Dynamic Effects of Monetary Policy Shocks on Macroeconomic Volatility∗

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Abstract
We develop a VAR that allows the estimation of the impact of monetary policy shocks on volatility. Estimates for the US suggest that an increase in the policy rate by 1% is associated with a rise in unemployment and inflation volatility of about 15%. Using a New Keynesian model, with search and matching labour frictions and Epstein-Zin preferences we show that these volatility effects are driven by the coexistence of agents’ fears of unemployment and concerns about the (in)ability of the monetary authority to reverse deviations from the policy rule with the impact magnified by the agents’ preferences.

Keywords: DSGE, Non-Linear SVAR, New Keynesian, Search and Matching Frictions, Epstein-Zin preferences, Stochastic Volatility

JEL Classification: E30, E40, E52, C11, C13, C15, C50

1 Introduction
Uncertainty shocks can cause business cycle fluctuations and drive policy, but policy changes themselves can lead to changes in uncertainty. This paper investigates empirically and theoretically to what extent monetary policy shocks can affect the volatility of macroeconomic variables. It finds an important transmission channel from monetary policy to endogenous uncertainty, both in the data and using a model where households react to the the anticipated risk of long unemployment spells. Using a structural VAR with stochastic volatility (extended to allow for feedback from the endogenous variables to the volatility), we show that monetary policy shocks increase macroeconomic volatility and the results are robust across identification schemes. It is also shown here that the monetary policy shock is responsible for about 40% to 50% of the forecast error variance contribution of all level shocks to the volatility of the endogenous variables. These volatility contributions are substantially higher than monetary policy shock’s shares of explaining the level series (as it is commonly found in the literature and illustrated again here).

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To understand how volatility is affected when monetary authorities decide to deviate unexpectedly from their Taylor type reaction function, we employ a stylised New Keynesian DSGE model, with search & matching labour frictions and Epstein-Zin preferences. The model is estimated using limited information impulse response matching techniques. Although the literature has questioned the ability of “simple” search and matching New Keynesian models to jointly replicate the dynamics of both unemployment and inflation (Krause and Lubik (2007) and Gertler et al. (2008)), our estimated model reproduces VAR responses remarkably well.

Simulations from the theoretical model suggest that the transmission of the policy shock to volatility depends on three modelling features: (i) the presence of labour market real frictions, (ii) the monetary authorities’ desire for gradual policy adjustments and (iii) the existence of Epstein-Zin preferences. It is the coexistence of agents’ fears about being prolonged unemployment and policymakers’ preference for interest rate smoothing that causes volatility to increase significantly. It is only in this scenario that Epstein-Zin preferences have a quantitatively meaningful role. From an economic point of view, households acknowledge the real risk of becoming unemployed and the fact that during the unemployment spells additional adverse shock may occur. However, it is the combination of these risks together with the policy-rate smoothing parameter that causes monetary policy to have significant volatility effects and not the shock per-se. In other words, agents are not overly concerned that authorities are able to deviate unexpectedly from their objective function but they significantly price the fact that the central bank cannot fully undo such actions resulting in prolonged unemployment spells where they are vulnerable to further adverse shocks and future uncertainty rises.

As in the empirical and theoretical literature on the impact of uncertainty shocks (Bloom (2009) and Fernandez-Villaverde et al. (2015)), our paper highlights the importance of these type of disturbances. However, the focus and results of our analysis are novel in one key respect. Unlike the bulk of the uncertainty literature, this paper attempts to model the transmission of monetary policy shocks to economic volatility and thus takes a step towards treating economic volatility as endogenous.

Regarding our empirical contribution, the study of Ludvigson et al. (2015) is the closely related. Ludvigson et al. (2015) develop a procedure that separates movements in volatility caused by primitive (first order) shocks and by uncertainty shocks. However, crucially, they do not identify the source of the primitive shocks. In contrast, our focus is on the impact of monetary policy shocks on economic volatility.

The papers closest to our theoretical work are the studies of Rudebusch and Swanson (2012) and Swanson (2015), who use a similar theoretical setup to the one employed here to understand the asset pricing implications of volatility effects caused by level shocks. Cacciatore and Ravenna (2016) develop a real business cycle model, with labour search and matching frictions and an occasionally binding constraint on downward wage adjustment to understand the effect from a negative productivity shock on volatility. Our paper is also related to the work of Bikbov and Chernov (2013) and Campbell et al. (2014), who uses macro-finance models to understand the

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1Cacciatore and Ravenna (2016) also use their framework to understand the state dependent amplification mechanism of exogenous uncertainty shocks.
relationships between monetary policy and bond risk premia. Petrosky-Nadeau et al. (2018) illustrate that when real business cycle models with search & matching friction are calibrated and solved carefully, then they can generate endogenous disasters. Our finding support fully their analysis, we illustrate below that only the version of the model with these labour frictions can give rise to endogenous disasters.\(^2\)

Finally, our work is related to the heterogeneous agents (HA) literature that introduces unemployment into these incomplete markets models either to understand how different fiscal policies are transmitted to the economy (McKay and Reis (2016)) or to develop models that can account for extreme economic phenomena such as the Great Recession without employing large and persistent exogenous shocks (Ravn and Sterk (2017), Den Haan et al. (2018)). Agents in these models cannot fully insure against idiosyncratic unemployment risk and, therefore, they are concerned about their consumption level if the become unemployed. So when an adverse shock takes place, they act in a precautionary manner and increase savings. These concerns are more elevated in bad times as unemployment spells last longer making the agents’ responses state dependent. In our setting, the specification of the utility function leads to state-dependence of responses. However this feature is now driven by the difference between current and steady-state consumption with agents responding by more in states where consumption is below the steady state.

The paper is organised as follows, Section 2 presents the empirical model and discusses the data and empirical results. Section 3 reviews the theoretical model, its calibration and presents the impulse response analysis. The final section concludes.

## 2 Empirical results

on second moments of key macroeconomic variables, we estimate an extended structural VAR model with stochastic volatility. The observation equation of the model is given by:

\[
Z_t = c + \sum_{j=1}^{P} \beta_j Z_{t-j} + \sum_{k=1}^{K} b_k \tilde{h}_{t-k} + \Omega_t^{1/2} e_t, e_t \sim N(0, I_N) \tag{1}
\]

In equation (1) \(Z_t\) is \(N \times 1\) vector of endogenous variables and \(\tilde{h}_t\) denotes the \(N \times 1\) vector of log stochastic volatilities. The coefficients are denoted by the \(N \times N\) matrices \(\beta_j\) and \(b_k\) while \(I_N\) is a \(N \times N\) identity matrix. The covariance matrix of the VAR residuals is time-varying and factored as:

\[
\Omega_t = A^{-1} H_t A^{-1}' \tag{2}
\]

\[
H_t = \text{diag}(\exp(\tilde{h}_t)) \tag{3}
\]

and the \(N \times N\) diagonal matrix \(H_t\) holds the stochastic volatility of the orthogonalised shocks on the main diagonal \(\tilde{h}_t = [h_{1,t}, h_{2,t}, ..., h_{N,t}]\). The structure of the \(A\) matrix is chosen by the

\(^2\)In our framework the endogenous disasters are caused by monetary policy shocks and not by productivity shocks as it is the case in Petrosky-Nadeau et al. (2018). Endogenous disasters in our model illustrate that the monetary policy could cause highly adverse economic conditions and this is why agents in the model: (i) are concerned and (ii) try to insure against these outcomes.
econometrician to model the contemporaneous relationship amongst the reduced-form shocks. We discuss our choice of the structure of the $A$ matrix in section 2.3 below.

The transition equation for the stochastic volatilities is given by the following VAR model:

$$\hat{h}_t = \alpha + \theta \hat{h}_{t-1} + \sum_{j=1}^{K} d_j Z_{t-j} + \eta_t, \eta_t \sim N(0, Q), E(e_t, \eta_t) = 0$$  \hspace{1cm} (4)

The constants and coefficients on lags are denoted by the $N \times 1$ and $N \times N$ matrices $\alpha$ and $\theta$, respectively. Following standard practice in the literature on stochastic volatility models (see for e.g. Kim et al. (1998)), we allow $\hat{h}_t$ to depend on its first lag. However, the $N \times N$ coefficient matrices $d_j$ also allow lagged endogenous variables to affect the log variances. If these coefficients are non-zero, then shocks to equation 1 have an impact on $\hat{h}_t$ and consequently on $\Omega_t$ and measures of the unconditional variance of $Z_t$. Note also that the stochastic volatility in mean formulation of equation 1 allows feedback from lagged volatilities to the endogenous variables.

