Taxation under Oligopoly in a General Equilibrium Setting*

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Abstract

Taxation under oligopoly is analysed in a general equilibrium setting where the firms are large relative to the size of the economy and maximise the utility of their shareholders. Assuming that preferences are either identical and homothetic or identical and quasi-linear, then the oligopoly model is an aggregative game, which greatly simplifies the comparative statics for the effects of taxation. This novel analysis of taxation leads to a number of counterintuitive results that challenge conventional wisdom in microeconomics. A lump-sum tax may increase the price of the oligopolistic good and decrease welfare whereas a profits tax may decrease the price of the oligopolistic good and increase welfare. A profits tax is shown to be superior to a lump-sum tax. Furthermore, in line with conventional wisdom, total tax revenue is always higher with an *ad valorem* tax than with a specific tax that leads to the same price for the oligopolistic good.

Keywords: Oligopoly; General Equilibrium; Aggregative Games; *Ad Valorem* Taxes; Specific Taxes; Profits Taxes; Lump-Sum Taxes.

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1. Introduction

The analysis of taxation under oligopoly is usually undertaken in a partial equilibrium setting that assumes the oligopolistic firms are small relative to the size of the economy and ignores the income effect on demand. However, there is evidence that industries are becoming more concentrated and that firms are becoming more dominant with the rise of superstar firms.[1] In a general equilibrium setting where firms are large relative to the size of the economy, a number of additional effects of the decisions of oligopolistic firms have to be considered. Firstly, the decisions of an oligopolistic firm will affect the income of consumers through their effect on the profits of the firm and through their effect on the profits of its competitors, which will affect the demand facing the firm. Secondly, the decisions of an oligopolistic firm will affect the prices paid for the firm's output by its shareholders, which will affect the utility of the shareholders. This article will show that if preferences are assumed to be either identical and homothetic or identical and quasi-linear then oligopoly in a general equilibrium setting can be analysed as an aggregative game allowing comparative statics for the effects of taxation to be derived quite easily.

The modelling of oligopoly in a general equilibrium setting when oligopolistic firms are large relative to the size of the economy has proved to be problematic for a number of reasons as discussed by Bonanno (1990).[2] Firstly, since the demand of consumers depends upon their income, which includes the profits of the oligopolistic firms paid as dividends, there will be a feedback effect into the objective of the oligopolistic firms that will be a function of their own profits and the profits of their competitors, which complicates the optimisation

[1] See Autor *et al.* (2017) and the report by Tim Harford, Undercover Economist, This is the Age of the Microsoft and Amazon Economy, Financial Times, 19th May 2017. Also, according to The Nokia Effect, The Economist, 25th August 2012, some firms have revenues that are substantial in comparison to the GDP of their country of domicile.

[2] An alternative approach to the modelling of oligopoly in a general equilibrium setting has been to assume that firms are large in their industry, but that the industries are infinitesimally small in the economy as in Neary (2003). Then, the output decisions of an oligopolistic firm have an infinitesimally small effect on prices facing shareholders.

problem facing the firms. To avoid this complication, the optimisation problem has been solved by assuming that there is no income effect as in Hart (1982) or that profits are taxed at 100% as in Guesnerie and Laffont (1978) or that firms take the profits of competitors as given as in Myles (1989). Secondly, if the oligopolistic firms are assumed to maximise profits then the choice of the numeraire good can have a real effect on the equilibrium outcome as shown by Gabszewicz and Vial (1972). Finally, it has been argued that profit maximisation may not be a valid objective for the firms. As explained by Dierker and Grodal (1998, 1999), profit maximisation is only a valid assumption if the firms are price-takers or if the shareholders of the firm do not consume the firm's product. A view supported by Kreps (2013, pp. 200-201):

> *'it is worth noting that the assertion that owners prefer profit maximization is very bound up in the assumption that the firm has no impact on prices. When firms affect prices, and when owners of the firm consume (or are endowed with) the goods whose prices the firm affects, it is no longer clear that the owners either should or do prefer profit-maximizing choices by firms.'*

Similarly, Hart (1985, pp. 106-107) argues for owner utility maximisation as the objective:

> *'The reason is that the owners of a firm are interested not in monetary profits per se, but rather in what this profit can buy. ... This argument suggests that we should substitute owner utility maximization for profit maximization as the firm's goal.'*

When shareholders consume the product of the oligopolistic firm, Dierker and Grodal (1998, 1999) also argue that the objective of the firm should be the maximisation of the real wealth of the shareholders rather than the maximisation of nominal profits, and in this case the choice of numeraire good does not matter so there is no price normalisation problem.[3]

The analysis of taxation under oligopoly has generally been undertaken in a partial equilibrium setting. For example, the incidence of taxes under oligopoly has been analysed by

[3] One might argue that the price normalisation problem is a symptom of the real problem that profit maximisation is not valid when firms can affect the prices facing their shareholders, and it is cured by the use of a valid objective function for the firms such as shareholder utility maximisation.

Seade (1985), Stern (1987), and Dierickx *et al.* (1988) who showed that consumption taxes may lead to price over-shifting (under-shifting) when price increases are larger (smaller) than the tax increase, which is a possibility that does not arise under perfect competition.[4] Another difference between perfect competition and imperfect competition is that specific and *ad valorem* taxes are equivalent under perfect competition, but that equivalence breaks down under imperfect competition. Under monopoly, Suits and Musgrave (1953) showed that an *ad valorem* tax is superior to a specific tax that results in the same tax revenue. This result was extended to the case of oligopoly by Delipalla and Keen (1992) while Anderson *et al.* (2001) showed that an *ad valorem* tax would yield higher tax revenue than a specific tax that results in the same price. Under monopoly, Skeath and Trandel (1994) show that a specific tax can be replaced by Pareto-superior *ad valorem* tax. However, in a general equilibrium setting with a 100% profits tax, Blackorby and Murty (2007) show that the set of Pareto optima with a specific tax is identical to the set of Pareto optima with an *ad valorem* tax. Myles (1996) considers the optimal combination of *ad valorem* and specific taxes. In a general equilibrium setting, where oligopoly is modelled as a strategic market game, Grazzini (2006) claims to show that a specific tax can dominate an *ad valorem* tax, but the result is driven by the effect on income distribution. Only a few authors have analysed taxation under oligopoly in a general equilibrium setting, notably Myles (1989) who derived the Ramsey taxes.

