Probabilistic Abstract Dialectical Frameworks

Sylwia Polberg[1]* and Dragan Doder[2]**

[1] Vienna University of Technology, Institute of Information Systems
Favoritenstraße 9-11, 1040 Vienna, Austria
[2] University of Luxembourg, Computer Science and Communications
Rue Richard Coudenhove-Kalergi 6, L-1359 Luxembourg

Abstract. Although Dung's frameworks are widely approved tools for abstract argumentation, their abstractness makes expressing notions such as support or uncertainty very difficult. Thus, many of their generalizations were created, including the probabilistic argumentation frameworks (PrAFs) and the abstract dialectical frameworks (ADFs). While the first allow modeling uncertain arguments and attacks, the latter can handle various dependencies between arguments. Although the actual probability layer in PrAFs is independent of the chosen semantics, new relations pose new challenges and new interpretations of what is the probability of a relation. Thus, the methodology for handling uncertainties cannot be shifted to more general structures without any further thought. In this paper we show how ADFs are extended with probabilities.

Keywords: abstract argumentation, abstract dialectical frameworks, probabilistic argumentation frameworks

1 Introduction

Within the last decade, argumentation has emerged as a central field of Artificial Intelligence [1]. One of its subfields is the abstract argumentation, at the heart of which lies the Dung's argumentation framework (AF) [2]. Although quite powerful, for many applications Dung's AFs appear too abstract in order to conveniently model all aspects of an argumentation problem. This has led to the development of their numerous enrichments [3]. One of AF's shortcomings is the insufficient handling of the levels of uncertainty [4], an aspect which typically occurs in domains, where diverging opinions are raised. This calls for augmenting AFs with probabilities [4, 5]. They serve as a basis to generate AF–subgraphs, which naturally represent the possible situations induced by the uncertainties in a given probabilistic framework (PrAF). From them we obtain extensions and their associated uncertainty coming from the subgraphs. Consequently, the uncertainty layer is independent of the underlying semantics and of the framework itself, which is considered one of its greatest strengths.

* The author is supported by the Vienna PhD School of Informatics and the FWF project I1102. We would also like to thank Stefan Woltran for his valuable comments.
** The author is supported by the National Research Fund (FNR) of Luxembourg through project PRIMAT.

Argument and attack uncertainties had proved to be a useful concept. Therefore, it is not unreasonable that an AF enrichment should also incorporate them [6]. Due to the independency of the probability layer, it was claimed that it can be done easily [5]. However, it is natural to expect that the probability of a positive relation between the arguments may be interpreted in different ways. While we may doubt if e.g. a positive interaction between a and b will be carried out, we can also question whether b requires a (or only a) to hold. Thus, the conditions for accepting an argument might be uncertain. Those two interpretations of relation probabilities are modeled in exactly the opposite way. Assuming that the relation does not occur, in the first case b would not be acceptable, while in the latter it would not be a problem. Generating the subgraphs in the usual manner would allow us to model only one of the scenarios at a time. Thus, new relations pose new challenges and this research should not be dismissed so easily.

Unfortunately, AFs permit only binary conflict. Among the most general structures addressing this issue [3] are abstract dialectical frameworks (ADFs) [7]. They assign acceptance conditions to arguments, which can be seen as a Boolean functions stating if its "owner" can be accepted or not w.r.t. given arguments. Although various other frameworks that can handle positive relations were proposed [8–10], our preliminary findings show that they can be expressed within ADFs. Thus, ADFs make a good base for probabilistic frameworks that would allow us to model various uncertain relations, not limited to attack or support only. In this paper we create a framework joining both the uncertainty and the relation research – the probabilistic abstract dialectical framework. We show that it generalizes ADFs as well as PrAFs. Our goal is to model situations when the requirements to accept an argument might be uncertain. We achieve it by assigning not a single acceptance condition to an argument, but a number of them. We then adopt the subgraph approach to our new setting. Consequently, we are able to generalize the methodology introduced by PrAFs to handle different interpretations of probability. Finally, we discuss other possible methods of augmenting ADFs with uncertainties and give pointers for future work.

