tolerate inflation within a certain band; namely Australia, New Zealand and Canada. Therefore, it is more than clear that all the major central banks in advanced economies do not view zero inflation as their objectives, with 2 percent as the most popular choice.

[1] In the Swiss case, the central bank targets 2 percent as a maximum not a midpoint.

Table 1: Inflation Targets of Major Central Banks

Central Banks	Inflation Targets (percentage)
Developed Economies	
Federal Reserve	2
European Central Bank	2
Bank of England	2
Bank of Japan	2
Reserve Bank of Australia	2-3
Reserve Bank of New Zealand	1-3
Sveriges Riksbank (Sweden)	2
Norges Bank (Norway)	2
Swiss National Bank	<2
Bank of Canada	1-3
Emerging Economies	
Bank of Korea	2
Czech National Bank	2
Banco Central do Brasil	4.5
Banco Central de Chile	2-4
Banco de la República Colombia	3 ± 1
South African Reserve Bank	3-6
National Bank of Poland	2.5 ± 1

Source: various central bank websites, see Appendix C

For the emerging economies, these figures tend to be even higher, the central bank of Brazil has a target of 4.5 percent. Both the central banks of Chile and South Africa target a band, with the former pursues 2 to 4 percent, and the latter chases 3 to 6 percent. Both Poland and Colombia set their target to 2.5 and 3.5 percent, while permitting plus or minus one percent departure from the target. The targets are relatively lower in the Czech Republic and

South Korea, but are still significantly above zero. Again, no central bank that conducts explicit inflation targeting actually target zero. Thus, the assumption of central bankers should model the economy based zero inflation steady state for monetary stabilisation policy studies is clearly unreasonable and misleading.

The overlook of positive inflation steady state (or trend inflation, the two are interchangeable throughout this thesis) creates both theoretical flaw and practical inconsistency. Fortunately, this issue started to attract academic attention in the research of monetary economics in the last decade. Early researches can trace back to Ascari (2004), Amano, Ambler and Rebei (2007), and Ascari and Ropele (2007). More recent breakthrough attributes to Cogley and Sbordone (2008), Ascari and Ropele (2009), Coibion and Gorodnichenko (2011), Ascari and Sbordone (2014), and Ascari, Phaneuf and Sims (2018).

The Issues

The major issues with regards to the presence of trend inflation and the difference it can make to modern New Keynesian models that documented in existing literature include the following: first, the presence of trend inflation alters the macroeconomic dynamics of various macro variables and optimal monetary response.

Second, due to the presence of trend inflation, the probability of satisfying determinacy drops considerably as trend inflation rises, this is true even under different monetary policy set-up, extensive discussions can be found from Ascari and Ropele (2009), and Coibion and Gorodnichenko (2011). Therefore, conclusions from the literature on optimal stabilisation that assume zero steady state inflation may become invalid. Under certain parametrisation, by satisfying the *Taylor principle*, which is defined as a more than one-to-one response of nominal interest rate to inflation change, is not a sufficient condition for the model to satisfy determinacy condition. In particular, Coibion and Gorodnichenko (2011) finds that failure to satisfy the Taylor principle is not the only reason that the Federal Reserve failed to anchor inflation during the *Great Inflation* time. As a result of the higher level of trend inflation experienced in the 1970s, which causes the determinacy region to shrink dramatically, the Fed's policy was well inside the indeterminacy region for much of the 1970s and it required the Fed to respond much more aggressively to inflation fluctuations than it was required had trend inflation not been so high.

Third, as a result of non-zero inflation steady state, for central banks that are inside the determinacy region, the optimal monetary stabilisation policy also change. This is well-documented in Ascari and Ropele (2007), and Coibion and Gorodnichenko (2011). In particular, the latter finds that under moderate trend inflation, interest smoothing, aggressive response to output growth, price level targeting all make positive contributions to good monetary stabilisation policy. Nevertheless, heavy responses to output gap produce a huge destabilising effect to the economy rather than stabilising.

Fourth, in the aftermath of the financial crisis in 2007-08, a number of economists (see Blanchard, Dell Ariccia and Mauro (2010); Ball (2013)) argue that the Federal Reserve should raise its inflation target from 2 percent to 4 percent in order to reduce the probability of encountering liquidity trap in future recessions; and this can provide more room for the Fed to cut interest rate in order to stimulate the economy. However, by raising the inflation target, this could bring two by-products at the same time. One is the possibility of failing to anchor inflation in normal time and bring the economy subject to self-fulfilling expectations-driven fluctuations. The other is the potential welfare cost of rising inflation target in normal time, which may well exceed the gain from rising it for the preparation of bad time.

Another critical issue in modern macroeconomic policy research based on New Keynesian framework is the tendency and attempts among academics to make particular assumptions and sometimes unreasonable ones to obtain favourable results in terms of matching empirical evidence for macro variables, in particular, the inflation persistence observed in postwar United States. In order to do so, researchers introduce some types of backward looking terms into the New Keynesian Phillips Curve; the most popular two methods are: 1) some forms of indexation in price setting behaviours of intermediate firms and wage setting behaviour of labour unions. This can be found in Christiano, Eichenbaum and Evans (2005), and Smets and Wouters (2007). 2) a rule-of-thumb approach as introduced by Clarida, Galí and Gertler (2000). Nonetheless, the assumption of price indexation has been heavily criticised by economists for its lack of empirical evidence and theoretical contradiction with microeconomic foundation. Put it in simple terms, once indexation is allowed, then all the prices and wages change every period, and this clearly violates the nature of nominal rigidity. These criticisms are documented in Chari, Kehoe and McGrattan (2009), Cogley and Sbordone (2008), Dixon and Kara (2010), and Woodford (2007) .

In fact, the nature of inflation persistence itself is also subject huge debate among economists. Cogley and Sbordone (2008) incorporates a time-varying inflation trend with a drift into the NKPC and find that once the drift in inflation trend is taken into account, the observed inflation persistence of inflation gap, which is defined as inflation deviation from the time-varying trend, is much less persistent, and the observed persistence is mainly attributed to the persistence in the trend component. Furthermore, they conclude that the change in trend inflation over time is mainly caused by switches in monetary policy conduct in the postwar US. Researches share the same implications include Clarida, Galí and Gertler (2000), Lubik and Schorfheide (2004). On the contrary, some other literature found that there is no such "regime change" in monetary conduct; for example, Cogley, Primiceri and Sargent (2010), Sims and Zha (2006).

The Structure

Based on all the facts, evidence and problems discussed above, there is imperative demand for more researches to be done in this area. This thesis attempts to thoroughly investigate the issue of trend inflation and inflation persistence by formulating three independent chapters on a step-by-step basis. More specifically, the introductory *Chapter 1* focuses on the issue of trend inflation in a small-scale New Keynesian model, which is based on the well-known Clarida, Galí and Gertler (1999) study of monetary policy. The purpose is to check how trend inflation alters the final equations used to study monetary policy when the model is log-linearised around a non-zero inflation steady state. Also, this chapter re-examines how trend inflation alters macroeconomic dynamics and optimal monetary policy response when the economy experiences high levels of trend inflation, and evaluate whether the key results from Clarida, Galí and Gertler (1999) still hold once trend inflation is taken into account. *Chapter 2* goes one step further by introducing trend inflation to a medium-scale New Keynesian DSGE model with a wide range of nominal and real rigidities. The chapter reevaluates the prominent Smets and Wouters (2007) model but based on trend inflation, again, the model is log-linearised around a non zero inflation steady state. The model is kept as close to the original model as possible in order to test the difference that trend inflation makes in the most direct way. The new generalised version of this medium scale model is then estimated using a Bayesian technique, the same method as the original Smets and Wouters (2007), in order to examine whether this generalised model can fit well with macro data and capture empirical facts for macro variables. This is followed by the reinvestigation of how macroeconomic dynamics may be impacted by the presence of trend inflation in such medium scale New Keynesian model in the same chapter. Last, *chapter 3* relaxes the heavily-criticised assumption of backward indexation, and re-estimate this new model using the same Bayesian technique to see how well this revised version of the generalised model can fit macroeconomic data. The second major research question of *chapter 3* is to detect whether trend inflation can replace backward indexation as the generator of inflation persistence in such medium-scale New Keynesian models.

The Results

The major results and contributions of this thesis include the following. First, as similar to a number of previous literature, *Chapter 1* finds that a small scale NK model exhibits new features when it is log-linearised around a non-zero inflation steady state, agents become more forward looking as trend inflation increases due to the flattening New Keynesian Phillips curve. The additional term of price dispersion in the NKPC implies the divine coincidence no longer hold in such generalised NK model. Furthermore, trend inflation has huge impacts on the macroeconomic dynamics of such a model. *Chapter 1* finds some new quantitative and qualitative implications for the optimal monetary response under positive trend inflation; when trend inflation increases, there are two countering effects for optimal stabilisation policy. On the one hand, due to the high level of trend inflation, it requires more output sacrifice for the central bank in order to stabilise inflation fluctuations and this makes the central bank respond *less* aggressively to inflation. On the other hand, as trend inflation rises, the monetary authority is less concerned about output deviation, as reflected by the declining weight on output stabilisation in the central bank's objective function. This second effect is amplified by the rising price dispersion, which makes inflation deviations more costly for the central bank, and together the second effect makes the central bank to counter inflation *more* aggressively. The overall effects somehow depend on which effect is larger and also the parametrisation assumed in the model. This result is complementary to existing literature on this issue (e.g., Ascari and Ropele (2007) and Alves (2014)) In addition, a central bank can make considerable gains from commitment since price setting firms are now more forward looking with trend inflation. In addition, most of the results from Clarida, Galí and Gertler (1999) are still valid.

Chapter 2 finds that the features shown in *Chapter 1*'s small scale model still holds in this medium scale Generalised New Keynesian (GNK) model. A hybrid NKPC exhibits more forward looking property when trend inflation increases; a flattening NKPC due to trend inflation. Some additional and interesting features are also observed from this medium scale GNK model: 1) while inflation becomes more forward looking, the market wage rate somehow become more backward looking as trend inflation rises; 2) optimal reset wage for labour union is more concerned with expectation of optimal reset wage and less concerned with expectation of inflation as trend increases in inflation. Bayesian estimation shows that this GNK model can capture macroeconomic evidence of the postwar US, and the Bayesian posterior mean indicates that trend inflation is around 3.2 percent on average for the sample period from 1970 to 2017, this is in line with existing literature. A sub-sample estimation finds that the trend inflation fluctuates considerably across three sub-periods: the *Great Inflation*, the *Great Moderation*, and the *Great Recession*, with trend inflation 6.96%, 2.28%, and 2.0%, respectively. In addition, trend inflation still has huge alternating effects on the macroeconomics dynamics in this medium scale GNK model even with indexation, which is proven to have muting effects on trend inflation. As trend inflation increases, inflation deviates less on impact but is much more persistent after a monetary policy shock with higher levels of trend inflation.

Chapter 3 finds that a medium scale GNK model with trend inflation but no indexation is still able to capture the macro facts of the postwar US economy. Furthermore, the most valuable finding of this chapter (if not the entire thesis) is that trend inflation with 4 percent or higher can actually generate a hump-shaped response for inflation after a very transitory monetary policy shock ($\rho = 0.10$). Furthermore, inflation reacts less on impact but becomes much more persistent as trend inflation increases. This indicates the incorporation of trend inflation in such GNK model with no backward indexation can dramatically improve the model fit in the sense that such model can capture the movement of inflation after monetary shock in a much closer way to the observed evidence. This has both policy and theoretical implications. On the policy front, it implies that improper account of trend inflation can lead to huge miscalculation for the timing of maximum policy effect and damaging consequence for the optimal monetary response, and this finding has serious policy implications for central banks. The greater fluctuations caused by higher level of trend inflation should serve as a warning sign for central banks who think about raising inflation target in order to reduce the likelihood of being caught up by zero lower bound. On the theoretical front, it shows that once trend inflation is properly taken into account, one does not need backward indexation to generate a hump-shaped response for inflation with Calvo price setting. To my knowledge, this is the first paper to do so without additional assumptions[2].

[2] Ascari, Phaneuf and Sims (2018) uses a similar model without indexation, but adding the roundabout structure of production and extended borrowing, and they attribute the hump shape to the additional assumptions.

Chapter 1

Optimal Monetary Policy and Inflation Targeting in a New Keynesian Model with Moderate Trend Inflation

1.1 Introduction

The better understanding of monetary stabilisation policy and business cycle fluctuations is one of the most fruitful and productive areas of economic research in the last few decades. Both the design of micro-found macroeconomic models that are rich enough to answer the central question of Lucas critique and the methodology of how to estimate such complicated and medium scale dynamic stochastic general equilibrium (DSGE, henceforth) models have advanced considerably. Macroeconomists from both New Classical and New Keynesian schools of thought have largely converged on method, model design, reduced-form shock, and principles of policy advice (Chari, Kehoe and McGrattan, 2009). Early works on how to evaluate monetary policy based on a New Keynesian framework include: Clarida, Galí and Gertler (1999), Goodfriend and King (1997), and Rotemberg and Woodford (1997).

The well-known study from Clarida, Galí and Gertler (1999) has been regarded as one of the most important literature of monetary policy researches in New Keynesian economics, and it provides some valuable insights of how should the central bank respond to various exogenous shocks in the most welfare-desirable way, and how change in monetary policy conduct can affect the macroeconomic dynamics under New Keynesian framework. However, one thing seems common in these early New Keynesian models: long run inflation or trend inflation is assumed to be zero and therefore all the models are approximated linearly around a zero inflation steady state, despite the overwhelming evidence that zero inflation neither empirically grounded nor a practical objective for central banks in major economies (Ascari and Ropele, 2007). Unfortunately, this issue has attracted some attention and a number of researches have been done to directly address this issue in recent years. Early works can be found in Ascari (2004), Ascari and Ropele (2007), Ascari and Ropele (2009), Amano, Ambler and Rebei (2007).

The earliest and one of the most cited works on the role of trend inflation dates back to Ascari (2004), the study finds that steady state level of output is very much sensitive to the steady state rate of money growth, and very mild levels of trend inflation imply large changes in the steady state output level. He also shows that short run dynamics of a small scale NK model is hugely impacted by the change in the level of trend inflation. In addition, the presence of positive trend inflation has serious implications for the dynamic behaviour following a disinflation policy. While Ascari (2004) mainly focuses on the role of trend inflation on model dynamics and disinflation with Calvo and Taylor pricing, Amano, Ambler and Rebei (2007) extend the analysis by examining the movements of deterministic steady states and the stochastic means when trend inflation is present based on three different pricing schemes: Calvo, truncated-Calvo, and Taylor. They show that regardless of pricing mechanism, trend inflation leads to a reduction in the stochastic means of output, consumption and employment, and increase in the stochastic mean of inflation beyond its deterministic steady state levels. The mechanism that is responsible for increasing the spread between deterministic steady states and stochastic means operates via the price dispersion variable. Their results also show that with an annualised trend inflation of 4 percent, the stochastic mean of inflation is always greater than 4 percent regardless of the pricing scheme. This implies that adopting a positive inflation target leads to an outcome where inflation systematically exceeds its target unless the monetary authority runs a policy where output, on average, is forced to fall short of its deterministic steady state value. The discussion of the importance of price dispersion in a model with non zero inflation steady state can be found in Schmitt-Grohé and Uribe (2007a) and Schmitt-Grohé and Uribe (2007b).

Ascari and Ropele (2007) is probably the first paper that thoroughly examines the potential change in optimal monetary policy under positive trend inflation. Their research finds that under discretionary policy, the optimal condition shows the sacrifice of output contraction for inflation stabilisation actually drops as trend inflation rises, and the efficient policy deteriorates and there is no guarantee of determinacy. Furthermore, targeting non-zero inflation can lead to substantial welfare losses even under commitment. The analysis provides some very useful thoughts on the optimal stabilisation policy under trend inflation, however, the fact that their analysis fails to apply a non zero inflation based quadratic loss function makes some of their conclusion vulnerable. In addition, Ascari and Ropele (2009) finds that positive trend inflation shrinks the determinacy region of a basic NK DSGE model when monetary policy is conducted by a contemporaneous interest rate rule. Neither the Taylor principle nor the generalised Taylor principle is a sufficient condition for local determinacy of equilibrium. Therefore, they argue regardless of the theoretical set-up, the monetary literature on interest rate rules cannot disregard the average long run inflation in both theoretical and empirical analysis.

Given all the literature listed above, I believe it is important to start this thesis with a relatively small-scale New Keynesian DSGE model with trend inflation and use this model to re-examine the well-known CGG study of monetary policy and to check whether these key results from CGG still holds under this NK model with trend inflation. The analyses and results of this *Chapter 1* confirm some of the conclusion from previous literature and complement to previous findings. The findings can be summarised as the following: first, the generalised New Keynesian Phillips curve (GNKPC) becomes flatter as trend inflation increases where inflation reacts less to change in output or output gap as trend inflation increases, and agents become more forward looking. Second, trend inflation alters macroeconomic dynamics in a significant way, these alterations depend on the nature of shocks, the calibrated value of parameters, and also the persistence levels of shocks. However, in contrast to previous literature, by applying a welfare loss function that is based on non-zero inflation steady state, the weight of output stabilisation now depends on the level of trend inflation. This generates two countering effects for optimal monetary response under high levels of trend inflation. On the one hand, as the weight for output stabilisation drops due to higher trend inflation, and combined with the fact that inflation fluctuations become much more costly for the central bank, the central bank should respond to inflation deviation *more* aggressively. On the other hand, as trend inflation increases, more output needs to be sacrificed in order to keep inflation stable as indicated by the optimal condition, this makes the central bank to react *less* aggressively to inflation fluctuations and make the inflation-output trade-off even more serious. The overall net effect really depends on the parametrisation of the model. In addition, the well-known inflationary bias is still present under trend inflation, while welfare loss associated with trend inflation arises under both discretionary and commitment regimes.

The rest of this *chapter 1* is structured as the following: section 1.2 derives the generalised version of the baseline model, which is largely based on CGG. I show how key equations change due to the incorporation of trend inflation.

Section 1.3 provides the calibration of structural parameters and examines the how the macroeconomic dynamics are affected by trend inflation based on a standard Taylor rule. Then, section 1.4 analyses the effect of trend inflation on optimal monetary policy under discretion and commitment. Section 1.5 concludes this chapter.

1.2 The Generalised New Keynesian Model

This section presents the baseline model used in this chapter for the studies of the effects of trend inflation. This model is largely based on Clarida, Galí and Gertler (1999) (CGG henceforth) model, and it is completed by log-linearising the system of equation around a positive inflation steady state.

1.2.1 Household's Decision Making

A representative household makes decision by maximising the utility function subject to the budget constraint

$$max\ U_t = E_t \sum_{j=0}^{\infty} \beta^j \left[\frac{C_{t+j}^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-b} \left(\frac{M_{t+j}}{P_{t+j}} \right)^{1-b} - \chi \frac{N_{t+j}^{1+\eta}}{1+\eta} \right]$$

Subject to

$$C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} = \left(\frac{W_t}{P_t} \right) \cdot N_t + \frac{M_{t-1}}{P_t} + (1 + i_{t-1}) \cdot \left(\frac{B_{t-1}}{P_t} \right) + \Pi_t$$

Among these variables, M_t and B_t are the money and bond held by the household, N_t is labour supply in hours, Π_t is the profits distributed to households, and i_t is the gross return on bonds holding.

First order conditions for present and future consumptions give the Euler equation, which characterises household's consumption behaviour over time.

$$\frac{1}{C_t^\sigma} = \beta E_t \left[\left(\frac{P_t}{P_{t+1}} \right) (1 + i_t) \left(\frac{1}{C_{t+1}^\sigma} \right) \right] \tag{1.1}$$

The intra-temporal optimal condition, which set MRS equals real wage

$$\frac{W_t}{P_t} = \chi N_t^\eta C_t^\sigma \tag{1.2}$$

This is also interpreted as the labour supply equation.

1.2.2 Final Goods Producers

At a given point in time t, a perfectly competitive final goods firm produces the final consumption good Y_t by combining a continuum of intermediate goods $Y_{i,t}$, according to the technology

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\epsilon-1}{\epsilon}} \, di \right]^{\frac{\epsilon}{\epsilon-1}}$$

where $Y_{i,t} \in (0, 1)$.

The profit maximisation and the zero profit condition imply that the price of the final good, P_t is a CES aggregate of the prices of the intermediate goods $P_{i,t}$.

$$P_t = \left[\int_0^1 P_{i,t}^{1-\epsilon} \right]^{-\epsilon}$$

The demand for intermediate good i is therefore given by

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t} \right)^{-\epsilon} Y_t$$

1.2.3 Intermediate Goods Producers

The production function faced by a representative intermediate goods producer takes a standard Cobb-Douglas form but without capital, labour is the sole input factor, and total factor productivity is represented by A_t:

$$Y_{i,t} = A_t N_{i,t}$$

The cost minimisation problem yields the real marginal cost for intermediate goods producers

$$mc_t = \frac{w_t}{A_t} \tag{1.3}$$

Price Setting Behaviour

The representative firm maximises its profit by setting its price, subject to the market demand function. In this model, since labour is the only input in production, hence, wage is the only source of production cost for this firm, as reflected by the marginal cost equation (1.3).

$$max\ E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} \theta^j \left[\left(\frac{P^*_{i,t}}{P_{t+j}} \right) Y_{i,t+j} - \frac{W_t}{P_t} \frac{Y_{i,t}}{A_t} \right]$$

subject to

$$Y_{i,t+j} = \left(\frac{P^*_{i,t}}{P_{t+j}} \right)^{-\epsilon} \cdot Y_{t+j}$$

Where $\Lambda_{t,t+j} \equiv \beta^j \frac{\lambda_{t+j}}{\lambda_t}$ is the stochastic discount factor, in which λ_t represents the marginal utility of consumption for households in period t. $P^*_{i,t}$ is the optimal reset price for a monopolistic firm. Furthermore, the Calvo parameter θ is the measure of nominal rigidity, and it implies the proportion of firms that cannot reset their prices in a given period (Calvo, 1983).

First order condition of $P^*_{i,t}$ yields the following optimal condition

$$=> \frac{P^*_{i,t}}{P_t} = \frac{\epsilon}{\epsilon - 1} \cdot \frac{E_t \sum_{j=0}^{\infty} \theta^j \Delta_{t,t+j} Y_{t+j} \pi^{\epsilon}_{t,t+j} MC_{t+j}}{E_t \sum_{j=0}^{\infty} \theta^j \Delta_{t,t+j} Y_{t+j} \pi^{\epsilon-1}_{t,t+j}} \tag{1.4}$$

Hence, the gross price mark-up is given by $\mu = \frac{\epsilon}{\epsilon - 1}$

The aggregate price level in the economy evolves according to

$$P_t = \left[(1-\theta)P_{t-1}^{1-\epsilon} + \theta {P_{i,t}^*}^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$$

$(1-\theta)P_{t-1}^{1-\epsilon}$ represents the firms these are not able to reset their prices in period t, hence, it is the general price level carried out from $t-1$, there is no price indexation in this model; while $\theta {P_{i,t}^*}^{1-\epsilon}$ represents the firms these have the chance to reset their prices in period t. Therefore, θ is a measure of the nominal rigidity in this economy.

Dividing both sides by P_t and letting $\pi_t = P_t/P_{t-1}$, we get the real optimal reset price

$$p_{i,t}^* = \left[\frac{1-\theta\pi_t^{\epsilon-1}}{1-\theta}\right]^{\frac{1}{1-\epsilon}}$$

This indicates that the relative optimal reset price is a function of inflation rate, and $p_{i,t}^* = \frac{P_{i,t}^*}{P_t}$ represents the real optimal price.

Now, in order to solve this equation, two auxiliary variables are further defined

$$\psi_t = E_t \sum_{j=0}^{\infty} \theta^j \Delta_{t,t+j} Y_{t+j} \pi_{t,t+j}^{\epsilon} MC_{t+j}$$

$$\phi_t = E_t \sum_{j=0}^{\infty} \theta^j \Delta_{t,t+j} Y_{t+j} \pi_{t,t+j}^{\epsilon-1}$$

So they can also be expressed in recursive forms:

$$\psi_t = w_t A_T^{-1} Y_t^{1-\sigma} + \theta\beta E_t\left[\pi_{t+1}^{\epsilon}\psi_{t+1}\right]$$

$$\phi_t = Y_t^{1-\sigma} + \theta\beta E_t\left[\pi_{t+1}^{\epsilon-1}\phi_{t+1}\right]$$

Now, the real optimal reset price can be expressed as:

$$p_{i,t}^{*} = \frac{\epsilon}{\epsilon - 1} \cdot \frac{\psi_t}{\phi_t}$$

1.2.4 The Central Bank

The central bank in this economy conducts monetary policy by following a standard Taylor rule

$$\left(\frac{1+i_t}{1+\bar{i}}\right) = \left(\frac{\pi_t}{\bar{\pi}}\right)^{\phi_\pi} \cdot \left(\frac{y_t}{y_t^n}\right)^{\phi_y} + m_t$$

The equations implies that the central bank adjusts its nominal interest rate whenever inflation deviates from its target level, which is in line with the trend inflation, and also when output deviates from its natural rate. A shock to the term m_t represents the monetary policy shock in the economy.

Aggregation and Price Dispersion

To closely follow the original set-up in Clarida, Galí and Gertler (1999), this model assumes there is neither investment nor government spending in the economy. Therefore, the total amount of consumption is equivalent to the total production generated. The aggregate resource constraint is therefore given by

$$Y_t = C_t$$

This constraint implies that consumption at the households level can be drawn equivalent to the total output at the firm level.

Assume there are a number of differentiated labour supplied by a continuum of households, $i \in [0, 1]$. The market labour demand function can be written as:

$$
\begin{aligned}
N_t^d &= \int_0^1 N_{i,t}^d \, di = \int_0^1 \left(\frac{Y_{i,t}}{A_t} \right) \, di \\
&= \int_0^1 \left(\frac{P_{i,t}}{P_t} \right)^{-\epsilon} di \left(\frac{Y_t}{A_t} \right) = s_t \left(\frac{Y_t}{A_t} \right)
\end{aligned}
$$

So $s_t = \int_0^1 \left(\frac{P_{i,t}}{P_t} \right)^{-\epsilon} di$ indicates the level of price dispersion in the economy.

According to Schmitt-Grohé and Uribe (2007a) , this price dispersion s_t is bounded below by 1. It represents the resource costs due to relative price dispersion arising from positive long-run inflation: the higher s_t, the more labour is required to produce a given level of output. The impact on the overall economy due to the presence of this price dispersion is further discussed in the following sections. It can also be written in recursive form as

$$
s_t = (1 - \theta) \left(\frac{P_{i,t}}{P_t} \right)^{-\epsilon} + \theta \pi_t^{\epsilon} s_{t-1}
$$

1.2.5 Deterministic Steady State

Assume that $\bar{\pi}$ as the trend inflation rate. Therefore, if $\bar{\pi} = 1.00$, it means the steady state inflation is zero. If $\bar{\pi} > 1.00$, then it implies there is positive trend inflation. In steady state, we have the following set of deterministic paths:

$$\bar{\Pi} = \beta(1+i) \tag{1.5}$$

$$w = \chi N^{\eta} Y^{\sigma} \tag{1.6}$$

$$p_i^* = \left[\frac{1-\theta\bar{\pi}^{\epsilon-1}}{1-\theta}\right]^{\frac{1}{1-\epsilon}} \tag{1.7}$$

$$p_i^* = \frac{\epsilon}{\epsilon-1}\cdot\frac{\psi}{\phi} \tag{1.8}$$

$$\psi = \frac{wA^{-1}Y^{1-\sigma}}{1-\theta\beta\bar{\pi}^{\epsilon}} \tag{1.9}$$

$$\phi = \frac{Y^{1-\sigma}}{1-\theta\beta\bar{\pi}^{\epsilon-1}} \tag{1.10}$$

$$N = s\left[\frac{Y}{A}\right] \tag{1.11}$$

$$s = \frac{1-\theta}{1-\theta\bar{\pi}^{\epsilon}}(p_i^*)^{-\epsilon} \tag{1.12}$$

$$mc = \frac{w}{A} \tag{1.13}$$

First of all, according to equation (1.7), one can see that optimal reset price increases as trend inflation $\bar{\pi}$ rises. Hence, higher $\bar{\pi}$ pushes up the optimal reset price in steady state.

Substitute (1.8) into (1.13), price dispersion can be expressed in terms of the steady state inflation rate, namely, trend inflation.

$$=> \ s = \frac{1-\theta}{1-\theta\bar{\pi}^{\epsilon}}\cdot\left(\frac{1-\theta\bar{\pi}^{\epsilon-1}}{1-\theta}\right)^{\frac{\epsilon}{\epsilon-1}}$$

A clear feature from this equation is that the level of price dispersion in steady state depends on three parameters: the elasticity of substitution of consumption ϵ, the Calvo rigidity parameter θ and trend inflation $\bar{\pi}$. One can see that when prices are fully flexible ($\theta = 0$), the steady state price dispersion collapse to 1, which implies no price dispersion. When prices are fully flexible, there should be no dispersion in prices in the steady state. Furthermore, when trend inflation $\bar{\pi}$ is 1, the expression on the right hand side collapses to 1 and no price dispersion is present in steady state. However, when trend inflation $\bar{\pi} > 1$, which implies positive trend inflation, as $\bar{\pi}$ increases, the steady state level of price dispersion also increase. This indicates that the cost of price dispersion should not be ignored when positive trend inflation is present.

The gross mark-up in steady state also depends on the level of trend inflation now

$$\mu = \frac{1}{mc} = \left[\frac{1-\theta\bar{\pi}^{\epsilon-1}}{1-\theta}\right]^{\frac{1}{\epsilon-1}} \left[\frac{\epsilon}{\epsilon-1} \cdot \frac{1-\beta\theta\bar{\pi}^{\epsilon-1}}{1-\beta\theta\bar{\pi}^{\epsilon}}\right]$$

1.2.6 Log-Linearised Equations

Now, the model is log-linearised around a non-zero inflation steady state. A variable with a hat represents its log deviation from the steady state: $\hat{X}_t = lnX_t - ln\bar{X}$. The completed system of linearised equations are

$$\hat{Y}_t = E_t\hat{Y}_{t+1} - \frac{1}{\sigma}(\hat{i}_t - E_t\hat{\pi}_{t+1}) \tag{1.14}$$

$$\hat{w}_t = \eta\hat{N}_t + \sigma\hat{Y}_t \tag{1.15}$$

$$\hat{Y}_t = \hat{A}_t + \hat{N}_t - \hat{s}_t \tag{1.16}$$

$$\hat{i}_t = \phi_\pi\hat{\pi}_t + \phi_Y\hat{Y}_t + v_t \tag{1.17}$$

$$\hat{p}^*_{i,t} = \hat{\psi}_t - \hat{\phi}_t \tag{1.18}$$

$$\hat{p}^*_{i,t} = \frac{\theta\bar{\pi}^{\epsilon-1}}{1-\theta\bar{\pi}^{\epsilon-1}}\hat{\pi}_t \tag{1.19}$$

$$\hat{\psi}_t = (1-\theta\beta\bar{\pi}^{\epsilon})[\hat{w}_t - \hat{A}_t + (1-\sigma)\hat{Y}_t] + \theta\beta\bar{\pi}^{\epsilon}(\epsilon E_t\hat{\pi}_{t+1} + E_t\hat{\psi}_{t+1}) \tag{1.20}$$

$$\hat{\phi}_t = (1-\sigma)(1-\theta\beta\bar{\pi}^{\epsilon-1})\hat{Y}_t + \theta\beta\bar{\pi}^{\epsilon-1}[(\epsilon-1)E_t\hat{\pi}_{t+1} + E_t\hat{\phi}_{t+1}] \tag{1.21}$$

$$\hat{s}_t = (\theta\bar{\pi}^{\epsilon} - 1)\hat{p}^*_{i,t} + \theta\bar{\pi}^{\epsilon}(\epsilon\hat{\pi}_t + \hat{s}_{t+1}) \tag{1.22}$$

Among these, equation (1.14) is the dynamic IS equation in this GNK model. Also, to have a closer look at the equation (1.16), one can realise that there is a huge potential issue of under-production in the economy under high level of trend inflation. Due to the presence of substantial price dispersion (increasing function of $\bar{\pi}$), the equilibrium level output with positive trend inflation will always be less than the equilibrium output level arises under no price dispersion. This is also true to this log-linearised expression; for a given level of technology, more labour is always required to produce the same level of output when trend inflation exists in the steady state. Without growth in labour input, the total production under high level of trend inflation could be substantially lower than what it would be under zero trend inflation.

Starting from the expression of optimal reset price in terms of two auxiliary variables, one can eliminate $\hat{\phi}_t$, and upon replacing the expression of real marginal cost, the New Keynesian Phillips curve can be written in terms of output

$$\hat{\pi}_t = \lambda(\bar{\pi})\hat{Y}_t + b_1(\bar{\pi})\ E_t\hat{\pi}_{t+1} + \kappa(\bar{\pi})[\eta\hat{s}_t - (\eta+1)\hat{A}_t] + b_2(\bar{\pi})\ [(1-\sigma)\hat{Y}_t - E_t\hat{\psi}_{t+1}] \quad (1.23)$$

This is the Generalised New Keynesian Phillips curve (GNKPC) in terms of output, the reason it is called the generalised version is because it obtained by log-linearising the model around a non-zero inflation steady state. Therefore, it is a generalisation of the standard New Keynesian Phillips curve, so the model is called a Generalised New Keynesian (GNK) model. This follows the same terminology of Cogley and Sbordone (2008) and Ascari and Sbordone (2014). Where $\kappa(\bar{\pi})$, $\lambda(\bar{\pi})$, b_1, and b_2 are composite coefficients that are consist of structural parameters in the economy.

$\kappa(\bar{\pi}) \equiv \frac{(1-\theta\bar{\pi}^{\epsilon-1})(1-\theta\beta\bar{\pi}^{\epsilon})}{\theta\bar{\pi}^{\epsilon-1}}$, when $\bar{\pi} = 1.00$, $\kappa(\bar{\pi}) = \frac{(1-\theta)(1-\theta\beta)}{\theta}$

$\lambda(\bar{\pi}) \equiv \kappa(\bar{\pi})(\eta+\sigma)$, when $\bar{\pi} = 1.00$, $\lambda(\bar{\pi}) = \frac{(1-\theta)(1-\theta\beta)(\eta+\sigma)}{\theta}$

$b_1(\bar{\pi}) \equiv \beta[1+\epsilon(\bar{\pi}-1)(1-\theta\bar{\pi}^{\epsilon-1})]$, when $\bar{\pi} = 1.00$, $b_1(\bar{\pi}) = \beta$

$b_2(\bar{\pi}) \equiv \beta\,(1-\theta\bar{\pi}^{\epsilon-1})\,(1-\bar{\pi})$, when $\bar{\pi} = 1.00$, $b_2(\bar{\pi}) = 0$

There are a few properties that can be immediately spotted from this

GNKPC. According to the equation, we can see that as trend inflation $\bar{\pi}$ increases, the value of $\lambda(\bar{\pi})$ declines, and this clearly makes the GNKPC flatter; inflation respond less to change in output. This is one of the most important features that trend inflation introduces to this Generalised New Keynesian (GNK) model.

In addition, auxiliary variable $\hat{\psi}_t$ evolves according to the following equation.

$$\hat{\psi}_t = (1 - \theta\beta\bar{\pi}^{\epsilon})[(\eta + 1)(\hat{Y}_t - \hat{A}_t) + \eta\hat{s}_t] + \theta\beta\bar{\pi}^{\epsilon}(\epsilon E_t\hat{\pi}_{t+1} + E_t\hat{\psi}_{t+1}) \quad (1.24)$$

1.2.7 The Flexible Equilibrium

The flexible economy is defined by removing all the nominal rigidities, a state where $\theta = 0$, and $s = p_i^* = 1$. Following Ascari and Sbordone (2014), here I define the output gap under trend inflation steady state as the following:

$$\hat{x}_t = \hat{Y}_t - \frac{\eta + 1}{\eta + \sigma} + \bar{x} \quad (1.25)$$

This output gap $\hat{x}_t$ is the output gap based on non-zero inflation steady state, and $\bar{x}$ is defined as the long run output gap due to non zero long run inflation, as in Ascari and Sbordone (2014). The value of $\bar{x}$ can be derived from the steady state conditions listed in section 1.2.4.

One can now use the relationship between output gap and output to substitute actual output, and the system of equations can be rewritten in terms of output gap.

1.2.8 The Completed Generalised New Keynesian Model

- Dynamic IS Equation

$$\hat{x}_t = E_t\hat{x}_{t+1} + \frac{1}{\sigma}(\hat{i}_t - E_t\hat{\pi}_{t+1}) + \hat{g}_t \tag{1.26}$$

The term $\hat{g}_t$ represents an ad hoc demand shock. Here, in order to keep the model as close to the original CGG model as possible, I do not introduce the natural interest rate. A similar case based on trend inflation with natural interest rate can be found from Alves (2014), and Coibion, Gorodnichenko and Wieland (2012).

- Generalised New Keynesian Phillips Curve (GNKPC)

$$\begin{aligned}\hat{\pi}_t = \lambda(\bar{\pi})\left[\hat{x}_t + \frac{\eta+1}{\eta+\sigma}\hat{A}_t - \bar{x}\right] + \kappa(\bar{\pi})(\eta\hat{s}_t - (\eta+1)\hat{A}_t) + +b_1(\bar{\pi})E_t\hat{\pi}_{t+1} \\ + b_2(\bar{\pi})\left[\left(\hat{x}_t - \bar{x} + \frac{\eta+1}{\eta+\sigma}\hat{A}_t\right)\cdot(1-\sigma) - E_t\hat{\psi}_{t+1}\right] + u_t \end{aligned}\tag{1.27}$$

The u_t is a cost push that represents a shock to the supply side of the economy. Price dispersion increases the persistence of output and inflation because there is mutual feedback between inflation and price dispersion, whose strength is governed by the parameter η.

- Price Dispersion

$$\hat{s}_t = \epsilon\theta\bar{\pi}^{\epsilon-1}\left[\frac{\bar{\pi}-1}{1-\theta\bar{\pi}^{\epsilon-1}}\right]\hat{\pi}_t + [\theta\bar{\pi}^{\epsilon}]\hat{s}_{t-1} \tag{1.28}$$

- The Definition of ψ

$$\hat{\psi}_t = (1-\theta\beta\bar{\pi}^{\epsilon})\left[\eta\hat{s}_t + (\eta+1)\left(\hat{x}_t + \bar{x} - \frac{1-\sigma}{\sigma+\eta}\cdot\hat{A}_t\right)\right] + \theta\beta\bar{\pi}^{\epsilon}[E_t\hat{\psi}_{t+1} + \epsilon E_t\hat{\pi}_{t+1}] \quad (1.29)$$

- Shocks

$$\hat{A}_t = \rho_A\hat{A}_{t-1} + e_{A_t} \quad (1.30)$$

$$\hat{g}_t = \rho_A\hat{g}_{t-1} + e_{g_t} \quad (1.31)$$

$$u_t = \rho_u u_{t-1} + e_{u_t} \quad (1.32)$$

$$m_t = \rho_m m_{t-1} + e_{m_t} \quad (1.33)$$

This completes the entire small-scale Generalised New Keynesian (GNK) model.

1.3 Macroeconomic Dynamics Under Moderate Trend Inflation

This section examines how the presence of positive trend inflation affects the macroeconomic dynamics in this small scale GNK model. Along with the analysis, I also directly evaluate some of the major results from CGG regarding monetary policy response to various exogenous shocks. In this section, the monetary policy conduct is characterised by a standard contemporaneous Taylor rule.

$$\hat{i}_t = \psi_\pi \hat{\pi} + \psi_y \hat{x}_t + m_t \tag{1.34}$$

This is the Taylor rule corresponding to equation 7.1 [1] in CGG, but in a slightly different way. The rule suggests that the central bank responds to the current inflation deviation from its non-zero steady state level $\bar{\pi}$. In contrast to the one in 7.1 of CGG, the inflation target $\bar{\pi}$ does not appear in the Taylor rule and inflation deviation from the central bank's target does not need to be written as $\hat{\pi}_t - \bar{\pi}$, because $\hat{\pi}_t$ is already a measure of the inflation deviation from the trend level, which is assumed to be equal to the central bank's inflation target. This is why this entire GNK model is formed in a consistent way as how trend inflation and inflation target should be modelled. Moreover, equation 1.34 suggests the central bank should also react to output gap, which is represented by the deviation of actual output from its natural level under trend inflation. In addition, m_t captures the monetary policy shock in this economy.

[1] Equestion (7.1) on page pp.1695 in CGG: $i_t^* = \alpha + \gamma_\pi(\pi_t - \bar{\pi}) + \gamma_x x_t$

1.3.1 Baseline Calibrations

There are total seven structural parameters in this small model plus the trend inflation term $\bar{\pi}$. The calibration of this model mainly follows chapter 3 in **?**. The elasticity of marginal utility of consumption σ is assumed to be 1.5. At the same time, the elasticity of labour supply η is set at 1.0. The discount factor β is assumed to be 0.995, which means the steady state real interest rate is 1.25% on a quarterly basis. The Dixit-Stiglitz elasticity of substitution among goods ϵ is set at 9, which implies a gross mark-up of 1.15. The Calvo parameter θ for price setting is fixed at 0.75, which implies an average price duration of four quarters. For trend inflation $\bar{\pi}$, it is set at four different levels, in order to check how changes in trend inflation can affect the macroeconomic dynamics of the model. The four levels are 1.000, 1.005, 1.010, and 1.015, and they correspond to annualised trend inflation of 0%, 2%, 4%, and 6%, respectively. The inflation response parameter in the Taylor rule ψ_π is assumed to be 1.5, and this ensures that the Taylor principle is satisfied. Central bank's response to output ψ_y is set at 0.125 on a quarterly basis. Table 1.1 summarises these calibrated parameters.

Table 1.1: Calibrated Parameters

Description	Parameter	Calibrated value
Elast. of marginal utility of con.	σ	1.5
Elast. of labour supply	η	1.0
Elast. of substitution among goods	ϵ	9
Calvo probability	θ	0.75
Discount factor	β	0.995
Inflation response	ψ_π	1.50
Output gap response	ψ_y	0.125

1.3.2 Equilibrium Dynamics

Changes in the level of trend inflation considerably affect the macroeconomic dynamics. In order to see what these effects are, this section analyses the model dynamics in case of three exogenous shock: shock to total factor productivity, monetary policy shock and a shock to the GNKPC. The persistence parameter of all three shocks are set at 0.50, i.e., $\rho_a = \rho_m = \rho_u = 0.50$, which implies moderately persistent shocks.