The model in equations 1 and 4 contains two innovations relative to the standard BVAR with stochastic volatility (see Clark (2011)). First, it allows the elements of $\hat{h}_t$ to co-move while most of the previous literature assumes an independent AR or random walk process for each log variance. The specification used here thus captures the possibility that volatility of shocks to macroeconomic and financial variables may move together – a phenomenon that may be important during periods of recession and financial stress. Secondly, unlike previous applications of this model (see Muntaz and Theodoridis (2015)), the terms $\sum_{k=1}^{K} b_k \hat{h}_{t-k}$ and $\sum_{j=1}^{K} d_j Z_{t-j}$ in equations 1 and 4 allow a dynamic relationship between the level and volatility of the endogenous variables.$^3$ One way to see this is to re-write the observation and transition equations jointly as an expanded VAR system:

$$\begin{pmatrix} Z_t \\ \hat{h}_t \end{pmatrix} = \begin{pmatrix} c \\ \alpha \end{pmatrix} + \begin{pmatrix} \beta (L) & b (L) \\ d (L) & \theta L \end{pmatrix} \begin{pmatrix} Z_t \\ \hat{h}_t \end{pmatrix} + \begin{pmatrix} u_t \\ \eta_t \end{pmatrix}$$  \hspace{1cm} (5)

$$\text{var} \begin{pmatrix} u_t \\ \eta_t \end{pmatrix} = \begin{pmatrix} A^{-1} H_t A^{-1'} & 0 \\ 0 & Q \end{pmatrix}$$  \hspace{1cm} (6)

where $\beta (L), b (L)$ and $d (L)$ denote lag polynomials of order $P, K$ and $K$ respectively. As discussed above, our interest lies in investigating the possible impact of monetary policy shocks on the second moments of the endogenous variables. The specification above enables us to calculate the impulse response of $\hat{h}_t$ and thus $\text{var} (Z_t)$ to a monetary policy shock identified via an appropriate structure for $A$.

Equation 5 reveals two restrictive features of the benchmark model. First, the coefficient matrices are time-invariant and the model does not directly account for structural change. Second, the error covariance matrix (equation 6) is assumed to be block diagonal and level (volatility) shocks have a lagged impact on volatility (levels). In the sensitivity analysis below (see Section

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$^3$ An exception is the univariate stochastic volatility in mean model of Chan (2017) that allows lagged effects from the data in the transition equation.
3.1.1), we relax these assumptions and show that our main results are qualitatively similar in the extended versions of the benchmark model.

2.1 Data

The model is estimated using US data on the civilian unemployment rate, annual CPI inflation, an interest rate representing the policy instrument and the spread of 10 year government bonds over the three month T-bill rate. The data is monthly and, in the benchmark case, runs from 1947m1 to 2007m12, with the last few years dropped as they represent the period of unconventional monetary policy. The first ten years are used as a training sample with estimation carried out over the period 1957m1 to 2007m12. In the benchmark model, we use the three month T-bill rate as a proxy for the policy instrument. In an additional model specification we identify the monetary policy shock using an external instrument approach. As explained in section 2.3 below, this version of the model uses a one year government bond yield as the policy instrument. The data on the unemployment rate, CPI and the three month T-Bill rate is obtained from FRED, while the 1 and 10 year bond yield is obtained from Global Financial Data.

2.2 Estimation and impulse responses

The model is estimated using Bayesian methods. In the on-line appendix we state in detail the Gibbs sampling algorithm used to approximate the posterior distribution. In short, the algorithm is an extension of the MCMC methods used to estimate Bayesian VARs with stochastic volatility, presented for example in Cogley and Sargent (2005). The prior distribution for the VAR coefficients in equation 1 are based on existing studies and ‘shrink’ the VAR coefficient matrix towards an AR specification for each endogenous variable. We employ a similar prior for the transition equation and thus assume apriori that each log stochastic volatility follows an AR process and that there is no feedback from \( Z_{t-j} \).

The impulse responses of \( \log \text{var}(Z_t) \) to a monetary policy shock are calculated via Monte-Carlo integration. In particular, the impulse responses are defined as the difference between the following conditional expectations

\[
IRF_t = E(\ln \text{var}(Z_{t+k}) | \Psi_t, Z_{t-1}, \mu) - E(\ln \text{var}(Z_{t+k}) | \Psi_t, Z_{t-1})
\]

where \( \Psi_t \) denotes the parameters and state variables of the model and \( \mu \) is the monetary policy shock. The first term in equation 7 denotes a forecast of the log volatility conditioned on one of the structural shocks \( \mu \). Note that, the volatility of the endogenous variables depends on the structural shocks through equation 4 above. The second term is the baseline forecast of the log variance, i.e. conditioned on the scenario where the shock equals zero. Koop et al. (1996) describe how to approximate these conditional expectations via a stochastic simulation of the non-linear VAR model. We use 100 simulations to calculate \( IRF_t \) repeating this for 500 retained Gibbs draws. In order to account for history dependence of the non-linear responses,

\footnote{We use a particle Gibbs sampler (see Andrieu et al. (2010). This is described in the technical appendix.}
the calculation is done for $t = 1, 12, \ldots, T$ i.e. every 12\(^{th}\) month in the sample and the mean across time is reported in the figures below.

### 2.3 Model specification and identification

We set the lag length in the VAR model to 12 and use 3 lags of the endogenous variables in the transition equation 4 and 3 lags of the stochastic volatilities in the observation equation 1. As shown in the sensitivity analysis, the main results are very similar for longer lag lengths.

We consider three schemes to identify the monetary policy shock. The schemes are implemented by placing restrictions on the column of the $A^{-1}$ matrix corresponding to the equation for the policy instrument. The remaining columns of the matrix correspond to a triangular structure.

The benchmark identification scheme uses contemporaneous sign restrictions to identify the monetary policy shock. We assume that a contractionary policy shock increases the short-term interest rate on impact and leads to a rise in unemployment and a fall in CPI inflation. The second scheme assumes a recursive structure and implies that monetary policy shocks have no contemporaneous impact on unemployment and inflation but can affect the term spread immediately. Finally, we follow Gertler and Karadi (2015) and identify the monetary policy shock using an external instrument. This version of the model uses the 1 year government bond yield as the measure of the policy rate and the estimation sample runs to 2012 M6. Gertler and Karadi (2015) argue that the use of the 1 year rate accounts for unconventional policy such as forward guidance. We use the benchmark instrument employed in Gertler and Karadi (2015) – i.e. surprise changes in three month ahead fed funds futures rate on FOMC dates. As discussed in Mertens and Ravn (2013), under the assumption that the instrument is relevant and uncorrelated with other structural shocks, the impulse vector to a unit shock can be recovered by a regression of the reduced form residuals on the instrument.\(^5\)

### 3 Results

#### 3.1 Impulse response to a monetary policy shock

Figure 1 presents the impulse response to a contractionary monetary policy shock normalised to increase the T-Bill rate by 100 basis points. The unemployment rate rises by about 0.2 percentage points at the two year horizon. Inflation displays a persistent decline of about 0.3 percentage points. Finally, the term spread falls by about 70 basis points on impact.

The last three rows of the figure present the response of the unconditional volatility to this shock. It is clear from the figure that the volatility of all endogenous variables rises in response to this shock. This is reflected in the measure of overall volatility, the log determinant of the covariance matrix of the endogenous variables which shows a persistent increase. The response

\(^5\)Gertler and Karadi (2015) present a detailed evidence that suggests that three month ahead fed funds futures rate innovations provide a strong instrument to identify monetary policy shocks.
of volatility is persistent lasting for about 2 years with the magnitude of the response of interest rate and inflation volatility slightly larger than the remaining variables.

Figure 2 presents the response of the volatility of the endogenous variables estimated using the three identification schemes discussed above. The second row of the figure shows the recursive identification schemes produces results very similar to the benchmark case. Similarly, when the external instrument is used to identify the monetary policy shock the impulse responses still suggest that volatility rises after a monetary contraction.

