A Long-Commodity-Cycle Model of the World Economy Over a Century and a Half — making bricks with little straw

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Abstract

This paper explores the world business cycle using unfiltered data from 1870 and looks for a theory that could account for the long wave commodity cycle in the world economy. We build a simple DSGE model that includes a long time-to-build constraint in the commodity sector. We find that this model can produce long cycles in output and commodity prices as introduced by Kontradieff (1925) and Schumpeter (1935). Our findings show that these long business cycles are produced by the long gestation of commodity capacity which causes very large swings in commodity prices.

Keywords: Long waves; commodities; DSGE model; Indirect Inference
JEL classification: E10; E32; E52

1 Introduction: the world business cycle and the long wave

The goal of this paper is to consider the world business cycle and specifically relate it to two main shocks: from materials productivity, and from money supply policy. Most macroeconomists think of the business cycle as of fairly short duration, perhaps between 5 and 10 years from peak to peak. This could well be true at a national level because of the variety of shocks that can trigger recession for a smaller, national, unit. One can use data filters, such as the Hodrick-Prescott filter, with a filter window such that cycles are generated at such frequencies. Such filters can do the same thing at the level of the world economy. However, if one looks at unfiltered world data from around 1870 to the present day, which is our data sample here, one can discern a series of very long output cycles, with the length on the upswing of some 20 years' length. Figure 1 shows world output annual growth from 1870; if one takes uninterrupted spells of positive annual growth (i.e spells with no year of negative growth) there are four such upswings in the sample, 1877-1892, 1894-1907, 1947–1974 and 1983–2008. The omitted periods are two downswings, 1973-1982 and 2009-2012; and also the period including the two world wars, 1908-1945, a period of sharp and mostly short swings, including the 1930s Great Depression. What we see from this data, excluding that war period, is that on this admittedly crude definition of a recession as annual negative growth, we have an average upswing of 21 years and an average downswing of about 7 years, a total average cycle length of 28 years. The war period, with its several short cycles, brings down this average. However, what we draw attention to here is the presence of several long cycles.

Two prominent economists in the past two centuries have urged the presence of long cycles; both have been largely ignored in recent macroeconomics. One, Kondratieff (1925; see also Kondratieff and Stolper,1935), wrote in the Soviet Union. The other, Schumpeter (1935), was for many years professor at Harvard where he wrote several books about growth and the business cycle. Two of his big macroeconomic ideas were the role of 'creative destruction' in productivity growth and the effect of commodity long waves on the business cycle. One can see creative destruction at work in commodity innovation where new materials simply push aside the old one, as iron and steel pushed away wood, or polypropylene sisal and jute; but here we do not focus on this idea about the source of growth as our productivity

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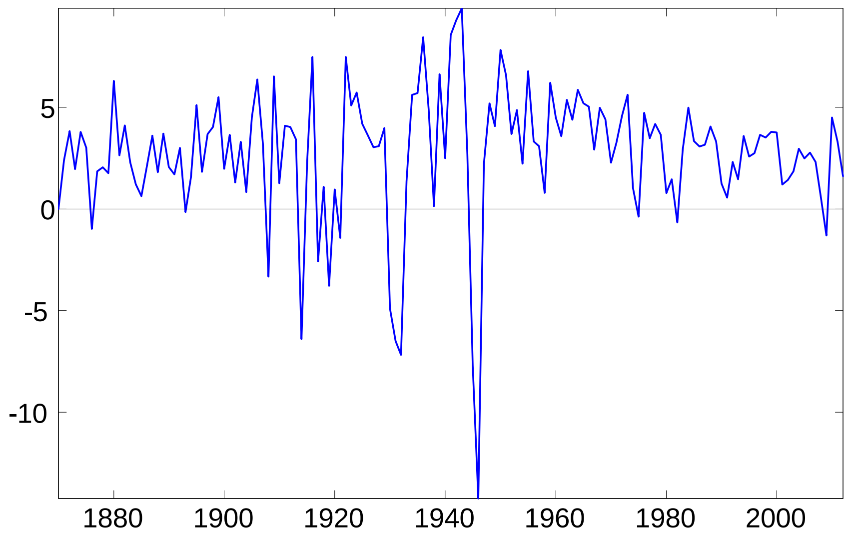

Figure 1: World GDP Growth (Source: Maddison, 2010)

change in the model is entirely exogenous. Both these economists connected the business cycle and the long cycles in it to the behaviour of commodity prices where indeed one can see plain evidence of long cycles, the focus of this paper. Figure 2 shows the data for real non-oil and oil commodity prices over the same sample. If one takes the non-oil as representative throughout the period, one can discern five peaks, 1873, 1917, 1951, 1974 and 2011, implying a peak to peak average cycle length of around 35 years. If one examines real oil prices after WW2, there are two peaks, 1974 and 2011, implying just one long cycle of 37 years peak to peak. There are also short periods of cyclical movement within these long cycles.

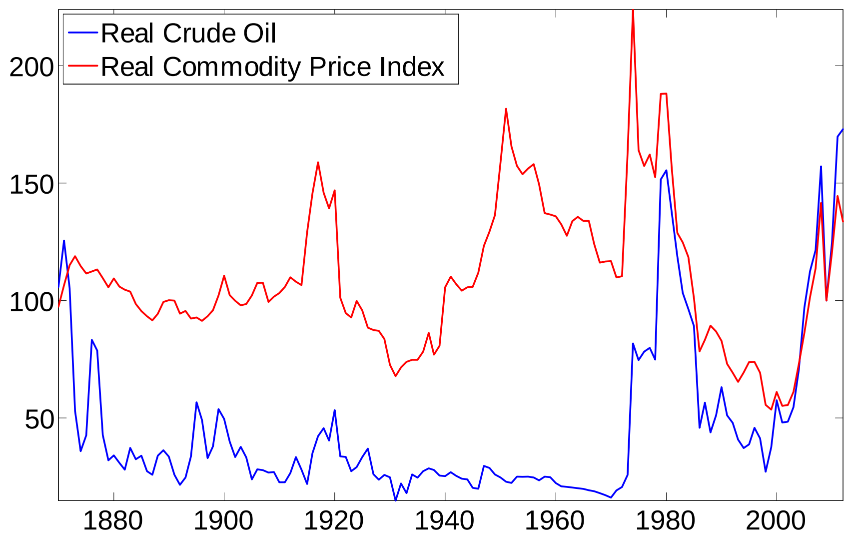

Figure 2: Commodities (Source: Mitchell 1971 and World Bank)

It is tempting to believe that there is a strong connection between these two long cycles, in output and in commodity prices.

There is much previous work on the behaviour of the world economy and long cycles. On the one hand, there is an extensive literature devoted to identifying a long wave cycle. The initial decomposition method used by Goldstein (1988), for example, aimed at isolating major fluctuations in the deviations of a macroeconomic variable around its trend through a combination of detrending procedures and smoothing techniques. This method however was criticised for relying heavily on ad-hoc assumptions about the form of the trend or the predefinition of historical phase periods. Amongst many alternatives, spectral analysis appears as one of the dominant methods. This method can simultaneously break down any time series into a set of cyclical components having different frequencies. Using this method Van Ewijk (1982) finds evidence of Kondratieff waves. In particular, Van Ewijk finds the existence of a long wave in prices, but not in real variables for the UK, US, France and Germany. The other dominant method for detecting long waves is the filter design approach. In this method, as in spectral analysis, a time series is considered as the result of the summation of different frequencies and the task of the filter consists of determining the filter coefficients so as to isolate specific frequencies and to show the course of pre-specified frequency components in the time domain (Baxter and King, 1999; Christiano and Fitzgerald, 2003). There is also a vast literature that takes the existence of long wave cycles as given and tries to explain the long wave cycle. Rostow (1978, 1980) constructs a model where a scarcity or surplus of basic commodities generated a Kondratieff type wave in prices, GNP and real wages. The swings depend on the assumption of parameters for the elasticity of substitution between capital, labour, and raw materials. Forrester (1979) used a complicated "National Model" and finds that each major expansion grows around a highly integrated and mutually supporting combination of technologies. He finds that the long wave process culminates in excess debt and overbuilding of capital sectors, followed by depression when the excess capital plant is worn out and financially depreciated. Further, Graham and Senge (1980) find that near the peak of the wave, major industries suffer decreasing returns to capital and are vulnerable to government regulation. As the economy slides into depression, opportunities arise for waves of inventions. That is, there is a close correlation between primary energy source substitution and waves of innovation. Around the same time, Marchetti (1980) shows that the prices of fuels rise in coincidence with the apex of each long wave cycle. Gordon, Weisskopf and Bowles (1983) develop a theoretical model to link the two types of fluctuations, the business cycle and long swing expansions and crises. The model is based on the idea that swings are due to the social institutions in place: these determine economic and political conditions, which in turn drive profit expectations and capital accumulation. They find that the long swing is associated with the non-reproductive cycle, i.e. the cycle in which the downturn does not correct itself endogenously to restore rapid accumulation conditions for profitability and thus requries institutional changes.