Productivity Shock

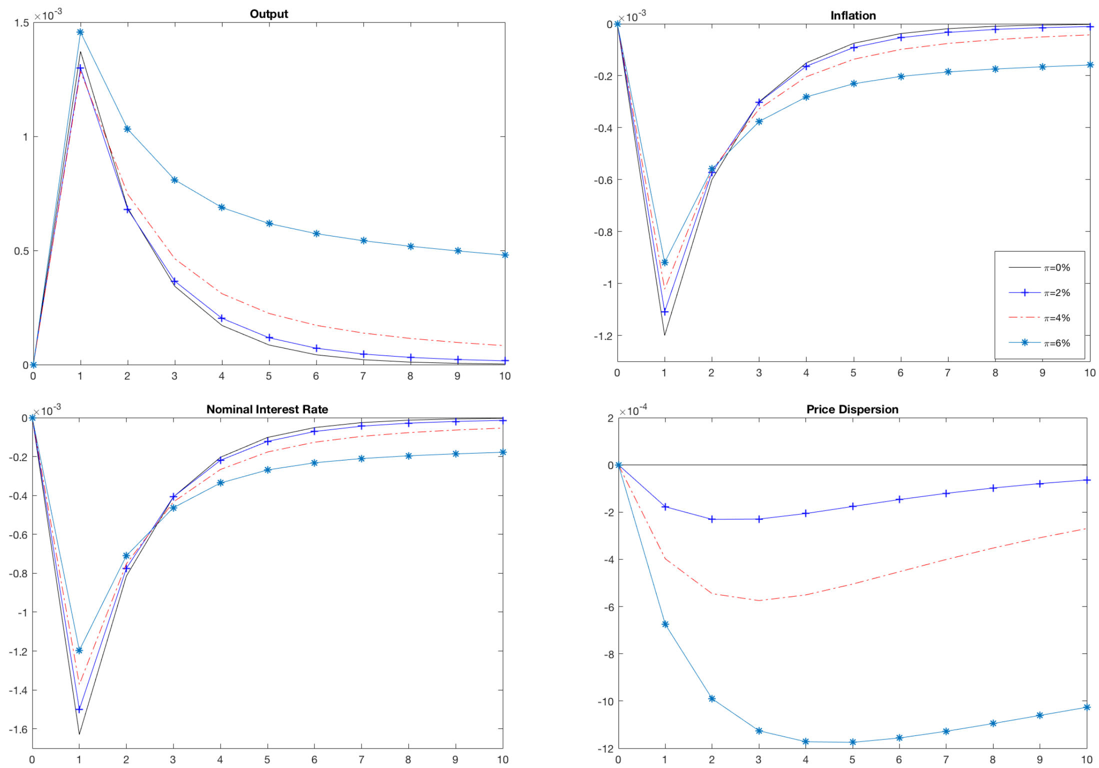

Figure 1.1: Impulse response functions to a one percent productivity shock ($\rho_a = 0.50$)

Figure 1.1 shows the impulse response functions (IRFs) of output, inflation, nominal interest rate and price dispersion after a positive one percent shock to total factor productivity. In general, there are two offetting effects of trend inflation after a technology shock. The first thing to realise is that trend inflation has a dampening effect that technology shock makes on output and inflation, due to the property that NKPC is flatter with a higher level of trend inflation. This is why inflation actually deviates less with higher levels of trend inflation immediately after TFP shock. However, there is a second effect emerging from higher trend inflation: price setting agents become more forward looking due to rise in trend inflation, therefore, trend inflation amplifies this the impact that this productivity shock can make to output and inflation. This explains why output with higher inflation reacts more to this shock with higher trend inflation throughout the entire forecast horizon (10 quarters).

Ascari and Sbordone (2014) demonstrates this issue by evaluating the IRFs after two extreme productivity shocks: a purely transitory shock ($\rho_a = 0$) and a highly-persistent shock ($\rho_a = 0.95$), their IRFs show exactly these two extreme scenarios. When $\rho_a = 0$, the flattening Phillips curve dominates; while when $\rho_a = 0.95$, the forward looking effect dominates. This analysis is a complement to Ascari and Sbordone (2014)'s studies. Here, I show that when persistence level is moderately persistent ($\rho_a = 0.50$, a more neutral scenario), output is dominated by the forward looking effect from the beginning; as it shows greater response with higher trend inflation. The response of inflation shows that the flattening Phillips curve impact outplays the other effect at the start, but soon the situation is reversed after two quarters. As one can see from the IRF of inflation, inflation generates an increasingly high level of fluctuations with higher trend inflation towards the end of the forecast horizon.

In addition, there is an extra feedback effect on inflation from price dispersion, but it is not captured by inflation at the beginning. As price dispersion is about to reach its maximum deviation in the third and fourth quarter, this feedback effect is strong enough so that inflation is pushed to a greater level of deviation with high trend inflation.

Monetary Policy Shock

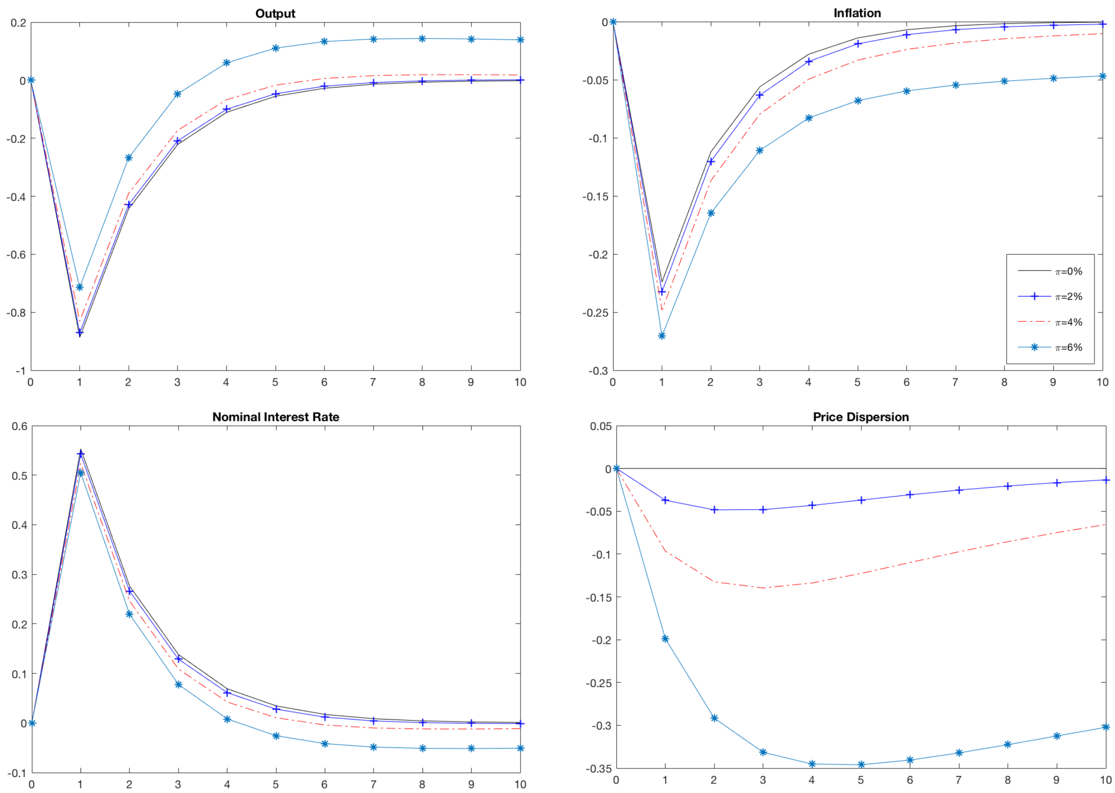

Figure 1.2: Impulse response functions to a one percent contractionary monetary policy shock ($\rho_m = 0.50$)

Figure 1.2 reports the impulse response functions (IRFs) of four key macroeconomic variables: output, inflation, nominal interest rate, and price dispersion, after a one percentage point contractionary monetary policy shock. Under higher level of trend inflation, agents are less concerned with the current state of the economy, and become more forward looking. Hence, agents feel the signal of contraction in the economy and lower their expectation of inflation to a greater extent with higher trend inflation, this lowers value of $E_t\hat{\pi}_{t+1}$ and leads to a higher output level according to the IS equation. Given a $\rho_m = 0.50$ persistence level, inflation initially drops to a lower level with higher trend inflation due to more forward looking generated by trend inflation. Then, large negative deviation in price dispersion feeds back to inflation and leads to even lower inflation with higher trend inflation. Combined with the flattening Phillips curve effect, inflation becomes more independent from output. This decline in inflation is so great with higher levels of trend inflation that the central bank needs to relax the rise in nominal interest rate with higher trend inflation in order to stabilise the economy, as suggested by the Taylor rule. Therefore, this ultimately leads to a lower decline in output and a smaller rise in nominal interest rate with higher levels of trend inflation throughout the entire forecast horizon.

Cost Push Shock

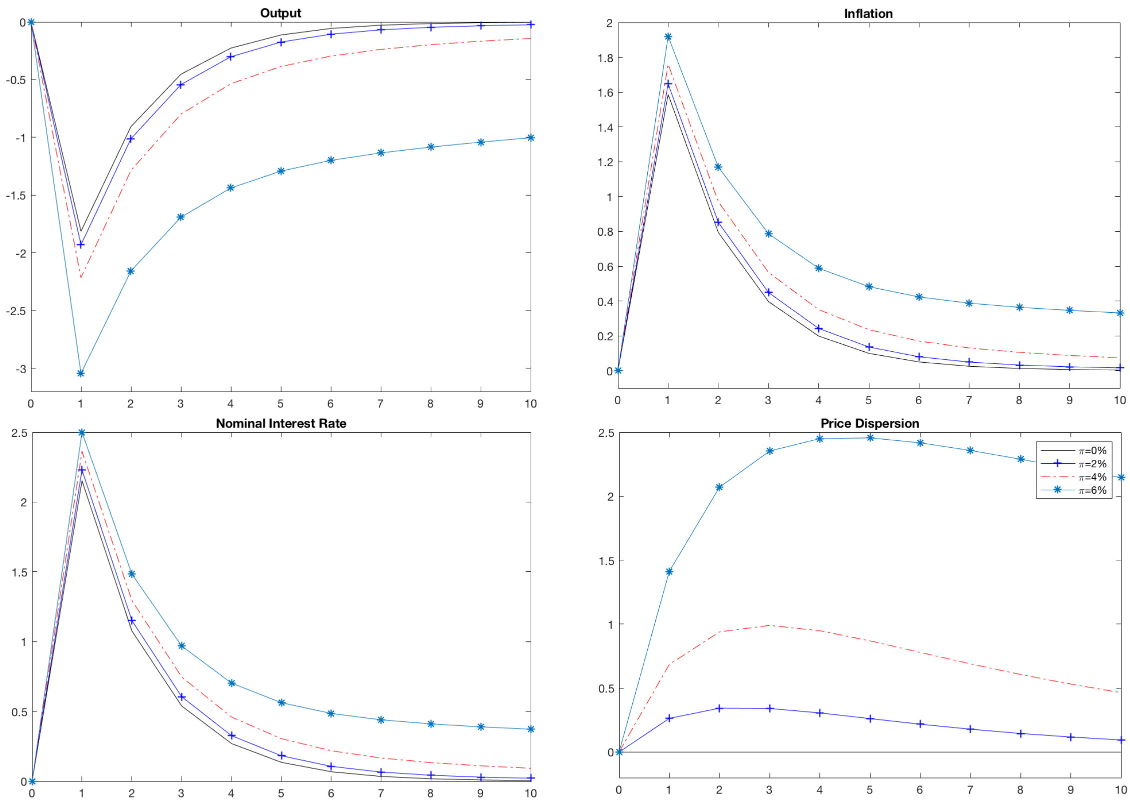

Figure 1.3: Impulse response functions to a one percent cost push shock ($\rho_u = 0.50$)

Figure 1.3 summarises the impulse response functions of the same four macro variables after a one percent cost push shock. At initiation, inflation is pushed up further with higher trend inflation due to agents become more forward looking, and a cost push shock certainly makes price setting firms more concerned about future price level growth. This is reflected by higher expected inflation $E_t\hat{\pi}_{t+1}$ and higher current inflation, as suggested by the GNKPC with ϵ_t^u. Output is dampened further with higher trend due to higher $E_t\hat{\pi}_{t+1}$ as indicated by the dynamic IS equation. Price dispersion is still making damaging effects on inflation, and the central bank needs to raise nominal interest rate to a higher level with greater trend inflation in order to stabilise the economy. Furthermore, in this cost push case, it is not hard to find that the alterations of equilibrium dynamics are asymmetrical to change in trend inflation levels: increase in macro variables fluctuations due to rise in trend inflation are much higher when trend inflation is already high (4 percent to 6 percent) than increase in fluctuations when trend inflation is at a lower level (2 percent to 4 percent).

To sum up, as the above studies show us, trend inflation can make considerable impacts on the macroeconomic dynamics of this GNK model. As a result of the change in inflation-output trade-off faced monetary authority, this can leads to further effects on the optimal monetary policy practice, and this is what the next section focuses.

1.4 Optimal Monetary Policy

The prominent Clarida, Galí and Gertler (1999) study of monetary policy produces a comprehensive survey and evaluation of the optimal stabilisation policy under New Keynesian framework. This section attempts to re-examine the key results and implications of CGG and investigate whether these key results still hold under a similar-in-scale, but Generalised New Keynesian (GNK) model with moderate trend inflation.

The exercise starts with the description of monetary policy conduct of a representative central bank, then followed by separate discussions and evaluations of optimal monetary stabilisation policy under different regimes. The two monetary conducts examined in this chapter include the discretionary policy and rule-based commitment policy.

1.4.1 Policy Specifications

As in a series of literatures and standard textbooks on monetary economics, for example, Woodford (2003) and Walsh (2017); a central bank that adopts optimisation-based monetary policy rule seeks to maximise its objective function, which is typically represented by a quadratic loss function, subject to the constraint, which is usually characterised by a New Keynesian Philips curve. As in Ch.3 of Woodford (2003), I assume a quadratic loss function of the monetary authority, which is approximated from a representative household's utility function, it takes the following form:

$$\omega = \frac{1}{2}E_t\sum_{j=0}^{\infty}\beta^j[\chi\hat{x}_{t+j}^2 + \hat{\pi}_{t+j}^2]$$

where χ represents the weight assigned by the central bank to output stabilisation relative to inflation stabilisation, and this can be seen as an indicator of the hawkishness of this central bank. Output gap $\hat{x}_t$ is the log-difference of actual output from the natural output level, therefore, it assumes the central bank views the natural output level as the target. This $\hat{\pi}_t$ now stands for the inflation gap, which is defined as the inflation deviation from its non zero steady state level[2]. Since it is linearised around trend inflation, hence it directly implies by how much inflation deviate from the central bank's target, a correct and consistent measure of welfare loss from inflation, but it is not the case in the original CGG where inflation stands for deviation from zero steady state. This issue becomes even more flawed when one adds an inflation target to the loss function in a standard NK model, which means the bank targets

[2]This definition can be easily found from other literature on trend inflation and inflation persistence. For example, Cogley, Primiceri and Sargent (2010), Cogley and Sbordone (2008)

positive inflation while the economy is modelled around zero inflation steady state, a completely illogical practice.

Another issue arises with the presence of trend inflation. This original quadratic loss function from Woodford (2003) is indeed approximated around a zero inflation state, hence, it is not an accurate measure of the loss in social welfare when steady state inflation is non-zero, as noted by Ascari and Ropele (2007). There are a couple of issues with studying optimal monetary policy based on a quadratic loss function that is approximate around a zero inflation steady state, as summarised by Alves (2014): first, a second order approximation around zero inflation SS underestimates the true curvature of the welfare function when the SS inflation is indeed around a positive trend. Second, such second order approximation based on a non-zero inflation SS fails to endogenise the welfare loss incurred when trend inflation is indeed positive. He also pointed out that when the optimal monetary policy is achieved by stabilising inflation at this positive value through inflation targeting, the above issues suggest that a more accurate policy evaluation is obtained when such approximation is done around a non-zero inflation steady state. Alves (2014), therefore, develop a second order approximation of households' utility function around a non-zero inflation steady state, the major difference with the Woodford (2003) loss function is the weight for output gap stabilisation χ. Based on the parameter and utility function, it can be expressed as:

$$\chi(\bar{\pi}) = \frac{1 - \theta\bar{\pi}^{\epsilon-1}}{1 - \theta\bar{\pi}^{\epsilon}} \cdot \frac{\lambda(\bar{\pi})}{\epsilon}$$

To have a closer look at this composite coefficient $\chi(\bar{\pi})$, the term trend inflation $\bar{\pi}$ now appear in the output gap weight, and therefore it affects the weight directly. More specifically, as the level of trend inflation grows, the value of $\chi(\bar{\pi})$ drops, this effect is amplified by the decline in $\lambda(\bar{\pi})$ caused by increasing trend inflation, as discusses in previous sections. This implies that the central bank should take stabilising inflation more seriously when trend inflation is present than when it is absent, also, there is more demand for less response to the output gap. Thus, the weight χ from the central bank's objective function is replaced by this $\chi(\bar{\pi})$ for the rest of the optimal monetary policy studies. One needs to realise that any composite coefficient followed by a $\bar{\pi}$ in the bracket, directly relies on the level of trend inflation.

1.4.2 Optimal Monetary Policy under Discretion

To follow the original CGG paper closely, this chapter also mainly focus on two optimisation-based monetary policy rules: discretionary policy and commitment policy. The former is hardly a rule because the central bank is not constrained from the past behaviour in any credible way, and the latter takes the specific mechanism of what Woodford (2003) defines as the *timeless perspective* commitment rule. As CGG points out that to better understand the gains from commitment policy-making, it is necessary to understand how inflation may affect the optimal response under a benchmark case with no commitment.

A discretionary central bank minimises its loss function subject to the GNKPC for the current period, and find the optimal condition (Walsh, 2017). Then plug this optimality condition back into the dynamic IS equation to obtain the implied monetary policy feedback rule. Due to the lack of credibility to the public, a discretionary central bank re-optimises when it arrives every single period, practically, the bank is only concerned with the current economic circumstances. This behaviour is usually described as "lean against the wind" by economists.

In order to exercise the study in a more straightforward manner, I set the elasticity of substitution in consumption $\sigma = 1$ (which implies log utility for consumption), therefore, the GNKPC becomes

$$\hat{\pi}_t = \kappa(\bar{\pi})\eta\hat{s}_t + \lambda(\bar{\pi})(\hat{x}_t - \bar{x}) + b_1(\bar{\pi})E_t\hat{\pi}_{t+1} - b_2E_t\hat{\psi}_{t+1} + u_t \tag{1.35}$$

Since a discretionary central bank only focuses on the current period, therefore, this becomes a single period static choice for the bank. In each period, the central bank chooses a set of three variables $\hat{x}_t, \hat{\pi}_t, \hat{i}_t$, two targeting variables and the policy instrument, to maximise the objective function subject to the GNKPC and the dynamic IS equation. Practically, this optimisation problem consists of two states: in the first stage, the central bank obtains the optimal value of $\hat{x}_t$ and $\hat{\pi}_t$ that maximise its objective, subject to constraints. In the second stage, conditional on the optimal values of $\hat{x}_t$ and $\hat{\pi}_t$, the bank determines the value of $\hat{i}_t$ implied by the dynamic IS equation. More specifically, since the bank is not able to make any credible promise to the public, hence it is in its best interests to find the optimal value of $\hat{x}_t$ and $\hat{\pi}_t$ that maximise its objective in the current period only. In addition, the central bank has to take the private sector expectations as given as it has no control over public beliefs. The static optimisation problem can be written as the following

$$L = \frac{1}{2}(\chi(\bar{\pi})\hat{x}_t^2 + \hat{\pi}_t^2) + \Gamma_t\left[\hat{\pi}_t - \kappa(\bar{\pi})\eta\hat{s}_t - \lambda(\bar{\pi})(\hat{x}_t - \bar{x}) - b_1E_t\hat{\pi}_{t+1} + b_2\hat{\psi}_{t+1}\right] \tag{1.36}$$

The first order conditions and solution to the first stage problem yield the following optimal condition

$$\hat{x}_t = -\frac{\lambda(\bar{\pi})}{\chi(\bar{\pi})} \cdot \hat{\pi}_t$$

The solution is very similar to the one obtain in original CGG and Ascari and Ropele (2007). However, they are different in the way that both $\lambda(\bar{\pi})$ and

$\chi(\bar{\pi})$ now depend on the value of $\bar{\pi}$. The CGG does not take into account the effect of trend inflation, while the Ascari and Ropele (2007) study uses a fixed χ, which does not take into account the effect of $\bar{\pi}$. Recall the definition of $\chi(\bar{\pi}) = \frac{1-\theta\bar{\pi}^{\epsilon-1}}{1-\theta\bar{\pi}^{\epsilon}} \cdot \frac{\lambda(\bar{\pi})}{\epsilon}$, one can replace $\chi(\bar{\pi})$ in the above optimality condition and obtain that

$$\hat{x}_t = -\frac{(1-\theta\bar{\pi}^{\epsilon}) \cdot \epsilon}{1-\theta\bar{\pi}^{\epsilon-1}} \cdot \hat{\pi}_t$$

This expression becomes much more straightforward in terms of the effect from rising trend inflation, it implies that as trend inflation $\bar{\pi}$ increases, the coefficient for inflation $\hat{\pi}$ becomes larger in absolute value, i.e., more negative. Therefore, this implies more output needs to be sacrificed in order to stabilise inflation deviation from the central bank's target. This is not completely in line with the results from Ascari and Ropele (2007), as they fail to take into account the change in the definition of χ due to the presence of trend inflation.

There are now two very important countering effects here: on the one hand, as the coefficient for inflation in the optimal condition become larger with higher trend inflation, the central bank should respond less to inflation deviation from target because the sacrifice of output is dramatically increased compared with no trend inflation. This suggests the monetary authority should be *less* aggressive to inflation fluctuations. On the other hand, since $\chi(\bar{\pi})$ becomes smaller with higher trend, therefore, the stabilisation of output becomes much less important for the central bank with higher trend inflation, and inflation fluctuations become much more costly; the central bank should be much less concerned with the decline in output compared with the increase in inflation, and this suggests the central bank should be *more* aggressive to a inflationary shock.

Furthermore, inflation deviation is more costly under higher trend inflation due to an additional channel from price dispersion caused by higher trend inflation and the mutual feedback between inflation and price dispersion, and this trade-off between stabilising inflation and stabilising output becomes even greater here as a result of trend inflation. The total effects may yield a situation where the central bank to cut output level by a greater amount with higher trend inflation in order to keep inflation at a lower rate at the cost of output contraction, as the cost of inflation is hugely amplified. But these overall effects are largely determined by the value of parameters in the model.

Stabilisation under Discretion - Cost Push Shock

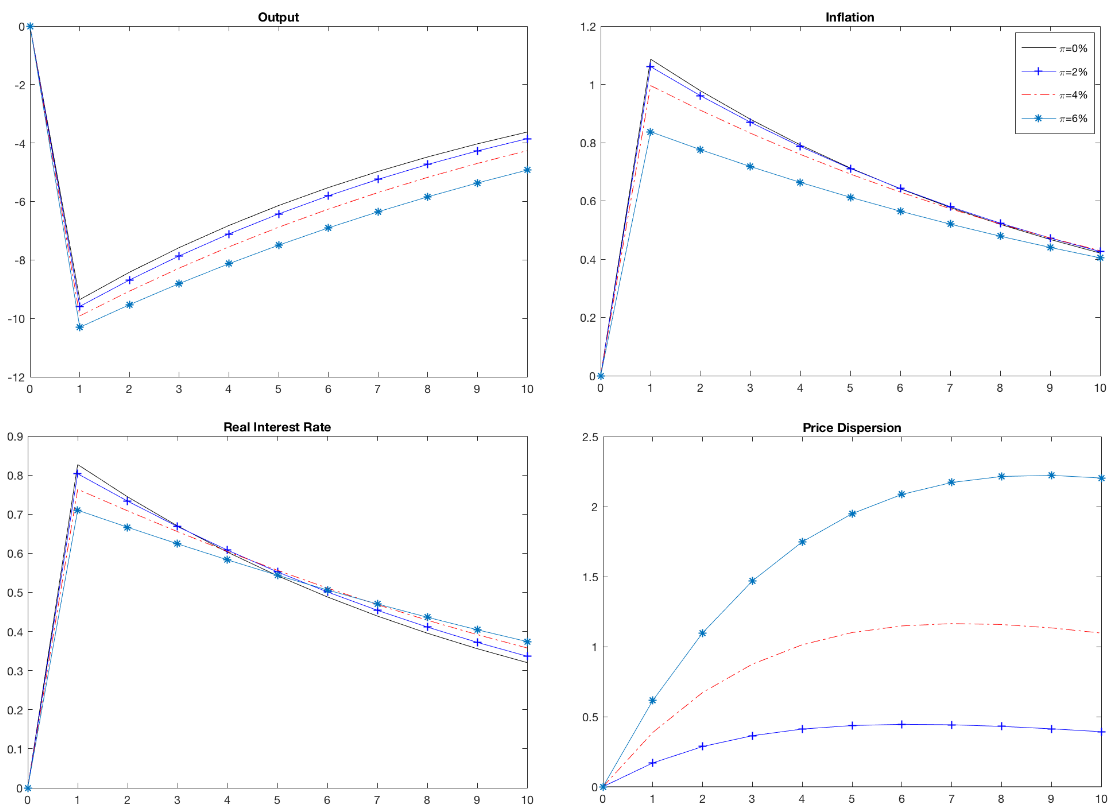

Figure 1.4: Impulse response functions to a one percent cost push shock under discretionary policy ($\rho_u = 0.90$)

Figure 1.4 shows the impulse response functions of output, inflation, real interest rate and price dispersion after a highly persistent cost push shock ($\rho_u = 0.90$). Clearly, the trade-off between stabilising output gap and inflation is still present, it means the **Result 1** from CGG still hold with trend inflation.

The IRFs in Figure 1.4 confirm the initial inference: the rise in inflation is indeed lower for higher levels of trend inflation because it is too costly for the central bank to tolerate inflation now. Quantitatively, inflation with 6 percent trend inflation is 23 percent lower than inflation with 0 percent trend inflation in period 1. Also, in order to control inflation with a higher trend, output is dampened by a greater amount (10 percent more decline in output for 6 percent trend compared with zero trend) with higher trend inflation, as indicated by the IRF of output.

Stabilisation under Discretion - Productivity Shock

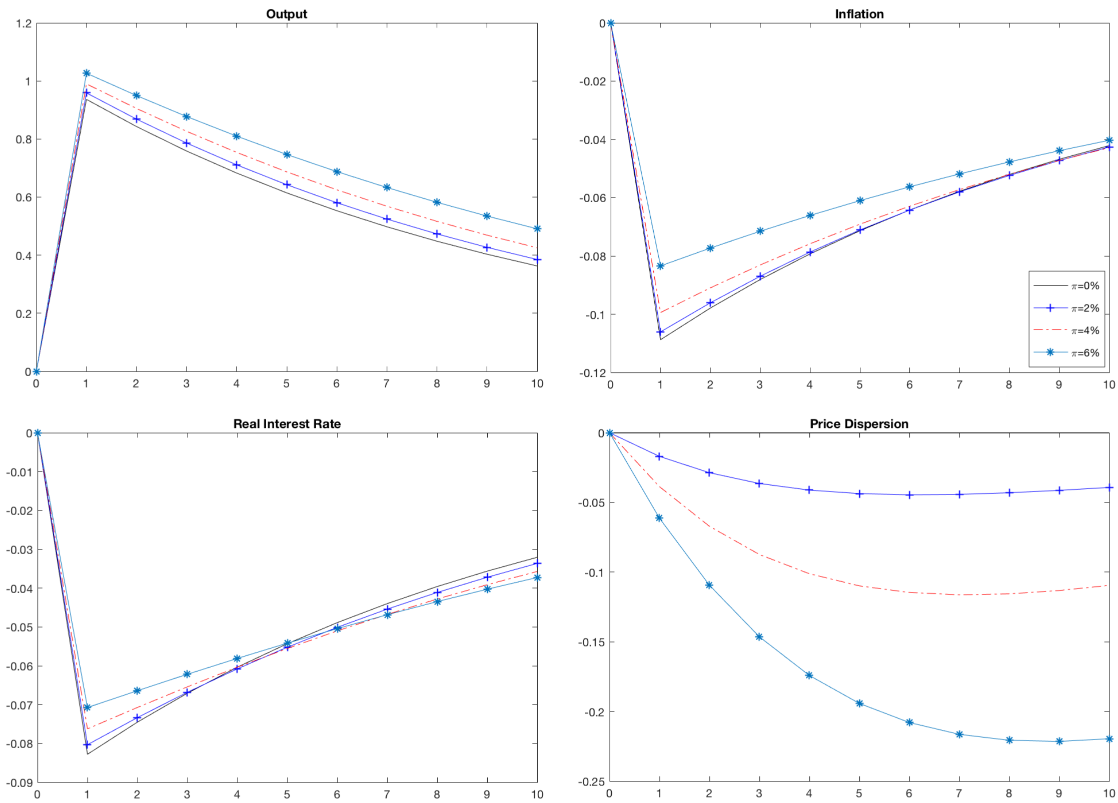

Figure 1.5: Impulse response functions to a one percent productivity shock under discretionary policy ($\rho_a = 0.90$)

Figure 1.5 shows exactly the opposite scenario for the same story. When the central bank is faced with deflationary pressure after a shock to total factor productivity, the central bank stabilises the economy by reducing the drop in inflation due to deflation is now too costly for the bank combined with large negative price dispersion. At the same time, output is allowed to deviate to a greater extent with higher levels of trend inflation. These responses of output and inflation justify the analysis in previous paragraphs. In addition, the two sets of IRFs demonstrate that the **Result 2** from CGG still hold; the optimal policy incorporates inflation targeting in the sense that it requires to aim for convergence of inflation to its target over time.

The **Result 3** of Taylor principle in CGG is also valid in this GNK model. In fact, due to the presence of trend inflation, satisfying the *Taylor Principle* becomes a necessary but not sufficient condition for determinacy. As documented in Ascari and Ropele (2009) and Coibion and Gorodnichenko (2011), this extension of the Taylor Principle breaks down with positive trend inflation because the slope of the NKPC becomes negative for extremely high levels of trend inflation. Small but positive responses to the output gap lead to lower minimum responses to inflation to achieve determinacy, as it was the case under zero inflation steady state (Coibion and Gorodnichenko, 2011).

The Classic Inflationary Bias Problem

The **Result 5** of CGG describe the presence of inflationary bias under a discretionary central bank. As many New Keynesian papers and textbooks of New Keynesian economics; a discretionary central bank always makes extra welfare loss due to the short term temptation of pursuing positive output gap.

Suppose a discretionary central bank pursues a positive output gap target, which implies the monetary authority intentionally keeps actual output above its natural level. The new objective function for the central bank can be rewritten as

$$\omega = \frac{1}{2} E_t \sum_{j=0}^{\infty} \beta^j [\chi(\bar{\pi})(\hat{x}_{t+j} - \xi)^2 + \hat{\pi}_{t+j}^2]$$

where ξ is the output gap target, and it satisfies $\xi > 0$.

Optimisation yields the following first order condition

$$\hat{x}_t^{\xi} = -\frac{\lambda(\bar{\pi})}{\chi(\bar{\pi})} \cdot \hat{\pi}_t^{\xi} + \xi$$

The superscript ξ stands for the variables in optimal solution under the scenario when $\xi > 0$. According to this optimal condition, ξ term is still in the optimal condition and the inflationary bias is still present in this GNK model. This indicates that if a central bank pursues a policy that pushes output above its natural level, then it has to accept the fact that inflation would be substantially above its target at the same time with no gain in output.

Subsequently, this implies no matter what level of inflation the central bank targets, as long as the bank seeks to create a positive output gap in the short run, there is no way it can meet its own target, even when the target is zero. Thus, the action of ignoring trend inflation in the model becomes inevitably fatal.

1.4.3 Optimal Monetary Policy under Commitment

A commitment central bank conducts monetary policy by making credible commitments to the public and strictly follow these promises. Since the central bank is credible, hence, it is not only concerned about the current state of the economy, but all the future periods, and a lifetime optimisation has to be taken into account when it makes decisions. The central bank now maximises its objective function for its lifetime and subject to constraints in all the future periods. A dynamic optimisation problem can be written as

$$\begin{aligned} L = \frac{1}{2}E_t\{\beta^j[\chi(\bar{\pi})\hat{x}_{t+j}^2 + \hat{\pi}_{t+j}^2] \\ + \Gamma_{t+j}\left[\hat{\pi}_{t+j} - \kappa(\bar{\pi})\eta\hat{s}_{t+j} - \lambda(\bar{\pi})\hat{x}_{t+j} - b_1E_t\hat{\pi}_{t+j+1} + b_2\hat{\psi}_{t+j+1}\right] \end{aligned} \quad (1.37)$$

In this example, I solve the optimal policy by appyiyng the *timeless perspective* method from Woodford (2003). The *timeless perspective* commitment rule is designed to solve the problem of time inconsistency, which arises under commitment and it suggests the central bank should ignore the optimality for the initial period and apply the optimal conditions for following periods, as viewing the problem from infinite future. The optimal solution of *timeless perspective* yields

$$\hat{x}_{t+i} - \hat{x}_{t+i-1} = -\frac{\lambda(\bar{\pi})}{\chi(\bar{\pi})} \cdot \hat{\pi}_{t+i}$$

for $i = 1, 2, 3, ...$

This optimal condition is almost identical to equation (4.18) in CGG, the only difference is that the two parameters in front of inflation $\hat{\pi}_{t+1}$ directly rely on the level of trend inflation. The distinction with the optimal condition under discretion is the central bank now stabilise inflation by controlling the change of output over two consecutive periods. The same rationale obtained under discretion still hold: as trend inflation increases, a commitment central bank stabilise inflationary pressure by creating a negative change of output gap, in other words, a drop of output gap from the previous period. However, due to the credibility it obtains under commitment, the central bank can control targeting variables by influencing future expectations in the economy in the sense that it can make credible promises regarding central bank's future stabilisation activities. The amplified trade-off between stabilisations of inflation and output is still present according to this optimality, the way of central bank response is different, nonetheless. As trend inflation increases, the price setting firms become more forward looking, this facilitates the policymaker to use more forward guidance to influence future expectations.

Stabilisation under Commitment - Cost Push Shock

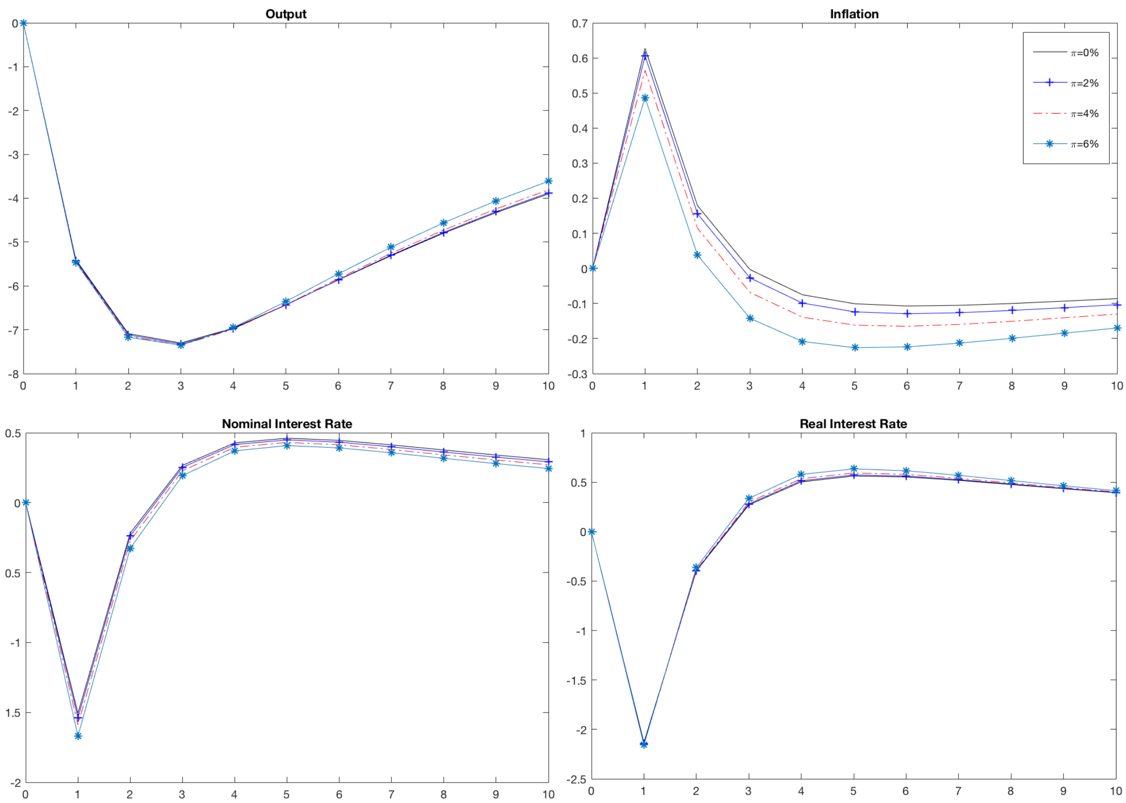

Figure 1.6: Impulse response functions to a one percent cost push shock under timeless perspective commitment policy ($\rho_u = 0.90$)

With credibility to the public, a central bank under commitment can better control inflation by influencing expectations of the future. The best way to study stabilisation performance and potential gains from commitment policy is to see how the central bank react after a cost push shock. Figure 1.6 shows the impulse response functions for a highly persistent cost push shock. Similar to the case under discretion, inflation jumps on impact by less with higher levels of trend inflation. This lower inflation for higher trend is due to the fact that inflation becomes much more costly with higher trend through its impact on $\chi(\bar{\pi})$, as demonstrated under the previous case of discretion.

Quantitatively, the initial deviations of inflation is much lower than that under discretion for all four levels of trend inflation, huge welfare gains from commitment on inflation can be observed. This is achieved by making credible promises to the public that the central bank is to conduct disinflationary policy for the future. The price-setting firms are reluctant to raise prices immediately after a cost push shock if they expect the monetary authority is to respond with this promised contractionary policy. For output, the initial decline is only around half of that under discretion with 6 percent trend inflation; a considerable improvement from the discretionary policy. This is also achieved by responding to change in output gap rather than directly react to output gap; inflation expectation is anchored in the short run, and according to the optimal condition under commitment, the contraction in output does not need to be as large as it is under discretion. Overall, the monetary authority achieves a much better inflation-output trade-off under commitment by sufficiently utilising its ability to influence inflation expectations. In fact, the nominal interest is cut to negative value from its steady state, and this allows the central bank to offer more accommodations to output and then soon to raise it as it promises to the public. This is in contrast toAscari and Ropele (2007) where they assume a constant χ, which does not depend on the value of $\bar{\pi}$. This eliminates one of the two channels in this monetary stabilisation process, and it implies the central bank does not become less concerned with output deviation as trend inflation rises. Therefore, this study produces very different quantitative and qualitative implications.

Stabilisation under Commitment - Productivity Shock

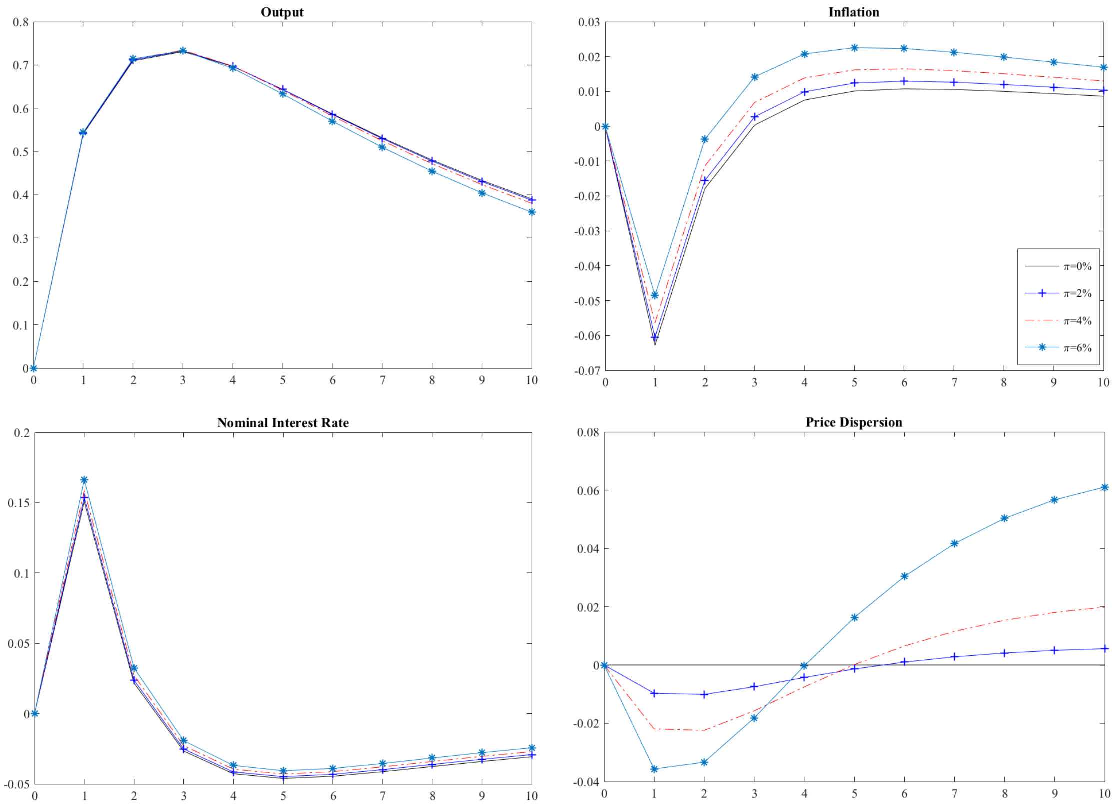

Figure 1.7: Impulse response functions to a one percent productivity shock under timeless perspective commitment policy ($\rho_a = 0.90$)

Figure 1.7 offers the IRFs of output, inflation, nominal interest rate, and price dispersion after a productivity shock. Again, due to the higher cost of inflation (in this case, deflation), the central bank responds more aggressively to inflation with higher trend and lifts inflation to a higher level compared with inflation with lower trend. Combined with the credibility to the public, commitment produces a better stabilisation trade-off than discretion, as the IRF of output also suggests. Both inflation and output deviate quantitatively much less than they do under discretion. In addition, by comparing IRFs of output and inflation under commitment with that under discretion, one can notice that macroeconomic fluctuations are much less persistent under commitment after shocks with the same persistence level as compared with discretion.