3.1.1 Robustness checks

Time variation As noted above, the benchmark model restricts the VAR coefficients to be fixed over time. To check the structural stability of the estimated impulse responses we extend the benchmark model to allow the coefficients to be time-varying. In particular, we estimate the following version of the model:

\[
Z_t = c_t + \sum_{j=1}^{p} \beta_{t,j} Z_{t-j} + \sum_{k=1}^{K} b_{t,k} \tilde{h}_{t-k} + \Omega_t^{1/2} e_t \tag{8}
\]

\[
\Omega_t = A^{-1} H_t A^{-1'} \tag{9}
\]

\[
\tilde{h}_t = \alpha_t + \theta_t \tilde{h}_{t-1} + \sum_{j=1}^{K} d_{t,j} Z_{t-j} + \eta_t \tag{10}
\]

where \( \text{var}(\eta_t) = Q \). Letting \( \Theta_t \sim N(NP+NK+1) \times 1 \) and \( \Psi_t \sim N(N+NK+1) \times 1 \)

\[
\Theta_t = \Theta_{t-1} + \tilde{Q}_{1}^{1/2} v_{1t} \tag{11}
\]

\[
\Psi_t = \Psi_{t-1} + \tilde{Q}_{2}^{1/2} v_{2t} \tag{12}
\]

where \( (e_t', v_{1t}', v_{2t}')' \sim N(0, I_{\tilde{N}}) \) with \( \tilde{N} = N(NP+NK+1) + N(N+NK+1) + N \). The model can be estimated using an extended version of the Gibbs algorithm summarised above. The extension is described in the technical appendix.

The time-varying impulse responses of volatility to a 1 unit monetary contraction are shown in Figure 3. As in the benchmark case, sign restrictions are used to identify the policy shock. As noted in previous studies, there is some evidence suggesting that the impact of monetary policy on the real economy has declined over time (see Boivin and Giannoni (2006)). As in Boivin and Giannoni (2006) the inflation response becomes positive at medium horizons in the earlier part of the sample. While there is some weak evidence to suggest that the response of volatility may have been slightly larger during the 1970s and the first half of the 1980s, the impact on volatility remains positive and persistent throughout the sample period.

Further sensitivity checks The technical appendix provides a range of further checks. The benchmark model in equations 5 and 6 does not allow a contemporaneous relationship between
level and volatility shocks. Following Alessandri and Mumtaz (2018), we extend the model and relax this assumption. Impulse responses from this version of the model (see Section 4.1, pages 14-17 in the appendix) support the conclusion that volatility rises after a monetary contraction. In addition, we show that the results survive if a longer lag length is used in the benchmark model. Similarly, versions of the model that include the Federal Funds rate, industrial production or stock returns produce results similar to the benchmark case. Finally, positioning the short-term interest rate first in the recursive order or using the Romer and Romer (2004) measure of monetary policy shocks as an instrument produces responses of volatility that support the results depicted in the second and third rows of Figure 2.

3.2 Variance decomposition

To investigate the importance of the monetary policy shock we construct the forecast error variance (FEV) decomposition for the benchmark model using the method described in Lanne and Nyberg (2016) for non-linear models. Table 1 presents the contribution of the monetary policy shock and compares it with the contribution of all 4 level shocks in the VAR model. The third and fourth columns of the table display the contribution to the FEV of volatility of the variables while the final two columns display the contribution to the FEV of the level. The final column of the table shows that, as highlighted by several previous studies, the monetary policy shock makes a modest contribution to future movements in the unemployment rate and inflation. As in Bernanke et al. (2005), the contribution to the FEV of the unemployment rate is about 10 percent while the contribution to inflation FEV does not exceed 5 percent. The contribution to the interest rates is higher, especially at shorter horizons. When compared to the contribution of all level shocks jointly (column 5 of the table), the monetary policy shock does not appear to be the most important component.

Column four of table 1 shows that the contribution of the monetary policy shock to the volatility of the variables is also modest in absolute terms and ranges from about 5 to 7 percent. However, in relative terms, the monetary policy shock appears to be important, especially at the one year horizon. For example, the total contribution of the level shocks to the FEV of unemployment volatility at this horizon is 16 percent. Almost half of this contribution comes from the monetary policy shock. Similarly, the monetary policy component in the contribution of level shocks to the FEV of inflation, interest rate and spread volatility accounts for 40 to 50 percent at the one year horizon. However, the relative importance of this shock declines at the 60 month horizon suggesting that other level or second moment shocks may play a role in the long run.

To investigate the economic importance of monetary policy transmission via volatility, we estimate a version of the benchmark model that restricts the effects of level shocks on second moments to be equal to zero (by setting $d_j = 0$ in equation 4). In Figure 4, we compare the

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6In the technical appendix we present results using the proxy VAR of Gertler and Karadi (2015) extended to include the measure of uncertainty developed by Jurado et al. (2015). Results from this model, which allows for a contemporaneous impact of monetary policy shock on uncertainty, support our key conclusions.

7Based on the deviance information criterion (DIC) of Spiegelhalter et al. (2002) the benchmark model is preferred to this restricted model. The DIC for the restricted model is -4526.68, while the corresponding estimate for the benchmark model is smaller (-4560.82) indicating an improved fit.
impulse response to a monetary policy shock in the restricted and the benchmark model. It is clear from the figure that the response of unemployment and inflation is less persistent in the restricted case. This implies that the cumulated change in these variables is estimated be much smaller if the effect of policy shocks on volatility is assumed away.

4 Theoretical Analysis

In order to investigate the transmission of monetary policy shocks to volatility, we build and estimate a New Keynesian DSGE model. We first describe the building blocks of the key sectors of the DSGE model and consider how time-varying volatility arises in this set-up. We then use an estimated version of the model to calculate the response of the key variables and their volatility to monetary policy shocks.

4.1 DSGE Model

**Households:** The economy is populated by a continuum of households \( h \in [0, 1] \) that attain utility from consumption \( \tilde{C}_t (h) \) and leisure \( 1 - L_t (h) \), where \( L_t (h) \) denotes the fraction of the household that is employed. Household’s preferences are separable

\[
    u \left( \tilde{C}_t (h), \tilde{Z}_t, L_t (h) \right) = \frac{\left( \tilde{C}_t (h) - b \tilde{C}_{t-1} \right)^{1-\sigma_C}}{1 - \sigma_C} - \chi_0 \tilde{Z}_t^{1-\sigma_C} L_t (h)^{1+\sigma_L} \frac{1}{1 + \sigma_L} \tag{13}
\]

where \( \sigma_L \) is the inverse of the Frisch elasticity, \( \sigma_C \) stands for the inverse of intertemporal elasticity of substitution and \( \tilde{Z}_t = Z \tilde{Z}_{t-1} \left( \frac{\tilde{Z}_{t-1}}{\tilde{Z}_{t-2}} \right)^{\rho_z} e^{\sigma_z \omega_{z,t}} \) denotes the non-stationary productivity process (the tilde indicates that the variable is non-stationary) where \( Z \) is the steady-state value of the productivity growth, \( \rho_z \) indicates the degree of persistence and \( \sigma_z \) is the standard deviation of the productivity growth process.

The empirical analysis above illustrates convincingly that the changes in the policy instrument have an impact on the level and volatility of endogenous variables. This evidence points to the existence of important non-linearities in the data that give rise to these effects. Given our stylised facts are related to volatility, it seems a natural starting point to investigate whether these non-linearities are due to agents’ preferences. The analysis of Rudebusch and Swanson (2012) and Swanson (2015) suggest that when agents form recursive preferences (Epstein and Zin (1989)) then a productivity level shock induces the stochastic volatility of the series in the model to vary. We proceed, therefore, by assuming that agents have preferences of this form:

\[
    V_t (h) = u \left( \tilde{C}_t (h), \tilde{Z}_t, L_t (h) \right) + \beta \left( E_t V_{t+1} (h)^{1-\gamma} \right)^{\frac{1}{1-\gamma}} \tag{14}
\]

The attractive feature of Epstein-Zin preferences is that the coefficient of relative risk aversion decouples from the intertemporal elasticity parameter. The parameter \( \gamma \) illustrates the degree of agents’ desire for an early resolution of uncertainty over future consumption. Household maximises its utility function subject to its budget constraint which is:

\[
    \tilde{P}_t \tilde{C}_t (h) + \tilde{D}_t (h) + T_t (h) = \tilde{P}_t \tilde{W}_t L_t (h) + (1 - L_t (h)) \tilde{P}_t \tilde{B}_t + \tilde{D}_{t-1} (h) + \tilde{\Xi}_t (h) \tag{15}
\]
where $\tilde{P}_t$ is the price index, $\tilde{D}_t(h)$ is the one period risk free government debt, $R_t$ is the return on investing on the government debt, $\tilde{W}_t$ stands for the real wage, $T_t(h)$ is the lump sum taxes, $\tilde{B}_t$ is the unemployment benefit and $\tilde{\Xi}_t(h)$ denotes firms’ profits.

The budget constraint reveals the existence of real labour market frictions that lead some members of the household to become unemployed. However, they enjoy the same consumption levels as the employed members due to our complete markets assumption. The structure of the labour market is discussed below. Here we mention two pieces of evidence supporting the argument that search and matching frictions could be a vital part of the mechanism relating monetary policy shocks and second moments. Firstly, the empirical exercise undertaken in Section 3.2 (Figure 4) reveals that the impact of monetary policy on unemployment is enhanced by the impact of this shock on volatility. This is indicative of the existence of frictions in the labour market that re-enforce the effects of volatility.

