Modelling (and learning from) inventory inaccuracies in e-retailing/B2B contexts

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Abstract

Practical experience and scientific research show that there is scope for improving the performance of inventory control systems by taking into account the discrepancies between the actual physical inventory levels and those recorded in the information system (IS). Such discrepancies, which are often referred to as inventory (record) inaccuracies, are a major concern in contemporary supply chains where commitments to orders are usually made based on IS records only. Empirical data obtained in two case studies motivate the development of a multi-period inventory control model that explicitly accounts for the differences between physical inventory levels and IS stock records. Numerical experiments help derive some key managerial insights. We find that previous important results on the behavior of the optimal order quantity in the retailing environment do not necessarily apply in an e-retailing/Business-To-Business (B2B) context. We adjust and apply the zero balance walk technique to the e-retailing/B2B case and deduce a simple and efficient learning mechanism about the errors' distributions. We close with an agenda for further research in this area.

Keywords: Inventory inaccuracies; Periodic inventory control; Learning; E-retailing; B2B.

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1. Introduction

One implicit assumption frequently made in the inventory control literature is that the physical flow of products in the system is free from uncertain defects. The same is commonly assumed for the associated (inventory) information flow resulting in a hypothesized match between physical and information system (IS) inventories. In practice, many factors may generate discrepancies between inventory records shown in the IS and the physical inventory that is actually available in the store or warehouse (DeHoratius and Raman, 2008). Such discrepancies are referred to as inventory record inaccuracies (IRI).

IRI may affect the operational and financial performance of a company in various ways (e.g., Rinehart, 1960, Hardgrave et al., 2009, Chuang et al., 2016). If the IS overcounts the available inventory, the system may fail to order at the right point in time, causing stockouts and a reduction in service levels. A lower IS inventory record than what is actually available, in turn, leads to hidden inventory in the system that cannot be used for its intended purpose, and consequently unnecessary orders. The number of Stock Keeping Units (SKUs) affected by inaccurate stock records may be substantial. Prior empirical research reported inaccuracy rates ranging between 20% and 67% of the SKUs kept in stock (Rinehart, 1960, Kang and Gershwin, 2005, DeHoratius and Raman, 2008). Kull et al. (2013), who analyzed IRI at a multi-channel retailer, also showed that discrepancies may, on average, be larger in direct sales channels than in traditional brick-and-mortar channels. Cannella et al. (2015) demonstrated in a simulation experiment how in collaborative supply chains even very small inventory inaccuracies may lead to significant unnecessary costs. Empirical evidence finally also suggests that IRI may cause profit losses that range between 3% and 10% (Raman et al., 2001, DeHoratius and Raman, 2008).

The source of IRI may be errors affecting the physical inventory and/or errors that distort the IS records, with examples including replenishment errors, database errors, poor or incomplete data synchronization, counting errors, as well as customer and employee theft (DeHoratius and Raman, 2008). IRI can lead to significant problems in traditional/in-store retail supply chains by triggering inventory stock-outs or unnecessary inventory costs, but their effect may become even more dramatic in electronic/internet retail supply chains. In the latter case, not only ordering but also sales decisions are made solely based on the inventory level displayed in the IS. As a result, the company may make a sales commitment assuming that the inventory level is sufficient to fulfil the demand, then to find out that it is unable to deliver when the shipment is due. Problems resulting from IRI do not only affect retailers, but usually also upstream stages of the supply chain that may also face discrepancies between physical inventory levels and stock records themselves, or that may be affected by IRI of downstream stages.

Conditioning inventory replenishment decisions to IRI is not a straightforward exercise. Companies may of course just ignore such errors and continue their operations as if no errors have occurred. Alternatively, if some information on the behavior of these errors is available, companies may attempt to include it explicitly in the calculation of order quantities and replenishment intervals to improve their performance.

The aim of our work is to study and model the implications of inventory record inaccuracies in a supply chain. Unlike previous research in this area, we do so by considering a multi-period system subject to generalized and realistic assumptions. In more detail, there are three main points of departure from the current state of knowledge:

1. *Modelling*: in contrast to the majority of investigations dealing with inventory control subject to inaccuracies in the retailing context (business-to-customer, B2C), we extend the scope of the problem to e-retailing/business-to-business (B2B) (wholesaling) environments. As discussed above, the implications of inaccuracies in B2B go beyond the on-shelf availability observed in B2C. Besides, all types of IRI are considered in order to model all sources of errors and as such realistically reflect their collective impact.

2. *Empirical motivation*: few studies have been conducted in this area involving empirical data, and all of them have been concerned with the retailing context. Our work is motivated by two case studies that are accompanied by empirical data on customer claim reports. Interestingly, these cases enable us to investigate additional error sources over and above those that have been reported in the literature, and provide a comprehensive mapping of such sources on the Supply Chain Operations Reference (SCOR) model. The empirical data also allow the derivation of a learning technique to estimate IRI distributions.

3. *Theoretical contribution*: we consider a multi-period model subject to all types of inaccuracies (including their additive and multiplicative settings). In addition to cost optimization, we also consider service level constraints. This better reflects operational practices in the presence of IRI, but also facilitates a tractable formulation of the problem. Most importantly, in contrast

to the big majority of investigations where IRI error distributions are assumed to be given/known, we build a mechanism that allows learning about them over time.

Our findings defy, collectively, various intuitively appealing expectations in inventory theory and enable the generation of some key insights. In particular, we find that some known results pertaining to the B2C context (such as the monotonic behavior of the order quantity with the errors' parameters or the monotonic link between service level and the order quantity) do not apply in an e-retailing/B2B environment. Further, under the e-retailing/B2B context, inventory managers may capitalize on customer claim reports to improve their knowledge about the distribution of the errors. We make use of such information to deduce a simple and efficient learning mechanism about IRI errors. Such contributions should be of interest to academics and practitioners alike.

The remainder of the paper is organized as follows. The next section gives an overview of prior research on inventory record inaccuracies followed by some interesting empirical evidence on this issue obtained at two case study organizations. Section 3 presents our research framework and analytical developments. In Section 4, we present our simulation experiment and derive managerial insights. The paper concludes in Section 5 with an outlook on future research opportunities.

2. Research background and empirical context

2.1 Research background

The literature on inventory record inaccuracies may be divided into two main streams of research: empirical and analytical/simulation-based research. The former, which primarily investigates correlations between operations and the presence of IRI, is perhaps best represented by DeHoratius and Raman (2008). The authors analyzed data from an inventory audit of 37 stores of a major US retailer and found that only 35% of 370,000 SKUs had inventory records that matched the physical inventory found in the store. The other 65% were subject to inaccuracies due to a number of factors, including replenishment errors, employee theft, customer shoplifting, unrecorded damaged merchandise, and imperfect audits.

Other empirical work in the area of retailing includes that of Chuang and Oliva (2015), who assessed the relationship between labor availability and IRI using longitudinal data from five stores in a global retail chain. The researchers came up with some strong evidence that full-time labor reduces IRI, whereas part-time labor fails to alleviate the problem. Using four years of data from stores of a large retailer, Ton and Raman (2010) showed that increasing product variety and inventory levels has an indirect negative effect on store sales through their impact on *phantom* products—products that are physically present at the store, but only in storage areas where customers cannot find or purchase them. Other authors, such as Hardgrave et al. (2009, 2013) or Bertolini et al. (2015), investigated how the technology used for updating stock records can contribute to reducing IRI. Field experiments reported in these studies showed that radio frequency identification (RFID) technology, as compared to barcodes, can substantially reduce both IRI and manual adjustments of inventory records.

Despite the increasing interest in omni-channel logistics, it is worthwhile noticing that the implications of IRI in e-retailing/B2B contexts have received very little attention from the academic community. Daily data collected from a multi-channel retailer were used to ground a discrete-event simulation experiment through which Kull et al. (2013) investigated the daily variation of errors and their impact on operational performance. The study revealed that brick-and-mortar and direct channels are impacted differently. Our paper not only provides additional evidence that the inaccuracy issue exists in the e-retailing/B2B context, but that it also leads to more penalizing consequences (as compared to traditional retailing) directly affecting customer demand satisfaction and, generally, resulting in customers' claims as will be shown in the next section.

The analytical research stream is more extensive, but is also mainly concerned with the retailing context, and in the great majority of cases, IRI errors' distributions are supposed to be given/known, independent of the stock policy and following a simple random-walk pattern over time (Kull et al., 2013). There are two important variations when it comes to modelling. One relates to the sources of errors (where the errors come from) and the other to the hypothesized error settings (how the errors behave).

With regards to the former, the main sources of errors discussed in the literature are the following (e.g., Kang and Gershwin, 2005; DeHoratius and Raman, 2008):

- *Shrinkage:* Inventory shrinkage refers to the loss of products (say due to theft) between the point of manufacture or purchase from the supplier and the point of sale.
- *Misplacement:* Some products may become temporarily unavailable for sales because of their wrong place within the store or the warehouse. Misplacement errors occur when a fraction of the inventory is misplaced and, as such, it is not available to meet customers' demand until it is found.
- *Transaction errors:* Transaction errors are unintentional errors occurring during inventory transactions. Such errors arise while counting the inventory, receiving an order or checking out at the cash register.
- *Other:* Errors occurring in the production system or in the presence of an unreliable supply system delivering a wrong quantity lead to IRI.

With regards to the error settings, there are two types of assumptions usually made in the literature on the behavior of errors; additive and multiplicative:

- Under the *additive setting*, errors are independent of the order levels. Transaction and human errors could generally be modeled as additive since the errors being made (and their magnitude) do not depend on the quantity recorded in the IS.
- Under the *multiplicative setting*, the error realization is a function of the order level. Theft, and more generally shrinkage errors, may reasonably be modelled as such since it makes sense to assume that the quantity subjected to internal theft or damage during transportation increases with the quantity ordered. In random yield theory (as well as in inventory modeling subject to unreliable supply systems), the multiplicative setting is known as the proportionally random yield problem (Yano and Lee, 1995).

Table 1 lists the main analytical/simulation inventory-type investigations dealing with IRI. In particular, we classify them based on the supply chain (SC) context under study (retailing, e-retailing, multi-echelon supply chains), the error source (shrinkage, misplacement, transaction errors, supply errors or any mixture), the inventory framework under investigation, and the error assumptions (deterministic, random, additive, multiplicative, given, or learned about). The table is organized around three parts: the first part is dedicated to the retailing context; the second part extends the scope to different echelons of the supply chain; and the third part is concerned with an interesting issue we deal with in this paper, i.e. learning about the errors' distributions.

