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## A Decentralized Voltage Control Strategy of Soft Open Points in Active Distribution Networks

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### Abstract

With the integration of high shares of distributed generators (DGs), it is increasingly difficult to cope with the risk of voltage violations and puts forward a higher requirement for the operational flexibility in active distribution networks (ADNs). Soft open point (SOP) is a novel power electronic device which can realize accurate power flow control and continuous voltage regulation. Currently, the centralized control strategy is mainly used to operate SOPs. However, the heavy burden of communication and complex global optimization processing will hinder its fast response to the frequent voltage fluctuations. This paper proposes a decentralized voltage control strategy of SOPs in ADNs. Based on the results of network partition, the alternating direction method of multipliers (ADMM) algorithm is applied to realize the decentralized optimization of the transmission power of SOPs among connected areas. The near-global optimal solution can be obtained without the huge calculation burden. The potential benefits of SOPs are fully explored to reduce power losses and improve the voltage profile of ADNs. Finally, the effectiveness of the decentralized voltage control strategy of SOPs is validated on the PG&E 69-node distribution system.

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*Keywords:* active distribution network (ADN); distributed generation (DG); soft open point (SOP); decentralized voltage control

### 1. Introduction

With the integration of high shares of distributed generators (DGs), it is increasingly difficult to cope with the risk of voltage violations and puts forward a higher requirement for the operational flexibility in active distribution networks (ADNs). However, the current regulation devices such as switchable capacitor banks (CBs) and static var

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Nomenclature		
<b>Sets</b>		
$\mathcal{N}$	set of all nodes except source node	$P_i^{\text{DG}}, Q_i^{\text{DG}}$ active/reactive power capacity of DG
$\mathcal{L}$	set of all branches	$P_i^{\text{SOP}}, Q_i^{\text{SOP}}$ active/reactive power injection by SOP
<b>Parameters</b>		
$N_a$	set of nodes in area $a$	$N_N$ total number of the nodes
$N_o$	set of overlapping nodes	$\alpha, \beta$ weight coefficients
<b>Indices</b>		
$i, j, h, k$	indices of nodes, from 1 to $N_N$	$r_{hk}, x_{hk}$ resistance/reactance of branch
$a$	indices of areas, from 1 to $S$	$u_{\text{thr}}^{\text{max}}, u_{\text{thr}}^{\text{min}}$ upper/lower limit of desired squared voltage range
<b>Variables</b>		
$u_i, l_{ij}$	squared voltage/current magnitude	$P_i^{\text{LOAD}}, Q_i^{\text{LOAD}}$ active/reactive power consumption
$P_i, Q_i$	total active/reactive power injection	$u^{\text{max}}, u^{\text{min}}$ upper/lower limit of squared voltage
$P_i^{\text{SOP,L}}$	active power losses of SOP	$l^{\text{max}}$ upper limit of squared current
$P_{ji}, Q_{ji}$	active/reactive power flow of branch	$A_i^{\text{SOP}}$ loss coefficient of SOP
		$S_i^{\text{SOP}}$ capacity limit of SOP
		$\varepsilon$ the predetermined tolerance

compensator (SVC) mainly aim at reactive power adjustment. As the voltage distribution is also significantly impacted by the active power flow, it is difficult for ADNs to mitigate voltage violations just relying on the traditional reactive regulation devices [1]. Soft open point (SOP) is a novel power electronic device enabling the flexible connection between feeders [2]. The frequent voltage violation due to the fluctuation of DG outputs can be effectively eliminated by the accurate and fast power flow control of SOPs. Thus, it is of great significance to make full use of the voltage regulation potential of SOP in ADNs.

Currently, the centralized control strategy is mainly used to operate SOPs. However, the heavy burden of communication and complex global optimization processing will hinder its fast response to the frequent voltage fluctuations [3]. By using the local information of each area and boundary interaction among connected areas, the decentralized control method has significant advantages of high computation efficiency and strong reliability. Therefore, the decentralized control strategy is more suitable for the real-time response of SOPs to the fluctuations.

Thus, a decentralized voltage control strategy of SOPs in ADNs is proposed in this paper. Based on the results of network partition, the alternating direction method of multipliers (ADMM) algorithm is applied to realize the decentralized optimization of the transmission power of SOPs among connected areas. The near-global optimal solution can be obtained without the huge calculation burden. The potential benefits of SOPs are fully explored to reduce power losses and improve the voltage profile of ADNs. Finally, the effectiveness of the decentralized voltage control strategy of SOPs is validated on the PG&E 69-node distribution system.

