A Regional Computable General Equilibrium Model of Wales for Tax Policy Analysis

By

Long Zhou

A Thesis Submitted in Fulfilment of the Requirements for the Degree of Doctor of Philosophy of Cardiff University

Economics Section of Cardiff Business School, Cardiff University

May 2019

ABSTRACT

Under the background of ongoing regional tax devolution in Wales, the development of new regional economic models has been needed to understand tax policy variations. This thesis develops a Computable General Equilibrium model of Wales for tax policy analysis. This model is a static, multi-sector and single-regional model. A Social Accounting Matrix is also developed as the benchmark database for the model. It features 21 sectors, 1 representative household, 2 external agents, 7 types of taxes and 3 production factors, and is balanced with various methods. Unknown model parameters are calibrated by the data information contained in the SAM. The model can be solved to replicate the benchmark SAM and the simulation is conducted regarding three taxes: Stamp Duty Land Tax, Corporation Tax and Income Tax; and three time frames: short, medium and long run. The time frames are defined according to different degrees of factor mobility. The whole simulation is also run with sensitivity analysis that three elasticity values regarding substitution between production factors are examined: 0.5, 1 and 1.5. For all the taxes, the simulation results generally give negative effects in the short run, and only in the medium to long run there appears expected reasonable results. The results of SDLT variation effects generally suggest that narrowing the gap between residential and non-residential SDLT rates has slightly more impact than simply cut of both rates. In this case, the mutual drag between residential and non-residential property prices can be observed. Generally, a relatively lower elasticity of substitution tends to deliver more significant economy-wide effects than a high elasticity. This implies that it is important to avoid over-substitution effect between production factors so as to produce better results in response of a tax variation shock.

ACKNOWLEDGEMENT

This thesis would not have been possible without the continuing support of my supervisor Prof. Max Munday and his truthful dedication to the academic and supervisory work. Notably, his actual accomplishment is not only to provide advice when required, but to establish a relaxing environment where the productivity can flourish. My sincerest gratitude belongs to him. In addition, I would like to thank Prof. Kent Matthews for his always warm and encouraging smile and unconditional trust in me. His valuable experience and suggestions on the modelling work has given me massive confidence. This certainly constitutes a critical juncture in my life. Great thanks also go to Prof. Patrick Minford. As an influential economist, he led me into the world of macroeconomics whose advice is always gratefully appreciated. Without his help and encouragement I cannot finish this thesis with peace of mind. I would also like to express my gratitude to Dr. Andrew Crawley. He provides so generous technical help on my modelling practice. I owe gratitude to Dr. Annette Roberts and Neil Roche, for your continuous support and encouragement. This thesis is funded by the Welsh Government and I truly appreciate for the generous financial support.

The final words belong to my family. I am particularly grateful to my mother. The meaning and influence of her in my life extends far beyond my ostensible academic achievements. I am so grateful for my beloved wife’s support and patience during these years, especially for her understanding for all the days and nights I am absent. At last, this dissertation is dedicated to my son, Baichen, as you are my power and strength. I keep looking forward to our joint future.

DECLARATION

This work has not been submitted in substance for any other degree or award at this or any other university or place of learning, nor is being submitted concurrently in candidature for any degree or other award.

Signed …………Long Zhou ………………………………… (candidate)
Date ……………29/05/2019……..…………….………

STATEMENT 1

This thesis is being submitted in partial fulfillment of the requirements for the degree of …PhD……(insert MCh, MD, MPhil, PhD etc, as appropriate)

Signed …………Long Zhou……… …………………….……………. (candidate)
Date ……………29/05/2019……………….……………

STATEMENT 2

This thesis is the result of my own independent work/investigation, except where otherwise stated, and the thesis has not been edited by a third party beyond what is permitted by Cardiff University's Policy on the Use of Third Party Editors by Research Degree Students. Other sources are acknowledged by explicit references. The views expressed are my own.

Signed …………… Long Zhou………………………….…………..…… (candidate)
Date ……………29/05/2019…………..…………………

STATEMENT 3

I hereby give consent for my thesis, if accepted, to be available online in the University's Open Access repository and for inter-library loan, and for the title and summary to be made available to outside organisations.

Signed …………… Long Zhou…………………………..…...….. (candidate)
Date ……………29/05/2019……………………………

STATEMENT 4: PREVIOUSLY APPROVED BAR ON ACCESS

I hereby give consent for my thesis, if accepted, to be available online in the University's Open Access repository and for inter-library loans **after expiry of a bar on access previously approved by the Academic Standards & Quality Committee.**

Signed ……………Long Zhou……………………..……… (candidate)
Date ……………29/05/2019…………………………….

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1 INTRODUCTION

1.1 Background

Public spending in Wales is largely funded through a block grant from UK Treasury, and with changes in public funding linked to the Barnett formula. This adjusts the amounts of public spending given to Wales to link to changes in expenditure given to public services in England or Great Britain. Then in the recent past, there are limited links between tax revenues gathered in Wales and monies available for the Welsh Government to spend on public services. However, following a series of Commissions[1] in Wales, there has been more pressure for economic powers to be devolved to the region. For example, from 2015 business rates were fully devolved. Moreover, 2018 has witnessed an end to centrally-set Stamp Duty Land Tax (SDLT). The Welsh Government also gains some powers over Income Tax.

However, with devolving tax raising powers there comes a need for more economic data and the development of new economic models through which to understand the effects of tax changes. These needs have been recognised by the Commissions that preceded the devolution of more tax powers to Wales.

1.2 Economic models of Wales

Within Wales, there has been some history of developing economic models through which to understand changes in regional economic activity. For example, the region has benefited from a series of Input-Output tables. Input-Output tables were produced at irregular intervals prior to devolution from 1966 through to 2007. These tables have been used to assess the significance of different industrial activities, and to examine issues of infrastructure improvement in Wales. They have also been used to support policy development in the region. More generally Input-Output tables are important elements of national accounting frameworks. Importantly they are a building block for constructing social accounting matrices (SAMs) which show the flows of all economic transactions occurring within an economy and more detailed interconnections between households, firms and government. Together Input-Output tables and SAMs are very important should one wish to develop more complicated economic modelling processes.

[1] For example, see Holtham Commission (2010) and Silk Commission (2012).

Unfortunately, while Input-Output tables are very useful they are limited in application without a role of price when used in economic modelling. This is largely because they represent a demand-driven framework and this assumes supply is exogenous. Then for Wales, Input-Output tables are not really a useful means of exploring the effects of tax changes.

1.3 Modelling the Welsh effects of regionally-set tax changes

Following from the above, to better elucidate the effects of changes in regionally-set tax rates more complex economic models are required. In this thesis, a Computable General Equilibrium (CGE) model of Wales is developed and used to examine the effects of changes in SDLT, Corporation Tax and Income Tax. The research for the thesis was funded by the Welsh Government. While the project was initially focused around developing a framework in which to understand the economic effects in Wales of changes in SDLT it is expected that the model developed would have a wider implication to other types of tax variation in the region, for example, Corporation Tax and Income Tax. In short, this thesis comprises:

- The development of a Social Accounting Matrix for Wales.
- The development of a regional CGE model through which tax variation can be understood.
- A simulation of how a change in SDLT, Corporation Tax and Income Tax would have effects on the regional economy.

In terms of wider contribution, I believe the economic modelling framework developed here will have wider application to a series of other taxes in the regional economy.

1.4 Structure of the thesis

The main part of the work is subdivided into six further chapters.

Chapter 2 presents the background information for understanding and modelling the devolved tax variation in Wales. Basic economic facts and regional variations of the Welsh economy against the rest of the UK is investigated. Fiscal context is discussed regarding to the tax devolution process and current public fiscal status of Wales. The final part of the chapter devotes to the tax structure and tax base composition of those devolved taxes in Wales.

Chapter 3 comprises a review of CGE modelling and its fiscal applications. Here I provide a methodological literature review of CGE modelling and simulations, particularly in a regional context, and in application to a wide range of policy issues. The chapter explores economic models developed so far to understand changes in activity in the Welsh economy (Input-Output approaches, and econometric I-O models of Wales). The strengths and weaknesses of existing approaches are examined. For example, in the case of I-O, their use in modelling applications are limited by the general limitation of the demand-driven input-output model. In particular, the absence of price variation makes such approaches incapable of considering substitution effects due to relative prices change. This makes it very difficult to use such models for analysis of tax variation. I then make a case for developing a CGE to consider issues around tax. The chapter then turns to focus on CGE models applied in a regional context. The basic classification of regional CGE models is discussed, followed by reviews of major model characteristics and specifications adopted in different studies. In addition, a case study of the AMOS model (A Macro-Micro Model of Scotland - Strathclyde University CGE model) is conducted and placed in Appendix I.

By commenting on and summarizing different model elements designed for different research topics, I draw implications for understanding regional CGE models and how they can be used for modelling and simulation in the cases of tax issues.

Chapter 4 considers the theoretical basis of economic behaviour within CGE models. This chapter explains the theory of CGE modelling and its various specifications in describing the economic behaviours of different types of economic agents. The chapter looks at how we understand production behaviour, government and household behaviour, trade behaviour, and then macroeconomic closures. I show here that as a general equilibrium model, the CGE model describes both the supply and demand side as well as the trade behaviour of an open economy. Correspondingly, the decision-making process regarding consumption, production, export and import goods are modelled by equations of various functional forms. Only once these relationships are understood can one lay a foundation for the construction of a regional CGE model for the Welsh economy.

Different functional forms including the Cobb-Douglas function, the Leontief Input-Output (I-O) function, the Constant Elasticity of Substitution (CES) function and the Constant Elasticity of Transformation (CET) function are introduced in this Chapter. I show that functions are not only used to model consumption and production. For example, the CES functions and the related CET functions have also been used as aggregation of composite goods and transformation of domestic goods in a small open economy. Derivations and applications of these functional forms are illustrated in this part of my thesis.

The final part of Chapter 4 shows the development of a CGE model and the needs of macroeconomic closures to 'close' the model. In short, these macro closures are about the choices of endogenous variables against exogenous variables among all the variables in the model. Those endogenous variables represent the 'inside' of the model, while the exogenous variables represent the 'outside'. Therefore, there is a clear boundary distinguishing between the 'inside', which is depicted by the model for only part of the real economy, and the 'outside', which is assumed to be the remainder of the economy.

Chapter 5 describes the development of the Social Accounting Matrix (SAM) for Wales. This I believe is a key component of the work and an important contribution of the thesis. In this chapter the development of the CGE model database which is organized in the form of a Social Accounting Matrix is described. A SAM records the value of all circular flows of economic transactions in an economy. It records transaction data over a specified period of time, usually a calendar year. In this sense, it can be seen as a 'snapshot' describing comprehensively the economic structure and activities of economic agents in a particular period. The SAM developed and used throughout my thesis focuses on the Welsh economy during the 2013 calendar year.

Chapter 5 shows that the SAM table is a logical framework to arrange the transaction data of agents. These typically include industrial sectors, factors, households, government, investment, and foreign sectors. The data sources range from national accounts, Input-Output (I-O) tables, industry statistics, government fiscal statistics, trade statistics, surveys and Census. These data are organized into a square matrix using the double entry book-keeping principle. This means that each economic agent has both a column account recording its expenditure structure, and a row account recording its sources of income. Therefore, each number in the table represents a single transaction as the payment from an agent's column account to an agent's row account, and the number locates in the intersected cell between the row and column accounts. Total expenditure must equal total income for each agent, and the corresponding column account total must equal the row account total.

Based on the balancing of each agent account, the SAM table is then automatically balanced. A SAM must only be balanced to represent the equilibrium condition of the whole economy such that every market clears. Only then is the SAM a benchmark dataset ready for subsequent model simulation. The SAM then acts as the dataset foundation to present interactions of economic variables in the model and to calibrate the model parameters. It also provides a description of the target economy as an equilibrium benchmark for comparison against the post shock (tax variation) economic status. The SAM dataset is the foundation for model calibration, simulation and analysis of results implication in the later parts of the thesis.

In summary the material in Chapter 5 is organized to show: the basic theory of the SAM; and then the compilation of the SAM with data developed from a large number of sources. I then present material on the balancing methods and the balancing process applied in this case. The last section focuses on the calibration of the model parameters.

An important part of Chapter 5 relates to estimation issues with real estate sectors. The SAM comprises 3 real estate ownership sectors, but with no direct data sources of intermediate inputs at this level of aggregation. Therefore, estimations are made based on relevant real estate transaction and construction statistics. The chapter details different approaches to balancing the SAM, and justifies the method employed. The resulting balanced SAM table forms the benchmark dataset for the CGE model in use, and represents an equilibrium Welsh economy in 2013 as a starting point.

Chapter 6 focuses on the simulations. The main simulation focuses on the variation of SDLT, while Corporation tax and Income tax effects are also investigated. In this modelling, the SDLT rate is modelled as two separate rates: residential and non-residential effective rates. In reality the tax rates are both systems of multiple rates corresponding to multiple real estate transaction price bands, it can then only be modelled by the single effective rate for each system. The Corporation tax and Income tax are also simulated with single effective rate. The chapter shows how the simulation is designed to observe the impact of a policy change of the tax rate, which is equivalent to a particular amount of an expansion or reduction of the tax revenue. The impact of the SDLT variation is generated through price representing the true transaction cost of real estate properties regardless of types of property rights. The price change will effect both the supply and demand side of the real estate sector. It will also spread further to the supply and demand of all other sectors through the system of input-output inter-sectoral connections in the product market. In the factor market, the land factor is treated as the non-residential land input and the price change of this may also trigger factor substitution behaviour in the production process. The impact of Corporation tax is generated mainly from the supply side as capital using cost. The Income tax effects are realized through impacts on both supply side as labour cost and demand side as via household revenues and expenditures. As a result, the ramifications on major macroeconomic indicators following the policy variations can be investigated.

The simulation captures the tax effects in the short, medium, and the long run. In the short run, all factors are immobile across sectors; they are assumed fixed in each sector and their factor prices may vary across sectors following the policy shock. In the medium run, labour and capital are mobile and the total stock for each factor is regionally fixed; factor prices converge across sectors to a new level and there is only one price for one factor. The land factor is still sectorally fixed as in the short run. In the long run, the labour and capital stock are unconstrained and they can freely move across the region border, so that factor prices recover to their original level as in the benchmark. The only exception is land where only non-residential land is flexible across sectors and fixed in total.

Chapter 7 draws conclusions for the whole thesis.

CHAPTER 2 ECONOMIC AND FISCAL BACKGROUND OF DEVOLVED TAX VARIATION IN WALES

2.1 Introduction

This chapter explores the economic and fiscal background of tax variation issues in Wales. This is important for two reasons. First, to study the devolved tax variation effects on Wales, it is important to understand socio-economic conditions of the region as the modelling context. A comprehensive perception of the Welsh economy will help capture the characteristics of the economy, such as labour use, productivity, sectoral structure and regional productivity differentials with the rest of the UK. In developing the modelling framework we need to understand the evolution of the regional economy. These will form the base for the development of the benchmark database for the CGE modelling and simulation in the following chapters.

Second, as the thesis focuses on the case of devolved tax variation, the devolution process and the current fiscal status of the Welsh economy and Welsh Government also need to be examined. Details of all those devolved taxes are illustrated as well as the underlying tax bases. Although the Stamp Duty Land Tax is the main focus in this study, the modelling framework aims to be functionally general in terms of type of taxes. Given regional differences in terms of economic structure, fiscal arrangements and relative tax bases, it is important to formulate tax variation policy tailored to the Welsh tax base. The fiscal framework and the use of tax varying powers will profoundly impact the regional fiscal budget and the tax base, both of which will play important roles in shaping the Welsh economy in the short and long run.

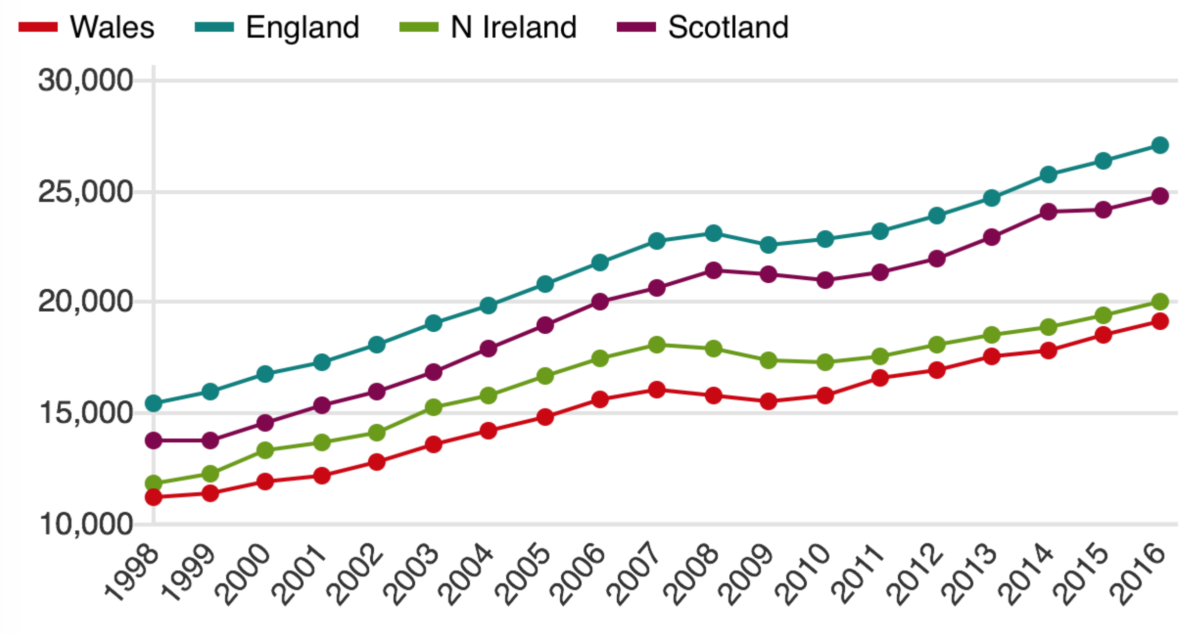
Hence, the examination of both the economic and fiscal background will enable us to better understand the baseline data for the modelling and the simulation of devolved tax variation under a Welsh context. This chapter is organised as follows. Section 2.2 presents the recent economic performance of the Welsh economy and its internal disparity. The history of devolution to Wales is outlined in Section 2.3. Section 2.4 discusses the current overall fiscal status of Wales. As part of the fiscal framework, possible methods of adjusting the block grant following tax devolution are also explained. Section 2.5 focuses on the taxes already devolved to Wales and those under consideration. Their underlying tax bases are also analysed as they are important determinants of the Welsh public budget. Section 2.6 concludes.

2.2 Basics of the Welsh economy

Wales is a constituent nation of the UK. It had a population of approximately 3.1 million in 2015 settled in a region of 20779 km^2. Although it has a terrestrial borderline with England to the east, it has a close link with the rest of the United Kingdom, and also the wider European Union area. According to Office for National Statistics (2017a), the total gross value added of Welsh economy was £59.6 billion in 2016, ranked tenth largest among the UK's twelve Nomenclature of Territorial Units for Statistics (NUTS) level 1 regions, just ahead of the North East of England and Northern Ireland.

When shifting to the per capita level, the Welsh GVA per capita was £19,140 in 2016 (72.7% of the UK average), again amongst the lowest in the UK regions (Office for National Statistics 2017a). Compared to the devolved nations, it is behind Northern Ireland of £19,997 per head, Scotland on £24,800 and England with £27,108 in 2016, and remains bottom since 1997. As shown in Figure 2.1, Wales still fails to close the gap with the UK as a whole (Dickins, 2017).

Figure 2.1 Gross Value Added per head of UK Nations (£), 1998-2016

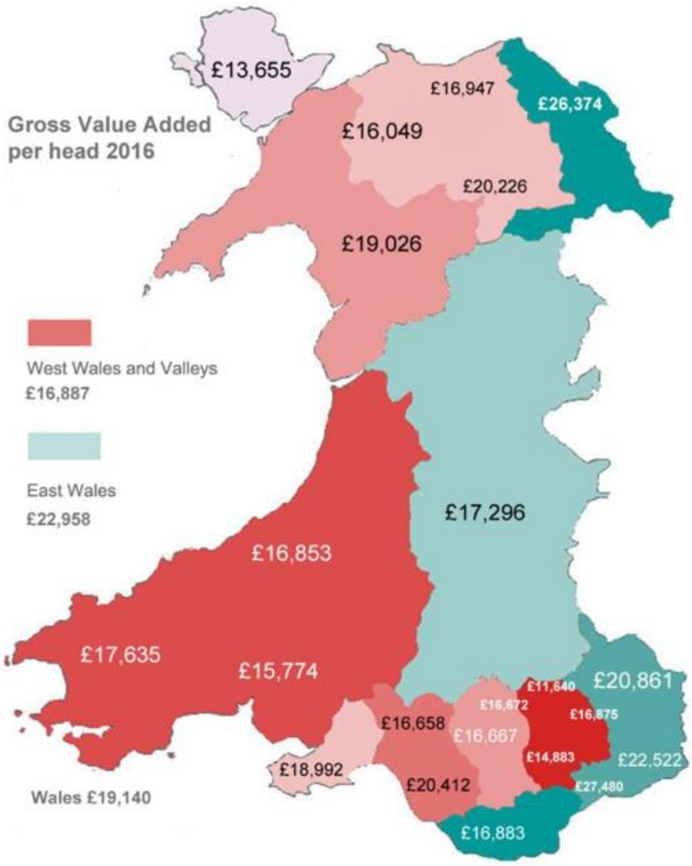


Source: Office for National Statistics (2017a)

However, the corresponding recent growth rates of Wales are relatively higher on average. Total GVA of Wales in 2016 grew 4%, compared to 3.7% of the UK against 2015. GVA per head increased 3.5% in the same period, and this is the second highest growth over the year of all UK regions. If the factor of inflation is taken into account, the annual growth rate of Welsh GVA per head in real terms was 1.3% in 2016, and the figure is the joint fifth highest of all UK regions (Office for National Statistics 2017c).

There is significant variation in GVA per head across the Welsh regions. Figure 2.2 shows gross value added per head for each Welsh region at both NUTS level 2 and 3. At NUTS level 2, Wales is divided into West Wales and the Valleys, and East Wales. According to Office for National Statistics (2017a), in 2016 GVA per head of East Wales is estimated as approximately 87.2% of the UK average, whilst West Wales and Valleys accounts only for 64.1%, and approximately half of Welsh live in each region.

Figure 2.2 Gross Value Added per head of Regions in Wales 2016



Source: Office for National Statistics (2017a)

Figures for West Wales and the Valleys have revealed the Isle of Anglesey with the lowest GVA per head in the UK at £13,655 in 2016, and Gwent Valleys with £14,759. The highest GVA per head in East Wales are found in Newport and Flintshire and Wrexham, which are £27,480 and £26,374 respectively, and with these close to the UK average of £26,327.

These figures show considerable economic disparities across Wales and this no doubt implies the challenge faced of rebalancing the Welsh regional economy. The regional imbalance of Welsh economy may be attributed to the mixed effect of geographic and historic factors during its economic development history.

Over the last several hundred years, Wales has transformed from a predominantly agricultural nation at first, then to an industrial, and now a post-industrial economy (Day 2002). Before the mid-18th century, the development of the Welsh economy was largely limited by its peripheral location and sparse population. Apart from the livestock droving economy, Welsh trade contacts were limited in the coastal ports with regular commerce with the English cities of Bristol and Liverpool (Falkus and Gillingham 1987).

The industrial development in Wales was benefitted and stimulated from its rich mineral deposits from the mid-18th century. During the Industrial Revolution and until the post-war era, heavy industry was dominant in this regional economy. Coal mining and exporting became the pillar industry with ever increasing demand for Welsh coal from local metallurgical industries, and overseas. Merthyr, the northern rim of the South Wales Coalfield, emerged as the UK's most important iron-producing district, while around Swansea, the southwest of the coalfield became an important non-ferrous metal smelting and tinplate production centre. Cardiff, yet to be a capital at that time, was once the largest coal-exporting port in the world. At the peak of the coal production before the Great War, nearly 233,000 people worked in the South Wales Coalfield, mining 56 million tons of coal. By the 1920s, over 40% of the male Welsh population were employed in heavy industry: either in mining and quarrying (mainly coal) or in Metal Manufacture (Williams 2003).

The export economy of Wales collapsed during the inter-war recession as a result of the emergence of oversea competitors under the international background of increasing protectionism. The following Great Depression in 1930s then caused serious economic deprivation to the whole Wales due to its narrow economic base overwhelmingly dependent on heavy industries and exploitation of natural resources. Between 1921 and 1939, the South Wales Valleys experienced an unprecedented level of unemployment and the exodus of 400,000 people (Day 2002).

In the post-war era, the Welsh economy was massively restructured with a shrinking of heavy industries. For the coal industry, the number of pits in South Wales reduced from 115 in 1953 to 34 in 1981, and coal production fell from almost 21 million tons to 7.7 million tons (Jenkins 1992). Large numbers of jobs disappeared from heavy industry and this was gradually replaced by employment in light industries and services. Pembrokeshire and Swansea Bay became centres of the petrochemical industry and other new light industries were attracted throughout Wales. From 1970s onwards, Wales has been successful in attracting foreign direct investment (FDI) with an above average share of that into the UK, though many of the new plants established by foreign enterprises essentially offered relatively low-paid jobs in manufacturing plants or call centres, while most highly paid job opportunities are retained elsewhere out of Wales (Massey 2009; Morgan 2002).

There was also a shift of the Welsh economy in the post-war decades towards service sector which now accounts for the majority of jobs. Public sector employment increased as various national bodies had been brought to Wales, including Royal Mint to Llantrisant, Companies House to Cardiff, Driver & Vehicle Licensing Agency to Swansea and Office for National Statistics to Newport. Tourism also played an increasingly vital role in the regional economy, especially for the rural areas, attributed to its unique cultural and geographic resources (Jenkins 1992).

In the early 1980s the recession had more adverse effect on Wales than the rest of the UK. Some 130,000 jobs were lost between 1979 and 1982 and the employment rate dropped to 62%. This left a long term legacy of serious unemployment of older men, especially in the South Wales Valleys. Due to the lack of high value added employment, the GVA per head, representing economic output per head, is relatively lower than other regions of the UK as mentioned in the preceding section. This has been potentially attributed to a comparable low activity rate and a weaker agglomeration effect. More recently, Wales has endeavoured to attract and develop high value added employment in sectors such as finance, business services and research (Welsh Government 2005).

2.3 Devolution in Wales

This section focuses on the process of tax devolution to Wales which also witnesses the founding process of Welsh constituent legislative and executive bodies.

Welsh devolution refers to the statutory granting of powers from the Parliament of the United Kingdom to the Welsh legislature, the National Assembly for Wales, and its associated executive body, the Welsh Government. In practice it is the process of establishing limited self-government for Wales, accompanied with a number of taxes devolved to the Welsh Government so that it can determine tax administration in a Welsh context.

The roots of devolution in Wales in modern times can be traced to the end of the nineteenth century. In 1881 the Sunday Closing (Wales) Act 1881 was passed, the first such legislation exclusively concerned with Wales. The devolution process began with the transfer of certain limited administrative functions of government to Wales. For example, the Central Welsh Board was established in 1896 and a separate Welsh Department of the Board of Education was formed in 1907. The Agricultural Council for Wales was set up in 1912, and the Ministry of Agriculture and Fisheries had its own Welsh Office from 1919.

The Council for Wales and Monmouthshire was established in 1949 to advise the UK Government on matters of Welsh interest. A post of Minister of Welsh Affairs was created in 1951 and the post of Secretary of State for Wales and the Welsh Office were established in 1964 leading to the abolition of the Council for Wales and Monmouthshire. From 1964 onwards, significant decisions relating to how Wales was run were made in Wales. At first, the Secretary of State only had responsibility for housing, local government and roads. Other areas including education and training, health, trade and industry, and the environment and agriculture were gradually added over the following years.

Following Royal Commission on the Constitution (1973) which recommended the creation of elected bodies for both Scotland and Wales, the proposal for the creation of a Welsh Assembly was rejected by the Welsh public in the 1979 referendum. It was in the 1997 referendum that devolution was approved. The following Government of Wales Act 1998 established the Welsh

National Assembly as a single corporate body with the executive (the government) and the legislature (the Assembly itself) operating as one. At this stage, the powers transferred were broadly equivalent to those previously held by the Secretary of State for Wales, as it limited the National Assembly to the making of secondary legislation in specified areas, including agriculture, fisheries, education, housing and highways.

However, the National Assembly's powers are too limited then and being as a single corporate body of this nature proved to be problematic and highlighted the need for constitutional change and stability. In 2002, the Commission on the Powers and Electoral Arrangements of the National Assembly for Wales (Richard Commission) was established by the Welsh Government. It recommended that the Welsh National Assembly should have powers to legislate in certain areas, whilst other areas would remain the preserve of UK Parliament. It also recommended the legal separation of the executive and legislature as individual legal entities within the framework of the 1998 Act (Richard Commission 2004). This formally came into force by Government of Wales Act 2006 which clarified the role of the Welsh Government responsible for making and implementing decisions, policies and subordinate legislation, while the National Assembly for Wales makes laws and hold its ministers to account.

In spite of the constitutional change, the Richard Commission also considered the financing of devolution in Wales and the option for tax varying powers. It concluded that it would be "desirable, though not essential, to confer tax varying powers" on a legislative National Assembly for Wales (Richard Commission 2004). The issue of further financing and funding settlements was focused on by an Independent Commission on Funding & Finance for Wales (Holtham Commission) established by the Welsh Government (Holtham Commission 2009; 2010).

In the Holtham Commission (2010) possible alternative funding mechanisms were considered, including the scope for the Welsh Government to have tax varying powers as well as greater powers to borrow. It proposed four broad models that could be applied to fund devolved government in Wales:

- Current Model: Welsh taxes pooled at UK level with the Welsh Government's budget almost entirely comprised of a block grant from the UK Government.

- Tax Assignment Model: The block grant could be partly replaced by revenue raised by certain taxes in Wales.
- Tax Devolution Model: The block grant could be partly replaced by revenue raised by certain taxes in Wales with Welsh Ministers given powers to vary the devolved tax rates.
- Fiscal Autonomy Model: Wales would be given extended taxing powers, in addition to support from the UK budget through grants. At the limit, Wales would be wholly reliant on its own resources, with no transfers between the Welsh and the UK government.

The Commission favoured the principles of the Tax Devolution Model and proposed taxes that could potentially be devolved to Wales, including larger taxes (Corporation and Income tax), property taxes (Non-Domestic Rates, Stamp Duty Land Tax and Capital Gains Tax), minor taxes (Landfill Tax, Aggregates Levy and Air Passenger Duty) and potential new additional taxes. The Holtham Commission also recommended limited borrowing powers should be devolved to the Welsh Government to finance capital expenditure which should be capped at an agreed amount.

The referendum held in 2011, known as the Welsh devolution referendum on law-making powers, supported more legislative powers being transferred from the UK parliament to the Welsh Assembly. Following the referendum, the Assembly gained primary law-making powers in relation to specific subjects, and the UK Government established the Commission on Devolution in Wales (Silk Commission) to consider the future of the devolution settlement in Wales. Two reports were then published, which reviewed and made recommendations on the arrangements of the Assembly’s financial powers and future legislative powers respectively (Silk Commission 2012; 2014).

The Silk Commission (2012) considered the Assembly's current taxation and borrowing powers. The report agreed on the same funding model as the Holtham Commission and recommended the Tax Devolution Model. The Silk Commission believed this model would best meet sound principles for funding the Welsh Government.

Regarding the devolved taxes, the Silk Commission recommended the Assembly should be given powers to introduce new taxes in addition to a range of taxes that should be devolved to Wales including: Stamp Duty Land Tax, Landfill Tax, Aggregates Levy, Air Passenger Duty and Non-Domestic Rates. These were considered to be smaller taxes by the Commission who felt the Assembly should be responsible for raising a more substantial proportion of its spending if its financial accountability was to be improved sufficiently.

Having looked at the four larger taxes (Corporation Tax, Income Tax, National Insurance Contributions and Value Added Tax), the Commission concluded that income tax would be appropriate for partial devolution subject to a referendum while the costs of devolving corporation tax, which the Holtham Commission recommended investigating, were at this stage considered to outweigh the benefits.

The Silk Commission also recommended additional borrowing powers for the Welsh Government to increase capital investment, sourced from the National Loans Fund and commercial sources. As mentioned in Holtham Commission (2010), this borrowing should be limited. It further suggested the Welsh Government should be able to issue its own bonds.

In its response to Silk Commission (2012), the Welsh Government (2012) commented it was 'open-minded on the case for tax devolution' in areas with a significant degree of devolved responsibility and where devolution would provide an additional lever for Welsh Government to deliver policy objectives.

As the response, the UK Government (2013) accepted most of the recommendations from Silk Commission (2012) and fully agreed with the Commission's key recommendation that the funding model of a block grant and some devolved taxes best meets sound principles for funding the Welsh Government, and that part of its budget should be funded from devolved taxation under its control. The document set out the UK Government's plans to:

- Partially devolve Income Tax to Wales subject to a referendum
- Devolve smaller taxes such as Non-Domestic Rates, Stamp Duty Land Tax and Landfill Tax
- Intend to devolve the Aggregates Levy subject to the resolution of legal challenges
- Provide the Assembly with powers to introduce specified taxes
- Introduce capital borrowing powers

The UK Government, however, was not convinced of the case for devolving air passenger duty to Wales at this stage given the 'potential effect across the country as a whole' (UK Government 2013).

Later in March 2014, the Command Paper, UK Government (2014), provided further details on the implementation and operation of the Assembly's new tax and borrowing powers. The aim to fully devolve Non-Domestic Rates (NDR) was also outlined in the paper. The Wales Act 2014 became legislation in December and made provision for the devolution of taxation powers to the Assembly and borrowing powers to the Welsh Government. The devolved taxes specified in the Act include Stamp Duty Land Tax, Landfill Tax and Income Tax subject to a referendum. These provisions will need to be commenced by Order made by the UK Government. The Command Paper UK Government (2014) contains explanatory information on how many of the powers in the Act will be implemented, and sets the target date for the commencement of these provisions as April 2018.

In February 2015, the Command Paper of UK Government (2015) was published, which included the UK Government's commitment to enable the Welsh Government to issue bonds to borrow for capital expenditure and reconsider the case to devolve Air Passenger Duty. Non-Domestic Rates were fully devolved to Wales in this year and again, were to be set the rate multiplier by the Welsh Government from April 2018. Together with Stamp Duty Land Tax and Landfill Tax (will be renamed as Land Transaction Tax and Landfill Disposals Tax respectively), these are first taxes to be controlled by the Welsh Government who is responsible for setting the taxable rate and redistributing the total revenues to local government.

In its Spending Review 2015, the UK Government announced its intention to remove the requirement for a referendum for partial devolution of income tax, and it was confirmed in the redrafted Wales Bill 2015, which finally received Royal Assent in January 2017 and became the Wales Act 2017. Based on the legislative recommendations proposed in Silk Commission (2014), the Wales Act 2017 gave extra powers to the National Assembly for Wales and the Welsh Government, and recognized them as permanent among UK's constitutional arrangements, with a referendum required before either can be abolished. The Act has also recognized that there is a body of Welsh law and it established the position of President of Welsh Tribunals.

The provisions regarding tax devolution issues in the Wales Act 2017 include, apart from the removal of the referendum requirement for the partial devolution of Income Tax, the ability of the National Assembly for Wales to borrow up to £1 billion per year, the creation of Welsh Revenue Authority as a tax authority for Welsh devolved taxes, and the ability to raise or lower income tax by up to 10p in the pound which will come on stream from April 2019.

A fiscal framework for Wales is then intended to be implemented to reflect the tax devolution. Detailed discussions will need to take place between both the Welsh and UK Governments given the complexities involved in agreeing a fair and transparent fiscal framework for Wales. A key area for negotiation will be the mechanism for adjusting the Welsh block grant reflecting the tax varying powers devolved to Wales, known as the 'block grant adjustment' (BGA), and this will be discussed more in the succeeding section.

2.4 Current fiscal status of Wales

This section discusses the current fiscal status in Wales, including the public sector fiscal balance and the possible block grant adjustment mechanisms which as part of the future fiscal framework, will also significantly affect the Welsh budget size in practice.

The current fiscal framework of Wales is based on the situation that most of the public sector revenues payable by Welsh residents and corporates are pooled at the UK level by HM Revenue and Customs, and forms part of the UK revenues. With the taxes that have already been devolved and administrated by local authorities, the revenue available to Welsh Government's expenditure is rather limited. Consequently, the Welsh Government receives a block grant additionally from the UK Government to finance most of its expenditure, which is set annually by HM Treasury based on the Barnett Formula. These two parts of the revenues jointly fund the devolved expenditure by the Welsh Government.

However, the devolved expenditure by the Welsh economy is far insufficient for the Welsh economy, which also receives direct expenditure from the UK Government regarding general public services, defence and social protection. This non-devolved expenditure accounts for as much as half of all public expenditure for Wales (This will be illustrated later). Therefore, the amount of revenue collected in Wales has been mostly disconnected from the funds available for the Welsh Government and, furthermore, the expenditure for the whole Wales by both Welsh Government and UK Government.

In fact, the Welsh economy has long been, and is still currently in a deficit condition that the public sector revenue collected in Wales is significantly less than all the public expenditure for Wales. To see this, Table 2.1 presents the current fiscal balance of Wales across 7 consecutive fiscal years, which is calculated as current revenue minus current expenditure and depreciation. The deficit peaked at 2011/12 fiscal year, then declined gradually, and remained above £10 billion. This implies Wales is financially reliant on and subsidized by the UK Government.

Table 2.1 Current Fiscal Balance in Wales: 2010/11 to 2016/17

£ million	2010/11	2011/12	2012/13	2013/14	2014/15	2015/16	2016/17
Current Revenue	20771	21604	21939	22524	23321	24716	26086
Current Expenditure	33079	33954	33599	34089	34570	35250	35828
Depreciation	1543	1737	1821	1870	1934	2014	2085
Current Fiscal Balance	-13851	-14087	-13481	-13435	-13183	-12548	-11827

Source: Office for National Statistics (2018a)

If we decompose the figures in detail, both revenue and expenditure are composed of the devolved and non-devolved parts. As provided in Poole et al. (2016), the current public sector revenue in 2014/15 fiscal year can be decomposed as shown in Table 2.2.

Table 2.2 Decomposition of Current Public Sector Revenue from Wales: 2014/15

£ million	2014/15
Current Revenue	23321
Devolved Revenue	4265
Council Tax	1277
Non-Domestic Rates	854
Land Transaction Tax	168
Landfill Disposals Tax	49
Welsh Rate of Income Tax (partial)	1877
Air Passenger Duty (under consideration)	10
Aggregates Levy (under consideration)	30
Non-Devolved Revenue	19056
Value Added Tax	5152
Income Tax	4634
National Insurance Contribution	3950
Corporation Tax	1000

Other Non-Devolved Revenue	4320

Source: Poole et al. (2016)

The figures of most devolved revenues are estimated since most of the devolved taxes had not been devolved at 2014/15 fiscal year. Assuming they have already been devolved to the Welsh Government, the incurred devolved revenue in total accounts only for less than one fifth of all revenue generated from Wales. Meanwhile, the breakdown of the public expenditure for Wales is shown in Table 2.3 below.

Table 2.3 Decomposition of Current Public Sector Expenditure for Wales: 2014/15

£ million	Current Expenditure	
	Devolved	Non-Devolved
General Public Services		
Public and Common Services	455	283
International Services	0	517
Public Sector Debt Interest	0	1596
Defence	4	1744
Public Order and Safety	848	576
Economic Affairs		
Enterprise and Economic Development	420	114
Science and Technology	44	180
Employment Policies	3	177
Agriculture, Forestry and Fisheries	432	12
Transport	822	181
Environment Protection	564	164
Housing and Community Amenities	586	6
Health	6429	39
Recreation, Culture and Religion	401	352
Education and Training	4086	22
Social Protection	3066	11141
Accounting Adjustment	1975	779
Total	20133 (53%)	17883 (47%)

Source: Poole et al. (2016)

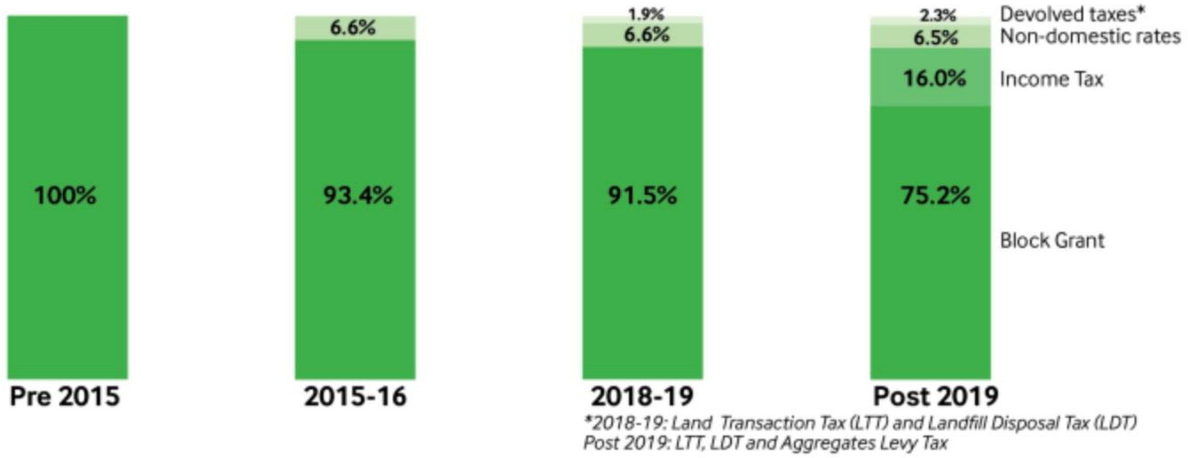
The largest portion of the Welsh Government's devolved expenditure is for health, which makes up almost a third of total. This is followed by education, which accounts for over one fifth. Other policy functions inputted more by devolved expenditure than non-devolved expenditure include housing, environment protection, agriculture and transport, while non-devolved expenditure of UK Government focused more on general public services and social protection, including pensions and benefits expenditure. The devolved expenditure administrated by the Welsh Government only meets slightly more than half (53%) of demand from the whole Welsh economy, and the another half is fulfilled directly from the UK Government. While the devolved revenue for the Welsh Government is far insufficient to finance the devolved expenditure, a block grant is received to fulfil the gap as shown in Table 2.4. Hence, the public sector expenditure funded by the Welsh Government is determined by the revenue of devolved taxes generated in Wales and more, by the block grant support from the UK Government.

Table 2.4 Fiscal Balance of the Welsh Government: 2014/15 (£ million)

Devolved Expenditure	20133	⟵	Devolved Revenue	4265
			Block Grant	15868
20133		Total	20133	

Under this fiscal arrangement, the block grant received by the Welsh Government is set annually by HM Treasury primarily using the Barnett Formula. However, after the devolution of the series of taxes, downward adjustments shall be made to the block grant to account for the Treasury's lost tax revenue as a key part of forming a fiscal framework for Wales. To see this, Figure 2.3 describes the estimated portion of devolved taxes versus block grant in Welsh Government's budget.

Figure 2.3 Adjustment of Block Grant to Compensate for Devolved Taxes



Source: Office for Budget Responsibility (2016)

According to Office for Budget Responsibility (2016), once Income Tax has been partially devolved to Wales, it is forecasted to constitute 16% of the Welsh Government's total budget. As shown in Figure 2.3, while Non-Domestic Rates stabilizes at around 6.5%, the combination of other devolved taxes (Land Transaction Tax, Landfill Disposals Tax and Aggregates Levy) will account for 2.3% after 2019. Consequently, this will require an adjustment to be made to the Welsh block grant, and the Welsh Government's budget will face uncertainty from the way the BGA is adjusted.

It is currently unclear how the adjustment will be made. However in the first year of taxes being devolved, the adjustment is straightforward that it will simply equal the amount of revenue immediately being devolved to Wales, ensuring the Welsh Government's budget neither better nor worse off immediately after devolution. This adjustment would not be appropriate for subsequent years and further adjustment mechanism will be subject to negotiations between the Welsh and UK Government.

The principle, the BGA should reflect the amount of revenue that the UK Government would have collected in Wales if the devolution had not occurred, however, the devolved tax policy of Welsh Government may change the tax revenue and the underlying tax base in the following years after devolution. It is therefore basically impossible to obtain exactly what this opportunity cost of devolution would have been, and available indicators will be needed for the adjustment.

Given the experience from the Scottish devolution, there are several options to determine the BGA. The first one is Indexed Deduction (ID), which indexes the changes in the BGA to the percentage change in total comparable tax revenues in the rest of the UK. For instance, if comparable revenues in the rest of the UK grow by 10%, the BGA would also grow by 10%. In this way, relative population growth rate between Wales and the rest of the UK will play a significant role in determining whether Welsh budget is better off or risk-exposed.

The second option is Per Capita Indexed Deduction (PCID). In this method, the BGA is indexed to the percentage change in comparable revenues per person of the rest of the UK and also the rate of growth of the Welsh population. Therefore, the effect of relative population growth is neutralized.

The third approach refers to the Levels Deduction (LD), which calculates the change in the BGA as a population share of the change in comparable revenues in the rest of the UK. If for instance, the revenues in the rest of the UK grow by £ 1 billion, then BGA for Wales would increase by the population share, say 5%, around £ 50 million. The rationale for this option is that it is in line with the spending side of the Barnett Formula.

However, the problem of this method is that since the revenue per capita is far lower than that in the rest of the UK, Welsh revenue would need to grow much faster in percentage terms to reach a population share of changes in rest-of-UK revenue. Otherwise, the Welsh budget would face large shortfall. The fourth option, named Comparable Model, addresses the problem by introducing a 'comparability factor', which reflects the difference between per capita revenue in Wales and the rest of the UK. The method is then calculated as the population share of the rest-of-UK revenue change multiplied by this comparability factor.

No matter which BGA adjustment approach is agreed, or any other option is selected, a wrong mechanism could result in Wales losing hundreds of millions of pounds from the budget after all taxes have been devolved. The National Assembly will have a key role in reviewing the discussions between the Welsh and UK Governments and in ensuring the mechanism chosen is suitable for Wales (Tipples 2016).

While the BGA is concerned as an essential issue of the fiscal framework settlement, equal consideration should also need to be given to the underlying Welsh tax base of those devolved taxes. The relative performance of the Welsh tax base compared with the rest of the UK will have much more significant impact on the long-run sustainable funding of Welsh public services (Ifan and Poole 2018).

2.5 Devolved taxes of Wales

In this section, we discuss those fully and partially devolved taxes as well as their underlying characteristics and tax bases along the devolution timeline.

2.5.1 Council Tax

Council Tax is a local taxation system introduced in 1993 by the Local Government Finance Act 1992 and was devolved to the Welsh Government in 1999. While functioning for local service charge, this tax is levied on domestic dwellings, which are classified to nine bands (A to I) in Wales based on the property value, and in each band the corresponding tax payable is a fixed amount. However, the amount has to be adjusted annually by local authorities to keep up with inflation. The Council Tax bands are based on their market value assessed in 1 Apr 2003, and the assessment will not change until a revaluation takes place. Table 2.5 reports the Council Tax bands in Wales.

Table 2.5 Council Tax Valuation Bands in Wales

Council Tax Band	Ranges of Value
A	Up to £44,000
B	More than £44,000 and up to £65,000
C	More than £65,000 and up to £91,000
D	More than £91,000 and up to £123,000
E	More than £123,000 and up to £162,000
F	More than £162,000 and up to £223,000
G	More than £223,000 and up to £324,000
H	More than £324,000 and up to £424,000
I	More than £424,000

Source: StatsWales

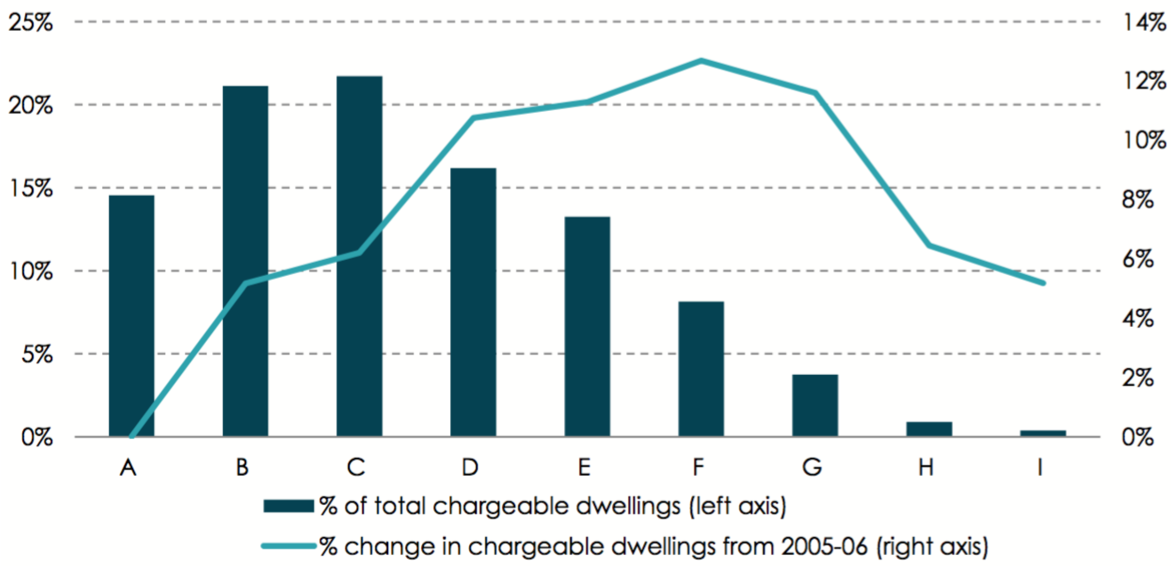
In 2017/18, around £1.45 billion tax revenue was raised from over 1.3 million properties. As the tax base of Council Tax, the property value of chargeable dwellings in Wales has formed a skewed distribution. This is shown in Figure 2.4 below.

As the rest of the UK, most dwellings in Wales are concentrated in the bottom bands. The bottom three bands A, B and C accommodate 57% of total number of chargeable dwellings, whilst the top three bands G, H and I only cover 5%. However, the higher bands have witnessed more rapid growth of numbers than lower bands from 2005/06.

Compared to England, the total number of dwellings in Wales has increased relatively slower. The average annual growth rate in Wales is 0.6% since 2006/07, while it is around 0.75% in England. However, the tax revenue in Wales has increased more rapidly than that in England. From 2009/10 to 2016/17, as an average representative, the band D revenue has grown faster in Wales since 2005/06, partly offsetting some of the cuts in grant funding (Luchinskaya

2017).

Figure 2.4 Proportion of chargeable dwellings by band in 2017/18 and number change from 2005/06: Wales



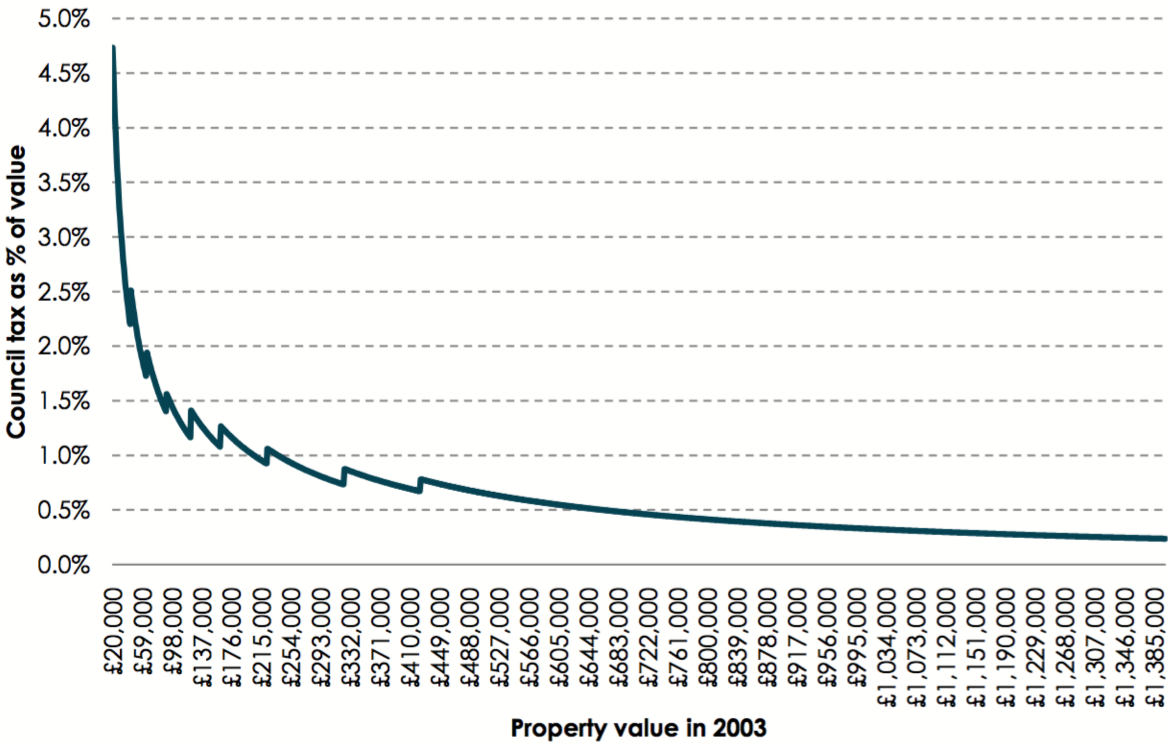
Source: StatsWales

The current Council Tax system has attracted criticism mainly for three issues (Ifan and Poole 2018). First, due to the Council Tax being largely detached from the up-to-date market value of the underlying chargeable dwellings, the increase of band D revenue, as an average of the total, is very weakly correlated with the increase of property prices. Besides, it is even negatively correlated, though weakly, with changes in disposable household income. Therefore, the situation implies a disconnection between the Council Tax revenue and the underlying tax base.

The second criticism concerns the banding system as a slab style that it does not distinguish between property values in the same band. A chargeable property just above a threshold is subject to the same levy as those in the same band but worth much more. This can be observed on those zigzag corners shown in Figure 2.5.

The third and last refers to its regressiveness of the band system: the more value the property is worth, the less is payable for Council Tax as a proportion of the property value. This is shown as the curve trend in Figure 2.5. While the tax levy in a top band is higher than that in a bottom band, the property values in comparison could have delivered a much larger gap. In this way, the Council Tax takes a higher share of disposable income from, and hence causes a greater burden to those poorer and younger households, even taking Council Tax Reduction Scheme in consideration (Welsh Government 2017f). As a result, it is a strong case suggested for a comprehensive reform for this tax system.

Figure 2.5 Effective Council Tax Rates in Wales Based on 2003 Valuation



Source: Ifan and Poole (2018)

2.5.2 Non-Domestic Rates

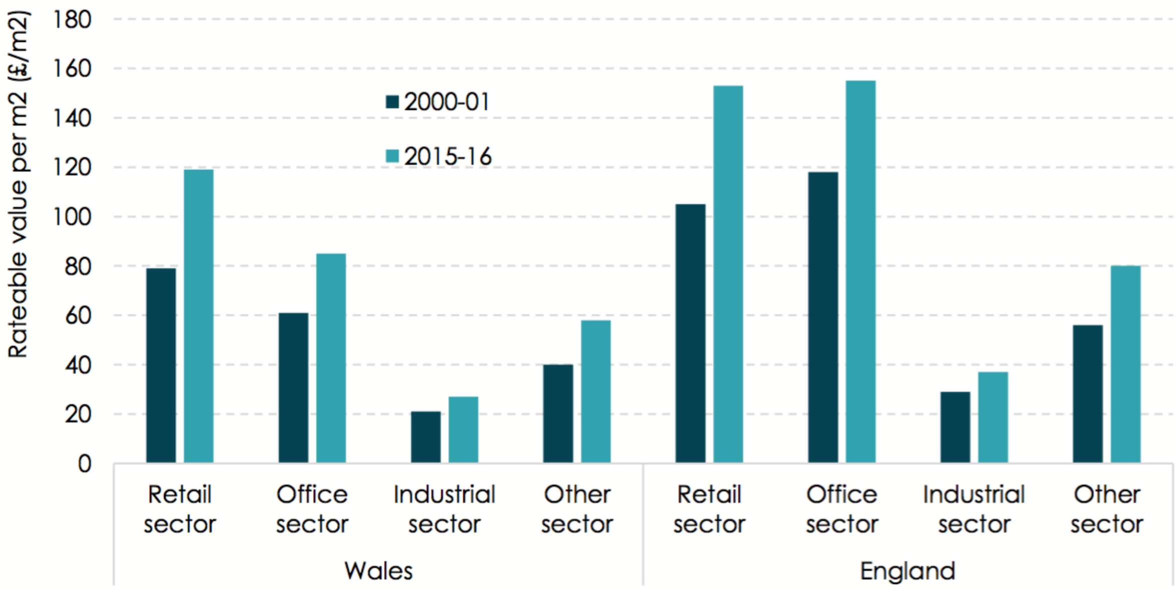
As opposed to Council Tax, Non-Domestic Rates (NDR) are levied on the value of non-residential properties as a way of contribute towards the costs of local authority services, to which they are pooled and then redistributed as part of the local government revenue settlement each year. The financial management of NDR has been fully devolved to the Welsh Government in 2015. In 2017/18, receipts of NDR totalled almost £949 million levied on around 109000 properties in Wales.

NDR are calculated by taking the Rateable Value (RV) of a property multiplied by the NDR multiplier. The Valuation Office Agency (VOA), which is independent of the Welsh

Government, values and assigns the RV of properties normally every five years. The multiplier is set by the Welsh Government each year, and from April 2018 the Welsh Government set the multiplier according to the Consumer Price Index (CPI) and for the fiscal year 2018/19 the multiplier is 0.514.

The revenue of NDR in Wales has grown slightly more slowly than England while the total number of rateable properties has been broadly in parallel with England since 2003/04. Hence, this implies relatively lower average rateable value per square metre in Wales. As shown in Figure 2.6, the rateable value differential exists across all sectors and the largest comes from the office sector.

Figure 2.6 Average Rateable Value per Square Metre by Sector in Wales and England: 2000/01 versus 2015/16

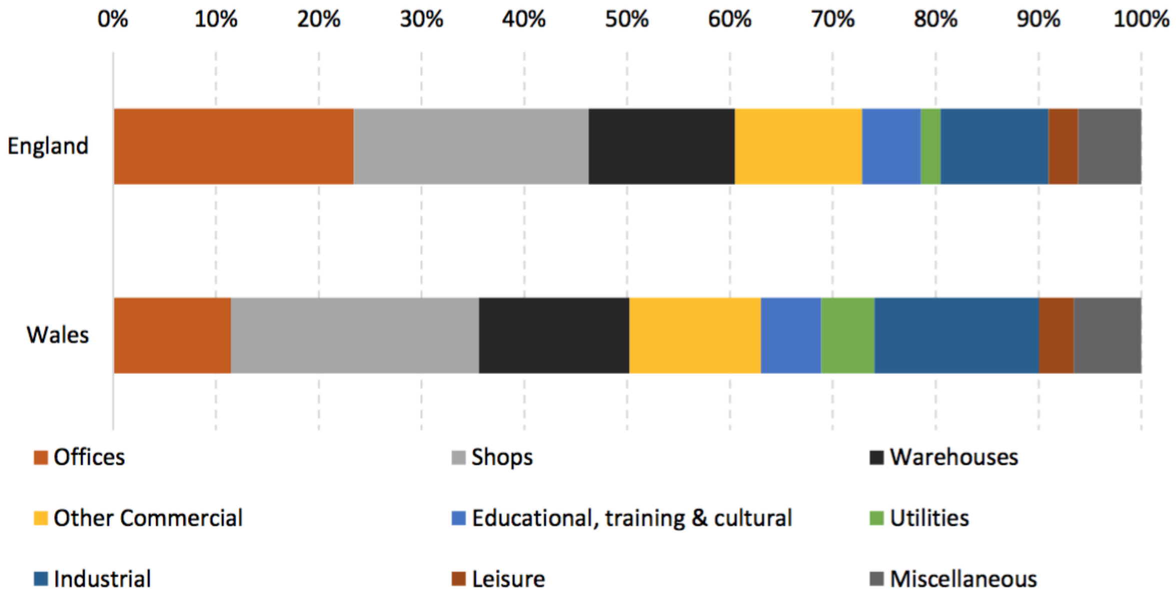


Source: Valuation Agency Office

In terms of total rateable value across property types, it is still the office property where Wales has a significantly lower share, whilst the shares of utilities and industrial properties are higher than their counterparts in England. Shares of other types are approximately matched. It is shown in Figure 2.7.

In terms of local authorities, Cardiff accounts for around one fifth of total rateable value in Wales and the share is still increasing, which implies the importance of the capital in determining the performance of tax base for NDR. From another dimension, properties with high rateable value are also of crucial importance, as they have a large bearing on overall trends in the tax base (Jones et al. 2017).

Figure 2.7 Share of Rateable Value by Property Type in Wale and England: 2015/16



Source: Valuation Agency Office

2.5.3 Land Transaction Tax

Land Transaction Tax (LTT) is the devolved tax as a replacement to the Stamp Duty Land Tax (SDLT) from April 2018. Unlike Council Tax and NDR that are levied on the stock properties, LTT, as well as the SDLT before devolution, is paid on the purchase, lease or transfer of properties including land with values over a certain threshold. This devolved tax is now collected through the newly formed Welsh Revenue Authority by the Welsh Government. In 2017/18 fiscal year, the tax revenue was totalled approximately as £260 million, in which £160 million collected from 56,000 residential property transactions, and £100 million generated from 6,000 non-residential transactions (Office for National Statistics 2018b).

The SDLT used to be imposed based on a 'slab' band structure, where the tax was charged as the transaction value multiplied by the single rate of the band that the value falls in. After 3 December 2014, the SDLT is reformed to a 'slice' system, that is, the total tax payable of a property is calculated as the sum of each of its value portion falling within each band multiplied by the rate of that band. Currently after devolution, the Welsh Government still follows the system and is able to determine its own bands, thresholds and rates to contribute to its budget.

Table 2.6 presents the prevailing main residential rates and bands of LTT in Wales and SDLT in England and Northern Ireland. The band structure presented is compromised between the two tax systems for an explicit comparison.

Table 2.6 Current Residential Main Rates and Bands: LTT versus SDLT

<table>
<tr><th>Price Threshold</th><th>LTT</th><th>SDLT</th></tr>
<tr><td>Up to £125,000</td><td rowspan="2">0%</td><td>0%</td></tr>
<tr><td>£125,001 ~ £180,000</td><td rowspan="2">2%</td></tr>
<tr><td>£180,001 ~ £250,000</td><td>3.5%</td></tr>
<tr><td>£250,001 ~ £400,000</td><td>5%</td><td rowspan="3">5%</td></tr>
<tr><td>£400,001 ~ £750,000</td><td>7.5%</td></tr>
<tr><td>£750,001 ~ £925,000</td><td rowspan="2">10%</td></tr>
<tr><td>£925,001 ~ £1,500,000</td><td>10%</td></tr>
<tr><td>Over £1,500,000</td><td>12%</td><td>12%</td></tr>
</table>

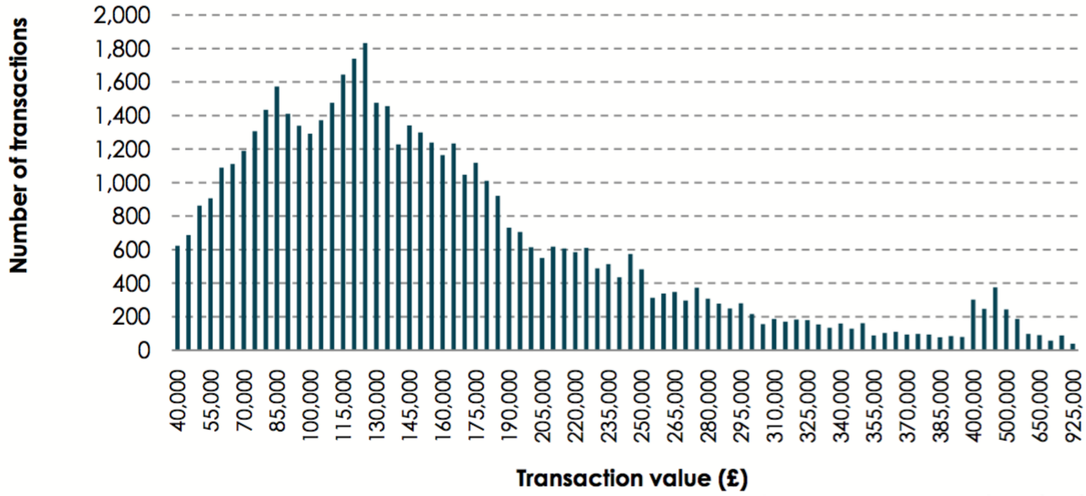
These tax rates apply to both freehold and leasehold residential properties. For leasehold properties, the SDLT rates apply to the purchase price of the lease (the ‘lease premium’), and apply additionally to the total rent over the life of a new lease only (known as the ‘net present value’). The additional rate is 1% on the net present value portion over £125,000. The LTT system in Wales, however, does not impose the additional rate on the newly granted residential lease. Both LTT and SDLT do not impose the additional rate on the existing, or assigned, residential lease.

On top of the main residential rates, a higher residential rate 3% is applied in both systems when the buyer has already owned one or more residences unless the residence to purchase is used to replace the main residence. Some special rules are also shared between the two systems which are not enumerated here. The first-time-buyer relief (FTBR) still remains under SDLT but has been abolished under the LTT system, which is only eligible for properties valued no more than £500,000 and the first time buyers don’t pay any tax up to £300,000 and 5% on the portion from £300,001 to £500,000.

Figure 2.8 shows the price distribution in terms of the number of residential transactions in

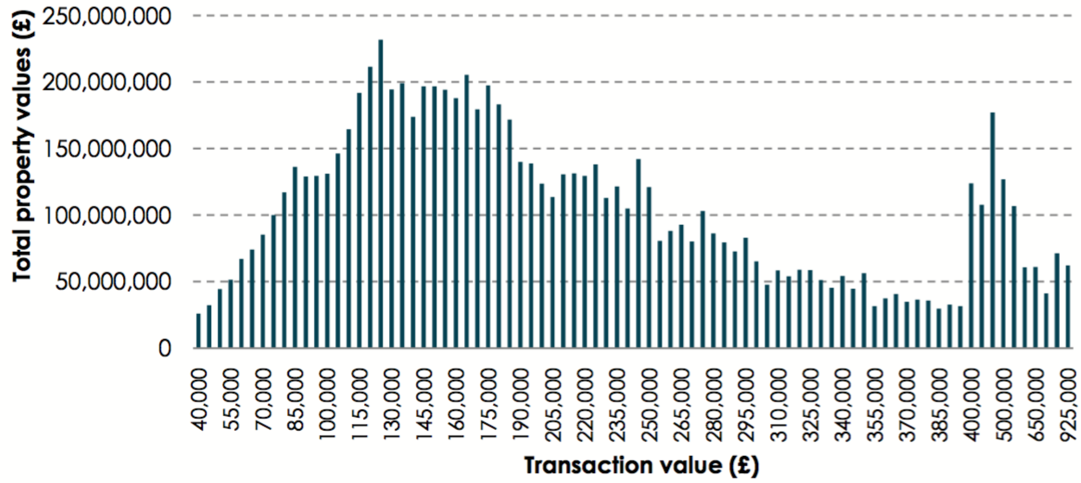
2015/16. It is estimated that 42% of transactions were valued below the starting price threshold £125,000 under SDLT, while 69% were below the starting threshold of LTT system, £180,000. There were only 14% of transactions above the value of £250,000 and 3% were above £400,000. However, the higher valued transactions contributed a much larger share of the total transaction value that the 14% of transactions above £250,000 accounted for 31% of the total transaction value and the 3% over £400,000 accounted for 11%. This is shown in Figure 2.9 as the right side of the distribution tail significantly humps compared to the same part in Figure 2.8.

Figure 2.8 Price Distribution in terms of Number of Residential Property Transactions in Wales: 2015/16



Source: Jones et al. (2017)

Figure 2.9 Price Distribution in terms of Value of Residential Property Transactions in Wales: 2015/16



Source: Jones et al. (2017)

Given the price distribution of the chargeable residential transactions and the comparison of the band structure between LTT and SDLT, it can be concluded that the Welsh Government has given 27% of transactions below £180,000 a tax exemption, a further 28% of transactions between £180,000 and £400,000 a mild tax cut, and the top 3% above £400,000 a tax hike. To see this, we can calculate the break-even price points that align the generated tax revenues in each band of the two systems.

For residential properties valued below £125,000, the corresponding 42% of the transactions would neither be benefitted nor worse off under LTT, while those transactions of £125,001 ~ £180,000 are apparently better off by a further tax exemption under LTT, and these transactions account for 27% (69% - 42% = 27%).

The band £180,001 ~ £250,000 does not give the answer straight away as the LTT rate suddenly rise to 3.5% compared to 2% remained under SDLT. This can be however sorted by finding the breakeven price, which equals approximately £253,333. As this price falls slightly above the threshold £250,000, this implies any transactions valued below £250,000 would still be better off under LTT.

Considering when the price enters the band £250,001 ~ £400,000 both rates are raised to 5%, the upper boundary of the tax cut under LTT can be promoted further to £400,000, as under the ‘slice’ style of the tax systems, the tax revenues from the portion that falls into this band are equal, only those below the band determine the relative size the revenues. This can be verified by the break-even point calculated as £402,000, which implies the threshold £400,000 would be the divide that determines whether a transaction would be imposed more tax or not under LTT compared to SDLT.

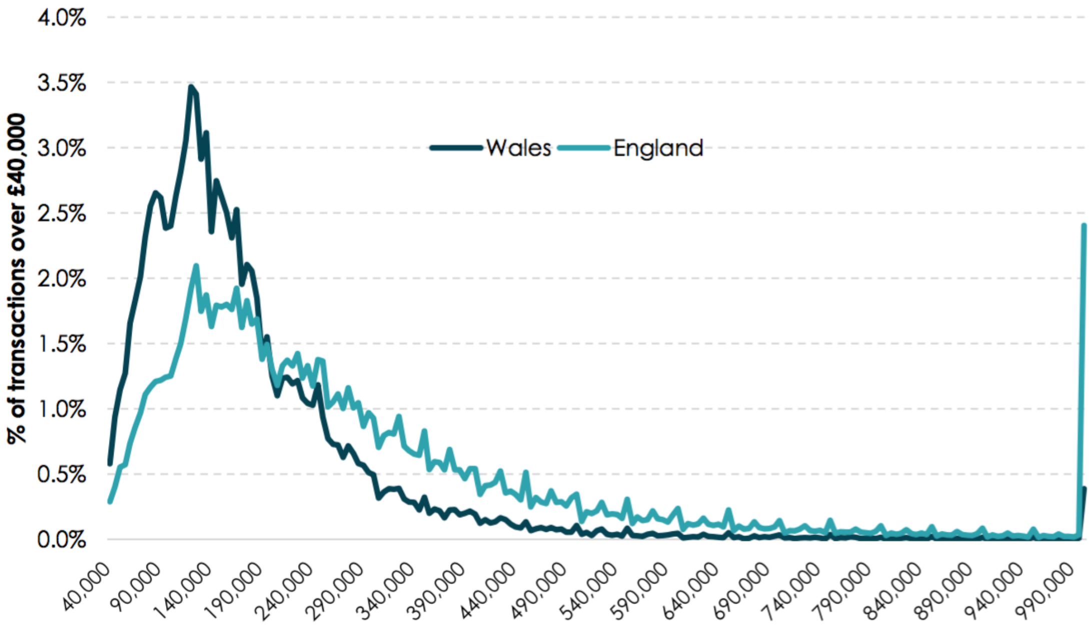
Although the tax rates for those above £925,000 are identical under two systems, the rates for the band £400,001 ~ £925,000 are much higher under LTT, the accumulation effect would be a tax hike for those valued above £400,000 which accounted for 3% of transaction numbers and 11% of total transaction value.