On the other hand there has recently arisen a literature of 'world macro models' mostly involving two- or three-country models (Laxton and Pesenti, 2003; Erceg, Guerrieri and Gust, 2006; Benigno and Thoenissen, 2003; Murchison, Rennison and Zhu, 2004; Adolfson et al. 2014). This work has mainly focused on explaining the fluctuations in recent data from the post-war period, and has not explored a role for long cycles. To our knowledge there is no DSGE model linking the two long cycles of commodity prices and output and generating this variety of long and occasional short cycles in output at the level of the world economy. This is what we aim to provide in this paper: a DSGE model in which people invest in commodity and final production capacity, consume final output and hold money controlled by the government's monetary policy, the key policy element integrated into our model (where we follow Friedman and Schwarz,1963, by focusing on money creation). Our further aim is to check whether such a theoretically-based DSGE can account statistically for the data behaviour we observe, inclusing the long and short cycles we observe.

We are quite limited in our available data (the 'straw' for creation), with the sample starting from 1870. Our model (the 'bricks' we create) is therefore sparing in its use of data and abstracts boldly from reality, taking quite literally Friedman's adjuration (Friedman, 1953) that models are 'as if' creations designed to be tested solely on their ability to explain the facts of interest. The data of interest to us is the cyclical behaviour of output, commodity prices, inflation and money supply growth. We use 'cyclical' to include long swings in the cycle which originate from non-stationary shocks; and we do not filter our data at all, so that it also includes deterministic as well as stochastic trends.

Our key findings are that long cycles are produced by the long gestation of commodity capacity. Because of this long gestation, commodity prices are capable of very large swings; these swings in turn cause swings in costs for non-commodity industries which act as a source of real business cycle shocks to output. Investment in the non-commodity sector is subject to quite short lags and so these shocks cause shorter investment booms and busts in this sector which, in turn, generate short booms and busts in general output. Monetary policy has some, but rather limited, ability to stabilise these cycles, even though rigidity in final prices enables monetary policy feedback to be effective; but it also has a definite ability to destabilise, as notably in the Great Depression, and so to generate additional short cycles.

The paper is organised as follows. We describe the model in outline and provide its details in section 2; in section 3 we estimate and test the model, using the not hitherto much used method of indirect inference which has considerable power in small samples; in section 4 we review the model's implications for the frequency of different cycle lengths; in section 5 we analyse the possibility of more effective monetary policy rules than the one we uncover during the sample; section 6 checks the robustness of our results, before finally concluding.

2 The model

2.1 The model outlined

The structure of the economy is as follows. Households maximise utility by consuming final output and holding money. They also produce final value-added and raw material/commodity and receive the total income from these productions. Both final value-added and material output are produced solely by capital. Labour input and rewards are ignored, i.e. the owning households get total income independent of any labour input and do not value leisure. Then if labour input is required to deliver the assumed output members of the household agree to share it equally. We also assume that the world population, the source of labour supply, is endogenous, responding positively to the supply of resources — a weaker version of Malthus (1798) where he argued that population would expand to absorb the available supply of resources. Plainly the world's great cities, where the mass of the world population (now 7 billion) lives, could not exist without the support of mass food production, transit, distribution and energy systems, all of them the product of innovation in primary production.

Households determine their investment for each production so that the expected marginal product of capital equals the real interest rate. However, there is a time-to-build constraint (following Kydland and Prescott, 1982) so that the capital stock reflects the expected conditions at the time of decision to build. For the final value-added production the time-to-build is short — one year. For the commodity production it is long: we set it at a representative 10 years, even though the facts indicate there is a wide variety of times to build depending on commodity, from as little as a few years for a synthetic fibre plant to as much as twenty years or more for some copper mines. Clo (2000) states that conventional discoveries for oil and gas can take 30–40 years to develop. In contrast Arezki et al. (2017) show that for giant oil and gas discoveries the delay between discovery and production averages 5.4 years. For mining the lead times are longer. Schodde (2014) reports the average delay is 12.4 years, but different commodities have different average delays. For gold the average delay is 10 years, copper 17.1 years, zinc and lead 15.0 years and uranium 14.7 years; the time delay for copper and gold can range from 1 year to 60 years. Our assumed average of 10 years is subject to our empirical tests and we can vary it in estimation.

The key element in the expected marginal product is expected commodity prices. When these prices are expected to be high the return is high for commodity investment but for final output investment it is low because of high relative input costs. The problem for these households' assessments is that both productivity in each production and commodity prices are non-stationary variables. In the simplest case where they are pure random walks the best forecast is the current value. It is this combined with the long time-to-build that creates the long commodity variation or 'long cycle': the capital stock today reflects commodity prices 10 years ago, yet since then accumulated shocks could have moved demand for commodities sufficiently to move commodity prices well away from those originating values. In particular, suppose they were high 10 years ago: this will have depressed final output investment so that the demands will have tended to fall subsequently. So by today commodity demand may well be depressed while capacity is high. Commodity prices will tend to be depressed triggering opposite tendencies for the following ten years. This is, however, just an example. Because the shocks are all unpredictable it is possible that after ten years the boom will have got even greater.

Households sell final value-added goods and commodities at the perfect competitive prices, at marginal cost, to the bundling producers, who bundle these household outputs into final output in fixed Leontief proportions. These firms are imperfectly competitive and they set their prices in a Calvo manner, at which they sell this final output to households. Households' demands get translated into bundled output at these prices and must be satisfied: the bundling firms transmit these demands back to producing households who supply them at rising marginal cost by varying the use of capacity, given their capital as set by previous investment decisions. We think of this capacity utilisation as consisting of using more shifts on existing capital. The profits of the imperfectly competitive bundling firms are paid in dividends to the owning households.

Households' consumption demand depends on total income and the real interest rate. Households' utility function also implies a demand for money in terms of money income and interest rates. Money is supplied by a world central bank (which we can think of as the average of the major 'money centre' central banks in the developed world, notably the UK and the US, through most of the sample period) and associated banking system according to a money growth feedback rule: the growth responds to inflation and to output growth, compared with some targets which may be moving over time.

The demand for commodity output is derived from the demand for final output, and hence is a function of real (i.e. relative to the price of final value-added) commodity prices and total income. The demand for capital equates the expected marginal product once on stream with the current and expected real interest rate over the period of construction.

There are five markets in the model: for commodities, for final value-added output, for gross final output, for money and for bonds. By Walras' Law, we require four market-clearing conditions. We focus as usual on the market for commodities, for final value-added output, for final output and for money. Commodity prices equate demand with a function of capacity (where the supply price equals marginal costs, rising with capacity utilisation). With prices being set in imperfect competition, the supply of final output moves to satisfy demand for capital construction plus consumption. The final value-added output prices is at marginal cost, the final prices less the mark-up, to equate its demand from the bundler with a production function with capacity utilisation. The interest rate equates the demand and supply of money. There are rational expectations.[1]

2.2 The model equations

We now turn to a detailed description of the model's equations. We assume that the world representative household receives the net income from both sectors, which equals the value of final value-added plus the value of raw materials output; and it spends this on investment in both sectors sector (i_F and i_M, where $F=$ final sector; $M=$commodity (materials), consumption (c), bond purchase (b) and real money balances ($m = M/P$). The household maximises

$$\sum_{t=0}^{\infty} \beta^t U(c_t, m_t) \tag{1}$$

subject to

$$c_t + i_{Ft} + i_{Mt} + M_t/P_t - M_{t-1}/P_t + b_t - (1 + r_{t-1})b_{t-1} + \tau_t = (\Omega + \epsilon_t)(Y_{Ft} - Y_{Mt}) + p_{Mt}Y_{Mt} + \Pi_t$$

where $U(c_t, m_t) = \frac{c_t^{1-\rho}}{1-\rho} + \theta_t \frac{m_t^{1-\xi}}{1-\xi}$,$\theta_t$is an error in liquidity preference, $P_t$is the price of final output (used as the general deflator and consumer price index), and $\Omega + \epsilon_t$is the inverse of the bundler mark-up on value-added (since final value-added is supplied to bundlers at marginal cost, namely the final price less this mark-up, $P_t[\Omega + \epsilon_t]$, where $\epsilon_t$reflects the mark-up's fluctuations, assumed to be random). So all resources are measured in units of final output. $p_{Mt}$is the price of raw materials relative to this deflator. $\Pi_t$are the bundling firms' profits deflated by P_t ($\Pi_T = Y_{Ft} - (\Omega + \epsilon_t)(Y_{Ft} - Y_{Mt}) - P_{Mt}Y_{Mt}$). But households supply these bundlers atomistically in a perfectly competitive market and so do not individually affect bundlers' profits. $Y_{Ft} - Y_{Mt} = K_{Ft}^{\phi} - u_{Ft}$ and $Y_{Mt} = a_t K_{Mt}^{\mu} - u_{Mt}$.where a_t is productivity in the use of materials capital and we will denote $a_t K_{Mt}^{\mu} = Y_{MCAPt}$, that is materials output capacity at t, planned at $t-10$. The terms in u are capacity non-utilisation rates. These arise because price-setting final-goods firms must meet demand; they do so with capacity already installed due to previous expectations, and use it more or less intensively by varying the labour input (e.g. by varying shift patterns). These capacity utilisation terms enter the Phillips curve as set out below.

The final output 'bundling' firm produces gross final output (Y_{Ft}) using the final value-added and the material output (Y_{Mt}) in a Leontief production function:

$$Y_{Ft} = (Y_{Ft} - Y_{Mt}) + Y_{Mt} \tag{2}$$

where $Y_{Mt} = vY_{Ft}$. The bundling firms which are imperfectly competitive set prices for this output in a Calvo manner.

[1] It might be asked how one can get cyclical tendencies under rational expectations. The reason is that, although it is impossible to do better in forecasting, the rigidities of time-to-build create opportunities for shocks to accumulate and so create booms or busts before investors can respond by changing capacity.