## 2. Network partition based on sensitivity analysis

Due to the large scale of distribution networks and the severely uneven distribution of DGs in the spatial dimension, the optimization of whole network is increasingly complex. Thus, the partition of whole network can simplify the network topology in each partition area, which is beneficial to the solving procedure of optimization problems.

### 2.1. Principle of network partition

With the high penetration of DGs, the topology and operation of distribution networks have been changed significantly. Regard DGs as the center of each area and determine the belonging of neighboring nodes by the threshold theory. If the voltage change of node caused by DG outputs is greater than the threshold, the node is included in the area where this DG belongs to. It means that the node is evidently affected by the DG of this area. Otherwise, the node is excluded from this area [4].

In a radial distribution network, there is only one path between node  $i$  and source node. For each node  $i \in \mathcal{N}$ ,  $\mathcal{L}_i \subseteq \mathcal{L}$  is defined as a line set of the unique path from node  $i$  to the source node. The overall line resistance  $R_{ij}$  and line reactance  $X_{ij}$  of the unique path from node  $i$  to the source node are introduced:

$$R_{ij} := \sum_{(h,k) \in \mathcal{L}_i \cap \mathcal{L}_j} r_{hk}, X_{ij} := \sum_{(h,k) \in \mathcal{L}_i \cap \mathcal{L}_j} x_{hk} \quad (1)$$

$R_{ij}, X_{ij}$  can be also regarded as the voltage-to-injected power sensitivity factors, namely the impact of power injection at node  $j$  on the voltage at node  $i$ .

$$R_{ij} = \partial V_i / \partial P_j, X_{ij} = \partial V_i / \partial Q_j \quad (2)$$

When the active and reactive power injection of each node varies in distribution networks, the corresponding voltage change at node  $i$  can be expressed as:

$$\Delta V_i = \sum_{j=1}^{NN} (R_{ij} \cdot \Delta P_j + X_{ij} \cdot \Delta Q_j) \quad (3)$$

The node voltages vary from time to time due to the fluctuation of DGs and loads in different time periods. The network partition can be dynamically updated according to the voltage changes of nodes.

## 2.2. Procedure of network partition

The procedure of network partition is as follows:

1) The sensitivity factors of nodes are analysed and the voltage change of each node caused by a single DG output is obtained;

2) Regard DGs as the center of each area and determine the belonging of each node by the threshold theory. If the voltage change of node caused by DG outputs is greater than the threshold, the node is included in the area where this DG belongs to. Otherwise, the node is excluded from this area.

3) If there are overlaps in the partition results obtained by step 2), the nodes belong to the area which causes the maximum voltage change. When the influence range of area contains the DGs of another area, two areas are merged into a larger area;

4) The network partition can be dynamically updated according to the fluctuation of DGs and loads.

5) Repeat step 2) until the end of the partitioning process.

## 3. Decentralized voltage control problem formulation with SOPs

In this section, the centralized voltage control model with SOPs is reviewed firstly. Based on the results of network partition, the ADMM algorithm is applied to realize the decentralized optimization of the transmission power of SOPs, which effectively reduces power losses and improves the voltage profile of ADNs.

### 3.1. Original centralized optimization model with SOPs

Based on fully controlled power electronics, SOPs realize the flexible connection between feeders. By adjusting the power flow between the connected feeders in real time, SOP provides effective voltage and reactive control of the connected feeders to improve voltage profile. The centralized voltage control model with SOPs is established.

#### 1) Objective function

Considering the economic operation and voltage profile of ADNs simultaneously, a linear weighted combination of minimum total power losses and voltage deviations is taken as the objective function.

$$\min f = \alpha f_L + \beta f_V$$

$$f_L = \sum_{ij \in \mathcal{L}} r_{ij} l_{ij} + \sum_{i=1}^{NN} P_i^{\text{SOP,L}} \quad (4)$$

$$f_V = \sum_{i=1}^{NN} |u_i - 1|, \quad u_i \geq u_{\text{thr}}^{\text{max}} \parallel u_i \leq u_{\text{thr}}^{\text{min}}$$

where  $f_L$  denotes the power losses and  $f_V$  denotes the extent of voltage deviation. The weight coefficients  $\alpha$  and  $\beta$  satisfy  $\alpha + \beta = 1$ .