However, the comparison between the two systems is merely based on the band structure and the transaction distribution, it does take those special rules, reliefs and exemptions into account.

Generally, it is still too early to judge the devolved policy effect on the tax revenues and the underlying tax base performance.

The price distribution difference between Wales and the rest of the UK (mainly England) may have been accounted for in designing the LTT bands and thresholds. This can be seen in Figure 2.10. This figure clearly shows the significant variation between the two regions' price distributions, where a much larger share of properties in Wales are sold at lower prices, whilst England, on the contrast, depends more on higher-valued transactions. The spike at far right of the distribution represents the transactions valued over £1 million that generate over a third of tax revenues in England but only a negligible portion in Wales. The intersection point between the two distributions is just located roughly at £180,000, which is the upper threshold of the starting band of LTT. Hence, a regionally tailed LTT structure has aimed to reduce burden of those lower valued transactions that form a much larger share in Wales.

Figure 2.10 Price Distribution of Residential Property Transactions in Wales and England: 2016



Source: HM Land Registry Price Paid Data

For those non-residential property transactions, the bands and rates are largely different with those of residential transactions. Table 2.7 presents the current band structures of main non-residential rates under LTT and SDLT.

Table 2.7 Current Non-Residential Main Rates and Bands: LTT versus SDLT

<table>
<tr><td>Price Threshold</td><td>LTT</td><td>SDLT</td></tr>
<tr><td>Up to £150,000</td><td>0%</td><td>0%</td></tr>
<tr><td>£150,001 ~ £250,000</td><td>1%</td><td>2%</td></tr>
<tr><td>£250,001 ~ £1,000,000</td><td>5%</td><td rowspan="2">5%</td></tr>
<tr><td>Over £1,000,000</td><td>6%</td></tr>
</table>

Like residential main rates, the non-residential main rates apply to both the price of freehold and leasehold non-residential properties. However, for newly granted leases, the net present value (NPV) of rent is liable for tax under both systems. The band structures for NPV are listed in Table 2.8.

Table 2.8 Current Non-Residential NPV Rates and Bands: LTT versus SDLT

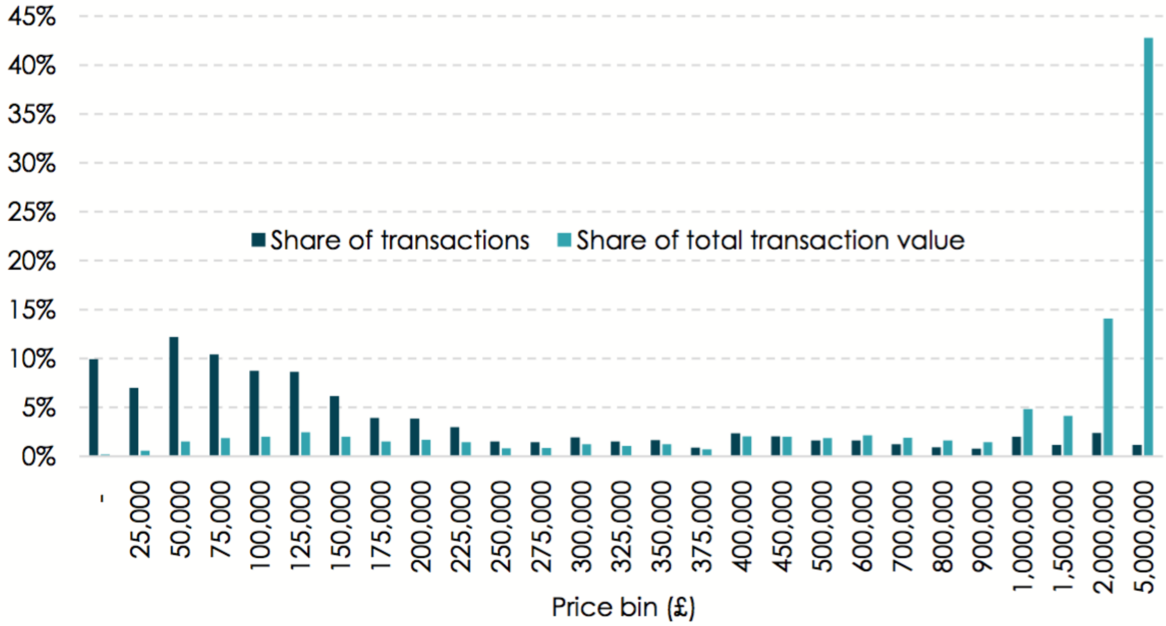
<table>
<tr><td>NPV Threshold</td><td>LTT</td><td>SDLT</td></tr>
<tr><td>Up to £150,000</td><td>0%</td><td>0%</td></tr>
<tr><td>£150,001 ~ £2,000,000</td><td>1%</td><td rowspan="2">1%</td></tr>
<tr><td>£2,000,001 ~ £5,000,000</td><td rowspan="2">2%</td></tr>
<tr><td>Over £5,000,000</td><td>2%</td></tr>
</table>

Compared to the residential transactions, the non-residential property transactions are far more concentrated on higher value transactions as shown in Figure 2.11. This figure presents the share distribution of all non-residential property transactions in terms of their numbers and prices in 2015/16.

The distributions of transaction number and price of non-residential properties shows a significant mismatch. While 96% of transactions below value of £2 million accounted only for 26% of total transaction value, the 200 transactions over £2 million accounted for 74%, in which the only 45 transactions over £5 million accounted for as high as 43%.

Given the band structure between the two systems, we can calculate the break-even price point as £1.1 million. Therefore, compared to SDLT, the LTT in Wales gives tax cut for those transactions below £1,100,000 which accounts for approximately 93% of transactions but only one fifth of total value, and a tax hike for those high value transactions above £1,100,000. The liable tax is also increased for those newly granted leases as the highest rate 2% is imposed on from the NPV value £2 million, while the threshold is more than twice higher as £5 million under SDLT.

Figure 2.11 Price Distribution in terms of the Share of Non-Residential Property Transaction Number and Value in Wales: 2015/16



Source: Jones et al. (2017)

With the significant price distribution differences, there is no surprise to see different paths of SDLT revenues tax bases in Wales and England in the past years. After peaking in 2006/07, SDLT revenue in England suddenly fell sharply because of the financial crisis, and gradually recovered until 2013/14 when it fully recovered its peak level. However, SDLT revenue in Wales has recovered much more slowly that it has only recovered until very recently. Considering that the number of transactions in Wales has been roughly kept pace with England, it is the property price which failed to recover quickly that caused the larger and larger receipts gap.

Besides, the different compositions of the tax bases in Wales and England also delivered divergent policy effects. As a result, the tax system often tended to have raised more revenues in England which has a much larger share of higher value properties imposed by higher rates. While this portion of revenue is relatively safe in England, it may not necessarily be so in Wales. A large share of revenue generated by a relatively small share of higher value transactions which are facing relatively higher LTT rates implies the devolved revenues, especially those incurred from non-residential transactions, may turn out to be highly volatile. However, the devolution of SDLT is still necessary for Welsh tax policies tailored for the Welsh economy, and for shaping the future tax base in determining the regional budget.

2.5.4 Landfill Disposals Tax

The Landfill Disposals Tax (LDT) was also fully devolved to Wales from 1 April 2018 by Wales Act 2014 to replace the original Landfill Tax. This tax applies to the waste disposed to landfill and aims to reduce its amount. It is paid by around 20 landfill sites in Wales and charged by weight with 3 tax rates shown in Table 2.9.

Table 2.9 LDT Rates: 2018/19 and 2019/20

LDT	2018/19	2019/20
Lower Rate	£2.80 per tonne	£2.90 per tonne
Standard Rate	£88.95 per tonne	£91.35 per tonne
Unauthorised Disposals Rate	£133.45 per tonne	£137.00 per tonne

The lower rate is charged on non-hazardous and low polluting waste and the standard rate is for all other material. Both rates will remain consistent with those in the rest of the UK for 2 years across 2018/19 and 2019/20. Unauthorised disposal rate is for taxable disposals made at places other than authorised landfill sites. The tax revenue has remained broadly stable in recent years around £50 million, however, the amount of standard rate waste has been declining and by Jones et al. (2017) the tax revenue is forecasted as around £28 million in 2018/19, with a further decline of one fifth by 2021/22. The relative small amount of revenue collected from the tax will have little risk to the overall size of the Welsh budget, while it mainly provides an environmental policy lever for the Welsh Government.

2.5.5 Welsh Rates of Income Tax

The next devolved tax will be Income Tax. From 2019/20 fiscal year, the tax will be partially devolved to the Welsh Government which was set out in the Wales Act 2017. Following the partial devolution, the tax collected from Welsh taxpayers will be shared between the Welsh and UK Governments. The Welsh government will then have the power to set Welsh Rates of Income Tax (WRIT) in each tax band, and receive revenues incurred from these rates. The current Income Tax bands and rates are listed in Table 2.10.

Table 2.10 Current Income Tax Bands and Rates in England and Wales

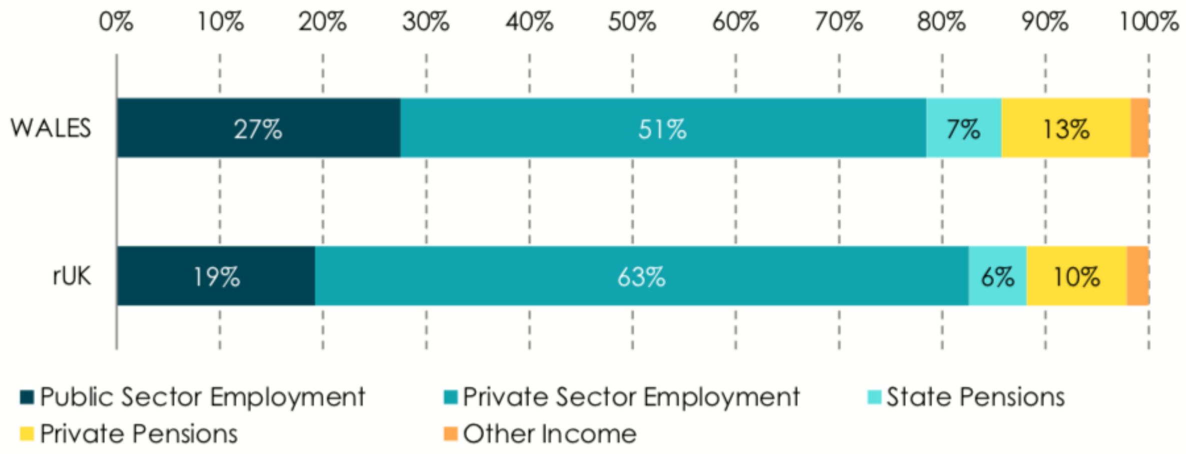
Band	Taxable Income	Tax Rates
Personal Allowance	Up to £11,850	0%
Basic Rate	£11,851 ~ £46,350	20%
Higher Rate	£46,351 ~ £150,000	40%
Additional Rate	Over £150,000	45%

Based on the current bands and rates, each of the three UK tax rates will be reduced by 10p in the pound paid by Welsh taxpayers. If the Welsh Government introduce its own rates as a 10p rate in each band by approval of the National Assembly, the UK government will collect the remainder. Specifically, in the basic rate band, Welsh taxpayers will pay 10% of their income to the Welsh Government and another 10% to the UK Government; for higher rate and additional rate bands, Welsh taxpayers will still pay 10% respectively to the Welsh Government, while pay 30% and 35% to the UK Government respectively. Office for Budget Responsibility (2016) forecasted that the Welsh Government budget will be directly funded by around £2.1 billion in 2019/20 given this arrangement that keeps the tax rates paid by Welsh taxpayers actually unchanged.

Income Tax paid on savings and dividends incomes will not be devolve and hence the corresponding tax base in Wales is mainly composed of employment income and pensions. Figure 2.12 presents a breakdown of the tax base in Wales as compared to the rest of the UK (rUK).

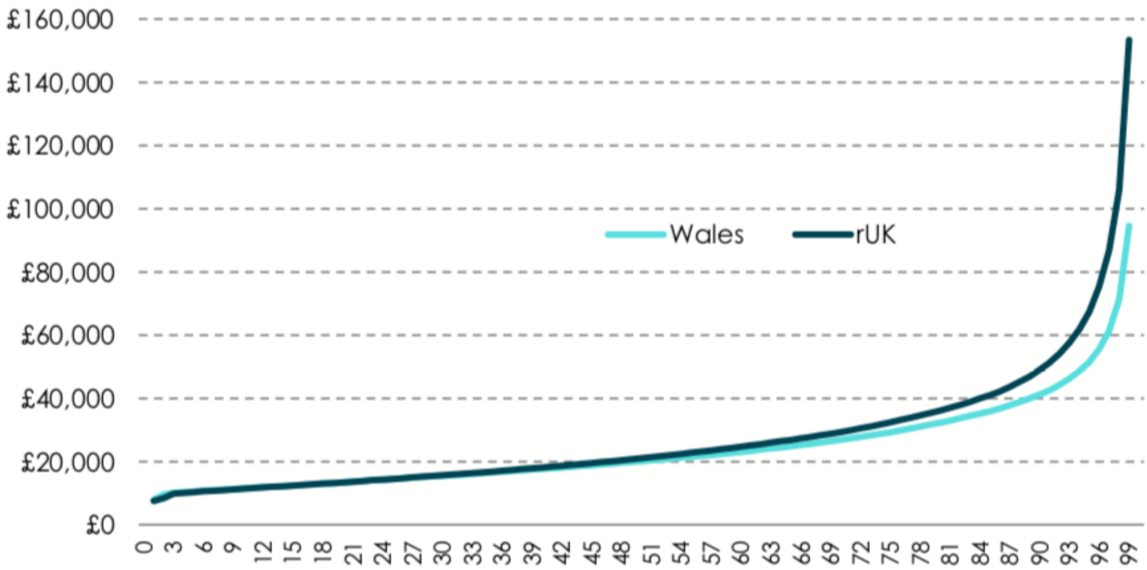
Compared to the rest of the UK, the income from public sector employment in Wales accounts for a much larger share, while the private sector employment income is relatively smaller. State and private pensions combined also occupied a larger share in Wales. The tax base difference reflects somehow the sectoral and labour age structure difference between the two regions. This also implies a significant divergence regarding the income distribution. As shown in Figure 2.12 and Figure 2.13, although the median taxpayer income basically coincides, a more advanced and larger private sector in rUK makes higher income possible. According to HMRC (2018), at the 90th percentile the taxpayer income is £48,900 in the rUK, higher than £41,200 in Wales. The gap is much larger as shown at 99th percentile, where the taxpayer income is as high as £153,400 in rUK compared to only £94,600 in Wales.

Figure 2.12 Income Tax Base Breakdown in Wales and rUK: 2015/16



Source: HMRC (2018)

Figure 2.13 Taxpayer Income Distribution in Wales and rUK: 2015/16



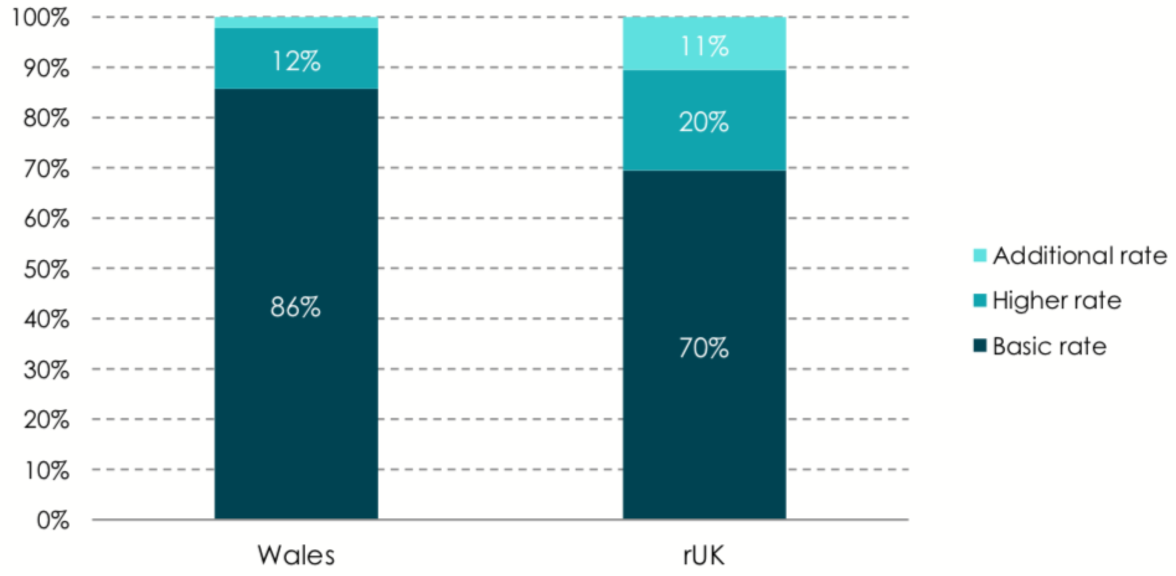
Source: HMRC (2018)

Structuring and fitting the income distributions into the tax bands gives the tax revenue by band as shown in Figure 2.14.

The divergent style of income distribution between Wales and rUK results in the different structures of tax revenues derived from each tax band. Most of the tax revenues are generated from the basic rate band, while for rUK the share is 70%, in Wales it is as high as 86%.

Oppositely, the revenue shares of higher rate and additional rate in Wale are much lower than rUK for about 8 to 9 percent, implying a less dependence of tax revenue on the higher-end income in Wales. Hence, it is necessary for the Welsh Government to formulate the devolved rates of Income Tax tailored for the characteristics of Welsh tax base, otherwise the Welsh economy may suffer disproportionate effects from universal tax policy throughout UK, such as the increase of the personal allowance. It is also reasonable for the fiscal framework between the Welsh and UK Government to establish separate BGA for each tax band to account for the tax base difference.

Figure 2.14 Income Tax Revenue by Tax Bands in Wales and rUK: 2015/16



Source: HMRC (2018)

2.5.6 Other taxes considered for devolution

Apart from those taxes already devolved, there are other taxes that are still under consideration of the UK Government. The most possibly devolved taxes are Air Passenger Duty (APD) and Aggregates Levy.

APD is an excise duty charged on the carriage of passengers from airports, of which short and long-haul APD combined generated an estimated £10 million in Wales in 2014/15. The recommended part by the Silk Commission is the long-haul APD which would only accounts for a small portion as only around £1 million. The UK Government has also considered devolving the Aggregates Levy, which is a tax on the commercial exploitation of rock, sand and gravel. In 2014/15, its revenue is estimated at around £30 million in Wales. Both taxes are minor taxes with small scale of receipts and hence will not largely affect the size of the Welsh budget (Poole et al. 2016).

The Wales Act 2014 also enabled the Welsh Government to introduce specific new taxes in Wales under the agreement of both Westminster and the Welsh Assembly. In October 2017, the Cabinet Secretary for Finance announced 4 tax ideas that the Welsh Government would develop further: a vacant land tax to encourage the increase of residence supply and commercial development, a social care levy to meet growing care demand of aging population; a disposable plastics tax to encourage re-use behaviour and environment protection, and a tourism tax to help provide sustainable and quality tourism services.

Other types of taxes that are devolved or under consideration in other regions include VAT in Scotland and Corporation Tax in Northern Ireland. However, the same VAT rates will continue to apply across Scotland and the rUK with half of its receipts directly assigned to the Scottish Government, and the plan to devolve Corporation Tax has been postponed indefinitely. The devolution of Corporation Tax was supposed to deal with the relatively low rate of Republic of Ireland, however its devolution within the UK is not much discussed as it may give rise to unnecessary tax competition between UK regions (Seely 2018).

2.6 Conclusion

This chapter presents the background information for understanding and modelling the devolved tax variation in Wales. Basic economic facts and regional variations of the Welsh economy against the rest of the UK is investigated. Fiscal context is discussed regarding to the tax devolution process and current public fiscal status of Wales. The final part devotes to the tax structure and tax base composition of those devolved taxes in Wales.

In summary, the Welsh economy is a relatively lagged regional economy within UK, and due to historic and geographic factors, it differs largely against the rest of the UK in terms of economic structure and production factor characteristics. Even within Wales the regional economic disparity is still significant. It has a relatively large public sector and used to depend largely on the heavy industry. Currently, relatively low agglomeration level and inactivity rate marks the employment base of the economy. Financial and banking, high-tech and high value added sectors still have great potentials in future development. The overall economic condition has also generated different composition of tax bases against other UK nations. As a result, Wales is actually under a public sector deficit status, which implies it is financially subsidized by the rest of the UK. As a result, the tax devolution process and the establishment of Welsh constituent bodies has delivered chances to formulate fiscal policies tailored to the Welsh economic characteristics and shape and enhance the underlying tax bases.

The public revenue and expenditure for Wales are largely detached that while the UK Government directly funds for Wales, it also allocates the block grant annually to the Welsh Government to form part of its budget. Along the tax devolution process carrying forward, both fully and partially devolved taxes will incur block grant adjustment which may bring risks to the Welsh budget size. The chosen adjustment mechanism would inevitably generates complexity to the tax devolution and its economic effects on the Welsh economy. Considering the undetermined status of the BGA mechanism and its exogeneity with regard to Wales, this study will neutralize the change of funding from the UK Government so as to isolate the economic effects from only internal tax variations within Wales.

CHAPTER 3 LITERATURE REVIEW OF CGE MODELLING AND ITS FISCAL POLICY APPLICATIONS

3.1 Introduction

With an ever-expanding increase in the demand for policy evaluation across a variety of issues, computational approaches have been applied as the workhorse of simulating and assessing the impacts of policy shocks. While empirical econometric analyses have been important in the field of economic research, computable general equilibrium (CGE) models have been developed to meet the demands of policymakers for more complex economic analysis.

This chapter aims to provide a methodological literature review of CGE modelling and simulations, particularly in a regional context, in application to a wide range of policy issues. The review is conducted in order to help inform this study of quantitatively evaluating the devolved tax varying power of Wales, a small open regional economy.

In the following sections, we begin by exploring economic models developed so far to understand changes in activity in the Welsh economy. This reveals the strengths and weaknesses of these approaches, and makes the case for developing a CGE to consider issues around tax. We then proceed with focusing on CGE models applied in a regional context. The basic classification of regional CGE models is discussed, followed by reviews regarding several major model characteristics and specifications adopted in different studies. By commenting on and summarizing different model elements designed for different research topics, implications are drawn for understanding regional CGE models and how they can be used for modelling and simulation in the cases of tax issues that will be examined in this thesis. A case study is further dedicated to explore the Scottish AMOS model for better understanding regional CGE simulations in Appendix I.

3.2 Economic modelling developed for Wales and applications

While a number of economic models have been developed for the UK national economy, there are a smaller number of models developed specifically to deal with sub-national problems. The situation applies especially to Wales, which features a relatively high level of economic integration with the English economy. In this regard, there has been a tendency in the past to believe that Welsh economy effects can be generalised from the findings of UK economy models. However, the process of devolution has brought forth new demands for economic data and more policy-led demands for bespoke analysis of the Welsh economy, and to this has been added the need for economic analysis to understand and support policy interventions on tax varying at the regional level. As to the Welsh economy alone, there were early attempts to develop models and understand the effects of structural change in Wales using input-output techniques. These early economic models were based on a historical series of input-output tables, and with this accompanied by a series of general limitations relating to general assumptions on prices, technical coefficients and scale.

The series of input-output tables for Wales have been developed following pioneering work by Nevin et al. (1966). This early input-output table was constructed for 1960 detailing the transactions between 31 regional industries. Most of the industries were assumed to feature the same sectoral intermediate transaction patterns with their UK counterparts, except five key industries, including locally important coal and steel sectors. In these cases, regional specific data was built into to the tables framework to allow some scenario modelling.

Ireson and Tomkins (1978) built on the foundation of Nevin et al. (1966) and developed 1968 input-output tables for Wales. These tables were based on a large-scale project leading to the production of local social accounts. A more comprehensive time series of input-output tables were produced by Cardiff Business School for the years 1994-1996, 2000 and 2007 (Hill and Roberts, 1996; Brand et al., 1998; Hill and Roberts 2001; Bryan et al. 2004, 2010). These regional economic modelling works occur with applications to industrial studies, tourism, and the environment.

However, in their application these input-output frameworks were applied to demand-led modelling assuming fixed input proportions, fixed prices and no flexibility in the supply side. While useful in a series of applications these general assumptions made the input-output modelling unsuitable for tax modelling.

However, there have been more complex economic models of Wales produced. For example, several dedicated econometric models have been developed to focus on the Welsh economy. This includes works by Wanhill (1980), Ioannidis et al. (1995), Minford and Riley (1995), and Foreman-Peck and Lungu (2009). Ioannidis et al. (1995) presented a structural vector autoregression (VAR) model, while both Minford and Riley (1995) and Foreman-Peck and Lungu (2009) developed economic models with an emphasis on the supply-side. These developments tended to result in highly aggregated models and this limited practical applications. Wanhill (1980) however, constructed a more complex econometric model for annual forecasting with national accounts data categorized by Standard Industrial Classification (SIC).

The structural VAR model (Ioannidis et al., 1995) used seasonally adjusted quarterly data covering the period 1977-1992. They investigated the impact of extra-regional shocks on the Welsh economy. Key components of the regional economy included in the model were housing prices, manufacturing output and unemployment. The regional GDP is unavailable for a quarterly frequency therefore unemployment was used as a proxy.

They assumed that the three Welsh variables are only passively impulsed by their national counterparts, national financial shocks and internationally sourced shocks, but not the other way around. The results were taken to show the openness of the regional economy. The simulation results confirmed that while house prices between Wales and the UK behave similarly in response to financial shocks, that Welsh manufacturing output is relatively more volatile when exposed to exchange rate shocks as opposed to interest rate shocks. This was taken as evidence of the relative openness of the regional economy. In general, Welsh manufacturing features much larger relative variance against its national UK counterpart in response to external stimuli. Indeed, these issues have been brought to the forefront again with current interest in how Welsh manufacturing will respond to BREXIT compared to the wider UK economy.

Foreman-Peck and Lungu (2009) compare the economic ramifications between balanced government spending expansion and a reduction. This is in the context of tax devolution and the growing fiscal dependency of Wales. Their model is a three sector Hecksher-Ohlin style model formulated following the Minford et al. (1994) model of Merseyside. Using annual data from 1971 to 2001, the structural econometric model only deals with a small sample of 30 data points for estimation. In this model non-traded sector output is demanded by both the traded sector and government sector. The traded sector output is assumed to be determined only by the sector employment without any benefits generated from government sector demand. All the private sectors produce with prevailing factor and product prices which are exogenously fixed.

The simulation results from Foreman-Peck and Lungu (2009) show that a government spending increase funded by a 1% increase of income tax is not as good as a fiscal contraction of 1% income tax reduction in terms of GDP. This is because the tax reduction yields more private sector jobs and more total employment than the fiscal expansion. The results indicate that the number of jobs created in the government sector due to fiscal expansion, do not exceed the employment rise stimulated by the tax reduction. This then implies a crowding-out effect that private sector resources are drawn from private sectors. Such results are in fact not beyond expectation with the model specification and the underlying assumptions.

Based on these results, it was argued that given the high fiscal dependency of Wales, that an excessive net inflow or transfer from central government may cause deflationary pressure on the regional private industrial sector. This is because local taxpayers do not value public goods as highly as private goods and then consumption demand is diverted to imports from outside the region. Such a situation can be explicitly accounted for by the region's current account balance proposed in the paper i.e. the Welsh current account deficit equals the net fiscal subsidy for Wales. In this sense, net central government transfers to some extent allow more extra-regional imports and the crowding-out effect may be even exacerbated by such a central government transfer in the case of a negative shock. However, we should note that this balance of payment identity may merely be a theoretical demonstration rather than an empirical one. It implies that the balance of payments does not necessarily hold through only monetary quantity variation, while the exchange rate between Wales and the rest of UK is fixed by using the mutual currency.

To summarize the current situation on economic modelling for Wales is limited. The developed econometric models are typically hindered by the lack of well-maintained, continuous and disaggregated time series data. This weakens model accountability and model forecast capacity. In addition, only one round of effects tends to be traced in the model frameworks. This conclusion is particularly applicable to any modelled consequences of tax variation. For example induced effects following on from direct responses are not explicitly considered. For example, Foreman-Peck and Lungu (2009) show that tax reductions could boost major regional macroeconomic variables. From these changes the tax base and the corresponding tax revenue could increase, so as to offset part of the initial tax reduction shock. However, these effects are not explicitly considered in their approach.

In terms of the input-output modelling frameworks developed for the Welsh economy, they have served to lay a strong foundation for understanding the transactions between Welsh industries, and have vastly improved regional accounting. However, while these approaches have had a wide application in impact assessments in Wales, their use in modelling applications are limited by the general limitation of demand driven input-output model. In particular, the absence of price variation makes such approaches incapable of considering substitution effects due to relative prices change. This makes it very difficult to use such models for analysis of tax variation.

CGE models, in comparison, can to some extent overcome the weakness of both types of models reviewed above, while retaining selected advantages. They have gained increased acceptance due to their capacity to deal with a wide variety of issues and due to the fact that they do not require the long statistical series that are necessary for econometric models (Menezes, et al. 2006). Principally they only utilize one year's data to form a Social Accounting Matrix (SAM). Even though constructing social accounting matrices is problematic, it is easier, in many cases, than obtaining relevant long time series data (Bayar et al., 2006). The SAM is based on disaggregated data from national accounts and input-output tables, as a baseline equilibrium for comparison against simulation results. Manipulation of the CGE model can then allow us to consider both supply-side and demand-side effects simultaneously and observe responses to exogenous shocks with relative prices varying. Moreover, the general equilibrium structure allows such a framework to capture rounds of shock effects until a new economic equilibrium converges. These characteristics qualify the CGE model as an ideal simulation tool for studying tax policy changes and helping inform fiscal policy formulation.

3.3 Regional CGE models

CGE models can be applied in a flexible context in terms of geographic scope. CGE modellers can focus on a single area which can be as small as a region or as large as a nation, or even a series of national economies. They can also focus on the linkages between different but related regions or nations and observe their interactions simultaneously. The scope and structure of modelling must be tailored to particular research needs, and provides a mode of model classification.

There are several ways in which regions are defined in CGE models. Some models disaggregate an economy into rural and urban regions (for example, see Bautista and Thomas, 2000; Jung and Thorbecke, 2001). Others identify specific states, cities, provinces, towns, and even villages in a country. Levantis (2006), for example, disaggregates Australia into its 6 states and 2 territories. On the other hand, Domingues and Haddad (2002) divide Brazil into San Paolo and the rest of the country. In some instances, a region in a country is disaggregated further into sub-regions. For example, Nakayama and Kaneko (2003) have identified the rural and urban regions of Beijing and Shanghai in China.

Existing regional CGE models can be grouped into two classes. The first are region-specific or single-regional models which focus on a particular area in a country. These models assume that changes in the region do not have an impact on the economy as a whole. The other class is multi-regional CGE models. These models specify a country which is divided into two or more regions. These models are more data intensive than single-regional models. This is because they require explicit specification of the behaviours of households and industrial sectors at the regional level, and also explicit treatment of inter-regional trade. Lack of either would make these model results weak in terms of accounting for feedback from the regional to national level. This section focuses on the single-regional models across various applications.

A Single-regional model is very similar to models of a country in terms of model structure. It identifies households, industrial sectors, government and foreign agents in a region. Moreover, the behaviour of these agents is also specified in the same way as in standard models. Finally, the base dataset is always the SAM.

The most significant difference between a national and region-specific CGE is the treatment of the foreign sector. In a national CGE model, the foreign sector represents the rest of the world. In contrast, the foreign sector in a single-regional CGE model is composed of the rest of the country and other countries in the rest of the world.

For the single-regional CGEs, a regional model is not only distinguished with a national one by the scope of borders, but also by other characteristics uniquely featured in a relatively small regional economy. For instance, for a region that is geographically part of a larger national system and is generally highly integrated with the rest of the nation, there is typically a more limited range of policy levers to gain macroeconomic adjustments. Moreover, regional authorities typically have little or no monetary policy options, nor do they have a full set fiscal policy options.

In such a regional context, both product and factor markets will typically feature a higher degree of mobility and openness compared to those at the national level (Liebeg et al., 2007). According to Lecca et al. (2014), the reasons for this include a higher proportion of regional output tending to serve export demands, lower transaction costs and ease of substitution. The reason for factor market mobility, especially the labour market, lays in the fact that inter-regional labour movements are significantly easier compared to international migrations.

Single-regional CGEs are also useful in examining issues which tend to be ignored in national models. These may be local concerns which are not relevant at the national level. These may also be important issues at the national level but simulations in national models are not expected to have noticeable impacts. In such instances, the costs of modelling the issue in national CGE are high relative to the usefulness of returns.

Horridge (1999) has provided an example of a regional issue that was examined using a single-regional CGE. The paper analyzed the effects of higher transport costs (which may be due to higher fuel taxes or road tolls) in Melbourne, Australia. It examines the impacts on the proportion of residents who work in the same zone, average distance commuting from home to office and the proportion of residents who live in high density housing, i.e. flats. Another example is the work of Holden et al. (2005) for the Ethiopian highlands. In this paper, one of the experiments involves examining the impact of removing fertilizer subsidies on land degradation in the region.

Single-regional CGE models are not only confined to the evaluation of policies for a particular region. The analysis of national policies is actually quite common in these models. For example, the aforementioned experiment with fertilizer subsidies (Holden et al., 2005) could easily be a national policy, although it may simply be the case that the interest of the study is on its regional impacts only.

In evaluating the impacts of a national policy, single-regional models are sometimes used in connection with a national CGE. For example, San et al. (2000) have examined the effects of a devaluation on the Sumatra region of Indonesia. In implementing the analysis, the authors followed a two-stage process. The first stage implemented the devaluation in a CGE model of Indonesia. In the second stage, the impacts on prices from the simulation were used as inputs into the single-regional model.

Kuiper and van Tongeren (2004) have conducted an even broader set of experiments. This study examined the impacts of removing tariffs and other import barriers of OECD countries on a specific village in Jiangxi, China. The authors initially implemented the experiment in a global model. The impacts on prices and labour demand from the simulations were then used as inputs in the CGE model for the village.

Among the single-regional models, the size of the regions in interest differs. Horridge (1999) and Nakayama and Kaneko (2003) have constructed models for relatively large cities (Melbourne, Beijing and Shanghai). In contrast, Kuiper and van Tongeren (2004) have used a model for a village in China that is composed of less than one thousand households. Similarly, the model of Stroombergen and Stuart (2003) represents a region in New Zealand that has a population of 73,000 only.

Models also differ in the way and degree of disaggregation. The number of commodities can range from 2 (Horridge, 1999) to 37 (Floros and Failler, 2004). At the level of households, Andre et al. (2004) had only one representative household for the region while de Miguel and Manresa (2004) had 11. The models of Nakayama and Kaneko (2003), San et al. (2000) and Horridge (1999) also included a regional disaggregation in their models: the first two models contained an urban-rural disaggregation while the third divided the Melbourne region into 9 zones. San et al. (2000) and de Miguel and Manresa (2004) disaggregate households according to location (rural-urban), income and age. These are categories usually found in models of a country. On the other hand, some models use classifications which appear to be more relevant to the region being studied. For example, Holden et al. (2005) classify households in the Ethiopian Highlands according to their ownership of oxen and Aryal (2005) disaggregate households in the Mardi Watershed of Nepal according to caste.

The key advantage of single-regional CGE models is their ability to simulate the impacts of policies and events, both regional and national, at the regional level. This type of assessment is valuable to regional authorities in terms of policy formulation and evaluation. The main constraint in constructing a single-regional model is data availability. SAM or Input-Output tables are often not officially available at the regional level on a continuous basis, especially for developing countries. However, the constraint is even larger for multi-regional models which requires data for all regions included in the model, as well as the accurate information regarding inter-regional trade. For this thesis, however, the shortage of inter-regional trade statistics and disaggregated regional data does exist, hence a single-regional model is more appropriate for the Welsh economy.

3.4 CGE applications and the general effects of tax changes

This section considers several empirical applications of CGE modelling in specific tax policy issues.

Fullerton et al. (1983) looked at the impact of replacing the 1973 U.S. tax system with a progressive consumption tax. They found both tax systems to be distortionary. Also, they show that sheltering more savings from the tax system could improve economic efficiency, even if marginal tax rates increase to maintain government revenue.

Chowdhury (1991) estimated the welfare and distributional effect of VAT and excise taxes of equal yield on Bangladesh. He found that poorer households were adversely affected by this reform and there was then a need for a different rate structure to improve the efficiency of the indirect tax system.

Frankel et al. (1991) examined the macroeconomic effects of VAT harmonization in Europe. Their result indicated that VAT harmonization lead to internal conflicts among countries as effects were not spread evenly across income groups, generations and countries.

Corporate tax variation has also been a field of study where CGE models have been employed. Corporation Tax variation has been criticized for creating differential rates of return to capital in different industries and countries. Specifically, it is argued that allocation of investment in the economy is distorted in favour of poorly incorporated sectors. It also doubly taxes income at both personal and corporate levels. Therefore, some have proposed integrating the two tax systems. Pereira (1993) looked at inter-temporal and inter-sectoral efficiency and distributional effects of integrating corporate and personal income taxes. The model is specialized to the US economy. It accommodates optimal inter-temporal investment decisions and allocation across sectors, inter-temporal household consumption and savings, government deficits, and crowding out. The results show that eliminating corporate tax and replacing it by increased income tax rates would yield long-run benefits that are at best 0.17% of the present value of future consumption and leisure. Also, average long run gains were three times larger than average short-run gains. The study also finds that partial integration yields negative gains, and in its distributional effects, it is shown that, with integration, highly incorporated sectors undertake more capital formation and low-income households become worse off.

Bogart and Gentry (1995) studied the relation between the marginal tax rates on capital gains and revenue realizations in the Washington, D.C. and surrounding states. They examined an elasticity assumption which implies a decreasing capital-gains tax rate would lead to an increase in revenue from capital-gains taxation. They found an elasticity value of -0.65. This meant that cuts in capital gains tax rates do not lead to sufficient generation of revenue to offset the losses from tax cuts.

Kraybill and Pai (1995) evaluated the effects of a job tax credit program that Ohio began in 1992. According to the program, the state government is permitted to decrease the state corporate tax liability of new or expanding firms by an amount equal to 100% of the personal income tax withheld for every new employee for a period lasting 10 years. Some of the features of the model include endogenously determined labour supplies and capital stock, inclusion of investment multipliers and a state and local government balanced-budget requirement. The initial credit was the creation of 32,000 jobs in the manufacturing sector. The state output growth, investment, and exports differ according to whether or not there is a retaliatory program from neighbouring states. When surrounding states do not introduce tax abatement programs similar to Ohio's, the study found that real output went up by 1.6% annually, investment increased by 2.6%, and exports expanded by 3.6%. If there was full retaliation, however, the growth rates were 0.27% for real output, 0.1% for exports, and 0.6% for investments. Furthermore, annual wages declined for all skill categories compared to the case when there is no tax retaliation.

Waters (1997) studied an event in 1990 when Oregon in the US passed a ballot measure that placed a ceiling on local property tax rates at 1.5% of their market value, and that any resulting shortfalls in local education expenditures were to be met with transfers from general state funds at the expense of other programs. The study aimed to investigate the impact in state fiscal year 1996, if assessed property values remained at 1990 levels. According to the results, education tax revenues decreased by 74%, while compensating transfers to education from state general funds increased by 90%. At the same time, state non-education tax revenues went up by 1.1%-1.2%.

Rege (2002) presented CGE model for India using Leontief and Cobb-Douglas production function and found that replacing existing indirect tax of India with VAT will lead to a reduction in welfare.

Go et al. (2004) quantified welfare, revenue and distributional effects resulting from South African tax reform. They took 2003 as base year and set 10 households income deciles for the study. They included 6 sectors and 49 commodities in the model with various elasticities applied from the existing literature. Four production factors such as capital, high- skilled, semi-skilled and unskilled labour were included in the model. They made four simulations for the analysis such as removal of VAT, increase in VAT by 50%, zero VAT for food and replacing existing tariffs with uniform VAT. They found that VAT negatively affects the welfare of low income households.

Giesecke and Tran (2009) developed a general equilibrium framework to analyze macroeconomic and sectoral effects of the tax system in Vietnam. They suggested single rate instead of three tax rates of VAT and found that private real consumption would increase with this single rate VAT system.

Aviststant et al. (2011) compared three proposals i.e. current indirect tax, VAT with some services and uniform VAT by using a CGE model calibrated on 1995 national accounts data. They found that welfare effect is correlated with number of goods and services included in VAT. VAT with only goods lead to more welfare than VAT with some services, and the uniform VAT results in superior welfare.

The above reveals that CGE models have had a wide application in the field of tax analysis, but with rather fewer studies examining land taxes and land sales taxes, which is the subject of this thesis. In the next part of this chapter we turn to review some aspects of CGE model specifications.

3.5 CGE model time perspectives

For research that focus on tracing system-wide impact of a particular shock towards the long-run equilibrium, it is often useful to distinguish different time perspectives in terms of the conceptual periods of the model in use. This is in order to improve understanding of how the response of the economy evolves. To explicitly define time intervals from short to long run, it is important to consider how long a conceptual period represents in any model simulation. Typically this is regarded as one year (McGregor et al.,1996). This is because data employed for a CGE model is normally annual data. This is also a typical time period as a Social Accounting Matrix is based on a year, and is the benchmark database for model parameter calibration. Hence, CGE modellers are able to establish a connection between the virtual and practical time periods to improve their understanding of simulation results.

As an example of defining and identifying the difference between the time perspectives in model simulation, Ferguson et al. (2007) distinguish the short, medium and long run according to the degree that supply constraints are relaxed. In their study, population and capital stock are both fixed in the short run, while the medium run is marked by fully adjusted population because of inter-regional migration and a fixed capital stock. It is only in the long run that the capital stock, as well as population, is fully optimized. Such an arrangement is also coherent with an earlier study of McGregor et al. (1996), which argued that infinitely elastic labour supply is a phenomenon much more credible at regional rather than national level, as characterized by the existence of regional excess labour supply at the ruling wage and high inter-regional, or intra-national labour mobility through conventional regional migration functions (Greenwood et al., 1991; Layard et al., 1991; McGregor et al., 1993; Treyz et al., 1993). Therefore, in the short run both capacity and labour supply are constrained. However in the long run, both elasticity of factor supplies increases to eliminate the constraint, especially for the regional labour supply that precedes capital stock in the process of full adjustment.

Focusing on a long run time context characterized by full factor stock adjustment, McGregor et al. (1996) show that the equilibrium solutions of the CGE model just replicate the results of the Input-Output model in response to final demand disturbances. The conditions include linear homogeneous production technologies, fixed interest rate and import prices, full factor adjustment and limited joint-production (Stiglitz, 1970; Johansen, 1972). These are all standard neoclassical settings of CGE models in a regional context. Small region financial and labour markets are highly integrated with their national counterparts and hence are conventionally price-takers. In this case the interest rate is exogenously set and the wage rate maintained at the existing level to ensure a perfectly elastic labour supply (McGregor et al., 1996).

McGregor et al. (1996) demonstrate how the long run equilibrium replicates I-O solutions using the case of a 10% stimulus in manufacturing export demand. They begin by simulating the effects with perfectly elastic labour supply but with a capital constraint - this corresponds to the medium run categorized by Ferguson et al. (2007). Then as the model is run forward the capital constraint is gradually eased and then the simulated variables indefinitely converge to their I-O 'solution' counterparts.

The reason why the long run equilibriums simulated by CGE replicate the I-O figures is made clear by McGregor et al. (1996): with the conditions mentioned above satisfied, final demand disturbances do not generate price change of inputs and commodities. This is because in the long run the factor supply constraint is fully eased to reinstate the prices to the original base level. While the relative prices maintain constant in the long run, even a substitution-mechanism-available CGE will operate as if by fixed coefficients typically featured by an I-O system. This implicitly assumes no factor constraints and relative price change. Hence, in the long run, the CGE is in fact simulated with the major characteristics of I-O and renders I-O type results. The implication of their study is that the long run solution of the CGE can be conveniently generated in an I-O framework. Clearly, one of the strengths of I-O lays in its comparative ease of implementation. It tends to give prominence to the comparative advantage of CGE in the short-term observation of the regional economy, with the availability of relative price change and substitution mechanism only activated in a constrained capacity context.

3.6 CGE and regional fiscal institutions

Regional fiscal arrangements within a national system vary around the world. Among those systems, the UK case attracts some research interest because of the evolving devolution settlement (McGregor and Swales, 2005). In Wales the prospect of greater fiscal power enhancement has existed since the work of the Richard Commission (2004). While there is a growing research literature examining the regional effects of UK fiscal devolution much of the work is concentrated on the Scottish economy (see Ferguson et al., 2003; McGregor et al., 2007;

Lecca et al., 2014). The focus of research is on related issues of spatial equity in funding distribution across regions, sustainable regional development under devolution, ‘spillover’ effects, and greater fiscal autonomy (Ferguson et al., 2003; McGregor and Swales, 2005). There is rather less consideration of the systematic effects of the level of the regional budgets in the context of the Barnett formula. Ferguson et al. (2007) is one exception which is next reviewed.

Before proceeding to the review of the work, it is useful to explain the working mechanism of the Barnett formula. The Barnett formula is used by the Treasury in the UK to automatically adjust the amounts of public expenditure allocated to Northern Ireland, Scotland and Wales to reflect changes in spending levels allocated to public services in England, England and Wales or Great Britain. If the central government department funding covers England only, the principle is that any increase or reduction in expenditure in England will automatically lead to a proportionate increase or reduction in resources for the devolved governments in Wales, Scotland and Northern Ireland. The formula can be expressed as below.

$$\begin{array}{c}\text{Extra funding}\\ \text{in Scotland, Wales}\\ \text{or Northern Ireland}\end{array} = \begin{array}{c}\text{Extra funding}\\ \text{in England}\end{array} \times \begin{array}{c}\text{Population proportion}\\ \text{compared to England}\end{array} \times \begin{array}{c}\text{The extent to which the relevant English}\\ \text{departmental programme is comparable with the}\\ \text{services carried out by the devolved administration}\end{array}$$

According to Edmonds (2001), an extra amount of funding to Scotland, Wales and Northern Ireland consists of a baseline plus increases, based on the increases in public spending in England in comparable programmes, applied in proportion to current populations. The formula applies only to expenditure on issues for which the devolved administrations are responsible.

Given the structure of the Barnett formula, the study of Ferguson et al. (2007) contributes to the literature of regional fiscal institution effects in that it establishes a bi-directional connection between regional population and government expenditure. In this case, both variables are interacted and endogenized such that the population is available to enter into the determination of regional government expenditure via the Barnett formula. Meanwhile there will always be a corresponding equilibrium population level for any given level of government expenditure. Within this framework, the study investigated the economic consequences of a rigorously implementing Barnett formula i.e. until the per capita public expenditure converges across Scotland and England, which implies a significant reduction of Scottish assigned budget for public expenditure in the long term. This final point of the convergence is the ‘Barnett equilibrium' where the endogenous Scottish government expenditure and population are mutually underpinned in equilibrium while the exogenous English counterparts remain constant. Therefore in contrast to prior research (see McGregor et al., 2007; Allan et al., 2014; Lecca et al., 2014), Ferguson et al. focus on a regional fiscal policy effect where the regional government expenditure can be set endogenous to regional population through population weight of the Barnett formula.

In Ferguson et al. (2007) the time perspective setting reviewed in the preceding section is consistent with the needs of modelling an endogenous regional population where the inter-regional migration responds to macroeconomic indicators, accompanied with a capital stock update. They find that in the short-run where factors are fixed, the contractionary effect of imposing the Barnett formula is relatively small. However, in the long run with regional population updated for government expenditure, effects in terms of the scale of the reduction to GDP and employment could be more than doubled. With the detailed disaggregation of 25 industrial sectors, rather than normally 3 aggregated sectors in 'Tartan tax' effect literature, the results reveal that the sectors most affected are non-traded sectors, especially those providing public goods such as public administration and defence, education, health and social work.

Their work extends the literature of understanding the overall regional effect of the Barnett formula, while it is still not clear whether the Departmental Expenditure Limits (DEL) as representing government expenditure in the analysis, is the real limit for regional government in practice, or includes other expenditures directly from central government which could account for a sizable portion of public expenditures for Scotland.

3.7 CGE: parameterization and sensitivity analysis

As in other macroeconomic and econometric models, it is very important to pin down all the parameters required by the model specification before proceeding to any simulation. This process is even more important for CGE models. This is because CGE models require generating solution values for all the endogenous variables, and one needs to ensure all the values replicating the benchmark dataset are established for the chosen base year. This is necessary before introducing any disturbance or shocks to the models. Therefore the applied parameters mutually determine the magnitude and direction of the variable transmission mechanism implied by the model specification, and subsequently affect the extent of deviation of post-shock counterfactual solutions from the benchmark.

Berck and Dabalen (1995) emphasized the importance of parameter estimation and calibration in their review. They stress the necessity to take into consideration the issue of model parameterization when conducting policy change analysis. They divide the CGE model parameters into two broad groups: observable and unobservable. The first includes, for example, factor shares and input proportions in production technologies, depreciation rates etc. These can be calibrated through the benchmark dataset. The latter (unobservable) concerns those elasticities and preference parameters which are not easily observable directly from the base year economy. Usually these hard-to-observe parameters are 'best guess' (e.g., McGregor et al. 1996) because the related regional data for econometric estimation is often subject to availability problems. This is why sensitivity analysis around the key model parameters is an important step for better understanding the model simulation and then potentially informing relative policy formulations.

3.8 CGE and regional tax effects in the UK

CGE has found some application in the study of regional tax varying in the UK. (For a wider review of the use of CGE in this respect see section 3.4 above).

McGregor et al. (2007) and Lecca et al. (2014) use CGE models to examine the regional impact of varying the rate of income tax, or so called the "tartan tax" in Scotland. They both investigate the mixed effects of varying income tax rates on aggregate economic variables in Scotland. Whilst McGregor et al. (2007) choose 1997 as base year, Lecca et al. (2014) update the analysis based on a 2004 Scottish SAM. The simulated policy shock is assumed to augment balanced budget government spending financed by activating the full Scottish Variable Rate. This is allowed in the context of the regional devolution arrangement for Scotland. The increased government expenditure is capped by an adjustment of up to three pence in the pound of the basic rate of income tax, equivalent to a 1.45% rise in average personal income tax, which is estimated as £450 million at 1997 prices in McGregor et al. (2007). In Lecca et al. (2014) using 2004 budget estimates, it is a 1.52% rise and an estimated £810 million at 2004 prices.

The core of their research is that they not only focus on the positive demand-side stimulus effect through the mechanism of conventional Keynesian balanced budget multiplier, but also explore the supply-side effects. This is called the “competitiveness effect” through the labour market, and captures the net impact of the assumed policy shock. The possible negative competitive effect through employment and population is noted as an “inverted Haavelemo effect” (see Knoester and van der Windt, 1987). The mixed effect is examined by incorporating a shift term standing for a tax rate change into both the zero-net migration function and regional bargained real wage function. Both terms are multiplied by the coefficient β measuring the residents and potential migrants’ tax amenity. In particular, the tax amenity indicates how the households value public consumption set against their own private consumption. The regional bargaining function also includes the coefficient α which represents the level of social-wide bargaining, or the extent to which the amenity is incorporated into the wage bargaining process, as multiplied by the tax amenity parameter. In this way, both studies assume an imperfect labour market. This gives an opportunity for regional tax variation to directly effect regional wage determination. For example, the existence of a monopoly union can fully internalize the amenity effect for the bargaining behaviour ($\alpha=1$). On the contrary, for a perfectly competitive labour market, the individual and atomized workers are unable to impact the scale of the amenity effect ($\alpha=0$), which is in most cases of the fiscal federalism literature. Hence, the direct connection between the income tax rate change and labour market aggregate variables is built through the influence from amenity parameters.

The full range of macroeconomic variables, especially those labour market aggregates, are simulated in the Macroeconomic model of Scotland (AMOS) in response to the balanced public expenditure rise, with different combinations of amenity parameter values as a sensitivity analysis. The model updates through the net migration function and records the long-run percentage change of the macroeconomic variables against their initial base year values. The results clearly show that the coefficients α and β significantly determine whether there is an overall positive net impact or a negative one given the same fiscal policy stimulus.

A typically ideal outcome dwells on the combination that both the coefficients equal 1. In this case the augmented government expenditure can boost a positive effect on the regional economy without spurring price and wage variables. This is because households treat public consumption as a perfect substitute as their own, without the need to bargain a higher nominal wage to cover their higher income tax payment. However, whenever one of the coefficients is zero, the Keynesian multiplier effect is heavily offset by the negative supply-side effect as nominal wages and prices rises to reduce regional competitiveness. The research also includes sensitivity analysis towards price elasticity of export demand on two moderate value sets of α and β combination, *ceteris paribus*. It shows that the higher the elasticity, or more open the economy, the more likely that the negative supply-side effect will occur because the regional economy with greater export shares suffers a more adverse effect when the consumption for regionally produced commodities is substituted due to the labour cost and price increase.

In addition to Lecca et al. (2014), McGregor et al. (2007) also express an open attitude to discussion regarding concerns with their major labour market assumptions. Complementing their core analysis and results, they consider replacing the migration function with a labour demand function and imposing zero labour mobility in response to the concern of low UK labour mobility. The results prove to be a relative weakened effect on employment under the parameter combination $\alpha=1$ and $\beta=2$.

Another alternative assumption concerns the exogeneity of the regional nominal wage. It is argued that the regional wage is basically in line with the national counterpart through national bargaining or nation-wide multi-regional firms (McGregor et al., 2007). In this case, the paper experiments with the exogenously fixed nominal wage line instead of the regional bargained real wage function in simulating the employment effects. However, as the zero net-migration function is still in position then regional residents attach less value on public consumption against their private value. Here then there should be, predicted by this alternative setting, employment rising, possibly also accompanied by outmigration simultaneously, for which the AMOS model fails to produce. Besides, in spite of the discussion above under each alternative assumption, it could also be constructive to consider the two alternative assumptions simultaneously (exogenous regional nominal wage and zero labour mobility) with plausible amenity parameter combinations, in addition to this line of researches.

Allan et al. (2014) also undertook research in this broad and applied a regional bargained real wage function, and a net migration function describing regional labour market variation and update (following Layard et al., 1991). The application covered was in terms of a carbon tax. This research focused on the economic and environmental impact of an ecological policy in Scotland. The paper examined the impacts of reducing CO_2 emissions by 42% in 2020 (compared to a 1990 base year). The policy scenario is the imposition of an *ad valorem* carbon tax on the use of three energy sources: coal, oil and gas, as intermediate inputs.

The CGE model applied in this study is an energy-economy-environmental variant of the AMOS macroeconomic model. There were 17 industrial sectors including 13 energy sectors in the model setting. The energy sectors covered a comprehensive range of energy generating industries with various input natural sources. Calibrated on a base year 2000, three scenarios were simulated with the model: the first one explores the tax revenues being spent in the rest of UK; the second one investigates the tax revenues used for local government spending expansion; and the last assumes income tax reduction funded by the carbon tax revenues.

The rapid reduction of CO_2 emissions in the short run were easily captured by each simulation. Unemployment and post-tax real wage always returned to their initial level in the long-run, which is an unsurprising converging movement predicted by the model's migration function. For all three scenario simulations, only the last one presents a positive effect on the regional economy. The first two both result in a reduction in economic activity because the carbon tax levy directly affects regional industrial production. The second scenario with carbon tax revenue spent inside the regional economy only partially mitigates the overall contraction effect. Although the last scenario simulation focuses on the income tax cut, the authors did not apply the augmented migration function of McGregor et al. (2007) and Lecca et al. (2014) to explore any possible inverse tax amenity effect of a tax rate cut as opposed of tax rate rise. Neither did they investigate the tax amenity against the public expenditure in the second scenario. The study relied on the standard functional form for modelling labour market and migration based on the approach of Layard et al. (1991). It therefore did not examine the effect of the combination of the alternative labour market assumptions – exogenous regional nominal wage and low labour mobility - as mentioned above, either. However, while Allan et al. (2014) did not incorporate the element of tax amenity, the exogenously fixed nominal wage would only imply fixed labour demand and population in this case.

3.9 CGE: different labour market closures

The degree of factor mobility plays an important part in the specification of a regional CGE model and differs from one model to another. In general factor mobility increases with the time span of analysis. It is quite common to find fixed factor supplies at least in the short term. With imperfect factor mobility, factor returns may not be equal across regions, and factor productivity differentials across regions may persist over time.

To specify the closure of the regional labour market, the majority of the models use the neoclassical assumption that wages are endogenous and flexible so wages adjust in order to equate labour demand and labour supply. There are also regional models incorporating Keynesian features that wages and prices are fixed at predetermined exogenous levels and there may be factor underutilization or unemployment due to this wage rigidity. Table 3.1 shows how different models have dealt with factor mobility and labour market closure.

Table 3.1 Regional Literature, Factor Inputs, and Labour Market Closure

Literature	Inputs	Closure
Berck et al. (1991)	K, L, M, R, water; K, L fixed	Endogenous wages
Berck et al. (1996)	K, L, M; L flexible	Endogenous wages
Buckley (1992)	K, L, M; K, L fixed	Endogenous wages
Condrad and Schroder (1993)	K, L, M, E; K, L fixed	Exogenous wages
Despotakis and Fisher (1988)	K, L, M, E; K, L fixed	Endogenous wages
Gazel (1996)	K, L; K fixed, L free	Endogenous wages
Gazel et al. (1995)	K, L; K, L fixed	Endogenous wages
Haddad (1999)	K, L, M; K, L fixed	Wage differentials
Hertel and Mount (1985)	K, L, M, E; K fixed, L free	Exogenous wage bill
Hoffman et al. (1996)	K, L, M; Several cases	Several cases
Jones and Walley (1989, 1990)	K, L, M, R; K, L free	Endogenous wages
Kimbell and Harrison (1984)	K, L, M; K free, L fixed	Endogenous wages
Koh et al. (1993)	K, L, M, R; L free	Several cases
Li and Rose (1995)	K, L, M, E; K, L fixed	Several cases
Liew (1984)	K, L, M, R; L fixed	Endogenous wages
Morgan et al. (1989)	K, L, R; K, L free	Endogenous wages
Morgan et al. (1996)	K, L, R; K, L free	Endogenous wages
Peter et al. (1996)	K, L, M; K fixed, L free	Several cases
Rickman (1992)	K, L, R; K fixed, L free	Several cases
Schreiner et al. (1999)	K, L, R, M; K fixed, L free	Endogenous wages
Walley and Trela (1986)	K, L, R; K, L free	Endogenous wages

Note: K – capital, L – labour, M – intermediate goods, E – energy, R – natural resources.

In the UK case, especially in respect of research in Scotland, the setting of labour market closures is more complicated. In fact, the bargained real wage function applied in the aforementioned Scottish literature reflects only one of the noted regional labour market closures and is not initially incorporated in the AMOS framework. The system-wide impacts of other labour market closures initially considered in AMOS are illustrated in detail in Harrigan et al. (1992). This research examined two sets of effects from both the supply-side and demand-side of the economy under the introduction of 5 alternative labour market closures. The closures reflect different descriptions of the operational mechanism of labour market aggregates, of which the ‘competitive’ and ‘real wage resistance’ represent the flexible and fixed real wage respectively; ‘Keynesian’ stands for the exogenously determined fixed nominal wage and ‘sticky wages’, or ‘regional Phillips curve’ stands for flexible nominal wage responsive to regional consumer price index and unemployment. The last, ‘exogenous labour-supply’, corresponds to a fixed total employment on the base year level, which in this paper calibrates on the year 1979.

In Harrigan et al. (1992), the simulation starts from tracing the effects from the supply-side, for which they set a 5% increase in nominal and real wage separately. Since in a CGE model, the simulated shock can only dwell on the arbitrary value change of exogenous variables or parameters, the supply-side effect simulations imply that the nominal wage shock is performed under ‘Keynesian’ closure, while the real wage shock is performed under ‘real wage resistance’ closure. Both wage shocks cause a decline of GDP and as expected, the real wage shock results in more, as 1.95% and 3.06% respectively. While the 5% increase in real wage is reached by the nominal wage increasing 5% larger as exceeding CPI in the simulation result, the 5% increase in nominal wage compromises the real wage as CPI increases by 1.78%. The research also considers varying the degree of commodity market integration between the Scottish economy and the rest of UK (RUK) and world (ROW) under real wage resistance closure. This is represented in value by the corresponding commodities’ elasticity of substitution. The sensitivity analysis shows employment falls with higher degree of integration, implying the regionally produced commodities are substituted away by foreign demand as more sensitive to higher regional commodity prices due to the real wage hike.

The supply-side simulation considers the effect of 10% increase in export demand under all 5 labour market closures. All closures report positive impacts on GDP, of which the largest increase of 2.03% appears in the ‘Keynesian’ case, and 1.32% for ‘real wage resistance’ case. Effects from the rest are more moderate. Despite the common assumptions of the AMOS application in the Scottish economy, such as perfect competition in commodity markets and standard neoclassical production functions, all the simulation results of Harrigan et al. (1992) are based on the aggregate fixity of capital and labour, where perfect sectoral mobility within the region is only allowed with labour. Such an assumption is consistent with the static modelling adopted by the study, where labour migration modelling is implicit. In fact, the interregional integration of the labour market has already been implied by two of the closures: 'Keynesian' and 'real wage resistance', as exogenously determined regional wages have to be sustained by interregional migration flow. Moreover, as argued by Harrigan et al. (1992), the two closures can successively capture perspectives from the short-run onwards in a sequence of economic adjustment processes, no matter whether the regional labour market is assumed as spatial insularity or integration. While the regionally exogenous nominal wage of 'Keynesian' closure is appropriate to appear in the short-run for both extreme regional integration forms, the exogenous real wage of 'real wage resistance' appropriately captures medium-run in spatially isolated market and long-run in spatially integrated market.

3.10 Conclusion

This chapter has considered prior research on regional CGE modelling, different aspects of CGE models and why they are important. A specific review of the Scottish AMOS model in Appendix I is also presented to better understand these in the form of a case study. The previous literature and the dedicated case study will inform the modelling and simulations undertaken in the following chapters. Before introducing the theory framework of CGE models, the next chapter will first focus on the modelling object, the Welsh economy, and the relevant fiscal background.

CHAPTER 4 THEORY AND SPECIFICATION OF ECONOMIC BEHAVIOURS IN CGE MODELLING

4.1 Introduction

This chapter explains the theory of CGE modelling and its various specifications in describing the economic behaviours of different economic agents. As a general equilibrium model, the CGE model describes both supply and demand, as well as the trade behaviour of an open economy. Consequently, the decision-making processes regarding consumption, production, and the export and import of goods, are modelled by equations of various functional forms. Understanding and formulating these equations is the first stage in the construction of a regional CGE model for the Welsh economy. Various functional forms are introduced in this chapter as the behavioural equations across both producers and consumers.

The most frequently used functional forms include the Cobb-Douglas function, the Leontief Input-Output (I-O) function, the Constant Elasticity of Substitution (CES) function, and the Constant Elasticity of Transformation (CET) function. These functions are not only used to model consumption and production, as the CES functions and the related CET functions have also been used in the context of the aggregation of composite goods and the transformation of domestic goods in a small open economy. Derivations and applications of these functional forms are illustrated in this chapter, and this theoretical background provides a foundation for the development of the CGE model for Wales.

The structure of this chapter is organized as follows. Section 4.2 and 4.3 present various functional forms used in CGE models, by examining production behaviour in the supply side, and household behaviour in the demand side. Section 4.4 introduces the nest structure organizing and connecting the optimization behaviours of different agents in the whole economy. Section 4.5 presents the concept of model closures as different combinations of macroeconomic balance constraints. The regional CGE model is then derived based on the model framework and characteristics of the Welsh economy in section 4.6. The final section 4.7 provides some conclusions.

4.2 Production behaviour

The fundamental behavioural model for enterprises is the production function. This is a mathematical specification of how factor services combine to transform resources and components into goods and services, and lies at the heart of CGE modelling. This section reviews a variety of widely accepted specifications for production functions and discuss how they can be implemented empirically.

4.2.1 Elasticity of substitution and transformation

The elasticity of substitution (EOS) and elasticity of transformation (EOT) are important parameters in CGE modelling equations. They determine the degree of substitutability and transformability between function inputs. Consider a standard convex production function in two inputs $Q = f(K, L)$. For a given isoquant representing some constant level of output Q, the firm can substitute labour for capital, thus moving along the isoquant without sacrificing or gaining output. The marginal rate of technical substitution of labour for capital ($\mathrm{MRTS_{LK}}$) is defined as - dK/dL. It shows the rate at which labour can be substituted for capital while holding output constant i.e. along the isoquant. In equilibrium, the $MRTS_{LK}$ is equal to the ratio of factor prices, (W_L/W_K). Any rational profit-maximising firm will alter its capital-labour ratio in response to a change in the factor price ratio. The EOS measures the proportionate change in the capital-labour ratio (K/L) relative to a proportionate change in the ratio of prices (W_L/W_K=$MRTS_{LK}$) (Nicholson 1992). Thus,

$$\sigma = \frac{\%\Delta(K/L)}{\%\Delta MRTS_{LK}} = \frac{d(K/L)}{d(MRTS_{LK})}\frac{MRTS_{LK}}{(K/L)} \tag{4-1}$$

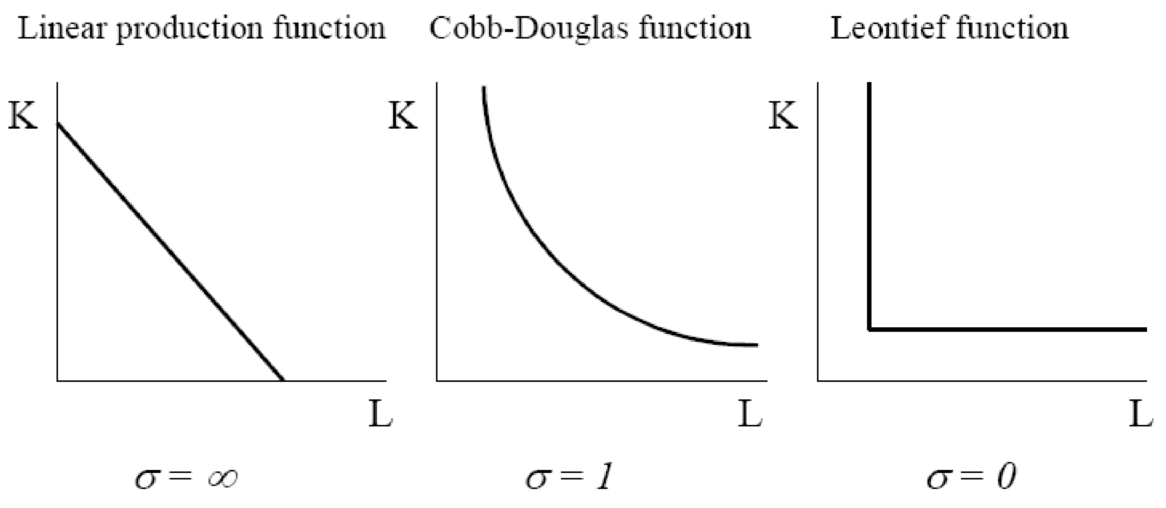
Note that the capital-labour ratio declines as labour is substituted for capital, i.e. labour is increased and capital is decreased. At the same time the slope of the isoquant ($MRTS_{LK}$) also declines, provided the production function is convex with respect to the origin. The EOS is therefore a ratio of the rate of decline of the capital-labour ratio and the rate of decline of the $MRTS_{LK}$. Most mathematical production and consumption functions used in economics are established in such a way that the EOS is constant, i.e. the rate of decline of the capital-labour ratio is equal to the rate of decline of the slope of the isoquant, as labour is substituted for capital. Note that the EOT refers to the same concept relating to transformation functions, i.e.

functions that are concave with respect to the origin.

To illustrate the elasticities more explicitly, three basic functional forms are considered below, each with different constant elasticities of substitution. The same principle applies throughout to concave functions and the EOT. It is shown that the larger the value of σ, the easier factors can be substituted.

Firstly, a linear isoquant of the form $Q = aK + bL$, can be considered, this is characterised by perfect substitutability between the factors. For a given level of output $\bar{Q}$, the absolute value of the slope of this function is given by $|dK/dL| = b/a = MRTS_{LK}$. Since the slope is constant for all possible combinations of K and L, the change in the $MRTS_{LK}$ will always be zero. Consequently σ is undefined or "infinite" due to division by zero. Thus, for linear production functions characterised by perfect substitutability between factors, the EOS is infinite.

Figure 4.1 Shapes of the 3 Basic Functional Forms with Different EOS Values



On the other extreme the Leontief input-output function is characterised by zero substitutability, as inputs are used in fixed proportions to the level of output. Any profit-maximising firm will always produce at the corner point of this L-shaped function. Thus, as the ratio of factor prices change, the capital-labour ratio will remain unchanged. The percentage change in the capital-labour ratio is therefore always zero along a given Leontief input-output isoquant, and as a consequence, the EOS is also zero for functions with zero substitutability between factors.

The linear and Leontief production functions represent the two extremes for convex production functions, and hence the upper and lower boundaries of the EOS are defined by the expression $0 \leq \sigma \leq \infty$. Next, consider the Cobb-Douglas function, which is characterised by imperfect substitutability. The EOS of a standard Cobb-Douglas function is one. Figure 4.1 below shows these results graphically.

4.2.2 The Cobb-Douglas production function

The Cobb-Douglas function is the most widely used function in the general field of economics (Heathfield and Wibe 1987). It owes its name to Charles Cobb who suggested the functional form, and Paul Douglas who used manufacturing data for the United States, for the period 1899-1922, to infer its properties. Although the function was initially based on manufacturing data and two inputs from capital and labour, it can be extended to include multiple inputs. It can also be used to model consumption as utility functions. The two-input Cobb-Douglas production function has the following form:

$$Q = AK^{\alpha}L^{\beta} \tag{4-2}$$

The parameter A $(A > 0)$ is known as a shift or efficiency parameter. The parameters α and β $(0 \leq \alpha \leq \infty, 0 \leq \beta \leq \infty)$ are the function exponents. The function exponents determine the degree of homogeneity. If each factor is increased by a factor λ, total output will increase by $\lambda^{\alpha+\beta}$, as shown below.

$$Q' = A(\lambda K)^{\alpha}(\lambda L)^{\beta} = \lambda^{\alpha+\beta}AK^{\alpha}L^{\beta} = \lambda^{\alpha+\beta}Q \tag{4-3}$$

The Cobb-Douglas function will exhibit constant returns to scale if the function exponents are restricted to sum to unity. If $\alpha + \beta = 1$, β can be replaced by the expression $\beta = 1 - \alpha$, and hence we can rewrite the Cobb-Douglas function as $Q = AK^{\alpha}L^{1-\alpha}$. To illustrate an important characteristic of the Cobb-Douglas function, the marginal physical product (MPP_F) of each factor F is calculated below:

$$MPP_K = \alpha \cdot A \cdot K^{\alpha-1}L^{1-\alpha} = \frac{\alpha Q}{K} \Rightarrow \frac{MPP_K K}{Q} = \alpha \tag{4-4}$$

$$MPP_L = (1-\alpha) \cdot A \cdot K^{\alpha}L^{-\alpha} = \frac{\alpha Q}{L} \Rightarrow \frac{MPP_L L}{Q} = 1 - \alpha \tag{4-5}$$

The equations above show that, for a linearly homogenous Cobb-Douglas function, the function exponent of each factor represents the relative share of that factor in total output.