	Investigations	SC context	Sources of errors	Inventory framework	Error assumptions
IRI in a retailing context	Noori and Keller (1986); Inderfurth (2004); Rekik et al. (2007)	Retailing	Supply errors	Newsvendor	Random, additive or multiplicative and given
	Iglehart and Morey (1972); Morey (1985); Morey and Dittman (1986); Sandoh and Shimamoto (2001); Nachtmann et al. (2010)	Retailing	Transaction errors	Periodic Review	Random and given
	Rekik et al. (2008); Camdereli and Swaminathan (2010); Tao et al. (2018); Zhang et al. (2018 a,b)	Retailing	Misplacement	Newsvendor	Deterministic and given
	Xu et al. (2012); Fan et al. (2015); Wang et al. (2016)	Retailing	Shrinkage, misplacement	Newsvendor	Deterministic and given
	Gaukler et al. (2007)	Retailing	Shrinkage, misplacement, transaction errors	Newsvendor	Deterministic and given
	Atali et al. (2006)	Retailing	Shrinkage, misplacement, transaction errors	Periodic review	Random, additive and given
	Heese (2007)	Retailing	Shrinkage, misplacement	Newsvendor	Random, multiplicative and given
	Kang and Gershwin (2005); Agrawal and Sharda (2012)	Retailing	Shrinkage	Continuous Review	Random, additive and given
	Kök and Shang (2007)	Retailing	Shrinkage	inspection-adjusted base-stock policy	Random, additive and given

	Rekik et al. (2009)	Retailing	Shrinkage	Newsvendor	Deterministic and given
IRI in a multi-echelon supply chain context	Fleisch and Tellkamp (2005)	Multi-echelon SC	Shrinkage, misplacement, supply errors	Periodic review	Random and given
	Dai and Tseng (2012)	Multi-echelon SC	Shrinkage	Periodic review	Random, additive and given
	Rekik et al. (2015)	E-retailing	Transaction errors, shrinkage	Newsvendor	Random, additive and given
	Cannella et al. (2015)	Multi-echelon SC	shrinkage	Periodic Review	Deterministic, multiplicative and given
Methods for learning about IRI	Mersereau (2013)	Retailing	Shrinkage, misplacement, transaction errors	Periodic review	*Learning by*: Partially observed Markov decision process
	DeHoratius et al. (2008)	Retailing	Shrinkage, misplacement, transaction errors	Periodic review	*Learning by*: Bayesian inventory record and audit-triggering
	Bensoussan et al. (2007)	Retailing	Shrinkage, misplacement, transaction errors	Periodic Review	*Learning by*: Zero balance walk. Approximate conditional distribution
	Qiao et al. (2015)	Retailing	Shrinkage	Periodic Review	*Learning by*: Partially observed Markov decision process

Table 1: Key analytical/simulation inventory record inaccuracy investigations

In the majority of the inventory models developed in this area, the proposed way to tackle the inaccuracy issue is to integrate the errors' distributions in the inventory policy resulting in the calculation of an additional buffer stock. A prerequisite for the practical implementation of probabilistic inventory models is the characterization of the errors' distributions. However, and as indicated in Table 1, the errors' distributions have generally been assumed to be accurately known in the literature. Very few investigations have considered the estimation of, and learning about, the errors and their integration in the inventory policy. These contributions that are of special relevance to the work at hand are discussed in the following.

By using the zero-balance walk model, Bensoussan et al. (2007) assumed that the inventory levels are partially observed because of IRI and that a signal is captured only if the shelf is empty. The researchers showed that the problem has an infinite-dimensional state space, and established the existence of an optimal ordering policy when single period costs are bounded or when the discount factor is sufficiently small.

By using routine periodic audits, DeHoratius et al. (2008) developed a series of models that collectively predict the magnitude and variance of an item's IRI given SKU characteristics. The authors then considered a multi-period lost sales inventory system with additive errors drawn from an arbitrary discrete distribution. They proposed maintaining an explicit inventory belief referred to as the Bayesian inventory record, which is updated according to the Bayes rule using sales observations as signals of the underlying inventory levels.

Mersereau (2013) modelled the replenishment process under IRI by a partially observed Markov decision process (POMDP) and proposed an approximate POMDP replenishment algorithm. Similarly, Qiao et al. (2015) proposed two robust replenishment control policies: a POMDP-based policy and a Dynamic Programming (DP)-based policy for a production/inventory system with invisible stock loss and IRI. The POMDP-policy is based on the computation of the probability distribution of the physical inventory level, which cannot be observed accurately. The DP-based policy is developed by replacing the random stock loss rate and demand rate by their mean values. Based on a numerical analysis, the authors showed that the POMDP-based policy performs better than the DP-based one.

For an excellent review of the analytical literature in the area of inventory record inaccuracies, we refer interested readers to Chen and Mersereau (2015). This paper contains a comprehensive discussion of how analytical models can make best use of available information in the absence of inventory visibility afforded by tracking technologies or process improvement initiatives.

In this paper, we consider the implications of IRI in the e-retailing/B2B context where demand satisfaction is mainly performed based on inventory levels shown in the computer system. The sequence of events is different from the brick-and-mortar context where the demand is mainly satisfied from the inventory levels available on the selling floor. Consequently, and as will be illustrated in the next sub-section, additional signals of information are available for the inventory manager to estimate the error distributions over time. By extending the zero-balance walk principle to the computer system, we propose a simple learning mechanism based on some events occurring in the inventory system which are easily observable by the manager.

Further, we move beyond additive or multiplicative settings that facilitate mathematical tractability, but not necessarily reflect reality (DeHoratius and Raman, 2008). In the next section, we show that inventory systems may be subject to combinations of error types. Modelling all three types of errors is consistent with retail settings (Chen and Mersereau, 2015); we introduce some generic modeling of errors whereby both physical and computer record levels are subject to both additive and/or multiplicative settings of errors.

All the modeling assumptions considered in this paper contribute collectively towards closing the gap in the IRI literature both from an empirical and analytical perspective. They are in agreement with the open research area pointed out by the literature review conducted by Chen and Mersereau (2015): the need for more models enabling the detailed estimation of model parameters as well as the need for new developments coordinating better model realism and analytical tractability.

2.2 Empirical context

Serving customer demand in an e-retailing/B2B context based on inaccurate information leads to unsatisfied commitments which, in turn, result in customers' claims. Our following analysis based on such customers' claims shows the magnitude of the inaccuracy issue in the e-retailing/B2B context and motivates our subsequent analytical work.

Our research reflects work with two collaborating companies, both of which are facing inventory record inaccuracy problems in their everyday business. The first case organization (**Company A**) is part of a Japanese electronics manufacturer. The company represents the European head office for the manufacturer under concern. The European Parts Distribution Centre (EPDC) is the control tower for the central stock holding of spare parts and is situated in Germany. Whereas physical flows initiate in Germany, information is controlled in Manchester, UK, in a center that previously served also as the stock coordinating unit within Europe. The company serves the demand received through a typical after-sales service support network (subsidiaries, wholesalers, repair centers of various sizes) across ten European countries. The stock base consists of approximately 70,000 SKUs. Inventories are controlled through various forms of periodic review systems depending on the nature of the SKUs as defined based on an ABC-type classification scheme. This is a typical B2B operation with all orders received solely electronically and decisions being made based on the IS inventory levels. Commitments are all too often not satisfied resulting in the generation of customer claims which subsequently trigger an alignment between IS and Physical inventories (both being set to zero).

The empirical dataset provided by the company relates to claim reporting, including the sources of errors and the country where the claims originated, over a 25-month period. We have also been provided with the detailed claims reported for one big client (including the frequency/no. of claims per day, the source of the errors, the actions taken to rectify the problem, and the time needed to answer each claim).

The second case organization (**Company B**) is a wholesaler of engineering supplies stocking a wide range of valves and ancillary products and selling primarily to the construction sector. The company was founded in 1985 and is situated just outside the Manchester city center boundaries. Its customers throughout the UK range from small to medium-sized companies to government-funded bodies, universities and major engineering contractors, whereas they also sell products directly to final consumers. That is, the company is operating both in a B2B and B2C fashion. Orders are received electronically in both cases, resulting in the latter case in an e-retailing scenario essentially. The company sells an extensive range of stock items which are primarily stored in the warehouse ready for dispatch; it also accepts ‘special’ orders which often involve highly complex equipment sourced from various suppliers. The company’s supply base is quite vast, given the wide range of items available in its catalogue. At the time of the research, the stock base comprised 3,011 codes. Inventories are controlled based on a periodic inventory system with reviews taking place weekly.

A very comprehensive database with the weekly-detected errors and consequent adjustments to stock has been put together by this company for the purposes of our research. This database covers also a period of 25 months. Stock adjustments are performed if operational problems are detected and are complementary to the yearly adjustments done during a stock take. A big majority of these adjustments are supported by comments helping understanding their roots.

The sources of errors (IRI) are analyzed in a descriptive fashion to enable a greater understanding of pertinent issues in the case organizations. We start by clustering the causes of IRI in higher-level families / categories and mapping those to the Supply Chain Operations Reference (SCOR) delivery process / model (APICS, 2017). Process reference models integrate the well-known concepts of business process reengineering, benchmarking and process measurement into a cross-functional framework. Once a complex management process is captured in a standard process reference model form, it can be (amongst other purposes) measured, managed and controlled. Estampe et al. (2013) analyzed various models used to assess supply chains, and they developed a qualitative grid to help managers decide on the model that is most suited to their needs. The SCOR model has been used to map e-business applications, for example in Wall et al. (2007) and Asmuni (2014); to the best of the authors’ knowledge though, it has not yet been utilized to structure errors contributing to the IRI.

SCOR models five processes: planning, sourcing, manufacturing, delivery (of stocked products) and returns. The delivery process in particular allows us to: i) highlight the phase where commitments based on the IS records are being made on the part of the supplying company; and ii) map causes of IRI into the various stages of the process. The former attribute permits the distinction between e-retailing/B2B and classical retailing. The latter facilitates an extension of the SCOR delivery process to capture the issues of IRI.

In more detail, the commitment phase corresponds to the “SCOR Delivery” sub-processes D1.2-D1.5 inclusive (see Figure 1). The total number of inventory adjustment records was 1,083 and 1,532 for case companies A and B, respectively, over a period of 25 months. In Company A, adjustments were most often performed when a claim was made; according to our analysis, a claim is a consequence of the company not being able to respect its commitment. As will be discussed in the next section, this is a cost factor that has not been adequately discussed in the literature. Since the stock adjustments are performed upon the generation of a claim, a full explanation is provided with regards to the very cause of the inaccuracy.

On the other hand, the management of IRI in Company B was based on frequent inspections (i.e., the company employs a strategy that is more proactive in nature). This results in many adjustments being reported with no additional comments to enable an understanding of the causes of the inaccuracies. In addition, many inaccuracies can be attributed to design- and quality-related problems resulting in the SKUs being withdrawn. Quality-related issues are not being considered as part of this research.

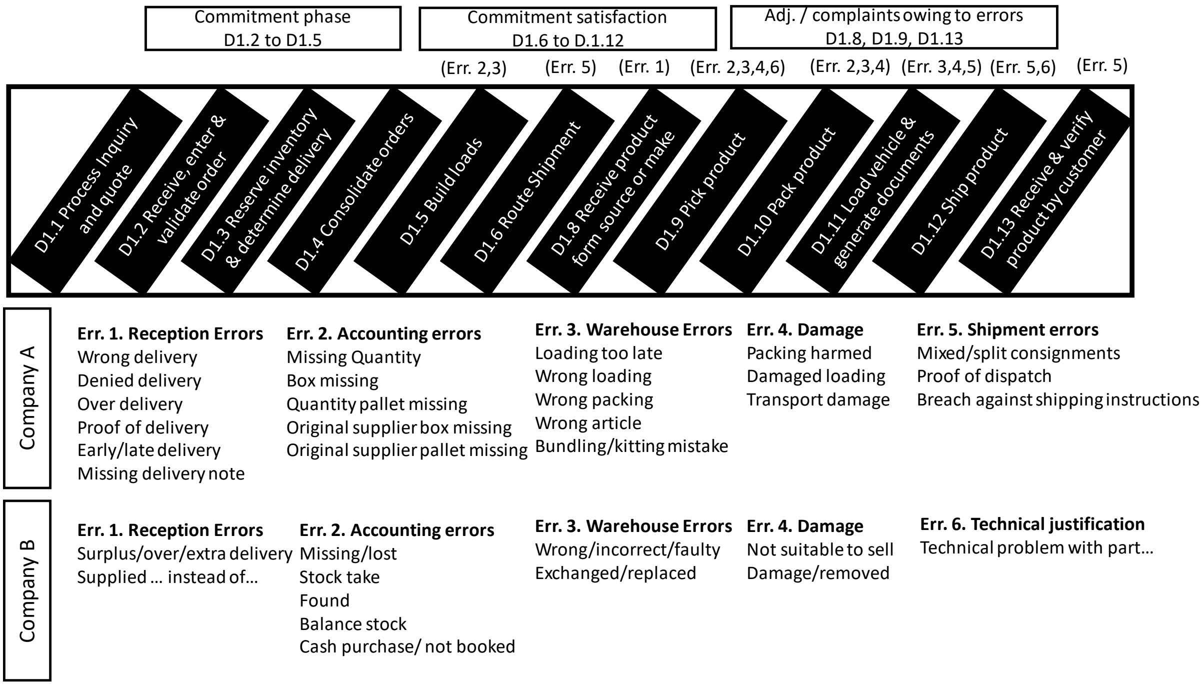


Figure 1: Mapping causes of inventory record inaccuracies to the SCOR delivery processes