2 Dung's Framework and its Probabilistic Extensions

Definition 1. *A* ***Dung's argumentation framework*** *(AF) is a pair $F = \langle A, R\rangle$ where A is a set of arguments and $R \subseteq A \times A$ is the attack relation.*

An argument $a \in A$ is ***defended*** *(in F) by $S \subseteq A$ if $\forall b \in A$ s.t. $(b, a) \in R$, $\exists c \in S$ s.t. $(c, b) \in R$. A set $S \subseteq A$ is:*

- ***conflict–free****, if there are no $a, b \in S$, such that $(a, b) \in R$.*
- ***stable****, if it is conflict–free and for all $a \in A \setminus S$, $\exists b \in S$, s.t.$(b, a) \in R$;*
- ***admissible****, if it is conflict–free and each $a \in S$ is defended (in F) by S;*
- ***complete****, if it is admissible and each a defended (in F) by S is in S;*
- ***grounded****, if it is the least w.r.t. $\subseteq$ complete;*
- ***preferred****, if it is maximal w.r.t. $\subseteq$ admissible.*

By $\sigma(F)$ we will denote the extensions of F under semantics σ listed above.

For an AF $F = (A, R)$ and a set $A' \subseteq A$, by $R_{A'}$ we denote the restriction of R to $A' \times A'$, i.e. $R_{A'} = \{(a, b) \in R \mid a, b \in A'\}$.

We will now recall the probabilistic frameworks [5]. In this setting, instead of asking if a set of arguments is an extension of a given semantics, one now expects to analyze the probability that it is. This is addressed by the idea of subgraphs, which express the possible interpretations of the original probabilistic framework F_{PR} in terms of AFs, whereby it is not sure that all arguments or attacks in F_{PR} actually appear in a given AF. The collection of such graphs represents the possible scenarios induced by the probabilities in the initial structure.

Definition 2. *A **probabilistic argumentation framework** (PrAF) F_{PR} is a tuple $\langle A, R, P_A, P_R\rangle$, where $\langle A, R\rangle$ is a Dung's framework, $P_A : A \longrightarrow (0, 1]$ and $P_R : R \longrightarrow (0, 1]$ are the probabilities of arguments and attacks.*

Definition 3. *Let $F_{PR} = (A, R, P_A, P_R)$ be a PrAF. A **subgraph** [3] G of F_{PR} (denoted $G \sqsubseteq F_{PR}$) is a pair (A', R') s.t. 1) $A' \subseteq A$ and $\{a \in A \mid P_A(a) = 1\} \subseteq A'$, and 2) $R' \subseteq R_{A'}$ and $\{(a, b) \in R \mid a, b \in A', P_A(a) = P_A(b) = 1, P_R(a, b) = 1\} \subseteq R'$. $s(F_{PR}) = \{G \mid G \sqsubseteq F_{PR}\}$ denotes the set of all subgraphs of F_{PR}.*

Given a semantics σ and its potential extension E, we determine the subgraphs of an F_{PR} that have E as their AF σ–extension. The sum of the probabilities of such subgraphs gives us the final probability that E is a σ–extension of F_{PR}.

Definition 4 ([5]). *Let $F_{PR} = \langle A, R, P_A, P_R\rangle$ be a PrAF and let $G = \langle A', R'\rangle \sqsubseteq F_{PR}$. Then the probability of G is:*

$$p_{F_{PR}}(G) = (\prod_{a\in A'} P(a))(\prod_{a\in A\setminus A'} (1-P(a)))(\prod_{r\in R'} P_R(r))(\prod_{r\in R_{A'}\setminus R'} (1-P_R(r))). \quad (1)$$

Theorem 1 ([5]). *The function $p_{F_{PR}}$ is a probabilistic distribution on the set $s(F_{PR})$, i.e., a nonnegative function s.t. $\sum_{G\sqsubseteq F_{PR}} p_{F_{PR}}(G) = 1$.*