In this article, taxation under oligopoly will be analysed in a simple general equilibrium setting where oligopolistic firms are large in the economy. The income feedback problem will be addressed by assuming that preferences of consumers (workers and shareholders) are either identical and homothetic, or identical and quasi-linear. The objective of the firms will be to maximise the utility of their shareholders, which is equivalent to the maximisation of the real wealth of shareholders in Dierker and Grodal (1998, 1999), and as a result there is no price

[4] See also Häckner and Herzing (2016), Lapan and Hennessy (2011), and Fujiwara (2016)

normalisation problem.[5] Since preferences are identical and homothetic, or identical and quasi-linear, there is no problem aggregating shareholder preferences as they can be modelled using a representative shareholder.[6] With both assumptions about preferences, it turns out that the model is an aggregative game so the equilibrium condition can be expressed as a function of the aggregate output of the oligopolistic industry. The model will be used to analyse the incidence of lump-sum taxes, profits taxes, specific taxes and *ad valorem* taxes, and to compare the revenue raised by specific and *ad valorem* taxes that result in the same price. In contrast to conventional wisdom, lump-sum taxes and profits taxes may affect the equilibrium price under oligopoly. A lump-sum tax will increase the price with homothetic preferences and a profits tax will decrease the price with quasi-linear preferences. Hence, it will be shown that a profits tax is superior to a lump-sum tax.

2. The Aggregative Game Model

Consider an economy with two goods: X and Y, and one factor of production: labour, L. Good X is a homogeneous product produced by an oligopolistic industry and its price is p_X, while good Y is produced by a perfectly competitive industry and its price is p_Y. Labour is supplied by L worker/consumers who each supply one unit of labour (total labour supply is L) and who each receive a wage w. The utility of the lth worker is: $u_{Ll} = u\left(x_{Ll}, y_{Ll}\right)$, for $l = 1,\ldots,L$, where x_{Ll} is consumption of good X and y_{Ll} is consumption of good Y. Utility maximisation yields the indirect utility function of the lth worker: $v_{Ll} = v\left(p_X, p_Y, m_{Ll}\right)$, where m_{Ll} is the income of the lth worker, which is equal to the wage in the absence of any transfers.

[5] According to Dierker and Dierker (2006, page 438) 'Birgit Grodal was determined to solve the question of how a firm should measure the wealth of its shareholders without reference to utility functions'. However, in order to have a tractable model to analyse taxation, maximisation of shareholder utility will be used and it will be assumed that all shareholders have identical preferences.

[6] When shareholders have heterogeneous preferences, there is the usual social choice problem of aggregating the preferences of shareholders, which firms solve by allowing shareholders to vote according to the number of shares that they hold and shareholders presumably vote according to their preferences.

The X industry is a Cournot oligopoly consisting of J firms (if $J=1$ then the industry is a monopoly) owned by K shareholder/consumers with each shareholder owning shares in only one of the oligopolistic firms hence each firm has K/J shareholders.[7] The shareholders are assumed to be price takers when making their consumption decisions, and the utility of the k th shareholder of the j th firm is: $u_{Kkj}=u\left(x_{Kkj},y_{Kkj}\right)$, for $k=1,\ldots,K/J$ and $j=1,\ldots,J$, where x_{Kkj} is the consumption of good X and y_{Kkj} is the consumption of good Y. Utility maximisation yields the indirect utility function of the shareholder/consumer: $v_{Kkj}=v\left(p_X,p_Y,m_{Kkj}\right)$, where m_{Kkj} is the dividend income of the shareholder.[8]

The unit labour input requirement in the X industry is c_X, and the unit labour input requirement in the Y industry is c_Y. The unit labour input requirements are constant in both industries hence there are constant returns to scale in both industries. The labour market is assumed to be perfectly competitive hence there will be full employment of labour, which implies that: $c_X X+c_Y Y=L$, where X is the total output of the X industry and Y is the total output of the Y industry. In the perfectly competitive Y industry, where firms are assumed to be owned by the workers, firms are price takers and maximise profits. Therefore, assuming that both goods are produced in equilibrium, the wage will be equal to the marginal product of labour in the Y industry, $w=p_Y/c_Y$. For simplicity, and without loss of generality as there is no price normalisation problem, the price of good Y will be normalised at unity, $p_Y=1$, and the relative price of the oligopolistic good will be denoted by $P=p_X/p_Y$. The marginal cost

[7] If shareholders each held shares in all of the oligopolistic firms then the maximisation of shareholder utility would lead to a collusive outcome that maximised the joint utility of shareholders. This would be equivalent to the monopoly case where $J=1$ so the results of this article for the effects of taxation would still hold. Also, the structure of shareholdings is exogenous but it is not necessary that all shareholdings are identical, and the interests of all shareholders will be congruent given the assumption of identical and homothetic preferences.

[8] If shareholders were also endowed with labour then there would be an additional effect due to the decisions of the firms affecting the value of the labour endowment and hence the utility of the consumers that would further complicate the analysis.

of producing good X is the labour input requirement multiplied by the wage, $c_X w = c_X / c_Y$, which is the opportunity cost of good X in terms of good Y.

The government can use a number of instruments to tax the oligopolistic industry: a lump-sum tax, T; a profits tax, κ; a specific consumption tax, t; and an *ad valorem* consumption tax, τ. In a general equilibrium setting, the expenditure and/or transfers financed by the tax revenue have to be explicitly modelled therefore, for simplicity, the tax revenue is assumed to be redistributed to the workers as lump-sum transfers.[9] With these various taxes, the profits of an oligopolistic firm in the X industry are:

$$\Pi_j = (1-\kappa)\left[\left(\frac{P}{1+\tau} - c_X w - t\right)X_j - T\right] \qquad j = 1,\ldots,J \tag{1}$$

Since $\sum_{j=1}^{J} X_j = X$, the aggregate profits of the oligopolistic firms is: $\Pi = \sum_{j=1}^{J} \Pi_j$:

$$\Pi(X) = (1-\kappa)\left[\left(\frac{P}{1+\tau} - c_X w - t\right)X - JT\right] \tag{2}$$

The total tax revenue collected by the government from the various taxes on the oligopolistic industry is:

$$R = \frac{\kappa}{1-\kappa}\Pi + tX + \frac{\tau}{1+\tau}PX + JT \tag{3}$$

The aggregate income of the shareholders is equal to the aggregate profits of the oligopolistic firms: $M_K = \Pi$, and the aggregate income of the workers is equal to their wage income plus the transfer they receive from the government, which is equal to total tax revenue:

[9] If the tax revenue was redistributed to the shareholders then the firms would realise that the taxes that they paid were being redistributed to their shareholders and this would lessen the effects of the taxes. In the extreme case of a monopoly where all the tax revenue was redistributed to the shareholders, taxes would not have any effect as the monopolist would realise that any taxes it paid would be received by its shareholders. Alternatively, the tax revenue could be used to finance a public good that benefits all consumers.