Secondly, there are a growing number of studies employing heterogeneous agents models that argue in favour of including unemployment into these incomplete market models in order to better understand extreme economic episodes such as the Great Recession (Ravn and Sterk (2017), Den Haan et al. (2018)). The outcome of these studies is further supported by the work of Petrosky-Nadeau et al. (2018) that illustrates search and matching frictions in a real business cycle could give rise to endogenous disasters when the model is calibrated and solved carefully. These two different type of approaches seems to indicate that these labour frictions may have very rich nonlinear implications. Finally, during the ZLB period and the introduction of the forward guidance policy by the FED and Bank of England, unemployment became the primary policy variable in terms of monetary authorities communicating the end date of the excess stimulus in the economy.\(^8\)

**Labour Market:** The existence of a real – search and matching – friction in the labour market (Mortensen and Pissarides (1994)) prevents all job-seekers ($U_t = 1 - (1 - \delta_N) L_{t-1}$) from being matched with vacancies ($\Upsilon_t$) posted by firms and they end up unemployed ($u_t = 1 - L_t$). The matching technology is described by the following Cobb-Douglas (expression 16)

$$M_t = \bar{\mu} U_t^{\mu} \Upsilon_t^{1-\mu}$$

$$L_t = (1 - \delta_N) L_{t-1} + Q_t \Upsilon_t$$

$$\Psi_t = \kappa \tilde{Z}_t \Upsilon_t$$

While employment evolves according to equation 17, where $\delta_N$ is the separation probability. This formulation incorporates the assumption that new hires start working in the same period they are hired (Blanchard and Gali (2010)). Furthermore, firms in order to be able to hire a worker they need to post a vacancy and this incurs a cost (expression 18, see Mortensen and Pissarides (1994)). In other words, the cost is a linear function of the vacancies posted. This is different set-up than the cost of hiring function used in Gertler et al. (2008). However, this particular formulation implies that the cost is paid after the vacancy is filled and it reflects

\(^{8}\text{In August 2013, the Bank of England augmented its policy toolkit with (state dependent) forward guidance. Unemployment became a forward guidance threshold variable and the Bank of England started publishing its fan chart in order to better communicate with public its projection about real economy.}\)
internal costs of adjusting the number of employees (such as training). This specification, thus, minimises the exposure of entrepreneur’s profits unsuccessful matches and, consequently, to uncertainty, since the cost is only paid after the vacancy is filled. This feature makes this formulation less suitable in our setting.

**Final Good Producer:** The next two paragraphs discuss the price Phillips curve. The set-up is quite standard and the nominal rigidities are introduced as a simple way to make output demand driven in the short run and to allow monetary policy to be able to affect the economic cycles. The final good is produced via the following production function

$$
\tilde{Y}_t = \left[ \int_0^1 \tilde{Y}_t(f) \frac{1}{f^\varepsilon} df \right] ^{1\over 1-\varepsilon}
$$

(19)

$$
\tilde{Y}_t(f) = \left( \frac{P_t(f)}{P_t} \right)^{-\varepsilon} Y_t
$$

(20)

where $\varepsilon$ denotes the elasticity of substitution between differentiated intermediate goods ($f \in [0,1]$).

The demand for intermediate goods (expression 20) results from profit maximisation and the assumption that the final good producer operates under perfect competition.

**Intermediate Good Producers:** Similar to Krause and Lubik (2007) and Krause et al. (2008) we assume that there is a continuum of firms ($f \in [0,1]$) that post vacancies, combine employment, fixed capital and employ the following technology:

$$
\tilde{Y}_t(f) = \tilde{Z}_t (L_t(f))^{1-\phi} K\phi
$$

(21)

to produce the intermediate good, where $\phi$ is the capital share in the production function. These producers solve a two-stage problem. In the first stage, taking the wage and the cost of filling a vacancy as given they decide how many vacancies to post and people to employ, these choices result from the maximisation of their profit function:

$$
E_t \sum_{j=0}^{\infty} M_{t+j} \beta^j \left\{ MC_{t+j}(f) \tilde{Y}_{t+j}(f) - \tilde{W}_{t+j}L_{t+j}(f) - R^K K - \kappa \tilde{Z}_{t+j} Y_{t+j}(f) + \Theta_{t+j}(f) \left( (1-\delta_N) L_{t+j-1}(f) + Q^*_j(f) \right) \right\}
$$

(22)

where $MC_t$ (the marginal cost), $\Theta_t$ (the shadow value of hiring an additional worker) are the Lagrange multipliers associated with the goods’ production function and the employment’s law of motion, respectively. Finally, $M_t$ denotes the stochastic discount factor

$$
M_{t+1} = \left[ \frac{V_{t+1}}{E_t V_{t+\gamma}^1} \right] ^{1-\gamma} \left( C_t(h) - b\tilde{C}_{t-1} \right) ^{\sigma C}
$$

In the second stage, producers set the price of the intermediate good that maximises their profits. The optimisation problem in this case reflects that prices are set in a staggered manner. This means that every period a fraction $(1-\xi)$ of firms receive a random signal and set prices optimally $\left( \tilde{P}_t(f) \right)$, while those firms who miss the signal set prices based on a rule of thumb backward looking indexation scheme $\left( \tilde{P}_t(f) = \Pi_{t-1} \tilde{P}_{t-1}(f) \right)$. As explained in Christiano et al. (2005), this pricing setup allows us to replicate the hump shaped response of inflation to the
monetary policy observed in the empirical section. The pricing problem is summarised by the following profit maximisation
\[
\max_{\Pi_t} \sum_{j=0}^{\infty} M_{t,t+j} (\beta \xi)^j \left[ \left\{ \frac{\dot{P}_t (f) \Pi_{t+j-1}}{P_{t+j}} - MC_{t+j} (f) \right\} \dot{Y}_{t+j} (f) \right] 
\]
subject to
\[
\dot{Y}_{t+j} (f) = \left( \prod_{s=0}^{j} \Pi_{t+s-1} \frac{P_t (f)}{P_{t+j}} \right)^{-\varepsilon} Y_{t+j}
\]

**Wage Determination:** The wage is determined by solving a Nash bargaining problem between workers and firms that takes place in order to decide how to split the surplus produced by a match (see Mortensen and Pissarides (1994) and Krause et al. (2008) amongst others). This simple framework is commonly used in the literature and assumes that newly hired workers get a match (see Mortensen and Pissarides (1994) and Krause et al. (2008)). To set the problem we need to define the value of the firm:
\[
J^F_t = MC_t \frac{(1-\phi) \dot{Y}_t}{L_t} - \dot{W}_t + \beta E_t M_{t+1} (1-\delta_N) \dot{J}^F_{t+1}
\]
On the other hand, the value of an employed and unemployed worker is given by:
\[
\begin{align*}
\dot{J}^W_t &= \dot{W}_t - \chi_0 \dot{Z}_t^{1-\sigma_c} \ell_t^{\sigma_c} \left( \dot{C}_t - b \dot{C}_{t-1} \right)^{\sigma_c} \\
&\quad + \beta E_t M_{t+1} \left\{ [1-\delta_N (1-Q_{t+1}^U)] \dot{J}^W_{t+1} + \delta_N (1-Q_{t+1}^U) \dot{J}^U_{t+1} \right\} \\
\dot{J}^U_t &= \dot{B}_t + \beta E_t M_{t+1} \left\{ Q_{t+1}^U \dot{J}^W_{t+1} + (1-Q_{t+1}^U) \dot{J}^U_{t+1} \right\}
\end{align*}
\]
and the wage results from the following bargaining problem:
\[
\dot{W}_t^{Nash} = \arg \max_{\dot{W}_t} \left( \dot{J}^W_t - \dot{J}^U_t \right)^{\eta} \left( \dot{J}^F_t \right)^{1-\eta}
\]
Similarly to Krause and Lubik (2007) and Leduc and Liu (2016), we allow for real wage rigidity via the following norm:
\[
\dot{W}_t = \dot{W}_t^{\pi w} \left( \dot{W}_t^{Nash} \right)^{1-\pi w}
\]

**Government and Aggregation:** The government in this economy runs a balanced budget:
\[
\dot{P}_t \dot{C}_t + D_{t-1} + (1-L_t) \dot{P}_t \dot{B}_t = T_t + \frac{D_t}{R_t}
\]
where \( \dot{C}_t = g_t \dot{Y}_t \) is government consumption and \( g_t = g \left( \frac{u_{t-1}}{g} \right)^{\rho_g} e^{\sigma_g \omega_{g,t}} \) is the of the government share in the economy. Monetary policy is set based on Taylor Type rule:
\[
\log (R_t) = r_t = \rho_R r_{t-1} + (1-\rho_R) \left\{ \zeta_{\Pi} \log \left( \frac{\Pi_t}{\Pi} \right) + \zeta_u \log \left( \frac{u_t}{u} \right) \right\} + \omega_{R,t}
\]
where \( \Pi \) is the inflation target, \( \rho_R \) is the interest rate smoothing parameter, \( \zeta_{\Pi} \) and \( \zeta_{\Pi} \) are the policy reaction coefficients to inflation and demand growth, respectively, and \( \omega_{R,t} = \rho_{\omega_R} \omega_{R,t-1} + \sigma_{\omega_R} \omega_{R,t} \) is the monetary policy shock. Finally, the market clearing condition is derived after a number of simple substitutions (see the Section 5.1, pages 21-22):
\[
\frac{\dot{Z}_t L_t^{1-\phi} K^{\phi}}{\Delta_t} = \dot{C}_t + \dot{G}_t + \dot{Y}_t
\]
The de-trended and steady-state calculations are discussed in the technical appendix (Section 5.1, pages 21-22).