Definition 5. *Let $F_{PR} = \langle A, R, P_A, P_R\rangle$ be a PrAF, $E \subseteq A$ a set of arguments, and $\sigma \in \{$conflict-free, admissible, complete, preferred, stable, grounded$\}$ a semantics. The set of subgraphs of F_{PR} for which E is a σ-extension is $Q^{\sigma}_{F_{PR}}(E) = \{G \in s(F_{PR}) \mid E \in \sigma(G)\}$. The probability that $E \subseteq A$ is in $\sigma(F_{PR})$ is defined as:*[4]

$$P^{\sigma}_{F_{PR}}(E) = \sum_{G\in Q^{\sigma}_{F_{PR}}(E)} p_{F_{PR}}(G). \quad (2)$$

3 Abstract Dialectical Frameworks

Abstract dialectical frameworks have been defined in [7] and further developed in [11–16]. Their main goal is to be able to express arbitrary relations, which is achieved by the use of acceptance conditions. They define what sets of arguments related to a given argument should be present for it to be accepted or rejected.

[3] In [5], subgraphs are called AFs induced from F_{PR}

[4] The definition from [5] is more general, it computes the probability that a set is a subset of a σ extension.

Definition 6. *An* ***abstract dialectical framework*** *(ADF) as a tuple* $\langle S, L, C\rangle$*, where* S *is a set of* ***arguments*** *(nodes),* $L \subseteq S \times S$ *is a set of* ***links*** *(edges) and* $C = \{C_s\}_{s\in S}$ *is a set of* ***acceptance conditions****, one condition per each argument. An* ***acceptance condition*** *is given by a total function* $C_s : 2^{par(s)} \to \{in, out\}$*, where* $par(s) = \{p \in S \mid (p, s) \in L\}$ *is the set of* ***parents*** *of* s*.*

One can also use the propositional representation, i.e. with $C = \{\varphi_s\}_{s\in S}$ where φ_s is a propositional formula over the parents of s. Since the links no longer define the nature of the connections between the arguments and can be easily extracted from the conditions, we can use shortened notation $D = \langle S, C\rangle$.

Instead of returning sets of accepted arguments, the semantics of ADFs from [11] produce three–valued interpretations in which arguments are assigned truth–values from $\{\mathbf{t}, \mathbf{f}, \mathbf{u}\}$. The values are compared w.r.t. precision (information) ordering $\leq_i$, defined as $\mathbf{u} \leq_i \mathbf{t}$ and $\mathbf{u} \leq_i \mathbf{f}$. It can be extended to interpretations: given two interpretations v and v' on S, $v \leq_i v'$ iff $\forall_{s\in S}\, v(s) \leq_i v'(s)$. In case v is three and v' two–valued (i.e. has only $\mathbf{f}$ and $\mathbf{t}$ mappings), we say that v' *extends* v. The set of all two–valued interpretations extending v is denoted $[v]_2$. The pair $(\{\mathbf{t}, \mathbf{f}, \mathbf{u}\}, \leq_i)$ forms a complete meet–semilattice with the meet operation $\sqcap$ defined as: $\mathbf{t} \sqcap \mathbf{t} = \mathbf{t}$, $\mathbf{f} \sqcap \mathbf{f} = \mathbf{f}$ and $\mathbf{u}$ in all other cases. $\sqcap$ can also be defined for interpretations: for interpretations v and v' on S, $v \sqcap v' = v''$ where $\forall_{s\in S}\, v''(s) = v(s) \sqcap v'(s)$. Meet simply checks whether two interpretations agree on assignments or not. Finally, we will use v^x to denote a set of arguments mapped to x by v, where $x \in \{\mathbf{t}, \mathbf{f}, \mathbf{u}\}$. We can now recall the ADF semantics from [11], which are based on the notion of a characteristic operator:

Definition 7. *Let* $D = \langle S, L, C\rangle$ *with* $C = \{\varphi_s\}_{s\in S}$ *be an ADF,* V_S *the set of all three–valued interpretations defined on* S*,* $s \in S$ *and* v *an interpretation in* V_S*. The* ***three–valued characteristic operator*** *of* D *is a function* $\Gamma_D : V_S \to V_S$ *s.t.* $\Gamma_D(v) = v'$ *with* $v'(s) = \bigsqcap_{w\in[v]_2} C_s(par(s) \cap w^{\mathbf{t}})$*. We say that* v *is:*
- ***admissible*** *iff* $v \leq_i \Gamma_D(v)$*;*
- ***complete*** *iff* $v = \Gamma_D(v)$*;*
- ***preferred*** *iff it is* $\leq_i$*–maximal admissible;*
- ***grounded*** *iff it is the least fixpoint of* Γ_D*.*

The stable semantics is a slightly different case, as formally we receive a set, not an interpretation. However, stability leaves nothing undecided, and we can just map arguments not in the set to $\mathbf{f}$. The definition uses the concept of a reduct. Reduction of an acceptance condition simply means that the occurrences of rejected arguments are replaced by $\mathbf{f}$ (one can also use $\perp$).

Definition 8. *Let* $D = \langle S, C\rangle$ *with* $C = \{\varphi_s\}_{s\in S}$ *be an ADF. We say that* $M \subseteq S$ *is a* ***model*** *of* D *iff* $\forall m \in M, C_m(S \cap par(m)) = in$ *and* $\forall\, s \in S,\ C_s(M \cap par(s)) = in$ *implies* $s \in M$*. A* ***reduct*** *of* D *w.r.t.* M *is* $D^M = (M, C^M)$*, where for* $m \in M$ *we set* $C^M_m = \varphi_m[b/\mathbf{f} : b \notin M]$*. Let* gv *be the grounded model of* D^M*. Model* M *is* ***stable*** *iff* $M = gv^{\mathbf{t}}$*.*

Finally, we recall that ADFs properly generalize AFs [11].

Definition 9. *For an $F = \langle A, R \rangle$, the associated ADF is $D_F = \langle A, R, \{\varphi_a\}_{a \in A} \rangle$ with $\varphi_a = \bigwedge_{b:(b,a) \in R} \neg b$ for $a \in A$. For an interpretation v, the set $E_v = \{a \in A \mid v(a) = \mathbf{t}\}$ defines the unique extension associated with v.*

Theorem 2. *Let F be an AF and D_F its associated ADF. An extension E is in $\sigma(F)$, where $\sigma \in \{$admissible, complete, preferred, stable, grounded$\}$, iff it is in $\sigma(D_F)$.*

4 Probabilistic Abstract Dialectical Frameworks

The probability of a positive interaction between arguments can be interpreted in several ways. First of all, it can happen that a given argument is actually supporting another one only with a certain probability, which canhave its source in e.g. ambiguity or incompletion. However, it is also possible that the requirements to accept a given argument change. This brings us to the idea of ADFs in which the acceptance conditions are assigned a level of uncertainty. In order to grasp the probabilities of different scenarios, instead of a single condition, an argument receives a block of acceptance conditions. Each member of the block is assigned a probability in a way that they all sum up to 1. The uncertainty of a condition should be understood as the uncertainty of the argument's requirements for acceptance. This also means that at a given point, only a single condition of a block can "happen". However, it does not mean that only one relation targeted at this argument can occur. If we consider an AF and its associated ADF (Definition 9), both augmented with probabilities, it is not the case that every condition and its probability would correspond to one attack and its probability in the original framework. ADF conditions provide a bigger point of view and express the general requirements of an argument to hold. Given an argument attacked by two others (with some probabilities), ADFs would model the situation with four conditions – when both, none, and only one of the attacks occur.

The idea of our method of determining the probability of an extension is similar to the one in PrAFs. Just like in PrAFs we generated AF subgraphs, in probabilistic ADFs we will create ADFs. This brings us to another reason why the total probability of a condition block has to be 1. In PrAFs, if we knew that a attacks b with a chance 0.3, then we also knew that a does not attack b with a chance 0.7. In ADFs, should a given acceptance condition be used with probability 0.3, what condition should occur with 0.7? The state of an argument is always defined by the condition, thus on any occasion one has to be assigned. Consequently, it is important that the total probability of a block is 1.

Let us now describe our party example and introduce the framework. We can observe that ADFs can express relations between arguments (see argument d) which go beyond the usual understanding of attack or support and that cannot, to the best of our knowledge, be conveniently modeled in any other framework.