$M_L = wL + R$. Therefore, since $\Pi + R = (P - c_X w)X \equiv \overline{\Pi}$, the aggregate income of all consumers is: $M = M_K + M_L = wL + (P - c_X w)X = wL + \overline{\Pi}$, which does not directly depend upon any of the tax rates but only indirectly through their effect on output.

The demand facing a firm will, in general, depend upon the profits of all the oligopolistic firms which depend upon the outputs of all the oligopolistic firms. Therefore, the payoff function of each firm depends upon the payoff functions of all the oligopolistic firms as well as the outputs of all the oligopolistic firms. This makes determining the equilibrium quite intractable without additional assumptions about preferences. However, if it is assumed that preferences are identical and homothetic (*H*) or identical and quasi-linear (*Q*) then the model turns out to be an aggregative game, which can be solved quite easily allowing comparative statics to be obtained. With identical and homothetic preferences (*H*), the Marshallian demand for good X can be written as a function of prices and the aggregate income of the consumers:

$$X = D(p_X)M = D(P)\left[wL + \overline{\Pi}(X)\right] \tag{4}$$

The slope of the Marshallian demand function, holding income constant, will be assumed to be negative, $\partial X/\partial P = M\,\partial D/\partial P < 0$. Clearly, income is not constant as the output decisions of the oligopolistic firms will affect income through their effect on profits. Taking account of the effect on profits of output, the Marshallian demand function (4) can be inverted to yield an inverse demand function for good X:

$$P = P(X; c_X, w, L) = P(X) \tag{5}$$

This inverse demand function $P(X)$ includes the effects of changes in the output of firms on profits and thereby on aggregate income. Obviously, this is not the same as the Marshallian inverse demand function due to the effect of output on profits. The slope of this new inverse demand function (5) can be obtained by totally differentiating (4), which yields:

$$P' = \frac{\partial P}{\partial X} = \frac{P}{X}\frac{1-\theta_{\Pi}}{\eta_X^M + \theta_X} \qquad (6)$$

where $\eta_X^M = P\partial D/\partial p_X/D < 0$ is the usual own price elasticity of the Marshallian demand function obtained by differentiating (4) with respect to P while holding M constant; $\theta_X \equiv PX/M$ is the share of good X in total expenditure; $\theta_{\Pi} = \overline{\Pi}/M$ is the share of profits from the oligopolistic industry in total income, and it follows that: $0 < \theta_{\Pi} < \theta_X < 1$. The slope of this new inverse demand function will be negative if $\eta_X^M + \theta_X < 0$. Henceforth, it will be assumed that the inverse demand function with identical and homothetic preferences is downward sloping, $P'(X) < 0$.

With identical and quasi-linear preferences (*Q*), the Hicksian/Marshallian demand for good *X* for an individual $\tilde{D}(P)$ does not depend upon their income, and aggregating over the $K+L$ individuals yields the aggregate demand: $X = (K+L)\tilde{D}(P)$. This can be inverted to give the inverse Marshallian/Hicksian demand function: $P = P(X)$, which is downward sloping, $\partial P/\partial X = P'(X)$, as the Hicksian demand is always downward sloping.

As the oligopolistic firms in the X industry are large relative to the size of the economy, they each independently and simultaneously choose their output to maximise the utility of their shareholders rather than to maximise profits.[10] If preferences are identical and homothetic (*H*), the utility of the shareholders can be modelled using a representative shareholder. The indirect utility of the representative shareholder of the j th firm is:

$$V_j = v(P)\Pi_j \qquad j = 1,\dots,J \qquad (7)$$

[10] An alternative assumption is that firms maximise real profits defined as nominal profits divided by the true cost of living index of the shareholders. This is equivalent to maximising shareholder utility in this case as the income of shareholders comes entirely from dividends paid by the oligopolistic firm.

Since $\partial V_j / \partial P = \Pi_j \partial v / \partial P$ and $\partial V_j / \partial M_j = v$, Roy's identity implies that demand for good X from shareholders is $X_{Kj} = -\Pi_j \partial v / \partial P / v$. Assuming an interior solution, where all firms produce positive quantities, the first-order conditions for the maximisation of shareholder utility are:

$$\frac{\partial V_j}{\partial X_j} = \Pi_j \frac{\partial v}{\partial P} P' + v \frac{\partial \Pi_j}{\partial X_j} = v \left[(1-\kappa)\left(\frac{P}{1+\tau} + \frac{X_j P'}{1+\tau} - c_X w - t \right) - X_{Kj} P' \right] = 0 \quad (8)$$

Hence, the maximisation of shareholder utility implies that the expression in square brackets is zero.[11] Since the preferences of all consumers are identical and homothetic, the fraction of good X consumed by the shareholders is the same as the fraction of total income received by the shareholders. Hence, consumption of good X by all the shareholders is:

$$X_K = \sum\nolimits_{j=1}^{J} X_{Kj} = \frac{\Pi}{wL + \left(P(X) - c_X w\right) X} X = \frac{\Pi(X)}{wL + \overline{\Pi}(X)} X \quad (9)$$

where $\Pi(X) \equiv (1-\kappa)\left[\left(P(X)/(1+\tau) - c_X w - t \right) X - JT \right]$ and $\overline{\Pi}(X) \equiv \left(P(X) - c_X w \right) X$. Demand for good X from shareholders can be expressed as a fraction of the total output of the oligopolistic industry, and the fraction is a function of total output of the oligopolistic industry.