Trend inflation has profound quantitative and qualitative impact on the optimal monetary stabilisation policy in the business cycle. Even though a majority of the results from CGG sill hold in this GNK model, the quantitative analyses change drastically once trend inflation is taken into account. Some of the results shown here are subject to a number of uncertainties. For example, the values of parameters, as shown in section 1.4. When an increase in trend inflation has offsetting effects on the optimal monetary response, the overall net effects depend on the calibrated values of structural parameters. Furthermore, in practice, it is hard for the central bank to accurately estimate the true value of the natural level of output, hence, whether the weight of output stabilisation $\chi(\bar{\pi})$ should be even lower in practice when estimating natural output becomes increasingly hard is subject to debate. At least, this section demonstrates that trend inflation plays a significant role in the New Keynesian Models.

1.5 Conclusion

This opening chapter forms a Generalised New Keynesian (GNK) model. Unlike the traditional New Keynesian model, this model is log-linearised around a non-zero inflation steady state, hence, it is a generalisation of the standard New Keynesian model. This GNK solves the empirical inconsistency that long run inflation is far from zero in the postwar industrialised countries, and also the practical incoherence that no major central bank in the world acutally pursue a zero inflation long run target. This GNK model exhibit a number of features. First, the generalised New Keynesian Phillips curve (GNKPC) becomes flatter as trend inflation increases where inflation reacts less to change in output or output gap as trend inflation increases. Second, the flattening GNKPC implies that agents are more forward looking as a result of an increase in inflation. Furthermore, I show that trend inflation alters macroeconomic dynamics in a significant way, these alterations depend on the nature of shocks, also the calibrated value of parameters.

For optimal monetary policy, the quadratic loss function for central bank's stabilisation practice changes when it is approximated around a non-zero inflation steady state. In particular, the weight of stabilisation for output or output gap relies on the value of trend inflation; monetary authority is less concerned with output stabilisation as trend inflation rises. The generates two countering effects for optimal monetary response under a high level of trend inflation. On the one hand, as the weight for output stabilisation drops due to higher trend inflation, and combined with the fact that inflation fluctuations become much more costly for the central bank, the central bank should respond to inflation deviation more aggressively. On the other hand, as trend inflation increases, more output needs to be sacrificed in order to keep inflation stable, this makes the central bank to react less aggressively to inflation fluctuations and make the inflation-output trade-off even more serious. The overall net effect really depends on the value of key parameters. In the examples shown in section 1.4, I demonstrate a case where central bank tolerate a huge drop in output in order to keep inflation stable at higher trend inflation level under discretion. However, a commitment monetary authority that can make credible promises to the public can achieve a much superior stabilisation outcome and better inflation-output trade-off by influencing expectations about the future.

This chapter provides a cornerstone and indicator for the rest of this thesis regarding the role of trend inflation in New Keynesian DSGE models. The next two chapters are to study and evaluate this issue in a much more sophisticated environment and complicated model set-ups.

Chapter 2

Shocks, Frictions, and Trend Inflation in a Medium-Scale Generalised New Keynesian Model: A Bayesian DSGE Approach

2.1 Introduction

The previous chapter has shown that trend inflation has a huge impact on the overall dynamics of macroeconomic variables and optimal monetary response in a small-scale New Keynesian model with Calvo price setting. This chapter goes one step further by extending the analysis to a medium-scale NK DSGE model. There is very limited literature that uses medium scale New Keynesian model for the purpose of studying trend inflation. There are a few reasons behind this tendency. First, for implementation-related motivations, it is practically challenging to log-linearise a system of equations based on a medium-scale model around a non-zero inflation steady state, even though the presence of trend inflation only affects equations for price and wage setting behaviour. Second, for a medium-scale GNK model, it is not possible to conduct determinacy analysis like Ascari and Ropele (2009) analytically, and the only solution is to rely on numerical methods. The effect of trend inflation on macro dynamics has been proven and evaluated in literature, however, there is no guarantee that such impact can be replicated in the same way based a much more complicated model. By changing one parameter ($\bar{\pi}$) with a tiny fraction, it may not impact a model that involve around 20 structural parameters in any significant way.

Previous literature with relevant work including Arias, Ascari, Branzoli and Castelnuovo (2018) where they use a medium-scale New Keynesian model based on Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007) but without price or wage indexation. They find that the probability of determinacy drops considerably conditional on model-free estimates of the monetary policy rule based on real-time data, and this is largely due to the heavy response to output gap in the estimated policy rule. However, their work does not include anything about the impact from trend inflation on macroeconomic dynamics. Ascari, Phaneuf and Sims (2018) apply a medium-scale NK model with trend inflation by adding a few new features: trend growth in investment-specific technology, extended working capital channel, roundabout production function structure. They argue that trend inflation plays a significant role in this model due to these newly-added features.

Based on this limited literature, I think it is important to look at the issue of trend inflation based an estimated medium-scale New Keynesian model and I believe the best way to execute this is to re-develop the famous Smets and Wouters (2007) (SW07 henceforth) based on trend inflation. This model is regarded as a state-of-the-art framework for New Keynesian monetary economics and has been widely cited in a large number of papers. Furthermore, as other authors have found that trend inflation interacts significantly with new additional features of such model (Ascari, Phaneuf and Sims, 2018), I think it is essential to keep the model as close to the original SW07 framework as possible, and just to correct its inconsistency with regards to the assumption of zero-inflation steady state.

Therefore, the primary research questions of this *Chapter 2* are: **1) whether a medium-scale New Keynesian model like Smets and Wouters (2007) with trend inflation can capture the macro data in postwar US?** and **2) what impact does trend inflation make to macro variables and dynamics in such a medium-scale Generalised NK model?**

In order to answer the first question, I also estimate this medium-scale GNK model using a Bayesian technique, the same method as original SW07. The model I develop in this chapter inherits most of the key features of the SW07, but based on a non-zero inflation steady state, which is absent from their original model. First, my model combines non-zero steady state inflation with nominal price and wage rigidities and also partial indexation in both price and wage settings. Second, there are several real frictions in this model: habit formation in consumption preferences, adjustment cost in investment, and variable capital utilisation rate. Third, it incorporates real output growth per capita generated from total factor productivity growth. Fourth, the Kimball (1995) aggregator is replaced by the Dixit and Stiglitz (1977) index, in order to obtain the assumption of CES. Fifth, the central bank conducts monetary policy by setting nominal interest rate according to an inertial Taylor-type policy rule.

The intention of this study is to keep the assumptions as close to the original SW07 in order to evaluate how much difference does trend inflation alone can make. Therefore, I keep the assumption of backward indexation in the model used in this chapter. However, as a few studies have shown that the degree of indexation has a huge influence on the effect of trend inflation in such models. Ascari and Branzoli (2015) point out that when indexation is zero, inflation persistence depends only on the level of trend inflation, the inflation gap is purely forward-looking and trend inflation affects the model dynamics inducing inertia in the adjustment through price dispersion which is a backward looking variable. Nevertheless, full indexation makes the inflation gap persistent, eliminates the effects of trend inflation on macroeconomic dynamics of the model. Thus, it is worth to look at the two different cases where indexation is present and not. For this reason, I add a new parameter to the model that controls the overall degree of indexation, so that in the next chapter I can switch indexation off and to check the difference that indexation makes on the role of trend inflation for the model dynamics.

The rest of this chapter is organised as the following: section 2.2 develops a medium-scale Generalised New Keynesian model, then followed by section 2.3 where the Bayesian estimation is presented and discussed. Section 2.4 conducts a sub-sample estimation in order to see whether the level of trend inflation changes over time. Then, section 2.5 focuses on the impact of trend inflation on macro dynamics. Finally, section 2.6 draws some concluding remarks.

2.2 The Medium Scale New Keynesian DSGE Model with Trend Inflation

This section presents the medium-scale Generalised New Keynesian (GNK) DSGE model with the explicit modelling of trend inflation. It is largely based on the Smets and Wouters (2007) model with a number of modifications, which are discussed in detail in the following sub-sections. Throughout this thesis, I refer this GNK model with indexation and trend inflation as the *benchmark model*, there are three more different versions derived from this benchmark model are formed for further studies in later sections and chapter.

2.2.1 Decision-Making by Firms, Households and the Government

Final Goods Producers

The final goods producers face a perfectly competitive market, where firms are price takers. Nevertheless, unlike the SW07 model where they use Kimball (1995) aggregator, here, I assume a Dixit and Stiglitz (1977) style aggregator, which implies a constant elasticity of substitution (CES). The subscript i represents individual intermediate good producer, and there is a continuum of intermediate goods firms, indexed by $i \in [0, 1]$. At a given point in time t, a perfectly competitive final goods firm produces the final consumption good Y_t by combining a continuum of intermediate goods $Y_t(i)$, according to the technology

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{1}{1+\lambda_{p,t}}} di \right)^{1+\lambda_{p,t}}$$

The problem for a representative final goods firm is to maximise its profit

$$max_{Y_t, Y_t(i)} \; P_t Y_t - \int_0^1 P_t(i) Y_t(i) d_i$$

where $P_t(i)$ is the price of the ith intermediate goods and P_t is the aggregate good price in the economy. The first order condition with respect to $Y_t(i)$ implies the following demand function for the ith intermediate goods:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} Y_t$$

and aggregate price index

$$P_t = \left[\int_0^1 P_t(i)^{-\frac{1}{\lambda_{p,t}}} \right]^{-\lambda_{p,t}}$$

The parameter $\lambda_{p,t}$ determines the degree of mark-up of intermediate goods firms and can be considered as a measure of market power in the intermediate goods market. A shock to $\lambda_{p,t}$ represents a price mark-up shock.

Intermediate Goods Producers

The intermediate goods sector is a monopolistic competition market and firms optimise their prices in each period subject to Calvo (1983) contract. In contrast to the original SW07 set-up, there is no labour-augmenting deterministic trend growth in the economy. F stands for the fixed cost of the production process, and the intermediate goods producer's production process can be characterised by a standard Cobb-Douglas function.

$$Y_t(i) = Z_t K_t^s(i)^{\alpha} L_t(i)^{1-\alpha} - F \tag{2.1}$$

where $K_t^s(i)$ is the capital service used by individual producer i. Z_t is the total factor productivity, and an exogenous shock to Z_t represents a technology shock.

Firm's profit function is given by:

$$\Pi_t = P_t(i)Y_t(i) - W_t L_t(i) - R_t^k K_t^s(i)$$

The first order condition of profit maximisation problem yields the following optimalities:

$$K_t^s = \left(\frac{\alpha}{1-\alpha}\right)\frac{W_t}{R_t^k} \cdot L_t \tag{2.2}$$

$$MC_t = \alpha^{-\alpha}(1-\alpha)^{\alpha-1} Z_t^{-1} \cdot W_t^{1-\alpha}(R_t^k)^{-\alpha} \tag{2.3}$$

Where MC_t is the nominal marginal cost for intermediate firms, and it is the same across firms, therefore, the subscript i can be dropped.

Price Resetting Firms

A minor change from the original SW07 model regarding the prices and wages indexation is the introduction of a new parameter, the total degree of indexation, which is governed by a separate parameter χ. The reason behind this assumption of partial indexation in price and wage settings is that it can be easily used to switch off indexation in later chapters by simply setting $\chi = 0$, this is just for practical convenience. This generalisation has further benefits when I conduct trend inflation exercise in later sections and chapter. As explained by Ascari and Branzoli (2015) and Ascari, Phaneuf and Sims (2018), full indexation would nullify the effects of trend inflation, and limits to second order effects of price dispersion, which is one of the most crucial variables in this model.

Even though SW07 claims that their set-up of indexation is partial as a result of the weighted average of past inflation and steady state inflation, the price would change in every period no matter how much weight is assigned to past inflation, the price is simply indexed to steady state inflation, which is similar to one of the two options in Christiano, Eichenbaum and Evans (2005) model. A similar way of indexing to steady state inflation can also be found in Erceg, Henderson and Levin (2000) and Yun (1996).

The optimal price reset problem for intermediate firms can be written as

$$max_{\tilde{P}_t(i)}\, E_t \sum_{s=0}^{\infty} \zeta_p^s \left[\frac{\beta \Xi_{t+s} P_t}{\Xi_t P_{t+s}} \right] \cdot \left(\tilde{P}_t(i) \prod_{l=1}^{s} (\pi_{t+l-1}^{l_p} \bar{\pi}^{(1-l_p)})^{\chi} - MC_{t+s} \right) Y_{t+s}(i)$$

subject to

$$Y_{t+s}(i) = Y_{t+s} \left(\frac{P_{t+s}(i)}{P_{t+s}} \right)^{-\frac{1+\lambda_{p,t+s}}{\lambda_{p,t+s}}}$$

$$P_{t+s}(i) = P_t(i) \prod_{l=1}^{s} (\pi_{t+l-1}^{l_p} \bar{\pi}^{(1-l_p)})^{\chi} = P_t(i) X_{t,s}$$

Where $\tilde{P}_t(i)$ is the optimal reset price in a given period t, and ζ_p is the Calvo nominal rigidity parameter and it defines the probably that a firm is not allowed to reset its price in a given period. $\Pi_t = \frac{P_t-1}{P_t}$ is defined as the gross inflation rate. $\frac{\beta \Xi_{t+s} P_t}{\Xi P_{t+s}}$ is the stochastic discount factor for firms.

First order condition with respect to $\tilde{P}_t(i)$ yields

$$E_t \sum_{s=0}^{\infty} \zeta_p^s \left[\frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} \right] \cdot Y_{t+s}(i) \cdot \left(\frac{1}{\lambda_{p,t+s}} \right) [(1+\lambda_{p,t+s}) MC_{t+s} - \tilde{P}_t(i) X_{t,s}] = 0 \tag{2.4}$$

The aggregate price index in the economy is given by

$$P_t = \left[(1-\zeta_p) \tilde{P}_t^{-\frac{1}{\lambda_{p,t}}} + \zeta_p ((\pi_{t-1}^{l_p} \bar{\pi}^{1-l_p})^{\chi} P_{t-1})^{-\frac{1}{\lambda_{p,t}}} \right]^{-\lambda_{p,t}}$$

2.2.2 Households

There is a continuum of households, indexed by $j \in [0,1]$, and the representative household attempts to maximise the lifetime utility

$$E_t \sum_{s=0}^{\infty} \beta_p^s \left[\frac{1}{1-\sigma_c} (C_{t+s}(j) - \lambda C_{t+s-1}(j))^{1-\sigma_c} \right] \cdot exp \left(\frac{\sigma_c - 1}{1+\sigma_l} \cdot L_{t+s}(j)^{1+\sigma_l} \right)$$

Subject to the corresponding lifetime budget constraint

$$C_{t+s}(j) + I_{t+s}(j) + \frac{B_{t+s}(j)}{R_{t+s} P_{t+s}} - T_{t+s}$$

$$= \frac{B_{t+s-1}(j)}{P_{t+s}} + \frac{W_{t+s}(j) L_{t+s}(j)}{P_{t+s}} + \frac{R_{t+s}^k U_{t+s}(j) K_{t+s}(j)}{P_{t+s-1}}$$
$$-a(u_{t+s}(j) K_{t+s-1}(j) + \frac{Div_{t+s}}{P_{t+s}}$$

The utility function takes exactly the same form as the original SW07 model, where λ is the parameter that governs habit formation in consumption preference. B_t is the government bonds holding, with a gross return rate of R_t. Div_{t+s} is the profits redistributed to households for the share they hold in firms. Note, utilities in consumption and leisure are non-separable.

Meanwhile, the capital accumulation process can be defined as the following:

$$K_t(j) = (1-\delta) \cdot K_{t-1}(j) + \left[1 - \mu_t S\left(\frac{I_t(j)}{I_{t-1}(j)}\right)\right] \cdot I_t(j)$$

$S(\cdot)$ is the capital adjustment cost function, with the property of $S(1) = 1$, $S'(1) = 1$, and $S''(\cdot) > 1$. μ_t represents the shock to the relative price of investment to consumption goods, and it can also be interpreted as the marginal efficiency of investment, as described by Justiniano and Primiceri (2008). A shock to this μ_t represents a marginal efficiency of investment (MEI) shock.

Since all the households make the same optimal decision, thus, the subscripts (j) can be dropped from now on. First order conditions of households utility maximisation problem gives the marginal cost equation

$$\frac{W_t^h}{P_t} = -\frac{1}{\Xi_t}\left[\frac{1}{1-\sigma_c}(C_t - \lambda C_{t-1})^{1-\sigma_c}\right] exp\left(\frac{\sigma_c - 1}{1+\sigma_l} \cdot L_{t+s}^{1+\sigma_l}\right) \cdot (1+\sigma_l) L_t^{\sigma_l}$$

This implies that $MRT = MRS$; the right hand side is the marginal disutility of labour, which is identical across all households.

Intermediate Labour Unions and Wage Setting

In contrast to Erceg, Henderson and Levin (2000), where households are assumed to provide heterogenous labour. This model assumes households supply homogeneous labour to labour unions, which they have monopolistic power over firms. There is a continuum of labour unions in labour market, indexed by $l \in [0,1]$. Labour union differentiates labour services and set wages subject to Calvo (1983) contract, then sell these labour to labour packers. Labour used by the intermediate goods producers L_t is a composite:

$$L_t = \left[\int_0^1 L_t(l)^{\frac{1}{1+\lambda_w}} dl\right]^{1+\lambda_w}$$

This is also the labour supply, unlike SW07, this set-up for labour markets follows Del Negro et al. (2007) and Justiniano and Primiceri (2008). λ_w is now a fixed parameter rather than a variable, and this model does not consider shock to wage mark-up.

Following the differentiation and wage setting from labour unions, labour packers purchase the labour from unions, package L_t, and resell these labour to the intermediate goods producer. These labour packers attempt to maximise their profits in a perfectly competitive market. The first order conditions for labour packers yield the labour demand equation

$$L_t(l) = \left(\frac{W_t(l)}{W_t}\right)^{-\frac{1+\lambda_w}{\lambda_w}} \cdot L_t$$

Combining this with the zero profit condition (packers), one can obtain the wage cost expression for intermediate goods producers:

$$W_t = \left[\int_0^1 W_t(l)^{-\frac{1}{\lambda_w}}, dl\right]^{-\lambda_w}$$

λ_w is the wage mark-up charged labour unions, and it implies the amount that unions charge firms over what they actually pay the households. Labour packers buy labour from the unions. Here, the unions serve as an intermediary between households and labour packers. The unions have market power by differentiating and allocating labour services from households. They can choose wage subject to the labour demand equation and Calvo (1983) probability.

The probability that a union can readjust its wages in a given period is

$1-\zeta_w$, hence, ζ_w is the likelihood that a union is stuck during the resetting process. For those who cannot readjust wages, $W_t(l)$ adjusts with a weighted average of the steady state inflation $\bar{\pi}$, and the inflation rate carried out from last period π_{t-1}.

For those who have the chance to re-optimise, the problem is to choose a wage $\tilde{W}_t(l)$ such that it can maximise the wage income in all states of nature where the union is stuck with that wage in the future.

$$max\ E_t \sum_{s=0}^{\infty} \zeta_w^s \left[\frac{\beta \Xi_{t+s} P_t}{\Xi_t P_{t+s}}\right] \cdot [W_{t+s}(l) - W_{t+s}^h] L_{t+s}(l)$$

subject to

$$L_{t+s}(l) = \left(\frac{W_{t+s}(l)}{W_{t+s}}\right)^{-\frac{1+\lambda_w}{\lambda_w}} \cdot L_{t+s}$$

with

$$W_{t+s}(l) = \tilde{W}_t(l) \cdot \prod_{l=1}^{s} (\pi_{t+l-1}^{l_w} \bar{\pi}^{1-l_w})^{\chi} = \tilde{W}_t(i) X_{t,s}$$

Since the wage setting decision is identical across individuals, therefore, the subscript (l) can be dropped, optimisation problem yields the following first order condition:

$$E_t \sum_{s=0}^{\infty} \zeta_w^s \left[\frac{\beta \Xi_{t+s} P_t}{\Xi_t P_{t+s}}\right] \cdot L_{t+s}(l) \left(\frac{1}{\lambda_{w,t+s}}\right) [(1+\lambda_{w,t+s}) W_{t+s}^h - X_{t,s} \tilde{W}_t(l)] = 0$$

The aggregate wage index is given by

$$W_t = \left[(1-\zeta_w) \tilde{W}_t^{-\frac{1}{\lambda_{w,t}}} + \zeta_w ((\pi_{t-1}^{l_w} \bar{\pi}^{1-l_w})^{\chi} W_{t-1})^{-\frac{1}{\lambda_{w,t}}}\right]^{-\lambda_{w,t}}$$

This above setting permits the model to capture the behaviour of wage setting, which later plays a crucial role in determining the effects of trend inflation on model dynamics.

2.2.3 The Central Bank

Finally, the monetary authority conducts monetary policy by following a Taylor style rule, which is characterised by the following equation. Nominal interest rate is adjusted in response to deviation of inflation from its steady state level, and output deviation from the natural rate, also the change in output gap over time.

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\bar{\pi}}\right)^{\psi\pi} \left(\frac{Y_t}{Y^n}\right)^{\psi_y}\right]^{1-\rho_R} \left(\frac{Y_t/Y_{t-1}}{Y_t^n/Y_{t-1}^n}\right)^{\psi_{\Delta y}} \cdot M_t$$

where

$\bar{R}$ is the steady sate nominal interest rate in gross term

$\bar{\pi}$ is the steady state inflation

Y^n is the natural output level

ρ_r is the degree of interest rate smoothing

M_t captures the monetary policy shock

Here, the central bank supplies money that demanded by households to support the desired nominal interest rate in the market.

2.2.4 Resource Constraint, Aggregations and Price Dispersion

For the final goods market, market clearing condition is obtained by integrating the household budget constraint across all households and combine it with the government's budget constraint. Then, aggregation yields

$$P_tC_t+P_tI_t+P_tG_t = \Pi_t+\int W_t(j)L_t(j)dj+R_t^k \int K_t(j)dj-P_ta(u_t) \int K_{t-1}(j)dj$$

Recall that $\Pi_t = \int \Pi_t(i)di = \int P_t(i)Y_t(i)di - W_tL_t - R_t^kK_t$, where $L_t = \int L_t(i)di$ is the total labour supplied by the labour packers.

Replacing the definition of Π_t in the household budget constraint, and combining it with the capital accumulation process and labour packers' zero-profit condition, we get

$$W_t L_t = \int W_t(j) L_t(j) dj = \int W_t^h(j) L_t(j) dj + Div_t$$

where $W_t(j)$ is the wage rate paid by firms, and $W_t^h(j)$ is the wage rate received by households, the difference between the two is reflected by level of wage markup.

Combining this with the zero-profit condition for final goods market, one can obtain the real term expression

$$C_t + I_t + G_t + a(u_t) \cdot K_{t-1} = Y_t$$

This is the aggregate resource constraint.

Price Dispersion

One of the most important variables of this GNK model is the level of price dispersion. It is expressed as the following

$$s_t = \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\frac{1+\lambda_p}{\lambda_p(1-\alpha)}} di$$

According to Schmitt-Grohé and Uribe (2007a), the relative price dispersion (with Calvo contract) s_t is bounded below by one, and it represents the resource costs as a consequence of relative price dispersion with long-run inflation.

By accounting for the presence of price dispersion, one can obtain the true production level given all the inputs:

$$L_t = \left(\frac{Y_t}{A_t {K^s_t}^{\alpha}} \right)^{\frac{1}{1-\alpha}} \cdot s_t$$

The higher the level of price dispersion is, more labour is needed in order to produce the same amount of output. Therefore, for the same level of output, price dispersion cause equilibrium real wage to increase and so does the real marginal cost of the firm (Ascari and Sbordone, 2014).

Under the assumption of Calvo (1983) price setting mechanism, the expression for s_t can be rewritten in a recursive form:

$$s_t = \zeta_p \tilde{p}_t(i)^{-\frac{(1+\lambda_p)}{\lambda_p(1-\alpha)}} + (1-\zeta_p)\bar{\pi}^{-\frac{(1+\lambda_p)\chi(1-l_p)}{\lambda_p(1-\alpha)}} \pi_{t-1}^{-\frac{(1+\lambda_p)\chi l_p}{\lambda_p(1-\alpha)}} \pi_t^{\frac{(1+\lambda_p)}{\lambda_p(1-\alpha)}} s_{t-1} \qquad (2.5)$$

where $\tilde{p}_t(i)$ is the real optimal reset price. Also, $\tilde{p}_t(i)$ satisfies the condition of

$$1 = [(1-\zeta_p)\tilde{p_p}^{\frac{1}{\lambda_p}} + \zeta_p(\pi_{t-1}^{lp\chi}\bar{\pi}^{(1-lp)\chi}\pi_t^{-1})^{\frac{1}{\lambda_p}}]^{\lambda_p}$$

.

When price indexation is removed ($\chi = 0$), the expression of price dispersion collapses to

$$s_t = \zeta_p \tilde{p}_t(i)^{-\frac{(1+\lambda_p)}{\lambda(1-\alpha)}} + (1-\zeta_p)\pi_t^{\frac{(1+\lambda_p)}{\lambda_p(1-\alpha)}} s_{t-1}$$

According to Schmitt-Grohé and Uribe (2007a), this state variable s_t represents the resource costs triggered by the inefficient dispersion of price of price in equilibrium and it must satisfy three conditions: first, s_t is bounded below by 1, this implies that price dispersion is always distortionary and costly to the overall economy [1]. Second, in an economy where the non-stochastic level of inflation is zero (i.e., $\bar{\pi} = 1$) or where prices are fully indexed to any variable such as ω_t with the property that its deterministic steady-state level equals the deterministic steady state value of inflation (i.e., $\bar{\omega} = \bar{\pi}$), then the variable of price dispersion $s_t = 1$ follows, up to first order, the univariate autoregressive process $\hat{s}_t = \alpha\hat{s}_{t-1}$. In such cases, the studies that restrict attention to

[1] To see the proof of this condition, please refer to Schmitt-Grohé and Uribe (2007a) and Schmitt-Grohé and Uribe (2007b) for detailed derivations and discussion.

linear approximations to the equilibrium conditions are justified to ignore this variable s_t if the model features no price dispersion in the deterministic steady state. However, even though the model is approximated up to first order in this study, the fact and assumption of positive trend inflation (i.e., $\bar{\pi} > 1$) clearly show the second condition does not apply in this model with trend inflation, and this variable must not be ignored here. Last, when prices are fully flexible, $\zeta_p = 0$, which implies $\tilde{p}_t(i) = 1$ and we will have $s_t = 1$, because there is no price dispersion in a fully flexible economy (Schmitt-Grohé and Uribe, 2007a).

In addition, based on equation (2.5), one can see that this term provides some extra dynamics to the model, as price dispersion depends on it lagged own term s_{t-1}.

2.2.5 Exogenous Shocks

In this model, there are totally five shocks, which are two less than that of the original SW07 model. The shocks that are ignored include wage mark-up shock and shock to investment premium. The ignorance of these two shocks are due to their lack of microeconomic foundation, as discussed by Chari, Kehoe and McGrattan (2009).

All the five variables, total factor productivity, government spending, monetary policy term, price mark-up, and investment relative price follow a first order autoregressive process, except government spending and price mark-up, which both follow a first order ARMA (1,1) process. Unlike Del Negro et al. (2007) and Justiniano and Primiceri (2008), I assume the shocks are all stationary in this model, subsequently, stationary data is used for estimation. The following equations specify the motions of these five variables related to exogenous shocks.

- Technology Shock

$$lnZ_t = \rho_z lnZ_{t-1} + \varepsilon_t^z \tag{2.6}$$

- Government Spending Shock

$$lng_t = \rho_g lng_{t-1} + \rho_{gz} lnZ_t - \rho_{gz} Z_{t-1} + \varepsilon_t^g \tag{2.7}$$

- Monetary Policy Shock

$$lnM_t = \rho_m lnM_{t-1} + \varepsilon_t^m \tag{2.8}$$

- Price Mark-up Shock

$$ln\lambda_{p,t} = (1-\rho_p)ln\lambda_p + \rho_p ln\lambda_{p,t-1} - \theta_p \varepsilon_{t-1}^p + \varepsilon_t^p \tag{2.9}$$

- Investment Relative Price

$$ln\mu_t = \rho_\mu ln\mu_{t-1} + \varepsilon_t^\mu \tag{2.10}$$

All five ε_t's satisfy $\varepsilon_t \sim N(0, \sigma^2_{\varepsilon_t}), i.i.d.$

2.2.6 The Log-Linearised Model

$$\hat{y}_t = \frac{\bar{c}}{\bar{y}} \cdot \hat{c}_t + \frac{\bar{i}}{\bar{y}} \cdot \hat{i} + \frac{\bar{g}}{\bar{y}} \cdot \hat{g}_t + \frac{\bar{r}^k k}{\bar{y}} \cdot \hat{u}_t \tag{2.11}$$

This is the aggregate resourse constraint, aggregate output ($\hat{y}_t$) in the economy is made of consumption ($\hat{c}_t$), private investment ($\hat{i}_t$), government spending ($\hat{g}_t$) and capital utilisation costs ($\hat{u}_t$). The steady state fraction in front of each variables represents their corresponding shares in the economy.

$$\hat{c}_t = \frac{1}{1+\lambda} E_t \hat{c}_{t+1} + \frac{\lambda}{1+\lambda} \hat{c}_{t-1} - \frac{1-\lambda}{\sigma_c(1+\lambda)} \cdot [\hat{R}_t - E_t \hat{\pi}_{t+1}] - \frac{(\sigma_c - 1)\bar{w}^h \bar{L}/\bar{c}}{\sigma_c(1+\lambda))} [E_t \hat{L}_{t+1} - \hat{L}_t] \tag{2.12}$$

This equation defines the dynamics of consumption. According to the equation, current consumption depends on a weighted average of past and expected future consumptions. It is also negatively correlated with the change in labour hours ($E_t\hat{L}_{t+1} - \hat{L}_t$) and the real interest rate ($\hat{R}_t - E_t\hat{\pi}_{t+1}$).

$$\hat{i}_t = \frac{1}{1+\beta}\hat{i}_{t-1} + \frac{1}{1+\beta}E_t\hat{i}_{t+1} + \frac{1}{(1+\beta)\psi} \cdot \hat{Q}_t^k \tag{2.13}$$

This equation define the dynamics of investment. It depends on both the past and future investments.

$$Q_t^k = \frac{1-\delta}{\bar{r^k}} \cdot E_t Q_{t+1}^k + \frac{\bar{r}^k}{\bar{r}^k + 1 - \delta} \cdot E_t \hat{r}_{t+1}^k \tag{2.14}$$

Q_t^k is a measure of capital stock. The current level of this capital stock is positively correlated with expected future real interest rate and the expected future value of itself.

$$\hat{y}_t = \alpha\hat{k}_t^s + (1-\alpha)(\hat{L}_t - \hat{s}_t) + \hat{Z}_t \tag{2.15}$$

Output ($\hat{y}_t$) is produced by using capital service ($\hat{k}_t^s$), labour (hours) ($\hat{L}_t$), and also determined by the level of total factor productivity ($\hat{Z}_t$). In addition, it is negative affected by the level of price dispersion in the economy, and this variable plays a crucial part of the dynamics in the model.

$$\hat{k}_t^s = \hat{u}_t + \hat{k}_{t-1} \tag{2.16}$$

The installed capital used in production process depends on the past capital stock and also utilisation level of capital service.

$$\hat{k}_t = (1-\delta)k_{t-1} + \frac{\bar{\bar{i}}}{\bar{y}} \cdot \hat{i}_t \tag{2.17}$$

Capital stock is a function of past undepreciated capital and new investment

$$\hat{u}_t = \frac{1-\eta}{\eta} \cdot \hat{r}^k_t \tag{2.18}$$

Capital utilisation equation is determined by capital rental rate.

$$\hat{r}^k_t = \hat{L}_t - \hat{k}_t + \hat{w}_t \tag{2.19}$$

The rental rate of capital is a function of labour hour, capital and real wage rate.

$$\begin{aligned}\hat{s}_t = -\frac{1+\lambda_p}{\lambda_p(1-\alpha)} \cdot \left[1 - \zeta_p \bar{\pi}^{\frac{(1-\chi)(1+\lambda_p)}{\lambda_p(1-\alpha)}}\right] \cdot \hat{\tilde{p}}_t + \zeta_p \bar{\pi}^{\frac{(1-\chi)(1+\lambda_p)}{\lambda_p(1-\alpha)}} \\ \cdot \left[\left(\frac{1+\lambda_p}{\lambda_p(1-\alpha)}\right) \cdot \hat{\pi}_t - \left(\frac{1+\lambda_p}{\lambda_p(1-\alpha)}\right) \cdot l_p \chi \hat{\pi}_{t-1} + \hat{s}_{t-1}\right]\end{aligned} \tag{2.20}$$

where

$$\hat{\tilde{p}}_t = \frac{\zeta_p \bar{\pi}^{\frac{1-\chi}{\lambda_p}}}{1 - \zeta_p \bar{\pi}^{\frac{1-\chi}{\lambda_p}}} \cdot [\hat{\pi}_t - l_p \chi \hat{\pi}_{t-1}]$$

This equation indicates that the overall price dispersion in the economy depends not only on the Calvo probability of price reset ζ_p as the basic price stickiness theory suggests, but also on the current and past inflation rates, as well as the past value of price dispersion itself $\hat{s}_{t-1}$. The presence of $\hat{\pi}_{t-1}$ and $\hat{s}_{t-1}$ introduces more overall dynamics into the model. This is consistent with all the previous literature on trend inflation, for example, Amano, Ambler and Rebei (2007). As trend inflation increases, its corresponding marginal effects on price dispersion from all variables are reinforced, and therefore making price dispersion a more serious issue for a given set of values of the variables.

To analyse this equation further, one can find the effect of different levels of trend inflation $\bar{\pi}$ is governed by a few parameters: the total degree of indexation χ, price mark-up λ_p, and the capital share of production cost α. More specifically, a lower degree of indexation, a lower level of price mark-up and a larger share of capital in production function all amplify the effects of trend inflation on price dispersion, and therefore, a higher degree of price dispersion. Among all three, a full indexation $\chi = 1$ can actually kill off the effect of trend inflation, and a lower level of indexation can strengthen the effect of trend inflation on the overall macroeconomic dynamics. This property of price indexation regarding trend inflation is extensively discussed in Ascari and Branzoli (2015) and Ascari, Phaneuf and Sims (2018).

$$\hat{w}_t^h = \frac{1}{1-\lambda} \cdot \hat{c}_t - \frac{\lambda}{1-\lambda} \cdot \hat{c}_{t-1} + \sigma_l \hat{L}_t \tag{2.21}$$

This is the marginal product of labour (MPL) and it represents the real wage rate that is received by the representative household, which is different from the real wage that is paid by firms due to the fact labour unions charge a mark-up over what they pay households.

$$\hat{mc}_t = (1-\alpha)\hat{w}_t + \alpha \hat{r}_t^k - \hat{Z}_t \tag{2.22}$$

This is the real marginal cost. It depends positively on the real wage paid by firms and the rental rate of capital. Meanwhile, an increase in total factor productivity reduces the real marginal cost of firms.

$$\hat{\pi}_t = \frac{l_p\chi}{1+\zeta_p\beta l_p\chi+\beta l_p\chi\bar{\pi}^{\frac{(\chi-1)(1+\lambda_p)}{\lambda_p}}-\zeta_p\beta l_p\chi\bar{\pi}^{\chi-1}}\cdot\hat{\pi}_{t-1}$$
$$+\frac{\zeta_p\beta+\beta\bar{\pi}^{\frac{(\chi-1)(1+\lambda_p)}{\lambda_p}}-\zeta_p\beta\bar{\pi}^{\chi-1}}{1+\zeta_p\beta l_p\chi+\beta l_p\chi\bar{\pi}^{\frac{(\chi-1)(1+\lambda_p)}{\lambda_p}}-\zeta_p\beta l_p\chi\bar{\pi}^{\chi-1}}\cdot E_t\hat{\pi}_{t+1}$$
$$+\frac{(1-\zeta_p\beta)(1-\zeta_p\bar{\pi}^{\frac{1-\chi}{\lambda_p}})}{\left[1+\zeta_p\beta l_p\chi+(1-\zeta_p)\beta l_p\chi\bar{\pi}^{\frac{(\chi-1)(1-\lambda_p)}{\lambda_p}}\right]\cdot\zeta_p}\cdot\hat{mc}_t+\hat{\lambda}_{p,t} \quad (2.23)$$

This is the hybrid version Generalised New Keynesian Phillips Curve (GNKPC) based on positive steady state inflation (the term GNKPC and NKPC are interchangeable in the rest of this thesis). It does not look considerably different from the prototype from Galí and Gertler (1999) and the more sophisticated SW07 version. The first thing to notice is that once the trend inflation term $\bar{\pi}$ is set equal to one (no trend, zero steady state inflation), this expression collapses to the standard case as it is in the original SW07 model. Moreover, one needs to pay attention to this new definition of inflation gap $\hat{\pi}$ (again, this is interchangeable with inflation for the rest of this thesis [2]), which is the deviation of inflation from its non-zero steady state level. As it is referred to in various trend inflation literature [3].

In addition, due to the presence of trend inflation $\bar{\pi}$, as trend inflation increases, the weights of terms in the NKPC also change accordingly. In particular, the weight on expected inflation $E_t\hat{\pi}_{t+1}$, which is represented by a composite coefficient in front of $E_t\hat{\pi}_{t+1}$ increases, while the weights on backward-looking inflation $E_t\hat{\pi}_{t-1}$ and current real marginal cost $\hat{mc}_t$, both drop. Therefore, agents become more forward looking in a world with trend inflation, and the GNKPC becomes flatter, this is the same property as shown in the small

[2] In fact, when $\bar{\pi}=1$, inflation gap is equivalent to inflation.

[3] See Sbordone (2007), Cogley and Sbordone (2008), Ascari and Sbordone (2014) for the initial definition and discussion

scale GNK model in *chapter 1*. This gives a clear look of how the level of trend inflation affects the whole economy, and this has been confirmed by a number of previous literature. (See Ascari and Ropele (2007); Ascari and Sbordone (2014); Cogley and Sbordone (2008)). As analysed by Ascari and Ropele (2007), the optimal price setting under positive trend inflation reflects future economic conditions or expectations more than the short-run economic fundamentals and cyclical fluctuations. Price re-optimising firms clearly become more forward-looking than they are without trend inflation.

$$\hat{w}_t = (1 - \zeta_w \bar{\pi}^{\frac{1-\chi}{\lambda_w}})\hat{\tilde{w}}_t + \zeta_w \bar{\pi}^{\frac{1-\chi}{\lambda_w}}[l_w \chi \hat{\pi}_{t-1} - \hat{\pi}_t + \hat{w}_{t-1}] \tag{2.24}$$

This equation shows the aggregate market wage in the economy, and there are a few things need to be noticed. First of all, as trend inflation $\bar{\pi}$ increases, the composite coefficient in front of the current period optimal reset wage decreases, which means the market wage will put fewer weights on the current optimal reset wage $\hat{\tilde{w}}_t$. Nonetheless, as trend inflation rises, the composite coefficient in front of the square bracket also increase, this implies that the current inflation is more weighted to the part of the economy that is not able to re-optimise its prices and therefore is only able to index to the past inflation. This overall effect makes the aggregate real wage rate more backward looking, which is in contrast to inflation where an increase in trend inflation makes inflation more forward looking.

However, one should also realise that once trend inflation $\bar{\pi}$ becomes too high and exceeds a certain threshold, the coefficient for optimal reset wage $1 - \zeta_w \bar{\pi}^{\frac{1-\chi}{\lambda_w}}$ could turn to negative, and the market wage rate $\hat{w}_t$ can even be negatively correlated with the current optimal reset wage $\hat{\tilde{w}}_t$. It is not hard to see that $1-\zeta_w \bar{\pi}^{\frac{1-\chi}{\lambda_w}}$ can easily go negative for high level of trend inflation when wage mark-up λ_w and the degree of wage indexation are small. In fact, as $\bar{\pi}$ is always tied with Calvo wage setting parameter ζ_w, for a given level of ζ_w, an increase in $\bar{\pi}$ significantly increases the probability of wage being stuck in a given time, and ultimately makes the market wage rate more backward looking. In turn, this also indicates that the optimal price re-optimised today will have a larger impact on the market wage tomorrow, and therefore introduce more dynamics into the entire model.

$$\hat{\tilde{w}}_t = (1 - \zeta_w \beta) \hat{w}_t^h + \zeta_w \beta E_t \hat{\tilde{w}}_{t+1} + \zeta_w \beta \bar{\pi}^{\chi - 1} (E_t \hat{\pi}_{t+1} - l_w \chi \hat{\pi}_t) \qquad (2.25)$$

This is the log-linearised version of the optimal wage resetting decision. According to (2.25), the only way that trend inflation can affect optimal reset wage is through the difference between expected future inflation and the indexed current inflation rate, however, this effect from inflation differential shrinks as the level of trend inflation grows. In fact, as trend inflation increases, the marginal effect of inflation differential $E_t \hat{\pi}_{t+1} - l_w \chi \hat{\pi}_t$ on optimal reset wage declines. Unions or households are more concerned with the current economic fundamentals such as the marginal product of labour $\hat{w}_t^h$ and the expected optimal reset price $\hat{\tilde{w}}_{t+1}$. The possible interpretation of this can be drawn from the nature of Calvo pricing contract. As Dixon and Kara (2010) explained, with Calvo scheme, price or wage setting agents are usually more forward looking since they do not know what is the next time they will be able to update their prices or wages, therefore, as trend inflation increases, the potential inflation differential in the very near future tends to be less important as they see through to the infinite future. Together these effects make wage resetting agents to shift their focuses to more fundamental issues. Here, I give an opposite interpretation as Amano, Ambler and Rebei (2007) where they argue as trend inflation rises, firms become more sensitive to fluctuations in inflation and relatively less sensitive to fluctuations in macroeconomic conditions and this is the reason that inflation becomes more persistent as trend inflation increases. Later in this thesis, I show inflation indeed become more persistent as the trend goes up and so does optimal reset wage, but this reinforced persistence in inflation is largely driven by the backward looking nature of real market wage and great persistence in price dispersion with higher levels of the trend.

$$R_t = \rho_R R_{t-1} + (1-\rho_R)(\psi_\pi \hat{\pi}_t + \psi_y(\hat{y}_t - \hat{y}_{t-1})) + \psi_{\Delta y}(\hat{y} - \hat{y}_{t-1} - (\hat{y}^{flex} - \hat{y}^{flex}_{t-1})) + m_t \quad (2.26)$$

This is the log-linearised Taylor rule, the central bank stabilises the economy by responding to inflation deviation from steady state, output gap, and also the change of output gap. A shock to the term m_t represents a monetary policy shock.

2.2.7 The Flexible Economy

In order to obtain the second best level of output, or the natural level, a flexible version of the economy needs to be defined. In this flexible economy, monopolistic competition in intermediate goods and labour unions still exist, nonetheless, prices and wages are allowed to adjust freely and hence all nominal frictions are removed.