The investment technology requires time to build new capital. The following identities are for investment and the capital stock from the time-to-build set-up, where s is an 'investment project' undertaken that takes in the case of final capital one year to build and with materials capital ten years, producing the capital addition at the end:

$$i_{Ft} = s_{Ft};\; i_{Mt} = 0.1(s_{Mt} + ... + s_{Mt-9})$$

$$K_{Ft} = (1-\delta_F)K_{Ft-1} + s_{Ft-1};\; K_{Mt} = (1-\delta_M)K_{M_{t-1}} + s_{Mt-10}. \tag{3}$$

The final and material capital stocks depreciate at rate δ_F and δ_M, respectively.

A government prints money and issues (real one-period) bonds, b_t, to finance its existing stock of nominal liabilities and its spending net of its (lump-sum) taxes, τ_t:

$$(1 + r_{t-1})P_t b_{t-1} + M_{t-1} = G_t - \tau_t + M_t + P_t b_t.$$

We can write the Lagrangean for the household problem,

$$\max_{c_t, m_t, b_t, s_{Ft}, s_{Mt}, K_{Ft+1}, K_{Mt+10}} L = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} [\frac{c_t^{1-\rho}}{1-\rho} + \frac{m_t^{1-\xi}\theta_t}{1-\xi}] + \lambda_t[c_t + s_{Ft} + 0.1(s_{Mt} + ... + s_{Mt-9}) \\ +m_t - m_{t-1}(P_{t-1}/P_t + b_t - (1 + r_{t-1})b_{t-1} \\ +\tau_t - (\Omega + \epsilon_t)K_{Ft}^{\phi} - p_{Mt} a_t K_{Mt}^{\mu} + u_{Ft} + p_{Mt} u_{Mt}] \\ +q_t\left(K_{Ft} - (1-\delta_F)K_{Ft-1} - s_{Ft-1}\right) \\ +\psi_t\left(K_{Mt} - (1-\delta_M)K_{M_{t-1}} - s_{Mt-10}\right) \end{array} \right\}$$

where $q_t$and $\psi_t$denote the Kuhn-Tucker multipliers associated with the equality constraints (3) .The first order conditions are:

$$c_t : 0 = \frac{\delta L}{\delta c_t} = c_t^{-\rho} + \lambda_t \tag{4}$$

$$m_t : 0 = \frac{\delta L}{\delta m_t} = m_t^{-\xi}\theta_t + \lambda_t - \beta\frac{E_0\lambda_{t+1}}{1+\pi_{t+1}} \tag{5}$$

$$b_t : 0 = \frac{\delta L}{\delta b_t} = -\lambda_t + \beta E_0 \lambda_{t+1}(1 + r_t) \tag{6}$$

$$s_{Ft} : 0 = \frac{\delta L}{\delta s_{Ft}} = \lambda_t - \beta q_{t+1} \tag{7}$$

$$K_{Ft} : 0 = \frac{\delta L}{\delta K_{Ft+1}} = -\beta\lambda_{t+1}\phi(\Omega + \epsilon_{t+1})K_{Ft+1}^{\phi-1} + \beta q_{t+1} - \beta^2(1-\delta_F)\,q_{t+2} \tag{8}$$

$$s_{Mt} : 0 = \frac{\delta L}{\delta s_{Mt}} = 0.1\lambda_t + 0.1\beta\lambda_{t+1} + ... + 0.1\beta^9\lambda_{t+9} - \beta^{10}\psi_{t+10} \tag{9}$$

$$K_{Mt} : 0 = \frac{\delta L}{\delta K_{Mt+10}} = -\beta^{10}\lambda_{t+10}\mu K_{Mt+1}^{\mu-1} p_{Mt+10} a_{t+10} + \beta^{10}\psi_{t+10} - \beta^{11}(1-\delta_M)\,\psi_{t+11} \tag{10}$$

where $\pi_t = P_t/P_{t-1} - 1$. Notice that together conditions (7& 8) and (9& 10) imply that investment equates the expected utility-discounted marginal product of capital (including the effect as it depreciates) with the current marginal utility of the investment cost. Also as a proxy for all $i - year - ahead$ interest rates at t we use the current (approximately one year) interest rate, $1 + r_t$. Hence, from these conditions we obtain:

$$c_t = E_t c_{t+i}\left[\beta(1 + r_t)\right]^{-\frac{1}{\rho}}; \tag{11}$$

$$m_t = c_t^{\frac{\rho}{\xi}}\left(\frac{R_t}{1+R_t}\right)^{-\frac{1}{\xi}} \theta_t^{\frac{1}{\xi}}; \tag{12}$$

$$K_{Ft+1} = (\phi\Omega)^{\frac{1}{1-\phi}}(r_t + \delta_F)^{\frac{-1}{1-\phi}}; \tag{13}$$

$$K_{Mt+10} = \left(\frac{\mu}{\varphi}\right)^{\frac{1}{1-\mu}}\left[\frac{(1-(1+r_t))\,E_t p_{Mt+10} a_{t+10}}{\left(1-(1+r_t)^9\right)(r_t+\delta_F)}\right]^{\frac{1}{1-\mu}} \tag{14}$$

From the capital demand equations we can obtain the capital project and investment equations as

$$s_{Ft} = i_{Ft} = \{1 - (i - \delta_k)L\}K_{Ft+1}$$
$$\text{and } s_{Mt} = \{1 - (1 - \delta_M)L\}K_{Mt+10}$$

so that

$$i_{Mt} = 0.1\left\{1 + L + ... + L^9\right\}\{1 - (1 - \delta_M)L\}K_{Mt+10}$$

We can also determine the capacity that will prevail in materials in ten years' time in a log form as

$$\ln Y_{MCAPt+10} = \mu \ln K_{Mt+10} = \frac{\mu}{1-\mu}E_t\left[p_{Mt+10} + a_{t+10} - \psi r_t\right] \tag{15}$$

where $\psi = \left[\frac{1}{r+\delta_M} - \frac{1}{r} - \frac{(1+r)^8}{1-(1+r)^9}\right]$. Here any covariance between p_{Mt} and a_t would be merged with the constant term. This is the plan made in this period based on its cost of capital and the expected material price when the plant has been built.

The firms to which households delegate production have only two tasks: to set prices of final goods and to produce the output demanded at these prices, namely the gross final output (which also determines the material output). Thus output is demand-determined in the short run; and there is a New Keynesian Phillips Curve for inflation, obtained under the usual Calvo set-up. The inflation equation for final output is therefore:

$$\pi_t = \beta E_t\pi_{t+1} + \zeta(\ln Y_{Ft} - \ln Y_{MCAPt}) + \eta_t \tag{16}$$

Here the marginal cost term, $\zeta(\ln Y_{Ft} - \ln Y_{MCAPt}) = \zeta\{(1-\nu)[\ln(K^{\phi}_{Ft}) - \ln Y_{MCAPt}] - u_{Ft}/Y_{Ft} - \nu u_{Mt}/Y_{Mt}\}$, where ν is the share of materials in final output. The latter two terms are the effects of rising capacity utilisation on costs. The first term measures the pressure on resources, notably land, in countries where final output is produced; expansion faster than the growth rate in materials capacity, proxying the growth rate of local resources, adds to costs. The error term in the Phillips Curve, η_t, represents other real marginal costs. Typically these will be correlated with current movements in real commodity prices and this would be picked up empirically through our vector bootstrapping procedure, whereby all current innovations are drawn as a vector, so preserving any correlations found in the innovations extracted from the sample data.

The government follows a money supply growth rule similar to a Taylor Rule (Taylor, 1993):

$$\Delta \ln M_t = \alpha\pi_t + \gamma(\ln Y_{Ft} - \ln Y_{MCAPt}) + \epsilon_t \tag{17}$$

The error here represents both the behaviour of the banking system and the monetary judgements of the authorities that cause deviations from its normal behaviour. While in the post-war period from the 1980s onwards central banks may well have obeyed a Taylor Rule, for much of our long sample period the world was on the Gold Standard, certainly from 1860 to 1914 and for much of the inter-war period; furthermore under Bretton Woods the US maintained a link from the dollar to the price of gold, severed only in 1971. The Gold Standard enjoined central banks whose gold reserves were low because of excess demand or inflation, to contract the money supply growth rate in order to cause an inflow of gold. Hence if across the world as a whole there is excess demand or inflation we should expect central banks in aggregate to be contracting the money supply growth rate. Nevertheless we must also remember that the banking system responds to the state of the economy, essentially in a pro-cyclical way; when inflation rises in a period of strong growth, asset prices will rise and banks are encouraged to lend. For this money supply rule to lean against inflation, α should be less than unity; we expect γ to be negative so that monetary policy leans against excess demand.

Taken with the demand for money function from households this money supply function determines interest rates, R_t, where the demand for money is given by households' first order conditions as:

$$\ln M_t/P_t = \frac{\rho}{\xi}\ln c_t - \frac{1}{\xi\overline{R}}R_t + \frac{1}{\xi}\ln\theta_t \tag{18}$$

We now turn to the market-clearing equations. As we have seen the money market determines R_t.We then define as usual as $r_t = R_t - E_t\pi_{t+1}$.