If preferences are identical and quasi-linear (*Q*), the utility of the shareholders can be modelled using a representative shareholder. Summing the utility of all the shareholder of the jth firm yields the utility of the representative shareholder of the jth firm: $V_j = K\upsilon(P)/J + \Pi_j$. Assuming an interior solution, where all firms produce positive quantities, the first-order conditions for the maximisation of shareholder utility are:

[11] The existence of a Nash equilibrium when firms maximise the real wealth of shareholders is proved in theorem three of Dierker and Grodal (1999).

$$\frac{\partial V_j}{\partial X_j} = \frac{K}{J}\frac{\partial \upsilon}{\partial P}P' + \frac{\partial \Pi_j}{\partial X_j} = (1-\kappa)\left(\frac{P}{1+\tau} + \frac{X_j P'}{1+\tau} - c_X w - t\right) - X_{Kj}P' = 0 \tag{10}$$

With quasi-linear preferences, Roy's identity implies that $\partial\upsilon/\partial P = -\tilde{D}(P)$ so $X_{Kj} = K\tilde{D}(p_X)/J$, and hence $X_K = K\tilde{D}(p_X) = KX/(K+L)$.

Aggregating the expression in square brackets in (8) over all the oligopolistic firms then using (9) yields the equilibrium condition with homothetic preferences (*H*). Similarly, aggregating (10) yields the equilibrium condition with quasi-linear preferences (*Q*):

$$\begin{aligned}
&\Omega(X) \equiv M\Pi(X) - MX(X) = 0 \\
&\text{where } M\Pi(X) \equiv (1-\kappa)\left[J\left(\frac{P(X)}{1+\tau} - c_X w - t\right) + X\frac{P'(X)}{1+\tau}\right] \\
&\text{and } MX(X) = \begin{cases} \dfrac{\Pi(X)}{wL + \bar{\Pi}(X)} XP'(X) < 0 & \text{if } H \\[2ex] \dfrac{K}{K+L} XP'(X) < 0 & \text{if } Q \end{cases}
\end{aligned} \tag{11}$$

Thus, since the equilibrium condition depends upon aggregate output and not the output of the individual firms, the game is aggregative as in Bergstrom and Varian (1985), which will simplify the comparative static analysis.[12] The function $\Omega(X)$ is assumed to be positive when aggregate output is zero if $P(0) > (c_X w + t)(1+\tau)$. Assuming that both goods are produced in equilibrium (incomplete specialisation) then there will be an equilibrium where $\Omega(X^*) = 0$ for some $X^* \in [0, L/c_X]$. A sufficient condition for a unique equilibrium is that $\Omega(X)$ is everywhere strictly decreasing in X, and a necessary condition is that $\Omega(X)$ is strictly decreasing at every equilibrium. Henceforth, it will be assumed that there is a unique

[12] Recently, Cornes and Hartley (2012) and Acemoglu and Jensen (2013) have generalised the concept of aggregative games, which may allow this model to be extended to the case of differentiated products. For applications of aggregative games to oligopoly in a partial equilibrium setting see Anderson *et al.* (2016) and Nocke and Schutz (2018).

equilibrium and that $\Omega(X)$ is strictly decreasing in equilibrium so $\partial\Omega(X^*)/\partial X < 0$.[13] Therefore, it is possible to solve (11) for the unique equilibrium aggregate output as a function of the various tax rates: $X^*(\kappa, t, \tau, T)$.

The term $M\Pi$ in (11) is the *marginal profit effect*, an increase in the output of the firm decreases the utility of the shareholder through its effect on profits, and this is the marginal cost of increasing the output of the firm. The term MX in (11) is the *marginal expenditure effect*, an increase in the output of the firm increases the utility of the shareholder by reducing the price of good X (there is a decrease in the minimum expenditure required by the shareholders to reach a given level of utility), and this is the marginal benefit of increasing the output of the firm. In equilibrium, the *marginal profit effect* is equal to the *marginal expenditure effect*, as shown in figure 1, and the *marginal expenditure effect* is clearly negative so the *marginal profit effect* must also be negative.[14] Marginal profits $M\Pi$ are obviously decreasing in the output of the oligopolistic industry, and marginal expenditure MX is less steep than $M\Pi$ since $\partial\Omega/\partial X = \partial M\Pi/\partial X - \partial MX/\partial X < 0$. Note that the MX unlike $M\Pi$ depends upon the preferences of the shareholders, and that in the case of quasi-linear preferences (*Q*) it is independent of the tax rates. Taxes will shift the $M\Pi$ curve through their effect on revenue and costs of the oligopolistic firms whereas, with homothetic preferences (*H*), they will shift the MX curve through their income effect on shareholders' consumption of the oligopolistic good.

[13] Since the model satisfies the assumptions of a nice aggregative game in Acemoglu and Jensen (2013), their theorem 6 implies that even if there were multiple equilibria the comparative static results for taxes would hold for the smallest and largest equilibrium aggregate outputs

[14] An equivalent condition is obtained by Dierker and Grodal (1998, page 175) when firms maximise the real wealth of shareholders, but price is the strategic variable in their case. Therefore, *marginal profits* and *marginal expenditure* are derivatives with respect to price rather than output, and hence are positive in equilibrium.

3. The Effects of Taxes under Oligopoly

This section will use the aggregative game model of oligopoly in a general equilibrium setting to analyse the effects of a lump-sum tax, T, a profits tax, κ, a specific consumption tax, t, and an *ad valorem* consumption tax, τ. Aggregate welfare, ignoring concerns about income distribution, can be obtained by summing the utility of all consumers since preferences are identical and homothetic. With homothetic preferences, aggregate welfare is $V = v(P)\left[wL + \bar{\Pi}\right]$ so the effect of a change in a tax is obtained by differentiating welfare with respect to $\sigma = \kappa, \tau, t, T$ and using Roy's identity, which yields:

$$\frac{\partial V}{\partial \sigma} = \left(\left[wL + \bar{\Pi}\right]\frac{\partial v}{\partial p}P' + v\left[\left(P - c_X w\right) + XP'\right]\right)\frac{\partial X}{\partial \sigma} = v\left(P - c_X w\right)\frac{\partial X}{\partial \sigma} \tag{12}$$

With quasi-linear preferences, aggregate welfare is $V = (K + L)\upsilon(P) + wL + \bar{\Pi}$ so the effect of a change in a tax is obtained by differentiating welfare with respect to $\sigma = \kappa, \tau, t, T$ and using Roy's identity, which yields:

$$\frac{\partial V}{\partial \sigma} = \left[(K + L)\frac{\partial \upsilon}{\partial p}P' + \left(P - c_X w\right) + XP'\right]\frac{\partial X}{\partial \sigma} = \left(P - c_X w\right)\frac{\partial X}{\partial \sigma} \tag{13}$$

Regardless of preferences, if the change in tax increases (decreases) the output of the oligopolistic industry then welfare will increase (decrease) since it will decrease (increase) the deadweight loss from oligopoly.