4.2 Heteroscedasticity

The novel part of our analysis is that we focus on the volatility implications of the monetary policy shock. With the term volatility or measured uncertainty we refer to the heteroscedastic response of a variable, say $x_t$, defined as in Basu and Bundick (2017) and Swanson (2015)

$$
\hat{\sigma}_{x,t} = 100 \ln \left( \frac{\sigma_{x,t}}{\sigma_x} \right)
$$

where

$$
\sigma_{x,t} = \text{var}_t(x_t) = \mathbb{E}_t(x_{t+1} - \mathbb{E}_t x_{t+1})^2
$$

and $\sigma_x$ is the stochastic steady-state standard deviation of the variable $x_t$.

It is perhaps important to highlight that equation (33) coincides with the definition of volatility studied by Jurado et al. (2015). As explained in Rudebusch and Swanson (2012) and Swanson (2015), the higher moments of economy’s endogenous state vector are time-varying ($\sigma_{x,t}$) due to (i) the additive separability of consumption in the period utility function and (ii) the Epstein-Zin preferences. According to these authors the additive separability property of consumption makes the model non-homogeneous and this is what induces a small degree of heteroscedasticity, which is further enhanced by the risk aversion parameter ($\gamma$).

The economic intuition behind these two technical conditions is actually quite simple. The additive separability property of consumption makes agents’ responses to economic shocks depend on the current level of or the state of the economy. For instance, when the current level of consumption is low (or the marginal utility of consumption is high) then consumption uncertainty is higher (relative to the case where the initial level of consumption/output is high) and this reflects agents’ elevated concerns about future shocks. As an adverse shock that lowers output further is going to induce a proportionally a larger reduction in utility relative to the case where the initial level of consumption/output was high. This channel is the further enhanced by Epstein-Zin preferences as the risk parameter reflects how much agents dislike elevated uncertainty (Rudebusch and Swanson (2012)).

Loosely speaking, agents in this economy price adverse shocks more heavily in ‘bad times’ when compared to ‘good times’. This behaviour induces a wedge between the mode of the distribution $x_t$ and its mean as the latter captures these elevated concerns. Figures 5 illustrates this phenomenon. We use the model developed in this study (and the estimates discussed below) to simulate the data. Panel A of Figure 5 shows the probability density function of unemployment rate, unemployment expected duration, labour income, GDP and annual inflation as deviations from their stochastic steady states when the monetary policy shock is drawn from its estimated distribution.\textsuperscript{10} It is apparent that even under one standard deviation monetary policy shocks

\textsuperscript{9}In the text below, we use the term uncertainty to refer to ‘measured uncertainty’.

\textsuperscript{10}Both the non-stationary productivity and government spending shocks are switched off for the rest of the analysis. To be precise, although the values of the latter two shocks are set to zero, their standard deviations are not. This affects peoples average behaviour as their expectations are based on the distribution of the two shocks
the probability density functions displays a ‘downward risk’, meaning that the average unemployment rate and the average expected duration of being unemployed are higher than their modes. Similarly, the average labour income, GDP and inflation fall to left of their modes.

We repeat the same exercise in Panel B of Figure 5, however, we apply larger shocks this time (two times their standard deviation). As expected, the asymmetry becomes more pronounced indicating that agents economic behaviour is also a function of the state of the economy.

The above simulations illustrate that under certain conditions monetary policy actions can have quite dramatic implications for the economy; for instance, unemployment rate could rise 6 percentage points (pps) above the stochastic steady state, while inflation, GDP and labour income could fall 10% below the stochastic steady state, respectively. These adverse economic conditions are taken into account by agents when they form their decisions optimally and try to minimise their exposure to these downward risks.

**Monetary Policy and Volatility:** To understand the role of monetary policy in inducing volatility it is instructive to consider the solution of the model approximated to the third order:

\[
\begin{align*}
    z_t &= h_z z_{t-1} + H_{zz} (z_{t-1} \otimes z_{t-1}) + H_{zzz} (z_{t-1} \otimes z_{t-1} \otimes z_{t-1}) \\
    &+ \frac{3}{6} h_{\sigma \sigma z} \sigma^2 z_{t-1} + \frac{1}{2} h_{\sigma \sigma} \sigma^2 + \frac{1}{2} h_{\sigma \sigma \sigma} \sigma^3 + \sigma \eta_t
\end{align*}
\]

where \( z_t \) is the state vector of the economy and \( \epsilon_t \) is the vector of structural shocks. The matrices \( h_z, H_{zz}, H_{zzz}, h_{\sigma \sigma}, h_{\sigma \sigma \sigma} \) denote the derivatives of the system with respect to the state and/or shock vectors for different orders and evaluated at the non-stochastic steady-state (see Andreasen et al. (2018) for the exact details). The one step ahead expectation of this expression (ignoring constant terms for simplicity) can be written as:

\[
E_t z_{t+1} = h_z z_t + H_{zz} var_t (z_t) + H_{zzz} skew_t (z_t)
\]

where \( var_t (z_t) = E_t (z_t \otimes z_t) \) is the column stacked covariance matrix of \( z_t \) at time \( t \) and \( skew_t (z_t) = E_t (z_t \otimes z_t \otimes z_t) \) is approximately the column stacked skewness matrix of \( z_t \) at time \( t \).

Thus, along with the current state, \( z_t \), the higher moments of the system directly affect the one step ahead expectation and these moments are time-varying due to the specification of the agents’ preferences. As discussed earlier, Epstein-Zin agents have a preference for an early resolution of uncertainty with the magnitude of this preference determined by the terms

\[
H_{zz} var_t (z_t) + H_{zzz} skew_t (z_t).
\]

In other words, this preference for an early resolution of uncertainty is a function of the location of the current state of the economy relative to its distribution. Moreover, the non-linearity of equation 35 implies that the agents’ aversion to uncertainty is larger when the shock is negative and pushes the economy below the steady state.

Because of this non-linearity, when a contractionary monetary policy shock occurs, agents start forming expectations that are more heavily influenced by the possibility of future adverse shocks.
This asymmetry increases the dispersion of their forecast errors meaning that volatility (see equation 33) increases. Loosely speaking, as the economy contracts, households and firms start hedging against the worst case scenario through their expectations. While this reduces their exposure, the dispersion of their forecast errors increases as they are concerned about events that do not necessarily arise.

Table 2 compares a set of data estimated disaster statistics reported by Petrosky-Nadeau et al. (2018), with those predicted by the model that is subject to a monetary policy shock. The aim of this second exercise is to use data evidence to quantify the risks to which the agents in this economy are exposed and, consequently, to understand why they want to hedge against them. As in Petrosky-Nadeau et al., we apply the peak-to-trough method discussed in Barro and Ursua (2008) to to identify rare disasters. The disasters are defined as cumulative fractional declines in per capita output of at least 10%.

When the economy is exposed to one standard deviation policy shocks, then the disaster probability is significantly less than 1%. However, if this highly unlikely event ever takes place then it lasts for almost 7 years and growth reduces by almost 12%.\(^{13}\) Not surprisingly, when the economy is perturbed with larger policy shocks, then the disaster probability and the size of output collapse increase significantly and non-linearly, while the duration of disaster state decreases: The probability of disaster and its size rise to 5% and 14% respectively, while the duration of the disaster shortens to 4 years. Unlike in the case of one standard deviation shocks these estimates are significant closer to those obtained using actual data. Finally, the comparison of the second and third columns of Table 2 illustrates again that agents’ responses to monetary policy shocks are different at different stages of the cycle.

