Enhanced Multi-User DMT Spectrum Management

Using Polynomial Matrix Decomposition Techniques

Patrick Kunle Adebayo

School of Engineering
Cardiff University

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Dedicated to my late father Mr. John Julius Adebayo.

Declaration

I hereby declare that except where specific reference is made to the work of others, the contents of this dissertation are original and have not been submitted in whole or in part for consideration for any other degree or qualification in this, or any other university. This dissertation is my own work and contains nothing which is the outcome of work done in collaboration with others, except as specified in the text and acknowledgements.

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Abbreviations and Acronyms

MIMO	Multiple Input Multiple Output
SISO	Single Input Single Output
ADSL	Asymmetrical Digital Subscriber Line
VDSL	Very high-speed Digital Subscriber Line
DMT	Discrete Multi-Tone
DSLAM	Digital Subscriber Line Access Multiplexer
CSI	Channel State Information
CSIR	Channel State Information at Receiver
CSIT	Channel State Information at Transmitter
SVD	Singular Value Decomposition
PMD	Polynomial Matrix Decomposition
FEXT	Far-End CrossTalk
NEXT	Near-End CrossTalk
FFT	Fast Fourier Transform
OFDM	Orthogonal Frequency Division Multiplexing
PSD	Power Spectral Density
DFT	Discrete Fourier Transform
DSM	Dynamic Spectrum Management

MMSE	Minimum Mean Square Error
FIR	Finite Impulse Response
ZF	Zero Forcing
CP	Cyclic Prefix
IDFT	Inverse Discrete Fourier Transform
BER	Bit Error Rate
SNR	Signal to Noise Ratio
SINR	Signal to Tnterference plus Noise Ratio
MLE	Maximum Likelihood Estimation
PSVD	Polynomial Singular Value Decomposition
PEVD	Polynomial EigenValue Decomposition
MCM	Multi-Carrier Multiplexing
ISI	Inter-Symbol Interference
IBI	Inter-Block Interference
ICI	Inter-Channel Interference
SIR	Signal to Interference Ratio

Notations and Fixed Symbols

Notations

x	Scalar quantity
$\mathbf{x}$	Vector quantity
$\boldsymbol{X}$	Polynomial Matrix
$\mathbf{X}$	Matrix quantity
$\bar{\mathbf{x}}$	Mean vector
$\mathscr{X}$	Set of Matrices
$\mathscr{R}$	Set of Real Matrices
$\mathscr{C}$	Set of Complex Matrices
$\hat{\mathbf{x}}$	Estimate of original quantity $\mathbf{x}$
$(.)^T$	Transpose operator
$(.)^H$	Hermitian transpose operator
$\tilde{(.)}$	Paraconjugate operator
$(.)^{-1}$	Matrix inverse
$(.)^*$	Complex conjugate operator
$\|.\|$	Matrix determinant
$\mathbf{trace}\{.\}$	Trace of a matrix
$\|\|.\|\|_2, \quad \|\|.\|\|$	Euclidean norm

$\|\|\mathbf{X}\|\|_F$	Frobenius Norm of **X**
$\mathbf{diag}\{X\}$	Diagonal component of a matrix
$\mathbf{offdiag}\{X\}$	Off-diagonal component of a matrix
$\mathbf{I}_A$	The identity matrix of size **A**
$0_{A\times B}$	Matrix of zeros of dimension $A \times B$
x_i	ith element of vector x

Fixed Symbols

$\mathbf{A}$	Generic scalar Matrix
$\mathbf{H}$	Instantenous MIMO channel Matrix
$\boldsymbol{H}(z)$	Polynomial MIMO channel Matrix
$\boldsymbol{C}(z)$	Polynomial MIMO transfer Matrix
β	Number of iteration & condition of ending state for processing
$\mathbf{P_T}$	Total transmit power & available radio resources
$\mathbf{D}$	Discrete Fourier transform matrix
E_{rel}	Relative error (crosstalk)
$\mathbf{P}_i^k$	Total transmit power for ith user at the transmitter over a tone k
$\boldsymbol{F}(z)$	Transmitter FIR Matrix (with Pre-coding)
$\boldsymbol{G}(z)$	Receiver FIR Matrix (with Post-coding)
j	Complex operator
k	Tone index
i	User index
M	Number of receive tones

N	Number of transmit tones
$\mathbf{U}$	Unitary Matrix from SVD decomposition & Jacobi rotation Matrix
$\boldsymbol{U}(z)$	ParaUnitary Matrix from PSVD decomposition
$\mathbf{V}$	Unitary Matrix from SVD decomposition
$\boldsymbol{V}(z)$	ParaUnitary Matrix from PSVD decomposition
$\boldsymbol{X}_i(z)$	Transmit signals as a polynomial vector
$\boldsymbol{Y}_i(z)$	Receive signals as a polynomial vector
σ_i^2	The variance of Gaussian white noise for user i
$\mathbf{A}^{\dagger}$	The pseudo-inverse of $\mathbf{A}$

Abstract

This thesis researches the increasingly critical roles played by intelligent resource management and interference mitigation algorithms in present-day input multiple output (MIMO) communication systems. This thesis considers the application of polynomial matrix decomposition (PMD) algorithms, an emerging broadband factorisation technology for broadband MIMO access networks. Present DSL systems' performance is constrained by the presence of interference (crosstalk) between multiple users sharing a common physical cable bundle. Compared to the traditional static spectrum management methods that define their survival to the worst-case scenarios, DSM methods provides some degree of flexibility to both direct channel and noise parameters to improve evolvability and robustness significantly. A novel crosstalk-aware DSM algorithm is proposed for the efficient management of multi-user DSL systems. Joint power allocation procedures are considered for the proposed single-channel equalisation method in DSL access networks.

This thesis then shows that DSM can also benefit overdetermined precoding-equalisation systems, when the channel state information (CSI) parameters call for a specific decision feedback criterion to achieve a perfect reconstruction. A reasonable redundancy is introduced to reformulate the original multi-user MIMO problem into the simplest case of power management problem. DSM algorithms are primarily applied to solve the power allocation problem in DSM networks with the aim of maximising the system attribute rather than meeting specific requirements. Also, a powerful PMD algorithm known as sequential matrix diagonalisation (SMD) is used for analysing the eigenvalue decomposition problem by quantifying the available system resource including the effects of the crosstalk and its parameters. This analysis is carried out through joint precoding and equalisation structures.

The thesis also investigates dynamic interference mitigation strategies for improving the performance of DSL networks. Two different mitigation strategies through a decision feedback equalisation (DFE) criterion are considered, including zero-forcing (ZF) and minimum mean square error (MMSE) equalisers. The difference between ZF and MMSE equalisations is analysed. Some experimental simulation results demonstrate the performance of both ZF and MMSE equalisation under the DFE equalisation constraint settings. Model

reduction on the MMSE equalisation is thus applied to balance the crosstalk interference and enhance the data-rate throughput.

Finally, the thesis studies a multi-user MIMO problem under the utility maximisation framework. Simulation results illustrate that the power allocation of multi-user DSL transmission can be jointly controlled and the interference can often be mitigated optimally on a single user basis. Driven by imperfect CSI information in current DSL networks, the research presents a novel DSM method that allows not only crosstalk mitigation, but also the exploitation of crosstalk environments through the fielding of versatile, flexible and evolvable systems. The proposed DSM tool is presented to achieve a robust mitigating system in any arbitrary overdetermined multi-user MIMO environment. Numerical optimisation results show that the mitigation of crosstalk impairment using the proposed DSM strategy. The design and implementation of the proposed DSM are carried out in the environment of MATLAB.

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Chapter 1

Introduction

1.1 Motivation

Recent advances in communication industries are moving towards the realisation of a globally infrastructure, where access to information becomes available at our fingertips, i.e., when it is needed, where it becomes required, and in whatever form it is necessary. These communication industries can provide the information through a single communication network to provide high-speed services to various users and they can now be implemented, for example, through wireless access networks and digital subscriber line (DSL) systems. However, these systems cannot support the current requirements and demand [1, 2].

Over recent years, several digital modulation technologies including orthogonal frequency division multiplexing (OFDM) for the wireless systems and discrete multi-tone (DMT) modulation for DSL networks have been investigated and developed to meet the demand. The success of these modulation techniques lies in their potentially flexibility, robust against noise and interference, high resolution, low signal attenuation, low signal distortion, low transmit power utilisation and low-cost usage [3].

One fundamental problem associated with DMT modulation system is that, as more and more users continue using the DSL network, and as the development evolves with high-frequency network applications such as data browsing on the World Wide Web (WWW), Java applications, video conferencing, etc., the DMT modulation suffers significantly from interference limitations imposed by crosstalk. Therefore, there is a call for healthy alternatives with very high-frequency transportation network facilities, whose capabilities are much beyond those that existing DMT multiplexing techniques can provide. The challenge remains that there is not enough bandwidth in today's network facilities to support the exponential growth in traffic demands. Given that the benefit associated with DMT's bandwidth is higher than current achievable data rates, there is a strong motivation to tap into the bandwidth management potential [4].

Realising also that the maximum possible performance which an end user can achieve from the network is restricted by exploiting power allocation strategy. The key to designing robust modulation schemes for efficiently bandwidth potential is to introduce an alternating dynamic spectrum management (DSM) mechanism. This can be accomplished using polynomial matrix decomposition (PMD) procedures with the aim of allocating optimal transmit power to various users at each user's own bit-rate and to support different networks, protocols and architectures.

From the literature, DMT modulation systems are analysed and implemented in two domains; time and frequency. Frequency domain system analysis is somewhat futuristic for technologies today. Under the time domain, each DSL line is synchronised and controlled within a single slot. The achievable data rate leads to the aggregation rate over all the independent bands for available transmitting DSL lines in the network, so the frequency domain mechanism may offer much higher data rate than each user's data rate. As a result, the study introduces and present the transmission on the polynomial domain, which combines the other two domains and can achieve much higher throughput than the frequency domain. Thus, frequency and time domain signal analysis attract high complexity where the combined field is required. Individually, polynomial mechanisms and their model interpretations are currently the favourite modulation method for designing a complex multiple-input multiple-output (MIMO) system [**?** 5, 6]. The main idea includes allocating the transmit power centrally and operating only at the bit rate of a single-mode MIMO system.

The network model can be chosen arbitrarily and corrects the impact of system imbalances. Hence, the available research for controlling crosstalk at the reception devotes much effort to developing and representing MIMO modulation in the frequency domain. This study aims at turning the promise of full multiplexing benefit into reality using polynomial representations for a group of DSL users in order to meet the service requirements of the future generation.

1.2 The DSL Access Network

DSL network is a collection of high-speed communication technologies that allow potentially large broadband services to be delivered to various end-user subscribers over the traditional twisted pair copper telephone lines [7]. The evolution of DSL networks started with the use of splitter modems to separate the plain old telephone services (POTS) from high-speed data services by using the unused POTS frequency channels/lines. The fundamental reason for the coexistence of DSL networks with other POTS such as integrated service digital network is

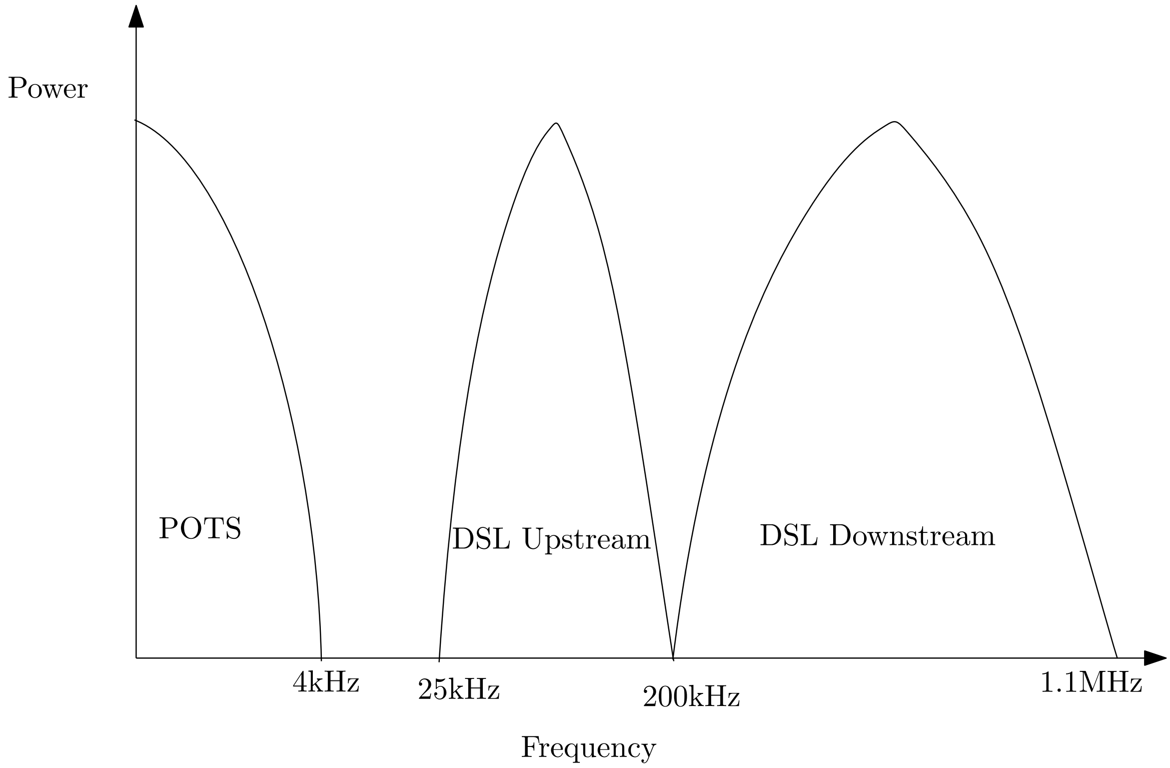


Figure 1.1 The frequency sharing between POTS and DSL Network

the use of different frequency bands. Any POTS service, for example, uses frequency bands up to $4kHz$ for maximum power spectral density (PSD) allocation. The most common DSL network, ADSL, exploits the PSD above $25kHz$. Fig. 1.1 illustrates the frequency bands used for POTS and DSL.

DSL has increased the connection speed of copper wire from 14.4 kbps in 1994 to 1.5 Mbps today.

As of OECD[1] 2013 [8], there are more than 379 million fixed wire-line subscribers worldwide, accounting for over 70% of the share of DSL access network subscribers. Broadband using cable modems is one of the leading competitive wire-line networks with over 109 million users or about 20% of the market share. The growth in wireless has become a significant challenge to DSL networks over the last two decades by continuously installing new smart technology for mobile subscribers.

With many present-day hybrid technologies that employ DSL networks as a "last kilometre" access technology to connect broadband customers' households to telecommunication service terminals, more improved connections are now hosted at a node, curb or basement of a building. The growth of hybrid DSL networks, even up to the Fiber to the cabinet (FTTC),

[1]OECD stands for Organisation for Economic Co-operation and Development Countries

technology has passed through evolution rather than revolution. Instead of installing full fibre to the end users, DSL networks are now connected between the cabinet and the building.

The newest hybrid network, full fibre to the building (FTTH), provides a high-speed connection through an optical fibre direct to the subscribers. In a hybrid system, fibre can be laid to the end of each street where an optical network unit (ONU), also known as a remote terminal (RT) is installed to connect each customer premise equipment (CPE) to the central office (CO). Following the evolutionary approach of hybrid DSL networks, operators are now focusing on replacing the copper wires entirely in the full fibre to the building (FTTB) technology.

DSL has been one of the most successfully marketed broadband access technologies over the last decade and will sustain this dominance. Asymmetric Digital Subscriber Line (ADSL), an early DSL standard, was initially developed in 1987 and supports connectivity from the CO to the CPE providing downstream data rates of up to 6 Mbps and upstream data rates up to 800 Kbps on short lines [9]. However, as the demand for higher data-rate increases, the channel's achievable gain decreases noticeably. With increased user demand for triple play services, a combination of video, high-speed internet and voice applications, new DSL technologies such as ADSL2, ADSL2+, VDSL, and VDSL2 are being developed to allow copper-based systems to compete with alternative access network technologies [10].

ADSL standards are designed to use frequencies from 0 to 1.104 MHz for high-speed transmission while VDSL2 standards extend the spectrum of ADSL up to as much as 52 MHz [7]. As this higher frequency band, VDSL users are connected by optical fibre to optical network units (ONUs) from the CO, thereby shortening the copper telephone line length significantly. VDSL experiences some adjacent lines interference, also known as crosstalk, which then becomes the primary limiting factor in achieving higher data rates.

Crosstalk interference is one of the primary sources of performance limitation in DSL systems. The term “noise ”has now gradually broadened in many applications and refers to interference between any communication circuits. This kind of noise includes both near-end crosstalk (NEXT) and far-end crosstalk (FEXT) components.

The NEXT components are imposed by transmitters interfering with receivers on the same side of the bundle and can be avoided by using non-overlapping transmit and receive spectra (frequency division duplex; FDD) or disjoint time intervals (time division duplex; TDD). On the other hand, FEXT components are experienced when the transmitters on opposite sides of the bundle interfered with one another. It is shown [11] that NEXT interference is about 10 to 20 decibel larger than the background noise.

Recent developments in DSL systems have expanded the use of hybrid fibre bandwidth. A new type of DSL networks, the so-called very high-speed digital subscriber line (VDSL-2), enables more convenient fibre-optic broadband services by addressing the problems outlined above. It is evident that full-fibre based networks ensure the optimal, low-attenuation transmission, but such a system experiences high maintenance cost. VDSL-2 appears as an alternative to the use of a high-cost full fibre network. A typical VDSL-2 system is shown in Fig. 1.2.

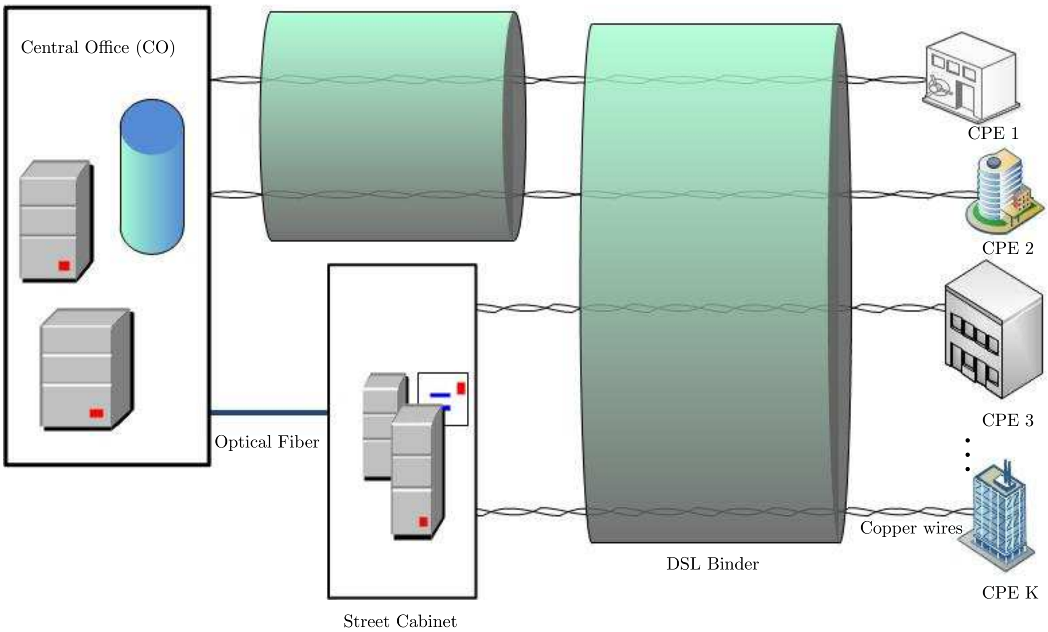


Figure 1.2 DSL transmission model.

Within the DSL loop, it is generally well-known that NEXT noise [**?**], opposed to the FEXT noise, presents the dominant challenge. The issue becomes severe when DSL services are operated on different lines in the same cable bundle. It is critical to minimise or even cancel the crosstalk imposed by a DSL transmission. Doing so can significantly improve the data-rate performance of DSL systems and increases loop throughput.

Power and bit allocation are also factors that determine the performance of DSL systems. They are most commonly raised issues concerning the development of discrete multitone (DMT) multiplexing standards. DMT is a coordination mechanism that estimates the bit and power loading to a group of subcarriers within the same DSL loop, and it can be at either the customer premises equipment (CPE) or the central office (CO) location. In the CO, the mechanism is termed “digital subscriber line access multiplexing (DSLAM)” [7]. A typical DSL system is illustrated in Figure 1.3, where four CPEs are transmitting in both directions through CO equipment and in the presence of crosstalk.

Additionally, there are other limiting factors in the literature, which have deterred the growth of DSL services: in particular- rate, reach and symmetry.

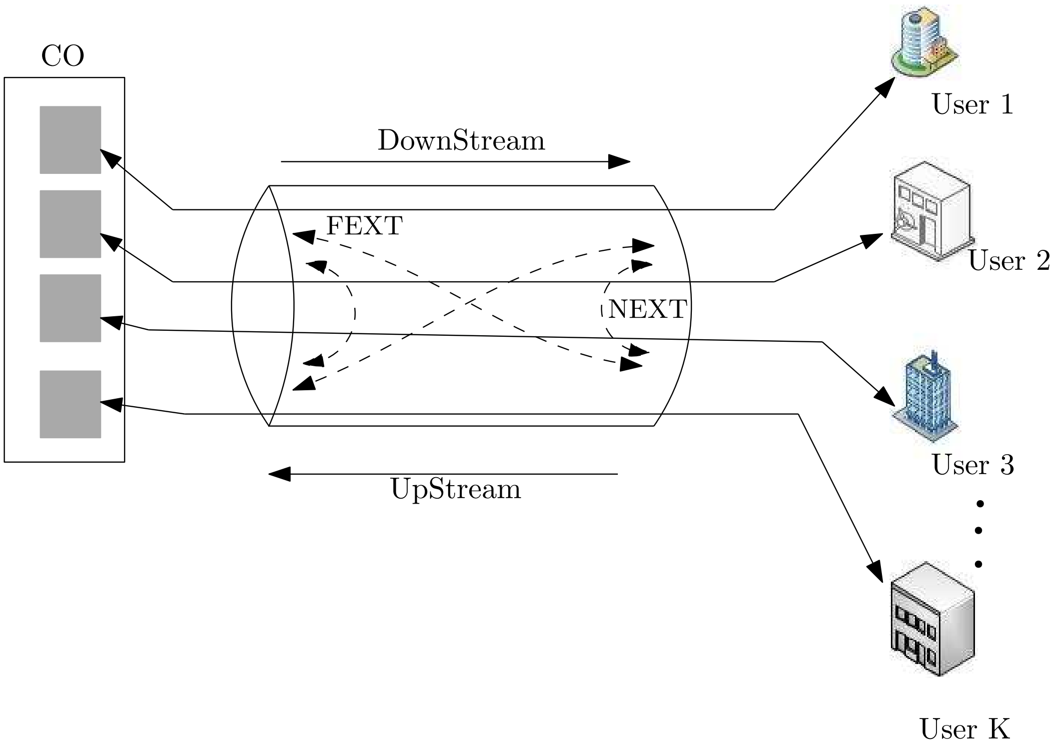


Figure 1.3 DSL transmissions with crosstalk

Rate

The global market for DSL services had witnessed tremendous growth over the recent years, even as new connection technologies are promising to emerge. This success is driven by the desire for higher bit-rate connections at both upstream and downstream. For example, delivering duplex channels at 12 Mbps per channel, plus high-speed Internet at 10 Mbps, plus a voice/music channel of 1 Mbps requires a 35 Mbps service. Today's ADSL networks can only offer 3 Mbps in high-density urban areas, and their access rates in suburban and rural areas are somehow less than 256 kbps [2]. Increasing the available rate at the remote areas is the fundamental challenge and has become a vital litmus test for the state of DSL markets. This issue is crucial to the competitive challenge from wireless communication networks, which continue upgrading their facilities for higher access rates [12]. Rate competition among providers enables users in urban areas a good choice between wired and wireless options.

Technically, hybrid fibre networks provide a superior medium to full twisted pairs. Today's hybrid network is only limited to the switching speed between the terminal interfaces and the central office, as illustrated in Figure 1.2 for VDSL-2 systems. A fibre network is a trunked medium that connects all the CPE lines and switches each of the transmitting lines at the full rate of the fibre. As such, full-fibre systems are more expensive to manufacture than for DSL. This slight advantage will soon change as Moore's law decreases the cost of computing power. Hence it is imperative for DSL service providers to offer increasing access rates in order to remain competitive.

Reach

DSL subscribers in suburban and rural areas are often situated far from the CO. As a result, these subscribers on long attenuation links suffer from poor quality because of the twisted-pair medium. This reach problem limits the penetration rate covered by DSL services. It is notably more severe in geographically sparse countries like the USA and Australia where DSL penetration is less than 5%. Compare this with countries like Korea, which has a penetration of 29%, and it is evident that DSL marketplaces have not achieved their full potential for revenue [1].

Symmetry

Another limiting factor facing the growth of DSL services is the difference in the upstream (US) and downstream (DS) bit-rate connections. The existing DSL technologies use asymmetric connection mode, i.e., providing a higher rate for the DS than the US. As a result, these technologies are inefficient for modern applications such as web-browsing and video-streaming. Also, the growth of peer-to-peer file-sharing of music and movies, video conferencing and teleworking are now increasing the demand for equal US data-rate.

Providing a symmetric connection remains a very significant challenge for future DSL deployments. All three of these issues - rate, reach and symmetry - are promising to be addressed with the use of a hybrid DSL network. In such fibre-based networks, copper technology will cover a short distance, leading to a lower channel attenuation and providing higher data-rates. However, the deployment of full fibre-equipment to the end of each street is costly.

1.3 Spectrum Management Techniques

Spectrum management (SM) techniques in DSL systems are powerful tools used for addressing system uncertainties in communication systems. The techniques are the basic tool for analysing system architectures with the goal of maximising the overall system throughput rather than meeting specific requirements. SM itself is not a crosstalk analysis tool but instead performs the analysis of the system information including the effects of the crosstalk and its parameters. The idea behind its innovation is that if the effects of crosstalk and its associated interference can be quantified, then, the associated risk may be easily mitigated. These techniques can also be defined as risk management tools for multidisciplinary systems. They are particularly well suited to types of optimisation problem that are difficult to quantify instantaneously but may become better known later.

It is shown in [13] that the spectrum management problem associated with DSL systems can be solved by using the adaptive feedback equalisation from receiver to the transmitter, mainly when the system information about target signals and its interference parameter appears as an optimisation problem.

The first SM technique began in literature with work of Peter Chow [14]. Chow analysed what is now known as the *optimal discrete power-and bit-loading optimisation problem* (PBLP), i.e. how to allocate an appropriate transmit power density function (PDF) to an arbitrary number of multiple different users in a noisy environment.

It is evident that the water-filling method is the optimum solution for a single user scenario [9]. Consequently, the distributed iterative water-filling (IWF) algorithms employed for addressing the DSL optimisation problem in [15, 16] do not support the required centralised standards. Apart from the fundamental performance limitation of IWF, an additional requirement of PBLP is the spectra compatibility for multi-user non-homogeneous systems; that is, allocating the PDF required to transmit for various users; the user should not have more power than necessary to achieve their target data rates.

Two competing methods are known to achieve the spectral compatibility of DSL networks at both the transmitters and receivers. These are spectrum and signal coordination [9, 17, 18]. In some cases, spectrum combination methods may be used as crosstalk avoidance tools. On the other hand, signal coordination methods are typically defined for the cancellation of crosstalk. More details of these methods will be provided in Section 2.4.

The signal coordination methods, in particular, are well suited for types of crosstalk interference that are difficult to quantify at the transmitter end but become better fully-known at the receiver. A spectrum coordination design for link-to-link power control is well established in [19]. Managing the power consumption at the system-level is usually constrained by the large number of users, leading to some specific compromise between the achievable data-rate and complexity [20]. In particular, signal coordination allows more systematic characterisation of uncertainties during the lifetime of propagation processes. Using space-time analysis of this complex problem allows the system-link to be quantified.

Signal coordination method can also introduce model reduction concepts to explore only the required opportunities and possible address the additional risk. Reduction in power consumptions through the network management central via DSLAM have been reported in [21].

Much attention is currently focused on multi-user signal coordination techniques that can adaptively meet the need of the newest VDSL-2 system architectures. Again, spatial-temporal analysis and its exploration become the dominant tool to evaluate multiple structures based not just on their suitability at the transmitter but for balancing at the receiver.

This spatial-temporal method has been applied to some multi-carrier systems. Some specific areas of interest in MIMO communication include: (1) transmit optimisation with limited feedback mitigation [22]; (2) space-time coding to achieve better channel capacity [23]. This analysis requires a complete knowledge of the optimisation problem. Conceptually, this gives the system architect a tool to deal with overdetermined MIMO problems.

The well-known VBLAST algorithm requires few decision feedback parameters and performs quite well in a suitably structured environment. However, with rank deficient matrices, the VBLAST performance is far from optimal [24]. Similar space-time coding methods can be implemented in the form of vectoring transmission [25–27], where the signal of various users can be jointly coordinated at both transmitter and receiver to reduce the effects of crosstalk interference, and thus improve the performance of systems characterised by VBLAST. Vectored technologies typically require significantly higher complexity than spectrum-level coordination.

Vectoring treats the optimisation problem as a multiple-input multiple-output (MIMO) system where various users can be grouped together and then jointly processed. Each user within the group is coordinated at the signal-level, to effectively mitigate the induced crosstalk from adjacent users, through successive decoding of the aggregate data-rate across all lines [28].

A vectored method cannot only enable error-free transmission for multi-user components but also perform a single carrier system with equalisation. The method always searches for robust codes (pre-coding function) that allow coding across different spatial dimensions. The pre-coding function provides greater transmit diversity and is more robust in the presence of crosstalk.

To this end, many efforts are focusing on the joint optimisation strategy with interference control procedure [21, 17]. A similar example of such a strategy is the joint precoding and decoding design with decision feedback equalisation (DFE) [29]. In general, there are two different approaches to DFE equalisation, perfect channel state information and imperfect channel information. The perfect channel equalisation for mitigating channel interference is referred to as a zero-forcing method [22]. This concept addresses how to improve the performance of DMT systems in the frequency domain by the introduction of guard bands. The approach involves estimating the coherent equalisation parameters that represent synchronisation references, assuming that actual channel information is entirely known to both the transmitter and the receiver.

The other method, and focus of this thesis, is imperfect channel DFE design. DFE solution can be implemented without considering the additive bands, making the approach more flexible and capable of providing superior performance over DSL transmissions. Overdetermined precoding-equalisation is required when channel state information (CSI) for the multi-user PBLP problem is not entirely available at the transmitter, leading to an imbalance in spectral synchronisation between the transmitter and the receiver. The principal advantage of overdetermined equalisation designs lies in the inability to mitigate channel impairments too severe for the linear equalisation method. These DFE designs can be implemented as a set of simple Fourier impulse response (FIR) filters but at the expense of extreme in high computational load.

It is also worth noting that with per-user channel equalisation procedures, the DFE solution has received a great deal of attention in recent years. Such designs are slightly more mature than linear methods with regards to imbalanced and overdetermined complex problems.

The resource management technique proposed in this thesis focuses attention on the implementation of vectoring transmission with overdetermined channel equalisation. The optimisation problem considers a multi-user multiplexing standard between the transmitters and the receivers. The design also employs a joint power control scheme and exploits some system reduction procedures to shorten the length of copper twisted pair loops, with the aim of improving performance by mitigating the interference on each of the lines.

Two fundamental steps are followed during the process. The first step involves the application of polynomial matrix decomposition (PMD) techniques to the DMT architecture for allocating the optimal joint transmitting power to a group of user in noisy DSL environment. The PMD algorithm is used to separate the channel information into the available signals and the noise signals.

An essential feature of the proposed design includes the power allocation which is carried out in a distributive manner through the PMD algorithm. The resulting method can be viewed as a vectored combination of users across the DSL network, in contrast to other spectrum coordination schemes discussed in [30, 31]. Secondly, the transmission error associated with the approach can be managed by optimising the expected vectored parameter with respect to some decision feedback criterion. As the level of inter-user communication is reduced, the performance degrades gracefully toward the single-user channel equalisation solution in the limit of communicative isolation. The proposed algorithm also involves a fresh look at IWF. A novel algorithm called interference-aware spectrum management approach through iterative processes is considered that simplifies overdetermined channel equalisation approaches and enjoys low complexity implementation.

1.4 Why Polynomial Matrix Decomposition (PMD) Algorithms?

Polynomial matrix decomposition (PMD) algorithms are a family of multi-carrier factorisation techniques. These algorithms provide a spatial-temporal solution to a complex multi-user channel problem and apply various mitigation concepts including pre-processing, linear transformation and equalisation to improve the systems' throughput. PMD is suitable for solving generalisations of the single channel equalisation problem typically by computing the eigenvalue decomposition (EVD) and exploiting subspace decomposition [5].

These algorithms are now viewed as powerful optimisation tools for broadband or convolutive blind source separation (BSS), although in range of application is now much broader. The classical EVD approach to a single-carrier problem begins by exploiting second-order statistics to generate an instantaneous mixing matrix and performing independent components analysis (ICA) [32, 33]. ICA is usually obtained through factorisation of the instantaneous mixing matrix using unitary decomposition methods, such as the singular value (SVD) or eigenvalue decomposition (EVD) [34, 35].

In the polynomial matrix framework, the broadband MIMO problem is represented as a convolutive mixing matrix or a set of finite impulse response (FIR) filters. The transfer function of such a set of FIR filters forms a polynomial matrix, which can accurately represent the mixing multi-carrier problems. Typically, polynomial matrix decompositions estimate a reduced subspace and provide a more accurate physical representation of the mixing parameters. They usually incorporate a-priori knowledge of the mixing problem and address statistical aspects of the problem [5].

Polynomial matrices have been applied in a wide range of multi-disciplinary fields, including adaptive control systems [36], principal component analysis [37] and adaptive lossless filters [38]. Several PMD processes are formulated using the Smith–Macmillan form [39] and many others using polynomial matrix factors that are para-unitary [40]. The goal is to optimise a specific objective function for known channel state information and to form a space-time coding analysis. This type of coding analysis is always derived from auto- and cross-correlation terms, whose symmetries are created by a para-Hermitian polynomial matrix [41, 42].

The space-time coding analysis is applied here as an adaptive power allocation tool to solve the multi-carrier DSL problems. It has been shown in [43] that being able to jointly allocate a given transmit power efficiently to a various users in a noisy environment can significantly improve the intelligibility of a communication system. To the best of the knowledge, polynomial matrix approach has not been previously considered in the context of DSL spectrum management framework[2].

1.5 Outline and Contributions

The fundamental limitation in vectored transmission systems includes the exhaustive search for the precoding and equalisation thresholds, which are used to choose the best-suited power allocation. By employing the conventional EVD algorithm, one can derive the optimal power allocation functions for minimising induced crosstalk within a narrow frequency range subject to data-rate and spectral power constraints and hence the required bandwidth but experience high computational complexity arises as the number of users increases.

In an attempt to reduce the complexity, an iterative EVD equalisation algorithm known sequential matrix diagonalisation (SMD) suggested in [41] will be employed. This decomposes a complex optimisation problem into a set of SISO problems, and formulates joint precoding and equalisation thresholds to achieve the possible optimal power allocation solution.

The assumption of allocating the PSD function only to essential information is implemented, as suggested in [41], by reducing the optimisation problem to a reasonable setting, which considerable improves the average spectral efficiency and can be achieved with low-complexities. Ta and Weiss in [44] stated that, subject to the MMSE constraint and the peak power allocation constraint, the joint precoding and equalisation strategy through the use of polynomial singular value (PSVD) algorithm as an exhaustive search provides the optimal choice on power consumption minimisation and data-rate maximisation for broadband MIMO systems.