Example 1. Julia is throwing a dinner party and is deciding with her husband Mark which of their friends – Anne, Bernard, Cecilia and David – to invite. Bernard and Cecilia are taking care of their sick mother and the two of them will not be able to come at the same time. Mark was told that Anne had to reject one of Bernard's projects at work and he might not want to meet with her now. Julia believes that David is still angry at Cecilia for their bad break up and will not come if she is invited unless Anne, who is his current girlfriend, also shows up. However, Mark thinks that David is fine with it now and Cecilia's presence should not be a problem, but he might prefer to come with Anne anyway since she is leaving for a business trip soon. Finally, they both agree that even though Anne would like to come, she might not be able to due to the travel preparations. We now construct arguments a, b, c and d representing Anne, Bernard, Cecilia and David coming to the dinner, and their possible acceptance conditions. The condition of a is just $\top$, since Anne's decision does not depend on anyone else. However, since she is busy, a is assigned a probability of 0.5. The condition of b might be just $\neg c$ – since Bernard cannot come together with Cecilia – but it can also be $\neg a \wedge \neg c$ due to issues with Anne. We give both of them a 0.5 chance. Similarly, c is assigned $\neg b$. Finally, we have that condition of d might be $a \vee \neg c$, reflecting Davids problem with Cecilia, or just a in case he sorted it out and just wants time with Anne. We assign to them probabilities 0.7 and 0.3 respectively.

Definition 10. *A* ***probabilistic abstract dialectical framework*** *(PrADF) is a tuple* $D_{PR} = \langle A, \{C_a\}_{a \in A}, P_A, \{P_{C_a}\}_{a \in A} \rangle$*, where* A *is a set of* ***arguments****,* $C_a = \{\varphi_{a,i} \mid i = 1, \ldots, n_a\}$ *is a set of possible* ***acceptance conditions*** *of* a*,* $P_A : A \to (0,1]$ *is the* ***probability of arguments*** *and* $P_{C_a} : C_a \to (0,1]$ *s.t.* $\sum_{\varphi_{a,i} \in C_a} P_C(\varphi_{a,i}) = 1$*, is the* ***probability of acceptance conditions****.*

We can now continue with the definition of a subframework in our new setting.We first choose an arbitrary subset of arguments – the only restriction is that it contains the ones that are certain to happen. We then assign each argument an acceptance condition from its block and thus obtain our subframework. However, it can happen that an argument occurring in the condition no longer appears in our set. Therefore, what needs to be performed is the reduction of the conditions (see Definition 8). This brings us to the definition of a subframework:

Definition 11. *Let* $D_{PR} = \langle A, \{C_a\}_{a \in A}, P_A, \{P_{C_a}\}_{a \in A} \rangle$ *be a PrADF and* $A' \subseteq A$ *a set of arguments s.t.* $\{a \in A \mid P_A(a) = 1\} \subseteq A'$*. Given a collection of indices* $\{i_a\}_{a \in A'}$*, the induced* ***subframework*** *is* $D' = \langle A', \{\varphi^{A'}_{a,i_a}\}_{a \in A'} \rangle$*. The set of all subframeworks of* D_{PR} *is denoted by* $s(D_{PR})$*.*

Note that it is possible that two acceptance conditions of an argument a that are initially different in a PrADF, i.e. $\varphi_{a,i} \neq \varphi_{a,j}$ for some $j \neq i$, become equivalent in some subframework $D' = \langle A', \{\varphi^{A'}_{a,i}\}_{a \in A'} \rangle$ (*i.e.* $\varphi^{A'}_{a,i} = \varphi^{A'}_{a,j}$[5]). Thus, the definition of the probability of D' has to take this situation into account.