Furthermore, under conditions of perfect competition, the employer will hire additional units of a factor as long the additional factor price is less than the additional revenue generated by that factor. The firm will continue to do so until $W_F = MRP_F = MPP_F.P$ for all factors, where W_F stands for the factor price, MRP_F stands for the marginal revenue of product, and P represents price of output Q. In equilibrium, we have:

$$Q \equiv K\frac{\partial Q}{\partial K} + L\frac{\partial Q}{\partial L} = K \cdot MPP_K + L \cdot MPP_L = K \cdot \frac{MRP_K}{P} + L \cdot \frac{MRP_L}{P} \tag{4-6}$$

$$P \cdot Q = K \cdot W_K + L \cdot W_L \tag{4-7}$$

This result shows that the shares of total product paid to the factors capital and labour respectively, i.e. the wage times the employment level, exhaust the total product if W_F equals the MRP_F. It only applies in the long run equilibrium and under conditions of perfectly competitive product and factor markets where zero economic profits are made.

In the meantime, the equilibrium wage will be a fixed share of the average revenue product (ARP_F) of that factor, where the ARP_F is defined as ($AP_F = Q/F$) multiplied by price P:

$$\frac{MRP_K K}{P \cdot Q} = \alpha \Rightarrow W_K = \frac{\alpha \cdot P \cdot Q}{K} = \alpha \cdot ARP_K \tag{4-8}$$

$$\frac{MRP_L L}{P \cdot Q} = 1 - \alpha \Rightarrow W_L = \frac{(1-\alpha) \cdot P \cdot Q}{L} = (1-\alpha) \cdot ARP_L \tag{4-9}$$

The equations above are typically used as the first order conditions for profit maximization, also known as the factor demand equations. The factor demand equations satisfy the optimal input-ratio, which is derived using the standard approach to profit maximization. If the firm wishes to maximise profits a profit function (Π) needs to be defined as the total revenue (TR) minus total cost (TC):

$$\begin{aligned} \Pi &= TR - TC \\ &= P \cdot Q - W_K \cdot K - W_L \cdot L \end{aligned}$$

$$= P \cdot A \cdot K^{\alpha} L^{1-\alpha} - W_K \cdot K - W_L \cdot L \tag{4-10}$$

Differentiation with respect to *K* and *L* gives the following two equations, both of which should be simultaneously set equal to zero:

$$\frac{\partial \Pi}{\partial K} = \alpha \cdot P \cdot A \cdot K^{\alpha-1} L^{1-\alpha} - W_K = 0 \tag{4-11}$$

$$\frac{\partial \Pi}{\partial L} = (1-\alpha) \cdot P \cdot A \cdot K^{\alpha} L^{-\alpha} - W_L = 0 \tag{4-12}$$

This gives the optimal ratio of employment in equilibrium. Then it can be verified that the result obtained in equation (4-7) and (4-8) satisfies this equilibrium condition:

$$\frac{K}{L} = \frac{W_L}{W_K} \cdot \frac{\alpha}{1-\alpha} \tag{4-13}$$

The factor demand conditions for multiple-input Cobb-Douglas production function is generalized below. Suppose the function has n inputs and takes the following form:

$$Q = A \cdot {x_1}^{\alpha_1} {x_2}^{\alpha_2} \cdots {x_n}^{\alpha_n} \tag{4-14}$$

The first-order condition for profit maximization for this function is derived below. As before, multiplication of each factor by a factor λ will cause output to increase by a factor $\lambda^{\alpha_1+\alpha_2+\cdots+\alpha_n}$. Thus, if $\alpha_1 + \alpha_2 + \cdots + \alpha_n = 1$, the function is linearly homogenous. The parameter α_F represents, as before, the share of total product accruing to each factor, as is shown for factor *F* below:

$$\begin{aligned} x_F MPP_F / Q &= x_F \alpha_F A {x_1}^{\alpha_1} \cdots {x_F}^{\alpha_F - 1} \cdots {x_n}^{\alpha_n} / Q \\ &= \alpha_F A {x_1}^{\alpha_1} \cdots {x_F}^{\alpha_F} \cdots {x_n}^{\alpha_n} / Q \\ &= \alpha_F \end{aligned} \tag{4-15}$$

Since $MRP_F = MPP_F.P$ and $W_F = MRP_F$ in equilibrium, the following equilibrium condition can be derived as the first-order condition for profit maximization:

$$W_F = \frac{\alpha_F \cdot P \cdot Q}{x_F} \tag{4-16}$$

4.2.3 The Leontief production function

Another extreme case of the production function is the Leontief function which simply has the substitution of zero. The function can be expressed as below:

$$Q = min\left(\frac{x_i}{\alpha_i}\right) \tag{4-17}$$

In this case, there are no substitutions between any production inputs. The factor inputs change only in connection with the output level and in a proportionate relationship, regardless of the change in input prices. It can be derived to the forms below:

$$P = \sum_{i=1}^{n} \alpha_i P_i \tag{4-18}$$

$$x_i = \alpha_i Q \tag{4-19}$$

Here the unit cost is a simple weighted average of the input prices where the weights are given by the share parameters. The demand function clearly shows that input demand is invariant to changes in input prices.

4.2.4 The CES function

Although widely used, both the Cobb-Douglas function and the Leontief function have a major drawback: the EOS always takes on a pre-determined value. The CES function, first developed by Arrow, Chenery, Minhas and Solow (Nicholson 1992), allows for greater flexibility, in that the modeller can choose the value of the EOS. This function takes on the following form in the two-input case:

$$Q = A[\delta K^{-\rho} + (1-\delta)L^{-\rho}]^{-\frac{\varepsilon}{\rho}} \tag{4-20}$$

The parameter *A* $(A > 0)$ is the efficiency or shift parameter as in the Cobb-Douglas function. The parameter δ $(0 \leq \delta \leq 1)$ is a distribution or share parameter. It permits the relative importance of the inputs to vary, thus operating in much the same way as the function exponents of the Cobb-Douglas production function. The parameter ρ $(-1 \leq \rho \leq \infty, \rho \neq 0)$ is the substitution parameter or function exponent. The relationship between this parameter and the EOS is explained below. Finally, ε $(0 \leq \varepsilon \leq \infty)$ determines the degree of homogeneity of the function. The function can exhibit increasing, decreasing or constant returns to scale depending on the value of ε. Multiplying each factor by a constant λ changes the level of output as follows:

$$\begin{aligned} Q' &= A[\delta(\lambda K)^{-\rho} + (1-\delta)(\lambda L)^{-\rho}]^{-\frac{\varepsilon}{\rho}} \\ &= \lambda^{\varepsilon} A[\delta K^{-\rho} + (1-\delta)L^{-\rho}]^{-\frac{\varepsilon}{\rho}} \\ &= \lambda^{\varepsilon} Q \end{aligned} \tag{4-21}$$

Thus, if $\varepsilon < 1$, the function exhibits decreasing returns to scale, if $\varepsilon = 1$, there are constant returns to scale and if $\varepsilon > 1$ there are increasing returns to scale. Among them, the linearly homogenous (constant return to scale) function is mostly used in CGE models. The following discussion will then be based on the case of constant return to scale. The relationship between the EOS of a linearly homogenous CES function and the function exponent can be shown to be the following:

$$\sigma = \frac{1}{1+\rho}, 0 \leq \sigma \leq \infty \tag{4-22}$$

The range of the function exponent was given as $-1 \leq \rho \leq \infty, \rho \neq 0$. The CES function is not defined for $\rho = 0$ due to division by zero. However, using L'Hôpital's Rule it can be shown that as $\rho \rightarrow 0$, the linearly homogenous CES production function approaches the linearly homogenous Cobb-Douglas function (Chiang 1984). In general, the CES encompasses all of the functional forms as shown in Figure 4.1. The linear production function, Cobb-Douglas production function and Leontief production function can all be regarded as special cases of the generalized CES function. The flexibility of the CES function has contributed to its popularity in CGE modelling.

The first-order condition for profit-maximization can be derived directly by substituting the profit maximizing condition ($W_F = MRP_F$) into the equation for the MPP_F:

$$MPP_K = (-\frac{1}{\rho})A[\delta K^{-\rho} + (1-\delta)L^{-\rho}]^{-\frac{1}{\rho}-1}[\delta(-\rho)K^{-\rho-1}] \quad (4\text{-}23)$$

$$\frac{MRP_K}{P} = A[\delta K^{-\rho} + (1-\delta)L^{-\rho}]^{-\frac{1}{\rho}-1}[\delta K^{-\rho-1}] \quad (4\text{-}24)$$

$$\Rightarrow W_K = P \cdot A[\delta K^{-\rho} + (1-\delta)L^{-\rho}]^{-\frac{1}{\rho}-1}[\delta K^{-\rho-1}] \quad (4\text{-}25)$$

The equilibrium factor demand equation for L can also be derived similarly.

$$W_L = P \cdot A[\delta K^{-\rho} + (1-\delta)L^{-\rho}]^{-\frac{1}{\rho}-1}[(1-\delta)L^{-\rho-1}] \quad (4\text{-}26)$$

These equations can be shown to satisfy the first-order condition for profit maximization that can be derived by defining a profit function (Π) as TR minus TC:

$$\Pi = P \cdot A \cdot K^{\alpha} L^{1-\alpha} - W_K \cdot K - W_L \cdot L \quad (4\text{-}27)$$

Taking the first order partial differentials of the profit function with respect to K and L respectively and solving these simultaneously yields the following profit equilibrium condition:

$$\frac{\delta}{1-\delta}\left(\frac{L}{K}\right)^{\rho+1} = \frac{W_K}{W_L} \quad (4\text{-}28)$$

The CES function can be easily extended to include multiple inputs. The generalised multiple-input CES function $Q = f(x_1, ..., x_n)$ takes the following form:

$$Q = A[\delta_1 {x_1}^{-\rho} + \delta_2 {x_2}^{-\rho} + \cdots + \delta_n {x_n}^{-\rho}]^{-\frac{\varepsilon}{\rho}} \quad (4\text{-}29)$$

As before, multiplication of each of the factor x_F by a factor λ will increase output by a factor λ^{ε}. Thus, this function will be linearly homogenous for $\varepsilon = 1$. The first order condition for profit maximization looks similar to those derived for the two-input function:

$$W_F = P \cdot A[\delta_1 {x_1}^{-\rho} + \delta_2 {x_2}^{-\rho} + \cdots + \delta_n {x_n}^{-\rho}]^{-\frac{1}{\rho}-1}[\delta_F F^{-\rho-1}] \quad (4\text{-}30)$$

4.2.5 The CET function

Related to the CES function is the Constant Elasticity of Transformation (CET) function. A CET function has a similar functional form, but is concave with respect to the origin. While the CES function is typically used to choose an optimal combination of demands subject to either a CES production technology or a CES utility function, the CET function is used to optimally allocate supplies across markets subject to a CET production technology. As the derivation of a CET problem is highly similar to that of a CES one, for simplicity both the production efficiency parameter and the degree of homogeneity parameter are set to one as a default in the following discussion. Hence, a CET problem is solved by the following formulation:

$$max \sum_{i=1}^{n} P_i x_i \tag{4-31}$$

$$subject\ to \qquad Q = [\gamma_1 {x_1}^{\nu} + \gamma_2 {x_2}^{\nu} + \cdots + \gamma_n {x_n}^{\nu}]^{\frac{1}{\nu}} = [\sum_{i=1}^{n} \gamma_i {x_i}^{\nu}]^{\frac{1}{\nu}} \tag{4-32}$$

In this optimization problem, the supplier desires to maximize revenues across all markets, subject to the transformation frontier, where *xi* represents supply to market *i* at price *Pi*, and *Q* is aggregate supply. The supply to market *i* and the corresponding price are given by:

$$P = [\sum_i \gamma_i^{-\omega} P_i^{1+\omega}]^{\frac{1}{1+\omega}} \tag{4-33}$$

$$x_i = \gamma_i^{-\omega} \left(\frac{P_i}{P}\right)^{\omega} Q \tag{4-34}$$

where the following relation between the CET transformation elasticity and the function exponent applies:

$$\omega = \frac{1}{\upsilon - 1}, 0 \leq \sigma \leq \infty \tag{4-35}$$

The range of the function exponent was given as $1 \leq \upsilon \leq \infty$, $\upsilon \neq 0$. In the supply equation, the component price, *Pi*, is given in the numerator. This is desirable, since a rise in the price in one market being supplied (compared to the average market price), would naturally encourage suppliers to increase supply to that market.

4.3 Household behaviour

In this section, functional forms commonly used in CGE models to describe private household behaviours are surveyed. Private households in CGE models are assumed to be utility maximizers who allocate their income across commodities based on their preferences and subject to their budget and commodity prices. These preferences are described by a utility function which is an equation that quantifies how much utility consumers derive from any given combination of consumption goods. Given their utility function, consumers select the basket of goods that generates the maximum achievable satisfaction given the prices of the goods and their budgets.

Economic shocks in CGE models usually lead to changes in income and in relative prices. Consumers respond by changing the quantities of goods and services that they purchase, depending on their subjective preferences. Therefore, it is useful to study the functional forms commonly used to describe consumer preferences in CGE models and to understand the practical implications for their model results.

4.3.1 The CES utility function

The consumer demand derived from the CES utility function is generated by the following framework:

$$\max U = \left[\sum_{i=1}^{n} a_i C_i^{\rho}\right]^{\frac{1}{\rho}} \quad (4\text{-}36)$$

$$subject\ to \quad \sum_{i=1}^{n} P_i C_i = Y \quad (4\text{-}37)$$

where C is the vector of consumer demand for goods and services, P is the vector of consumer prices, and Y is disposable income. The a parameters are share parameters and will be interpreted below. The solution of the optimisation leads to the following demand equations:

$$C_i = a_i^{\sigma} \left(\frac{P}{P_i}\right)^{\sigma} \frac{Y}{P} \tag{4-38}$$

$$P = [\sum_{i=1}^{n} a_i^{\sigma} P_i^{1-\sigma}]^{\frac{1}{1-\sigma}} \tag{4-39}$$

where the relationship between the EOS and the exponent is given as as $\sigma = \frac{1}{1-\rho} \geq 0$. The same logic also applies to the derivation process of Cobb-Douglas utility function, as similar in the production function case.

The indirect utility function, $v(P,Y)$ is derived by inserting the optimal consumption function into the utility function. This yields the following indirect utility function:

$$v(P,Y) = \frac{Y}{P} \tag{4-40}$$

It can be verified that the Marshallian demand function can be derived from the indirect utility function:

$$C_i(P,Y) = -\frac{\partial v/\partial P_i}{\partial v/\partial Y} = -\frac{-YP^{-2}(\partial P/\partial P_i)}{1/P} = -\frac{\partial P}{\partial P_i}\frac{Y}{P} = a_i^{\sigma} \left(\frac{P}{P_i}\right)^{\sigma} \frac{Y}{P} \tag{4-41}$$

The expenditure function is the solution to the following minimisation problem, where u and P are exogenous:

$$\min E = \sum_{i=1}^{n} P_i C_i \tag{4-42}$$

$$subject\ to \quad [\sum_{i=1}^{n} a_i C_i^{\rho}]^{\frac{1}{\rho}} = u \tag{4-43}$$

This yields:

$$E(P,u) = uP \tag{4-44}$$

The compensated or Hicksian demand function is then given by:

$$H_i(P,u) = \frac{\partial E}{\partial P_i} = a_i^{\sigma} \left(\frac{P}{P_i}\right)^{\sigma} u \tag{4-45}$$

4.3.2 The Stone-Geary/ Linear Expenditure System function

The linear expenditure system (LES), also known as the Stone-Geary demand system owing to the early development of the system by Stone (1954) and Geary (1950). The LES function is derived from a rather simple modification of the Cobb-Douglas utility function. The modification permits the income elasticity for each demanded commodity to differ from unity. The LES utility function has the following form:

$$U = \prod_{i=1}^{n} (C_i - \theta_i)^{\mu_i} \tag{4-46}$$

where the parameter μ_i satisfies:

$$\sum_{i=1}^{n} \mu_i = 1 \tag{4-47}$$

where U is utility, C is the vector of consumption goods, and μ and θ are consumer demand parameters which are interpreted below. The reason for the normalization constraint on the μ parameters will also be explained below. There are n consumer goods. The consumers solves the following problem:

$$\max U = \prod_{i=1}^{n} (C_i - \theta_i)^{\mu_i} \tag{4-48}$$

$$subject\ to\ \sum_{i=1}^{n} P_i C_i = Y \tag{4-49}$$

where P is the vector of consumer prices, and Y is disposable income (after taxes and disposition of household saving). The first order conditions are:

$$\frac{\mu_i}{C_i - \theta_i} u = P_i \lambda \tag{4-50}$$

$$\sum_{i=1}^{n} P_i C_i = Y \tag{4-51}$$

By substitution the demand function can be derived as:

$$C_i = \theta_i + \frac{\mu_i}{P_i} Y^* \tag{4-52}$$

$$Y^* = Y - \sum_{j=1}^{n} P_j \theta_j \tag{4-53}$$

The usual interpretation of the demand function is that consumer demand is the sum of two elements. The first part is the so-called subsistence minima, θ (also referred to as the floor consumption). The second element is a share of disposable income after the purchase of the aggregate subsistence minima (the μ parameter is sometimes called the marginal propensity to consume). The expression of Y^* is referred to as the supernumerary income, it is the value of residual disposable income after purchases of the subsistence minima. From the demand equation the income and price elasticities can be derived:

$$\eta_i = \frac{\partial C_i}{\partial Y} \frac{Y}{C_i} = \frac{\mu_i Y}{P_i C_i} = \frac{\mu_i}{s_i} \tag{4-54}$$

$$\varepsilon_{ii} = \frac{\partial C_i}{\partial P_i} \frac{P_i}{C_i} = \left[-\frac{\mu_i}{P_i^2} Y^* - \frac{\mu_i}{P_i} \theta_i \right] \frac{P_i}{C_i} = \left[-\frac{1}{P_i} (C_i - \theta_i) - \frac{\mu_i \theta_i}{P_i} \right] \frac{P_i}{C_i} = \frac{\theta_i (1 - \mu_i)}{C_i} - 1 \tag{4-55}$$

$$\varepsilon_{ij} = \frac{\partial C_i}{\partial P_j} \frac{P_j}{C_i} = -\frac{\mu_i \theta_j P_j}{P_i C_i} = -\frac{\mu_i \theta_j P_j}{s_i \mathrm{Y}} \tag{4-56}$$

where η_i are the income elasticities, ε_{ii} are the own price elasticities, and ε_{ij} are the cross-price elasticities. The income elasticity is the ratio of the marginal propensity to consume out of supernumerary income over the average budget share, s_i.

The Lagrangian multiplier, λ, is the marginal utility of income, i.e. it represents the increment to utility by relaxing the budget constraint. Placing the demand function back to the first order condition gives the expression for λ.

$$\lambda = \frac{U}{Y^*} \tag{4-57}$$

This can also be expressed in terms of the variable P which defines the dual price index of the consumer prices:

$$\lambda = \frac{1}{P} = \frac{U}{Y^*} = \frac{U}{Y^{*\mu_1 + \mu_2 + \cdots + \mu_n}} = \frac{\prod_{i=1}^{n} (C_i - \theta_i)^{\mu_i}}{\prod_{i=1}^{n} Y^{*\mu_i}} = \prod_{i=1}^{n} \left(\frac{C_i - \theta_i}{Y^*}\right)^{\mu_i} = \prod_{i=1}^{n} \left(\frac{\mu_i}{P_i}\right)^{\mu_i} \tag{4-58}$$

The indirect utility function is immediately derived as:

$$\upsilon(P,Y) = \prod_{i=1}^{n}(C_i - \theta_i)^{\mu_i} = \prod_{i=1}^{n}\left(\frac{\mu_i}{P_i}Y^*\right)^{\mu_i} = Y^*/P \tag{4-59}$$

The indirect utility function represents the maximum level of utility obtainable given income and the vector of prices. It is also possible to derive the Marshallian demand function starting from the indirect utility function:

$$C_i(P,Y) = -\frac{\partial\upsilon/\partial P_i}{\partial\upsilon/\partial Y} \tag{4-60}$$

where

$$\frac{\partial\upsilon}{\partial P_i} = \frac{1}{P}\frac{\partial Y^*}{\partial P_i} - \frac{Y^*}{P^2}\frac{\partial P}{\partial P_i} = -\frac{\theta_i}{P} - \frac{\mu_i}{P_i}\frac{Y^*}{P} \tag{4-61}$$

$$\frac{\partial\upsilon}{\partial Y} = \frac{1}{P}\frac{\partial Y^*}{\partial Y} = \frac{1}{P} \tag{4-62}$$

$$\frac{\partial P}{\partial P_i} = \frac{\mu_i}{P_i}P \tag{4-63}$$

The resulting expression is equivalent to the demand equation. The expenditure function is derived by minimizing the cost of achieving a given level of utility u. This is established as:

$$\min \sum_{i=1}^{n} P_i C_i \tag{4-64}$$

$$subject\ to \quad \prod_{i=1}^{n}(C_i - \theta_i)^{\mu_i} = u \tag{4-65}$$

The first order conditions for the expenditure function are:

$$\lambda u = \frac{P_i}{\mu_i}(C_i - \theta_i) \tag{4-66}$$

$$\prod_{i=1}^{n}(C_i - \theta_i)^{\mu_i} = u \tag{4-67}$$

Combing the two equations yields

$$\lambda = P \tag{4-68}$$

The expenditure function is then derived as:

$$E(P,u) = \sum_{i=1}^{n} P_i C_i = \sum_{i=1}^{n} P_i \theta_i + uP \tag{4-69}$$

Where

$$P = \prod_{i=1}^{n} \left(\frac{P_i}{\mu_i}\right)^{\mu_i} \tag{4-70}$$

The expenditure function represents the minimum level of expenditure required to achieve the level of utility *u* with the given vector of prices. The Hicksian (compensated) demand functions are given by the derivative of the expenditure function with respect to *P*:

$$H_i(P,u) = \frac{\partial E}{\partial P_i} = \theta_i + u\frac{\partial P}{\partial P_i} = \theta_i + u\prod_i^n \mu_i \left(\frac{P_i}{\mu_i}\right)^{\mu_i - 1} \frac{1}{\mu_i} = \theta_i + \frac{\mu_i}{P_i} uP \tag{4-71}$$

Hicksian equivalent variation (EV), a measure of welfare is given by the following formula:

$$EV = E(P^1, u^1) - E(P^0, u^1) \tag{4-72}$$

i.e. the value of expenditure necessary to compensate a consumer at base year prices to achieve the new level of utility. If *EV* is positive, there is a net welfare loss. The compensated own-price elasticities are given by:

$$\begin{aligned} \xi_{ii} &= \frac{\partial H_i}{\partial P_i}\frac{P_i}{H_i} = \left[u\frac{\mu_i}{P_i}\frac{\partial P}{\partial P_i} - u\frac{\mu_i}{{P_i}^2}P\right]\frac{P_i}{H_i} = u \cdot \frac{\mu_i}{H_i} \cdot \frac{P}{P_i}(\mu_i - 1) \\ &= \mu_i(\mu_i - 1)\frac{Y^*}{P_i H_i} = (\mu_i - 1)\frac{\mu_i Y^*}{s_i Y} \end{aligned} \tag{4-73}$$

and the cross-price elasticities by:

$$\xi_{ij} = \frac{\partial H_i}{\partial P_j} \cdot \frac{P_j}{H_i} = \left[u \cdot \frac{\mu_i}{P_i} \cdot \frac{\mu_j}{P_j} \cdot P\right]\frac{P_j}{H_i} = \mu_j \frac{\mu_i Y^*}{s_i Y} \tag{4-74}$$

The two formulas can be combined to yield:

$$\xi_{ij} = [\mu_j - \delta_{ij}] \frac{\mu_i Y^*}{s_i Y} \tag{4-75}$$

where δ_{ij} is the Kronecker product, i.e. equal to 1 when *i* = *j*, otherwise equal to 0.

4.3.3 The Extended Linear Expenditure System

Household savings behaviour has not been included so far in the discussion on consumer demand systems. Many models assume separability in household decision-making between saving and current consumption. Lluch et al. (1977) introduced a relatively straightforward extension of the LES to include the saving decision simultaneously with the allocation of income on goods and services, and this has become known as the Extended Linear Expenditure System (ELES). The ELES is based on consumers maximising their intertemporal utility between a bundle of current consumption and an expected future consumption bundle represented in the form of savings. The utility function of the ELES has the following form:

$$U = \prod_i (C_i - \theta_i)^{\mu_i} \left(\frac{S}{P^s}\right)^{\mu_s} \tag{4-76}$$

with

$$\sum_{i=1}^{n} \mu_i + \mu_s = 1 \tag{4-77}$$

where *U* is utility, *C* is the vector of consumption goods, *S* is household saving in value, *Ps* is the price of saving, and μ and θ are ELES parameters similar to LES. In ELES, the saving can be assumed to be the (n^{th}+1) good, whose minimum value is assumed to be zero. The demand equations are derived similarly to above, so the consumer solves the following problem:

$$max \prod_i (C_i - \theta_i)^{\mu_i} \left(\frac{S}{P^s}\right)^{\mu_s} \tag{4-78}$$

$$subject\ to\ \sum_{i=1}^{n} P_i C_i + S = Y \tag{4-79}$$

Then the demand functions below are derived from the first order conditions:

$$C_i = \theta_i + \frac{\mu_i}{P_i}\left(Y - \sum_{j=1}^{n} P_j\theta_j\right) \tag{4-80}$$

$$S = \mu_s\left(Y - \sum_{j=1}^{n} P_j\theta_j\right) = Y - \sum_{j=1}^{n} P_j C_j \tag{4-81}$$

From the demand equation the income and price elasticities can be derived:

$$\eta_i = \frac{\mu_i Y}{P_i C_i} = \frac{\mu_i}{s_i} \tag{4-82}$$

$$\eta_s = \frac{\mu_s Y}{S} = \frac{\mu_s}{s} \tag{4-83}$$

$$\varepsilon_i = \frac{\theta_i(1-\mu_i)}{C_i} - 1 \tag{4-84}$$

$$\varepsilon_s = -1 \text{ (3-70)}$$

$$\varepsilon_{ij} = -\frac{\mu_i P_j \theta_j}{P_i C_i} = -\frac{\mu_i P_j \theta_j}{s_i Y} \tag{4-85}$$

$$\varepsilon_{sj} = -\frac{\mu_s P_j \theta_j}{sY} = -\frac{P_j \theta_j}{Y^*} \tag{4-86}$$

where s is the average propensity to save.

With the addition of saving, the indirect utility function is given by:

$$v(P,Y) = \prod_i \left(\frac{\mu_i}{P_i} Y^*\right)^{\mu_i} \left(\frac{\mu_s}{P^s} Y^*\right)^{\mu_s} \tag{4-87}$$

$$v(P,Y) = \frac{Y^*}{P},\ where\quad P = \prod_i \left(\frac{P_i}{\mu_i}\right)^{\mu_i} \left(\frac{P^s}{\mu_s}\right)^{\mu_s} \tag{4-88}$$

The expenditure function is derived by minimising the cost of achieving a given level of utility:

$$\min \sum_{i=1}^{n} P_i C_i + S \tag{4-89}$$

$$subject\ to \qquad \prod_i (C_i - \theta_i)^{\mu_i} \left(\frac{S}{P^s}\right)^{\mu_s} = u \tag{4-90}$$

The final expression for the expenditure function is derived as:

$$E(P,u) = \sum_{i=1}^{n} P_i\theta_i + uP \quad (4\text{-}91)$$

where

$$P = \prod_i \left(\frac{P_i}{\mu_i}\right)^{\mu_i} \left(\frac{P^s}{\mu_s}\right)^{\mu_s} \quad (4\text{-}92)$$

4.3.4 The Almost Ideal Demand System

The Almost Ideal Demand System (AIDS) was proposed by Deaton and Muellbauer (1980), which is able to replicate a wider range of income and price elasticities than the LES demand system. It is of the class of flexible functional forms. It is derived from the following expenditure function:

$$E(P,u) = e^{a(P)}e^{ub(P)} \quad (4\text{-}93)$$

where

$$a(P) = \alpha_0 + \sum_{i=1}^{n} \alpha_i \ln(P_i) + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \gamma_{ij}^{*} \ln(P_i)\ln(P_j) \quad (4\text{-}94)$$

$$b(P) = \beta_0 \prod_{i=1}^{n} P_i^{\beta_i} \quad (4\text{-}95)$$

The Hicksian demand function can be derived by taking the partial derivative of *E* with respect to *Pi*:

$$H_i(P,u) = \frac{\partial E}{\partial P_i} = e^{a(P)}e^{ub(P)}\frac{\partial a(P)}{\partial P_i}$$

$$= E\left[\frac{\alpha_i}{P_i} + \frac{1}{2P_i}\sum_{j=1}^{n} \gamma_{ij}^{*} \ln(P_j) + \frac{1}{2P_i}\sum_{j=1}^{n} \gamma_{ji}^{*} \ln(P_j)\right] + E\left[ub(P)\frac{\beta_i}{P_i}\right]$$

$$= \frac{E}{P_i}\left[\alpha_i + \sum_{j=1}^{n} \gamma_{ij} \ln(P_j) + u\beta_i b(P)\right] \quad (4\text{-}96)$$

where the γ coefficients are defined by:

$$\gamma_{ij} = \frac{1}{2}\left(\gamma_{ij}^{*} + \gamma_{ji}^{*}\right) = \gamma_{ji} \tag{4-97}$$

Replacing *u* by (*ln*(*E*) - *a*)/*b*, and multiplying both sides by the factor (P_i/*E*) yields:

$$s_i = \alpha_i + \sum_{j=1}^{n} \gamma_{ij} \ln\left(P_j\right) + \beta_i\left(\ln(E) - a(P)\right) \tag{4-98}$$

where *si* is the budget share allocated to commodity *i*. At the optimum, *E* is identically equal to the budget *Y*, and a price index *P* can be defined by:

$$\ln(P) = \alpha_0 + \sum_{j=1}^{n} \alpha_j \ln\left(P_j\right) + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \gamma_{ij} \ln(P_i)\ln(P_j) \tag{4-99}$$

Then the budget share equation has the following reduced form:

$$s_i = \alpha_i + \sum_{j=1}^{n} \gamma_{ij} \ln\left(P_j\right) + \beta_i \ln(Y/P) \tag{4-100}$$

This equation is almost linear in the logs of price and real income which is relatively convenient for estimation purposes.

4.4 Structure of production and commodity markets in an open economy

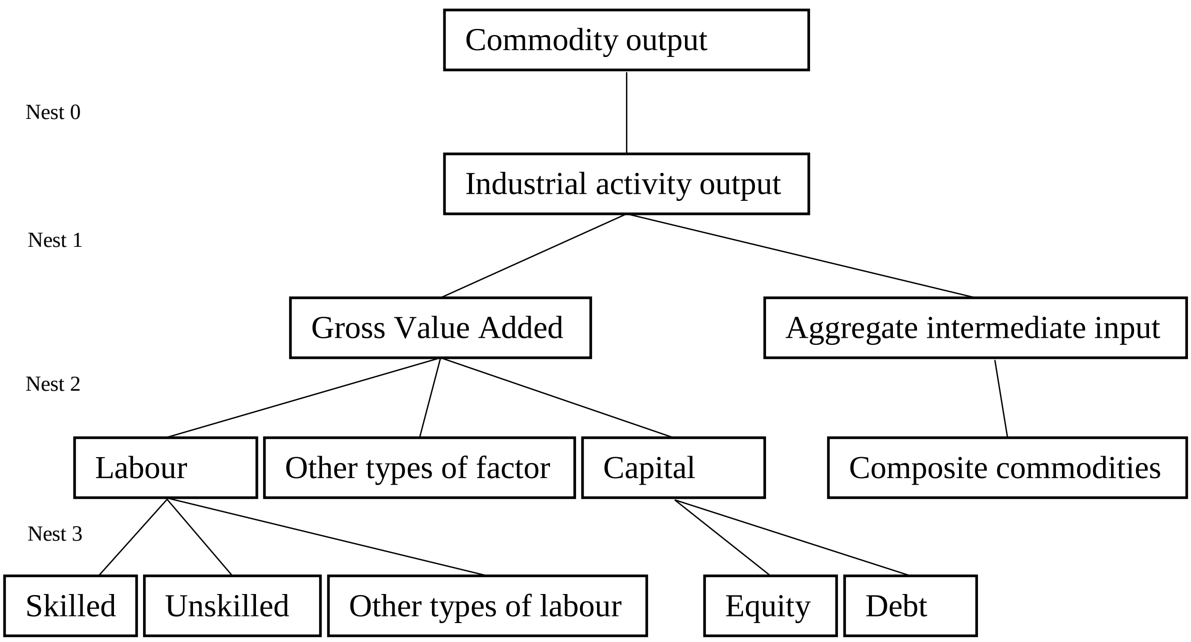
In a CGE model, the output production process of each sector is not completed in one step, but rather through a nesting structure. In such a structure, each component utilized in the production has its own production process characterized by a specific technology and degree of substitutability. For instance, each representative producer, represented by the industrial activity of the sector, is assumed to maximize profits, defined as the difference between revenue earned and the cost of factors and intermediate inputs. Meanwhile, firms minimise the costs of production arising from the use of factors, intermediate goods and taxes for a given level of output. The figure below shows the nesting structure depicting the production function of each representative producer. Note the stages are determined simultaneously, and the nesting structure allows for different elasticities of substitution among different inputs.

At the very top nest (Nest 0), producers turn their activity output into commodities. Producers in the model have the ability to produce more than one commodity type. If the prices of commodities change, firms are able to change their output mix to maximise profits. This is captured by the CET function, with the elasticity of transformation determining the ease with which sectors can change their output mix in response to changes in relative commodity prices.

In Nest 1, producers combine aggregate intermediate inputs with Gross Value Added (GVA). The EOS between these inputs are usually zero, implying it is a Leontief production function exhibiting fixed proportions among its inputs. Alternatively, the CES function is preferable for particular sectors if empirical evidence suggests that available techniques permit the aggregate mix between GVA and aggregate intermediate inputs to vary.

In Nest 2, producers combine aggregate capital and aggregate labour according to a CES/Cobb-Douglas production function to form the valued added composite. The EOS in this nest determines the ease with which firms can change their capital-labour ratio in response to changes in relative prices and governs the elasticity of demand for labour and capital. The choice of intermediate goods a producer uses is governed by a Leontief production function, where composite intermediate commodities form aggregate intermediate inputs.

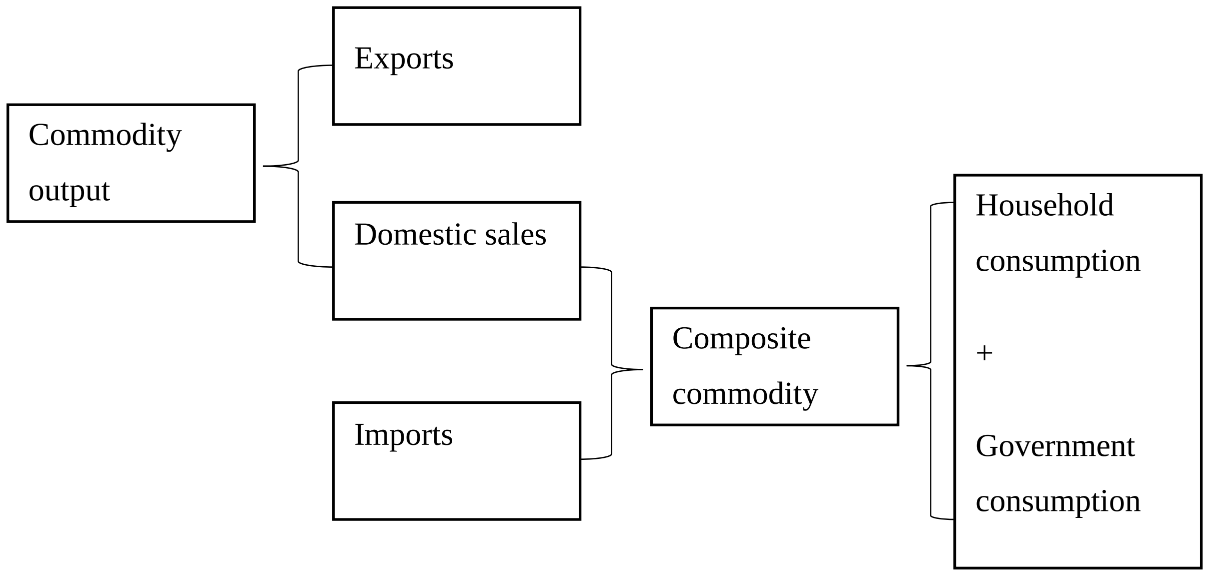
Figure 4.2 Nest Structure of Production



The final nest (Nest 3) shows producers use a combination of different labour and capital types as inputs, with associated EOS parameters determining the substitutability between types. This is important as a change in corporation tax, for example, will affect the relative price of equity and debt (affecting the cost of capital), which may lead producers to adjust their gearing ratio.

After the production process, all commodities enter commodity markets with the exception of those home-consumed outputs. Similar to the production nests, the absorption of marketed commodities is not immediate after production activities, but through disaggregation and aggregation with exported and imported commodities. The flow chain describing this is shown in Figure 4.3.

Figure 4.3 Flow Chain of Marketed Commodities



Before entering into the trade market, it is usually assumed that the object economy is a small open economy in the sense that it is small compared to global markets and that its world export and import prices are given and fixed. This assumption implies that the economy's world market share is too small for changes in its export and import quantities to affect global prices. Another important assumption relates to the imperfect substitutability between goods domestically produced and those imported/exported. That is, domestic-made products are supposed to be similar to, but slightly different from, their traded counterparts, otherwise there cannot be both exports and imports for the same goods simultaneously, known as two-way trade. The degree of difference/similarity between goods can be measured by a parameter such as the elasticity of substitution in CES functions. If they are significantly different from each other, the elasticity tends to be small, and vice versa. Such function is also referred to as an

Armington function, named after Paul Armington (1969) who introduced imperfect substitutability between imports and domestic commodities in economic models.

At the first stage, domestic commodity output is allocated between exports and domestic sales on the assumption that suppliers maximize sales revenue for any given aggregate output level, subject to imperfect transformability between exports and domestic sales, expressed by a constant elasticity of transformation (CET) function. In the international markets, export demands are infinitely elastic at given world prices. The price received by domestic suppliers for exports is expressed in domestic currency and adjusted for the transaction costs to the border and export taxes if any. The supply price for domestic sales is equal to the price paid by domestic demanders minus the transaction costs of domestic marketing from the supplier to the demander per unit of domestic sales. If the commodity is not exported, total output is passed to the domestic market.

To the extent that a commodity is imported, all domestic market demands are for a composite commodity made up of imports and domestic output, the demands for which are derived on the assumption that domestic demanders minimize cost subject to imperfect substitutability. This is captured by an Armington CES aggregation function in the middle stage.

The derived demands for imported commodities are met by international supplies that are infinitely elastic at given world prices. The import prices paid by domestic demanders also include import tariffs and the transaction cost. Similarly, the derived demand for domestic output is met by domestic suppliers. The prices paid by the demanders include the cost of transactions services, in this case reflecting that the commodity was moved from the domestic supplier to the domestic demander. The prices received by domestic suppliers are net of these transaction costs. Flexible prices equilibrate demands and supplies of domestically marketed domestic output.

At the final stage, domestic demand for composite commodities comprises the sum of demands for household consumption, government consumption, investment and intermediate inputs.

Compared with the alternative assumptions of perfect substitutability and transformability, the assumptions of imperfect transformability between exports and domestic sales of domestic output and imperfect substitutability between imports and domestically sold domestic output, permit the model to better reflect the empirical realities of most economies. The assumptions used give the domestic price system a degree of independence from international prices and prevent unrealistic export and import responses to economic shocks. At the disaggregated commodity level, these assumptions allow for a continuum of tradability and two-way trade, which is commonly observed even at very fine levels of disaggregation.

4.5 Macroeconomic closures

A general equilibrium model, including the CGE model, requires macroeconomic closures to 'close' the model. In short, these macro closures are about the choices of endogenous variables against exogenous variables among all the variables in the model. Those endogenous variables represent the 'inside' of the model, while the exogenous variables represent the 'outside'. Therefore, there is a clear boundary between the 'inside', which is depicted by the model for only part of the real economy, and the 'outside', which is assumed to be given as the rest of the economy.

The macroeconomic closures are characterised by the economic constraints or balance rules of the markets in an economy. Usually, they cover government balance, the external balance (the balance of payments constraints), the savings-investment balance and the labour market closure. In CGE modelling, the choices made have no influence on the solution to the base simulation but will typically influence the results for other simulations. The closures are summarized in Table 4.1.

For the government balance, the first closure (GOV1) is that government surplus/deficit (the difference between government revenue and government expenditure) is a flexible residual while the government expenditure and all tax rates are fixed. When the tax bases change following a policy shock, government revenue will be changed followed by a change in its surplus/deficit. In the second alternative closure (GOV2), both government revenue and expenditure are fixed and to maintain it, the tax rates are flexibly adjusted. Under the third alternative government closure (GOV3), government surplus/deficit and tax rates are both fixed and government expenditure is the adjusting variable. For example, when the tax bases expand, government receives more tax revenue and increase government expenditure.

Table 4.1 Basic Macroeconomic Balances

		Endogenous (flexible)	Exogenous (fixed)
Government balance	GOV1	government revenue, fiscal surplus/deficit	government expenditure, tax rates
	GOV2	tax rates	government revenue, government expenditure, fiscal surplus/deficit
	GOV3	government revenue, government expenditure	fiscal surplus/deficit, tax rates
Balance of payments	BOP1	real exchange rate	foreign savings
	BOP2	foreign savings	real exchange rate
Savings-Investment	SI1	savings, savings rates	investment
	SI2	savings, investment	savings rate
Labour market closure	LAB1	wage rate	employment
	LAB2	employment	wage rate

In an open economy, there are two more relevant variables compared to a closed economy model: foreign savings (or current account deficit) and the foreign exchange rate, and one additional equation, the balance of payments. In this equation, we have to choose either of the two variables as exogenous or endogenous. For example, in most developing countries the availability of foreign savings is actually limited and in this case the foreign savings must be made exogenous while the real exchange rate is flexible as in the first external balance closure (BOP1). If, *ceteris paribus,* foreign savings are below the exogenous level, a depreciation of the real exchange rate would correct this situation by simultaneously (i) reducing spending on imports (a fall in import quantities at fixed world prices) and (ii) increasing earnings from exports (an increase in export quantities at fixed world prices). Under the alternative closure (BOP2), the real exchange rate is fixed while foreign savings is flexible. It must be noted that foreign savings are not really savings or loans but merely a transfer from the external world. In essence, the domestic agents get external funds but have no 'next period' to pay them back.

For the Savings-Investment balance, it is usually assumed that either of investment or savings (or key parameters characterizing them) is exogenously given and that the other, as an endogenous variable, adjusts itself to establish equality between them. Hence, it is either the investment-driven closure, as in SI1, or savings-driven as in SI2. In the investment-driven closure, investment quantities are fixed and exogenous. In order to generate savings that finance the investment, the base-year savings rates of agents are adjusted. In the savings-driven closure, all savings rates are fixed, and savings are first determined by income that an increase in income leads to an increase in savings and then investment.

In addition, there are more balance rules related to other variables, such as those in the labour market. The most basic labour closures include two types: the closure LAB1 where the wage rate is flexibly adjusted to achieve full employment in the labour market, and the closure LAB2 where the role of the two variables are exchanged. While the former is more of the neoclassical tradition, assuming zero unemployment, the latter is of Keynesian perspective, useful to simulate an economy in depression.

The appropriate choice between the different closures depends on the context of the analysis, and should be compatible with the research objective and focus. For example, with the closure of fixed government surplus/deficit, it is not possible to investigate the variation of fiscal status as a result of policy shock. Similarly, a constant foreign savings cannot be used for analyses of policies aimed at improvement of current account deficits. Equally, a model with a fixed foreign exchange rate cannot be used to measure how a policy directly affects this foreign exchange rate and how changes in a certain economic variable affect the economy through changes in the exchange rate. In addition, savings-driven closure would be suitable for quantifying the impact of certain policy changes or exogenous shocks, through changes in savings, exerted on other economic variables. On the other hand, the investment-driven closure would be useful for analysis that quantifies the impact of those shocks through changes in investment. For the labour market closure, this should be chosen to reflect the labour market status of the target economy.

Table 4.2 schematically presents how the different macroeconomic closures relate to the assumptions on the factor market, and on the balance identity between savings and investments. This table also highlights which variables are fixed and which are not.

In terms of the closure rules in Table 4.2, a combination of BOP1 with SI1 and either of GOV1 or GOV2 is known as ‘Johansen closure’ in the literature. The Johansen closure assumes exogenous investments and endogenous consumption, whose volume adjusts to liberate sufficient savings to finance investment. Another macro closure often used in applied work is the savings-driven ‘Neoclassical closure’, in which investment is determined by the sum of private, government, and foreign savings. It is mainly distinguished from the Johansen closure in that it uses SI2 instead of SI1.

Table 4.2 Summary of the Four Macroeconomic Closures

	Neoclassical	Keynesian	Johansen	Kaldorian
Labour market	Full employment	Unemployment	Full employment	Full employment
Savings-Investment balance	Savings-driven	Investment-driven	Investment-driven	Investment-driven
Variables				
Employment	Fixed		Fixed	
Investment		Fixed	Fixed	Fixed
Wage		Fixed		Fixed
Savings rate	Fixed	Fixed		Fixed

Both the savings-driven Neoclassical closure and the investment-driven Johansen closure are useful when looking at the historical experience of countries adjusting to macroeconomic shocks. If the analysis aims at exploring the role for complementary policies, it is generally preferable to impose a relative balanced closure with simultaneous adjustments in the all components of absorption. Under these circumstances, a combination of GOV3 and SI1is also an option.

However, all the macroeconomic closures assume no link between macroeconomic variables and aggregate employment. It is feasible in CGE modelling to specify a ‘Keynesian closure’ in which the labour market element is also considered. This closure is an example of a structuralist macroeconomic model of the type advocated by Lance Taylor (1990). In this Keynesian closure, investment is fixed in quantity. An increase in exogenous investment will generate an increase in employment, an increase in income, and an increase in savings to finance the increased investment.

Another macroeconomic closure supposes that employment and output are fixed but income redistribution takes place and frees the necessary savings, which is known as the 'Kaldorian closure'. In this closure, the wage rate is not determined by the marginal product of labour, and the total investment is exogenous. This assumes that different type of labour possesses different savings rate, then the income allocation across different types of labour is adjusted so as to match the savings level with total investment.

4.6 Development of the CGE model for the Welsh economy

4.6.1 General outline of the model

This thesis aims to provide a general equilibrium modelling platform of the Welsh economy, using the functional forms and the conceptual framework of CGE models. This section derives the model according to the agents' optimization behaviours in each block, and the regional economic characteristics of Wales. Specifically, it is a static, single-region and multi-sector CGE model that incorporates the behaviour of five economic agents: firms, households, regional government, rest of the UK and the rest of the world.

In this model, both supply and demand side behaviours are subject to relevant optimizations in terms of distribution of production cost, or maximizing profits, and allocation of consumption goods and services. All economic agents conduct optimizing behaviours subject to relevant constraints in a perfect competition context.

The model includes 21 sectors in total, and each of these produce one homogeneous commodity. This implies the classification of the commodities is consistent with that of the sectors. The Welsh economy is treated as a small open economy that it is a price taker, and has no significant impact on the UK and world market prices. However, it assumes Armington's (1969) assumption that regionally produced/consumed commodities and those imported/exported are imperfect substitutes. That is, they are supposed to be similar to, but slightly different from, each other in use. Such assumption allows and accounts for the existence of two-way trade in practice, which is normally reported in the actual data (i.e. that the imported and exported goods are same goods given a certain level of disaggregation). Hence, the imported, exported and regionally produced goods are differentiated from each other even though they are classified in the data as the same category.

This model is calibrated on the regional Social Accounting Matrix (SAM) for Wales in the 2013 calendar year. The year 2013 is the latest year for which all the necessary data became available when the SAM development work commenced. The SAM, which is the model's benchmark database, is derived and presented in the next chapter. The whole model is calibrated and solved using the software General Algebraic Modelling System (GAMS) with the solver PATHNLP.

This model is neoclassical in spirit, and is developed in terms of three time perspectives: short run, medium run and long run. The development of this model references to the work of Marcouiller et al. (1999), Bayar (2006) and Ferguson et al. (2007), however minor modifications are made to merge their set-up of the different time frames.

In the short run, the model is characterized by a sectorally fixed stock of factors. In this time perspective no factors can move freely across sectors. Hence, they are also fixed in the total level within the regional economy, and the factor price changes in each sector will vary in response to a policy shock.

The medium-run perspective features free mobility of factors across sectors, with total stock still fixed. A natural consequence of this perspective is the economy-wide factor price formed for each factor, as free mobility of factors will erase their factor return gaps between sectors. At the same time, the land factor is still maintained as sectorally fixed.

The long run time frame will display factor prices recovering to their initial benchmark equilibrium level, as the factor stocks are fully adjusted to meet the demand. The essence of this time frame is that labour and capital prices are fixed as their benchmark level, and their factor stocks are sufficiently flexible allowing for potential inter-regional factor mobility. The land factor is slightly different with the other two as it is separated as residential used and non-residential use, and this time frame only allows free mobility within its own category. This implies residential properties are not allowed to be renovated for non-residential use and vice versa.

In the model description below, the following conventions are adopted. The subscript *i* labelled on the model variables represent the set of sectors that receive income or monetary inflow, and the subscript j refer to those sectors that have expenditure or monetary outflow. The two subscripts are actually equivalent to each other as they just represent the views from symmetrical row and column sectors in the SAM, and they are balanced between total income and total expenditure. Hence, a variable with subscript i or j is only distinguished regarding in which economic stage they refer to in the SAM.

For example, on one hand, those variables subscribed with j usually appear in the functions describing the production process in the supply side, since at this stage all sectors spend money for intermediate and factor inputs along the columns of the SAM as necessary for producing outputs. On the other hand, subscript i usually appear in the sale stage of the demand side, that the expenditure of the firms in each sector have been transformed into commodities for sale to meet intermediate and final demand. At this stage, firms receive monetary payments along the rows of the SAM for selling outputs. In summary, sectors in set i are completely identical to those in set j, but only distinguished for convenience of reflecting the underlying economic value realization stage.

To distinguish the variables and parameters, and endogenous and exogenous variables, the naming rule is set as followed. All endogenous variables are written by letters in upper case, while all parameters are in lower case. Exogenous variables are all set with with upper case letters with a bar on top.

The optimization behaviour of each agent in the model is described in the following sections. The model equations, together with sector sets, parameters, endogenous and exogenous variables are listed in Appendix II.

4.6.2 Production behaviour

In the bottom nest of the production process, input factors $F_{h,j}$ are used for the production of a composite factor, or value added, VA_j. A Cobb-Douglas production function[2] is initially assumed for this nest ($\sum_h af_{h,j} = 1$). Then for each representative industrial activity sector j, it

[2] A CES function as its generalized form is also constructed to facilitate sensitivity analysis where alternative elasticity of substitution between factors can be tested. The CES form and its derived equations are listed in Appendix II.

maximizes the production profit as below for a Cobb-Douglas case.

$$maximize\ \pi_j = PVA_j \cdot VA_j - \sum_h PF_h \cdot F_{h,j} \tag{4-101}$$

$$subject\ to\ VA_j = \prod_h F_{h,j}^{af_{h,j}} \tag{4-102}$$

where PVA_j is the price of value added of sector j, and PF_h is the price of the factor h in that sector. Solving the maximization problem yields:

$$VA_j = \prod_h F_{h,j}^{af_{h,j}} \tag{4-103}$$

$$F_{h,j} = \frac{af_{h,j} \cdot PVA_j \cdot VA_j}{PF_h} \tag{4-104}$$

Specifically, if the input factors in this regional CGE model are capital, land and labour, the optimal conditions in the short run can be re-described as:

$$PVA_j \cdot VA_j = PK_j \cdot \bar{K}_j/(1-tk) + \left(1 + tsd\_nr\_int_j\right) \cdot PT_j \cdot \bar{T}_j + (1+tlr) \cdot PL_j \cdot \bar{L}_j/(1-ty) \tag{4-105}$$

$$PK_j \cdot \bar{K}_j/(1-tk) = ak_j \cdot PVA_j \cdot VA_j \tag{4-106}$$

$$\left(1 + tsd\_nr\_int_j\right) \cdot PT_j \cdot \bar{T}_j = at_j \cdot PVA_j \cdot VA_j \tag{4-107}$$

$$(1+tlr) \cdot PL_j \cdot \bar{L}_j/(1-ty) = al_j \cdot PVA_j \cdot VA_j \tag{4-108}$$

The first zero-profit equation is an alternative for the production function for value added in the modelling system. The three factors capital, labour and land are represented by $\bar{K}_j$, $\bar{L}_j$ and $\bar{T}_j$, which are all exogenous variables in the short run time frame. Parameters ak_j, at_j, al_jrepresent the factor share parameter for capital, land and labour respectively. Parameter tlr represents the National Insurance Contribution (NIC) rate payable by employer applied to the labour cost. The parameter $tsd\_nr\_int_j$ stands for the non-residential SDLT rate applied to the land factor price.

In the medium run, all factors are fixed in total stock rather than within each sector. Factor prices are then all uniform prices across sectors. The optimal equations are then presented as below:

$$PVA_j \cdot VA_j = PK \cdot K_j/(1-tk) + \left(1 + tsd\_nr\_int_j\right) \cdot PT_j \cdot \bar{T}_j + (1+tlr) \cdot PL \cdot L_j/(1-ty) \tag{4-109}$$

$$PK \cdot K_j/(1-tk) = ak_j \cdot PVA_j \cdot VA_j \tag{4-110}$$

$$\left(1 + tsd\_nr\_int_j\right) \cdot PT_j \cdot \bar{T}_j = at_j \cdot PVA_j \cdot VA_j \tag{4-111}$$

$$(1+tlr) \cdot PL \cdot L_j/(1-ty) = al_j \cdot PVA_j \cdot VA_j \tag{4-112}$$

In the long run, the equations are adjusted as below where capital and labour factors are both endogenously flexible, while their prices are both exogenously fixed as in initial equilibrium. The land factor is noted here to be separated to residential use and non-residential use in production. While the residential land is employed only to the sector of residential rental (with the sector name R_R as subscript), the land factor of non-residential use is employed in all sectors excluding the residential rental sector (*jnr* is the set containing these non-residential-rental sectors). The corresponding economy-wide non-residential land return is represented by PT_NR which implies the non-residential land use can be freely altered across all non-residential-rental sectors. As the residential rental sector is the only sector using residential land factor and this factor is the only component in this sector's factor input, the corresponding land price is then naturally the economy-wide price and equals the sector's value added price, VA_{R_R}. The sector settings concerned above will be discussed in detail in the next chapter for constructing a SAM for the model.

$$\begin{aligned} PVA_j \cdot VA_j = \overline{PK}_j \cdot K_j/(1-tk) + \left(1 + tsd\_nr\_int_{jnr}\right) \cdot PT\_NR \cdot T_{jnr} \\ + PVA_{R\_R} \cdot VA_{R\_R} + (1+tlr) \cdot \overline{PL}_j \cdot L_j/(1-ty) \end{aligned} \tag{4-113}$$

$$\overline{PK}_j \cdot K_j/(1-tk) = ak_j \cdot PVA_j \cdot VA_j \tag{4-114}$$

$$\left(1 + tsd\_nr\_int_{jnr}\right) \cdot PT\_NR \cdot T_{jnr} = at_{jnr} \cdot PVA_{jnr} \cdot VA_{jnr} \tag{4-115}$$

$$(1+tlr) \cdot \overline{PL}_j \cdot L_j/(1-ty) = al_j \cdot PVA_j \cdot VA_j \tag{4-116}$$

The total factor income for capital, land and labour is then derived straightforward. For the short run,

$$YK = \sum_j PK_j \cdot \bar{K}_j \tag{4-117}$$

$$YT = \sum_j PT_j \cdot \bar{T}_j \tag{4-118}$$

$$YL = \sum_j PL_j \cdot \bar{L}_j \tag{4-119}$$

In the medium run,

$$YK = \sum_j PK \cdot K_j \tag{4-120}$$

$$YT = \sum_j PT_j \cdot \bar{T}_j \tag{4-121}$$

$$YL = \sum_j PL \cdot L_j \tag{4-122}$$

In the long run,

$$YK = \sum_j \overline{PK}_j \cdot K_j \tag{4-123}$$

$$YT = \sum_{jnr} PT\_NR \cdot T_{jnr} + PVA_{R\_R} \cdot VA_{R\_R} \tag{4-124}$$

$$YL = \sum_j \overline{PL}_j \cdot L_j \tag{4-125}$$

For the middle nest, a Leontief production function is assumed for the gross regional output Y_j, implying that the composite factor inputs VA_j and intermediate inputs $INT_{i,j}$ are proportionately combined for regionally produced output. This choice of functional form has the great practical advantage of reducing considerably the number of parameters necessary for implementing the model. In this nest, the maximization problem of gross regional output is solved subject to its Leontief production function:

$$maximize\ \pi_j = PY_j \cdot Y_j - \left(PVA_j \cdot VA_j + \sum_i PQ_i \cdot INT_{i,j}\right) \tag{4-126}$$

$$subject\ to\ Y_j = \min\left(\frac{INT_{i,j}}{aint_{i,j}}, \frac{VA_j}{ava_j}\right) \tag{4-127}$$

where $INT_{i,j}$ is the composite intermediate inputs produced by sector *i* and employed by sector *j*, and PQ_i is the corresponding price. The intermediate good, as so called 'Armington composite good', which is implied by Armington's assumption that both supply and demand side do not directly consume or employ imported goods but rather the composite goods comprising imports and the corresponding regionally produced goods. Solving this maximization problem yields:

$$INT_{i,j} = aint_{i,j} \cdot Y_j \tag{4-128}$$

$$VA_j = ava_j \cdot Y_j \tag{4-129}$$

$$Y_j = \min\left(\frac{INT_{i,j}}{aint_{i,j}}, \frac{VA_j}{ava_j}\right) \tag{4-130}$$

However, the Leontief production function Y_j in (3-130) generates rectangular isoquants that will cause difficulty in numerical computations. To deal with such a computational problem, this is replaced with a zero-profit condition for competitive firms.

$$\pi_j = PY_j \cdot Y_j - \left(PVA_j \cdot VA_j + \sum_i PQ_i \cdot INT_{i,j}\right) = 0 \tag{4-131}$$

Then substituting VA_j and $INT_{i,j}$ using equations (3-128) and (3-129), we can derive equation (4-131) as:

$$PY_j \cdot Y_j = PVA_j \cdot VA_j + \sum_i PQ_i \cdot INT_{i,j} \tag{4-132}$$

$$\Rightarrow PY_j \cdot Y_j = PVA_j \cdot ava_j \cdot Y_j + \sum_i PQ_i \cdot aint_{i,j} \cdot Y_j \tag{4-133}$$

$$\Rightarrow PY_j = ava_j \cdot PVA_j + \sum_i aint_{i,j} \cdot PQ_i \tag{4-134}$$

4.6.3 Households behaviour

The representative household optimizes the behaviour by allocating its net income on different type of goods and services to maximize the consumption utility. The maximization problem subject to its income budget can be defined as below:

$$maximize\ U = \prod_i C_i^{ac_i} \tag{4-135}$$

$$subject\ to\ \sum_i PQ_i \cdot C_i = YH - TY - TLE - TCCL - SH \tag{4-136}$$

where YH is the households total annual revenue, and it comprise income from factors and social protection transfer received, $\overline{TRFH}$. The model assumes that the regional total labour income *YL* is entirely occupied by the households, while the regional total capital return *YK* and land return *YT* are distributed to the households and Welsh enterprises separately and proportionately. The value of these shares in detail is discussed in the next chapter. The capital and land return that households receive are determined by the fixed share parameter *skh* and *sth* respectively, and the marginal propensity of saving is represented by parameter *mps*. Thus, households income and saving are described in the following two equations:

$$YH = skh \cdot YK + sth \cdot YT + YL/(1 - ty) + \overline{TRFH} \tag{4-137}$$

$$SH = mps \cdot (YH - TY - TLE - TCCL) \tag{4-138}$$

where *TY* denotes the induced income tax payment. *TLE* and *TCCL* represent the National Insurance Contribution (NIC) payable by employee and Council tax payment by households.

Solving the households optimization problem gives the solution:

$$PQ_i \cdot C_i = ac_i \cdot (YH - TY - TLE - TCCL - SH) \tag{4-139}$$

This equation describes the optimal intra-temporal consumption demand for each type of commodities, C_i, on the consumer price PQ_i, with the sum of allocation share equal unity, $\sum_i ac_i = 1$.

4.6.4 Regional government behaviour

A significant difference between a regional and a national CGE model lies in the fiscal arrangement of the regional government. Unlike a national central government, normally a sub-national government is not responsible for all the tax revenue generated within its administrative region. In most cases, there is not a symmetry between public expenditure for the region and public revenue levied by the sub-national government from the region. In practice, while the sub-national government's share of public spending has increased in a majority of OECD countries, the share in general government revenues (excluding grants) has failed to rise correspondingly, and has even declined in several cases (Journard and Kongsrud 2003).

In the case of Wales, the regional government has only a very limited number of devolved taxes in the current fiscal framework. The level of public expenditure by the Welsh government and local government in Wales is mostly unconnected to the amount of revenues collected in Wales (Poole et al. 2016). In fact, most public expenditure in Wales depends highly on finance from central UK government, in the form of a block grant and direct central payment on the spending programmes, which is exogenous to the Welsh government. Under this fiscal background, the model is set to ensure that only revenue changes in devolved taxes determine the change in public revenue, and then expenditure for Wales. All non-devolved tax revenues are centralized to the UK government and hence have no influence on the Welsh government's fiscal revenue. The financial transfer is also exogenously isolated from any change of Welsh government's fiscal revenue.

The public sector revenue sources of Welsh government include devolved taxes, non-devolved taxes and factor income. To simulate the current fiscal situation of Welsh government, the fiscal revenue *YG* is described by the equation below:

$$YG = YG\_DT + YG\_F + YG\_NDT + TRFG \qquad (4\text{-}140)$$

where *YG_DT* stands for revenue from devolved taxes, *YG_F* for factor income in Wales, and *YG_NDT* for non-devolved taxes. *TRFG* is assumed to represent all the financial transfer received by Welsh government from central UK government, including block grant, direct public expenditure by central government for spending programmes in Wales, and all the other public sector revenues incurred in Wales. This term is then modelled by the equation (4-141) below which ensures that only those devolved taxes and factor revenues directly administrated by the Welsh government determine any changes of total fiscal revenue given any policy shock.

$$TRFG = \overline{YG0} - \overline{YG\_DT0} - \overline{YG\_F0} - YG\_NDT \qquad (4\text{-}141)$$

where $\overline{YG0}$, $\overline{YG\_DT0}$, and $\overline{YG\_F0}$ are all benchmark value of total fiscal revenue, devolved revenue and factor revenue in the 2013 base year. Substituting equation (4-141) to equation (4-140) gives:

$$YG = YG\_DT + YG\_F + YG\_NDT + TRFG$$

$$= YG\_DT + YG\_F + YG\_NDT + \overline{YG0} - \overline{YG\_DT0} - \overline{YG\_F0} - YG\_NDT$$
$$= \overline{YG0} + (YG\_DT - \overline{YG\_DT0}) + (YG\_F - \overline{YG\_F0}) \quad (4\text{-}142)$$

Equation (4-142) shows that in responding the policy shock, any changes of fiscal revenue compared to its benchmark value, is brought by the changes from devolved taxes and factors revenue generated in Wales. These three revenue components are described in the following three equations.

$$YG\_DT = TSD\_R\_NS + TSD\_NR\_NS + TSD\_R\_RS + TSD\_NR\_RS + TP + TCCL \quad (4\text{-}143)$$
$$YG\_NDT = TY + TK + TLR + TLE + TC \quad (4\text{-}144)$$
$$YG\_F = skg \cdot YK + stg \cdot YT \quad (4\text{-}145)$$

The devolved taxes revenue in this model involves SDLT in terms of different category of property transaction, net production taxes *TP*, and Council tax *TCCL*. Non-devolved taxes revenue included in this model cover Income tax *TY*, Corporation tax *TK*, NIC payment *TLR* (payable by employer) and *TLE* (payable by employee), and net product taxes *TC*. The government's factor income regards to the gross operating surplus and rent income incurred from capital and land factors, which are both determined by their corresponding fixed shares, *skg* and *stg*.

Next, the equations are presented which describe that all these tax revenues are generated by their tax rates multiplying the underlying tax bases. As most of the taxes are not single-band rate types, or a mix of multiple taxes net subsidies, the tax rates applied in the model are not supposed to be the real rates in practice, but rather effective rates that are all single rates calculated as total tax revenue actually collected, rather than payable, divided by the corresponding tax base.

The tax revenue of SDLT originates from both residential and non-residential property transactions. In this model, it is further split to four revenue components: *TSD_R_NS*, revenue from new residential property sale; *TSD_NR_NS*, revenue from new non-residential property sale; *TSD_R_RS*, revenue from residential property resale and *TSD_NR_RS*, revenue from non-residential property resale.

As all the variables in a CGE model are required to account for all the values in the model's benchmark database – SAM, the set-up of a CGE model always needs to take into account the set-up of the underlying SAM. The tax bases for new residential and non-residential SDLT here are easy to map into the SAM, which are just the gross regional output of the two sectors: residential new sale and non-residential new sale, respectively. However, the residential and non-residential property resale value is not supposed to appear in a SAM as they are not counted into GDP or national accounting. Hence, they are indirectly modelled by setting proxies in the SAM and then in the model.

The proxy for the property resale value is the actual annual rents generated by the underlying properties. It is assumed that the annual rent value reflects the whole property value in the property market. Thus, the tax base of residential SDLT from resale transactions is gross regional output of the residential rental sector in the SAM. For non-residential SDLT from resale, the tax base maps to the non-residential rentals as land factor inputs by non-residential sectors (these sectors belong to the set *jnr* mentioned above). Therefor the SDLT revenue equations are listed below in order.

$$TSD\_R\_NS = tsd\_r\_fnl \cdot PY_{R\_NS} \cdot (1 + tp_{R\_NS}) \cdot Y_{R\_NS} \tag{4-146}$$

$$TSD\_NR\_NS = tsd\_nr\_fnl \cdot PY_{NR\_NS} \cdot (1 + tp_{NR\_NS}) \cdot Y_{NR\_NS} \tag{4-147}$$

$$TSD\_R\_RS = tsd\_rr\_fnl \cdot PY_{R\_R} \cdot (1 + tp_{R\_R}) \cdot Y_{R\_R} \tag{4-148}$$

$$TSD\_NR\_RS = \sum_{jnr} tsd\_nr\_int_{jnr} \cdot PT_{jnr} \cdot \bar{T}_{jnr} \tag{4-149}$$

$$TSD\_NR\_RS = \sum_{jnr} tsd\_nr\_int_{jnr} \cdot PT\_NR \cdot T_{jnr} \tag{4-150}$$

$$TSD\_NR\_RS = \sum_{j} tsd\_nr\_int_{j} \cdot PT \cdot T_{j} \tag{4-151}$$

The equation (4-149) above is for short run, while (4-150) and (4-151) is for medium and long run respectively. All equations are in the form of tax rate multiplying the tax base. *R_NS*, *NR_NS* and *R_R* are sector names for residential new sale, non-residential new sale and residential rental respectively.

Another devolved tax revenue equation in the model is Council tax revenue determination equation which is expressed as the effective Council tax rate multiplied by the resale value of the whole dwelling stock. Since in this model the residential rental price acts as the proxy of residential resale price, the equation is expressed as:

$$TCCL = tccl \cdot PY_{R\_R} \cdot \overline{Q\_R\_STOCK} \tag{4-152}$$

where $\overline{Q\_R\_STOCK}$ represents the total stock of the regional dwellings, which equals £212046.42 million. This number is estimated by the whole UK stock, obtained by Mitchell (2014), apportioned by the population ratio between Wales and the UK.

The revenue from non-devolved taxes are also calculated straightforward by the following equations:

$$TY = ty \cdot YH \tag{4-153}$$

$$TK = tk \cdot YK/(1 - tk) \tag{4-154}$$

$$TLR = tlr \cdot YL/(1 - ty) \tag{4-155}$$

$$TLE = tle \cdot YL/(1 - ty) \tag{4-156}$$

$$TP = \sum_j tp_j \cdot PY_j \cdot Y_j \tag{4-157}$$

$$TC = \sum_j tc_j \cdot PY_j \cdot (1 + tp_j) \cdot Y_j \tag{4-158}$$

Given the government fiscal revenue, the government expenditure consists of disposable budget for current consumption and social protection payment, $\overline{TRFH}$. The optimal spending by the government is then given by solving the maximization problem:

$$maximize\ UG = \prod_i G_i^{ag_i} \tag{4-159}$$

$$subject\ to\ \sum_i PQ_i \cdot G_i = YG - \overline{TRFH} \tag{4-160}$$

The optimal demand equation for each type of commodity is:

$$PQ_i \cdot G_i = ag_i \cdot (YG - \overline{TRFH}) \tag{4-161}$$

where ag_i is the spending share with $\sum_i ag_i = 1$.

4.6.5 Trade behaviour

This regional CGE model is developed to model the Welsh economy, which is a small open economy. This implies that the regional economy does not have a significant impact on the rest of world (including rest of the UK), and that the export and import prices are exogenously given for this economy. Besides, Armington's assumption is also applied that foreign-produced goods are similar and imperfectly substitutable with regionally produced goods. The degree of difference/similarity is measured by the elasticity of substitution in CES function and the elasticity of transformation in CET function. The more similar they are from each other, the larger the elasticities become (i.e., elastic), and vice versa.

In similarity to the production behaviour, the trade behaviour also features a nested structure regarding different destinations here. The first nest points to the rest of UK (RUK) and the second points to the rest of world (ROW). In detail, for exports, we assume that firms first transform the gross regional output into commodities sold in international markets and in the domestic UK markets, then transform further the domestic commodities into Welsh markets and RUK markets. Both transformation processes also assume imperfect substitution (or, transformation) between exports and the regional supply.

Consider the first transformation process that relates to the final stage in a production process of the representative firm. This process determines the supply ratio between the international and domestic UK markets described by a CET function. Depending on the relative price between exports and domestic goods, the supply ratio changes. The larger the elasticity of transformation is, the more similar between export and domestic good is, and the export-domestic supply ratio tends to be more sensitive to the change of relative price.