The materials market works as follows. Mines producing a raw product from the earth's crust, such as oil or ore are assumed to obey a model following Hotelling (1931) and Nordhaus (1974): there is some expensive substitute that will be used when this product runs out, and the raw product's price is set today at the discounted value of that 'backstop' price. Demand is supplied at this price (which is effectively the price at which it just pays to keep the product in the ground as an investment asset); the higher the demand and the lower the capacity the faster the product runs out and so the sooner the backstop price arrives. One can think of this a market-clearing mechanism for resources in fixed supply. Thus higher demand now and in the future set against this fixed supply raises prices via bringing the backstop price into operation earlier. For raw materials that are produced from harvesting a renewable resource, here assumed to be under perfect competition, the price clears the market where the marginal cost of production equals the price: this too implies that as demand increases or capacity decreases the price rises with rising marginal cost and rising capacity utilisation. Hence in both cases the market-clearing price rises as demand increases and as capacity decreases.

$$\ln p_{Mt} = \psi_1 \ln Y_{Ft} - \psi_2 \ln Y_{MCAPt} \tag{19}$$

For final output, market-clearing is done by output adjusting to demand given prices that have been set.

$$Y_{Ft} = c_t + i_{Ft} + i_{Mt} + G_t \tag{20}$$

We can make use of the consumption Euler equation to substitute for c in terms of output and investment terms, so yielding us the usual forward-looking IS curve. This is done by a) loglinearising the market-clearing equation (where $\widetilde{c}$ is the share of consumption in GDP, k_F is the capital-output ratio for final output and k_M that for materials output) and using the expressions for i_{Ft} and i_{Mt} above, to obtain

$$\begin{aligned}\ln Y_{Ft} &= \widetilde{c} \ln c_t + k_F \left[\ln K_{Ft+1} - (1-\delta_F) \ln K_{Ft}\right] \\ &\quad + 0.1 k_M (1 + L + ... + L^9) \left(\ln K_{Mt+10} - (1-\delta_M) \ln K_{Mt+9}\right) + g_t\end{aligned}$$

and b) substituting from the consumption Euler equation for $\ln c_t$ to obtain:

$$\ln Y_{Ft} = E_t \ln Y_{Ft+1} - \frac{\overline{c}}{\rho} r_t + (1 - B^{-1})(X_t + g_t) \tag{21}$$

$$\begin{aligned} X_t &= 0.1 k_M (1 + L + ... + L^9) \frac{1}{1-\mu} \left[\begin{array}{c} E_t \left(\ln p_{Mt+10} + a_{t+10} - \psi r_t\right) \\ -(1-\delta_M) E_t \left(\ln p_{Mt+9} + a_{t+9} - \psi r_{t-1}\right) \end{array} \right] \\ &\quad - \frac{k_F}{(1-\phi)(\overline{r} + \delta_F)} \left[r_t - (1-\delta_F) r_{t-1}\right] \end{aligned} \tag{22}$$

B^{-1} is the forward operator leading the variable but not the date of expectation. Since the expected future change will be converging from the current change towards zero, $(1 - B^{-1})$ dampens the variables it premultiplies; assuming that the model converges at some approximate constant rate, then this will act like a simple constant multiplier of less than unity. So the IS curve RHS contains the current level of interest rates as well as strings of current and lagged changes of interest rates; and also a string of current and lagged expected, up to 10 years ahead, future changes in raw material prices. g_t is the residual of this aggregate demand equation, in principle the ratio of government (or non-private) spending to world GDP, and is treated as an error process. This also allows us to rewrite the money demand equation (18) in term of output as

$$\ln \frac{M_t}{P_t} = -\frac{1}{\xi \left(1 + \overline{R}\right) \overline{R}} R_t + \frac{1}{\xi} \ln \theta_t + \frac{\rho}{\xi} \frac{1}{\overline{c}} [\ln Y_{Ft} - X_t - g_t] \tag{23}$$

2.3 Loglinearised model

Our model will therefore consist of the following 7 (loglinearised) equations in 7 variables, $\pi(P)$, Y_F, R, r, P_M, Y_{MCAPt}, M.

Equation (19) determines materials prices this period. The equation depends upon an unobservable variable, Y_{MCAPt}. We reveal this as the (negative of the) residual of this equation:

$$\ln P_{Mt} = c_1 + a_1^{pm} \ln Y_{Ft} + \epsilon_{1t} \tag{24}$$

where $\epsilon_{1t} = -\psi_2 \ln Y_{MCAPt}$. Notice that as we have no data on Y_{MCAP} we use this error which we extract as our measure (up to a multiple) of it.

Equation (15) determines this period's materials capacity:

$$\ln Y_{MCAPt} = c_2 + a_1^{ym} E_{t-10} \ln P_{Mt} - a_2^{ym} r_{t-10} + \epsilon_{2t} \tag{25}$$

Here the error term is the ten years' ago expectation of today's productivity.

Equation (21), the forward-looking IS curve, gives:

$$\ln Y_{Ft} = c_3 + E_t \ln Y_{Ft+1} - a_3^y r_t + (X_t - E_t X_{t+1}) + (1-\rho_g)\epsilon_{3t}$$

$$X_t = a_2^y [(E_t \ln P_{Mt+10} + ... + E_{t-9} \ln P_{Mt+1}) - (1-\delta_M)(E_t \ln P_{Mt+9} + ... + E_{t-9} \ln P_{Mt})] \tag{26}$$

$$- a_1^y [(r_t + ... + r_{t-9}) - (1-\delta_M)(r_{t-1} + ... + r_{t-10})] - a_6^y [r_t - (1-\delta_F) r_{t-1}] \tag{27}$$

Here the error term is the effect of government spending. However, this IS curve is unwieldy and finding coefficients for it that can fit the data behaviour has proved an elusive task. We therefore simplify it by representing the solution for X_{t+1} as a truncated ARMA(1,1) process namely $X_{t+1} = jX_t + \eta_{t+1} + m\eta_t$ to give $X_t - E_t X_{t+1} = (1-j)X_t + m\eta_t$, with η_t swept into the IS error term.

We then use (17) and (23) to yield the demand and supply of money as:

$$\ln(M_t/P_t) = c_4 + a_1^{md}(\ln Y_{Ft} - X_t - \epsilon_{3t}) - a_2^{md} R_t + \epsilon_{4t} \tag{28}$$

$$\Delta \ln M_t = c_5 + a_1^{ms} \pi_t - a_2^{ms}(\ln Y_{Ft} - \ln Y_{MCAPt}) + \epsilon_{5t} \tag{29}$$

These two equations solve for R_t and M_t respectively.

We use equation (16) for inflation as:

$$\pi_t = c_6 + \beta E_t \pi_{t+1} + a_2^{\pi}(\ln Y_{Ft} - \ln Y_{MCAPt}) + \epsilon_{6t}$$

(The price level is derived from the inflation rate.) The error term here is the shifting Calvo mark-up and also any error in our marginal cost specification. β is firms' time preference rate which we expect to be substantially lower than households' because of firm entry and innovation.

Finally, we have the identity linking real and nominal interest rates:

$$r_t = R_t - E_t \pi_{t+1}$$

Rational expectations prevail for the expected terms.

We can represent the resulting model as the following 7 (loglinearised) equations:

$$\text{Materials Prices: } \ln P_{Mt} = c_1 + a_1^{pm} \ln Y_{Ft} - \ln Y_{MCAPt} \tag{30}$$

$$\text{Materials Capacity: } \ln Y_{MCAPt} = c_2 + a_1^{ym} E_{t-10} \ln P_{Mt} + \epsilon_{2t} \tag{31}$$

$$\text{IS Curve: } \ln Y_{Ft} = c_3 + E_t \ln Y_{Ft+1} + a_2^y X_t + a_3^y r_t + \epsilon_{3t} \tag{32}$$

$$\text{Money Demand: } \ln(M_t/P_t) = c_4 + a_1^{md} \ln Y_{Ft} + a_2^{md} R_t + \epsilon_{4t} \tag{33}$$

$$\text{Money Supply: } \Delta \ln M_t = c_5 + a_1^{ms} \pi_t + a_2^{ms}(\ln Y_{Ft} - \ln Y_{MCAPt}) + \epsilon_{5t} \tag{34}$$

$$\text{Phillips Curve: } \pi_t = c_6 + \beta E_t \pi_{t+1} + a_2^{\pi}(\ln Y_{Ft} - \ln Y_{MCAPt}) + \epsilon_{6t} \tag{35}$$

$$\text{Fisher Equation: } r_t = R_t - E_t \pi_{t+1} \tag{36}$$

$$\text{where: } X_t = (E_{t-1} P_{Mt+9} + ... + E_{t-10} P_{Mt}) \tag{37}$$

3 Model Estimation

3.1 Data

Using unfiltered data from $1870-2008$ (see Figure 3) we estimate the model using the method of Indirect Inference as in Le et al. (2011)[2]. As the data is non-stationary we use a VECM as the auxiliary model with output, inflation and materials prices as endogenous variables and the non-stationary materials capacity shock and a deterministic time trend as exogenous variables. In the calculation of the Wald statistic we include the parameters on the exogenous terms in order to match the long-run trends seen in the data.