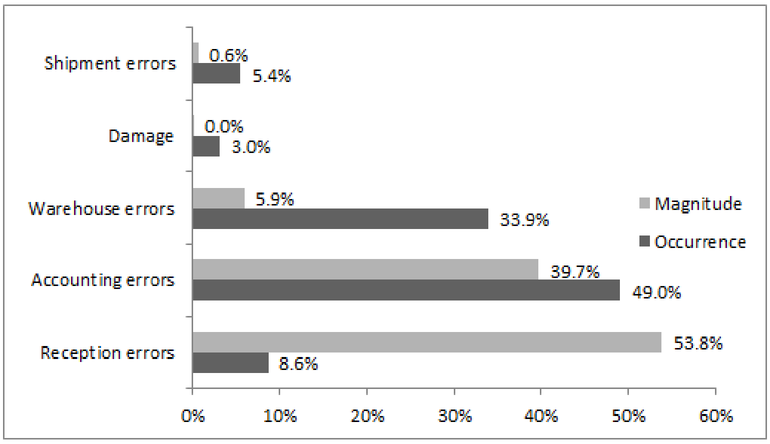
Upon representing the process description given by SCOR, and considering the comments found in the claim reports, all inaccuracy errors are summarized under six main categories. The objective here is not to be prescriptive, but rather to allow for a qualitative summarization of the information in hand. Further, the categories have been confirmed as accurate high-level summary descriptors by the companies. The categories are the following: i) reception errors; ii) accounting errors; iii) warehouse errors; iv) damage-related errors; v) shipment errors; vi) design- and quality-related errors – provided through some technical commentaries. Figure 1 provides the output of this exercise. For each category of errors, we provide examples of comments found in the claim reports. And for each category, a linkage is provided to the SCOR sub-processes by means of utilizing some appropriate numbering. This figure provides, we believe, some very much needed (additional) empirical evidence on factors generating IRI.

We note that the errors presented in Figure 1 are not specific to the e-retailing context. Instead, these are general error types that could occur both in an e-retailing context as well as in brick-and-mortar stores, even though we expect that the relative importance of the error types may differ between these environments. One major difference between these environments is the physical presence of the customer in traditional retail stores that may lead to sources of IRI that are not the same with an e-retailing warehouse (customer theft, the customer misplacing items, etc.). In the latter type of warehouse, these error types would only be triggered by the employees working in the warehouse (employee theft, employees misplacing items, etc.). The important point though is that the same error types would still be present. Quantifying the relative contribution of the error types to IRI and the performance of the warehouse (say in terms of service level), both in a regular retailing and in an e-retailing context, would be an interesting and relevant topic for future research.

The frequency distributions of the various causes of IRI for the case companies A and B are presented in Figures 2 and 3, respectively. The causes are grouped under the categories discussed above. The percentage occurrence out of the total number of inaccuracies is indicated for each category along with the percentage contribution to the total absolute discrepancy between physical and IS stock. All percentages are rounded to the first decimal place.

Discrepancies are recorded as: IS stock (IS) and physical stock (PH). When a customer claim or a stock take is performed, an adjustment of the stock is recorded where both physical and IS stocks are tracked. Figures 2 and 3 also show the average and the standard deviation of the recorded discrepancies. Using the comment accompanying each stock adjustment, we assign them to an error source.

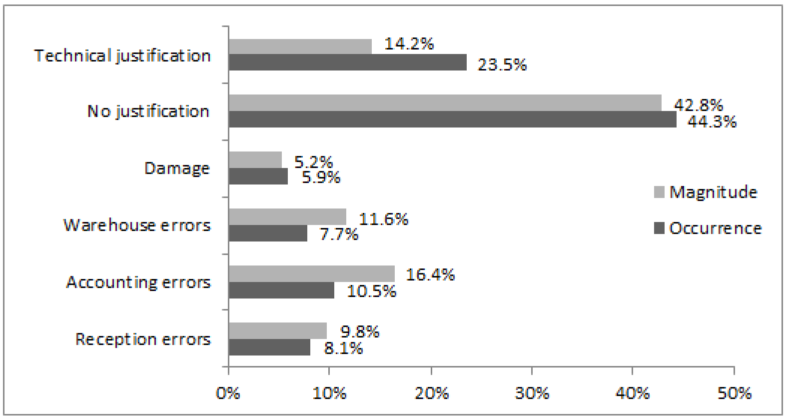
A total of 1,087 recorded inventory record inaccuracies resulting in a total absolute discrepancy of 5,318 units.



Company A	Average	Standard Deviation
Shipment errors	-2.13	3.89
Damage	-0.08	0.28
Warehouse errors	-1.69	3.08
Accounting errors	-5.09	19.07
Reception errors	5.50	9.23

Figure 2: Frequency distribution of IRI causes – case company A

A total of 1,613 recorded inventory record inaccuracies resulting in a total absolute discrepancy of 13,590 units.



Company B	Average	Standard Deviation
Technical justification	-6.09	23.95
No justification	1.42	22.71
Damage	4.26	16,28
Warehouse errors	1.69	32.32
Accounting errors	5.94	27.16
Reception errors	2.85	18.98

Figure 3: Frequency distribution of IRI causes – case company B

The summary results regarding the magnitude of the errors across the various categories for the two case organizations are presented in Figure 4. Please note that, for presentation purposes, the y-axes (scales) are not the same for both figures.

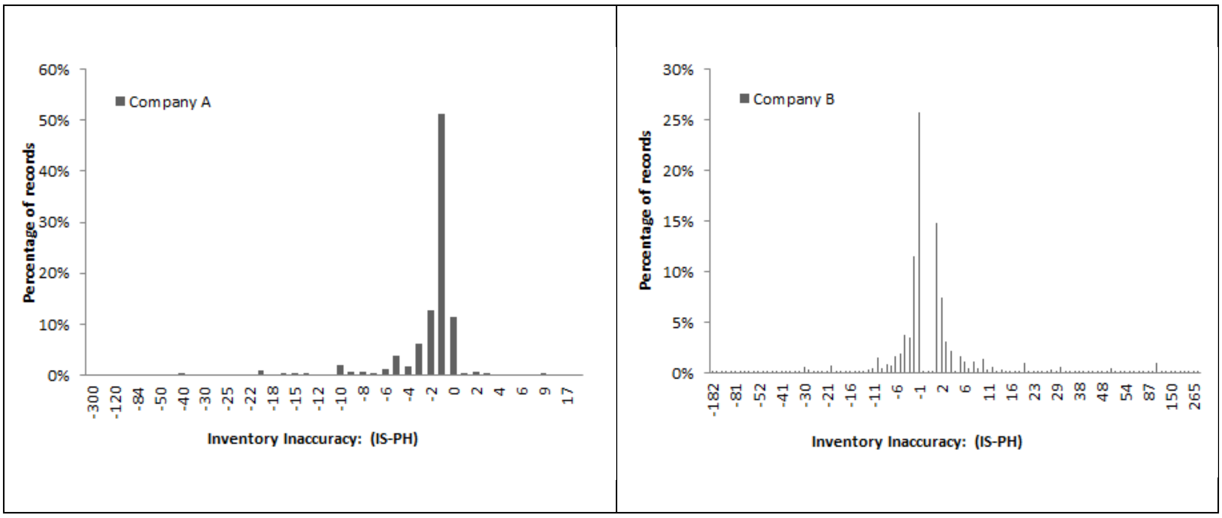


Figure 4: Histogram of the signed inventory record inaccuracies – case company A/B

As previously discussed, Company A reconciles the stock only upon the receipt of claims. This results exclusively in negative discrepancies, meaning that the physical stock is less than the IS stock. The situation is different in Company B, associated with both positive and negative discrepancies, since the reconciliation procedure here is based on frequent stock takes.

Given the claim records and the frequent stock take adjustments, it is not a straightforward exercise to infer either an additive or multiplicative behavior of the discrepancies. Our theoretical framework will assume a mixture of additive and multiplicative settings for errors, and we will show that the penalty of using the multiplicative assumption when in fact the error mechanism is additive is less than using the additive assumption when in fact errors are multiplicative.

3. Analytical framework

3.1 Sequence of events, assumptions and notations

We consider a periodic order-up-to level review inventory system subject to inaccuracies on both the physical (PH) and the information system (IS) inventory levels. The inventory context refers to either e-retailing or B2B (wholesaling), where demand is satisfied based on the IS records. In this case, orders arrive electronically and are compared against the IS inventory levels. Commitments to the customers are then made based on this comparison. Because of inaccuracy-related issues (the PH level might be different from the IS one), the company may not be able to satisfy such commitments when the delivery of the product to the customer is due. The system operates under the objective of minimizing the total expected variable inventory cost subject to a service level constraint. The cycle service level

(CSL), defined as the percentage of cycles running with no stock-outs[1], is a common service target and also the one assumed for the purposes of our research.

The sequence of events for each ordering cycle (also illustrated in Figure 5) is as follows:

1. At the beginning of each ordering cycle *k*, the IS inventory position x_k^{IS} (i.e., the IS inventory level + pending deliveries) is checked and an order is placed to replenish up to a certain level y_k.
2. The initial physical stock x_{PH}^k is not perfectly known to the inventory manager, but s/he is able to characterize the random variable $\varepsilon_k^a = x_k^{IS} - x_k^{PH}$ which measures the accumulated discrepancy between the IS and the PH levels at the beginning of period *k*.
3. The order quantity is received after a random lead time *L* (being normally distributed).
4. During the selling cycle, customer orders are electronically recorded and PH and IS errors may occur leading to the IS level, y_k^{IS}, and the PH inventory level, y_k^{PH}, differing from the order-up-to level y_k and from one another.
5. At the end of the selling cycle (review period *T* + lead time *L*), the total customer demand, modeled by a random variable D_k, is summed up, and it is satisfied based on the IS inventory level y_k^{IS}. That is, a commitment $C_k = Min(y_k^{IS}, D_k)$ is made based on the IS inventory level and the total received demand.
6. When delivering the products to the customers, the company may not be able to fulfill all commitments if the PH inventory level at the end of the cycle is lower than the commitments made. The delivered quantity is the minimum of the commitment quantity and the physically available inventory: $S_k = Min(y_k^{PH}, C_k)$.

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[1] Strictly speaking, the target is formulated as the maximum allowable percentage of cycles running with a stock-out (P_1), resulting in a service level target of $1-P_1$.