The profit maximization problem for the representative firm of the i-th sector in transforming the gross regional output into international exports $EROW_i$ and domestic goods WD_i can be expressed as follows:

$$maximize\ \pi_i = \overline{PEROW_i} \cdot EROW_i + PWD_i \cdot WD_i - (1 + tmx_i) \cdot PY_i \cdot Y_i \quad (4\text{-}162)$$

$$subject\ to\ Y_i = \theta w_i \cdot \left[serow_i \cdot EROW_i^{\rho w_i} + (1 - serow_i) \cdot WD_i^{\rho w_i}\right]^{1/\rho w_i} \quad (4\text{-}163)$$

Notations involved are:

$\overline{PEROW}_i$: price of the i-th ROW export good,

PWD_i: price of the i-th regionally produced domestic good,

$EROW_i$: ROW exports of the i-th good,

WD_i: supply of the i-th domestic good,

tmx_i: mixed tax rate on the i-th gross regional output, (tmx_i=$(1+tp_i)\cdot(1+tc_i)\cdot(1+tsd\_r\_fnl)\cdot(1+tsd\_nr\_fnl)\cdot(1+tsd\_rr\_fnl)\cdot(1+tsd\_nr\_int_i)-1$)

θw_i: scaling coefficient of the i-th international transformation,

$serow_i$: ROW export share coefficient for the i-th international transformation, ($0\leq serow_i\leq 1$)

ρw_i: parameter defined by the elasticity of transformation, ($\rho w_i=\frac{\sigma w_i+1}{\sigma w_i},\ \rho w_i\geq 1,\ \sigma w_i\geq 0$)

σw_i: elasticity of transformation of the i-th commodity's international transformation, ($\sigma w_i\equiv\frac{d(EROW_i/WD_i)}{EROW_i/WD_i}\Big/\frac{d(\overline{PEROW}_i/PWD_i)}{\overline{PEROW}_i/PWD_i}=\frac{1}{\rho w_i-1}$)

Solving the maximization problem yields the supply functions for exports and domestic goods:

$$EROW_i=Y_i\cdot\left[\theta w_i^{\rho w_i}\cdot serow_i\cdot(1+tmx_i)\cdot PY_i/\overline{PEROW}_i\right]^{-\sigma w_i}\qquad(4\text{-}164)$$

$$WD_i=Y_i\cdot\left[\theta w_i^{\rho w_i}\cdot(1-serow_i)\cdot(1+tmx_i)\cdot PY_i/PWD_i\right]^{-\sigma w_i}\qquad(4\text{-}165)$$

Furthermore, the profit maximization problem for the representative firm of the i-th sector in transforming the domestic supply into RUK exports $ERUK_i$ and regional goods W_i can be expressed as follows:

$$maximize\ \pi_i=\overline{PERUK}_i\cdot ERUK_i+PW_i\cdot W_i-PWD_i\cdot WD_i\qquad(4\text{-}166)$$

$$subject\ to\ WD_i=\theta k_i\cdot\left[seruk_i\cdot ERUK_i^{\rho k_i}+(1-seruk_i)\cdot W_i^{\rho k_i}\right]^{1/\rho k_i}\qquad(4\text{-}167)$$

Notations involved are:

$\overline{PERUK}_i$: price of the i-th RUK export good,

PW_i: price of the i-th regionally produced good for regional supply,

$ERUK_i$: RUK exports of the i-th good,

W_i: regional supply of the i-th regional good for regional demand,

θk_i: scaling coefficient of the i-th domestic transformation,

$seruk_i$: RUK export share coefficient for the i-th domestic transformation, $(0 \leq seruk_i \leq 1)$

ρk_i: parameter defined by the elasticity of transformation, $(\rho k_i = \frac{\sigma k_i+1}{\sigma k_i}, \rho k_i \geq 1, \sigma k_i \geq 0)$

σk_i: elasticity of transformation of the i-th commodity's domestic transformation, $(\sigma k_i \equiv \frac{d(ERUK_i/W_i)}{ERUK_i/W_i} / \frac{d(\overline{PERUK}_i/PW_i)}{\overline{PERUK}_i/PW_i} = \frac{1}{\rho k_i - 1})$

Solving the maximization problem yields the supply functions for RUK exports and regional goods:

$$ERUK_i = WD_i \cdot \left[\theta k_i^{\rho k_i} \cdot seruk_i \cdot PWD_i / \overline{PERUK}_i\right]^{-\sigma k_i} \tag{4-168}$$

$$W_i = WD_i \cdot \left[\theta k_i^{\rho k_i} \cdot (1 - seruk_i) \cdot PWD_i / PW_i\right]^{-\sigma k_i} \tag{4-169}$$

The import behaviour relates to the construction of the 'Armington composite good' through combining the regionally produced and demanded commodities with RUK import input in the first nest and the second nest with ROW input. To describe the process of constructing 'Armington composite good', it is assumed that representative firms maximize profits by choosing a suitable combination of regional goods and RUK imported goods, and then ROW imported goods. The process is described by CES functions with elasticity of substitutions, which indicate the percentage changes in the input factor ratio caused by a 1% change in relative input prices.

The optimization problem for the i-th composite good in the first nest with RUK imports can be written as:

$$maximize\ \pi_i = PQW_i \cdot QW_i - (\overline{PMRUK}_i \cdot MRUK_i + PW_i \cdot W_i) \tag{4-170}$$

$$subject\ to\ QW_i = \gamma k_i \cdot \left[smruk_i \cdot MRUK_i^{\eta k_i} + (1 - smruk_i) \cdot W_i^{\eta k_i}\right]^{1/\eta k_i} \tag{4-171}$$

Notations involved are:

$\overline{PMRUK}_i$: price of the i-th RUK import good,

PQW_i: price of the i-th domestic composite good,

$MRUK_i$: RUK imports of the i-th good,

QW_i: the i-th domestic composite good combining regional good and RUK imports,

γk_i: scaling coefficient of the i-th domestic substitution,

$smruk_i$: RUK import share coefficient for the i-th domestic substitution, ($0 \leq smruk_i \leq 1$)

ηk_i: parameter defined by the elasticity of substitution, ($\eta k_i = \frac{\omega k_i - 1}{\omega k_i}, \eta k_i \leq 1, \omega k_i \geq 0$)

ωk_i: elasticity of substitution of the i-th commodity's domestic substitution, ($\omega k_i \equiv -\frac{d(MRUK_i/W_i)}{MRUK_i/W_i} / \frac{d(\overline{PMRUK}_i/PW_i)}{\overline{PMRUK}_i/PW_i} = \frac{1}{1-\eta k_i}$)

Solving the maximization problem yields the demand functions for RUK imports and regional goods:

$$MRUK_i = QW_i \cdot \left[\gamma k_i^{\eta k_i} \cdot smruk_i \cdot PQW_i / \overline{PMRUK}_i\right]^{\omega k_i} \quad (4\text{-}172)$$

$$W_i = QW_i \cdot \left[\gamma k_i^{\eta k_i} \cdot (1 - smruk_i) \cdot PQW_i / PW_i\right]^{\omega k_i} \quad (4\text{-}173)$$

The optimization problem further for the i-th composite good in the second nest with ROW imports is expressed as:

$$maximize\ \pi_i = PQ_i \cdot Q_i - (\overline{PMROW}_i \cdot MROW_i + PQW_i \cdot QW_i) \quad (4\text{-}174)$$

$$subject\ to\ Q_i = \gamma w_i \cdot \left[smrow_i \cdot MROW_i^{\eta w_i} + (1 - smrow_i) \cdot QW_i^{\eta w_i}\right]^{1/\eta w_i} \quad (4\text{-}175)$$

Notations involved are:

$\overline{PMROW}_i$: price of the i-th ROW import good,

PQ_i: price of the i-th composite good,

$EROW_i$: ROW imports of the i-th good,

Q_i: the i-th composite good combining domestic good and ROW imports,

γw_i: scaling coefficient of the i-th international substitution,

$smrow_i$: ROW import share coefficient for the i-th international substitution, ($0 \leq smrow_i \leq 1$)

ηw_i: parameter defined by the elasticity of substitution, ($\eta w_i = \frac{\omega w_i - 1}{\omega w_i}, \eta w_i \leq 1, \omega w_i \geq 0$)

ωw_i: elasticity of substitution of the i-th commodity's international substitution, ($\omega w_i \equiv -\frac{d(MROW_i/QW_i)}{MROW_i/QW_i} / \frac{d(\overline{PMROW}_i/PQW_i)}{\overline{PMROW}_i/PQW_i} = \frac{1}{1-\eta w_i}$)

Solving the maximization problem yields the demand functions for ROW imports and domestic goods:

$$MROW_i = Q_i \cdot \left[\gamma w_i^{\eta w_i} \cdot smrow_i \cdot PQ_i/\overline{PMROW_i}\right]^{\omega w_i} \tag{4-176}$$

$$QW_i = Q_i \cdot \left[\gamma w_i^{\eta w_i} \cdot (1 - smrow_i) \cdot PQ_i/PQW_i\right]^{\omega w_i} \tag{4-177}$$

Finally, the Armington composite good Q_i combining regional supply, RUK and ROW imports is supplied for the whole demand side.

4.6.6 Investment and saving behaviour

This model applies the Neoclassical closure, as the essence of this closure is saving-driven, which implies the investment, including new dwellings and non-residential properties, is driven by saving as a binding constraint through fixed saving rate. All other closures are investment-driven, which require fixed investment or a fixed relationship of investment with the total demand. This does not meet one of the research subject - real estate properties, which should be passively driven rather than a driving force. Hence, the total savings should be given first by:

$$S = (1 - skh - skg) \cdot YK + (1 - sth - stg) \cdot YT + SH + \overline{SRUK} + \overline{SROW} \tag{4-178}$$

Here the total savings are composed of the rest portions of capital and land factor income, which can be seen as firms savings, households savings, and fixed extra-regional savings with RUK and ROW. The extra-regional savings are essentially the balance of payment terms to balance the current trade surplus/deficit.

Given the total savings, the demand for investment by type of commodities is modelled again with the maximization problem below:

$$maximize\ UI = \prod_i I_i^{ai_i} \tag{4-179}$$

$$subject\ to\ \sum_i PQ_i \cdot I_i = S \tag{4-180}$$

The investment demand equation is derived as:

$$PQ_i \cdot I_i = ai_i \cdot S \tag{4-181}$$

Then the price of the composite investment good PI is given by:

$$PI = \sum_i (1 + tmx_i) \cdot PQ_i \cdot ai_i \tag{4-182}$$

4.6.7 Market-clearing conditions

Market-clearing conditions are imposed to ensure that good markets and factor markets all clear to reach equilibrium that demands equals supply at the prevailing prices. On good markets, the sum of demand for intermediate inputs, of demand for households and government consumption and investment goods must equal the supply of the composite good in each sector:

$$Q_i = C_i + G_i + I_i + \sum_j INT_{i,j} \tag{4-183}$$

Factor market equilibriums for all factors are only imposed for the medium run perspective, while in the short run, factor stocks are already fixed in each sector and do not need further constrained in the total level. In the long run, capital and labour supply are perfectly elastic and do not need total stock constraints either. Only land is constrained by its total stock scale across residential and non-residential categories. Hence, the factor market clearing conditions in the medium run are listed below:

$$\sum_j L_j = \bar{L} \tag{4-184}$$

$$\sum_j K_j = \bar{K} \tag{4-185}$$

In the long run, there is only land factor market clearing condition as below:

$$\sum_{jnr} T_{jnr} + \bar{T}_{R\_R} = \bar{T} \tag{4-186}$$

On the foreign markets with RUK and ROW, balance of payments conditions are dropped to avoid over-determination of the model. According to Walras' law if (n-1) markets are cleared, the n-th one is cleared as well. Considering additionally that the exchange rate with RUK and ROW and foreign saving terms are all exogenously fixed, balance of payments conditions are completely redundant.

4.7 Conclusion

Based on the investigation of the theory of CGE modelling and various functional form specifications describing agents' optimization behaviours, this chapter has explained the development of a regional CGE model for Wales as a small open economy. Nested Cobb-Douglas function, Leontief function, CES and CET functions are adopted to describe the optimal decision-making processes for production, consumption, investment and trade. Suitable macroeconomic closure and balance constraints regarding different time frames are discussed and selected based on the research objects and characteristics of the Welsh economy.

The model features 21 sectors and commodities, 3 types of factors and 6 economic agents including 1 category of households. The model describes production, consumption and investment behaviours through presenting economic agent activities of firms, households, government, trade and saving-investment. The model aims to provide an opportunity to simulate how they interact with each other and the resulted macroeconomic effects in response to a tax variation policy shock in a general equilibrium context. This model is currently a static CGE model, which simulates a counter-factual consequence compared to the initial economic state, with 2013 as the benchmark year. Therefore, it is not assumed to present the detail route of endogenous variables from their base values toward their states ex-post. However, further extension regarding model dynamics would be possible if relevant regional data information becomes available, as well as further disaggregation of sectors and households groups. The construction of the benchmark database for the model, a 2013 Social Accounting Matrix for Wales, is presented in detail in the next chapter.

CHAPTER 5 THE DEVELOPMENT OF A WELSH SOCIAL ACCOUNTING MATRIX

5.1 Introduction

This chapter describes the development of the CGE model database which is organized in the form of Social Accounting Matrix (SAM). As the model database, the SAM acts as the dataset foundation to present interactions of economic variables in the model and to calibrate the unknown model parameters. Besides, it also provides a description of the target economy as an equilibrium benchmark for comparison against the post shock economic status. Thus, by presenting this chapter we are able to make the dataset ready for model calibration, simulation and analysis of results implication in the next chapter.

The SAM presented in this chapter aims to give a snapshot of the Welsh economy in calendar year 2013. To develop the SAM, we first introduce the basic concepts and framework to establish the knowledge base, then propose a schematic structure of SAM as a guidance for organizing the data required. The details of the acquisition and assembly of relevant data which fulfil the schematic SAM block by block is illustrated. As the preliminarily constructed SAM is unbalanced, we will introduce several methods to balance it and the balancing process will be discussed.

The chapter is organized as followed. Section 5.2 explains the basic theory of SAM and proposes the schematic SAM. Section 5.3 records the compilation of the SAM with data achieved from multiple sources. The balancing methods and their principles are presented in section 5.4, and section 5.5 explains the balancing process applied with these methods. Section 5.6 focuses on the calibration of parameters contained in the model and finally section 5.7 draws the conclusion.

5.2 Basic theory of SAM

5.2.1 Introduction of SAM

CGE modelling requires a consistent and coherent database in the form of a Social Accounting Matrix (SAM). A SAM records the value of all circular flows of economic transactions in an economy. It records transaction data over a specified period of time, usually a calendar year. In this sense, it can be seen as a 'snapshot' describing comprehensively the economic structure and activities of economic agents in a particular period. The SAM developed and used throughout this dissertation focuses on the Welsh economy during 2013 calendar year.

The SAM table is a logical framework to arrange the transaction data of agents. These typically include industrial sectors, factors, households, government, investment, and foreign sectors. The data sources range from national accounts, Input-Output (I-O) tables, industry statistics, government fiscal statistics, trade statistics, surveys and census. These data are organized into a square matrix using double entry bookkeeping principle. This means that each economic agent has both a column account recoding its expenditure structure, and a row account recording its sources of income. Therefore, each number in the table represents a single transaction as the payment from an agent's column account to an agent's row account, and the number locates in the intersected cell between the row and column accounts. Total expenditure must equal total income for each agent, and the corresponding column account total must equal the row account total.

Based on the balancing of each agent account, the SAM table is then automatically balanced. A SAM must only be balanced to represent the equilibrium condition of the whole economy such that every market clears. Only then is the SAM a benchmark dataset ready for subsequent model simulation. The balanced SAM, as a description of economic activities of the whole economy, reflects the Walrasian general equilibrium principle that income equals expenditure for every market clearing.

5.2.2 System of national accounts, circular flow of income

Theoretical development of the SAM originates from the work of Richard Stone (1947) who established the basic framework for the standardized system of national accounts (SNA) and reconciled it with economic production accounting. The SNA is the internationally agreed standard system for macroeconomic accounts, which forms the construction basis for an integrated I-O framework and SAM extensions. National accounts are the main data source of both I-O and SAM tables, and also provide an organized way of dealing with complications arising from collection of data and recording the economic value flows. This is accomplished by means of ‘T’ style double entry bookkeeping ‘balance sheets’ for the key agents and sectors of the economy. In such a balance sheet, all accounts are summarized into balanced debit and credit entries. Subsequently, the two-dimensional I-O and SAM table can be derived from the nature of the transactions recorded in national accounts balance sheets. Specifically, each transaction flow of national accounts is recorded in each cell of the table, and the cell’s corresponding column account represents the flow source and its row account represents the flow destination.

Figure 5.1 Circular Flow of Income

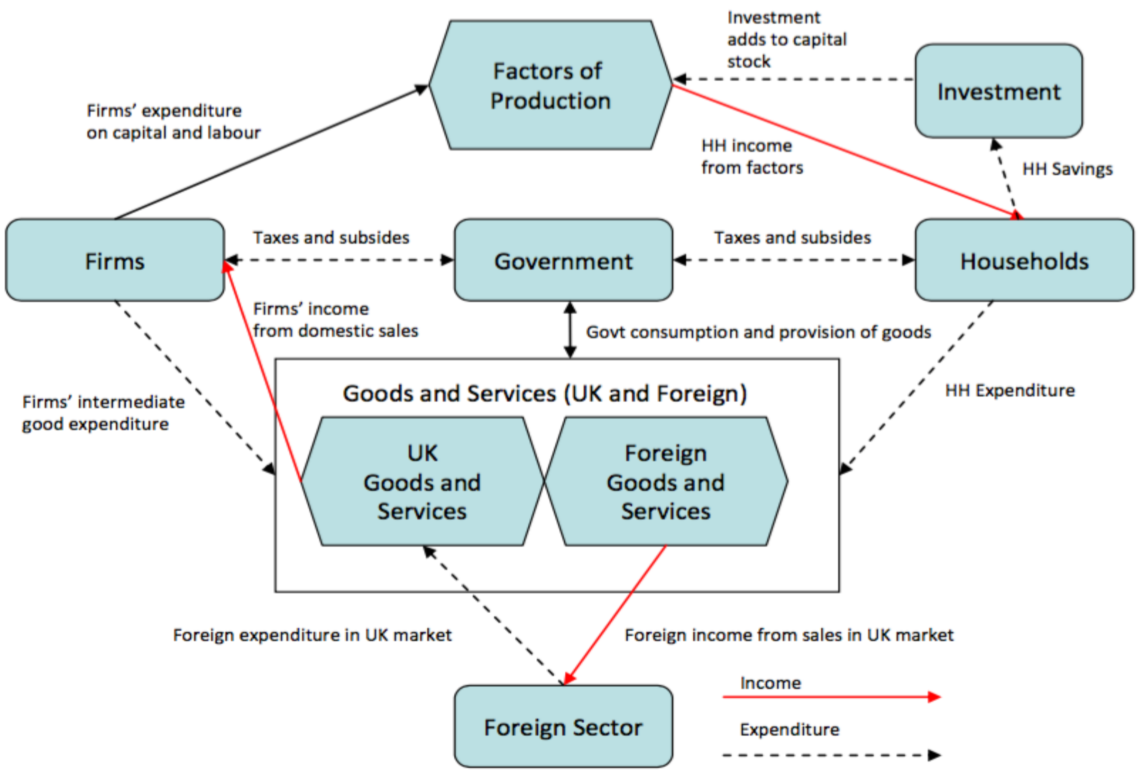


Figure 5.1 shows the connections between national accounts and agent’s transaction behaviours. At the heart is the circular flow of income on which general equilibrium theory is based. The transaction flows discussed above circulate within the economy, and also through borders between different economies. The diagram shows how economic agents receive income and make spending, and how supply and demand clear simultaneously in different markets through

these flows. The figure below is an example of circular flow of income which focuses on the UK as an open economy.

It shows in Figure 5.1 that households receive income by providing production factors for consuming goods and services, and pool the rest of their income through the capital market to provide capital investment for firms. UK firms' factor and intermediate payments are financed by selling goods and services both domestically and abroad. Both domestically produced and imported goods and services are demanded by UK households, firms and government, while the government receives tax revenues from both households and firms to provide them public goods and distributes transfer payments. Hence, the whole economy operates with monetary flows circulating accompanied with transactions of goods and services, and the figure can be seen as a visualized representation of a SAM table.

5.2.3 Input-Output framework, the price system, and the need for social accounting matrices

The I-O table mentioned above also originates from the organized summary of national or regional accounts, and at the same time it forms a basis for constructing a corresponding SAM. In practice, a SAM table can be developed from an existing I-O table and this avoids a great deal of primary data collection. To develop a regional SAM a regional I-O table is a critical component. Hence, it is important to explain the I-O framework before we proceed to the procedures to SAM development.

First, it is worth explaining the price system used in I-O, SAM and CGE modelling. In general equilibrium modelling, prices are core variables to drive the economic propagation following diversified shocks. The prices adopted in the national accounts system reflect the stage at which goods and services are produced and what price components they include by that stage. Below we introduce the definitions of three fundamental prices and, as the incurred taxes are common components of these prices, two basic type of taxes are also explained.

The first type of prices refers to the basic price, which is the most commonly used price in various national accounts statistics. Basic prices reflect the following:

> These prices are the preferred method of valuing gross value added and output. They reflect the amount received by the producer for a unit of goods or services minus any

> taxes payable plus any subsidy receivable on that unit as a consequence of production or sale (that is the cost of production including subsidies). As a result, the only taxes included in the basic price are taxes on the production process – such as business rates and any vehicle excise duty paid by businesses – which are not specifically levied on the production of a unit of output. Basic prices exclude any transport charges invoiced separately by the producer. (Office for National Statistics 2015f, p.134)

Therefore, the basic price only covers the intermediate inputs, value added inputs and production taxes less subsidies in producing one unit of output (net taxes on production). Besides, according to the World Bank, for imported products, the equivalent price is the c.i.f. (cost, insurance and freight) value, that is, the value at the border of the importing country.

The second price is the producer price. This type of price considers the price 'at the factory gate'. According to the World Bank, the producer price is the amount receivable by the producer inclusive of taxes on products except deductible value added tax and exclusive of subsidies on products. The equivalent price for imported products is the c.i.f. value plus any import duties or other taxes on imports (minus any subsidies on imports). Its relation with basic price is then:

Producer price = Basic price + taxes on products (excluding VAT) - subsidies on products

The definition of taxes on products as a component of producer price, other than the taxes on production explained in basic price, is defined in the European system of national and regional accounts as below:

> Taxes on products are taxes that are payable per unit of some good or service produced or transacted. The tax may be a specific amount of money per unit of quantity of a good or service, or it may be calculated as a specified percentage of the price per unit or value of the goods and services produced or transacted. Taxes assessed on a product, irrespective of which institutional unit pays the tax, are to be included in taxes on products, unless specifically included in another heading. Statistical Office of the European Communities and European Commission 2013, p.92)

Taxes on products comprise value added tax, taxes and duties on imports and exports and other taxes on products (e.g. excise duties, stamp taxes on the sale of specific products, such as alcoholic beverages or tobacco, car registration taxes, taxes on lotteries, taxes on insurance premiums).

The last price introduced is the purchaser price. The purchaser price is the amount payable by the purchaser. This includes trade margins realized by wholesalers and retailers (by definition, their output) as well as transport margins (that is, any transport charges paid separately by the purchaser) and non-deductible VAT. Hence, the purchaser price is derived as:

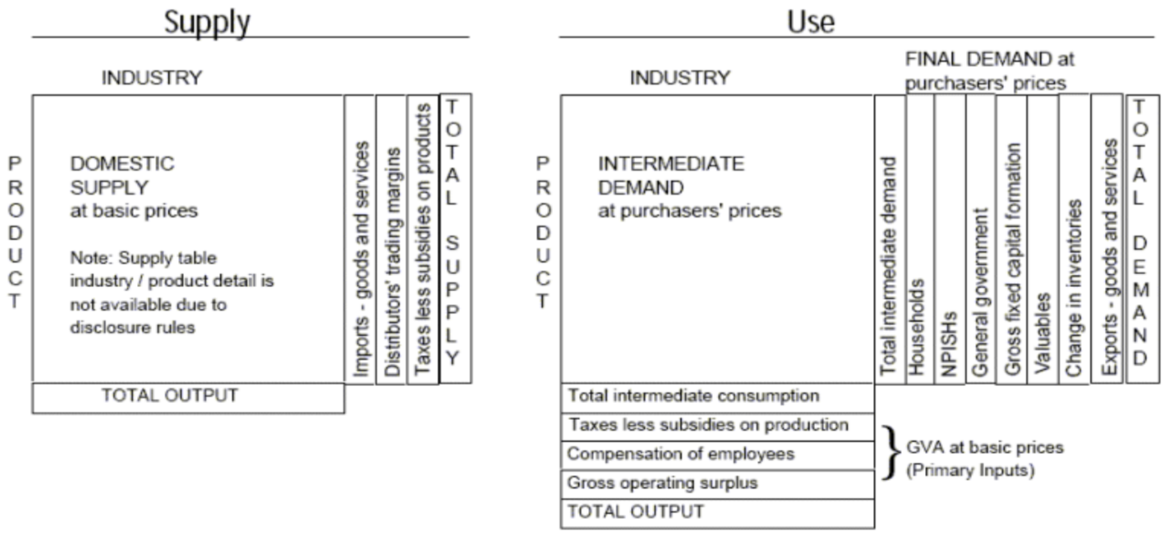
Purchaser price = Producer price + trade and transport margins + non-deductible VAT

Therefore, the taxes included in basic price are only taxes on production, whereas taxes on products are included in both producer price and purchaser price, the only difference of which is whether the value added tax is deductible. The purchaser price is normally paid by consumers as final demand price, so the value added tax is non-deductible to them and hence a compulsory payment. However, for producers it may not be the case as their payment of value added tax can be deductible and hence eliminated from the producer price actually received by the producers.

So far, we have reviewed the three important types of prices covering different stages throughout production and sale. The distinction of these prices is explicitly adopted and displayed in I-O analysis. I-O analysis is also known as inter-industry analysis and the focus is on the interdependence of industrial sectors in an economy. The interdependent relations between industries are reflected in two foundational types of I-O tables: the Make (or Supply) table and the Use table. The Make table shows the production of commodities supplied by domestic industries where the domestic supply is marked by basic prices, while the Use table shows the purchases of commodities by domestic industries and final demand institutions, with the intermediate and final demand marked by purchaser price. The structures of the tables are shown in Table 5.1.

The two tables can be combined to give Supply and Use Table (SUT) characterized by a commodity-by-industry format and valued in purchaser prices. These are published annually for the UK by the Office for National Statistics (ONS). However, for economic analysis, it is useful to develop SUTs further to construct analytical I-O tables. Analytical transactions tables feature three main differences from SUTs, and each of these differences is important in the facilitation of multi-sectoral economic modelling.

Table 5.1 Structure of Make and Use Tables



First, analytical I-O tables are balanced for each industry, such that each industry's output equals all the inputs to that industry. Second, the main inter-industry transaction block is converted to symmetric form to be consistent with the balance. Therefore, it can be of commodity-by-commodity format or industry-by-industry format. Third, the analytical I-O tables are valued by basic prices, so all the distributors' margins and taxes less subsidies on products are stripped out.

Based on these characteristics, the structure of a schematic analytical I-O table for a UK region can be considered as consisting of four quadrants isolated by double solid lines as shown below. The top-left quadrant records the expenditure of regional producers on intermediate inputs produced by regional producers. The bottom-left quadrant lists regional producers' inputs that are not produced by regional producers. These include imported products, net taxes on products, and primary inputs such as labour and capital return, which together with net taxes on production comprise gross value added (GVA). Here we have obtained the column total through the two quadrants on the left-hand-side as the total inputs of regional producers, and it can be seen as the total supply.

On the right-hand-side, the top-right quadrant records the final demand consumption for regional products. The demand comes from households, government, investment, and external sectors. Because the table is structured for a regional framework, the external sectors include not only the rest of the world exports (ROW), but also the rest of UK exports (RUK). As the top-left quadrant represents sales to intermediate demand and the top-right quadrant represents sales to final demand, the whole upper side of the table reflects the total demand for the regional products, and is balanced with the total supply. The bottom-right quadrant concerns final consumption purchases on imports and payment on product taxes.

Table 5.2 Structure of Analytical I-O Tables

Expenditure / *Income*	Industries	Households	Government	Investment	Rest of UK	Rest of World	*Total*
Industries	Regional demand for regional output	Households demand for regional output	Government demand for regional output	Investment demand for regional output	RUK demand for regional output	ROW demand for regional output	*Total Demand*
Factors	Gross value added						*Gross value added*
Taxes	Net taxes on products	Net taxes on products	Net taxes on products	Net taxes on products	Net taxes on products	Net taxes on products	*Total net taxes on products*
Rest of UK	Imports from RUK	Imports from RUK	Imports from RUK	Imports from RUK	Imports from RUK	Imports from RUK	*Total Outflows to RUK*
Rest of World	Imports from ROW	Imports from ROW	Imports from ROW	Imports from ROW	Imports from ROW	Imports from ROW	*Total Outflows to ROW*
Total	*Total Supply*	*Total Household Expenditure*	*Total Government Expenditure*	*Total Investment*	*Total Inflows from RUK*	*Total Inflows from ROW*	

As explained earlier, one of main features of I-O framework is to present more fine grained national accounts and organize major macroeconomic variables in a logical way. Through this framework, major economic variables can be modelled not only for the whole economy but also for a series of sectors. To see this, we fill the schematic I-O table above with variables as shown below in Table 5.3.

Table 5.3 Input-Output Table in Summary

Expenditure / *Income*		Industries 1	Industries 2	Households	Government	Investment	Exports RUK+ROW	*Total*
Industries	1	Z11	Z12	C1	G1	I1	E1	X1
	2	Z21	Z22	C2	G2	I2	E2	X2
Factors	Labour	L1	L2	CL	GL	IL	EL	L
	Capital	K1	K2	CK	GK	IK	EK	K
Taxes		T1	T2	CT	GT	IT	ET	T
Imports	RUK+ROW	M1	M2	CM	GM	IM	EM	M
Total		X1	X2	C	G	I	E	Y

In Table 5.3, variables *Z, L, K, T,* and *M* represent intermediate inputs, labour inputs, capital inputs, tax payments and import inputs respectively. *C, G, I, E* stand for final demands from institutions - households, government, investment and exports respectively. Combinations of the two series of variables in the bottom-right quadrant mean the final demand expenditure on the factors, taxes and imports, although normally the final demands on factors are zero as institutions are not producing sectors. The table has two production industries. For each of the two industries, X is total supply from the column summation and is simultaneously total demand from the row summation. Y is gross output of the whole economy, covering both intermediate and final realizations of value.

In summary, the gross output can be calculated from the expenditure or income basis. In the first case, the gross output equals intermediate expenditures of industrial sectors and final demand expenditures of institutions such that:

$$Y = X1 + X2 + C + I + G + E \tag{5-1}$$

From an income basis, the gross output equals income of intermediate products sales and income of products for final consumption such that:

$$Y = X1 + X2 + L + K + T + M \tag{5-2}$$

Connecting the two equations enables us to cancel out X1 and X2 so we can obtain the regional economy's gross domestic product (GDP) at purchaser prices:

$$X1 + X2 + C + I + G + E = X1 + X2 + L + K + T + M \tag{5-3}$$
$$C + I + G + E = L + K + T + M \tag{5-4}$$
$$C + I + G + (E - M) = GDP = L + K + T \tag{5-5}$$

Therefore, it can be explicitly shown that the left-hand-side of the equation represents the expenditure method of calculating GDP and the right-hand-side represents the income method.

So far, the I-O table has shown how economy income and expenditure are generated, and the composition of the economy's GDP through describing the production structure. However, this basic framework can be further developed with a SAM. A SAM addresses selected issues with the basic I-O framework.

Firstly, an analytical I-O table is not entirely symmetrical but rather symmetrical only in terms of its inter-industrial transaction block. All other institution accounts, including factors, taxes, households, government, investment and external sectors, do not have their row or column counterparts. This implies the most significant structural difference between I-O and SAM tables concentrate in the bottom-right quadrant. In this quadrant in a SAM we can observe how the income and expenditure are allocated, transferred, and distributed among the agents. For example, such information could contain factor income distribution, government budget conditions, government transfers, savings pooling, tax payments and funds transfer across borders. These are all key components of a general equilibrium system.

Secondly, an I-O table provides limited information on inter-institutional transaction flows. This means it is of limited use in investigating the macro-economic closures required to close a CGE model. This includes how the government budget is financed, how investment is balanced with savings, and the balance of payments between inflows to and outflows from the target economy.

Thirdly, a SAM provides the possibility of separating industrial production and commodities. That is, based on different degrees of disaggregation of industrial sectors and commodities, different sectors can produce the same commodity and different commodities can also be produced by the same sector. Comparatively, in an analytical I-O table for economic modelling, the production structure is highly symmetric only in terms of the sectors, commodities, and degree of disaggregation. It is always a one-to-one match between industries for an industry-by-industry I-O structure, or between commodities for a commodity-by-commodity I-O structure.

Last, there are no places to locate production-irrelevant taxes in an I-O table. For example, with no income sources displayed for agents, there are no cells to fill in taxes whose tax bases are their income, such as income tax paid by households and corporation tax paid by corporates. Similar situations apply to many other types of taxes whose tax bases do not explicitly appear in the table. Hence, the I-O framework is generally not suitable for tax modelling analysis.

Summarizing the differences discussed above, the SAM is actually a general form of analytical I-O table. Therefore, to properly model and analyze overall economic behaviours, there is a need to incorporate more comprehensive information regarding transaction flows so as to describe the whole economy in equilibrium. This requirement can be fulfilled by developing a SAM as a benchmark dataset for general equilibrium modelling.

5.2.4 A schematic SAM and accounts setting

Based on the discussion in the above sections, we present a schematic SAM below which models a national economy. Based on this schematic SAM, each economically significant sub-matrix block incorporated will be explained. This framework serves as a base for our Welsh regional version of the SAM in the following sections.

The accounts in a SAM are set up based primarily on the agents that pursue economic activities in an economic system. As discussed above, these economic agents include producers, consumers, government and external sectors, which are represented in the schematic SAM table by the accounts 'Industries', 'Households', 'Government', 'Rest of UK' (RUK) and 'Rest of World' (ROW) respectively.

The accounts also normally include market elements such as labour, capital, saving-investment, as well as various types of taxes. These are represented by the account 'Factors', the equivalent 'Saving' and 'Investment' accounts, and the interim account 'Taxes' respectively in the table. Therefore, a SAM does not only record the sale of products and services between the supply side and demand side, but also involves other transactions, such as purchase and use of intermediate and primary inputs during the production process, transfer and tax payment as inter-institutional transactions between households and government, conversion from saving to investment through capital market, and trade and transfer with external sectors.

As shown in Table 5.4, each economically significant sub-matrix block is labelled with a name which primarily describes the transaction behaviour between the corresponding row and column accounts. We start from the column account of 'Industries' which describes all of industries' expenditures on the inputs used in the whole production process. For the 'Intermediate input' block, it records the intermediate inputs that are all 'Commodities'. All 'primary inputs, including wages, rents, profits, and tax expenditures during production are recorded in the 'Gross value added' block with 'Factors' as the corresponding row account. The column sum of 'Industries' is the total expenditure of regional industries, as well as the value of their gross output from the row's summation.

The row account of ‘Industries’ records where industries sell their output. It is usually assumed that the entire output, apart from those directly exported to RUK and ROW, is sold to the ‘Commodities’ account, which along its column can be thought of as wholesalers who purchase products from regional producers and package them with imports and relative product taxes to create composite bundles for demand. Therefore, the row account of ‘Commodities’ shows where the commodities are sold: it is intermediate demand if they are sold to producers and it is final demand if the destinations are households consumption, government purchase and fixed capital formation as investment.

As mentioned before, for the production behaviour of each industrial sector, there is normally no distinction between ‘Commodities’ account and production ‘Industries’ account in an I-O table, as it is implicitly assumed that one industry sector produces only one type of commodity. In a SAM table, however, it is possible to distinguish between the two accounts. Such separation provides advantages in describing production segment. For example, one sector may produce several commodities while one type of commodity may be produced by different sectors. Additionally, the ‘Industries’ account can be classified according to corporate scales, for example to large enterprises and SMEs, and apparently, these different scaled enterprises may also produce the same commodity. It is also useful to separate the two accounts in identifying the relevant tax-induced price changes as the prices in ‘Industries’ account are basic prices, while in ‘Commodities’ account they are purchaser prices.

The ‘Factors’ account usually includes labour and capital, and sometimes land etc. The row account of each factor records the income return received from producers for the amount employed. The column account of factors describes how the factor income net relative tax payment is allocated and distributed among households and government as factor owners. The ‘Taxes’ row account in the SAM describes the amount and sources of various types of taxes levied, and its column account records the tax revenues pooling into ‘Government’ row account. The ‘Government’ column account then describes the government purchase on different commodities in ‘Government expenditure’ block, and government social protection expenditure to other institutions, merely households here, in the ‘Government transfer’ block.

The ‘Rest of UK’ and ‘Rest of World’ accounts both represent external institutions outside the region of interest. They capture not only the trade flows but also financial flows across the regional border. The row accounts of them show the regional spending on imports and monetary outflow, while the column accounts report the export sales and monetary inflow, and also the balance of trade to the total social savings. The balance of trade is the difference between exports and imports. A trade deficit condition corresponds to a positive net monetary inflow as the economy in this case is considered as borrowing externally and the foreign savings form part of social savings. On the contrary, the foreign savings would be negative when the economy runs a trade surplus, implying lending to external sectors as outflows.

The ‘Saving-Investment’ account, as mentioned above, deals with the savings of institutions. This could be positive as consumption can be covered by income or a budget surplus, or negative if the budget is overspent. The table does not include government savings as the regional government is assumed to be, although partially devolved, dependent on funding transferred from central government. This can be observed in the the ‘Central government transfer’ block whose corresponding row account source is ‘Rest of UK’.

The ‘Saving-Investment’ account is highly critical in CGE modelling as it is the bridge to connect the demand side and supply side, and is useful for analyzing macroeconomic issues and closures for the model. In fact, it is an important balance in national accounting that total investment equals total saving. It is summarized by two major identities of national accounting:

$$Y = C + I + G + X - M \tag{5-6}$$
$$Y = C + S + T \tag{5-7}$$

Table 5.4 Social Accounting Matrix Summary Structure

Income \ Expenditure	Industries	Commodities	Factors	Taxes	Households	Government	Investment	Rest of UK	Rest of World	*Total*
Industries		Regional demand for regional output						Exports to RUK	Exports to ROW	*Gross Regional Output*
Commodities	Intermediate input				Household consumption	Government expenditure	Gross fixed capital formation			*Total Demand*
Factors	Gross value added				Rental payment					*Total Factor Income*
Taxes		Product tax, SDLT	Corporation tax		Income tax, NIC, Council tax					*Total Tax Revenue*
Households			Household earning			Government transfer				*Total Household Income*
Government			Government factor revenue	Government tax revenue				Central government transfer		*Total Government Revenue*
Saving					Household saving			RUK saving	ROW saving	*Total Saving*
Rest of UK		Imports from RUK								*Total Outflows to RUK*
Rest of World		Imports from ROW								*Total Outflows to ROW*
Total	*Total Regional Industry Expenditure*	*Total Supply*	*Total Factor Expenditure*	*Total Tax Revenue*	*Total Household Expenditure*	*Total Government Expenditure*	*Total Investment*	*Total Inflows from RUK*	*Total Inflows from ROW*	

where Y, C, I, G, X, M, S, T represent output, consumption, investment, government, export, import, saving and tax revenue respectively. Combining the two equations, we have

$$I = S + (T - G) + (M - X) \tag{5-8}$$

Here S stands specifically for household saving, the term *(T - G)* represents government saving and *(M - X)* represents net foreign saving, such as net financial capital inflow. From the balance of payments perspective, net export/import corresponds to net capital outflow/inflow to maintain the balance in the long run. The total investment here includes gross fixed capital formation, newly formed inventories and newly built real estate properties. Note that both government saving and net foreign saving can be negative terms, implying fiscal account deficit and current account surplus respectively, and in this case household saving may succeed social investment to maintain the national accounting balance.

In general, the accounts included in SAMs often differ in applications such as CGE modelling. They may differ in dimensions; that is, in their number and scale of industries and commodities, factors of production, or household types. Given the accounts structure setting, researchers can further aggregate or disaggregate the accounts regarding their internal classification and research focus. One may need to disaggregate some accounts due to the relevant statistical data calibre being too aggregated, or may aggregate some less important accounts compared to those of research interest. New accounts can then be correspondingly created by merging, being split out from original accounts, or combination of both kinds of the adjustment.

The figures in each cell of a SAM are usually non-negative, as negative values may cause problems in balancing, parameter calibration and model simulation. However, many figures can be negative with economic significance, such as negative government saving when the government is in deficit, or negative foreign saving implying a current account surplus that exports surpass imports and financial capital flows out. There could also exist various subsidies or equivalent tax credits etc., shown as negative tax payments in the SAM. The situation can be sorted by rearranging the location of the negative figures to their symmetric counterpart locations. For example, if the negative figure appears in the cell intersected by the *ith* row and *jth* column, it can be rearranged to the symmetric cell of *jth* row and *ith* column with positive sign. Although such solutions will affect the relevant row and column sums, it will not bias the table balance, and can be reversed when balancing, calibration or simulation processes are finished. We will see this clearly in the following sections regarding Welsh SAM development.

5.3 Welsh SAM: construction and data issues

After introducing the background, basic theory and principles of SAM, in this section we describe the data required for compiling a preliminary SAM table, and issues in data availability and collection.

All data are collected referring to the 2013 calendar year, which is the benchmark year of the Welsh SAM and the developed CGE model. 2013 is chosen because it is the latest year in which it was possible to obtain all the required regional data.

The construction of a SAM table is data intensive. Due to time constraints and the lack of primary survey-based regional information, a non-survey based method was adopted. In this case, most of the economic data are officially sourced. For official data such as those published by ONS, some have the calibre directly consistent with the intended sector aggregation level, so they can be applied straightforward. In the meantime, some other data may have different aggregation level that there is a need to estimate them based on reasonable assumptions in order to calibrate them with the table's aggregation level. In the following sub-sections, we will illustrate the data sources, estimation and compilation process to work out a preliminary SAM for Welsh economy in 2013.

As the data is derived from many different sources the preliminary version of the compiled SAM is normally unbalanced. The balancing methods involved and their applications are reported later in this chapter.

5.3.1 Dimension of the SAM

The dimension of the SAM concerns the size and number of industries, the type and number of commodities and factors, and also the classification of households. Dimensions depend largely on the quality and aggregation of regional industry.

This SAM for Welsh economy of 2013 was based on 21 sectors and commodities, 3 production factors and one aggregate household sector. The reason for selecting this structure is a result of two considerations. One is to highlight real estate relevant sectors which links to the tax modelling objectives of the thesis. The other lays in the convenience and consistency of adopting the published intermediate input and value added data as control totals, based on the UK Standard Industrial Classification of Economic Activities 2007 (SIC 2007). In the meantime, we also hope that the table could represent a reasonable disaggregation level as a base and convenience for future Welsh regional economic modelling. The sectors in the Welsh SAM are shown in Table 5.5.

Table 5.5 Sectors in 2013 SAM of Wales

Sectors in 2013 SAM of Wales	Sections & Divisions in SIC 2007
Agriculture, forestry & fishing	A 01-03
Mining & quarrying	B 05-09
Manufacturing	C 10-33
Energy & utility	D 35 & E 36-39
Construction (excl. new ownership)	F 42-43
New residential ownership	F 41
New non-residential ownership	F 41
Wholesale & retail	G 45-47
Transport & storage	H 49-53
Accommodation & food service	I 55-56
Information & communication	J 58-63
Finance & insurance	K 64-66
Residential rental	L 68.2
Real estate - agencies & management	L 68.3
Professional, scientific & technical services	M 69-75
Administrative & support services	N 77-82
Public administration & defence services	O 84
Education services	P 85
Human health & social work services	Q 86-88
Arts, entertainment & recreation	R 90-93

Other services	S 94-96

Table 5.5 shows that it was necessary to gain a finer disaggregation of real estate sectors in line with the requirements of later modelling of Stamp Duty Land Tax change. New residential and non-residential ownership, as representing the development and sale of new residential and non-residential properties, are split out from the original ‘Construction’ sector. In detail, they correspond to both SIC (2007) group 41.1 ‘Development of building projects’ and group 41.2 ‘Construction of residential and non-residential buildings’ included in division 41 ‘Construction of buildings’ of section ‘Construction’. Here the outputs of the two new sectors account for the transaction volumes of the new residential and non-residential real estate properties respectively, which are assumed as accounting for part of regional construction output.

For the resales of the properties, their transaction values are not supposed to appear in a national-accounting-based SAM or any GDP statistics due to the second-hand nature. Their economic value was already delivered on the day sold for the first time. Therefore, they are indirectly represented by their residential and non-residential proxies. The proxy for the residential property resale value is the actual total annual rent generated by the underlying dwelling stock. It is assumed that the actual total annual rent value is closely related to the total imputed rent implied by the total dwelling stock, which is further proportionately connected to the total property value of the dwelling stock. The total property stock value is then linked to the market value of those resold on the market. Similarly, the proxy for the non-residential resale is the non-residential land rental as factor input by each sector.

The residential rental sector corresponds to the residential part of the group 68.2 ‘Renting and operating of own or leased real estate’, while the non-residential land rental is presented in the form of factor inputs and is not formed as a specific sector in the SAM. The ‘Real estate - agencies & management’ sector corresponds to the group 68.3 ‘Real estate activities on a fee or contract basis’.

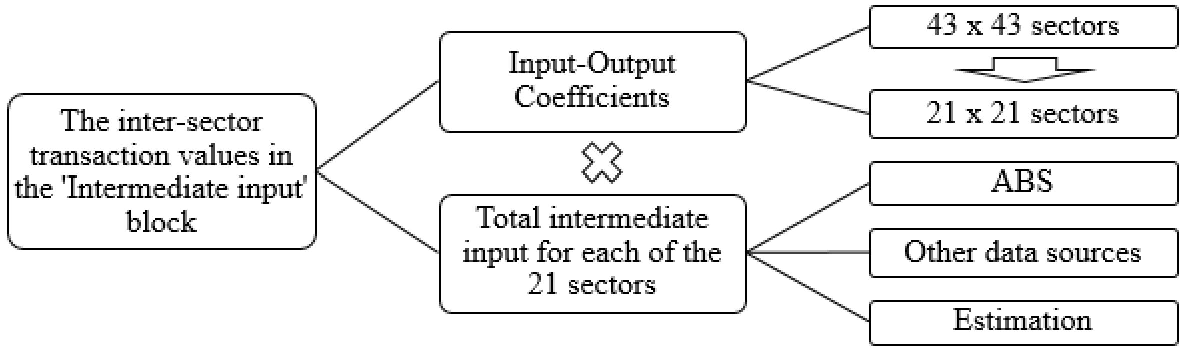
The input factors are classified to three components: labour, capital and land. The labour returns are further disaggregated as discussed in later sections. The land factor is included to suit the research interest in SDLT policy modelling. The capital factor is more difficult to classify. Normally its return is calculated as the value added residual apart from other factor returns. In a more aggregate way of input factor structure, the land factor is incorporated in the capital factor, so the capital return is in fact all the operating profits other than labour compensation expenditure, so called ‘Other value added’ or ‘Gross operating surplus’. In this Welsh SAM, the land rental is split out from it thanks to the detailed components listed in ‘Gross operating surplus’ term in Office for National Statistics (2015e)[3].

The households type is not further disaggregated due to the lack of regional differentiated household expenditure data, so the consumption structure is assumed to be uniform across the whole Wales.

5.3.2 The ‘Intermediate input’ block

This sub-section is devoted to the construction of the ‘Intermediate input’ block of the SAM as shown in Table 5.4. The input-output coefficients, or technical coefficients, are essential elements determining the structure of the linkages between sectors in the ‘Intermediate input’ block. The inter-sector transaction values for Wales are not officially published. An indirect way of derive these figures is through multiplying total intermediate inputs of each sector by derived input-output coefficients. The total intermediate input is obtained mainly through the ‘Total purchases of goods, materials and services’ in the ‘Annual Business Survey (ABS) – 2013 Regional Results’, and for the coefficients we refer to the analytical ‘Input-Output Table 2013 Wales’ (see Appendix III) generously offered by the Welsh Economy Research Unit[4] as a starting point. The development process of this block is shown in Figure 5.2.

Figure 5.2 Schematic Diagram for the Development of the 'Intermediate input' block



[3] See ‘Information’ and ‘Contents’ tabs in ‘Regional Gross Value Added (Income Approach) reference tables’.
[4] This is the IO table for 2013 developed from the 2007 IO table produced by the Welsh Economy Research Unit. The unpublished 2013 IO table was derived by Dr Andrew Crawley as part of an EU funded Marie Curie project.

In the developed ‘Input-Output Table 2013 Wales’, there are 43 sectors covering all Welsh industries. In this case, the I-O table must be aggregated first to the SIC 2007 section level (see Table 4.5) in order to meet the ‘21x21’ dimension of the SAM. The aggregation matrix method was used as a systematic way of accomplishing the task. This procedure is outlined below.

Suppose the original unaggregated matrix is defined as a $n \times n$ matrix Z where n is the number of unaggregated sectors, and the aggregated matrix is defined as a $k \times k$ matrix Z^* where k is the number of sectors after aggregation. The aggregation matrix S can then be denoted as a $k \times n$ matrix of zeros and ones. The location of ones in S determines the order of the aggregated sectors and which sectors are to be aggregated. The order of each unaggregated sector in matrix Z is reflected in which column the unaggregated sector, represented by one, is located in matrix S. The row order of the ones determines the order of the newly aggregated sectors. If two or more ones are located in the same row, it implies the unaggregated sectors represented by those ones will be aggregated into one sector in the new matrix Z^*. The matrix Z^* is then derived as:

$$Z^* = SZS' \tag{5-9}$$

Take the case where $k = 3, n = 4$ for an example, one would be aggregating a four-sector matrix into a three-sector matrix. Both of these matrices are square. Suppose we aggregate the original sector 2 and 3 to a new sector ranked as sector 1, the original sector 1 becomes the third sector and the original sector 4 is reordered to the sector 2 in the new aggregated matrix. The matrix Z^* is calculated as:

$$\begin{aligned} Z^* = SZS' &= \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} z_{11} & z_{12} & z_{13} & z_{14} \\ z_{21} & z_{22} & z_{23} & z_{24} \\ z_{31} & z_{32} & z_{33} & z_{34} \\ z_{41} & z_{42} & z_{43} & z_{44} \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} z_{22} + z_{23} + z_{32} + z_{33} & z_{24} + z_{34} & z_{21} + z_{31} \\ z_{42} + z_{43} & z_{44} & z_{41} \\ z_{12} + z_{13} & z_{14} & z_{11} \end{bmatrix} \end{aligned} \tag{5-10}$$

The example is shown through symmetric unaggregated and aggregated matrices, but it can also be applied on matrices with any dimensions, e.g. a vector matrix. Therefore, not only the intermediate input transaction matrix can be aggregated using this method but also other non-square matrices such as the final demand matrix and gross value added matrix.

The next step is to map the sectors in the I-O table into the SAM sectors. The mapping table is shown below.

Table 5.6 Sectoral Aggregation Scheme: Mapping Scheme

43 Sectors of 'Input-Output Table 2013 Wales'	21 Sectors of 2013 SAM of Wales
Crop, fish and animal production	Agriculture, forestry & fishing
Forestry and logging	
Mining and extraction	Mining & quarrying
Manufacture of food and beverages	Manufacturing
Manufacturing of apparel and textiles	
Manufacture of wood and paper products	
Printing and reproduction of recorded media	
Manufacturing of petroleum and chemical, pharmaceutical products	
Manufacturing of rubber and non-metallic mineral products	
Manufacture of basic metals and fabricated products	
Manufacture of computer and electrical equipment	
Motor vehicles	
Furniture	
Other manufacturing	
Electricity energy	Energy & utility
Water collection treatment and supply	
Construction	Construction (excl. new ownerships)
	New residential ownership
	New non-residential ownership
Wholesale	Wholesale & retail
Retail	
Accommodation	Accommodation & food service
Restaurants	

Railways	Transport & storage
Road transport	
Sea and air transport	
Transport services	
Travel agents	
Postal services	
Telecoms	Information & communication
Business services	Finance & insurance
Real estate	Real estate - agencies & management
Renting of movables	Administrative & support services
Computer and related activities	Information & communication
R&D	Professional, scientific & technical services
Public admin	Public administration & defence services
Education	Education services
Health and social work	Human health & social work services
Museums & galleries	Arts, entertainment & recreation
Attractions, gardens & other entertainment etc.	
Theme parks and stadia	
Other recreation, media & film	
Sanitary services	Energy & utility
Other services	Other services

The mapping table shows that not all the I-O sectors are aggregated to SAM, and the sectors of both sides are not all mapped in SIC order. Some I-O sectors that should be aggregated are not aligned together, such as ‘Telecoms’ and ‘Computer and related activities’ both belonging to ‘Information & communication’ sector, and ‘Electricity energy’, ‘Water collection treatment and supply’ and ‘Sanitary services’ all belonging to ‘Energy & utility’ sector.

Note here in the table that the ‘Construction’ sector of ‘Input-Output Table 2013 Wales’ is shown as disaggregated into new real estate ownership sectors. This does not imply that the aggregation procedure involves such disaggregation but rather reflects the final structure of SAM sectors.

For the 2 new real estate ownership sectors in the SAM, there is limited information about their sectoral input coefficients. Here we assume that they share the same input structure as the sector they are split out from. Hence, the new residential and non-residential ownership sectors have the same technical input coefficients with the construction sector. The input structure of the residential rental sector is simply the basic housing structure in terms of maintenance expenditure, mortgage interest payments and payments to real estate agency services. These inputs are incurred for property owners in earning actual rents, hence exclude energy bills that are normally paid by tenants. These input figures are obtained from Office for National Statistics (2014a)[5], covering inputs to construction, finance and insurance services and real estate agencies and management services, and totalled as £ 528.91 million.

It should be noted here that new real estate properties do not provide intermediate goods to any sectors but only form fixed capital. This implies that there are no sectors purchasing intermediate goods from new residential and non-residential ownership sectors. The output structure of the residential rental sector is similar, that it does not provide fixed capital formation but is rather demanded by households.

According to the mapping table, the aggregation matrix can be constructed by aggregating and reordering the I-O sectors. The technical input coefficients can then be derived from the aggregated intermediate input matrix that each input of the aggregated sector is divided by the sector's total intermediate inputs along the column in the aggregated matrix. Finally, each sector's input coefficients multiplying the total intermediate inputs figures below will yield the estimation of the whole intermediate input block.

Figures for sectoral total intermediate inputs in this SAM are derived from official dataset or estimation. As mentioned earlier, the official figures of total intermediate inputs are mainly from the 'Annual Business Survey (ABS) – 2013 Regional Results' of Office for National Statistics (2015a). However, the ABS covers only non-financial business sectors, and some sectors' data are incomplete as not all the economic activities are disclosed. Therefore, the missing data are either sought from other sources or proportionately estimated. Table 5.7 below lists the data sources of total intermediate inputs for each SAM sector.

[5] See 'Table 2.5 Housing expenditure by UK Countries and regions 2013' of 'Family Spending – 2014: reference tables'

Table 5.7 Data Sources for Total Intermediate Inputs for SAM Sectors

Sectors in 2013 SAM of Wales	Total Intermediate Inputs (£ million)	Data Sources
Agriculture, forestry & fishing	Agriculture: 1202	Department for Environment, Food and Rural Affairs et al. (2015)[6]
	Forestry: 47	Office for National Statistics (2015a)[7]
	Fishing: 24	Office for National Statistics (2015a)[7]
Mining & quarrying	289	Office for National Statistics (2015a)[7]
Manufacturing	29095	Office for National Statistics (2015a)[7]
Energy & utility	3025.05	'Input-Output Table 2013 Wales'
Construction (excl. new ownership)	2506.65	Office for National Statistics (2015a[7]; 2015d[8]), HM Land Registry (2015a)[9]
Construction - new residential ownership	347.73	
Construction - new non-residential ownership	906.62	
Wholesale & retail	21552	Office for National Statistics (2015a)[7]
Transport & storage	1409	Office for National Statistics (2015a)[7]
Accommodation & food service	1405	Office for National Statistics (2015a)[7]
Information & communication	1204	Office for National Statistics (2015a)[7]
Finance & insurance	2247.76	'Input-Output Table 2013 Wales'
Residential rental	528.91	Office for National Statistics (2014a)[5]
Real estate - agencies & management	234.38	'Input-Output Table 2013 Wales'
Professional, scientific & technical services	1630	Office for National Statistics (2015a)[7]
Administrative & support services	1833	Office for National Statistics (2015a)[7]

[6] See 'Table 3.2 Summary measures by country at current price (a)'

[7] See figures along the column 'Total purchases of goods, materials and services' in the tab 'Region by Section-Division' of the dataset 'Annual Business Survey (ABS) – 2013 Regional Results'

[8] See the tab 'Table 6' of the dataset 'Output in the Construction Industry'

[9] The dataset is downloaded using the standard reports application on the website landregistry.data.gov.uk. To configure this aggregated price data report from the website, follow the steps: Standard reports – create a standard report – Average prices and volumes – Region – Wales – Don't aggregate, just show the total – 2013 all year – New-build properties only – Generate report – choose report format to download.

Public administration & defence services	1778.86	‘Input-Output Table 2013 Wales’
Education services	1318.8	Office for National Statistics (2014b)[10]
Human health & social work services	3732.14	‘Input-Output Table 2013 Wales’
Arts, entertainment & recreation	1067	Office for National Statistics (2015a)[7]
Other services	343	Office for National Statistics (2015a)[7]

The total intermediate inputs for sectors that are exclusive or incomplete in the ABS are either referenced from other sources or estimated by reflation method. For example, the total intermediate input of education services sector is estimated based on the UK’s ratio between the sector’s total intermediate inputs and GVA in the 2010 UK I-O analytical table from Office for National Statistics (2014b)[10]. The total intermediate input of the education sector in Wales can be then reflated by multiplying the UK ratio (25378/82919 = 0.306) with the Welsh sector’s GVA[11] (4309) given in Office for National Statistics (2015e). This is shown in Table 5.8.

Table 5.8 Estimation of the Education Sector's Total Intermediate Input in Wales

Education sector (£ million)	Wales	UK	shared TII/GVA ratio
Total Intermediate Input (TII)	1318.8	25378	0.306/1
GVA	4309	82919	

For the 2 new real estate ownership sectors, there are no direct data source of intermediate inputs at this level of aggregation. Estimation of the 2 figures in Table 4.7 is made based on relevant real estate transaction and construction statistics. This is elucidated below.

Firstly, the transaction volume, which accounts for the output of ‘New residential ownership’, is acquired from HM Land Registry classified by residential property type. The original data is organized by average price and number of sales of each type of residential real estates shown in Table 5.9 below. The total transaction value is then achieved by summing total sale of each property type or directly multiplying overall average price of residential properties by total sales. This gives a total of £635.91 million.

[10] See ‘Domestic Use Table at basic prices’ of ‘United Kingdom Input-Output Analytical Tables, 2010’.
[11] See the tab ‘Table 6’ of the dataset ‘Regional Gross Value Added (Income Approach) reference tables’.

Table 5.9 2013 New Residential Sales in Wales

Overall average	Total sales	Detached	Sales	Semi-detached	Sales	Terraced	Sales	Flat/maisonette	Sales
£0.19m	3290	£0.25m	1492	£0.16m	793	£0.16m	558	£0.13m	447
Total: £635.91m		£365.58m		£124.16m		£89.09m		£57.08m	

Secondly, the output of 'New non-residential ownership' is obtained through the dataset 'Output in the Construction Industry' table (Office for National Statistics 2015d)[8], rather than from HM Land Registry as there is no data for the new non-residential sales from this source. The dataset contains all new building construction output in Wales in 2013. We choose those new works excluding housing and infrastructure construction – totalling £1,658 million.

Finally, given the whole construction sector's total output £6,878 million as the sum of £3,761 million total intermediate inputs and £3,117 million GVA, the two new ownership sectors' total intermediate inputs are estimated as proportionate with their outputs ratios against the whole construction sector's output. This gives £347.73 million for new residential ownership and £906.62 million for new non-residential ownership in Table 5.10.

Table 5.10 Estimation of the New Ownership Sectors' Total Intermediate Inputs in Wales

Wales (£ million)	New residential ownership sector	New non-residential ownership sector	shared TII/TO ratio with the construction sector
Total Intermediate Input (TII)	347.73	906.62	0.547/1
Total Output (TO)	635.91	1658	

By now, we have obtained all the sectors' total intermediate inputs that can be multiplied by the technical input coefficients to work out the input structure of each sector, which presents a full picture of the inter-sectoral transactions in the Welsh economy. After fulfilling this intermediate input block of the SAM, it is necessary to develop the GVA block so that we can identify the regional output of each sector and drop the anchor for constructing the rest parts of the SAM.

5.3.3 The 'Gross value added' block

This sub-section focuses on the construction of the ‘Gross value added’ block. The GVA data for each sector are adopted from the official source: ‘Regional Gross Value Added (Income Approach) reference tables’ from Office for National Statistics (2015e). The original data in the table comprises mainly four categories: Compensation of Employees (CoE), Gross Operating Surplus (GOS), Mixed Income, and Taxes less Subsidies on Production (ToPN), and they are classified by SIC 2007 sections, which is consistent with the scale of Welsh SAM sectors.

However, to be consistent with the research focus, we need to adjust further the GVA categories. The adjustment of the GVA structure is shown in Table 5.11.

Table 5.11 Categories Mapping of Welsh GVA

Categories in Regional GVA Table	Categories in 2013 SAM of Wales	
Compensation of Employees	CoE	NIC (payable by employers)
Mixed Income		Labour income
Gross Operating Surplus	GOS	Land rental
		Other value added
Taxes less Subsidies on Production	ToPN	Taxes less subsidies on production (excl. SDLT)
		SDLT - non-residential

To isolate the income tax base, we merge the Mixed Income into Compensation of Employees, as the income tax is not only levied on wages of employees but also simultaneously levied on the mixed income of those self-employed sole traders, which is considered a mixture of profits and self-paid wages. In the meantime, as the income tax is not levied on the National Insurance Contributions (NIC) paid by employers, the employer-paid NICs need to be split out from CoE. We should note that the NICs paid by employers themselves are subject to income tax so we do not need to split them out.

The rental portion of GOS is split out as a separate factor: land rental. All the remaining part of GOS is incorporated in Other value added (OVA). For the Taxes less subsidies on production, part of the non-residential SDLT are isolated from them for the sectors that have non-residential land as factor input. The reason SDLT is included in ToPN in the production stage is implied in the concept of ToPN. According to ‘SNA 2008’ published by United Nations et al. (2009), taxes on production are compulsory taxes paid for example on the production and import of goods and services, the employment of labour, and the ownership or use of land, buildings or other assets used in production. They are payable whether or not a profit is made.

At last, because SDLT has different rate system for residential and non-residential property transactions by the time of 2013, we need to disaggregate it to residential and non-residential SDLT revenue respectively.

After adjusting the GVA categories for the Welsh SAM, it is straight forward to fulfil the GVA block roughly by the 3 aggregational categories: CoE, GOS, ToPN for each sector in Table 5.13. For those 2 new real estate ownership sectors split out from the 'Construction' sector, the GVA is disaggregated to them by the GVA components ratio of the whole construction sector as shown in Table 5.12. The residential rental sector is relatively special in terms of its GVA structure. Residential land input is the only GVA component of this sector, which is just the residual of actual gross residential rent (£2081.96 million)[5] minus total intermediate housing inputs (£528.91 million).

Table 5.12 Disaggregation of the New Ownership Sectors' GVA Components in Wales

Wales (£ million)	New residential ownership	New non-residential ownership	shared CoE/GOS/ToPN ratio with the whole constructin sector
CoE	215.27	561.28	26.976/15.87/1
GOS	126.62	330.13	
ToPN	7.98	20.79	

Table 5.13 Aggregated GVA of All Sectors in the Welsh SAM (£ million)

Sectors in 2013 SAM of Wales	GVA		
	CoE	GOS	ToPN
Agriculture, forestry & fishing	38.00	289.00	6.00
Mining & quarrying	78.00	129.00	7.00
Manufacturing	6074.00	2810.00	90.00
Energy & utility	859.00	1247.00	86.00
Construction (excl. new ownership)	1698.44	151.25	5.23
New residential ownership	215.27	126.62	7.98

New non-residential ownership	561.28	330.13	20.79
Wholesale & retail	4145.00	1066.00	308.00
Transport & storage	1367.00	387.00	56.00
Accommodation & food service	1123.00	452.00	83.00
Information & communication	891.00	507.00	43.00
Finance & insurance	963.00	1087.00	86.00
Residential rental		1553.05	
Real estate - agencies & management	223.58	730.66	1.43
Professional, scientific & technical services	1434.00	619.00	78.00
Administrative & support services	1262.00	427.00	36.00
Public administration & defence services	3003.00	1176.00	0.00
Education services	3820.00	477.00	12.00
Human health & social work services	4776.00	679.00	10.00
Arts, entertainment & recreation	565.00	154.00	21.00
Other services	929.00	142.00	14.00

Next, it is necessary to disaggregate each GVA categories to suit the categorical structure of Welsh SAM. The first step regards stripping the NICs paid by employers from the whole CoE in each sector. We obtain this by deducting the NIC paid by employees[12] from the whole NIC paid[13]. The whole NIC is calculated as a weighted average of the two neighbouring fiscal year: 2012/13 and 2013/14, with the weight 0.25 and 0.75 respectively. In fact, all the government expenditure and revenue data from Poole et al. (2016) are calculated as weighted averages this way because the 2012/13 fiscal year obviously covers only the first quarter of the calendar year 2013, and the 2013/14 fiscal year covers the remaining 3 quarters. The total NIC paid by employers is then calculated as approximately £2255.92m shown in Table 5.14. Divided by the total CoE of all sectors (£34146 million), the NIC payable by employers accounts for approximately 0.066 in Wales. Simply assuming this ratio is prevalent across all sectors, we can derive NIC payable by employers of each sector by multiplying this ratio with CoE of each sector. The rest of the CoE apart from NIC payable by employers remains labour income. This is shown in Table 5.20.

[12] See ‘Table A35 Detailed household expenditure by UK countries and regions, 2012 to 2014’ of ‘Family Spending – 2015: reference tables’ published by Office for National Statistics (2015b).
[13] See along the row of ‘National insurance contributions’ of ‘Table 2.3: Current Revenue: Wales 1999-00 to 2014-15’ in Poole et al. (2016).

Table 5.14 Split of NIC by Type of Payers (£ million)

Fiscal year	2012/13	2013/14
Weight	25%	75%
NIC	3743	3844
	3818.75	
NIC paid by employees	1562.83	
NIC paid by employers	2255.92	

We can also see from the table that employees contribute relatively less than employers. This is consistent with the contribution rate system in 2013, as although the NIC rate system is different across employers and employees and across different contribution categories, the employees' contribution rates are generally lower than those of employers, especially when we look at the major category A.

The second step is simply isolating the rental portion from GOS, and form it as non-residential land input factor for each sector. As the rental figures for each sector are already listed in the Regional GVA tables of Office for National Statistics (2015e), we do not need to estimate it except for those for the 2 new real estate ownership sectors. As these sectors are both split from the original construction sector, their land rental is estimated as the same with the ratio between rental and GOS of the construction sector. The estimation is shown in Table 5.15.

Table 5.15 Estimation of the New Ownership Sectors' Land Rental in Wales

Wales (£ million)	New residential ownership	New non-residential ownership	shared LR/GOS ratio
Land Rental (LR)	14.87	38.78	0.117
GOS	126.62	330.13	

The final step is to estimate the SDLT payment regarding each type of property transaction in the SAM. As the SDLT rate system is different for residential and non-residential property transactions, we need to calculate the two lines of total SDLT payment in calendar year 2013 in the first place. All SDLT receipts of the two consecutive fiscal years here in Table 5.16 are obtained from HMRC (2016)[14].

[14] See 'Section 3.2' of the 'Annual Stamp Tax Statistics 2015-16'.

Table 5.16 Split of SDLT Revenues (£ million)

Fiscal year	2012/13	2013/14
Weight	25%	75%
Residential SDLT	70	87.5
	83.125	
Non-residential SDLT	35	57.5
	51.875	

Given the total SDLT payment for both residential and non-residential property transactions, we can separate the SDLT payment further for new sale and resale of the two property types. To do this, the resale value of the second-hand properties as the tax bases have to be obtained first. Similar to the new residential ownership sector, the transaction volume of second-hand residential properties is also obtained from HM Land Registry (2015b)[15] listed in Table 5.17 below.

Table 5.17 2013 Second-hand Residential Sales in Wales

Overall average	Total sales	Detached	Sales	Semi-detached	Sales	Terraced	Sales	Flat/maisonette	Sales
£0.15m	33006	£0.22m	9733	0.14m	9969	£0.11m	11055	£0.11m	2249
Total: £4977.67m		£2125.15m		£1364.15m		£1230.47m		£257.91m	

The resale value of non-residential properties is calculated by the transaction volume of all non-residential property minus the new portion of it. The former is obtained from HMRC (2015a)[16] as £1915 million, while the latter is already obtained above as £1658 million[8]. Their difference then gives £257 million.

The SDLT is finally disaggregated by the proportional ratio listed in the bracket below each type of transaction as shown in Table 5.18. While residential new sale (£635.91 million) accounts for 11.33% of all residential property sale (£5613.58 million), the SDLT receipts from

[15] The dataset is downloaded using the standard reports application on the website landregistry.data.gov.uk. To configure this aggregated price data report from the website, follow the steps: Standard reports – create a standard report – Average prices and volumes – Region – Wales – Don't aggregate, just show the total – 2013 all year – Old properties only – Generate report – choose report format to download.
[16] See 'Section 6' of 'Annual UK Property Transaction Statistics'.

residential new sale is then assumed to account for the same proportion of SDLT receipts incurred from total residential sale (£83.125 million), which is estimated as £9.416 million. In a similar way, other SDLT receipts incurred from residential resale, non-residential new sale and resale are also estimated according to their underlying transaction proportions.

Table 5.18 Disaggregation of SDLT (£ million)

<table>
<tr><th colspan="2"></th><th>Gross Fixed Capital Formation</th><th>Resale Purchasers</th><th>Total</th></tr>
<tr><td rowspan="2">Residential</td><td>new sale</td><td>635.91
(11.33%)</td><td>0</td><td rowspan="2">5613.58
(100%)</td></tr>
<tr><td>resale</td><td>0</td><td>4977.67
(88.67%)</td></tr>
<tr><td rowspan="2">Non-residential</td><td>new sale</td><td>1658
(86.58%)</td><td>0</td><td rowspan="2">1915
(100%)</td></tr>
<tr><td>resale</td><td>0</td><td>257
(13.42%)</td></tr>
<tr><td colspan="2">Residential SDLT incurred</td><td>9.416
(11.33%)</td><td>73.709
(88.67%)</td><td>83.125
(100%)</td></tr>
<tr><td colspan="2">Non-residential SDLT incurred</td><td>44.913
(86.58%)</td><td>6.962
(13.42%)</td><td>51.875
(100%)</td></tr>
</table>

The locations to accommodate these SDLT figures in the SAM is discussed as follows, and the disaggregated GVA of all sectors is fully shown at the end of this sub-section in Table 5.20.

Firstly, the tax bases of the first 3 types of SDLT receipts £9.416 million, £73.709 million and £44.913 million are outputs of sectors ‘New residential ownership’, ‘Non-residential ownership’ and ‘Residential rental’ respectively. They are not included in the GVA block as they are not incurred from factor inputs, but rather incurred when the outputs are sold as commodities to meet final demand. As a result, these taxes are no longer paid by the sector producers, but should be paid by successors of the property transactions, in this case, the Households & NPISH and Gross Fixed Capital Formation. However, the payments of these taxes are not shown as directly paid by these two institutions, but realized by being included in the final price of the real estate commodities as their transfer costs. Therefore, these 3 SDLT receipts are placed on top of the output/commodity value of the 3 sectors and will be located along the corresponding commodity columns in the 'Product tax, SDLT' block of the SAM as shown in Table 5.4.

Such treatment is consistent with the GFCF-relevant concepts. According to the clause 10.49 of United Nations et al. (2009, p.200-201), 'Costs incurred on acquisition of an asset are treated as an integral part of the value of that unit's gross fixed capital formation'. In clause 10.52, 'All these costs of ownership transfer are treated as gross fixed capital formation'. Of all the costs of ownership transfer mentioned here, one of the kinds is listed in clause 10.51 as 'All taxes payable by the unit acquiring the asset on the transfer of ownership of the asset'. Obviously, these SDLT receipts just fall into this range.