[2] For a full account of indirect inference methods and available computer programmes see Le et al. (2016a)

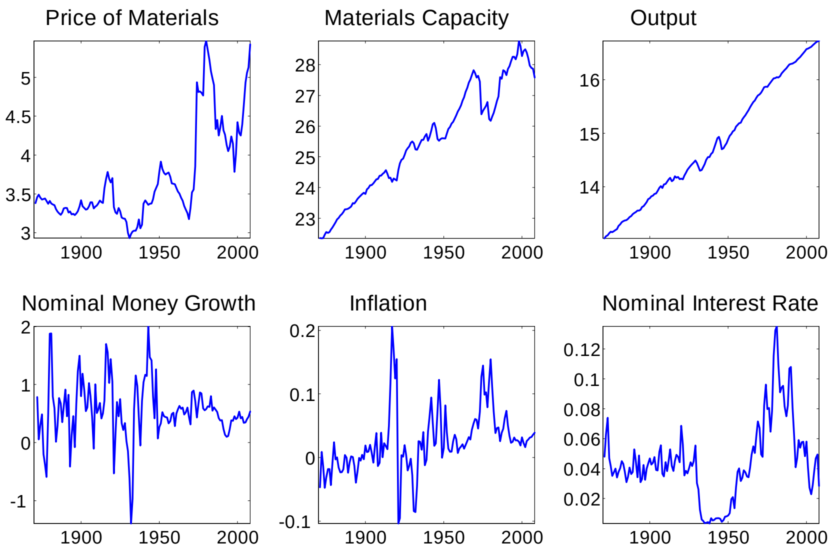

Figure 3: World Data (1870-2008)

A full set of data sources is in the Table 1. The commodity prices are based on widely used indices, deflated into real terms. Output, money, interest rates and consumer prices are a weighted average of UK and US data as these two countries were the two main world trading economies and money centres throughout the period. Plainly this raises issues of the role of omitted countries and we address this in a robustness test where we substitute world weighted series for all these, where all developed countries are included with GDP weights; the results with these series are virtually unchanged. We take this as indirect confirmation of our assumption that the world environment was essentially shaped by the two major money and trade centres.

Variable	Source
Output	Historical Statistics of the World Economy: 1-2008 AD (Copyright Angus Maddison)
Inflation	Average of US CPI and UK RPI (measuringworth.com)
Interest Rate	Average of US and UK short term rate (contemporary) (measuringworth.com)
Money Supply	UK: "The UK recession in context — what do three centuries of data tell us?" Ryland Thomas US: "Monetary Statistics of the United States". Milton Friedman and Anna Jacobson Schwartz
Commodities Prices	"British Historical Statistics" B. R. Mitchell for pre 1960 World Bank for post 1960

Table 1: Data Sources

3.2 Estimation

Table 2 shows the key parameters as estimated as well as the Wald statistic and its p-value. The Wald statistic tests whether the model can jointly match the coefficients on the VECM produced by the data. We find that the model fits adequately, with the p-value of 0.06 showing the model is well away from being rejected by the data.

The IS curve has the unit forward coefficient on future output and since the real interest rate is highly persistent this implies that its effect inclusive of the forward effect is rather high; if one assumes that it has an average AR coefficient of 0.9, then the long run slope of the IS curve ($\partial R/\partial y$) would be -0.4. The LM curve's short run and long run slope is 0.5.

The model can therefore be understood as having a forward-looking IS curve that reacts positively to expected future raw material prices via increased commodity investment, and negatively to real interest rates. The LM curve is shifted by the growth of the real money supply. The Phillips Curve is of normal

Key Parameters		Estimated
Elasticity of Materials Prices to final output	a_1^{pm}	0.97
Elasticity of Materials Capacity to expected materials prices	a_1^{ym}	0.35
IS curve - semi-elasticity to future materials prices	a_2^{y}	0.09
- semi-elasticity to real interest rate	a_3^{y}	-0.22
Money Demand - elasticity to consumption	a_1^{md}	1.73
- semi-elasticity to interest rates	a_2^{md}	-3.50
Money Supply growth - semi-elasticity to inflation	a_1^{ms}	0.17
- elasticity to output gap	a_2^{ms}	-0.01
Phillips Curve - response to expected inflation	β	0.58
- response to output gap	a_2^{π}	0.62
Shocks: ΔMaterials Productivity	ρ^{pm}	0.13
IS Curve	ρ^{y}	0.92
Money Demand	ρ^{md}	0.99
Money Supply Growth	ρ^{ms}	0.55
Phillips Curve	ρ^{π}	0.93
Wald statistic		37.08
p-value		0.06

Table 2: Estimated Parameters

dimensions, with a high response to expected future inflation and a fair-sized response to the output gap. There is a 10-year delay before any commodity investment changes raw material capacity; once this delayed capacity is triggered, it shifts the Phillips Curve to the right, lowering inflation and stimulating demand/output, pushing the IS/LM intersection rightwards towards the new higher capacity.

Table 3 shows the coefficients from the auxiliary model alongside their 95% bounds from the simulations. The auxiliary model is a VECM with productivity and a deterministic trend as the drivers of the data ensuring cointegration. We also include the variances of the residuals from the VECM in our set of coefficients we are trying to match. We find that as well as the model jointly matching the coefficients they, on the whole, also match individually. The variance of the output residual is lower than the bounds, and the variance of the inflation residual is just outside the lower bound.

	Actual	Lower Bound	Upper Bound	IN?
Y_Y(-1)	0.94264	0.51935	1.04548	IN
PI_Y(-1)	0.02258	−0.04615	0.42267	IN
PM_Y(-1)	0.18834	−0.35910	0.88359	IN
Y_PI(-1)	−0.06170	−0.41401	0.01390	IN
PI_PI(-1)	0.64408	0.50019	0.86760	IN
PM_PI(-1)	0.29192	−0.55938	0.49092	IN
Y_PM(-1)	0.00389	−0.13735	0.40853	IN
PI_PM(-1)	0.00253	−0.31545	0.14406	IN
PM_PM(-1)	0.89104	0.08809	1.35801	IN
Y_A(-1)	−0.01080	−0.10572	0.40925	IN
PI_A(-1)	0.00676	−0.36819	0.06432	IN
PM_A(-1)	−0.01037	−0.76949	0.39937	IN
Y_T	0.00144	−0.00213	0.01979	IN
PI_T	−0.00032	−0.01814	0.00191	IN
PM_T	−0.00433	−0.03672	0.01553	IN
var(Y)	0.00114	0.00225	0.00498	OUT
var(PI)	0.00119	0.00121	0.00287	OUT
var(PM)	0.02420	0.00615	0.03409	IN

Table 3: Auxiliary Model (VECM) coefficients and bounds

3.3 Residuals and Innovations Extracted

Figure 4 shows the single equation residuals that result from the parameter estimates. We treat materials productivity as being non-stationary, and the rest as trend-stationary (apart from money supply which is stationary). The persistence of the shocks are shown in Table 2.

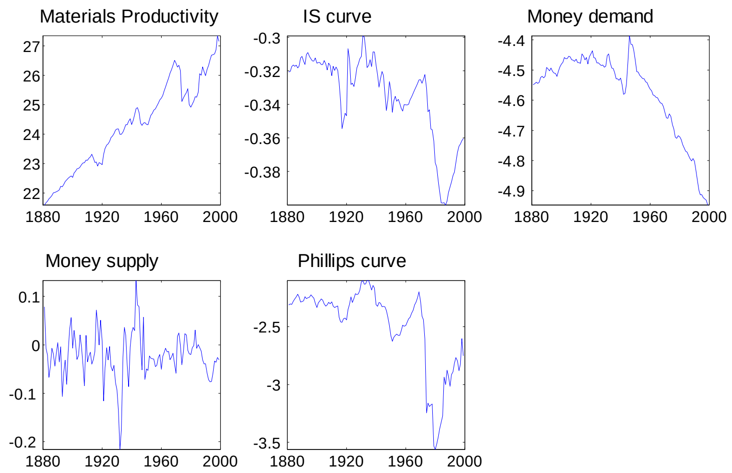

Figure 4: Residuals

The innovations we use to shock the model are shown in Figure 5. Over such a long sample we find there are periods of calm, and periods of high volatility. The materials capacity and IS curve shocks are highly volatile during the 1970s. This is caused by the sharp rise in the price of materials which was mainly due to the shortage of oil. Similarly the Phillips Curve shock is highly volatile during the same period. Money supply growth and money demand shocks are, however, biggest before WW2.

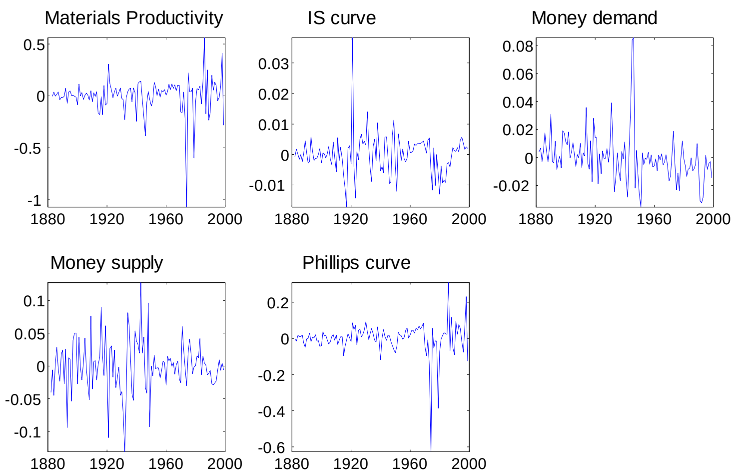

Figure 5: Innovations

Notice that when we bootstrap these shocks we do so by time vector, that is to say we draw all the innovations for one period together when we randomly select shocks. This preserves any simultaneous correlation between them which may well be important because a single event source can trigger shocks all over the economy — think of the recent financial crisis.