In light of the intensive computational complexities required hitherto, the proposed procedure is novel only require low-level single channel equalisation, and can achieve comparable performance to other existing exhaustive search techniques.

This thesis will analyse two important problems in robust spectrum management designs under the polynomial matrix framework; the implementation of joint optimal precoders

[2]Polynomial matrix decomposition algorithm can be more useful to DSL network to control the presence of crosstalk interference. First, with the joint precoding structure equalisation, the transmission can be balanced with the single-user decision feedback optimisation estimator at the receiver. Perfect reconstruction of the transmitted signal should seem more feasible when using the overdetermined channel equalisation at the receiver.

and equalisers, and the development of a robust decision feedback equalisation structure for overdetermined multi-user DSL systems. The influence of crosstalk on performance is investigated through the coherent equalisation with zero-forcing (ZF) equalisation and overdetermined channel equalisation using a decision feedback detector.

The differences in average spectral efficiency are presented for different channel state information (CSI). It is of interest to develop a generalised crosstalk-aware dynamic spectrum management (CA-DSM) scheme whereby crosstalk can be efficiently identified and isolated at the transmitters for any multi-user DSL system.

1.6 Thesis Overview

The outline of this thesis is as follows: Chapter 2 provides an overview of multi-carrier modulation systems; in particular, it discusses the discrete multi-tone (DMT) modulation for DSL systems. It also addresses the DSL interference channel, and the evolution of dynamic spectrum management algorithms.

Chapter 3 discusses some polynomial matrix decomposition (PMD) algorithms proposed in the literature for multi-user MIMO systems. They include SBR2 (second-order sequential best rotation) and SMD (sequential matrix diagonalisation). In particular, it focuses on SMD algorithm that is fast, robust and incurs low control overhead in multi-user MIMO network where it is used to develop the polynomial singular value decomposition (PSVD) framework [45, 5].

Chapter 4 introduces the application of the resulting PEVD algorithm to DMT modulation. Some precoding and equalisation framework based on PSVD algorithms are also discussed. These frameworks include precoding and equalisation for single-user DSL transmission, joint precoding and equalisation for multi-user DSL transmission and block precoding and equalisation for multi-user DSL transmission. The chapter concludes with some worked numerical examples.

Chapter 5 analyses joint precoding and equalisation with decision feedback constraints for overdetermined channel DSM problems. The joint optimal precoders and equalisers allow linear vectored transmission schemes. Several interference mitigation algorithms based on the decision feedback equaliser strategies are also proposed for further improvement of the DSM systems' data transmission performance with per-subchannel power constraints. Comparisons with a common linear zero-forcing strategies show that the proposed MMSE model performs better considering a set of joint optimal precoding and equalisation measures with the corresponding lower complexity and higher accuracy.

Chapter 5 also provides some simulation result for imperfect CSI MIMO channel using the MMSE-DFE equaliser at the receiver to increase the bit-rate at the transmitting end. The divergence between zero-forcing assumptions and minimum mean square (MMSE) statistics under decision feedback equalisation criteria is analysed. Numerical simulation helps evaluate the performance of multi-user spectrum management algorithms under decision feedback equalisation conditions.

Chapter 6 provides the conclusions and suggestions for future work.

Chapter 2

Overview of Multi-carrier Modulation Systems

This chapter first provides a brief introduction to multi-carrier modulation systems and then presents an overview of discrete multitone (DMT) modulation. It also presents some brief background information on the DSL interference channel, which requires some additive samples for propagation. In the remaining part of this chapter, we attempt to provide an overview of the key algorithms, including the iterative water-filling method and other state of the art dynamic spectrum management methods.

2.1 Multi-Carrier Modulation Systems

The application of multi-carrier modulation has dramatically advanced in modern communication technologies and enabled many intelligent resource management algorithms for communication systems. A brief review of multi-carrier modulation techniques is provided in this section.

Modern communication systems such as DSL networks suffer from inter-symbol interference (ISI) and inter-carrier interference (ICI) when successive symbols or blocks interfere with each other - especially for a high–speed communication system that exploits a narrow spacing between symbols/blocks in the time domain. Single-carrier systems use simple equalisers to reduce both ISI and ICI, and treat the channel as an equivalent additive white Gaussian noise (AWGN) process. A simple example is a single-user zero-forcing (ZF) equaliser, which performs an inversion on the channel gain in the frequency domain and forces the interference between symbols to zero. Figure 2.1 shows a measured the channel transfer function of a single-carrier transmission.

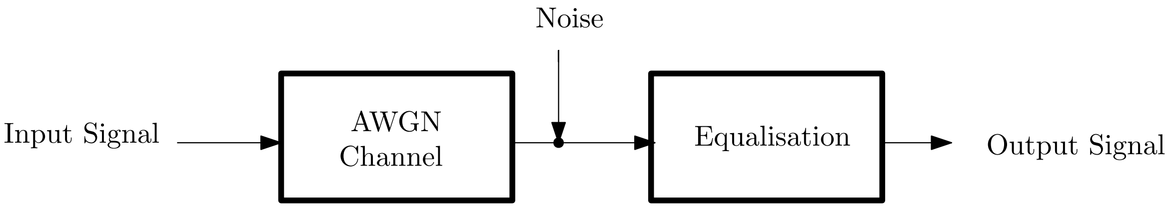


Figure 2.1 Single-User System

To flatten the AWGN channel, extremely complicated equalisation procedures need to be implemented. Also, there are notches in some different frequency ranges where the common channel inversion process would introduce a substantial noise.

Other equalisation methods for a single-carrier system experience similar problems and are either significantly reducing the performance of the system or not implementable in practice. This led to the introduction of multi-carrier modulation systems. Two of the popular multi-carrier modulations of today's system are the orthogonal frequency division multiplexing (OFDM) for wireless communication and discrete multi-tone (DMT) for digital subscriber line networks.

2.2 Discrete Multitone (DMT) Modulation

Similar to OFDM in wireless systems, DMT is best suited for high-speed multi-user transmission over fast-fading frequency selective channels. DMT based systems always partition the available channel bandwidth into multiple parallel sub-channels, each with narrowband frequency tone. Each sub-channel can be viewed as an AWGN channel. The difference is that the transmitter in DMT always optimises energy allocation over the parallel sub-channels instead of putting an equal amount of energy on all sub-channels. Also, DMT performs bit-loading by transmitting different constellations on each sub-channel based on its SNR. DMT is widely adopted in various DSL systems with a slowly varying channel, where the receiver often provides specific feedback of channel information to the transmitter by jointly coordinating and optimising the power allocation. In DSL channel transmissions the channel state information (CSI) may be difficult to estimate at the transmitter and is often unavailable.

A good example of DMT transmission is illustrated in Figure 2.2, where **(a)** shows how a DMT based system partitions the available bandwidth $s(t)$ into multiple parallel sub-channels with each sub-channel allocated for narrowband signal $x_i(t)$. The input signal $s(t)$ in **(b)** is then converted into several digital bit streams using the inverse discrete Fourier transform (IDFT) operation to form the discrete-time signal $x_i(t)$. This operation calculates the inverse fast Fourier transformation (IFFT) elements of $s(t)$ and then conveys each through

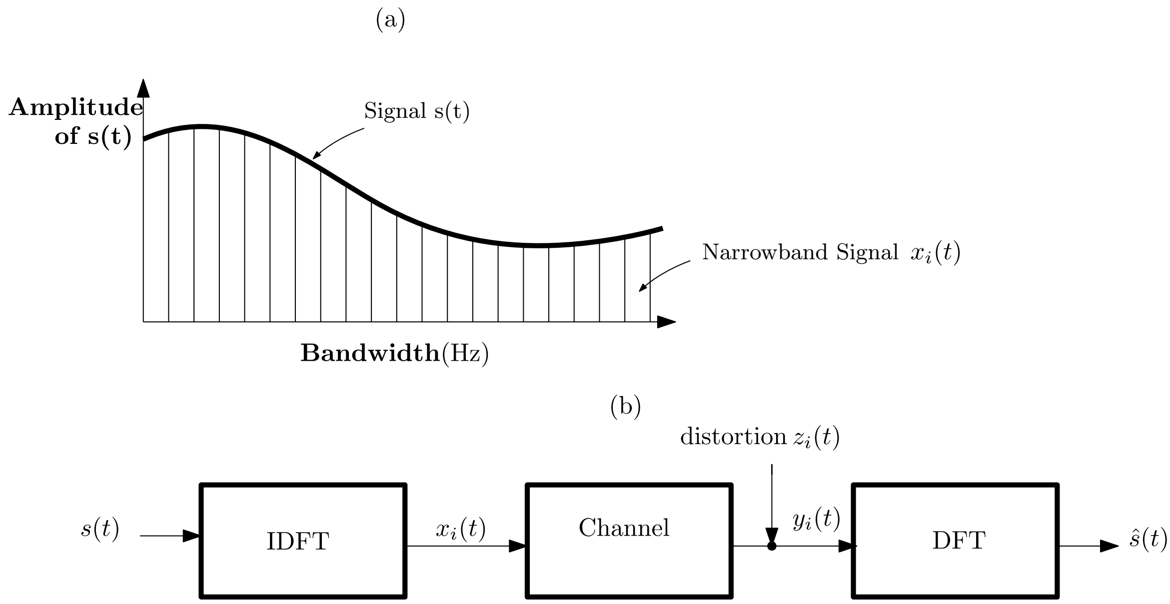


Figure 2.2 DMT based transmission

a DSL channel. During the propagation, channel distortions $z_i(t)$ are invariably introduced. The received signal $y_i(t)$ becomes corrupted, and the overall performance may be severely affected. The desired signal $\hat{s}(t)$ at the receiver would appear quite different from the input signal $s(t)$. The task of the receiver is then to perform another fast Fourier transform (DFT) by calculating the fast Fourier transformation (FFT) of the received discrete signal to equalise the effect of these distortions by reproducing $s(t)$ with little or no difference.

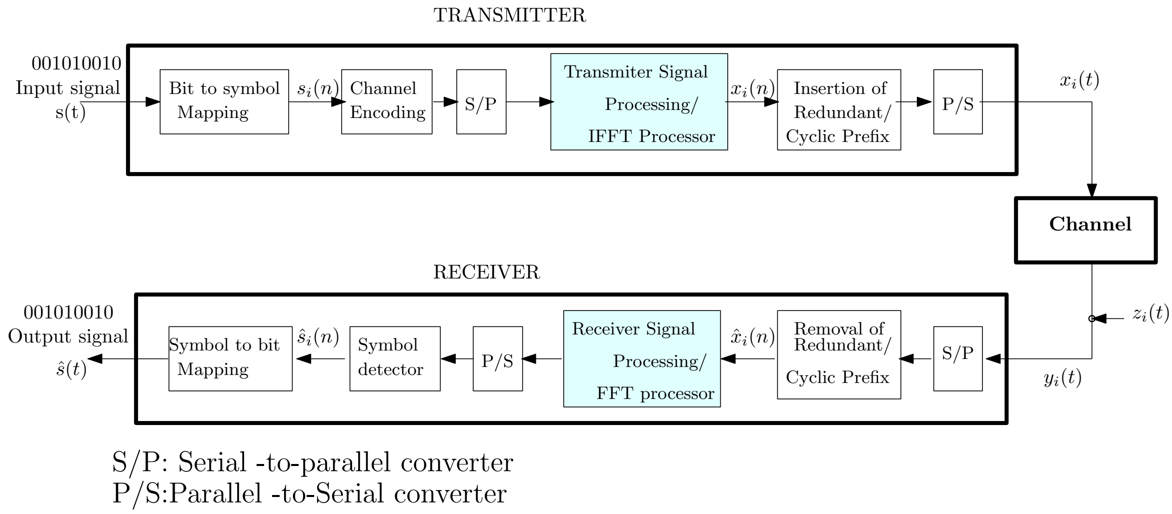


Figure 2.3 DMT transformation processes

All existing DSL networks, from ADSL standards onwards to the newest G.fast systems, use DMT as the core multiplexing modulation technique. DMT is a class of multi-carrier modulation to the wireless OFDM technique. In contrast to OFDM, DMT can adapt the modulation structure of each tone irrespective of the level of change in channel conditions. This is, in fact, the main reason why various DMTs are standardised for high-speed data transmission over twisted copper wire services, mainly, in situation where the channel information is almost static.

In DSL, there exist some transmission distortions due to network imbalances, but these variations occur slowly over time. Because of the mostly static channel, adaptive modulation for each tone can deliver substantial gains in possible performance. However, for current high-speed communication systems such as VDSL-2, the channel variations occur very fast and so adaptive per-tone modulations are much crucial.

A general block diagram of DMT transmission is given in Figure 2.3, employing the IDFT operation at the transmitter and DFT operation at the receiver after converting the channel state information into discrete input signals. In principle, the Fourier transformation converts the continuous-time input signal $s(t)$ to a discrete-time signal after obeying the Nyquist sampling rate, and performs the inverse operation to produce the output signal $\hat{s}(t)$.

Practical DMT architectures employ single-carrier water-filling procedures to maximise the data-rate adaptively subject to a total power constraint. The optimisation problem is represented as:

$$\begin{aligned} \text{maximize:} \quad & \frac{1}{2}\sum_{n=1}^{N}\log_2\left(1+\frac{\Xi_n g_n}{\Gamma\sigma_n^2}\right) \\ \text{subject to} \quad & \sum_{n=1}^{N}\Xi_n \leq \Xi_x, \end{aligned} \tag{2.1}$$

where Ξ_n and σ_n^2 respectively represent the transmit and noise power density of each sub-carrier n, "g_n" is the channel gain, and Γ represent the SNR gap of digital modulation for a achievable probability of error per bit P_e [7, 46]. The Lagrangian decomposition of the problem into several sub-problems can be expressed as:

$$L(\lambda,\Xi) = \frac{1}{2}\sum_{n=1}^{N}\log_2\left(1+\frac{\Xi_n g_n}{\Gamma\sigma_n^2}\right) + \lambda\left(\sum_{n=1}^{N}\Xi_n - \Xi_x\right). \tag{2.2}$$

Taking derivatives of $L(\lambda,\Xi)$ with respect to Ξ_n and setting them to zero, the optimal condition can be formed as:

$$\begin{aligned} &\Xi_n + \frac{\Gamma\sigma_n^2}{g_n} = Constant \\ &\Xi_n \geq 0,\ n = 1,\ldots,N. \end{aligned} \tag{2.3}$$

This expression indicates the sum of the transmitted PDF and noise power spectra normalised to the channel gain. The optimal power allocation of (2.1) follows a water-filling pattern [7]. The water-filling transmit power allocation can be approximated by a flat discrete transmission on virtual DSL carriers with little or no effect on overall performance, as long as the PDF parameters are used [14]. A detailed discussion of such a discrete transmission is available in [47].

2.3 The DSL Interference Channels

A typical optimisation model is similar to the DSL interference channels described in Figure 1.2, where K users share a binder and crosstalk interference arises when transmitting input data from the customer premise equipment (CPE) to a single central office (CO). The FEXT interference in Figure 1.3 can be mitigated efficiently by allocating none-overlapping power spectra for the upstream and downstream transmission.

We use K to denote the total number of users, N to denote the total number of DMT frequency tones, $\mathbf{H}_{kj}^n$ denotes the channel transfer function from user j to user k on tone n, ω_k the weight assigned to user k, S_k^n the power spectral density allocated to user k on tone n. The number of bits available for user k on tone n, b_k^n is

$$b_k^n = \log_2 \left(1 + \frac{g_k^n}{\Gamma}\right) \tag{2.4}$$

where Γ is the product of the gap-to-capacity of the code and the SINR margin used for protection against unexpected noise [7]. g_k^n represents the signal-to-interference-plus-noise ratio (SINR) of user k on tone n and can be expressed as:

$$g_k^n = \frac{|\mathbf{H}_{kk}^n \tilde{\mathbf{H}}_{kk}^n| S_k^n}{\sum_{j \neq k} \mathbf{H}_{kj}^n \tilde{\mathbf{H}}_{kj}^n S_j^n + (\sigma_k^n)^2}. \tag{2.5}$$

The data rate for user k is defined as

$$R_k^n = \sum_{n=1}^{N} b_k^n. \tag{2.6}$$

The maximum weighted sum rate (WSR) power allocation problem can be defined as

$$\begin{aligned} \text{maximise:} \quad & \omega_k R_k^n \\ \text{subject to} \quad & \sum_{n=1}^{N} S_k^n \leq P_k, \qquad \forall k \\ & 0 \leq S_k^n \leq M_k^n \qquad \forall k,n \end{aligned} \tag{2.7}$$

where P_k is the achievable transmit power for user k and M_k^n is the regulatory power mask for user k on tone n. The weighed function, ω_k represents the fairness weight for all users and can be adjusted to improve a fairness in the power control. The fairness weight can be viewed economically as a "monetary budget" for user k in the power allocation while the channel condition can be considered to be a price factor for spectrum management in the DSL system [26].

A largest ω_k provides tone k the ability to compete for more transmit power in the DSL network even if the price (channel condition) is not favourable to the user. A largest competitive value of ω_k is incrementally granted to user k when it is in need of more power allocation and data rate. The objective function can be represented as the difference of two convex functions $c(x) - d(x)$, where

$$c(x) = \sum_{k=1}^{K} \omega_k \sum_{n=1}^{N} \log_2 \left(\mathbf{H}_{kk}^n \tilde{\mathbf{H}}_{kk}^n S_k^n + (\sigma_k^n)^2 + \sum_{j \neq k} \mathbf{H}_{kj}^n \tilde{\mathbf{H}}_{kj}^n S_j^n \right) \tag{2.8}$$

$$d(x) = \sum_{n=1}^{N} \omega_k \sum_{n=1}^{N} \log_2 \left((\sigma_k^n)^2 + \sum_{j \neq k} \mathbf{H}_{kj}^n \tilde{\mathbf{H}}_{kj}^n S_j^n \right). \tag{2.9}$$

The optimisation problem is non-convex and requires difference-of convex programming procedure. Another equivalent formulation of the optimisation problem for tracking the transmit power allocation function on a multi-user network is to decouple the non-convex problem into a set of minimum requirements and propagate each simpler problem through a channel on its data rate.

2.4 Dynamic Spectrum Management Algorithms

Early work in the deployment of DSM is uses of the iterative water-filling (IWF) algorithm. This repeatedly performs water-filling to allocate the transmit power to each user while considering the crosstalk from adjacent users as noise [48]. While fast in algorithmic speed, IWF is a fully distributed and autonomous algorithm with reasonable computational complexity, leading to a sub-optimal performance in near-far scenarios DSL transmission

[7]. Since the design of the distributed IWF algorithm, many recent efforts have focused on either spectrum or signal coordination methods to solve the underlying optimisation problem appearing in DSL networks. Unfortunately, this optimisation problem is non-convex and problematic. This rest of this section is devoted to analysing coordination methods and several other DSM algorithms.

2.4.1 Spectrum Coordination

Many different spectrum coordination algorithms have been proposed. Cendrillon et. al. proposed the optimal spectrum balancing (OSB) algorithm that performs a exhaustive-search to find the optimal power allocation to a predetermined quantization of user powers [30, 49]. To solve the non-convex optimisation problem in (2.7), the OSB algorithm uses Lagrange multipliers to enforce constraints that are coupled over frequencies [50]. The dual of a non-convex optimization problem is always convex, even if the primal problem is not convex [50]. If the duality gap is zero or negligible, optimizing the dual problem reaches the same optimal value as the primal problem. OSB searches for the optimal solution to the per-user power constraint decoupled dual problem. Consequently, OSB is a fully centralised algorithm and suffers from an exponential complexity in the number of users. However, finding the optimal Lagrange multipliers can become more complex when more than two users are involved in the optimisation problem. Another centralised algorithm, iterative spectrum balancing (ISB) proposed in [51] extends the application of Lagrange multipliers by iteratively computing the optimal solution to the dual non-convex problem for reduced algorithmic complexity. ISB avoids the exponential complexity by using a sub-gradient search. However, the stepsize of the sub-gradient has to be small to guarantee convergence, leading to a high computation cost. The description of OSB and ISB algorithms is analysed below. The Lagrangian dual

$$L(\lambda, S) = \sum_{k=1}^{K} \omega_k \sum_{n=1}^{N} \log_2 \left(1 + \frac{\mathbf{H}_{kk}^n \tilde{\mathbf{H}}_{kk}^n S_k^n}{(\sigma_k^n)^2 + \sum_{j \neq k} \mathbf{H}_{kj}^n \tilde{\mathbf{H}}_{kj}^n S_j^n}\right) + \sum_{k=1}^{K} \lambda_k \left(P_k - \sum_{n=1}^{N} S_k^n\right). \quad (2.10)$$

The dual objective function $g(\lambda)$ is defined as:

$$g(\lambda) = \underset{S}{\text{argmax}} \quad L(\lambda, S). \quad (2.11)$$

The dual optimisation problem of (2.7) can be expressed as:

$$\begin{aligned} \text{minimise:} \quad & g(\lambda) \\ \text{subject to} \quad & \lambda \geq 0 \end{aligned} \quad (2.12)$$

The difference between OSB and ISB is the calculation of the sub-gradient $g(\lambda)$. OSB either jointly discretises the power allocation S_k^n (power-loading) or discretises the bit allocation b_k^n(bit-loading) to approximate a finite set of the possible power allocation matrices. ISB exhaustively searches the set of power allocation matrices to find the value of $g(\lambda)$ over each tone, i.e., the maximum value of $L(\lambda, S)$ over the set of possible energy matrices on each tone.

If the discretisation size is M, OSB has a complexity of $O(NKM^K)$ on the approximation of $g(\lambda)$. ISB, instead, introduces a coordinate descent approximation procedure to compute $g(\lambda)$, thus effectively reducing the dual optimisation complexity. A pseudocode description of ISB algorithms is summarised in Algorithm 1. ISB computes the $g(\lambda)$ compared to the exhaustive OSB search. However, ISB does not always reach an optimal value of $L(\lambda, S)$, even for the convex $g(\lambda)$. Coordinate descent's inaccurate evaluation of $g(\lambda)$ may harm the update of the dual variable λ, and thus ISB may diverge and reach a suboptimal solution. Therefore, OSB is still widely used as a benchmark on the rate region achievable for a given DSL optimisation problem [11].

Algorithm 1 Pseudo-code for sub-gradient $g(\lambda)$ calculation for ISB Algorithm

Repeat
for `n=1 to N` **do**
  for `K=1 to K` **do**
    $S_k^n = \underset{S_k^n}{\text{argmax}} \sum_{k=1}^{K} \left(\omega_k \log_2 \left(1 + \frac{\mathbf{H}_{kk}^n \tilde{\mathbf{H}}_{kk}^n S_k^n}{(\sigma_k^n)^2 + \sum_{j \neq k} \mathbf{H}_{kj}^n \tilde{\mathbf{H}}_{kj}^n} \right) - \lambda S_k^n \right)$
  end for
end for
Until *S converges.*

While ISB reduces complexity compared to OSB, it is still unclear how well-suited they are for practical implementation.

Other spectrum coordination algorithms include; Autonomous Spectrum Balancing (ASB) [52, 53, 31], Selective Iterative Water-filling (SIW) [54], Successive Convex Approximation for Low complExity (SCALE) [11, 55], Band Preference Spectrum Management (BPSM) [56], Iterative Power Pricing (IPP) [57], Grouping Spectrum Management (GSM) [58], Semi-Blind Spectrum Balancing (2SB) [59], Distributed Spectrum Balancing (DSB) [60, 15, 61], and successive convex approximation for water-filling (SCAWF) [11].

2.4.2 Signal Coordination

Signal coordination, also know as vectored DSM, involves considering the problem appearing in (2.7) as a multiple-input multiple-output (MIMO) problems. Previous work includes [26, 62–64, 25, 65–68]. With vectored DSM, the implementation requirements on infrastructure are considerably higher. For vectoring transmission, various users have to have some form of physically co-located at the transmitting side and the receiver side, where the knowledge of all direct signals, crosstalk signal and channel gains involved are usually available. On the one hand, vectored transmission can deliver substantial gains in comparison to spectrum coordination, mitigating most if not all crosstalk and even benefiting from the crosstalk. It is Worthing that the fact that crosstalk is beneficial can be interpreted as the spatial-temporal gain.

This kind of coordination can provide support for all variations of spectrum efficient, vectored VDSL-2 systems. It consists of joint transmit and joint receive processors. When connected to a standard network management centre device such as the DSLAM, this combination allows the implementation of a complete point-to-point vectoring transmission. The vectored transmission supports all features defined in, ITU G.993.2 (VDSL2), G.993.5 (G.vector) and ITU G.997 (G.ploam). This coordination solution is enabled by decoupling the system architecture and resources into a set of linear multiple vectored lines. This high level of scalability, coupled with the compelling features offered by the linear and independent vectored set, lead to a solution that is ideally suited to a wide range of applications such as heterogeneous DSLAM systems. A typical scenario for joint signal coordination with

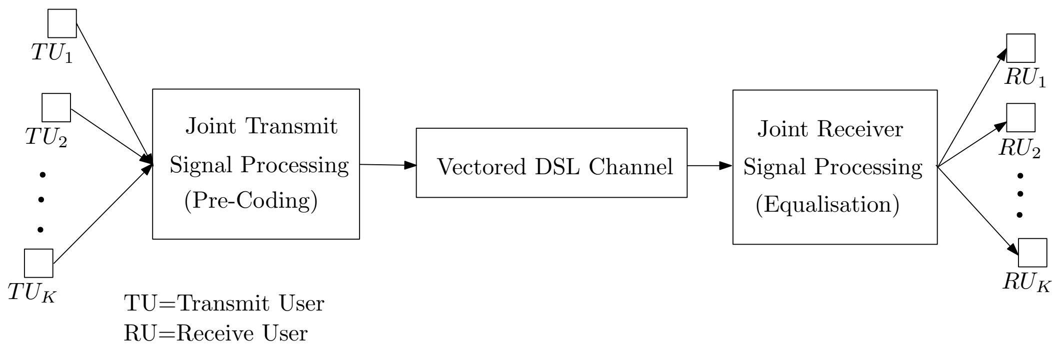


Figure 2.4 This is the most common scenario considered in the signal coordination of DSL system, where we have joint signal processing (Pre-coding) at the transmit side, and we have joint signal processing (Equalisation) at the receive side.

a vectored DSL channel is illustrated in Figure 2.4. The redundant precoding potential of the signal coordination coupled with the low power consumption allows vectored DSM technology to comply with the parallel transmission standard for overdetermined MIMO systems. The flexibility of the redundant precoding simplifies portfolio diversification addressing multiple applications while ensuring that such systems are proven for next-generation networks. The vectored transmission offers elegant technical solutions to crosstalk containment. The common and current implementations are with an interference mitigation scheme.

Hitherto most previous work in the literature have applied vectored DSM in two distinct situations. The first situation is one with full two-sided coordination. In our nomenclature, this is joint equalisation with a zero-forcing condition. The second situation is the one that is considered by the majority of papers in the DSM equalisation literature. It is the one where each user independently equalises crosstalk impact at the receiver, and there is full joint coordination of all users at the transmitter side but no common signal processing at the receiver side.

This situation arises when the DSL channel optimisation problem is spread into several independent signals, and each signal has a unified transmitting channel from the transmitter to the receiver. In the literature, this independent user equalisation requires specific feedback information to minimise the mean square estimated error (MSE). In this thesis, we consider applying signal coordination to only two-sided vectored transmission scenarios.

2.4.3 Two-Sided Vectored Signal Processing

This section will analyse a two-sided vectored signal processing approach that preconditions a mixture of signals before transmission, then transmits only the actual signal through a DSL channel, and also equalises the resulting transmission errors at the receiver. In the MIMO channel literature, a two-sided vectored signal processing is usually performed by decomposing the channel state information into completely independent channels via singular value decomposition (SVD).

Figure 2.5 represents a general two-sided vectored DSM model for DSL systems , where the model assumes a DSL channel representation in the frequency domain, ω. The

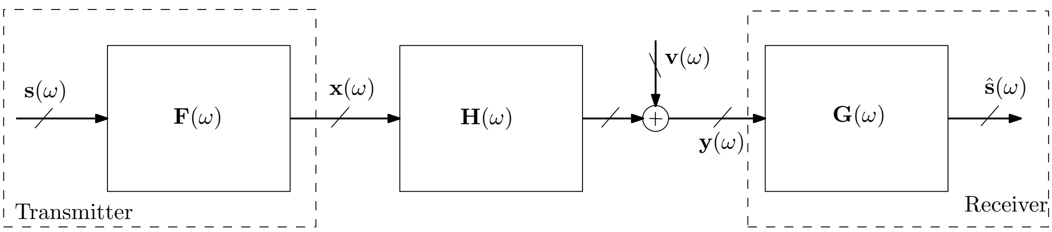


Figure 2.5 This is the generalised two-sided vectored DSM model of DSL system, with the precoder $\mathbf{F}(\omega)$ at the transmitter, the channel $\mathbf{H}(\omega)$ and equaliser $\mathbf{G}(\omega)$ at the receiver.

received signals $y_k(\omega), k = 1, \cdots, K$ are represented by the vector $\mathbf{y}(\omega) = [y_1(\omega), \cdots, y_K(\omega)]$. Consequently, the signal $\mathbf{y}(\omega)$ can be described by

$$\mathbf{y}(\omega) = \mathbf{H}(\omega)\mathbf{x}(\omega) + \mathbf{v}(\omega) \tag{2.13}$$

where the mixing input signals in a vector $\mathbf{s}(\omega) = [s_1(\omega), \cdots, s_K(\omega)]^T$ are converted to the transmit vector $\mathbf{x}(\omega) = [x_1(\omega), \cdots, x_K(\omega)]^T$ through the precoder matrix $\mathbf{F}(\omega)$. The vectored signal $\mathbf{x}(\omega)$ is then transmitted through the DSL channel $\mathbf{H}(\omega)$ in the presence of a noise vector $\mathbf{v}(\omega) = [v_1(\omega), \cdots, v_K(\omega)]^T$. The channel matrix $\mathbf{H}(\omega) \in \mathbb{C}^{K\times K}$ defines both the real channel information (diagonal elements) and the crosstalk information (off-diagonal elements). Converting the channel matrix to a convolutional model, therefore, $\mathbf{H}(\omega)$ can be modelled as:

$$\mathbf{H}(\omega) = \begin{bmatrix} \mathbf{H}_{K-1}(\omega) & \mathbf{H}_{K-2}(\omega) & \cdots & \mathbf{H}_0(\omega) & 0 & \cdots & 0 \\ 0 & \mathbf{H}_{L-1}(\omega) & \cdots & \mathbf{H}_1(\omega) & \mathbf{H}_0(\omega) & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{H}_{K-1}(\omega) & \mathbf{H}_{K-2}(\omega) & \cdots & \mathbf{H}_0(\omega) \end{bmatrix} \tag{2.14}$$

The current OFDM and DMT based systems employ a form of cyclic prefix (CP) or zero padding (ZP) to act as a buffer[22, 69], where the $\mathbf{H}_{padded}(\omega)$ is the $K \times K$ Toeplitz matrix with

$$\mathbf{H}_{padded}(\omega) = \begin{bmatrix} \mathbf{H}_0(\omega) & 0 & \cdots & 0 \\ \mathbf{H}_1(\omega) & \mathbf{H}_0(\omega) & \cdots & 0 \\ \ddots & \ddots & \ddots & \vdots \\ \mathbf{H}_{K-1}(\omega) & \mathbf{H}_{K-2}(\omega) & \cdots & \mathbf{H}_0(\omega) \end{bmatrix} \tag{2.15}$$

As analysed in [22, 70], the importance of the cyclic prefix is to convert the linear convolution of the channel into a circular convolution. As will be shown in the following section, the circular convolution in the time domain is equivalent to multiplication in the frequency domain, provided the additional CP ensure that all transmitting tones are orthogonal during their propagation through the channel.

By applying SVD, the channel matrix $\mathbf{H}_{padded}(\omega)$ at frequency tone (ω) can be defined as:

$$\mathbf{H}_{padded}(\omega) = \mathbf{P}(\omega)\Lambda(\omega)\mathbf{Q}^{\mathrm{H}}(\omega) \tag{2.16}$$

where $\mathbf{P}(\omega)$ and $\mathbf{Q}(\omega)$ are unitary matrices and $\Lambda(\omega)$ is diagonal with real and non-negative values. The real channel information with no crosstalk interference is represented by $\Lambda(\omega)$, each value of $\Lambda(\omega)$ can be used independently for vectored transmission.

To benefit from the diagonally vectored channel, the mixing input signals $\mathbf{s}(\omega)$ that are preconditioned by pre-coding matrix $\mathbf{Q}(\omega)$ to form transmit vectored signal $\mathbf{x}(\omega)$ can be expressed as

$$\mathbf{x}(\omega) = \mathbf{Q}(\omega)\mathbf{s}(\omega). \tag{2.17}$$

The vectored received signal $\mathbf{y}(\omega)$ is compensated through the equalisation matrix $\mathbf{P}^{\mathrm{H}}(\omega)$ at the receiver, where

$$\begin{aligned}\mathbf{P}^{\mathrm{H}}(\omega)\mathbf{y}(\omega) =&\mathbf{P}^{\mathrm{H}}(\omega)\Big(\mathbf{H}_{padded}(\omega)\mathbf{x}(\omega)+\mathbf{v}(\omega)\Big)\\ =&\mathbf{P}^{\mathrm{H}}(\omega)\mathbf{H}_{padded}(\omega)\mathbf{Q}(\omega)\mathbf{s}(\omega)+\mathbf{P}^{\mathrm{H}}(\omega)\mathbf{v}(\omega)\\ =&\Lambda(\omega)\mathbf{s}(\omega)+\mathbf{P}^{\mathrm{H}}(\omega)\mathbf{v}(\omega).\end{aligned} \tag{2.18}$$

Here, we have chosen $\mathbf{G}^{\mathrm{H}}(\omega)$ to be the optimal $\mathbf{P}^{\mathrm{H}}(\omega)$ and $\mathbf{F}^{\mathrm{H}}(\omega)$ to be the optimal $\mathbf{Q}^{\mathrm{H}}(\omega)$. Also, we also assumed that the noise parameters in $\mathbf{P}^{\mathrm{H}}(\omega)\mathbf{v}(\omega)$ are white and additive Gaussian noise.

To reconstruct the original input signals, the output signal can be simply estimated by inverting $\Lambda(\omega)$[46] (provided the channel is invertible) i.e,

$$\begin{aligned}\hat{\mathbf{s}}(\omega) =&\Lambda^{-1}(\omega)\mathbf{P}^{\mathrm{H}}(\omega)\mathbf{y}(\omega)\\ =&\mathbf{s}(\omega)+\Lambda^{-1}(\omega)\mathbf{P}^{\mathrm{H}}(\omega)\mathbf{v}(\omega).\end{aligned} \tag{2.19}$$

The above analysis utilises a joint signal equalisation of the receivers to counteract the effect of the noise matrix $\Lambda^{-1}(\omega)\mathbf{P}^{\mathrm{H}}(\omega)\mathbf{v}(\omega)$. But in practice with imperfect channel matrices, a suitable channel estimation procedure is required to continuously mitigate the effect of overdetermined MIMO transmission and update the transmit matrices occasionally.

The benefit of this channel estimation over the static joint signal equalisation standards is that error propagation from channel to channel is avoided. Accordingly, the complexity normally associated with the presence of crosstalk is reduced as a result of its implementation.