Secondly, the figure of SDLT payment incurred from non-residential resale, £6.962 million, is further disaggregated for all sectors proportionately with their non-residential land factor inputs which are, as discussed earlier in sub-section 5.3.1, the proxy tax bases of non-residential-resale-incurred SDLT. In addition, these disaggregated SDLT figures are split out from the GVA component: Taxes less subsidies on production. According to the definition of taxes on production explained previously, they are compulsory taxes relating to the production and import of goods and services, the employment of labour, or the ownership or use of land, buildings or other assets in production. Therefore, these SDLTs belong to the case of 'the ownership or use of land', and are split out from Taxes less subsidies on production for all the sectors. This is shown in Table 5.19.

The only absences in Table 5.19 are sector 'Residential rental' and sector 'Public administration & defence services' which neither feature non-residential land input nor incur production taxes. For the sector 'Residential rental', the only GVA component is the land input which, however, is essentially residential land rather than non-residential land. This is basically the reward return for renting the dwellings and generating residential rental output, hence does not incur any SDLT payment. For the sector 'Public administration & defence services', it does not involve any land input at all, and involve no production taxes as well.

Table 5.19 Estimation of Sectors' Non-residential SDLT Input in 2013 SAM of Wales

£ million	Land Rental (LR)	shared ratio of SDLT - non-residential/ LR	SDLT - non-residential (1)	Taxes less subsidies on production (excl. SDLT) (2)	Taxes less subsidies on production (1)+(2)
Agriculture, forestry & fishing	50	0.00926	0.46	5.54	6
Mining & quarrying	2		0.02	6.98	7
Manufacturing	23		0.21	89.79	90
Energy & utility	8		0.07	85.93	86
Construction (excl. new ownerships)	117.35		1.09	4.14	5.23
New residential ownership	14.87		0.14	7.84	7.98
New non-residential ownership	38.78		0.36	20.43	20.79
Wholesale & retail	61		0.56	307.44	308
Transport & storage	16		0.15	55.85	56
Accommodation & food service	15		0.14	82.86	83
Information & communication	34		0.31	42.69	43
Finance & insurance	278		2.57	83.43	86
Real estate - agencies & management	6		0.05	27.95	28
Professional, scientific & technical services	23		0.21	77.79	78
Administrative & support services	37		0.34	35.66	36
Education services	4		0.04	11.96	12
Human health & social work services	13		0.12	9.88	10
Arts, entertainment & recreation	7		0.06	20.94	21
Other services	4		0.04	13.96	14
Total	752		6.962	991.038	998

The disaggregated GVA of all sectors for the SAM is presented in Table 5.20.

Based on the intermediate inputs and GVA figures, then we can acquire the sum of them along the columns as total industry expenditure for each sector. Since in a square SAM the column total equals the row total of each sector, this also gives gross regional output for each sector along the angle of rows. Deducting the exports to the RUK and ROW, the remaining of the regional output of each sector is the portion left to supply within the region. These regionally produced and regionally supplied goods are combined with imports from the RUK and ROW to form the composite goods available to the region.

Table 5.20 Disaggregated GVA of All Sectors in 2013 SAM of Wales (£ million)

Sectors in 2013 SAM of Wales	GVA					
	Compensation of Employees		Gross Operating Surplus		Taxes less subsidies on production	
	NIC (payable by employers)	Labour income	Land rental	Other value added	Taxes less subsidies on production (excl. SDLT)	SDLT - non-residential
Agriculture, forestry & fishing	2.51	35.49	50.00	239.00	5.54	0.46
Mining & quarrying	5.15	72.85	2.00	127.00	6.98	0.02
Manufacturing	401.29	5672.71	23.00	2787.00	89.79	0.21
Energy & utility	56.75	802.25	8.00	1239.00	85.93	0.07
Construction (excl. new ownerships)	112.21	1586.23	117.35	33.90	4.14	1.09
New residential ownership	14.22	201.05	14.87	111.75	7.84	0.14
New non-residential ownership	37.08	524.20	38.78	291.35	20.43	0.36
Wholesale & retail	273.85	3871.15	61.00	1005.00	307.44	0.56
Transport & storage	90.31	1276.69	16.00	371.00	55.85	0.15
Accommodation & food service	74.19	1048.81	15.00	437.00	82.86	0.14
Information & communication	58.87	832.13	34.00	473.00	42.69	0.31
Finance & insurance	63.62	899.38	278.00	809.00	83.43	2.57
Residential rental	0	0	1553.05	0	0	0
Real estate - agencies & management	22.73	321.27	6.00	148.00	27.95	0.05
Professional, scientific & technical services	94.74	1339.26	23.00	596.00	77.79	0.21
Administrative & support services	83.38	1178.62	37.00	390.00	35.66	0.34
Public administration & defence services	198.40	2804.60	0.00	1176.00	0.00	0.00
Education services	252.38	3567.62	4.00	473.00	11.96	0.04
Human health & social work services	315.54	4460.46	13.00	666.00	9.88	0.12
Arts, entertainment & recreation	37.33	527.67	7.00	147.00	20.94	0.06
Other services	61.38	867.62	4.00	138.00	13.96	0.04

5.3.4 The trade with RUK and ROW

In this sub-section, we will discuss the fulfilment of the blocks regarding the regional trade with extra-regional sectors. This covers the four blocks ‘Imports from RUK’, ‘Imports from

ROW', 'Exports to RUK', and 'Exports to ROW'. The Welsh exports are firstly discussed and then followed by the Welsh imports.

The exports in the Welsh SAM consists of two elements: exports to the rest of UK (RUK) and exports to the rest of world outside of the UK (ROW). As we are not able to find any official data for the exports to the RUK, these figures are directly referenced from the aggregated version of the table 'Input-Output Table 2013 Wales' utilizing the aggregated matrix method mentioned above in sub-section 5.3.2. As these data are not officially sourced, they are temporarily enclosed and will be open to balancing adjustment in a later stage.

The exports to ROW, on the contrary, are sourced officially from HMRC (2015b)[17]. However, the export figures are classified by the Standard International Trade Classification (SITC) section, rather than SIC 2007 section that is more consistent with the SAM sector aggregation level. SITC, maintained by the United Nations, is a classification of goods used to classify the exports and imports of a country to enable comparing different countries and years. Therefore, we need to map the SITC goods to the Welsh SAM sectors/commodities based on the output range details explained in hierarchical structure of SIC 2007. The mapping is shown in the table below.

Table 5.21 Classification Mapping of Welsh Exports to ROW (£ million)

Exports by SITC Section in 2013		Corresponding SIC 2007 Classification	
Food and Live Animals	305.36	39.83	C-10 Manufacture of food products
		265.53	A-01.4 Animal production
Beverages and Tobacco	9.30	9.30	C-11 Manufacture of beverages C-12 Manufacture of tobacco products
Crude Materials	52.91	52.91	B Mining & quarrying
Mineral Fuels	3887.50	3887.50	C-19 Manufacture of coke and refined petroleum products
Animal and Vegetable Oils	1.84	1.84	C-10.4 Manufacture of vegetable and animal oils and fats
Chemicals	1468.36	1468.36	C-20 Manufacture of chemicals and chemical products
Manufactured Goods	2076.84	2076.84	C Manufacturing
Machinery and Transport	6067.12	6067.12	C-28 Manufacture of machinery and equipment n.e.c. C-29 Manufacture of motor vehicles, trailers and semi-trailers C-30 Manufacture of other transport equipment

[17] See the tab 'WA' of 'UK Regional Trade in Goods Statistics: Quarter 4, 2014 Press Release'

Miscellaneous Manufactures	878.48	878.48	C Manufacturing
Other commodities	37.46	37.46	C Manufacturing

We can find in the table that most of the exports are manufactured goods, where only live animals and crude materials can be seen as raw materials without processing. An additional step to disaggregate the food and live animal goods has to be conducted before the mapping, since the food is manufactured good and live animals belong to the output of the 'Agriculture, fishing & forestry' sector. This is displayed in Table 5.22. The ratio splitting them is derived from the 2007 Welsh I-O Table produced in Bryan et al. (2010), because this table has a detailed structure of food and agriculture exports to ROW.

Table 5.22 Disaggregation of Food and Live Animals Exports (£ million)

Exports to ROW in 2007 I-O Wales		Ratios	Disaggregation	
Agric & fish	135.9	87.06%	265.53	Live Animals
Meat	7.2	12.94%	39.83	Food
Dairy	2.8			
Fish, vegetables, grain mill products	3.7			
Bread & biscuits	4			
Misc foods	1.3			
Confectionery	1.2			
Total	156.1	100%	305.36	Total

Finally, the Welsh exports to ROW is concentrated on the first 3 sectors as shown below.

Table 5.23 Exports to ROW in 2013 Welsh SAM (£ million)

Agriculture, forestry & fishing	265.53
Mining & quarrying	52.91
Manufacturing	14466.47

Given the exports and regional output of each sector, the regional demand for regional output can be obtained straight-forward that it equals the regional output exclusive of its portion demanded from outside of the region. Thus, the 'Regional demand for regional output' block is obtained simply by the sectors' row or column totals deducting their exports to RUK and

ROW. These figures are then located on the diagonal of the block, reflecting the assumption that one regional sector produces only one type of commodity to meet both regional and extra-regional demands.

Similar to the exports, the construction of Welsh imports part of the SAM also follows the same procedure. While the data of imports from RUK is directly referenced from the table 'Input-Output Table 2013 Wales', the derivation of imports from ROW is sourced from HMRC (2015b)[17] again and the process is shown in order below.

Table 5.24 Disaggregation of Food and Live Animals Imports from ROW (£ million)

Imports from ROW in 2007 I-O Wales		Ratios	Disaggregation	
Agric & fish	75.2	21.68%	91.72	Live Animals
Meat	101.2	78.32%	331.36	Food
Dairy	56.4			
Fish, vegetables, grain mill products	27.8			
Bread & biscuits	40.1			
Misc foods	41.4			
Confectionery	4.7			
Total	346.8	100%	423.08	Total

Table 5.25 Classification Mapping of Welsh Imports from ROW (£ million)

Imports by SITC Section in 2013		Corresponding SIC 2007 Classification	
Food and Live Animals	423.08	331.36	C-10 Manufacture of food products
		91.72	A-01.4 Animal production
Beverages and Tobacco	4.53	4.53	C-11 Manufacture of beverages C-12 Manufacture of tobacco products
Crude Materials	1580.05	1580.05	B Mining & quarrying
Mineral Fuels	320.39	320.39	C-19 Manufacture of coke and refined petroleum products
Animal and Vegetable Oils	7.71	7.71	C-10.4 Manufacture of vegetable and animal oils and fats
Chemicals	1396.42	1396.42	C-20 Manufacture of chemicals and chemical products
Manufactured Goods	1363.62	1363.62	C Manufacturing
Machinery and Transport	1828.94	1828.94	C-28 Manufacture of machinery and equipment n.e.c.

			C-29 Manufacture of motor vehicles, trailers and semi-trailers C-30 Manufacture of other transport equipment
Miscellaneous Manufactures	588.22	588.22	C Manufacturing
Other commodities	26.64	26.64	C Manufacturing

Table 5.26 Imports from ROW in 2013 Welsh SAM (£ million)

Agriculture, forestry & fishing	91.72
Mining & quarrying	1580.05
Manufacturing	5868.28

5.3.5 'Taxes on products' blocks

The taxes on products are an aggregation of those payable on product transactions and sales. They may include various duties and the VAT, among which the VAT is a major component. However, there is still a deficiency of the detailed official data on these taxes for each type of Wales-produced commodity. Thus, we reference these data directly from the aggregated version of the 'Input-Output Table 2013 Wales'.

In applying these data to the Welsh SAM, we should note that these taxes do not include those paid on real estate properties. In fact, all residential properties are exempt from VAT no matter whether they are freehold or leasehold. Besides, the sale or lease of a commercial property is also generally exempt from VAT except those freeholds built within three years. In this case, the seller can opt to tax if there is a need to neutralize VAT costs regarding, e.g., renovations or refurbishments taking place. However, opting to tax is not always appropriate as some businesses are unable to recover VAT incurred on costs. These are VAT adverse businesses and include businesses in financial & insurance, health, welfare and charitable sectors. Hence, for simplicity and manageability we assume no VAT charged on all property transactions.

Although there is no VAT incurred from the final demand on the real estate properties, the sales of these properties are still subject to SDLTs, which are displayed on top of the regional outputs and product taxes along the commodity columns in the SAM because they are not classified as part of VAT at all.

By far, we have settled with the whole supply side of the Welsh economy and the extra-regional demand side for the Welsh output. In particular, the column total of each sector describes the sectoral production process and it represents each sector's total output produced in Wales, which is also reflected by each sector's row total. Deducting the output exported for extra-regional demand, the combination of the imports, product taxes and the regional output supplies the commodities available for the whole Wales to meet both regional intermediate and final demand. In the following sub-sections, we will move to the construction of the blocks concerning how the factor income is allocated among institutions and agents and their regional expenditure on final demand subject to their income.

5.3.6 The allocation of GVA and tax revenues

In this sub-section, all the factor returns involved in GVA are allocated towards local final demand institutions: households, government and saving. This process therefore includes the construction of the 4 blocks of the schematic SAM in Table 5.4: 'Corporation tax', 'Household earning', 'Government factor revenue' and 'Government tax revenue' block. The allocation result is shown in Table 5.27. Here, all the tax revenues, including SDLTs and NICs, are centralized to the government, and all the labour income belongs to households.

Table 5.27 The Allocation of GVA and Tax Revenues (£ million)

	Compensation of Employees & Self-employed		Gross Operating Surplus		Taxes less subsidies on production (excl. SDLT)	Stamp duty land tax - residential	Stamp duty land tax - non-residential
	NIC (payable by employers)	Labour income	Land rental	Other value added			
Corporation tax				886.75			
Households & NPISH		31890.08	1931.46	854.77			
Government	2255.92		150.50	1939.75	991.04	83.13	51.88
Saving/Investment			222.80	7976.73			

The allocation of gross operating surplus is less straightforward. Its derivation starts with obtaining the government revenues from Corporation tax, other value added as well as land rental shown in Table 5.28. Relevant data in this table are sourced from Poole et al. (2016). The corporation tax is directly deducted from other value added which mainly represents the economy-wide corporation profits realized in one year, and although not described in Table 5.27, it is finally centralized to the government tax revenues in the full SAM.

Table 5.28 The Government Revenues from Corporation Tax, OVA and Land Rental (£ million)

Fiscal year	2012/13	2013/14
Weight	25%	75%
Corporation tax[18]	874	891
	886.75	
Other value added[19]	1933	1942
	1939.75	
Land rental[20]	161	147
	150.5	

The land rental allocated to households is then estimated as the whole households' rental payments deducting those made for the government. The figure of households' rental payment in Wales is derived again from Office for National Statistics (2014a)[5], which is totalled as approximately £2081.96 million. Deducting the £150.5 million receipts by the government, the rest of it, £1931.46 million, goes to the households as inter-households rent transfer. The remaining portion of the land rental, £222.80 million, is centralized into the social saving.

The other value added allocated to households, £854.77 million, is obtained from Office for National Statistics (2015c)[21]. Finally, the remaining portion of other value added apart from those allocated for corporation tax, households and government is left into social saving.

[18] It corresponds to the slot under the row 'Corporation tax' and column 'Other value added' in Table 5.27. For relevant data see along the row of 'Corporation tax (excl North Sea)' of 'Table 2.3: Current Revenue: Wales 1999-00 to 2014-15' in Poole et al. (2016).
[19] It corresponds to the slot under the row 'Government' and column 'Other value added' in Table 5.27. For relevant data see along the row of 'Gross operating surplus' of 'Table 2.3: Current Revenue: Wales 1999-00 to 2014-15' in Poole et al. (2016).
[20] It corresponds to the slot under the row 'Government' and column 'Land rental' in Table 5.27. For relevant data see along the row of 'Rent and other current transfers' of 'Table 2.3: Current Revenue: Wales 1999-00 to 2014-15' in Poole et al. (2016).
[21] See 'Table A41: Income and Source of Income by UK Countries and Regions' of 'Family Spending – 2015: reference tables' published by Office for National Statistics (2015c).

5.3.7 Households spending

Given the households income obtained from GVA, the households spending is then constrained by it and the remaining income forms the households saving. The households spending consists of 3 blocks in the schematic SAM: ‘Households consumption’, ‘Rental payment’ and ‘Income tax, NIC, Council tax’. The ‘Households consumption’ block refers to the households consumption on all the commodity bundles regionally available and the corresponding data information is given in Office for National Statistics (2015b)[12]. NIC payable by employees is also recorded in Office for National Statistics (2015b)[12]. The figures of Income tax[22] and Council tax[23] paid by households are from Poole et al. (2016) and adjusted with their calibre from fiscal years around 2013 to the calendar year 2013. Apart from the expenditure on commodities and taxes, the leftover forms households’ saving. Finally, the households spending structure is summarized below.

Table 5.29 Households Spending and Saving (£ million)

Agriculture, forestry & fishing	0
Mining & quarrying	0
Manufacturing	0
Electricity, gas, water supply & waste management	2271.40
Construction (excl. new ownership)	963.89
New residential ownership	0
New non-residential ownership	0
Wholesale & retail	12595.32
Transport & storage	549.64
Accommodation & food service	2198.55
Information & communication	1357.54
Finance & insurance	1267.73
Residential rental	2155.67
Real estate - agencies & management	26.19

[22] See along the row of ‘Income tax’ of ‘Table 2.3: Current Revenue: Wales 1999-00 to 2014-15’ in Poole et al. (2016).
[23] See along the row of ‘Council tax’ of ‘Table 2.3: Current Revenue: Wales 1999-00 to 2014-15’ in Poole et al. (2016).

Professional, scientific & technical services	13.24
Administrative & support services	1317.81
Public administration & defence services	0
Education services	735.06
Human health & social work services	311.24
Arts, entertainment & recreation	629.10
Other services	0
Income tax	4468.75
NIC (payable by employees)	1562.83
Council tax	1198.50
Saving/Investment	14781.35

5.3.8 Government fiscal structure

This sub-section includes the construction process regarding the 3 SAM blocks: ‘Government expenditure’, ‘Government transfer’ and ‘Central government transfer’. The first block accounts for the government current expenditure on the commodities in the level of regional government. The second block concentrates on the government transfer that is basically social protection. Both data sources are from Poole et al. (2016)[24] and all relevant data are adjusted for the calendar year caliber. Similar to the classification mapping of Welsh trade statistics, we also need to map the public sector current expenditure for Wales recorded in Poole et al. (2016) to the Welsh SAM sectors/commodities based on the hierarchical structure of SIC 2007. The mapping is shown in Table 5.30 below.

Table 5.30 Classification Mapping of Government Spending for Wales (£ million)

<table>
<tr><th colspan="2">Government spending for Wales in 2013</th><th colspan="2">Corresponding SIC 2007 Classification</th></tr>
<tr><td colspan="2">General public services</td><td rowspan="4">7821.5</td><td>O Public administration and defence;
Compulsory social security</td></tr>
<tr><td>Public and common services</td><td>643.25</td><td>O-84.11 General public administration activities</td></tr>
<tr><td>International services</td><td>431.25</td><td>O-84.21 Foreign affairs</td></tr>
<tr><td>Public sector debt interest</td><td>1787.75</td><td>O-84.11 General public administration activities</td></tr>
</table>

[24] See along the column of ‘Current’ of ‘Table 4.2: Total Expenditure: Wales, 2010-11 to 2014-15’ in Poole et al. (2016).

Defence	1471		O-84.22 Defence activities
Public order and safety	1354.5		O-84.24 Public order and safety activities
Economic affairs			O-84.13 Regulation of and contribution to more efficient operation of businesses
Enterprise and econ development	384.75		
Science and technology	0		
Employment policies	175.5		
Agriculture, forestry and fisheries	421.75		
Transport	479		
Environment protection	538.75		O-84.12 Regulation of the activities of providing health care, education, cultural services and other social services, excluding social security
Housing and community amenities	134		
Health	5933.5	5933.5	Q Human health and social work activities
Recreation, culture and religion	629	629	R Arts, entertainment and recreation
Education and training	3912.5	3912.5	P Education
Social protection	13727.5	13727.5	N/A (to Households & NPISH)

It is clear in the mapping table that most of the expenditure programmes are aggregated on the public administration services, while the social protection is the largest expenditure programme which is mainly transferred for households rather than on industrial services.

The last block 'Central government transfer' is particularly set up in accordance with the Welsh fiscal condition that tax revenues levied in Wales is insufficient to finance all the expenditure on Wales. In particular, according to Poole et al. (2016), Wales' current budget balance in 2013-14 fiscal year was a deficit of £13.4 billion, accounting for approximately 39.4% of the whole current expenditure for Wales, and such level of deficit had been fairly stable throughout years since 2010. The Welsh SAM we are developing here is consistent with this status that Wales needs net fiscal funding from the UK government on top of all the tax revenues contributed by Wales.

We also simply assume here that all the taxes levied from Wales that appear in the SAM are directly centralized to Welsh government, rather than as in reality most of them directly to the UK central government and being partly returned to Welsh government for spending on Wales. Based on the fiscal figures in the SAM, the net fiscal funding from UK government is calculated by total government spending deducting total government tax revenues as £10744.93 million. In the Welsh SAM, this figure is located as funds inflow for 'Government' from 'RUK'.

5.3.9 The 'Gross fixed capital formation' block

The Gross Fixed Capital Formation (GFCF) statistically measures the value of acquisitions of new or existing fixed assets of the business sectors, governments and households less disposals of fixed assets. It is called 'gross' because the measure does not make any adjustments to deduct the consumption or depreciation of fixed capital from the investment figures. As a component of GDP which measures the net new output added to the existing stock of wealth, the GFCF should definitionally include only newly produced fixed assets, not second-hand assets. In addition, the acquisition value of it should also include corresponding acquisition taxes and fees and measures 'all-up' costs of fixed investment.

The data in fulfilling the 'GFCF' block is multiply sourced. First, the total value of GFCF is referenced from Office for National Statistics (2016b)[25]. However, the detail investment structure in terms of each sector is not adopted from it. The reason behind lies in a fact that is likely mistaken. In an I-O or SAM table, the investment demand for the industrial sectors represents the purchase of capital asset products those sectors produce, rather than investment expenditure spent by those sectors. In fact, what Office for National Statistics (2016b) contains is investment expenditures from the sectors so those data do not fit to our purpose.

For example, the GFCF figure corresponding to the manufacturing sector in a SAM or I-O table represents the machinery and equipment products that the manufacturing sector produces. These products can be demanded by all sectors including the manufacturing sector itself. In the meantime, the investment expenditure of the manufacturing sector may include multiple types of capital goods ranging from buildings, factories and structures to intellectual property products produced by other sectors.

[25] See the tab 'GFCF NUTS1' of 'Regional GFCF 2000-2013'. The figure of Wales' total GFCF was disclosed initially as £10605.69 million, however it was revised and updated to £11763.55 million later in 'Regional GFCF 2000-2014', so the newer figure is adopted.

Therefore, while we only adopt the total GFCF value for the Welsh economy in Office for National Statistics (2016b), we refer to Office for National Statistics (2016a)[26] for the components of total GFCF. However, it is a UK level table and the capital asset types in the table have to be re-categorized to fit the SAM. Therefore, we firstly conduct the re-categorization of the capital assets. The mapping between the GFCF asset types and SAM commodities with their estimated values is shown below.

Table 5.31 Classification Mapping of GFCF (£ million)

GFCF categorized by asset	Corresponding commodities and values in SAM	
Transport equipment	6369.88	Manufacturing
ICT equipment and other machinery and equipment		
Dwellings	645.32	Construction - new residential ownership
Other buildings and structures and transfer costs	1702.91	Construction - new non-residential ownership
	716.61	Construction (excl. new ownership)
Intellectual property products	2328.67	Professional, scientific & technical services
Total	11763.55	

In estimating the GFCF values across different asset types, the first settled is new dwellings which obviously corresponds to the new residential ownership sector. As the only final demand component, the value of the new residential properties formed as fixed capital in year 2013 just equals to its transaction value, or sector output, including the SDLT incurred as the transfer cost. Similarly, all new non-residential properties transacted with SDLT involved also form GFCF demand for the new non-residential ownership sector, which is part of 'Other buildings and structures and transfer costs' under the GFCF asset categories. The two figures are calculated as £645.32 million and £1702.91 million[27], both of which also balance their corresponding sectors in the SAM.

The other part of 'Other buildings and structures and transfer costs' corresponds to the rest of the new construction output apart from those new residential and non-residential buildings. The value is derived again from the term 'All new work' in Office for National Statistics (2015d)[8],

[26] See the last tab 'G16 CP NSA A levels and %' of 'Gross fixed capital formation – by sector and asset'.
[27] See Table 5.18, £645.32m = £635.9m + £9.42m, and £1702.91m = £1658m + £44.91m

deducting the figures of the new residential and non-residential GFCF above. It is calculated as £716.61 million.

The next type of GFCF estimated is intellectual property products that are produced by professional, scientific & technical services in Wales. The value is estimated as £2328.67 million, which is proportionately consistent with its counterpart in the UK. It is shown in Table 5.32.

Table 5.32 Estimation of Intellectual Property Products in GFCF of 2013 SAM of Wales

£ million	Wales	UK	shared IPP/GFCF ratio
Intellectual property products (IPP)	2328.67	64407	0.229
GFCF	11763.55	280224	

The last estimated GFCF type is manufacturing corresponding to transport equipment, ICT equipment and other machinery and equipment. This value is simply estimated as the residual of the total GFCF deducting the values of all real estate and intellectual properties, which is £6369.88 million and occupies the largest portion in the whole GFCF.

5.3.10 Balance of trade

This sub-section deals with the last bits in constructing the whole preliminary SAM: the balance of trade with RUK and ROW. As illustrated in sub-section 5.3.4, the difference between extra-regional exports/monetary inflow and imports/monetary outflow settles in the row for ‘Saving/Investment’ account in a SAM, representing the net foreign saving as part of the whole regional saving.

In terms of the trade with ROW, the Welsh economy was in an absolute current account surplus status that its exports succeeded its imports. In this case, the net foreign saving is negative, and calculated as -£7245 million. For the monetary flow with RUK, however, it is more complicated. If we focus on the trade itself, the Welsh imports exceeded its exports to RUK based on the unofficial data, making its current account deficit. Nevertheless, there is another source of monetary inflow transferring from UK central government as discussed in sub-section 5.3.8, which is the net fiscal transfer to support Welsh government spending. Taking this into account, the total monetary inflow would then exceed the outflow to RUK and the corresponding net foreign saving would be negative.

This figure balancing monetary flow with RUK is rather essential in balancing the whole SAM table, because it simultaneously determines whether the saving-investment balance identity is finally achieved. As this figure locates in the intersection of the row account of Saving/Investment and the column account of RUK, it is highly difficult to reach balance for the two accounts simultaneously in this last step of fulfilling a preliminary SAM. Normally when the RUK account reaches balance, the figure could lead to saving unmatched by investment, and vice versa.

In this section, we do find that the negative balance of trade with RUK does not make even between saving and investment. As in this stage we cannot make even of the RUK and Saving/Investment accounts simultaneously, we temporarily leave the figure with the value able to balance Saving/Investment account but unable to balance RUK account. After all, all the data regarding trade with RUK are unofficially sourced and open to further adjustment towards table balance, which may finally give chance to let the figure balance the RUK account after balance.

To summarize, the raw SAM table we have constructed by now is an unbalanced table. The unbalanced accounts concentrate on those commodities accounts and the RUK account. The whole table needs to be further adjusted with balancing methods so that all accounts reach balance. We will discuss the balancing details in the following sections.

5.3.11 Data issues

Before the end of this section, this sub-section summarizes the data issues in terms of their availability and quality in order for us to realize the limitation of this SAM as the model foundation, and gain some insight into possible works subject to relevant data improvement in the future.

Table 5.33 summarizes the datasets used in developing the SAM and their corresponding references. Each data source is rated by different colours according to their availability and reliability. The dark green colour marks the datasets with highest reliability and available for direct application without obvious defects. The light green colour implies less reliability with various reasons and being subject to further improvement. The yellow colour marks the datasets with the lowest level of data integrity or feasibility and subject to discretion in use.

Table 5.33 Data Sources Involved for SAM Development and Quality Rating

Dataset	Reference	
Regional Gross Value Added (Income Approach): reference tables	Office for National Statistics (2015e)	
Annual Business Survey – 2013 Regional Results	Office for National Statistics (2015a)	
Family Spending – 2014: Table 2.5: housing expenditure by UK countries and regions, 2013	Office for National Statistics (2014a)	
Agriculture in the United Kingdom 2014	Department for Environment, Food and Rural Affairs et al. (2015)	
Input-Output Table 2013 Wales	Unpublished, from Dr. Andrew Crawley of Welsh Economy Research Unit	
Output in the Construction Industry	Office for National Statistics (2015d)	
Customized Standard Reports from Land Registry	HM Land Registry (2015a; 2015b)	
United Kingdom Input-Output Analytical Tables, 2010	Office for National Statistics (2014b)	
Family Spending – 2015: Table A35: detailed household expenditure by UK countries and regions, 2012 to 2014	Office for National Statistics (2015b)	
Government Expenditure and Revenue Wales 2016	Poole et al. (2016)	
Annual Stamp Tax Statistics 2015-16	HMRC (2016)	
Annual UK Property Transaction Statistics	HMRC (2015a)	
UK Regional Trade in Goods Statistics: Quarter 4, 2014 Press Release	HMRC (2015b)	
Family spending – 2015: Table A41: income and source of income by UK countries and regions, 2012 to 2014	Office for National Statistics (2015c)	
Regional GFCF 2000-2013	Office for National Statistics (2016b)	
Gross Fixed Capital Formation – by sector and asset	Office for National Statistics (2016a)	

Among all the datasets with dark green marks, Office for National Statistics (2014b; 2016a) only focus on data with UK calibre and hence cannot be directly adopted. They are only used for indirect estimation for relevant regional figures. Their data quality and reliability for purpose of studying relevant UK topics, however, is already adequate. For the regional level, Office for National Statistics (2015e) is a pillar dataset available for UK countries and regions. Regional GVA values presented are available with GVA components, and this provides a solid foundation for constructing a regional I-O or SAM table.

Other dark green level datasets mainly concentrate on real estates and fiscal data, apart from one report for agriculture data. The report of Department for Environment, Food and Rural Affairs et al. (2015) provides complete and detail quantitative description of the UK agriculture sector. It also provides a simple presentation of main output variables in terms of UK countries, which largely compensates the non-disclosed parts of data from Office for National Statistics (2015a).

For the construction and real estate data, Office for National Statistics (2015d) focuses on construction data in a quarterly rolling way in terms of UK regions. It provides detailed data classified with new and maintenance works. It would be much more applied for relevant studies if it can provide data classifications consistent with SIC2007. HM Land Registry (2015a; 2015b) are datasets based on the property transaction registrations with HM Land Registry. These datasets simply summarize the first-hand raw data and hence of high reliability.

Poole et al. (2016) is a fiscal data source under the dark green level, which focuses on the detailed presentation of Wales' fiscal status. It covers both revenue and expenditure data of the public sector in Wales, however it is not published annually, or even periodically, for future possible reference.

For those light green levelled data, HMRC (2015a; 2016) both provide comprehensive information of property transactions available for various types of classification, while HMRC (2016) also provides SDLT receipts incurred from relevant transactions. HMRC (2015a) is available by type of property and price range, and by type of buyer and price range across calendar years, however they are only classified to 2 big regions: Scotland, and England & Wales. There is no such classifications of data specifically for Wales alone. The only available classification for Wales in HMRC (2015a) is by type of property, however it does not list the SDLT receipts incurred from the transaction of each type of property. HMRC (2016), on the other way, provides such data classification and incurred SDLT receipts, but only available for fiscal years. Estimation has to be made to adjust to calendar years. Similar to HMRC (2015a), it is also available by type of property and price range, and by type of buyer and price range, however, again, such classifications are only available for fiscal years and not available for UK regions. Besides, the price ranges set for transactions are relatively too much biased towards high prices where much fewer transactions happened compared to lower price ranges. The classification of type of buyer in this dataset is even more aggregated than HMRC (2015a) and hence is of very little use. In summary, they are reliable data sources, however, the way they present the data significantly limits their value to be applied in regional modelling around real estate topics. A more disaggregated segmentation of the type of buyers, more reasonable price ranges presented and more classifications available to UK countries and regions would strongly aim the relevant regional studies.

Office for National Statistics (2014a; 2015b; 2015c) are all datasets from Family Spending surveys. These datasets are based on survey samples of households and hence there will inevitably be biases against true population values. Besides, the housing expenditure items commonly listed in Office for National Statistics (2014a; 2015b) clash with each other, and this may be one of the consequences of sampling bias. Therefore, these datasets are levelled with light green. In practice, it would be more convenient if the classification of consumption items in Office for National Statistics (2014a; 2015b) follows SIC2007, and the classification of income sources in Office for National Statistics (2015c) is more disaggregated.

The 'Input-Output Table 2013 Wales' is only graded as the middle level simply as it is an unpublished piece of work without further academic scrutiny, therefore this dataset is only used as backup source for those table values without more reliable reference sources. HMRC (2015b) provides trade statistics by UK regions, however it only covers trade data of each region with other areas in the world but no data for inter-regional trade flows.

Office for National Statistics (2015a) is used mainly for estimating intermediate inputs of Welsh industries, however it only covers non-financial sectors which only accounts for approximately two thirds of the UK economy in terms of GVA, not to mention there are many parts of data are suppressed to avoid disclosure. Besides, the GVA data of Welsh sectors it provides also frequently clash with other sources including regional GVA tables of Office for National Statistics (2015e). As the regional GVA table is made specifically for GVA figures, we reference Office for National Statistics (2015e) for GVA values of each Welsh sector, and only reference Office for National Statistics (2015a) for intermediate inputs when any better source is not yet available.

Another yellow levelled dataset falls on Office for National Statistics (2016b) for two reasons. One is that it may mislead users. What it includes is actually the investment demand of each sector, rather than investment goods that each sector produces as fixed capital formation. An extra transpose matrix is necessary to convert to each other (investment demands and investment goods). To construct an I-O or SAM table, the gross fixed capital formation classified by the output of investment goods is highly needed. Second, as it is noted in the dataset, the data included ‘are not official UK statistics and should only be regarded as estimates’, and there are ‘serious concerns over the quality of source data used to compile these estimates’.

In summary, the data sources referenced are diversified, however these sources can hardly be seen as adequate to construct a regional SAM table without estimation, especially when some sectors need to be disaggregated. Trade statistics regarding inter-regional exports and imports is always absent, while this is a key component in a SAM for a small open regional economy. Depending on the current status of data availability for a regional economy, necessary compromise has always to be made between aggregation level and how often the estimation is needed. More comprehensive and officially available regional datasets can significantly reduce the scale of data estimation and hence further enhance the reliability and quality of a regional SAM and the CGE modelling embedded.

5.4 SAM balancing methods

In this section, we discuss firstly what the balancing procedure means and why it is necessary, then how it can be implemented through introducing several balancing methods that could be technically and manually operated. Lastly, an ultimately balanced SAM for Welsh economy in 2013 will be presented following the procedure.

5.4.1 Principles of SAM balancing

As a SAM table is normally a square matrix, the total number of rows equals the total number of columns. Suppose the total number of rows or columns is noted by *n*, and each of the matrix elements, or each of the cell of the SAM, is noted by Q_{ij} , then the mathematical expression of the SAM Q is then presented as followed:

$$Q = [Q_{ij}], \quad i = 1, \dots, n, \quad j = 1, \dots, n \tag{5-11}$$

Since that the row and column number equal is due to the fact that each institutional account has both row and column accounts, the SAM structure as a whole has to follow the social accounting rule of double entry, that balance holds between debit and credit. Therefore, as same in an I-O table, the total of the figures in a row has to equal that of the corresponding column. Mathematically, it holds as:

$$\sum_i^n Q_{ik} = \sum_j^n Q_{kj}, \quad \forall\ k \in i, j \tag{5-12}$$

However, in the practice of development of a SAM table, it is usually unbalanced after the table is filled with original data from various sources. Because the statistical calibre and quality, publication stage (whether provisional or revised), whether official or not, and data estimation methods applied may all differ across different sources, it is then not surprising that table is not automatically balanced after all data pooling together. Hence, as the final step of constructing a SAM table, it is necessary to revise and calibrate the filled raw figures so as to equate row and column totals of each institutional account. Such process is just SAM balancing.

There are several methods to achieve a balanced SAM. Those frequently used include RAS method, least squared method, direct entropy method, as well as manual adjustment. In fact, the balancing process normally not only depends on technical devices but also relies on researchers' experience, judgement and the understanding of the data and the model behind. So it is also a reasonable practice that they are applied combining together. In the following sections, both technical and manual methods are discussed and compared to help derive a balanced Welsh SAM based on our raw data acquired.

5.4.2 RAS method

RAS method is a popular technique capable of working with both I-O and SAM tables. It is also known as a bi-proportional matrix balancing technique which best describes its working mechanism. This technique can be applied not only on balancing but also on regionalizing from a table of a larger geographical scale and updating tables given new data information. The realization of these functions are all based on its mechanism characteristic that the adjustment process is proportionally evolved in both row and column directions across the entries. That how the method works in detail is shown below.

Initially the RAS method was utilized on balancing I-O tables that the procedure focuses on adjusting the intermediate input matrix – the conventionally noted A matrix. Suppose the unbalanced or original table's intermediate matrix is denoted as *A0* and the balanced or targeted one by *A1*. The purpose of the method is to finally equate the row totals and column totals of *A0* with their targeted counterparts in *A1*. In this context, the row totals are defined as values of total inter-industry sales of each row account and the column totals as total inter-industry purchases by each column account, and they are normally already predetermined before balancing by data from a more recent time period, a more down-scaling geographic area, or just more reliable publication sources.

The first step is to start adjusting arbitrarily from one direction, and we choose to start from the rows. If the target values of row totals differ from those of the original ones, we could multiply the ratio of target values divided by original values to each of the figures in the original rows. Denoting the targeted *ith* row total of *A1* as $\overline{A1}_i$, and the superscripts as the balancing step orders, then

$$A0_{ij}^{1} = A0_{ij} \frac{\overline{A1}_i}{\sum_j^n A0_{ij}} \quad \forall\, i = 1, \dots, n \tag{5-13}$$

This is the logic of the row adjustment. After this first step, the row totals of the matrix must have been equated to the target values, but the column totals may still probably be biased from target column totals. Then the second step is to adjust toward the columns. Similarly, if we denote the targeted *jth* column total of *A1* as $\overline{A1}_j$, then

$$A0_{ij}^{2} = A0_{ij}^{1} \frac{\overline{A1}_j}{\sum_i^n A0_{ij}^{1}} \quad \forall\, j = 1, \dots, n \tag{5-14}$$

The third step is to redo from rows as the row totals may have diverged again against the target values after column adjustment. Then the process continues until we reach the target matrix from both row and column directions and the divergences drop into an acceptable minimal range. Suppose the convergence procedure takes *p* iterated steps to hit the targets, then finally it is:

$$A0_{ij}^{p} = A1_{ij} \tag{5-15}$$

and

$$\sum_{i}^{n} A0_{ij}^{p} = \overline{A1}_{j}, \;\; \sum_{j}^{n} A0_{ij}^{p} = \overline{\mathrm{A1}}_{i} \tag{5-16}$$

Here if we use r_i^1 to denote the *ith* row adjustment ratio in the first step, and we use s_j^2 to denote the *jth* column adjustment ratio in the second step, then the mathematical expression can be simplified to:

$$A0_{ij}^{1} = A0_{ij} r_i^1 \tag{5-17}$$

$$A0_{ij}^{2} = A0_{ij}^{1} s_j^2 \tag{5-18}$$

Additionally, further simplification of notation can be made through writing with matrix form. Suppose we have the diagonal matrix R^1 made up of the ratio r_i^1 in *ith* row, S^2 made up of s_j^2 in *jth* column, and the superscripts still refer to the steps in the procedure, then the first step is equivalent to be written as pre-multiplying *A0* matrix by R^1, and the second step as post-multiplying further by S^2:

$$\mathbf{A0^2} = \mathbf{R^1 A0\ S^2} \tag{5-19}$$

The following steps are given by:

$$\mathbf{A0^4} = \mathbf{R^3 A0^2\ S^4} = \mathbf{R^3 R^1\ A0\ S^2\ S^4} \tag{5-20}$$

$$\mathbf{A0^6} = \mathbf{R^5 A0^4\ S^6} = \mathbf{R^5 R^3 R^1\ A0\ S^2\ S^4\ S^6} \tag{5-21}$$

.

.

.

$$\mathbf{A0^{p} = A1} \tag{5-22}$$

Ignoring the superscripts and the numbers (0 and 1) denoting the states before and after adjustment, we have 'RAS' in the procedures and this is just the origin of the method name.

In fact, the method has currently not been limited to balancing intermediate transaction matrix, but also been extended to the whole I-O tables given target row totals including final demand and column totals including GVA of each account. To this point, it could also be applied to balance a SAM table as long as the target value of row and column totals are known. The mathematical expression of SAM table Q is then shown as:

$$\mathbf{Q0^{2} = R^{1}Q0\,S^{2}} \tag{5-23}$$

$$\mathbf{Q0^{4} = R^{3}Q0^{2}\,S^{4} = R^{3}R^{1}\,Q0\,S^{2}\,S^{4}} \tag{5-24}$$

$$\mathbf{Q0^{6} = R^{5}Q0^{4}\,S^{6} = R^{5}R^{3}R^{1}\,Q0\,S^{2}\,S^{4}\,S^{6}} \tag{5-25}$$

.

.

.

$$\mathbf{Q0^{p} = Q1} \tag{5-26}$$

While most researchers view the RAS method as a pure mathematical technique, such technique, by Stone (1961), is endowed with economic implications. The row adjustment process is described as reflecting the economic phenomena of 'substitution effects' and the column adjustment process as 'fabrication effects'. The former refers to the substitution between production inputs of an economic sector, as some inputs have to be adjusted larger along the row they dwell in to meet the target row total, while other inputs may have to be shrunk towards smaller row totals and the input structure is then changed towards the balanced or updated table. The latter term accounts for the expansion or contraction of the total intermediate inputs when some economic sectors have to increase the scale of intermediate inputs while others do the contrary to meet new targets. This is a common phenomenon when industrial sectors depend more on high technological capital equipment and/or skilled labour over time, consequently the GVA occupies more weight in producing one unit of product. In fact, the fabrication effect can, to some extent, be considered as another type of substitution effect – this time between intermediate inputs and value added inputs. Along the process of balancing, updating or regionalizing, the adjustment changes are assumed to be distributed throughout the economy to fit the original data structure into the new economic context.

The advantages of this method lie in the reasonability of balance starting from the proportional relations between the table figures, and the capability of flexible application in non-square rectangular matrices. However, it also features limitation that it may not converge even after a large number of iterations. Such situation often occurs in a matrix containing too many zero entries, so the entire uniform proportional adjustment burden is forced onto the remaining non-zero entries. Depending on the aggregation level of the table and the relative locations between zeros and non-zeros, the balancing task may not always be accomplished purely with this technique. Besides, most significant among others, it requires known target values for each row and column total, which may not be the case in practice of balancing. Therefore, to accomplish the balancing task, it is common to seek other methods as complementarity. The GAMS code conducting RAS method is presented in Appendix IV-3.

5.4.3 Least squares method

Implied by the name, the least squares method is based on the same principle with the OLS econometric regression, only different on application objects. If we treat the figures in the SAM analogous as sample points for regression, then the balanced figures are just equivalent to estimated sample values that deliver the minimal sum of squared divergences. The objective function of this method is therefore formed. Additionally, there is always the constraint that has to be posed to ensure the balance: the row total equals column total for each account, and that is equation (4-12) brought in again.

Denoting the squared sum of divergence between balanced and unbalanced values as z, the balancing problem is solved below as:

$$\min_{Q1_{ij}} z = \sum_i^n \sum_j^n (Q1_{ij} - Q0_{ij})^2 \tag{5-27}$$

$$\text{subject to} \quad \sum_i^n Q1_{ik} = \sum_j^n Q1_{kj} \quad \forall\, k \in i, j \tag{5-28}$$

$$and \quad Q1_{ij} \geq 0 \qquad i = 1, \dots, n \quad j = 1, \dots, n \tag{5-29}$$

In circumstances that figures in the SAM are heavily divergent in terms of their scales, the objective function could also be applied in the following form:

$$\min_{Q1_{ij}} z = \sum_i^n \sum_j^n (\frac{Q1_{ij}}{Q0_{ij}} - 1)^2 \quad (5\text{-}30)$$

The logic of least squares method is straight-forward, however there is no underlying theoretic consistence or economic implications like in RAS method. Although the objective is explicit, it remains obscure in terms of the adjustment process, and what happens to the figures in the table is somewhat unpredictable. Based on the non-proportional operating mechanism, by itself we cannot control the emergence of zero and non-zero entries. Some entry values may be removed while some zero entries turn out to be positive when updated or balanced. Hence, manual interventions have sometimes to be imposed in order to maintain the original data structure, and such additional interventions could well be miscellaneous and requires much extra coding work. The GAMS code for this method is shown in Appendix IV-1.

5.4.4 Direct cross-entropy method

This method is a popular modern technique in use of balancing SAM. It is developed through reference to statistics and informatics, etc. In information economics, the information entropy is used as an indicator measuring the information intensity resulted from new information. Assume we have a prior probability distribution of an event as $\mathrm{p} = (p_1, \dots, p_n)$, and a posterior distribution as $\mathrm{s} = (s_1, \dots, s_n)$ after the arrival of a new piece of information, then the expected entropy intensity of this new message is:

$$\mathrm{z} = \sum_i^n s_i \log \frac{s_i}{p_i} \qquad 0 \le p_i \le 1, 0 \le s_i \le 1;\ \sum_i^n p_i = 1, \sum_i^n s_i = 1 \quad (5\text{-}31)$$

If each couple of prior and posterior probabilities are identical, then the expected entropy intensity is zero, implying the incoming message brings nothing new information. On the contrary, the more divergence lies between them, the larger the expected entropy is. Similarly, just like the least square method connecting with econometric OLS, each figure in the unbalanced original SAM can also be analogously mapped to the prior probability, whereas balanced figures correspond to posterior probabilities. From this point of view, Robinson et al.

(2001) applied this expected entropy method as method for balancing SAM. The principle is to minimize the expected entropy value of the entries before and after balance, in order to make the figures after balance as close as possible to the original values while the balance condition is still met.

Before conducting the SAM balancing problem, we have to convert the entry values from absolute value to ratios against the total of all the figures in the SAM. We can label the value of whole total as *H*, then the unbalanced one is denoted as *H0* and *H1* for balanced one.

$$H0 = \sum_i^n \sum_j^n Q0_{ij} \qquad H1 = \sum_i^n \sum_j^n Q1_{ij} \tag{5-32}$$

The ratio of each SAM entry is then:

$$a0_{ij} = \frac{Q0_{ij}}{H0} \qquad a1_{ij} = \frac{Q1_{ij}}{H1} \tag{5-33}$$

The objective function is then derived as below:

$$\begin{aligned}
\text{z} &= \sum_j^n \sum_i^n a1_{ij} log \frac{a1_{ij}}{a0_{ij}} \\
&= \sum_j^n \sum_i^n \frac{Q1_{ij}}{H1} \log\left(\frac{Q1_{ij}}{H1} \Big/ \frac{Q0_{ij}}{H0}\right) \\
&= \frac{1}{H1} \sum_j^n \sum_i^n Q1_{ij} \left[\log\left(\frac{Q1_{ij}}{Q0_{ij}}\right) + \log\left(\frac{H0}{H1}\right)\right] \\
&= \frac{1}{H1} \sum_j^n \sum_i^n Q1_{ij} \log\left(\frac{Q1_{ij}}{Q0_{ij}}\right) + \frac{1}{H1} \sum_j^n \sum_i^n Q1_{ij} \log\left(\frac{H0}{H1}\right) \\
&= \frac{1}{H1} \sum_j^n \sum_i^n Q1_{ij} \log\left(\frac{Q1_{ij}}{Q0_{ij}}\right) + \log\left(\frac{H0}{H1}\right)\left(\frac{1}{H1} \sum_j^n \sum_i^n Q1_{ij}\right)
\end{aligned} \tag{5-34}$$

As the term $\frac{1}{H1} \sum_j^n \sum_i^n Q1_{ij}$ equals one by definition and hence cancel out, the direct cross-entropy method is to solve:

$$\min_{Q1_{ij}} z = \frac{1}{H1} \sum_{j}^{n} \sum_{i}^{n} Q1_{ij} \log\left(\frac{Q1_{ij}}{Q0_{ij}}\right) + \log\left(\frac{H0}{H1}\right) \tag{5-35}$$

$$\text{subject to} \quad \sum_{i}^{n} Q1_{ik} = \sum_{j}^{n} Q1_{kj} \quad \forall\, k \in i, j \tag{5-36}$$

$$and \quad Q1_{\mathrm{ij}} \geq 0 \qquad i = 1, \ldots, n \quad j = 1, \ldots, n \tag{5-37}$$

Now the function resembles the expected entropy function except for an additional rear term $\log(H0/H1)$. For this term we have to add some further constraint because for *H0*, it is a constant given number representing the total sum of all the unbalanced SAM entries, while for *H1*, it could tend to move towards infinity in the process of minimization adjustment. An ad hoc solution for this is to limit the value of the term $H0/H1$ within a reasonable range. In the present thesis we choose it to be $[0.5, 2]$, implying the total sum of all the entries after balance is set double size the value before balance in maximum and half the size in minimum. It is a rather wide range as can be imagined that the ultimate divergence should turn out to be much smaller. Finally, for the last step, additional conditions may have to be involved to ensure unchanged table structure as like introduced in least square method. The GAMS code for this method is shown in Appendix IV-2.

5.4.5 Imposition of additional constraint conditions

As mentioned in the previous sub-sections, we may have to intervene the non-RAS methods by imposing additional conditions so as to maintain the table structure and some critical economic relations between accounts. Such procedure in most occasions requires understanding of economic theories and judgement of actual data status such as quality and reliability, and utilizing all the relevant information obtained.

Sometimes, the data value of the same account may vary across different statistic sources and even within the same source, vary across different acquisition or estimation approaches. For example, GVA data is published by both ABS and ONS, and ONS delivers two versions based on income approach and production approach respectively, while the latter is experimental statistics that would require user's discretion. ABS delivers estimated GVA and is the only option for total intermediate inputs of each sector. Hence the adoption of income based GVA has priority over the other two and should be bound fixed when conducting balancing method.

In the meantime, the intermediate inputs value may be endowed more flexibility on whichever total sums or input structure when balancing.

Trade data is normally reliable whereas inter-regional information always suffers scarcity and may set open to full adjustment. Figures of tax and other income as government revenue sources is definite while expenditure figures are relatively less easy to be classified and distributed clearly, especially for regional economies. Household consumption expenditure data is usually unbalanced with and more reliable than income data, especially in emerging economies where the grey income accounts for some portion that is not reckoned in official statistics. In this case, that which side dominates, or choose a somewhat mediate value remains under judgement. Investment demand is less accurate than household and government demand figures and may be subject to adjustment when there are not enough figures to bear the adjustment burdens.

In summary, purely relying on the technical methods may not be enough to achieve a reasonable and logical balanced SAM or I-O style table, we may also need extra practical experience and information analysis to make judgement and adjustment, and impose corresponding constraint conditions inside the balancing programmes. In few cases, we may also resort to manual adjustment in the balancing process, which is discussed in the following sub-section.

5.4.6 Manual balancing method

This last method we refer to is normally used on the original table where there is not much discrepancy between row and column totals, and there are always relatively ‘unimportant’ figures on the right locations to be adjusted. Just because this, different researchers may bear different views on the relative importance rating based on their experience and perception, and such subjective factor may finally result in different balanced tables from the same unbalanced one.

The manual balancing somehow relies on identifying less important figures to bear the adjustment burden. Sometimes it is relatively easy to identify based on sufficient data information, while sometimes the ‘unimportant’ figures are just arbitrarily located only because they are large to absorb relatively small adjustment. The procedure normally starts from the first row and column. First we should work out the discrepancy between the row total and the column total, and decide from which side (row or column), or both, to pick up the entries pending adjustment towards the discrepancy. The discrepancy could be wholly allocated to one entry or proportionally distributed to a series of entries, but not to the intersection entry of the row and column as otherwise the discrepancy still remains. The adjustment in first step will result in changes of the entry values of the rest part of the table – it could be better towards balance, or probably not. Suppose it is not and we proceed to second step with the same principle on the second row and column, until they are balanced. Note that all entries of rows and columns in previous steps will be exempted from further adjustment in later steps, unless we spot errors and have to redo the process. So it is obvious that the number of entries available for adjustment drops fast along the process. The process continues until the penultimate row and column, and after their balance, it will leave the last entry unnecessary to change to promise a whole balanced table.

An alternative way of manual balance is to find out two pairs of rows and columns every time who present the largest and second largest discrepancies with opposite signs in each step. The strategy is to identify the two intersected entries across different pairs and decide which one, or both, to be adjusted towards the discrepancies. Sometimes the intersections may refer to entries that are originally blank and hence unable to change, in this case we should seek at least one intersection being originally economically meaningful and non-zero. This way could enable both discrepancies to shrink simultaneously from opposite directions. The rest discrepancy portion that is not cancelled out can be left to form pairs with other opposite signed discrepancies. The process can continue until no opposite signed pair remains, and if there are still rows and columns left for balance, we can resort to the previously mentioned alternative way.

Next, we draw a simple example to show the second way of manual balance. Note that the example table does not represent the real SAM table structure we develop, but is only an abstract one for illustration convenience. Suppose we have the unbalanced Table 5.34 as shown below.

We can see from Table 5.34 that the discrepancies between row totals and column totals have both positive and negative numbers. We proceed to pick up the largest discrepancies in absolute values and with opposite signs to start, which are 65 of Sector2 account and -60 of Household account. To making both to shrink simultaneously, we need to identify the two intersections across different accounts, which are (Sector2, Household) and (Household, Sector2). We do not need to manipulate the other two intersections (Sector2, Sector2) and (Household, Household), as they don't offer the opportunity to shrink the discrepancies simultaneously. We could either decrease the number 95 in (Sector2, Household), or increase the vacant cell in (Household, Sector2). However, in this case there is no economic meaning for the entry (Household, Sector2) as industrial production activities do not directly pay households, but delivers factor payment which is then transferred to Households. So we cannot change the vacant cell and only decrease the number 95 by 60. As a result, the discrepancy of Household account is eliminated and the discrepancy of Sector2 shrink to 5 as shown in Table 5.35. The underlined italic font marks the number for adjustment in each step.

Table 5.34 Unbalanced Example Table in Original Condition

	Sector1	Sector2	Factor	Household	Row total	Discrepancy
Sector1	50	40		150	240	-15
Sector2	95	45		95	235	65
Factor	110	85			195	10
Household			185		185	-60
Column total	255	170	185	245		

Table 5.35 Unbalanced Example Table after First Adjustment

	Sector1	Sector2	Factor	Household	Row total	Discrepancy
Sector1	50	40		150	240	-15
Sector2	95	45		35	175	5
Factor	110	85			195	10
Household			185		185	0
Column total	255	170	185	185		

After the first adjustment, the largest discrepancies are of Sector1 and Factor accounts. Again, we can either increase (Sector1, Factor), which is not economically meaningful, or decrease (Factor, Sector1) by 10. Therefore, Sector1's factor input decreases to 100 and Factor account's discrepancy is also cancelled out shown in Table 5.36 below.

Table 5.36 Unbalanced Example Table after Second Adjustment

	Sector1	Sector2	Factor	Household	Row total	Discrepancy
Sector1	50	40		150	240	-5
Sector2	95	45		35	175	5
Factor	100	85			185	0
Household			185		185	0
Column total	245	170	185	185		

There are only discrepancies of Sector1 and Sector2 accounts left for balancing after two steps, which happen to be the same in absolute value. This time both intersections (Sector1, Sector2) and (Sector2, Sector1) have economic meanings and we can change either or both of them. As we have not enough information to determine the preference, we spread the discrepancy to both of them evenly that 40 increases by 2.5 to 42.5 and 95 decreases by 2.5 to 92.5. Finally all the discrepancies are removed and the table is balanced below.

Table 5.37 Balanced Example Table after Third Adjustment

	Sector1	Sector2	Factor	Household	Row total	Discrepancy
Sector1	50	42.5		150	242.5	0
Sector2	92.5	45		35	172.5	0
Factor	100	85			185	0
Household			185		185	0
Column total	242.5	172.5	185	185		

The manual balancing methods discussed above are all prioritized on pure mathematical efficiency, however in practice the economic theory, common knowledge and prior information have to be carefully considered to avoid unreasonable adjustment based on researchers' personal judgement. For example, if we deem that consumption expenditures are most reliable data, the inter-industry transaction data is relatively moderate, while factor input expended by production sectors and factor income received by households are less reliable, the adjustment order as well as the outcome could be totally different like shown below. We still start from the unbalanced Table 5.38. The first step is to equate the factor income received by household to the total of household consumption expenditures as the latter is more reliable.

Table 5.38 Unbalanced Example Table in Original Condition (Alternative)

	Sector1	Sector2	Factor	Household	Row total	Discrepancy
Sector1	50	40		150	240	-15
Sector2	95	45		95	235	65
Factor	110	85			195	10
Household			185		185	-60
Column total	255	170	185	245		

The table then becomes:

Table 5.39 Unbalanced Example Table after First Adjustment (Alternative)

	Sector1	Sector2	Factor	Household	Row total	Discrepancy
Sector1	50	40		150	240	-15
Sector2	95	45		95	235	65
Factor	110	85			195	-50
Household			245		245	0
Column total	255	170	245	245		

Then we find the largest discrepancies are 65 and -50, and we choose to increase the value in (Factor, Sector2) by 50, not otherwise in (Sector2, Factor) as there is no economic explanation for that. The table turns out to be:

Table 5.40 Unbalanced Example Table after Second Adjustment (Alternative)

	Sector1	Sector2	Factor	Household	Row total	Discrepancy
Sector1	50	40		150	240	-15
Sector2	95	45		95	235	15
Factor	110	135			245	0
Household			245		245	0
Column total	255	220	245	245		

At the last stage, again we spread the remaining discrepancy evenly to the two intersections and make the balanced table as following:

Table 5.41 Balanced Example Table after Third Adjustment (Alternative)

	Sector1	Sector2	Factor	Household	Row total	Discrepancy
Sector1	50	47.5		150	247.5	0
Sector2	87.5	45		95	227.5	0
Factor	110	135			245	0
Household			245		245	0
Column total	247.5	227.5	245	245		

We can observe obvious difference between the two balanced tables: Table 5.37 and Table 5.41. If a researcher has another different judgement of the data set quality, the balanced table could be different again. Hence, we can see there are some contingency and arbitrariness incorporated in the manual adjustment, and it is best practice not to depend solely on it but to combine with other balancing techniques.

5.5 Balancing 2013 Welsh SAM

This section records the balancing process using the methods introduced above. As the RAS method requires pre-determined column or row total of each account, it is not applied in the first step of balancing that is conducted by applying the least squares method and direct cross-entropy method. Rather, the RAS method as well as some manual adjustment may be utilized to smooth the balancing adjustment to the figures given by the first step.

As the most important step of balancing procedure, the least squares method and direct cross-entropy method are firstly applied on the original SAM table successively. While the balancing codes are run in the GAMS system, a series of restrictions are imposed to ensure those reliable figures maintain fixed throughout the balancing process. The principles of setting these restrictions are summarized as below.

First, all those zeros of the empty blocks in the SAM are set to be maintained zeros after balancing. For example, the whole industries-by-industries block and commodities-by-commodities block are both kept full of zeros, and all the off-diagonal figures in the industries-by-commodities block are maintained zeros. Second, those figures that have reliable official data sources remain as their original values while those unofficial figures are left open to balancing adjustment. For example, the GVAs, tax revenues and factor incomes are kept fixed, whereas taxes on products and the trade with RUK are set adjustable. Third, some figures are not officially sourced as individual ones but have an official total value in a group level. For example, in the intermediate input block, the intermediate inputs of each sector are all estimated while their sectoral total intermediate values are all official figures. Therefore, in the balancing process these figures are open to adjustment as long as their sectoral total values are fixed. All restriction impositions are recorded as the side notes in balancing GAMS codes in Appendix IV.

After balancing we find that the structure of the preliminary table is maintained much more complete by direct cross-entropy method than least square method. We can observe that in the least-square-method-balanced table there are a mass of figures wiped to zeros and also zeros turned into non-zero figures. Such outcome has basically distorted the original SAM structure and hence not accepted.

On the contrary, only a few figures have altered between zero and non-zero value in the table balanced by the direct cross-entropy method. By this method, the original structure of the preliminary SAM is properly reserved.

Another standard to select the direct cross-entropy method is to compare the scale of adjustment against the original table. Here we choose the least sum of squares of deviations between the unadjusted and adjusted figures as the comparable variable. It is calculated as 1.301167×10^9 for least square method and 3.489104×10^8 for direct cross-entropy method, which clearly indicates that the latter only amounts to a quarter of the former. Therefore, no matter to which reason, the table balanced by the direct cross-entropy method is accepted as the appropriate one.

The second step of balancing procedure is to smooth the adjusted figures in the balanced table. This step is conducted due to the disproportion of the balancing adjustment. By observing the locations of the cells in which the figures are adjusted, they concentrate on the trade with RUK and intermediate input block. Most of the adjusted figures change drastically as becoming prominent within a proportional context. For example, for some commodities, the exports to and imports from RUK may change disproportionately that while exports are wiped to zero, the imports stay still; or imports decrease to zero while exports are unchanged. In fact, the balancing adjustment change on the RUK trade figures can be split proportionately to exports and imports according to their preliminary proportional relationship, in order to avoid the balancing-led disproportion. It is then conducted manually as it is relatively straightforward to do so.

For those adjusted intermediate inputs, the RAS method is applied to smooth them within the intermediate inputs block. Since at this stage the total intermediate inputs (column total of the intermediate inputs block) and the total intermediate outputs (row total of the intermediate inputs block) are already determined in the balanced table, the RAS is available to work that all the intermediate input figures can be re-adjusted and hence smoothed while the column and row totals of the block is fixed. By this way, the balancing adjustment to the intermediate inputs can be distributed more dispersedly rather than centralized in several figures within the block. This process can be completed by GAMS codes which are included in Appendix IV.

Finally, the SAM table for Welsh economy in 2013 is developed via multiple data sources and balanced by different balancing methods. The structure of the table is consistent with the CGE model specification and will act as the benchmark database for parameter calibration of the model as well as model operation and simulation. The final balanced table is shown in Appendix V.

5.6 Parameterization

The preceding sections detailed the development of a balanced social accounting matrix (SAM) for Wales. This section is devoted to the parameterization of the CGE model based on this SAM. This implies estimation of coefficients, parameters and exogenous variables of the model. The parameterization process generally applies econometric estimation methods, however, they can hardly be directly used in the parameterization of CGE models. This is due to the usual situation that only a limited number of observations is available compared to a relatively large number of parameters and exogenous variables to estimate. Insufficient degrees of freedom will be resulted if econometric regression methods are applied. In the case of this study, we have only one set of observations which is the SAM of 2013 and hence statistical econometric methods are not even considered. Therefore, we employ the method of calibration for model parameterization to overcome this problem.

5.6.1 The basic concept of calibration

This sub-section introduces the basic concept of the calibration method in pinning down the unknown parameters adopted in a CGE model. This is conducted based on the values in the initial equilibrium state observed in the SAM. Suppose each endogenous variable has its initial equilibrium value in the SAM which serves as the benchmark compared to the counter-factual value given by simulation run. The CGE model system can be then expressed as the function *F* in vector form:

$$F(Y, X, a) = 0 \tag{5-38}$$

where *Y* denotes the endogenous variable vector, *X* the exogenous variable vector, and *a* the parameter vector. The model can only be solved for *Y* in the simulation run when exogenous variables in *X* and parameters in *a* are known. The calibration process, in the contrary, solves the unknown parameters in advance given exogenous variables *X* and the initial equilibrium value of *Y* as *Y0*, which must be a solution of the model equations if there is no shock. Therefore *Y0* satisfies:

$$F(Y0, X, a) = 0 \tag{5-39}$$

and the unknown parameters can be derived while the above equations hold.

Using this method, the values of unknown parameters and exogenous variables can be calibrated, however they cannot be statistically tested due to the fact that there is only one benchmark dataset as the sample observation. Additionally, there are two cases that this technique cannot be adopted. One is that some parameter values have to be assumed based on prior information from outside of the SAM, such as elasticity parameters in various CES/CET functions. Though theoretically elasticities can be statistically estimated, their values are normally estimated with data sources other than SAM due to reasons stated above, or directly borrowed from literature based on similar research context and practice of ‘best guess’.

Another case regards to the situation that the number of parameters in *a* exceeds the number of independent equations in the model system. This case can be solved by reducing the number of unknowns by, again, external information.

A simple example is shown below to illustrate the essence of calibration method. Consider the revenue determination function of a specific type of tax. The function is expressed as the tax revenue equals the corresponding tax rate multiplying its underlying tax base.

$$T = t \cdot P \cdot Q \tag{5-40}$$

In the equation above, *T* denotes the tax revenue of the government, *t* denotes the tax rate, and *P* denotes the commodity price multiplied by the commodity quantity *Q*, which is the commodity value as tax base. In the initial equilibrium, the benchmark version of the equation can be written as:

$$\overline{T0} = t \cdot \overline{P0} \cdot \overline{Q0} \tag{5-41}$$

where variables' names followed by 0 denote their initial equilibrium values in SAM and the bars on top imply that such benchmark values are fixed. The tax rate parameter can be calibrated as below.

$$t = \overline{T0} / (\overline{P0} \cdot \overline{Q0}) \tag{5-42}$$

Here we should note that the tax base here is the value expressed as price multiplying quantity, so it is important to distinguish them with a consistent way. This is discussed in the next sub-section.

5.6.2 Normalization of price and quantity

As data figures in the SAM are only recorded in terms of values, it is useful to separate the value data into prices and quantities for the purpose of calibration and model initialization. Consider again the tax base in the preceding sub-section as the value equalled to price multiplying quantity as below:

$$V = P \cdot Q \tag{5-43}$$

Given price data, quantity data can be easily extracted, and vice versa, however, if only quantity data are available. Therefore, it is convenient to normalize the price and quantity data by redefining their measurements. As the quantity represents the number of physical units of the specific commodity and the price represents the unit value of the commodity, then the price can be assumed as unity if the quantity is also redefined as a new number of virtual units, each of which delivers just unity price. This procedure is so-called 'normalization' of the price and quantity data. In this way, the following holds:

$$V = 1 \cdot Q' \;\; \Rightarrow \;\; Q' = V \tag{5-44}$$

In other words, the value data in the SAM can be now considered as quantity figures, while the price and quantity data are so-called normalized prices and quantities.

The normalization of price and quantity is useful in pinning down the benchmark equilibrium value of some tax-inclusive endogenous variables to initialize the model simulation. The model initialization is very important prior to solving the model, which is done by setting a starting point for each endogenous variable. This step is important because the CGE model is solved by a numerically computational convergence process that requires staring points to search solutions and reach convergence. In practice, the starting point is set as the initial equilibrium value. While some tax-exclusive endogenous variables can be simply initialized as their initial equilibrium quantity figure, most of the benchmark values of tax-inclusive variables cannot be identified directly in the SAM. However, they can be determined simply applying equations show above. Consider the commodity again whose tax-inclusive value is assumed as V^T:

$$V^T = P^T \cdot Q^T \tag{5-45}$$

It can be re-written based on the tax-exclusive-value-form as:

$$V^T = (1 + t) \cdot P \cdot Q \tag{5-46}$$

Equalizing (4-45) and (4-46) yields:

$$P^T \cdot Q^T = (1+t) \cdot P \cdot Q \tag{5-47}$$

The equation (4-47) expressed in the initial equilibrium is then:

$$\overline{P0}^T \cdot \overline{Q0}^T = (1+t) \cdot \overline{P0} \cdot \overline{Q0} \tag{5-48}$$

According this equation, it may cause confusion in initializing the quantity of tax-inclusive commodity Q^T without considering price normalization. To see this, the conventional view of tax-inclusive price would give this relation that:

$$\overline{P0}^T = (1+t) \cdot \overline{P0} \tag{5-49}$$

so that

$$\overline{Q0}^T = \overline{Q0} \tag{5-50}$$

which may result in misleading initialization of Q^T. However, this can be modified by normalizing both the tax-inclusive and tax-exclusive prices as unity such that:

$$\overline{P0}^{T\prime} = \overline{P0}' = 1 \tag{5-51}$$

The variable Q^T can be then correctly initialized as:

$$\overline{Q0}^{T\prime} = (1+t) \cdot \overline{Q0}' \tag{5-52}$$

Now this initialization procedure will improve the chance of numerical convergence of the model in yielding solution of endogenous variables that replicates the figures in the SAM prior to simulation run.

5.6.3 Calibration procedure of the parameters

This sub-section will deal with the calibration of all parameters in the CGE model presented in Chapter 4. All parameters shown in the model are listed in Appendix II.

The calibration equations derived below are all based on the variables and model equations presented in the preceding chapter, and all endogenous variables used in the equations are actually their initial equilibrium values in the SAM, which are all suffixed with '0' by the notation rule.

First calibrated parameters are effective rates of all devolved and non-devolved taxes following the logic stated in the preceding sub-section 5.6.2.

$$tp_j = \overline{TP0}_j / \overline{Y0}_j \tag{5-53}$$

$$tc_j = \overline{TC0}_j / [\overline{Y0}_j (1 + tp_j)] \tag{5-54}$$

$$tccl = \overline{TCCL0} / \overline{Q\_R\_STOCK} \tag{5-55}$$

$$tk = \overline{TK0} / \overline{YK0} \tag{5-56}$$

$$ty = \overline{TY0} / \overline{YH0} \tag{5-57}$$

$$tle = \overline{TLE0} / \overline{YL0} \tag{5-58}$$

$$tlr = \overline{TLR0} / \overline{YL0} \tag{5-59}$$

$$tsd\_r\_fnl = \overline{TSD\_R\_NS0} / [\overline{Y0}_{R\_NS} \cdot (1 + tp_{R\_NS})] \tag{5-60}$$

$$tsd\_nr\_fnl = \overline{TSD\_NR\_NS0} / [\overline{Y0}_{NR\_NS} \cdot (1 + tp_{NR\_NS})] \tag{5-61}$$

$$tsd\_rr\_fnl = \overline{TSD\_R\_RS0} / \overline{Y0}_{R\_R} \tag{5-62}$$

$$tsd\_nr\_int_j = \overline{TSD\_NR\_RS0}_j / \overline{T0}_j \tag{5-63}$$

In the calibration equations above, tp_j and tc_j are effective net rates of production and product taxes respectively; *tccl*, *tk*, *ty*, *tle*, *tlr* represent effective rates of Council tax, Corporation tax, Income tax, NIC payable by employee and employer respectively; tsd_r_fnl, tsd_nr_fnl, tsd_rr_fnl and $tsd\_nr\_int_j$ stand for effective SDLT rates for residential new sale value, non-residential new sale value, residential resale value and non-residential resale value respectively.