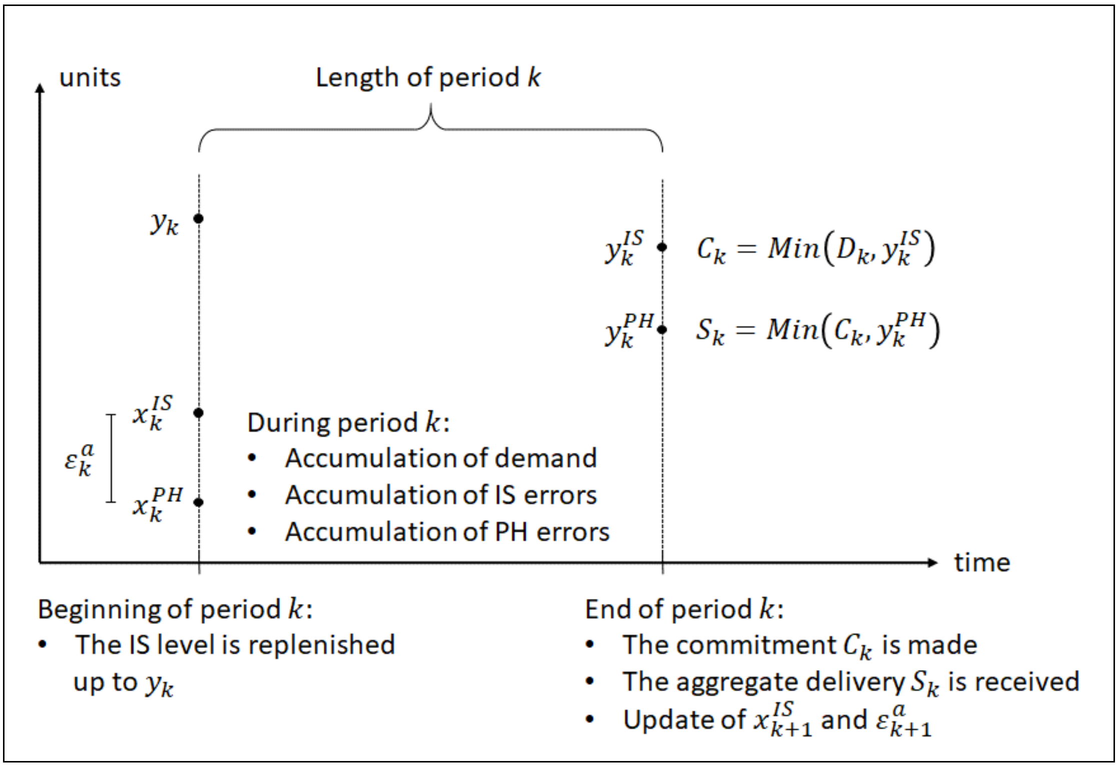

Figure 5: Sequence of events

With respect to **inventory cost**, we consider overage (h) and underage (u_1) unit costs used in classical error-free inventory problems to which we add a new unit cost (u_2) associated with non-satisfied commitments:

- h: the unit overage cost paid when the physical inventory level is positive after deliveries to the customers; this occurs when $y_k^{PH} \geq C_k$.
- u_1: type 1 unit shortage cost that refers to penalties that accrue when demand is lost because it cannot be satisfied based on the IS inventory level; this occurs when $D_k > y_k^{IS}$.
- u_2: type 2 unit shortage cost which constitutes the penalty that has to be paid when a commitment is not respected; this occurs when $y_k^{PH} \leq C_k$. Type 2 penalties account for sales that are lost (as in the previous case) in addition to losses in customer goodwill that result from unfulfilled commitments. Because of the decision already committed, u_2 should be higher than u_1.

Regarding the **errors structure**, we consider a mixture of additive and multiplicative errors. With an initial discrepancy at the beginning of period k measured using the random variable $\varepsilon_k^a = x_k^{IS} - x_k^{PH}$, we consider both additive and multiplicative error structures occurring in each period, and we model the inventory levels y_k^{IS} and y_k^{PH} as follows:

$$y_k^{IS} = \gamma_k^{IS} y_k + \lambda_k^{IS} \tag{1}$$

$$y_k^{PH} = \gamma_k^{PH} y_k + \lambda_k^{PH} + \varepsilon_k^a \tag{2}$$

where

- γ_k^{IS} (γ_k^{PH}): random variable describing the multiplicative errors of the IS (PH, respectively) level occurring in period k ;
- λ_k^{IS} (λ_k^{PH}): random variable describing the additive errors of the IS (PH, respectively) level occurring in period k ;
- ε_k^a : random variable describing the accumulation of errors up to period k . It measures the difference between the IS and the PH level at the beginning of period *k*, i.e., $\varepsilon_k^a = x_k^{IS} - x_k^{PH}$.

Without loss of generality, we assume that the additive and multiplicative errors on both IS and PH records are i.i.d. The following additional notations are used for the remainder of the paper:

- $f\,(\mathrm{F})$, the PDF (CDF, respectively) of the random variable D ;
- $\mathrm{g}_j^M\,(\mathrm{G}_j^M)$ the PDF (CDF, respectively) of the multiplicative errors γ_k^j , $j = IS, PH$;
- $\mathrm{g}_j^A(\mathrm{G}_j^A)$ the PDF (CDF, respectively) of the additive errors λ_k^j , $j = IS, PH$;
- $\gamma_{\mathrm{IS}}\,\mathrm{h}_k\,(\mathrm{H}_k)$, the PDF (CDF, respectively) of the errors accumulated prior to period *k*, ε_k^a .

By denoting $\gamma_k = \gamma_k^{IS} - \gamma_k^{PH}$ and $\lambda_k = \lambda_k^{IS} - \lambda_k^{PH} - \varepsilon_k^a$, the difference between the IS and PH level in period k could be written as $y_k^{IS} - y_k^{PH} = \gamma_k y_k - \lambda_k$.

Regarding the IS errors, since the inventory manager is assumed to track his/her past replenishment decisions by recording, for instance, the order-up-to level determined in the current period, one could assume that before committing to an order, the inventory manager cancels the impact of the unobserved IS errors by manually setting $y_k^{IS} = y_k$ (i.e., by setting $\gamma_k^{IS} = 1$ and $\lambda_k^{IS} = 0$). Even though such a verification may seem obvious, its application in practice is not straightforward because the IS level y_k^{IS} could actually be different from the order-up-to level y_k , particularly if an inspection was performed when receiving the items into inventory. Such an inspection could itself be subject to counting errors (see, e.g., Sahin, 2004). For the sake of generality, this paper assumes that y_k^{IS} is not manually updated before the commitment is made. The case of a manual update of y_k^{IS} could be accounted for in our investigation by setting $\gamma_k^{IS} = 1$ and $\lambda_k^{IS} = 0$. For the remainder of this paper, we assume that the IS level is only manually updated in the case where the inventory on hand exceeds the period order-up-to level after demand satisfaction. Such a scenario occurs when the period IS error realization is "uncommonly" higher than the demand realization. Besides, this assumption permits to prove that a myopic policy is optimal since we implicitly assume that a replenishment should be performed in each selling period. Hence, the multi-period problem could be seen as multiple single-period optimizations problem.

The optimal order-up-to level takes into account all the errors' distributions in addition to the demand distribution. Consequently, the resulting commitment considers all the available statistical data about errors. However, the commitment is made when the demand is revealed to the inventory manager. One can assume that in the light of the demand realization, the inventory manager can manually adjust the IS inventory level before the commitment is made. The commitment in this case would be $C_k = Min(y_k^{IS} - adj_k^{IS}, D_k)$, where adj_k^{IS} is a positive or negative adjustment value decided by the inventory manager. We will not incorporate this adjustement option into our numerical analysis because manual manipulations of the inventory record are regarded as one of the factors increasing the innacuracy risk. However, our theoritical framework permits the introduction of this adjustement option. If the adjustement is allowed, a second round of optimation, very similar to the one proposed in the following section, should be performed before the commitment is made, where the demand distibution is replaced by its realisation value(s).

In the following, we first consider a single-period optimization problem assuming that the error distributions are known. We present a comprehensive analytical study for this problem by proposing a cost as well as a service level approach. We then show how these single-period results could be used in a multi-period setting by stressing the dynamics between periods in the presence of errors. In fact, to apply an optimal ordering strategy, the random variables describing the accumulated errors should be characterized in each period. The latter study permits us to derive a learning mechanism about errors depending on the events that occurred during the past selling period. The application and the contribution of the proposed learning technique will be analyzed in a simulation study in Section 4.

3.2 Single-period problem

This section proposes two models, one that minimizes the expected inventory cost and one that pursues a service level target. In classical inventory theory (i.e., error-free inventory systems), the Cycle Service Level (CSL) is used to derive an optimal ordering policy according to which the target CSL is achieved at minimum expected variable costs. The CSL approach needs to be adjusted in the e-retailing /B2B context under IRI. The service level could be measured using two indicators:

- The **IS service level** CSL^{IS} : it measures the capability to satisfy customer demand based on the inventory level shown in the IS. As the classical CSL measure, it can be calculated by referring to the probability that the demand realization is lower than the IS level y_k^{IS} .

- The **PH service level** CSL^{PH} : it measures the capability to respect the commitment by delivering the right quantity to the customer. It can be calculated by referring to the probability that the PH level y_k^{PH} is enough to cover the committed quantity .

Consequently, for given target IS and PH service levels denoted by CSL_0^{IS} and CSL_0^{PH} , respectively, the optimization problem can be described as follows:

$$\begin{cases} \text{Minimise Expected } C\text{ost} \\ subject\ to \\ CSL_k^{IS} \geq CSL_0^{IS} \\ CSL_k^{PH} \geq CSL_0^{PH} \end{cases} \qquad (3)$$

Three order quantities can be deduced from the optimization problem: a first one, $y_k^{\cos t*}$, minimizing the expected cost and a tuple (y_k^{CSLIS*}, y_k^{CSLPH*}) achieving IS and PH target service levels. The expressions of the three order quantities are provided in the following result.

Result 1

1. If an optimal solution $y_k^{\cos t*}$ minimizing the expected cost function exists, it should solve:

$$K_1(y_k^{\cos t*}) = K_2 \tag{4}$$

where:

$$\begin{aligned}K_1(y) = &(u_1+h)\int_{\gamma^{IS}=0}^{+\infty}\gamma^{IS}F_a(\gamma^{IS}y)g_m(\gamma^{IS})\\ &-(u_2+h)\int_{\gamma^{IS}=0}^{+\infty}\int_{\gamma=0}^{+\infty}\int_{\lambda=-\infty}^{\gamma y}\left[H'(\gamma^{IS}y)-\left\{(\gamma^{IS}-\gamma)+\frac{\lambda}{y}\right\}H'((\gamma^{IS}-\gamma)y+\lambda)\right]g_m(\gamma^{IS})p_m(\gamma)p_a(\lambda)\\ &+(u_2+h)\int_{\gamma^{IS}=0}^{+\infty}\int_{\gamma=0}^{+\infty}\gamma P_a(\gamma y)g_m(\gamma^{IS})p_m(\gamma)\end{aligned} \tag{5}$$

$$K_2 = (u_2+h)\int_{\gamma^{IS}=0}^{+\infty}\int_{\gamma=0}^{+\infty}\gamma g_m(\gamma^{IS})p_m(\gamma)+u_1\int_{\gamma^{IS}=0}^{+\infty}\gamma^{IS}g_m(\gamma^{IS})-u_2E[\gamma] \tag{6}$$

$$H(x) = F(x)(x-\mu_D)+\sigma_D f(x). \tag{7}$$

2. The optimal order-up-to level achieving CSL_0^{IS} satisfies:

$$\int_{\gamma_k^{IS}=0}^{+\infty}F_a(\gamma_k^{IS}y_k^{CSLIS*})g_m(\gamma_k^{IS})d\gamma_k^{IS} = CSL_0^{IS} \tag{8}$$

3. If it exists, the optimal order-up-to level achieving CSL_0^{PH} satisfies:

$$\int_{\gamma_k^{IS}=0}^{+\infty}\int_{\gamma_k^{PH}=0}^{+\infty}\int_{\lambda_k=-\infty}^{(\gamma_k^{PH}-\gamma_k^{IS})y_k^{CSLPH*}}F_a(\gamma_k^{PH}y_k^{CSLPH*}+\lambda_k)\,g_{IS}^{M}(\gamma_k^{IS})\,g_{PH}^{M}(\gamma_k^{PH})\,p_a(\lambda_k) = 1-CSL_0^{PH} \tag{9}$$

Proof: cf. *Appendix A*[2]

3.2.1 Numerical study: impact of errors on the optimal ordering strategy

The IRI literature is mature, containing a set of proven results mainly on the impact of errors on the ordering strategy and the economic performance of the inventory system. We avoid in the following

[2] The Appendices appear in an electronic supplement accompanying our manuscript.

reproducing known results and focus on new findings related to the comprehensive modeling of errors in our framework.