There exist some studies on vectored DSM systems where decision feedback equalisation (DFE) is independently applied to mitigate the impact of crosstalk on each of the received tones. The most common DFE approach for cancelling crosstalk at the receiver is probably zero-forcing (ZF). In [71–76], crosstalk is mitigated at the transmitter using a per-tone equalisation scheme, where the received signal is defined as

$$\mathbf{y}_k(\omega)=\frac{1}{\beta_k(\omega)}\mathbf{H}_{padded}(\omega)\mathbf{x}_k(\omega)+\mathbf{v}_k(\omega) \tag{2.20}$$

and $\beta(\omega)$ is the feedback parameter chosen so that the best vectored linear signal is achieved. Thus, the output vectored signal becomes

$$\hat{\mathbf{s}}_k(\omega)=\beta_k(\omega)\Lambda_k^{-1}(\omega)\mathbf{P}_k^{\mathrm{H}}(\omega)\mathbf{y}_k(\omega). \tag{2.21}$$

The scalar feedback parameter may be chosen so that the bit-error rate of the vectored transmission is optimised by setting

$$\beta(\omega) = \underset{\mathbf{s}(\omega)}{\operatorname{argmax}} ||\mathbf{H}_{padded}(\omega)^{\mathrm{H}}\mathbf{H}_{padded}(\omega)||. \tag{2.22}$$

In general, most of the algorithms available in DSL environments to minimise (2.22) are somewhat suboptimal, because the scalar parameters $\beta(\omega)$ require finite power constraints at all transmit paths. Because of the sub-optimality of the existing DFE equalisation under the zero-forcing constraints, there is an extra benefit in using MMSE and polynomial channel representation. The analysis of DSL problems modelled as polyphase (polynomial) matrices has been addressed in [39, 77]. It is also shown in that decision feedback, and polynomial matrix decomposition (PMD) system architectures [78] are required to enable a complete vectoring transmission. A similar approach has been suggested for MIMO systems in [44, 22].

It is, therefore, the purpose of this research to investigate the problems associated with DSL systems in the polynomial domain and provide a novel alternative approach to vectored DSM implementation. More particularly, this research aims to provide a tractable practical system without being overly complicated. The main difficulty with known solutions of the multi-carrier interference mitigation technique lies in the fact that the optimisation problem of current DSL systems, particularly VDSL-2, is inherently an overdetermined MIMO problem that requires single channel equalisation. This research also aims to develop a simple per-user equalisation method through a decision feedback mechanism which involves sending information from the receiver back to the transmitter for each user. Consequently, we are to show that a polynomial matrix based DSM approach can provide a capability for improving the performance over DSL communication systems, providing several optimal vectored channels over which each input bit is transmitted and received by respective users.

The proposed method follows the following steps: (1) collecting channel state information about the DSL line to encapsulate the signal and interference characteristics of a group of users in the polynomial domain; (2) determining the crosstalk line, background noise signal and other interference characteristics which exist within a group of DSL cable binders through the sequential matrix diagonalisation (SMD) algorithm; (3) dynamically allocating transmit power to a group of DSL lines taking into consideration the actual line, signal and interference characteristics and nose weight of the channel information; (4) exclusively optimising the expected received information at the receiver by minimising the crosstalk information and providing decision feedback to enable the maximisation of data-rates for each of the respective users.

2.5 Concluding Remarks

Vectored DSM algorithms have the potential to provide feedback information to improve the performance of overdetermined MIMO systems. This section first discussed the revolution of multi-carrier modulation in DSL networks from the standardised DMT system to the innovation of signal coordination DSM algorithms. These algorithms led to vectored transmission incorporated with channel estimation. This section also illustrated how to reconstruct the original input signal at the receiver by minimising the MSE constraints.

Chapter 3

Polynomial Matrix Decomposition Techniques

3.1 Introduction

This section will discuss the principle of a polynomial matrix formulation and its Eigenvalue decomposition. The study focuses in particular on the sequential matrix diagonalisation (SMD) technique to compute the polynomial matrix eigenvalue decomposition (PEVD) for a multi-carrier MIMO system. SMD is introduced to formulate the polynomial singular value decomposition (PSVD) structures for the system. The SMD algorithm is a natural extension of the second-order sequential best rotation (SBR2) method, which was devised for the blind separation of signals that have been mixed in a convolutive manner [41].

Just as many instantaneous mixing models exploit the standard EVD or SVD as a second order pre-processing step, the proposed PSVD algorithm applied to a convolutive mixing model, where polynomial matrices are used to represent the channel state information. The SMD algorithm is used to compute the necessary PSVD algorithm.

The PSVD can be implemented by applying a two-sided SMD decomposition to a para-Hermitian polynomial matrix in order to create a diagonal polynomial matrix. It can, therefore, be considered as an extension of the conventional Jacobi algorithm suitable for scalar Hermitian matrices. Consequently, this PSVD algorithm can be applied for a scalar Hermitian matrix by merely diagonalising the matrix via Jacobi's algorithm [79]. The idea of SMD was first established by Redif et al. [41], by performing a direct calculation of the PEVD for MIMO signal processing using the well-established the (SBR2) algorithm. The reader is referred to [5, 45**?** , 80, 81] for more details about this variant.

Finally, this chapter analyses and describes how the SMD algorithm operates to shorten the order of the optimisation problem and form vectored diagonal matrices. Convergence of the algorithm is proven, and its application to formulate joint optimal pre-coding and channel equalisation is highlighted using a simple numerical example. Other applications of the algorithm include multichannel spectral factorisation and the design of filter banks for optimal data suppression and perfect reconstruction. Our interest of course is the application of polynomial matrix technique to DSM designs in order to alleviate the performance degradation due to impaired channel state information.

3.2 Polynomial Matrices

Polynomial matrices arise in many different situations; a pertinent example is the case of multi-user MIMO systems. Polynomial matrices can be defined as either a polynomial with matrix-valued coefficients or a matrix with polynomial entries [**?** 82]. In general, the polynomials required are Laurent polynomials with positive and negative powers.

3.2.1 Laurent Polynomial

A Laurent polynomial involves a combination of positive and negative powers of the independent variable with fixed coefficients. Laurent polynomials differ from ordinary polynomials in that they may have terms of negative degree.

A Laurent polynomial matrix takes the form

$$\boldsymbol{H}(z) = \mathbf{H}_{-V_1} z^{V_1} + \ldots + \mathbf{H}_{-1} z^{1} + \mathbf{H}_0 + \mathbf{H}_1 z^{-1} + \ldots + \mathbf{H}_{V_2} z^{-V_2} \tag{3.1}$$

more compactly,

$$\boldsymbol{H}(z) = \sum_{v=-V_1}^{V_2} \mathbf{H}_v z^{-v}, \tag{3.2}$$

where $V_1 > 0$, $V_2 > 0$, and z is the indeterminate variable. The complex coefficient $\mathbf{H}_v$ is a matrix with M inputs and N outputs. For the purpose of this thesis, we assume that the coefficient entries are complex and so(3.2) represents a complex Laurent polynomial.

3.2.2 Properties of a Polynomial Matrix

Order and Degree

The dimension of $\boldsymbol{H}(z)$ is $M \times N$, with the polynomial order dependent on the support of the auto- and cross-correlation sequences contained within it. For the example in (3.1) the order is $(v_2 - (-v_1)$. The degree of the polynomial matrix is defined as the number of delay elements required to implement it as an FIR filter; therefore, it is only defined for causal polynomial matrices. The degree is quite different to the order of a polynomial matrix and to avoid confusion degree will not be discussed any further in this thesis.

Parahermitian Transpose

The term "paraHernitian transpose" (sometimes referred to as "para-conjugate") is an extension of the Hermitian transpose from scalar matrices to polynomial matrices. The parahermitian transpose of a polynomial matrix involves a time reversal of the polynomial elements, i.e. $\tilde{\boldsymbol{H}}(z) = \boldsymbol{H}^{\mathrm{H}}(z^{-*})$ where $\{\tilde{.}\}$ is used to denote the paraHermitian transpose operation. Note that in the case where the polynomial matrix is of order zero, the paraHermitian transpose is equivalent to the Hermitian transpose.

ParaHermitian Property

The idea of Hermitian symmetry can be extended to the polynomial matrices. A polynomial matrix is paraHermitian if it is equal to its paraHermitian transpose, i.e. $\tilde{\boldsymbol{H}}(z) = \boldsymbol{H}(z)$. One implication of this is that a paraHermitian polynomial matrix has the same number of positive and negative powers and also that $M = N$. This means therefore, that taking the z-transform of a multi-user transfer function leads to a paraHermitian structure.

Paraunitary Property

The paraunitary property for polynomial matrices is equivalent to the unitary (energy preserving) property for scalar matrices. Applying the meaning of unitary matrices in the polynomial domain, a polynomial matrix is paraunitary if, when applied to its para hermitian transpose, it gives the identity transformation, i.e. $\boldsymbol{H}(z)\tilde{\boldsymbol{H}}(z) = \mathbf{I}_M$ and $\widetilde{\boldsymbol{H}(z)}\boldsymbol{H}(z) = \mathbf{I}_N$ where $\mathbf{I}$ denotes the identity matrix as usual.

Frobenius Norm

The Frobenius norm of the polynomial matrix $\boldsymbol{H}(z)$ is defined to be

$$||\boldsymbol{H}(z)||_F = \sqrt{\text{trace}\{[\boldsymbol{H}(z)\tilde{\boldsymbol{H}}(z)]|_0\}} \tag{3.3}$$

where $[.]_0$ defines the coefficient matrix of z^0 in the polynomial matrix.

3.3 Polynomial Matrix Decomposition Algorithms

The idea of polynomial matrix decomposition arises from a family of iterative space-time covariance algorithms that can factorise a complex multi-user matrix to generate an approximate polynomial matrix eigenvalue decomposition (PEVD) [82].

The iterative algorithm extends the concept of the EVD from narrowband to broadband problems via SBR2 factorisation, which has found application in subband coding [83] and broadband communication [82, 5, 82], channel coding [84, 85] transmit and receive beamforming transmission for a broadband MIMO channel [86, 87], angle of arrival estimation [88, 89], spectral factorisation [90–92], and joint optimal pre-coding and equalisation [93, 94, 6].

This type of decomposition also allows the channel data to be compressed at the preprocessing stage for de-noising [84], decorrelation [82] and optimum subband decompositions [83], or improving the system data rate [5].

In this section, we begin with iterative EVD algorithm and discuss how the state-of-the-art algorithmic developments for PEVD, and then provide a comprehensive discussion of existing PEVD algorithms from the following two perspectives: (i) what algorithms are available for computing the PSVD and (ii) how it works differently for shortening the multi-user polynomial matrix.

3.3.1 Sequential Best Rotation Algorithm (SBR2)

The EVD of a complex Hermitian matrix $\mathbf{H}[n] \in \mathbb{C}^{M\times M}$ is given in 3.4 where $\mathbf{D}$ is a diagonal matrix consisting of eigenvalue and $\mathbf{Q}$ is a unitary matrix such that $\mathbf{Q}^{\mathrm{H}}\mathbf{Q} = \mathbf{Q}\mathbf{Q}^{\mathrm{H}} = \mathbf{I}_M$ and

$$\mathbf{D} = \mathbf{Q}\mathbf{H}[n]\mathbf{Q}^{\mathrm{H}} \tag{3.4}$$

The diagonalisation of $\mathbf{H}[n]$ is achieved for example using an iterative sequence of Jacobi rotation matrices [82]. First, on each iteration, the dominant (largest in magnitude) off-diagonal element of $\mathbf{H}[n]$ is located. Assume that this is the element $[\mathbf{H}]_{ij} = h_{ij}$. Note that the search can be restricted to either the lower triangular or upper triangular part of $\mathbf{H}[n]$ due to its Hermitian symmetry property. The rotation angle θ which is used to annihilate the dominant off-diagonal element h_{ij} is chosen such that

$$\cot 2\theta = \frac{h_{jj} - h_{ii}}{2h_{ij}} \tag{3.5}$$

The corresponding Jacobi rotation matrix $\mathbf{Q}_l$ can then be expressed as

$$\mathbf{Q}_l = \begin{bmatrix} 1 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & & \vdots & & \vdots \\ 0 & \cdots & c_{ii} & \cdots & -s_{ij} & \cdots & 0 \\ \vdots & & 0 & \ddots & 0 & & \vdots \\ 0 & \cdots & s_{ji} & \cdots & c_{jj} & \cdots & 0 \\ \vdots & & 0 & & 0 & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 & \cdots & 1 \end{bmatrix} \tag{3.6}$$

where $c = \cos\theta$ and $s = \sin\theta$. The subscripts represent the corresponding element location. This matrix is unitary by construction. The diagonalising EVD involves the Hermitian using a sequence of such Jacobi rotation matrices. This eventually leads to a similarity transformation of the form

$$\mathbf{D}' = \mathbf{Q}\mathbf{H}[n]\mathbf{Q}^{\mathrm{H}} \tag{3.7}$$

where

$$\mathbf{Q}^{\mathrm{H}} = \mathbf{Q}_1^{\mathrm{H}}, \mathbf{Q}_2^{\mathrm{H}}, \mathbf{Q}_3^{\mathrm{H}} \cdots \mathbf{Q}_L^{\mathrm{H}}$$

The matrices $\mathbf{Q}_l$ denote the successive Jacobi rotation matrices and L is the number of iterations required. Thus, $\mathbf{H}[n]$ is diagonalized by performing a unitary similarity transformation as required.

SBR2 operation for PEVD Algorithm

Analogous to the scalar EVD, the problem under investigation requires factorisation of a paraHermitian polynomial matrix, which can be expressed as

$$\boldsymbol{H}(z) = \sum_{n=-n_{max}}^{n_{max}} z^{-n}\mathbf{H}[n], \tag{3.8}$$

where $\mathbf{H}[n] \in \mathbb{C}^{M\times M}$ and n denotes the polynomial index. The paraHermitian property implies that

$$[h_{ij}[n] = \mathbf{H}[n]]_{ij} = [\mathbf{H}[-n]]^*_{ji} = h_{ji}[n] \qquad \forall j,i. \tag{3.9}$$

The diagonalization of $\boldsymbol{H}(z)$ using SBR2 corresponds to a PEVD as in 3.10, where $\boldsymbol{D}(z)$ is a diagonal polynomial matrix, $\boldsymbol{G}(z)$ is a paraunitary matrix and $\tilde{\boldsymbol{G}}(z)$ is its paraconjugate.

$$\boldsymbol{G}(z)\boldsymbol{H}(z)\tilde{\boldsymbol{G}}(z) = \boldsymbol{D}(z). \tag{3.10}$$

SBR2 extends the Jacobi rotation matrix in the EVD approach to para-Hermitian matrices. First, the dominant (largest in magnitude) off-diagonal element of $\boldsymbol{H}(z)$ is located, i.e., the off-diagonal element whose importance is most significant. This search can either be restricted to the lower triangular or upper triangular part of $\boldsymbol{H}(z)$ due to its para-Hermitian property. A delay matrix, $\boldsymbol{B}(z)$, is achieved, which shift the dominant off-dominant element to the zero position, z^{-0}. An elementary unitary matrix $\mathbf{Q}$ is then designed to drive the off-diagonal elements to zero leading to a new polynomial matrix, $\boldsymbol{H}'(z)$.

Using $\boldsymbol{G}_i(z)$ to denote the product of the elementary unitary matrix $\mathbf{Q_i}$ and the delay matrix $\boldsymbol{B}_i(z)$ for the *ith* iteration, i.e. $\boldsymbol{G}_i(z) = \boldsymbol{Q}_i(z)\boldsymbol{B}_i(z)$, the updated paraHermitian polynomial matrix is given explicitly by

$$\boldsymbol{H}_i'(z) = \boldsymbol{G}_i(z)\boldsymbol{H}_i(z)\tilde{\boldsymbol{G}}_i(z). \tag{3.11}$$

Assuming that the dominant element is found in the j,k location of coefficient matrix for z^{-t}, the delay matrix $\boldsymbol{B}_i(z)$ is defined as the $M \times M$ polynomial identity matrix with the exception of the j,k diagonal element, which is z^t, i.e.

$$\boldsymbol{B}_i(z) = \begin{bmatrix} \boldsymbol{I} & 0 & 0 \\ 0 & z^{-t} & 0 \\ 0 & 0 & \boldsymbol{I} \end{bmatrix} \tag{3.12}$$

The elementary paraunitary transformation process represented by $\boldsymbol{G}_i(z)$ is repeated iteratively until the magnitude of the maximum off-diagonal coefficient becomes approximately zero, to within some target precision. This termination factor is determined by the ratio, denoted ε, of the magnitude of the off-diagonal element to the Frobenius norm of the diagonalized matrix, $||\text{diag}\{\underline{\boldsymbol{D}}(z)\}||_F$, or alternatively to $||\text{diag}\{\underline{\boldsymbol{H}}(z)\}||_F$ see [81, 82], for which a proof of convergence of the algorithm is given.

The resulting paraunitary transformation $\boldsymbol{G}(z)$, is given by 3.10 where

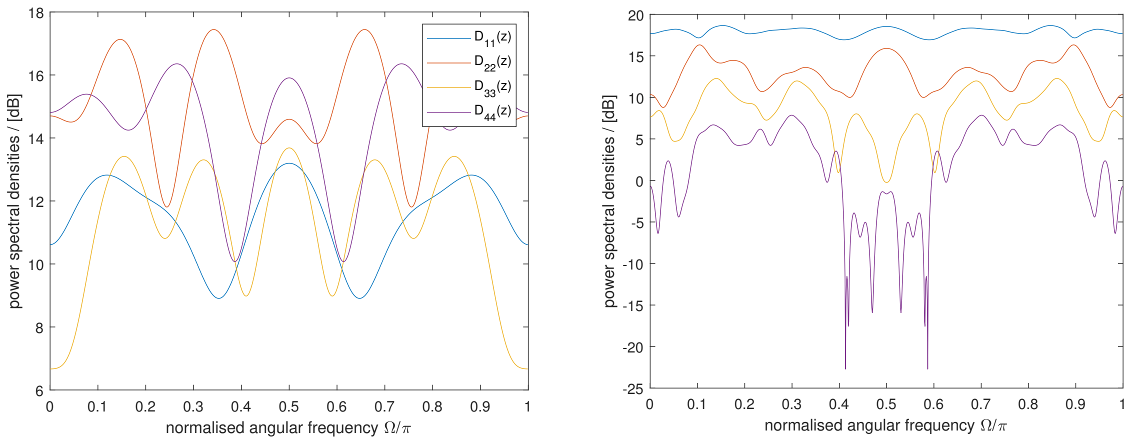
$$\tilde{\boldsymbol{G}}(z) = \tilde{\boldsymbol{G}}_1(z)\tilde{\boldsymbol{G}}_2(z)\tilde{\boldsymbol{G}}_3(z)\cdots\tilde{\boldsymbol{G}}_\beta(z) \tag{3.13}$$

and β represents the unspecified number of iterations. To a good approximation, $\underline{\boldsymbol{D}}(z)$ is a diagonal polynomial matrix represented by

$$\boldsymbol{D}(z) = \text{diag}\{\boldsymbol{D}_1(z)\boldsymbol{D}_2(z)\cdots\boldsymbol{D}_M(z)\}. \tag{3.14}$$

Because of its diagonal structure, the elements of $\boldsymbol{D}(z)$ constitute the (polynomial) eigenvalues of $\boldsymbol{H}(z)$ and they may be ordered such that for all z on the unit circle and $\alpha = 1, 2.....M-1$

$$\boldsymbol{D}_{\alpha+1}(z) \leq \boldsymbol{D}_{\alpha}(z) \tag{3.15}$$



(a) **(b)**

Figure 3.1 Example of spectral allocation **(a)** of the primary multi-user MIMO system **(b)** with order majorisation after diagonalisation.

This ordering of the eigenvalues is referred to as spectral majorisation [95]. An examples of spectral majorisation is shown in Figure 3.1, where the power spectral of the multi-user MIMO problem and the ordered power spectral in the diagonalised matrix is described respectively in Figures 3.1(a) and 3.1(b).

In a multi-user MIMO system context, the power allocation is directly influenced by the spectral properties of the independent input sources. Even if the input sources are not spectrally ordered, the iterative PEVD algorithms are known to impose spectral majorisation on their decomposition. Another essential property of PEVD is the dynamic range or the difference between the highest and lowest power spectral densities (PSDs). The importance of the majorisation on the multi-user MIMO problem is discussed in chapter 5, showing the effectiveness of applying PSVD algorithms with different types of channel information.

3.3.2 Sequential Matrix Diagonalisation Algorithms

The sequential matrix diagonalisation (SMD) is an improved iterative algorithm that diagonalises a paraHermitian matrix to approximate its PEVD. Other versions of SMD include maximum element SMD (ME-SMD) and multiple shift maximum element SMD (MSME-SMD). Unlike PEVD algorithms, the SMD family extends the SBR2 algorithms to eliminate the dominant off-diagonal elements entirely at each iteration, and transfer the squared L_2 norm of its off-diagonal elements onto the main diagonal of the lag-zero coefficient matrix. In addition, SMD require an initialisation step to compute a full EVD of the para-Hermitian matrix $\boldsymbol{H}[0]$, i.e.,

$$\mathbf{D}^{(0)}[0] = \mathbf{Q}^{(0)}(z)\mathbf{H}[0]\tilde{\mathbf{Q}}^{(0)}(z) \tag{3.16}$$

where $\boldsymbol{Q}^{(0)}(z)$ denotes the scalar EVD of $\mathbf{H}[0]$ and each iteration brings a new row and column to $\boldsymbol{D}^{(i)}[0]$, whose energy is then transferred onto the diagonal by a scalar EVD.

This diagonalization of $\boldsymbol{H}(z)$ is achieved using paraunitary matrices $\boldsymbol{Q}^{(i)}(z)$ [41]. At each iteration, the largest (in magnitude) off-diagonal element of $\boldsymbol{H}(z)$ is located. This search can either be restricted to the lower triangular or upper triangular part of $\boldsymbol{H}(z)$ due to its para-Hermitian property. The rotation angle θ which is used to demolish the off-diagonal element of matrix $\boldsymbol{H}(z)$ is chosen such that the SMD parameter selection in the ith iteration can be expressed as

$$\arg\max ||\tilde{d}_k^{(i-1)}[n]||_2 \qquad \forall n \tag{3.17}$$

where the notation $||.||_2$ denote the Frobenius norm, and the vector $\tilde{d}_k^{(i-1)}[n]$ contains all corresponding elements in the k-th column of $\mathbf{D}^{(i)}[n]$.

The corresponding diagonal polynomial matrix can be expressed as

$$\boldsymbol{H}^{(i)}(z) = \boldsymbol{Q}^{(i)}(z)\boldsymbol{H}(z)\tilde{\boldsymbol{Q}}^{(i)}(z) \tag{3.18}$$

where

$$\tilde{\boldsymbol{Q}}^{(i)}(z) = \tilde{\boldsymbol{Q}}^{(1)}(z), \tilde{\boldsymbol{Q}}^{(2)}(z)\cdots \tag{3.19}$$

and the $\boldsymbol{Q}^{(i)}(z)$s denotes the successive paraunitary matrices. To maximise the reduction in off-diagonal energy, the diagonal element selection of the SMD algorithm is the geometric mean.

The convergence of SMD is proven in [41], with a specific stopping criterion. SMD has been shown to diagonalise para hermitian matrices with a lower number of iterations than conventional SBR2 because more energy is transferred from off-diagonal to on-diagonal elements.

As the scalar EVD at each iteration has to be calculated, the unitary matrix no longer has the sparse structure of the elementary transformation but allows a full matrix multiplication at every power position. Despite this, the SMD algorithm produces paraunitary polynomial matrices of a lower order than SBR2. Therefore, SMD enables better performance with lower order paraunitary rotation matrix than SBR2.

3.3.3 Multiple Shift Maximum Element SMD (MSME-SMD) Algorithm

An alternative version of SMD search method considered in the study is termed multiple-shift maximum element SMD (MSME-SMD). The MSME-SMD algorithm [45] is primarily proposed for the purpose of further reducing the search cost in the ME-SMD algorithm. It converges and transfers more energy than a maximum element search. The only difference between them is that the ME-SMD algorithm find a single dominant off-diagonal element at each iteration while the MSME-SMD algorithm [45, 96] imposes a "multiple-shift" strategy to search for more off-diagonal elements and shift them to their power position. One of the fundamental drawbacks of this novel algorithm is the additional time required to apply the paraunitary matrices at each iteration.

3.4 Polynomial Matrix Singular Value Decomposition via SMD Algorithms

The PSVD of a polynomial channel matrix, $\boldsymbol{H}(z) \in \mathbb{C}^{p\times q}$, can be expressed as

$$\boldsymbol{H}(z) = \boldsymbol{U}(z)\boldsymbol{D}(z)\tilde{\boldsymbol{V}}(z) \tag{3.20}$$

where $\boldsymbol{U}(z) \in \mathbb{C}^{q\times p}$, $\tilde{\boldsymbol{V}}(z) \in \mathbb{C}^{q\times q}$ are paraunitary polynomial matrices, such that $\tilde{\boldsymbol{V}}(z)\boldsymbol{V}(z) = \mathbf{I}$ and $\tilde{\boldsymbol{U}}(z)\boldsymbol{U}(z) = \mathbf{I}$, and $\boldsymbol{D}(z) \in \mathbb{R}^{p\times q}$ is a diagonal polynomial matrix.

By applying the SMD algorithm to $\boldsymbol{H}(z)$, the paraunitary structure of $\boldsymbol{U}(z)$ and $\boldsymbol{V}(z)$ are obtained respectively from the para-Hermitian matrix decomposition of $\boldsymbol{H}(z)\tilde{\boldsymbol{H}}(z)$ and $\tilde{\boldsymbol{H}}(z)\boldsymbol{H}(z)$. As discussed in Section 3.3.2, the PSVD of $\boldsymbol{H}(z)$ can be formulated by separately post and pre-multiplying the channel matrix with its para-conjugate, such that

$$\begin{aligned}\boldsymbol{D}(z)\tilde{\boldsymbol{D}}(z) &= \boldsymbol{U}(z)\boldsymbol{H}(z)\underbrace{\tilde{\boldsymbol{V}}(z)\boldsymbol{V}(z)}_{\mathbf{I}}\tilde{\boldsymbol{H}}(z)\tilde{\boldsymbol{U}}(z) \\ &= \boldsymbol{U}(z)\underbrace{\boldsymbol{H}(z)\tilde{\boldsymbol{H}}(z)}_{\boldsymbol{R}_1(z)}\tilde{\boldsymbol{U}}(z)\end{aligned} \tag{3.21}$$

and similarly,

$$\begin{aligned}\tilde{\boldsymbol{D}}(z)\boldsymbol{D}(z) &= \boldsymbol{V}(z)\boldsymbol{H}(z)\underbrace{\tilde{\boldsymbol{U}}(z)\boldsymbol{U}(z)}_{\mathbf{I}}\tilde{\boldsymbol{H}}(z)\tilde{\boldsymbol{V}}(z)\\ &= \boldsymbol{V}(z)\underbrace{\tilde{\boldsymbol{H}}(z)\boldsymbol{H}(z)}_{\boldsymbol{R}_2(z)}\tilde{\boldsymbol{V}}(z)\end{aligned} \tag{3.22}$$

Forming the para-Hermitian input to SMD by post-multiplying the polynomial channel matrix with its para-conjugate, i.e $\boldsymbol{R}_1(z) = \boldsymbol{H}(z)\tilde{\boldsymbol{H}}(z)$ yields the paraunitary polynomial matrix $\boldsymbol{U}(z)$. Performing a similar operation with the paraHermitian input to SMD formed by the premultiplication of the para-conjugate of the polynomial channel matrix and the polynomial channel matrix itself, i.e., $\boldsymbol{R}_2(z) = \tilde{\boldsymbol{H}}(z)\boldsymbol{H}(z)$ produces the paraunitary polynomial matrix $\boldsymbol{V}(z)$ as in (3.22).

Finally, the polynomial diagonal matrix $\boldsymbol{D}(z)$ is calculated from (3.20) such that

$$\boldsymbol{D}(z) = \tilde{\boldsymbol{U}}(z)\boldsymbol{H}(z)\boldsymbol{V}(z) \tag{3.23}$$

Therefore a true PSVD has been performed on $\boldsymbol{H}(z)$ by applying the SMD algorithm for PEVD twice. This decomposition can be summarised as follow;

Algorithm 2 Summary of the PSVD decomposition via SMD Algorithm

Input: Polynomial matrix $\boldsymbol{H}(z) \in \mathbb{C}^{p\times q}$
Specify: the convergence parameters, the truncation parameter, and the maximum number of iterations per column-step of the algorithm (MaxIter).
Set: $\mathbf{U} \Leftarrow \boldsymbol{I}_M$, $\boldsymbol{V} \Leftarrow \boldsymbol{I}_N$, $iter \Leftarrow 0$, $g \Leftarrow 1+\varepsilon$,
while $iter < MaxIter$ and $g > \varepsilon$ **do**
  Find: i, j and n where $i \neq j$ such that $|h_{ij}[n]| \geq |h_{mn}n|$ holds for $m = 1, \cdots, M$, $n = 1, \cdots, N$ such that $m \neq n$ and $\forall n \in \mathbf{Z}$. Set $g \Leftarrow |h_{ij}[n]$
  if $g > \varepsilon$ **then**
    iter $\Leftarrow$ iter +1
    Set: para-Hermitian Matrices $\boldsymbol{R}_1(z) \Leftarrow \tilde{\boldsymbol{H}}(z)\boldsymbol{H}(z)$, and $\boldsymbol{R}_2(z) \Leftarrow \boldsymbol{H}(z)\tilde{\boldsymbol{H}}(z)$
    Calculate the SMD of $\boldsymbol{R}_1(z)$: $\boldsymbol{U}_1\boldsymbol{R}_1(z) = \boldsymbol{D}_1(z)$
    Set $\boldsymbol{R}_1(z) \Leftarrow \tilde{\boldsymbol{D}}_1(z)$ and $\boldsymbol{U} \Leftarrow \boldsymbol{U}_1\tilde{\boldsymbol{U}}$
    Calculate the SMD of $\boldsymbol{R}_2(z)$: $\boldsymbol{V}_1\boldsymbol{R}_2(z) = \boldsymbol{D}_2(z)$
    Set $\boldsymbol{V} \Leftarrow \boldsymbol{V}_1\boldsymbol{V}$ and $\boldsymbol{R}_2(z)(z) \Leftarrow \boldsymbol{D}_2$
    Truncate $\boldsymbol{H}(z)$, $\boldsymbol{V}$ and $\boldsymbol{U}$ according to the appendix.
  end if
end while
Until $\boldsymbol{D}(z) = \boldsymbol{H}(z)$ *converges*.

3.5 Channel Matrix Decomposition via PSVD algorithms

The SVD constitutes one of the fundamental matrix factorisation tools in MIMO signals processing and has found application in numerous multidisciplinary scientific fields, such as in artificial intelligence, social and organisation sciences, distributing computing, natural language processing, philosophy, etc. [37]. The tool helps to perform a principal component analysis and compute the singular components of any arbitrary matrix after centring the original problem around the mean [22, 24].

Many other methods of matrix decomposition and analysis, such as factor analysis are also available [97, 98]. As discussed earlier, SVD computes the pre-fixed receiver solution to a closed-form channel problem and provides one of the most popular tools used for multi-user MIMO systems [99–101]. In principle, SVD diagonalises the complex multi-user problem to obtain the most straightforward single-user problem (i.e., the singular vectors) and their unique values (i.e., the singular values).

This robust method has been exploited in numerous image-processing applications, such as in the calculation of eigenvalues to provide an efficient representation of the channel state information (CSI) [102, 103]. It is also crucial for theoretical developments, such as reference tracking methods for disturbance rejection in dynamical systems [104]. It has been applied in channel reduction [105], and bit-rate maximisation problem of broadband MIMO systems [5]. Computing the SVD for perfect signal reconstruction is still expensive, but there are available algorithms that offer lower computational complexity (see, for example, [106]) as well as near perfect reconstruction algorithms via the EVD [107].

There exist some improved polynomial EVD algorithms for higher-dimensional factorisations using Tensor decomposition in[108]; these have transformed computational multi-linear algebra over the last decade. This study proposed PSVD structures and exploits the best single-value components of a complex multi-user MIMO system. The study extends work on the PSVD framework in [109] to vectored DSM transmission systems, using the sequential matrix diagonalisation (SMD) algorithm but focuses predominantly on its application to DSL access multiplexing (DSLAM) technique.

3.6 Numerical Analysis

This section provides some examples regarding the properties of PSVD algorithm. For the first example, we presents an analytical solution that takes into account the polynomial channel matrix of a non-square multi-user system. The optimal precoder and equaliser of the system is them provided using PSVD algorithm for non-square channel information, making all coefficient associated with polynomal off-diagonal elements equal to zero in order to limit the impact of the noise on the transmission. Example 2 presents a simple application of using the PSVD by SMD algorithm to compute the joint precoding and equalisation filters for DSM designs.

3.6.1 Worked Examples

Example 1:

Consider here MIMO model involving six users using two transmitter and four receiver $(M = 2, N = 4)$ We assumed that the system has only two non-zero coefficients situated beneath the diagonal of the polynomial matrix to eliminate. The polynomial matrix applied to describe the dynamics of the propagation environment can assume the form

$$\boldsymbol{H}(z) = \frac{1}{\sqrt{8}} \begin{bmatrix} z^{-2}+z+1 & -z^{-1}+z^2+z \\ -z^{-3}+1+z^{-1} & z^{-2}+z+1 \\ z^{-2}+z+1 & -z^{-1}+z^2+z \\ -z^{-3}+1+z^{-1} & z^{-2}+z+1 \end{bmatrix}. \tag{3.24}$$

The PSVD of $\boldsymbol{H}(z) \in \mathbb{C}^{4\times 2}$ as discussed in Eqn. (3.23), achieves a spectral factorisation $\boldsymbol{D}(z) = \tilde{\boldsymbol{U}}(z)\boldsymbol{H}(z)\boldsymbol{V}(z)$, where all the optimal power control parameters can be defined as $\boldsymbol{V}(z) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & z \\ -z^{-1} & 1 \end{bmatrix}$, $\boldsymbol{U}(z) = \frac{1}{2} \begin{bmatrix} 1 & z & 1 & z^2 \\ -z^{-1} & 1 & -z^{-2} & 1 \\ 1 & z & -1 & -z^2 \\ -z^{-1} & 1 & z^{-2} & -1 \end{bmatrix}$, and the resulting diagonal transmission components as $\boldsymbol{D}(z) = \begin{bmatrix} z^{-2} & 0 \\ 0 & z+1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$. Here, all coefficient associated with polynomial off-diagonal elements of $\boldsymbol{H}(z)$ are equal to zero.

To measure the accurate decomposition, the inversion decomposition is formulated as $\boldsymbol{H}(z) = \tilde{\boldsymbol{U}}(z)\boldsymbol{D}(z)\boldsymbol{V}(z)$ and the relative error is computed as:

$$E_{rel} = ||\boldsymbol{H}(z) - \boldsymbol{U}(z)\boldsymbol{D}(z)\tilde{\boldsymbol{V}}(z)||_F^2 / ||\boldsymbol{H}(z)||_F^2 \tag{3.25}$$

This measure was found to equal to zero for the example above. These results are quoted to the standard accuracy given by the PolyX toolbox. Of course, it is worth noting the decomposition can be compared with the performance of other existing decomposition methods.

Example 2:

The model considered here is a similar polynomial channel problem to that applied to the Poly-X analytical solution in Example 1, but instead a polynomial matrix decomposition algorithm is used. The number of the transmitter and receiver modem are N = 2 and M = 4, respectively. This leads to a polynomial matrix $\boldsymbol{H}(z) \in \mathbb{C}^{4\times2\times6}$ with independent polynomial channel information $\boldsymbol{H}_j(z)$, $j = -3, -2, \cdots, 1, 2$ are represented graphically as in Figure3.2. One can then analyse each of the user information as

$$\boldsymbol{H}_{-3}(z) = \frac{1}{\sqrt{8}}\begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad \boldsymbol{H}_{-2}(z) = \frac{1}{\sqrt{8}}\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad \boldsymbol{H}_{-1}(z) = \frac{1}{\sqrt{8}}\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$\boldsymbol{H}_{0}(z) = \frac{1}{\sqrt{8}}\begin{bmatrix} 0 & -1 \\ 1 & 0 \\ 0 & -1 \\ 1 & 0 \end{bmatrix}, \quad \boldsymbol{H}_{1}(z) = \frac{1}{\sqrt{8}}\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \boldsymbol{H}_{2}(z) = \frac{1}{\sqrt{8}}\begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 0 & 0 \\ -1 & 0 \end{bmatrix}, \tag{3.26}$$

such that the corresponding convolutional mixing matrices for the model can be represented as showing in Figure. The stem plots in Figure 3.2 represent the magnitude of the series of coefficients for each of the polynomial channel information $\boldsymbol{H}_j(z)$. The position of each stem plot corresponds to the index of polynomial channel information $\boldsymbol{H}_j(z)$.