3.1 A Lump-Sum Tax on the Oligopolistic Industry

Conventional wisdom from partial equilibrium analysis says that a lump-sum tax on a monopoly or an oligopoly will have no effect on the output or price set by the firms. To obtain the comparative static result for the effect of a lump-sum tax in this general equilibrium setting, totally differentiate (11), which yields:

$$\frac{\partial X^*}{\partial T} = -\frac{\partial \Omega / \partial T}{\partial \Omega / \partial X} = \begin{cases} -\dfrac{1}{\partial \Omega / \partial X}\left[\dfrac{XP'}{wL+\overline{\Pi}}\dfrac{\partial \Pi}{\partial T}\right] = -\dfrac{(1-\kappa)}{\partial \Omega / \partial X}\left[\dfrac{XP'}{wL+\overline{\Pi}}J\right] < 0 & \text{if } H \\ 0 & \text{if } Q \end{cases} \tag{14}$$

With homothetic preferences, a lump-sum tax leads to a decrease in the aggregate output of the oligopolistic industry thereby leading to an increase in the price of the oligopolistic good and a decrease in welfare. With quasi-linear preferences, a lump-sum tax has no effect on aggregate output of the oligopolistic industry thereby leading to no effect on the price of the oligopolistic good and no effect on welfare. These results lead to the following proposition:

Proposition 1: *In a general equilibrium setting, a lump-sum tax on a monopoly or a Cournot oligopoly will result in an increase in the price and a decrease in welfare with homothetic preferences, but no effect on price or welfare with quasi-linear preferences.*

In figure 2, an increase in the lump-sum tax does not shift the $M\Pi$ curve, since it does not affect the marginal profitability of the firms. With homothetic preferences, the MX curve shifts upwards to MX_H^T as the lump-sum tax reduces the income of shareholders and hence the oligopolistic industry will decrease their output from X^* to X_H^T. With quasi-linear preferences, since there is no income effect, the MX does not shift so there is no change in aggregate output. The result with homothetic preferences is contrary to conventional wisdom in a partial equilibrium setting and is due to the income effect of the lump-sum tax on consumption of the oligopolistic good by the shareholders.

3.2 A Profits Tax on the Oligopolistic Industry

Conventional wisdom from partial equilibrium analysis says that a profits tax on a monopoly or oligopoly will have no effect on the output or price set by the firms. To obtain the comparative static result for the effect of a profits tax in this general equilibrium setting, totally differentiate (11) and evaluate at the equilibrium, which yields:

$$\frac{\partial X^*}{\partial \kappa} = -\frac{\partial \Omega / \partial \kappa}{\partial \Omega / \partial X} = \begin{cases} \dfrac{1}{\partial \Omega / \partial X} \dfrac{\Omega\left(X^*\right)}{1-\kappa} = 0 & \text{if } H \\ \dfrac{1}{\partial \Omega / \partial X}\left[J\left(\dfrac{P(X)}{1+\tau} - c_X w - t\right) + X\dfrac{P'(X)}{1+\tau}\right] > 0 & \text{if } Q \end{cases} \tag{15}$$

With homothetic preferences, a profits tax has no effect on the aggregate output of the oligopolistic industry thereby leading to no effect on the price of the oligopolistic good and no effect on welfare. With quasi-linear preferences, a profits tax increases the output of the oligopolistic industry thereby leading to a decrease in the price of the oligopolistic good and an increase in welfare. These results lead to the following proposition:

Proposition 2: *In a general equilibrium setting, a profits tax on a monopoly or a Cournot oligopoly will result in no effect on the price or welfare with homothetic preferences, but will decrease the price and increase welfare with quasi-linear preferences.*

In figure 3, it can be seen that the $M\Pi$ curve swivels anti-clockwise around the profit-maximising output to $M\Pi^{\kappa}$ as marginal profits are multiplied by $\left(1-\kappa\right)$. With homothetic preferences, the MX curve shifts upwards to MX_H^{κ} as the profits received by the shareholders are also multiplied by $\left(1-\kappa\right)$, and hence there is no change in aggregate output. With quasi-linear preferences, since there is no income effect, the MX curve does not shift, and hence aggregate output increases from X^* to X_Q^{κ}.

In a partial equilibrium setting, a lump-sum tax and a profits tax are equivalent as they both raise tax revenue without any deadweight loss. In this general equilibrium setting, propositions 1 and 2 have shown that a profits tax is superior to a lump-sum tax. With homothetic preferences, a profits tax has no effect on welfare whereas a lump-sum tax decreases welfare. With quasi-linear preferences, a profits tax increases welfare whereas a lump-sum tax has no effect on welfare.

3.3 Ad Valorem and Specific Consumption Taxes on the Oligopolistic Industry

To obtain the comparative static result for the effect of a specific consumption tax in this general equilibrium setting, totally differentiate (11) and evaluate at the equilibrium, which yields:

$$\frac{\partial X^*}{\partial t} = -\frac{\partial \Omega/\partial t}{\partial \Omega/\partial X} = \begin{cases} \dfrac{(1-\kappa)}{\partial \Omega/\partial X}\left(J - \dfrac{X^2 P'}{wL + \overline{\Pi}}\right) < 0 & \text{if } H \\ \dfrac{(1-\kappa) J}{\partial \Omega/\partial X} < 0 & \text{if } Q \end{cases} \tag{16}$$

A specific tax leads to a decrease in the aggregate output of the oligopolistic firm thereby leading to an increase in the price and a decrease in welfare.