#### 2) System power flow constraints

$$\sum_{ji \in \mathcal{L}} (P_{ji} - r_{ji} l_{ij}) + P_i = \sum_{ik \in \mathcal{L}} P_{ik}$$

$$\sum_{ji \in \mathcal{L}} (Q_{ji} - x_{ij} l_{ij}) + Q_i = \sum_{ik \in \mathcal{L}} Q_{ik}$$

$$u_i - u_j + (r_{ij}^2 + x_{ij}^2) l_{ij} = 2(r_{ij} P_{ij} + x_{ij} Q_{ij}) \quad (5)$$

$$l_{ij} u_i = P_{ij}^2 + Q_{ij}^2$$

$$P_i = P_i^{\text{DG}} + P_i^{\text{SOP}} - P_i^{\text{LOAD}}$$

$$Q_i = Q_i^{\text{DG}} + Q_i^{\text{SOP}} - Q_i^{\text{LOAD}}$$

#### 3) Secure operation constraints

$$u^{\text{max}} \leq u_i \leq u^{\text{min}}, \quad l_{ij} \leq l^{\text{max}} \quad (6)$$

#### 4) SOP operation constraints

The operational constraints of SOPs mainly include the active power transmission and reactive power compensation of SOPs.

$$\begin{aligned}
 P_i^{\text{SOP}} + P_j^{\text{SOP}} + P_i^{\text{SOP,L}} + P_j^{\text{SOP,L}} &= 0 \\
 P_i^{\text{SOP,L}} &= A_i^{\text{SOP}} \sqrt{(P_i^{\text{SOP}})^2 + (Q_i^{\text{SOP}})^2}, P_j^{\text{SOP,L}} = A_j^{\text{SOP}} \sqrt{(P_j^{\text{SOP}})^2 + (Q_j^{\text{SOP}})^2} \\
 -S_i^{\text{SOP}} &\leq P_i^{\text{SOP}} \leq S_i^{\text{SOP}}, -S_j^{\text{SOP}} \leq P_j^{\text{SOP}} \leq S_j^{\text{SOP}} \\
 (P_i^{\text{SOP}})^2 + (Q_i^{\text{SOP}})^2 &\leq 2 \frac{P_i^{\text{SOP,L}}}{\sqrt{2}A_i^{\text{SOP}}} \frac{P_i^{\text{SOP,L}}}{\sqrt{2}A_i^{\text{SOP}}}, (P_j^{\text{SOP}})^2 + (Q_j^{\text{SOP}})^2 \leq 2 \frac{P_j^{\text{SOP,L}}}{\sqrt{2}A_j^{\text{SOP}}} \frac{P_j^{\text{SOP,L}}}{\sqrt{2}A_j^{\text{SOP}}}
 \end{aligned} \tag{7}$$

As a consequence, constraints (4)-(7) form the centralized optimization model of voltage control with SOPs. The proposed model is mathematically a mixed integer second-order cone programming model, which can be effectively solved by commercial optimization software such as CPLEX and MOSEK.

### 3.2. Decentralized voltage control strategy of SOPs based on ADMM

To meet the requirement of voltage control in large-scale ADNs, the ADMM algorithm is applied to realize the decentralized optimization of the transmission power of SOPs among connected areas. The near-global optimal solution can be obtained without the huge calculation burden, which is suitable for the fast response to the voltage fluctuations.

In Section 2, the distribution network is divided into  $S$  areas, which are interconnected by connecting lines or SOPs. Let  $N_a$  denote the node set in area  $a$ . For any area  $a$ ,  $N'_a$  comprises both  $N_a$  and the other end of connecting lines or SOPs. The SOP-based voltage control model in Section 3 can be expressed in the following compact form:

$$\begin{aligned}
 \min f &= \min \sum_{a=1}^S f_a(\mathbf{x}) \\
 \text{s. t. } &\begin{cases} \mathbf{g}_a(\mathbf{x}) \geq 0 \\ \mathbf{h}_a(\mathbf{x}) = 0, \quad a = 1, 2, \dots, S \\ \underline{\mathbf{x}} \leq \mathbf{x} \leq \bar{\mathbf{x}} \end{cases}
 \end{aligned} \tag{8}$$

where  $\mathbf{x} := (x_1, \dots, x_S)^T$  is a global state vector.  $\mathbf{g}_a(\mathbf{x})$  and  $\mathbf{h}_a(\mathbf{x})$  denotes the inequality constraints and the equality constraints in area  $a$ , respectively.  $f_a(\mathbf{x})$  represents the objective function corresponding to area  $a$ .