The paraHermitian matrices $\boldsymbol{R}_1(z) = \boldsymbol{H}(z)\tilde{\boldsymbol{H}}(z)$ and $\boldsymbol{R}_2(z) = \tilde{\boldsymbol{H}}(z)\boldsymbol{H}(z)$, which provide a symmetric spectral mask with the order is usually higher than $\boldsymbol{H}(z)$. The stem plots show that the two paraHermitian matrices which produce the paraunitary matrices after decomposition indeed behave quite differently. The $\boldsymbol{R}_2(z)$, which produces $\boldsymbol{V}(z)$ paraunitary matrix has the channel information with its slight reduce cost, despite retaining the inherent diagonal property. Of the $\boldsymbol{R}_1(z)$ that results to $\boldsymbol{U}(z)$ utilises full channel information with with consederable the same off-diagonal information as $\boldsymbol{R}_2(z)$. The use of SMD algorithm to yield $\boldsymbol{V}(z)$ and $\boldsymbol{U}(z)$ matrices results in spectrally majorisation of $\boldsymbol{D}_1(z)$ and $\boldsymbol{D}_2(z)$. Spectral majorisation is similar to channel decomposition [95].

The resulting paraunitary matrices $\boldsymbol{U}(z)$ and $\boldsymbol{V}(z)$ are presented in Figure 3.5.

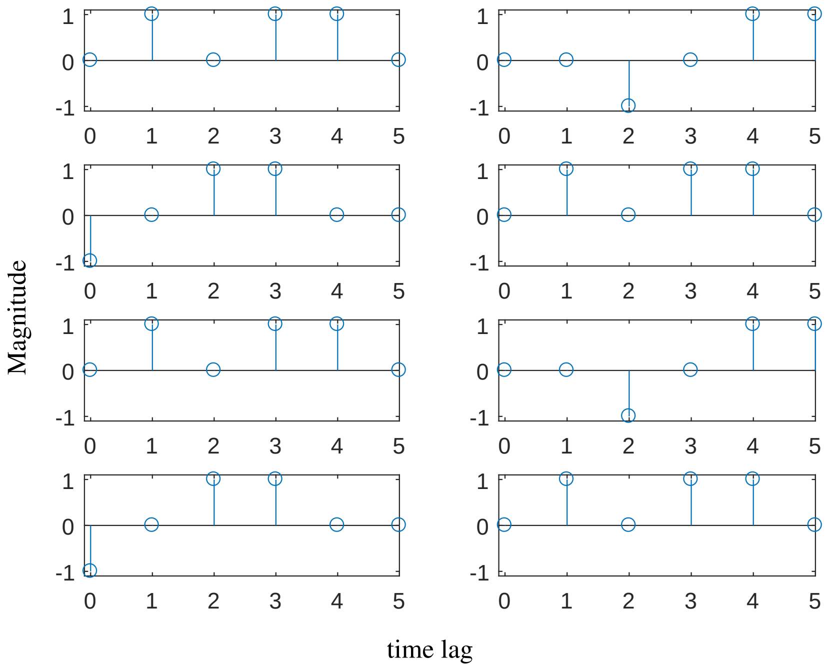


Figure 3.2 The stem plot representation of the polynomial channel model matrix for the six users.

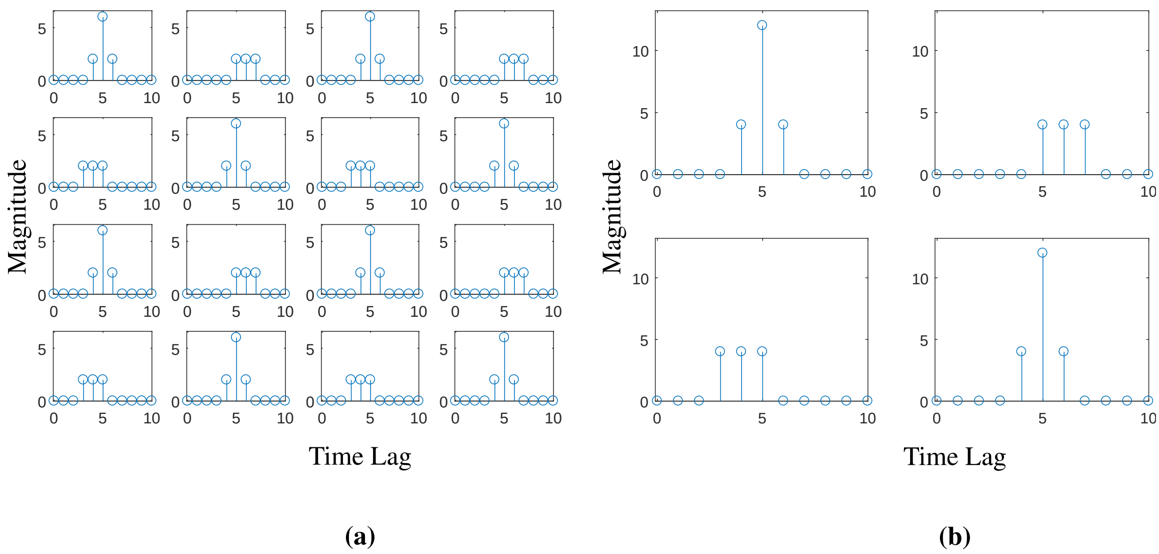
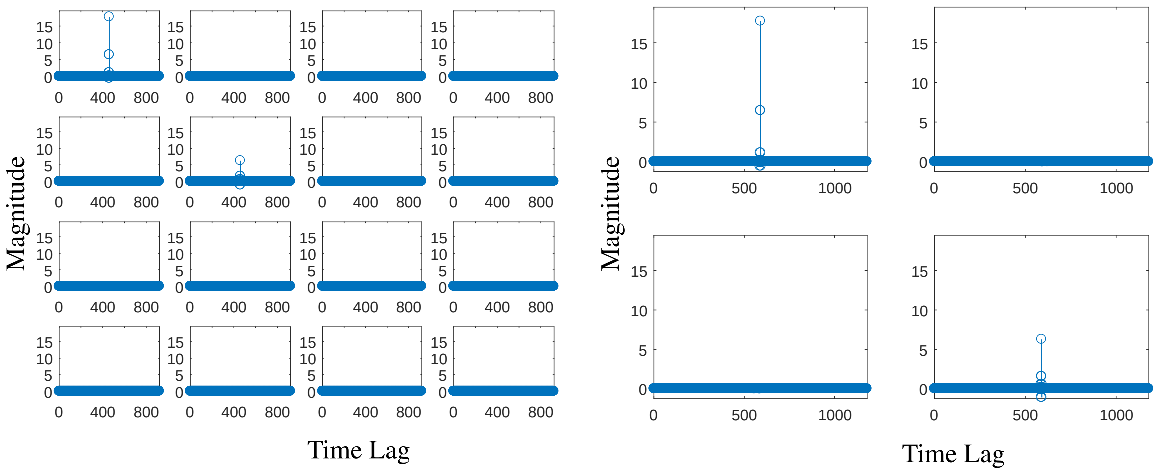


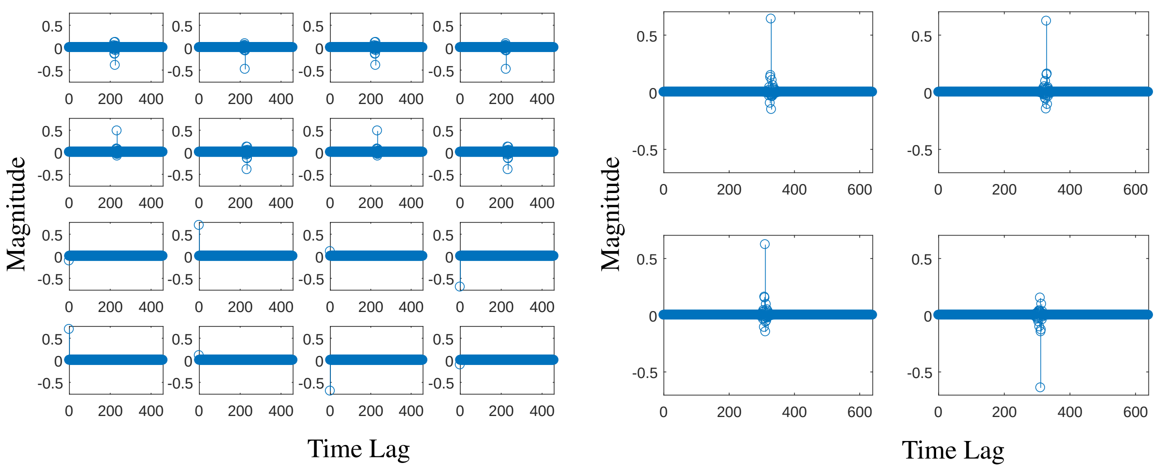
Figure 3.3 Stem plot representation of the Para-Hermitian matrices obtained **(a)** for $\boldsymbol{R}_1(z)$ and **(b)** for $\boldsymbol{R}_2(z)$ against the time lag.

Aiming at determining the generalisation decomposition strength of the proposed SMD model versus the common PolyX model, Figure 3.6 shows the singular values versus the normalized frequency of each diagonal matrix obtained by the SMD algorithm and compared with that of PolyX values. it can be seen that the PolyX-obtained singular values that have are



(a) **(b)**

Figure 3.4 The diagonal polynomial matrix **(a)** for $\mathbf{R}_1(z)$ and **(b)** for $\mathbf{R}_2(z)$.



(a) **(b)**

Figure 3.5 The paraunitary stem plot **(a)** for $\boldsymbol{U}(z)$ and for **(b)** $\boldsymbol{V}(z)$.

analytic solution and SMD provides spectrally majorised singular values that in the context of power allocation, spectral majorisation, i.e. a strict ordering of the subchannels is desirable.

3.7 Chapter Summary

This chapter first introduces a brief overview of polynomial matrix decomposition techniques and its properties. Next the different types of PMD algorithms including PEVD, SBR2 and PSVD are discussed. The PEVD was introduces followed by established pre-post based

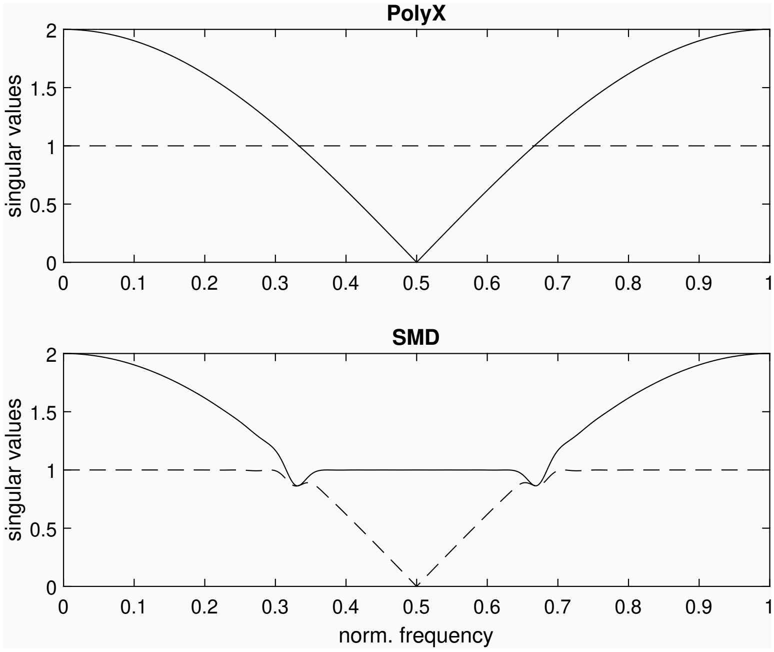


Figure 3.6 singular values against normalised frequency for both Polyx and SMD algorithm

PSVD algorithms using two different SMD decomposition. Following that of analytical-based Polyx solution utilising the PolyX decomposition method, with was later compared with the SMD approach.

Different from analytical solution, the SMD-based solution of PSVD introduces a new class of decomposition, termed spectral majorisation, that produces ordering of the subchannels. As a result, more energy is transferred onto the main diagonal per iteration, leading to a effective distribution of power spectral. However, since the ordering is form the highest magnitude, the lower power lever can be discarded for allocation, hereby significantly reducing the process complexity. Without any implementation tricks, the specral majorisation is significantly more desirable in terms of independent channel control.

Chapter 4

Novel Framework for DSM Techniques

Research in DSM involves developing a novel technique to extract the actual channel knowledge that in a compact, less abstract, but understandable form which is useful for perfect reconstruction. As discussed in Section 2.4.3, DSM related problems can be defined as joint transmit power allocation issues based on error minimisation objective. In this chapter, DSM refers to the overall process of formulating joint optimal transmitter and receiver matrices for a group of users, i.e. extracting valid and important information from complex channel model.

Singular value decomposition (SVD), as a generalisation of factor analysis, will be employed to define the optimal channel parameters in order to minimise the occurrence of transmission interference. However, the definition of these parameters for a multi-user MIMO system in the presence of transmission errors usually involves expressing the finite impulse response (FIR) channel model (polynomial matrix) as a distinct linear combination. As a result, the exiting SVD methods do not reliably reveal the vital structure of overdetermined channel problems.

This chapter introduces and extends the polynomial singular value decomposition algorithm to take into account the relevance of evaluating the independent component parameters inherent in DSM optimisation problems. The objective of doing this is to achieve vectored transmission through a PSVD framework in order to improve the conventional method and to mitigate the presence of interference in DSL systems.

4.1 Introduction

There are different ways of extracting valid information and enhancing the performance of multi-user complex problems. In this chapter, we focus on three strategies: pre-processing procedures, joint power allocation, and even error minimisation mechanisms. The most straightforward approach to describing this problem is as a vectoring procedure, incorporating a joint pre-processing function to define the joint post-equalisation process. These pre-processing functions provide useful knowledge to reveal the important parameters for joint power allocation. These parameters can be obtained through PEVD algorithms.

For simplicity, this study will focus on polynomial channel representation through the space-time covariance matrix and explore the geometric mean of the channel model instead of the arithmetic mean. The exploitation of such a polynomial description enables the system variance to be controlled so that the stability problem associated with the multi-user system (which is more significant than that for single-user cases) can be captured with the PSVD algorithms.

This kind of representation is termed convolutive mixing matrix. Here the spectrum analysis can be derived by separating the impaired error function from the actual information [105]. The independent component is used for the power allocation method rather than the original signal. The method being investigated therefore enhances the data-rate performance without affecting the spectrum management efficiency of the model reconstruction process.

It is shown in [22] that by compromising some of the general ability of the overall channel optimisation problem, the reduced polynomial based model can be employed to improve the reconstruction process over other available methods. Finally, we explore a reduced-model mitigation strategy to control the reconstruction of the original channel model from the resulting independent components. The technique referred to as joint precoding, and equalisation is an extension of the mitigation technique in complex multi-input multiple-output (MIMO) systems [6].

Reduced post-processing that decodes the pairwise linear combination in the actual system is incorporated into the model reconstruction. By making use of this reduced post-processing, the mitigation model emphasises the single-user error detection process and has slightly better performance than the joint-user mitigation model.

In principle, the DSM process is iterative having multiple steps with, as stated above, the interference mitigation process being one of them. Before the actual mitigation process, problem selection, pre-processing and transmit power allocation can be done, while afterwards channel equalisation is required to achieve the transmission gains. In general, the entire chain of DSM consists of the following three steps as depicted in Figure 4.1

This chapter will analysis a DSM system through the application of a reduced polynomial model. This reduced model allows obtaining accurate estimations to be obtained even when a complete CSI information is not available. The model reduction situation typically occurs in the early stages of the transmission. The aim is to achieve low complexity, reduced channel information, and to predict transmission costs with significant precision.

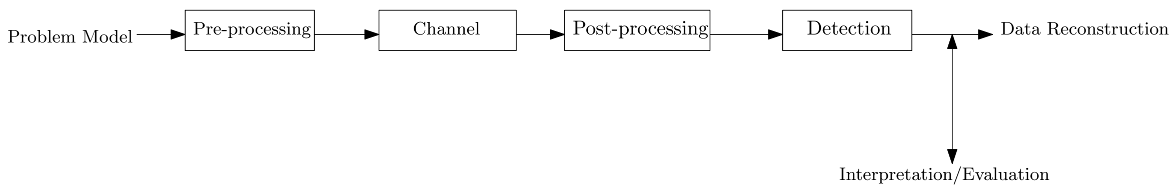


Figure 4.1 The DSM process

4.2 Channel Model and Problem Formulation

Before any processing, the first step is to formulate the multi-user DSL problem by identifying the goal of the perfect reconstruction process from the input user's perspectives. Note that the channel model presented in this section is similar to the DSL problem described for a vectored transmission scheme, where the channel is considered as a MIMO system.

We consider linear transmission over a bundle of K DSL lines/users, consisting of twisted copper pairs, which may induce severe cross-talk between adjacent channels. The transmission also assumes the application of a synchronous discrete multi-tone (DMT) modem with M frequency-spaced tones. The set of users can be denoted by $k = 1,...,K$, the set of transmit tones per user by $j = 1,...,M$ and the set of receive tones per user by $i = 1,...,N$.

In general, a DSL system can be further modelled as a matrix, where the mixed input signals are grouped as the row vectors and their output counterparts as the columns. Let the kth's user channel matrix be $\mathbf{H}[n]$; the channel matrix with N transmitting tones having M receiving tones could then be represented as

$$\mathbf{H}[n] = \begin{bmatrix} \mathbf{h}_{1,1}[n] & \mathbf{h}_{1,2}[n] & \cdots & \mathbf{h}_{1,j}[n] & \cdots & \mathbf{h}_{1,N}[n] \\ \mathbf{h}_{2,1}[n] & \mathbf{h}_{2,2}[n] & \cdots & \mathbf{h}_{2,j}[n] & \cdots & \mathbf{h}_{2,N}[n] \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{h}_{M,1}[n] & \mathbf{h}_{M,j}[n] & \cdots & \mathbf{h}_{M,j}[n] & \cdots & \mathbf{h}_{M,N}[n] \end{bmatrix} \in \mathbb{R}^{M \times N} \tag{4.1}$$

In general, the problem can be represented as a matrix of frequency selective channels, each with a finite impulse response (FIR). This structured representation is the most straightforward method to use for the problem of interest and makes explicit the goal of the perfect reconstruction process from an input user's perspective. As a result, we adopt an FIR model that can be represented in polynomial matrix form and incorporates the important feature of convolutive mixing into the DSM process. The each user polynomial channel matrix $\boldsymbol{H}_k(z)$ can be written as

$$\boldsymbol{H}_k(z) = \sum_{n=0}^{L_c-1} \mathbf{H}[n] z^{-n} \tag{4.2}$$

where $(L_c - 1)$ is the maximum support length of the linear-time invariant FIR filter $\boldsymbol{C}(z)$.

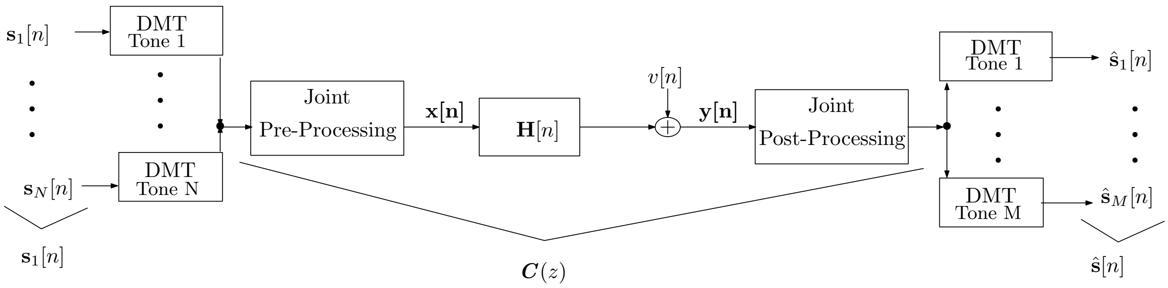


Figure 4.2 General Joint Pre-Post Processing System

Figure 4.2 shows a general DMT transmission system with each user having a channel matrix $\mathbf{H}[n] \in \mathbb{R}^{M\times N}$, input signals $\mathbf{s}[n]$, additive white Gaussian noise (AWGN) $\mathbf{v}[n]$ and received signals $\hat{\mathbf{s}}[n]$. It operates with joint pre-and post-processing operations performed respectively, at transmitter and receiver. The joint pre-processing operator multiplexes N input signals of $\mathbf{s}[n] = \{\mathbf{s}_1,...,\mathbf{s}_N\}^{\dagger}$ onto the transmit tones $\mathbf{x}[n]$. In the receiver, a joint post-processing operator attempts to extracts M transmit signals from the received tones $\mathbf{y}[n]$ to form the output signal $\hat{\mathbf{s}}[n] = \{\hat{\mathbf{s}}_1,...,\hat{\mathbf{s}}_M\}^{\dagger}$.

In most cases, DSL transmissions usually occur in the presence of a noise signal $\mathbf{v}[n] = \{\mathbf{v_1},...,\mathbf{v_k}\}^{\dagger}$. We further assume that the transmit tone $\mathbf{x}[n]$ and receive tone $\mathbf{y}[n]$ are emerged and multiplexed into K transmission lines. One can define the K-user multiplexing transmit and receive symbols at discrete time instance n as

$$\mathbf{x}[n] := \begin{bmatrix} \mathbf{x}_1[nK] \\ \vdots \\ \mathbf{x}_N[nK] \\ \mathbf{x}_1[nK+1] \\ \vdots \\ \mathbf{x}_N[nK+1] \\ \vdots \\ \mathbf{x}_1[nK+K-1] \\ \vdots \\ \mathbf{x}_N[nK+K-1] \end{bmatrix}, \quad \mathbf{y}[n] = \begin{bmatrix} \mathbf{y}_1[nK] \\ \vdots \\ \mathbf{y}_M[nK] \\ \mathbf{y}_1[nK+1] \\ \vdots \\ \mathbf{y}_M[nK+1] \\ \vdots \\ \mathbf{y}_1[nK+K-1] \\ \vdots \\ \mathbf{y}_M[nK+K-1] \end{bmatrix}. \tag{4.3}$$

The resulting multi-user linear transfer process of the system can then be formulated as

$$\hat{\boldsymbol{S}}(z) = \boldsymbol{C}(z)\boldsymbol{S}(z) \tag{4.4}$$

whereby $\hat{\mathbf{s}}[n] \circ\!\!-\!\!\bullet \hat{\boldsymbol{S}}(z)$, $\mathbf{s}[n] \circ\!\!-\!\!\bullet \boldsymbol{S}(z)$ and the overall transfer transfer matrix $\boldsymbol{C}(z)$ takes a block pseudo-circulant form

$$\boldsymbol{C}(z) = \begin{bmatrix} \boldsymbol{C}_0(z) & z^{-1}\boldsymbol{C}_{K-1}(z) & \cdots & z^{-1}\boldsymbol{C}_1(z) \\ \boldsymbol{C}_1(z) & \boldsymbol{C}_0(z) & \cdots & z^{-1}\boldsymbol{C}_2(z) \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{C}_{K-1}(z) & \boldsymbol{C}_{K-2}(z) & \cdots & \boldsymbol{C}_0(z) \end{bmatrix}. \tag{4.5}$$

The matrices $\boldsymbol{C}_k(z)$, $i = 1, \cdots, K-1$, are the K polyphase components of $\boldsymbol{C}(z)$ matrix such that

$$\boldsymbol{C}(z) = \sum_{k=0}^{K-1} \boldsymbol{C}_k(z^K) z^{-k}. \tag{4.6}$$

Two important questions arise from the general channel model above. The first is how best to construct a joint precoding model. The second is how to use his pre-processing function to derive the corresponding equalisation functions. As a result, several joint precoding and equalisation schemes are revisited. For more information, the reader is referred to [78] and [110].

4.3 Precoding and Equalisation for a Single-User DSL Transmission

In the general case, a structured FIR problem is not directly suitable for perfect reconstruction. It may contain some uncontrollable noise, irregularities, inconsistencies and crosstalk and so on. Therefore, the transmission requires a model reduction technique that minimises the interference by choosing some pre-processing procedures to benefit from crosstalk and other irregularities (see, e.g., using a post-compensation method for problem selecting and re-ordering the system complexities and eventually, estimating the channel uncertainties before transmission).

In the DSM literature, some model reduction techniques proposed are developed through precoding and equalisation schemes. Therefore, one can focus on joint precoding and equalisation channel representation as a filtering scheme for a single-user DMT transmission. The description is similar to the proposed approach is discussed in [107] by Scaglione et al.

As discussed in Section 2, we consider transmitting redundancies in the form of cyclic-prefix intervals to mitigate both inter-carrier interference (ICI) and inter-symbol interference (ISI) caused by the channel sparsity. The approach also utilises joint optimal precoding and equalisation filtering schemes to isolate the effect of the crosstalk and include the background noise. The optimisation criteria are minimum mean square error (MMSE) function under per-user power constraint and maximum achievable data rate.

The discrete-time DSL transmission is illustrated in Figure 4.3. The channel model, in this case, is for a single-user and corresponds to the model presented in Section 4.2 with K=1.

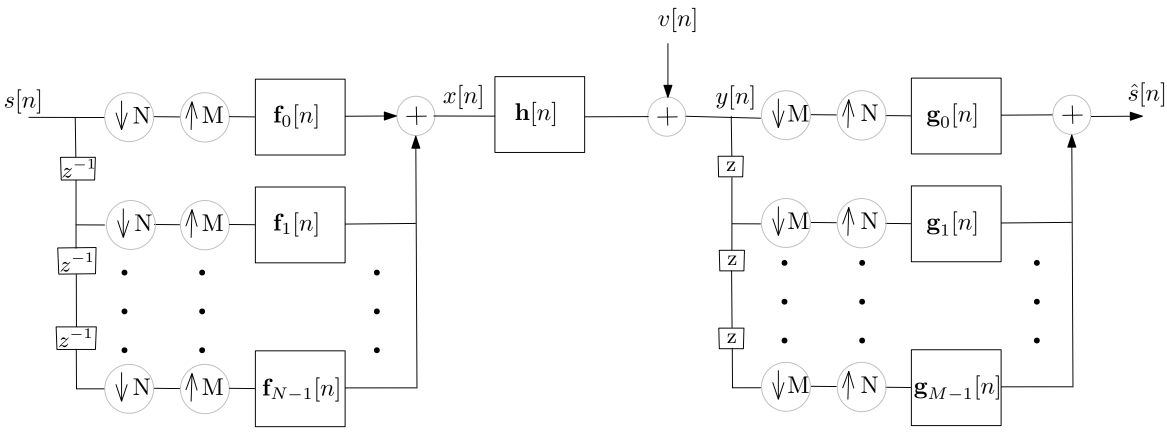


Figure 4.3 Precoding and equalisation for a single-user DSL transmission

The input signal $\mathbf{s}[n]$ is converted into a sequence of blocks of size N. A guard interval is inserted to mitigate both ISI and ICI through the upsamplers by M where $M \geq N$. Thus, the input blocks of size N are mapped into blocks of size M using the precoder $\mathbf{f}[n]$. The output of $\mathbf{f}[n]$ is the transmitted through the frequency selective channel with impulse response $\mathbf{h}[n]$. During the transmission, some distortions such as background noise $v[n]$ are inevitably introduced into the channel. As a result, the received blocks of size M are mapped back to a sequence of a block of size N through the equaliser $\mathbf{g}[n]$. The input and output signal vector can be defined as

$$\mathbf{s}[n] := \begin{bmatrix} \mathbf{s}[nN] \\ \mathbf{s}[nN+1] \\ \vdots \\ \mathbf{s}[nN+N-1] \end{bmatrix}, \qquad \hat{\mathbf{s}}[n] = \begin{bmatrix} \hat{\mathbf{s}}[nN] \\ \hat{\mathbf{s}}[nN+1] \\ \vdots \\ \hat{\mathbf{s}}[nN+N-1] \end{bmatrix}.$$

We can also denote the noise vector of the transmission and the input vector of the equaliser respectively as

$$\mathbf{v}[n] := \begin{bmatrix} \mathbf{v}[nM] \\ \mathbf{v}[nM+1] \\ \vdots \\ \mathbf{v}[nM+M-1] \end{bmatrix}, \qquad \mathbf{y}[n] = \begin{bmatrix} \mathbf{y}[nM] \\ \mathbf{y}[nM+1] \\ \vdots \\ \mathbf{y}[nM+M-1] \end{bmatrix},$$

It is shown in [6] that the relationship between input and output vectors of precoder, equaliser and channel can be written as

$$\mathbf{x}[n] = \sum_{i=-\infty}^{\infty} \mathbf{F}_i \mathbf{s}[n-i] \tag{4.7}$$

$$\hat{\mathbf{s}}[n] = \sum_{i=-\infty}^{\infty} \mathbf{G}_j \mathbf{y}[n-j] \tag{4.8}$$

$$\mathbf{y}[n] = \sum_{i=-\infty}^{\infty} \mathbf{H}_l \mathbf{s}[n-l] + \mathbf{v}[n] \tag{4.9}$$

where

$$\mathbf{F}_i = \begin{bmatrix} \mathbf{f}_0[iM] & \mathbf{f}_1[iM] & \cdots & \mathbf{f}_{N-1}[iM] \\ \mathbf{f}_0[iM+1] & \mathbf{f}_1[iM+1] & \cdots & \mathbf{f}_{N-1}[iM+1] \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{f}_0[iM+M-1] & \mathbf{f}_1[iM+M-1] & \cdots & \mathbf{f}_{N-1}[iM+M-1] \end{bmatrix} \tag{4.10}$$

$$\mathbf{G}_j = \begin{bmatrix} \mathbf{g}_0[jN] & \mathbf{g}_1[jN] & \cdots & \mathbf{g}_{M-1}[jN] \\ \mathbf{g}_0[jN+1] & \mathbf{g}_1[jN+1] & \cdots & \mathbf{g}_{M-1}[jN+1] \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{g}_0[jN+N-1] & \mathbf{g}_1[jN+N-1] & \cdots & \mathbf{g}_{M-1}[jN+N-1] \end{bmatrix} \tag{4.11}$$

$$\mathbf{H}_l = \begin{bmatrix} \mathbf{h}[lM] & \cdots & \mathbf{h}[lM-M+1] \\ \vdots & \ddots & \vdots \\ \mathbf{h}[lM+M-1] & \cdots & \mathbf{h}[lM] \end{bmatrix}. \tag{4.12}$$

The output vectors $\hat{\mathbf{s}}[n]$ of the system can be expressed as

$$\hat{\mathbf{s}}[n] = \sum_{j,l,i=-\infty}^{\infty} \mathbf{G}_j \mathbf{H}_l \mathbf{F}_i \mathbf{s}[n-l-i-j] + \sum_{j=-\infty}^{\infty} \mathbf{G}_j \mathbf{v}[n-j] \tag{4.13}$$

With the following assumptions

1. The channel is of order L with independent transmit vector $\mathbf{h}[n] = \{\mathbf{h}[0], \cdots, \mathbf{h}[L-1]\}$; $(\mathbf{h}[0], \cdots, \mathbf{h}[L-1] \neq 0)$
2. The parameters M, N, and L are chosen so that $M = N + L$.
3. The precoder functions $\{\mathbf{f}_i[n]\}_{i=0}^{N-1}$ are casual ($\mathbf{f}_i[n] = 0$ for $i < 0$) and These functions are selected do that the rank $(\mathbf{F}_0) = N$.

The pseudo-circulant matrix $\boldsymbol{C}(z)$ in (4.5) can now be simplified to

$$\boldsymbol{C}(z) = \boldsymbol{C}_0 \delta[l] + \boldsymbol{C}_1 \delta[l-1] \tag{4.14}$$

where $\delta[.]$ denotes the usual delta function and

$$\boldsymbol{C}_0 = \begin{bmatrix} \mathbf{c}[0] & 0 & 0 & \cdots & 0 \\ \vdots & \mathbf{c}[0] & 0 & \cdots & 0 \\ \mathbf{c}[L] & \cdots & \ddots & \cdots & \vdots \\ \vdots & \ddots & \cdots & \ddots & 0 \\ 0 & \cdots & \mathbf{c}[L] & \cdots & \mathbf{c}[0] \end{bmatrix} \tag{4.15}$$

$$\boldsymbol{C}_1 = \begin{bmatrix} 0 & \cdots & \mathbf{c}[L] & \cdots & \mathbf{c}[1] \\ \vdots & \ddots & 0 & \ddots & \vdots \\ 0 & \cdots & \ddots & \cdots & \mathbf{c}[L] \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & \cdots & 0 & \cdots & 0 \end{bmatrix}. \tag{4.16}$$

The precoding and equalisation functions can be modelled as

$$\boldsymbol{F}_i = \boldsymbol{F}_0 \delta[i] \tag{4.17}$$

$$\boldsymbol{G}_j = \boldsymbol{G}_0 \delta[j] \tag{4.18}$$

Comparing with the joint precoding and equalisation channel model in Figure 4.2, one can see that the system being considered here is a specific case of the pseudo-circulant model with $K = 1$ and the transfer matrix $\boldsymbol{C}(z)$ is expressed as $\boldsymbol{C}(z) = \boldsymbol{C}_0 + \boldsymbol{C}_1 z^{-1}$. One can rewrite (4.13) as

$$\hat{\mathbf{s}}[n] = \boldsymbol{G}_0 \boldsymbol{H} \boldsymbol{F}_0 \mathbf{s}[n] + \boldsymbol{G}_1 \boldsymbol{H} \boldsymbol{F}_1 \mathbf{s}[n-1] + \boldsymbol{G}_0 \mathbf{v}[n] \tag{4.19}$$

From the above equation (4.19), the term $\boldsymbol{G}_1 \boldsymbol{H}_1 \boldsymbol{F}_1 \mathbf{s}[n-1]$ represents the channel interference (both ISI and ICI) and therefore it is a requirement to have $\boldsymbol{G}_1 \boldsymbol{H} \boldsymbol{F}_1 = 0$. By considering the structure of $\boldsymbol{H}$ matrix in Eqn (4.12), there are two available approaches in the literature to satisfy such a condition.

The first approach is to set the last L components in the precoding function to be zero so that $\boldsymbol{F}_i = (\boldsymbol{F}_0 \quad 0)^T$, where $\boldsymbol{F}_1 = 0$ is an $L \times N$ block of zeros and $\boldsymbol{F}_0$ is an $N \times N$ matrix. This approach is termed the *trailing zero (TZ)* approach. The second involves setting the first receive equalisation function to be zero so that $\boldsymbol{G}_j = (0 \quad \boldsymbol{G}_0)$, where $\boldsymbol{G}_1 = 0$ is an $N \times L$ block of zeros and $\boldsymbol{G}_0$ is an $N \times N$ matrix. This approach is termed the *leading zero (LZ)* approach. For more information about these approaches, the reader is referred to [111] and [78].