The flexible economy can be summarised as the following set of equations

$$\hat{y}^{flex}_t = \frac{\bar{c}}{\bar{y}} \cdot \hat{c}^{flex}_t + \frac{\bar{i}}{\bar{y}} \cdot \hat{i} + \frac{\bar{g}}{\bar{y}} \cdot \hat{g}_t + \frac{\bar{r}^k k}{\bar{y}} \cdot \hat{u}^{flex}_t \quad (2.27)$$

$$\hat{c}^{flex}_t = \frac{1}{1+\lambda} E_t \hat{c}^{flex}_{t+1} + \frac{\lambda}{1+\lambda} \hat{c}^{flex}_{t-1} - \frac{1-\lambda}{\sigma_c(1+\lambda)} \cdot \hat{R}^{flex}_t - \frac{(\sigma_c - 1)\bar{w}^h \bar{L}/\bar{c}}{\sigma_c(1+\lambda))} [E_t \hat{L}^{flex}_{t+1} - \hat{L}^{flex}_t] \quad (2.28)$$

$$\hat{i}_t^{flex} = \frac{1}{1+\beta}\hat{i}_{t-1}^{flex} + \frac{1}{1+\beta}E_t\hat{i}_{t+1}^{flex} + \frac{1}{(1+\beta)\psi} \cdot Q_t^{\hat{k,flex}} \tag{2.29}$$

$$\hat{y}_t^{flex} = \alpha\hat{k}_t^{s,flex} + (1-\alpha)\hat{L}_t^{flex} + \hat{Z}_t \tag{2.30}$$

$$\hat{k}_t^{s,flex} = \hat{u}_t^{flex} + \hat{k}_{t-1}^{flex} \tag{2.31}$$

$$\hat{k}_t^{flex} = (1-\delta)k_{t-1}^{flex} + \frac{\bar{i}}{\bar{y}} \cdot \hat{i}_t^{flex} \tag{2.32}$$

$$\hat{u}_t^{flex} = \frac{1-\eta}{\eta} \cdot \hat{r}_t^{k,flex} \tag{2.33}$$

$$\hat{r}_t^{k,flex} = \hat{L}_t^{flex} - \hat{k}_t^{flex} + \hat{w}_t^{flex} \tag{2.34}$$

$$(1-\alpha)\hat{w}_t^{flex} + \alpha\hat{r}_t^{k,flex} = \hat{Z}_t \tag{2.35}$$

Note, when prices are fully flexible, the marginal cost expressed in terms of deviation from the steady state becomes zero.

$$\hat{w}_t^{flex} = \frac{1}{1-\lambda} \cdot \hat{c}_t^{flex} - \frac{\lambda}{1-\lambda} \cdot \hat{c}_{t-1}^{flex} + \sigma_l \hat{L}_t^{flex} \tag{2.36}$$

In a fully flexible economy, the market wage equals the optimal reset wage, which in turn equal the wage rate that received by households.

2.3 Bayesian Estimations

This completed medium-scale GNK model with trend inflation is then estimated using a Bayesian technique with four essential macroeconomic variables in the US economy as observables: real gross domestic product (GDP), effective federal fund rate, inflation rate, which is based on the US GDP deflator, and labour hours that measured as the non-farm business sector average weekly hours [4].

The reason behind the selection of these four particular observables are three-folded. First, as the presence of trend inflation is the core of this study, inflation is undoubtedly the most crucial observable. Second, as this thesis is designed to detect how changes in trend inflation affect macro dynamics and monetary policy response, hence the inclusion of nominal interest rate is also essential. Last, as labour hour is closely associated with price dispersion, which is the variable that trend inflation is most likely to affect, hence, labour hour is also added to the list of observables.

The data covers from the first quarter of 1970, which is a slightly later starting year compared with SW07 where they use data back to 1964. The ending observation is the fourth quarter of 2017, and this is the longest time span can be possibly covered at the time this chapter is written. Furthermore, compared with the SW07, this is a longer time spell and it is then further divided into three periods for sub-sample studies, which is discussed in section 2.4.

2.3.1 The Bayesian Method

In the last two decades, Bayesian method for DSGE model estimation has become the most popular and dominant choice among economists. The reasons behind this trend and the theoretical advantages of Bayesian estimation can be summarised as the following. First, the likelihood of DSGE models is

[4] Please refer to the Appendix B for detailed data description.

a highly dimensional object with a large number (in a medium-scale DSGE model) of parameters. Any research in such a high dimensional function would be extremely concerning, especially, likelihoods of DSGE models are full of local maxima and minima and of almost flat surfaces. Therefore, even the most sophisticated optimisation algorithms like simulation annealing or the simplex method are likely to run into serious difficulties when maximising the likelihoods of such dynamic models (Fernández-Villaverde, 2010). Moreover, the standard errors of the estimates are recognisably difficult to compute and the usual sample size of DSGE estimation does not satisfy the asymptotic distribution requirement and therefore yield a very poor approximation. Nonetheless, Markov Chain Monte Carlo methods have a much more comfortable time exploring the likelihood of DSGE models and generate a comprehensive view of the object of interests (Fernández-Villaverde, 2010).

The technique of Bayesian estimation is based on the fundamental theorem of Bayesian inference. Recall the basic elements in Bayes' theorem; first, there are some data set $y^T = y^T_{t,t=1} \in \Re^{N \times T}$. Second, there is a model that is based on economic theory, for instance, a completed DSGE model. There are some restrictions to the model either due to statistical factors or economic factors. For example, the inflation response parameter in the Taylor style monetary policy rule needs to be greater than one, in order to satisfy the Taylor Principle and ensure the model meets its determinacy. Hence, the model is consist of three elements Koop, Poirier and Tobias (2007):

1. A parameter set, $\Theta_i \in \Re^k$, which defines the upper and lower bounds of the parameters that index the functions in the model. Some of the restrictions come from statistics. For instance, standard deviation must be positive. Others may come from economics, for instance, Calvo parameters must be bounded by zero and one in order to have any economic meaning for nominal rigidities.

2. A likelihood function $p(y^T \mid \theta, i) : \Re^{N \times T} \times \Theta_i \to \Re^+$, which implies the

probability that the model assigns to each observation conditional on some parameter values.

3. A prior distribution $\pi(\theta \mid i) : \Theta_i \rightarrow R^+$, which captures pre-sample beliefs about the true value of the parameters to be estimated.

According to the Bayes's theorem, the posterior distribution of the parameters is given by:

$$\pi(\theta \mid y^T, i) = \frac{p(y^T \mid \theta, i)\pi(\theta \mid i)}{\int p(y^T \mid \theta, i)\pi(\theta \mid i)d\theta}$$

Consequently, one only need to know the distribution and therefore the likelihood of the parameter of interests, then provide the prior, and one can obtain the posterior.

Furthermore, the practical attractiveness of Bayesian estimation for DSGE model are summarised by Fernández-Villaverde (2010) as the following: first, Bayesian econometrics provides a number of answers that are directly related to researchers' questions. Particularly for policymakers, they would be able to interpret the results for policy makings. With Bayesian estimation results, a central bank governor would be able to know the probability of making right decision by cutting nominal interest rate by fifty basis points, conditional on the observed sample. Second, for most of the research in New Keynesian DSGE models, the pre-sample and therefore the priors are really rich and considered useful by macroeconomists, hence, if it would not be rational to simply ignore such pre-sample information. This is not going to deny the fact that the researcher needs to pay careful attention when these micro evidence are converted into macro priors, negligence could lead to huge misinterpretation of the estimated posteriors. Yes, there are academics on the opposite side arguing that Bayesian inference relies too heavily on the priors, however, how can one argue that this is worse than the situation when one does not even have any information about one's parameter of interests. Third, Bayesian estimation allows a direct computation of many objects of interest while capturing in these computed objects all the existing uncertainties regarding parameter values. Last, Bayesian estimation can deal with misspecified models in a relatively natural way. In theory, there is no model which is perfectly specified, all models are misspecified to some extent, but some are practically useful for policymaking. Thus, being able to generate some good description of the data is practically useful.

2.3.2 Data Treatment and Detrending

All the raw data for these four macro observables have been de-trended by applying a one-sided Hodrick and Prescott (1997) (H-P henceforth) filter and thereafter all pass the augmented Dicky-Fuller test for stationarity. The reason why this method is used rather than the first differencing technique as SW07 originally does is due to the intention of using stationary macro data. For the sample period I estimate (1970 Q1 to 2017 Q4), first differencing detrending cannot generate stationary results. After a couple of experiments on both first differencing and second differencing, I found that in no occasion can the filtered data pass a ADF test, thus, an alternative method has to be applied and luckily a one-sided H-P filter does just that. The method of polynomial detrending (up to 4th order) is also tested but it does not produce any good result either. The combination of the two sample periods with few structural breaks in later 1970s and 2000s may be to blame for this non-stationarity trouble. After the detrending of raw data, each of the detrended observations enters the measurement equation based on a one-to-one relationship with these four variables defined in the model.

2.3.3 Prior Distribution of Parameters

The priors for some of the parameters and stochastic processes in my estimation follow the same priors as SW07, but some others are based on the posterior means from SW07 estimation results. This should not be an unreasonable practice as their results have been cited in a large number of papers.

All the standard deviations of these five innovations are assumed to follow an inverse-gamma distribution with standard deviations 2.0, which are the same as the volatility assumed by SW07, these are rather loose priors. The priors of all the shock standard errors are based on the posterior means of SW07 estimation. The persistence parameters of these stochastic shocks are to follow a beta distribution with mean 0.50 and standard deviation of 0.20, these are exactly the same as SW07, except the feedback of productivity shock on government spending, which is set with mean 1/3 and standard deviation of 0.10, in order to limit the interaction between productivity shock and government spending shock.

On the households side, the habit formation parameter λ is also assumed to follow a Beta distribution with mean 0.71, which is the posterior mean of SW07 estimation, and standard deviation of 0.15. The inverse elasticity of substitution for consumption goods (σ_c) and labour supply elasticity (σ_l) are to follow a normal distribution with mean 1.50 and standard deviation 0.375 for consumption and mean 2.0 and standard deviation of 0.75 for labour supply, exactly the same as SW07.

On the firm side, the production share of capital α is allowed to vary, and it is assumed to follow a Beta distribution with mean equals 1/3 and standard deviation 0.5, in contrast to SW07, which α is fixed at 0.19. The degree of total indexation χ, cost of capital utilisation η are both assumed to follow a beta distribution with means 0.50 and 0.54, respectively, and standard deviations both equal 0.15. The relative weights of price and wage indexation on past inflation l_p and l_w are both to follow a Beta distribution with a standard deviation of 0.10. While l_p is assumed to have a prior mean of 0.40, l_w is believed to have a prior mean of 0.60, both are consistent with the most literature on empirical studies as most of them find indexation is a relatively more plausible phenomenon for wages than for prices. Finally, the Calvo parameters for price rigidity ζ_p and wage rigidity ζ_w are both to follow a Beta distribution with standard deviation of 0.05, the former has a prior mean of 0.50 and the latter has a mean of 0.70, nonetheless. Again, this is what empirical literature suggests regarding the evidence that wages tend to be more rigid than prices. For example, Gertler, Sala and Trigari (2008), Justiniano and Primiceri (2008), and Smets and Wouters (2007).

The most important parameter of all, trend inflation ($\bar{\pi}$), is to be estimated along with other structural variables, and it is assumed to follow a normal distribution with a mean of 1.0078, which correspond to an annual rate of 3.12% inflation in steady state and standard deviation of 0.10. This prior mean is slightly less than the arithmetic average of inflation rate in these 192 quarters from Q1 1970 to Q4 2017. For the monetary policy rule, the response parameter for inflation deviation from steady state ψ_π, output gap ψ_y, and change in output gap $\psi_{\Delta y}$ all follow a normal distribution but with different means and standard deviations. In particular, ψ_π has a mean of 1.75 and standard deviation of 0.25, both ψ_y and $\psi_{\Delta y}$ have mean 0.12 and standard deviation of 0.10 for ψ_y and 0.05 for $\psi_{\Delta y}$. All follow the same priors as SW07. A summary of prior distributions and descriptions of the structural parameters and shock processes is presented in table 2.1 and 2.2.

Table 2.1: Prior Distribution of Structural Parameters and Shock Processes

Description	Parameter	Distribution	Prior Mean	St. Dev.
Capital Share	α	Beta	1/3	0.05
Habit Formation	λ	Beta	0.71	0.15
Elas. of Sub in Con	σ_c	Normal	1.50	0.375
Elastic of Lab Sub	σ_l	Normal	2.00	0.75
Adjustment Cost	Ψ	Normal	5.74	1.50
Degree of Indexation	χ	Beta	0.50	0.15
Capital Utilisation	η	Beta	0.54	0.15
Price Indexation	l_p	Beta	0.40	0.10
Wage Indexation	l_w	Beta	0.60	0.10
Calvo for Price	ζ_p	Beta	0.50	0.05
Calvo for Wage	ζ_w	Beta	0.70	0.05
Trend Inflation	$\bar{\pi}$	Normal	1.0078	0.10
Inflation Response	ψ_π	Normal	1.75	0.25
Output Response	ψ_y	Normal	0.12	0.10
Output Gap Response	ψ_{Δ_y}	Normal	0.12	0.05

Table 2.2: Prior Distribution of Shocks Processes

Description	Parameter	Prior Distribution	Prior Mean	St. Dev.
Tech. shock	ρ_z	Beta	0.50	0.20
Gov. spending shock	ρ_g	Beta	0.50	0.20
Mnetary solicy shock	ρ_m	Beta	0.50	0.20
Price mark-up shock	ρ_p	Beta	0.50	0.20
g_t to z_t response	ρ_{gz}	Beta	1/3	0.10
MA(1) mark-up shock	θ_p	Beta	0.50	0.20
Investment shock	ρ_{inv}	Beta	0.50	0.20
Tech. shock	ε_z	Inverse Gamma	0.45	2.0
Gov. spending Shock	ε_g	Inverse Gamma	0.53	2.0
Monetary policy shock	ε_m	Inverse Gamma	0.24	2.0
Price mark-up shock	ε_p	Inverse Gamma	0.14	2.0
Investment shock	ε_{inv}	Inverse Gamma	0.45	2.0

In addition to the priors set above, there are seven parameters fixed during the estimation process and therefore not estimated. They are discount factor β, which is fixed at 0.995 and this is a standard setup and it implies that the real interest rate is around 1.25% in steady state on a quarterly basis. Then, the depreciation rate δ is pre-set at 0.025 and this is assumed on a quarterly basis. The exogenous government spending to GDP ratio $\frac{\bar{g}}{\bar{y}}$ is fixed at around 0.20, and this is consistent with empirical evidence of this ratio for the time of this estimation period. Correspondingly, the consumption to GDP ratio is set at around 0.65, and again this is consistent with what is observed during these 192 quarters. Finally, the degree of interest rate smoothing parameter ρ_i is set at 0.81, which is what SW07 posterior mean suggests. Finally, the implied gross price and wage mark-ups are set at 1.25, these two values are broadly consistent with literature with a similar framework, such as Gertler, Sala and Trigari (2008), Lindé, Smets and Wouters (2016). All the above information regarding calibrated parameters is summarised in table 2.3.

Table 2.3: Calibrated Parameters

Description	Parameter	Calibrated value
Discount Factor	β	0.995
Capital Depreciation Rate	δ	0.025
Government Spending Ratio	$\bar{g}/\bar{y}$	0.20
Consumption Spending Ratio	$\bar{c}/\bar{y}$	0.65
Interest Rate Smoothing	ρ_i	0.81
Price Mark-up	λ_p	0.25
Wage Mark-up	λ_w	0.25

2.3.4 Posterior Estimates of Parameters in Benchmark Model

The posterior distribution is estimated using the Metropolis-Hastings algorithm, after 250,000 iterations with two Markov Chains (totally 500,000 iterations), the main results of Bayesian estimation are summarised in table 2.4 and 2.5. The acceptance ratio of the two chains are: 24.56% for chain 1 and 24.29% for chain 2, and 200,000 draws per chain were kept. Table 2.4 and 2.5 show the posterior means, the standard deviations of the posterior mean, and 5 and 95 percentile of the posterior distribution of the 15 structural parameters, 7 persistence parameter and 5 exogenous shock processes. In addition, the last columns of table 2.4 and 2.5 are the Posterior means of SW07.

Table 2.4: Posterior Distribution of Structural Parameters

Parameter	Post. Mean	Post. St.D	5%	95%	SW Mean
α	0.259	0.038	0.194	0.324	0.19†
λ	0.698	0.054	0.632	0.761	0.71
σ_c	2.499	0.378	2.114	2.951	1.38
σ_l	1.259	0.568	0.251	2.115	1.83
Ψ	6.869	1.260	4.838	8.820	5.74
η	0.396	0.105	0.450	0.734	0.54
χ	0.594	0.116	0.236	0.548	1.00†
l_p	0.293	0.094	0.147	0.432	0.24
l_w	0.589	0.100	0.430	0.763	0.58
ζ_p	0.773	0.028	0.729	0.816	0.66
ζ_w	0.748	0.054	0.652	0.847	0.70
$\bar{\pi}$	1.008	0.009	1.762	2.356	1.0078‡
ψ_π	2.069	0.185	1.718	2.327	2.04
ψ_y	0.185	0.066	0.079	0.293	0.08
$\psi_{\Delta y}$	0.201	0.037	0.139	0.260	0.22

† Pre-fixed parameters in SW07

‡ Estimated in a different way

According to table 2.4, the capital share in production α has a mean of 0.259, which comes with no big surprise, however, this parameter is fixed at 0.19 in SW07. The habit formation parameter λ is estimated to be around 0.698, and this is extremely close to the SW07 estimate of 0.71; this implies a similar level of persistence in households' consumption preference. The inverse elasticity of substitution for consumption goods is at 2.499, which is much larger than the SW07 estimate of 1.38. At the same time, the elasticity of labour supply is at 1.259, which in turn to be much smaller than the SW07 result, and it suggests a much more inelastic labour supply for the sample period.

The degree of price stickiness is estimated to be much larger than the SW result of ζ_p; according to the estimation result, 77.3% of the firms cannot re-optimise their prices in a given quarter, compared to 66% in SW estimation. This measure implies an extremely high level of rigidity in the intermediate good sector. For the labour market, the estimated mean of ζ_w is 0.748, which indicates that around 74.8% of the labour unions are not able to re-optimise their wages in a given period, this is also a higher value compared with the SW07 estimate of 70%, but not by a significant margin. Therefore, the estimates suggest that both prices and wages are more rigid compared with the SW07 model, however, one needs to remember the fact that given the assumption of price and wage indexations, the interpretations of both Calvo parameter are subject to caution. This is merely a measure of how many firms and unions are able to re-optimise their prices and wages in a given quarter, but not a legitimate measure of the overall rigidity in the economy.

The adjustment cost of investment Ψ is estimated to be 6.869, which is significantly higher than the SW07 estimate of 5.74, and it suggests that households would find it more costly to adjust their investment over time. The estimate for capital utilisation rate of firm η is 0.396, this is a lower value compared with SW07, and it indicates higher level of efficiency in capital utilisation process.

The overall degree of price and wage indexation χ is estimated to be around 0.594, which is close to the prior mean of 0.50. The estimate of weight assigned to past inflation is around 0.293 for price setting and 0.589 for wage setting. This result confirms what is usually obtained in literature; wage indexation is generally more plausible than price indexation. Nonetheless, due to the extra layer of indexation χ, the same value of l_p or l_w would imply a much less overall effect of indexation to past inflation. Furthermore, given that l_p is less than 0.50, one can interpret that firms view trend inflation more than they view past inflation when they index their prices. Meanwhile, as l_w is greater than

0.5, hence it seems that labour unions view past inflation more than trend inflation. Trend inflation tends to play a significant role in firms' price setting behaviour.

The most crucial parameter in this study, steady state inflation rate $\bar{\pi}$ has an estimated value of 1.0080, which corresponds to an annualised inflation rate of 3.20%, and also close to its prior. This result suggests that the annual level of inflation in steady state has been annually 3.2% on average from 1970 to the end of 2017. This is roughly in line with the literature, SW07 estimate a value of 1.0078, which converts to an annual rate of 3.12%. However, one needs to realise that SW07 models this trend inflation in a completely different way as this model; it treats the difference between observable inflation and inflation from the model as the source of trend inflation, but does not model it explicitly. Hence, from a theoretical point of view, their result does not tell much about the long run trend. Moreover, to better analyse this estimated trend inflation, this chapter will further conduct a sub-sample estimation across three different time spans: the *Great Inflation*, the *Great Moderation*, and the *Great Recession*. Further studies find that this trend varies considerably across these three periods that had very different macroeconomic fundamentals, and this is discussed extensively in section 4 of this chapter.

For the Taylor rule, the inflation response parameter is estimated to be 2.069, which indicates a roughly similar toughness of central bank towards inflation deviation as the SW07 result suggests. Furthermore, the response parameter of change in output is estimated to be 0.185, and this is a much higher value than the SW07 result, which is 0.08. This implies a more hawkish view from the Federal Reserve towards the output deviation from its steady state. Finally, the response parameter of change in output gap is around 0.201, which is very close to the SW07 result of 0.22.

Table 2.5: Posterior Distribution of Shocks Processes

Parameter	Posterior Mean	Post. St. Dev	5%	95%	SW Mean
ρ_z	0.989	0.043	0.979	0.9997	0.95
ρ_g	0.856	0.030	0.810	0.910	0.97
ρ_m	0.097	0.047	0.026	0.162	0.15
ρ_p	0.758	0.055	0.655	0.865	0.89
ρ_{gz}	0.587	0.355	0.518	0.667	0.52
θ_p	0.406	0.166	0.138	0.673	0.69
ρ_{inv}	0.514	0.194	0.184	0.837	0.71
ε_z	0.487	0.024	0.446	0.529	0.45
ε_g	0.395	0.029	0.347	0.441	0.54
ε_m	0.197	0.011	0.179	0.215	0.24
ε_p	0.190	0.066	0.097	0.285	0.14
ε_{inv}	0.361	0.439	0.105	0.643	0.45

For the estimated results of the shock processes, which are presented in table 2.5, these results suggest that the size of three of the shocks are relatively close to the SW07 result: productivity, monetary policy and price mark-up shocks. However, the other two shocks are considerably different from the SW07 results: government spending and investment-specific technology shock. The size of the shock to productivity is around 0.487, which is in line with the estimate from SW07 of 0.45 and the monetary shock is 0.197 is not far from the SW07 result of 0.24. The price mark-up shock of 0.19 is also not significantly different from the SW07 estimate of 0.14. On the contrary, the exogenous government spending stock is about 0.395, which is much lower than the SW07 result of 0.53. Furthermore, the standard deviation of the investment relative price shock is significantly higher than the estimate of the volatilities of all the other four shocks; this is consistent with Justiniano and Primiceri (2008)'s conclusion that shocks to investment are extremely volatile in the postwar US data.

2.3.5 Estimation When Trend Inflation is Removed

After estimating the benchmark model with trend inflation, I estimate the entire model again but without the explicit modelling of trend inflation. This is a rather simple process, one can construct a "new version" of the *benchmark model* without trend inflation by setting the steady state inflation rate $\bar{\pi}$ equals 1; it implies that the gross inflation rate is 1 in steady state and trend inflation is zero in steady state. From here onwards, I refer to this new version as the *benchmark model without trend inflation.* The estimation results are shown in table 2.6 and 2.7. Compared with the estimation results in the previous case where trend inflation is assumed to be positive, one can find that although most of the parameters stay similar to the benchmark model, few of them change quite dramatically.

Table 2.6: Posterior Distribution of Structural Parameters in Benchmark Model with No Trend Inflation

Parameter	Mean ($\bar{\pi} > 1$)	Mean $\bar{\pi} = 1$	SDs $\bar{\pi} > 1$	SDs $\bar{\pi} = 1$
α	0.259	0.214	0.036	0.034
λ	0.698	0.768	0.037	0.037
σ_c	2.499	2.185	0.259	0.280
σ_l	1.259	1.198	0.405	0.704
Ψ	6.869	6.916	1.146	1.215
χ	0.395	0.377	0.083	0.093
η	0.594	0.473	0.072	0.090
l_p	0.293	0.303	0.069	0.090
l_w	0.589	0.570	0.098	0.102
ζ_p	0.773	0.758	0.027	0.029
ζ_w	0.748	0.817	0.055	0.048
$\bar{\pi}$	1.008	1.000	0.012	NA
ψ_π	2.069	1.883	0.177	0.184
ψ_y	0.185	0.174	0.062	0.062
$\psi_{\Delta y}$	0.201	0.139	0.029	0.033

SDs stand for standard deviations

More specifically, the capital share production cost α drops from 0.26 to 0.21, which implies a lower share of capital in the production function. The habit formation parameter λ increases from 0.698 to 0.768, this indicates a greater persistence in households' consumption preference. Both the inverse elasticity of substitution for consumption goods and labour supply elasticity decline, while all the three parameters associated with price and wage indexation, χ, l_p, and l_w plus the capital adjustment cost Ψ stays roughly the same.

The Calvo probability price setting drops from 77.3% to 75.8%, on the contrary, the probability of labour union not being able to reset wages increases from 74.8% to 81.7%.

For parameters in the Taylor rule, the response parameter of central bank ψ_π to inflation deviation from steady state drops to 1.883, the response to output deviation ψ_y also decrease slightly to 0.174. In addition, the response to changes in output gap ψ_{Δ_y} decline to 0.139, and therefore, once trend inflation is removed or artificially fixed to zero, central bank responds less to all measurements.

Table 2.7: Posterior Distribution of Shocks Processes without Trend Inflation

Parameter	Mean with $\bar{\pi}$	Mean no $\bar{\pi}$	SDs $\bar{\pi}$	SDs no $\bar{\pi}$
ρ_z	0.989	0.640	0.007	0.053
ρ_g	0.856	0.844	0.027	0.030
ρ_m	0.097	0.100	0.044	0.043
ρ_p	0.758	0.786	0.060	0.056
ρ_{zg}	0.587	0.646	0.046	0.019
θ_p	0.758	0.362	0.083	0.151
ρ_{inv}	0.514	0.498	0.179	0.199
ε_z	0.487	0.466	0.025	0.024
ε_g	0.395	0.434	0.030	0.028
ε_m	0.197	0.195	0.011	0.011
ε_p	0.190	0.171	0.058	0.052
ε_{inv}	0.476	0.400	0.211	0.317

SDs stand for standard deviations

The estimates of most of the shock processes do not change much across the two versions of the model, the only exception is the persistence parameter for productivity shock ρ_z and government spending shock ϵ_g. The shock to productivity becomes much less persistent in the new model (without trend inflation), while the size of the shock to government purchases increases considerably in this new estimation.

2.3.6 Cross Model Comparison

Bayesian Factors

In order to examine whether such an assumption of trend inflation would make any significant difference in terms of DSGE modelling, I have to compare the model with trend inflation with the one without this set-up. One of the methods to compare them is the statistic called Bayesian factors. The usual statistic that economists use is the Log data density computed by using Laplace approximation, which is reported in the Bayesian estimation results. The rule of thumb is that the greater this log density is, the better the modelling is from a Bayesian estimation point of view.

Table 2.8: Bayesian Factors Comparison

Log data density	With trend inflation	Without trend inflation
Laplace Approximation	-297.9649	-298.1405

The statistical measures of Bayesian factors for the two models are summarised in the table above. It can be found seen that for the Bayesian factor computed by Laplace approximation, the trend inflation model is preferred to the model with no trend inflation, even just marginally.

2.3.7 Applications of the Benchmark Model

Shock Decomposition: What are the main driving forces over the business cycle?

Figure 2.1 and figure 2.2 provide the historical shock decomposition for output and inflation.

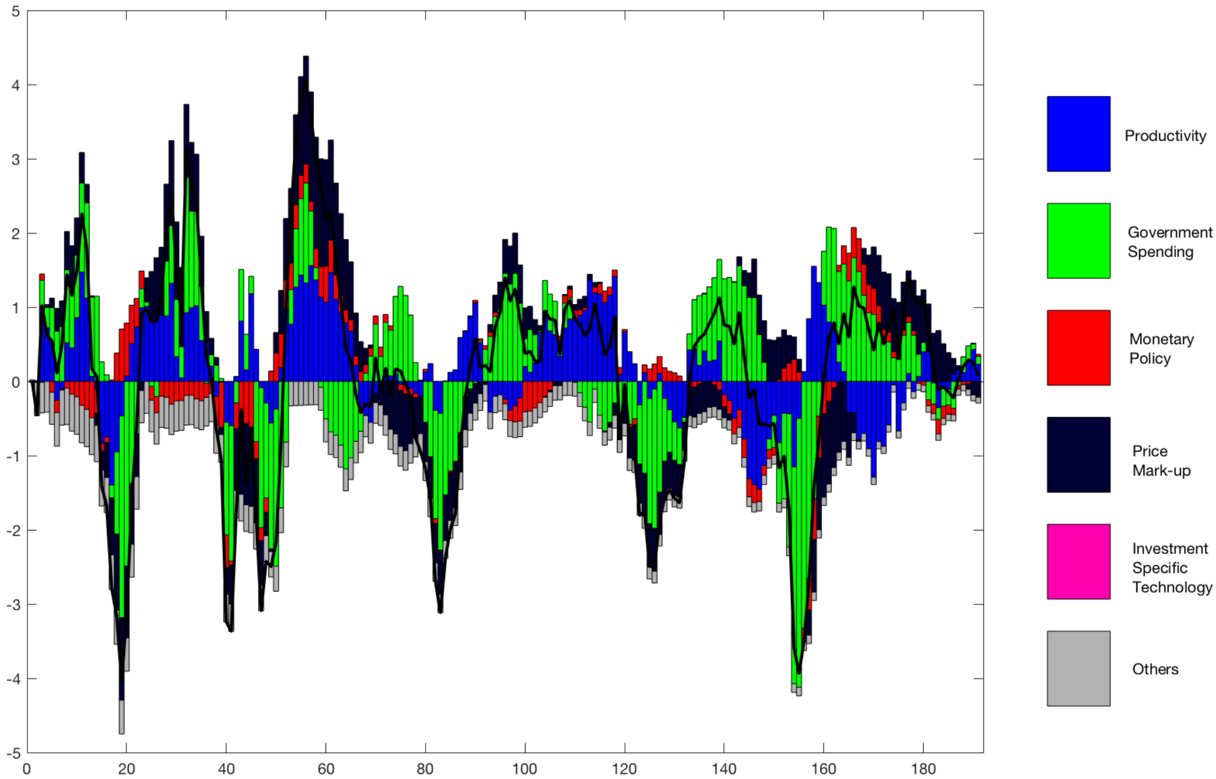

Figure 2.1: Historical Shock Decomposition for Output

According to figure 2.1 and 2.2, the recession in the early 1970s (5th-20th quarters) is led by monetary policy shock, which may reflect a slack in the Federal Reserve's policy at the time. Then the recession is worsened by productivity and demand shocks (exogenous government spending) and then pushed to the bottom by the supply shock, which may reflect the soaring oil price at the time. Overall, this is a recession initiated by loose monetary policy and by the rocketing oil price. While the recession in the early 1980s (40th quarter) brings a very mixed picture of three types of shocks, the recession in early 1990s (80th quarter) and the beginning of the new millennium (120th quarter) happens initially due to demand and mark-up shocks.

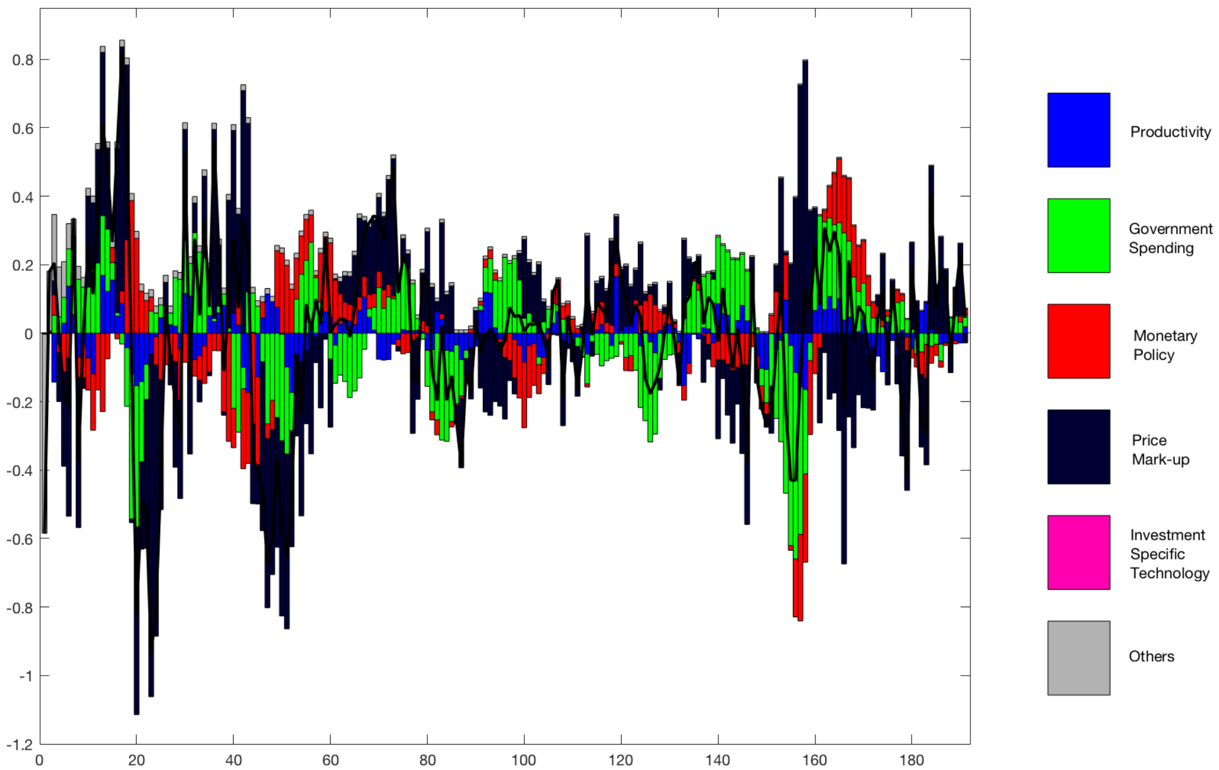

Figure 2.2: Shock Decomposition for Inflation

2.3.8 Macroeconomic Dynamics of the Benchmark Model

It is also worth to look at how the macroeconomic variables perform under different exogenous shocks, and compare them with the dynamics in the SW07 model. Figure 2.3 through to figure 2.6 give the impulse response functions of four macro variables when the economy is hit by four exogenous shocks: productivity shock, government expenditure shock, monetary policy shock and shock to price mark-up. The solid lines are the impulse response functions and the shadow areas represent the 5% and 95% confidence interval of the posterior estimation.

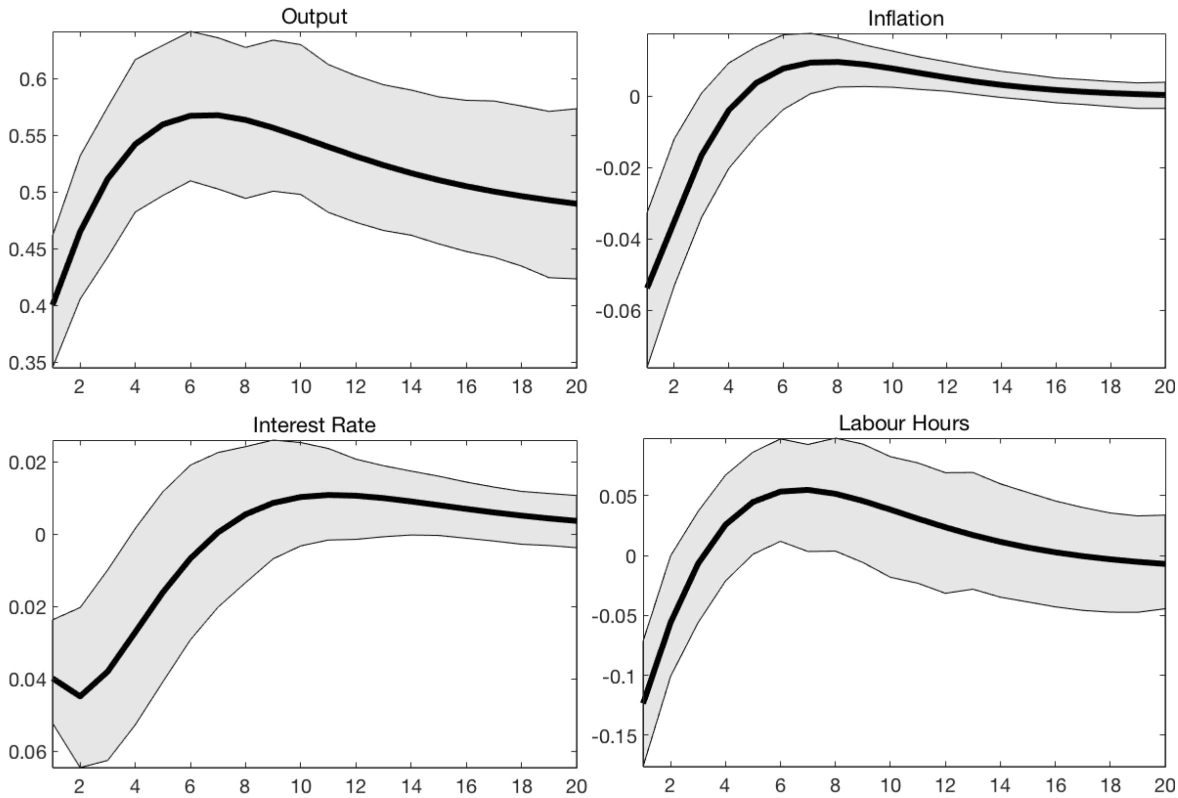

Figure 2.3: The estimated IRFs to a productivity shock

According to figure 2.3, a productivity shock of 0.487 creates a positive output deviation from its steady state. The initial deviation is around 0.4, which is slightly larger than the SW07 value, and then it peaks in the 7th quarter, similar to the SW07 peak of 8th quarter, and the response in output in my model is slightly more persistent than the SW07 one. Inflation initially deviates from the steady state by just over 0.05, which is relatively less than the SW07, however, this should not be hard to understand. Due to the fact that this inflation gap is now the deviation of inflation from a non-zero steady state, hence, the deviation should be smaller than the case that steady state is assumed to be zero for inflation. Moreover, the inflation gap shows less persistence compared with SW07; it bounds back to the steady state after just 5 quarters, in comparison with nearly 10 quarters in the SW07. In addition, a productivity shock in this model can generate a much smaller decline in hours worked. The reason behind this is that the presence of trend inflation in this model primarily serves as a negative productivity shock in macroeconomic dynamics (Schmitt-Grohé and Uribe, 2007a). As Ascari and Sbordone (2014) explained, this is done through the channel of price dispersion, and this negative productivity effect of trend inflation counteracts with the positive shock in productivity and makes the overall effect on labour hour fluctuations considerably less.

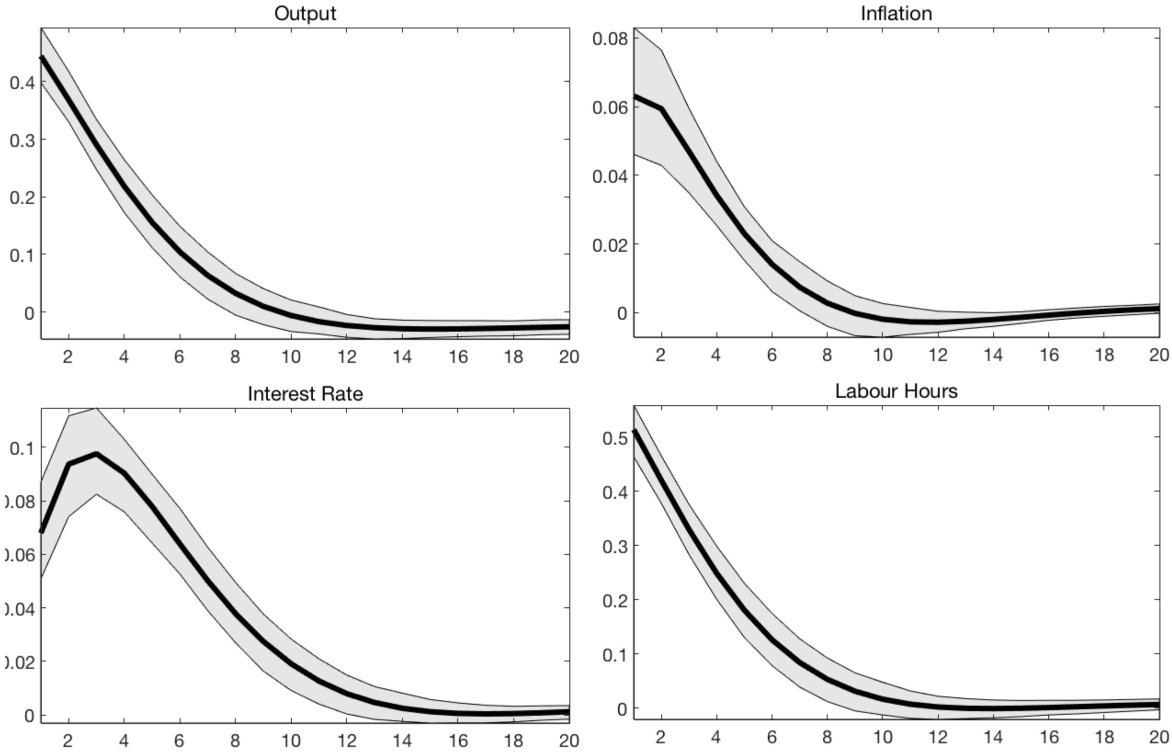

Figure 2.4: The estimated IRFs to a government spending shock

Figure 2.4 gives the impulse response functions of a exogenous government spending shock of 0.395. All four variables exhibit very similar patterns and magnitude as the SW07.