It is well known in the classical inventory control literature and the IRI literature that:

- The optimal order quantity increases with the unit shortage penalty (Silver et al., 2017);
- The optimal order quantity increases with the inaccuracy error average and variability (Kök and Shang, 2007; Camdereli and Swaminathan, 2010);
- The service level behaves monotonically in the order quantity (Silver et al., 2017).

Some of these well-known results are contradicted below. Figure 6 illustrates that the optimal order quantity could increase or decrease in the unit shortage cost u_2 depending on the position of the IS multiplicative error γ_k^{IS} when compared with the position of the PH error γ_k^{PH}. The same behavior is also observed when contrasting the position of the additive IS error λ_k^{IS} with the position of the PH additive error λ_k^{PH}. When mixing different values of the averages of the multiplicative errors with different values of the additive errors' averages, we can observe a non-monotonic behavior of the optimal order quantity in u_2 (cf. Figure 7).

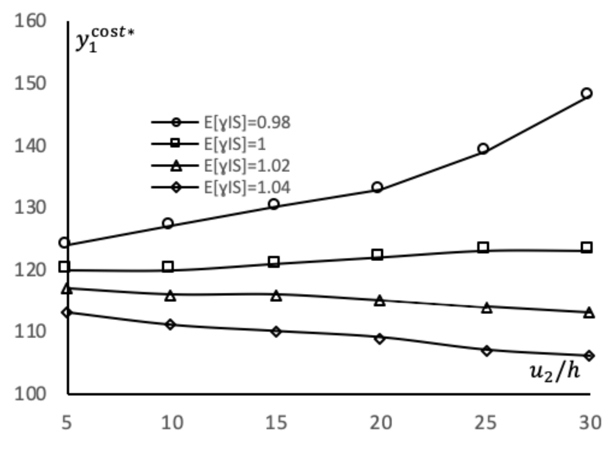

Figure 6: Order-up-to level as a function of u_2/h and $E[\gamma_{IS}]$ - $\frac{u_1}{h} = 1, E[\gamma_{PH}] = 1, Std[\gamma_{IS}] = 0.01, Std[\gamma_{IS}] = 0.02, no\ additive\ error$

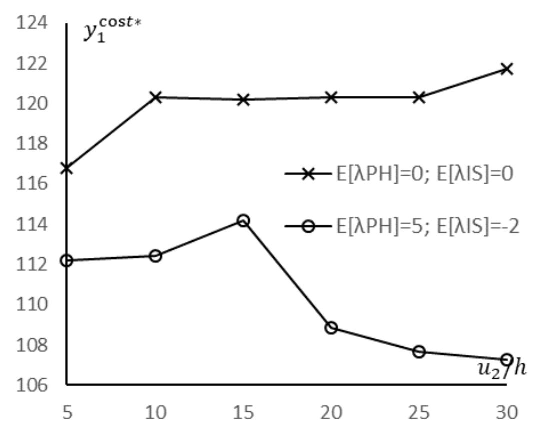

Figure 7: Order-up-to level as a function of u_2/h and $E[\lambda_{IS}]$ and $E[\lambda_{PH}]$ - $\frac{u_1}{h} = 1$, $Std[\lambda_{IS}] = Std[\lambda_{PH}] = 1, E[\gamma_{PH}] = E[\gamma_{IS}] = 1, Std[\gamma_{IS}] = Std[\gamma_{PH}] = 0.01$

To understand how this behavior contradicts well-known results according to which the order-up-to level always increases in the unit shortage cost, we note that:

- When the type 2 shortage cost u_2 increases, it is obvious that the inventory manager should enlarge the order quantity to increase the PH stock and consequently satisfy the commitment as much as possible.

- However, a higher order quantity may lead to a higher commitment quantity itself. When ordering more, the inventory manager tends to accept more incoming customer orders based on his/her observation of the IS record.

- Consequently, one way to avoid the cost associated with higher u_2 values is either to order a higher quantity to increase the PH level, or to order a lower quantity to decrease the IS level and as such to commit less. The difference between the two strategies depends on the relative position of the IS additive and multiplicative errors when compared to the PH additive and multiplicative errors.

The same behavior can be observed when studying the evolution of the optimal order quantity in the variability of the multiplicative errors. To understand how our findings contradict some well-known results from the literature, we notice that:

- It is well known in the inventory control literature that when the system is subject to more variability, the inventory manager tends to order more as a way of tackling the uncertainty issue.

- However, ordering more in our case leads to an increase of the multiplicative IS and PH errors. Increasing variabilities of the errors may induce the inventory manager to decrease the order quantity to lower the magnitude of the multiplicative errors. Here again, the strategy that is adopted to tackle the uncertainty issue depends on the positions of the errors' distributions.

We also note that in contrast to all IRI investigations where the behavior of the optimal order quantity in the model parameters can be predicted, our extended model shows that these behaviors are actually unpredictable because they are very sensitive to the stochastic comparison between the random distributions modelling the demand, the two multiplicative errors and the two additive errors. For instance, a managerial result like "the optimal order quantity increases with the variability of the multiplicative IS error" can be changed to its exact opposite by only changing the sign of the average of the PH additive errors (cf. Figure 8).

With regard to the optimization approach based on the service level, it is well known that ordering more leads to a better service level. This is true for the classical CSL where the IS and PH levels are assumed to be reconciled. This is also true for the IS service level where a higher order-up-to level results in a higher IS level. However, this is not always true for the PH service level. As illustrated in Figure 9, the PH service level might decrease in the order-up-to level. In some particular situations, depending on the IS and PH errors' moments, ordering less results in a lower commitment and consequently in a higher PH service level. We notice that a solution is always guaranteed for the IS service level optimization (Eq. (8)), but that there may be situations where the PH service level optimization (Eq. (9)) leads to an infeasible solution (cf. Figure 10). It is also important to note that the service level solution could be driven by the IS CSL optimization or the PH CSL optimization depending on the target values that the manager wishes to achieve (cf. Figure 11).

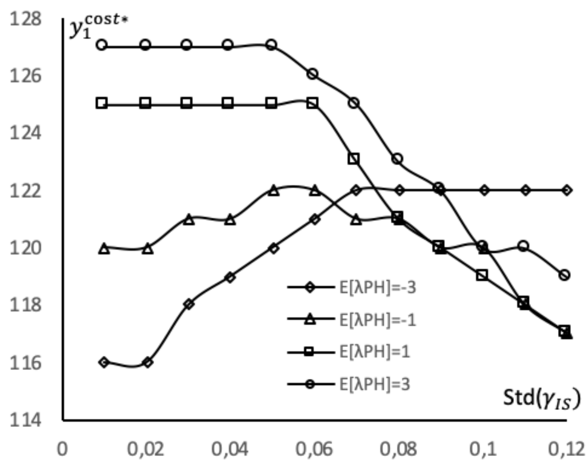

Figure 8: Order-up-to level as function of $Std[\gamma_{IS}]$ and $E[\lambda_{PH}]$ - $E[\lambda_{IS}] = 0$, $E[\gamma_{IS}] = 1$, $Std[\lambda_{IS}] = Std[\lambda_{PH}] = 1$, $Std[\gamma_{PH}] = 0.02$

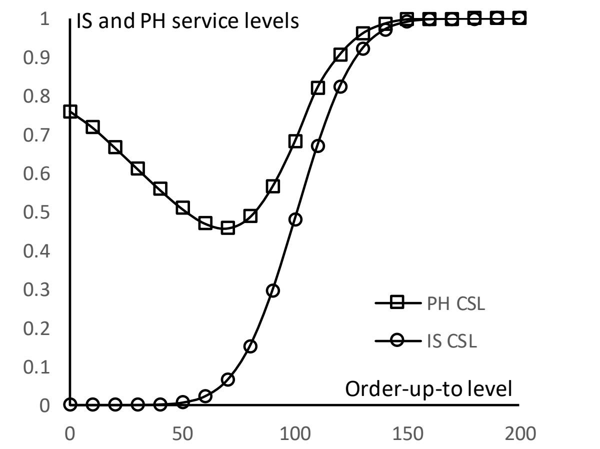

Figure 9: IS and PH service levels as functions of the order-up-to level - $E[\lambda_{PH}] = 0$, $E[\lambda_{IS}] = 1$, $Std[\lambda_{PH}] = Std[\lambda_{IS}] = 1$, $E[\lambda_{IS}] = 0.98$, $E[\lambda_{PH}] = 1$, $Std[\lambda_{IS}] = Std[\lambda_{PH}] = 0.01$

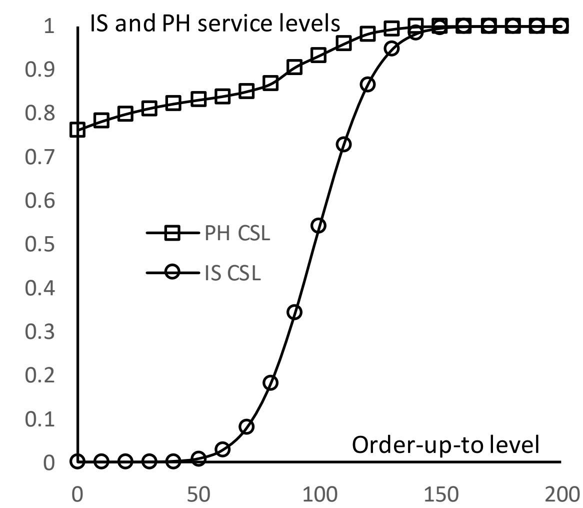

Figure 10: IS and PH service levels as functions of the order-up-to level - $E[\lambda_{PH}] = 0$, $E[\lambda_{IS}] = 1$, $Std[\lambda_{PH}] = Std[\lambda_{IS}] = 1$, $E[\lambda_{IS}] = 1.01$, $E[\lambda_{PH}] = 1$, $Std[\lambda_{IS}] = Std[\lambda_{PH}] = 0.01$

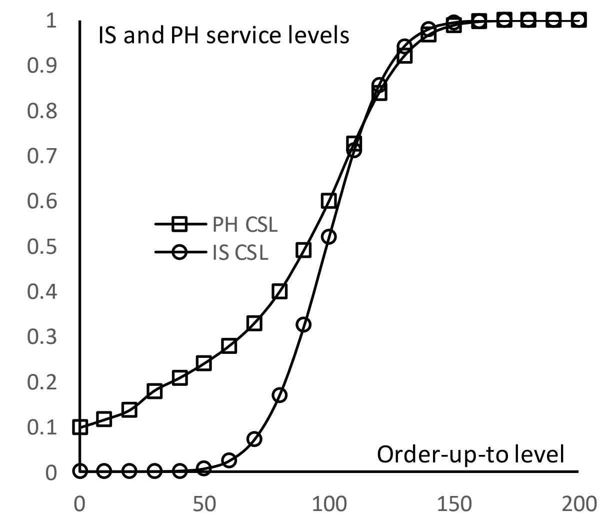

Figure 11: IS and PH service levels as functions of the order-up-to level - $E[\lambda_{PH}] = 2$, $E[\lambda_{IS}] = -1$, $Std[\lambda_{PH}] = Std[\lambda_{IS}] = 2$, $E[\lambda_{IS}] = 1.01$, $E[\lambda_{PH}] = 0.98$, $Std[\lambda_{IS}] = Std[\lambda_{PH}] = 0.01$

3.2.2 Numerical study: equivalence between cost and service level approaches

The unit inventory holding cost, h , is a measure that practitioners are able to calculate relatively easily. The traditional unit shortage cost, u_1 , is a measure composed of a quantifiable margin loss for units short to which a subjective goodwill penalty is added. The unit commitment non-satisfaction cost, u_2 , is also composed of a unit margin loss and a more important goodwill measure that exceeds the goodwill loss in u_1 .