To obtain the comparative static result for the effect of an *ad valorem* consumption tax in this general equilibrium setting, totally differentiate (11) and evaluate at the equilibrium, which yields:

$$\frac{\partial X^*}{\partial \tau} = -\frac{\partial \Omega/\partial \tau}{\partial \Omega/\partial X} = \begin{cases} \dfrac{(1-\kappa)}{(\partial \Omega/\partial X)(1+\tau)^2}\left[JP + XP' - \dfrac{X^2 PP'}{wL + \overline{\Pi}}\right] & \text{if } H \\ \dfrac{(1-\kappa)}{(\partial \Omega/\partial X)(1+\tau)^2}\left[JP + XP'\right] & \text{if } Q \end{cases} \tag{17}$$

As the *marginal profit* effect is negative in equilibrium, marginal revenue of each firm may be negative so $JP + XP'$ may be negative, and hence the effect of an *ad valorem* tax is ambiguous for the case of quasi-linear preferences. For the case of homothetic preferences, using the equilibrium condition (11) to sign (17) and assuming that all the taxes are positive, yields:

$$\frac{\partial X^{*}}{\partial \tau}=\frac{(1-\kappa)}{(\partial \Omega/\partial X)(1+\tau)}\left[J\left(c_{X}w+t\right)-\frac{XP'}{wL+\overline{\overline{\Pi}}}\left(\left(c_{X}w+t\right)X+T\right)\right]<0 \tag{18}$$

With homothetic preferences, an *ad valorem* tax decreases the aggregate output of the oligopolistic industry thereby leading to an increase in the price and a decrease in welfare. With quasi-linear preferences, if $JP+XP'>(<)0$ then an *ad valorem tax* decreases (increases) aggregate output of the oligopolistic industry thereby leading to an increase (decrease) In price and a decrease (increase) in welfare. These results lead to the following proposition:

Proposition 3: *In a general equilibrium setting, a specific tax will increase the price and decrease welfare with homothetic and quasilinear preferences. With homothetic preferences, an ad valorem tax will increase the price and decrease welfare. With quasi-linear preferences, if* $JP+XP'>(<)0$ *then an ad valorem tax will increase (decrease) price and decrease (increase) welfare.*

One question that arises in the analysis of taxation under oligopoly is whether consumption taxes are over-shifted or under-shifted. A consumption tax is over(under) shifted if $\partial P/\partial t>(<)1$ for a specific tax and $(1+\tau)(\partial P/\partial \tau)/P>(<)1$ for an *ad valorem* tax. It can be shown using (16) and (17), when evaluated at the same initial equilibrium, that $\partial P/\partial t>(1+\tau)(\partial P/\partial \tau)/P$, and therefore a specific tax is more likely to be over-shifted than an *ad valorem* tax, which is consistent with the results of Dierickx *et al.* (1988) in a partial equilibrium setting.

Another question that arises in the analysis of taxation under oligopoly is whether an *ad valorem* tax is superior to a specific tax in the sense that if both taxes lead to the same price then the *ad valorem* tax yields higher tax revenue. Compare the situation when there is a specific consumption tax t (and a zero *ad valorem* tax) with the situation when there is an *ad valorem* consumption tax τ (and a zero specific tax) that both result in the same market price (and aggregate output) in the oligopolistic industry. Assume that the lump-sum tax is equal to zero, $T=0$, and that the profits tax κ is the same in both situations. Conjecture, as in Anderson *et al.* (2001), that setting the specific tax $t=\tau c_X w$ will yield the same aggregate output X in the two situations. Setting $\tau=0$ and $T=0$ yields aggregate profits with the specific tax, denoted Π^t, while setting $t=0$ and $T=0$ yields aggregate profits with the *ad valorem* tax, denoted Π^τ, then using the relationship: $t=\tau c_X w$, yields:

$$\begin{aligned}
&\Pi^t=(1-\kappa)\left(P(X)-c_X w-t\right)X \\
&\Pi^\tau=\frac{(1-\kappa)}{(1+\tau)}\left(P(X)-c_X w(1+\tau)\right)X=\frac{(1-\kappa)}{(1+\tau)}\left(P(X)-c_X w-t\right)X=\frac{\Pi^t}{1+\tau}
\end{aligned} \tag{19}$$

As in Anderson *et al.* (2001), the aggregate profits of the oligopolistic industry with an *ad valorem* tax are lower than with a specific tax. Now consider the equilibrium condition (11) in the situation when there is a specific tax:

$$\Omega^t\left(X^*\right)=(1-\kappa)\left[J\left(P\left(X^*\right)-c_X w-t\right)+X^*P'\left(X^*\right)\right]-\frac{\Pi^t\left(X^*\right)}{wL+\bar{\Pi}\left(X^*\right)}X^*P'\left(X^*\right)=0 \tag{20}$$

To confirm the conjecture that aggregate output is the same in the two situations when $t=\tau c_X w$, one has to show that the same aggregate output X^* solves the equilibrium condition with an *ad valorem* tax. From (11), and using (19), the equilibrium condition in the situation when there is an *ad valorem* tax:

$$\begin{aligned}
\Omega^\tau(X)&=\frac{(1-\kappa)}{1+\tau}\left[J\left(P\left(X^*\right)-c_X w(1+\tau)\right)+X^*P'\left(X^*\right)\right]-\frac{\Pi^\tau\left(X^*\right)}{wL+\bar{\Pi}\left(X^*\right)}X^*P'\left(X^*\right) \\
&=\frac{(1-\kappa)}{1+\tau}\left[J\left(P\left(X^*\right)-c_X w-t\right)+X^*P'\left(X^*\right)\right]-\frac{1}{1+\tau}\frac{\Pi^t\left(X^*\right)}{wL+\bar{\Pi}\left(X^*\right)}X^*P'\left(X^*\right) \\
&=\frac{\Omega^t\left(X^*\right)}{1+\tau}=0
\end{aligned} \tag{21}$$

If the aggregate output X^* solves the equilibrium condition when there is a specific tax then it solves the equilibrium condition when there is an *ad valorem* tax so both taxes yield the same market price. Now compare the total tax revenue of the government, including the profits tax, in the two situations. Total tax revenue when there is a specific tax, denoted R^t, and total tax revenue when there is an *ad valorem* tax, denoted R^τ, are:

$$R^t = \frac{\kappa}{1-\kappa}\Pi^t + tX \qquad\qquad R^\tau = \frac{\kappa}{1-\kappa}\Pi^\tau + \frac{\tau P}{1+\tau}X \tag{22}$$

Using $t = \tau c_X w$ and (19), the difference in tax revenue with the two taxes can be shown to be:

$$R^\tau - R^t = \frac{\kappa}{1-\kappa}\left(\Pi^\tau - (1+\tau)\Pi^\tau\right) + \tau\left(\frac{P}{1+\tau} - c_X w\right)X = \Pi^t - \Pi^\tau \geq 0 \tag{23}$$

Hence, an *ad valorem* consumption tax yields higher tax revenue than a specific consumption tax that results in the same market price.