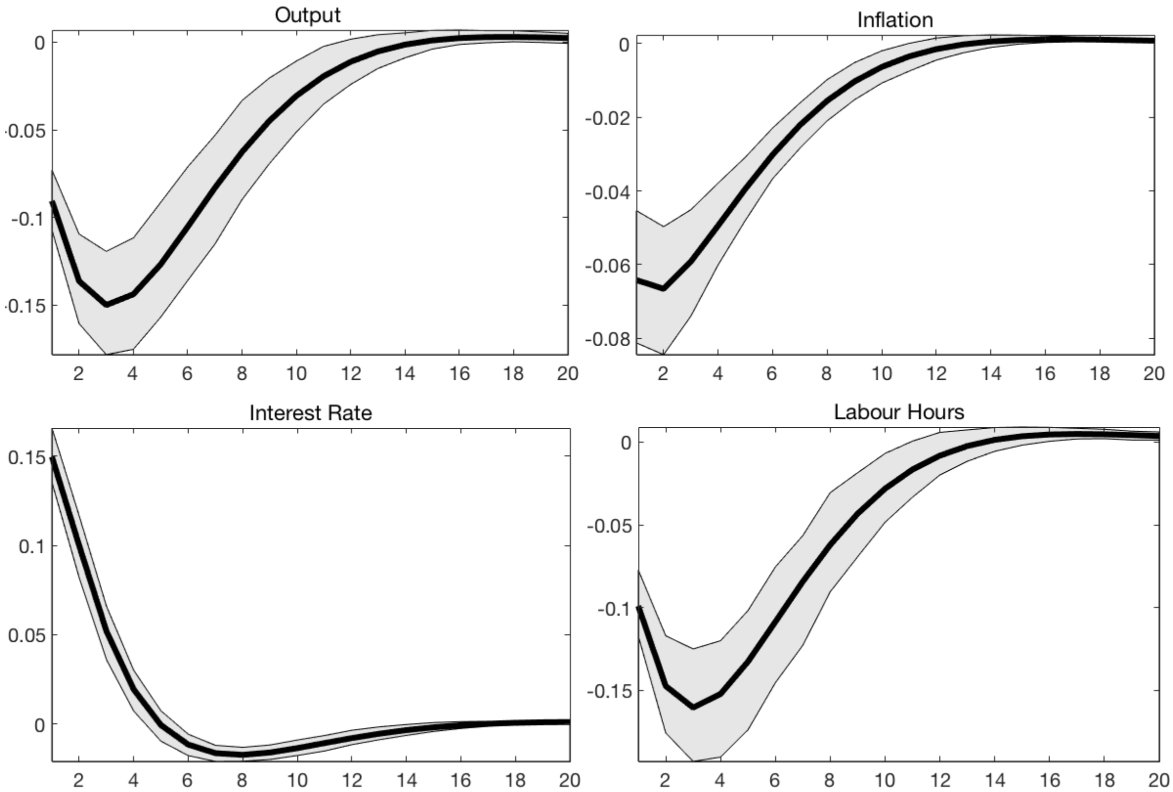

Figure 2.5: The estimated IRFs to a monetary policy shock

The main focus of the contractionary monetary policy shock is captured by Figure 2.5. The impulse response functions of a 0.197 shock to the Taylor rule show output drops initially by 0.09 from its steady state level and the deviation peaks at -0.15 in the third quarter, which is the same time as SW07, however, the magnitude is much smaller than the SW07, albeit with a very similar level of persistence in output. Inflation dives to over -0.06 on impact, and this is actually a larger impact compared with SW07 of -0.045. This difference is primarily due to the difference in the size of the monetary shock in my estimation (0.197) and SW07 estimation (0.14). This GNK model is able to generate a hump-shaped response for inflation and it peaks in the second quarter after the shock, one quarter earlier than SW07. This difference may attribute to the effect of indexation on past inflation and trend inflation. As illustrated by Ascari and Branzoli (2015), one of the major effects of indexation on the macroeconomic dynamics of such GNK model is that the persistence in the inflation gap relies directly on the overall degree of indexation. The higher the level of price indexation, the lower the inflation persistence in inflation gap, partly due to the reduced persistence in price dispersion term. The degree of indexation is estimated to be 0.589 and thus it is reasonable to imagine that inflation persistence becomes smaller compared with SW07. Furthermore, the impact of the shock on inflation dies out after about 14 quarters, whereas SW07's inflation persists to nearly until the 20th quarter.

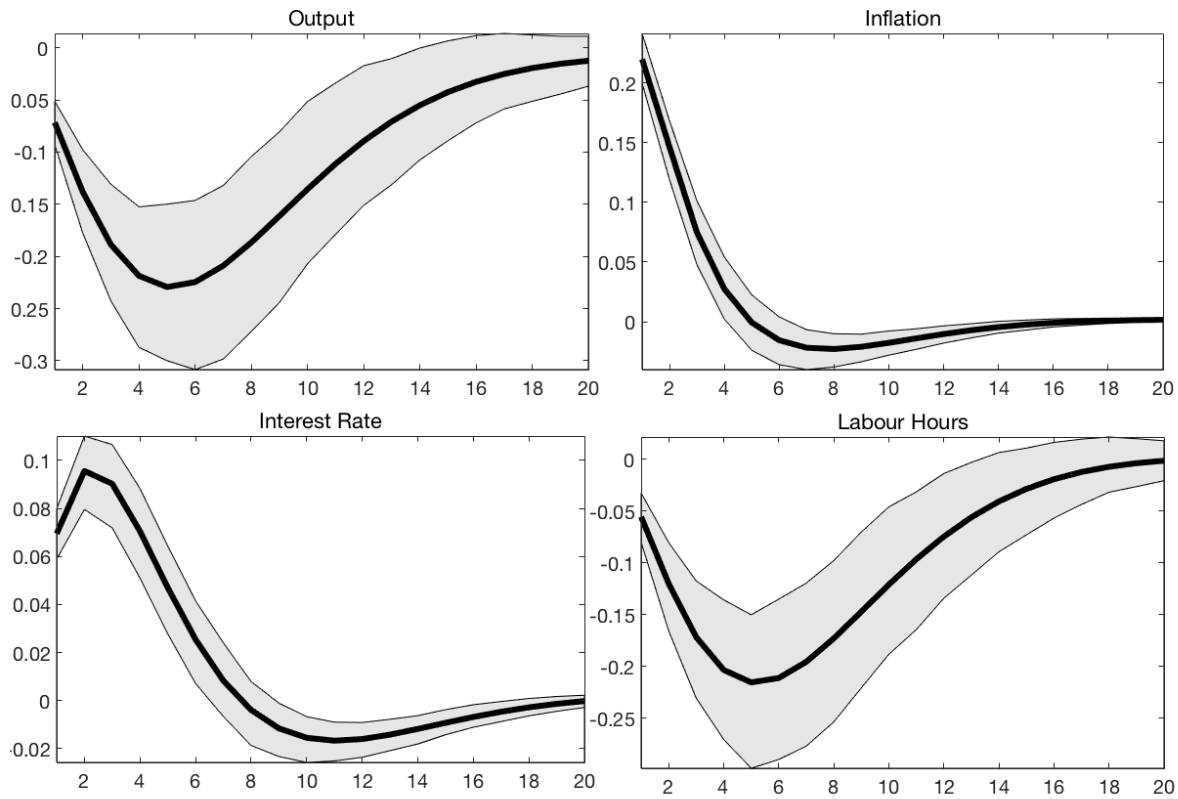

Figure 2.6: The estimated IRFs to a price mark-up shock

Last, the impulse response functions of a price mark-up is summarised in figure 2.6. All the four variables look very close to SW07.

2.4 Sub-Sample Analysis: the Great Inflation, the Great Moderation, and the Great Recession

Since the time covered in Bayesian estimation stretched cross almost five decades; 1970s-2010s. During the five decades, the 1970s and early 1980s experienced highly volatile fluctuations in price level and it is regarded as the *Great Inflation* period by economists, and the later saw much more stable price level and lower inflation rate on average and labelled by macroeconomists as the *Great Moderation* time. The new term of the *Great Recession* only came in more recent years as a description of the time following the 2007-08 financial crisis. Thus, it is worth to consider estimating the model again by dividing the sample into three sub-sample periods and analyse them accordingly. The three sub-sample periods are divided as the *Great Inflation* which last from the first quarter of 1970 to the last quarter of 1983. The *Great Moderation* which stretch from the first quarter of 1985 until the second quarter of 2007. The *Great recession* covers from the third quarter of 2007 to the last quarter of 2017. The Bayesian estimation results of key structural parameters and shocks are presented in the following two tables:

Table 2.9: Posterior distribution for sub-sample estimates

Parameters	Great Inflation	Great Moderation	Great Recession
α	0.285	0.257	0.213
λ	0.748	0.783	0.712
σ_c	1.843	1.965	1.040
σ_l	1.887	1.341	1.000†
Ψ	5.965	7.160	5.362
χ	0.484	0.420	0.481
η	0.393	0.579	0.589
l_p	0.384	0.320	0.383
l_w	0.595	0.581	0.592
ζ_p	0.635	0.706	0.752
ζ_w	0.723	0.745	0.720
$\bar{\pi}$	1.0174	1.0057	1.0040
ψ_π	2.095	1.818	1.125†
ψ_y	0.159	0.194	0.125†
$\psi_{\Delta y}$	0.135	0.099	0.105†

† Pre-fixed parameters

Table 2.10: Posterior Distribution of Shocks Processes

Parameters	Great Inflation	Great Moderation	Great Recession
ρ_z	0.507	0.666	0.717
ρ_g	0.761	0.852	0.835
ρ_m	0.097	0.357	0.100
ρ_p	0.853	0.778	0.435
ρ_{gz}	0.561	0.587	0.601
θ_p	0.369	0.417	0.376
ρ_{inv}	0.492	0.503	0.499
ε_z	0.538	0.400	0.395
ε_g	0.639	0.331	0.256
ε_m	0.335	0.088	0.050
ε_p	0.354	0.168	0.225
ε_{inv}	0.422	0.598	0.429

Table 2.10 and 2.11 summarise the main results of this sub-sample estimation and provides a clear look of how these parameters and stochastic shocks evolve across the three periods. It clearly shows that the most crucial parameter in this thesis $\bar{\pi}$ is estimated to be 1.0174, which corresponds to an annualised trend in inflation of 6.96% in the *Great Inflation* time from 1970s to early 1980s. This figure comes with no big surprise if one recalls how high inflation was in the 1970s. However, the measure drops considerably to 1.0057, which implies an annual rate of 2.28% during the *Great Moderation* time. It reflects a much more stable and moderate level of inflation rate during the time and reflecting why it is characterised as a moderation period by economists. Finally, the steady state inflation declines even further during the *Great Recession* time, the estimate of 1.0040 for $\bar{\pi}$ translates to an annual trend inflation of just 1.6%. Nevertheless, these results are in sharp contrast with the SW07 estimation where they find the rate merely dropped from 2.9% in the *Great*

Inflation to 2.6% in the *Great Moderation*. This may due to the completely different methodology of how this trend inflation is incorporated in their model and in this GNK model, where the trend is explicitly modelled.

For the structural parameters, there are some interesting points to be noticed. While the inverse elasticity of substitution in consumption goods increases from the *Great Inflation* to the *Great Moderation*, the elasticity of labour supply drops significantly in the same time. This is not what SW07 finds as both parameters increase over time in their estimation. The adjustment cost of investment rises dramatically from the *Great Inflation* to the *Great Moderation*, this is consistent with the SW07 finding. However, it drops again during the *Great Recession* time. All the three price and wage indexation parameters χ, l_p, and l_w all stay quite stable across the three sub-sample periods, so does the utilisation rate of capital η. As shown in a number of literature, both Calvo parameters ζ_p and ζ_w increased from the *Great Inflation* to the *Great Moderation*, while ζ_p rises further in the *Great Recession* and ζ_w starts to decline in the same time.

By examining the change of response parameters in the Taylor rule, one would notice that as the level of trend inflation declines from the *Great Inflation* to the *Great Moderation* time, the central bank's estimated inflation response and output gap response both drop at the same time. This result confirms what Arias et al. (2018), Ascari and Ropele (2009), and Coibion and Gorodnichenko (2011) find; as trend inflation declines, the Taylor rule should respond less aggressively towards inflation, though the Taylor rule used here is slightly different from their trend inflation immune Taylor rule (TIIT).

Furthermore, the most significant differences between these three sub-sample periods are the standard errors of the stochastic processes. Especially the size of exogenous government spending and monetary policy shock. These two shocks have continuously declined in size throughout these three sub-sample periods. This result is also confirmed by SW07 sub-sample analysis. The interesting thing here is that the productivity shock also decreases considerably from the *Great Inflation* to the *Great Moderation*, however, it stays almost unchanged from the time onwards to the *Great Recession*. The shock to price mark-up has captured a U-shape change, it drops from the first sub-sample periods to the second, but it climbs again in the last sub-sample time. Overall, one should pay less attention to the *Great Recession* estimates due to both its limited sample and number of pre-fixed parameter values.

2.5 The Effects of Trend Inflation on Macroeconomic Dynamics in the Benchmark Model

Next, the focus turns to another core research question of this thesis: how different levels of trend inflation or potential levels of inflation target for central would alter the macroeconomic dynamics when the economy is hit by various shocks. To conduct this exercise in line with the baseline estimation of the *benchmark model*, I set the shocks' persistence parameters to the Bayesian posterior means, but the standard deviation of each shock to 1, in order to see the same magnitude of shocks that happen to the economy. Then, by changing the level of trend inflation, I examine how the key macroeconomic variables respond differently to these shocks under different levels of inflation steady state. The macro variables to be analysed including output, inflation, nominal interest rate, real market wage rate, and price dispersion. The levels of trend inflation to be tested are 0%, 2%, 4%, 6% and 8%.

2.5.1 Productivity Shock

Figure 2.7 shows the impulse response functions (IRFs henceforth) of 1 standard deviation productivity shock with persistence parameter equals 0.989, therefore, a highly persistent shock to the total factor productivity.

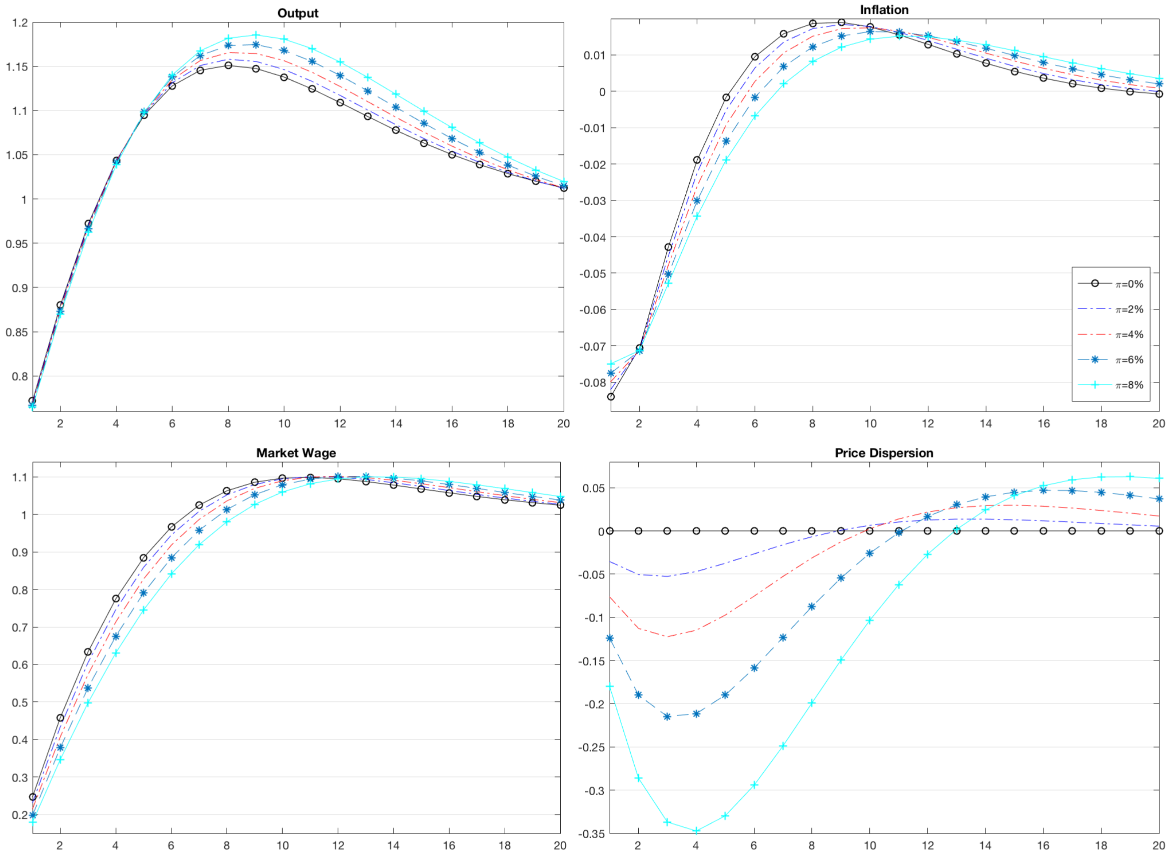

Figure 2.7: IRFs to a productivity shock ($\varepsilon_z = 1, \rho_z = 0.989, \rho_{zg} = 0.587$)

The first thing to notice is that under higher level of trend inflation, productivity shock always causes a greater degree of price dispersion on both sides of the steady state. Second, the IRF of output illustrates that output responds to a TFP shock with roughly the same magnitude under all levels of trend inflation up to the 5th quarter. However, since the 6th quarter, output reacts more to the shock under higher level of trend inflation, and this discrepancy keeps enlarging and reaches its maximum at around 12th quarter, 3 years after the shock. Then the gap between output reactions to different levels of trend inflation starts to shrink and remains open for this 20 quarters forecast horizon. The overall picture for output is that output reacts more heavily to productivity shock after one and half years and the reaction is more persistent with higher level of trend inflation.

The IRF of inflation provides a more interesting picture; inflation responds less on impact with higher level of trend inflation. Nonetheless, it becomes significantly higher soon after the second quarter, and the size of inflation deviation under 8 percent becomes about twice as large as it is with 0 percent in the fourth quarter. Moreover, inflation goes back to the steady state level after the fifth quarter with zero trend inflation but persists until the seventh quarter with 8 percent, hence a half year delay. The same story happens to later time as inflation bounds above steady state, inflation is more persistent under higher level of trend inflation throughout the entire forecast horizon. This IRF also looks very similar to the IRF of price dispersion, and this confirms their mutual feedback effect in this case.

Next, as demonstrated in section 2 that aggregate wage rate becomes more backward looking as trend inflation increases, the IRF of market wage rate clearly confirms this nature of wage innertia. Again, market wage rate responds less on impact, however, it persists much longer under higher level of trend inflation. Combined with the IRF of inflation, it is fair to say that this significant inertia in wage rate caused by higher trend inflation has apparent propagation effects to the rest of the supply side of the economy.

2.5.2 Government Spending Shock

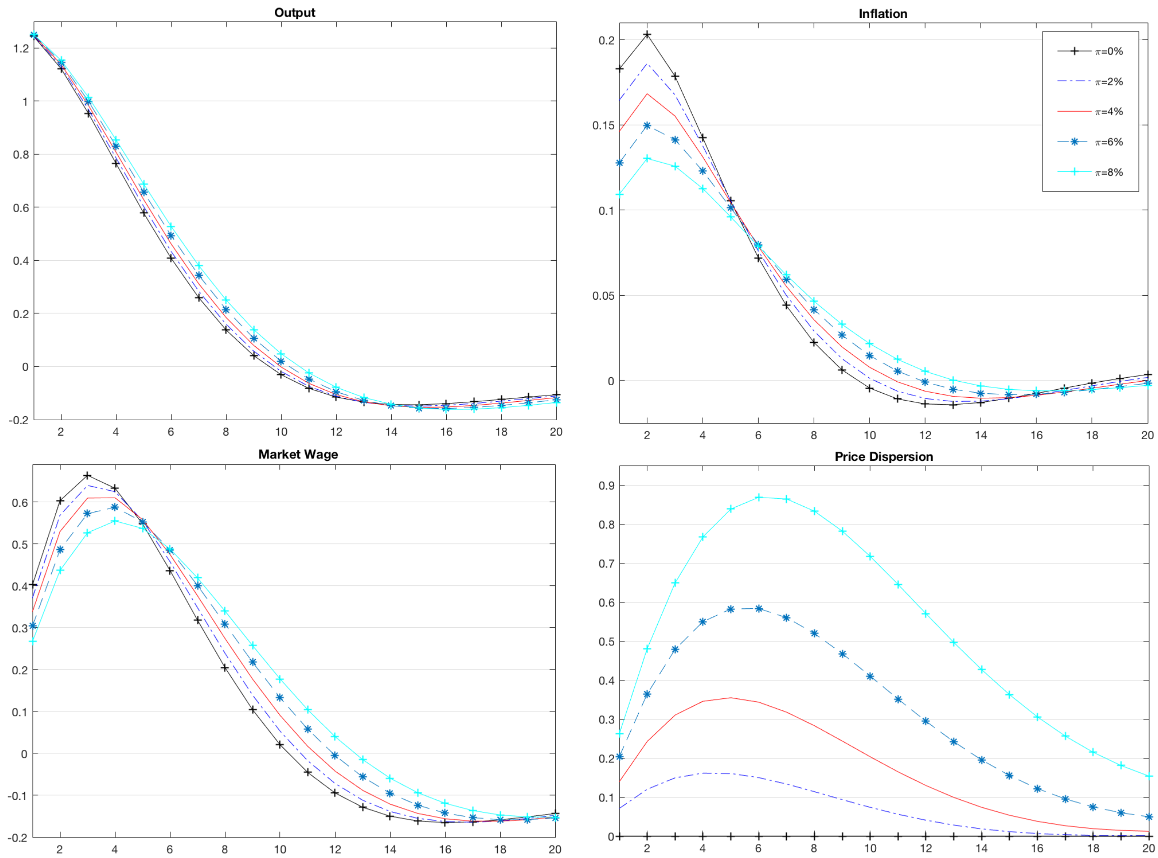

Figure 2.8: IRFs to a government spending shock ($\varepsilon_g = 1, \rho_g = 0.856$)

According to figure 2.8, after a government spending shock, the similar story happens to output, inflation, and market wage rate as in the previous case of productivity shock. Initially, output responds by roughly the same magnitude under all levels of trend inflation, however, the difference starts to emerge among different trend inflation and output becomes more persistent with higher trend inflation, even though the difference is trivial. Only a two quarters delay of going back to steady state under 8 percent trend inflation compared with zero trend inflation.

Both inflation and market wage rate react less on impact with higher trend inflation, the patterns are reversed after five quarters for wage rate and six quarters for inflation. Both inflation and market wage rate are much more persistent under higher inflation steady state than they are when steady state inflation is zero. The backward looking nature of market wage still plays a crucial part in the aggregate economy.

2.5.3 Monetary Policy Shock

The impulse response functions of a 1 standard deviation expansionary monetary policy shock in Figure 2.9 captures a more interesting story.

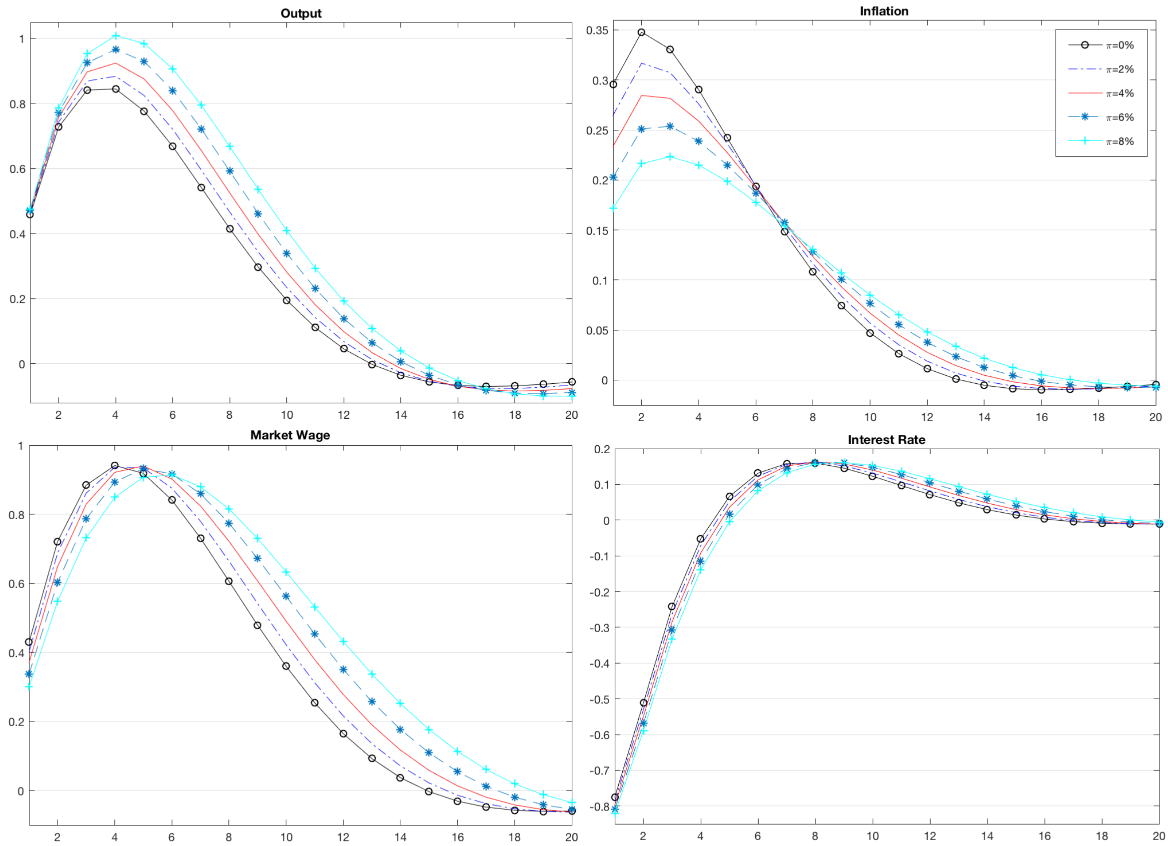

Figure 2.9: IRFs to a monetary policy shock ($\varepsilon_m = 0.197, \rho_m = 0.097$)

Output responds to the same level for almost all five different levels of trend inflation on impact. Nonetheless, in distinction to the previous productivity and government spending shock cases, this obvious discrepancy of output reaction among different levels of trend inflation starts to emerge after just two quarters and this dispersion remains considerable for almost three years. Then output reverts to steady state after around thirteen quarters under zero trend inflation, but persists until the fifteenth quarter with 8 percent trend, so a half year delay of returning to the origin. Again, the timing and persistence of discrepancy differ from the productivity and demand shocks, this IRF shows that output deviation is larger and more persistent under higher level of trend inflation compared with assuming zero inflation steady state.

It is always worth to look at the IRF of the market wage rate as it is one of the main sources of inertia in this medium scale GNK model. The discrepancy of market wage among different levels of trend inflation begins to present after just two quarters and keeps enlarging to a great extent. Market wage rate is obviously more persistent with even just moderate trend inflation 4 percent, represented by the red solid line in Figure 2.10. More precisely, real market wage goes back to steady state after the fifteenth quarter with zero trend, and it persists until just before the nineteenth quarter, an entire year of delay is captured here. For the forecast horizon of 20 quarters observed here, this dispersion of response under different trend levels remains significantly large.

The IRF of inflation indicates that inflation reacts much less on impact under higher trend inflation. The initial deviation of inflation under 8 percent trend inflation is just over a half of the initial deviation of inflation under zero trend inflation. Therefore, a central bank would assume a much larger effect of monetary policy change on impact when it believes the long run steady state of inflation is zero, compared with the same central bank but who can actually realise its long run inflation is obviously above zero. Moreover, according to the baseline calibrations based on Bayesian posteriors, inflation deviation peaks in the second quarter under zero, two and four percent inflation steady states. However, under both 6, and 8 percent trends, inflation peaks in the third quarter after the shock, which is more consistent with empirical evidence.

In addition, by comparing the black line with circles and the cyan line with crosses, one should realise that inflation returns to the steady state level in the thirteenth quarter under zero trend inflation, nevertheless, inflation persists until the eighteenth quarter when the trend is assumed to be 8 percent. An astonishing five quarters delay with just a $\rho_m = 0.10$ persistence level monetary policy shock. The overall picture this IRF tells us is that under high level of long run inflation steady state, a monetary policy shock causes much less response of inflation on impact compared with the case with very low or zero trend inflation, however, this response from inflation is much more persistent in high trend environment than it is the case under zero trend. This quantitative result captures a long-believed wisdom in monetary economics study: the effect on inflation after a monetary policy shock does not happen on impact, and this effect is also highly persistent (Mankiw and Reis, 2002).

To briefly sum up what is learnt from the IRFs of this expansionary monetary policy shock, this study provides some new enrichments to the current understanding of inflation persistence in monetary economics by showing that higher trend inflation can explain this empirical norm from a different standpoint (through introducing more inertia in market wage), and central banks should not ignore the presence of trend inflation when they conduct policy research.

2.5.4 Price Mark-up Shock

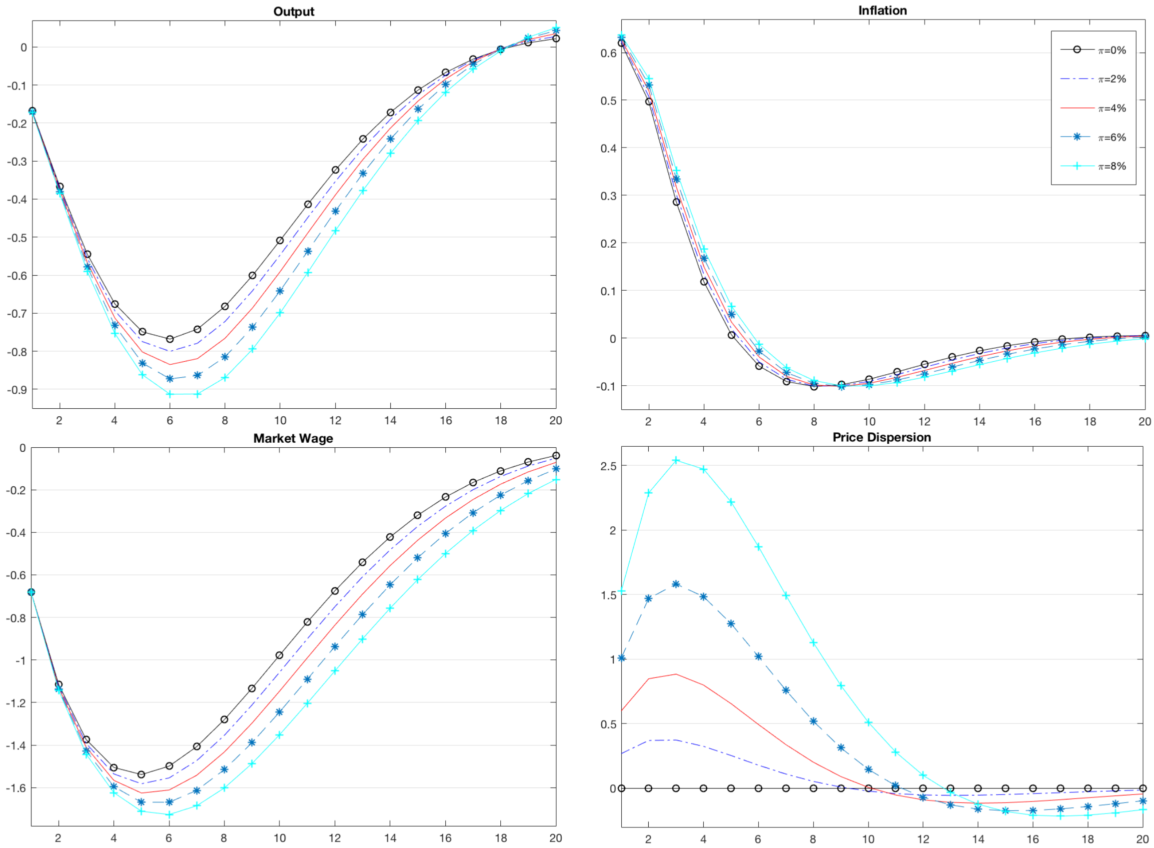

Figure 2.10: IRFs to a price mark-up shock ($\varepsilon_m = 1, \rho_p = 0.758$)

A supply side shock is captured by a shock to the Generalised NKPC, namely, price mark-up shock. Figure 2.10 summarises the IRFs after a price mark-up shock. A similar story to the previous monetary policy shock can be seen here. In general, output reacts much more under higher trend inflation, and this discrepancy enlarges from the third quarter while reaches it maximum around the ninth quarter. The only difference is that under baseline calibration, output with different levels of trend inflation all goes back to the steady state at around the same time, hence persistence of output is quite the same across all trend levels. But higher trend inflation generates much more volatility in output and presumably more welfare cost.

Inflation reacts more to this price mark-up shock with higher level of trend inflation from the beginning of the shock until inflation returns to its steady state level. The gap between the timing that inflation reaches steady state under zero and 8 percent trend is one quarter, which means inflation persistence is fairly close in this case compared with the previous monetary policy shock case. Nevertheless, the persistence of real market wage is still drastically larger with high trend inflation as the previous cases. It confirms the backward looking nature of this variable presents in every single case of this study.

Overall, under the baseline calibration of a moderately high degree of price indexation ($\chi = 0.60$), the impact of a change in trend inflation level on macroeconomic dynamics can be clearly observed in all exogenous shocks, even though they are very small in some cases. In particular, inflation is much more persistent with a high level of trend inflation after a monetary policy shock. A clear delay of the maximum effect can be found at 6 and 8 percent trend inflation case, which confirms the conventional monetary economic wisdom that the effects of monetary policy shock do not happen on impact, and also effects are quite persistent. One of the main contributions to this improved persistence of macro variables can be found through the effects of trend inflation on market wage rate, as confirmed by the IRFs of wage rate in all four cases.

2.6 Conclusion

This chapter constructs a medium-scale GNK model based on the work from Smets and Wouters (2007), but with the explicit modelling of trend inflation. The model is log-linearised around a non-zero inflation steady state while keeping most of the feature from the original SW07 model.

This new framework, the *benchmark model* shows some very interesting properties. The Generalised NKPC becomes flatter as trend inflation increases; the weights assigned to expected inflation becomes larger, while the weights on real marginal cost and past inflation become smaller, and making the GNKPC more forward looking. Moreover, as trend inflation increases, the real market wage rate in the aggregate economy is tied more to the past wage carried forward, or those who are not able to re-optimise their wages, and less to the current optimal reset wage. Therefore, it makes the market wage rate more backward looking and introduces more inertia to the wage movements. The overall effects on the whole model is a huge amount of increased dynamics.

This Generalised New Keynesian model is then estimated using a Bayesian technique with US quarterly data from 1970 Q1 to 2017 Q4. The estimation results, the impulse response functions and one period ahead forecast all indicate that this GNK model is able to capture the main macro features in the post-war United States. In addition, the Bayesian posterior on trend inflation shows the long run steady state inflation is about 3.2% annually during the time. An estimate that is largely in line with previous literature, but with the explicit modelling of trend inflation.

Based on the baseline calibration from Bayesian posteriors, the simulations demonstrate that rising trend inflation from zero to 6 or 8 percent can have a profound impact on the overall macroeconomic dynamics. More specifically, both output and inflation become more persistent as trend inflation rises; this is associated with a substantial increase in the persistence of market wage rate at the same time. Particularly for monetary policy shock, at the baseline calibration of 0.10 level of persistence, inflation can persist 5 quarters longer under 8 percent trend inflation than it can under zero trend inflation. In addition, the peak of the hump in inflation is delayed for one quarter with 6 percent trend inflation or higher, a substantial improvement in terms of capturing the macro evidence.

All the above result are obtained based on a calibrated degree of indexation of $\chi = 0.60$. As reflected in previous literature on trend inflation that indexation has a muting effect on the role that trend inflation can play to macroeconomic dynamics [5]. While as the degree of indexation becomes smaller the

[5] More discussion can be found in Ascari and Branzoli (2015); Ascari, Phaneuf and Sims (2018)

impact that trend inflation makes on equilibrium dynamics turns larger, declining degree of indexation can also cause reduced persistence and backward looking feature in the model. Thus, it is worth to look at how macro dynamics are changed once indexation is removed from this GNK model, and this is one of the central research questions of *Chapter 3*.

Chapter 3

Trend Inflation, Price Indexation and Inflation Persistence in a Generalised New Keynesian Model

3.1 Introduction

The model's implication that individual prices should continuously adjust in response to changes in prices elsewhere in the economy flies in the face of the survey evidence that many (though not all) individual prices remain unchanged in money terms for several months, or even longer—that has always provided one of the main arguments for supposing that prices are not continuously re-optimized.

Michael Woodford (2007) on price indexation

For a long period of time, there is a general agreement among leading macroeconomists that inflation has been showing a high degree of persistence in the post-war United States, and to find a model with features that can generate enough persistence in inflation became an essential criteria of evaluating whether one's model can successfully capture the empirical fact. Put it differently, whether it is a good DSGE model. Nonetheless, since the early 2000s, many researchers started to study whether this observed inflation persistence is intrinsic in inflation or it is just due to some unobserved component in the inflation dynamics. Prominent literature regarding this issue includes: Cogley and Sargent (2001), Cogley and Sargent (2005), Cogley and Sbordone (2008), Primiceri (2006), Cogley, Primiceri and Sargent (2010), and Stock and Watson (2007).

Earlier works such as Cogley and Sargent (2001), Cogley and Sargent (2005) were fruitful in terms of raising the issue but was unsuccessful in terms of distinguishing between inflation and time-varying inflation gap, as acknowledged in their own later work Cogley, Primiceri and Sargent (2010). The latter, however, successfully address this distinction. By defining a measure of persistence in terms of inflation gap predictability [1], they find that inflation gap, defined in their own term as the difference between inflation and its long run steady state or Federal Reserve's long run target, is weakly persistent when the effects of shocks die out quickly, and it is strongly persistent when these shocks' effects decay slowly. Therefore, based on their study, the persistence of inflation completely depends on how persistent the shocks that affect inflation are. In other words, inflation is intrinsically persistence because it inherits some persistence from the shocks.

One of the key assumptions required by macroeconomists to deliver such inflation persistence in DSGE models is the pricing behaviour of backward indexation. Therefore, the first issue I want to look at in this chapter is **the importance of backward indexation in this medium-scale Generalised New Keynesian (GNK) model**. Put it differently, whether the removal of indexation can cause the results obtained from *Chapter 2* become invalid. As illustrated in the previous chapter that the major motivations behind the design of price and wage indexation is to generate some backward-looking terms in a hybrid NKPC and therefore it can produce enough persistence in

[1] This predictability is defined as the fraction of total inflation gap variation j quarters ahead that is due to past shocks.

macroeconomic variables, particularly in inflation, as observed in empirical data.

Backward indexation states that for these price setting firms (or wage setting unions or households) who are not able to reset their prices (or wage), simply index the new prices as a function of last period inflation or steady state inflation rate, this can be found from Christiano, Eichenbaum and Evans (2005), Smets and Wouters (2007). An alternative mechanism of some rule-of thumb-behaviour approach is used in Clarida, Galí and Gertler (2000). However, such backward indexation assumption received heavy criticism from a wide range of researchers, from both theoretical and empirical standpoints. Chari, Kehoe and McGrattan (2009) argue that this feature is flatly inconsistent with the US data and counterfactual. These empirical analyses can be traced back to Golosov and Lucas Jr (2007), as well as Midrigan (2011). Furthermore, Dixon and Kara (2010) argue that a model with full or even partial indexation implies that every firm adjusts its price every period, and this clearly falls foul of the micro data. Therefore, a model with such backward indexation assumption can lead to huge trouble with the estimate of the Calvo probability of changing a price in an economy in which, because of backward indexation, all prices change in every period so that Calvo probability becomes an invalid estimate of overall rigidity in the economy. Just as reflected in the quote at the beginning of this chapter, Woodford (2007) points out that the assumption of indexation is simply inconsistent with micro survey data. In addition, Cogley and Sbordone (2008) criticise indexation's lack of microeconomic foundation by stating that "specifications of the Calvo model involving an indexation component are hard to reconcile with their evidence. When indexation is assumed, every firm changes price every quarter, some optimally rebalancing marginal benefit and marginal cost, others mechanically marking up prices in accordance with the indexation rule. Unless the optimal rebalancing happened to result in a zero price change, or lagged inflation was exactly zero, conditions that are very unlikely, no firm would fail to adjust its nominal price."

To make this trouble even worse in a GNK model, a high degree of indexation would mute the dynamic effects of trend inflation in a model where steady state inflation is correctly assumed to be non-zero in the long run. A number of researchers have found that the effects of trend inflation are dramatically reduced once a high degree of backward indexation is assumed. Both Ascari and Branzoli (2015), and Ascari, Phaneuf and Sims (2018) find a strong negative relationship between the degree of indexation and the magnitude that trend inflation can make to macroeconomic dynamics. Hence, the assumption of indexation can only make things worse in terms of a model's theoretical consistency, once we realise how important trend inflation is. The reason why indexation is so crucial to these macro models with trend inflation is the fact that when prices are fully flexible due to full indexation ($\chi = 1$), price dispersion disappears in the deterministic steady state (Schmitt-Grohé and Uribe, 2007b). When ($\chi < 1$), any deviation from zero inflation entails price dispersion, and the lower the degree of indexation, the higher is the price dispersion associated with a given level of inflation.

Therefore, it is essential for a model, which is designed to study the effects of trend inflation, to exclude the assumption of backward price indexation. For both theoretical consistency with micro foundation and empirical consistency with micro survey data, and also for the proper account of the effects of trend inflation in the economy.

Cogley and Sbordone (2008) formulate a time-varying drift in the trend inflation term and log-linearise a New Keynesian DSGE model around this time-varying inflation trend. They find that their estimates of the backward looking indexation parameter concentrate on zero, which implies that indexation is unnecessary once drift in trend inflation is taken into account. Furthermore, their model provides a good fit to the inflation gap, and the estimates of price adjustment frequency are broadly consistent with micro level studies. In addition, as trend inflation increases, the weight on forward-looking terms is enhanced, while that on current marginal cost is effectively muted, all the features that are confirmed by the GNKPC developed in *Chapter 2*. Therefore, it is worth to study **whether such medium scale GNK model with time-invariant trend inflation can also capture the observed inflation persistence without the sacrifice of assuming backward indexation**.

The last issue I want to focus on in this chapter is the **optimal response of monetary policy under a high level of trend inflation**. A typical contemporaneous Taylor rule usually involves the response to targeting variables such as inflation, output gap, output growth, and interest rate smoothing. Works by Coibion and Gorodnichenko (2011) find that heavy response to inflation, output growth and also a high degree of interest rate smoothing can improve the probability of securing the uniqueness of rational expectation equilibrium (REE), and avoid sunspot self-fulfilling fluctuations. This paper tests the optimal monetary response to these targeting variables conditional on satisfying the uniqueness of REE when the economy is experiencing a high level of trend inflation (8 percent).

The structure of this chapter is divided as the following: section 2 gives the general description of this new *benchmark model without indexation* and explains what difference these modifications are expected to make to the model. This is followed by section 3, which provides the Bayesian estimation results of this new model and the estimated equilibrium dynamics of the estimation. Section 4 conducts a simulation exercise of how the change in the level of trend inflation affects the macroeconomic dynamics when the economy is hit by various shocks. Section 5 focuses on the optimal monetary response under a high level of trend inflation conditional on determinacy. The last section draws some concluding remarks.