Being aware that calculating the subjective components of u_1 and u_2 is not a straightforward exercise, we provide below an equivalence study between the unit costs and the associated service levels (IS and PH) measures. In the classical inventory literature, there exist an analytical equivalence between the CSL measure and the overage/underage unit penalties expressed by the ratio $u_1/(u_1+h)$. In the presence of four random variables disturbing the IS and PH flows, the analytical equivalence is not straightforward to derive. For this reason, we illustrate the equivalence in a numerical study.

For a given vector (h,u_1,u_2), we deduce the corresponding ratio vector $(\frac{u_1}{h},\frac{u_2}{h})$ and its associated optimal order-up-to level $y_k^{\cos t^*}$ minimizing the expected cost. Then, we deduce the associated equivalent IS and PH service levels by calculating $CSL_k^{IS}(y_k^{\cos t^*})$ and $CSL_k^{PH}(y_k^{\cos t^*})$, respectively.

The first observation we make is that the unpredictable and non-monotonic behavior of the optimal order quantity in the problem parameters shown in the previous numerical study also applies to the behavior of the equivalent service levels. The latter are also very sensitive to the comparison of the relative positions of the five random distributions describing the demand and the errors.

A well-known result from classical inventory theory according to which the target service level increases in the shortage penalty is contradicted by our numerical experiment for some particular cases of the error parameters. For a setting where $\gamma_{PH} \sim N(1.01, 0.01)$; $\lambda_{IS} \sim N(5,1)$; $\lambda_{PH} \sim N(-2,1)$; $Std[\gamma_{IS}] = 0.01$ and $\frac{u_1}{h} = 5$, Figure 12 illustrates different behaviors of the equivalent CSL^{PH} (for three values of the ratio $\frac{u_2}{h}$) as a function of the average of the multiplicative IS errors.

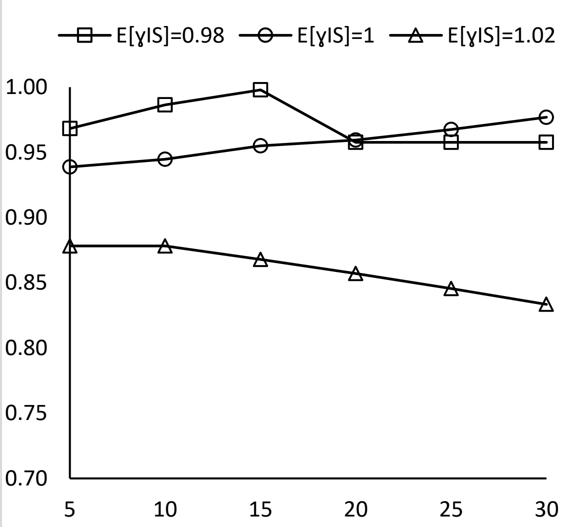

Figure 12: Equivalent PH CSL as a function of $\frac{u_2}{h}$ for different values of $E[\gamma_{IS}]$

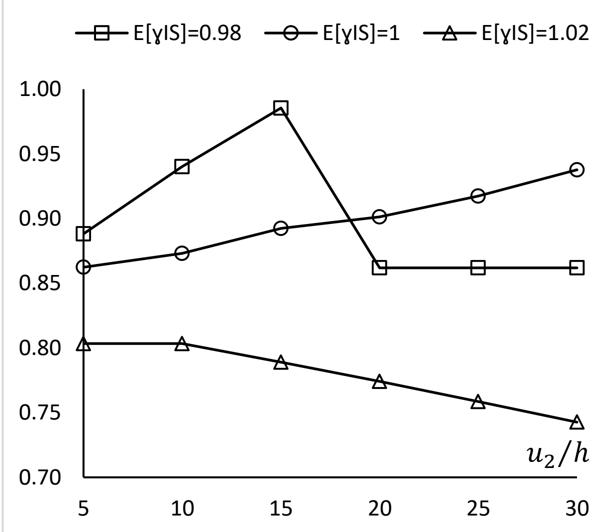

Figure 13: Equivalent IS CSL as a function of $\frac{u_2}{h}$ for different values of $E[\gamma_{IS}]$

The equivalent $\boldsymbol{CSL^{IS}}$, illustrated in Figure 13, is similar in terms of behavior, but lower in terms of value. For instance, if $\frac{u_2}{h}=15$ and $E[\gamma_{IS}]=1$, optimizing the cost problem is equivalent to solving the service level problem by having a target IS CSL equal to 89% and a target PH CSL equal to 95%.

3.3 Multi-period problem

Inventory managers utilize an inventory policy using information on the IS records that may differ from the PH inventory levels. The latter is unknown and can be automatically aligned with the IS record upon the occurrence of some specific events that we will describe later in this section. As stated in the sequence of events, the case of a manual update of the IS record occurs when the period IS error realization is "uncommonly" higher than the demand realization. Such an assumption enables us to prove that a myopic policy is optimal since we implicitly assume that a replenishment should be made in each selling period. Hence, the multi-period problem could be modeled as a collection of multiple single-period optimization problems.

The following information is required to manage the multi-period problem:

- The transition of the system in-between two periods: we need to describe the evolution of the accumulated random error variable, ε_k^a, as well as the initial IS inventory level, x_k^{IS}, which could be manually updated during some particular events.
- The estimation of the distributions' moments associated with the four random error variables. We assume that the inventory manager is aware of the presence of errors in the inventory system. However, in contrast to the majority of investigations in the IRI literature where the errors' distributions $\gamma_k^{IS}, \gamma_k^{PH}$, λ_k^{IS} and λ_k^{PH} are assumed to be accurately known by their moments and a hypothetical distribution (Normal, for instance), we assume that the errors' moments are dynamically estimated over time.

With regard to the evolution of ε_k^a and x_k^{IS} over time, we propose in Table 2 an event-based mechanism that allows us to estimate the moments of the former and the value of the latter as a function of the events occurring during period $k-1$.

With regard to the error distributions' moments estimation, we propose a learning technique utilizing events when the inventory manager collects discrepancy values in order to update his/her knowledge of the IS errors' distributions of γ^{IS} and λ^{IS} by contrasting the observed value y_k^{IS} with the order-up-to level y_k. S/he does the same by contrasting the value of $y_k^{IS}-y_k^{PH}$ with y_k in order to estimate the distributions of γ_k and λ_k.

As for the initial IS inventory level x_k^{IS} and the random variable ε_k^a measuring the accumulated discrepancy in the beginning of period k, we propose a mechanism that allows the estimation of the value of the former and the distribution of the latter based on what occurred in period $k-1$:

	No u_2 event occurred in period $k-1$		A u_2 event occurred in period $k-1$	
	Situation 1	Situation 2	Situation 3	Situation 4
	No u_1 in period $k-1$	A u_1 in period $k-1$	No u_1 in period $k-1$	A u_1 in period $k-1$
Commitment	$C_{k-1} = d_{k-1}$	$C_{k-1} = y_{k-1}^{IS}$	$C_{k-1} = d_{k-1}$	$C_{k-1} = y_{k-1}^{IS}$
Delivery	$L_{k-1} = d_{k-1}$	$L_{k-1} = y_{k-1}^{IS}$	$L_{k-1} = y_{k-1}^{PH}$	$L_{k-1} = y_{k-1}^{PH}$
x_k^{IS} observed value update	$x_k^{IS} = y_{k-1}^{IS} - d_{k-1}$	$x_k^{IS} = 0$ Then manually count and set: $x_k^{IS} = y_{k-1}^{PH} - y_{k-1}^{IS}$	$x_k^{IS} = y_{k-1}^{IS} - d_{k-1}$ Then manually set: $x_k^{IS} = 0$	$x_k^{IS} = 0$
x_k^{PH} non-observed value update	$x_k^{PH} = y_{k-1}^{PH} - d_{k-1}$	$x_k^{PH} = y_{k-1}^{PH} - y_{k-1}^{IS}$	$x_k^{PH} = 0$	$x_k^{PH} = 0$
Estimation of ε_k^a	Cf. moments estimations in Eqs. (11) and (12)	0	0	0

Table 2: Learning events

3.3.1 Estimation of ε_k^a moments

From Table 2, Situations 2, 3 and 4 where a u_1 and/or u_2 event takes place, may result in opportunities, referred to as 'learning events', where the inventory manager may collect some data about the discrepancies and update the IS level manually (as illustrated in the 6th row of Table 2). We will explain these opportunities in detail in Section 3.3.2. Under situation 1, the errors accumulate over subsequent periods in the absence of a reconciliation of the IS and PH levels. The moments of ε_k^a depend on the number of periods, *N*, elapsed since the last learning event (i.e., the last time a u_1 or u_2 event occurred). When errors accumulate over these *N* periods, the difference between the IS and the PH level is written as:

$$\varepsilon_k^a = (\gamma_k^{IS} - \gamma_k^{PH}) \sum_{i=k}^{k-N} y_i + N(\lambda_k^{IS} - \lambda_k^{PH}) \tag{10}$$

Under the i.i.d. assumption for the errors' distributions, the average and the variance of ε_k^a may consequently be calculated as follows:

$$E\left[\varepsilon_k^a\right] = (E\left[\gamma_k^{IS}\right] - E\left[\gamma_k^{PH}\right]) \sum_{i=k}^{k-N} y_i + N(E\left[\lambda_k^{IS}\right] - E\left[\lambda_k^{PH}\right]) \tag{11}$$

$$Var\left[\varepsilon_k^a\right] = (Var\left[\gamma_k^{IS}\right] + E\left[\gamma_k^{PH}\right])\left(\sum_{i=k}^{k-N} y_i\right)^2 + N(Var\left[\lambda_k^{IS}\right] + Var\left[\lambda_k^{PH}\right]) \tag{12}$$

For some particular situations (IS errors being more important than PH errors, for instance), the accumulation of ε_k^a can lead to a continuously increasing optimal order quantity satisfying the PH service level. Figures 14 and 15 illustrate that situations may occur where the accumulated errors lead to a diverging behavior between the optimal order quantity satisfying the IS service level and the one satisfying the PH service level. With increasing accumulated errors, the order-up-to level satisfying the PH service level might continuously increase with *N* (number of periods since the last reconciliation) as shown in the two examples illustrated in Figures 14 and 15.

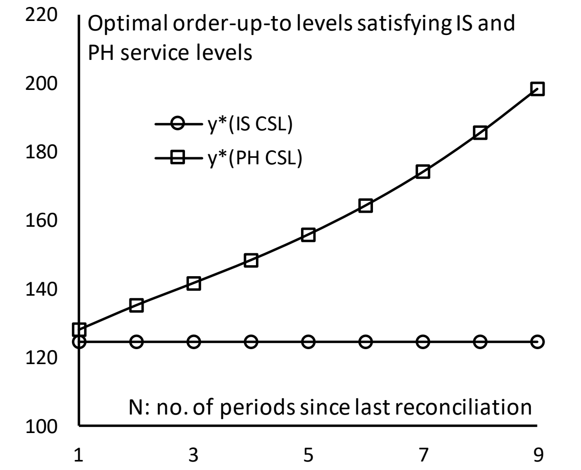

Figure 14: Service level optimal order-up-to level as a function of number of periods since the last reconciliation – case 1: $\gamma_{IS} \sim N(1.02, 0.01)$; $\gamma_{PH} \sim N(0.98, 0.01)$; $\lambda_{IS} \sim N(-1, 2)$; $\lambda_{PH} \sim N(2, 2)$.