Definition 12. *Let* $D_{PR} = \langle A, \{C_a\}_{a \in A}, P_A, \{P_{C_a}\}_{a \in A} \rangle$ *be a PrADF and* $D' = \langle A', \{\varphi^{A'}_{a,i_a}\}_{a \in A'} \rangle$ *its subframework.The probability of* D' *is defined as:*

$$p_{D_{PR}}(D') = (\prod_{a \in A'} P_A(a))(\prod_{a \in A \setminus A'} (1 - P_A(a)))(\prod_{a \in A'} \sum_{j : \varphi^{A'}_{a,j} = \varphi^{A'}_{a,i}} P_{C_a}(\varphi_{a,j})). \quad (3)$$

[5] We identify the equivalent formulas, since they induce the same acceptance functions.

Theorem 3. *Given a PrADF $D_{PR} = \langle A, \{C_a\}_{a\in A}, P_A, \{P_{C_a}\}_{a\in A}\rangle$, the function $p_{D_{PR}}$ is a probabilistic distribution on the set $s(D_{PR})$, i.e., a nonnegative function s.t.* $\sum_{D'\in s(D_{PR})} p_{D_{PR}}(D') = 1$.

We will now proceed with PrADF semantics, focusing on the extensions associated with ADF interpretations.

Definition 13. *Let $D_{PR} = \langle A, \{C_a\}_{a\in A}, P_A, \{P_{C_a}\}_{a\in A}\rangle$ be a PrADF and $E \subseteq A$. The set of all subframeworks D' of D_{PR} s.t. E is a σ extension of D_{PR}, where $\sigma \in$ {admissible, complete, preferred, stable, grounded}, is:*

$$Q^{\sigma}_{D_{PR}}(E) = \{D' \sqsubseteq D_{PR} \mid \exists v \in \sigma(D') \text{ s.t. } v^{\mathbf{t}} = E\}. \tag{4}$$

The probability that E is a σ-extension of D_{PR} is defined as:

$$P^{\sigma}_{D_{PR}}(E) = \sum_{D'\in Q^{\sigma}_{D_{PR}}(E)} p_{D_{PR}}(D'). \tag{5}$$

Example 2. Let us now construct a PrADF D for our scenario from Example 1. Our arguments are $\{a, b, c, d\}$, where $P_A(a) = 0.5$ and since there are no reasons against, $P_A(b) = P_A(c) = P_A(d) = 1$. As discussed before, $\varphi_a = \top$. This is the only condition of a and thus $P_{C_a}(\varphi_a) = 1$. For b we have $\varphi_{b_1} = \neg c$ and $\varphi_{b_2} = \neg a \wedge \neg c$ with probabilities $P_{C_b}(\varphi_{b_1}) = P_{C_b}(\varphi_{b_2}) = 0.5$. In the case of c, $\varphi_c = \neg b$ and has chance of 1 just like a. Finally, for d we have $\varphi_{d_1} = a \vee \neg c$ and $\varphi_{d_2} = a$ with chances $P_{C_d}(\varphi_{d_1}) = 0.7$ and $P_{C_d}(\varphi_{d_2}) = 0.3$. We obtain 6 possible subframeworks: $D_{G_1} = \langle\{a, b, c, d\}, \{\varphi_a = \top, \varphi_b = \neg c, \varphi_c = \neg b, \varphi_d = a\}\rangle$, $D_{G_2} = \langle\{a, b, c, d\}, \{\varphi_a = \top, \varphi_b = \neg a \wedge \neg c, \varphi_c = \neg b, \varphi_d = a\}\rangle$, $D_{G_3} = \langle\{b, c, d\}, \{\varphi_b = \neg c, \varphi_c = \neg b, \varphi_d = \neg c\}\rangle$, $D_{G_4} = \langle\{a, b, c, d\}, \{\varphi_a = \top, \varphi_b = \neg a \wedge \neg c, \varphi_c = \neg b, \varphi_d = a \vee \neg c\}\rangle$, $D_{G_5} = \langle\{a, b, c, d\}, \{\varphi_a = \top, \varphi_b = \neg c, \varphi_c = \neg b, \varphi_d = a \vee \neg c\}\rangle$, and $D_{G_6} = \langle\{b, c, d\}, \{\varphi_b = \neg c, \varphi_c = \neg b, \varphi_d = \bot\}\rangle$. Their probabilities and extension are listed in Table 1. Note D_{G_3} and D_{G_6} can be induced in two ways, as reducing φ_{b_1} and φ_{b_2} w.r.t. $\{b, c, d\}$ leads to equivalent formulas.