The following equations account for calibration of consumption share parameters ac_i, ag_i and ai_i in terms of households consumption, government spending and investment demand respectively. The corresponding calibration equations are derived straight forward based on the optimal consumption demand equations:

$$ac_i = \overline{C0}_i/(\overline{YH0} - \overline{TY0} - \overline{TLE0} - \overline{TCCL0} - \overline{SH0}) \tag{5-64}$$

$$ag_i = \overline{G0}_i/(\overline{YG0} - \overline{TRFH}) \tag{5-65}$$

$$ai_i = \overline{I0}_i/\overline{S0} \tag{5-66}$$

What follows are calibration equations for various share parameters in those nested production equations.

$$ak_j = \overline{K0}_j/\overline{VA0}_j \tag{5-67}$$

$$al_j = (1 + tlr) \cdot \overline{L0}_j/\overline{VA0}_j \tag{5-68}$$

$$at_j = \left(1 + tsd\_nr\_int_j\right) \cdot \overline{T0}_j/\overline{VA0}_j \tag{5-69}$$

$$ava_j = \overline{VA0}_j/\overline{Y0}_j \tag{5-70}$$

$$aint_{i,j} = \overline{INT0}_{i,j}/\overline{Y0}_j \tag{5-71}$$

The share of factor income attributed to households and government (*skh, skg, sth, stg*) are simply calibrated as their portion divided by total factor income of each factor. The marginal propensity of saving of households is calibrated as:

$$mps = \overline{SH0}/(\overline{YH0} - \overline{TY0} - \overline{TLE0} - \overline{TCCL0}) \tag{5-72}$$

The calibration of trade-relevant parameters is relatively complicated compared to those tax rates and share coefficients calibrated above. For each trade block, there are always 4 unknown parameters and only 3 equations available for calibration, which implies we cannot solve all of them using the calibration technique. Therefore, we have to assume a value for one of these 4 parameters and reduce the number of unknown parameters so as to equal the number of available equations. The value usually assumed is elasticity, based on previous studies or databases.

The elasticity of transformation (EOT) and substitution (EOS) with respect to ROW is referenced to Piggott and Whalley (2009), where world trade elasticities are categorized regarding a set of more disaggregated sectors than our model. Hence, arithmetic mean value is calculated for both EOT and EOS regarding corresponding sectors. For the EOT and EOS with respect to RUK, however, there are no appropriate values to reference in the literature. Considering the fact that goods and services produced in Wales are highly similar to those produced in the RUK, the degree of substitutability between them should be must higher than between regional goods and foreign produced goods. Therefore, we choose EOT and EOS with respect to RUK both to be ten times higher than their ROW counterparts.

We start from the export trade block with respect to the ROW. The relevant equations are (3-163), (3-164) and (3-165), with now only three unknown parameters, ρw_i, $serow_i$ and θw_i, given EOT with respect to ROW σw_i pinned down. The calibration equations are derived as follows.

At first, the parameter defined by EOT is calculated as:

$$\sigma w_i = \frac{1}{\rho w_i - 1} \Longrightarrow \rho w_i = \frac{\sigma w_i + 1}{\sigma w_i} \tag{5-73}$$

The ROW export share parameter $serow_i$ is obtained by dividing equation (3-164) by (3-165), which gives:

$$\frac{EROW_i}{WD_i} = \left[\frac{serow_i \cdot PWD_i}{(1 - serow_i) \cdot \overline{PEROW}_i}\right]^{-\sigma w_i}$$

$$\Longrightarrow serow_i = \frac{\overline{PEROW}_i \cdot EROW_i^{-\frac{1}{\sigma w_i}}}{\overline{PEROW}_i \cdot EROW_i^{-\frac{1}{\sigma w_i}} + PWD_i \cdot WD_i^{-\frac{1}{\sigma w_i}}}$$

$$\Longrightarrow serow_i = \frac{\overline{EROW0}_i^{(1-\rho w_i)}}{\overline{EROW0}_i^{(1-\rho w_i)} + \overline{WD0}_i^{(1-\rho w_i)}} \tag{5-74}$$

The scaling coefficient θw_i is then derived directly from equation (3-163) as:

$$\theta w_i = \frac{Y_i}{\left[serow_i \cdot EROW_i^{\rho w_i} + (1-serow_i)\cdot WD_i^{\rho w_i}\right]^{1/\rho w_i}}$$

$$\Rightarrow \theta w_i = \frac{\overline{Y0}_i}{\left[serow_i \cdot \overline{EROW0}_i^{\rho w_i} + (1-serow_i)\cdot \overline{WD0}_i^{\rho w_i}\right]^{1/\rho w_i}} \tag{5-75}$$

Similarly, parameters contained in the export trade block with respect to RUK are calibrated as:

$$\sigma k_i = \frac{1}{\rho k_i - 1} \Rightarrow \rho k_i = \frac{\sigma k_i + 1}{\sigma k_i} \tag{5-76}$$

$$\frac{ERUK_i}{W_i} = \left[\frac{seruk_i \cdot PW_i}{(1-seruk_i)\cdot \overline{PERUK}_i}\right]^{-\sigma k_i}$$

$$\Rightarrow seruk_i = \frac{\overline{PERUK}_i \cdot ERUK_i^{-\frac{1}{\sigma k_i}}}{\overline{PERUK}_i \cdot ERUK_i^{-\frac{1}{\sigma k_i}} + PW_i \cdot W_i^{-\frac{1}{\sigma k_i}}}$$

$$\Rightarrow seruk_i = \frac{\overline{ERUK0}_i^{(1-\rho k_i)}}{\overline{ERUK0}_i^{(1-\rho k_i)} + \overline{W0}_i^{(1-\rho k_i)}} \tag{5-77}$$

$$\theta k_i = \frac{WD_i}{\left[seruk_i \cdot ERUK_i^{\rho k_i} + (1-seruk_i)\cdot W_i^{\rho k_i}\right]^{1/\rho k_i}}$$

$$\Rightarrow \theta k_i = \frac{\overline{WD0}_i}{\left[seruk_i \cdot \overline{ERUK0}_i^{\rho k_i} + (1-seruk_i)\cdot \overline{W0}_i^{\rho k_i}\right]^{1/\rho k_i}} \tag{5-78}$$

For parameters contained in the import trade block with respect to RUK, they are calibrated with the same procedure as:

$$\omega k_i = \frac{1}{1-\eta k_i} \Rightarrow \eta k_i = \frac{\omega k_i - 1}{\omega k_i}$$

$$\frac{MRUK_i}{W_i} = \left[\frac{smruk_i \cdot PW_i}{(1-smruk_i)\cdot \overline{PMRUK}_i}\right]^{\omega k_i}$$

$$\Rightarrow smruk_i = \frac{\overline{PMRUK}_i \cdot MRUK_i^{\frac{1}{\omega k_i}}}{\overline{PMRUK}_i \cdot MRUK_i^{\frac{1}{\omega k_i}} + PW_i \cdot W_i^{\frac{1}{\omega k_i}}}$$

$$\Rightarrow smruk_i = \frac{\overline{MRUK0}_i^{(1-\eta k_i)}}{\overline{MRUK0}_i^{(1-\eta k_i)} + \overline{W0}_i^{(1-\eta k_i)}} \tag{5-79}$$

$$\gamma k_i = \frac{QW_i}{\left[smruk_i \cdot MRUK_i^{\eta k_i} + (1-smruk_i)\cdot W_i^{\eta k_i}\right]^{1/\eta k_i}}$$

$$\Rightarrow \gamma k_i = \frac{\overline{QW0}_i}{\left[smruk_i \cdot \overline{MRUK0}_i^{\eta k_i} + (1 - smruk_i) \cdot \overline{W0}_i^{\eta k_i}\right]^{1/\eta k_i}} \tag{5-80}$$

Finally, parameters contained in the import trade block with respect to ROW are calibrated as:

$$\omega w_i = \frac{1}{1 - \eta w_i} \Rightarrow \eta w_i = \frac{\omega w_i - 1}{\omega w_i} \tag{5-81}$$

$$\frac{MROW_i}{QW_i} = \left[\frac{smrow_i \cdot PQW_i}{(1 - smrow_i) \cdot \overline{PMROW}_i}\right]^{\omega w_i}$$

$$\Rightarrow smrow_i = \frac{\overline{PMROW}_i \cdot MROW_i^{\frac{1}{\omega w_i}}}{\overline{PMROW}_i \cdot MROW_i^{\frac{1}{\omega w_i}} + PQW_i \cdot QW_i^{\frac{1}{\omega w_i}}}$$

$$\Rightarrow smrow_i = \frac{\overline{MROW0}_i^{(1 - \eta w_i)}}{\overline{MROW0}_i^{(1 - \eta w_i)} + \overline{QW0}_i^{(1 - \eta w_i)}} \tag{5-82}$$

$$\gamma w_i = \frac{Q_i}{\left[smrow_i \cdot MROW_i^{\eta w_i} + (1 - smrow_i) \cdot QW_i^{\eta w_i}\right]^{1/\eta w_i}}$$

$$\Rightarrow \gamma w_i = \frac{\overline{Q0}_i}{\left[smrow_i \cdot \overline{MROW0}_i^{\eta w_i} + (1 - smrow_i) \cdot \overline{QW0}_i^{\eta w_i}\right]^{1/\eta w_i}} \tag{5-83}$$

So far, all model parameters have gained value based on the benchmark data contained in the SAM with the calibration method. The CGE model is then prepared to be solved given the known calibrated coefficients and exogenous variables.

5.7 Conclusion

This chapter focuses on developing a SAM for 2013 Welsh economy and calibrating unknown parameters based on the SAM. By introducing the basic concepts and background knowledge, we have laid the foundations for constructing a SAM table. The data adopted for producing the preliminary SAM are multiply sourced. Due to the data caliber and estimation across diversified sources, the preliminary table fulfilled by those data is highly possible in imbalance.

The unbalanced SAM table can be dealt with several balancing methods which can be implemented manually or by GAMS programming. By comparing the structure and scale of deviation against the preliminary unbalanced table, the direct cross-entropy method balanced table is adopted and is further smoothed based on the proportional relations of the original table.

The balanced SAM table forms the benchmark dataset for the CGE model in use, and represents an equilibrium Welsh economy in 2013 as a starting point. Most of the model parameters can be calibrated by the variables' quantitative relationships implied by the SAM. The model can only be solved to replicate the SAM first so as to simulate the policy effect. In the next chapter, the equilibrium value of each endogenous variable contained in the balanced SAM will be compared with its value after policy variation shock. Policy implications can then be drawn through investigating such changes. Hence, the SAM constructed in this chapter has helped laid a solid foundation for cognizing the regional economy of Wales and its inter-regional exchanges of funds and commodities, and appraising economy-wide effect of tax variation policies.

CHAPTER 6 MODEL SIMULATION AND RESULTS ANALYSIS

6.1 Introduction

This chapter deals with the model simulations and interpretation of the results. Tax variation effects are investigated for three types of taxes: SDLT, Corporation tax and Income tax. As the model programming environment, different software packages are introduced and we show why GAMS software is applied in this study. The programming procedures required in the context of GAMS are illustrated. This is followed by a crucial step before running simulations: replication of the benchmark data in SAM by solving the model without shocks. Reliability of the model is checked in this step, which is a foundation for simulation in the next step. The chapter shows how the simulation is designed to observe the impact of a rise or cut of the tax rate, which is equivalent to a particular amount of an expansion or reduction of the tax revenue. The impact of the SDLT variation is generated through price representing the true transaction cost of real estate properties regardless of types of property rights, while the impact of Corporation tax is generated mainly from the supply side as capital using cost. The Income tax effects are realized through impacts on both supply side as labour cost and demand side as via households revenues and expenditures. As a result, the ramifications on major macroeconomic indicators following the policy variations can be investigated. The simulation captures the tax effects in the short, medium, and the long run. Tax effects in each time frame are compared and discussed.

This chapter is organized as follows. Section 6.2 deals with the introduction of GAMS as CGE model programming software. Section 6.3 presents the modelling procedure. Section 6.4 solves the model to replicate the benchmark. Section 6.5, 6.6, 6.7 conduct simulation of SDLT, Corporation tax and Income tax variation respectively. Section 6.8 summarizes results and draws conclusion.

6.2 Software for CGE programming

To solve CGE models and make simulations, software for numerical computations is required as those models are normally formulated as a complicated system of non-linear simultaneous equations. For such computations, we use GAMS in this study. This sub-section discusses the

GAMS software and its graphical user interface GAMS IDE (Integrated Development Environment) out of various other software choices for CGE modelling.

As what the first word ‘Computable’ of the acronym ‘CGE’ implies, CGE models can be solved with numerical method using computational algorithms. Such a numerical method usually corresponds to adjusting with possible solution values until a final solution is obtained to reach the equilibrium. Normally the computation process has to rely on fast and powerful computation capacity. The popularization of modern computers and development of relevant programs have gradually provided such facility. Nowadays, CGE models have become a standard tool, owing to development of shared databases as well as diversified and specifically designed software packages catering to different research and policy need.

Technology developments have aided CGE modelling with a greater choice of computer software which are designed for large non-linear general equilibrium systems. The advantage of using software lies in the relative ease of model building and solution finding. The most frequently used software in modelling CGE include RunGTAP, which is based on GTAP and primarily aimed for trade policy research, GEMPACK (General Equilibrium Modelling PACKage), MPS/GE(Mathematical Programming Software/General Equilibrium), GAMS (Generalized Algebraic Modelling System) and many derivations based on various model languages.

Many researchers use the shared CGE model database developed by Global Trade Analysis Project (GTAP). It is a publicly available resource developed and maintained by researchers from Purdue University. It provides the core data sets of CGE models, including the values of a variety of exogenous variables and parameters, the initial equilibrium values of endogenous variables, various input-output tables, and all other data that comprise the SAMs of CGE models. This database is regularly renewed with an update cycle of three to four years depending on global participation of data denoting users who combine a network in employing various databases (Narayanan et al. 2015).

In terms of all the software available for CGE modelling, GAMS is probably the most widely used software package. GAMS was originally developed by the World Bank aiming to analyze developing economies, and has been updated and distributed commercially by GAMS Development Corporation. The major merits of this software are:

(1) powerful algorithm of numerical computation
(2) flexible selection of solvers targeting diversified modelling needs
(3) extensive library of applications regarding mathematical optimization problem and general equilibrium modelling
(4) algebraically expressed programming syntax and programming transparency with consistent logic
(5) large world-wide community of users and developers in favour of mutual communication, problem sharing and solving.

We use the GAMS IDE software, equipped with a programme editor, in the MS Windows operating system. In developing and operating programme codes in GAMS, it allows us to compile input files, to run programmes, and to present the results. Thus, GAMS IDE supports all the procedures for numerical computation.

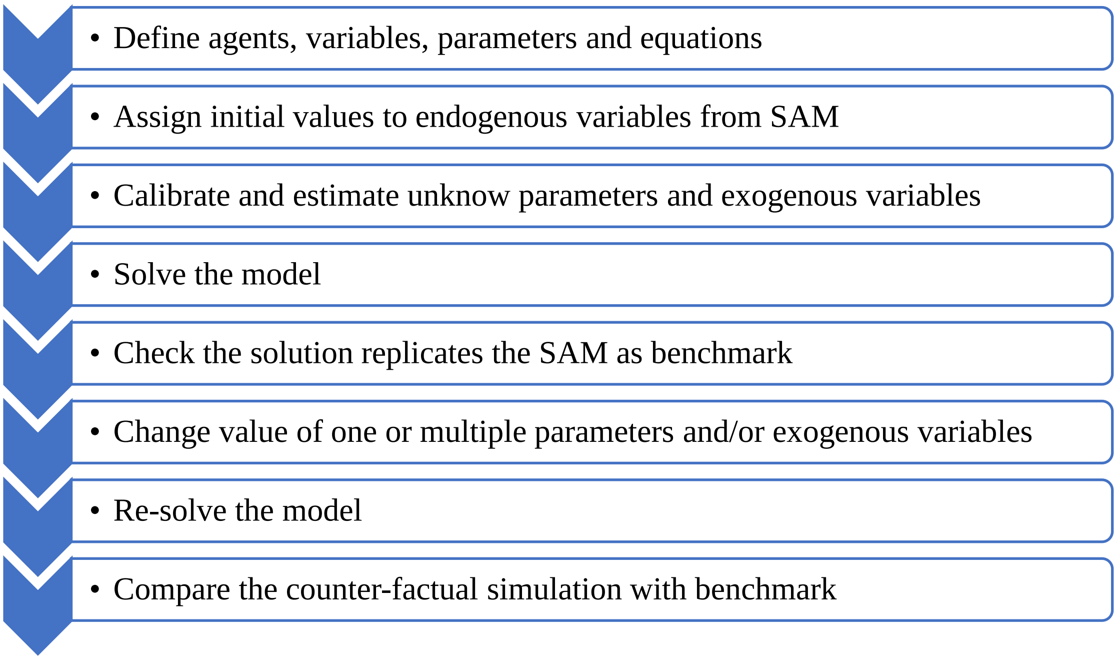
Since GAMS is commercially distributed by GAMS Development Corporation, its trial version available on the GAMS web site is strictly limited in terms of model size although it is free of charge. This situation also applies to the solver used for solving the model. As the CGE model developed in this study exceeds the trial version limit, both the software itself and the incorporated solver are purchased for the following simulation. The solver used in this study is PATHNLP which is specialized for large scale non-linear programming. This software-solver combination provides all that is required in terms of functional specification flexibility and solving power in dealing with SAM balancing, parameter calibration, model solving and simulation.

6.3 CGE modelling procedures

The CGE model solves following a line of procedures from defining the model framework, parameterization to solving the model and drawing the results. Each step is presented in the GAMS programing codes with relevant notes and comments. The procedure flow in running the model is presented below:

The first step regards to the set-up of the whole model framework which includes the number and scale of production sectors and other economic agents, endogenous and exogenous variables adopted and parameters in use, model equations describing optimal behaviours of agents from both supply and demand side, and appropriate macroeconomic closures for clearing the markets. This step is already accomplished in Chapter 4.

Figure 6.1 CGE Modelling Procedures



The second step is about initialization of all endogenous variables so that they are assigned initial equilibrium values based on their relevant data observations in the SAM. This gives a set of starting points for GAMS to search converging solutions. Calibration is conducted in the third step. These two steps are illustrated in Chapter 5.

The fourth and fifth steps will be accounted for in the next section, while the simulation and results analysis of the last three steps will be presented in this chapter.

In summary, these steps deliver a guideline of CGE modelling procedures for illustration use. It is more detailed in terms of the procedures when considering programming and running the model in the context of GAMS syntax. Subsequently, the procedure flow can be decomposed into what follows in programming the model:

(1) Declaration and definition of sets in terms of sectors and agents
(2) Installation of the SAM as the benchmark dataset for the model

(3) Retrieval of the data from the SAM for initialization of the endogenous variables

(4) Declaration and definition of exogenous variables and parameters

(5) Value assignment to the exogenous variables and elasticity parameters, and calibration of other parameters

(6) Declaration and definition of endogenous variables

(7) Declaration and definition of equations

(8) Initialization for all the endogenous variables to ease numerical convergence in computation

(9) Setting lower bounds for endogenous variables to avoid zero denominator

(10) Declaration and definition of the model

(11) Solving the model (base run)

(12) Replication of the benchmark SAM

(13) Assigning external policy shock

(14) Solving the model (simulation run)

(15) Recording the new solutions and calculating deviations against base run benchmark

By this section we have clarified the programming roadmap of the model, and the next section will deal with the base run solving of the model which targets at replicating the data contained in the SAM. All relevant programming codes are listed in Appendix VI.

6.4 Solving the model (base run)

The CGE model developed in this study is a static, single-regional model which captures economic-wide effects of major macroeconomic indicators by generating a counter-factual set of solutions to compare with the benchmark equilibrium given one or multiple external shocks. This process is known as the simulation process which, however, requires a precondition. That is, in the first place, the model is required to solve and yield the solutions that replicate the values contained in the SAM given no external shock disturbance. This base-replication step is essential to ensure the reliability of the model.

To check whether the model yields solutions that replicate the SAM, we record all the solution values of endogenous variables and calculate the percentage deviation with respect to their initial equilibrium value as in the form below:

$$dY = (Y/\overline{Y0} - 1) \cdot 100\% \tag{6-1}$$

Finally, all deviations of the variables are calculated for their arithmetic mean across all sectors and agents. This gives a scalar mean which equals 2.45%. It implies that the percentage deviation of each endogenous variable's solution against the benchmark is averaged as 2.45%. Therefore, the benchmark dataset of SAM is not perfectly replicated, however, the average bias is still acceptable given a 5% tolerance level. This level of deviation promises the reliability of the model and further simulation is possible. The next section will focuses on the simulations given policy shock and the counter-factual simulation results will be compared to these base run solutions.

6.5 Simulation with Stamp Duty Land Tax

This section will focus on the simulation of SDLT rate variation effects. The simulation is not only conducted with Cobb-Douglas production function which implies unity elasticity of substitution between labour, capital and non-residential land, but also gives chance to alternative elasticity values within a more general CES function. The alternative elasticities are typically chosen around unity as 0.5 and 1.5. The former represents a more complementary and less substitutable relation between the factors, while the latter implies even stronger substitutability than unity. The simulation with the sensitivity test is delivered across all the three time frames: short run (SR), medium run (MR) and long run (LR).

As the SDLT has two tax rate lines regarding residential and non-residential real estate properties, the simulation scenarios are conducted separately with the two property types. The first scenario explores the tax cut effects of both lines of rates. The second scenario considers the case where the effective tax rates of both lines are adjusted to approach each other. Since by calibration we have gained the effective SDLT rate for residential properties is 1.48%, and for non-residential properties it is approximately 2.71%, to narrow the gap the former has to rise and the latter needs to decline. Therefore the second scenario simulates a combination of the residential rate hike and non-residential rate cut. Both scenarios are simulated by 10p per Pound of the tax. The simulation scenarios are summarized as follows.

Scenario 1: Decrease both residential and non-residential SDLT effective rate by 10p per Pound (denoted by ‘R- NR-’).

Scenario 2: Increase residential SDLT effective rate by 10p per Pound, while simultaneously decrease non-residential SDLT effective rate by 10p per Pound (denoted by ‘R+ NR-’).

Results for both scenarios are presented and summarized below by 3 tables. Each table corresponds to an elasticity value.

Table 6.1 SDLT Simulation Results under Elasticity of 0.5 (%)

SDLT 10p	Elasticity of Substitution between Labour, Capital and Land = 0.5					
	Short Run		Medium Run		Long Run	
Major variables	R- NR-	R+ NR-	R- NR-	R+ NR-	R- NR-	R+ NR-
Government revenue	-0.0422	0.0098	-0.0315	0.0200	-0.0421	0.0099
Devolved revenue	-1.0130	0.2353	-0.7567	0.4797	-1.0111	0.2374
Residential SDLT	-10.0088	10.0039	-9.9810	9.7486	-9.9983	10.0212
Non-residential SDLT	-10.0030	-9.9791	-3.4682	-3.4979	-9.9703	-9.9556
Non-devolved revenue	-0.0070	0.0023	4.5380	6.0242	0.0122	0.0210
Government spending	-0.0738	0.0172	-0.0551	0.0349	-0.0737	0.0173
GVA	-0.0131	0.0035	1.6168	1.6233	0.0004	0.0218
GDP (basic price)	-0.0129	0.0034	1.6127	1.6128	0.0007	0.0218
GDP (consumer price)	-0.0114	0.0031	2.8201	3.3342	0.0043	0.0220
Households income	-0.0088	0.0038	-0.8132	-1.1826	0.0015	0.0166
Households income - net	-0.0088	0.0040	-0.8035	-1.1788	0.0017	0.0169
Households consumption	-0.0088	0.0040	-0.7958	-1.1712	0.0017	0.0169
GFCF	-0.0175	0.0075	6.8946	7.6947	0.0032	0.0376
Non-Residential property value	0.0884	0.0930	9.3367	4.5322	0.2252	0.1248
Residential property value	-0.0088	0.0040	-0.7999	-1.1752	0.0017	0.0169
Price	-0.0088	0.0040	1.8525	1.4672	6.3089	0.1961
Volume	0	0	-2.6042	-2.6042	-5.9329	-0.1788

Table 6.1 shows results of both scenarios across all time frames under elasticity of 0.5. In the short run, the non-residential price is stimulated to rise (+0.088%) given tax cut in scenario 1 and suppressed (-0.093%) given tax hike in scenario 2, although both scales are very small. On the contrary, the residential price always changes in an opposite way and the change scales are even smaller: -0.0088% and -0.004% respectively. Such small price changes do not bring significant changes to the property market and the whole economy. As a result, a tax cut does

not make the correspondent tax revenue recouped and even make it slightly less than the direct loss due to the cut.

Major economic variables show sharp changes in the medium run. Non-residential SDLT revenue only reduces by less than 3.5% in both scenarios. Considering it is a 10p cut in both scenarios, almost two thirds of the revenue loss is recouped. The large recoupment is resulted from the expansion of the non-residential SDLT tax base, which is the non-residential property value. In Scenario 1, non-residential property value rises by 9.34% and in Scenario 2 it rises by 4.53%. As in the medium run the non-residential land factor is fixed, all non-residential property value changes come from inter-industrial average price changes. The non-residential price change 4.53% in Scenario 2 is only half of that in Scenario 1 due to the drag from the residential price which is supposed to decline in response to residential SDLT rate hike in Scenario 2. In fact, the residential and non-residential price has mutual effects to each other, so that the residential price in Scenario 2 does not fall but still increase by 1.47% due to support from the non-residential price. The residential price rises by 1.85% in Scenario 1, but falls by 2.6% on turnover in both Scenario 1 and 2.

Both scenarios witness matchable rise of output, especially with basic price. There are basically two reasons for this. One is the structural optimization of the three production factors on the supply side. In the medium run, both labour and capital are free to move across all sectors. The tax cut for non-residential land transactions will temporarily benefit all sectors with non-residential land input, and this will further drive labour and capital to move and relocate according to new profitability level of each sector. This optimization process of the factor structure in each sector generates more value added and hence promote output. The second reason lies in the demand side that more fixed capital forms because of the expansion of both residential and non-residential properties. This is accounted for by, on one side, the property price rise directly effected by the tax variation, and on the other side, the quantity increase of the fixed capital investment enabled by higher value added produced. As the fixed capital, or GFCF here in the table, is a main component of GDP, the expansion of GFCF supports the rise of GDP.

In the long run, however, the expansion is basically wiped away as the labour and capital stock is completely relaxed. This may cause labour and capital stock to adjust while their factor prices return to the equilibrium level. The stimulation effect from property prices cannot be achieved without the optimized factor structure of each sector. Additionally, the non-residential land as input also becomes flexible across sectors and hence largely smooths its factor price.

The price changes of residential and non-residential properties still follow the style of that in the medium run. In Scenario 2, the two prices drag each other that the non-residential price does not achieve the amount of rise as in Scenario 1, and the residential price does not fall but turns out to rise a bit. The residential price change is about 1.85% rise against base in the medium run and 6.31% rise in the long run. As these are responses of a 10p cut of residential SDLT, the elasticity of price change per 1p is then calculated as the percentage figures divided by ten: -0.185 in the medium run and -0.631 in the long run. These results are basically consistent with those of Davidoff and Leigh (2013). Their research is based on econometric regression and indicates a range of -0.151 to -0.372 for as far as three years. Their research did not inform the long run effects, but the residential price change may well last and expand in the long run. However, the turnover response to the tax variation is only partially consistent with the research. In their three-year accumulation impacts on house sales, the elasticity ranges from -0.479 to -0.634, while results here (+0.26 to +0.593) only captures their second-year impact as from +0.435 to +0.891.

Table 6.2 SDLT Simulation Results under Elasticity of 1 (%)

SDLT 10p	Elasticity of Substitution between Labour, Capital and Land = 1					
	Short Run		Medium Run		Long Run	
Major variables	R- NR-	R+ NR-	R- NR-	R+ NR-	R- NR-	R+ NR-
Government revenue	-0.04215	0.00979	-0.04215	0.00979	-0.04215	0.00979
Devolved revenue	-1.01233	0.23512	-1.01234	0.23512	-1.01233	0.23512
Residential SDLT	-10.00340	10.00198	-10.00349	10.00201	-10.00343	10.00199
Non-residential SDLT	-9.99347	-9.98335	-9.99359	-9.98331	-9.99350	-9.98334
Non-devolved revenue	-0.00139	0.00023	-0.00146	0.00025	-0.00140	0.00024
Government spending	-0.07378	0.01714	-0.07378	0.01714	-0.07378	0.01714
GVA	-0.00589	0.00092	-0.00600	0.00096	-0.00593	0.00094
GDP (basic price)	-0.00572	0.00089	-0.00583	0.00092	-0.00575	0.00090
GDP (consumer price)	-0.00463	0.00062	-0.00473	0.00065	-0.00466	0.00064
Households income	-0.00347	0.00194	-0.00356	0.00197	-0.00351	0.00196
Households income - net	-0.00344	0.00203	-0.00354	0.00206	-0.00348	0.00205
Households consumption	-0.00344	0.00203	-0.00354	0.00206	-0.00348	0.00205
GFCF	-0.00638	0.00348	-0.00653	0.00352	-0.00642	0.00350
Non-Residential property value	0.09520	0.09032	0.09518	0.09033	0.09522	0.09032
Residential property value	-0.00344	0.00203	-0.00354	0.00206	-0.00348	0.00205

Price	-0.00272	0.00182	-0.00281	0.00185	-0.00275	0.00184
Volume	-0.00072	0.00021	-0.00072	0.00021	-0.00073	0.00021

Table 6.2 shows the results under the elasticity of 1. Compared to the results under the elasticity of 0.5 as a whole, the scale of changes of major economic variables shrink much especially in the medium run, and the changing scale of each variable in both scenarios tends to converge across time frames. Although Scenario 2 gives positive responses of GDP in all time frames, Scenario 1 fails to see this. GDP still slightly falls given a cut on both residential and non-residential rates in Scenario 1. While the non-residential price always presents expected direction of change across both scenarios and time frames, the residential price and turnover do not. They always present opposite direction of changes against expected, although all the changes are extremely small. As there are generally no significant effects on all major macroeconomic variables, there is basically no recoupment to any tax cut or expansion to any tax hike. The reason behind may still lie in the elasticity value that relative changes of factors in response of the tax variation just to some extent cancel off with such a degree of substitution.

Table 6.3 SDLT Simulation Results under Elasticity of 1.5 (%)

SDLT 10p	Elasticity of Substitution between Labour, Capital and Land = 1.5					
	Short Run		Medium Run		Long Run	
Major variables	R- NR-	R+ NR-	R- NR-	R+ NR-	R- NR-	R+ NR-
Government revenue	-0.0422	0.0098	-0.0421	0.0098	-0.0421	0.0099
Devolved revenue	-1.0130	0.2354	-1.0115	0.2350	-1.0116	0.2374
Residential SDLT	-10.0089	10.0039	-9.9987	10.0012	-9.9995	10.0215
Non-residential SDLT	-10.0030	-9.9791	-9.9805	-9.9862	-9.9809	-9.9572
Non-devolved revenue	-0.0071	0.0023	0.0164	-0.0034	0.0118	0.0217
Government spending	-0.0738	0.0172	-0.0737	0.0171	-0.0737	0.0173
GVA	-0.0131	0.0035	0.0016	-0.0004	-0.0006	0.0222
GDP (basic price)	-0.0129	0.0035	0.0019	-0.0005	-0.0004	0.0222
GDP (consumer price)	-0.0115	0.0031	0.0064	-0.0015	0.0033	0.0225
Households income	-0.0088	0.0038	0.0004	0.0013	0.0003	0.0168
Households income - net	-0.0089	0.0040	0.0005	0.0014	0.0004	0.0171
Households consumption	-0.0089	0.0040	0.0005	0.0014	0.0004	0.0171
GFCF	-0.0175	0.0075	0.0086	0.0007	0.0021	0.0385
Non-Residential property value	0.0884	0.0930	0.1055	0.0890	0.1445	0.1058
Residential property value	-0.0089	0.0040	0.0005	0.0014	0.0004	0.0171
Price	-0.0089	0.0040	0.0001	0.0016	6.0336	0.2126
Volume	0	0	0.0004	-0.0002	-5.6899	-0.1950

Table 6.3 presents the simulation results when the elasticity becomes even larger as 1.5. In the short run, the result is nothing different with that under elasticity of 0.5. The value of elasticity does not make any difference when all the production factors are sectorally fixed. Hence, Scenario 1 still presents negative effects and Scenario 2 shows positive effects on the whole economy. However, residential price responses oppositely to the direction as expected in both scenarios. In the medium run, the situation is reversed that positive effects start to appear in Scenario 1 but Scenario 2 falls back and has residential price changed in the wrong direction. The results only becomes relatively reasonable given a long time horizon. The mutual drag between residential and non-residential prices appears again. GDP turns positive in both scenarios and Scenarios 2 has slightly higher scale (0.0225%) than another (0.0033%). This also happens in the medium run and long run when the elasticity equals 0.5. It seems narrowing the gap between residential and non-residential SDLT rates has slightly more impact than simply cut of both rates. At last, the effects are still trivial in scale across all time horizons, as under a high elasticity the land tends to over-substitute the other two factors which are supposed to be the main pillars for producing value added.

6.6 Simulation with Corporation Tax

This section will focus on the simulation of Corporation tax rate variation effects. Although the Corporation tax was still in a double-rates system back in 2013, the tax base is basically uniform in terms of the corporation types. The rates only differentiate with regard to the profit level. Hence, for this simulation, the capital factor income is the only tax base for imposed tax variation shock. As the Corporation tax is relatively a large-scale tax compared to SDLT, a 5p per Pound variation of tax rate is sufficient to generate explicit simulation results. The model ensures that only the tax revenue increment/loss caused by this 5p variation accounts for the change of fiscal status. The simulation scenario in the text only focuses on the tax cut case, as the results of tax hike are more or less mirror figures to those of tax cut. Again, sensitivity of the simulation is also delivered across the three time frames.

Scenario: Decrease effective Corporation Tax rate by 5p per Pound.

When conducting simulations in this scenario, we find that as the elasticity value becomes larger (especially larger than 1), the simulation results tend to be unstable. When the elasticity value equals as high as 1.5, the model does not deliver significant percentage changes against base values for the long run time frame. Hence, we ignore the long-run simulation results here for the highest elasticity value case, and only discuss the short and medium run results as comparable to those of 0.5 and unity elasticity value cases.

Table 6.4 Corporation Tax Simulation Results (%)

Corporation Tax -5p	Elasticity of Substitution between Labour, Capital and Land							
Major variables	0.5			1			1.5	
	SR	MR	LR	SR	MR	LR	SR	MR
Government revenue	-0.1381	-0.1410	-0.1409	-0.1387	-0.1388	-0.1388	-0.1382	-0.1374
Devolved revenue	-1.9920	-2.0343	-2.0320	-2.0010	-2.0014	-2.0015	-1.9934	-1.9816
Corporation Tax	-5.0065	-5.1206	-5.1108	-5.0234	-5.0241	-5.0243	-5.0066	-4.9806
Non-devolved revenue	-0.0089	0.0206	0.0407	-0.0227	-0.0236	-0.0236	-0.0089	0.0436
Net Production Taxes	0.0207	0.0524	0.1395	-0.0038	-0.0049	-0.0051	0.0208	0.0653
Net Product Taxes	-0.0033	0.0031	0.0414	-0.0214	-0.0221	-0.0223	-0.0033	0.0933
Income Tax	-0.0102	0.0380	0.0304	-0.0176	-0.0184	-0.0183	-0.0102	0.0011
NIC (employer)	-0.0262	0.0271	0.0255	-0.0362	-0.0373	-0.0372	-0.0264	-0.0122
NIC (employee)	-0.0262	0.0271	0.0255	-0.0362	-0.0373	-0.0372	-0.0264	-0.0122
Government spending	-0.2417	-0.2469	-0.2464	-0.2428	-0.2429	-0.2429	-0.2419	-0.2405
GVA	-0.0203	0.0068	0.0020	-0.0324	-0.0334	-0.0333	-0.0203	-0.0019
GDP (basic price)	-0.0195	0.0078	0.0048	-0.0318	-0.0328	-0.0328	-0.0195	-0.0006
GDP (consumer price)	-0.0173	0.0071	0.0098	-0.0304	-0.0313	-0.0313	-0.0173	0.0121
Households income	-0.0102	0.0380	0.0305	-0.0176	-0.0184	-0.0183	-0.0102	0.0011
Households income - net	-0.0099	0.0396	0.0315	-0.0174	-0.0182	-0.0181	-0.0098	0.0016
Households consumption	-0.0099	0.0396	0.0315	-0.0174	-0.0182	-0.0181	-0.0098	0.0016
GFCF	0.2619	0.2496	0.2435	0.2402	0.2387	0.2387	0.2620	0.2957
Total factor income - net	0.0814	0.1152	0.1071	0.0693	0.0759	0.0759	0.0815	0.1083
Land factor income	0.0170	1.0854	0.2110	-0.0085	-0.0094	-0.0154	0.0253	0.1116
Capital factor income	0.4048	0.2842	0.2945	0.3869	0.3861	0.3859	0.4046	0.4321
Labour factor income	-0.0262	0.0271	0.0256	-0.0362	-0.0373	-0.0372	-0.0264	-0.0122
Total exports to RUK	0.0075	-0.2221	0.0450	-0.0097	-0.0109	-0.0109	0.0081	0.0395
Total exports to ROW	-0.0060	-0.3947	-0.1128	-0.0211	-0.0213	-0.0217	-0.0076	0.1193
Total imports from RUK	-0.0288	0.4549	-0.0632	-0.0330	-0.0333	-0.0333	-0.0286	0.1291
Total imports from ROW	0.0589	-0.0382	0.0743	0.0522	0.0516	0.0514	0.0567	0.1455

The effective tax rate change does not only brings change of Corporation tax revenue, but also triggers revenue changes of other taxes because it is a general equilibrium model where one exogenous change will lead to changes of all variables including all the underlying tax bases. However, other tax revenues do not change significantly, leaving only Corporation tax revenue reduces around 5% across all elasticity values and time frames. This is consistent with the 5p-per-Pound change without large tax recoupment, as it does not significantly expand the tax base and the economy output.

The tax base of the Corporation tax is the gross profit totally generated by all the companies in the Welsh economy, which is marked by the term ‘Capital factor income’ here. This factor income has a positive response to the tax cut across all time frames and elasticities, although the increasing scale is quite limited ranging from about 0.28 to 0.43 percent. Hence, the strength of 5p per Pound cut of Corporation tax is not able to recoup itself as the effect on its tax base is not strong enough.

The tax cut effect on GDP is even proportionately smaller. It has only positive effect in the medium and long run with elasticity of 0.5 (0.0071% and 0.0098% respectively), and in the medium run with elasticity of 1.5 (0.0121%). All positive changes are trivial in scale no matter which price is used to measure GDP. If we look at the basic-price GDP without product taxes such as VAT, the positive changes occur even less, only 0.0078% in the medium run and 0.0048% in the long run for the 0.5 elasticity case.

While the GDP can be decomposed into four components: consumption, government spending, investment and net export, the pillar for the positive GDP response to the tax cut is only consumption. We can clearly see from the results table that the government spending remains slightly stable below -0.24%, and GFCF ranges around 0.25% across all time frames and elasticities. Only households consumption’s sign of change is highly consistent with the sign of GDP change in each case. Whenever GDP increases, there is correspondingly a rise in households consumption. As the representative household faces a balanced budget, the consumption change is also consistent with the households income change, which is mainly attributed to labour factor income. The labour income is raised slightly less than 0.03% in both medium and long run under elasticity of 0.5, while it declines in all other cases in response to the shock. Therefore, the direction and scale of labour income change dominates the regional GDP.

Focusing on both capital and labour factor income, we find their changes are relatively stable (capital factor income rises around 0.4% and labour factor income reduces around 0.03%) across all cases except in the cases where GDP rises (‘MR’ and ‘LR’ with 0.5 elasticity). In these two cases, they tend to change towards each other that capital factor income rises relatively less as around 0.29% and labour factor income changes become positive as around 0.026%. This implies in the medium to long run with low elasticity of substitution, the labour employed is not that easy to be substituted when more capital is introduced and installed due to Corporation tax cut stimulus. Compared to the cases where elasticity of substitution becomes higher and the labour employed tends to be more substitutable, the labour employed in the low elasticity case tends to be more skilled, higher value added, and more complemented (and less substitutable) to the capital utilized. As a result, more labour income is generated and hence raises the household income and consumption to sustain the economy. Hence, the simulation result implies that rather than simply expanding investment and introducing more capital with tax cut stimulation, it is important to be aware of the composition and quality of the regional labour resources, and also the industry and project to input capital so as to avoid large substitution effect brought by the capital on the local labour stock.

6.7 Simulation with Income tax

This section presents the simulation results of Income tax rate variation effects. The tax base of Income tax is basically income from employment, including self-employed profits, pensions, rental income and savings interest. Based on different income bands, this tax is levied via 3 rates: 20%, 40% and 45%. From April 2019, the UK government will reduce the 3 rates of Income tax paid by Welsh taxpayers: basic rate from 20% to 10%, higher rate from 40% to 30% and additional rate from 45% to 35%. The Welsh government will then decide how much to collect on top of the reduced rates and bear the fiscal consequence, which may directly affect the government budget. The simulation is set to ensure the variation of the Income tax rates will account for the government spending change as a result of a balanced budget. Similar to the setting of the Corporation tax simulation, the Income tax simulation is also set as a 5p per Pound tax cut.

Scenario: Decrease effective Income Tax rate by 5p per Pound.

As in the Corporation tax case, the simulation of Income tax also gives unstable results when the elasticity goes higher. When the elasticity equals one, the three time frames tend to give convergent results. Given the elasticity as high as 1.5, the model does not converge to significant solutions in the long run frame and hence it is not listed in Table 6.5.

Table 6.5 Income Tax Simulation Results (%)

Income Tax -5p	Elasticity of Substitution between Labour, Capital and Land							
Major variables	0.5			1			1.5	
	SR	MR	LR	SR	MR	LR	SR	MR
Government revenue	-0.7057	-0.7015	-0.6876	-0.7094	-0.7094	-0.7096	-0.7015	-0.6937
Devolved revenue	-3.8948	-3.8718	-3.7881	-3.9155	-3.9153	-3.9164	-3.8718	-3.8287
Income Tax	-5.0718	-5.0512	-4.9389	-5.0972	-5.0970	-5.0983	-5.0424	-4.9911
Non-devolved revenue	-0.0197	0.5290	0.3445	-0.0671	-0.0677	-0.0689	-0.0159	0.1850
Net Production Taxes	0.1343	0.4685	0.5372	0.0567	0.0565	0.0554	0.1247	0.4150
Net Product Taxes	0.0204	0.9070	0.4751	-0.0284	-0.0297	-0.0305	0.0096	0.3179
Corporation Tax	-0.0470	0.3580	0.2626	-0.0930	-0.0926	-0.0944	-0.0888	-0.2138
NIC (employer)	-0.1342	-0.1766	0.0505	-0.1710	-0.1707	-0.1727	-0.0870	-0.0499
NIC (employee)	-0.1342	-0.1766	0.0505	-0.1710	-0.1707	-0.1727	-0.0870	-0.0499
Government spending	-1.2351	-1.2278	-1.2031	-1.2417	-1.2416	-1.2420	-1.2278	-1.2142
GVA	-0.0901	0.0293	0.1312	-0.1295	-0.1292	-0.1311	-0.0662	-0.0316
GDP (basic price)	-0.0856	0.0381	0.1394	-0.1257	-0.1254	-0.1273	-0.0623	-0.0226
GDP (consumer price)	-0.0713	0.1558	0.1849	-0.1126	-0.1125	-0.1142	-0.0526	0.0235
Households income	-0.0755	-0.0539	0.0643	-0.1023	-0.1021	-0.1035	-0.0447	0.0094
Households income - net	0.4668	0.4916	0.6097	0.4394	0.4397	0.4382	0.4981	0.5546
Households consumption	0.4668	0.4916	0.6097	0.4394	0.4397	0.4382	0.4981	0.5546
GFCF	0.5612	0.8871	0.9536	0.4948	0.4953	0.4924	0.5723	0.5738
Total factor income - net	0.2564	0.3933	0.4155	0.2163	0.2166	0.2147	-0.0849	0.3154
Land factor income	0.0910	3.2973	0.6612	0.0136	0.0124	0.2994	0.0634	2.3887
Capital factor inco me	-0.0470	0.3580	0.2626	-0.0930	-0.0926	-0.0944	-0.0888	-0.2138
Labour factor income	0.3737	0.3311	0.3505	0.3367	0.3370	0.3350	-0.0870	0.4584
Total exports to RUK	-0.0652	-0.2042	0.4921	-0.1364	-0.1365	-0.1389	-0.0732	-1.4735
Total exports to ROW	-0.0161	0.7258	0.6846	-0.0438	-0.0490	-0.0460	-0.0457	-2.1344
Total imports from RUK	-0.1265	1.7890	0.3928	-0.1562	-0.1563	-0.1582	-0.1436	2.6872
Total imports from ROW	0.1192	0.8715	0.6445	0.0918	0.0820	0.0895	0.1076	-0.2318
GDP per capita (basic price)			0.0889			-0.4608		
GDP per capita (consumer price)			0.1344			-0.4477		
Households income per capita			0.0139			-0.4370		
Households income per capita - net			0.5590			0.1029		

The 5p-per-Pound cut delivers about 5% reduction of Income tax revenue and causes total government revenue to decrease around 0.7% across all cases. There is only trivial recoupment to the tax cut in the long run when the elasticity takes 0.5 and in the medium run when the elasticity takes 1.5. The reason is the same as in the Corporation tax case that the tax cut does not boost the economy significantly.

The tax base of the Income tax is mainly compensation of employee marked by the term 'Labour factor income' and from investments through a portion of capital and land factor income. Compared to the labour and capital factor income, the land factor income is much smaller in scale. Although its change against base fluctuates much more across each case than that of other two factors, its relatively small scale has limited its effect in determining output change. The capital factor income has only positive response in two cases: in the medium run (0.36%) and long run (0.26%) with 0.5 elasticity. The labour factor income has basically positive responses (0.33~0.46%) to the tax cut except reduces slightly (-0.09%) only in the short run with the elasticity of 1.5.

While the tax effect on GDP is mixed measured by different prices (-0.02% by basic price versus +0.02% by consumer price), it only increases in the medium run and long run when elasticity is low, no matter which price is used to measure it. This situation coincides with the Corporation tax case, and the reason lies again in the over-substitution effect between capital and labour. We can observe that only in these two cases that both factors have positive responses simultaneously. That is, increase of one factor's income stimulated by the tax cut shock does not substitute too much for the other factor's, otherwise this will result in proportional imbalance between factor incomes and encumber output of the regional economy through their connections with the demand side.

Compared to the Corporation tax simulation results, the capital factor income, rather than labour factor income, determines the regional GDP in this case. When the capital factor income is not substituted strongly and is ultimately raised, GDP change is always positive, and vice versa. This is also reflected in the changes of demand-side GDP components again. While the government spending remains stable around -1.22% and consumption around 0.5% across each case, GFCF gains highest rise (0.89% and 0.95%) only when both GDP and capital factor income rise. As we know, companies' proper profitability is necessary for new projects and investments to form GFCF. Therefore, the pillar for the GDP rise in response to the Income tax cut is the capital factor income (from the angle of income approach in calculating GDP) and GFCF (from the angle of expenditure approach in calculating GDP). More profit is generated and hence gives more chance to investment to form fixed capital and raise the output.

In addition to the results of total regional GDP, the simulation also presents effects on per capita GDP which is only shown in the long run, as in both short and medium run the total stock of labour is fixed and per capita changes would be the same as aggregate changes. The GDP per capita rises 0.0889% if measured by basic price and 0.1344% measured by consumer price when elasticity equals 0.5. Since these are changes brought by a 5p cut, we can convert it to 0.01778% and 0.02688% respectively per 1p cut. This result is basically consistent with Basic Rate cut result of Foreman-Peck and Zhou (2019). In their research, the rising speed of GDP per capita from short to long run is higher when the scale of tax cut is larger, however the efficiency of the stimulation is relatively low in terms of per 1p change. When the Basic Rate is cut from 20% to 10%, GDP per capita rises only 0.0142% per 1p in year 9 and 0.015% per 1p in year 10. Comparatively, when it is cut only to 17.5%, GDP per capita rises 0.01657% and 0.01812% per 1p respectively. Considering it can reach 0.06% for the Higher Rate and 0.01% for the Additional Rate along the same years, a comprehensive tax cut across all bands may generates greater but in no way far from around 0.02% rise, because either Basic Rate taxpayer income or tax revenue accounts for the majority (approximately 86%) of all bands in Wales[28]. The simulation results of GDP per capita in this section may still take around 10 years to reach applied with their results context.

Although the 5p cut of Income tax has positive economic effects only when in the medium and long run with low elasticity, a 5p tax hike would not make better results as positive effects only appear in the short run since the signs of effects are mirrored to the tax cut. When the elasticity equals 1, the tax hike generates mirror figures and could have positive effects across all time horizons, however the effect scales do not exceed those of tax cut in the medium to long run with 0.5 elasticity. Therefore, although a 5p Income tax cut only leads to quite a limited boost, an opposite 5p tax hike is not better off.

6.8 Conclusion

Based on the introduction of GAMS software and the modelling procedures required in its programming context, this chapter solves the model for both base run and simulation run. The regional CGE model developed in this study can be solved to replicate the benchmark of the SAM given a reasonable tolerance level, and can also be solved generally given tax policy shocks. SDLT, Corporation tax and Income tax are simulated in this chapter with different elasticity values and different tax rate variation scenarios.

[28] See Figure 2.13 and Figure 2.14 in Chapter 2.

For all the taxes, the simulation results generally give negative effects in the short run, and only in the medium to long run there appears expected reasonable results. Except that Corporation tax effects are relatively small, results of SDLT and Income tax are generally consistent with recent relative literature given low elasticity value.

The unity elasticity of substitution tends to make results converge across all time frames, while only under other elasticities the results diverge across time frames. The elasticity of 0.5 and 1.5 represent low and high substitutability respectively, and the unity elasticity may stands on a balance point that causes relative changes of factors tend to cancel off. When the elasticity becomes larger, it seems the model becomes more unstable, especially for the long run frame. It may be due to the existence of many non-linearity equations and distortions from relative scales between inter-sectoral transactions. Generally, the government revenue and spending are less sensitive to the elasticity, at the same time the GVA and GDP are relatively more sensitive due to sensitive change of their factor income components to different tax variations.

Compared to the elasticity of 1.5, the elasticity of 0.5 always generates more reasonable results no matter which tax effect is simulated. This implies that it is important to avoid over-substitution effect between production factors so as to produce better results in response of a tax variation shock. A lower elasticity represents lower substitutability and higher complementarity that one factor’s change does not substitute too much for other factors so that the value added structure may not be crippled given the shock. A less skilled labour stock may not be well matched but rather compromised by too much high-end capital investment and weakens consumption on the demand side, because unskilled labour implies high substitutability. With a high degree of substitutability, land-intensive industries may also over-substitute other two factors which are supposed to be pillars of the economy and hence cannot significantly enhance the economy given a stamp duty cut. Lower substitutability between factors seems more efficient in optimizing the factor structure in response to tax policy shocks and beneficial to the whole economy.

CHAPTER 7 CONCLUSION

Under the background of ongoing regional tax devolution in Wales, the development of new regional economic models has been needed to understand tax policy variations. For Wales, a series of Input-Output tables have been developed and employed to assess the significance of different industrial activities, to examine issues of infrastructure improvement and to support policy development in Wales.

However, Input-Output table are limited in economic modelling of tax effects as it lacks the role of price which is nexus in connecting taxes and major macroeconomic variables. As a result, in this thesis a Computable General Equilibrium model of Wales is developed to examine the tax effects. While the project was initially focused around developing a framework in which to understand the economic effects of SDLT policy variation in Wales, it is expected that the model developed would also have a wider implication to other types of tax variation in the region. Hence, this regional CGE model incorporates a number of devolved and non-devolved taxes as a basic model framework for possible investigation and analysis of other taxes in the future.

In this thesis, a Social Accounting Matrix is also constructed as the benchmark database required by the CGE model. It is a logical framework to arrange all the transaction data of economic agents. It can be seen as a 'snapshot' describing comprehensively the economic structure and activities of economic agents in a particular period. The base year for the SAM is 2013. The SAM developed and used in this thesis features 21 sectors, 1 representative household, 2 external agents, 7 types of taxes and 3 production factors. Several methods have been applied to balance the raw SAM table which compiles various sources of data. Unknown model parameters are calibrated by the data information contained in the SAM.

Base on the SAM, the regional CGE model is developed concerning production behaviour, government and household behaviour, trade behaviour, and macroeconomic closures that depicts both supply and demand side of the economy as a general equilibrium model. Different functional forms including the Cobb-Douglas function, the Leontief Input-Output (I-O) function, the Constant Elasticity of Substitution (CES) function and the Constant Elasticity of Transformation (CET) function are adopted in the model.

The model can be solved to replicate the benchmark SAM. This provides a solid foundation for further progress into simulations. The simulation is conducted regarding three taxes: SDLT, Corporation tax and Income tax. The SDLT rate is modelled as two separate rates: residential and non-residential effective rates. In reality the tax rates are both systems of multiple rates corresponding to multiple real estate transaction price bands, it can then only be modelled by the single effective rate for each system. The Corporation tax and Income tax are also simulated with their single effective rate. The simulation designs different tax variation scenarios to observe the impact. The impact of the SDLT variation is generated through price representing the true transaction cost of real estate properties regardless of types of property rights. The price change will effect both the supply and demand side of the real estate sector. It will also spread further to the supply and demand of all other sectors through the system of input-output inter-sectoral connections in the product market. The impact of the Corporation tax is generated mainly from the supply side as capital using cost. The Income tax effects are realized through impacts on both supply side as labour cost and demand side as via households revenues and expenditures. As a result, the ramifications on major macroeconomic indicators following the policy variations can be investigated.

The simulation captures the tax effects in the short, medium, and long run. In the short run, all factors are immobile across sectors; they are assumed fixed in each sector and their factor prices may vary across sectors following the policy shock. In the medium run, while the land factor is still sectorally fixed, labour and capital are free to move across regional sectors and the total stock for them is regionally fixed; labour and capital prices converge across sectors to a new level and there is only one price for each of them. In the long-run, the stock for labour and capital is unconstrained and they can freely move across region borders, so their factor prices recover to their original level as in the benchmark. The only exception is land where total land is still fixed but non-residential land is flexible across sectors.

The results of SDLT variation effects generally suggest that narrowing the gap between residential and non-residential SDLT rates has slightly more impact than simply cut of both rates. In this case, the mutual drag between residential and non-residential property prices can be observed given opposite tax variation direction for them. Significant results tend to appear in the medium run and then resolve in the long run when elasticity of substitution is low. A high elasticity only presents weak effects in the long run. As in the long run both labour and capital are free to move across regional borders, a stimulation for land use may over-substitute them which are supposed to be pillars for the whole economy. The factor structure then loses balance and is incapable to maintain any significant effects. From the policy view, such tax variation effect may not last in a long time horizon, however, this is simulation results from a single shock and it is assumed there is no other shock emerging across all time frames. In reality, such situation will not happen since any policy or economic shock could kick in any time. Therefore, from a practical perspective the medium run results are even more referable in informing tax policies.

Generally, the results of all taxes imply that a relatively lower elasticity of substitution tends to deliver more significant economy-wide effects than a high elasticity. This may relate to the factor structure within value added. An example of low elasticity of substitution is high value-added employment, such as services or skilled manufacturing. With a low elasticity, other factors are not that easy to be over-substituted by the factor that is stimulated by a tax cut, and vice versa. Otherwise, this will result in proportional imbalance between factor incomes and encumber output of the regional economy through their connections with the demand side.

For example, highly skilled labour may corresponds to lower elasticity of substitution that they are less substitutable and more complemented to profitable high-end capital investment. On the contrary, if the region has large portion of unskilled labour, they could be heavily substituted and raise unemployment. This will further affect household consumption and the whole economy. Hence, only in the low elasticity case can factors relatively complement each other to avoid a crippled economy. High capital income matched with low labour income may increase the economic inequality and weakens the demand side, and low capital income matched with high labour income may hinder industrial escalation and dampens the supply side. A balanced economy has more chance to sustain and develop.

From the policy view, a mixed tax policy combining variations of different taxes may generate more significant effects. Such policy combination should ensure the policy objects are matchable and avoid over-substitution between factors. For example, as a Corporation tax cut may attract more investment on more profitable projects which requires more skilled labour force, it may be better matched with tax cut on Higher Rate of Income tax which those skilled labour benefits from. This can also be matched with non-fiscal labour market policies.

This model is a static, single-regional CGE model. As it is the outcome of the first independent attempt in developing the model and the associated SAM for tax variation analysis in Wales, the work still possesses large space for further improvement and generalization. Dynamics is considered to be an option adding into the model, which will deliver the time path of the model in response of policy shocks. Inter-regional mobility of factors plays a key role in such a dynamic model, in which the scale of labour and capital migration across the regional border resulted from inter-regional tax rate gap needs to be quantified. Then the regional factor stock will evolve to trigger further changes of all endogenous variables along the time path. However, Wales has been a relatively new devolved economy that has little experience in manipulating devolved taxes and hence insufficient historical data to use. A specific regression model addressing the dynamic relationship between factors' inter-regional migration and all devolved taxes will further contribute to this research area in the future.

This thesis has developed and applied a relatively generalized CGE model to address multiple tax variation issues. However, this is to some extent a compromise to regional data deficiency, especially which relates to tax bases. Without such data constraint the model can be adjusted in terms of scale, dimension, and even model types to account for unique characteristics of each tax. For example, if there is sufficient aggregate transaction value data regarding the buyer and seller types and the sectors they belonged for each property type, the model dimension can be significantly shrunk to highlight activities of those property-transaction-relative sectors and agents. This can be based on a simplified SAM, or a particular real estate satellite account, or even no SAM but only a detached model. Given current data and theoretical constraints, the model has to be imposed a number of relative assumptions. This should be borne in mind for results explanation and the results might be rather directionally referable for their policy implication.

The Income tax simulation can be largely improved given detailed household expenditure by different taxable income band. Based on this, the sector dimension can be reduced but dimensions of the demand side can be accounted for more accurately. With only one representative household and one consumption style in this model, the simulation results might be limited in accuracy but provides more insight in structural effects on the economy. Both Corporation tax and Income tax may brings migration effect on capital and labour movement, a bi-regional model might also be a choice subject to data sufficiency. In addition, a breakdown of VAT-in and VAT-out for sectors in Wales is preferred to get an insight into the high value-added sectors of the economy.

The CGE modelling is a useful tool in the sense that it bridges the I-O analysis and general equilibrium setting by activating the role of price. This may provide deeper insight when it is applied for tax policy analysis, as a tax takes effect on prices in most cases. Yet for the CGE modelling in a regional context, there are always practical difficulties given data constraints. The accuracy of simulation results and dynamics are compromised to some extent, making it deliver limited impact on regional tax policy development. Although the regional CGE modelling is less powerful than modelling a national economy without data constraints, however, it still delivers great value especially in revealing the story behind the supply side under a price shock. This makes possible the interaction between the supply side and demand side, and delivers unique information rather than from only one side of the economy.

In particular, this research is only a rough attempt in regional CGE modelling of the Welsh economy regarding tax variation issues. Although the simulation results may be weakened by some compelling model settings and assumptions due to data deficiency, we can still draw some policy implication from structural effects on the value added through cross-elasticity comparisons. It is expected that this study may have a chance in helping evaluate and recognize the potential economic incidence of policy variation of these taxes, and sheds some light on tax policy development in the new devolved tax regime.

REFERENCE

Allan, G. et al. 2014. The economic and environmental impact of a carbon tax for Scotland: a computable general equilibrium analysis. *Ecological Economics* 100, pp. 40–50. doi: 10.1016/j.ecolecon.2014.01.012.

Arndt, C. and Lewis, J.D. 2001. The HIV/AIDS pandemic in South Africa: sectoral impacts and unemployment. *Journal of International Development* 13(4), pp. 427–449.

Babiker, M. et al. 2009. Forward-looking versus recursive-dynamic modeling in climate policy analysis: a comparison. *Economic Modelling* 26(6), pp. 1341–1354. doi: 10.1016/j.econmod.2009.06.009.

Bandara, J.S. 1991. Computable general equilibrium models for development policy analysis in LDCs. *Journal of economic surveys* 5(1), pp. 3–69.

Barry, S. 2018. *Wales' booming construction sector predicted to create 12,000 new jobs over the next five years*. Cardiff: WalesOnline. Available at: https://www.walesonline.co.uk/business/business-news/wales-booming-construction-sector-predicted-14244679.

Bayar, A. et al. 2006. A Computable General Equilibrium Modeling Platform for the Azorean Economy: A simple approach with international trade.

Berck, P. and Dabalen, A. 1995. A CGE model for California tax policy analysis: a review of literature. *Department of Agricultural & Resource Economics, UCB* . Available at: http://escholarship.org/uc/item/7zv6b61x.pdf [Accessed: 20 September 2016].

Brand, S. et al. 1998. *Welsh input-output tables for 1995*. Cardiff: University of Wales Press.

Bryan, J. et al. 2004. *Welsh input-output tables for 2000*. Cardiff: Welsh Economy Research Unit, Cardiff University.

Bryan, J. et al. 2010. *The input-output tables for Wales 2007*. Cardiff: Welsh Economy Research Unit, Cardiff University.

Chumacero, R.A. and Hebbel, K.S. 2004. General equilibrium models: an overview. *Documentos de Trabajo (Banco Central de Chile)* (307), p. 1.

Cox, A. et al. 2013. *National strategic skills audit for Wales 2012*. Caerphilly: Welsh Government Social Research. Available at: https://gov.wales/docs/caecd/research/130228-national-strategic-skills-audit-2012.pdf.

Crawley, A. 2015. *Economic data and modelling for Wales in a changing union: FSB Wales Round Table Seminar Series No. 3.* Available at: https://nls.ldls.org.uk/welcome.html?ark:/81055/vdc_100038568021.0x000001 [Accessed: 3 February 2017].

Davidoff, I. and Leigh, A. 2013. How do stamp duties affect the housing market? *Economic Record* 89(286), pp. 396–410.

Day, G. 2002. *Making sense of Wales*. Cardiff: University of Wales Press.
Department for Environment, Food and Rural Affairs et al. 2015. *Agriculture in the United Kingdom 2014*.

Devarajan, S. and Robinson, S. 2002. The influence of computable general equilibrium models on policy. In: *Proceedings of the conference on frontiers in applied general equilibrium modeling*. Yale University, New Haven: Cowles Foundation. Available at: http://anciensite.pep-net.org/fileadmin/medias/pdf/DevarajanRobinson.pdf [Accessed: 20 July 2015].

Dickins, S. 2017. *Wales still bottom of UK's Gross Value Added table*. London: BBC. Available at: https://www.bbc.co.uk/news/uk-wales-42425797#.

Drakeford, M. 2017. *Repositioning of the national procurement service and value: Wales*. Cardiff: Welsh Assembly Government. Available at: http://www.assembly.wales/ministerial%20statements%20documents/repositioning%20of%20the%20national%20procurement%20service%20and%20value%20wales/170921%20-%20md%20-%20written%20statement%20-%20repositioning%20of%20the%20national%20procurement%20service%20and%20value%20wales%20-%20english.doc.

Econtech, K. P. M. G. 2010. *CGE Analysis of the Current Australian Tax System. Final Report.* Available at: http://taxreview.treasury.gov.au/content/html/commissioned_work/downloads/KPMG_Econtech_Efficiency%20of%20Taxes_Final_Report.pdf [Accessed: 10 October 2015].

Ergas, H. and Pincus, J. 2014. Have mining royalties been beneficial to Australia? *Economic Papers: A Journal of Applied Economics and Policy* 33(1), pp. 13–28. doi: 10.1111/1759-3441.12068.

Ezaki, M. 2006. CGE model and its micro and macro closures. *Computable General Equilibrium Approaches in Urban and Regional Policy Studies," World Scientific* , pp. 9–24.

Falkus, M. and Gillingham, J. 1987. *Historical atlas of Britain*. London: Kingfisher.

Ferguson, L. et al. 2003. The regional distribution of public expendictures in the UK: an exposition and critique of the Barnett formula. In: *Regional Public Finance, European Research in Regional Science*. London: Pion Ltd, pp. 191–207.

Ferguson, L. et al. 2007. The impact of the Barnett formula on the Scottish economy: endogenous population and variable formula proportions. *Environment and Planning A* 39(12), pp. 3008–3027. doi: 10.1068/a38254.

Foreman-Peck, J. and Lungu, L. 2009. Fiscal devolution and dependency. *Applied Economics* 41(7), pp. 815–828. doi: 10.1080/00036840601019182.

Foreman-Peck, J. and Zhou, P. 2019. Devolving fiscal policy: migration and tax yields. *Regional Studies* 53, pp. 1–10. doi: https://doi.org/10.1080/00343404.2019.1602256.

Giesecke, J. 2003. Targeting regional output with state government fiscal instruments: a dynamic multi-regional cge analysis. *Australian Economic Papers* 42(2), pp. 214–233.

Giesecke, J.A. 2011. Development of a large-scale single US region CGE model using IMPLAN data: a Los Angeles County example with a productivity shock application. *Spatial Economic Analysis* 6(3), pp. 331–350. doi: 10.1080/17421772.2011.586722.

Gilmartin, M. et al. 2013. Regional policy spillovers: the national impact of demand-side policy in an interregional model of the UK economy. *Environment and Planning A* 45(4), pp. 814–834. doi: 10.1068/a44355.

Groenewold, N. et al. 2001. *Competitive federalism: a political-economy general equilibrium approach*. Department of Economics, University of Western Australia.

Harrigan, F. et al. 1991. AMOS: a macro-micro model of Scotland. *Economic Modelling* 8(4), pp. 424–479. doi: 10.1016/0264-9993(91)90028-M.

Harrigan, F. et al. 1992. Imperfect competition in regional labour markets: a computable general equilibrium analysis. *Environment and Planning A* 24(10), pp. 1463–1481.

Harrigan, F. and McGregor, P.G. 1989. Neoclassical and Keynesian perspectives on the regional macro-economy: a computable general equilibrium approach. *Journal of Regional Science* 29(4), pp. 555–573. doi: 10.1111/j.1467-9787.1989.tb01244.x.

Harrigan, F.J. et al. 1980. Simulating the structure of a regional economy. *Environment and Planning A* 12(8), pp. 927–936.

HESA. 2018. *Who's studying in HE?* Available at: https://www.hesa.ac.uk/data-and-analysis/students/whos-in-he.

Higgs, G. ed. 2003. *Rural services and social exclusion*. London: Pion.

Hill, S. and Roberts, A. 1996. *Welsh input-output tables for 1994*. Cardiff: University of Wales Press.

Hill, S. and Roberts, A. 2001. Welsh input-output tables for 1996. Cardiff: South East Wales Economic Forum

Hirte, G. 1998. Welfare effects of regional income taxes Results of an interregional CGE analysis for Germany. *The Annals of Regional Science* 32(2), pp. 201–219.

HM Land Registry. 2015a. *Data derived from: Land Registry Price Paid Data - Standard Reports*. Available at: http://landregistry.data.gov.uk/app/standard-reports/download-report?utf8=✓&report=avgPrice&areaType=region&area=WALES&aggregate=none&period%5B%5D=2013&age=new.

HM Land Registry. 2015b. *Data derived from: Land Registry Price Paid Data - Standard Reports*. Available at: http://landregistry.data.gov.uk/app/standard-reports/download-report?utf8=✓&report=avgPrice&areaType=region&area=WALES&aggregate=none&period%5B%5D=2013&age=old.

HMRC. 2015a. *Annual UK property transaction statistics*. Available at: https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/438425/2015_AUKPTS_circ.pdf.

HMRC. 2015b. *UK regional trade in goods statistics: quarter 4, 2014 press release*. Available at: https://www.uktradeinfo.com/Statistics/RTS/RTS%20Releases/RTS_Q4_2014.xls.

HMRC. 2016. *Annual Stamp Tax statistics 2015-16*. Available at: https://www.google.co.uk/url?sa=t&rct=j&q=&esrc=s&source=web&cd=1&cad=rja&uact=8&ved=2ahUKEwiOr9Kyz_rfAhVKI1AKHb_NCT8QFjAAegQIChAC&url=https%3A%2F%2Fwww.gov.uk%2Fgovernment%2Fuploads%2Fsystem%2Fuploads%2Fattachment_data%2Ffile%2F556321%2FAnnualStampTaxes-Release-Sep16.pdf&usg=AOvVaw3RPlQ7g6TI99uWYIi6hcmC.

HMRC. 2018. *Personal incomes statistics: 2015-16*. Available at: https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/694507/National_Statistics_T3_12_to_T3_15a_publication_2015_16_-_FINAL.pdf.

Holmøy, E. 2014. The equilibrium relationship between public and total employment. The importance of endogenous non-labour income. *Discussion Paper No.779* . Available at: http://ideas.repec.org/p/ssb/dispap/779.html.

Holtham Commission. 2009. *Funding devolved government in Wales: Barnett & beyond*. Available at: https://gov.wales/docs/icffw/report/090708barnettfullen.pdf.

Holtham Commission. 2010. *Fairness and accountability: a new funding settlement for Wales*. Available at: https://gov.wales/docs/icffw/report/100705fundingsettlementfullen.pdf.

Ifan, G. and Poole, E.G. 2018. *The Welsh tax base: risks and opportunities after fiscal devolution*. Cardiff: Wales Governance Centre.

Ioannidis, C. et al. 1995. Business cycle analysis and forecasting with a structural vector autoregression model for Wales. *Journal of Forecasting* 14(3), pp. 251–265.

Ireson, R. and Tomkins, C. 1978. *Inter-regional input output tables for Wales and the rest of the UK, 1968*. London: HMSO.

Jefferis, K. et al. 2008. Macroeconomic and household-level impacts of HIV/AIDS in Botswana. *Aids* 22, pp. S113–S119.

Jenkins, P. 1992. *A history of modern Wales, 1536-1990*. Harlow: Longman.

Johansen, L. 1960. *A multi-sectoral study of economic growth*. North-Holland Publishing Company. Available at: http://books.google.co.uk/books?id=JhTRAAAAMAAJ.

Jones, E.T. et al. 2017. *Independent scrutiny and assurance of devolved tax forecasts for Wales: final report*. Available at: http://senedd.assembly.wales/documents/s66969/FIN524-17%20P2%20Bangor%20University%20Independent%20scrutiny%20and%20assurance%20of%20devolved%20tax%20forecasts%20for%20W.pdf.

Keogh-Brown, M. et al. 2009. Global CGE modelling of antibiotic resistance: an application of the GLOBE model. In: *Proceedings of the 12th annual conference on global economic analysis*. Santiago, Chile

Lecca, P. et al. 2013. Forward-looking and myopic regional computable general equilibrium models: how significant is the distinction? *Economic Modelling* 31, pp. 160–176. doi: 10.1016/j.econmod.2012.11.010.

Lecca, P. et al. 2014. Balanced budget multipliers for small open regions within a federal system: evidence from the Scottish variable rate of income tax. *Journal of Regional Science* , p. n/a-n/a. doi: 10.1111/jors.12113.

Lisenkova, K. et al. 2006. Macro-economic impacts of demographic change in Scotland. *Final Report to the ESRC (Grant No. RES-342-25-0002), Department of Economics, University of Strathclyde, Glasgow, June* . Available at: http://www.researchgate.net/profile/Peter_Mcgregor/publication/268341213_Macroeconomic_Impacts_of_Demographic_Change_in_Scotland/links/54fec71d0cf2eaf210b3df0f.pdf [Accessed: 20 July 2015].

Lisenkova, K. et al. 2010. Scotland the grey: a linked demographic–computable general equilibrium (CGE) analysis of the impact of population ageing and decline. *Regional Studies* 44(10), pp. 1351–1368. doi: 10.1080/00343400701654228.

Livesey, D.A. 1974a. Reviewed work: The computation of economic equilibria. by H. Scarf. *The Economic Journal* 84(336), pp. 987–989. doi: 10.2307/2230583.

Livesey, D.A. 1974b. The importance of numerical algorithms for solving economic optimization problems. *International Journal of Systems Science* 5(5), pp. 435–451. doi: 10.1080/00207727408920112.

Livesey, D.A. and Scarf, H. 1974. The computation of economic equilibria. *The Economic Journal* 84(336), p. 987. doi: 10.2307/2230583.

London Economics. 2002. *Investigating the use of regional economic forecasting models in Wales*.

Luchinskaya, D. et al. 2017. *Welsh government budgetary trade-offs: looking forward to 2021-22*. Cardiff: Wales Public Services 2025.