The method used with homothetic preferences, as in Anderson *et al.* (2001), does not work with quasi-linear preferences so an alternative method must be employed. Comparing the equilibrium condition (11) for an *ad valorem* tax with that for a specific tax, it can be shown that the specific tax that yields the same market price is:

$$t = \frac{\tau}{1+\tau}\frac{JP + XP'}{J} \tag{24}$$

Since $JP + XP'$ may be negative, the specific tax that is equivalent to a positive *ad valorem* tax may be negative (a subsidy). Comparing profits for the case of an *ad valorem* tax with profits for the case of a specific tax, it can be shown that the difference in profits is:

$$\Pi^t - \Pi^\tau = -(1-\kappa)\frac{\tau}{1+\tau}\frac{X^2 P'}{J} > 0 \tag{25}$$

Profits are higher with the specific tax than with the *ad valorem* tax so there will be more revenue from the profits tax with the specific tax than with the *ad valorem* tax. Comparing the total tax revenue from a specific tax with that from an *ad valorem* tax, and using (24) and (25), it can be shown that the difference in total tax revenue is:

$$R^{\tau} - R^{t} = \left(\frac{\tau}{1+\tau}P - t\right)X + \frac{\kappa}{1-\kappa}\left(\Pi^{\tau} - \Pi^{t}\right) = -\left(1-\kappa\right)\frac{\tau}{1+\tau}\frac{X^{2}P'}{J} > 0 \tag{26}$$

Hence, an *ad valorem* consumption tax yields higher total tax revenue than a specific consumption tax that results in the same market price. Note that the difference in total tax revenue is equal to the difference in profits so $R^{\tau} - R^{t} = \Pi^{t} - \Pi^{\tau}$. This leads to the following proposition:

Proposition 4: *In a general equilibrium setting with identical and homothetic preferences or identical and quasi-linear preferences, and no lump-sum tax, an ad valorem consumption tax yields higher total tax revenue than a specific consumption tax that results in the same price.*

This proposition extends the partial-equilibrium results of Delipalla and Keen (1992) and Anderson *et al.* (2001) to a general equilibrium setting. Also, it extends these results by considering the tax revenue from a profits tax in addition to the tax revenue from the consumption tax.[15] The intuition for this result is that both taxes shift the demand curve facing a firm downwards, but an *ad valorem* tax makes the demand curve less steep and more elastic than a specific tax. This implies that the price-cost margin is smaller and hence the tax imposed by the government can be higher with the *ad valorem* tax than with the specific tax. The same intuition explains why a specific tax is more likely to be over-shifted than an *ad valorem* tax.

[15] Note that if the profits tax rate is 100%, $\kappa = 1$, as in Blackorby and Murty (2007, 2013) then the difference in total tax revenue is zero since $\Pi^{\tau} = 0$.

4. Conclusions

Taxation under Cournot oligopoly has been analysed as an aggregative game in a general equilibrium setting where firms are large relative to the economy and oligopolistic firms maximise the utility of shareholders. This novel analysis of taxation has led to a number of counterintuitive results that challenge conventional wisdom in microeconomics. A lump-sum tax was shown to lead to an increase in the price of the oligopolistic good in the case of homothetic preferences, and to have no effect on the price in the case of quasi-linear preferences. The former result is counterintuitive and contrasts with conventional wisdom. A profits tax was shown to have no effect on the price of the oligopolistic good for the case of homothetic preferences, and to lead to a decrease in the price in the case of quasi-linear preferences. The latter result is counterintuitive and contrasts with conventional wisdom. With both assumptions about preferences, a profits tax was shown to be superior to a lump-sum tax. Furthermore, in line with conventional wisdom, it was shown that total tax revenue is always higher with an *ad valorem* tax than with a specific tax that leads to the same price of the oligopolistic good with both homothetic and quasi-linear preferences.

The tractable analysis of taxation under Cournot oligopoly in a general equilibrium setting has been possible because it was modelled as an aggregative game, which required some simplifying assumptions about preferences. The assumption of identical and homothetic preferences or identical and quasi-linear preferences allowed consumer preferences to be aggregated and allowed the preferences of shareholders to be aggregated so that the firms could maximise the utility of a representative shareholder. Without these assumptions about preferences, the model would not be an aggregative game and the analysis of taxation under oligopoly in a general equilibrium setting would probably be intractable. However, given that the partial equilibrium analysis of taxation assumes that preferences are quasi-linear, extending the analysis to the case of homothetic preferences seems like a step forward.

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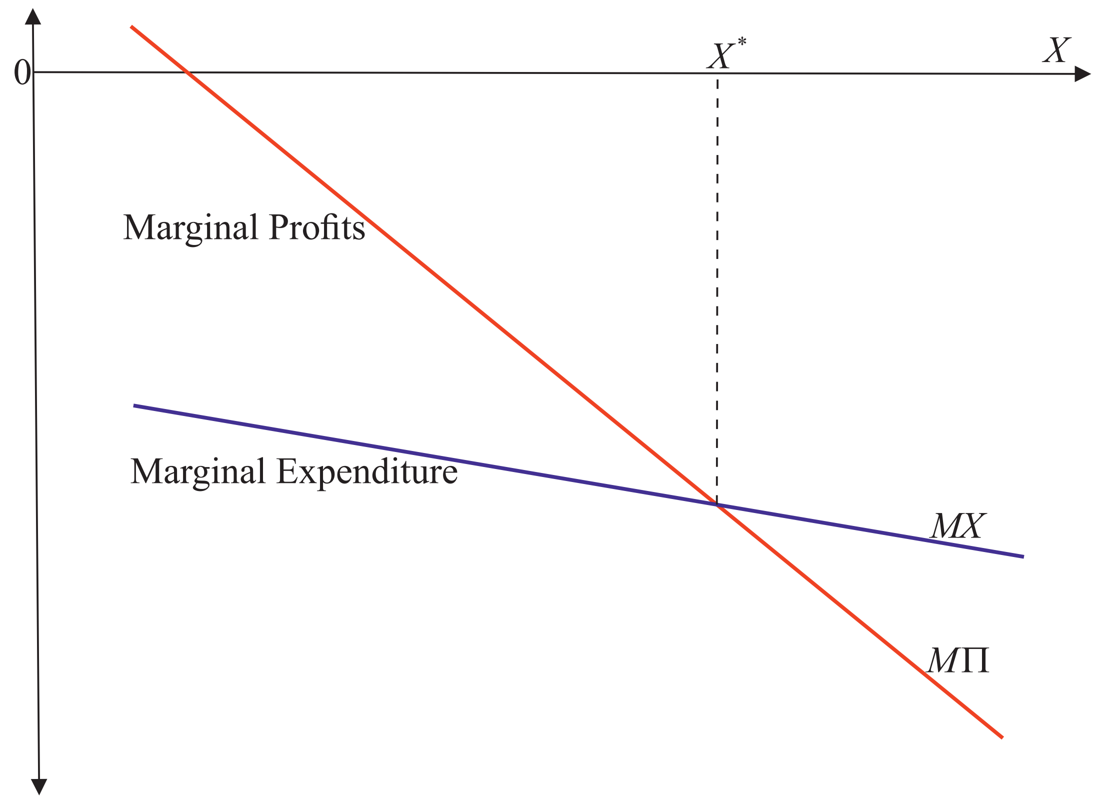