As expected, there is no possibility of inviting everyone. The next options in which we get the most friends are extensions $\{a, b, d\}$ and $\{a, c, d\}$. The first one has probability $p_D(D_{G_1}) + p_D(D_{G_5}) = 0.25$ if we assume preferred or complete semantics, but 0 in case of grounded.The other set occurs in $D_{G_1}, D_{G_2}, D_{G_4}$ and D_{G_5}, which yields probability 0.25 w.r.t. grounded semantics and 0.5 otherwise. Inviting just Anne and David, i.e. $\{a, d\}$, would have a chance of 0.5 in admissible semantics ($D_{G_1}, D_{G_2}, D_{G_4}$ and D_{G_5}), 0.25 in complete and grounded (D_{G_1} and D_{G_5}), and would not be possible at all in preferred and stable cases. Going just for the manly team - $\{b, d\}$ would give us 0 probability in the grounded case, 0.52 in admissible and 0.35 in any other.

Note that by setting argument probability to 1 and using single element acceptance condition blocks, we easily retrieve ADFs from PrADFs. We close this section by showing that PrADFs properly generalize PrAFs.

Definition 14. *The PrADF associated to the PrAF $F_{PR} = \langle A, R, P_A, P_R\rangle$ is $D_{F_{PR}} = \langle A, \{C_a\}_{a\in A}, P_A, \{P_{C_a} \mid a \in A\}\rangle$, where:*

Table 1. Subframeworks of D and their extensions.

$s(D)$	p_D	*stb*	*grd*	*adm*	*prf*	*com*
D_{G_1}	0.075	{a, b, d}, {a, c, d}	{a, d}	∅, {a}, {b}, {c},{a, b}, {a, c}, {a, d}, {a, b, d}, {a, c, d}	{a, b, d}, {a, c, d}	{a, d}, {a, b, d}, {a, c, d}
D_{G_2}	0.075	{a, c, d}	{a, c, d}	∅, {a}, {c}, {a, c}, {a, d}, {a, c, d}	{a, c, d}	{a, c, d}
D_{G_3}	0.35	{c}, {b, d}	∅	∅, {b}, {c}, {b, d}	{c}, {b, d}	∅, {c}, {b, d}
D_{G_4}	0.175	{a, c, d}	{a, c, d}	∅, {a}, {c}, {a, c}, {a, d}, {a, c, d}	{a, c, d}	{a, c, d}
D_{G_5}	0.175	{a, b, d}, {a, c, d}	{a, d}	∅, {a}, {b}, {c}, {a, b}, {a, c}, {a, d}, {b, d}, {a, b, d}, {a, c, d}	{a, b, d}, {a, c, d}	{a, d}, {a, b, d}, {a, c, d}
D_{G_6}	0.15	{b}, {c}	∅	∅, {b}, {c}	{b}, {c}	∅, {b}, {c}

- $C_a = \{\bigwedge_{(b,a)\in R'} \neg b \mid R' \subseteq R\}$
- $P_{C_a}(\bigwedge_{(b,a)\in R'} \neg b) = (\prod_{(b,a)\in R'} P_R((b,a)))(\prod_{(b,a)\in R\setminus R'}(1 - P_R((b,a))))$

Theorem 4. *Let $F_{PR} = \langle A, R, P_A, P_R\rangle$ be a PrAF and let $D_{F_{PR}}$ be its associated PrADF. Then $P^\sigma_{F_{PR}}(E) = P^\sigma_{D_{F_{PR}}}(E)$.*

5 Discussion and future work

One of the most interesting observations we have made in our research is the fact that the probabilities of acceptance conditions allow us to express the probabilities of arguments. This method is unique to ADFs and is possible thanks to the fact that we can have a $\varphi_s = \bot$ condition, which is simply interpreted as *s does not exist.* Consequently, an argument–based PrADF can be transformed into an acceptance condition based one. Given an argument a assigned probability arg_1 and conditions $C_1, ..., C_n$ with probabilities $p_1, ..., p_n$, we can shift the argument uncertainty into a condition. We produce an additional formula $C_{n+1} = \bot$ with probability $1-arg_1$ and alter the probabilities of existing conditions by multiplying them by arg_1. Consequently, PrADFs can be improved and that a simpler, cleaner formulation can be created. We would like to fully develop this idea in our future work and create an approach without the independency assumption.