### 4.3 Calibration

The model is estimated using limited information impulse response matching techniques (Christiano et al. (2005), Christiano et al. (2010)). However, the value of a small number of parameters is decided prior to the estimation. To be precise, the share of capital in the production ($\phi$) and its depreciation rate have been calibrated to 0.36 and 0.025, numbers typically used in the literature (Christiano et al. (2005)). The steady-state unemployment ($u$) is set equal to 5.8% (the sample mean), while the steady-state value of output ($y$) to 1. The time discount factor ($\beta$) equals 0.995, while the both the steady-state value of inflation and productivity growth have been set to 2%. The last three parameters imply that the non-stochastic steady-state of the annual policy rate is 6%. Similar to Smets and Wouters (2007), the government spending and investment to GDP ratios are calibrated to 0.18 and 0.2, respectively. Finally, the steady-state value of the probability filling the vacancy is 70% (Hagedorn and Manovskii (2008)).

The parameters $\sigma_C$, $\sigma_L$, $b$, $\gamma$, $\xi$, $\ell$, $\zeta_{\Pi}$, $\zeta_{R}$, $\zeta_{\eta}$, $\mu$, $\eta$, $\delta_N$, $\tilde{B} = \frac{B}{\Pi_{Z}}$, $\ell_w$, $\Phi$, $\rho_{\varepsilon_{Z}}$, $\rho_{\varepsilon_{R}}$, $\rho_{\varepsilon_{G}}$, $\rho_{\varepsilon_{H}}$, $\sigma_{\varepsilon_{Z}}$, $\sigma_{\varepsilon_{G}}$ and $\sigma_{\varepsilon_{H}}$ are selected to match the nonlinear VAR responses to an identified monetary

\(^{13}\)In the online Appendix (Section 5.3) we develop a version of the model without search and matching real labour frictions but with sticky nominal wages. Although, this version of the model replicates the cyclical dynamics remarkably well, it fails to produce an endogenous disaster even when the economy is hit with 2 standard deviation policy shocks.
policy shock using the benchmark identification scheme. The model is solved using third-order perturbation methods and the impulse responses are calculated relative to the stochastic steady state (Cacciatore and Ravenna (2016)).

The objective of the estimation is for the model to be able to replicate not only the empirical responses to a monetary policy shock illustrated in Figure 1 but also the contribution of the ‘uncertainty channel’ to these responses. This is defined as the difference between the unrestricted and restricted responses plotted in Figure 4 and discussed in Section 3.2. The latter set of targets ensures that the parameters that control the size of the uncertainty channel in the model are calibrated carefully and the predictions of the model about the importance of that channel are in line with those observed in the data. As it will become apparent later in our analysis, the last set of moments acts also as a natural metric that allows us to assess which part of the transmission mechanism is responsible for the existence of the endogenous uncertainty channel. The process used to estimate the DSGE contributions of uncertainty follows closely the steps employed for the empirical models. To be precise, the uncertainty contribution is defined as the difference between the responses obtained using the third order solution model minus those by using only the first order component of the solution.

4.4 Estimation Results

Figure 5 in the online appendix (page 30) illustrates the ability of the theoretical model to replicate the identified empirical responses (Panel A) as well as the contribution of the uncertainty channel to these responses (Panel B), respectively. Even though the literature has questioned ability of a “simple” search and matching New Keynesian model to jointly replicate the dynamics of both unemployment and inflation (see the discussion in Krause and Lubik (2007) and Gertler et al. (2008)), our estimated model seems to be robust to this criticism as it reproduces the data dynamics remarkably well.

In this study, however, our results go further. The model is capable of replicating: (i) the VAR based stochastic volatility responses to a level monetary policy shock and, (ii) the empirically identified contribution of the uncertainty channel. This is a new set of results that further supports the analysis of Petrosky-Nadeau et al. regarding the aptitude of DSGE models with search and matching labour market frictions of reproducing highly non-linear dynamics seen in the data.

Returning to the discussion of the estimated parameters (Table 1, online appendix, page 31), the remaining discussion illustrates that the model can replicate the data features mentioned in the previous paragraph by relying on a set of parameter values that have been extensively

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14 No pruning is applied in our calculations, which have been implemented using Dynare 4.4.3. The model and replication files can be downloaded from authors’ webpages. We also check if our results are robust if we apply Koop et al. (1996) methodology to calculate the IRFs and we find that our results are almost identical. We choose to employ the first method to calculate the IRFs as it dramatically reduces the estimation time.

15 We would like to thank an anonymous referee for proposing this exercise.

16 While the VAR-based volatility responses are all hump shaped, this feature is less evident in the model. This is likely to be because the model incorporates simplifying assumptions and does not capture all aspects of the data.
used in the literature. Perhaps the less common parameter estimate is that for the Epstein-Zin risk coefficient ($\gamma = 115.57$) with the estimated value indicating that agents dislike future uncertainty. This value is similar to the one used in the Rudebusch and Swanson (2012) and Swanson (2015). As explained in Swanson (2015) – and it is further illustrated in the next section – this parameter only has a small effect on the stochastic volatility of macroeconomic variables and therefore the large magnitude is innocuous.\footnote{This is not the case for asset price variables as the they are functions of the stochastic discount factor. The risk parameter has a great influence on the second term of the stochastic discount factor that captures how agents ‘trade’ uncertainty across time. As it is discussed briefly in the next section, we discovered that the risk parameter variations have perhaps a larger impact on the stochastic steady-state than the cyclical dynamics regarding the macro variables.}

In terms of the utility kernel, the model demands some curvature ($\sigma_C = 2.15$, Chen et al. (2012)) and ($\sigma_L = 4.00$, Christiano et al. (2010)) in order to reconcile the predicted dynamics with those in the data. The degree of consumption smoothing ($b = 0.57$), is close to the estimates reported by Christiano et al. (2005).

The parameters that govern the labour market block of the model, the Cobb-Douglas matching parameter ($\mu = 0.40$), the job separation rate ($\delta_N = 0.18$), the income replacement ratio ($100\bar{B} = \frac{B}{W_L} = 54\%$) and the steady-state, the bargaining power for workers ($\eta = 0.50$) search and matching friction ($100\Phi = 1\%$) are again consistent with those in the literature (see for instance, Krause and Lubik (2007) and Krause et al. (2008), Gertler et al. (2008) and Hagedorn and Manovskii (2008) among others).

Consistent with the analysis Krause and Lubik (2007) and Leduc and Liu (2016) (among others), the model requires a high degree of real wage rigidity ($\tau_w = 0.89$) in order to be able to match the volatility of unemployment in the data (Figure 5, Panel A, page 30, online appendix). As explained by these authors and it can be seen from equation (28), under this calibration paid wages decouple from productivity and this leads to high surplus for firms (relative to the situation of no real wage rigidity) stimulating vacancy creation.

Similar to Christiano et al. (2005) a high degree of price indexation ($\iota = 0.72$) and a small probability of resetting prices optimally ($1 - \xi = 0.03$) are needed for the model to match the response of inflation after a monetary policy shock. Furthermore, the steady-state value of firms’ markup is 50\% ($\varepsilon = 3.0$) a value similar to Smets and Wouters (2007) and Gertler et al. (2008). Finally, the estimates of the policy reaction coefficients are similar to those reported in the literature (Krause et al. (2008), Gertler et al. (2008), Leduc and Liu (2016)).

### 4.5 Impulse Response Analysis

The aim of this section is to discuss the transmission mechanism of monetary policy shocks to macroeconomic volatility. We illustrate this via impulse response analysis. We first describe the results in the benchmark model and then investigate the features of the model that drive the transmission of the shock.
4.5.1 Benchmark results

The blue solid line in Panel A of Figure 6 shows the agents’ responses to a monetary policy shock estimated using the benchmark version of the model.

**Households:** Starting from the household side, as consumption moves away from its steady-state level due to the adverse policy shock, agents start becoming concerned about the fact that another adverse economic shock is going to take their current consumption even further away from its steady-state. For the agents, this is more costly in utility terms than if consumption had been above its steady-state prior to the adverse shock (see Rudebusch and Swanson (2012) and Swanson (2015)). To insure themselves against this downward risk they act in a precautionary manner and reduce current consumption by a larger amount (Basu and Bundick (2017) and Fernandez-Villaverde et al. (2015)).

Furthermore, in an economy with search and matching frictions, the agents face additional risks: i.e. the risk of job separation and unemployment. This enhances households’ concerns about their expected consumption plan and intensifies their desire to hedge against this uncertainty. An adverse shock that reduces consumption when the latter is already below its steady state would have a larger detrimental impact when agents are unemployed. We consider the importance of these risks below.