3.2 The Benchmark Model without Indexations

As a result of the flexible set-up of this GNK *benchmark model*, the feature of price and wage indexations can be easily switched off by setting $\chi = 0$. Upon removing the two indexations, expression of price dispersion becomes

$$\hat{s}_t = -\frac{1+\lambda_p}{\lambda_p(1-\alpha)} \cdot \left(1 - \zeta_p \bar{\pi}^{\frac{1+\lambda_p}{\lambda_p(1-\alpha)}}\right) \cdot \hat{\tilde{p}}_t + \zeta_p \bar{\pi}^{\frac{1+\lambda_p}{\lambda_p(1-\alpha)}} \cdot \left[\left(\frac{1+\lambda_p}{\lambda_p(1-\alpha)}\right) \cdot \hat{\pi}_t + \hat{s}_{t-1}\right] \quad (3.1)$$

where

$$\hat{\tilde{p}}_t = \frac{\zeta_p \bar{\pi}^{\frac{1}{\lambda_p}}}{1 - \zeta_p \bar{\pi}^{\frac{1}{\lambda_p}}} \cdot \hat{\pi}_t$$

Based on this new expression for price dispersion, one can easily find that past inflation does not appear in the equation anymore, hence, past inflation no longer has any impact on the current price dispersion. Furthermore, the past inflation is also missing from the optimal reset price equation $\hat{\tilde{p}}_t$. The only effect of past inflation on current price dispersion is through its impact on past price dispersion $\hat{s}_{t-1}$. Therefore, price dispersion loses some backward looking feature as a result of the removal of indexation.

From this new expression of price dispersion, one can detect the role that played by several key parameters on the magnitude of trend inflation on price dispersion in this equation. Due to the exclusion of price indexation, the marginal impact of trend inflation becomes much larger (power changes from $1 - \chi$ to 1). This implies for the same values of parameters λ_p, α, and ζ_p, as trend inflation increases, the mutual feedback effect between price dispersion $\hat{s}_t$ and inflation $\hat{\pi}_t$ becomes greater, and therefore, for the same level of inflation, a higher level of trend inflation now lead to a larger degree of price dispersion in the economy, this causes severe welfare loss. In addition, the same effect happens to the feedback coefficient of past price dispersion $\hat{s}_{t-1}$, therefore, even though past inflation $\hat{\pi}_{t-1}$ disappears from this expression after indexation is removed, the motion of $\hat{s}_t$ becomes no less dynamic.

The Generalised New Keynesian Phillips curve becomes

$$\hat{\pi}_t = \left[\zeta_p + \bar{\pi}^{-\frac{(1+\lambda_p)}{\lambda_p}} - \zeta_p \bar{\pi}^{-1}\right] \beta \cdot E_t \hat{\pi}_{t+1} + \frac{(1-\zeta_p \beta)(1-\zeta_p \bar{\pi}^{\frac{1}{\lambda_p}})}{\zeta_p} \cdot \hat{mc}_t + \hat{\lambda}_{p,t} \tag{3.2}$$

The first thing to notice from this new version of the GNKPC is that it becomes a purely forward looking Phillips curve in terms of inflation, the backward looking term $\hat{\pi}_{t-1}$ disappears once price indexation χ is set to zero. However, the same effects of trend inflation still hold in this new GNKPC. As the level of trend inflation $\bar{\pi}$ increases, even though the composite coefficients in front of both expected inflation $E_t \hat{\pi}_{t+1}$ and real marginal cost $\hat{mc}_t$ drop, the relative weight on π_{t+1} becomes much larger relative to the weight on $\hat{mc}_t$, again, the GNKPC becomes more forward looking and flatter as trend inflation $\bar{\pi}$ rises. This result is consistent with the GNKPC based on time-varying trend inflation in Cogley and Sbordone (2008) and also Ascari and Sbordone (2014)'s finding.

The equation for aggregate real wage rate changes to

$$\hat{w}_t = (1 - \zeta_w \bar{\pi}^{\frac{1}{\lambda_w}}) \hat{\tilde{w}}_t + \zeta_w \bar{\pi}^{\frac{1}{\lambda_w}} [l_w \chi \hat{\pi}_{t-1} - \hat{\pi}_t + \hat{w}_{t-1}] \tag{3.3}$$

The same conclusion from the original GNKPC of the previous chapter also hold, and the effect of trend inflation on the dynamics of real market wage rate clearly becomes larger as χ disappear. In general, the market wage become more backward looking after indexation is removed, this now introduces tremendous inertia in real market wage.

The optimal reset wage for labour union becomes

$$\hat{\hat{w}}_t = (1 - \zeta_w \beta)\hat{w}_t^h + \zeta_w \beta \hat{\hat{w}}_{t+1} + \frac{\zeta_w \beta}{\bar{\pi}} E_t \hat{\pi}_{t+1} \tag{3.4}$$

Again, the term of current inflation is gone after indexation is switched off. This implies the optimal reset wage is no longer affected by current inflation, and agents are more concerned with the current marginal product of labour, expected future optimal reset price and less concerned about the expected inflation when they have the chance to re-optimise their wages. This can be interpreted as that due to the presence of trend inflation, wage-setting agents feel inflation is harder to predict with accuracy and instead focus on more the fundamental factor, such as, the productivity of labour and expected reset wage.

3.3 Bayesian Estimation of the Benchmark Model Without Indexation

Now, the new GNK model without indexation is re-estimated using a Bayesian technique based on the same sample period from the first quarter of 1970 to the fourth quarter of 2017 with the same observables. The estimation is performed in exactly the same way as the one in *Chapter 2*: two models are estimated separately with one has $\bar{\pi}$ being estimated and the other one has $\bar{\pi}$ pre-fixed to 1.000, which implies zero inflation steady state. The results are reported in Table 3.1 and 3.2.

Table 3.1: Posterior Distribution of Parameters in Benchmark Model With No Indexation

Parameter	Mean ($\bar{\pi} > 1$)	St.Dev ($\bar{\pi} > 1$)	Mean ($\bar{\pi} = 1$)	St.Dev ($\bar{\pi} = 1$)
α	0.261	0.042	0.261	0.040
λ	0.696	0.041	0695	0.039
σ_c	2.494	0.253	2.523	0.252
σ_l	1.273	0.617	1.367	0.626
Ψ	6.943	1.230	6.899	1.216
η	0.586	0.089	0.588	0.087
ζ_p	0.776	0.026	0.778	0.026
ζ_w	0.740	0.060	0.751	0.057
$\bar{\pi}$	1.0073	0.012	1.000†	NA
ψ_π	2.082	0.189	2.069	0.194
ψ_y	0.185	0.062	0.185	0.061
$\psi_{\Delta y}$	0.197	0.038	0.202	0.035

† Pre-fixed parameters in SW07

Table 3.1 and 3.2 show the posterior estimates of the structural parameter for both versions of the same model. The first two columns report the Bayesian posterior means and standard deviations of the *benchmark model without indexation*, the last columns summarise the means and standard deviations of the *benchmark model with no indexation and no trend*, the direct comparison between the two models can be seen from the table.

Capital share of production α stays similar to the *benchmark model* estimate of 0.261, and it is exactly the same across two versions. The habit formation parameter λ is estimated to be around 0.696 in the benchmark without indexation model. Both α and λ are very close to the estimates of benchmark model. While both the inverse elasticity of substitution for consumption goods σ_c and labour supply elasticity σ_l stays close to the estimates in benchmark model, both parameters become higher in this new model with no trend inflation. The adjustment cost is relatively large in both model, with an estimate of 6.94 in the trend model and 6.90 in the no trend version. The capital utilisation rate is roughly the same across two models.

Interestingly, after removing price and wage indexations, the estimates of ζ_p and ζ_w can now be interpreted as the true measures of nominal rigidities in price and wage, since for those who are not able to re-optimise their prices and wages, now they would simply carry on with previous prices and wages without any indexing behaviour. The price rigidity ζ_p is estimated to be 0.78, along with ζ_w of 0.74 implies very similar estimates of these two Calvo probabilities with the *benchmark model*. Even though the estimates look similar to the previous results, the economic interpretation is completely different.

For the trend inflation estimates, the estimate of $\bar{\pi}$ becomes 1.0073, which translates to a 2.92% annual rate. A slight decline from the previous estimate of *benchmark model*, possible interpretation can be that once indexation is removed, the model exhibits less price flexibility at the aggregate level, therefore, the overall prices and wages do not change as frequently as the *benchmark model*, subsequently, this may lead to a slightly lower value of estimated steady state inflation.

For the monetary policy response parameters, the central bank's response to inflation becomes slightly larger, 2.082, compared with 2.069 in the *benchmark model*. Furthermore, the central bank's responses to output gap and change in output gap do not change much across these four versions.

Table 3.2: Posterior Distribution of Shocks Processes with No Indexation

Parameter	Mean ($\bar{\pi} > 1$)	St.Dev ($\bar{\pi} > 1$)	Mean ($\bar{\pi} = 1$)	St.Dev ($\bar{\pi} = 1$)
ρ_z	0.986	0.035	0.991	0.008
ρ_g	0.857	0.029	0.858	0.028
ρ_m	0.100	0.044	0.100	0.045
ρ_p	0.802	0.049	0.801	0.050
ρ_{zg}	0.587	0.051	0.586	0.050
θ_p	0.376	0.160	0.383	0.172
ρ_{inv}	0.492	0.199	0.509	0.199
ε_z	0.467	0.026	0.488	0.025
ε_g	0.497	0.029	0.393	0.028
ε_m	0.191	0.011	0.196	0.011
ε_p	0.169	0.054	0.180	0.081
ε_{inv}	0.380	0.269	0.389	0.267

The estimates of shock processes and persistence parameters provide some positive results. After removing the backward indexation assumption in both price and wage settings, the estimated shock standard deviations and persistence parameters do not change much. The shock to total factor productivity has a persistence parameter close to one (0.986), the monetary policy shock is as persistent as it in the benchmark model (0.10). The only noticeable change is the standard deviation government expenditure becomes much larger, 0.497, compared with 0.395 in the *benchmark model* estimates. All other estimates stay roughly the same. A similar result is obtained here: investment-specific technology shock has a much larger estimated volatility than all other shocks. A result confirmed by Justiniano and Primiceri (2008).

Importance of Key Features

Different features in DSGE model plays a very different role in terms of generating these observed macroeconomic dynamics. Among these set-ups, price stickiness plays a far less significant role than wage stickiness in terms of generating these macroeconomic dynamics. The removal of price stickiness makes almost no significant difference to the dynamics of the model, however, the exclusion of wage stickiness wipes out a large part of the dynamics. This is in line with most of the literature on similar models; both Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007) draw a similar conclusion. Variable capital utilisation is crucial in terms of generating observed persistence in output and inflation inertia. The degree of steady state price and wage mark-ups play no significant role in producing the macro dynamics. Again, all these findings are consistent with existing literature.

Macroeconomic Dynamics

One of the motivations behind the introduction of backward indexation is its assistance for generating enough persistence in macro variables, particularly output and inflation. Hence, it is worth to look at how macroeconomic dynamics change after indexation is removed.

Figure 3.1 and 3.2 summarise the estimated impulse response functions (IRFs) of the same four macro variables: output, inflation, nominal interest rate, labour hours, after two exogenous shocks: productivity shock and monetary policy shock.

Figure 3.1 shows that both output and inflation have similar patterns in their IRFs as in *Chapter 2*. More specifically, both output and inflation have less response immediately after the TFP shock, they both become more persistent now, nevertheless. Output peaks in the tenth quarter and stays at around 0.5 deviations after twenty quarters, while inflation returns to steady state after ten quarters, this is actually much closer to the SW07 estimation.

Furthermore, both nominal interest and labour supply produce better results in terms of matching SW07 result than the benchmark model in *Chapter 2*.

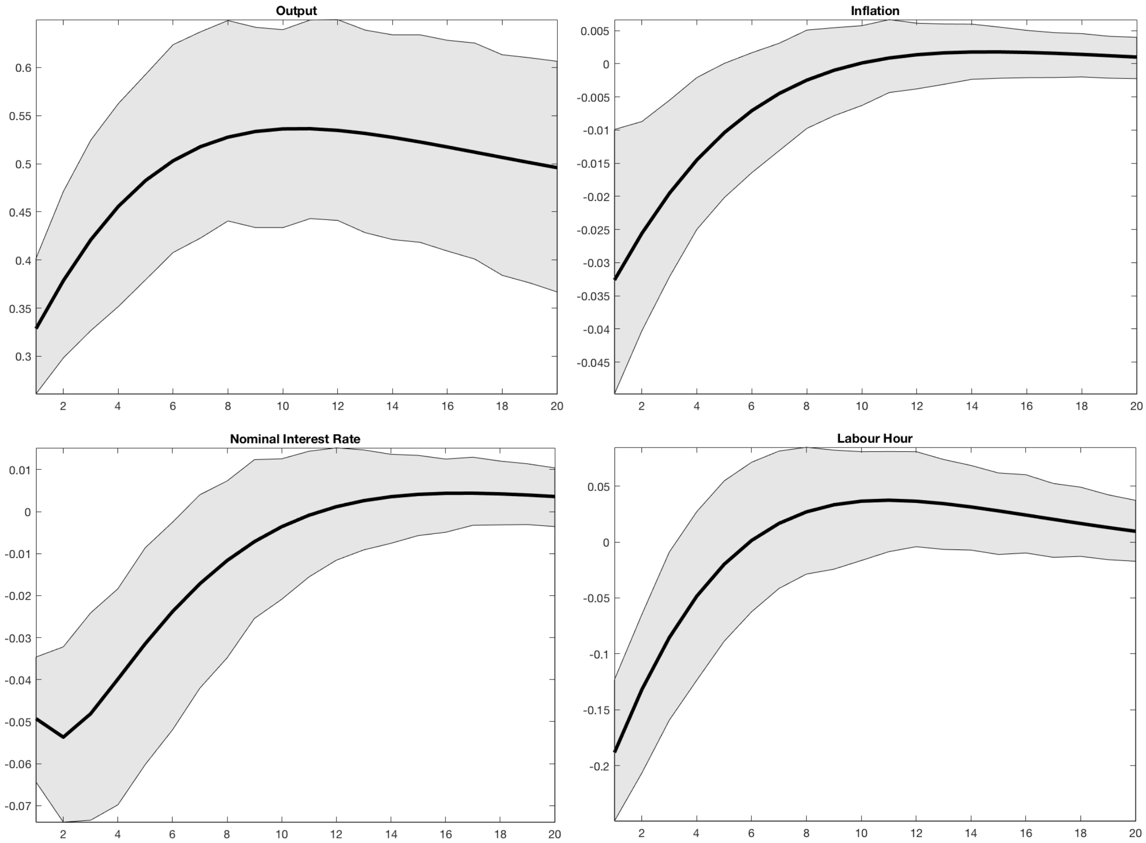

Figure 3.1: Impulse Response Functions to a 0.485 Productivity shock ($\rho_z = 0.986$)

[1]Shadow area represents the 90 percent confidence interval

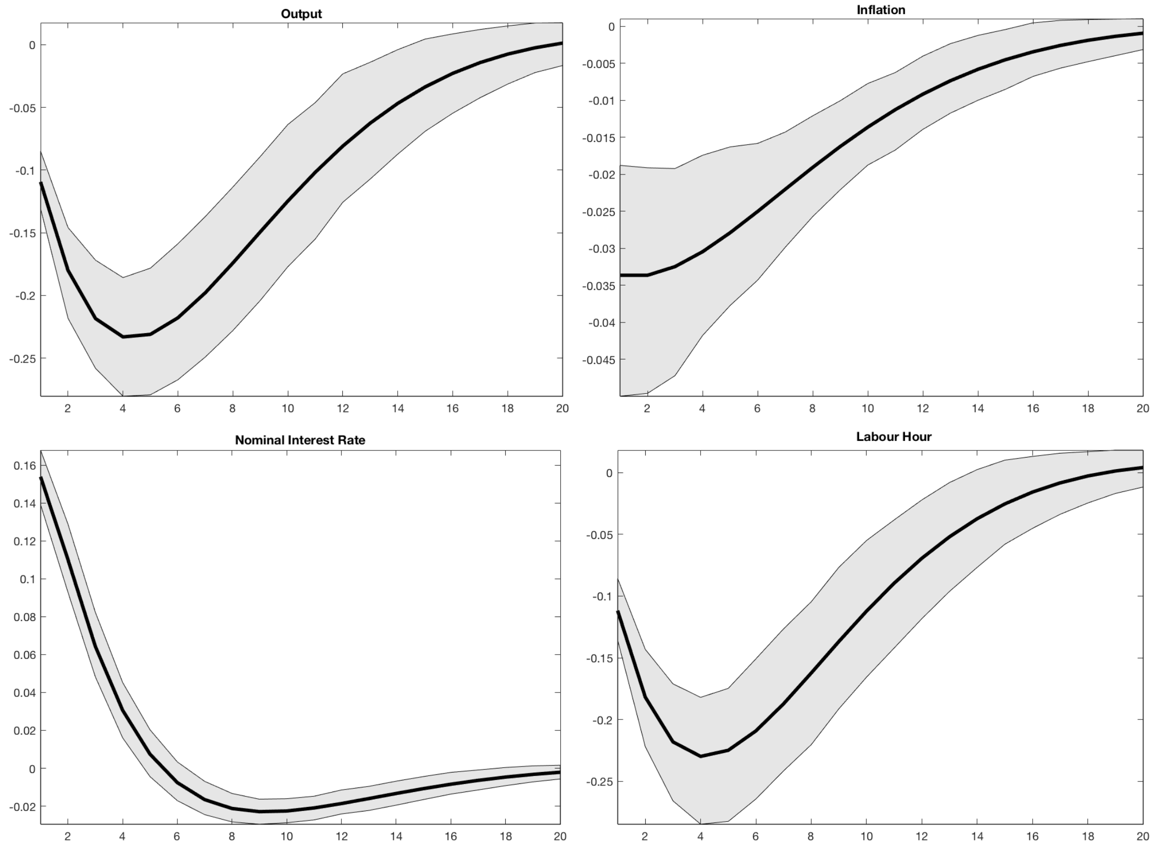

Figure 3.2: Impulse Response Functions to a 0.197 Monetary Policy shock $\rho_m = 0.100$

Figure 3.2 shows that IRFs after a contractionary monetary policy shock. Output and inflation again show very similar responses as the *benchmark model* response. Output drops downwards by slightly more than 0.1 on impact, then decline further to around -0.23 in the fourth quarter, which means the maximum effect in this estimated IRF is even later than the SW07 IRF. The effect of this monetary policy shock on output persists longer than the *benchmark model* estimation, it bounds back to the steady state after twenty quarters, which is exactly the same as SW07's estimation. Moreover, the IRF of inflation also shows some promising result. Even though this IRF does not generate an obvious hump-shaped response for inflation, it can be seen that the maximum effect inflation does not happen on the first quarter and then soon die out. The maximum impact persists to the second quarter and then inflation gradually returns to steady state after more than twenty quarters, again, a much similar picture as SW07 than the *benchmark model*. Overall, this GNK model without indexation produces some very promising results in terms of replicating the macro dynamics estimated from SW07 without the hugely-flawed assumption of backward indexation.

3.4 The Effects of Trend Inflation on Macroeconomic Dynamics without Indexation

Chapter 2 shows the presence of trend inflation can have some moderate effects on the model dynamics with an intermediate degree of price and wage indexation ($\chi = 0.60$). This section is to conduct the same exercise but based on this GNK model without indexation. In order to do so, I test how macroeconomic variables react to exogenous shock differently under different levels of long run inflation steady state.

Figure 3.3 through to Figure 3.7 summarise how output, inflation, nominal interest rate, market wage, investment, and price dispersion, react to five shocks under five different level of trend inflation: 0%, 2%, 4%, 6% and 8%. The first thing to realise from the initial glance at these five sets of impulse response functions is that the impact of trend inflation on macroeconomic dynamics is undoubtedly more noticeable than the IRFs when indexation is present, like in *Chapter 2*.

3.4.1 Productivity Shock

According to Figure 3.3, output with higher trend inflation responds slightly less on impact, and the gap between outputs under high and low trend inflation begins to enlarge after two quarters. After eight quarter, output with high trend inflation becomes dominant and generates a much higher level of deviations compared with its lower trend inflation counterparts. It also shows a much greater degree of persistence with high trend inflation throughout the 20 quarters forecast horizon.

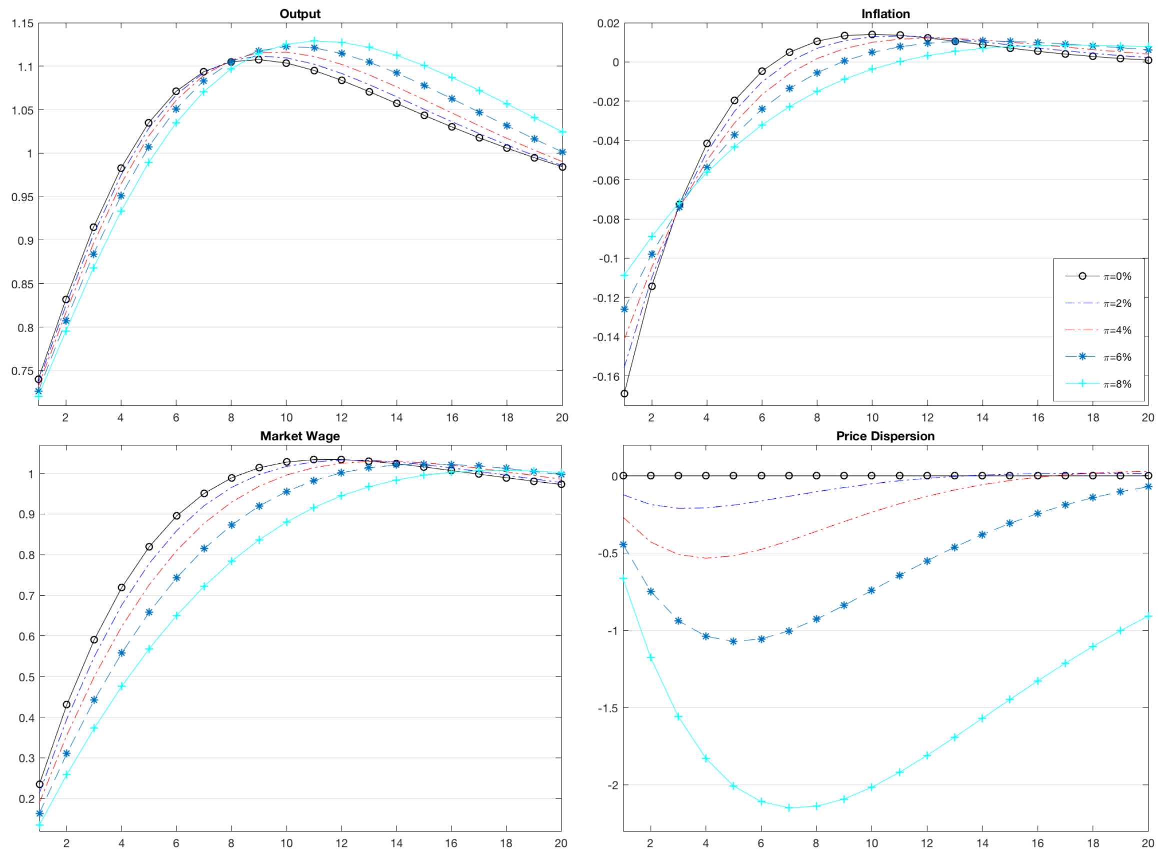

Figure 3.3: Impulse response functions to a productivity shock($\varepsilon_z = 1$, $\rho_z = 0.986, \rho_{zg} = 0.587$)

The similar story happens to inflation, inflation with higher trend inflation reacts much less on impact after this productivity shock, however, the pattern soon reverse after three quarters. High trend inflation clearly produces more inflation fluctuations than lower trends and inflation also come back to steady state later. More specifically, inflation with zero trend inflation returns to the origin after just six quarters, in the 8 percent trend inflation, it comes back to steady state in the eleventh quarter, an extraordinary five quarters of delay.

Market wage has a slightly different story, the wage rate jumps less on impact and clearly shows more inertia with higher level of trend inflation, as the wage equation exhibits more backward looking feature with higher trend inflation. This lag of response in real wage is present until the sixteenth quarter, after which it becomes more volatile and this pattern persists for the rest of the forecast horizon.

3.4.2 Government Spending Shock

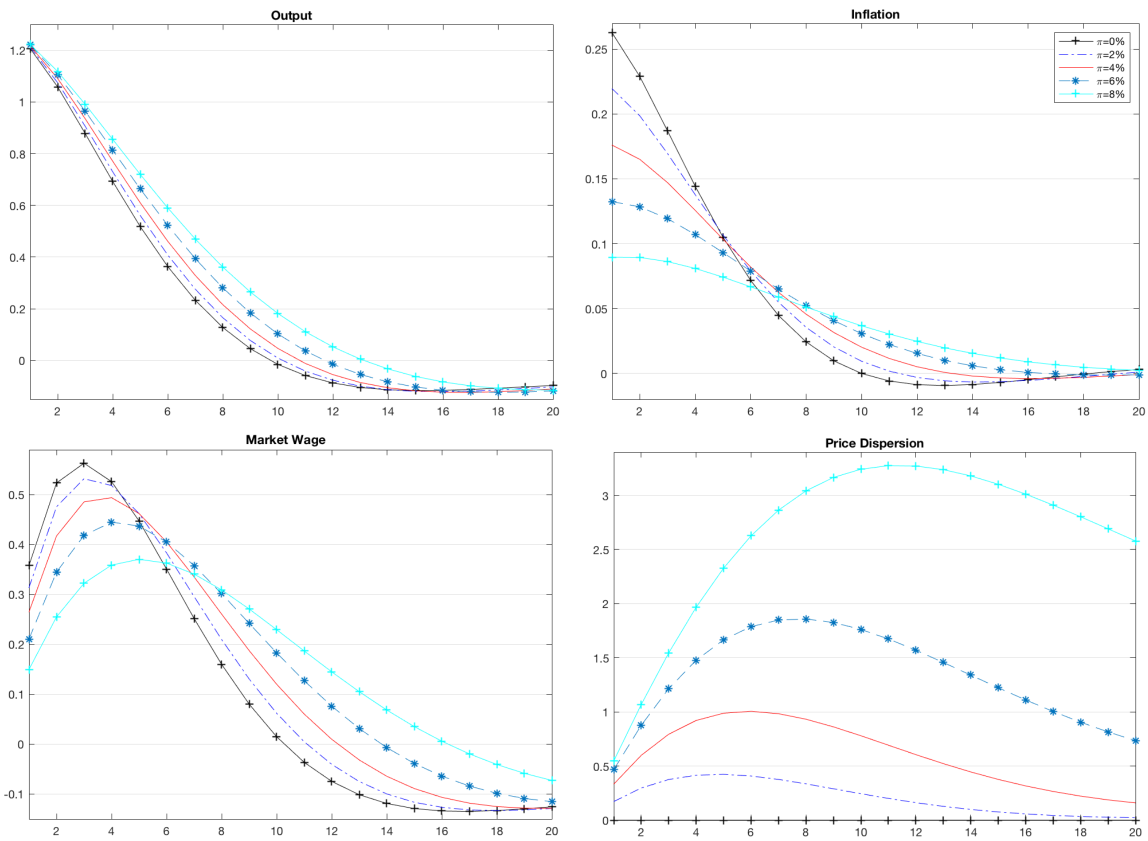

Figure 3.4: Impulse response functions to a government spending shock($\varepsilon_g = 1, \rho_g = 0.857$)

The response of output after a government spending shock is more straightforward. Output with all levels of trend inflation respond by almost exactly the same amount on impact, then the discrepancy starts to emerge and enlarge, and finally reaches its maximum in the ninth quarter. Output with higher level of trend inflation is more persistent: output goes back to steady state after 13 quarters for 8% trend inflation, and it returns to the origin after only 10 quarters.

Inflation jumps less but is much more persistent with higher levels of trend inflation. With 8 percent trend, inflation reacts by just a third of its reaction with zero percent (0.09 v.s. 0.27), however, the latter returns to steady state after just 10 quarters, and the former comes back after 20 quarters. Therefore, a stunning 10 quarters delay for inflation happens here, with the baseline calibration. A similar story can be found from the IRF of the market wage rate, even though the delay is not as large as it is for inflation.

3.4.3 Monetary Policy Shock

The IRFs of monetary policy shock produce yet another very interesting set of pictures. According to Figure 3.5, output jumps by roughly the same amount for all levels of trend inflation immediately after the shock, however, the movements start to disperse soon after the second quarter, then keeps expanding. The discrepancy reaches its maximum at the sixth quarter and is kept until the tenth quarter before it starts to narrow. Quantitatively, output with zero percent trend inflation returns to its steady state level after fourteen quarters, while the inflation with 4 percent trend inflation comes back only in the 17th quarter, so exactly a year of lag between the two zero and four percent trends. For the entire forecast horizon with positive output deviation, output with higher trend inflation always fluctuates more and generates significantly larger persistence. Subsequently, the economy experiences a much greater welfare loss due to higher level of trend inflation.

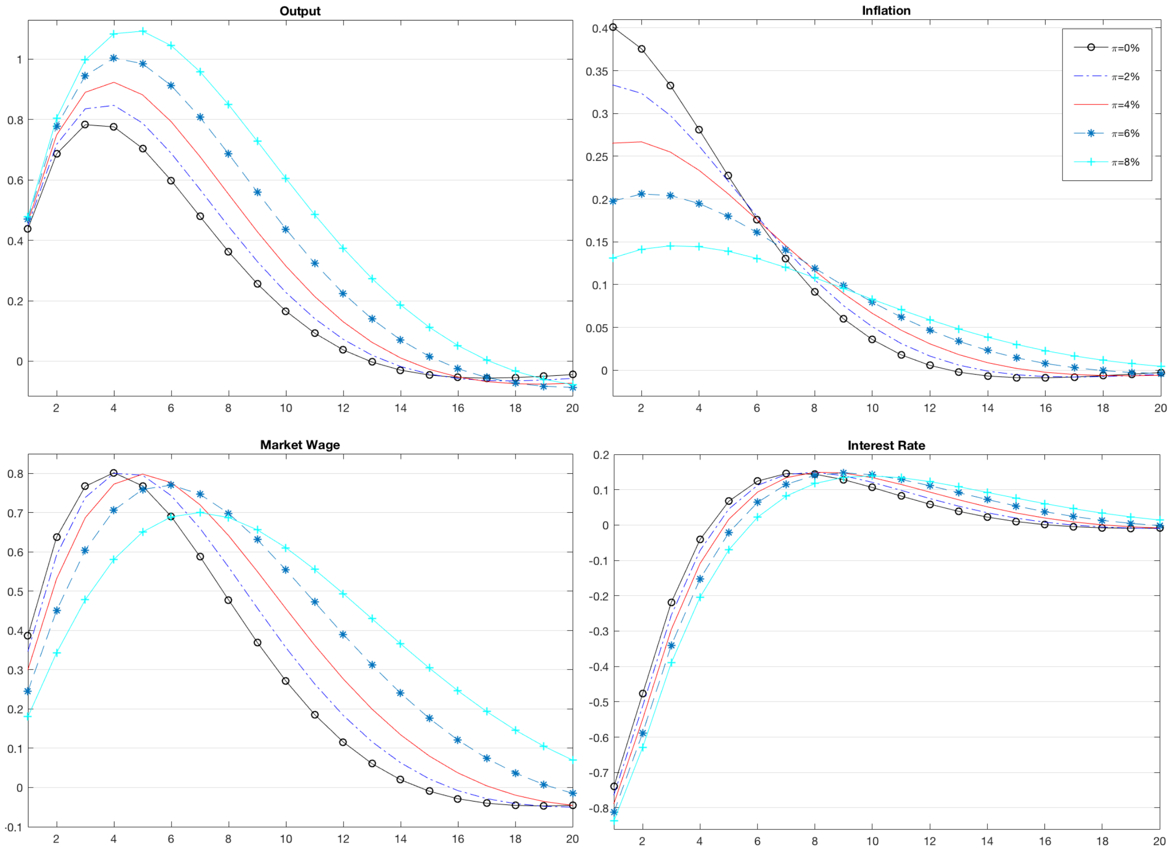

Figure 3.5: Impulse response functions to a monetary policy shock($\varepsilon_m = 1$, $\rho_m = 0.10$)

The IRF of inflation in Figure 3.5 yields one of the most valuable (if not the) results in this thesis. Overall, inflation with higher trend reacts to a monetary policy shock much less on impact compared with inflation under zero trend or very moderate trend inflation levels, but it persists for a much longer period. More specifically, by comparing the dashed navy blue line with star and the solid black line with circle one can find that inflation with 6 percent trend (0.20) reacts by just half of the reaction from inflation with 0 percent trend inflation (0.40) on impact. Then, inflation starts to decline with absolutely no comeback with 0 and 2 percent trend inflation. However, for inflation with trend inflation of 4 percent or above, the largest impact is delayed until the second quarter for 4 and 6 percent trend inflation. Most strikingly, inflation with 8 percent trend inflation peaks between the third and fourth quarter, a clear and beautiful hump can be observed here for 4, 6, 8 percent of trend inflation.

Regarding the hump shape of inflation, Dixon and Kara (2010) analyse four different types of price setting behaviour and find that without indexation behaviour (also no trend inflation in their model), a Calvo price setting model cannot generate a hump shape, the largest response of inflation to a monetary policy shock always happen on impact. However, here I successfully generate a hump-shaped response of inflation to a very transitory ($\rho_m = 0.10$) monetary shock using a model without the assumption of backward indexation behaviour or any other extra features, even just for moderate trend inflation of 4 percent. In fact, a further experiment to test the relationship between the effect of trend inflation and the persistence level of monetary policy shock find that the two are highly positively correlated. Figure 3.6 summarises that IRFs of inflation after a monetary policy with four different shock persistence levels: $\rho_m = 0.30$, $\rho_m = 0.60$, $\rho_m = 0.90$, $\rho_m = 0.99$. The first thing to notice from the IRFs is that with no trend inflation ($\bar{\pi} = 1$), inflation never exhibits a humped-shape after monetary policy under any persistence level. Second, with a highly persistent monetary shock ($\rho_m = 0.90$), even inflation with just 2 percent trend inflation can generate a slight hump which peaks in the second quarter, while the maximum effect of this shock on inflation with 4 percent trend is delayed to the fourth quarter after the shock. Therefore, it is clear to say that the timing of the maximum effect on inflation is highly positively correlated with the persistence level of this monetary policy.

On the monetary policy front, under high trend inflation environment, if a central bank fails to take into account the positive long run trend inflation, a central bank could completely miscalculate the time it takes for monetary policy to become effective on inflation and how long this effect can last. Therefore, ignoring trend inflation could generate devastating consequences to the stabilisation policy for economies that experience considerably above zero trend inflation. This result is in contrast to Ascari, Phaneuf and Sims (2018), where they apply a medium scale GNK model and find inflation with higher trend react more on impact but is less persistent than inflation with lower trends, the reverse is found in this model. The results shown here are actually consistent with the work of a small scale GNK model from Ascari and Sbordone (2014).

Again, the analysis of the IRF for market wage rate tends to provide us with some insights of how this model is able to generate so much inertia as trend inflation level rises. Figure 3.5 shows aggregate wage rate with 4 percent trend inflation reacts by less than a quarter (0.30) of the response of wage rate (0.39) with 0 percent trend inflation. So, a much less reaction on impact, the same story as inflation. Then this pattern is reversed after around two years, the deviation of the market wage rate becomes much larger with higher levels of trend inflation. Subsequently, inflation with zero percent trend comes back to steady state after 15 quarters while inflation with 4 percent trend inflation only returns to steady state after 18 quarters, hence, an obvious delay of 3 quarters for this $\rho_m = 0.10$ monetary policy shock. This IRF yet again show how much inertia can be generated by an increase in trend inflation on market wage rate, and then it turns out to impact the overall economy as a whole.

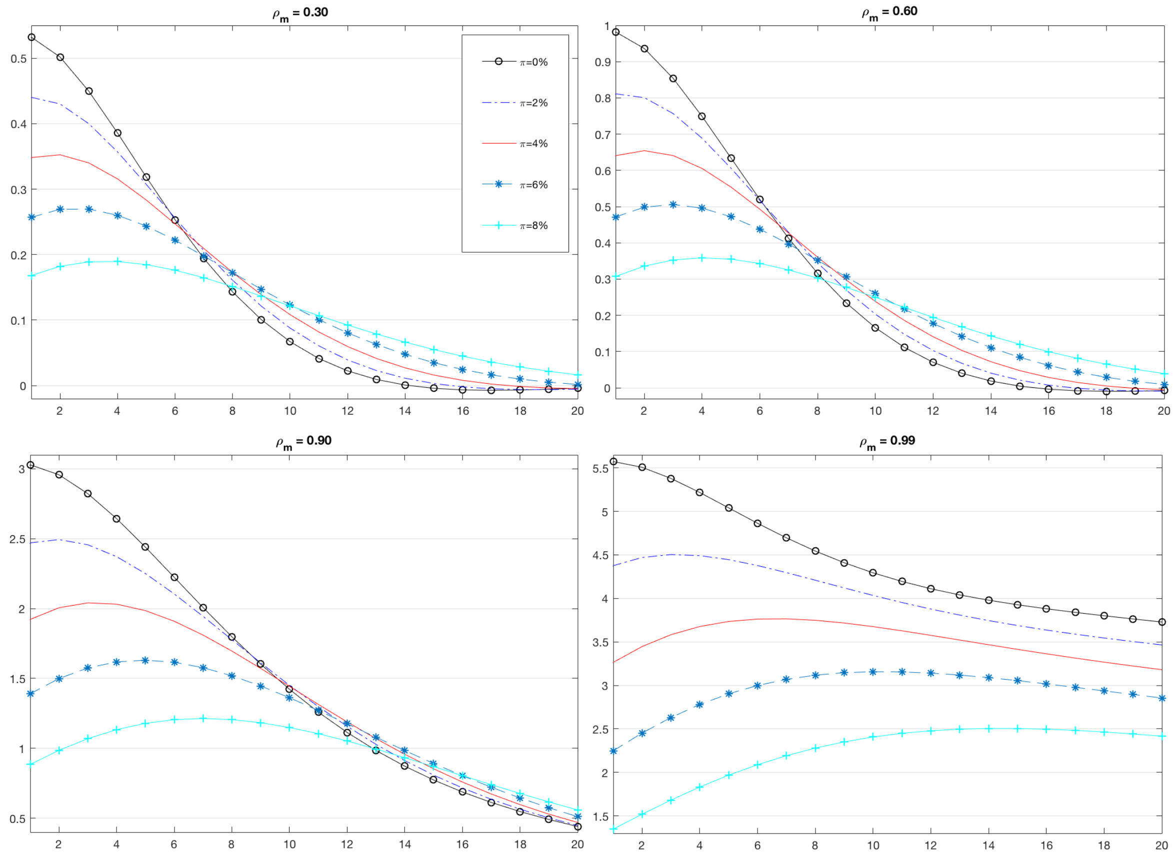

Figure 3.6: Impulse response functions to monetary policy shock with different persistence levels ($\varepsilon_m = 1, \rho_m = 0.30, 0.60, 0.90, 0.99$)

3.4.4 Price Mark-up Shock

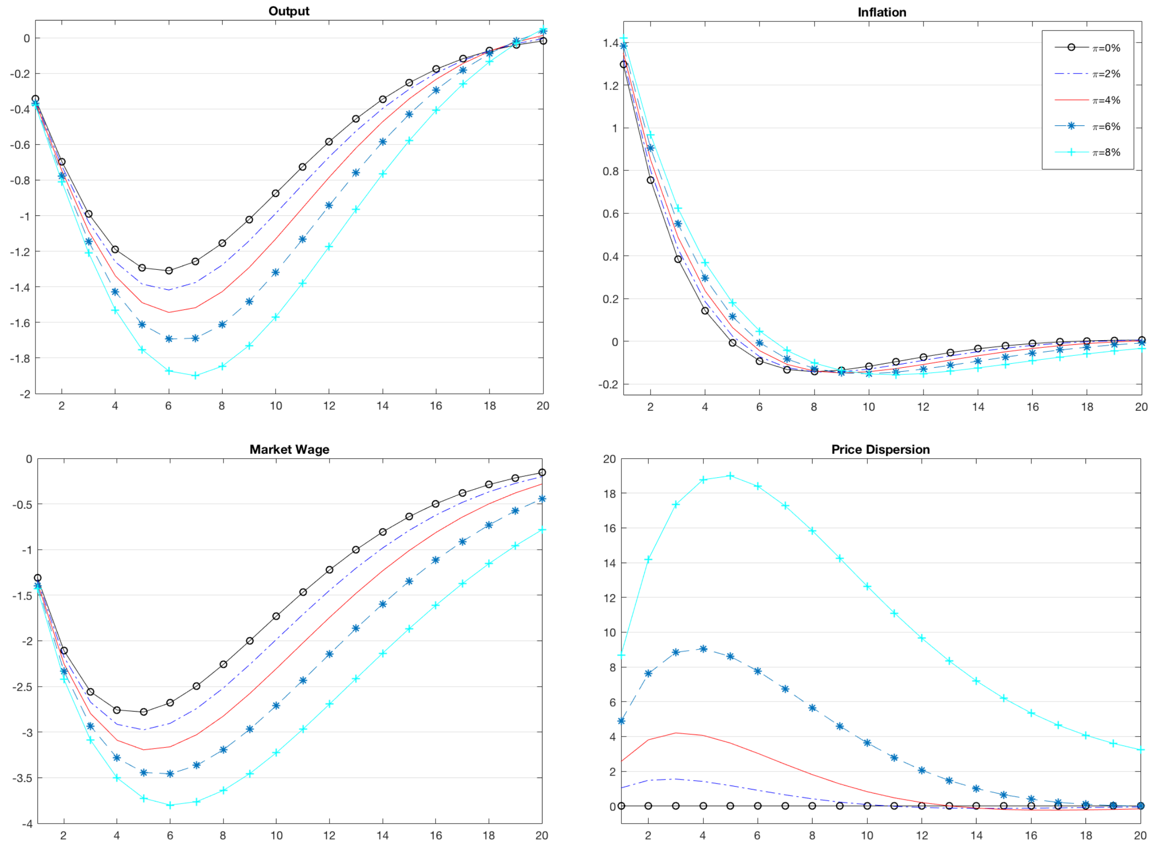

Figure 3.7: Impulse response functions to a price mark-up shock
($\varepsilon_p = 1, \rho_p = 0.802, \theta_p = 0.376$)

A shock to the supply side of the economy, which is reflected by a shock to the price mark-up in the GNKPC, yields the most straightforward analysis of output deviations among all five exogenous shocks. In Figure 3.7, the impulse response functions of both output and real market wage show that output and market wage rate both react by roughly the same level for all levels of trend inflation and soon they start to disperse. The only difference is that market wage is much more persistent with higher trend inflation, however, although output with higher trend inflation produces much more overall fluctuations, it does not show any greater persistence for the forecast horizon. Price dispersion responds much more with higher trend inflation than it is with lower trend inflation.