3.4 Why use Indirect Inference?

Indirect Inference is a relatively unfamiliar method of estimation and testing. We use it here because we need a method that will powerfully reject a mis-specified model in the small sample that we have (around 140 annual observations). The two main alternatives today are Bayesian estimation with strong priors or Maximum Likelihood (equivalent to Bayesian estimation with flat priors).

The former is an appropriate method when much is already known about the issue at hand, so that priors can be set out that command general assent; often the case in the physical sciences and indeed in some parts of the social sciences. However, this condition does not apply here: the macroeconomics of the world economy is not much explored and remains controversial.

Maximum Likelihood estimation is based on minimising the model's now-casting prediction errors and its associated test is based on the likelihood implied by these errors. The two main difficulties of this method are first that it exhibits high estimation bias in small samples and second that the power of the test in small samples is also rather limited and in particular its power to reject a mis-specified model is close to zero, because such a model can be fitted closely to the data, so creating small errors. Le et al (2016a) carried out a Monte Carlo comparison of this method with Indirect Inference, treating the widely used Smets and Wouters (2007) model of the US as the true model, and concluded that Indirect Inference offered very low bias and potentially large power. The method involves first describing the data behaviour in the sample by an 'auxiliary model', for which we use a VECM; and then simulating the DSGE model by bootstrapping its innovations to create many parallel samples (or histories) from each of which implied auxiliary model coefficients are estimated, generating a distribution of these coefficients according to the DSGE model. We then ask whether the VECM coefficients found in the actual data sample (actual history) came from this distribution with a high enough probability to pass the test threshold (which we put at 5%).

In our VECM auxiliary model we use three variables — total output, inflation and raw materials prices. We choose these three variables after a check on the power of our test for different sets of variables. We find (Table 4) that the test's power with this three-variable VECM is high — rejecting a model 100% of the time if it is only on average 13% false (we generate this increasing x-percentage falsity by alternately lowering and raising parameters from their true values by x%). Hence adding any more variables would raise the power to unacceptably high levels at which we risk never finding a tractable model that fits. As far as different three-variable sets are concerned, we have found in other work that the results are robust to any shifts in choice; what matters is now many variables are used (Meenagh,Minford, Wickens and Xu, 2018).[3]

Falseness	Rejection Rate
1%	5.38%
3%	7.00%
5%	14.7%
7%	47.16%
10%	94.82%
13%	99.96%
15%	100.00%

Table 4: Power of the Indirect Inference Test

4 Impulse Response Functions and the model's workings

Figures 6–9 shows the IRFs for the main shocks. The materials productivity shock is non-stationary and therefore has a permanent effect on all variables. The Phillips Curve shock to inflation is stationary but will have a non-stationary effect on money and prices. Similarly the money supply growth shock is stationary but also has a permanent effect on money and prices.

First, the rise in capacity lowers inflation and raw material prices and through both stimulates demand, shifting the IS and LM curves to the right. As final output converges on raw material capacity raw material prices return to the same equilibrium. No further capacity is invested in therefore.

Secondly, the rightward shift of the IS curve along the LM curve raises output, and via the Phillips Curve raises inflation; with money supply growth rising little as it reacts negatively to the output gap,

[3]Including four endogenous variables in the auxiliary model results in a model being only 5% false getting rejected 100% of the time.

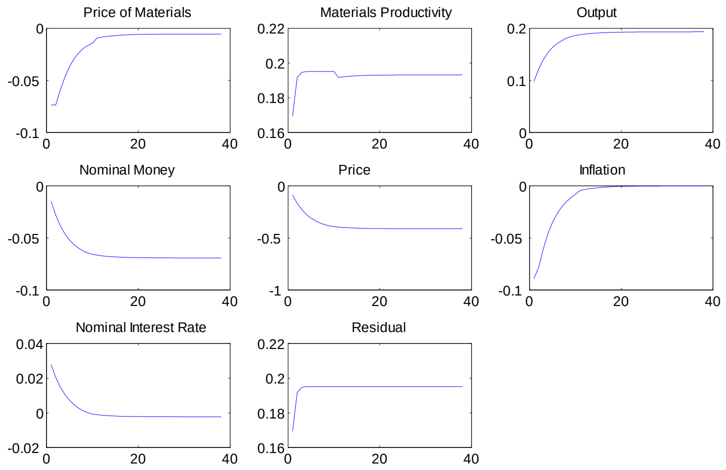

Figure 6: IRFs for a Materials Productivity Shock (nonstationary)

the fall in real money balances raises interest rates which reverses the output rise within five years. The rise in rates persists because the rise in prices is reversed only slowly by mild disinflation; commodity prices rise but fall back close to zero and hardly disturb commodity investment. In effect the IS curve shift is 'crowded out' rapidly by higher interest rates.

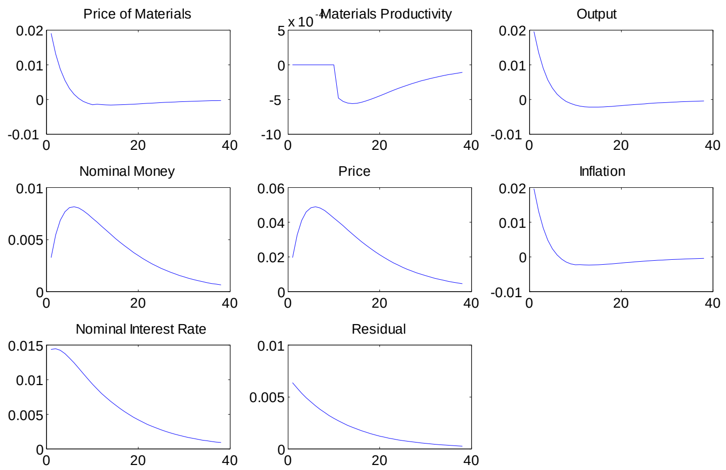

Figure 7: IRFs for an IS curve shock

Third, the rise in money supply growth shifts the LM curve to the right along the IS curve, both stimulating inflation via the Phillips curve and so shifting the IS curve to the right by lowering real interest rates. Raw material prices rise but this rise dies out quickly as output also falls back, with prices catching up with money to reduce real balances back to equilibrium. Commodity capacity therefore hardly moves; nominal interest rates also hardly move, with both IS and LM curves reverting to equilibrium quickly. Monetary policy is therefore powerful as in Friedman and Schwarz (1963), directly feeding into output and inflation. (The money demand shock has qualitatively the same effects but the innovation variance is very small so that it is of little importance quantitatively.)

Notice that all the shocks we have considered so far have effects that die out rather quickly; within 5 years output is back close to equilibrium (this is permanently changed with the productivity shock; but the point is that output quickly achieves this new position).

Finally, in the case of the Phillips Curve shock, which is to be thought of as a persistent rise in

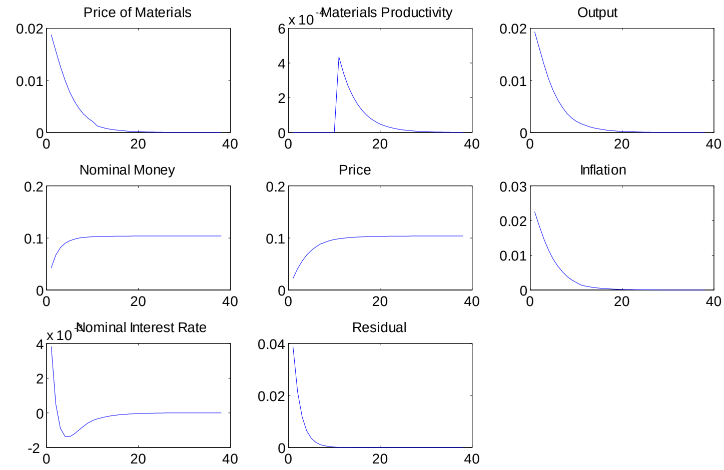

Figure 8: IRFs for a Money Supply Growth shock

mark-up or other costs, inflation rises driving down the real money supply, shifting the LM curve sharply to the left. At this point the model searches for a response of output that can accommodate both the rise in costs and the contractionary effect of lower real balances. We can think of this as a shift in the forward-looking IS curve to the left. Output falls persistently and drives down raw material prices setting off a fall in commodity investment, culminating in lower raw material capacity in ten years' time. The output contraction drives interest rates, both nominal and real, downwards. The fall in commodity capacity eventually restores commodity prices to equilibrium, and output too reverts to equilibrium as the shock dies away. But this whole process takes over 40 years.

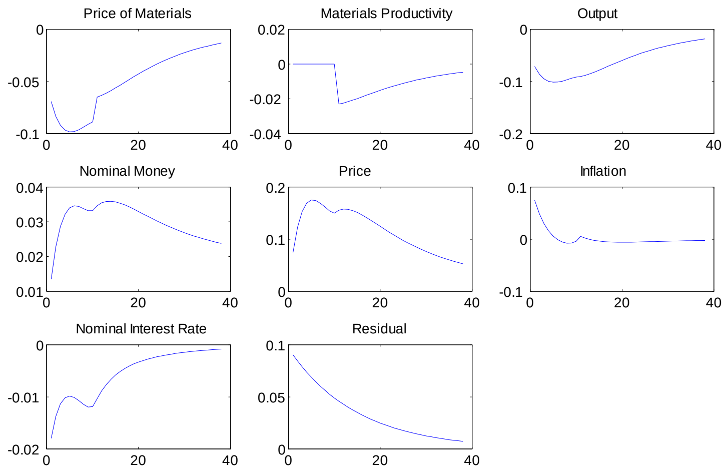

Figure 9: IRFs for a Phillips Curve shock

One can see from these IRFs that output responds most to the materials and Phillips curve shocks — as does inflation. The IS curve shock has its main effect on inflation and interest rates. These responses are reflected in the variance decomposition and the historical timelines, to which we now turn.