**Firms:** Firms are owned by households and so they use the same stochastic discount factor to weight expected profits. This means that a profit reduction is more costly when the previous period profits had been below rather than above their steady-state. As it is explained carefully in Swanson (2015), these preferences induce entrepreneurs to devote more attention to generating profits in bad times. However, firms who face adverse economic conditions are exposed to: (i) paying a wage higher than worker’s productivity (taking into account future forgone costs of hiring) due to high wage rigidity and to, (ii) low expected demand for their output.

To mitigate these exposures firms pause hiring. As explained in Bloom (2009) and Leduc and Liu (2016) filling a vacancy is an irreversible decision that has long-term implications. Therefore, entrepreneurs act more cautiously and post even less vacancies pushing up unemployment.

Moreover, the firm recognises that a lower price could lead to a higher demand for its output and, consequently, more profits. Despite the fall in the marginal cost (caused by the fact that supply exceeds labour demand), the firm has an incentive to set an even lower price during bad times in order to secure more demand and, consequently, hedge itself against future more adverse economic outcomes.

As inflation falls and monetary authorities reduce the policy rate only gradually (due to their preferences of avoiding injecting too much interest rate volatility) the real interest rate remains persistently positive and this enhances the desire for saving.
Impact on volatility: As shown in the last row of Panel A of Figure 6, volatility increases after a monetary policy shock. As discussed in Section 4.2 above, the volatility of a variable can be viewed as a wedge between the mean and the mode of the distribution. When the adverse monetary policy shock occurs, agents in the economy form expectations about future events. Households take into account the probability of longer unemployment spells during which they may be exposed to additional negative shocks. Similarly, firms’ expectations about profits are skewed downwards. As discussed in Section 4.2, this leads to an increase in the dispersion of forecast errors and volatility rises. However, as the time evolves and no further shocks are realised this wedge between expectations and what actually happens eases, and so does the volatility in the economy.

Contribution of uncertainty: The blue circle line in Panel B of Figure 6 isolates the contributions of uncertainty in the transmission of monetary policy. The uncertainty contribution is defined as the difference between the response derived by using the full third order solution of the model minus the responses produced using only the first term of that solution. For instance, unemployment increases 0.3 pps after one standard deviation monetary policy shock (Panel A of Figure 6, blue solid line/left y-axis) and 1/3 (30%) of this increase is due to the uncertainty (Panel B of Figure 6, blue circle line/left y-axis). Furthermore, uncertainty seems to account for almost 50% of the output, labour income and inflation fall. This exercise reveals that: (i) the endogenous uncertainty channel plays a sizeable role in the transmission of the monetary policy shock and (ii) it manifests itself as a demand type shock (Leduc and Liu (2016)).

4.5.2 Key Features of the model

It is interesting to consider the features of the model that drive the impact of monetary policy on second moments. In particular, the model contains three ingredients that play a role: (1) Search and Matching labour market frictions, (2) interest rate smoothing by the monetary authorities and (3) Epstein-Zin preferences. In order to gauge the role of these features of the model, we derive the impulse responses under the counterfactual scenario where these channels are switched off one by one.

Labour Market Frictions We pursue two exercises: (i) we consider what happens when non-stochastic steady-state probability of finding a job ($Q^U$) is (almost) equal to one (or the non-stochastic steady-state of unemployment is almost equal to zero) and (ii) what happens when the friction is removed completely but nominal wages adjust only gradual (sticky nominal wages, Christiano et al. (2005), Smets and Wouters (2007)). Due to the space required for the development of the second model, the latter exercise is conducted in the online Appendix (Section 5.3), but the intuition of the results coincides with the first experiment and it is, therefore, discussed here briefly.

Panel A of Figure 6 compares the agents’ responses to a monetary policy shock derived by the benchmark version of the model (blue solid line) against the responses when implicitly there is
no risk that agents will ever become unemployed \((Q^U = 1\), red dashed line)\(^{18}\). Panel B of Figure 6 on the other hand identifies the contribution of the uncertainty channel for both versions of the model. Without the search and matching labour market real friction the contribution of the endogenous uncertainty channel to the economy is substantially smaller (red cross line/right y-axis). This evidence seems to indicate that households fear that they can remain unemployed for an extended period of time (the expected average duration is given by \(\frac{1}{1-(1-Q^U)}\)). Their fear is magnified by the possibility that, during this period, another adverse shock that moves their consumption away from its steady-state might arrive. To insure themselves against this uncertainty, they reduce consumption by more (relative to the situation where they move from unemployment to employment almost instantaneously) when the monetary shock takes place.

We develop in the online appendix (Section 5.3) a version of the model without search and matching frictions but with sticky nominal wages (Christiano et al. (2005), Smets and Wouters (2007)) and Epstein-Zin preferences (SW). The estimated version of the SW model again replicates the cyclical responses remarkably well. However, the steady-state value of the output stochastic volatility is almost zero and the model fails to produce distribution skewness and endogenous disasters.

These two quite different exercises seems to converge to the same conclusion that search and matching frictions are important. This is a result that seems to go hand in hand with the finding of the heterogenous agents literature. To be precise, Ravn and Sterk (2017) and Den Haan et al. (2018) (amongst others) argue convincingly about the necessity of incorporating search and matching friction (along with nominal price or/and wage rigidities) into these incomplete markets model in order to produce quantitative sizeable results without replying on large and very persistent shocks. It also coincides with the analysis of Petrosky-Nadeau et al. (2018), who argue that DSGE models with search and matching frictions can generate endogenous disasters.

**Interest Rate Smoothing** Taking the real labour frictions as given, we consider how discretionary monetary actions could cause uncertainty to increase endogenously and significantly.

The systematic part of the monetary policy consists of the two parts: (i) the response to deviation from FED’s inflation and unemployment gap objectives and (ii) the interest rate smoothing. Our investigation seems to suggest that it is the policymaker’s desire to ‘smooth’ changes in the policy rate that actually causes uncertainty to rise after a monetary policy shock\(^{19}\). To be precise, when the interest rate smoothing coefficient is set to zero \((\rho_R = 0)\), the uncertainty channel disappears and the impact of monetary policy on second moments is close to zero. (Figure 7). This happens as the lack of preference for interest rate smoothing allows authorities to loosen policy very quickly in order to restore both the inflation and unemployment targets. In other words, agents do not seem to be particularly concerned about the ability of monetary

---

\(^{18}\)This is implemented in the model by lowering the non-stochastic steady-state unemployment rate to 0.01%.

\(^{19}\)In the online Appendix (Section 5.1) we illustrate that if the reaction coefficient to unemployment gap is increased beyond empirically plausible values (such as greater than one) then the uncertainty channel diminishes significantly. Although, this simulation lacks empirical support (as we do not observed such high values for \(\zeta_u\) in the literature) the results further enforce the message of this paragraph. The higher policy response to unemployment countervails the smoothing parameter and the policy rate is decreased faster in order to support the recovery of the economy.
authorities to discretionary deviate from their objective function as long as they can reverse their actions and restore economy’s steady-state.

**Epstein-Zin Risk Coefficient** Figure 8 compares the responses (Panel A) and the uncertainty contributions (Panel B) in the benchmark case and the counterfactual case where the risk coefficient is set to zero \((\gamma = 0)\). With \(\gamma = 0\) the importance of the endogenous uncertainty channel declines and the uncertainty contributions are substantially smaller. Moreover, the response of volatility in the counterfactual case is smaller than the benchmark case.

However, these results also suggest that in relative terms the dramatic reduction in \(\gamma\) from 115 to 0 does not lead to effects that are extreme. As the economies’ agents have a high desire of early resolution of future uncertainty, the central bank is expected to keep policy expansionary for longer to meet its objectives. As a result, the long-term interest rate falls by more than 30 bps and stays below its stochastic steady-state for more than a year. Finally, this exercise is an additional evidence in favour of endogenous uncertainty acts as a demand channel (shock).\(^{20}\)

## 5 Conclusion

This study investigates the response of macroeconomic volatility to an unexpected increase in the policy rate. For this purpose we develop an empirical model that allows us to estimate the response of macroeconomic volatility to a monetary policy shock. To investigate the transmission channel of the shock, we build a simple New Keynesian model, with search and matching labour frictions and Epstein-Zin preferences.

The empirical model suggests that a 100 basis points increase in the policy rate causes unemployment and inflation volatility to rise by around 10\% above its unconditional value. The theoretical model has been calibrated to match the SVAR responses. Simulations from the theoretical model suggest that it is the coexistence of agents’ fears about being prolonged unemployment and monetary authorities’ desire for gradual policy adjustments that causes volatility to increase to levels observed empirically. In other words, households understand the risks of becoming unemployed and the fact that during the unemployment spells additional adverse shock may occur. However, it is the combination of these risks together with the policy-rate smoothing parameter that causes monetary policy to have significant volatility effects and not the shock per-se. When these two conditions pre-exist, only then the Epstein-Zin preferences play a significant role.