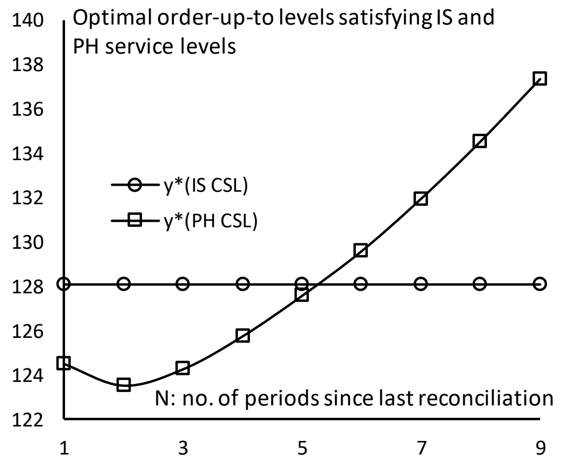

Figure 15: Service level optimal order-up-to level as a function of number of periods since the last reconciliation – case 2: $\gamma_{IS} \sim N(0.99, 0.01)$; $\gamma_{PH} \sim N(0.98, 0.01)$; $\lambda_{IS} \sim N(-1, 2)$; $\lambda_{PH} \sim N(2, 2)$.

The continuously increasing order-up-to level occurs in cases where the PH errors positively dominate the IS errors. The service level shows the opposite "freezing" behavior discussed by Kang and Gershwin (2005) in the retail sector with backlogging, where the PH level continuously decreases without being detected by the IS level.

To avoid a continuously increasing order-up-to level, we propose a mechanism where the PH CSL solution is not selected if its impact on the effective IS service level is higher than a certain percentage of its target value. In such a case, the IS CSL solution is selected even if its value is lower than the value of the PH CSL. Such a mechanism will push the system towards a u_2 event in the forthcoming periods and will avoid very costly inventory levels.

The PH CSL equation provided in Eq. (9) is also changed into a conditional solution:

Calculate y_k^{CSLPH*} solving:

$$\int_{\gamma_k^{IS}=0}^{+\infty}\int_{\gamma_k^{PH}=0}^{+\infty}\int_{\lambda_k=-\infty}^{(\gamma_k^{PH}-\gamma_k^{IS})y_k^{CSLPH*}} F_a(\gamma_k^{PH} y_k^{CSLPH*}+\lambda_k)\, g_{IS}^M(\gamma_k^{IS})\, g_{PH}^M(\gamma_k^{PH})\, p_a(\lambda_k) = 1 - CSL_0^{PH}$$

If $CSL_k^{IS}(y_k^{CSLPH*}) > \alpha CSL_0^{IS}$, then ignore y_k^{CSLPH*}

3.3.2 Learning about the errors' distributions

Regarding the **learning process about the errors' distributions**, there may be three types of events from which the inventory manager could collect data about IRI:

1. *Learning opportunity with zero cost*: without any physical inspection, the inventory manager could contrast the IS and the PH levels in Situations 3 and 4. Under both situations, a u_2 event occurs, and the PH level is assumed to be zero. If the IS level is different from zero (Situation 3), then the value shown in the IS record could be collected to update the learning process about errors. If the IS level is equal to zero because a u_1 event occurred (Situation 4), the commitment quantity and the delivered quantities could be contrasted to update the learning process about errors.

2. *Learning opportunity with a negligible cost*: under Situation 2, a u_1 event occurs leading to an IS record equal to zero. In an error-free inventory system, the PH level should be equal to zero in this case as well. Without a u_2 event, if the inventory manager notices any remaining items after fulfilling the commitment, s/he could easily count the remaining items and update the IS level accordingly. The counting process should be associated with a negligible cost if few items remain in the PH stock, which should be the case when a u_1 event occurred.

3. *Learning opportunity with an inspection cost*: one could assume that an inspection is performed regularly or periodically, in particular when there are no u_1 and u_2 events (i.e., under Situation 1). In such a case, the actual IS and the PH levels could be contrasted with the order-up-to level to update the learning process about the errors' distributions.

In the presence of both additive and multiplicative errors (as stated in Eqs. (1) and (2)), it is not straightforward for the inventory manager to estimate the moments of the errors' distributions when s/he collects some data about errors in different learning opportunities, as discussed earlier. If, for instance, the observed y_k^{IS} is 110, while the period order-up-to level y_k is 100, it is not straightforward, statistically speaking, to use this inaccuracy observation to explain its root from the multiplicative γ_k^{IS} or the additive λ_k^{IS} errors. We assume that the actual error is a mix of both additive and multiplicative errors, and we compare the two learning assumptions which could be set by the inventory manager:

1. *The additive error and learning assumptions*: in each learning event, the inventory manager calculates the **difference** between the observed IS level and the order-up-to level to update his/her knowledge about the additive error distribution λ_k^{IS}. Then, calculating the difference between the observed IS and PH levels permits estimating the moments of the PH error λ_k^{PH} by taking into account the accumulation of only additive errors since the last IS and PH reconciliation. The

inventory manager then uses this updated error distribution to calculate the next order-up-to level by assuming that the inventory system is only subject to additive errors. In other words, the inventory manager is aware that the system is subject to both additive and multiplicative errors, but uses the data collected from the learning events to hypothetically model the inventory system as being subject to additive errors only.

2. *The multiplicative error and learning assumptions*: in each learning opportunity, the inventory manager calculates the **ratio** between the observed IS level and the order-up-to level in order to update his/her knowledge about the multiplicative IS error distribution γ_k^{IS}. The moments of the PH multiplicative error γ_k^{PH} are then deduced by observing the distribution of the difference between the PH and IS errors and by assuming that this difference informs about the accumulation of multiplicative errors since the last IS and PH reconciliation. The inventory manager then uses this updated error distribution to calculate the next order-up-to level by assuming that the inventory system is only subject to multiplicative errors.

The learning procedure described above utilizes the fact that the IS level observed during a learning event is informative with regards to the IS errors that occurred in the previous selling period, whereas the observed PH level provides information about the accumulation of the PH errors that occurred since the last learning event where the PH and IS levels were reconciled. In fact, the IS level is updated by the inventory system during inbound and outbound logistics processes, whereas the PH level, which is not known, is checked when the manager manually interferes with the inventory system. Such a manual intervention can result from a learning event or from an audit of the inventory system.

The two learning procedures are easy to deploy in practice. We will analyze the performance of the inventory system under the additive and multiplicative learning assumptions, and we will compare it with the case where the errors' moments are accurately known in order to provide managerial insights about the 'quality' of the two learning procedures.

With regards to the estimation procedure of the errors' moments, it is worthwhile noting a helpful difference between the evolution of the IS and PH errors over time. Assume that the last learning opportunity was performed *N* selling periods before. The IS level collected in the following learning opportunity does not keep the IS error accumulations in memory, since the IS level is manually updated at the beginning of each period when a replenishment is made. In other words, when the IS level is collected during a learning event and contrasted with the previous order-up-to level, the impact of only one period of IS errors could be deduced from the comparison. Such a comparison could provide trustworthy data to accurately estimate the IS errors' distributions.

The PH level (which is only known when a learning event occurs) keeps the PH errors' accumulation since the last learning event in memory, and the comparison between the observed PH level and the observed IS level provides data about the accumulation of both IS and PH errors over the last *N* periods. Using the IS errors' distributions deduced from the first comparison, the PH errors' distributions could be derived.

Based on the last comparison between the IS and PH error dynamics over time, Table 3 illustrates the learning procedures for both the additive and multiplicative learning assumptions.

Additive Learning Assumption	**Multiplicative Learning Assumption**
In Learning Event j Collect the value $y_j^{IS} - y_j$ and $\frac{y_j^{IS}-y_j^{PH}}{N_j}$ N_j : nb of periods from event j-1 to j	In Learning Event j Collect the value y_j^{IS}/y_j and $\frac{y_j^{IS}-y_j^{PH}}{N_j}$ N_j : nb of periods from event j-1 to j
Realize a Likelihood Maximization (LM1) on all collected $y_i^{IS} - y_i$ for all Events $i = 1..j$ to derive E[LM1] and Var[LM1]	Realize a Likelihood Maximization (LM3) on all collected y_i^{IS}/y_i for all Events $i = 1..j$ to derive E[LM3] and Var[LM3]
Deduce the estimated moments of λ_j^{IS} as E[LM1] and Var[LM1]	Deduce the estimated moments of γ_j^{IS} as E[LM3] and Var[LM3]
Realize a Likelihood Maximization (LM2) on all collected $\frac{y_i^{IS}-y_i^{PH}}{N_i}$ for all Events $i = 1..j$ to derive E[LM2] and Var[LM2]	Realize a Likelihood Maximization (LM4) on all collected $\frac{y_i^{IS}-y_i^{PH}}{N_i}$ for all Events $i = 1..j$ to derive E[LM4] and Var[LM4]
Deduce the estimated moments of λ_j^{PH} as E[LM1]- E[LM2] and Var[LM1]-Var[LM2]	Deduce the estimated moments of γ_j^{PH} as E[LM3]- E[LM4]/y_j and Var[LM3]-Var[LM4]]/y_j^2

Table 3: Learning procedures for the additive and multiplicative error assumptions

4. Simulation study and managerial insights

This section performs a simulation study to derive managerial insights and recommendations for inventory managers facing IRI. The objectives of this exercise are to:

1. Compare possible actions that the inventory manager could employ to handle the errors, namely manage the system with the proposed cost-free actions enabled by learning opportunities.

2. Assess the benefit of our joint learning and replenishment policy as compared to the case where errors are partially/fully ignored.

3. Measure the performance of the two proposed learning procedures and derive insights on the best learning assumptions to adopt (additive or multiplicative assumption) in the presence of a mix of additive and multiplicative errors.

Please note that this is a theoretically generated data simulation experiment. Conducting simulation on empirical data would necessitate time series information on sales and inventory levels, something that neither of the companies participating in this research have shared with us. Having said that, managers from both companies were involved in the discussion both of the setup and the outcomes of our simulation confirming its valuable implications for their businesses.

4.1 Setup of the experiment

For 100 selling periods, we simulate a periodic review inventory policy under the presence of both IS and PH errors. We assume a normally distributed demand per period with an average of 100 units and a standard deviation of 20 units, and we further assume that the objective is to satisfy target IS and PH cycle service levels. Many simulation settings consider (collectively) this standard demand profile. Kök and Shang (2007) considered in their numerical study a normally distributed demand, N(20, 2 and 8), and errors. Normally distributed demand was also assumed by Atali et al. (2006) and Kang and Gershwin (2005). Our choice of 100 as an average for the demand is to enable a linkage between the magnitude of the errors (used in the simulation) and the demand average. The errors' average can be easily interpreted as a percentage of the demand average. Given the errors' average used in the simulation, the absolute value of inaccuracies represents, depending on the simulation scenario, 0 to 5% of the demand stream which is in alignment with the empirical finding of DeHoratius and Raman (2008).

The IS and PH errors are also assumed to be normally distributed with unknown moments. We apply the learning procedure described in the last section in order to estimate the moments of the errors and over time use them to derive in a dynamic way the associated order-up-to levels. In each learning event (resulting from u_1 and u_2 events as described in Table 2), IS and PH stock levels are collected and used to improve the estimation of the errors' moments.

As in the simulation study conducted by DeHoratius et al. (2008), we assume that the learning exercise is started after an audit / stock count. That is, the inventory system starts with no errors which then start to accumulate from the first period.