With either or both of these approaches, channel model interference can be mitigated, and equation (4.19) can then be expressed as

$$\hat{\mathbf{s}}[n] = \boldsymbol{G}_0 \boldsymbol{H} \boldsymbol{F}_0 \mathbf{s}[n] + \boldsymbol{G}_0 \mathbf{v}[n] \tag{4.20}$$

where the effective precoder $\boldsymbol{F} = \boldsymbol{F}_0$ and the effective equaliser $\boldsymbol{G} = \mathbf{G}_0$. In the TZ case, $\boldsymbol{H}$ is an $M \times N$ matrix expressed as

$$\boldsymbol{H} = \begin{bmatrix} \mathbf{h}[0] & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ \mathbf{h}[L] & \ddots & \ddots & 0 \\ 0 & \ddots & \ddots & \mathbf{h}[0] \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \mathbf{h}[L] \end{bmatrix} \tag{4.21}$$

and in the LZ case, $\boldsymbol{H}$ is an $N \times M$ matrix which can be expressed as

$$\boldsymbol{H} = \begin{bmatrix} \mathbf{h}[L] & \cdots & \mathbf{h}[0] & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \mathbf{h}[L] & \cdots & \mathbf{h}[0] \end{bmatrix}. \tag{4.22}$$

After minimising the interference (ICI, ISI or crosstalk) using the TZ or LZ approach, the output of the equaliser is further optimised under some criterion, such as MMSE or DFE. Here, the input signal $\mathbf{s}[n]$ and the noise $\mathbf{v}[n]$ are assumed to be mutually uncorrelated containing known covariance matrices $\mathbf{R}_{ss}$ and $\mathbf{R}_{vv}$ respectively. The optimum precoder under the mean square error (MSE) criterion is estimated to maximise the data-rate at the equaliser output subject to the zero-forcing (ZF) constraint $\boldsymbol{GHF} = \mathbf{I}$. The optimal equaliser is derived as a function of the precoder matrix subject to the condition of maximising the signal-to-noise ratio (SNR). The precoder matrix is then calculated from the ZF constraint.

Based on the following eigendecomposition

$$\begin{aligned}\mathbf{R}_{ss} &= \boldsymbol{U}(z)\Delta\hat{\boldsymbol{U}}(z)\\ \hat{\boldsymbol{C}}\mathbf{R}_{vv}^{-1}\boldsymbol{C} &= \begin{cases}\boldsymbol{V}(z)\Lambda\hat{\boldsymbol{V}}(z) & for \quad TZ \quad case\\ (\boldsymbol{V}(z),\boldsymbol{V}_n(z)\begin{pmatrix}\Lambda & 0\\ 0 & 0\end{pmatrix}(\hat{\boldsymbol{V}}(z),\hat{\boldsymbol{V}}_n(z)) & for \quad LZ \quad case\end{cases}\end{aligned} \tag{4.23}$$

where $\boldsymbol{U},\boldsymbol{V},\boldsymbol{V}_n$ are unitary matrices and Δ,Λ are diagonal matrices, the optimal precoder and equaliser under the MSE criterion are given by [107];

$$\boldsymbol{F}_{opt} = \frac{\sqrt{W}}{\sigma_v}\Lambda^{-(1/2)} \tag{4.24}$$

$$\boldsymbol{G}_{opt} = \sigma_v\sqrt{W}\Lambda^{-(1/2)}\hat{\mathbf{V}}\hat{\boldsymbol{H}}\mathbf{R}_{vv}^{-1} \tag{4.25}$$

where W is defined as the spatial gain which determines the transmit power, and σ_v^2 is the variance of the noise. If the input signal is considered white with the matrix $\mathbf{R}_{ss} = \sigma_s^2\mathbf{I}$ and the transmit power assumed to be P_0, W is given by [105]

$$W = \frac{P_0\sigma_v}{\sigma_s^2\sum_i\lambda_{ii}^{-1}} \tag{4.26}$$

Under the minimum MSE (MMSE) criterion, the optimal equaliser pair is designed to minimise the average error of the system subject to transmit power constraint. The optimal equaliser matrix is derived from the Wiener solution and the precoder is derived by a water-filling algorithm that finds the best λ_{ii}. The optimal pair of precoder and equaliser is given by [105]

$$\boldsymbol{F}_{opt} = \boldsymbol{V}\Phi \tag{4.27}$$

$$\boldsymbol{G}_{opt} = \mathbf{R}_{ss}\hat{\boldsymbol{F}}_{opt}\hat{\boldsymbol{H}}(z)\Big(\mathbf{R}_{vv}+\boldsymbol{H}(z)\boldsymbol{F}_{opt}\mathbf{R}_{ss}\hat{\boldsymbol{F}}_{opt}\hat{\boldsymbol{H}}(z)\Big)^{-1} \tag{4.28}$$

where Φ is the optimal transmit power containing diagonal elements obtained through a water-filling algorithm

$$|\Phi_{ii}|^2 = \max\left(\frac{P_0+\sum_{j=1}^{M}\lambda_{jj}^{-1}}{\sum_{j=1}^{M}(\delta_{jj}/\lambda_{jj})^{1/2}}\frac{1}{\sqrt{\lambda_{ii}\delta_{ii}}}-\frac{1}{\lambda_{ii}\delta_{ii}}, \quad 0\right). \tag{4.29}$$

For the maximum data-rate criterion [105], the precoder and equaliser pair is designed to minimise the MSE between the input and the output signals. The optimal data-rate precoder can be written as in (4.27). The power allocation achieves the minimum MSE according to a classical water-filling algorithm with a single water level [40], which results in the main diagonal elements of Φ as

$$|\Phi_{ii}|^2 = \max\left(\frac{P_0 + \sum_{j=1}^{M} \lambda_{jj}^{-1}}{M\delta_{ii}} - \frac{1}{\lambda_{ii}\delta_{ii}},\quad 0\right) \tag{4.30}$$

where M is the number of positive $|\Phi_{ii}|^2$.

The optimal equaliser in this case for a given optimal precoder $\mathbf{F}_{opt}$ can be derived under minimum MSE criteria as

$$\boldsymbol{G}_{opt} = \left(\mathbf{R}_{vv}^{-1}\boldsymbol{H}\mathbf{F}_{opt}\right)^{\dagger}\mathbf{R}_{vv}^{-1} \tag{4.31}$$

where the symbol $(.)^{\dagger}$ denotes the pseudo-inverse operation.

The optimal precoding and equalisation functions discussed in the thesis decouple the multichannel MIMO problem into a number of single channel problems. One should note that the water-filling algorithm for MMSE design differs from the water-filling algorithm for MaxIR design. In the former, the transmit power allocated for each channel can be a convex function λ_{ii} while in the later the transmit power allocated for each flat subchannel is always a monotonically increasing function of λ_{ii}.

4.4 Joint Precoding and Equalisation for Multi-user DSL Systems

This section describes the application of a joint precoding and equalisation approach proposed in [44] to mitigate inter-carrier interference (ICI), which maximises the data-rate in multi-user DSL systems. The approach is similar to the one proposed in [102] that uses block transmission, exploiting some redundancies instead of the guard interval to counteract several effects of interference. The method also utilises a joint channel diagonalisation algorithm to derive the optimal precoders and equalisers. Some design criteria were proposed in [107] which targeted the minimum MSE, and a common transmission power condition. This section will focus on the optimal designs under transmission power constraints.

Consider the generalised channel model and its mathematical formulation in Section 4.2. Assume that the channel is a discrete-time slotted system, indexed by n, consisting of K users, with linear precoder and equaliser. Further assume that N denotes the number of transmit tones and M represents the number of receive tones. One can assume that the MIMO channel between transmitter and receiver of ith user be represented by $\boldsymbol{H}_i$.

For simplicity, the channel matrix $\boldsymbol{H}(z)$ is written in this section as $\boldsymbol{H}(z) = \boldsymbol{H}$. Let the precoding matrix associated with the ith user be denoted by $\mathbf{F}_i \in \mathbb{C}^{N\times t_i}$ where t_i is the length of the vector of symbols $\mathbf{s}_i[n]$ which is devised for i user. The input vector $s_i[n]$ is linearly mapped through the precoder $\mathbf{F}_i$ to form vector $\mathbf{x}_i[n]$. The output of the precoder of each user is then propagated through the transmit matrix $\mathbf{H}_i$ with ith distortion v_i.

The received signal $y_i[n]$ can then be written as

$$\mathbf{y}_i[n] = \sum_{i=1}^{K} \mathbf{H}_i\mathbf{F}_i\mathbf{s}_i[n] + \mathbf{v}_i[n] \tag{4.32}$$

where $\mathbf{v}_i[n]$ denotes the additive Gaussian white noise and interference with covariance matrix $\mathbb{E}\{\mathbf{v}_i[n]\mathbf{v}_i^*[n]\} = \sigma_{\mathbf{v}}^2\mathbf{I}$. Thus, we can represent the operation of the whole system in matrix form, whereby the received data from all of the receivers are coexisted and represented as

$$\mathbf{y}[n] = \begin{bmatrix} \mathbf{y}_1[n] \\ \vdots \\ \mathbf{y}_K[n] \end{bmatrix} = \begin{bmatrix} \mathbf{H}_1 \\ \vdots \\ \mathbf{H}_K \end{bmatrix} \begin{bmatrix} \mathbf{F}_1 & \cdots & \mathbf{F}_K \end{bmatrix} \begin{bmatrix} \mathbf{x}_1[n] \\ \vdots \\ \mathbf{x}_K[n] \end{bmatrix} + \begin{bmatrix} \mathbf{v}_1[n] \\ \vdots \\ \mathbf{v}_K[n] \end{bmatrix}, \tag{4.33}$$

alternatively,

$$\mathbf{y}[n] = \boldsymbol{H}\boldsymbol{F}\mathbf{s}[n] + \mathbf{v}[n] \tag{4.34}$$

The precoders that remove the channel interference then be defined so that $\mathbf{H}_i\mathbf{F}_i \neq 0$ and $\mathbf{H}_j\mathbf{F}_i = 0$ for all $j \neq i$. Specifically, $\boldsymbol{HF}$ is termed the block diagonal matrix. Here, one can define matrix $\hat{\mathbf{H}}$ as follow

$$\hat{\mathbf{H}}_i = \begin{bmatrix} \mathbf{H}_1^T & \cdots & \mathbf{H}_l^T & \mathbf{H}_{l+1}^T & \cdots & \hat{\mathbf{H}}_K^T \end{bmatrix}^T \tag{4.35}$$

Assume $M > N$ and $\text{rank}\left\{\hat{\mathbf{C}}_i\right\} = L_i \leq (\min(M,N))$ holds, then the SVD follows

$$\hat{\mathbf{H}}_i = \hat{\boldsymbol{U}}_i\hat{\Sigma}_i \begin{bmatrix} \hat{\mathbf{V}}_{i,0}^{\mathrm{H}} \\ \vdots \\ \hat{\mathbf{V}}_{i,1}^{\mathrm{H}} \end{bmatrix}, \tag{4.36}$$

where $\hat{\boldsymbol{V}}_{i,0}^{\mathrm{H}}$ contains the first L_i rows that correspond to the non-zero singular value of $\hat{\boldsymbol{H}}_i$ and $\hat{\boldsymbol{V}}_{i,1}^{\mathrm{H}}$ defines the null space of $\hat{\boldsymbol{H}}_i$, containing the last $M - L_i$ rows. Thus, $\hat{\boldsymbol{V}}_{i,1}^{\mathrm{H}}$ involves the components of both ICI and ISI. One can then define the optimal precoder $\boldsymbol{F}$ for all users as

$$\boldsymbol{F} = \begin{bmatrix} \hat{\boldsymbol{V}}_{1,0} & \cdots & \hat{\boldsymbol{V}}_{K,0} \end{bmatrix}. \tag{4.37}$$

and the overall channel transfer matrix with the pre-processing function $\mathbf{F}$ becomes

$$\boldsymbol{HF} = \begin{bmatrix} \boldsymbol{H}_1\hat{\boldsymbol{V}}_{1,0} & & 0 \\ & \ddots & \\ 0 & & \boldsymbol{H}_K\hat{\boldsymbol{V}}_{K,0} \end{bmatrix}, \tag{4.38}$$

Assume further that $L_i' = \text{rank}\{\boldsymbol{H}_i\hat{\boldsymbol{V}}_{i,0}\}$ and the SVD

$$\boldsymbol{H}_i\hat{\boldsymbol{V}}_{i,0} = \boldsymbol{U}_i \begin{bmatrix} \Sigma_i & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{V}_{i,0}^{\mathrm{H}} \\ \boldsymbol{V}_{i,1}^{\mathrm{H}} \end{bmatrix} \tag{4.39}$$

where $\boldsymbol{V}_{i,0}^{\mathrm{H}}$ holds the first L_i' right singular vectors that correspond to the non zero singular in diagonal matrix $\Sigma_i \in \mathbb{C}^{L_i' \times L_i'}$. The diagonal elements of each user become

$$\Sigma_i = \boldsymbol{U}_i^{\mathrm{H}}\boldsymbol{H}_i\hat{\boldsymbol{V}}_{i,0}\boldsymbol{V}_{i,1} \tag{4.40}$$

By using the equaliser $\boldsymbol{G} = \boldsymbol{U}_i^{\mathrm{H}}$ and setting the precoder $\boldsymbol{F}$ in (4.37) as

$$\boldsymbol{F} = \begin{bmatrix} \hat{\boldsymbol{V}}_{1,0}\boldsymbol{V}_{1,1} & \hat{\boldsymbol{V}}_{2,0}\boldsymbol{V}_{2,1} & \cdots & \hat{\boldsymbol{V}}_{K,0}\boldsymbol{V}_{K,1} \end{bmatrix}, \tag{4.41}$$

where (4.40) represents

$$\begin{bmatrix} \Sigma_1 & & 0 \\ & \ddots & \\ 0 & & \Sigma_K \end{bmatrix} = \begin{bmatrix} \boldsymbol{G}_1 & & 0 \\ & \ddots & \\ 0 & & \boldsymbol{G}_K \end{bmatrix} \boldsymbol{CF}. \tag{4.42}$$

Here, equation (4.42) defines a diagonal matrix that can eliminate the ISI and ICI caused by the multi-user DSL transmission. The overall operation of the multi-user system can, therefore, be described as

$$\begin{aligned} \hat{\mathbf{s}}[n] &= \boldsymbol{GHF}\mathbf{s}[n] + \boldsymbol{G}v[n] \\ &= \Sigma\mathbf{s}[n] + \boldsymbol{G}\mathbf{v}[n] \end{aligned} \tag{4.43}$$

where $\Sigma = \text{diag}\{\Sigma_1 \cdots \Sigma_K\}$ represents the optimal joint transmit power and $\hat{\mathbf{s}}[n] = [\hat{\mathbf{s}}_1[n] \cdots \hat{\mathbf{s}}_K[n]]^T$ is the vector of the estimated symbols at the output of equaliser $\boldsymbol{G} = \text{diag}\{\boldsymbol{U}_1^{\text{H}} \cdots \boldsymbol{U}_K^{\text{H}}\}$.

In order to optimise the multi-user DSL system, a water-filling algorithm with single-user optimisation like approach is performed on the diagonal elements of Σ so that the transmit power is allocated accordingly. Therefore, the precoder $\boldsymbol{F}$ in (4.41) is rewritten as

$$\boldsymbol{F} = \begin{bmatrix} \hat{\boldsymbol{V}}_{1,0}\boldsymbol{V}_{1,0} & \hat{\boldsymbol{V}}_{2,0}\boldsymbol{V}_{2,0} & \cdots & \hat{\boldsymbol{V}}_{K,0}\boldsymbol{V}_{K,0} \end{bmatrix} \Lambda^{1/2} \tag{4.44}$$

where $\Lambda = \lambda_{11}, \cdots, \lambda_{KK}$ is a diagonal matrix with elements λ_{ii} derived by the water-filling algorithm.

Under the defined assumptions, the data rate performance of the system is given by

$$R_{rate} = \log_2 |\mathbf{I} + \frac{\Sigma^2\Lambda}{\sigma_v^2}| \tag{4.45}$$

In the case of a spectrum management problem that requires minimising the transmit power subject to achieving a desired transmission rate for each user, the joint precoder and equaliser matrices are derived using the steps defined above, except that the matrix Σ is defined by performing water-filling separately for each user, where the constrained transmit power for each user is scaled to achieve the required transmission rate.

In the case of overdetermined MIMO channels, (i.e. $M > N$), this described process enables the equaliser $\boldsymbol{G}$ to setting the first t_i left singular vectors of $\boldsymbol{H}_i$ and defines $\hat{\boldsymbol{G}}_1 = \boldsymbol{F}_i^{\text{H}}\boldsymbol{H}_i$ to yield $\boldsymbol{F}$ and new $\boldsymbol{G}_i$, finally the matrix $\boldsymbol{G}$ is defined as a product between the first t_i independent N singular vectors of $\boldsymbol{H}_i$ and the new $\boldsymbol{G}_i$.

The condition for deriving each new $\boldsymbol{F}_i$ depends on the number of independent transmitting tones

$$N \leq \sum_{i=1}^{K} t_i \tag{4.46}$$

4.5 Block Precoding and Equalisation for Multi-user DSL Systems

Consider the DSL channel model described in Section 4.2, where the channel contains K users with combined blocks of joint precoder and equaliser as illustrated in Figure 4.4. The input symbol blocks $\mathbf{s}[n] \in \mathbb{C}^N$ are mapped through a precoder $\boldsymbol{F}(z) \in \mathbb{C}^{M\times N}$ to produce transmit symbol blocks $\mathbf{x}[n] \in \mathbb{C}^M$, such that

$$\mathbf{x}[n] = \boldsymbol{F}\mathbf{s}[n]. \tag{4.47}$$

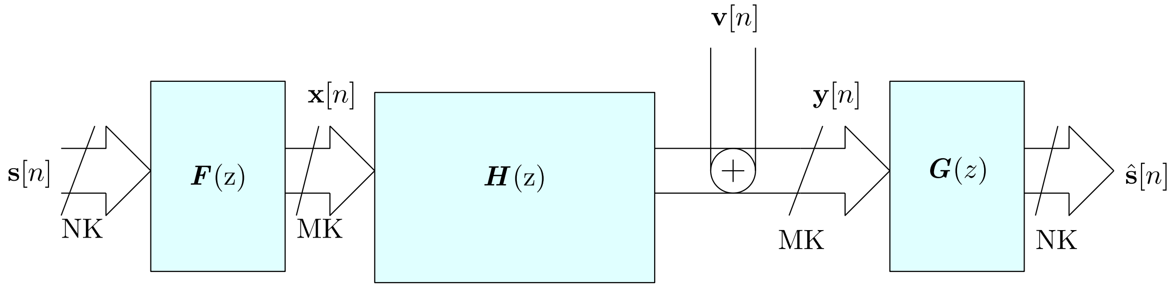


Figure 4.4 Joint precoding and equalisation for a multi-user DSL transmission

After mapping, each block $\mathbf{x}[n]$ contains N independent tones, which is then transmitted through N independent discrete-time channel matrix blocks $\boldsymbol{H}(z)$ in the presence of additive noise symbol blocks $\mathbf{v}[n] \in \mathbb{C}^M$. At the receiver, the received symbol blocks $\mathbf{y}[n] \in \mathbb{C}^M$ are reprocessed through the equaliser $\boldsymbol{G}(z) \in \mathbb{C}^{N\times M}$. The equaliser $\boldsymbol{G}(z)$ is defined to perform the inverse operation of $\mathbf{F}(z)$ to estimate the output symbol blocks $\hat{\mathbf{s}}[n]$.

As shown in Figure 4.4, the overall transmission problem can then be represented by a pseudo-circulant matrix $\boldsymbol{C}(z)$ as expressed in equation (4.5), where

$$\boldsymbol{C}(z) = \begin{bmatrix} \boldsymbol{C}_0(z) & z^{-1}\boldsymbol{C}_{MK-1}(z) & \cdots & z^{-1}\boldsymbol{C}_1(z) \\ \boldsymbol{C}_1(z) & \boldsymbol{C}_0(z) & \cdots & z^{-1}\boldsymbol{C}_2(z) \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{C}_{NK-1}(z) & \boldsymbol{C}_{NK-2}(z) & \cdots & \boldsymbol{C}_0(z) \end{bmatrix}.$$

Consider the transmission matrix $\boldsymbol{C}(z)$ as an FIR channel, therefore, one can express $\boldsymbol{C}_0$ and $\boldsymbol{C}_1$ of Eqn (4.12) in the following form.

$$\boldsymbol{C}_0 = \begin{bmatrix} \boldsymbol{C}_0(z) & 0 & 0 & \cdots & 0 \\ \vdots & \boldsymbol{C}_0(z) & 0 & \cdots & 0 \\ \boldsymbol{C}_{NK-1}(z) & \cdots & \ddots & \cdots & \vdots \\ \vdots & \ddots & \cdots & \ddots & 0 \\ 0 & \cdots & \boldsymbol{C}_{NK-1}(z) & \cdots & \boldsymbol{C}_0(z) \end{bmatrix} \tag{4.48}$$

$$\boldsymbol{C}_1 = \begin{bmatrix} 0 & z^{-1}\boldsymbol{C}_{NK-1}(z) & \cdots & \cdots & z^{-1}\boldsymbol{C}_0(z) \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \ddots & z^{-1}\boldsymbol{C}_{NK-1}(z \\ 0 & \cdots & 0 & \cdots & 0 \end{bmatrix}. \tag{4.49}$$

With $M > N$, the pseudo-circulant matrix $\boldsymbol{C}(z)$ is termed here a polynomial matrix and has unit order, in which the relation between the input and output block can be written as

$$\hat{\mathbf{s}}[n] = \boldsymbol{G}_0(z)\boldsymbol{H}(z)\boldsymbol{F}_0(z)\mathbf{s}[n] + \boldsymbol{G}_1(z)\boldsymbol{H}(z)\boldsymbol{F}_1(z)\mathbf{s}[n-1] + \boldsymbol{G}_0(z)\mathbf{v}[n] \tag{4.50}$$

where $\boldsymbol{G}_0(z)\boldsymbol{H}(z)\boldsymbol{F}_0(z)\mathbf{s}[n]$ and $\boldsymbol{G}_1(z)\boldsymbol{H}(z)\boldsymbol{F}_1(z)\mathbf{s}[n-1]$ are respectively the actual and the interference coefficient matrices of the transmission, and $\mathbf{v}[n]$ represents a block of noise samples.

As discussed in Section 4.3, the ICI and ISI in (4.50) is eliminated by setting the term $\boldsymbol{G}\boldsymbol{C}_1\boldsymbol{F}$ to zero. In the TZ approach, the last rows blocks in the precoder matrix are set to zero. In the LZ, however, the first columns blocks of the equaliser are set to zero.

The induced interference in the system is eliminated when (4.50) becomes

$$\hat{\mathbf{s}}[n] = \boldsymbol{G}_0(z)\boldsymbol{H}(z)\boldsymbol{F}_0(z)\mathbf{s}[n] + \boldsymbol{G}_0(z)\mathbf{v}[n] \tag{4.51}$$

where $\boldsymbol{H}$ is either, in TZ case

$$\boldsymbol{H}(z) = \begin{bmatrix} \boldsymbol{H}_0(z) & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ \boldsymbol{H}_{MK-1}(z) & \ddots & \ddots & 0 \\ 0 & \ddots & \ddots & \boldsymbol{H}_0(z) \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \boldsymbol{H}_{MK-1}(z) \end{bmatrix} \tag{4.52}$$

or, in the LZ case

$$\boldsymbol{H}(z) = \begin{bmatrix} \boldsymbol{H}_{MK-1}(z) & \cdots & \boldsymbol{H}_0(z) & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \boldsymbol{H}_{MK-1}(z) & \cdots & \boldsymbol{H}_0(z) \end{bmatrix}. \tag{4.53}$$

In the case when the MIMO channel is a full-rank matrix, $\boldsymbol{H}(z)$ is a blocked-banded matrix. It is note that to perfectly reconstruct the original input symbol blocks $\mathbf{s}[n]$ through $\boldsymbol{G}(z)$, it is necessary the $N = M$ and $\boldsymbol{H}(z)$ has full rank, i.e, $\text{rank}\{\boldsymbol{H}(z)\} = N$.

Considering the polynomial eigenvalue decomposition (PEVD) via SMD procedures discussed in Section 3.3.2, one can present the paraHermitian matrix decomposition as;

$$\begin{aligned} \boldsymbol{H}(z)\tilde{\mathbf{H}}(z) &= \mathbf{U}(z)\Lambda(z)\tilde{\Lambda}\tilde{\boldsymbol{U}}(z) \\ \tilde{\boldsymbol{H}}(z)\boldsymbol{H}(z) &= \boldsymbol{V}(z)\Lambda(z)\tilde{\Lambda}\tilde{\boldsymbol{V}}(z). \end{aligned} \tag{4.54}$$

This, in turn, defines the PSVD as

$$\Sigma_0^{1/2}(z) = \tilde{\boldsymbol{U}}_0(z)\boldsymbol{H}(z)\boldsymbol{V}_0(z) \tag{4.55}$$

where $\boldsymbol{V}_0(z)$ is a paraunitary transmit matrix corresponding to the first N columns of $\boldsymbol{V}(z)$ and $\Lambda(z) = \Sigma_0^{1/2}(z)$ is $Q \times Q$ optimal diagonal matrix with $Q = \text{rank}\{\tilde{\boldsymbol{H}}(z)\boldsymbol{H}(z)\}$, and $\boldsymbol{U}_0(z)$ is a paraunitary receive matrix corresponding to the first M columns of $\boldsymbol{U}(z)$.

By replacing $\boldsymbol{C}(z)$ using (4.55) and taking $\boldsymbol{G}(z)$ for $\tilde{\boldsymbol{U}}_0(z)$ and $\boldsymbol{F}(z)$ for $\boldsymbol{V}_0(z)$, the output signals in (4.50) can be defined as;

$$\begin{aligned} \tilde{\mathbf{s}}[n] =& \underbrace{\tilde{\boldsymbol{U}}_0(z)\boldsymbol{U}(\boldsymbol{z})_0}_{\mathbf{I_M}}\Sigma_0^{1/2}(z)\underbrace{\tilde{\boldsymbol{V}}_0(z)\boldsymbol{V}_0(z)}_{\mathbf{I}_N}\mathbf{s}[n] + \underbrace{\tilde{\boldsymbol{U}}_1(z)\boldsymbol{H}_1(z)\boldsymbol{V}_1(z)\mathbf{s}[n-1] + \tilde{\boldsymbol{U}}_1(z)\mathbf{v}[n]}_{interference+noise} \\ =& \Sigma_0^{1/2}(z)\mathbf{s}[n] + \tilde{\boldsymbol{U}}_1(z)\boldsymbol{H}_1(z)\boldsymbol{V}_1(z)\mathbf{s}[n-1] + \tilde{\boldsymbol{U}}_1(z)\mathbf{v}[n]. \end{aligned} \tag{4.56}$$

Here, we assume that the interference parameters in $\left(\tilde{\boldsymbol{U}}_1(z)\mathbf{H}_1(z)\boldsymbol{V}_1(z)\mathbf{s}[n-1] + \tilde{\boldsymbol{U}}_1(z)\mathbf{v}[n]\right)$ are white and additive Gaussian noise. To optimise the output signal, it is shown in [46] that the best method is the inversion of the channel matrix, provided that the overall transmission problem $\boldsymbol{H}(z)$ is perfectly stationary so that the initial matrix do not change throughout the transmission. The benefit of this channel inversion over the stationary channel information standards is being able to find the optimal precoders and equalisers under either the mean square error (MSE) minimisation criterion or the bit/data rate maximisation settings. Here, a minimum MSE (MMSE) criterion that minimises the mean error in the system to define the output symbol block $\tilde{\mathbf{s}}[n]$ closest to $\mathbf{s}[n]$ is considered.

Assume that the input symbol blocks $\mathbf{s}[n]$ are also uncorrelated mutually with noise symbol blocks $v[n]$. The known covariance matrices of the $\mathbf{s}[n]$ and $\mathbf{v}[n]$ can respectively be expressed in the following form

$$R_{ss} = \mathbb{E}\{\mathbf{s}[n]\mathbf{s}[n]^{\mathrm{H}}\} = \sigma_s^2\mathbf{I}, \tag{4.57}$$

and

$$R_{vv} = \mathbb{E}\{\mathbf{v}[n]\mathbf{v}[n]^{\mathrm{H}}\} = \sigma_v^2\mathbf{I}. \tag{4.58}$$

Under the MMSE criterion, the optimal equaliser under the MSE criterion is given by [6];

$$\boldsymbol{G}_{opt}(z) = \quad R_{ss}\tilde{\boldsymbol{F}(z)}\tilde{\boldsymbol{H}}(z)\left(R_{vv} + \boldsymbol{H}(z)\boldsymbol{F}_0(z)\boldsymbol{R}_{ss}\tilde{\boldsymbol{F}}(z)\tilde{\boldsymbol{H}}(z)\right)^{-1}. \tag{4.59}$$

The error covariance matrix can then be expressed as

$$R_{ee} = \sigma_s^2(\mathbf{I} + \sigma_s^2\tilde{\boldsymbol{F}}(z)\tilde{\boldsymbol{H}}(z)\boldsymbol{R}_{vv}^{-1}\boldsymbol{H}(z)\boldsymbol{F}_0(z))^{-1} \tag{4.60}$$

Under the MMSE condition, the error covariance matrix $\mathbf{R}_{ee}$ can be minimised subject to power constraint so that trace$(\boldsymbol{F}_{opt}(z)\tilde{\boldsymbol{F}}_{opt}(z)\sigma_s^2 = P_0$. One can then define the optimal MSE precoder as

$$\boldsymbol{F}_{opt}(z) = \quad \mathbf{V}(z)\Phi \tag{4.61}$$

where Φ represent the optimal transmit power containing $N \times N$ diagonal elements obtained through a water-filling algorithm

$$|\Phi_{ii}|^2 = \max\left(\frac{P_0 + \sum_{j=1}^{\hat{N}}\lambda_{jj}^{-1}}{\sigma_s^2\sum_{j=1}^{\hat{N}}(\sigma_{jj}/\lambda_{jj})^{1/2}}\frac{1}{\sqrt{\lambda_{ii}}} - \frac{1}{\lambda_{ii}\sigma_s}, \quad 0\right), \tag{4.62}$$

where $\hat{N} \leq N$ is the number of non-zero component of $|\Phi_{ii}|^2$, i.e, $|\Phi_{ii}|^2 > 0$ for $i \in [1, \hat{N}]$ and $|\Phi_{ii}|^2 > 0$ for $i \in [\hat{N}+1, N]$. For the maximum data-rate criteria [76], the optimal MSE precoder can be written as in (4.27). The maximum data-rate precoders are achieved as a function of the optimal power allocation using a classical water-filling algorithm with single-user approximation basis as described in [7], which leads to the main diagonal elements of Φ as

$$|\Phi_{ii}|^2 = \max\left(\frac{P_0 + \sum_{j=1}^{\hat{N}}\lambda_{jj}^{-1}}{\hat{N}\sigma_s} - \frac{1}{\lambda_{ii}\sigma_s}, \quad 0\right) \tag{4.63}$$

where M is the number of positive $|\Phi_{ii}|^2$.

4.6 Numerical Analysis

This section provides some examples regarding the properties of PSVD algorithm. For the first example, we presents an analytical solution that takes into account the polynomial channel matrix of a non-square multi-user system. The optimal precoder and equaliser of the system is them provided using PSVD algorithm for non-square channel information, making all coefficient associated with polynomal off-diagonal elements equal to zero in order to limit the impact of the noise on the transmission. Example 2 presents a simple application of using the PSVD by SMD algorithm to compute the joint precoding and equalisation filters for DSM designs.

To compute the data rate performance of the proposed DSM algorithm, three random independent BPSK source signals, each with 10000 symbols, were generated and convolutively mixed to form the input signal $\mathbf{S}(z)$. As each element of the mixing matrix is a fifth order FIR filter, the variance of the noise process was chosen to give a signal-to-noise ratio (SNR) at the receiver of 5 dB. The space-time covariance matrices for both input signal and noise signal were computed respectively according to (4.57) and (4.58). The modulation scheme used is BPSK for evaluation purposes since its extension to large constellations is straightforward. The number of time slots of the channel, N=2048. Initially, we assume imperfect CSI information at transmitter. The proposed model exploit DMT multiplexing scheme as a benchmark. DSL-DMT modulation is a scheme that uses inverse fast Fourier transform (IFFT) based transmission to decompose complex MIMO channel into several simpler SISO channels.

The input symbol block $\mathbf{S}(z)$ undergoes the same multiplexing process as the DSL-DMT based scheme. Prior to transmission, the optimal precoder $\mathbf{F}(z)$ transmitter performs an IFFT operation on the input symbol block and convert the available channel into vectored transmit channel.

Some redundancies such as cyclic prefixes are estimated prior to transmission. At the receiver, the available redundancies are discarded off and FFT operation is performed to the received signal at each linear vectored received channel via the optimal equaliser $\mathbf{G}(z)$. The standard SVD decomposition is the applied within each SISO channel. The iterative cancellation within the local CSI information of the received signal is performed on each tone individually subjected to MMSE-DFE constraints to detect and balance the resulting transmission error.

The existing DMT-DMT transmission incurs an average loss in spectral efficiency on account of the introduced cyclic prefix. If the CSI is fully known at the transmitter, this loss is negligible so this has not the target of this study. The Frobenius norm of the channel matrix is set to unity.

Here, the performance are carried-out over two different scenario, namely when K=1, and when K>1.