Let set  $A_i := \{a | i \in N'_a\}$  denote the areas that contain node  $i$ . Set  $N_0 := \{i | |A_i| > 1\}$  contains all the overlapping nodes. Let  $\varphi_i := \{P_i, Q_i, P_i^{\text{SOP}}, Q_i^{\text{SOP}}, u_i, P_{ji}, Q_{ji}, l_{ji} | \forall j: j \rightarrow i\}$  be a global state vector corresponding to node  $i$ . If area  $a$  belongs to the set  $A_i$ , the local variables  $x_{a,i}$  equal to the related global variables  $\varphi_i$  corresponding to area  $a$ . Model (8) can be reformulated as follows:

$$\begin{aligned}
 \min f &= \min \sum_{a=1}^S f_a(\mathbf{x}) \\
 \text{s. t. } &\begin{cases} \mathbf{g}_a(\mathbf{x}) \geq 0 \\ \mathbf{h}_a(\mathbf{x}) = 0, \quad a = 1, 2, \dots, S \\ \underline{\mathbf{x}}_a \leq \mathbf{x} \leq \bar{\mathbf{x}}_a \\ x_{a,i} = \varphi_i \quad \forall i \in N_0 \cap N'_a \end{cases}
 \end{aligned} \tag{9}$$

The above model can be effectively solved by the ADMM algorithm [5]. To further eliminate the global variables  $\varphi_i$ , auxiliary variables  $\omega_{a,i} = \varphi_i - \lambda_{a,i}/\rho$ ,  $\forall i \in N_0 \cap N'_a$  are introduced to achieve fully decentralized calculation.  $\lambda_{a,i}$  denotes the Lagrangian multiplier vector of equality constraints, and  $\rho$  is the penalty parameter of the ADMM algorithm. The decentralized voltage control model of SOPs can be described as follows:

$$\begin{cases} x_a^{k+1} = \arg \min_{x_a \in X^a} \left[ f_a(x_a) + \frac{\rho}{2} \sum_{\forall i \in N_0 \cap N'_a} \|x_{a,i} - \omega_{a,i}^k\|_2^2 \right] \\ \omega_{a,i}^{k+1} = \omega_{a,i}^k + x_{b,i}^{k+1} - \frac{x_{a,i}^k + x_{b,i}^k}{2}, \quad \forall i \in N_0 \cap N'_a, b = A_i \setminus a \end{cases} \tag{10}$$

where  $X^a$  is a feasible set for area  $a$ , and  $k$  denote the iteration index.  $x_a$  can be initialized as the power flow solution  $x_{a,i}^0$ , and the auxiliary variable  $\omega_{a,i}^0 = (x_{a,i}^0 + x_{b,i}^0)/2$ . The criterion for iterative convergence can be determined by the original residual and the dual residual, as shown in constraint (11).

$$\delta^k = \left\| \begin{pmatrix} r^k \\ d^k \end{pmatrix} \right\|_{\infty} < \varepsilon \quad (11)$$

where  $r^k$  is defined as the original residual of the feasibility of the original problem, and  $d^k$  is defined as the dual residual of the feasibility of the dual problem,  $\varepsilon$  is the given predetermined tolerance.

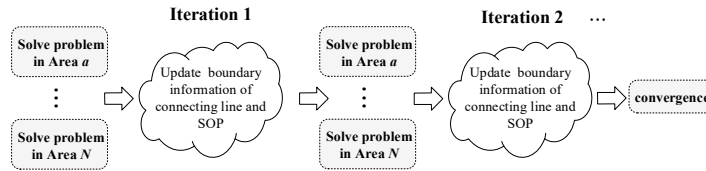


Fig. 1 Schematic of the decentralized solving procedure.

Fig. 1 shows the decentralized optimization process among areas, in which the power transmission of connecting lines and SOPs can be updated according to the latest boundary information. The paralleled calculation can be used for voltage control in each area to further improve the computational efficiency.

#### 4. Case study

In this section, the effectiveness of the proposed decentralized voltage control strategy of SOPs is verified on the modified PG&E 69-node system, as shown in Fig. 2. Ten photovoltaic generators (PVs) are integrated at node 3, 19, 20, 27, 34, 38, 48, 51, 54 and 66, with a capacity of 300kVA each. All the PVs are operated at a unit power factor. Two groups of SOPs with a capability of 1000kVA are installed. It is assumed that the loss coefficient of each inverter for SOP is 0.02. The lower and upper bounds of system voltage are set from 0.90 p.u. to 1.10 p.u. The weight coefficients  $\alpha$  and  $\beta$  are set to 0.7 and 0.3 by AHP. The penalty parameter  $\rho$  is set to 1.0 and the predefined precision  $\varepsilon$  in ADMM algorithm is set as 1.0e-3 to ensure the accuracy and speed of convergence.

##### 4.1. Network partition of modified PG&E 69-node system

Based on the network partition method described in Section 2, the modified PG&E 69-node system is divided into four areas. Take area 3 as an example, the PVs connected at node 19, 20 and 27 have the similar voltage influence range. The partition of whole network can simplify the network topology in each partition area, which facilitates to the solving procedure of optimization problems.