A particular line of research in abstract argumentation concerns the formalization of argumentation semantics in terms of logics. A uniform logical formalization for PrAFs using probabilistic logic was already developed in [17]. We believe that this approach may be further extended in order to logically formalize PrADFs. Finally, we would like to study the complexity of PrADFs and their semantics and possibly provide an implementation.

References

1. Bench-Capon, T.J.M., Dunne, P.E.: Argumentation in artificial intelligence. Artif. Intell. **171**(10-15) (2007) 619–641
2. Dung, P.M.: On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games. Artif. Intell. **77**(2) (1995) 321–358
3. Brewka, G., Polberg, S., Woltran, S.: Generalizations of Dung frameworks and their role in formal argumentation. Intelligent Systems, IEEE **29**(1) (Jan 2014) 30–38
4. Hunter, A.: A probabilistic approach to modelling uncertain logical arguments. Int. J. Approx. Reasoning **54**(1) (2013) 47–81
5. Li, H., Oren, N., Norman, T.J.: Probabilistic argumentation frameworks. In Modgil, S., Oren, N., Toni, F., eds.: Proc. TAFA. Volume 7132 of LNCS., Springer (2012) 1–16
6. Li, H., Oren, N., Norman, T.J.: Relaxing independence assumptions in probabilistic argumentation. In: Proceedings of ArgMAS 2013. (2013) to appear
7. Brewka, G., Woltran, S.: Abstract dialectical frameworks. In Lin, F., Sattler, U., Truszczyński, M., eds.: Proceedings of the 12th International Conference on Principles of Knowledge Representation and Reasoning (KR 2010), AAAI Press (2010) 780–785
8. Cayrol, C., Lagasquie-Schiex, M.C.: Bipolarity in argumentation graphs: Towards a better understanding. Int. J. Approx. Reasoning **54**(7) (2013) 876–899
9. Nouioua, F.: AFs with necessities: Further semantics and labelling characterization. In Liu, W., Subrahmanian, V., Wijsen, J., eds.: Proc. SUM '13. Volume 8078 of LNCS. Springer Berlin Heidelberg (2013) 120–133
10. Polberg, S., Oren, N.: Revisiting support in abstract argumentation systems. In: Proceedings of COMMA 2014. (2014) Forthcoming.
11. Brewka, G., Ellmauthaler, S., Strass, H., Wallner, J.P., Woltran, S.: Abstract dialectical frameworks revisited. In: Proc. IJCAI'13, AAAI Press (2013) 803–809
12. Polberg, S., Wallner, J.P., Woltran, S.: Admissibility in the abstract dialectical framework. In: Proc. CLIMA'13. Volume 8143 of LNCS., Springer (2013) 102–118
13. Strass, H.: Approximating operators and semantics for abstract dialectical frameworks. Artificial Intelligence **205** (2013) 39 – 70
14. Strass, H.: Instantiating knowledge bases in abstract dialectical frameworks. In: Proc. CLIMA'13. Volume 8143 of LNCS., Springer (2013) 86–101
15. Strass, H., Wallner, J.P.: Analyzing the Computational Complexity of Abstract Dialectical Frameworks via Approximation Fixpoint Theory. In: Proc. KR '14, Vienna, Austria (July 2014) Forthcoming.
16. Polberg, S.: Extension–based semantics of abstract dialectical frameworks. Technical Report DBAI-TR-2014-85, Institute for Information Systems, Technical University of Vienna (2014)
17. Doder, D., Woltran, S.: Probabilistic argumentation frameworks - a logical approach. In: Proc. SUM'14. LNAI, Oxford, UK, Springer (September 2014) Forthcoming.