4.6.1 Using an Adaptive Linear combiner at time instant t

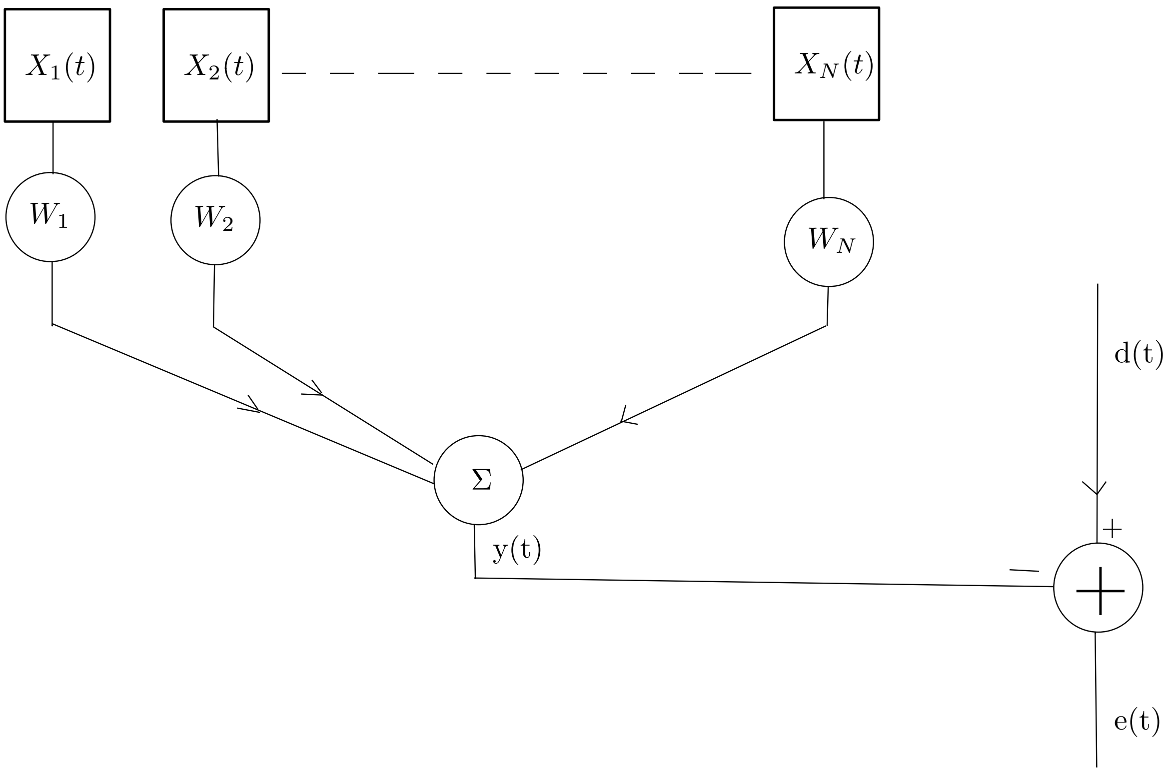


Figure 4.5 Adaptive Linear filter at instant time t

Consider the adaptive linear combiner in Figure 4.5. The output $\mathbf{y}(t)$ is given by

$$\mathbf{y}(t) = \sum_{i=1}^{N} \mathbf{w}_i \mathbf{x}_i(t) \tag{4.64}$$

and the error signal is

$$\mathbf{e}(t) = \mathbf{d}(t) - \mathbf{y}(t) = \mathbf{d}(t) - \sum_{i=1}^{N} \mathbf{w}_i \mathbf{x}_i(t) \tag{4.65}$$

where $\mathbf{d}(t)$ is the "desired" or "target" signal. the symbol t is used to denote a discrete time instant. It follow that

$$\mathbf{e}^2(t) = \mathbf{d}^2(t) - 2\mathbf{d}(t)\sum_{i=1}^{N}\mathbf{w}_i\mathbf{x}_i(t) + \sum_{k=1}^{K}\sum_{i=1}^{N}\mathbf{w}_i\mathbf{w}_k\mathbf{x}_i(t)\mathbf{x}_k(t). \tag{4.66}$$

It is assumed that the weights $\mathbf{w}_i$ is constant over the interval $t = \{1, 2, \cdots, T\}$ and introduce the vector notation

$\underline{\mathbf{w}}^T = \{\mathbf{w}_1, \mathbf{w}_1, \cdots, \mathbf{w}_N\}; \qquad \underline{\mathbf{x}}^T(t) = \{\mathbf{x}_1(t), \mathbf{x}_1(t), \cdots, \mathbf{x}_N(t)\}$

so that

$$\mathbf{e}(t) = \mathbf{d}(t) - \underline{\mathbf{w}}^T\underline{\mathbf{x}}(t) \tag{4.67}$$

and

$$\mathbf{e}^2(t) = \mathbf{d}^2(t) - 2\mathbf{d}(t)\underline{\mathbf{w}}^T\underline{\mathbf{x}}(t) + \underline{\mathbf{w}}^T\underline{\mathbf{x}}(t)\underline{\mathbf{x}}^T(t)\underline{\mathbf{w}}. \tag{4.68}$$

We aim to find the optimal weight vector, i.e. the one which minimises $r = \mathrm{E}\{\mathbf{e}^2(t)\}$, where $\mathrm{E}\{.\}$ here denotes the statistical average estimator given by

$$\mathrm{E}\{\mathbf{q}(t)\} = \frac{1}{T}\sum_{t=1}^{T}\mathbf{q}(t).$$

It is easy to show that

$$r = \mathrm{E}\{\mathbf{d}^2(t)\} - 2\sum_{i=1}^{N}\mathbf{w}_i\mathrm{E}\{\mathbf{d}(t)\mathbf{x}_i(t)\} + \sum_{k=1}^{K}\sum_{i=1}^{N}\mathbf{w}_i\mathbf{w}_k\mathrm{E}\{\mathbf{x}_i(t)\mathbf{x}_k(t)\}. \tag{4.69}$$

or alternatively

$$r = \mathrm{E}\{\mathbf{d}^2(t)\} - 2\sum_{i=1}^{N}\mathbf{w}_i\mathbf{p}_i + \sum_{k=1}^{K}\sum_{i=1}^{N}\mathbf{w}_i\mathbf{w}_k\mathbf{m}_{ik} = \mathrm{E}\{\mathbf{d}^2(t)\} - 2\underline{\mathbf{w}}^T\underline{\mathbf{p}} + \underline{\mathbf{w}}^T\mathbf{M}\underline{\mathbf{w}}. \tag{4.70}$$

where $\mathbf{p}_i = \mathrm{E}\{\mathbf{d}(t)\mathbf{x}_i(t)\}$ and $\mathbf{m}_{ik} = \mathrm{E}\{\mathbf{x}_i(t)\mathbf{x}_k(t)\}$ are elements of the cross-correlation vector $\underline{\mathbf{p}} = \mathrm{E}\{\mathbf{d}(t)\underline{\mathbf{x}}(t)\}$ and the covariance matrix $\mathbf{M} = \mathrm{E}\{\underline{\mathbf{x}}(t)\underline{\mathbf{x}}^T(t)\}$ respectively. In order to derive the optimal weight vector, we set $\frac{dr}{dw_j} = 0$ for $j = 1, \cdots, N$. Then from Equation (4.70), it follows immediately that

$$-2\mathbf{p}_j + 2\sum_{k=1}^{K}\mathbf{w}_k\mathbf{m}_{jk} = 0$$

for $j = 1, \cdots N$, so that we obtain a set of N equations in K unknowns (often referred to as the Gauss normal equations) which may be expressed succinctly in the form

$$\mathbf{M}\underline{\mathbf{w}} = \underline{\mathbf{p}}. \tag{4.71}$$

Introducing the temporal vectors $\underline{\mathbf{e}}^T = [\mathbf{e}(1), \mathbf{e}(2), \cdots, \mathbf{e}(T)]$, $\mathbf{y}^T = [\mathbf{y}(1), \mathbf{y}(2), \cdots, \mathbf{y}(T)]$ and $\mathbf{d}^T = [\mathbf{d}(1), \mathbf{d}(2), \cdots, \mathbf{d}(T)]$, we may write the statistical estimates $\mathbf{p}$ and $\mathbf{M}$ in the form

$$\underline{\mathbf{p}} = \frac{1}{T}\sum_{t-1}^{T} \underline{\mathbf{d}}(t)\underline{\mathbf{x}}(t) = \underline{\mathbf{d}}^T\underline{\mathbf{X}}$$

and

$$\mathbf{M} = \sum_{t=1}^{T} \underline{\mathbf{x}}(t)\underline{\mathbf{x}}^T(t) = \underline{\mathbf{X}}^T\underline{\mathbf{X}}$$

where

$$\underline{\mathbf{X}}^T = [\underline{\mathbf{x}}(1), \underline{\mathbf{x}}(2), \cdots, \underline{\mathbf{x}}(T)].$$

$\underline{\mathbf{X}}$ is sometimes referred to as the data matrix. In the case of a tapped delay line filter (convolution), the data matrix becomes

$$\underline{\mathbf{X}} = \begin{bmatrix} \mathbf{x}(n) & \mathbf{x}(n-1) & \mathbf{x}(n-2) & \cdots & \mathbf{x}(1) \\ \vdots & \ddots & \ddots & \vdots & \\ \vdots & \ddots & \ddots & \vdots & \\ \mathbf{h}[L] & \ddots & \ddots & 0 & \\ 0 & \ddots & \ddots & \mathbf{h}[0] & \\ \vdots & \ddots & \ddots & \vdots & \\ 0 & \cdots & 0 & \mathbf{h}[L] & \end{bmatrix} \tag{4.72}$$

Use, for example the model input in Figure 4.5. Assume the input signal $\{\mathbf{x}_1, \cdots, \mathbf{x}_N\}$ are known at the filter, even though this will probably not true in practice for PMD analysis.

4.6.2 Using Subchannel Equalisation with PSVD Algorithms

The polynomial transmission model follows a very similar procedure to the convolution data matrix. The space time covariance matrix $\mathbf{M}$ can be constructed as

$$\mathbf{M} = \mathrm{E}\{\mathbf{x}_i(t)\mathbf{x}_k(t)\}, \tag{4.73}$$

$\boldsymbol{R}_i(z) \circ\!\!-\!\!\bullet$ **M**. One can then compute the PEVD of paraHermitian $\boldsymbol{R}_i(z)$ and the polynomial eigenvalue of the true channel matrix $\Sigma_S(z)$ and the noise-plus-interference eigenvalues, $\Sigma_N(z)$.

$$\boldsymbol{R}(z) = \left[\tilde{\boldsymbol{Q}}_S(z)\tilde{\boldsymbol{Q}}_N(z)\right]\begin{bmatrix} \Sigma_S(z) & 0 \\ 0 & \Sigma_N(z) \end{bmatrix}\begin{bmatrix} \boldsymbol{Q}_S(z) \\ \boldsymbol{Q}_N(z) \end{bmatrix} \tag{4.74}$$

where $\tilde{\boldsymbol{Q}}_S(z)$ holds only the actual fractional delay that correspond to the diagonal matrix $\Sigma_S(z)$.

Results

This section examines the impact of applying two different equalisation methods- with and without per subchannel equalisations- to the outputs of the PSVD algorithm. The approach first uses the PSVD algorithm to compute several different levels of channel diagonalisation, and identify several subchannels that differ in spectral allocations.

Consider input signals for seven users in the form of sensor arrays. The signals are assumed to be transmitted signals via a channel with a corrupted independent and identically distributed complex Gaussian noise. The decomposition of the resulting channel matrix is carried out based on the two paraHermitian matrices obtained for the channel with the used of SMD algorithms at 100 iterations.

The polynomial channel model under consideration is depicted in Figure 4.6. The stem plots show the magnitude of the series of coefficients for each of the polynomial elements. The position of each stem plot corresponds to the index of the polynomial element, which it represents within the polynomial matrix. The two difference paraHermitian matrices constructed from the polynomial matrix are therefore given in Figure 4.7**(a)** and 4.7**(b)** respectively (each with channel order 13).

As expected, para-hermitian matrices $\boldsymbol{R}_1(z)$ and $\boldsymbol{R}_2(z)$ have an increase channel order than the polynomial channel matrix $\boldsymbol{H}(z)$ but the $\boldsymbol{R}_2(z)$ lower the its dimension by 2 based on the channel characteristics. Given the parameters in Table 4.1, the SMD algorithm on parahermitian matrices leads to the diagonalization of the polynomial channel matrix. The resulting diagonalized polynomial matrices $\boldsymbol{D}_1(z)$ and $\boldsymbol{D}_2(z)$ of $\boldsymbol{R}_1(z)$ and $\boldsymbol{R}_2(z)$ are shown in Figure 4.8**(a)** and 4.8**(b)** respectively. The corresponding para-unitary matrices $\boldsymbol{U}(z)$, and $\boldsymbol{V}(z)$ are stem plotted in Figure 4.9**(a)** and Figure 4.9**(b)**.

The two paraunitary matrices, although having increasing order of above 800 channel lengths, required a channel shortening. The SMD algorithm, however, restricts the required power equalisation parameters $\boldsymbol{G}(z)$ to only the first two columns of $\boldsymbol{U}(z)$. This reduced para-unitary model (of order 60) channel length was processed.

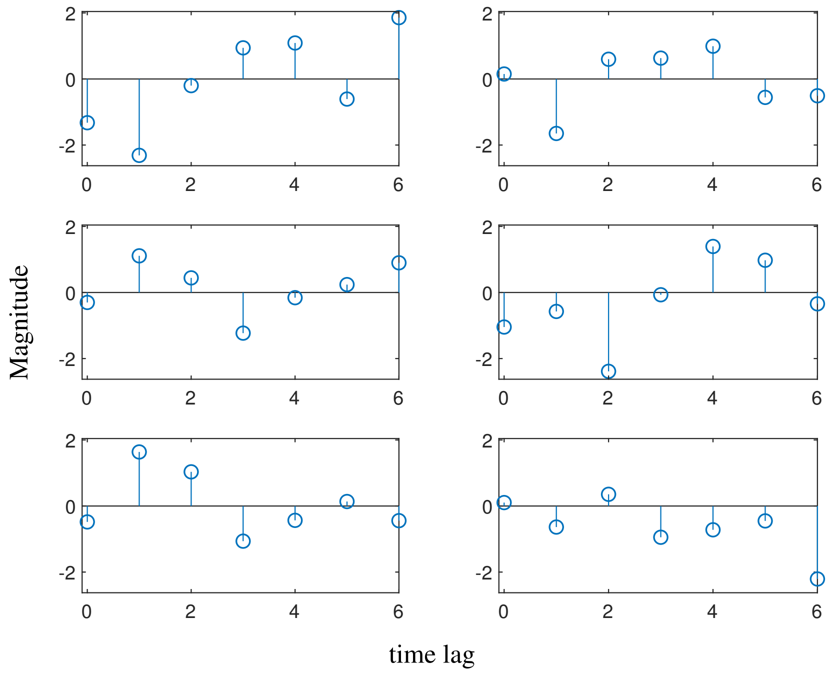
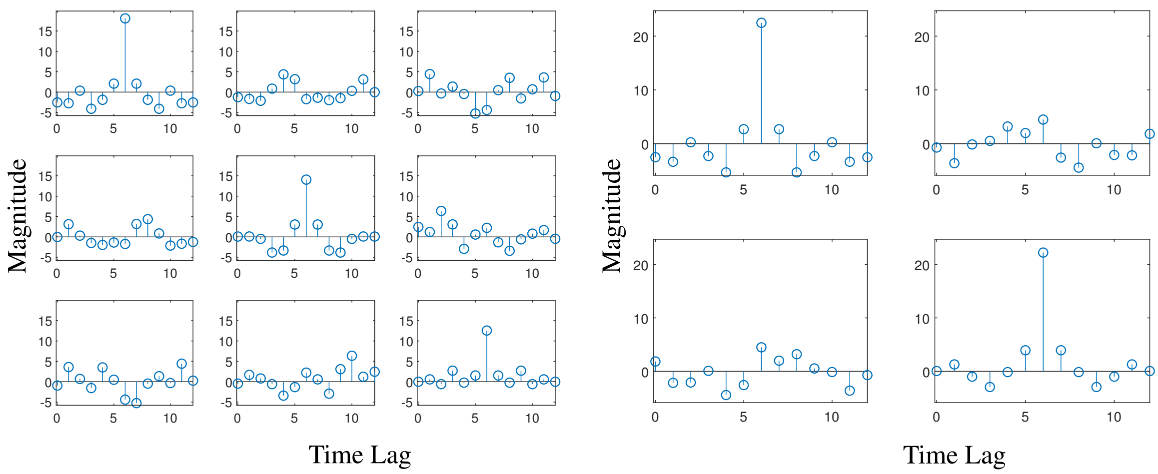


Figure 4.6 The stem plot representation of the 3×2 polynomial channel matrix.



(a) **(b)**

Figure 4.7 Stem plot representation of the Para-Hermitian matrices obtained **(a)** for $\boldsymbol{R}_1(z)$ and **(b)** for $\boldsymbol{R}_2(z)$.

The resulting channel shortening paraunitary matrices are presented in Figure 4.10. This result tells us that the computation cost can be reduced by 75% when the reduced-order structures are used in the computation.

Compared the effective diagonal matrix of using the actual and reduced-order transmission matrices, it is shown in Figure 4.11 that the two stem plots have similar features

Table 4.1 Model parameters

Parameter	Values
Number of transmitting modems	2
Number of receiving modems	3
Number of users	7
Number of transmitted bits	103776
Stopping criterion ε_r	10^{-3}
Maximum number of iterations (Maxiter)	400
Truncation parameter (μ)	10^{-6}

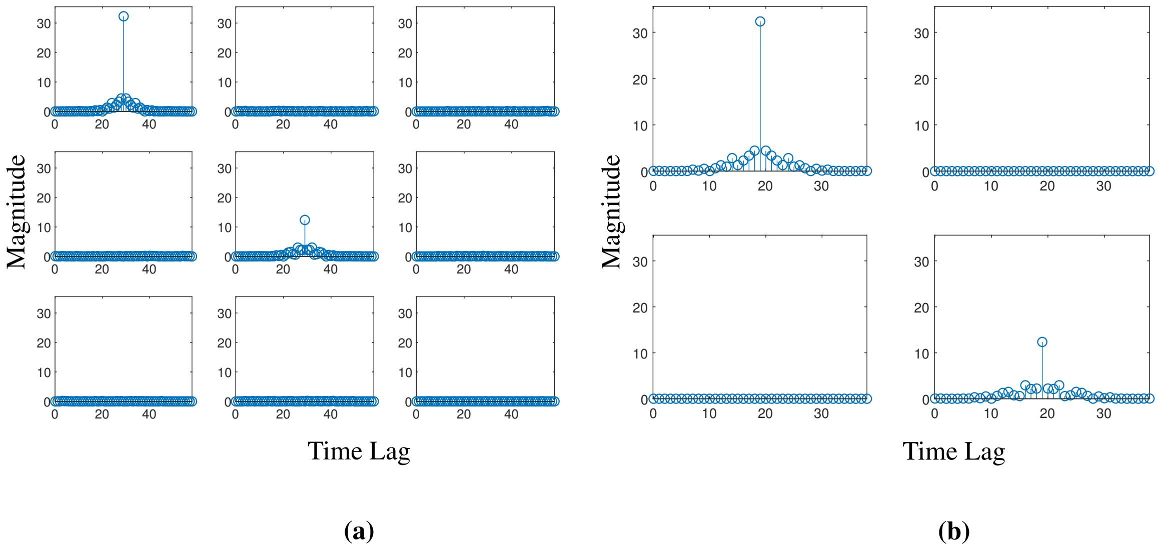
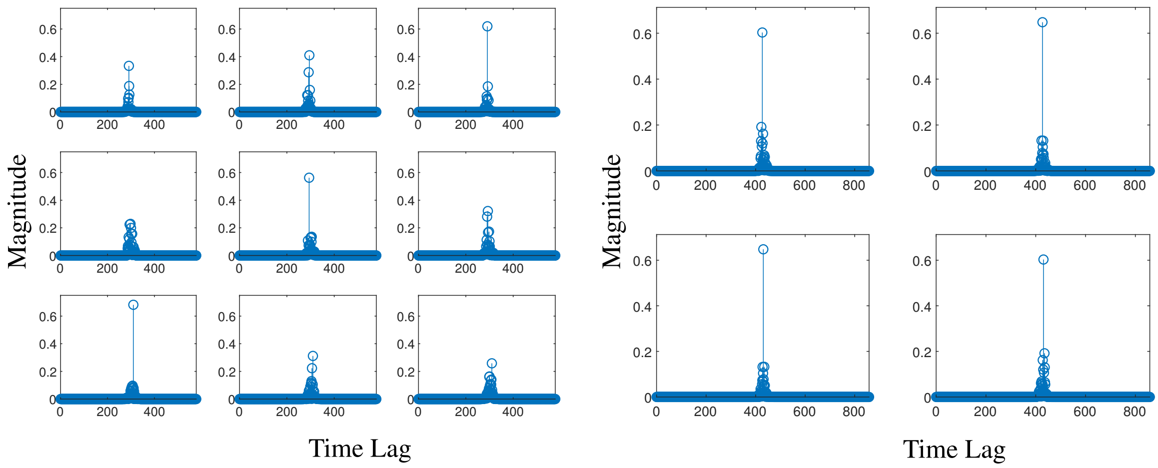


Figure 4.8 The diagonal polynomial matrix obtained when the SMD algorithm was applied **(a)** for $\mathbf{D}_1(z)$ and **(b)** for $\mathbf{D}_2(z)$.

and magnitudes but the one with reduced-order procedures exhibit low complexity by ratio 60:1000, indicating that the reduced computation cost also improves the achievable performance.

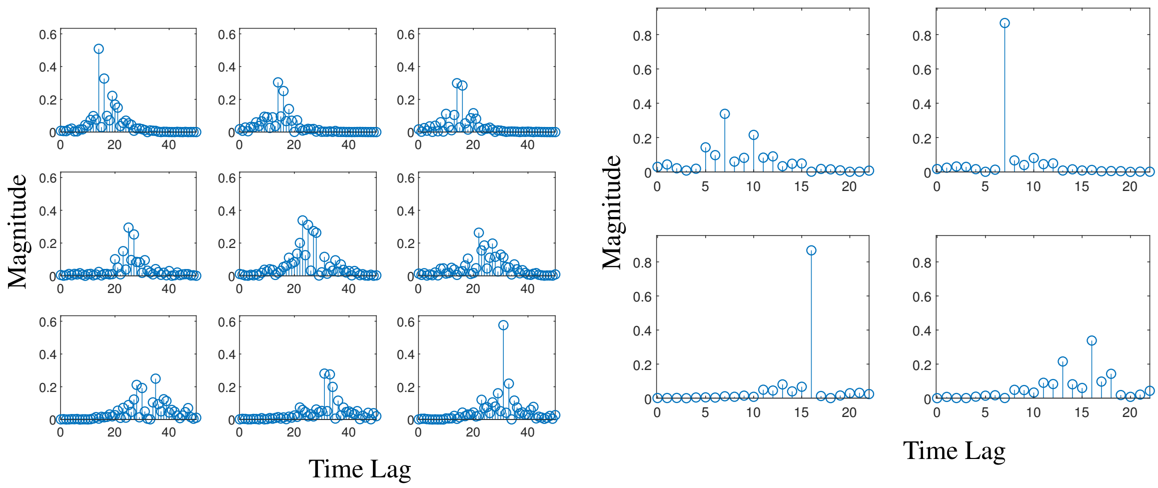
This reduced-order decomposition procedure provides the required power spectral for any given channel information. One can, then, regards the spectral power below 0dB as the interference signal. The estimation of each decomposed signal at the receiver can be balanced and equalised independently.

To compute the accuracy of this PSVD decomposition, the relative error in accordance to (3.25) as reported in [45] was computed to be -10.469760 dB, confirming that the transmission order shortening do not significantly compromise the decomposition. Note that if this measure is very very small. For the potential application of the decomposition a strictly upper triangular matrix is required, and so the relative error can again be calculated, however,



(a) **(b)**

Figure 4.9 The stem plot representation of the paraunitary matrix obtained after applying SMD algorithm **(a)** for $\boldsymbol{U}(z)$ and for **(b)** $\boldsymbol{V}(z)$.



(a) **(b)**

Figure 4.10 Representation of the optimal transmission matrices **(a)** for equalisation matrix **G** and for **(b)** for Precoder matrix **F**.

now setting all polynomial elements beneath the diagonal of the matrix equal to zero. This result confirms that a good approximate decomposition has been performed.

Furthermore, the simulation results of the output of the PSVD with independent subchannel equalisation and then without independent subchannel equalisation equivalently following the same system model are examined. The performance of these two equalisation settings over the iteration steps is shown in Figure 4.12. We see that the sum rate for per-channel

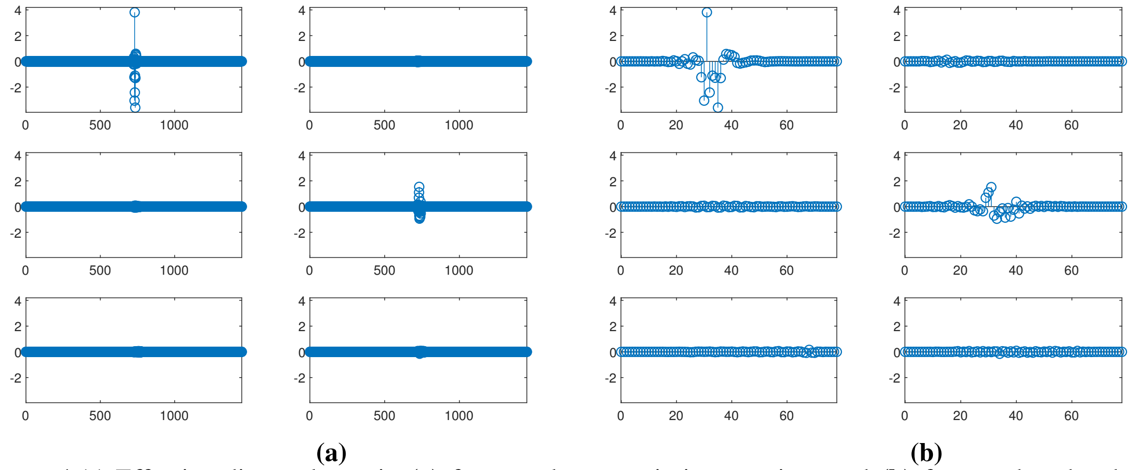


Figure 4.11 Effective diagonal matrix **(a)** for actual traansmission matrices and **(b)** for a reduced-order transmission matrices

equalisation converge significantly faster than for without independent. This gain is due to the enhanced energy gain in every step by each independent channel. Figure shows that the two equalisation methods (with and without per-user estimation) indeed behave quite differently. Of the without individual estimation, with its slightly reduced cost, initially converges slower, but attains a better convergence at higher iteration steps. Of the without per-channel equalisation algorithms, the basic reason is that it minimises the expected joint transmission errors, which optimises the collective power allocation gain instead.

The primary difference between the two equalisations is the independent equalisation algorithm: per-channel selection is restricted to improve each user rate within one of the paraunitary sets and thus has very low complexity, whereas the described without each user equalisation technique has no such restriction.

4.7 Concluding Remarks

In Section 4.2, the channel model and its problem formulation are presented to enable joint processing of users information at the transmitting end. The framework for precoding and equalisation with a single-user DSL transmission is illustrated in Section 4.3 The approach of joint precoding and equalisation for multi-user DSL transmission presented in Section 4.4 introduces a reduced-order precoding function to eliminate both ISI and ICI caused by channel interference. The use of block-based transmission over multi-user DSL channel that employed pseudo-circulant matrix is exploited in Section 4.5 to describe the polynomial matrix. PSVD is then applied to separate the induced interference from the direct channels. The chapter also derived the conditions for using the PSVD by SMD algorithm for the joint optimal precoding

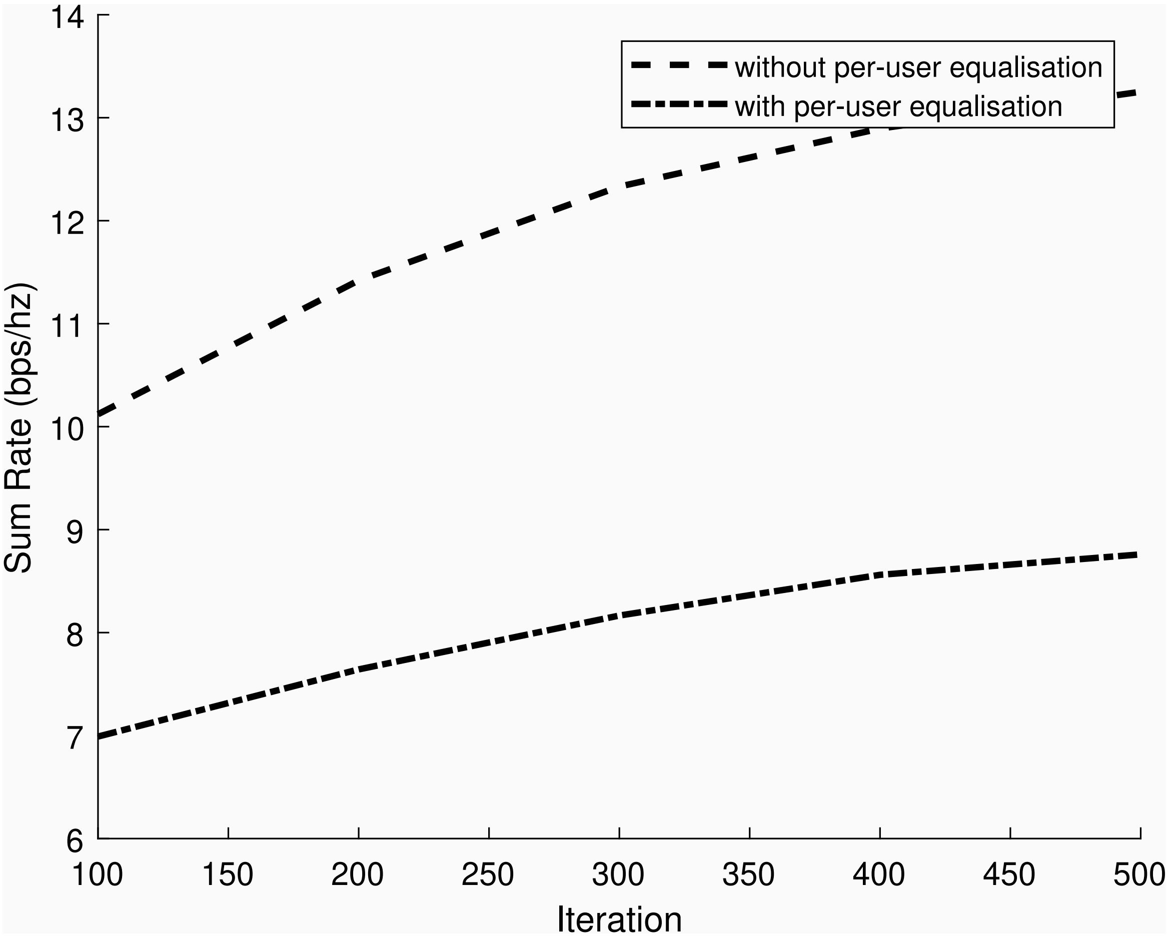


Figure 4.12 With and without per-user Equalisation

and equalisation of the multi-user MIMO system. It is noted that this particular method does not require perfect channel information state. Despite the overdetermined channel state information problem considered in the worked examples, the reduced-order paraunitary matrix can help reduce the computation cost. Through numerical experiments, we have demonstrated that the allocation of optimal power spectral independently using a PSVD by SMD algorithm to a group of users, separating the noise/interference (below 0dB level).

Finally, this chapter further examines the basic of whether with or without per- channel equalisation via PSVD algorithm provide a better sum rate in multi-user channels. This result simplifies to a comparison between with and without feedback and the surprising conclusion is that there is a very strong preference for independent channel estimation. Multi-user channels provide a performance gain that is proportional to the number of users. Although we have considered the decoupling of a multiuser channel matrix using PSVD algorithms. A multi-user channel with different diagonal elements is easier to describe and thus lower and control the power allocation to each sub-channel. Thus, PSVD further reinforces the preference towards accurate channel information. In terms of optimising power allocation, it is noted that a recent work has studied a closely related trade-off in the context of equalising the transmission error.

We proved the performance evaluation in the context of subchannels equalisation, where the comparison is made between with and without per-user equalisation.

In closing, it is worth emphasizing that the results here do not imply that spectral power allocation is worthless. On the contrary, it does provide a significant benefit. However, the model insight is that transmission error balancing at the receiver is required and used to obtain accurate power allocation by exploiting the multi-user channel information at the transmitter.

Chapter 5

Joint Precoding with Decision Feedback Equalisation

This chapter will discuss a novel approach to vectoring transmission for multi-user DSL channels through a joint precoding and equalisation scheme with decision feedback detection. The approach will first employ a polynomial singular value decomposition using the SMD algorithm to decouple the overall channel state information for a group of users into approximately independent single-user problems. In the second step, the resulting transmission error due to interference will be mitigated through a decision feedback equaliser (DFE) as suggested in [105, 112]. The proposed approach combine PSVD with either the zero-forcing (ZF) equaliser, which in fact is the common direct equalisation method, or the minimum mean square error (MMSE) equaliser.

The PSVD decomposition not only helps to estimate the induced crosstalk parameter in the transmissions but also allows a specific part of the transmission errors to be conserved with a minimal loss in space-time gain as will be demonstrated. The simulation results reported in this chapter show that the proposed method can enable a higher data-rate performance than that of other benchmark designs. Under a joint optimal precoding and equalisation constraint, the proposed DSM method enables the higher data-rate performance compare to the benchmark design to be achieved, while maintaining a relatively low-computational complexity.

5.1 Introduction

In DSM design, the vectoring transmission allows multiple transmit and receive users to be coordinated by means of either perfectly known or oversampled MIMO system [62, 113]. Such a design can offer better performance over single-user transmission scenarios, provided all the users to be combined coexist at a common location [78]. Recently, the treatment of channel state information (CSI) has become crucial as the number of both the user and the transmitting tones has increased [114, 22]. In a practical communication system, the perfect CSI at the transmitter (CSIT) is often impossible.

Several works including for some perfect CSIT systems in [24], provide exciting results by considering perfectly known beam-forming gains whose capacity is wholly characterised either through a feedback channel or based on the reciprocity of the channel. In other imperfect CSIT cases, where the CSI is only perfectly known at the receiver (CSIR), the transmitter only knows the mean or the covariance of the channel information. Several methods are also available for imperfect CSIT systems. These include the design of linear beamforming gains through a low-complexity DFE-like decoding scheme known as BLAST [24, 112], a space-time coding method [107], and an optimal capacity-achieving scheme based on multi-carrier transmission and eigenvalue decomposition analysis [115]. Most of these imperfect methods consider using the design problem required for joint precoding and equalisation signal processing with interference mitigation over a multi-user MIMO channel [105].

A considerable number of joint precoding and equalisation research publications focus on the case of a narrowband subchannel, where the multi-user MIMO problem can be modelled as a matrix, and the standard singular value decomposition (SVD) plays a central role in the joint design process to decouple the complex MIMO channel into several independent narrowband subchannels [105, 116, 117].

There are different ways to mitigate the error resulting from several narrowband independent subchannels with the standard SVD methods. In [22], worst-case designs scenario with zero-forcing designs are analysed. The method guarantees a specific system performance for any channel sufficiently close to the estimated one. The method as well leads to direct equalisation, which helps to achieve the absolute transmit power under the ideal transmission criteria. On the other hand, the CSIT problem can be modelled using averaging concepts. This guarantees a specific geometric mean performance over the channel realisations [112]. The overdetermined channel equalisation solution is used in the sequel.

Previous work on overdetermined channel cases has considered mean CSI with perfect CSIR. The concept of capturing the arithmetic average of nonlinear MIMO systems is addressed in [117, 118] for minimising the resulting mean square error (MSE). Another method combined the channel covariance, and the channel means of the system to address equalisation problem in [119], minimising an upper bound of the average pairwise error probability (PEP). A similar approach is exploited through eigenbeamforming, and in MISO

[115] and MIMO with SISO-MSE receiver [120] for maximising the mutual information by beamforming.

It is shown in [24] that robust designs can be achieved when both the mean, and covariance of the MIMO system is available. The problem of minimising the average MSE and maximising the average signal-to-noise ratio (SNR) in MISO channels is considered in [121], and in [69] for mitigating the total mean square error using an equivalent channel based on conditional channel mean and linear transceivers. With the increase of the MIMO channel information and the number of users, the existing standard SVD method can no longer be used for extracting the required independent narrowband subchannel problems from the complex problem, and research on massive heterogeneous MIMO signal method is gaining attention.

This has led to the suggestion of adopting a polynomial singular value decomposition (PSVD) technique. While PSVD is generally the best way of extracting the independent narrowband subchannels from extensive channel information. The application of PSVD in vectoring transmission designs leads to a much-simplified interference mitigation scheme.

Reference [44] describes two such approaches. Firstly, the technique can be used to decouple the complex multi-user problem into a large number of narrowband and parallel sub-channel problems. Secondly, the sub-channel approaches are now being formulated for the single-input-single-output case in [122], which can easily be extended to broadband MIMO transmission [93]. Palomar et al. in [102, 123] assume that the channel interference can be eliminated by the use of guard intervals. The paper presents an iterative algorithm that achieves joint optimal precoder and equaliser for multi-user MIMO channels, given a total power constraint for each user. Therefore, the precoder functions are first derived via the PSVD decomposition of the channel matrix and the equaliser functions are obtained as Wiener filter solutions, under different optimisation criteria that have been unified in the form of Schur-concave or Schur-convex functions. References [122, 124, 125] describe a block-transmission approach and introduce a certain amount of redundancy to eliminate transmission interference.

The redundancy introduced limits the spectral efficiency of the system due to increasing the transmit block size [3]. Under the total transmit power constraint, the energy per symbol can be decreased, and therefore the bit error rate performance becomes poorer. Reference [78] also addressed the use of guard intervals with extra degrees of freedom (DOF) equal to the channel order for cancelling crosstalk only, but these redundancies cannot be traded-off against ISI and noise amplification unless channel shortening is used. In [126], it is also established that the use of cyclic prefix intervals is not optimal in respect to the signal-noise ratio (SNR) performance.

Different from the joint precoding and equalisation DSL transmission analysis in [53, 30, 20], channel information shortening via polynomial eigenvalue decomposition was recently proposed in [95, 96] is employed to decouple the complex multi-user MIMO channel matrix into two-sided para-unitary matrices and a polynomial diagonal matrix. The extension of such a powerful PEVD to the DSM designs not only eliminate the crosstalk interference in the DSL system but also improve the overall throughput when combined with per-user interference cancellation schemes.