Luo, Y. et al. 2012. The overseas listing puzzle: Post-IPO performance of Chinese stocks and ADRs in the U.S. market. *Journal of Multinational Financial Management* 22(5), pp. 193–211. doi: 10.1016/j.mulfin.2012.06.008.

Massey, G. 2009. *Review of international business Wales*. Cardiff: Welsh Assembly Government.

Mcgregor, P. and Swales, K. 2005. Economics of devolution/decentralization in the UK: some questions and answers. *Regional Studies* 39(4), pp. 477–494. doi: 10.1080/00343400500128523.

McGregor, P.G. et al. 1996. A long-run interpretation of regional input-output analysis. *Journal of Regional Science* 36(3), pp. 479–501. doi: 10.1111/j.1467-9787.1996.tb01113.x.

McGregor, P.G. et al. 2007. The macroeconomic consequences of Scottish fiscal autonomy: inverted Haavelemo effects in a general equilibrium analysis of the tartan tax. In: *Proceedings of CPPR Conference-Fiscal Issues in Scotland: Lessons from Home and Abroad*. Blackwell. Available at: http://strathprints.strath.ac.uk/8748/1/macroeconomic_consequences.pdf [Accessed: 19 July 2015].

McGregor, P.G. et al. 2010. Innovations in regional computable general equilibrium (CGE) modelling. *Regional Studies* 44(10), pp. 1307–1310. doi: 10.1080/00343404.2010.530889.

Minford, P. et al. 1994. An econometric model of Merseyside: validation and policy simulations. *Regional Studies* 28(6), pp. 563–575. doi: 10.1080/00343409412331348486.

Minford, P. and Riley, J. 1995. Wales from the supply side: a classical model. *Welsh Economic Review* 8(1), pp. 65–71.

Morgan, K.O. 2002. *Rebirth of a nation: a history of modern Wales*. Oxford: Oxford University Press.

Narayanan, B. et al. eds. 2015. *Global trade, assistance, and production: the GTAP 9 Data Base*. West Lafayette: Center for Global Trade Analysis, Purdue University. Available at: http://www.gtap.agecon.purdue.edu/databases/v9/v9_doco.asp.

Nevin, E. et al. 1966. *The structure of the Welsh economy*. Wales University Press.

Nielsen, C.P. et al. 2001. Genetically modified foods, trade and developing countries. Available at: http://www.sciencedirect.com/science/article/pii/S0305750X01000407 [Accessed: 20 July 2015].

Nyasulu, A. 2015. Owner occupied dwellings - conceptual issues. Available at: http://www.unsiap.or.jp/e-learning/el_material/sna/1508_gdp_kor/Presentation/2_3A_Owner_Occupied_Dwelling_Conceptual_Issues.pdf.

Office for Budget Responsibility. 2016. *Economic and fiscal outlook: devolved taxes forecast*. Available at: https://cdn.obr.uk/March2016EFO.pdf.

Office for National Statistics. 1998. *United Kingdom: National Accounts concepts, sources and methods*. London: The Stationery Office.

Office for National Statistics. 2014a. *Family spending – 2014: Table 2.5: housing expenditure by UK countries and regions, 2013*. Available at:

https://www.ons.gov.uk/file?uri=/peoplepopulationandcommunity/personalandhouseholdfinances/incomeandwealth/compendium/familyspending/2014-12-02/familyspending2014referencetables/table25final2013_tcm77-383850.xls.

Office for National Statistics. 2014b. *UK Input-Output analytical tables - detailed, 2010*. Available at: https://webarchive.nationalarchives.gov.uk/20150906195520/http://www.ons.gov.uk/ons/publications/re-reference-tables.html?edition=tcm%3A77-346757.

Office for National Statistics. 2015a. *Annual business survey - 2013 regional results*. Available at: https://www.ons.gov.uk/businessindustryandtrade/business/businessservices/bulletins/uknonfinancialbusinesseconomyannualbusinesssurvey/2015-07-23.

Office for National Statistics. 2015b. *Family spending – 2015: Table A35: detailed household expenditure by UK countries and regions, 2012 to 2014*. Available at: https://www.ons.gov.uk/file?uri=/peoplepopulationandcommunity/personalandhouseholdfinances/incomeandwealth/compendium/familyspending/2015/familyspending2015referencetables/a35final2014_tcm77-423498.xls.

Office for National Statistics. 2015c. *Family spending – 2015: Table A41: income and source of income by UK countries and regions, 2012 to 2014*. Available at: https://www.ons.gov.uk/file?uri=/peoplepopulationandcommunity/personalandhouseholdfinances/incomeandwealth/compendium/familyspending/2015/familyspending2015referencetables/a41final2014_tcm77-423513.xls.

Office for National Statistics. 2015d. *Output in the construction industry*. Available at: https://www.ons.gov.uk/file?uri=/businessindustryandtrade/constructionindustry/datasets/outputintheconstructionindustry/current/previous/v1/outputintheconstructionindustryaugust2015_tcm77-419403.xls.

Office for National Statistics. 2015e. *Regional Gross Value Added (income approach) reference tables*. Available at: https://webarchive.nationalarchives.gov.uk/20160106003337/http://www.ons.gov.uk/ons/rel/regional-accounts/regional-gross-value-added--income-approach-/december-2015/rft-table-1.xls.

Office for National Statistics. 2015f. United Kingdom National Accounts, The Blue Book 2015 Edition. Available at: https://www.ons.gov.uk/economy/grossdomesticproductgdp/compendium/unitedkingdomnationalaccountsthebluebook/2015-10-30.

Office for National Statistics. 2016a. *Gross fixed capital formation - by sector and asset*. Available at: https://www.ons.gov.uk/economy/grossdomesticproductgdp/datasets/grossfixedcapitalformationbysectorandasset.

Office for National Statistics. 2016b. *Regional gross fixed capital formation, 2000 to 2013*. Available at:

https://www.ons.gov.uk/economy/grossvalueaddedgva/adhocs/005244regionalgrossfixedcapitalformation2000to2013.

Office for National Statistics. 2017a. *Graduates in the UK labour market: 2017*. Available at: https://www.google.co.uk/url?sa=t&rct=j&q=&esrc=s&source=web&cd=1&ved=2ahUKEwjVo-Tmgc7cAhUJXMAKHdPNCHgQFjAAegQICBAC&url=https%3A%2F%2Fwww.ons.gov.uk%2Femploymentandlabourmarket%2Fpeopleinwork%2Femploymentandemployeetypes%2Farticles%2Fgraduatesintheuklabourmarket%2F2017%2Fpdf&usg=AOvVaw2VBb0p9d6jJ7AFYQfP5nxF.

Office for National Statistics. 2017b. *Nominal regional gross value added (balanced) per head and income components*. Available at: https://www.ons.gov.uk/file?uri=/economy/grossvalueaddedgva/datasets/nominalregionalgrossvalueaddedbalancedperheadandincomecomponents/current/nominalregionalgvabperheadandincomecomponentsv2.xlsx.

Office for National Statistics. 2017c. *Regional gross value added (GVA) and sub-regional gross value added*. Available at: https://gov.wales/statistics-and-research/regional-gross-value-added-sub-regional-gross-value-added/?lang=en.

Office for National Statistics. 2018a. *Country and regional public sector finances, FYE 2017: net fiscal balance tables*. Available at: https://www.ons.gov.uk/file?uri=/economy/governmentpublicsectorandtaxes/publicsectorfinance/datasets/countryandregionalpublicsectorfinancesnetfiscalbalancetables/financialyearendingmarch2017/netfiscalbalancefinal.xlsx.

Office for National Statistics. 2018b. *UK Stamp Tax statistics 2017 to 2018 - tables*. Available at: https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/743872/ASTP-Release-Tables-Sept18.xlsx.

Owen, D. 2018a. The Welsh economy VI: the service sector. *State of Wales* 29 October. Available at: https://stateofwales.com/2018/10/the-welsh-economy-vi-the-service-sector/.

Owen, D. 2018b. The Welsh economy VIII: the public & social sectors. *State of Wales* 29 October. Available at: https://stateofwales.com/2018/10/the-welsh-economy-viii-the-public-social-sectors/.

Partridge, M.D. and Rickman, D.S. 1998. Regional computable general equilibrium modeling: a survey and critical appraisal. *International Regional Science Review* 21(3), pp. 205–248. doi: 10.1177/016001769802100301.

Petersen, T.W. 1997. An introduction to CGE-modelling and an illustrative application to Eastern Integration with the EU. *No 199701, DREAM Working Paper Series, Danish Rational Economic Agents Model, DREAM* . Available at: http://EconPapers.repec.org/RePEc:dra:wpaper:199801.

Poole, E.G. et al. 2016. *Government expenditure and revenue Wales 2016*. Cardiff: Wales Governance Centre.

Pratt, S. et al. 2013. Dynamic general equilibrium model with uncertainty: uncertainty regarding the future path of the economy. *Economic Modelling* 32, pp. 429–439. doi: 10.1016/j.econmod.2013.02.034.

Richard Commission. 2004. *Report of the Richard Commission*. Available at: https://webarchive.nationalarchives.gov.uk/20100404200945/http:/www.richardcommission.gov.uk/content/finalreport/report-e.pdf.

Roberts, D. et al. 2010. WP2 report: modelling approach, SAM construction and model calibration. *RURAL ECMOD Research Project*

Robinson, S. et al. 1999. From stylized to applied models: building multisector CGE models for policy analysis. *The North American Journal of Economics and Finance* 10(1), pp. 5–38. doi: 10.1016/S1062-9408(99)00014-5.

Royal Commission on the Constitution. 1973. *Royal Commission on the Constitution 1969-73*. London: HMSO.

Sarkissian, S. and Schill, M.J. 2009. Are There Permanent Valuation Gains to Overseas Listing? *The Review of Financial Studies* 22(1), pp. 371–412. doi: 10.1093/rfs/hhn003.

Scarf, H. 1967a. On the computation of equilibrium prices. In: Fellner, W. ed. *Ten essays in honor Irving Fisher*. New York: Wiley

Scarf, H. 1967b. The approximation of fixed points of a continuous mapping. *SIAM Journal on Applied Mathematics* 15(5), pp. 1328–1343. doi: 10.2307/2099173.

Scarf, H. 1973. *The computation of economic equilibria*. New Haven and London: Yale University Press. Available at: http://books.google.co.uk/books?id=sJ1EAAAAIAAJ.

Seely, A. 2018. *Corporation tax in Northern Ireland*. London: House of Commons. Available at: https://researchbriefings.parliament.uk/ResearchBriefing/Summary/SN07078.

Silk Commission. 2012. *Empowerment and responsibility: financial powers to strengthen Wales*. Available at: https://webarchive.nationalarchives.gov.uk/20140605075525/http://commissionondevolutioninwales.independent.gov.uk/files/2013/01/English-WEB-main-report1.pdf.

Silk Commission. 2014. *Empowerment and responsibility: legislative powers to strengthen Wales*. Available at: https://webarchive.nationalarchives.gov.uk/20140605075522/http://commissionondevolutioninwales.independent.gov.uk/files/2014/03/Empowerment-Responsibility-Legislative-Powers-to-strengthen-Wales.pdf.

Sim, A.B. and Pandian, J.R. 2003. Emerging Asian MNEs and Their Internationalization Strategies—Case Study Evidence on Taiwanese and Singaporean Firms. *Asia Pacific Journal of Management* 20(1), pp. 27–50.

Statistical Office of the European Communities and European Commission. 2013. *European system of accounts: ESA 2010*.

The World Bank. [no date]. What is the difference between purchaser prices, producer prices (VAP), and basic prices (VAB)? – World Bank Data Help Desk. Available at: https://datahelpdesk.worldbank.org/knowledgebase/articles/114947-what-is-the-difference-between-purchaser-prices-p [Accessed: 1 June 2017].

Tipples, C. 2016. *The future of Welsh Government funding*. Available at: https://seneddresearch.blog/2016/05/23/the-future-of-welsh-government-funding/.

TNS. 2015. *The GB tourist: statistics 2014*. Available at: https://gov.wales/docs/statistics/2015/150901-gb-tourist-statistics-2014-en.pdf.

Turner, K. et al. 2012. Econometric estimation of Armington import elasticities for a regional CGE model of the Illinois economy. *Economic Systems Research* 24(1), pp. 1–19. doi: 10.1080/09535314.2011.645800.

UK Government. 2013. *Empowerment and responsibility: devolving financial powers to Wales*. Available at: https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/259359/empowerment_and_responsibility_181113.pdf.

UK Government. 2014. *Wales Bill: financial empowerment and accountability*. Available at: https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/294470/Wales_Bill_Command_Paper_-_English.pdf.

UK Government. 2015. *Powers for a purpose: towards a lasting devolution settlement for Wales*. Available at: https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/408587/47683_CM9020_ENGLISH.pdf.

United Nations et al. eds. 2009. *System of national accounts 2008*. New York: United Nations.

Universities Wales. 2017. The economic impact of Welsh Universities. Available at: http://www.uniswales.ac.uk/wp/media/Economic-Impact-Infographic-WE_01.pdf.

Wanhill, S. 1980. An econometric model of Wales. In: Revell, J. ed. *Bangor occasional papers in economics*. Cardiff: University of Wales Press

Welsh Government. 2005. Wales: a vibrant economy. Available at: https://web.archive.org/web/20101216020108/http://wales.gov.uk/deet/publications/bande/wave/wavee.pdf?lang=en.

Welsh Government. 2012. *The Welsh Government's response to the call for evidence by the Commission on Devolution in Wales*. Available at: https://webarchive.nationalarchives.gov.uk/20140324151452/http://commissionondevolutioninwales.independent.gov.uk/files/2012/02/Welsh-Government-English.pdf.

Welsh Government. 2016a. Financial and professional services sector performance. Available at: https://gov.wales/docs/det/publications/140930-fps-sector-performance-en.pdf.

Welsh Government. 2016b. Rural development plan for Wales 2007-2013: the strategic approach. Available at: https://gov.wales/topics/environmentcountryside/farmingandcountryside/cap/ruraldevelopment/rural-development-plan-for-wales-2007/?lang=en.

Welsh Government. 2016c. *The review of higher education funding and student finance arrangements in Wales: final report*. Cardiff: Welsh Assembly Government. Available at: https://beta.gov.wales/sites/default/files/publications/2018-02/higher-education-funding-final-report-en.pdf.

Welsh Government. 2017a. Creation industries. Available at: https://gov.wales/topics/businessandeconomy/our-priority-sectors/creative-industries/?lang=en.

Welsh Government. 2017b. ICT sector performance. Available at: https://gov.wales/docs/det/publications/140930-ict-sector-performance-en.pdf.

Welsh Government. 2017c. Life sciences sector. Available at: https://gov.wales/topics/businessandeconomy/our-priority-sectors/life-sciences/?lang=en.

Welsh Government. 2017d. School census results. Available at: https://gov.wales/docs/statistics/2017/170726-school-census-results-2017-en.pdf.

Welsh Government. 2017e. Sectors delivery plan. Available at: https://gov.wales/docs/det/publications/130125deliveryplanen.pdf.

Welsh Government. 2017f. *Welsh tax policy report: Autumn 2017*. Available at: http://senedd.assembly.wales/documents/s66970/FIN524-17%20P3%20Welsh%20tax%20policy%20report.pdf.

Welsh Government. 2018a. Construction begins on Advanced Manufacturing Research Institute in Broughton. Available at: https://gov.wales/newsroom/businessandeconomy/2018/180516-construction-begins-on-advanced-manufacturing-research-institute-in-broughton/?lang=en.

Welsh Government. 2018b. Further education, work-based learning and community learning in Wales, 2016/17. Available at: https://gov.wales/docs/statistics/2018/180207-further-education-work-based-learning-community-learning-2016-17-en.pdf.

Welsh Local Government Association. 2017. Constitution. Available at: https://www.wlga.wales/SharedFiles/Download.aspx?pageid=62&mid=665&fileid=398.

Williams, P. 2003. *The psychology of distance. Wales: one nation*. Cardiff: Welsh Academic Press.

Williamson, D. 2017. Public sector and private sector pay compared - as nurses in Wales threaten strike action. Available at: https://www.walesonline.co.uk/news/politics/public-sector-private-sector-pay-13603971.

APPENDIX

Appendix I A case study of fiscal policy simulations with AMOS

This appendix presents a case study of three fiscal policy simulations using AMOS. The AMOS CGE model is 'A Macro-Micro Model of Scotland'. Initially it was calibrated on 1989 data for the Scottish economy, a small and open economy. Although it functions as a CGE model, Harrigan *et al* (1991), who first introduced it, rather regard it as a modelling and simulation framework because of its inclusion of a range of behavioural assumptions reflected in equations that can be configured in various ways. Put plainly, the behavioural assumptions allow a modeller the options of different model closures and parameter values for appropriate applications. In essence, what AMOS provides is more of a modelling environment than a single model (Harrigan *et al,* 1991).

Here the present case study investigates the fiscal policy application of the AMOS. The AMOS configuration is detailed in the original work of Harrigan *et al* (1991). Rather than applying the initial version of the AMOS which is a single region static modelling framework, we simulate the policy effect via a recursive dynamic variant of it, where the path of adjustment takes account of the aimed for policy change. Therefore, we are able to observe how the economy response evolves in the short-run, medium-run and long-run scenarios.

In this study, we aim to explore three cases of policy effects: pure government expenditure expansion, pure government revenue augmented through increasing the average income rate, and combining both under government budget balance. For the first two cases we are interested in the performance of the whole economy responding to the demand side expansion policy shock and supply side austerity policy shock, respectively. Finally the third case covers both a demand and supply side shock. That is increasing the average income tax rate and meanwhile facilitating additional fiscal expenditure with the balanced budget constraint. Besides, all the three cases are studied under three tax amenity parameter value settings: 0, 0.5 and 1. The parameter value of 0 and 1 are two extremes of the value range, where 1 means private households value government expenditure as a perfect substitute for their own. In contrast the value of 0 implies they do not consider government expenditure as a substitute but rather a complement.

AMOS Basics:

- Baseline equilibrium year: 1989
- 3 production sectors with relevant type of commodities: manufacturing, non-manufacturing traded, and non-traded/sheltered.
- 3 local transactors: households, firms and government.
- 2 external transactors: the rest of UK (RUK) and the rest of the world (ROW)
- 4 main components of final demand derived from the above transactors: household consumption, investment, government expenditure and exports.

Model Setup

The study model provided by Strathclyde University is wrapped into a software running with a menu style interface, hence the model details including functional forms, closures, constraints, parameters and simulation periods, etc., can all be set up via selecting available options and fulfilling intended parameter values. In this section we will introduce the common setup of the model for all the cases studied below. This is realized through the 'Set up' menu on the programme toolbar. There are basically five major blocks to set up: Closure, Technology, Trade, Technical progress and Time periods. We will check these sequentially below in more detail.

- Closure

This block concerns the macroeconomic closure of AMOS concerning different markets, constraints and identities to maintain balances and market as well as non-market clearing conditions.

a. Labour market. For this particular model all the labour market closures are applied with an aggregated labour force. As for the labour market, the AMOS provides six alternative closures, each of which reflects the type of labour market regime assumed to exist and the corresponding form of wage equations. The range of alternatives allows a modeller to make a decision over which one to choose according to regional labour market characteristics and conditions, and also to make comparisons between them.
    1) *Neo-classical.* The wage adjusts to equate labour demand and supply so as to realize continuous market clearing.
    2) *Keynesian*. This corresponds to a national bargaining regime where the nominal wage is determined exogenously out of the region and the aggregate labour supplies are of large excess capacity.
    3) *Real wage resistance*. The real wage is fixed through the nominal wage marking

up on the consumer price index.

4) *Sticky wages*. It should refer to the 'Regional Phillips curve' closure appeared in the original work of Harrigan *et al* (1991), that the nominal wage relates to consumer price index and unemployment.

5) *Regional BRW*. This is the one we apply throughout this case study, which is short for 'Regional Bargained Real Wage'. It assumes the regional real wage relies on workers' bargaining power and is inversely relating to the regional unemployment rate. The long-run elasticity of real wage with respect to unemployment is set as -0.113 and the speed of current real wage adjusting to the equilibrium level is set as 1.000 by default. The tax amenity parameter value is also specified here and as introduced above. We will experiment with three values in each case: 0, 0.5 and 1.

6) *Exogenous labour supply*. This is where a fixed proportional relation of the employment exists with working population.

b. Goods market. This study focuses on 'Competitive' goods market where price is simply equal to marginal cost, rather than 'Imperfect Competition' where prices are mark-ups over marginal costs. In fact, it is normal to assume perfect competition in the early stage of CGE models development. By far CGE models have been gradually not restricted to perfect competition assumption but also have accommodated imperfect competition conditions such as monopolistic competition. However, comparing the two goods market closures is not our emphasis here, hence we choose competitive market as default specification for simplicity.

c. Balance of payments constraint. This constraint is not our focus here so we simply set it to 'Passive' implying the balance of payments condition does not necessarily hold but is endogenously affected within the model.

d. Government budget constraint. This is where we can choose whether to hold the budget balance to facilitate this study of the three cases regarding fiscal policy. Therefore, this closure will be further specified individually in each of the following cases.

e. Economic growth. Here it concerns parameter values reflecting indigenous and migration-induced population growth and labour-augmenting technical progress. We will specify these elements further in detail.

f. Income tax increase. Again, we will specify the details further in the following cases.

- Technology

The technology block here refers to the nested multi-level production functions where cost-minimisation is imposed for the model to solve for the input factor choices that minimise production costs. The functional forms are generally of CES (Constant Elasticity of Substitution) form allowing substitution in response to relative price changes, with Cobb-Douglas and Leontief functions as special cases, whose elasticity of substitution equals 1 and 0, respectively.

The gross output is produced through value added and intermediate inputs, while the former is composed of labour and capital, the latter is composed of locally produced and imported goods and services. The setting of this block is through 'Individual' option to individually specify each nest of production. In this study, for gross output and value added, CES form is selected and the elasticity of substitution is set as 0.3 for all three sectors. For intermediate inputs, Leontief function is selected across local inputs.

- Trade

This block concerns exports to and imports from the RUK and ROW. The trade composites are linked through Armington elasticity, which are all set to 2.0 for all three sectors.

- Technical Progress

Here the default setting is maintained as Harrod Neutral (labour-augmenting technical process) out of Solow Neutral (capital-augmenting technical process) and Hicks Neutral (fixed ratio of marginal product of labour to marginal product of capital).

- Time Periods

This is where we can vary our simulation scenario to investigate the policy effect across different evolution stages. Note that whichever term we choose from short-run, medium or long-run, it is always a single-period simulation. Therefore, they are all one-off new equilibriums compared to the baseline that we cannot observe the adjustment process to any of the scenarios. Basically different terms correspond to different adjusting stages to labour force and capital stock. The adjustment mechanisms are explained below, and the detail specifications of the simulation windows will be arranged in each study case.

The adjustment process of the labour force to a new equilibrium is captured through net migration flows to update population stocks in each period. Again the adjustment periods are invisible and can only be observed in the multi-period simulation that can be configured in the 'Simulation' menu discussed later. The regional economy has no migration in the short-run so the zero net migration condition is in effect, while in the medium-run to long-run the net migration flows allow full adjustment of population to its new equilibrium so that the zero net migration condition is again reinstated. The net in-migration for each period is positively related to the real wage differential and negatively related to the unemployment rate differential between the regional and national economy. The migration model, which is based on that in the work of Harris and Todaro (1970), is presented as below:

$$m = \beta - 0.08(u_s - u_r) + 0.06(w_s - w_r)$$

where m is the net in-migration rate which is a proportion of the indigenous population, $(u_s - u_r)$ is the natural logarithms of the unemployment rate differential, and $(w_s - w_r)$ is the natural logarithms of the real wage differential. As for the parameters, β is pending calibrated and -0.08, 0.06 are the coefficient value as default of this model.

The capital adjustment process across periods is via investment equalling depreciation plus some fraction of the gap between the actual level, and the desired level of capital stock which is determined on cost-minimisation criteria. The mechanism of determining capital stock update is, similar to that of Tobin's q, by comparing between the capital rental rate and the user cost of capital, where the former represents the rental paid for physical capital in a competitive market, and the latter one is just the total cost to the firm of employing a unit of capital. If the user cost is exceeded by the rental rate, then there is an incentive to update the capital stock to a desired new level. The adjustment process lasts until the rental rate equals the user cost to restore the equilibrium in the long-run. Unlike labour force migration, the capital stock is only fully adjusted until in the long-run, while in the short-run and medium-run it remains fixed.

In terms of the model setting, the distinction of short, medium and long-run is realised through setting of investment, labour and capital. For short-run simulation, we select '<u>Short-term</u>', set '<u>Investment</u>' as endogenous and the parameter value for the speed of capital stock adjustment as default for all sectors, <u>0.3</u>. In '<u>Supplies</u>' we let '<u>Labour</u>' open with '<u>Migration</u>' '<u>On</u>' by '<u>Flow Adjustment</u>' while leave 'Capital' as default that capital market is isolated. The tax amenity parameter is again specified here in migration setting, and the coefficients in net in-migration function are set as mentioned above. In addition, in terms of multi-period simulation, the 'Short-term' option is also selected here in 'Time Periods', since each period of adjustment can be seen as a short-run simulation.

For the medium-run simulation, there is no independent menu option for it. It is done by selecting 'Long-run' and then 'Labour', with the detail setting the same as in the short-run, while again leaving capital isolated as default.

For the long-run simulation, select 'Long-run' and then 'Both', again with same setting for investment and labour.

Model Simulation

After specifying the model, this 'Simulation' menu option gives us the opportunity to choose how to simulate the model. One option is to simulate the model to replicate the baseline equilibrium, while the other one is to make change to exogenous variables to shock the model, in order to observe the response and convergence to a new equilibrium. Both options allow single-period and multi-periods simulations. The multi-period variant of the model here is not an inter-temporal optimisation model but a recursive dynamic model where the labour force and capital stock are updated period by period following the mechanisms mentioned above. Hence, the multi-period adjustment process is formed by a sequence of successive short-run periods generated by the recursive dynamics.

- Replicate Base

It is the initial and fundamental step for a CGE model simulation, that is, to run the model to recreate the base year data, which we refer to as 'calibration'. Through this first step the baseline equilibrium is replicated and ready to be compared with the post-shock economy. The calibration procedure should succeed regardless of model specifications in order to implement further simulation.

- Run Simulations

This option allows specifying the type and magnitude of the shock, including choices from exogenous prices and quantities, fiscal and regional policies, technical progress, and budget constraints. The available shocks cover demand and supply side shocks, efficiency shocks, and constraint shocks such as BoP/GDP ratio and Deficit/GDP ratio. The available range of shocks provides single and multiple choices to deliver a single shock or a combined group of shocks. We can also choose from here whether the shock is permanent or transitory, as well as on which period for the shock to emerge in case of doing multi-period simulation.

Model run

Following the model specification and simulation setup, the model running is realized by the menu option 'Run', where we can select whether doing 'Normal Run' or 'Sensitivity Analysis'. The latter one normally refers to changing the particular parameter value to examine the robustness of the targeted simulation result. Despite the sensitivity analysis, all we will do in this study focuses on 'Normal Run'.

Case 1: Fiscal expansion – government expenditure increasing by 20%

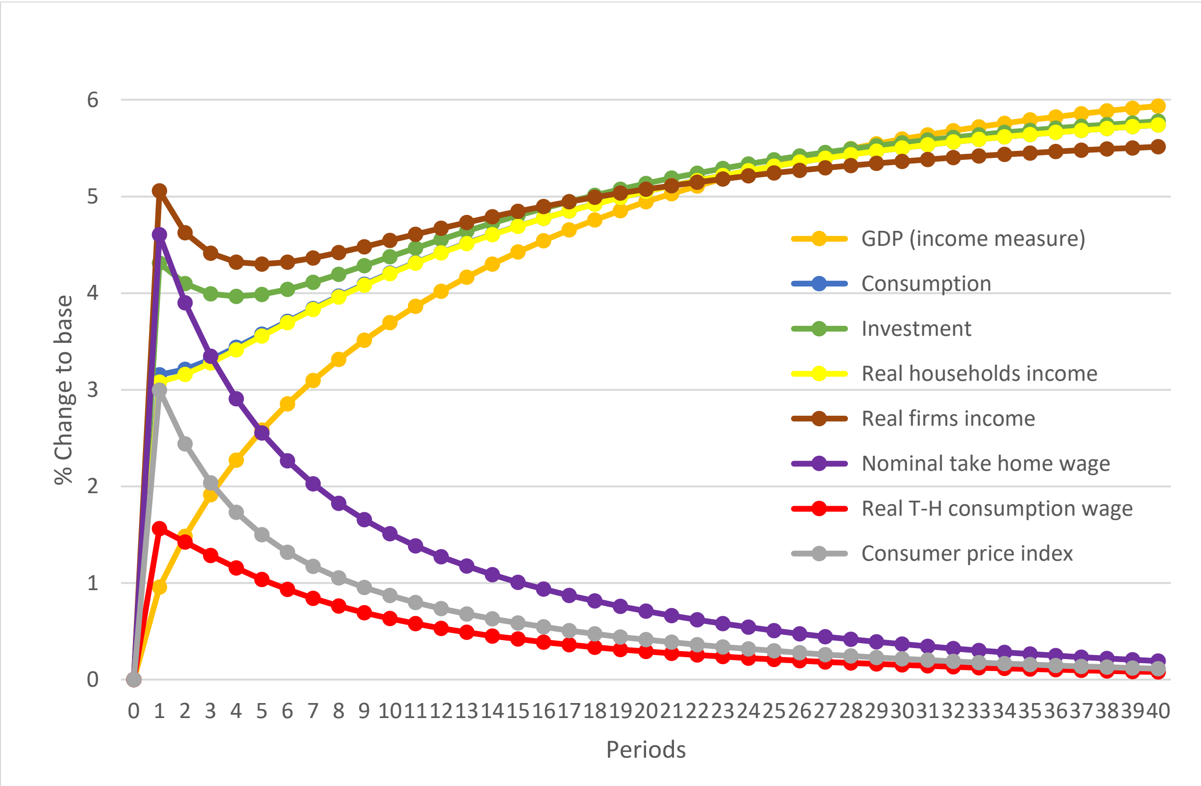
In this case we investigate the effect of government expenditure increasing by 20%, which we can set through 'Simulation' and then 'Run simulations': select 'Total government expenditure' of the 'Fiscal policies' group on the shock list, and input '020' standing for twenty percentage points. We can also choose to do 'Single Period' or 'Multiple Periods'. For multi-period simulation, we should additionally specify the number of periods and the nature of the shock. In our case, we will explore the effect of a 'Permanent' shock which emerges on the first period, in a 100-periods scenario. The single period short, medium and long-run simulation results are shown as below in Table A.1, where we find that the results are not affected at all by tax amenity value, so it is the common result regardless of the parameter value in the table. In Figure A.1, not all the 100 period movements of major variables are shown but only 40 periods window is intercepted because all the variables have already started to converge to the new equilibrium from then on.

Table A.1 Single Period Fiscal Expansion Simulation

Major Variables	Base Year	Short-run	Medium-run	Long-run
GDP (income measure)	60613.785	0.957	1.611	6.321
Consumption	42812.406	3.156	3.774	6.008
Government expenditure	16776.297	20	20	20
Investment	11195.96	4.31	5.441	6.027
Nominal before-tax wage	17.199	4.606	2.81	0
Nominal take home wage	13.507	4.606	2.81	0

Real Before-Tax consumption wage	17.199	1.562	0	0
Real Take-Home consumption wage	13.507	1.562	0	0
Total employment (000's)	2029.5	1.521	2.58	6.429
Unemployment rate (%)	8.354	-12.815	0	0
Total population (000's)	5120	0	2.58	6.429
Consumer price index	1	2.997	2.81	0
Real households income	55073	3.078	3.705	6.008
Real firms income	22158.354	5.059	6.202	5.683

Figure A.1 Impact of Fiscal Expansion



It is clear to see from the results that there are two main movement trends for the major variables: in the long-run, there are no changes to price variables, and quantity variables all increase compared to the base year value. The reason that prices including commodity prices, wages and capital rents return to their base level basically lies in the unconstrained supply side setting. The migration and investment towards desired capital level allows full adjustment of the production input factors so as to meet the additional demands triggered by government spending. The initial sharp increase of prices also implies that the local economy has no huge excess supply before the policy shock, so when excessive demand occurs, together with the suddenly relative short production supply, it pushes the wages and rents to increase sharply, which attract more labour force from outside the local region and more investment towards their regionally desired level. In the long-run, the adjustment process gradually lessens the upward pressure of prices until they converge to their initial level.

Such an effect can also be observed from Figure A.1 that the real firms income and investment deliver a 'U' shape covering a few periods after the initial increase. It is from then on that the prices initiate the downward course and are followed by firms investment and income due to less capital returns and sale prices. However, the huge demand effect will succeed the prices downward effect. After then, they proceed to benefit from the additional demands and rise towards the post-shock equilibrium level.

If we look further into the sectoral level of commodity prices, the most largely increased price comes from the sheltered sector, implying the structure of the government expenditure favours more to this sector, which also produces a temporary crowding out effect correspondingly. Higher demand and therefore higher wages and rents draw the input resources from other sectors in the short-run. Hence, there is temporary falls of outputs from other two sectors. In the long-run, again, the supply side relaxes sufficiently to allow higher outputs of all sectors relative to their pre-shock level.

Case 2: Fiscal austerity – average rate of income tax increasing by 10%

This case considers the effect of 10% increase in the average rate of income tax, which accounts approximately for additional 3p per Pound collected. Such a tax increase alone without any binding government spending change is simulated again through selecting 'Run simulations', but this time choosing 'Average rate of income tax' of the 'Fiscal policies' group on the shock list, and inputting '010' standing for ten percentage points increase. Other specifications are held unchanged.

Table A.2 Single Period Fiscal Austerity Simulation

Major Variables	Base Year	Tax Amenity 0			Tax Amenity 0.5			Tax Amenity 1		
		Short-run	Medium-run	Long-run	Short-run	Medium-run	Long-run	Short-run	Medium-run	Long-run
GDP (income measure)	60613.79	-0.431	-0.715	-2.586	-0.245	-0.407	-1.545	-0.061	-0.101	-0.484
Consumption	42812.41	-1.514	-1.806	-3.031	-1.47	-1.636	-2.381	-1.426	-1.467	-1.714
Government expenditure	16776.3	0	0	0	0	0	0	0	0	0
Investment	11195.96	-0.843	-1.332	-2.293	-0.535	-0.814	-1.388	-0.231	-0.3	-0.467
Nominal before-tax wage	17.199	0.767	1.491	2.131	0.244	0.658	1.062	-0.271	-0.168	0
Nominal take home wage	13.507	-0.664	0.051	0.682	-1.179	-0.771	-0.372	-1.686	-1.585	-1.419
Real Before-Tax consumption wage	17.199	0.798	1.44	1.44	0.35	0.717	0.717	-0.091	0	0
Real Take-Home consumption wage	13.507	-0.633	0	0	-1.074	-0.712	-0.712	-1.509	-1.419	-1.419
Total employment (000's)	2029.5	-0.682	-1.128	-2.785	-0.388	-0.644	-1.653	-0.096	-0.159	-0.5
Unemployment rate (%)	8.354	5.78	0	0	3.282	0	0	0.809	0	0
Total population (000's)	5120	0	-1.128	-2.785	0	-0.644	-1.653	0	-0.159	-0.5
Consumer price index	1	-0.031	0.051	0.682	-0.106	-0.059	0.342	-0.18	-0.168	0
Real households income	55073	-1.514	-1.806	-3.035	-1.47	-1.636	-2.381	-1.426	-1.468	-1.714
Real firms income	22158.35	-1.267	-1.755	-2.316	-0.962	-1.242	-1.581	-0.66	-0.73	-0.832

Figure A.2 Quantities Impact of Fiscal Austerity - Tax Amenity 0

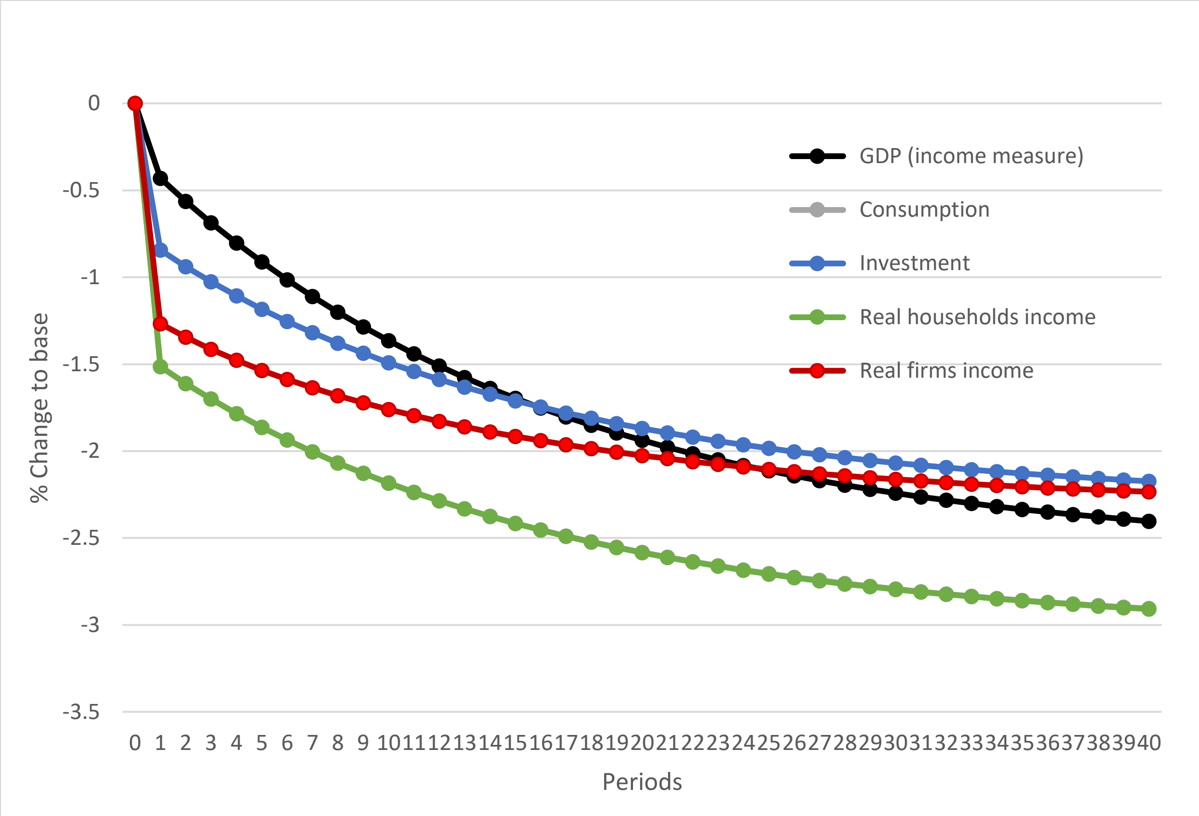


Figure A.3 Prices Impact of Fiscal Austerity - Tax Amenity 0

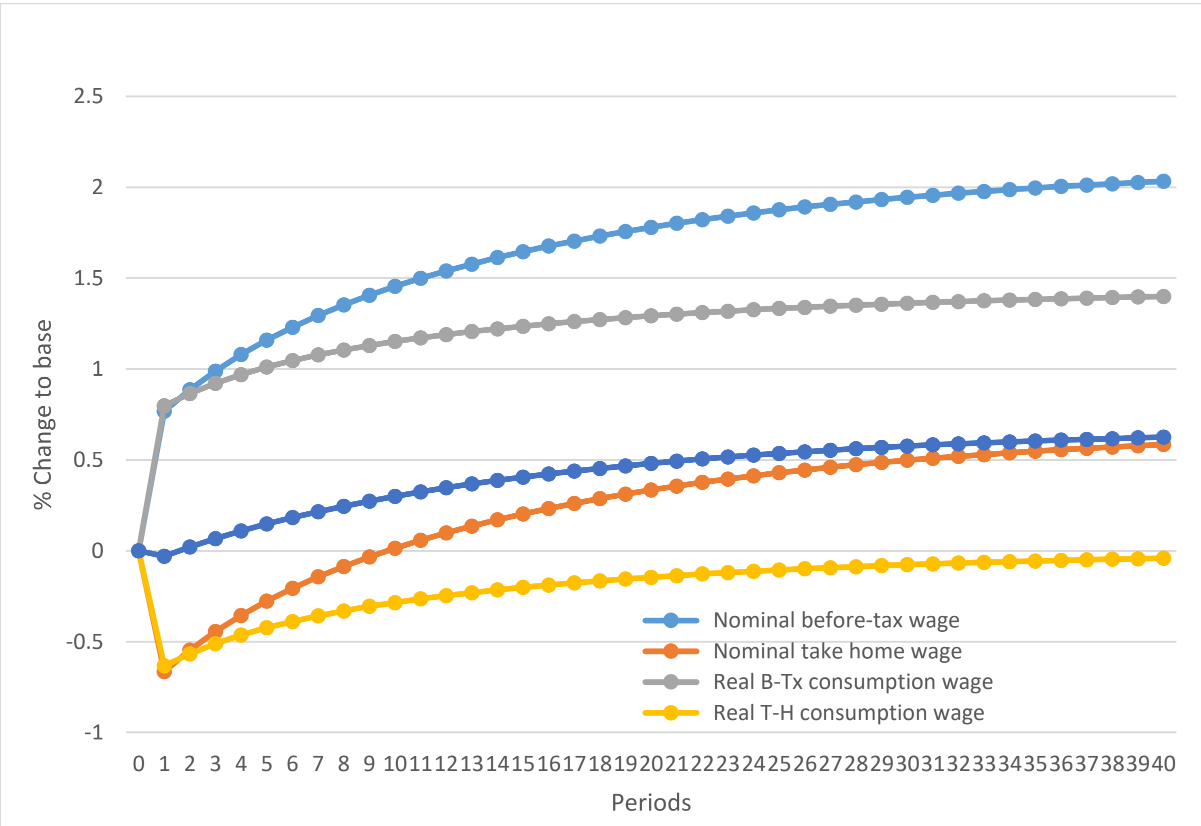


Figure A.4 Quantities Impact of Fiscal Austerity - Tax Amenity 0.5

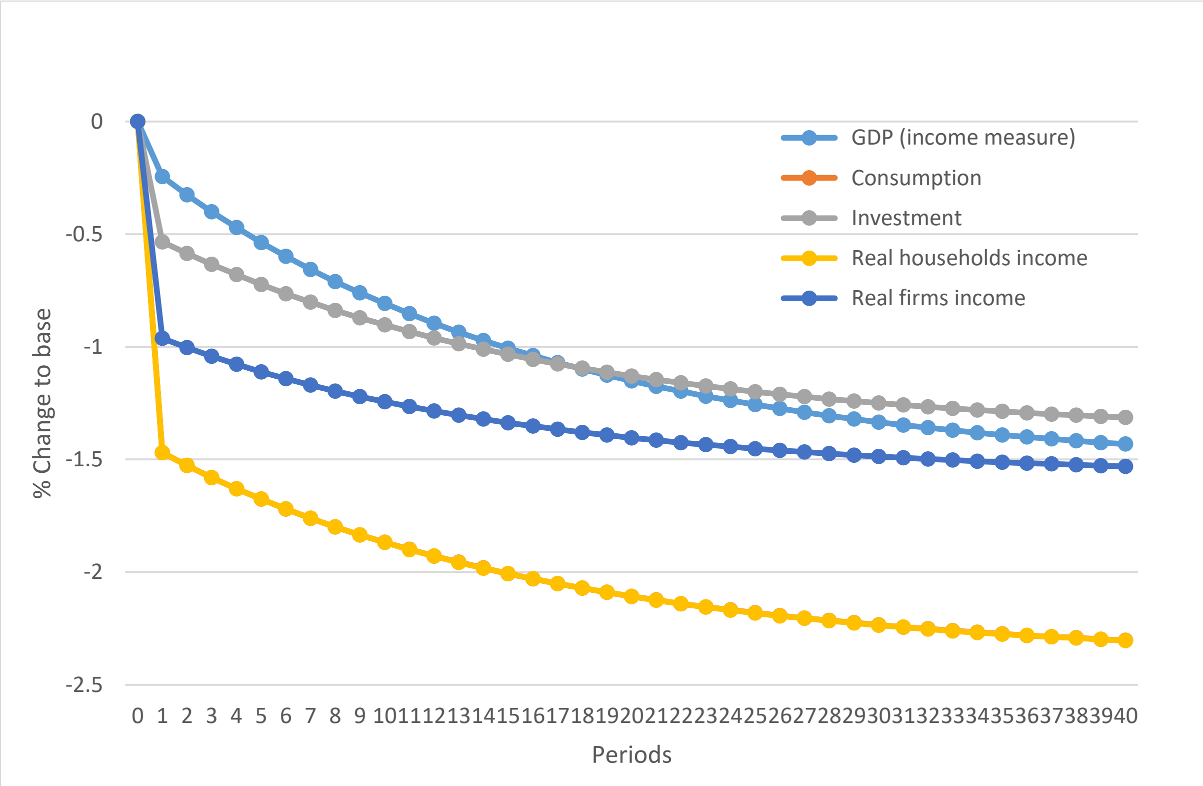


Figure A.5 Prices Impact of Fiscal Austerity - Tax Amenity 0.5

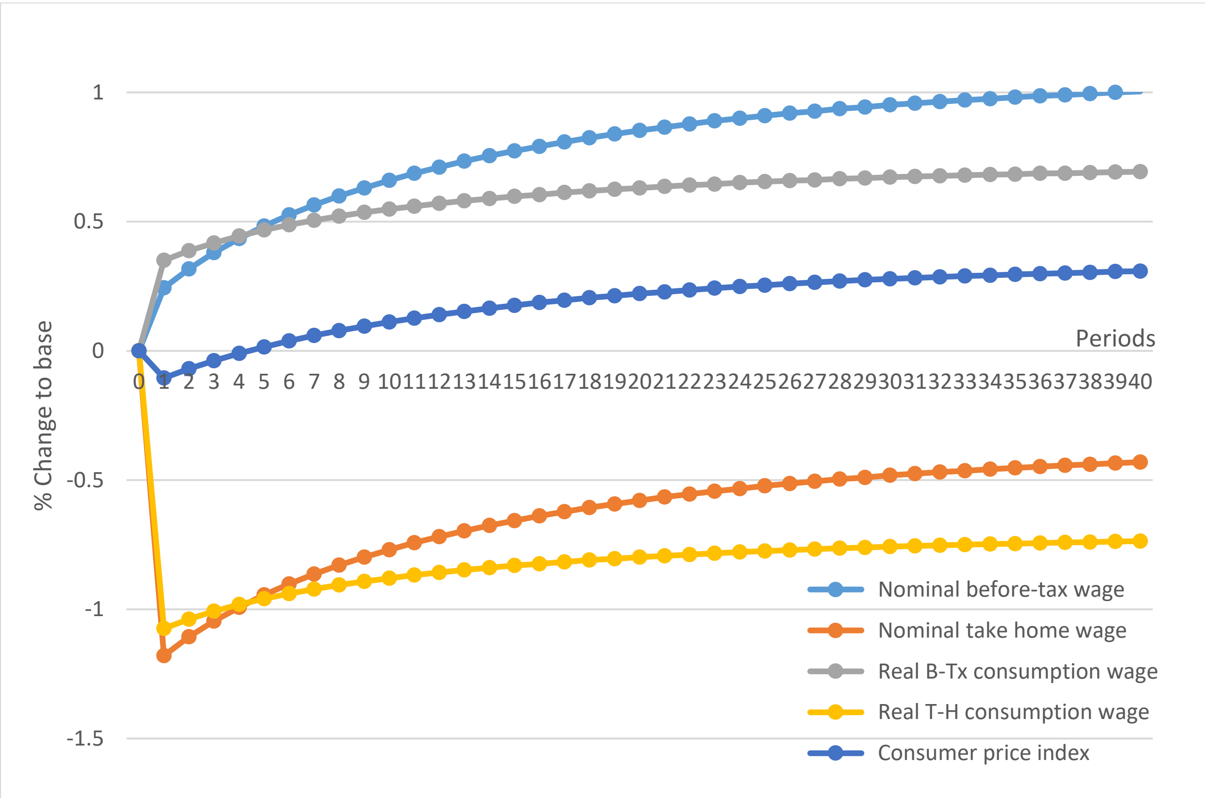


Figure A.6 Quantities Impact of Fiscal Austerity - Tax Amenity 1

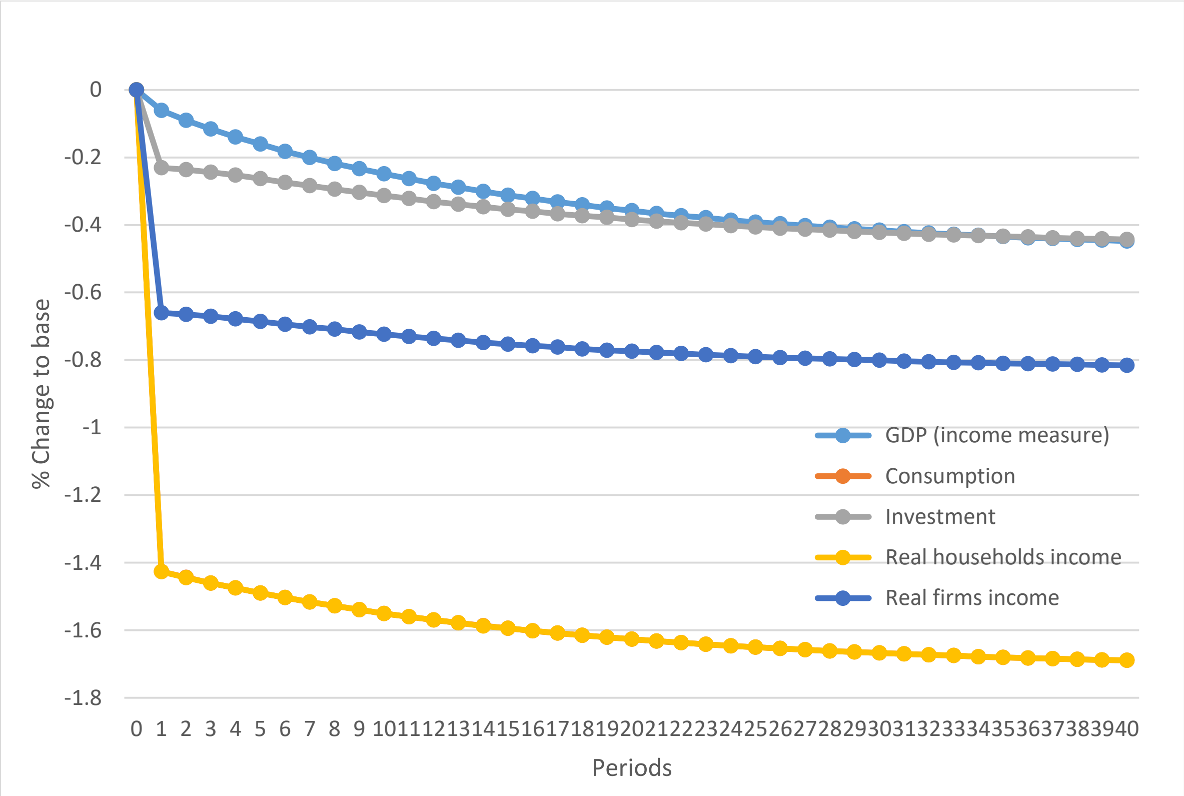
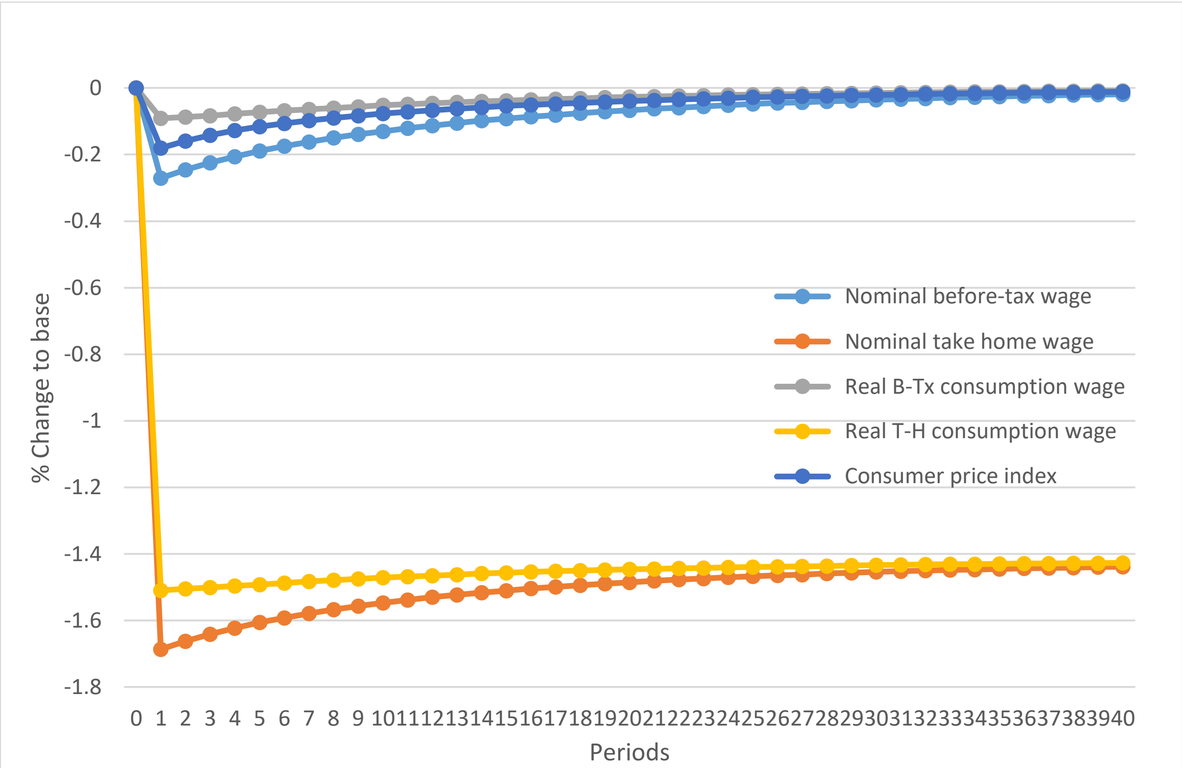


Figure A.7 Prices Impact of Fiscal Austerity - Tax Amenity 1



The most prominent effect in this case compared to fiscal expansion, as expected, are the overall decreasing quantity variables in the long-run because of the economy-wide austerity. Unemployment rate is unaffected in the long-run due to labour migration process. These are the evolving trends no matter what the tax amenity parameter value is. The only difference is that along the parameter value growing larger, the magnitude of all the macroeconomic quantity variables' decrease becomes smaller. Take GDP as an example, it falls 2.586% compared to its pre-shock level under tax amenity parameter equalling 0, while it falls only 0.484% when the parameter value takes 1.

For the prices, both nominal and real before-tax wage return to their initial levels in the long-run, as well as the consumer price index, when the tax amenity value takes 1. This is again due to the increase of factor capacity in the long-run, as well as the taxpayers' attitude towards more tax payment. As they do not regard the additional income tax payment as a loss, they only bargain for recovering before-tax wage, although their take-home wages do reduce finally due to the tax payment. The contractionary effect of austerity is even larger when the tax amenity value equals 0. In this case, taxpayers takes the additional income tax payment as pure loss against their income, and they bargain for their real take-home wage back to the base level. In the meantime, the commodity prices fall slightly in a short time as a result of households income and consumption downturn, then increase in the long-run as investment and production decline in response of less demand. When the amenity parameter takes the middle value of 0.5, the response of the economy displays moderately.

Case 3: Fiscal combination – average rate of income tax increasing by 10% with binding government expenditure

In this case, we combine the fiscal policies together, that is, increase the average rate of income tax by 10% as above again, while allowing to facilitate more government expenditure by the binding budget constraint. To set this up in the model, refer to the 'Closure' menu and choose 'Government budget constraint' 'Binding', then 'Expenditure' to be passive. Other setting remains the same as in case 2.

The effect of the fiscal combination yields the same responses of all wages and prices in the long-run with those in case 2 across different amenity parameter values. The only difference is the response process through the short and medium-run. All prices including wages have much more positive response compared to the pure austerity case. Apparently the government

Table A.3 Single Period Fiscal Combination Simulation

Major Variables	Base Year	Tax Amenity 0			Tax Amenity 0.5			Tax Amenity 1		
		Short-run	Medium-run	Long-run	Short-run	Medium-run	Long-run	Short-run	Medium-run	Long-run
GDP (income measure)	60613.785	-0.283	-0.955	-1.65	-0.091	-0.153	-0.439	0.098	0.15	0.795
Consumption	42812.406	-1.123	-2.684	-2.147	-1.063	-1.118	-1.34	-1.003	-0.956	-0.515
Government expenditure	16776.297	2.875	6.481	2.999	3.01	3.088	3.53	3.143	3.076	4.066
Investment	11195.96	-0.237	-1.059	-1.398	0.097	0.001	-0.332	0.426	0.508	0.752
Nominal before-tax wage	17.199	1.399	4.065	2.131	0.907	1.093	1.062	0.422	0.263	0
Nominal take home wage	13.507	-0.04	1.01	0.682	-0.525	-0.342	-0.372	-1.004	-1.16	-1.419
Real Before-Tax consumption wage	17.199	0.999	3.024	1.44	0.563	0.717	0.717	0.132	0	0
Real Take-Home consumption wage	13.507	-0.435	0	0	-0.865	-0.712	-0.712	-1.289	-1.419	-1.419
Total employment (000's)	2029.5	-0.464	-1.533	-1.834	-0.162	-0.259	-0.529	0.138	0.222	0.8
Unemployment rate (%)	8.354	3.929	0	0	1.368	0	0	-1.164	0	0
Total population (000's)	5120	0	-1.533	-1.834	0	-0.259	-0.529	0	0.222	0.8
Consumer price index	1	0.396	1.01	0.682	0.342	0.373	0.342	0.289	0.263	0
Real households income	55073	-1.125	-2.692	-2.151	-1.064	-1.12	-1.341	-1.004	-0.956	-0.515
Real firms income	22158.354	-0.647	-1.87	-1.474	-0.316	-0.412	-0.589	0.012	0.094	0.312

Figure A.8 Quantities Impact of Fiscal Combination - Tax Amenity 0

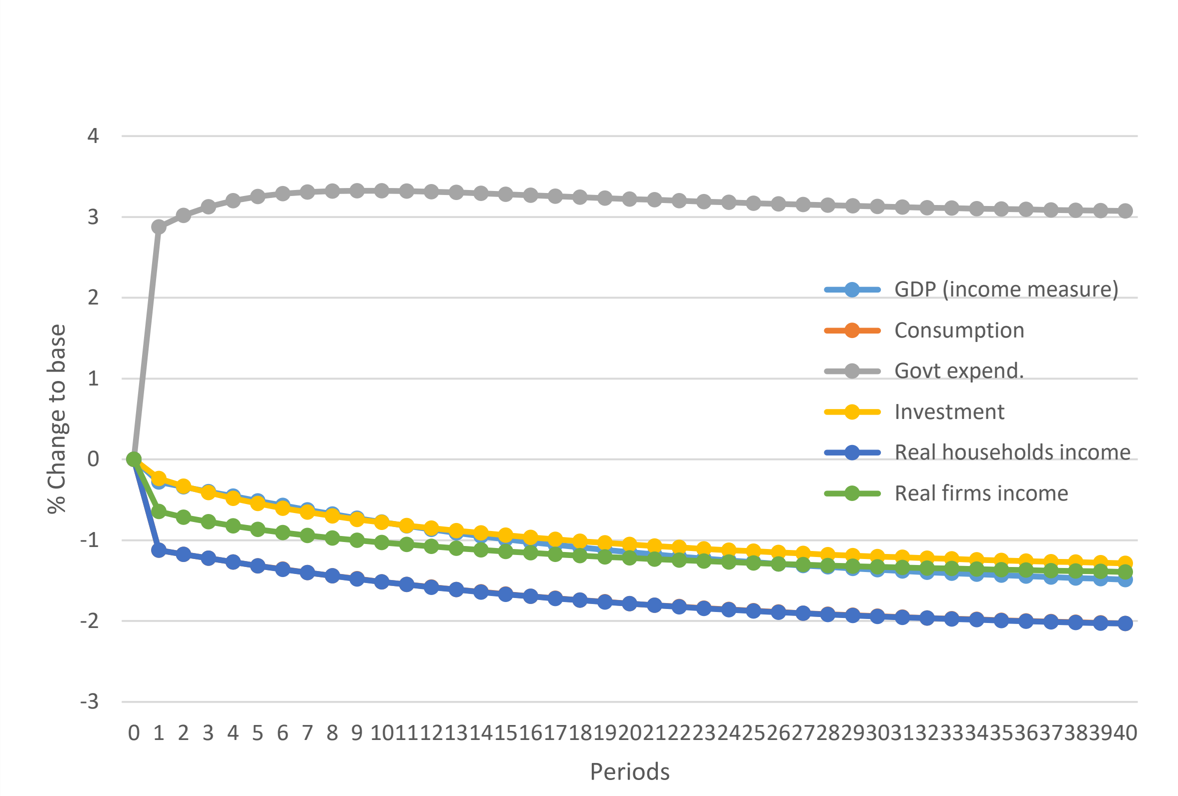


Figure A.9 Prices Impact of Fiscal Combination - Tax Amenity 0

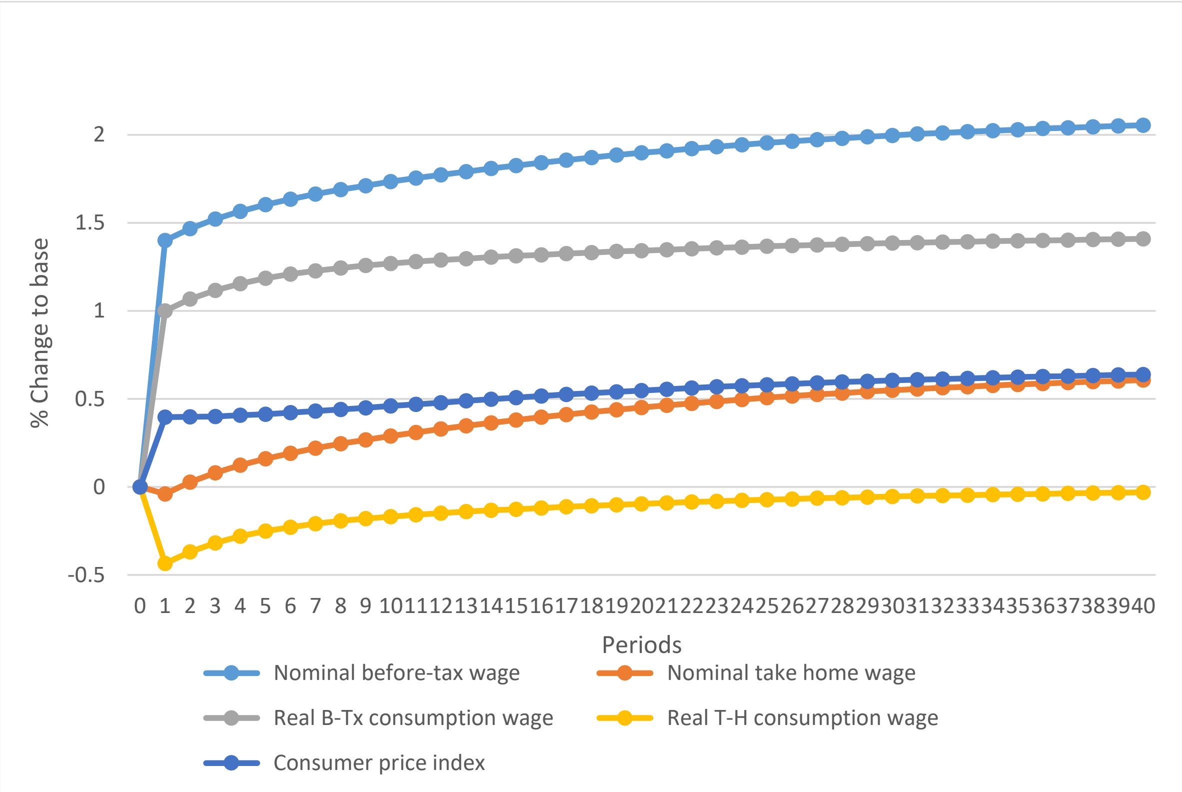


Figure A.10 Quantities Impact of Fiscal Combination - Tax Amenity 0.5

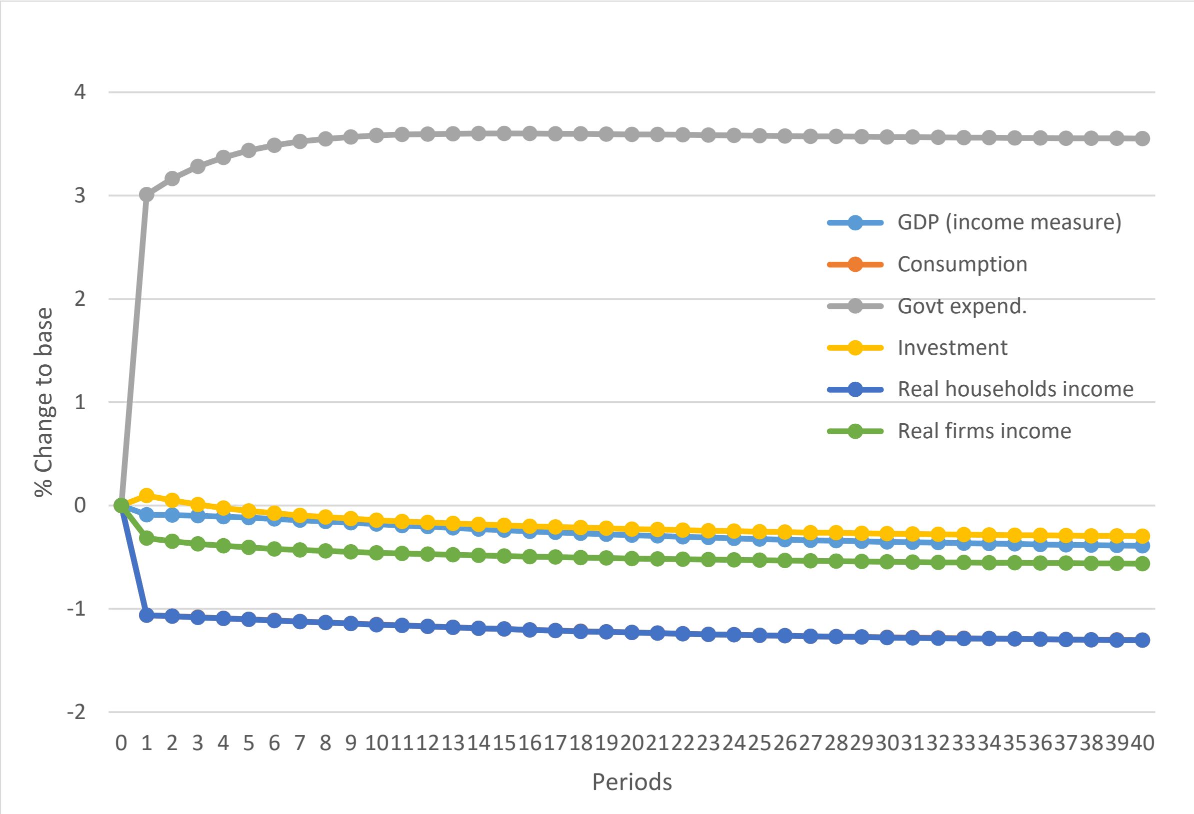


Figure A.11 Prices Impact of Fiscal Combination - Tax Amenity 0.5

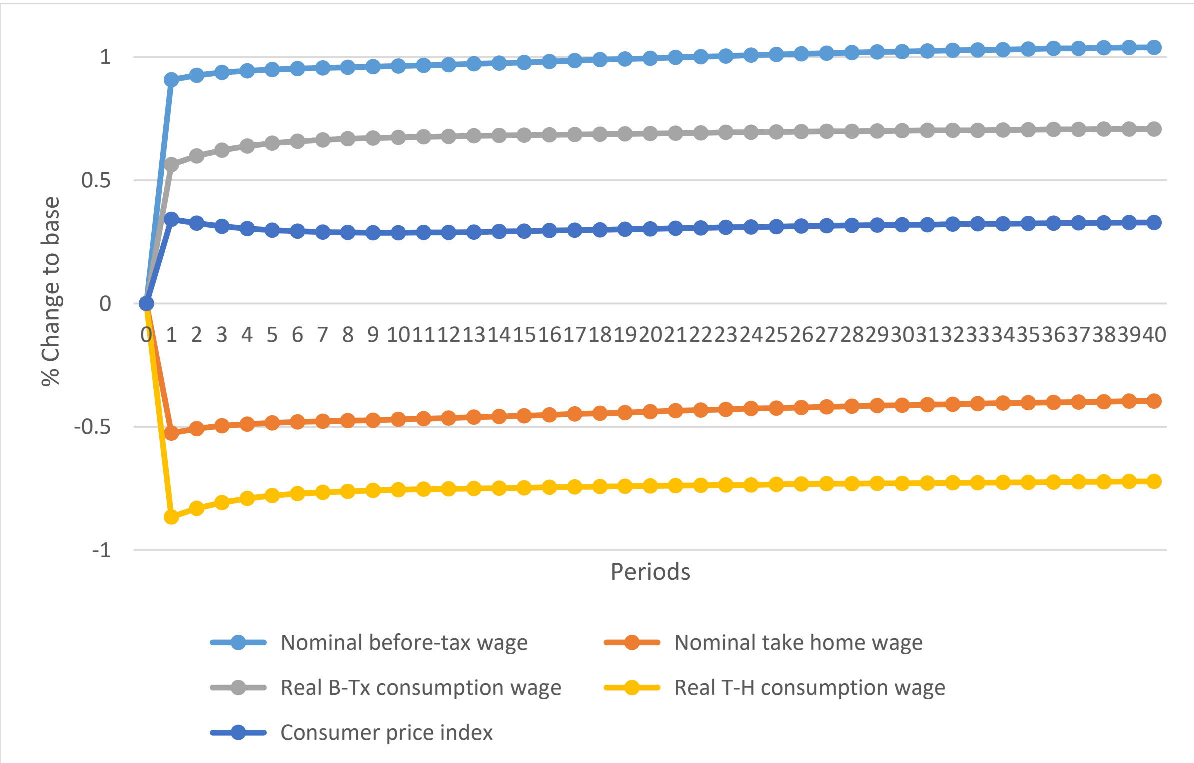


Figure A.12 Quantities Impact of Fiscal Combination - Tax Amenity 1

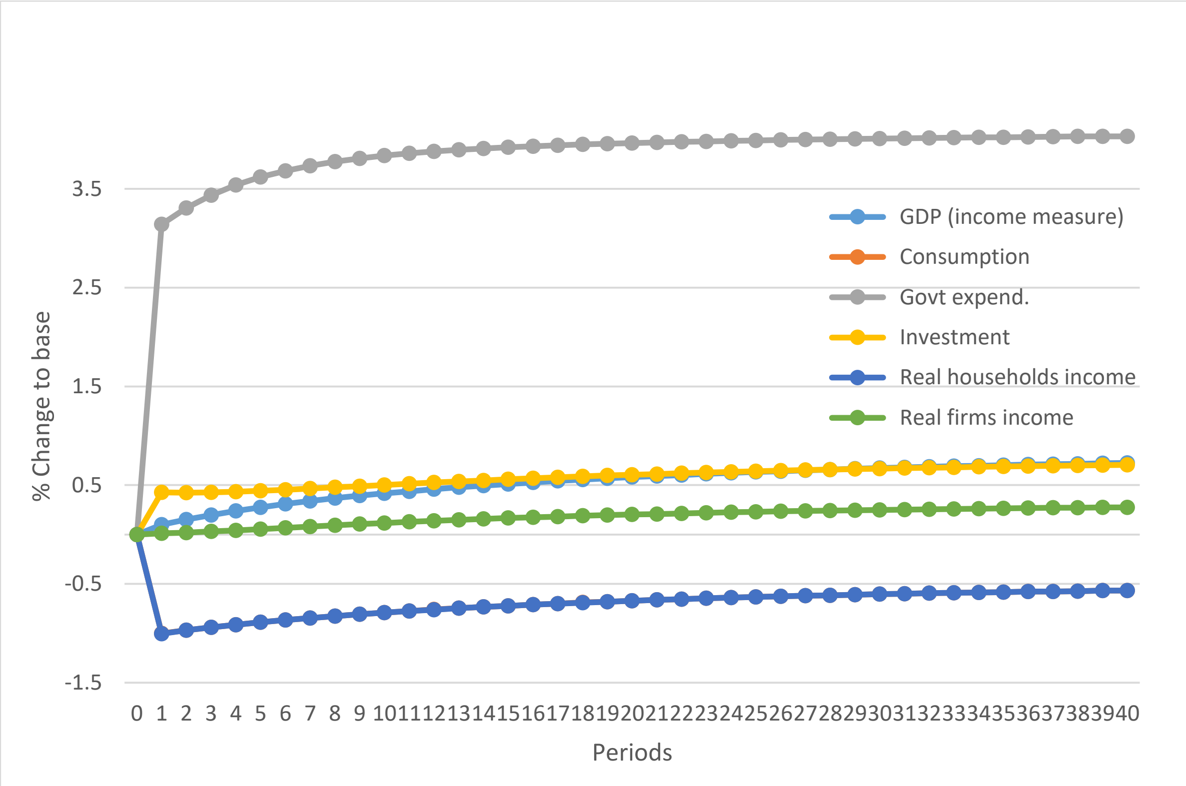
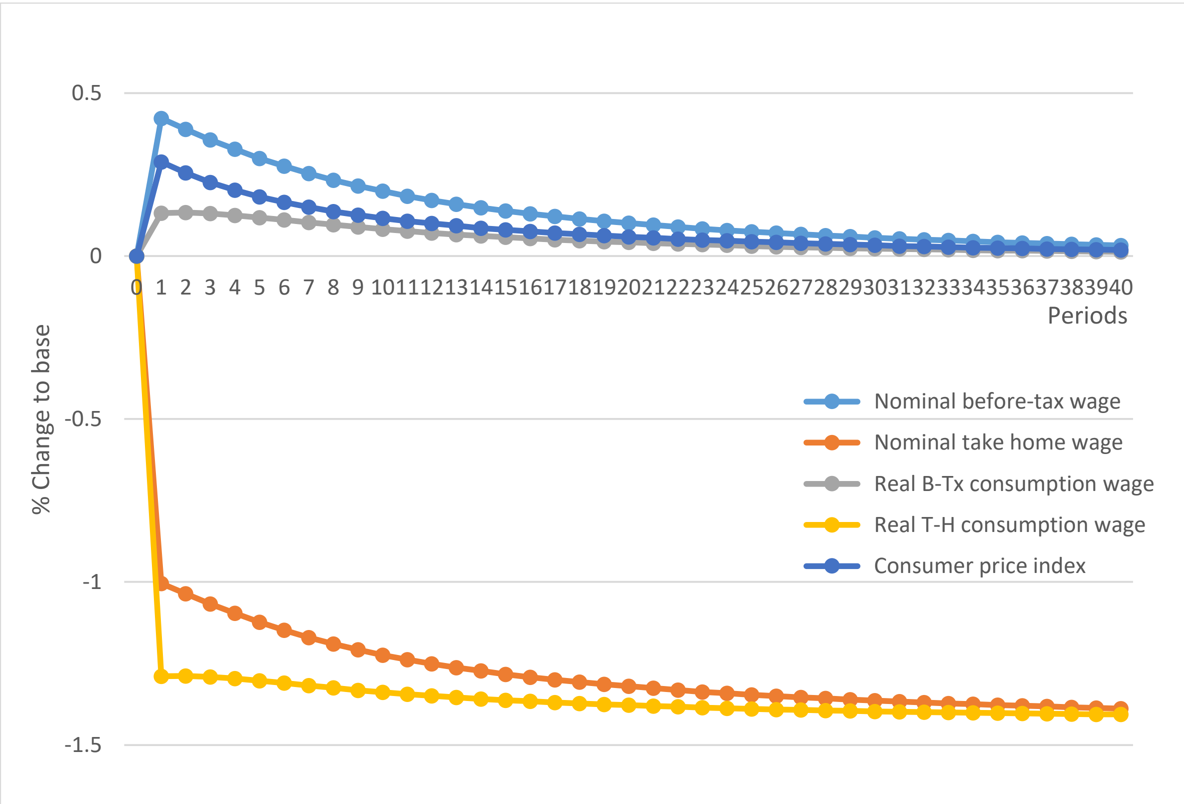


Figure A.13 Prices Impact of Fiscal Combination - Tax Amenity 1



expenditure in addition to the pure income tax hike has covered the shortage of households income and their consumption demand in the short and medium-run, while in the long-run only the degree of factor stock adjustment and tax amenity value determine whether the prices recover their base levels.