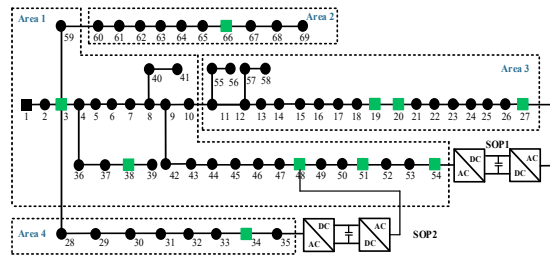


Fig. 2. Network partition of the modified PG&E 69-node system.

##### 4.2. Optimization result analysis

Three scenarios are adopted to verify the effectiveness of the proposed decentralized voltage control strategy of SOPs:

Scenario I: There is no control strategy conducted on SOPs, and the initial operation state of ADNs is obtained.

Scenario II: The proposed decentralized voltage control strategy is conducted on SOPs.

Scenario III: SOPs are regulated by the centralized voltage control strategy to realize global optimization.

Table 1. Optimization results of three scenarios.

	Scenario I	Scenario II	Scenario III
Power losses(kW)	108.09	21.98	21.95
Minimum voltage of ADN (p.u.)	0.9504	0.9811	0.9814
Maximum voltage of ADN(p.u.)	1.0054	1.0013	1.0008

The optimization results of the three scenarios are listed in Table 1. Compared with Scenario I, the proposed decentralized control strategy in Scenario II effectively mitigate voltage deviation and reduce power losses of whole networks. It can be seen that the proposed strategy has a similar performance to the centralized strategy in Scenario III, which fully utilizes the active and reactive outputs of SOPs. Considering the proposed strategy is based on less measurement information, it could reduce the computational burden as well as achieve the near-global optimal solution.

Table 2. Power transmission of SOPs in Scenario II and Scenario III.

Scenario	Active power transmission of SOP(MW)				Reactive power compensation of SOP(Mvar)			
	SOP1-27	SOP1-54	SOP2-35	SOP2-48	SOP1-27	SOP1-54	SOP2-35	SOP2-48
Scenario II	-0.3105	0.2970	-0.2209	0.2091	0.2975	0.4713	0.2095	0.5943
Scenario III	-0.3104	0.2972	-0.2213	0.2093	0.2972	0.4712	0.2093	0.5942

The power transmission of SOPs in Scenario II and Scenario III are shown in Table 2, which proves the accuracy of the proposed method. Fig. 3 shows the convergence of the residual described in (11). The residual drops rapidly at the beginning until the proposed D-SOCP converges to predefined accuracy  $\epsilon$ , and then decreases at a slower rate. An accuracy of  $1.0e-3$  is often sufficient for practical applications of ADNs. Relative to Scenario I, voltage profiles of ADNs are significantly improved in Scenario II and in Scenario III, as shown in Fig. 4.

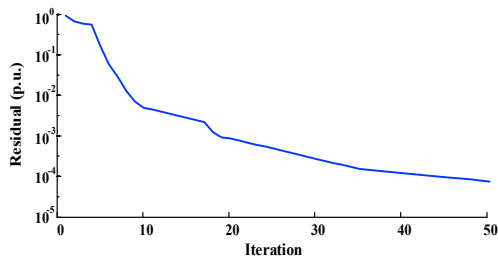


Fig. 3 Convergence of residual in Scenario II.

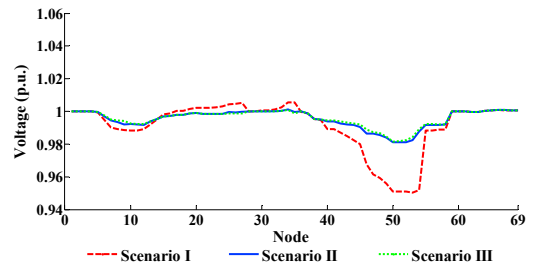


Fig. 4 Voltage profile in three scenarios.

## 5. Conclusion

The integration of high shares of DG causes severe voltage violations and puts forward a higher requirement for the operational flexibility in ADNs. This paper proposes a decentralized voltage control strategy of SOPs in ADNs. Based on the results of network partition, the ADMM algorithm is applied to realize the decentralized optimization of the transmission power of SOPs among connected areas. The near-global optimal solution can be obtained without the huge calculation burden. Then, the effectiveness of the decentralized voltage control strategy of SOPs is validated on the PG&E 69-node distribution system. The potential benefits of SOPs are fully explored to reduce power losses and improve the voltage profile of ADNs.

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