In the second step, the decoupled subchannel problems are further precoded and equalised using standard MMSE methods such as Weiner approach in [22]. Similar methods with non-coherent detection can be developed for cancelling crosstalk in a multiuser DSL system. In [104, 127, 122, 128], a per-user equalisation scheme is described, which outperforms linear beamforming and coherent equalisation approach. Since crosstalk can be eliminated partly with the help of the PSVD, this approach loosens the constraint of ISI elimination and provides the possibility of achieving a better spectral efficiency for the decoupled single-user sub-channels problem, thus leading to improved system performance.

Chapter outline

The remainder of this chapter is organised as follows. In Sections 5.2, the overall channel model and system characteristics are laid out. Section 5.3 addresses the first step in the design and introduces the applications of PSVD to the joint precoding and equalisation processing methods. The effectiveness of the methods for multi-user MIMO systems using single-user evaluation and detection concepts is presented in Section 5.7.2. Sections 5.8 discusses the generalisation ability of the proposed method, and comparison of the numerical results with existing method is shown. Finally, Section 5.9 concludes the chapter.

5.2 Channel Model and System Characteristics

5.2.1 DSL System Model

Here we consider a joint precoding and equalisation model for point-to-point multi-user DSL communication systems as described in Section 4.2. Assuming the system model consists of K users, each with N transmitting tones and M receiving tones. The tone here is referred to as the frequency or carrier. The channel length is L, and the transfer function of the each user may be expressed as a polynomial matrix of the form

$$\boldsymbol{H}_k(z) = \sum_{n=0}^{L} \mathbf{H}_k[n] z^{-n}, \tag{5.1}$$

where $\mathbf{H}_k[n] \in \mathbb{C}^{M\times N}$ denotes the CSI of kth user. It is also assumed that the exact CSI is known at the receiver side.

In the general multi-user case, we assume that the signal on each of the N inputs has resulted from a time-multiplexing of the K users, and that each of the M outputs is demultiplexed into K signals. Recall that the DSL transmission can be represented by the pseudocirculant matrix $\boldsymbol{C}(z) \in \mathbb{C}^{MK\times NK}$ as explained in Section 4.2.

$$\boldsymbol{H}(z) = \begin{bmatrix} \boldsymbol{H}_1(z) & z^{-1}\boldsymbol{H}_K(z) & \cdots & z^{-1}\boldsymbol{H}_2(z) \\ \boldsymbol{H}_2(z) & \boldsymbol{H}_1(z) & \cdots & z^{-1}\boldsymbol{H}_3(z) \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{H}_K(z) & \boldsymbol{H}_{K-1}(z) & \cdots & \boldsymbol{H}_1(z) \end{bmatrix}. \tag{5.2}$$

The matrices $\boldsymbol{H}_k(z) \in \mathbb{C}^{M\times N}$, $k = 1, \cdots, K$, are the K-user polyphase components of $\boldsymbol{H}(z)$ matrix such that

$$\boldsymbol{H}(z) = \sum_{k=0}^{K-1} \boldsymbol{H}_k(z^K) z^{-k} \tag{5.3}$$

or alternatively

$$\boldsymbol{H}_k(z) = \sum_{n=-\infty}^{+\infty} \mathbf{C}[nK+k] z^{-n} \tag{5.4}$$

In the following, a generic DSL system with joint precoder $\mathbf{E}(z) \in \mathbb{C}^{NK\times P}$, joint equaliser $\mathbf{B}(z) \in \mathbb{C}^{P\times MK}$ and transmit matrix $\mathbf{H}(z) \in \mathbb{C}^{MK\times NK}$ is illustrated in Figure 5.1.

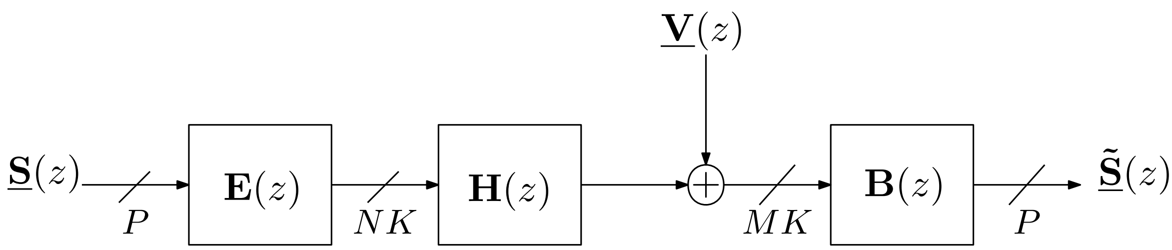


Figure 5.1 DSL system with channel matrix $\mathbf{H}(z)$, joint precoder $\mathbf{E}(z)$ and joint equaliser $\mathbf{B}(z)$ including a multiplexing by K.

5.3 Linear Precoder and Equaliser

When $K = 1$, $\boldsymbol{H}(z) = \boldsymbol{H}_1(z)$ is a $M \times N$ polynomial matrix. As the number of users K increases, the size of the polynomial matrix $\boldsymbol{H}(z)$ becomes more massive, but its polynomial order reduces in accordance with the shortening polyphase responses.

When $N = M$, the polyphase components $\boldsymbol{H}(z)$ are constants with no dependency on z. However, the block-pseudo-circulant form of $\boldsymbol{C}(z)$ in 4.5 ensures that for all $M > N$, the transmission matrix $\boldsymbol{C}(z)$ will be a first-order polynomial, which means that channel interference such as crosstalk can easily be isolated.

As discussed in Sections 4.3 and 4.5, to eliminate the transmission error, the transmission vectored signals rely on a space-time multiplex that is chosen under the overdetermined MIMO channel conditions, i.e. $M > N$ [78].

As a result, $\boldsymbol{C}(z)$ becomes a block diagonal matrix of only first order in z, as noted earlier. Specifically

$$\boldsymbol{C}(z) = \boldsymbol{C}_0(z) + \boldsymbol{C}_1(z) \tag{5.5}$$

where $\boldsymbol{C}_0(z)$ and $\boldsymbol{C}_1(z)$ are given in equations (4.48) and (4.49), respectively, for the MIMO case.

The polynomial order of the MIMO system matrix $\boldsymbol{H}(z)$ can be shortened by reducing $\boldsymbol{C}_1(z)$ to zero through either the TZ or LZ approach as mentioned in Sections 4.3 and 4.5. Thus the polynomial nature of $\boldsymbol{C}(z)$ has been eliminated, and the precoder and equaliser can be selected as non-polynomial matrices. However, one can see again that these TZ or LZ approaches, or even the multicarrier approach which uses cyclic prefix, always require at least the first L degrees of freedom (DOF) to be used only for ISI elimination.

5.4 Proposed Design

The proposed design has two components. First, the MIMO system matrix $\boldsymbol{H}(z)$ is decomposed into many independent subchannel problems using the recently proposed PSVD algorithm [109]. The decomposition of $\boldsymbol{H}(z)$ through PSVD can be written as

$$\boldsymbol{H}(z) = \boldsymbol{U}(z) \begin{bmatrix} \boldsymbol{D}(z) & 0 \\ 0 & 0 \end{bmatrix} \tilde{\boldsymbol{V}}(z) \tag{5.6}$$

whereby $\boldsymbol{D}(z) = \mathrm{diag}\{\boldsymbol{D}_{00}, \boldsymbol{D}_{11}, \cdots, \boldsymbol{D}_{T-1,T-1}\}$ and $\boldsymbol{U}(z)$ and $\tilde{\boldsymbol{V}}(z)$ are paraunitary matrices.

The PSVD decomposition in (5.6) motivates the use of a precoder $\boldsymbol{E}(z)$ containing the first T columns of $\boldsymbol{V}(z)$ and an equaliser $\boldsymbol{B}(z)$ containing the first T rows of $\tilde{\boldsymbol{U}}(z)$ such that the multi-user DSL channel matrix can be decomposed into $T \leq \min(MK, NK)$ independent subchannels.

One can write

$$\underline{\hat{\mathbf{S}}}(z) = \boldsymbol{B}(z)\boldsymbol{C}(z)\boldsymbol{E}(z)\underline{\mathbf{S}}(z) + \boldsymbol{B}(z)\underline{\mathbf{V}}(z) \quad (5.7)$$

$$= \boldsymbol{D}(z)\mathbf{S}(z) + \boldsymbol{B}(z)\mathbf{V}(z) \quad (5.8)$$

where $\underline{\mathbf{S}}(z) \in \mathbb{C}^P$ denotes the signal symbol blocks at the input of the precoder $\boldsymbol{E}(z)$, $\underline{\hat{\mathbf{S}}}(z) \in \mathbb{C}^P$ denotes the signal symbol blocks at the output of the equaliser $\boldsymbol{B}(z)$, and $\underline{\hat{\mathbf{V}}}(z) \in \mathbb{C}^{MK}$ contains the additive white Gaussian noise as indicated in Figure 5.1

PSVD provides the possibility of removing a specific part of the channel interference through $\boldsymbol{E}(z)$ at the transmitter side and $\boldsymbol{B}(z)$ at the receiver side. In some cases, where the decoupled sub-channels are dispersive and cause crosstalk, the second step of the proposed approach becomes the critical mitigation concept. Therefore in a second step, a soft decision mitigation concept is proposed for each decoupled sub-channel transmission so that the induced crosstalk can be eliminated. This soft decision mitigation approach relies on linear optimal precoders and equalisers [78] or nonlinear optimal precoders and equalisers [129] to reconstruct the input signals. Besides, the second step takes only the individual properties of each sub-channel — such as its SNR — into account. Note that K denotes both the number of polyphase components and the block size. For the single-user block transmission system described in Section 4.3, $K = 1$, while for the multi-user model considered here, K is greater than 1.

5.5 Vectored Transmission through SMD Algorithm

In the following, the PSVD described in [105], together with its single-input single-output pre-equalisation requirement is outlined. In contrast to the standard DFT employed in the current DMT architectures to diagonalise only considered the instantaneous optimisation problems associated with DSL systems, the proposed PSVD offers a powerful tool by diagonalising the complex multi-user problem in the polynomial domain. For the polynomial case, a PSVD decomposition can be obtained by implementing two polynomial eigenvalue decomposition (PEVD), in which a paraHermitian matrix $\underline{\boldsymbol{R}}_1(z) = \boldsymbol{H}(z)\boldsymbol{H}^{\mathrm{H}}(z)$ is decomposed such that

$$\begin{aligned}\underline{\boldsymbol{R}}_1(z) &= \underline{\boldsymbol{U}}(z)\Sigma(z)\underline{\boldsymbol{V}}^{\mathrm{H}}(z)\underline{\boldsymbol{V}}(z)\Sigma^{\mathrm{H}}(z)\underline{\boldsymbol{U}}^{\mathrm{H}}(z)\\ &= \underline{\boldsymbol{U}}(z)\Gamma^1(z)\underline{\boldsymbol{U}}^{\mathrm{H}}(z) \quad (5.9)\end{aligned}$$

where $\Gamma^1(z) = \Sigma(z)\Sigma^{\mathrm{H}}(z)$ is a diagonal matrix.

It can be seen that the matrix $\underline{\boldsymbol{R}}_1(z)$ is para-Hermitian, which means it satisfy the condition

$$\underline{\boldsymbol{R}}_1(z) = \underline{\boldsymbol{R}}_1^{\mathrm{H}}(z^*). \tag{5.10}$$

In an error-free case, $\underline{\boldsymbol{V}}(z)$ is paraunitary so that $\underline{\boldsymbol{V}}(z)\underline{\boldsymbol{V}}^{\mathrm{H}}(z) = \mathbf{I}$. As a result of SMD, $\Gamma^1(z)$ is spectrally majorised such that its diagonal elements

$$\Gamma^1_{00}(z), \Gamma^1_{11}(z), \cdots, \Gamma^1_{MK-1,MK-1}(z) \tag{5.11}$$

are ordered according to

$$\left|\Gamma^1_{kk}(z)\right| \geq \left|\Gamma^1_{k+1,k+1}(z)\right| \quad \forall(z) \quad \texttt{and} \quad k = 0, 1, \cdots, MK-1, \tag{5.12}$$

which is similar to the ranking of the singular values in a standard SVD decomposition. Note that the paraunitary matrix $\underline{\boldsymbol{U}}(z)$ conserves transmit power, which means trace$\left\{\Gamma^1[0]\right\} =$ trace$\{\underline{\boldsymbol{R}}_1[0]\}$ with $\Gamma^1[\tau] \circ\!\!-\!\!\bullet\, \Gamma^1(z)$ and $\underline{\boldsymbol{R}}_1[\tau] \circ\!\!-\!\!\bullet\, \underline{\boldsymbol{R}}_1(z)$.

In a similar operation, the paraHermitian matrix $\underline{\boldsymbol{R}}_2(z) = \boldsymbol{C}^{\mathrm{H}}(z)\boldsymbol{C}(z)$ is decomposed via EVD such that

$$\begin{aligned} \underline{\boldsymbol{R}}_2(z) &= \boldsymbol{V}(z)\Sigma^{\mathrm{H}}(z)\underline{\boldsymbol{U}}^{\mathrm{H}}(z)\underline{\boldsymbol{U}}(z)\Sigma(z)\underline{\boldsymbol{U}}^{\mathrm{H}}(z) \\ &= \underline{\boldsymbol{V}}(z)\Gamma^2(z)\underline{\boldsymbol{V}}^{\mathrm{H}}(z) \end{aligned} \tag{5.13}$$

where $\underline{\boldsymbol{U}}(z)$ assumes a paraunitary structure such that $\underline{\boldsymbol{U}}(z)\underline{\boldsymbol{U}}^{\mathrm{H}}(z) = \mathbf{I}$ and $\Gamma^2(z) = \Sigma^{\mathrm{H}}(z)\Sigma(z)$.

In this section, an iterative error mitigation procedure which uses a sequence of paraunitary operations to yield a near optimal solution of a joint precoding matrix $\underline{\boldsymbol{U}}(z)$ and a joint equalisation matrix $\underline{\boldsymbol{V}}(z)$ will be considered.

5.6 Precoder and Equaliser with Interference Mitigation

Applying the algorithm in Section 4.54 to decompose a multi-user channel matrix, this Section concentrates on the impact of channel estimation errors with regard to the application of the PSVD framework. In the literature, crosstalk impairment remains by far the most significant source of transmission errors in DSL systems. Usually, crosstalk occurs in two places, channel estimation and feedback equalisation. Noisy channel estimation can cause random interference in the detection which cannot be mitigated. On the other hand, feedback equalisation introduces a specific decision as well as errors in the detection criterion when passing certain part of information back from the receiver to the transmitter.

Usually, the feedback decision is of more interest, as the errors in detection can be avoided by using a low modulation order and powerful coding techniques. The feedback decision maintains the accuracy of the modulation order, as discussed in [78]. A practical approach to achieve the feedback decision channel balancing is to predict the single-input-single-output pre-equalisation through MMSE detection scheme, as proposed in [24]. First, the channel matrix to be decomposed can be expressed as

$$\boldsymbol{H}(z) = \underbrace{\underline{\boldsymbol{U}}(z)\boldsymbol{D}(z)\underline{\boldsymbol{V}}^{\mathrm{H}}(z^{-*})}_{\underline{\boldsymbol{H}}_0(z)} + \underline{\boldsymbol{M}}_{er}(z) \tag{5.14}$$

where $\boldsymbol{H}(z)$ represents the original matrix, $\underline{\boldsymbol{H}}_0(z)$ is the approximation true channel matrix given by the PSVD, and $\underline{\boldsymbol{M}}_{er}(z)$ is the associated errors including the crosstalk present in the estimated channel matrix.

As such, the absolute unitary errors can then be defined as

$$\begin{aligned} \underline{\boldsymbol{U}}^{\mathrm{H}}(z^{-*})\underline{\boldsymbol{U}}(z) &= \mathbf{I} + \underline{\boldsymbol{U}}_{er}(z) \\ \underline{\boldsymbol{V}}^{\mathrm{H}}(z^{-*})\underline{\boldsymbol{V}}(z) &= \mathbf{I} + \underline{\boldsymbol{V}}_{er}(z). \end{aligned} \tag{5.15}$$

=

The single-user approximative parameter for any given channel matrix takes the form:

$$\underline{\boldsymbol{U}}_k = \underline{\boldsymbol{U}}(e^{j2\pi k/N}) = \underline{\boldsymbol{U}}(z) \qquad \underline{\boldsymbol{V}}_k = \underline{\mathbf{V}}(e^{j2\pi k/N}) = \underline{\boldsymbol{V}}(z) \tag{5.16}$$

$$\boldsymbol{D}_k = \boldsymbol{D}(e^{j2\pi k/N}) = \boldsymbol{D}(z) \qquad \underline{\boldsymbol{M}}_k = \underline{\boldsymbol{M}}_{er}(e^{j2\pi k/N}) = \boldsymbol{M}_{er}(z) \tag{5.17}$$

$$\underline{\boldsymbol{U}}_{er,k} = \underline{\boldsymbol{U}}_{er}(e^{j2\pi k/N}) = \underline{\boldsymbol{U}}_{er}(z) \qquad \underline{\boldsymbol{V}}_{er,k} = \underline{\boldsymbol{V}}_{er}(e^{j2\pi k/N}) = \underline{\boldsymbol{V}}_{er}(z)$$

for $k = 0, ..., MK - 1$.

5.6.1 System Model

For a given multi-user DSL channel as described in Section 5.3, precoding and equalisation matrices obtained via PSVD give the receive model in Figure 4.4 as

$$\hat{\mathbf{S}}(z) = \underline{\boldsymbol{U}}^{\mathrm{H}}(z^{-*})\boldsymbol{H}(z)\underline{\boldsymbol{V}}(z)\boldsymbol{Q}^{1/2}(z)\mathbf{S}(z) + \underbrace{\underline{\boldsymbol{U}}^{\mathrm{H}}(z^{-*})\mathbf{V}_n(z)}_{W(z)}. \tag{5.18}$$

where $\boldsymbol{Q}(z)$ denotes the redundancy introduced to accommodate any possible channel imbalance at the transmitter and $\mathbf{V}_n(z)$ represents the resulting background noise of the channel estimate.

By using (5.14) and substituting $\boldsymbol{H}(z)$ into (5.18), the output of the received channel becomes

$$\begin{aligned}\hat{\mathbf{S}}(z) &= \underline{\boldsymbol{U}}^{\mathrm{H}}(z^{-*})\Big(\underline{\boldsymbol{U}}(z)\boldsymbol{D}(z)\underline{\boldsymbol{V}}^{\mathrm{H}}(z^{-*}) + \underline{\boldsymbol{M}}_{er}(z)\Big)\underline{\boldsymbol{V}}(z)\boldsymbol{Q}^{1/2}(z)\mathbf{S}(z) + W(z) \\ &= \Big(\underline{\boldsymbol{U}}^{\mathrm{H}}(z^{-*})\underline{\boldsymbol{U}}(z)\boldsymbol{D}(z)\underline{\boldsymbol{V}}^{\mathrm{H}}(z^{-*})\underline{\boldsymbol{V}}(z) + \underline{\boldsymbol{U}}^{\mathrm{H}}(z^{-*})\underline{\boldsymbol{M}}_{er}(z)\underline{\boldsymbol{V}}(z)\Big)\boldsymbol{Q}^{1/2}(z)\mathbf{S}(z) + W(z) \\ &= \Big(\boldsymbol{D}(z) + \boldsymbol{E}(z)\Big)\boldsymbol{Q}^{1/2}(z)\mathbf{S}(z) + W(z) \end{aligned} \tag{5.19}$$

where $\boldsymbol{D}(z)$ is a true diagonal matrix, assume the paraunitarity conditions of $\underline{\boldsymbol{V}}(z)$ and $\underline{\boldsymbol{U}}(z)$ hold, i.e., $\underline{\boldsymbol{V}}^{\mathrm{H}}(z^{-*})\underline{\boldsymbol{V}}(z) = \underline{\boldsymbol{V}}(z)\underline{\boldsymbol{V}}^{\mathrm{H}}(z^{-*}) = \mathbf{I}$ and $\underline{\boldsymbol{U}}^{\mathrm{H}}(z^{-*})\underline{\boldsymbol{U}}(z) = \underline{\boldsymbol{U}}(z)\underline{\boldsymbol{U}}^{\mathrm{H}}(z^{-*}) = \mathbf{I}$. The channel propagation error $\boldsymbol{E}(z) = \underline{\boldsymbol{U}}^{\mathrm{H}}(z^{-*})\underline{\boldsymbol{M}}_{er}(z)\underline{\boldsymbol{V}}(z)$due to the presence of crosstalk.

Accordingly, the generic linear independent communication processes in (5.20) can be rewritten in the polynomial form as

$$\hat{\mathbf{S}}(z) = \boldsymbol{B}(z)\boldsymbol{Q}^{1/2}(z)\mathbf{S}(z) + W(z) \tag{5.20}$$

where $\boldsymbol{B}(z) = \boldsymbol{D}(z) + \boldsymbol{E}(z))$ contains both the true channel and the error components of $\boldsymbol{H}(z)$.

To accommodate the presence of an estimated error, a vectored single-user level estimator should be used. This type of interference mitigation model will be presented in the polynomial domain, where the system preserving matrices $\underline{\boldsymbol{U}}(z)$ and $\underline{\boldsymbol{V}}(z)$ are paraunitary.

The input-output vectored relation for the system in (5.19) can be written in form.

$$\hat{\mathbf{S}}(z) = \sqrt{\gamma}_D(z)\boldsymbol{D}(z)S(z) + \sqrt{\gamma}_E(z)\boldsymbol{E}(z)\mathbf{S}(z) + W(z) \tag{5.21}$$

where $\gamma(z) = \boldsymbol{Q}^2(z)$ is introduced for each independent linear transmission as discussed in Eqn. 5.18. The second term $\sqrt{\gamma}_E(z)\boldsymbol{E}(z)\mathbf{S}(z)$ is the crosstalk interference. For the vectored DSM via PSVD system, it is optimal to use a set of a separate per-channel detector for every DMT tone. Doing so for the set of independent sub-problem will be sub-optimal as the system is not likely to be diagonalised perfectly. Despite this, if we were to use a set of separate SISO detectors, the performance of a given sub-optimal estimator would be prone to the interference caused by other sub-streams.

On the other hand, the computational complexity of per channel detection would be lower than that for joint detection. Assuming single user precoding and equalisation estimation through a PSVD system, the achievable rate of the received system model in (5.18) is given by solution of the optimisation problem of (4.63). This involves the rate objective function and can be expressed as

$$\begin{aligned} \underset{\boldsymbol{D}(z)}{\text{maximise}} \quad & \log\left(\mathbf{I} + \frac{\gamma_D(z)\boldsymbol{D}(z)}{\gamma_E(z)\boldsymbol{E}(z) + \sigma^2\mathbf{I}_w}\right) \\ \text{subject to} \quad & \underline{\boldsymbol{V}}(z)\underline{\boldsymbol{V}}^{\mathrm{H}}(z^{-*}) = \mathbf{I} \\ & \underline{\boldsymbol{U}}(z)\underline{\boldsymbol{U}}^{\mathrm{H}}(z^{-*}) = \mathbf{I} \end{aligned} \tag{5.22}$$

5.7 Channel Estimation and Equalisation

In the literature, various methods are employed to mitigate the estimated crosstalk from the single channel estimation in the multi-user MIMO transmissions. This section will consider two approaches, one is the direct linear joint precoding and channel equalisation technique as proposed in [6]. The other involves joint optimal precoding combined with a decision feedback equaliser (DFE) as presented in [78, 105]. The application of these approaches to multi-user DSL transmission will be considered as benchmarks to highlight the gain in performance of our proposed method.

5.7.1 Linear Precoding and Channel Estimation

Here, the linear precoding and equalisation methods proposed in [78], which were also reviewed in Section 4.3, will be applied to form SISO subchannels. The system arrangement for the ith vectored single-user transmission is shown in Figure 5.2, which is a generic version of the system illustrated in Figure 4.3.

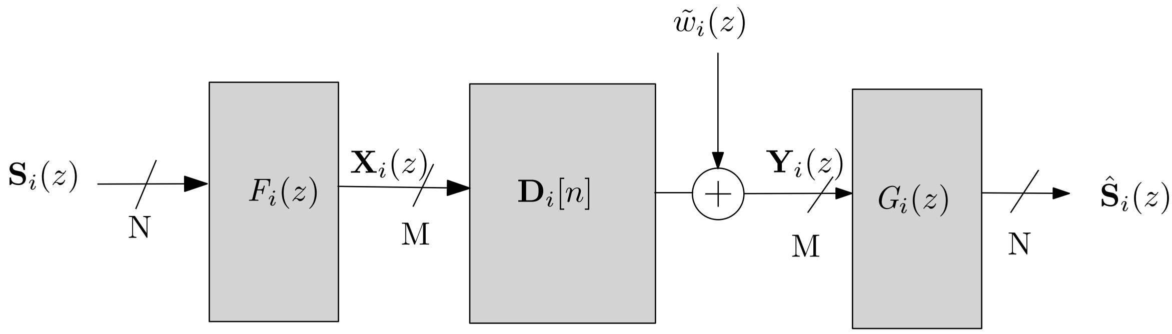


Figure 5.2 Linear joint precoding and equalisation for vectored single-user transmission

Here the symbol $\tilde{\mathbf{w}}_i(z)$, $i = 1, \cdots, K$ represents the noise components and $\boldsymbol{D}_i[n]$ is the channel block for ith-user. Considering that the channel has length $L_i + 1$, the transmit block size including the redundancy for cross-talk cancellation P_r is chosen such that M greater than L_i. The input block lenght assumes $N \leq M$, where the additive redundancy $P_r = M - N$. Without loss of generality, we exploit the TZ approach mentioned in Section 4.5 for mitigating the presence of crosstalk. This leads to express the optimal precoder in the following form

$$\boldsymbol{F}_i(z) = \begin{bmatrix} \boldsymbol{F}_{i,0}(z) \\ \boldsymbol{F}_{i,1}(z) \end{bmatrix} \tag{5.23}$$

with the is assigned a zero value and $\boldsymbol{F}_{i,0}(z) \in \mathbb{C}^{M \times N}$. The covariance matrix of the input signal $\mathbf{S}_i(z)$ is denoted by $\mathbf{R}_{\mathbf{S}_i\mathbf{S}_i} \in \mathbb{C}^{N \times N}$ and the covariance matrix arising from the noise signal $\tilde{\mathbf{w}}_\mathbf{i}(z)$ by $\mathbf{R}_{\tilde{\mathbf{w}}_i\tilde{\mathbf{w}}_i} \in \mathbb{C}^{M \times M}$. Note that the latter contains both the channel noise filtered by $\boldsymbol{F}_i(z)$ and the crosstalk components of the single-input single output equalisation problem characterised in Section 4.3.

Consider the use of a TZ precoder for mitigating crosstalk interference. The pseudo-circulant matrix $\boldsymbol{C}_i(z)$, similarly containing the finite impulse response of length $L_i + 1$ of the ith user, produced can be written as

$$\boldsymbol{H}_i(z) = \begin{bmatrix} \mathbf{D}_i[0] & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ \mathbf{D}_i[L-1] & \ddots & \ddots & 0 \\ 0 & \ddots & \ddots & \mathbf{D}_i[0] \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \mathbf{D}_i[L-1] \end{bmatrix} \tag{5.24}$$

where $\boldsymbol{C}_i(z) = \sum_{n=0}^{L-1} \mathbf{D}_i[n] z^{-n}$.

With important parameters of the transmission, such as $\boldsymbol{H}_i(z)$, $\mathbf{R}_{\mathbf{S}_i\mathbf{S}_i}$ and $\mathbf{R}_{\tilde{\mathbf{w}}_i\tilde{\mathbf{w}}_i}$ being well defined, one can perform the following eigendecomposition as discussed in (5.25)

$$\begin{aligned} \mathbf{R}_{\mathbf{S}_i\mathbf{S}_i} &= \boldsymbol{U}_i(z)\Delta_i(z)\boldsymbol{U}_i^{\mathrm{H}}(z) \\ \boldsymbol{H}^{\mathrm{H}}(z)\mathbf{R}^{-1}_{\tilde{\mathbf{W}}_i\tilde{\mathbf{W}}_i}\boldsymbol{H}(z) &= (\boldsymbol{V}_{0i}(z), \boldsymbol{V}_{1i}(z)) \begin{pmatrix} \Lambda_i(z) & 0 \\ 0 & 0 \end{pmatrix} (\boldsymbol{V}_{0i}(z), \boldsymbol{V}_{1i})^{\mathrm{H}}(z) \end{aligned} \tag{5.25}$$

with the diagonal matrices denoted by

$$\begin{aligned} \Delta_i(z) &= \mathrm{diag}\Big\{\delta_{00}^{(i)}(z), \delta_{11}^{(i)}(z), \cdots, \delta_{M-1,M-1}^{(i)}\Big\}(z) \\ \Lambda_i(z) &= \mathrm{diag}\Big\{\lambda_{00}^{(i)(z)}, \lambda_{11}^{(i)}(z), \cdots, \lambda_{M-1,M-1}^{(i)}(z)\Big\} \end{aligned} \tag{5.26}$$

As mentioned in Section 4.5, the precoder and equaliser under the MSE criterion are $\boldsymbol{V}_{0i}(z)$ and $\boldsymbol{U}_{0i}(z)$ respectively. The zero part of the precoder $\boldsymbol{V}_{1i}(z)$ occurs as a result of the presence of the interference and crosstalk.

In this section, we consider the MSE optimal precoders and equalisers as suggested in [78]. Let us assume that $\mathbf{V}_{0i}(z)$ contains the first L_i columns of $\mathbf{V}_i(z)$; then the optimum MMSE precoding filters can be expressed in the form

$$\boldsymbol{F}_{i,opt}(z) = \quad \boldsymbol{V}_{0,i}(z)\Phi_i \boldsymbol{U}_i(z) \tag{5.27}$$

where the elements of the diagonal matrix Φ represent the optimal transmit power obtained through the water-filling algorithm described in [130] and [131].

Under the MSE criterion, one can express the optimal MMSE equaliser as

$$\boldsymbol{G}_{i,opt}(z) = \quad \mathbf{R}_{\mathbf{S}_i\mathbf{S}_i}\boldsymbol{F}^{\mathrm{H}}_{i,opt}(z)\boldsymbol{H}^{\mathrm{H}}(z)\left(\mathbf{R}_{\tilde{\mathbf{W}}_i\tilde{\mathbf{W}}_i} + \boldsymbol{H}(z)\boldsymbol{F}_{i,opt}(z)\mathbf{R}_{\mathbf{S}_i\mathbf{S}_i}\boldsymbol{F}^{\mathrm{H}}_{i,opt}(z)\boldsymbol{H}^{\mathrm{H}}(z)\right)^{-1}. \tag{5.28}$$

The error covariance matrix can then be expressed as

$$\boldsymbol{R}_{ee}(z) = \sigma_s^2\left(\mathbf{I} + \boldsymbol{H}(z)\boldsymbol{F}_{i,opt}(z)\mathbf{R}_{\mathbf{S}_i\mathbf{S}_i}\boldsymbol{F}^{\mathrm{H}}_{i,opt}(z)\boldsymbol{H}^{\mathrm{H}}(z)\right)^{-1} \tag{5.29}$$

Therefore, the error covariance matrix $\boldsymbol{R}_{ee}(z)$ is minimised subject to the total power constraint so that $\mathrm{trace}(\boldsymbol{F}_{i,opt}(z)\boldsymbol{F}^{\mathrm{H}}_{i,opt}(z))\sigma_s^2 = P_0$. Note that in the design proposed here, the total transmit power P_0 for the whole MIMO system will be allocated by performing the water-filling algorithm for all $\Lambda_i(z)$, $(i = 1, \cdots, K)$ at the same time.

The use of the optimum precoding and equalisation filters given in (5.27) and (5.28) allows the decomposition of the multi-user MIMO channel into N single-input and single-output equalisation problem with different data-rates, where the value of $\bar{N}$ determines the water-filling algorithm.

With the input block size of $N(N \leq M)$ and each independent tone is assigned SNR of P_j with $j = 0 \cdots N$, the normalised mutual information β_i between the output and input of the ith user single-input single-output equalisation is given by

$$\beta_i = \sum_{j=1}^{N} \log_2\left(1 + SNR_i\right) \tag{5.30}$$

where $SNR = \frac{D_{ii}\Lambda_i}{\sigma^2_{W_i}}$ is the signal-to-noise ratio on each independent frequency tone.

Therefore, the average bit rate capacity R' of the overall MIMO system is given by

$$R' = \frac{1}{k}\sum_{i=1}^{K}\beta_i \tag{5.31}$$

5.7.2 Joint Precoding with Decision Feedback Equalisation

It is shown in [22] that joint optimal precoding and equalisation are the optimum mean squared error filters for a linear channel matrix. Unfortunately, the performance of these filters decreases significantly with the overdetermined channel matrix. To improve the performance throughput, many per-user MSE equalisation solutions are now available. Decision feedback equalisers (DFE) as well provide an excellent per-user MSE equalisation solution under specific feedback constraints. With the application of the joint optimal precoding filters, DFE solution can be designed. This DFE can be implemented as a combination of simple SISO precoding and equalisation filters.

In this subsection, the application of the joint optimal precoding and DFE procedures described in [6] for decomposing multi-user MIMO system into SISO precoding and equalisation problem is presented. The jointly optimal precoder and DFE equaliser are applied to minimise the transmission error between the input symbol and the output symbols.

The minumum MSE-based DFE equaliser makes use of previous decisions in attempting to estimate the current symbol with a symbol-by-symbol detector, where any tailing transmission error caused by a previous symbol is reconstructed and then subtracted.

This MMSE-DFE has inherently found application in the context of overdetermined channel matrix, but by assuming that all the previous decisions in process were coherent and linear. It is shown in [49] that the choice of MMSE-DFE design compromises the allowable computational complexity with the required performance gain.

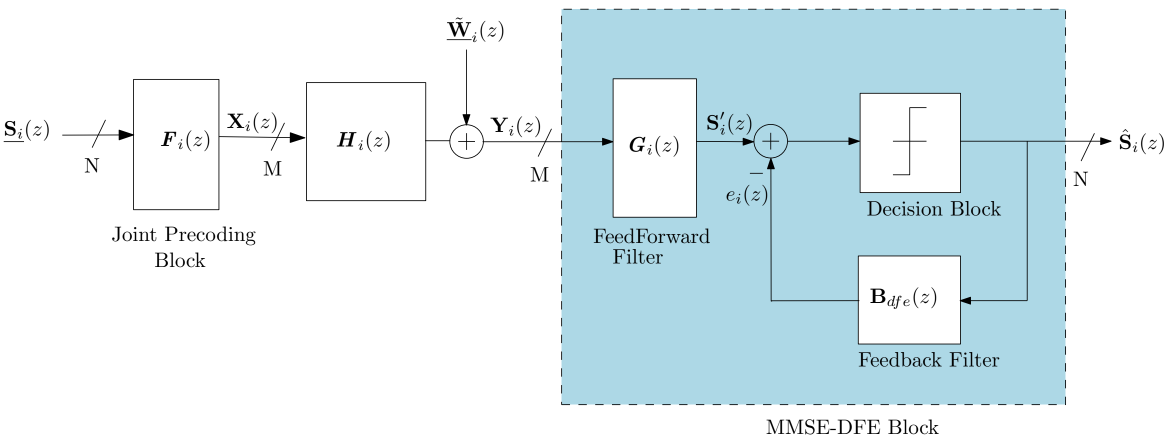


Figure 5.3 Joint precoding and MMSE-DFE block diagram

Figure 5.3 shows the block diagram of the joint precoding with MMSE-DFE block, where the MMSE-DFE block consists of a linear feed-forward filter $\boldsymbol{G}_i(z)$, linear feedback filter $\mathbf{B}_{dfe}(z)$ and a simple decision block. The input to the feedback filter is the decision of the previous symbol (from the decision device).