3.4.5 Marginal Efficiency of Investment Shock

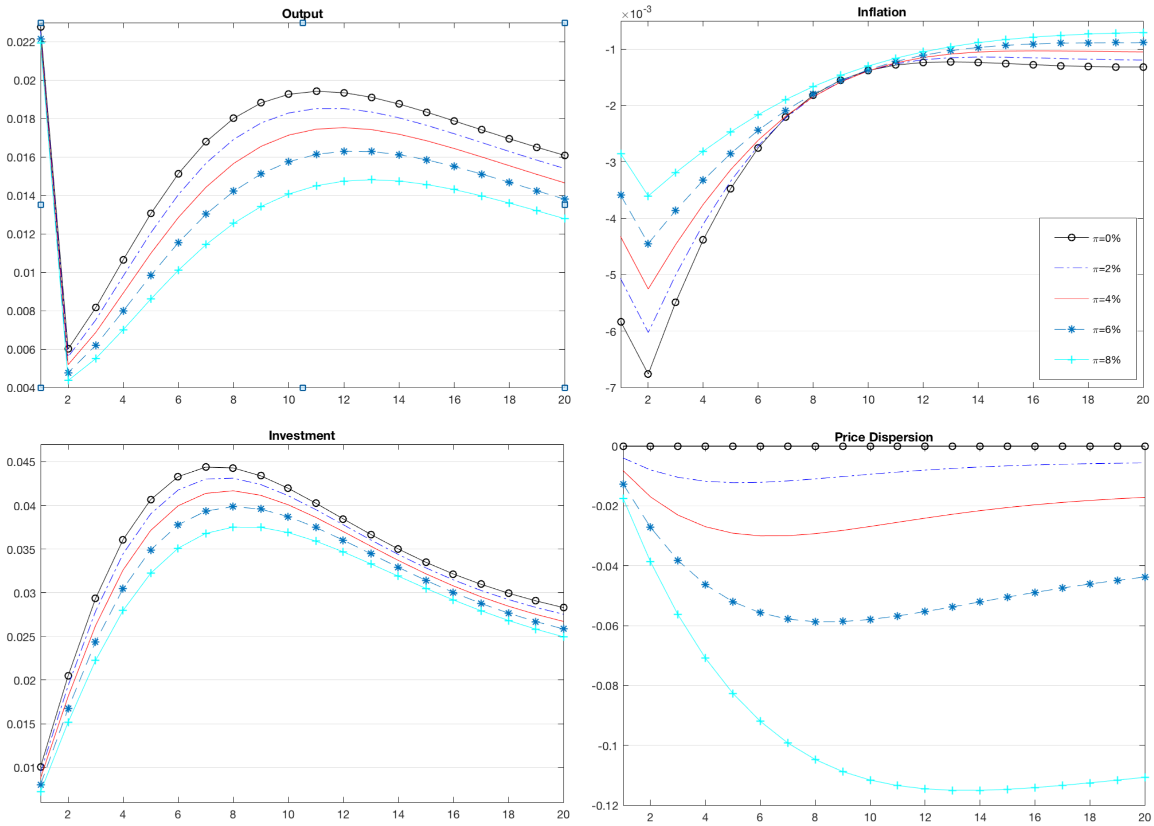

Figure 3.8: Impulse response functions to a investment-specific technology shock
($\varepsilon_{inv} = 1, \rho_{inv} = 0.492$)

Figure 3.8 gives the IRFs of an marginal efficiency of investment (MEI) shock. In contrast to most of the cases studies in this thesis so far, higher trend inflation actually generates much less response of output, inflation and investment after an MEI shock. As can be found from Figure 3.8 that output, inflation and investment's reactions are all less for higher levels of trend inflation than they are with lower trends. The price dispersion is the only exception as it is still worsened as trend inflation rises.

Overall, for the five shocks tested here, trend inflation interacts much more with these exogenous shocks than it does when price and wage indexation is present in *chapter 2*. This confirms previous studies from Ascari and Branzoli (2015), and Ascari, Phaneuf and Sims (2018). As indexation increases, the effect of trend inflation on macroeconomic dynamics declines. The IRF of inflation to a monetary policy shock clearly shows two important features: 1) inflation responds less on impact with higher levels of trend inflation, but persists for much longer time; 2) there is a hump shape shown in the IRF of inflation for 4 percent or above, which indicates that the largest effect of inflation after a monetary shock does not happen on impact, but it is delayed for a few quarters. This is consistent with macro evidence on monetary policy shock and it has profound implications for monetary policy studies. The rise of inflation persistence is again is mainly driven by the increases of persistence in price dispersion, which eventually feeds back on inflation.

3.5 Applications: Monetary Policy Response Under High Trend Inflation

The story of how the US economy has evolved from the *Great Inflation* to the *Great Moderation* and how the Federal Reserve's policy conduct has changed over time, so that the US economy moved from highly volatile inflation in the 1970s to the stable and moderate inflation time in the mid-1980s and throughout to the pre-financial-crisis have attracted much of the attention in both theoretical and empirical literature. However, the reasons behind this shift of monetary volatility of monetary stability are not generally agreed among economists. One the one hand, Taylor (1999), Clarida, Galí and Gertler (2000) and Lubik and Schorfheide (2004) have found that there is a regime change of how the Federal Reserve conducts monetary policy from the pre-1980s to post-Volcker time. On the other hand, Sims and Zha (2006) and Stock and Watson (2007) conclude that there was no such change happened, the major cause of moderate inflation movement post-Volcker was mainly due to the lower volatility of shocks for the later period. Coibion and Gorodnichenko (2011) find something more interesting. Their research suggests that the same response to inflation by the central bank can lead to determinacy at low levels of inflation but indeterminacy at higher levels of inflation. Therefore, it could be that the Volcker disinflation of 1979-1982 by lowering average inflation (consequently trend inflation), was enough to shift the US economy from indeterminacy to the determinacy region even with no change in the response of the central bank to macroeconomic variables.

This paper examines the best monetary policy response under high level of trend inflation, but in a very different way from the previous literature on trend inflation. First, I assume the determinacy is satisfied in the sense that local uniqueness of rational expectation equilibrium is guaranteed. Instead of finding the best response from monetary authority that maximises the chance of meeting determinacy, I examine what is the best response to different targeting variables in the Taylor rule that helps the central bank to stabilise output and inflation.

In order to do this, I take the posterior means of Bayesian estimation from the last section and calibrate the model based on these Bayesian posteriors, all the structural parameters and shock persistence follow the posterior means. I set the trend inflation parameter $\bar{\pi} = 1.02$, which implies an annualised trend inflation of 8%, the rate that the US economy suffered during on average during the *Great Inflation* in the 1970s. The next step is to hold all the parameters in line with the Bayesian posteriors, meanwhile change one of the value of four response parameters in the Taylor rule: response to inflation ψ_π, response to output gap ψ_y, and response to output growth ψ_{gy}, at one time, in order to check how much should the central bank responds to stabilise the economy. The Taylor rule that involves this four types of response has received most of the attention in general literature on monetary economics as well as literature on monetary stabilisation policy under trend inflation. In the rest of this section, I investigate how should the central bank respond to various targeting variables under high level of trend inflation.

3.5.1 Monetary Policy Response to Inflation

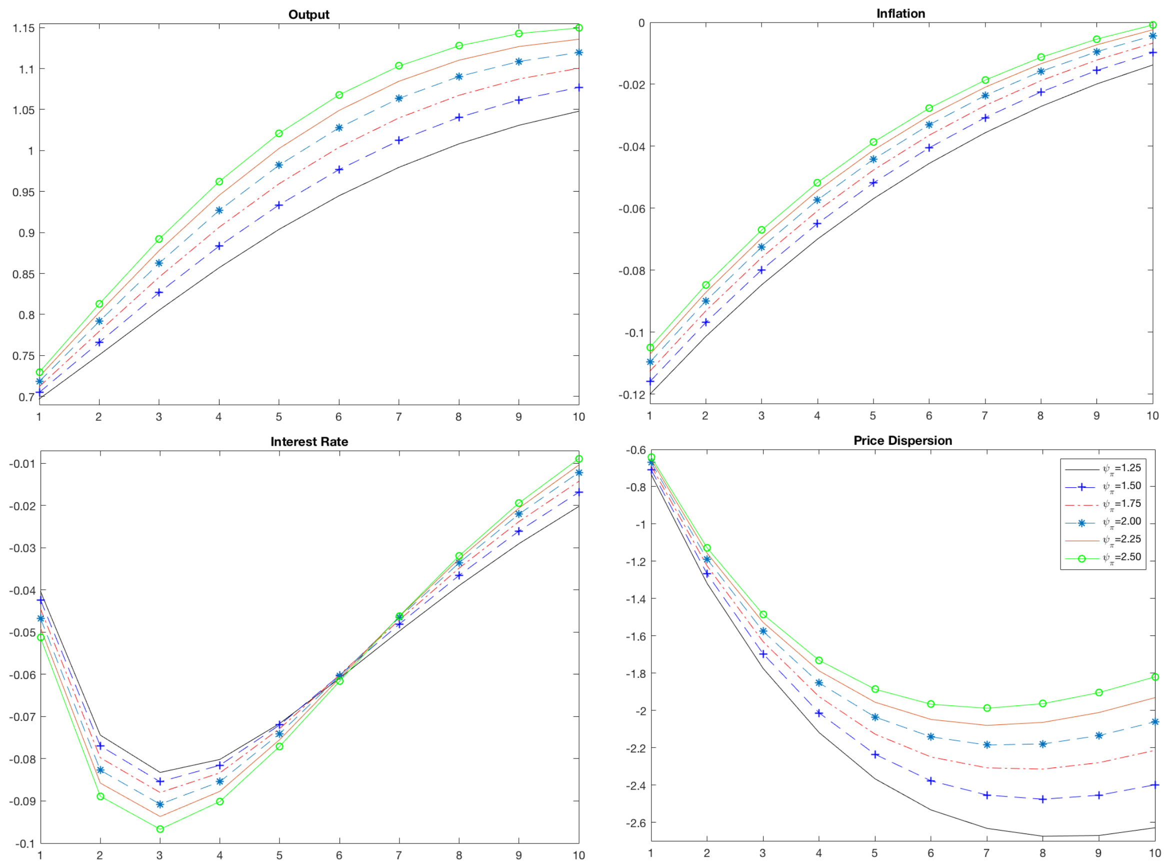

Figure 3.9: IRFs to a 1 standard deviation productivity shock under different response to inflation

The first case I evaluate is the parameter of inflation response in the Taylor rule. The question for investigation is how aggressively should a central bank respond to inflation under high level of trend inflation when the economy is hit by two exogenous shocks: the shock to total factor productivity and monetary policy shock. Recall that the productivity shock follows an AR(1) process with the persistence parameter ρ_z equals 0.986. Figure 3.5 shows that IRFs of four variables after a productivity shock under six calibrated value of inflation response parameter ψ_π: 1.25, 1.50, 1.75, 2.00, 2.25, and 2.50. The entire study is conducted upon an unique rational expectation equilibrium is guaranteed.

The top two IRFs of Figure 3.19 indicate that the central bank faces a clear trade-off between stabilising output and stabilising inflation after a productivity shock regarding whether it should respond heavily to inflation deviation from the target. The IRF of output shows that a higher value of ψ_π generates a larger deviation of output, however, the IRF of inflation shows the opposite, responding more heavily to inflation definitely helps the central bank to stabilise inflation. By responding more heavily to inflation does not necessarily generate the best outcome for both output and inflation stabilisations, therefore, the question is to what extent the central bank is concerned with inflation stabilisation relative to output stabilisation. Put it differently, how hawkish is this central bank to inflation? If the bank is more hawkish, then it should respond more to inflation and care less about the harm it makes for output. The reverse holds if the central bank is more dovish. Overall, this is a preference matter for the monetary authority, but the trade-off is present here.

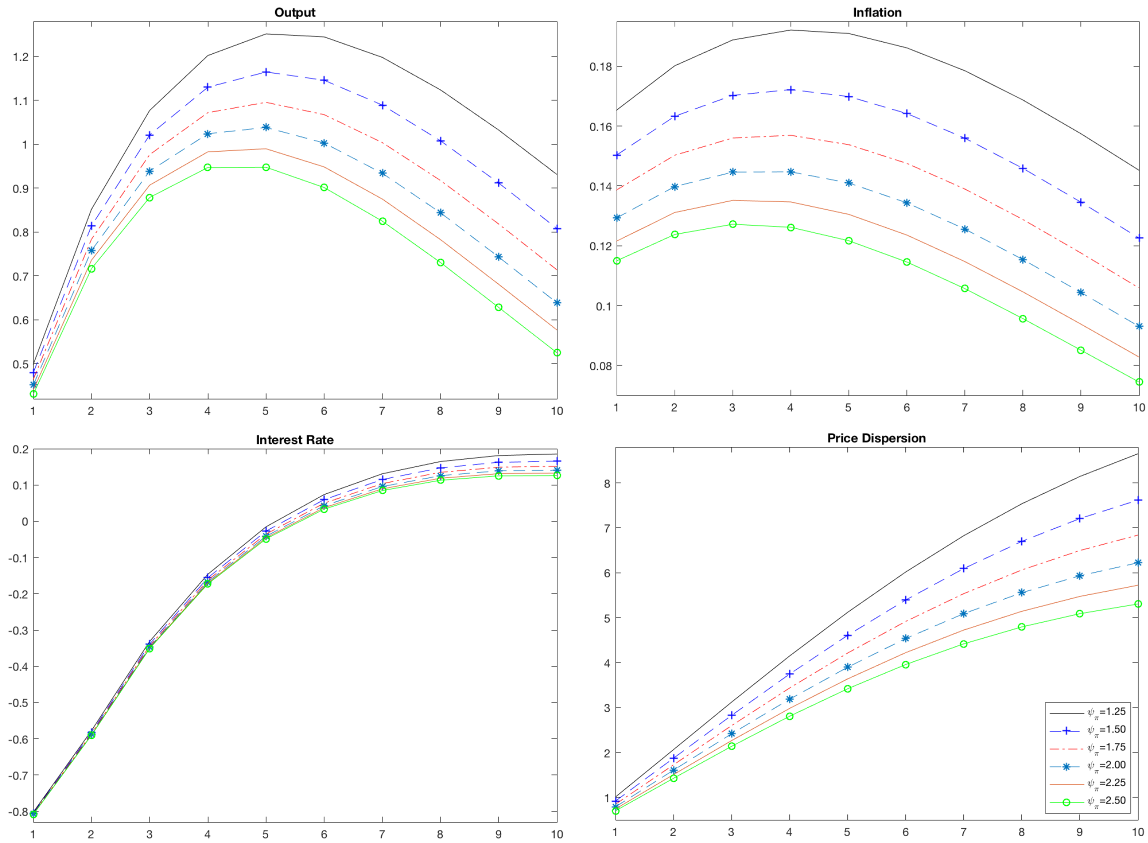

Figure 3.10: IRFs to a 1 standard deviation monetary policy shock under different response to inflation

Figure 3.10 provides the IRFs after a monetary policy shock ($\rho_m = 0.10$); this time output and inflation IRFs give a unified recommendation. By responding more aggressively to deviation in inflation helps the central bank to stabilise both output and inflation. Price dispersion is also kept much lower when the monetary authority reacts more to inflation fluctuations after a monetary shock. Apparently, the central bank does not face the same type of inflation-output trade-off as with the productivity shock.

Therefore, the overall picture of policy guidance under high trend inflation is the central bank should respond more aggressively to inflation after a monetary shock, however, the bank faces a clear trade-off between stabilising output and stabilising inflation after a shock to total factor productivity.

3.5.2 Responding to Output Gap

The debate of whether the central bank should respond to output gap has been ongoing in monetary policy studies for a long time. Under the assumption of positive steady state inflation, one agreement that is generally reached among existing literature is that responding more to output gap can lead the central bank to indeterminacy when the economy is experiencing high level of trend inflation. This has been well-documented in Ascari and Ropele (2009), Coibion and Gorodnichenko (2011), and Ascari, Florio and Gobbi (2017). However, whether a heavy response to output gap generates worse stabilisation outcome conditional on determinacy is another matter, and this section examines this issue.

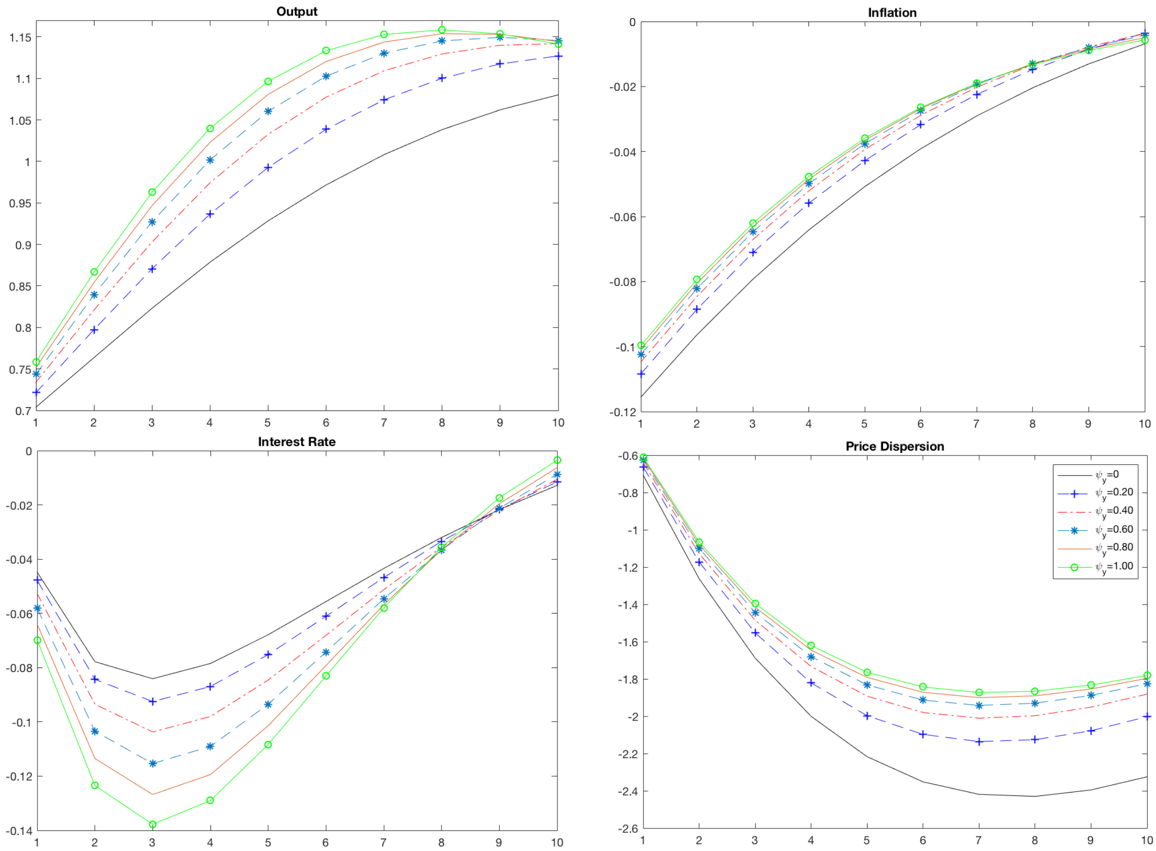

Figure 3.11: IRFs to a 1 standard deviation productivity shock under different response to output gap

The first thing to notice from Figure 3.11 is that responding more aggressively to output gap is definitely undesirable in terms of stabilising output, the black solid line represents $\psi_y = 0$, which implies not responding to output gap at all, clearly dominates the rest, and heavy response of one-to-one is extremely damaging. For inflation, it seems that responding more to output gap can also produce some superior result for the first two years, however, this benefit diminishes too soon afterwards. Also, as the response parameter ψ_y increases, the marginal benefit from stabilising inflation drops dramatically. Another potential gain from the response to output gap may come from its muting effects on price dispersion as it can be seen from the IRF. But the trade-off as in productivity shock study is still present here.

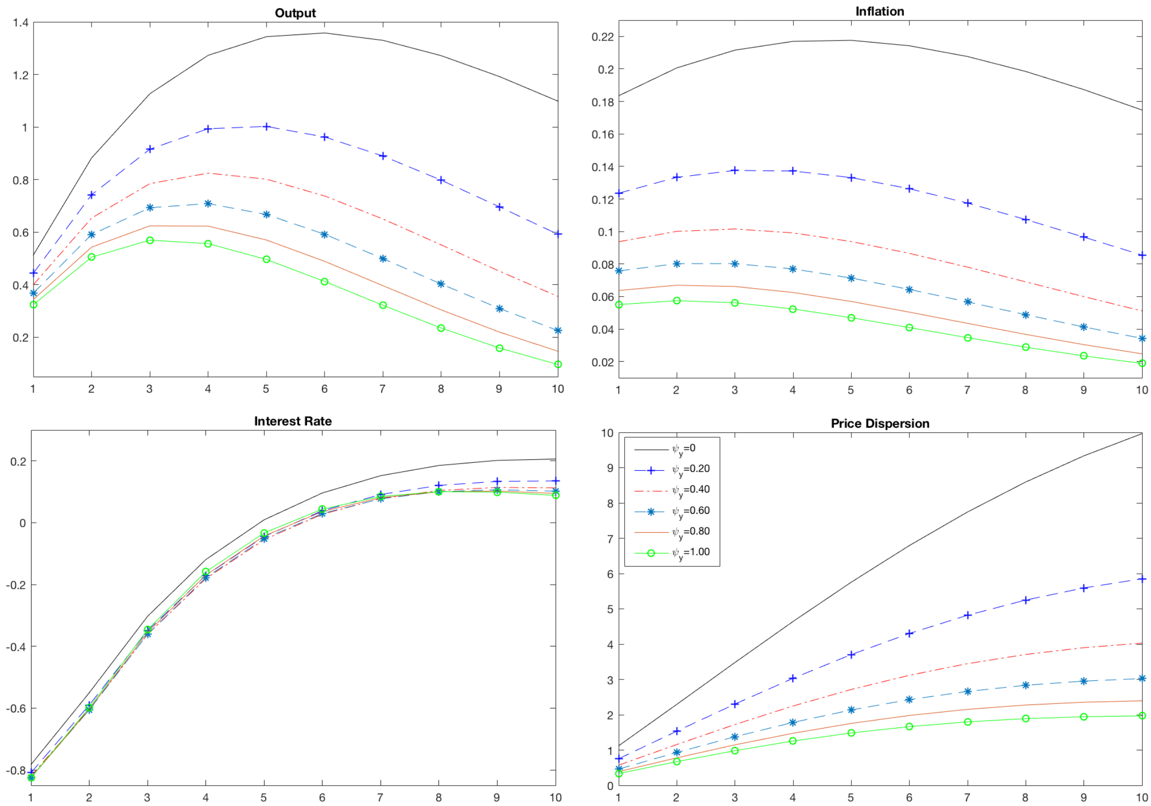

Figure 3.12: IRFs to a 1 standard deviation monetary policy shock under different response to output gap

Similar to the story of inflation response to monetary policy shock as shown in Figure 3.12; here, reacting to output gap is definitely helping the central bank to stabilise both output and inflation. More specifically, at the peak, responding one-to-one to output gap eliminates all the output and inflation fluctuations within eleven quarters, however, they would last much longer and to a much larger magnitude had the central bank chosen not to respond to output gap or respond really moderately. The price dispersion is also reduced by responding more strongly to output gap.

This mini counterfactual experiment concludes that even though it is generally agreed among researchers that a central bank should not respond heavily if at all to output gap in order to guarantee determinacy under positive trend inflation, nonetheless, it does not seem that responding to output gap generates an overwhelmingly inferior stabilisation outcome conditional on determinacy, particularly in the case of a large monetary policy shock. Certainly, here, the model assumes that the central bank can accurately estimate the natural level of output, this is hardly realistic in practice though.

3.5.3 Responding to Output Growth

In contrast to the results from the literature that focuses on determinacy under trend inflation, the previous case finds that responding to output gap can sometimes be desirable in terms of stabilisation. Hence, it is worth to check whether this contrast still exists for the response to another output targeting variable: output growth. Previous works from Walsh (2003), Orphanides and Williams (2006), Coibion and Gorodnichenko (2011) and Arias et al. (2018) all focus on the inclusion of output growth in the Taylor-type rule as it is shown that output growth targeting is extremely helpful for achieving determinacy. To determine how much a central bank should respond to this targeting variable, I conduct the following simulations.

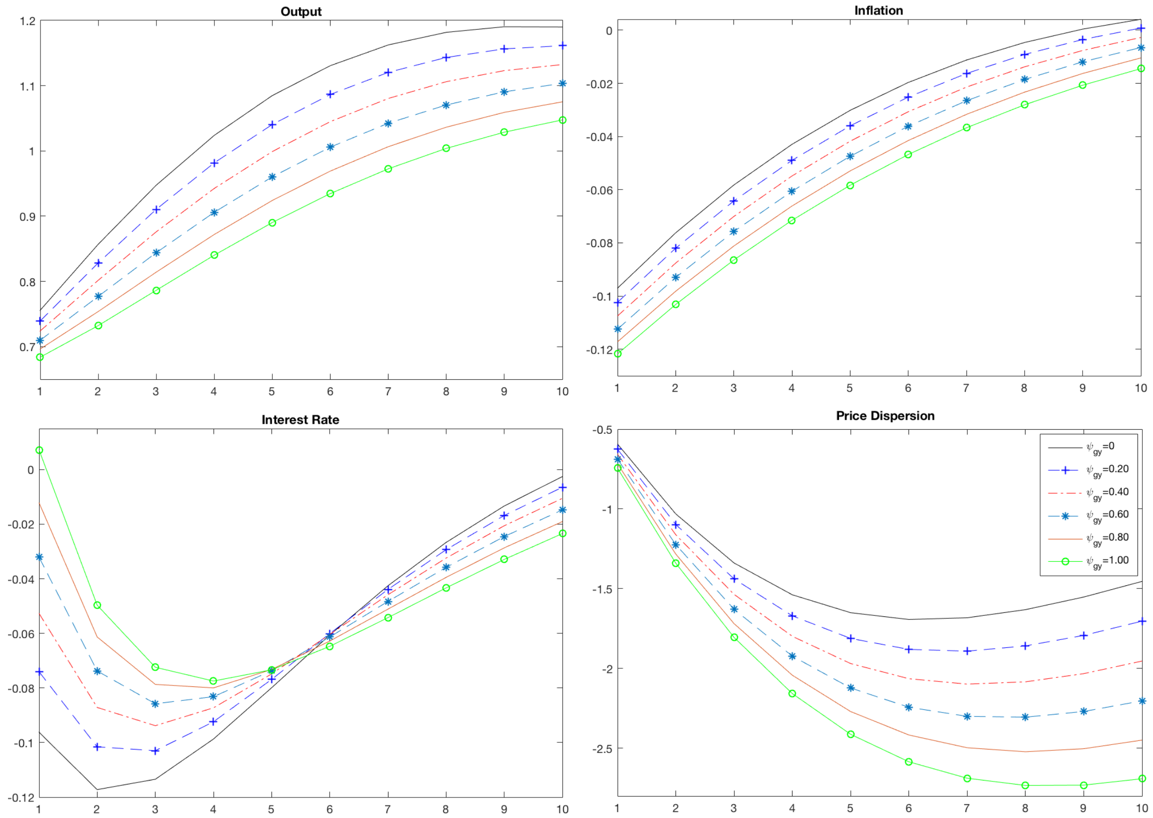

Figure 3.13: IRFs to a 1 standard deviation productivity shock under different response to output growth

Figure 3.13 provides the IRFs to a one standard deviation TFP shock with 8 percent trend inflation. Again, the IRFs for output and inflation implies an inflation-output trade-off between, however, in contrary to the previous two case where a heavy response to inflation and output gap helps the central bank to stabilise inflation but dampen the effectiveness of output stabilisation, the reverse appears in this case. On the one hand, output stabilisation is apparently enhanced by the aggressive response to output growth, one the other hand, inflation stabilisation is worsened by the heavy response to output growth under high level of trend inflation. Again, the central bank faces a trade-off here, but with a reverse dilemma.

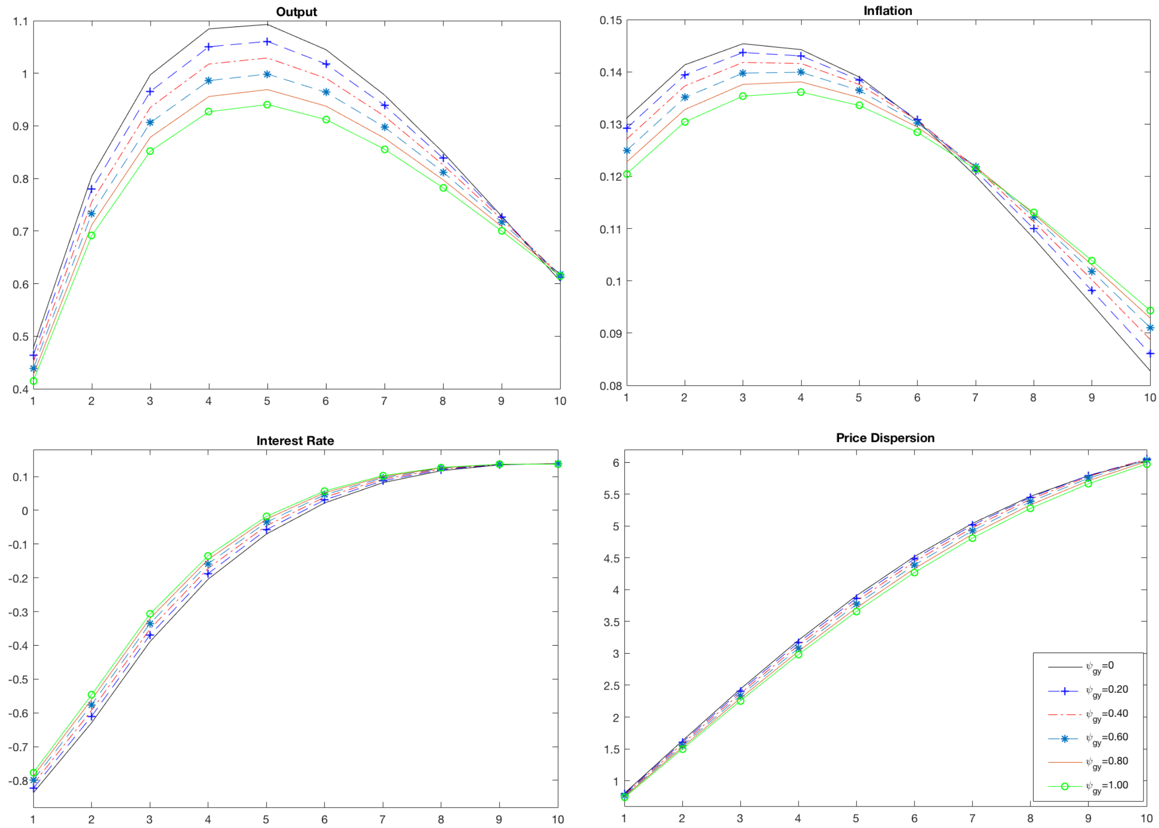

Figure 3.14: IRFs to a 1 standard deviation monetary policy shock under different response to output growth

Figure 3.14 provides the IRFs to a one standard deviation of monetary policy shock under 8 percent trend inflation. Both output and inflation stabilisations are enhanced by the strong response (represented by a large value of ψ_{gy}) to output growth, particularly in the short run. This superiority shrinks considerably after two years time for output, and the effect on inflation turns to negative after six quarters in which zero or weak response starts to dominate strong response to output growth.

The general arguments for a stronger response to output growth regarding determinacy are usually two-folded, as outlined by Coibion and Gorodnichenko (2011): first, responding to the output growth rate effectively makes the policy reaction function backward-looking since it relies on lagged variables. Second, responding to expected output growth amplifies the central bank's response to inflation. Both effects help the central bank to anchor inflation expectations. However, the results here indicate that stronger response to output growth does not always guarantee better stabilisation outcomes in output and inflation when the economy is experiencing high level of trend inflation, at least for the two shocks investigated here.

To sum up, this parsimonious counter-factual experiment provides some interesting results for monetary policy analysis under high level of trend inflation (8% in this case). The literature on trend inflation teaches us that determinacy is one of the (if not the) most crucial and fatal phenomena in trend inflation studies. As a series articles have documented, aggressive response to inflation, less or no response to output gap, and heavy response to output growth all help the central bank to improve the probability of finding unique rational expectation equilibrium (REE), the effectiveness of these targeting variables in stabilisation policy can have very different pictures.

More specifically, the heavy policy response to inflation generally provides very robust stabilisation performance for both output inflation after a monetary policy shock. However, in case of shocks to productivity, the central bank is likely to face a trade-off between stabilising output and stabilising inflation, as the change of policy response generally produces opposite effects for output and inflation. In contrast to the studies focusing on determinacy under high trend inflation, this experiment shows that variables that are believed to be overwhelmingly helpful for improving determinacy: output growth does not necessarily generate superior stabilisation results under high level of trend inflation conditional on determinacy. Variables that are treated as poor targeting variable such as the output gap can sometimes improve the central bank's stabilisation performance.

Nonetheless, this experiment is subject to many limitations: first, parameter uncertainty, the parameters are estimated in which it is subject to model misspecification; response parameters are fixed at Bayesian posterior means, there is no guarantee that once some of the other parameters in the Taylor rule are changed, how would this affect the results obtained here. Especially the value of ψ_π is fixed at 2.082, which is as high as it is always able to guarantee the local uniqueness of REE. A much lower value of ψ_π may produce significantly different results. Second, the result of the response obtained regarding output gap is based on the assumption that the central bank has perfect information about the natural output level, this is hardly realistic in practice, nevertheless. Further studies based on relaxed assumptions are highly desirable.

3.6 Future Studies

Unfortunately, this thesis did not have the chance to thoroughly investigate the issue of determinacy regarding trend inflation for this medium-scale GNK model, and this shall be the focus of my future research. Moreover, further studies of trend inflation based on more sophisticated assumptions are also in demand. For example, the role of trend inflation in models with leverage restrictions, where Christiano and Ikeda (2013) provides a useful framework. The account of unemployment in such GNK models is also highly desirable, and the recent work from Christiano, Eichenbaum and Trabandt (2016) may provide some positive indications. Alves (2018) also raises some interesting thoughts regarding the interaction between trend inflation and labour market variables.

3.7 Conclusion

This chapter modified the medium-scale Generalised New Keynesian (GNK) model constructed from the previous chapter by removing the heavily criticised assumption of backward indexation in both price and wage settings. The assumption of indexation is flawed due to its lack of evidence in micro survey data, and also the inconsistency with the microeconomic foundation of individual price and wage settings behaviour. Upon removing backward indexation, the GNKPC becomes more forward-looking, and the market wage equations become more backward looking, together they make the entire model more dynamic. A new response variable is added to the Taylor rule; the response to output growth, a variable that has been shown critical to the probability of determinacy under high level of trend inflation (Coibion and Gorodnichenko, 2011). Then a new estimation using Bayesian technique is performed on this modified *benchmark model without indexation.* This time the estimates for most of the parameters and shock processes stay fairly close to the previous *benchmark model* estimation results.

The IRFs and one period ahead forecast suggest the model can capture the macro data in the postwar United States. The performance for some variables is even closer to the SW07 result than the *benchmark model.* The only shortcoming is the IRF of inflation after a monetary policy shock lacks a hump shape response, but all other features are retained. Again, based on the Bayesian statistic, this new version of the model is slightly preferred to the same version but without positive trend inflation.

A study of the interaction between trend inflation and exogenous shocks find that change in the level of trend inflation drastically alters the dynamics of key macroeconomic variables after all five exogenous shocks. This is in contrast to some previous literature, for example, Ascari, Phaneuf and Sims (2018). They found that trend inflation does not significantly interact with productivity and monetary policy shock, but here I find it does. More specifically, both output and inflation exhibit a significantly higher degree of persistence with higher level of trend inflation than they do under zero or moderate trend inflation across all shocks. In particular, with baseline parametrisation, the largest effect of inflation with four percent trend inflation or above after a monetary policy shock does not happen on impact; there is a clear hump shape of inflation for higher trend inflation. Inflation response peaks only after 4th quarter with eight percent trend inflation after this transitory ($\rho_m = 0.10$) monetary policy shock. Interestingly, this Generalised New Keynesian (GNK) model without indexation can actually generate a hump shape response for inflation with a moderate level of trend inflation.

The monetary policy response exercise test what is the best policy response from the central bank to stabilise the economy after exogenous shocks when the economy is experiencing a very high level of trend inflation (8%). The results come with some surprises; targeting variables that are generally believed to have a great and positive influence on the determinacy of the model in high trend inflation trend to play a very different role when it comes to optimal monetary stabilisation. More specifically, in the two shocks tested: productivity and monetary shocks, responding heavily to output gap can actually help the central bank to stabilise the economy in general. This is in contrast to the conclusion regarding determinacy where academics find by responding less to output gap can help the monetary authority to achieve local uniqueness of REE. Moreover, heavy response to inflation, and output growth generally all help the central bank to stabilise the economy after a monetary shock under high level of trend inflation. However, the central bank faces a clear trade-off between stabilising output and stabilising inflation after a shock to TFP for all targeting variables.

Concluding Remarks

This thesis conducts three chapters that together focus on three major issues in monetary economics: trend inflation, inflation persistence, and optimal monetary policy. The starting point of this thesis is to correct the theoretical flawed, empirically unfounded, and practically inconsistent assumption of zero inflation steady state in modern DSGE models by showing that trend inflation makes huge impacts on the overall macroeconomic dynamics and optimal monetary response.

In *Chapter 1*, I derive a small-scale Generalised New Keynesian (GNK) model that is based on a non-zero inflation steady state. I find that due to the presence of trend inflation, the Generalised NKPC becomes flatter as trend inflation increases, while price setting firms become more forward looking. The model generates a higher level of deviations in output and inflation as trend inflation rises, this leads to a large welfare loss. The increase in price dispersion caused by trend inflation plays a significant role in the process as it mutually feeds back with inflation. An optimal monetary policy exercise shows some new findings: using a quadratic welfare loss function that is approximated based on non-zero inflation steady state, the weight assigned to output stabilisation drops as trend increases, and this makes two countering effects to monetary stabilisation policy. The overall effect very much depends on the model's parametrisation. Furthermore, the famous *divine coincidence* (Blanchard and Galí, 2007) disappears as price dispersion now appears in the GNKPC. In addition, most of the key results from the well-known paper of Clarida, Galí and Gertler (1999) on monetary policy study still hold under this GNK model, such as the inflationary bias under discretion.

Chapter 2 develops a medium-scale GNK model based on the state-of-the-art Smets and Wouters (2007) by log-linearising the model around a positive inflation state in order to study the effects of trend inflation in a more complicated environment. The model is then estimated using a Bayesian technique with four US quarterly times series, the estimated model can fit the macro data well even though with a few losses of persistence in some macro variables compared with SW07 results. This may due to the different selection of observables and different detrending methods. The estimated average trend inflation between 1970 and 2017 is around 3.2 percent annually, this is broadly consistent with the major literature. A subsequent sub-sample estimation finds this trend changes dramatically over time. It drops from 6.96 percent during the *Great Inflation* time to just 2.28 percent in the *Great Moderation* period, then further decline to just 1.6 percent in the *Great Recession* era. Further study finds that trend inflation can still alter the macroeconomic dynamics; some macro variables become more persistent after some shocks with higher trend inflation, even though the effects are not significant for moderate levels of trend inflation. This may due to the relatively high degree of estimated backward indexation ($\chi = 0.60$), which mutes the effect of trend inflation on model dynamics.

Chapter 3 further develop this medium-scale GNK model by removing the highly-criticised assumption of backward indexation for its inconsistency with micro level evidence and lack of microeconomic foundation regarding nominal rigidities. The model is re-estimated using a Bayesian technique and the estimated IRFs demonstrate that the model is capable of generating enough persistence in the output and inflation, although inflation does not show a hump-shaped response. The study of macro dynamics illustrates that once indexation is removed, trend inflation has much more significant effects on the model dynamics, as compared with the *Chapter 2* case. In particular, this model is able to produce a hump-shaped response for inflation with 4 percent trend inflation or above after a very transitory monetary policy ($\rho_m = 0.10$). Furthermore, inflation reacts less on impact but becomes much more persistent as trend inflation increases. This has both policy and theoretical implications. On the policy front, it implies that improper account of trend inflation can lead to huge miscalculation for the timing of maximum policy effect and damaging consequence for the optimal monetary response, and this finding has serious policy implications for central banks. On the theoretical front, it shows that once trend inflation is properly taken into account, one does not need backward indexation to generate a hump-shaped response of inflation with Calvo price setting. To my knowledge, this is the first paper has done so without additional assumptions.

Based on all the above findings, trend inflation makes a significant difference to modern macroeconomic DSGE models for all the theoretical, empirical and monetary policymaking reasons. Therefore, I believe economists should not ignore trend inflation in modern macroeconomic models.

Bibliography

Alves, S. A. L. (2014), 'Lack of divine coincidence in new keynesian models', *Journal of Monetary Economics* **67**, 33–46.

Alves, S. A. L. (2018), 'Monetary policy, trend inflation, and unemployment volatility', *Journal of Money, Credit and Banking* **50**(4), 637–673.

Amano, R., Ambler, S. and Rebei, N. (2007), 'The macroeconomic effects of nonzero trend inflation', *Journal of Money, Credit and Banking* **39**(7), 1821–1838.

Arias, J. E., Ascari, G., Branzoli, N. and Castelnuovo, E. (2018), 'Positive trend inflation and determinacy in a medium-sized new keynesian model'.

Ascari, G. (2004), 'Staggered prices and trend inflation: some nuisances', *Review of Economic dynamics* **7**(3), 642–667.

Ascari, G. and Branzoli, N. (2015), 'Inflation persistence, price indexation and optimal simple interest rate rules', *The Manchester School* **83**(S3), 1–30.

Ascari, G., Florio, A. and Gobbi, A. (2017), 'Transparency, expectations anchoring and inflation target', *European Economic Review* **91**, 261–273.

Ascari, G., Phaneuf, L. and Sims, E. R. (2018), 'On the welfare and cyclical implications of moderate trend inflation', *Journal of Monetary Economics* **99**, 56 – 71.

Ascari, G. and Ropele, T. (2007), 'Optimal monetary policy under low trend inflation', *Journal of Monetary Economics* **54**(8), 2568 – 2583.

Ascari, G. and Ropele, T. (2009), 'Trend inflation, taylor principle, and indeterminacy', *Journal of Money, Credit and Banking* **41**(8), 1557–1584.

Ascari, G. and Sbordone, A. M. (2014), 'The macroeconomics of trend inflation', *Journal of Economic Literature* **52**(3), 679–739.
URL: *http://www.aeaweb.org/articles?id=10.1257/jel.52.3.679*

Ball, L. M. (2013), The case for four percent inflation, Technical report, Working Papers, The Johns Hopkins University, Department of Economics.