Figure 1: Marginal Profits and Marginal Expenditure

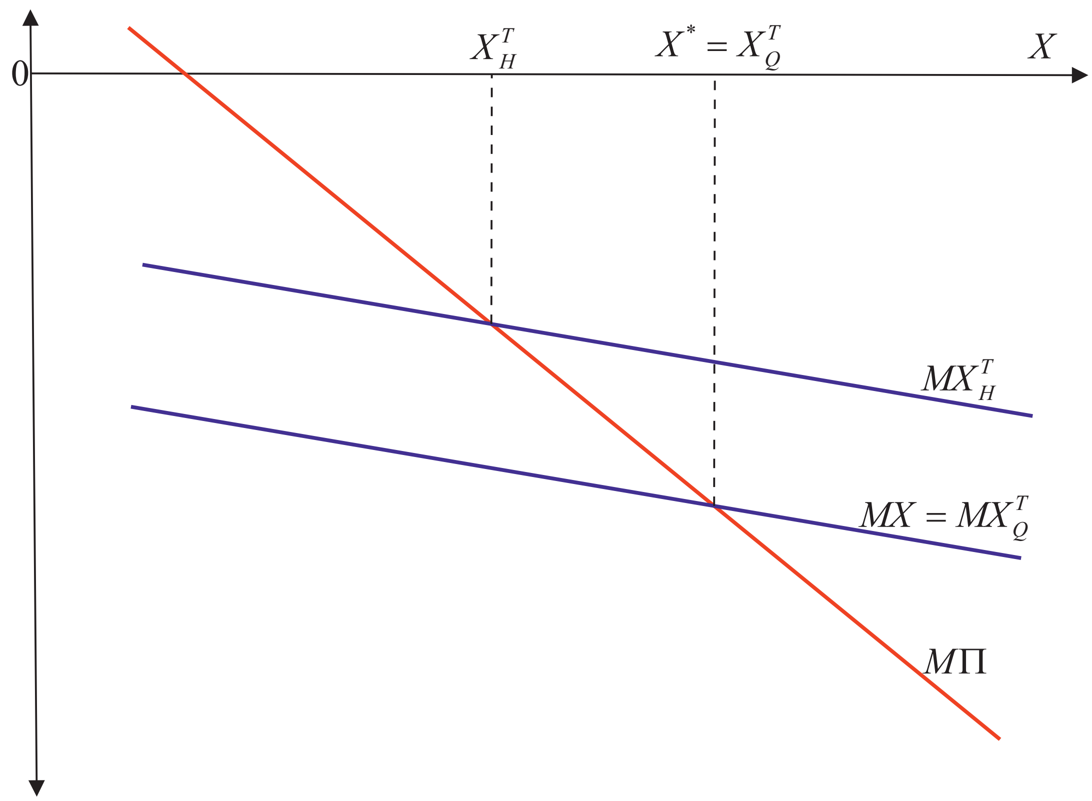

Figure 2: Lump-Sum Tax

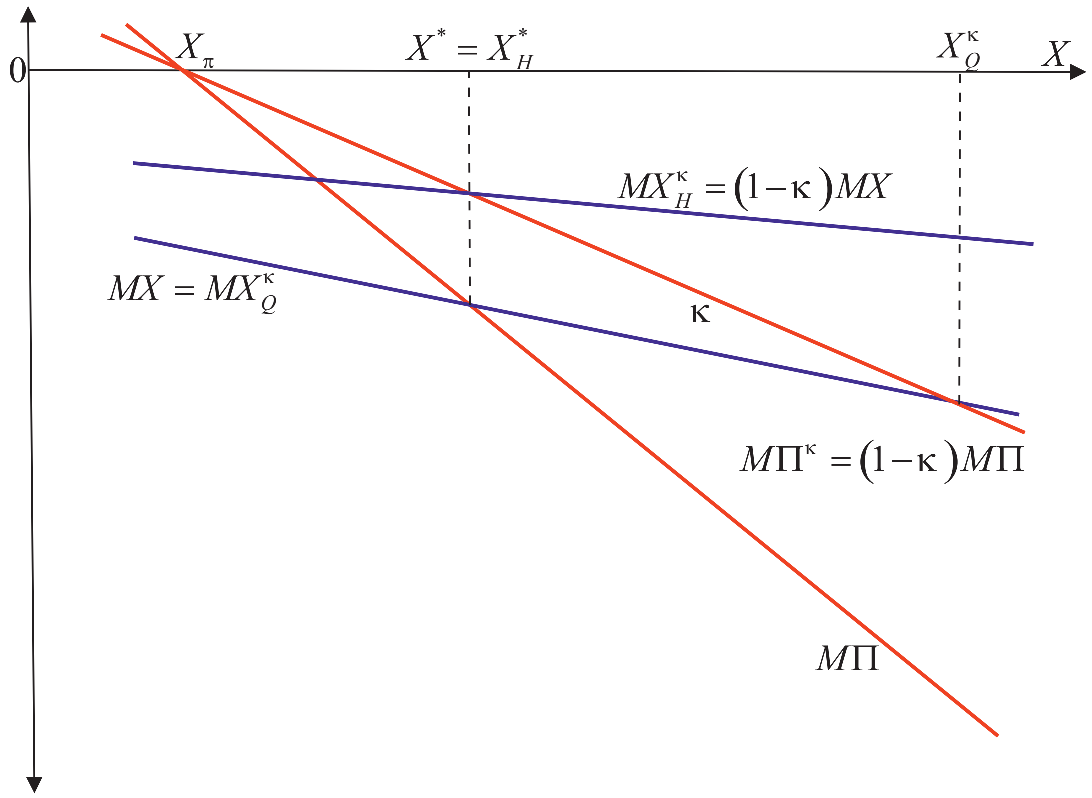

Figure 3: Profits Tax