The first process is to exploit PSVD to derive the vectored SISO subchannel through the joint optimal precoding block $\boldsymbol{F}_i(z)$. The output signals of the feed-forward filters are summed together before subtracting the error detected by the feedback filter block and passing it through the decision block.

The input signal $\underline{\mathbf{S}}_i(z)$ assumes to be white Gaussian noise (AWGN) with unit variance. As discussed in subsection 5.7.1 above, $\mathbf{R}_{\mathbf{S}_i\mathbf{S}_i} \in \mathbb{C}^{N\times N}$ denotes the covariance matrix of the input signal $\mathbf{S}_i(z)$ while the covariance matrix arising from the interference signal $\underline{\tilde{\mathbf{W}}}_i(z)$ is represented by $\mathbf{R}_{\underline{\tilde{\mathbf{W}}}_i\underline{\tilde{\mathbf{W}}}_i} \in \mathbb{C}^{M\times M}$. The interference signal is also assumed to comprise both the channel background noise filtered by optimal feed forward equaliser $\boldsymbol{G}_i(z)$ and the crosstalk components of ith single-user transmission channel.

The crosstalk elimination is performed by introducing redundancy.The additive transmit block length P is chosen such that $M = P + L_i$ where the definitions of M, L_i are similar to those in the previous sections and N is the length of the input transmitting block.

Consider the TZ case for a single user transmission, the precoding filters that eliminate crosstalk can be expressed in the form.

$$\boldsymbol{F}_i(z) = \begin{bmatrix} \boldsymbol{F}_{i,0}(z) \\ 0 \end{bmatrix}. \tag{5.32}$$

In a K-user block, one can define matrices $\boldsymbol{E}(z) \in \mathbb{C}^{MK\times NK}$, $\boldsymbol{B}(z) \in \mathbb{C}^{NK\times MK}$, $\boldsymbol{C}(z) \in \mathbb{C}^{MK\times MK}$, $\boldsymbol{R}_{\tilde{\mathbf{W}}\tilde{\mathbf{W}}}$, $\boldsymbol{R}_{\mathbf{SS}}$ and vector $\hat{\boldsymbol{W}} \in \mathbb{C}^{MK}$ as

$$\boldsymbol{E}(z) = \begin{bmatrix} \boldsymbol{F}_{1,0}(z) & 0 & \cdots & 0 \\ 0 & \boldsymbol{F}_{2,0}(z) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \boldsymbol{F}_{K,0}(z) \end{bmatrix}, \tag{5.33}$$

$$\boldsymbol{B}(z) = \begin{bmatrix} \boldsymbol{G}_1(z) & 0 & \cdots & 0 \\ 0 & \mathbf{G}_2(z) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \boldsymbol{G}_K(z) \end{bmatrix}, \tag{5.34}$$

$$\boldsymbol{H}(z) = \begin{bmatrix} \boldsymbol{H}_1(z) & 0 & \cdots & 0 \\ 0 & \boldsymbol{H}_2(z) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \boldsymbol{H}_K(z) \end{bmatrix}, \tag{5.35}$$

$$\mathbf{R}_{\mathbf{SS}} = \begin{bmatrix} \mathbf{R}_{\mathbf{S}_1\mathbf{S}_1} & & 0 \\ & \ddots & \\ 0 & & \mathbf{R}_{\mathbf{S}_K\mathbf{S}_K} \end{bmatrix}, \tag{5.36}$$

$$\mathbf{R}_{\underline{\tilde{\mathbf{W}}}\underline{\tilde{\mathbf{W}}}} = \begin{bmatrix} \mathbf{R}_{\underline{\tilde{\mathbf{W}}}_1\underline{\tilde{\mathbf{W}}}_1} & & 0 \\ & \ddots & \\ 0 & & \mathbf{R}_{\underline{\tilde{\mathbf{W}}}_K\underline{\tilde{\mathbf{W}}}_K} \end{bmatrix}, \tag{5.37}$$

$$\underline{\tilde{\mathbf{W}}} = \begin{bmatrix} \underline{\tilde{\mathbf{W}}}_1(z) \\ \vdots \\ \underline{\tilde{\mathbf{W}}}_K(z) \end{bmatrix}, \tag{5.38}$$

where $\boldsymbol{H}_i(z)$ $(i = 1, \cdots, K)$ is the pseudo-circulant matrix given by (5.24).

In general, the overall transfer function from the input signal $\underline{\mathbf{S}}(z)$ to the input of the decision block can be written as

$$\begin{aligned} \underline{\hat{\mathbf{S}}}(z) &= \sum_{i=1}^{K} \boldsymbol{G}_{i,0}(z)\boldsymbol{H}_i(z)\boldsymbol{F}_{i,0}(z)\underline{\mathbf{S}}_i(z) + \sum_{i=1}^{K} \mathbf{B}_{dfe_i}\mathbf{S}'_i(z) + \sum_{i=1}^{K} \boldsymbol{G}_{i,0}(z)\underline{\hat{\mathbf{W}}}_i(z) & (5.39) \\ &= \boldsymbol{B}(z)\boldsymbol{H}(z)\mathbf{E}(z)\underline{\boldsymbol{S}}(z) + \boldsymbol{B}_{dfe}(z)\boldsymbol{S}'(z) + \boldsymbol{B}(z)\underline{\hat{\mathbf{W}}}(z) & (5.40) \end{aligned}$$

where

$$\mathbf{S}'(z) = \begin{bmatrix} \mathbf{S}'_1(z) \\ \vdots \\ \mathbf{S}'_K(z) \end{bmatrix}. \tag{5.41}$$

The design of the $\boldsymbol{B}_{dfe}(z)$ matrix with a joint optimal precoder $\mathbf{E}(z)$ and equaliser $\boldsymbol{B}(z)$ is implemented under either the zero-forcing (ZF-DFE) or minimum mean square error (MMSE-DFE) criteria [22].

It is shown in [78] that the optimised DFE with the zero-forcing criterion achieves the optimum bit error rate under error-free transmission. The ZF-DFE design is only suitable for linear MIMO system. On the other hand, the proposed optimisation of joint precoding and equalisation filters under the MMSE-DFE constraints becomes more general, though more difficult.

The proposed design is designed to minimise the mutual information between the input and output signal in overdetermined transmissions.

Consider the channel model in Figure 5.3. The eigendecomposition of the channel information matrix can be defined as

$$\boldsymbol{C}^{\mathrm{H}}\mathbf{R}_{\tilde{\boldsymbol{W}}\tilde{\mathbf{W}}}^{-1}\boldsymbol{C} \;=\; \boldsymbol{V}\Lambda'\boldsymbol{V}^{\mathrm{H}} \tag{5.42}$$

with diagonal matrix

$$\Lambda' \;=\; \mathrm{diag}\{\lambda'_{11}, \lambda'_{22}, \cdots, \lambda'_{kk}\}.$$

According to [78], the optimisation of the MMSE-DFE block maximises the data rate for any channel information. Therefore, the optimal MMSE-DFE data-rate performance through a single-user water-filling algorithm can be expressed in the form.

$$|\phi'_k|^2 = \max\left(\frac{P_k + \sum_{i=1}^{\bar{N}} \lambda_i'^{-1}}{\bar{N}} - \frac{1}{\lambda'_k}\right) \tag{5.43}$$

where $\bar{N} = \min\{M, N\}$ is the number of positive $|\phi'_k|^2$ satisfying

$$\frac{1}{\lambda'_{\bar{N}}} < \max\left(\frac{P_k + \sum_{i=1}^{N} \lambda_i'^{-1}}{N}\right) \tag{5.44}$$

With the MMSE-DFE constraint, the optimal precoder $\boldsymbol{F}_{opt}(z)$, which minimises the mean square error, take the form

$$\boldsymbol{E}(z) = \boldsymbol{V}_{\bar{N}}(z)\Phi'\Theta' \tag{5.45}$$

where the matrix $\boldsymbol{V}_{\bar{N}}(z)$ contains the first $\bar{N}$ columns of $\boldsymbol{V}(z)$ and $\Theta = [\Theta' \quad 0_{\bar{N}\times(N-\bar{N})}]$ is a unitary matrix satisfying

$$\left(\mathbf{I}_{N\times N} + \Phi'^{T}\Lambda'_{\bar{N}}(z)\Phi'\right)^{1/2}\Theta' = \boldsymbol{U}(z)\mathbf{R}(z) \tag{5.46}$$

where $\Lambda_{\bar{N}}(z)$ is the upper left $\bar{N}\times\bar{N}$ matrix of $\Lambda(z)$, $\boldsymbol{U}(z)$ is a unitary and $\mathbf{R}(z)$ an upper triangular matrix with equal diagonal elements.

The optimal feedback and feed-forward matrices that minimise the mean square error between the joint precoder and equaliser are given by

$$\boldsymbol{B}_{dfe} = \sigma_e \mathbf{R}(z) - \mathbf{I}_{\bar{N}\times\bar{N}} \tag{5.47}$$

and

$$\boldsymbol{B}(z) = \sigma_e \mathbf{R}(z)\big(\boldsymbol{H}(z)\boldsymbol{E}(z)\big)^{\mathrm{H}}\Big[\big(\boldsymbol{H}(z)\boldsymbol{E}(z)\big)\big(\boldsymbol{H}(z)\boldsymbol{E}(z)\big)^{\mathrm{H}} + \mathbf{R}_{\underline{\tilde{\mathbf{W}}\tilde{\mathbf{W}}}}\Big]^{-1} \tag{5.48}$$

where

$$\sigma_e^2 = \bar{N}^{M/N}\left(P_0 + \sum_{i=1}^{\bar{N}} \lambda_i'^{-1}\right)^{-M/N} \prod_{i=1}^{\bar{N}} (\lambda_i')^{-1/N} \tag{5.49}$$

and P_0 is the constrained transmit power for the multi-user MIMO system. Equation shows that the water-filling algorithm is performed simultaneously across all the SISO transmissions.

Table 5.1 Procedure of computing the MMSE-DFE scheme through the PSVD algorithm

Step	Operation
1	Compute the SVD by SMD algorithm of $\boldsymbol{C}(z) = \boldsymbol{U}\boldsymbol{D}(z)\tilde{\boldsymbol{V}}$
2	Obtain the independent transmit power Σ_{ii} and $\mathbf{E}_0$ by solving equation **??**
3	Compute the Eigenvalue Decomposition $\begin{bmatrix} \boldsymbol{U}_{i,0} & \boldsymbol{U}_{i,1} \end{bmatrix}\begin{bmatrix} \boldsymbol{D}(z) & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \tilde{\boldsymbol{V}}_{i,0} \\ \tilde{\boldsymbol{V}}_{i,1} \end{bmatrix} = \mathbf{B}\mathbf{R}\tilde{\mathbf{E}}$ where $\mathbf{R}$ has diagonal elements Σ_{ii} from step 2
4	Obtain the optimal precoder as $\mathbf{E} = \boldsymbol{V}_{\bar{N}}\Phi'\Theta'$
5	Compute the transmission matrix, $\mathbf{T}_G = \boldsymbol{C}(z)\mathbf{E}$
6	Obtain the Feedforward Equaliser, $\mathbf{B} = \sigma_e \mathbf{R}\big(\boldsymbol{C}(z)\mathbf{E}\big)^{\mathrm{H}}\Big[\big(\boldsymbol{C}(z)\mathbf{E}\big)\big(\boldsymbol{C}(z)\mathbf{E}\big)^{\mathrm{H}} + \mathbf{R}_{\tilde{\mathbf{W}}\tilde{\mathbf{W}}}\Big]^{-1}$ and the Feedback Equaliser $\mathbf{B}_{dfe} = \sigma_e\mathbf{R} - \mathbf{I}_{\bar{N}\times\bar{N}}$, where the water-filling transmit power $\sigma_e^2 = \bar{N}^{M/N}\left(P_0 + \sum_{i=1}^{\bar{N}} \lambda_i'^{-1}\right)^{-M/N} \prod_{i=1}^{\bar{N}} (\lambda_i')^{-1/N}$ and $\mathbf{I}_{\bar{N}\times\bar{N}}$ is an identity matrix[114].

5.8 Simulations and Discussion

This section presents some numerical results for the optimisation of linear precoder and equaliser under the two variants of the DFE constraints: ZF-DFE scenario and MMSE-DFE scenario. Specifically, the considered optimisation model is for overdetermined DSL model and invokes the use of PSVD procedures. There are four aspects to this implementation.

Firstly, we construct the transmission channel of a multi-user DSL system as a statistical mixture model. Secondly, we apply the PSVD to factorise the statistical channel model, and to generate the joint optimal precoder and equaliser. Thirdly, we examine whether the proposed methods are also sufficient to improve the performance of a single-user MIMO system with polynomial procedures. Finally, we examine the generalisation ability of the proposed method from any arbitrary multi-user MIMO system.

5.8.1 Design with Linear Precoding and Equalisation

This section implements linear precoding and equalisation via the PSVD algorithm for a single user. The optimal linear precoding and equalisation for multi-user MIMO transmission proposed in [107] and later extended to broadband MIMO system in [6] is taken as benchmarks.

Setting the parameters of the benchmark designs to match the values required in [6], for example, a single-user data block consists of 175 symbols, of which $5L = 48$ symbols redundancy is considered. The input block size is chosen to be $N = 96$, and data rate is values at 0.57. The power loss is due to the presence of interference is $48/175 \approx 27\%$. The structure of linear precoder and equaliser pairs are computed according to subsection 4.36. Since the sampling rate are considered equal, P_i is chosen such that $P_1 = P_2 = P_3 = P_4 = p_5$. The channel gain parameter M_i of each subchannel is calculated from $M_i = P_i - L$ and one can have $M_1 > M_2 > M_3 > M_4 > M_5$. The most dominant subchannel with highest gain has largest M_i. The input block size N is chosen so that the data rate is equal to that of the benchmark design.

The power allocation for the proposed design is chosen to be static to the transmit power in the benchmark design and employs water-filling algorithms perform iteratively across all the scalar subchannels. This iterative water-filling algorithm allocates the portion of the bandwidth for each subchannel for transmission. Thus the parameter N_i is decided by water-filling algorithms under the constraint $\sum_i N_i = N$.

After averaging over 100 iterations, the average power gain share estimated at 52%, 34%, 9%, 4%, 1% for the first, the second, the third, the fourth and the fifth scalar subchannels, respectively.

In order to assess the result of the considered linear precoding and equalisation design, which scales the transmit power on the scalar subchannels to minimise the sum of MSE on all the scalar subchannels, one can measure the average BER in terms of SNR figure suggested in [123], which sets the afforded transmit energy per bit against the channel noise measured at the receiver.

$$SNR = \frac{P}{\sigma_v^2 N \lambda_b} \tag{5.50}$$

where λ_b is the number of bits per symbol. In this simulation, we consider a single-user transmission, and the BPSK modulation is chosen with $\lambda_b = 1$.

From the simulation results in Figure 5.4, it is shown that with the same transmit power, the proposed design has with a better BER performance than that of the benchmark.

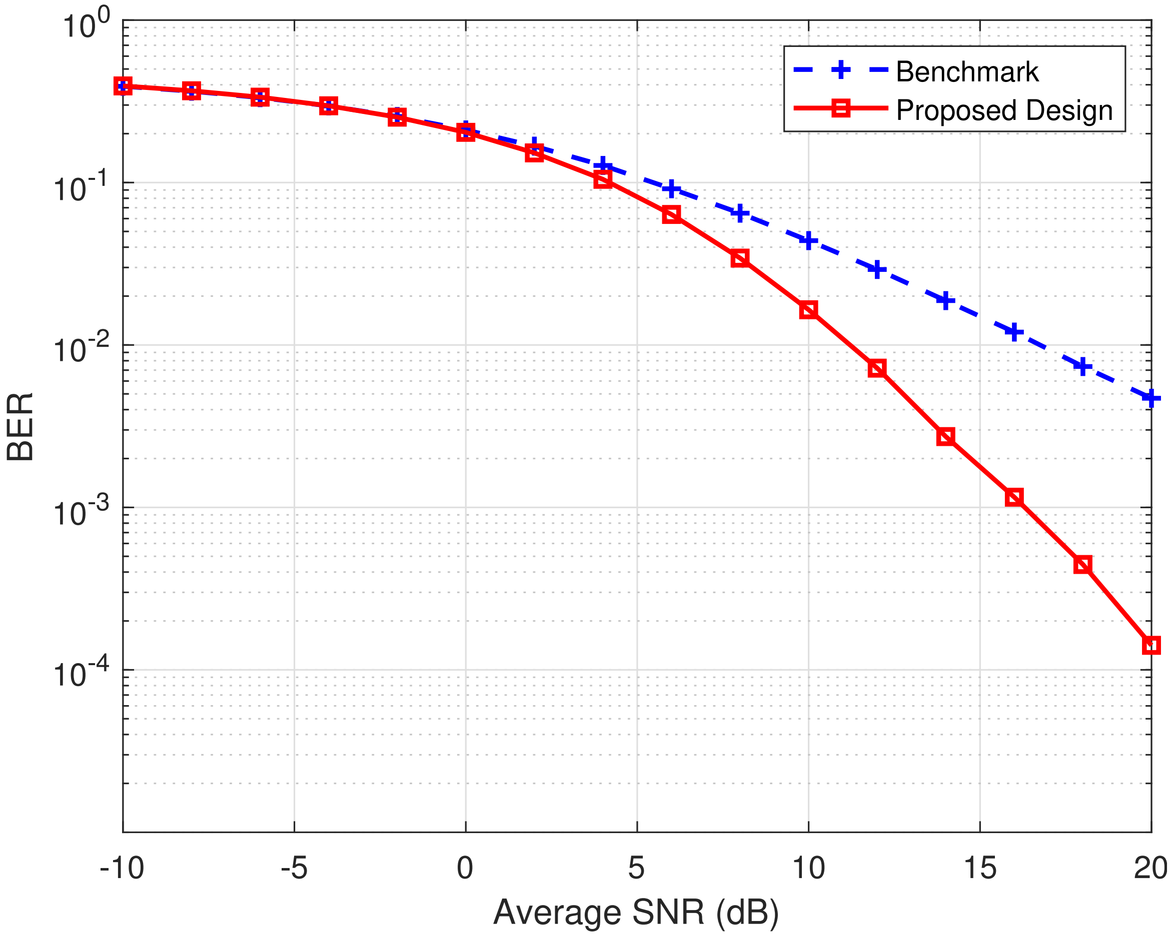


Figure 5.4 BPSK modulated, BER versus SNR for the proposed precoding-equalisation and the benchmark design

Also, the proposed design achieves same gain till BER of 0.9 with that of the benchmark design. One can see that the BER performance in the benchmark is poorer than the BER performance of the proposed design above 1dB. Next, the performance of the proposed design which uses the optimal joint precoding and equalisation is compared with the performance of the benchmark design under the same optimal criteria.

5.8.2 Design with Joint Precoding-Equalisation equalisation

To present a more comprehensive view on joint precoding-equalisation approach for four users, we compared with a design that do not make use of the joint precoding. The advantages in performance, power saving, and flexibility of the joint precoding-equalisation designs can justify the extra degree of freedom. Designs like the one in the linear process cannot be generalised for an arbitrary number of users and do not make use or take advantage of the diversity. In this section, the performance of the benchmark design with linear precoding and equalisation will be compared with the performance of the joint optimal pre-recording and equalisation proposed for multi-user MIMO.

Similar to the channel model used in 5.8.1, the transmitted data blocks contains 175 symbols, of which $5L = 48$ symbols.

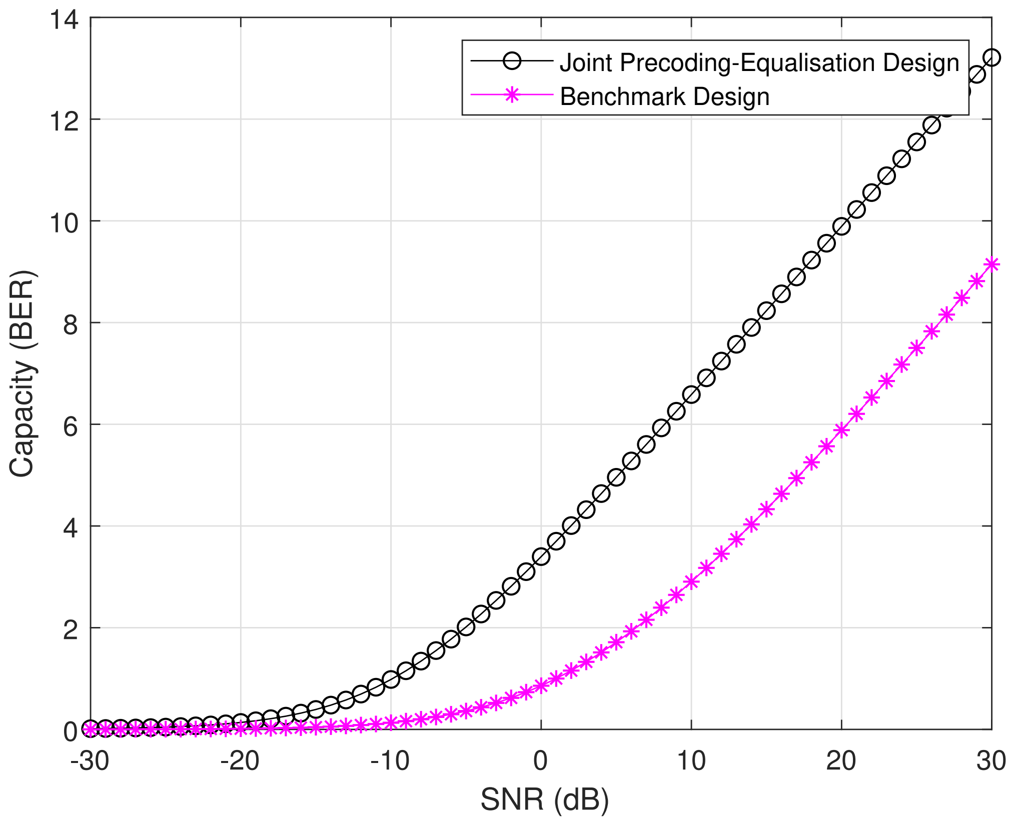


Figure 5.5 BER versus SNR for the proposed joint precoding-equalisation and for the benchmark design

The transmit power for each tone is chosen in accordance to the water-filling procedure across all the scalar subchannels. The parameter N_i is decided by water-filling algorithms under the constraint $\sum_i N_i = N$.

After averaging over 100 iterations, the average power gain share estimated at 52%, 34%, 9%, 4%, 1% for the first, the second, the third, the fourth and the fifth scalar subchannels, respectively. To assess the result of the considered joint precoding and equalisation design, we scale the transmit power into scalar subchannels and minimise the sum of mean square error on all the scalar subchannels for all users.

It is shown in Figure 5.5 that with nearly the same transmit power, the joint precoding-equalisation design has with a better BER performance than that of the benchmark design. For example, at the SNR of 10 the joint precoding-equalisation design can achieve as double BER gain (6 data rate) compared to the benchmark design with 3 data rate.

5.8.3 Joint Precoding with DFE equalisation

To validate and compare the performance of the proposed DFE model, consider a four-users MIMO channel with its element drawn from a complex Gaussian distribution with zero mean and unit variance. The size of transmitting block N = 3, the receiving block length of M = 4 applied a minimal amount of redundancy that is insufficient to allow the suppression of crosstalk through the precoding-equalisation system. The total transmit power is constrained to be constant and evenly distributed.

The channel is first decoupled into several independent subchannels. Each subchannel corresponds to a transmission block. At the receiver, the equaliser reweights every sub-channels block and estimate the transmission error. To balance the each received sub-channel block at the receiver, DFE is used to minimise the successive mean square error.

Simulation results in the term of BER performance over a PBSK transmission are investigated. The input parameters of three sets of input modulated symbols are considered. All the symbols are assumed to be transmitted in the presence of channel interference (crosstalk). The model is evaluated with two different DFE criteria: ZF-DFE and MMSE-DFE. Both of them applied decision feedback procedures. Begin with linear equalisation model. For a perfect known channel information, the receiver employs a simple optimal joint ZF-DFE and implements power balancing between the transmitter and receiver for improving the system throughput.

For imperfect channel condition, MMSE-DFE receiver applies an independent order linear least squares equaliser with per-user power constraints. This proposed MMSE-DFE receiver uses a joint equalisation with feedback weights, and also accounts for the increased number of transmission samples per block.

Figure 5.6 shows the BER performance against SNR over 100 randomised channel realisation of the different DFE criteria. It also shows the signal spectra of the linearly ZF and MMSE equalised signals.

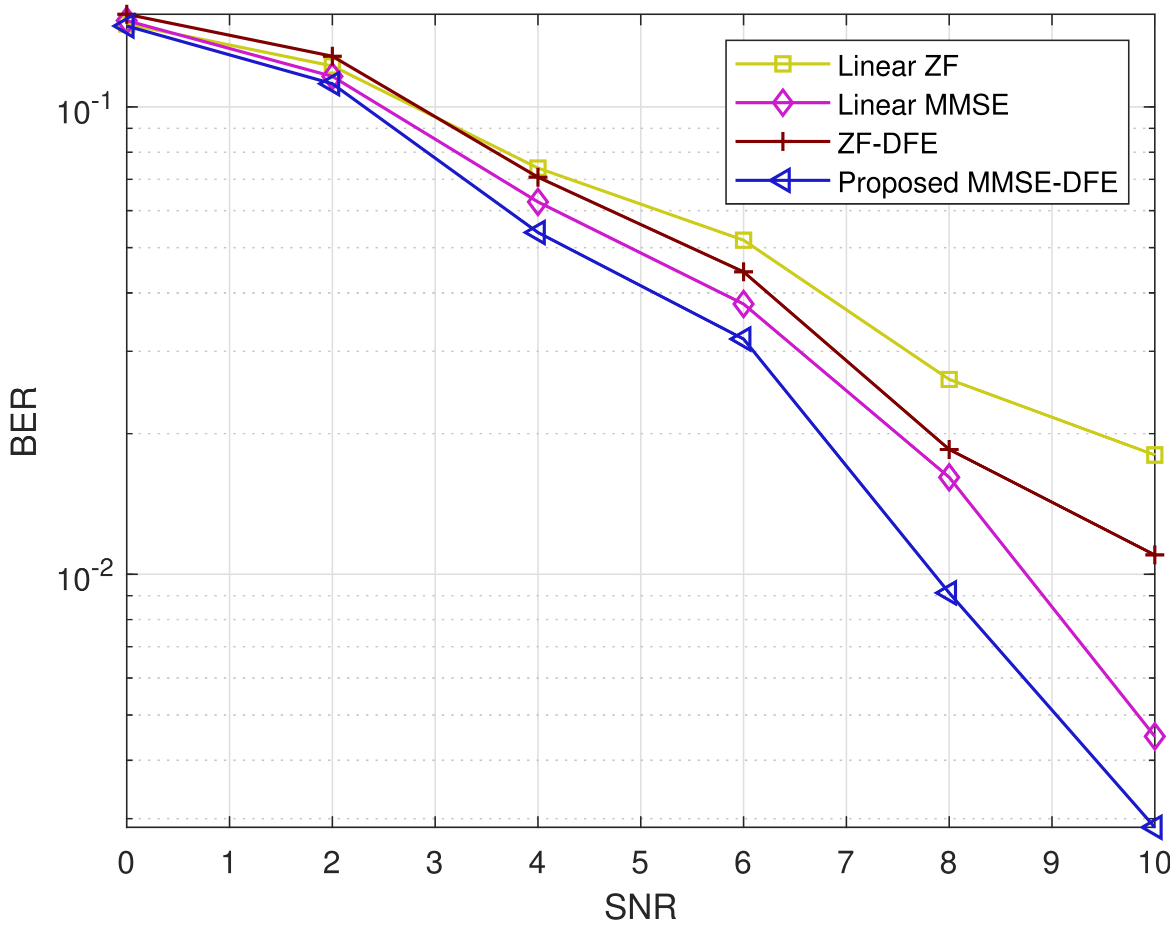


Figure 5.6 BER versus SNR for the proposed MMSE-DFE and for the linear Equalisation design

The linear precoding and equalisation are unsuitable since ZF-DFE designs such as in [102] is too rigid to combat the channel interference.

It is evident from the simulation that the proposed MMSE-DFE design can achieve a considerably higher BER performance than the BER of benchmark systems with linear MMSE designs. For example, at the SNR = 8dB, the proposed designs, MMSE-DFE can achieve a BER of about 10^{-2} while the benchmark designs (ZF-DFE) can only achieve a BER of 0.06.

5.9 Concluding Remark

In this chapter, we have shown how decomposition of multi-user MIMO system can be achieved using the PSVD algorithm. This approach relies on the geometrical information of the channel model, calculating the polynomial EVD. The applied PSVD algorithm iteratively allocates the optimal power spectra for various users at the same time, but it also accounts for the presence of transmission errors.

Section 5.1 present a novel linear joint precoding and equalisation model that includes a decision feedback scheme for multi-user DSL systems. The model can provide improved performance compared with the standard single-user benchmarks. In Section 5.3, we present a linear precoder and equaliser which requires only direct equalisation, where the CSI of the system such as in [123] is perfect. The considered approach here utilises a MMSE-DFE equaliser suggested in [6]. This study applied a similar step to develop a jointly optimal precoder with DFE structure to overcome the problem of imbalances (interference) in the multi-user DSL systems.

In Section 5.7, we have discussed two kinds of channel estimation and equalisation for mitigation crosstalk. In the direct linear joint precoding and channel equalisation design, the redundancy is evenly and jointly shared between the transmitter and receiver so that the interference can be eliminated by reducing the power allocation to the affected channel length. The required channel information rate is kept unchanged throughout the transmission so that the introduced redundancy can be jointly removed with low SNRs. Using the linear ZF approach, the length of the receiving symbol is set equal to the transmitting symbol P_i so that the optimal power allocation can be obtained and the vectored transmission is performed by the receiver only. The perfect channel synchronisation provided by the receiver is used for the selection of the active subchannels with the highest SNRs.

Finally, Subsection 5.8.3 presents joint precoding with DFE equalisation for mitigating the problem associated with imperfect multi-user MIMO channels. Here MMSE-DFE approach is incorporated with PSVD to provides some extra degrees of freedom to the multi-user MIMO system. The simulation results have shown that the MMSE-DFE helps to exploit the optimal power allocation better, leading to a better BER performance over that of the linear optimal ZF-DFE designs. However, evaluating the complexity of the approach is difficult, as the proposed method generally depends on the number of steps required by the PSVD algorithm to converge, as well as the resulting channel lengths of the single channel subsystems.

Chapter 6

Conclusions and Future Works

In this thesis, we explored joint precoding and equalisation techniques for the development of dynamic spectrum management of DSL systems. Due to the limited system's resource, the benefit associated with the application of polynomial matrix decomposition techniques offers a very promising way to improve the joint power allocation - allowing the control of multiple DSL transmission lines/users at a common location. However, due to the number of users involved as well as the tones, the solution to this problem is usually non-convex and thus are difficult to solve. The application of PSVD algorithms is also fascinating because the space-time channel representation of independent users can be exploited more efficiently.

Generally, in the literature, there are two directions for the optimal equalisation design considering the DFE condition. These are classified as the perfect CSI system and the imperfect CSI system.

For the perfect CSI model, the system information is usually available at both the transmitter and receiver, and then the interference can be jointly minimised at the receiver. The performance of such a system can be improved by means of ZF-DFE techniques.

MMSE techniques can be used to remove the interference caused by the imperfect CSI system. This provides more degrees of freedom for vectored transmission since the additive precoded samples need to discarded at the receiver. Note that the adopted MMSE approach in this study implements per-user power constraints.

Considering a polynomial channel matrix model, the PSVD method and MMSE-DFE technique are adopted. This PSVD method uses a two-sided SMD algorithm to calculates a number of vectored subchannels simultaneously and separates the channel gain of each independent user and the crosstalk. Consequently, there is still a need for improvement in the resulting precoding-equalisation design under the per-user power constraint.

With regard to the above issues, and motivated by the idea of crosstalk mitigation strategy, this thesis proposes a new dynamic spectrum management method by applying the MMSE-DFE procedures.

6.1 Summary

The research study is divided into two parts. The first part discusses the implementation of a vectored transmission framework through linear joint optimal precoding and equalisation. This is derived using the PSVD algorithm to eliminate the channel interference on each single-user channel. The use of PSVD based MIMO decomposition not only allocates the optimal transmission power to the multi-user MIMO channels but also helps to indirectly suppress that part of the interference caused by the crosstalk. Based on the iterative PEVD algorithm in [41], a pair of optimal precoding and equalisation matrices is implemented. These decompose the multi-user DSL channel into a number of independent single-user vectored channels.

The second part focuses on using the shortening of the channel matrix to minimise the mean square error. This channel shortening design is investigated from two recently reported methods: first, the received transmission error is mitigated using a joint optimal precoder-equaliser for the perfect MIMO channel; then a joint optimal precoder and MMSE-DFE for the imperfect MIMO channel.

In the vectored transmission design, the aims is to compensate the crosstalk via joint precoding-equalisation structure, which means that a more strict constraint on the single-channel equalisation design is imposed. This strategy results in simple structures for the single transmitter-receiver, at the expense of some more signal processing.

At each step, one subchannel with the largest channel gain is used to transmit the signal. Since the joint optimisation problem in the non-linear problem is still difficult to solve, we have employed an iterative technique. At each iteration, a single-user vectored channel is optimised with the precoding-equalisation link, and this method is shown to converge within a small number of iterations. The optimal result can be found at the expense of exponential computational complexity, which prohibits its implementation in practice.

The proposed methods have been shown to outperform the linear equalisation method. This MMSE-DFE design plays a crucial role in pre-compensating the transmission error for multiple users at the transmitter side and also balances its effect at the receiver end. It is shown that with perfect channel state information, the MMSE-DFE equaliser can perform better than a ZF-DFE. The key advantage is that MMSE-DFE offers some extra degrees of freedom, which provides additional flexibility to control the impaired symbol crosstalk or interference.

6.2 Future Work

Beyond the summary provided in Section 6.1, there are still many research challenges, which appear relevant in the context of multi-user MIMO communications. These include.

- **Implementations** Matlab implementation to simulate a multi-user DSL with more transmitters and receivers for the type proposed in this thesis; to see whether the conclusion for the power spectral allocation model holds.

  Evaluating the MMSE-DFE structure in the polynomial domain for a more involved channel model; to see whether the achievable performance holds.

- **Direct Kogbetliantz approach**

  Rather than calculating the PSVD via a two-sided SMD algorithm — noting that $\boldsymbol{H}(z)\tilde{\boldsymbol{H}}(z)$ and $\tilde{\boldsymbol{H}}(z)\boldsymbol{H}(z)$ will loose most if not all the sparsity of $\boldsymbol{H}(z)$ — Research on the use of a direct Kogbetliantz approach [40] could be advantageous.

- **Low Complexity**

  The current optimisation model with non-linear equalisation in a DSL system takes several minutes to converge. This solution still calls for a low-complexity algorithm.

- **Robust pre-equalisation Designs for imperfect Channel state information**

  This research considered the case where accurate CSI information is available at both sides of the link. While such an assumption may be acceptable for an ideal system, in many practical scenarios knowledge of the CSI is doomed to be imperfectly known at the transmitter due to channel uncertainties. Further research should consider how to apply the PSVD by SMD algorithm discussed in this thesis to robust designs for coping with imperfect CSI information.

- **Maximum Likelihood Estimators**

  In general, the vectored transmission considered in this thesis focused on low complexity and is tractable for practical implementation. An alternative is the maximum likelihood estimation approach which offers better performance was thought to be computationally intractable. It is now feasible to consider the application of a maximum-a-posterior estimator such as the maximum likelihood equaliser to achieve higher performance.

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