Blanchard, O., Dell Ariccia, G. and Mauro, P. (2010), 'Rethinking macroeconomic policy', *Revista de Economía Institucional* **12**(22), 61–82.

Blanchard, O. and Galí, J. (2007), 'Real wage rigidities and the new keynesian model', *Journal of money, credit and banking* **39**, 35–65.

Calvo, G. A. (1983), 'Staggered prices in a utility-maximizing framework', *Journal of monetary Economics* **12**(3), 383–398.

Chari, V. V., Kehoe, P. J. and McGrattan, E. R. (2009), 'New keynesian models: not yet useful for policy analysis', *American Economic Journal: Macroeconomics* **1**(1), 242–66.
URL: *http://www.aeaweb.org/articles?id=10.1257/mac.1.1.242*

Christiano, L. and Ikeda, D. (2013), Leverage restrictions in a business cycle model, Technical report, National Bureau of Economic Research.

Christiano, L. J., Eichenbaum, M. and Evans, C. L. (2005), 'Nominal rigidities and the dynamic effects of a shock to monetary policy', *Journal of Political Economy* **113**(1), 1–45.

Christiano, L. J., Eichenbaum, M. S. and Trabandt, M. (2016), 'Unemployment and business cycles', *Econometrica* **84**(4), 1523–1569.

Clarida, R., Galí, J. and Gertler, M. (1999), 'The science of monetary policy: a new keynesian perspective', *Journal of Economic Literature* **37**(4), 1661–1707.
URL: *http://www.aeaweb.org/articles?id=10.1257/jel.37.4.1661*

Clarida, R., Galí, J. and Gertler, M. (2000), 'Monetary policy rules and macroeconomic stability: evidence and some theory*', *The Quarterly Journal of Economics* **115**(1), 147–180.
URL: *http://dx.doi.org/10.1162/003355300554692*

Cogley, T., Primiceri, G. E. and Sargent, T. J. (2010), 'Inflation-gap persistence in the us', *American Economic Journal: Macroeconomics* **2**(1), 43–69.

Cogley, T. and Sargent, T. J. (2001), 'Evolving post-world war ii us inflation dynamics', *NBER macroeconomics annual* **16**, 331–373.

Cogley, T. and Sargent, T. J. (2005), 'Drifts and volatilities: monetary policies and outcomes in the post wwii us', *Review of Economic dynamics* **8**(2), 262–302.

Cogley, T. and Sbordone, A. M. (2008), 'Trend inflation, indexation, and inflation persistence in the new keynesian phillips curve', *American Economic Review* **98**(5), 2101–26.
URL: *http://www.aeaweb.org/articles?id=10.1257/aer.98.5.2101*

Coibion, O. and Gorodnichenko, Y. (2011), 'Monetary policy, trend inflation, and the great moderation: an alternative interpretation', *American Economic Review* **101**(1), 341–70.
URL: *http://www.aeaweb.org/articles?id=10.1257/aer.101.1.341*

Coibion, O., Gorodnichenko, Y. and Wieland, J. (2012), 'The optimal inflation rate in new keynesian models: should central banks raise their inflation targets in light of the zero lower bound?', *Review of Economic Studies* **79**(4), 1371–1406.

Del Negro, M., Schorfheide, F., Smets, F. and Wouters, R. (2007), 'On the fit of new keynesian models', *Journal of Business & Economic Statistics* **25**(2), 123–143.

Dixit, A. K. and Stiglitz, J. E. (1977), 'Monopolistic competition and optimum product diversity', *The American Economic Review* **67**(3), 297–308.

Dixon, H. and Kara, E. (2010), 'Can we explain inflation persistence in a way that is consistent with the microevidence on nominal rigidity?', *Journal of Money, Credit and Banking* **42**(1), 151–170.

Erceg, C. J., Henderson, D. W. and Levin, A. T. (2000), 'Optimal monetary policy with staggered wage and price contracts', *Journal of monetary Economics* **46**(2), 281–313.

Fernández-Villaverde, J. (2010), 'The econometrics of dsge models', *SERIEs* **1**(1-2), 3–49.

Galí, J. and Gertler, M. (1999), 'Inflation dynamics: a structural econometric analysis', *Journal of monetary Economics* **44**(2), 195–222.

Gertler, M., Sala, L. and Trigari, A. (2008), 'An estimated monetary dsge model with unemployment and staggered nominal wage bargaining', *Journal of Money, Credit and Banking* **40**(8), 1713–1764.

Golosov, M. and Lucas Jr, R. E. (2007), 'Menu costs and phillips curves', *Journal of Political Economy* **115**(2), 171–199.

Goodfriend, M. and King, R. G. (1997), 'The new neoclassical synthesis and the role of monetary policy', *NBER Macroeconomics Annual* **12**, 231–283.

Goodfriend, M. and King, R. G. (2001), The case for price stability, Working Paper 8423, National Bureau of Economic Research.

Hodrick, R. J. and Prescott, E. C. (1997), 'Postwar us business cycles: an empirical investigation', *Journal of Money, credit, and Banking* pp. 1–16.

Justiniano, A. and Primiceri, G. E. (2008), 'The time-varying volatility of macroeconomic fluctuations', *American Economic Review* **98**(3), 604–41.

Kimball, M. S. (1995), The quantitative analytics of the basic neomonetarist model, Technical report, National Bureau of Economic Research.

Koop, G., Poirier, D. J. and Tobias, J. L. (2007), *Bayesian econometric methods*, Cambridge University Press.

Lindé, J., Smets, F. and Wouters, R. (2016), Challenges for central banks' macro models, *in* 'Handbook of macroeconomics', Vol. 2, Elsevier, pp. 2185–2262.

Lubik, T. A. and Schorfheide, F. (2004), 'Testing for indeterminacy: an application to us monetary policy', *American Economic Review* **94**(1), 190–217.

Mankiw, N. G. and Reis, R. (2002), 'Sticky information versus sticky prices: a proposal to replace the new keynesian phillips curve', *The Quarterly Journal of Economics* **117**(4), 1295–1328.

Midrigan, V. (2011), 'Menu costs, multiproduct firms, and aggregate fluctuations', *Econometrica* **79**(4), 1139–1180.

Orphanides, A. and Williams, J. C. (2006), 'Monetary policy with imperfect knowledge', *Journal of the European Economic Association* **4**(2-3), 366–375.

Primiceri, G. E. (2006), 'Why inflation rose and fell: policy-makers' beliefs and us postwar stabilization policy', *The Quarterly Journal of Economics* **121**(3), 867–901.

Rotemberg, J. J. and Woodford, M. (1997), 'An optimization-based econometric framework for the evaluation of monetary policy', *NBER Macroeconomics Annual* **12**, 297–346.

Sbordone, A. M. (2007), 'Inflation persistence: Alternative interpretations and policy implications', *Journal of Monetary Economics* **54**(5), 1311–1339.

Schmitt-Grohé, S. and Uribe, M. (2007a), 'Optimal simple and implementable monetary and fiscal rules', *Journal of monetary Economics* **54**(6), 1702–1725.

Schmitt-Grohé, S. and Uribe, M. (2007b), 'Optimal inflation stabilization in a medium-scale macroeconomic model', *Monetary policy under inflation targeting* pp. 125–186.

Schmitt-Grohé, S. and Uribe, M. (2010), The optimal rate of inflation, *in* 'Handbook of Monetary Economics', Vol. 3, Elsevier, pp. 653–722.

Sims, C. A. and Zha, T. (2006), 'Were there regime switches in us monetary policy?', *American Economic Review* **96**(1), 54–81.

Smets, F. and Wouters, R. (2007), 'Shocks and frictions in us business cycles: A bayesian dsge approach', *American Economic Review* **97**(3), 586–606.

Stock, J. H. and Watson, M. W. (2007), 'Why has us inflation become harder to forecast?', *Journal of Money, Credit and banking* **39**, 3–33.

Taylor, J. B. (1999), A historical analysis of monetary policy rules, *in* 'Monetary policy rules', University of Chicago Press, pp. 319–348.

Walsh, C. (2003), 'Speed limit policies: the output gap and optimal monetary policy', *American Economic Review* **93**(1), 265–278.

Walsh, C. E. (2017), *Monetary theory and policy*, MIT Press.

Woodford, M. (2003), *Interest and prices: foundations of a theory of monetary policy*, Princeton University Press.

Woodford, M. (2007), 'Interpreting inflation persistence: comments on the conference on "quantitative evidence on price determination"', *Journal of Money, Credit and Banking* **39**, 203–210.

Yun, T. (1996), 'Nominal price rigidity, money supply endogeneity, and business cycles', *Journal of monetary Economics* **37**(2), 345–370.

Appendix A: Data Summary

All the data used for estimation in this thesis is available from the FEDERAL RESERVE BANK OF ST. LOUIS website: https://fred.stlouisfed.org/

The four observables used in the Bayesian estimation in *Chapter 2* and *Chapter 3* are output, inflation, nominal interest rate, and labour hours:

Output = ln ((GDPC1/GDPDEF)/CNP16OVIndex) * 100

Inflation = ln (GDPDEF/GDPDEF(-1)) * 100

Interest rate = FEDFUNDS/4

Labour hours = ln (PRS85006023*CE16OV/100)/CNP16OVIndex) * 100

GDPC1: Real Gross Domestic Product, Billions of Chained 2012 Dollars, Quarterly, Seasonally Adjusted Annual Rate

GDPDEF: Gross Domestic Product: Implicit Price Deflator, Index 2012=100, Quarterly, Seasonally Adjusted

FEDFUNDS: Effective Federal Funds Rate, Percent, Quarterly, Not Seasonally Adjusted

CNP16OV: Civilian Noninstitutional Population, Thousands of Persons, Quarterly, Not Seasonally Adjusted

CNP16OVIndex: CNP16OV(2012 Q2)=1

PRS85006023: Nonfarm Business Sector: Average Weekly Hours, Index 2012=100, Quarterly, Seasonally Adjusted

CE16OV: Civilian Employment Level, Thousands of Persons, Quarterly, Seasonally Adjusted

Appendix B: Bayesian Estimation

The final Bayesian estimation is performed on Dynare version 4.5.5, which is downloadable from http://www.dynare.org/download.

Here is a summary of the estimation-related graphs

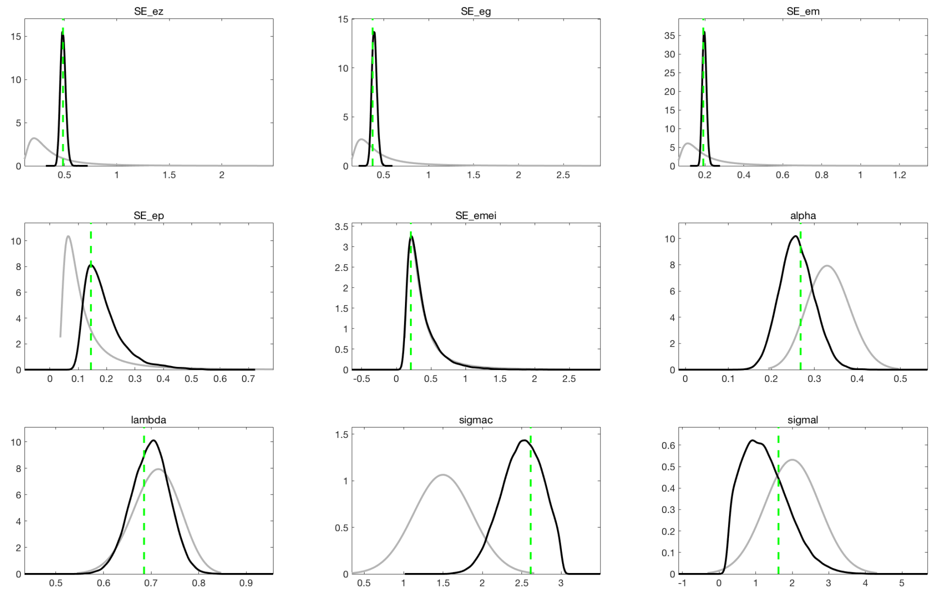

Figure 3.15: Priors and Posteriors

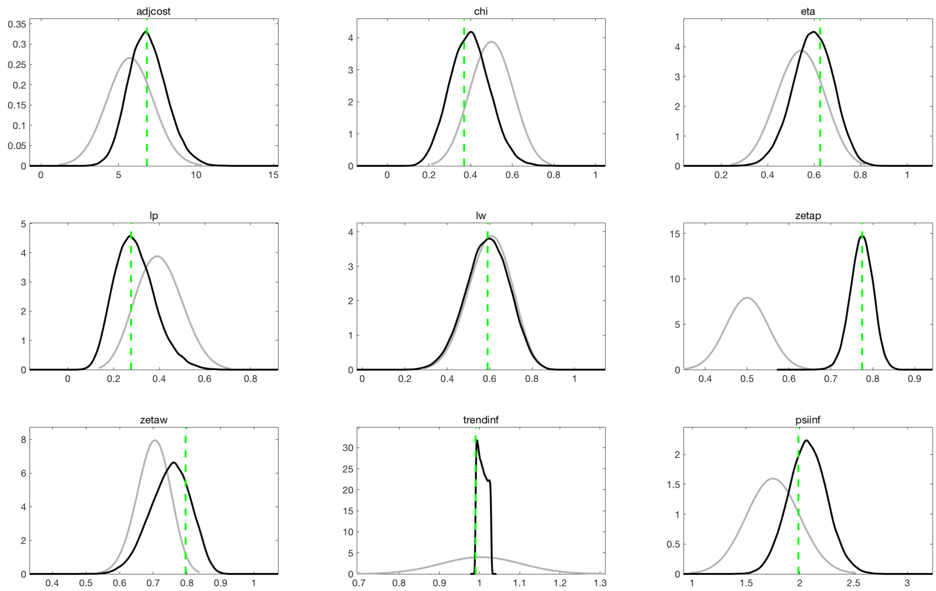

Figure 3.16: Priors and Posteriors

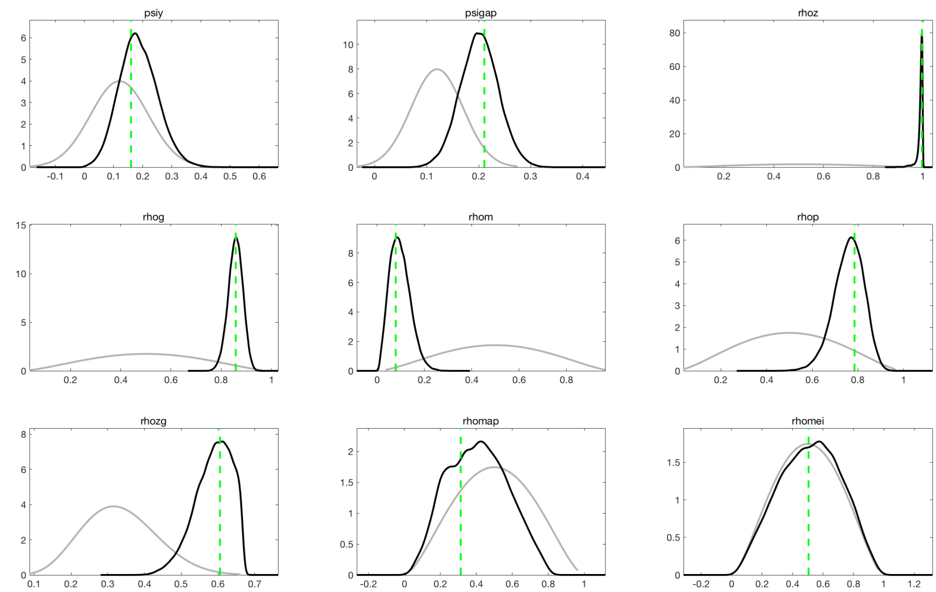

Figure 3.17: Priors and Posteriors

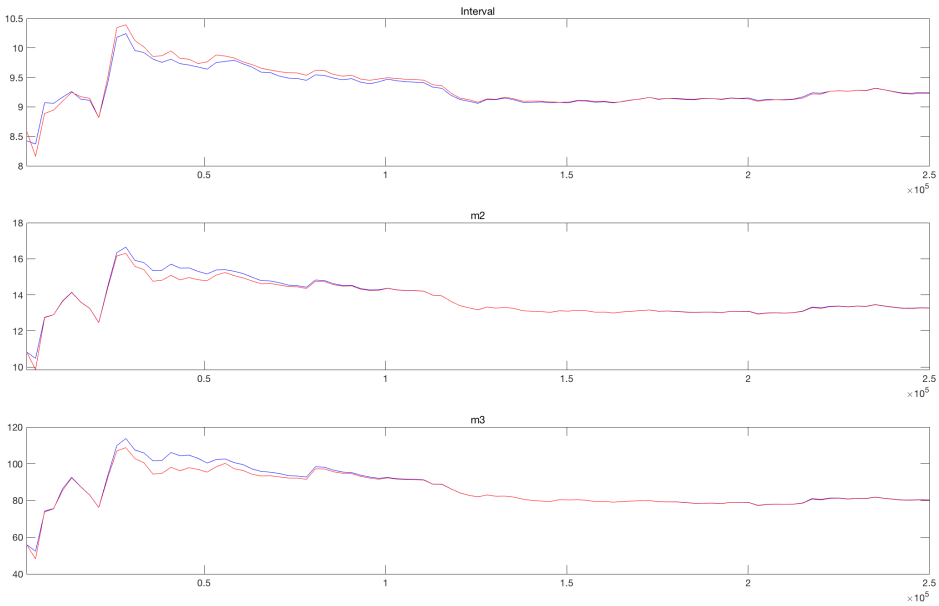

Figure 3.18: Multivariate Convergence Diagnostic

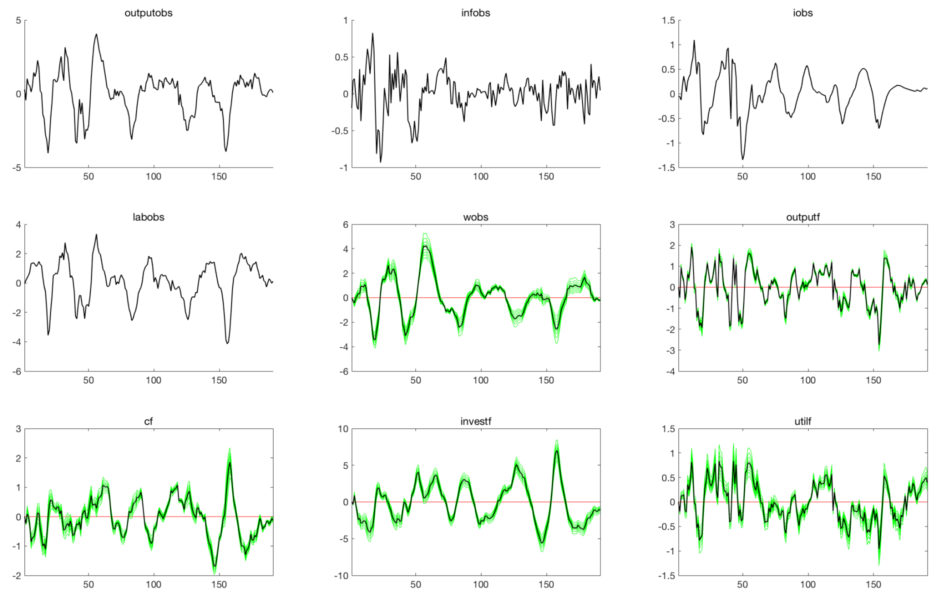

Figure 3.19: Updated Variables

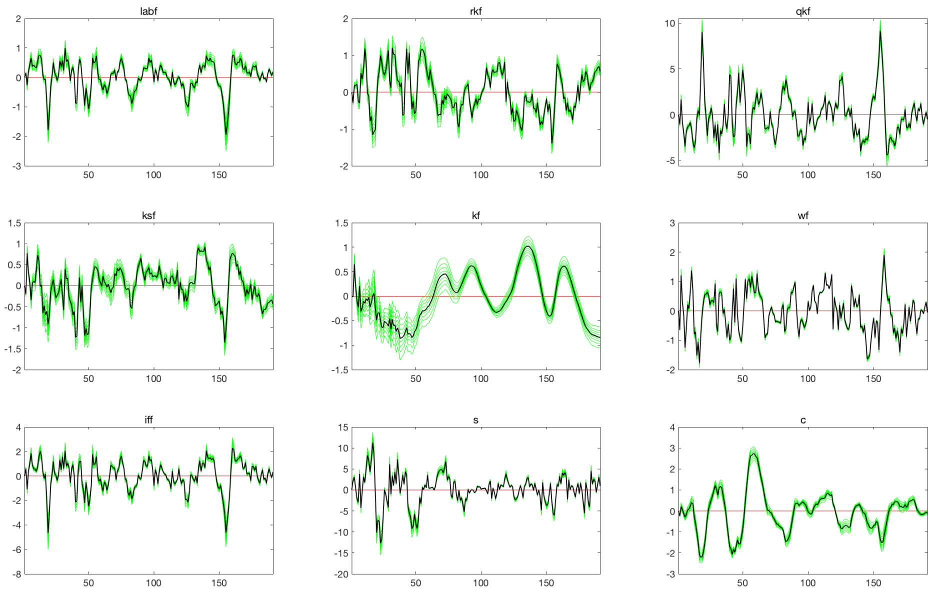

Figure 3.20: Updated Variables

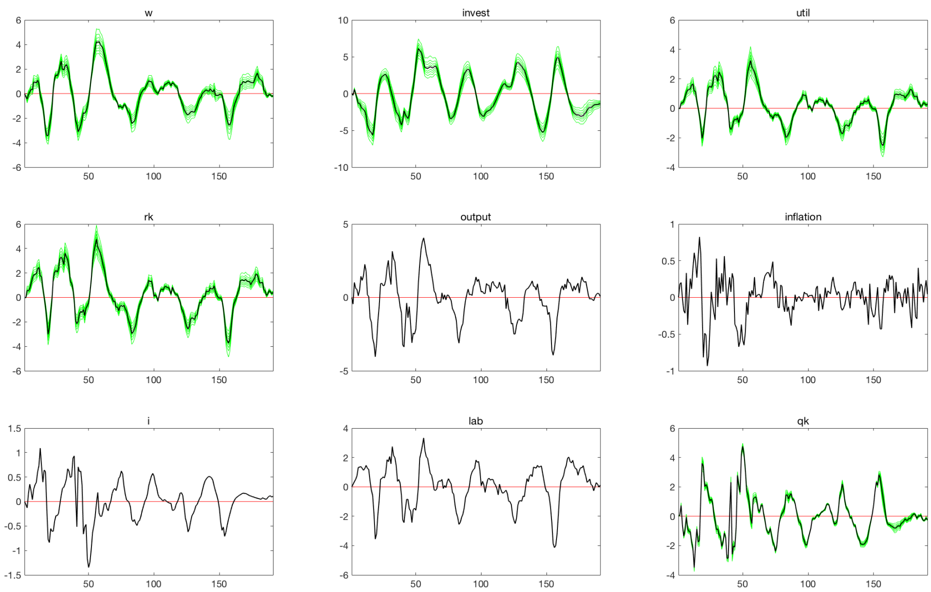

Figure 3.21: Updated Variables

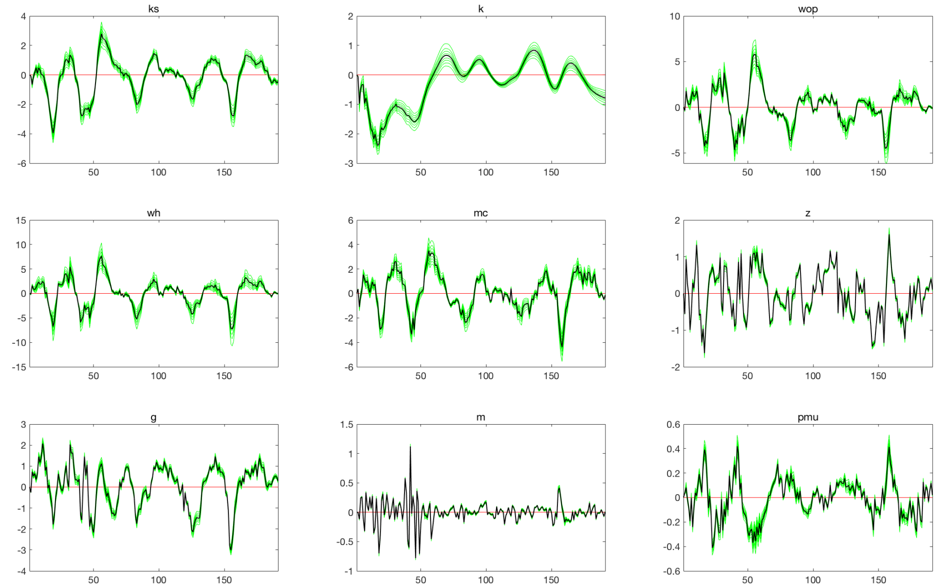

Figure 3.22: Updated Variables

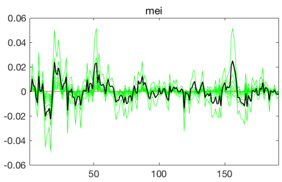

Figure 3.23: Updated Variables

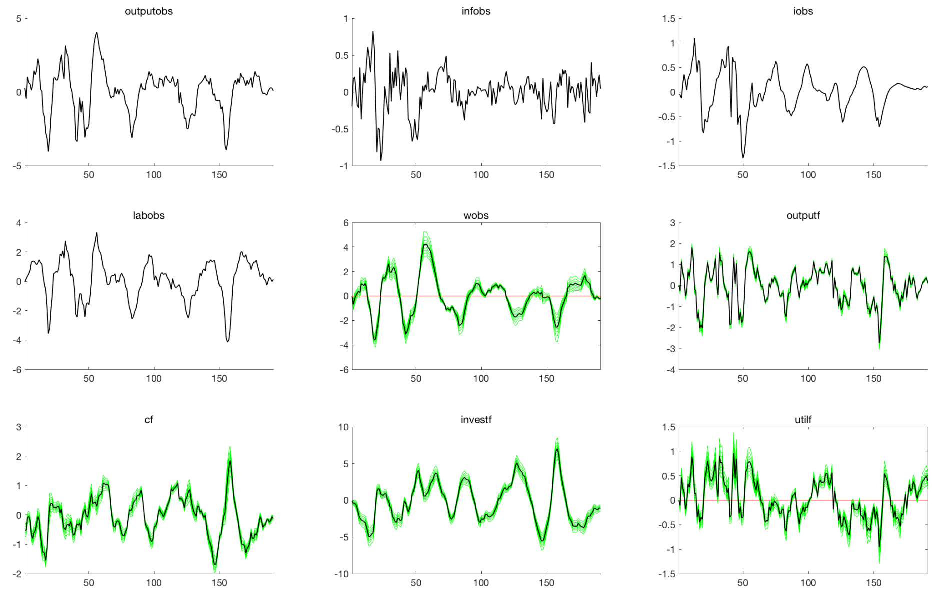

Figure 3.24: Smoothed Variables

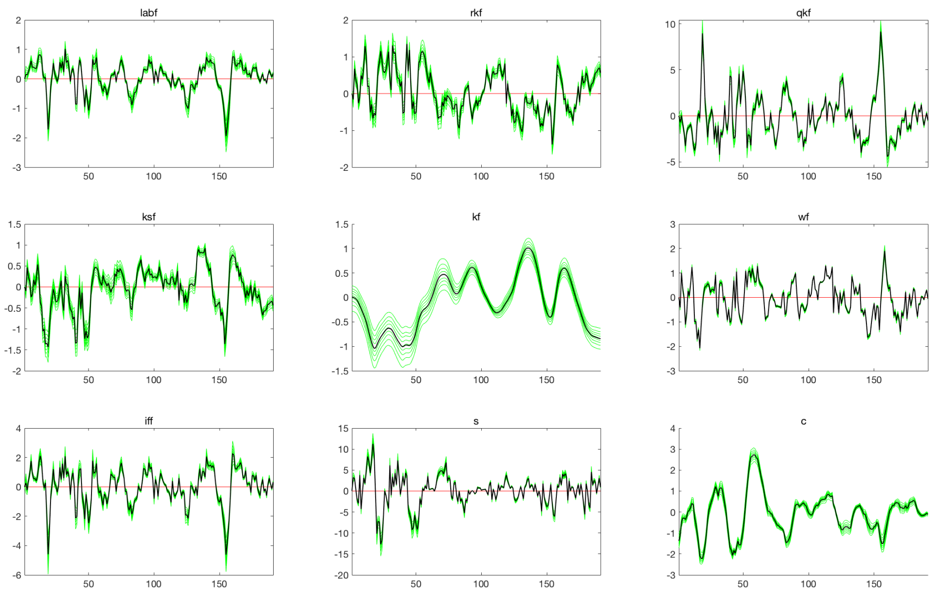

Figure 3.25: Smoothed Variables

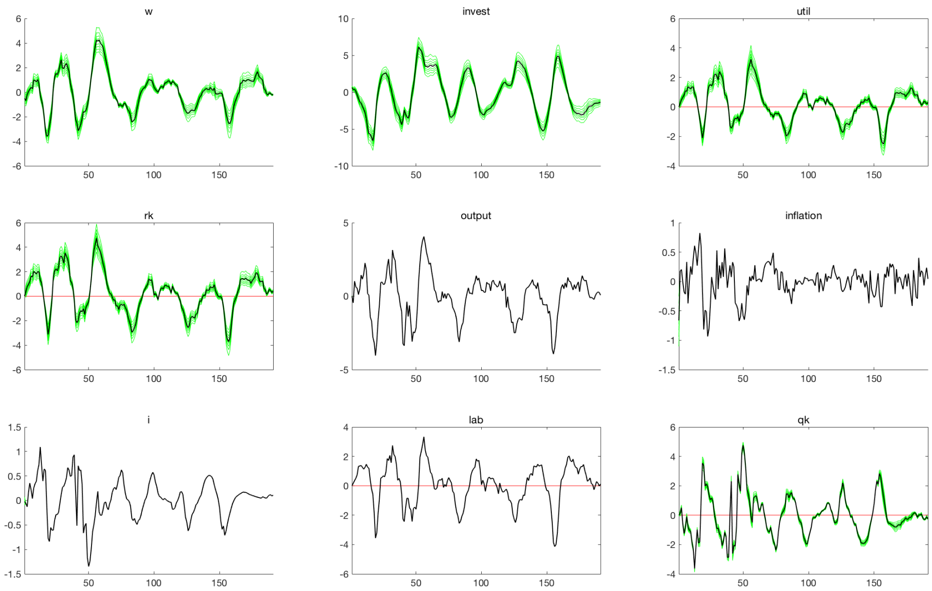

Figure 3.26: Smoothed Variables

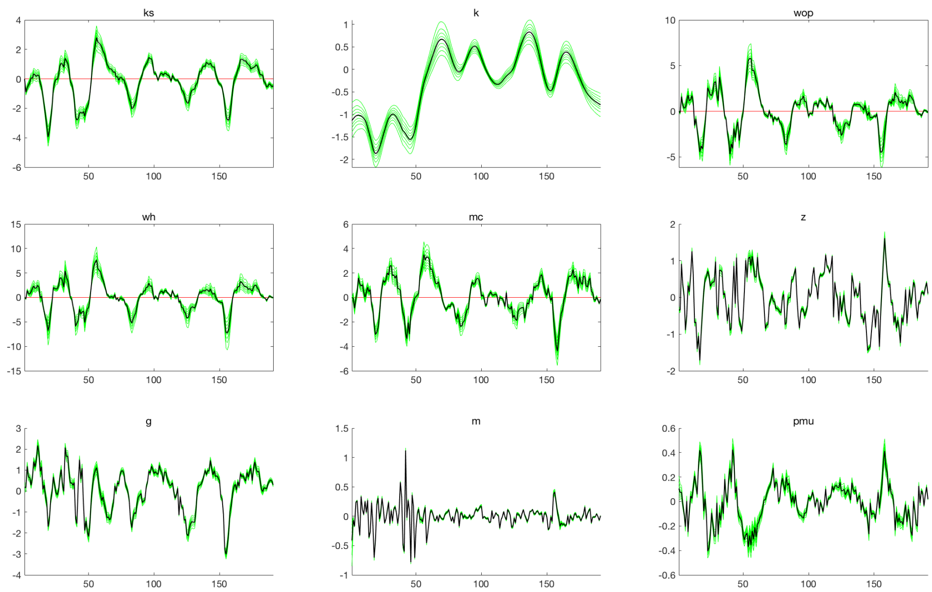

Figure 3.27: Smoothed Variables

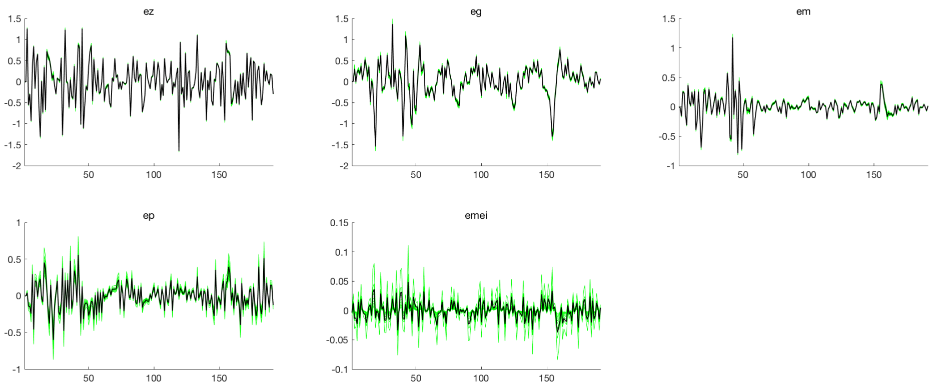

Figure 3.28: Smoothed Shocks

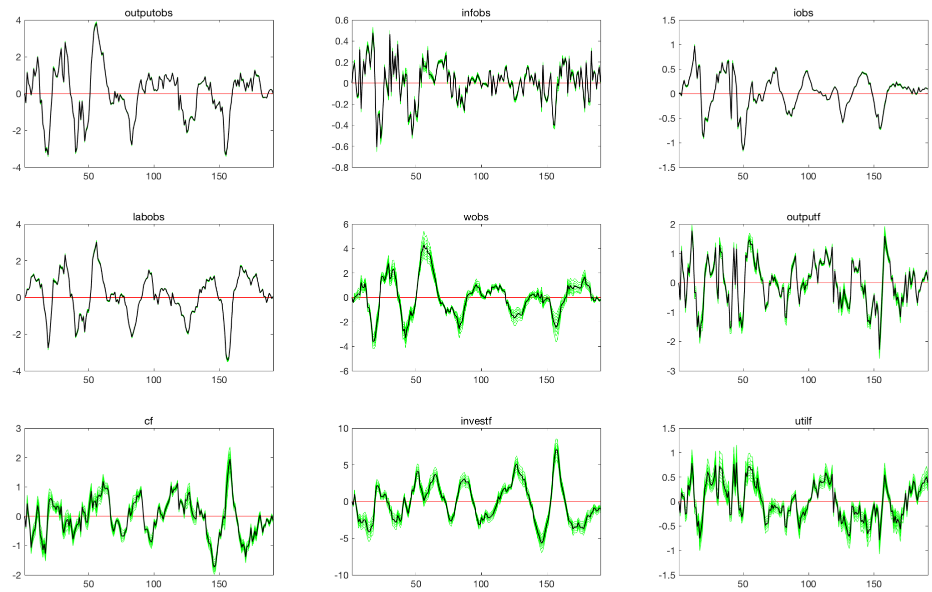

Figure 3.29: One Step Ahead Forecast

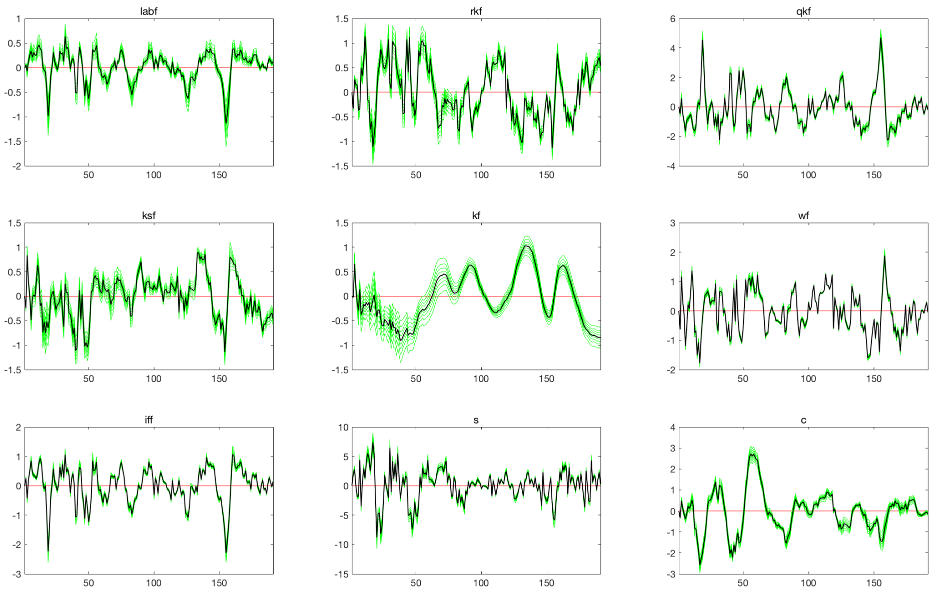

Figure 3.30: One Step Ahead Forecast

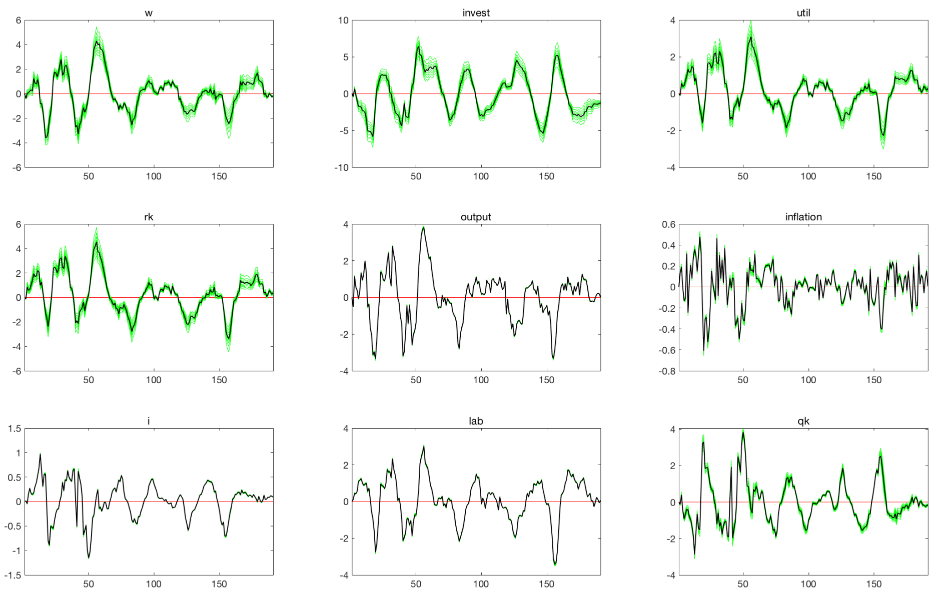

Figure 3.31: One Step Ahead Forecast

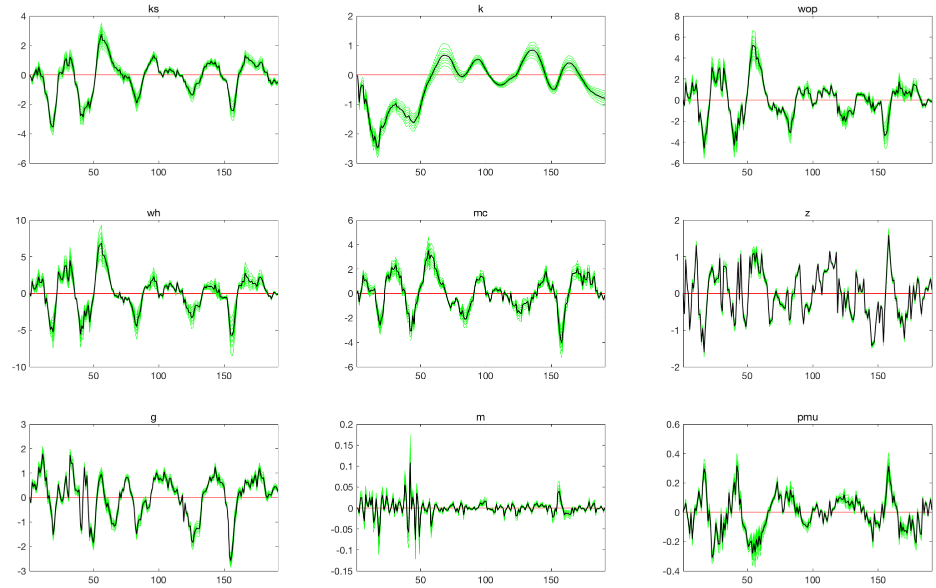

Figure 3.32: One Step Ahead Forecast

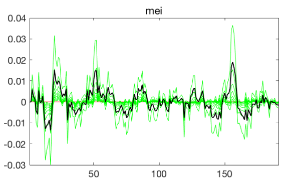

Figure 3.33: One Step Ahead Forecast

Appendix C: Inflation Target Sources

Here is the summary of sources used in *Introduction* for targeting level of central banks

United State:

https://www.federalreserve.gov/faqs/economy_14400.html

United Kingdom

https://www.bankofengland.co.uk/monetary-policy/inflation

Eurozone

https://www.ecb.europa.eu/mopo/strategy/pricestab/html/index.en.html

Japan

https://www.boj.or.jp/en/mopo/outline/qqe.htm/

New Zealand

https://www.rbnz.govt.nz/monetary-policy/policy-targets-agreements

Norway

https://www.norges-bank.no/en/about/Mandate-and-core-responsibilities/Monetary-policy-in-Norway/

Sweden

https://www.riksbank.se/en-gb/monetary-policy/the-inflation-target/

Switzerland

https://www.snb.ch/en/iabout/monpol/id/monpol_strat#t3

Australia

https://www.rba.gov.au/inflation/inflation-target.html

Canada

https://www.bankofcanada.ca/core-functions/monetary-policy/inflation/

Brazil

https://www.bcb.gov.br/en/#!/n/INFLATION

Chile

https://www.bcentral.cl/web/central-bank-of-chile/-/central-bank-of-chile-m

South Africa

https://www.resbank.co.za/MonetaryPolicy/DecisionMaking/Pages/InflationMeasures.aspx

South Korea

https://www.bok.or.kr/eng/main/contents.do?menuNo=400015

Colombia

http://www.banrep.gov.co/en/monetary-policy

Czech Republic

https://www.cnb.cz/en/monetary_policy/inflation_targeting.html