4.1 Variance decomposition and timelines

The variance decomposition (Table 5) shows that shocks to raw material capacity and the Phillips curve affect output and inflation the most. The IS curve shock also contributes to inflation variance and with the Phillips curve shock is important for interest rate variance.

	P_{Mt}	Y_{MCAPt}	Y_{Ft}	M_t	P_t	π_t	R_t
Materials Productivity shock	11.90	99.27	79.76	17.10	75.94	63.60	30.43
IS Curve shock	0.48	0.00	0.09	0.16	0.75	2.23	35.70
Money Demand shock	0.03	0.00	0.01	0.04	0.19	0.15	0.18
Money Supply growth shock	0.70	0.00	0.13	75.68	8.82	4.20	0.40
Phillips Curve shock	86.90	0.73	20.02	7.01	14.30	29.82	33.29

Table 5: Variance Decomposition

Next we look at the historical timeline analysis for the the main variables. First, we show two timelines for output: the first (Figure 10) includes the (data-based) deterministic trend, the second (Figure 11) abstracts from this, only including the shocks including the model-based stochastic trends. The role of money in stimulating the economy in the 1920s and deflating it in the 1930s can be seen in the yellow bars. However, this was an exceptional period; all the rest of the history output is dominated by the productivity shock (dark blue) and the cost shocks to the Phillips curve (dark green).

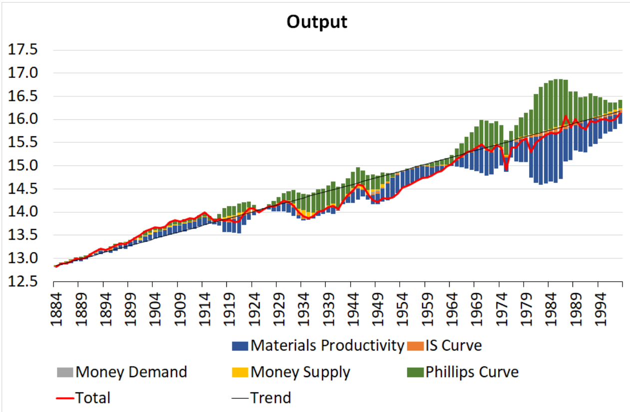

Figure 10: Timeline for Output with data-based trend

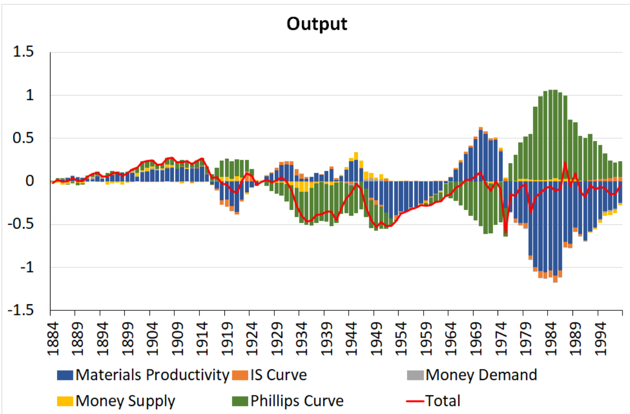

Figure 11: Timeline for Output

The timeline for inflation in Figure 12 shows how according to the model it was dominated by materials productivity and the Phillips Curve shock, itself probably related to raw material price movements; this is because these two shocks are intertwined — when commodity prices rise they act as an impulse shock to 'other costs' in inflation.

Inflation was quiescent until WW1 while the Gold Standard was operating: from that point it becomes disturbed by materials, cost, money supply and IS shocks which produce large fluctuations in used-up capacity, Y- YMCAP. The biggest fluctuation in this capacity measure is during the 1970s and early 1980s when the world experienced double-digit inflation and the oil/commodity crisis. These fluctuations even exceeded those in the interwar and WW2 periods.

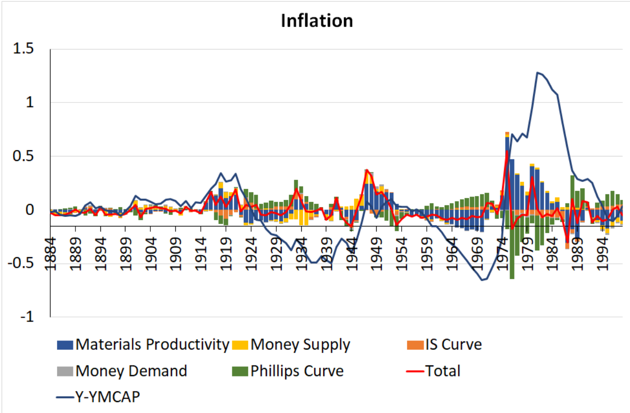

Figure 12: Timeline for Inflation

Figure 13 shows that interest rates were driven up in the 1970s by raw material shortage and IS demand stimulus (which affects interest rates more or less exclusively); and driven down in the 1930s by the same forces.

Interest rates are the equilibrating variable setting money demand equal to money supply: money supply growth accommodates inflation to some extent and leans somewhat against output, hence is largely exogenous. But as we have seen shocks to it have had little impact on the economy except in the thirties. It is fluctuations in money demand, responding to inflation and output, that have caused the fluctuations in rates- where the LM curve has shifted up and down the steep IS curve. Hence the shocks to the Phillips and IS curves as well as materials productivity have been the shocks driving rates.

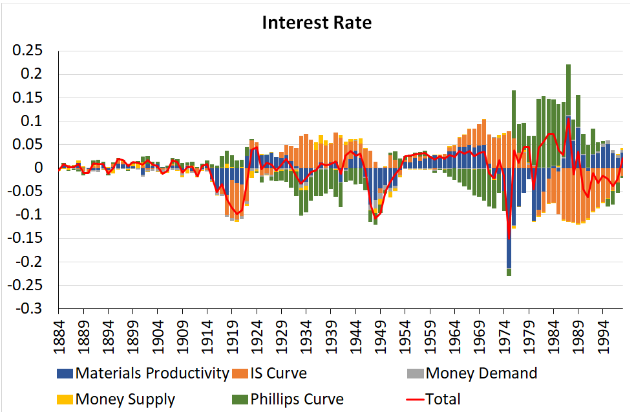

Figure 13: Timeline for Interest Rate

4.2 The length of cycles revealed by the model

What is the behaviour of the simulated samples, in terms of their cycle lengths and the propensity of the world economy to crises? We investigate this with the many 'bootstrap histories' we have created by simulating the DSGE model repeatedly: these histories taken as a whole can give us the pricture of what the world economy could have done according to the DSGE model and in particular the cycles it could have created. We find that cycle lengths vary in a way conjectured by Schumpeter. We get frequent long wave cycles, as well as shorter cycles. The frequency of the cycles is shown in various ways in Table 6 and Figure 14. Table 6 shows the percentage of cycles that are of length between [a,b], as well as the number of these cycles we would expect to find per 1000 years. This is then converted to show how many years we would spend in different cycles per 1000 years for both materials prices and output. We also report the average cycle length.

The most telling statistic that emerges from these tables is that according to this model we will spend just over two thirds of our lives, on average, in a very long cycle, whose length is around 42 years. The distribution of cycles for materials prices and output are quite different; those for output are slightly larger in length. We spend 785 years per thousand in a 49-year output cycle (the average length of long cycles over 22 years long), as against 608 years per thousand in a 36-year commodity price cycle. This means that two thirds of the time we are enjoying a 'long' cycle whose average full length, both down and up, is around 42 years. Of course there are a much higher percentage of shorter cycles, but as they last so much less time they are of little importance overall.

Materials Prices (Linear Detrended)		Percentage of cycles with length [a,b]							
Cycle Length	Average	[1,3]	[4,6]	[7,9]	[10,12]	[13,15]	[16,18]	[19,21]	22+
	18.22	9.87	13.66	10.80	9.11	9.32	8.20	7.04	32.00
Materials Prices (Linear Detrended)		Frequency of cycles with length [a,b]							
		[1,3]	[4,6]	[7,9]	[10,12]	[13,15]	[16,18]	[19,21]	22+
Number of cycles per 1000 years		5.16	7.14	5.65	4.76	4.87	4.29	3.68	16.73
Number of years spent in cycle of length		10.32	35.71	45.18	52.41	68.24	72.86	73.61	607.57

Output (Linear Detrended)		Percentage of cycles with length [a,b]							
Cycle Length	Average	[1,3]	[4,6]	[7,9]	[10,12]	[13,15]	[16,18]	[19,21]	22+
	30.92	8.73	9.22	6.18	5.37	5.48	5.13	4.50	55.39
Output (Linear Detrended)		Frequency of cycles with length [a,b]							
		[1,3]	[4,6]	[7,9]	[10,12]	[13,15]	[16,18]	[19,21]	22+
Number of cycles per 1000 years		2.53	2.67	1.79	1.55	1.59	1.49	1.30	16.05
Number of years spent in cycle of length		5.06	13.36	14.32	17.10	22.24	25.29	26.05	785.01

Table 6: Frequency of Cycles

Figure 14 shows the histogram of cycle lengths. From this we can clearly see there is an abundance of long cycles for both variables. The histograms also confirm that the output cycles are longer on average, with some cycles even being as long as 100 years.