For the same set of the generated random variables (demand and the four errors) and for the given target service levels, 30 simulation runs are performed and averaged in order to compare the performances of four stock control cases:

1. The case where errors are ignored: The order-up-to level coincides with the error-free solution. We distinguish between two sub-cases of ignoring errors:

    a. Partial ignorance (PI) of errors: the errors' distributions are not taken into account in the ordering strategy, but the inventory manager, aware of the presence of errors, uses the learning events (u_1 and u_2 event) introduced in Table 2 (i.e., Situations 1, 2 and 3) to manually update the inventory system by reconciling the IS inventory level.

    b. Full ignorance (FI) of the errors: the errors' distributions are not taken into account in the ordering strategy and the learning events are also ignored.

2. The case where errors are accurately known (K): the distributions of the errors are assumed to be accurately known. This is a benchmark case that permits us to evaluate the quality of the proposed additive and multiplicative learning procedures.

3. The additive learning assumption case (A): learning events are used to gather data to estimate the errors' moments by assuming that errors are additive.

4. The multiplicative learning assumption case (M): learning events are used to gather data to estimate the errors' moments by assuming that errors are multiplicative.

Applying the additive (A) and multiplicative (M) learning procedures separately will not lead to an accurate estimation of the actual moments of both additive and multiplicative errors because their impacts are mixed in the observed actual IS and PH levels. However, as will be shown in this section, the resulting estimators will lead to an inventory policy performing very close to the accurate errors case (K). Indeed, by applying our two learning assumptions, the resulting estimated moments of the additive (multiplicative, respectively) errors are overestimated or underestimated when compared to their actual values which compensates the fact of using only one error configuration (additive or multiplicative) and not the mixture of additive and multiplicative configurations.

Regarding the question of which error configuration should be adopted by the inventory manager when calculating the inventory order-up-to level, we notice that it is mathematically difficult to deduce the contribution of the additive and the multiplicative distributions on this observation from an observed IS or PH value. For this purpose, we propose a choice of the error configuration based on a comparison of the Andersen parameter associated with the Maximum Likelihood optimizations presented in Table 3. When the Anderson parameter associated with the ratio Maximum Likelihood Estimation is higher than the one associated with the difference, then the multiplicative error configuration is assumed, otherwise the additive one is chosen. The rationale behind this suggestion is to give priority to the error configuration that better fits the observed values.

We perform several runs of this experiment with different error parameter values, and we derive average results about the performance of each of the management cases described earlier. The performance of each control case, $\left(FI, PI, K, A, M\right)$, is measured by the following relative deviation from the target IS and PH service levels:

$$Performance_i^j = \frac{CSL_i^j - CSL_0^j}{CSL_0^j} \qquad i = \left\{FI, PI, A, M, K\right\} \quad j = \left\{IS, PH\right\} \tag{13}$$

The effective service levels obtained under each control case are not necessarily the same because they depend on the learning curve about the errors. For this reason, we complete the performance comparison by including the average holding cost, E(holding), associated with each control case.

4.2 Results

We run the experiment described in Section 4.1 for different scenarios of the errors' parameters and we evaluate the performance of the proposed learning procedures. We start by showing some examples of the evolution of the order-up-to levels over time the inventory manager may select if errors are accurately known (referred to as Y(K)), if errors are fully ignored (Y(FI)) or partially ignored (Y(PI)), if errors are assumed to be additive (Y(A)), and if errors are assumed to be multiplicative (Y(M)).

Figures B1 to B4 (in *Appendix B*) illustrate an example of the evolution of the four order-up-to levels (associated with the four control management approaches) over the 100 periods for different scenarios of errors modeled with the error vector $(\gamma_k^{IS}, \gamma_k^{PH}, \lambda_k^{IS}, \lambda_k^{PH})$. Table 4 presents the actual distribution parameters for each scenario with a description of which sources of errors could be modelled with these parameters.

Scenario	**Distribution of the vector** $(\gamma_k^{IS}, \gamma_k^{PH}, \lambda_k^{IS}, \lambda_k^{PH})$	**Type of errors modeled by the scenario**
Figure B1	(N(1.02,0.01),N(0.98,0.01),N(1,2),N(-1,2))	Shrinkage errors and positive (opposite) transaction errors
Figure B2	(N(0.98,0.01),N(0.98,0.01),N(1,2),N(-1,2))	Shrinkage errors slightly detected by the IS
Figure B3	(N(1,0.01),N(1,0.01),N(0,2),N(2,2))	Positive and additive supply system errors not detected by the IS
Figure B4	(N(1.02,0.01),N(1,0.01),N(-2,1),N(0,1))	Positive (M) and negative (A) transaction errors compensating each other

Table 4: Simulation scenarios

For each scenario, Table 5 summarizes the performance of the simulation by providing the effective IS and PH service levels as well as the average inventory level for the four control cases.

All investigated scenarios have as target service level the tuple (CSL_0^{IS}, CSL_0^{PH}) which is set equal to $(0.9, 0.9)$. In order to tackle the error accumulation increase and the divergence of the solution satisfying the PH service level, we set the upper stock level such that the IS service level is not higher than a tolerance of 2% of its target value. If the solution satisfying the PH service level leads to an effective IS service level higher than 2% of its target value, then the selected solution is the one derived from the IS service level optimization. Such a manipulation avoids the divergence of the solution and results generally in a u_2 event to reconcile the IS and PH levels and to start in a new cycle of errors accumulation. For this reason, the effective PH service levels for all figures (except Figure D3) are lower than the target $CSL_0^{PH} = 0.9$. If the tolerance value is higher than the 2% set during the experiment, the effective PH service level will be closer to the PH target value, but it will lead to a higher inventory level.

Scenario 1 (illustration in Figure B1 in *Appendix B*) could represent one of the worst scenarios that can occur in practice. The PH level is subject to negative errors (additive and/or multiplicative), whereas the IS level has indirectly been updated on the positive side. The discrepancy in each period is, as a consequence, not negligible and the accumulation of errors requires a variable order-up-to level function of the number of elapsed periods since the last stock reconciliation. The 2% tolerance value helps to avoid that the system accumulates errors for more than five periods. 11 u_1 events and 30 u_2 events occurred under the multiplicative control case as well as the additive control case leading to an efficient estimation of the error's moments since the performances of A and M are close to K in terms of service level and average inventory level. Partially ignoring the errors, i.e. ignoring the errors' distributions but manually reconciling the IS and PH levels in the case of u_1 and u_2 events, provides a good performance from a service level point of view, but the average inventory level is higher when the errors' distributions are estimated and taken into account in the inventory policy. Totally ignoring the errors, i.e. not manually reconciling the IS and PH levels in the case of u_2 events, leads to poor performance from a service level point of view. Comparing the average holding levels of the latter control case with the other cases is not relevant since the effective service levels are significantly different.

Scenario 2 (illustrated in Figure B2 in *Appendix B*) could represent a situation where the system is subject to negative supply errors (a supply system physically delivering less than the quantity ordered) and where a stock inspection is done when the products are delivered from the supply system. The delivery inspection is assumed subject to counting errors resulting in an IS stock update not exactly equal to the actual delivered quantity. To cope with the negative supply errors, the inventory manager should order more than the error-free model suggests. For this reason, both the FI and PI control cases performed poorly from a PH service level point of view. The learning technique is efficient here again since the performance of the M and A cases are close to case K.

Scenario 3 (illustrated in Figure B3 in *Appendix B*) shows a situation that is less frequent in practice where, on average, two extra items are delivered by the supply system for an average delivered quantity of 125 items. The extra items are not recorded by the IS system under the absence of a delivery audit. Since there is always more stock than what is shown in the IS, there are no u_2 cases in this scenario. However, the illustration of this scenario is motivated by the intention to show the very negative performance of the FI case where the PH level is continuously increasing due to the accumulation of positive PH errors. Without a manual update of the IS level when a u_1 event occurs, the inventory level increases considerably as shown in the results.

Scenario 4 (illustrated in Figure B4 in *Appendix B*) shows a situation subject to positive multiplicative and negative additive IS errors compensating each other. The results illustrate the efficiency of both learning assumptions (M and A) in handling a mixture of additive and multiplicative errors with a neutral effect.

	Scenario 1			**Scenario 2**			**Scenario 3**			**Scenario 4**		
	CSL IS	**CSL PH**	**E[holding]**	**CSL IS**	**CSL PH**	**E[holding]**	**CSL IS**	**CSL PH**	**E[holding]**	**CSL IS**	**CSL PH**	**E[holding]**
FI	0.9	0.61	11.84	0.9	0.78	16.63	0.91	1	158.12	0.87	0.88	25.89
PI	0.9	0.71	18.36	0.9	0.81	18.52	0.91	1	39.96	0.87	0.87	24.82
K	0.89	0.7	16.81	0.91	0.84	19.52	0.91	1	40.31	0.89	0.88	24.93
A	0.89	0.7	17.28	0.91	0.83	19.54	0.92	1	42.95	0.87	0.87	24.36
M	0.89	0.7	17.54	0.91	0.83	19.55	0.92	1	42.96	0.88	0.87	24.41

Table 5: Simulation results

5. Conclusion

Inventory record inaccuracies constitute a major concern in the modern business world. Various factors may lead to errors impacting the physical and/or the information system stock, which subsequently results in a discrepancy between them with detrimental effects in terms of costs and customer service level. This work has been concerned with the development of analytical solutions in a single- and multi-period review system in e-retailing/B2B contexts. Two case organizations operating in the B2B environment provided the empirical background to this work, and the analysis of their inventory record inaccuracies led to an elevated understanding of how errors may occur and how they can be modeled.

Our study considered different settings of errors, cost and service level optimization with a learning mechanism about the errors applicable in both retailing and e-retailing/B2B contexts. Some well-known managerial insights in the retailing IRI literature were revisited and found not to apply in an e-retailing/B2B context. In particular, we found that: i) the behaviour of the order quantity is very sensitive to the values of the IS and PH error moments; ii) inventory policies designed under the retailing context do not optimize the system under the e-retailing/B2B context; iii) ordering more does not necessarily improve the service level; iv) in both retailing and e-retailing/B2B contexts, a good estimation of the errors' distributions is an essential condition to effectively control the inventory system. The learning mechanism we propose extends the zero balance walk technique and takes advantage of 'computer' signals about the IRI issue.

Despite modelling five sources of uncertainty (four referring to the errors and one to demand), the solution proposed in our work is tractable and easy to deploy in practice, particularly with regards to the optimization based on the service level. There are some key operationalized suggestions emerging from our work: i) controlling the inventory system in an e-retailing/B2B context with only the classical IS service level is not enough to ensure customer satisfaction. An adjustment of the service level definition is needed to include the commitment satisfaction measure of performance; ii) it is important and possible to use each single signal coming from the inventory system to build a better knowledge about the errors' parameters; iii) redesigning the inventory policies by integrating the distributions' parameters is an efficient way to limit the impact of IRI.

Given the importance of IRI in modern business settings, and the insights our study revealed in electronic ordering contexts, research in the following areas would appear to be merited:

- First of all, there is a need for empirical investigations to assess the benefit of our proposed solutions on retailers' costs and profit in real world settings; this also includes an empirical analysis of the relative importance of the error sources identified in the customer claim reports provided by the two case companies in e-retailing & brick-and-mortar retailing environments.
- There is a natural extension of the work conducted here to other inventory policies. For example, amending the protection interval to account for demand over the lead time rather than lead time + review period, would extend the theory to re-order point, order quantity (s,Q) policies.
- Extending the scope of the study to the omni-channel context is also a natural next step of research to investigate the interaction of the IRI in online and offline businesses.
- Finally, the model proposed in this paper could be extended to permit a manual adjustment of the stock record by the decision maker. Given our experience with firms facing IRI issues where manual adjustments of stock records are often an additional source of inaccuracies, it would be interesting to investigate both the possible positive (fine-tuning the commitment in

light of the revealed demand) and the possible negative (introduce additional sources of transaction errors into the system) impacts of manual adjustments on the system.

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