### References


\(^{20}\)In the online appendix (Section 5.1) we investigate what happens to the economy when the inflation target increases from 2\% to 4\%. We find that the uncertainty effects from this policy change are small.


<table>
<thead>
<tr>
<th>Variable</th>
<th>Horizon</th>
<th>All Level shocks</th>
<th>Monetary Policy shock</th>
<th>All Level shocks</th>
<th>Monetary Policy shock</th>
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<tr>
<td></td>
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<td>Decomposition of volatility FEV</td>
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<td>Decomposition of level FEV</td>
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<tr>
<td>T-bill</td>
<td>12 mths</td>
<td>14.9 (7.88, 24.1)</td>
<td>6.45 (2.53, 11.8)</td>
<td>27.1 (20.36.5)</td>
<td>15.7 (11.7, 20)</td>
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<tr>
<td></td>
<td>24 mths</td>
<td>18 (11.3, 30.5)</td>
<td>5.93 (2.6, 11)</td>
<td>24.1 (16.6, 35.1)</td>
<td>7.98 (5.4, 11.1)</td>
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<td></td>
<td>60 mths</td>
<td>28.8 (18.2, 39.8)</td>
<td>5.8 (2.88, 11.2)</td>
<td>36.7 (23.5, 50.3)</td>
<td>6.6 (4.09, 10.8)</td>
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<tr>
<td>Unemployment</td>
<td>12 mths</td>
<td>15.6 (8.87, 24.3)</td>
<td>7.14 (3.35, 12.6)</td>
<td>57 (46.4, 67.3)</td>
<td>10.2 (6.04, 15.7)</td>
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<tr>
<td></td>
<td>24 mths</td>
<td>20 (12.6, 31.2)</td>
<td>6.59 (3.29, 11.8)</td>
<td>48.6 (35.9, 62.1)</td>
<td>10.9 (5.44, 17.4)</td>
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<tr>
<td></td>
<td>60 mths</td>
<td>30.4 (19.1, 41.7)</td>
<td>6.05 (3.43, 11.1)</td>
<td>48.4 (35.4, 60.4)</td>
<td>8.69 (4.59, 15.2)</td>
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<tr>
<td>Inflation</td>
<td>12 mths</td>
<td>17.8 (9.48, 28.9)</td>
<td>6.62 (2.76, 11.9)</td>
<td>72.2 (63.8, 80.6)</td>
<td>0.93 (0.27, 3.2)</td>
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<td>24 mths</td>
<td>22.7 (12.4, 34.7)</td>
<td>6.51 (3.11, 6.8)</td>
<td>57.6 (45.9, 69.6)</td>
<td>1.89 (0.41, 6.04)</td>
</tr>
<tr>
<td></td>
<td>60 mths</td>
<td>31 (19.6, 40.2)</td>
<td>6.93 (3.85, 12.1)</td>
<td>56.7 (41.3, 69.2)</td>
<td>4.04 (1.24, 11.4)</td>
</tr>
<tr>
<td>Term-Spread</td>
<td>12 mths</td>
<td>12.3 (7.05, 19.6)</td>
<td>5.83 (2.05, 10.8)</td>
<td>52.3 (44.5, 61.3)</td>
<td>18.6 (13.7, 24)</td>
</tr>
<tr>
<td></td>
<td>24 mths</td>
<td>15.7 (9.69, 25.9)</td>
<td>4.7 (1.9, 9.61)</td>
<td>39.1 (31.6, 48)</td>
<td>13.3 (9.85, 18.1)</td>
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<tr>
<td></td>
<td>60 mths</td>
<td>26.6 (16.5, 39.6)</td>
<td>4.77 (2.15, 9.43)</td>
<td>40.3 (33, 48.2)</td>
<td>12 (8.78, 16.8)</td>
</tr>
</tbody>
</table>

Table 1: Contribution to FEV of volatility and levels of endogenous variables. 68 percent error bands in parenthesis.
Table 2: Disaster Statistics

<table>
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<th>Metrics</th>
<th>Data</th>
<th>1 Standard Deviation</th>
<th>2 Standard Deviation</th>
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<tr>
<td>Probability</td>
<td>7.83</td>
<td>0.30</td>
<td>4.55</td>
</tr>
<tr>
<td>Size</td>
<td>21.99</td>
<td>11.84</td>
<td>14.46</td>
</tr>
<tr>
<td>Duration</td>
<td>3.72</td>
<td>6.35</td>
<td>3.76</td>
</tr>
</tbody>
</table>

Notes: The data disaster statistic estimates are those reported by Petrosky-Nadeau et al. (2018) (Table 4, pp. 2227). Starting from the stochastic steady state, the model is simulated for 50000 periods. Similar to Petrosky-Nadeau et al. (2018), we time-aggregate output into annual observations, and apply the peak-to-trough method to identify disasters as cumulative fractional declines in output of at least 10%. The disaster probabilities and average size are in percent, and the average duration is in terms of years.
B Figures
Figure 1: Impulse response to a monetary policy shock

Notes: The solid line is the median. The light shaded area is the 68% error band while the dark shaded area is the 90% error band.
Figure 2: Impulse response to a monetary policy shock

Notes: The solid line is the median. The light shaded area is the 68% error band while the dark shaded area is the 90% error band. The top row presents the responses in the benchmark model that uses sign restrictions. The second row presents results from a model that uses a Cholesky decomposition while the third row presents results from the model that uses fed fund futures rate surprises as an instrument to identify the shock.
Figure 3: Posterior median impulse response to a monetary policy shock using the model with time-varying coefficients

**Notes:** The impulse response is calculated every 12th month in the sample.
Figure 4: Impulse response to a monetary policy shock from the benchmark and restricted models

Notes: The light shaded area is the 68% error band while the dark shaded area is the 90% error band from the benchmark model. The thick black line shows the median response from the restricted model.
Figure 5: Downward Risk

Panel A: 1 Standard Deviation Shocks

Notes: Starting from the stochastic steady state, the model is simulated for 50000 periods. The histogram illustrates the distribution of $x_t$ as deviation from its stochastic steady-state. Unemployment and inflation are expressed in percentage points (x-axis), expected unemployment duration is measured in quarters, while the GDP and labour income are defined as percentage deviations.
Figure 6: Search and Matching Frictions

Panel A: Impulse Responses

Panel B: Uncertainty Contribution

Notes: The blue solid (Panel A) and blue circle (Panel B) line (left y-axis) represents the benchmark version of the model ($u = 5.8\%$ or $Q_U = 63\%$), while the red dashed (Panel A) and red cross (Panel B) line (right y-axis) is the responses of the model when the unemployment rate is set to (almost) zero or the probability of finding a job is (almost) one ($u = 0$ or $Q_U = 100\%$). Rates are reported in annual basis points, inflation in annual percentage rates, the job filling probability in percentage points, unemployment duration in quarters. The responses are calculated relative to the stochastic steady state. Panel B: The uncertainty contribution is defined as the difference between the response derived by using the third minus the responses produced using only the first order solution of the model.
Figure 7: Policy Rate Smoothing Preferences

Panel A: Impulse Responses

Panel B: Uncertainty Contribution

Notes: The blue solid (Panel A) and blue circle (Panel B) line (left y-axis) represents the benchmark version of the model ($\rho_R = 0.71$), while the red dashed (Panel A) and red cross (Panel B) line (right y-axis) denotes the responses of the model when the interest rate smoothing parameter is set to zero ($\rho_R = 0$). Rates are reported in annual basis points, inflation in annual percentage rates, the job filling probability in percentage points, unemployment duration in quarters. The responses are calculated relative to the stochastic steady state. 

Panel B: The uncertainty contribution is defined as the difference between the response derived by using the third minus the responses produced using only the first order solution of the model.
Figure 8: Epstein-Zin Risk Coefficient

Panel A: Impulse Responses

Panel B: Uncertainty Contribution

Notes: The blue solid (Panel A) and blue circle (Panel B) line (left y-axis) represents the benchmark version of the model (γ = 115.6), while the red dashed (Panel A) and red cross (Panel B) line (right y-axis) denotes the responses of the model when the Epstein-Zin Risk Coefficient is set to zero (γ = 0). Rates are reported in annual basis points, inflation in annual percentage rates, the job filling probability in percentage points, unemployment duration in quarters. The responses are calculated relative to the stochastic steady state. Panel B: The uncertainty contribution is defined as the difference between the response derived by using the third minus the responses produced using only the first order solution of the model.