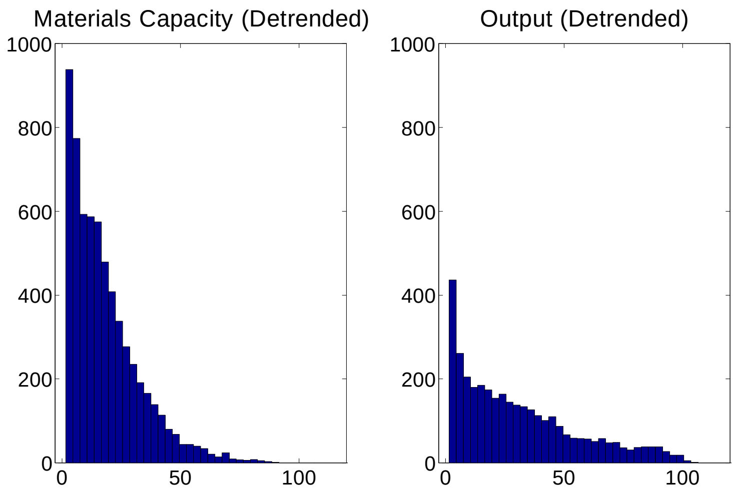

Figure 14: Histograms of Cycle Lengths

5 The role of new monetary rules

What reformed monetary rules could stabilise this world economy? Given that monetary policy permitted massive business cycles which caused considerable disruption, we consider the effectiveness of a straightforward Friedman k-percent rule and then go on to look at various Taylor Rules, including 'enhanced' ones targeting the Price Level and Nominal GDP (i.e differing by targeting the integral of inflation instead of the level; output is targeted in the same way in terms of the level of the output gap). It is noteworthy that the 'original' estimated money supply growth rule is highly accommodative of inflation (the response of money growth to the inflation rate is 1.24) but 'leans against' output (with a response of -0.009).

	var(output)	var(inflation)	Welfare Loss
Original	0.00346	0.00860	0.01206
Friedman k% Rule	0.00268	0.00612	0.00881
Taylor Rule	0.00729	0.00035	0.00764
NGDPT Rule*	0.01990	0.00389	0.02378
PLT Rule*	0.02056	0.00396	0.02452
MS Rule*	0.00411	0.00167	0.00578

Rules defined	
Taylor Rule	$r_t = 0.99r_{t-1} + 3.0\pi_t + 0.0(\ln Y_{Ft} - \ln Y_{MCAPt})$
NGDPT Rule	$r_t = 0.1r_{t-1} + 2.2(\ln Y_{Ft} + \ln P_t - (\ln Y_{MCAPt} + \ln P_t^*)$
PLT Rule	$r_t = 0.1r_{t-1} + 2.7(\ln P_t - \ln P_t^*)$
MS Rule	$\Delta \ln M_t = c_5 + a_1^{ms}\pi_t - a_2^{ms}(\ln Y_{Ft} - \ln Y_{MCAPt})$
	$-0.3(\ln P_t - \ln P_t^*) - 0.1(\ln Y_{Ft} + \ln P_t - (\ln Y_{MCAPt} + \ln P_t^*))$

*all these rules violated the zero lower bound on interest rates badly —see text

Table 7: Welfare Loss for Monetary Policy Rules

We measure their effectiveness in stabilising the economy with a crude welfare measure of volatilty, the unweighted variance of HP-detrended output and inflation. Using this measure, the Friedman Rule is more beneficial than the estimated rule. It slightly reduces both the variance of output and inflation as can be seen in Table 7.

If we supplement the estimated rule with a corrective response to a Nominal GDP or Price Level target, output volatility rises substantially with only a small improvement in inflation volatility. Furthermore, we must also note the behaviour of interest rates under these rules. It turns out that the apparently most successful rule — the MS rule which besides the usual money supply reactions found in the data, also responds to the level of prices and nominal GDP — pushes interest rates well into negative territory much of the time as it struggles to push the economy back towards full commodity capacity utilisation. The same is true of the other rules that respond to the price level or nominal GDP. Hence effectively we must ignore these rules as impossible to enforce due to the lower interest rate bound.

The Taylor Rule also appears to be successful in reducing macro variance; but on inspection it too is incredible as it involves a large short term response to inflation and very long persistence in this response, implying a long run response of 300! Not surprisingly it achieves a huge reduction of inflation variance but at the expense of a tripling of output variance compared with the Friedman Rule.

It turns out that this Friedman Rule, simply fixing the money supply growth at a constant is the best rule over our sample period. Interestingly, the Friedman Rule is not much different from the money supply rule we have estimated for the whole period. This only differs by having a small positive response (0.17) to inflation and a very small negative response (-0.01) to the output gap.

This greatly contrasts with the findings of Le et al. (2016b) for the recent post-war period since the mid-1980s where we found that a Taylor Rule targeting either the price level or Nominal GDP did by far the best job in stabilising the economy. However, of course this recent period, with its hugely developed markets in liquidity, represents a very different monetary world from the average over the last 150 years.

6 Robustness Checks

6.1 Robustness of the estimated model to the data definitions

As mentioned earlier our data for 'world' output, interest rates, money and consumer prices was limited to an average of US and UK data. However, plainly, the importance of other countries in the world totals was non-trivial and rose through the sample. Accordingly we created new replacement data series for these which included all countries on a GDP-weighted basis using data from the Jordà-Schularick-Taylor

Macrohistory Database (Jordà et al. 2017)[4]. Our objective was to see if our model would pass our stringent Indirect Inference Wald test on this reworked data. From the results shown in Table 8 it can be seen that the same model passes the test even at a slightly higher level of confidence. For good measure we show the variance decomposition in Table 9 which is more or less identical to our original variance decomposition in Table 5. There is therefore no reason to change the model from what we have already examined in some detail.

	Data	
	US-UK	Full Weighted
Wald	37.0794	35.0566
Transformed Wald	1.5145	1.4081
p-value	0.057	0.079

Table 8: Robustness Model Test

	P_{Mt}	Y_{MCAPt}	Y_{Ft}	M_t	P_t	π_t	R_t
Materials Productivity shock	11.60	99.24	79.22	21.21	77.56	64.09	28.985
IS Curve shock	0.50	0.00	0.10	0.22	0.81	2.42	36.04
Money Demand shock	0.02	0.00	0.00	0.02	0.08	0.12	0.35
Money Supply growth shock	0.53	0.00	0.10	69.61	6.61	3.22	0.30
Phillips Curve shock	87.35	0.76	20.58	8.94	14.94	30.15	34.33

Table 9: Robustness Variance Decomposition

6.2 Robustness of the policy results to potential model error

We can now use the results from our Monte Carlo power table above (Table 4) to carry out a robustness assessment of the model. We want to know for sure how misleading our results could be at worst if assailed by general inaccuracy. Plainly parameter estimation which we have carried out by indirect inference is subject to estimation bias in small samples; and so it is possible that our parameter estimates are not the true ones, even though we have found in general that this estimation bias is encouragingly small (Le et al, 2016a). Our power analysis indicates that the falsity of these parameters cannot exceed 13% or we would have rejected our model. We can now ask about our policy analysis over this historical period whether it could have been affected by this falsity being at its highest possible. So we redo our monetary policy rule evaluation with a set of parameters that are at this extreme of falsity. The results are shown in Table 10.

If we simply focus on the results for the Original estimated rule and the Friedman Rule, it can be seen that they hardly differ: both yield somewhat more instability, but the conclusions remain the same, that a Friedman Rule is the best.

	var(output)	var(inflation)	Welfare Loss
Original	0.00307	0.01279	0.01586
Friedman Rule	0.00244	0.00859	0.01103

Table 10: Welfare Loss for Monetary Policy Rules with Maximum False Model

7 Conclusions

In this paper we looked for a theory that could account for considerable evidence of a long wave commodity cycle in the world economy. We set up a DSGE model in which there is a long time (10 years)

[4]There are two hyper-inflation episodes (Germany 1920s, Japan 1940s) which if included considerably distort the consumer price series; both end with a currency reform which in principle returns the price level to parity with world prices. We have treated them as if throughout these two episodes prices were at world parity or 'dollarised'. Indeed currency substitution was likely to have been substantial; these countries' residents were substituting out of their home currency into either currency-substitutes or barter, and the 'home currency prices' therefore represent only a proportion of actual exchange transactions, possibly a small proportion, of which unfortunately we have no estimate.

needed to build a materials mine, a Phillips Curve, a forward-looking IS Curve and an LM curve. We estimated this model by indirect inference which passed the Wald test comfortably and found that the model generates a world in which we spend much of our time in long cycles. An example of such a cycle is the world economy from 1980 to 2007 where a commodity peak (1980) was followed by 3 decades of first falling, then very low, and finally sharply rising commodity prices, peaking in 2007. The two main shocks driving output are those for materials productivity (which dominates commodity production) and the Phillips Curve (which dominates inflation). The money supply growth shock was important for output in the 1930s but otherwise feeds mainly into inflation.

The estimated monetary policy rule was one in which the money supply growth rate slightly accommodates inflation and leans very slightly against the output gap. We found that Friedman's k-percent rule would have stabilised the world economy a bit better.

We conclude that this theory and evidence point to a need for macroeconomists to give a significant role in macro models to the long-gestation commodity sector.

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