All major macroeconomic quantity variables react more positively than in case 2 owing to the activation of government spending. However, they have positive changes compared to the base level only when tax amenity is 1. This implies that only when taxpayers regard the tax payment will be fully spent on themselves such that it is indifferent with their disposable income, the whole economy is slightly better-off to the fiscal combination policy. Nevertheless, if we set the amenity to a more realistic value, the whole economy will be worse-off. In this case, the crowding-out effect of the policy combination dominates that the fiscal intervention distorts the economic operation.

Appendix II CGE model sets, variables, parameters and equations

<u>Sector sets</u>

i/j	all sectors
ic	all sectors that produce commodities for households' consumption
ire/jre	all three real estate sectors
inre/jnre	all non-real-estate sectors
inns/jnns	all sectors excluding new real estate ownership sectors
inr/jnr	all sectors excluding residential rental sector

<u>Endogenous variables</u>

C_i	households' consumption demand by commodity
CPI	CPI calculated using Fisher index
CPI_L	CPI calculated using Laspeyres index
CPI_P	CPI calculated using Paasche index
WD_i	regional production supplied to domestic market
$EROW_i$	export supply to the ROW by sector
$ERUK_i$	export supply to the RUK by sector
G_i	fiscal expenditure demand by commodity
ID_i	investment demand by commodity
$INT_{i,j}$	intermediate inputs for regional production
K_i	capital demand by sector (factors fixed for total stock)
L_i	labour demand by sector (factors fixed for total stock)
$MROW_i$	import demand from the ROW by commodity
$MRUK_i$	import demand from the RUK by commodity
PWD_i	price of regional production supplied to domestic market
PI	average price of investment goods
PK	economy-wide capital return (factors fixed for total stock)
PK_i	capital return by sector (factors fixed for each sector)
PL	economy-wide labour wage (factors fixed for total stock)
PL_i	labour wage by sector (factors fixed for each sector)

PQ_i	price of regional sales of composites by commodity – net of product taxes
PQW_i	price of regional sales of domestic composites by commodity – net of product taxes
PT_NR	economy-wide non-residential land return (factors fixed for total stock)
PT_i	land return by sector (factors fixed for each sector)
PVA_i	price of value added by sector
PW_i	price of regional production supplied to regional market
PY_i	price of output of regional production by sector – basic price
Q_i	regional sales of composites combining regional production and all imports by commodity
QW_i	regional sales of domestic composites combining regional production and imports from RUK by commodity
S	total savings
SH	households' savings
SRUK	inter-regional savings from RUK
SROW	foreign savings from ROW
T_i	land demand by sector (factors fixed for total stock)
TC	product tax revenue
TCCL	council tax revenue
TK	corporation tax revenue
TLE	NIC revenue payable by the employee
TLR	NIC revenue payable by the employer
TP	production tax revenue
TRFG	fiscal transfer received by regional government from central government
TSD_R_NS	Stamp Duty Land Tax revenue from residential properties' new sale
TSD_NR_NS	Stamp Duty Land Tax revenue from non-residential properties' new sale
TSD_R_RS	Stamp Duty Land Tax revenue from residential properties' resale
TSD_NR_RS	Stamp Duty Land Tax revenue from non-residential properties' resale
TY	income tax revenue
U	households' utility level
VA_i	value-added bundle of factors by sector
W_i	regional production supplied to regional market

Y_i	output of regional production by sector
YG	total fiscal revenue
YG_DT	devolved tax revenue
YG_F	factor income of regional government
YG_NDT	non-devolved tax revenue
YH	households' income
YK	factor income of capital
YL	factor income of labour
YT	factor income of land

Exogenous variables

$\overline{K}$	capital stock (factors fixed for total stock)
$\overline{K}_i$	capital demand by sector (factors fixed for each sector)
$\overline{L}$	labour stock (factors fixed for total stock)
$\overline{L}_i$	labour demand by sector (factors fixed for each sector)
$\overline{PEROW}_i$	price of export supply to the ROW by sector in domestic currency
$\overline{PERUK}_i$	price of export supply to the RUK by sector in domestic currency
$\overline{PMROW}_i$	price of import demand from the ROW by sector in domestic currency
$\overline{PMRUK}_i$	price of import demand from the RUK by sector in domestic currency
$\overline{Q\_R\_STOCK}$	residential properties' regional stock
$\overline{T}$	land stock (factors fixed for total stock)
$\overline{T}_i$	land demand by sector
$\overline{TRFH}$	social protection transfer received by households from regional government

Parameters

ac_i	parameter with respect to households' demand for commodities
ag_i	parameter with respect to fiscal expenditure demand for commodities
aid_i	parameter with respect to investment demand for commodities
$aint_{i,j}$	Leontief parameter with respect to intermediate inputs by sector
ak_i	share parameter with respect to capital demand in Cobb-Douglas production function by sector

al_i	share parameter with respect to labour demand in Cobb-Douglas production function by sector
at_i	share parameter with respect to land demand in Cobb-Douglas production function by sector
ava_j	Leontief parameter with respect to value-added bundle by sector
$serow_i$	distribution parameter for export supply to the ROW by sector in the CET function
$seruk_i$	distribution parameter for export supply to the RUK by sector in the CET function
mps	marginal propensity of saving of households
$smrow_i$	distribution parameter for import demand from the ROW by sector in the Armington CES function
$smruk_i$	distribution parameter for import demand from the RUK by sector in the Armington CES function
sk_i	distribution parameter of capital demand by sector in the CES production function for GVA
sl_i	distribution parameter of labour demand by sector in the CES production function for GVA
st_i	distribution parameter of land demand by sector in the CES production function for GVA
skg	share of factor income of capital distributed to regional government
skh	share of factor income of capital distributed to households
stg	share of factor income of land distributed to regional government
sth	share of factor income of land distributed to households
tccl	effective council tax rate
tc_i	effective product tax rate by commodity
tk	effective corporation tax rate
tle	effective NIC rate payable by the employee
tlr	effective NIC rate payable by the employer
tmx_i	effective mixed tax rate combining product tax rate & residential & non-residential SDLT final rates
tp_i	effective production tax rate by sector
tsd_nr_fnl	effective non-residential Stamp Duty Land Tax rate on new sale
tsd_r_fnl	effective residential Stamp Duty Land Tax rate on new sale
tsd_rr_fnl	effective residential Stamp Duty Land Tax rate on residential rental

$tsd\_nr\_int_j$	effective non-residential Stamp Duty Land Tax rate on land factor
ty	effective income tax rate
γk_i	scaling coefficient in the Armington CES function for RUK imports
γw_i	scaling coefficient in the Armington CES function for ROW imports
γ_i	scaling coefficient in the CES production function for GVA
ηk_i	parameter defined by elasticity of substitution in the Armington CES function for RUK imports
ηw_i	parameter defined by elasticity of substitution in the Armington CES function for ROW imports
η_i	parameter defined by elasticity of substitution in the CES production function for GVA
θk_i	scaling coefficient in the CET function for RUK exports
θw_i	scaling coefficient in the CET function for ROW exports
ρk_i	parameter defined by elasticity of transformation in the CET function for RUK exports
ρw_i	parameter defined by elasticity of transformation in the CET function for ROW exports
σk_i	elasticity of transformation in the CET function for RUK exports
σw_i	elasticity of transformation in the CET function for ROW exports
ωk_i	elasticity of substitution in the Armington CES function for RUK imports
ωw_i	elasticity of substitution in the Armington CES function for ROW imports
ω_i	elasticity of substitution in the CES production function for GVA

<u>Equations (short run)</u>

$$PVA_j \cdot VA_j = PK_j \cdot \bar{K}_j/(1-tk) + \left(1 + tsd\_nr\_int_j\right) \cdot PT_j \cdot \bar{T}_j + (1+tlr) \cdot PL_j \cdot \bar{L}_j/(1-ty)$$

$$PK_j \cdot \bar{K}_j/(1-tk) = ak_j \cdot PVA_j \cdot VA_j \qquad \text{(Cobb-Douglas)}$$

$$\left(1 + tsd\_nr\_int_j\right) \cdot PT_j \cdot \bar{T}_j = at_j \cdot PVA_j \cdot VA_j \qquad \text{(Cobb-Douglas)}$$

$$(1+tlr) \cdot PL_j \cdot \bar{L}_j/(1-ty) = al_j \cdot PVA_j \cdot VA_j \qquad \text{(Cobb-Douglas)}$$

$$PK_j/(1-tk) = \gamma_j^{\eta_j} \cdot sk_j \cdot PVA_j \cdot \left(VA_j/\bar{K}_j\right)^{1-\eta_j} \qquad \text{(CES)}$$

$$\left(1 + tsd\_nr\_int_j\right) \cdot PT_j = \gamma_j^{\eta_j} \cdot st_j \cdot PVA_j \cdot \left(VA_j/\bar{T}_j\right)^{1-\eta_j} \qquad \text{(CES)}$$

$$(1+tlr)\cdot PL_j/(1-ty)=\gamma_j^{\eta_j}\cdot sl_j\cdot PVA_j\cdot\left(VA_j/\bar{L}_j\right)^{1-\eta_j}\qquad\text{(CES)}$$

$$YK=\sum_j PK_j\cdot\bar{K}_j$$

$$YT=\sum_j PT_j\cdot\bar{T}_j$$

$$YL=\sum_j PL_j\cdot\bar{L}_j$$

$$PT\_NR=\frac{\sum_{jnr}PT_{jnr}\cdot\bar{T}_{jnr}}{\sum_{jnr}\bar{T}_{jnr}}$$

$$TSD\_NR\_RS=\sum_{jnr}tsd\_nr\_int_{jnr}\cdot PT_{jnr}\cdot\bar{T}_{jnr}$$

Equations (medium run)

$$PVA_j\cdot VA_j=PK\cdot K_j/(1-tk)+\left(1+tsd\_nr\_int_j\right)\cdot PT_j\cdot\bar{T}_j+(1+tlr)\cdot PL\cdot L_j/(1-ty)$$

$$PK\cdot K_j/(1-tk)=ak_j\cdot PVA_j\cdot VA_j\qquad\text{(Cobb-Douglas)}$$

$$\left(1+tsd\_nr\_int_j\right)\cdot PT_j\cdot\bar{T}_j=at_j\cdot PVA_j\cdot VA_j\qquad\text{(Cobb-Douglas)}$$

$$(1+tlr)\cdot PL\cdot L_j/(1-ty)=al_j\cdot PVA_j\cdot VA_j\qquad\text{(Cobb-Douglas)}$$

$$K_j=VA_j\cdot\left[\gamma_j^{\eta_j}\cdot sk_j\cdot(1-tk)\cdot PVA_j/PK\right]^{1/(1-\eta_j)}\qquad\text{(CES)}$$

$$\left(1+tsd\_nr\_int_j\right)\cdot PT_j=\gamma_j^{\eta_j}\cdot st_j\cdot PVA_j\cdot\left(VA_j/\bar{T}_j\right)^{1-\eta_j}\qquad\text{(CES)}$$

$$L_j=VA_j\cdot\left[\gamma_j^{\eta_j}\cdot sl_j\cdot(1-ty)\cdot PVA_j/\left((1+tlr)\cdot PL\right)\right]^{1/(1-\eta_j)}\qquad\text{(CES)}$$

$$YK=\sum_j PK\cdot K_j$$

$$YT=\sum_j PT_j\cdot\bar{T}_j$$

$$YL=\sum_j PL\cdot L_j$$

$$PT\_NR=\frac{\sum_{jnr}PT_{jnr}\cdot\bar{T}_{jnr}}{\sum_{jnr}\bar{T}_{jnr}}$$

$$TSD\_NR\_RS=\sum_{jnr}tsd\_nr\_int_{jnr}\cdot PT_{jnr}\cdot\bar{T}_{jnr}$$

$$\sum_j L_j=\bar{L}$$

$$\sum_j K_j=\bar{K}$$

Equations (long run)

$$PVA_j \cdot VA_j = \overline{PK}_j \cdot K_j/(1-tk) + \left(1 + tsd\_nr\_int_{jnr}\right) \cdot PT\_NR \cdot T_{jnr}$$
$$+PVA_{R\_R} \cdot VA_{R\_R} + (1+tlr) \cdot \overline{PL}_j \cdot L_j/(1-ty)$$

$$\overline{PK}_j \cdot K_j/(1-tk) = ak_j \cdot PVA_j \cdot VA_j \qquad \text{(Cobb-Douglas)}$$

$$\left(1 + tsd\_nr\_int_{jnr}\right) \cdot PT\_NR \cdot T_{jnr} = at_{jnr} \cdot PVA_{jnr} \cdot VA_{jnr} \qquad \text{(Cobb-Douglas)}$$

$$(1+tlr) \cdot \overline{PL}_j \cdot L_j/(1-ty) = al_j \cdot PVA_j \cdot VA_j \qquad \text{(Cobb-Douglas)}$$

$$K_j = VA_j \cdot \left[\gamma_j^{\eta_j} \cdot sk_j \cdot (1-tk) \cdot PVA_j/\overline{PK}_j\right]^{1/(1-\eta_j)} \qquad \text{(CES)}$$

$$T_{jnr} = VA_{jnr} \cdot \left[\gamma_{jnr}^{\eta_{jnr}} \cdot st_{jnr} \cdot PVA_{jnr}/\left(\left(1 + tsd\_nr\_int_{jnr}\right) \cdot PT\_NR\right)\right]^{1/(1-\eta_{jnr})} \qquad \text{(CES)}$$

$$L_j = VA_j \cdot \left[\gamma_j^{\eta_j} \cdot sl_j \cdot (1-ty) \cdot PVA_j/\left((1+tlr) \cdot \overline{PL}_j\right)\right]^{1/(1-\eta_j)} \qquad \text{(CES)}$$

$$YK = \sum_j \overline{PK}_j \cdot K_j$$

$$YT = \sum_{jnr} PT\_NR \cdot T_{jnr} + PVA_{R\_R} \cdot VA_{R\_R}$$

$$YL = \sum_j \overline{PL}_j \cdot L_j$$

$$TSD\_NR\_RS = \sum_{jnr} tsd\_nr\_int_{jnr} \cdot PT\_NR \cdot T_{jnr}$$

$$\sum_{jnr} T_{jnr} + \overline{T}_{R\_R} = \overline{T}$$

Equations in common regardless time frames

$$INT_{i,j} = aint_{i,j} \cdot Y_j$$

$$VA_j = ava_j \cdot Y_j$$

$$PY_j = ava_j \cdot PVA_j + \sum_i aint_{i,j} \cdot PQ_i$$

$$YH = skh \cdot YK + sth \cdot YT + YL + \overline{TRFH}$$

$$SH = mps \cdot (YH - TY - TLE - TCCL)$$

$$PQ_i \cdot C_i = ac_i \cdot (YH - TY - TLE - TCCL - SH)$$

$$TY = ty \cdot YH$$

$$TK = tk \cdot YK/(1-tk)$$

$$TLR = tlr \cdot YL/(1-ty)$$

$$TLE = tle \cdot YL/(1-ty)$$

$$TCCL = tccl \cdot PY_{R\_R} \cdot \overline{Q\_R\_STOCK}$$

$$TP = \sum_j tp_j \cdot PY_j \cdot Y_j$$

$$TC = \sum_j tc_j \cdot PY_j \cdot (1 + tp_j) \cdot Y_j$$

$$TSD\_R\_NS = tsd\_r\_fnl \cdot PY_{R\_NS} \cdot (1 + tp_{R\_NS}) \cdot Y_{R\_NS}$$

$$TSD\_NR\_NS = tsd\_nr\_fnl \cdot PY_{NR\_NS} \cdot (1 + tp_{NR\_NS}) \cdot Y_{NR\_NS}$$

$$TSD\_R\_RS = tsd\_rr\_fnl \cdot PY_{R\_R} \cdot (1 + tp_{R\_R}) \cdot Y_{R\_R}$$

$$YG\_DT = TSD\_R\_NS + TSD\_NR\_NS + TSD\_R\_RS + TSD\_NR\_RS + TCCL$$

$$YG\_NDT = TY + TK + TLR + TLE + TC + TP$$

$$YG\_F = skg \cdot YK + stg \cdot YT$$

$$TRFG = \overline{YG0} - \overline{YG\_DT0} - \overline{YG\_F0} - YG\_NDT$$

$$YG = YG\_DT + YG\_F + YG\_NDT + TRFG$$

$$PQ_i \cdot G_i = ag_i \cdot (YG - \overline{TRFH})$$

$$Y_i = \theta w_i \cdot \left[serow_i \cdot EROW_i^{\rho w_i} + (1 - serow_i) \cdot WD_i^{\rho w_i}\right]^{1/\rho w_i}$$

$$EROW_i = Y_i \cdot \left[\theta w_i^{\rho w_i} \cdot serow_i \cdot (1 + tmx_i) \cdot PY_i / \overline{PEROW_i}\right]^{-\sigma w_i}$$

$$WD_i = Y_i \cdot \left[\theta w_i^{\rho w_i} \cdot serow_i \cdot (1 + tmx_i) \cdot PY_i / PWD_i\right]^{-\sigma w_i}$$

$$WD_i = \theta k_i \cdot \left[seruk_i \cdot ERUK_i^{\rho k_i} + (1 - seruk_i) \cdot W_i^{\rho k_i}\right]^{1/\rho k_i}$$

$$ERUK_i = WD_i \cdot \left[\theta k_i^{\rho k_i} \cdot seruk_i \cdot PWD_i / \overline{PERUK_i}\right]^{-\sigma k_i}$$

$$W_i = WD_i \cdot \left[\theta k_i^{\rho k_i} \cdot (1 - seruk_i) \cdot PWD_i / PW_i\right]^{-\sigma k_i}$$

$$QW_i = \gamma k_i \cdot \left[smruk_i \cdot MRUK_i^{\eta k_i} + (1 - smruk_i) \cdot W_i^{\eta k_i}\right]^{1/\eta k_i}$$

$$MRUK_i = QW_i \cdot \left[\gamma k_i^{\eta k_i} \cdot smruk_i \cdot PQW_i / \overline{PMRUK_i}\right]^{\omega k_i}$$

$$W_i = QW_i \cdot \left[\gamma k_i^{\eta k_i} \cdot (1 - smruk_i) \cdot PQW_i / PW_i\right]^{\omega k_i}$$

$$Q_i = \gamma w_i \cdot \left[smrow_i \cdot MROW_i^{\eta w_i} + (1 - smrow_i) \cdot QW_i^{\eta w_i}\right]^{1/\eta w_i}$$

$$MROW_i = Q_i \cdot \left[\gamma w_i^{\eta w_i} \cdot smrow_i \cdot PQ_i / \overline{PMROW_i}\right]^{\omega w_i}$$

$$QW_i = Q_i \cdot \left[\gamma w_i^{\eta w_i} \cdot (1 - smrow_i) \cdot PQ_i / PQW_i\right]^{\omega w_i}$$

$$S = (1 - skh - skg) \cdot YK + (1 - sth - stg) \cdot YT + SH + \overline{SRUK} + \overline{SROW}$$

$$PQ_i \cdot I_i = ai_i \cdot S$$

$$PI = \sum_i (1 + tmx_i) \cdot PQ_i \cdot ai_i$$

$$Q_i = C_i + G_i + I_i + \sum_j INT_{i,j}$$

Appendix III Input-Output Table 2013 Wales

2013 IO Wales	Crop, Fish and Animal Production	Forestry and logging	Mining and Extraction	Manufacture of Food and Beverages	Manufacturing of Apparel and textiles	Manufacture of wood and paper products	Printing and reproduction of recorded media	Manufacturing of petroleum and chemical, pharamaceutical products
Crop, Fish and Animal Production	94.2	0.5	0.0	446.0	0.0	2.8	0.1	1.2
Forestry and logging	0.1	4.8	0.0	0.4	0.0	23.0	0.0	0.1
Mining and Extraction	0.5	0.0	4.1	0.9	0.0	1.4	0.0	3.6
Manufacture of Food and Beverages	85.0	0.1	0.1	264.2	0.9	2.1	0.2	9.0
Manufacturing of Apparel and textiles	0.5	0.0	0.0	0.4	2.4	0.2	0.0	0.3
Manufacture of wood and paper products	3.1	0.5	0.1	23.6	0.6	76.4	2.4	7.1
Printing and reproduction of recorded media	1.7	0.4	0.1	10.1	0.5	3.5	8.1	11.3
Manufacturing of petroleum and chemical, pharamaceutical products	42.4	1.0	1.9	24.5	1.4	7.9	0.9	85.9
Manufacturing of rubber and non metallic mineral products	9.0	0.1	1.9	65.6	0.6	12.3	1.2	21.8
Manufacture of Basic Metals and fabricated products	3.4	0.2	2.5	47.8	1.3	13.4	2.3	30.9
Manufacture of Computer and Electrical Equipment	1.4	0.2	0.4	6.2	0.2	2.5	0.5	7.6
Motor Vehicles	3.4	0.6	0.6	10.9	0.4	3.6	0.5	12.0
Furniture	1.0	0.1	0.1	5.7	0.2	16.4	0.3	6.7
Other manufacturing	0.4	0.0	0.0	1.4	0.1	0.4	0.1	1.6
Electricity Energy	13.3	1.0	3.6	50.2	2.9	25.9	2.1	77.3
Water collection treatment and supply	5.9	0.0	0.2	8.4	0.4	1.4	0.2	10.8
Construction	35.2	1.8	0.8	22.6	1.4	8.8	0.9	30.4
Wholesale	17.9	2.9	1.3	78.6	3.0	25.6	3.2	119.9
Retail	1.7	0.2	0.2	5.7	0.4	2.0	0.5	7.3
Accomodation	1.1	0.2	0.1	3.4	0.3	0.9	0.2	3.2
Restaurants	1.5	0.3	0.1	4.2	0.4	1.3	0.2	4.2
Railways	0.5	0.1	0.3	2.0	0.2	0.6	0.4	3.5
Road transport	2.4	0.4	5.8	44.9	1.8	15.4	1.8	14.8
Sea and Air transport	0.1	0.0	0.1	0.6	0.1	0.2	0.3	1.5
Transport services	1.5	0.0	0.1	7.0	0.6	1.6	0.2	8.9
Travel Agents	0.0	0.0	0.0	0.3	0.0	0.1	0.0	0.2
Postal services	1.6	0.0	0.1	10.5	1.1	3.4	0.3	9.5
Telecomms	4.7	0.1	0.1	3.0	1.5	1.5	0.8	3.1
Business Services	75.8	0.2	5.5	166.3	11.2	45.1	60.3	134.4
Real estate	14.9	0.0	0.2	8.2	1.1	4.5	6.0	6.0
Ownership & Rental of Dwellings	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Renting of moveables	0.2	0.0	0.9	6.4	0.7	2.7	0.5	3.5
Computer and related activities	0.9	0.1	0.1	4.7	0.3	1.4	0.4	8.2
R&D	0.6	0.0	0.0	1.5	0.1	0.4	0.1	2.1
Public Admin	0.5	1.2	0.1	0.9	0.1	0.4	0.0	1.1
Education	0.8	0.0	0.0	3.6	0.2	0.9	0.1	4.2
Health and social work	10.9	0.0	0.0	3.1	0.2	0.9	0.1	4.4
Museums & Galleries	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.1
Attractions, Gardens & other entertainment nec.	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.1
Theme parks and stadia	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.1
Other Recreation, media & film	16.0	0.5	0.1	4.2	0.6	1.1	8.7	7.7
Sanitary Services	0.9	0.0	0.1	10.4	0.6	3.4	0.2	8.4
Other Services	1.9	0.0	0.0	0.0	0.0	0.0	0.1	0.0
Total Intermediate	457.1	17.6	31.9	1359.0	37.8	315.5	103.9	673.9
Imports RUK	271.6	10.1	39.4	1101.1	53.5	384.2	74.7	809.0
Imports ROW	80.5	2.1	14.5	410.8	43.7	328.3	27.4	2215.8
Taxes less subsidies on production	-9.0	-0.6	1.9	17.9	1.2	12.8	1.7	19.4
Compensation of employees	155.1	20.8	15.3	586.3	53.1	217.7	71.4	454.2
Gross operating surplus	408.2	22.3	15.0	478.6	26.3	198.9	45.7	395.9
Taxes on Products	42.5	1.6	2.1	74.4	3.2	41.5	6.3	110.8
TOTAL	1407.0	74.0	120.0	4028.0	219.0	1499.0	331.0	4679.0

Manufacturing of rubber and non metallic mineral products	Manufacture of Basic Metals and fabricated products	Manufacture of Computer and Electrical Equipment	Motor Vehicles	Furniture	Other manufacturing	Electricity - Coal	Water collection treatment and supply	Construction	Wholesale	Retail	Accomodation	Restaurants etc	Railways	Road transport
0.3	0.3	0.1	0.4	0.1	0.1	0.5	0.1	2.7	4.3	9.6	5.4	23.6	0.0	0.1
0.1	0.2	0.0	0.2	0.1	0.1	0.0	0.0	0.6	0.2	0.1	0.0	0.0	0.0	0.0
7.2	7.2	0.2	1.0	0.0	0.2	50.4	0.2	22.4	2.4	0.6	0.1	0.3	0.4	0.1
1.8	6.9	2.1	9.2	0.1	0.9	1.4	0.2	3.4	23.3	54.1	41.2	143.9	1.0	1.1
0.3	0.4	0.2	5.4	0.2	0.1	0.1	0.0	1.7	0.7	1.3	0.2	1.5	0.0	0.2
7.4	9.1	5.0	19.9	11.0	7.0	0.9	0.3	15.0	5.4	4.4	1.5	1.7	0.3	1.2
5.8	14.3	9.6	20.8	0.7	1.7	3.9	3.0	12.7	20.5	10.6	0.1	0.2	0.4	4.3
21.6	31.2	9.6	37.8	2.2	10.2	16.2	1.5	34.2	56.7	33.7	5.3	13.5	3.4	60.2
112.9	29.1	22.9	189.0	4.5	26.9	1.9	1.0	202.6	28.8	11.5	0.9	1.7	3.5	8.8
35.0	491.2	58.6	548.5	5.4	26.7	8.1	3.9	72.6	13.7	10.2	1.1	2.3	1.5	2.5
3.9	23.4	69.6	54.9	0.4	3.1	5.6	0.5	12.3	5.1	4.6	0.4	1.0	0.7	2.3
6.0	28.8	7.7	313.1	0.7	4.9	3.1	0.7	23.7	7.7	5.1	0.7	1.6	2.6	3.3
5.2	13.4	4.6	40.5	18.3	7.3	0.7	0.1	22.3	2.2	3.2	0.8	1.3	0.3	0.4
1.4	28.8	0.9	4.7	0.2	1.7	0.3	0.0	3.4	1.2	1.2	0.2	0.3	0.1	0.3
43.4	231.3	16.8	71.9	0.7	11.5	1855.0	18.5	25.1	14.7	33.1	25.7	4.4	6.6	5.1
2.7	10.3	1.4	6.7	0.1	1.0	0.7	0.4	6.1	1.9	2.6	6.6	0.2	0.2	0.5
9.0	52.7	9.8	79.0	1.2	3.0	28.5	51.1	720.9	52.5	43.0	9.8	10.6	8.9	23.5
32.4	185.2	63.9	168.1	4.8	20.2	30.5	1.1	54.5	12.4	13.5	3.9	9.9	2.0	3.7
2.9	12.7	4.1	10.8	0.3	1.6	2.3	0.4	12.5	5.8	12.2	0.7	1.7	2.5	2.2
1.2	4.8	1.6	4.7	0.1	0.7	0.9	0.6	5.0	3.8	29.9	0.7	0.6	0.6	1.7
1.8	7.0	2.3	6.5	0.1	0.9	1.2	0.3	3.5	5.2	43.2	1.8	2.1	1.5	1.7
2.5	23.6	1.9	12.4	0.1	1.1	1.7	0.0	4.4	7.6	3.4	0.6	3.0	2.0	0.7
48.2	95.0	8.4	39.3	2.0	7.5	0.8	0.3	38.3	144.1	44.8	2.9	8.8	2.1	16.6
0.9	20.3	0.6	3.7	0.0	0.4	0.0	0.0	2.2	3.3	1.1	0.4	1.2	0.4	0.5
4.6	8.6	6.2	28.3	0.5	1.1	2.7	2.4	17.5	216.0	25.7	4.2	10.9	100.8	42.8
0.2	0.6	0.2	0.6	0.0	0.1	0.1	0.1	1.7	2.4	0.9	0.9	0.0	1.4	0.7
3.9	12.8	1.0	8.4	0.2	0.7	0.6	0.1	7.0	24.2	10.8	1.7	6.5	0.8	2.6
2.4	7.1	3.1	4.0	0.3	1.6	2.3	1.1	10.0	26.7	10.6	2.5	9.7	1.3	3.7
72.6	268.6	88.8	287.5	7.1	35.4	38.1	365.4	257.9	245.5	252.1	40.4	57.2	12.8	55.2
7.9	13.5	6.6	16.3	0.8	4.5	14.4	2.2	86.0	90.7	171.9	5.1	12.3	1.0	6.3
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4.5	13.9	2.6	8.4	0.3	1.8	0.6	0.9	40.1	2.5	1.4	0.2	0.2	12.7	7.1
2.0	16.7	4.4	43.5	0.1	0.9	2.5	4.7	21.4	28.7	23.9	2.0	7.0	3.7	13.5
0.8	2.1	1.4	5.2	0.1	0.2	1.9	0.2	1.4	1.5	1.2	0.3	0.4	0.4	0.5
0.5	1.8	0.3	2.2	0.1	0.8	2.5	15.0	0.8	0.2	0.3	0.2	0.1	0.3	3.7
2.0	4.5	3.2	12.4	0.2	0.6	1.2	0.6	3.7	4.1	5.0	1.3	5.2	1.1	1.5
1.7	4.0	2.8	10.0	0.1	0.6	1.3	1.1	4.9	4.5	3.0	1.2	4.3	0.8	1.1
0.0	0.1	0.0	0.1	0.0	0.0	0.0	0.0	0.2	0.1	1.2	0.0	0.4	0.0	0.0
0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.3	0.1	0.2	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.3	0.1	0.2	0.0	0.0	0.0	0.0
2.9	11.6	5.3	16.2	0.2	2.2	0.1	0.2	5.3	6.2	6.4	3.4	17.1	1.3	2.5
4.4	6.0	2.0	11.2	0.3	3.5	1.3	82.1	3.4	2.3	3.4	1.8	2.7	0.6	0.9
0.0	0.1	0.0	1.2	0.0	0.0	1.0	17.7	1.5	1.9	2.9	1.5	5.7	0.7	0.9
464.6	1699.1	429.9	2104.2	63.4	192.7	2085.6	577.9	1765.1	1081.3	898.2	177.4	375.5	180.8	284.2
430.4	1877.1	466.2	2001.9	71.5	226.2	512.4	95.9	1223.2	763.3	810.8	139.7	344.2	152.7	204.9
282.4	1248.9	429.3	1846.6	47.5	153.4	344.3	22.3	334.8	174.8	132.5	4.9	0.6	12.5	52.9
15.5	101.2	11.5	31.4	2.7	8.1	109.6	24.3	51.3	49.5	184.2	39.7	39.4	1.3	15.2
413.0	1185.1	366.0	1228.8	59.1	148.2	163.6	21.7	1385.1	1082.9	1281.0	321.6	510.0	116.3	445.4
254.6	539.7	228.1	610.4	52.7	172.9	488.0	81.3	890.0	450.4	1014.5	114.1	353.6	20.4	129.5
28.4	98.9	31.1	195.6	7.0	15.5	70.6	14.6	145.0	86.7	78.7	12.6	32.6	8.9	70.9
1889.0	6750.0	1962.0	8019.0	304.0	917.0	3774.1	838.0	5794.6	3689.0	4400.0	810.0	1656.0	493.0	1203.0

Sea and Air transport	Transport services	Travel Agents	Postal services	Telecomms	Business Services	Real estate	Ownership of Dwellings	Renting of moveables	Computer and related activities	R&D	Public Admin	Education	Health and social work	Museums & Galleries
0.2	0.4	0.0	0.1	0.1	0.9	0.4	0.7	0.0	0.0	0.1	2.1	2.2	2.5	0.0
0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.1	0.0	0.0	0.0	0.2	0.0	0.1	0.0
0.0	6.8	0.0	0.0	0.2	0.6	0.2	0.8	0.0	0.0	0.0	2.0	0.4	1.0	0.0
0.4	4.0	0.3	1.4	1.6	13.6	0.2	0.3	0.2	0.5	0.2	8.9	3.6	29.1	0.0
0.0	0.2	0.0	0.1	0.1	1.6	0.1	0.1	0.1	0.0	0.1	3.4	1.0	0.4	0.0
0.0	1.5	0.2	0.3	0.6	11.1	0.9	1.1	0.6	0.3	0.1	25.2	2.8	23.1	0.0
0.3	12.9	1.9	1.1	2.1	92.4	3.2	4.7	0.7	1.6	0.2	34.1	10.8	8.2	0.2
6.4	9.6	0.4	2.5	5.2	34.5	2.2	4.7	4.5	1.9	0.7	41.1	8.6	104.2	0.0
0.2	5.7	0.2	0.9	12.2	10.9	0.7	1.1	0.5	0.8	0.1	15.9	4.4	13.2	0.0
0.3	4.0	0.5	0.8	2.8	18.5	1.8	2.6	1.9	1.8	0.1	46.0	2.3	11.9	0.0
0.2	2.1	0.2	0.6	11.7	6.5	0.9	1.3	1.8	1.6	0.1	39.6	1.9	48.4	0.5
2.6	8.9	0.3	0.7	2.0	10.5	1.1	2.3	2.4	0.9	0.1	132.4	1.1	9.9	0.0
0.0	1.0	0.1	0.2	0.5	5.7	0.4	0.6	0.6	0.2	0.0	10.1	1.9	3.4	0.0
0.0	0.4	0.1	0.1	0.4	1.9	0.2	0.1	0.1	0.1	0.0	2.0	0.5	0.9	0.0
0.7	6.2	0.9	2.5	4.2	31.5	2.2	6.7	4.2	2.7	0.8	47.1	8.9	40.1	0.3
0.1	1.0	0.1	0.2	0.2	2.7	0.5	2.3	0.3	0.1	0.1	16.0	2.5	8.2	0.1
1.5	38.1	9.6	4.5	13.6	82.3	62.5	337.9	22.3	5.7	0.5	290.1	14.3	35.9	0.5
0.4	3.7	0.3	0.8	17.0	21.3	0.8	1.0	1.6	1.3	0.2	47.6	4.1	35.2	0.1
0.3	3.6	0.6	0.5	2.7	12.9	1.3	0.7	0.7	0.4	0.1	7.9	1.1	5.5	0.0
0.8	3.5	0.3	0.8	1.1	17.3	0.8	2.6	0.3	0.6	0.0	12.1	0.8	4.2	0.1
1.2	4.9	0.5	1.1	1.4	23.8	1.5	2.1	0.1	0.9	0.0	16.2	1.6	8.5	0.1
0.1	42.2	0.3	7.7	0.5	9.5	0.6	0.6	2.5	0.1	0.2	31.6	4.4	9.0	0.1
0.2	19.0	0.5	6.4	1.9	29.5	1.0	1.3	7.6	1.1	0.2	13.1	7.8	40.8	0.1
2.3	5.0	0.2	2.4	0.4	12.3	0.5	0.6	0.5	0.4	0.0	1.9	0.3	0.6	0.0
8.8	167.9	0.5	3.6	5.2	75.6	3.0	4.5	5.3	0.9	0.3	8.5	5.0	4.9	0.0
0.2	3.8	0.0	0.1	0.1	1.1	0.1	0.1	0.1	0.0	0.0	0.3	0.1	0.2	0.0
0.4	3.5	3.2	9.8	3.3	150.0	0.8	1.3	1.7	0.7	0.3	44.9	3.7	10.8	0.0
0.6	11.9	3.9	2.8	26.7	114.9	2.9	4.3	3.1	2.1	0.2	28.7	2.9	26.0	0.0
10.2	144.6	30.3	19.4	24.3	1150.2	72.4	115.3	63.7	77.4	14.5	414.7	24.5	219.8	0.9
2.0	19.3	2.3	3.5	5.5	43.7	32.0	0.4	1.9	1.9	0.8	51.8	3.5	32.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1	9.5	1.3	0.6	0.3	6.5	0.4	0.6	2.1	0.4	0.1	2.4	2.5	2.4	0.1
4.1	31.9	2.2	6.6	8.9	85.1	6.1	9.2	3.6	4.6	0.5	72.0	15.0	37.2	0.0
0.1	1.1	2.8	0.2	0.3	4.8	0.3	0.5	0.1	0.8	0.3	11.9	2.5	21.8	0.0
0.1	0.0	4.1	0.2	0.0	4.9	26.5	0.0	0.0	0.0	0.1	16.5	0.2	0.3	0.8
0.6	6.8	5.4	2.4	4.0	65.2	2.7	4.0	0.3	3.0	2.8	137.4	45.7	23.7	0.1
0.3	1.7	3.2	0.7	1.3	11.3	0.2	0.2	0.4	0.4	0.6	2.5	1.4	2822.7	0.0
0.0	0.1	0.0	0.0	0.0	0.3	0.0	0.1	0.0	0.0	0.0	0.3	0.0	0.1	0.5
0.0	0.0	0.0	0.0	0.0	0.2	0.0	0.1	0.0	0.0	0.0	0.2	0.0	0.1	0.0
0.0	0.0	0.0	0.0	0.0	0.2	0.0	0.1	0.0	0.0	0.0	0.2	0.0	0.1	0.0
0.5	16.5	3.7	3.1	7.8	60.1	2.4	4.1	1.2	5.3	0.2	65.6	6.9	42.9	0.3
0.2	2.1	0.0	1.0	0.7	7.9	0.3	0.4	1.5	0.5	0.3	60.7	2.9	31.2	0.0
0.7	2.6	0.4	1.5	0.8	13.8	0.1	0.1	3.3	0.3	0.2	13.5	1.2	11.9	0.1
47.0	608.1	80.9	91.1	171.7	2247.8	234.4	521.3	142.0	121.4	24.9	1778.9	205.0	3732.1	4.8
38.9	712.7	53.1	68.7	192.9	1786.7	142.7	301.0	117.2	88.9	10.8	988.7	121.6	2087.3	2.6
30.7	58.6	5.4	28.5	108.8	394.4	19.4	29.2	25.6	75.9	13.4	926.7	51.3	575.4	0.1
1.8	34.0	4.8	4.7	5.8	100.2	1.4	-40.6	9.5	4.2	0.8	41.4	13.0	6.0	0.0
46.6	643.5	299.3	246.5	243.6	2771.6	223.6	220.8	207.9	252.5	48.7	3356.5	898.1	2911.4	72.2
28.0	272.6	49.1	49.3	240.0	1611.5	730.7	5672.2	157.8	101.2	20.5	594.0	71.5	894.5	0.0
12.0	22.5	5.5	4.3	20.2	196.6	15.8	50.0	13.2	11.9	1.8	187.3	14.5	100.7	0.3
205.0	2352.0	498.0	493.0	983.0	9108.7	1368.0	6754.0	673.0	656.0	121.0	7873.5	1375.0	10307.4	80.0

Attractions, Gardens etc.	Theme parks, stadia & professional sports teams	Other Recreation	Sanitary Services	Other Services	Intermediate	Consumers	NPISH	Central & Local Government	Exports	Stock2000	GFCF	Exports RUK	Total Demand
0.1	0.7	0.4	0.1	0.0	603.2	115.9	0.0	0.0	142.5	1.3	10.6	533.4	1406.9
0.0	0.0	0.0	0.0	0.0	30.6	0.4	0.0	0.0	0.9	1.9	0.4	39.8	74.0
0.0	0.0	0.1	1.0	0.1	116.4	0.1	0.0	0.0	0.6	0.0	0.1	2.7	120.0
0.2	1.9	7.9	0.8	0.2	727.5	1061.5	0.0	0.0	111.7	11.7	1.7	2113.9	4028.0
0.0	0.2	4.8	0.0	0.1	28.8	9.7	0.0	0.0	44.4	0.7	0.2	135.2	219.0
0.0	0.1	0.5	0.8	0.2	273.2	72.3	0.0	0.0	224.6	4.8	22.3	901.8	1499.0
0.0	0.0	0.4	2.1	2.4	323.7	1.3	0.0	0.0	0.9	0.0	0.0	5.1	331.0
0.0	0.3	12.9	14.6	3.7	761.1	152.0	0.0	0.0	1272.4	77.4	12.1	2403.9	4679.0
0.0	0.1	2.8	9.2	0.8	838.4	43.3	0.0	0.0	119.4	7.0	11.1	869.8	1889.0
0.3	0.2	1.8	3.6	0.7	1484.8	88.1	0.0	0.0	1736.6	29.5	112.5	3298.5	6750.0
0.5	0.4	15.0	1.2	0.6	341.6	65.1	0.0	0.0	1044.0	12.8	27.0	471.4	1962.0
0.1	0.4	2.2	2.0	0.6	621.9	186.7	0.0	0.0	1527.1	23.0	108.1	5552.2	8019.0
0.0	0.0	0.8	0.4	0.2	177.3	12.0	0.0	0.0	25.2	0.3	6.0	83.3	304.0
0.0	0.4	1.0	0.3	0.1	57.3	25.3	0.0	0.0	218.1	3.4	1.6	611.3	917.0
1.3	2.3	11.0	8.3	1.4	2723.4	645.0	0.0	0.0	7.3	32.3	30.4	335.4	3773.9
0.2	0.7	2.3	2.4	0.2	108.9	179.8	0.0	0.0	4.3	0.7	19.3	525.0	838.0
2.7	1.1	5.8	11.8	3.8	2150.0	237.4	0.0	0.0	93.1	12.2	2501.1	800.8	5794.6
0.2	4.8	5.1	4.4	1.5	1009.8	1802.9	0.0	0.0	111.6	204.7	29.1	530.9	3689.0
0.3	0.0	1.1	1.6	0.8	132.9	1953.9	0.0	0.0	704.6	-88.5	91.8	1605.4	4400.0
0.1	0.0	1.0	0.2	0.3	113.1	528.5	0.0	0.0	40.8	35.7	0.2	91.6	810.0
0.3	0.0	0.2	0.2	0.4	156.5	243.4	0.0	0.0	1067.7	20.8	28.7	138.9	1656.0
0.9	0.0	1.1	4.5	0.9	189.5	147.9	0.0	0.0	142.9	-5.8	1.5	17.0	493.0
0.7	0.0	0.8	6.7	1.5	686.4	266.4	0.0	0.0	267.4	-117.6	0.0	100.4	1203.0
0.0	0.0	7.1	0.8	0.2	73.6	59.9	0.0	0.0	35.0	0.0	2.6	33.8	205.0
0.0	0.0	5.1	4.6	2.5	798.6	775.4	0.0	0.0	466.0	-0.2	0.5	311.7	2352.0
0.1	0.0	0.4	0.3	0.1	17.9	26.6	0.0	0.0	5.0	-0.1	0.0	448.6	498.0
0.0	0.0	0.5	2.6	1.3	346.5	80.2	0.0	0.0	54.5	-2.5	0.1	14.3	493.0
0.0	0.1	5.9	2.6	3.3	343.8	144.0	0.0	0.0	20.7	-0.1	0.1	474.5	983.0
2.2	2.2	82.8	57.0	48.5	5158.1	1502.1	0.0	0.0	1507.8	10.2	23.2	907.3	9108.7
0.0	0.0	1.2	3.1	2.6	687.9	207.3	5.3	0.0	101.3	-9.6	39.2	336.6	1368.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	399.1	-9.5	0.0	6364.4	6754.0
0.0	0.1	3.0	6.5	2.7	153.5	482.3	0.0	0.0	0.0	0.4	0.0	36.8	673.0
0.0	0.0	11.6	6.3	5.6	501.7	25.6	0.0	0.0	17.8	0.3	0.0	110.5	656.0
0.6	0.0	0.1	6.2	0.0	76.6	0.8	0.0	0.0	6.7	-0.1	0.0	36.9	121.0
0.3	0.0	0.0	0.1	0.1	87.3	37.2	1664.9	0.0	180.9	-92.2	4.0	5991.4	7873.5
0.0	0.0	5.5	4.9	2.5	373.4	19.7	0.0	939.0	0.0	0.0	0.0	42.8	1375.0
0.1	0.0	1.4	4.4	3.6	2917.6	744.1	1146.1	4077.5	6.9	-3.0	0.1	1418.2	10307.4
0.4	0.0	0.8	0.1	0.0	5.2	1.3	2.7	69.5	0.2	0.0	0.0	1.1	80.0
0.0	0.0	14.4	0.0	0.0	16.3	6.3	0.0	28.1	48.7	0.0	0.7	1.0	101.0
0.0	7.2	14.7	0.0	0.0	23.9	3.4	0.0	16.8	61.2	0.2	0.7	0.9	107.0
10.5	2.4	108.1	6.0	12.2	479.6	167.9	0.0	23.6	508.2	156.2	9.2	111.4	1456.0
0.1	0.0	0.7	172.2	3.6	436.4	155.4	111.5	305.8	325.3	1.0	38.4	62.3	1436.1
0.0	0.0	14.6	7.6	6.0	115.9	70.2	0.0	257.1	2.0	-0.1	0.0	190.8	636.0
22.1	25.8	357.2	361.6	115.4	26270.3	12348.5	2930.4	5717.4	12655.6	319.3	3134.6	38063.0	101439.3
19.3	39.9	499.1	194.9	57.3	19589.3	8313.9	155.2	425.2	1155.1	144.4	1540.9	1548.7	32872.8
0.2	0.2	1.8	76.3	43.3	10676.2	5439.0	83.2	0.0	569.0	34.6	835.0	495.0	18131.9
4.6	1.1	0.0	35.0	8.3	966.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	966.2
43.5	46.5	410.5	472.9	249.9	23967.9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	23967.9
9.1	-8.9	146.2	267.0	152.6	18050.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	18050.0
2.1	2.4	41.3	28.4	9.2	1919.5	1871.6	16.1	75.8	1519.7	17.9	313.1	3062.5	8796.2
101.0	107.0	1456.0	1436.1	636.0	101439.3	27973.0	3185.0	6218.4	15899.4	516.2	5823.6	43169.2	204224.4

Appendix IV Balancing codes in GAMS

IV-1 Least Square Method

```
set ac  all accounts
        /i_agr,i_min,i_man,i_egy,i_con,i_r_ns,i_nr_ns,i_wr,i_trp,i_acm,i_inf,i_fin,
        i_r_rs,i_nr_rs,i_rtt,i_prf,i_adm,i_plc,i_edu,i_hlt,i_ent,i_oth,
        c_agr,c_min,c_man,c_egy,c_con,c_r_ns,c_nr_ns,c_wr,c_trp,c_acm,c_inf,c_fin,
        c_r_rs,c_nr_rs,c_rtt,c_prf,c_adm,c_plc,c_edu,c_hlt,c_ent,c_oth,
        nic1,lab,lnd,ova,t_pn,sdlt_r,sdlt_nr,t_pt,t_inc,nic2,t_cncl,t_cp,
        hh,gov,sav,ruk,row,total/
    a(ac) all accounts excl. totals
        /i_agr,i_min,i_man,i_egy,i_con,i_r_ns,i_nr_ns,i_wr,i_trp,i_acm,i_inf,i_fin,
        i_r_rs,i_nr_rs,i_rtt,i_prf,i_adm,i_plc,i_edu,i_hlt,i_ent,i_oth,
        c_agr,c_min,c_man,c_egy,c_con,c_r_ns,c_nr_ns,c_wr,c_trp,c_acm,c_inf,c_fin,
        c_r_rs,c_nr_rs,c_rtt,c_prf,c_adm,c_plc,c_edu,c_hlt,c_ent,c_oth,
        nic1,lab,lnd,ova,t_pn,sdlt_r,sdlt_nr,t_pt,t_inc,nic2,t_cncl,t_cp,
        hh,gov,sav,ruk,row/
    i(a) industrial activities
        /i_agr,i_min,i_man,i_egy,i_con,i_r_ns,i_nr_ns,i_wr,i_trp,i_acm,i_inf,i_fin,
        i_r_rs,i_nr_rs,i_rtt,i_prf,i_adm,i_plc,i_edu,i_hlt,i_ent,i_oth/
    c(a) commodities
        /c_agr,c_min,c_man,c_egy,c_con,c_r_ns,c_nr_ns,c_wr,c_trp,c_acm,c_inf,c_fin,
        c_r_rs,c_nr_rs,c_rtt,c_prf,c_adm,c_plc,c_edu,c_hlt,c_ent,c_oth/
    g(a) GVA components /nic1,lab,lnd,ova,t_pn,sdlt_r,sdlt_nr/
    t(a) tax categories /t_pt,t_inc,nic2,t_cncl,t_cp/
    n(a) all final demand institutions /hh,gov,sav,ruk,row/
;
alias (ac,ca),(a,b),(i,j),(c,k),(g,h),(t,s),(n,m);

set ik(i,k)    activity-commodity sub-matrix /#i.#k/     //These are used for excluding conditions
imposed later before solving
    ic(i,k)    diagonal cells of ik /#i:#k/
    tk(t,k)    tax-commodity sub-matrix /#t.#k/
```

```
    tptk(t,k)   product tax row of tk /t_pt.#k/
    nk(n,k)     institution-commodity sub-matrix /#n.#k/
    rukk(n,k)   RUK row of nk /ruk.#k/
    as(a,s)     taxes pooling sub-matrix /#a.#s/
    govtpt(a,s) product tax revenue for gov /gov.t_pt/
    im(i,m)     activity-institution sub-matrix /#i.#m/
    iruk(i,m)   RUK column of im /#i.ruk/
    cm(c,m)     final demand sub-matrix /#c.#m/
    csav(c,m)   investment demand column of cm /#c.sav/
    nm(n,m)     inter-institutional block /#n.#m/
    trf(n,m)    balancing terms in nm /ruk.sav, gov.ruk/
;

table SAM0(ac,ca)                                    //import excel
$ondelim
$include Unbalanced 2013 Welsh SAM4_positive.csv
$offdelim
;
display SAM0;

parameter Q0(a,b) intial value of each entry;
Q0(a,b)=sam0(a,b);



positive variable Q1(a,b);                              //!!!at last probably have to turn to set
as positive!!!
variable
*               Q1(a,b)
              z1 the objective function
              rtotal1 total value for each row
              ctotal1  total value for each column;

equation EQobjfn1    objective function equation
       EQrtotal1   calculate the total for each row
```

```
        EQctotal1   calculate the total for each column
        EQbalance1  balance condition for each pair of row and column
*$ontext
        EQfx1       1st fixing condition
        EQfx2       2nd fixing condition
        EQfx3       3rd fixing condition
        EQfx4       4th fixing condition
        EQfx5       5th fixing condition
*$offtext
;
EQobjfn1..       z1 =e= sum((a,b),power((Q1(a,b)-Q0(a,b)),2));
*EQobjfn1..       z1 =e= sum((a,b)$sam0(a,b),(Q1(a,b)-Q0(a,b))**2);   //seems not work; no
need for $sam0(i,j), since zero entries in sam0(i,j)
                                          //will not act as denominators here in the obj
function
EQrtotal1(a)..    rtotal1(a) =e= sum(b,Q1(a,b));
EQctotal1(b)..    ctotal1(b) =e= sum(a,Q1(a,b));
EQbalance1(a)..   rtotal1(a) =e= ctotal1(a);
*$ontext
EQfx1..          sum(c,Q1(c,'i_R_ns'))-Q1('c_R_ns','i_R_ns')-Q1('c_NR_ns','i_R_ns') //Fix the
total of R new sale's intermediate inputs
                         -Q1('c_R_rs','i_R_ns')-Q1('c_NR_rs','i_R_ns')     //excluding     the
inputs purchases from all kinds of real estate
          =e=sum(c,Q0(c,'i_R_ns'))-Q0('c_R_ns','i_R_ns')-Q0('c_NR_ns','i_R_ns')
                         -Q0('c_R_rs','i_R_ns')-Q0('c_NR_rs','i_R_ns');


EQfx2..        sum(c,Q1(c,'i_NR_ns'))-Q1('c_R_ns','i_NR_ns')-Q1('c_NR_ns','i_NR_ns') //Fix
the total of NR new sale's intermediate inputs
                         -Q1('c_R_rs','i_NR_ns')-Q1('c_NR_rs','i_NR_ns')  //excluding  the
inputs purchases from all kinds of real estate
          =e=sum(c,Q0(c,'i_NR_ns'))-Q0('c_R_ns','i_NR_ns')-Q0('c_NR_ns','i_NR_ns')
                         -Q0('c_R_rs','i_NR_ns')-Q0('c_NR_rs','i_NR_ns');
```

EQfx3..   sum(c,Q1(c,'i_R_rs'))-Q1('c_R_ns','i_R_rs')-Q1('c_NR_ns','i_R_rs') //Fix the total of R resale's intermediate inputs

-Q1('c_R_rs','i_R_rs')-Q1('c_NR_rs','i_R_rs') //excluding the inputs purchases from all kinds of real estate

=e=sum(c,Q0(c,'i_R_rs'))-Q0('c_R_ns','i_R_rs')-Q0('c_NR_ns','i_R_rs')

-Q0('c_R_rs','i_R_rs')-Q0('c_NR_rs','i_R_rs');

EQfx4..   sum(c,Q1(c,'i_NR_rs'))-Q1('c_R_ns','i_NR_rs')-Q1('c_NR_ns','i_NR_rs') //Fix the total of NR resale's intermediate inputs

-Q1('c_R_rs','i_NR_rs')-Q1('c_NR_rs','i_NR_rs') //excluding the inputs purchases from all kinds of real estate

=e=sum(c,Q0(c,'i_NR_ns'))-Q0('c_R_ns','i_NR_rs')-Q0('c_NR_ns','i_NR_rs')

-Q0('c_R_rs','i_NR_rs')-Q0('c_NR_rs','i_NR_rs');

EQfx5..   sum(c,Q1(c,'sav'))=e=sum(c,Q0(c,'sav'));

*$offtext

*Q1.l(a,b)=Q0(a,b);   //starting point - canNOT omit this sentence for the following fixing

*try to fix some parts

*$ontext

Q1.fx(i,j)=Q0(i,j);   //No entries between activity-activity

Q1.fx('c_R_ns',b)=Q0('c_R_ns',b);   //For fixing real estate sectors' totals.

Q1.fx('c_NR_ns',b)=Q0('c_NR_ns',b);

Q1.fx('c_R_rs',b)=Q0('c_R_rs',b);

Q1.fx('c_NR_rs',b)=Q0('c_NR_rs',b);

Q1.fx(g,j)=Q0(g,j);   //Fix GVA

Q1.fx(t,j)=Q0(t,j);   //No other tax payment during production

Q1.fx(n,j)=Q0(n,j);   //No direct payments to institutions during production

Q1.fx(i,k)$(ik(i,k) xor ic(i,k))=Q0(i,k);   //No entries between activity-commodity except those on diagonal

Q1.fx(c,k)=Q0(c,k);   //No entries between commodity-commodity

Q1.fx(g,k)=Q0(g,k);   //Fix GVA(SDLT)

Q1.fx(t,k)$(tk(t,k) xor tptk(t,k))=Q0(t,k);   //Only product taxes added on commodities

```
Q1.fx(n,k)$(nk(n,k) xor rukk(n,k))=Q0(n,k);    //Only imports added on commodities
Q1.fx(a,'c_R_ns')=Q0(a,'c_R_ns');              //For fixing real estate sectors' totals.
Q1.fx(a,'c_NR_ns')=Q0(a,'c_NR_ns');
Q1.fx(a,'c_R_rs')=Q0(a,'c_R_rs');
Q1.fx(a,'c_NR_rs')=Q0(a,'c_NR_rs');
Q1.fx(a,h)=Q0(a,h);                            //Fix Corporate Tax payment, factor income distribution
Q1.fx(a,s)$(as(a,s) xor govtpt(a,s))=Q0(a,s);  //Fix tax revenues for gov except product tax
Q1.fx(i,m)$(im(i,m) xor iruk(i,m))=Q0(i,m);    //Fix exports except RUK export
Q1.fx('i_R_ns',b)=Q0('i_R_ns',b);              //For fixing real estate sectors' totals.
Q1.fx('i_NR_ns',b)=Q0('i_NR_ns',b);
Q1.fx('i_R_rs',b)=Q0('i_R_rs',b);
Q1.fx('i_NR_rs',b)=Q0('i_NR_rs',b);
Q1.fx(c,m)$(cm(c,m) xor csav(c,m))=Q0(c,m);      //Fix all final demands except for GFCF
(investment demand), however it is fixed in total value
Q1.fx(g,m)=Q0(g,m);                            //Fix HH rental exp
Q1.fx(t,m)=Q0(t,m);                            //Fix HH tax exp
Q1.fx(n,m)$(nm(n,m) xor trf(n,m))=Q0(n,m);       //Fix inter-institutional transfers except the
balancing entries
*$offtext

model LSMethod /all/;
solve LSMethod using nlp minimizing z1;

parameter sam1f(ac,ca) the balanced SAM WITH some fixed original value;
sam1f(a,b)=Q1.l(a,b);
sam1f('total',b)=ctotal1.l(b);
sam1f(a,'total')=rtotal1.l(a);

display z1.l,sam0,Q1.l;
display sam1f;
```

IV-2 Direct Entropy Method

```
set ac  all accounts
        /i_agr,i_min,i_man,i_egy,i_con,i_r_ns,i_nr_ns,i_wr,i_trp,i_acm,i_inf,i_fin,
        i_r_rs,i_nr_rs,i_rtt,i_prf,i_adm,i_plc,i_edu,i_hlt,i_ent,i_oth,
        c_agr,c_min,c_man,c_egy,c_con,c_r_ns,c_nr_ns,c_wr,c_trp,c_acm,c_inf,c_fin,
        c_r_rs,c_nr_rs,c_rtt,c_prf,c_adm,c_plc,c_edu,c_hlt,c_ent,c_oth,
        nic1,lab,lnd,ova,t_pn,sdlt_r,sdlt_nr,t_pt,t_inc,nic2,t_cncl,t_cp,
        hh,gov,sav,ruk,row,total/
    a(ac) all accounts excl. totals
        /i_agr,i_min,i_man,i_egy,i_con,i_r_ns,i_nr_ns,i_wr,i_trp,i_acm,i_inf,i_fin,
        i_r_rs,i_nr_rs,i_rtt,i_prf,i_adm,i_plc,i_edu,i_hlt,i_ent,i_oth,
        c_agr,c_min,c_man,c_egy,c_con,c_r_ns,c_nr_ns,c_wr,c_trp,c_acm,c_inf,c_fin,
        c_r_rs,c_nr_rs,c_rtt,c_prf,c_adm,c_plc,c_edu,c_hlt,c_ent,c_oth,
        nic1,lab,lnd,ova,t_pn,sdlt_r,sdlt_nr,t_pt,t_inc,nic2,t_cncl,t_cp,
        hh,gov,sav,ruk,row/
    i(a) industrial activities
        /i_agr,i_min,i_man,i_egy,i_con,i_r_ns,i_nr_ns,i_wr,i_trp,i_acm,i_inf,i_fin,
        i_r_rs,i_nr_rs,i_rtt,i_prf,i_adm,i_plc,i_edu,i_hlt,i_ent,i_oth/
    c(a) commodities
        /c_agr,c_min,c_man,c_egy,c_con,c_r_ns,c_nr_ns,c_wr,c_trp,c_acm,c_inf,c_fin,
        c_r_rs,c_nr_rs,c_rtt,c_prf,c_adm,c_plc,c_edu,c_hlt,c_ent,c_oth/
    g(a) GVA components /nic1,lab,lnd,ova,t_pn,sdlt_r,sdlt_nr/
    t(a) tax categories /t_pt,t_inc,nic2,t_cncl,t_cp/
    n(a) all final demand institutions /hh,gov,sav,ruk,row/
;
alias (ac,ca),(a,b),(i,j),(c,k),(g,h),(t,s),(n,m);

set ik(i,k)     activity-commodity sub-matrix /#i.#k/      //These are used for excluding conditions
imposed later before solving
    ic(i,k)     diagonal cells of ik /#i:#k/
    tk(t,k)     tax-commodity sub-matrix /#t.#k/
    tptk(t,k)   product tax row of tk /t_pt.#k/
    nk(n,k)     institution-commodity sub-matrix /#n.#k/
    rukk(n,k)   RUK row of nk /ruk.#k/
```

```
   as(a,s)     taxes pooling sub-matrix /#a.#s/
   govtpt(a,s) product tax revenue for gov /gov.t_pt/
   im(i,m)     activity-institution sub-matrix /#i.#m/
   iruk(i,m)   RUK column of im /#i.ruk/
   cm(c,m)     final demand sub-matrix /#c.#m/
   csav(c,m)   investment demand column of cm /#c.sav/
   cmanprfsav(c,m) Man&R&D investment good /c_man.sav,c_prf.sav/
   nm(n,m)     inter-institutional block /#n.#m/
   trf(n,m)    balancing terms in nm /ruk.sav, gov.ruk/
;
table SAM0(ac,ca)                                   //import excel
$ondelim
$include Unbalanced 2013 Welsh SAM4_positive.csv
$offdelim
;
display SAM0;

parameter Q0(a,b) intial value of each entry;
Q0(a,b)=sam0(a,b);
parameter H0      sum of all the initial entries;
H0=sum((a,b),sam0(a,b));

positive variable Q1(a,b);                          //!!!again, probably turn to positive
variable
*               Q1(a,b)
               z1 the objective function
               rtotal1 total value for each row
               ctotal1  total value for each column
               H1      sum of all the entries in the balanced SAM
               Hratio  'H1/H0'
*                z11 least square
;

equation EQH1       calculate H1
```

EQHratio calculate Hratio
EQobjfn1 objective function equation
EQrtotal1 calculate the total for each row
EQctotal1 calculate the total for each column
EQbalance1 balance condition for each pair of row and column
EQfx1 1st fixing condition
EQfx2 2nd fixing condition
EQfx3 3rd fixing condition
EQfx4 4th fixing condition
EQfx5 5th fixing condition
* EQz11 calculate least square
;
*EQobjfn1.. z1 =e= sum((i,j),(1/H1)*Q1(i,j)*log(Q1(i,j)/Q0(i,j))-log(Hratio)); //without $sam0(i,j) there will be
//zero entries as denominator in Q1(i,j)/Q0(i,j)
EQobjfn1.. z1 =e= sum((a,b)$sam0(a,b),(1/H1)*Q1(a,b)*log(Q1(a,b)/Q0(a,b))-log(Hratio)); //!!!a must for $sam0(i,j)!!!
//such condition filters the zero entries to avoid them being as denominators
EQrtotal1(a).. rtotal1(a) =e= sum(b,Q1(a,b));
EQctotal1(b).. ctotal1(b) =e= sum(a,Q1(a,b));
EQbalance1(a).. rtotal1(a) =e= ctotal1(a);
EQH1.. H1 =e= sum((a,b),Q1(a,b)); //donnot have to be the same order as declaring equations,
EQHratio.. Hratio =e= H1/H0; //although should be ordered first logically, before calculating the obj function

EQfx1.. sum(c,Q1(c,'i_R_ns'))-Q1('c_R_ns','i_R_ns')-Q1('c_NR_ns','i_R_ns') //Fix the total of R new sale's intermediate inputs
-Q1('c_R_rs','i_R_ns')-Q1('c_NR_rs','i_R_ns') //excluding the inputs purchases from all kinds of real estate
=e=sum(c,Q0(c,'i_R_ns'))-Q0('c_R_ns','i_R_ns')-Q0('c_NR_ns','i_R_ns')
-Q0('c_R_rs','i_R_ns')-Q0('c_NR_rs','i_R_ns');

EQfx2.. sum(c,Q1(c,'i_NR_ns'))-Q1('c_R_ns','i_NR_ns')-Q1('c_NR_ns','i_NR_ns') //Fix the total of NR new sale's intermediate inputs

-Q1('c_R_rs','i_NR_ns')-Q1('c_NR_rs','i_NR_ns') //excluding the inputs purchases from all kinds of real estate

=e=sum(c,Q0(c,'i_NR_ns'))-Q0('c_R_ns','i_NR_ns')-Q0('c_NR_ns','i_NR_ns')

-Q0('c_R_rs','i_NR_ns')-Q0('c_NR_rs','i_NR_ns');

EQfx3.. sum(c,Q1(c,'i_R_rs'))-Q1('c_R_ns','i_R_rs')-Q1('c_NR_ns','i_R_rs') //Fix the total of R resale's intermediate inputs

-Q1('c_R_rs','i_R_rs')-Q1('c_NR_rs','i_R_rs') //excluding the inputs purchases from all kinds of real estate

=e=sum(c,Q0(c,'i_R_rs'))-Q0('c_R_ns','i_R_rs')-Q0('c_NR_ns','i_R_rs')

-Q0('c_R_rs','i_R_rs')-Q0('c_NR_rs','i_R_rs');

EQfx4.. sum(c,Q1(c,'i_NR_rs'))-Q1('c_R_ns','i_NR_rs')-Q1('c_NR_ns','i_NR_rs') //Fix the total of NR resale's intermediate inputs

-Q1('c_R_rs','i_NR_rs')-Q1('c_NR_rs','i_NR_rs') //excluding the inputs purchases from all kinds of real estate

=e=sum(c,Q0(c,'i_NR_ns'))-Q0('c_R_ns','i_NR_rs')-Q0('c_NR_ns','i_NR_rs')

-Q0('c_R_rs','i_NR_rs')-Q0('c_NR_rs','i_NR_rs');

EQfx5.. sum(c,Q1(c,'sav'))=e=sum(c,Q0(c,'sav'));

*EQfx5(j).. sum(c,Q1(c,j))=e=sum(c,Q0(c,j)); //Fix total intermediate inputs of each sector

*EQz11.. z11 =e= sum((a,b),power((Q1(a,b)-Q0(a,b)),2)); //outcome seems less realistic than parameter z11

Q1.l(a,b)=Q0(a,b); //starting point - canNOT omit this sentence for the following fixing

H1.l=H0;

Hratio.lo=0.5;

Hratio.up=2;

*try to fix some parts as same as the corresponding original values in unbalanced SAM

Q1.fx(i,j)=Q0(i,j);   //No entries between activity-activity
Q1.fx('c_R_ns',b)=Q0('c_R_ns',b);   //For fixing real estate sectors' totals.
Q1.fx('c_NR_ns',b)=Q0('c_NR_ns',b);
Q1.fx('c_R_rs',b)=Q0('c_R_rs',b);
Q1.fx('c_NR_rs',b)=Q0('c_NR_rs',b);
Q1.fx(g,j)=Q0(g,j);   //Fix GVA
Q1.fx(t,j)=Q0(t,j);   //No other tax payment during production
Q1.fx(n,j)=Q0(n,j);   //No direct payments to institutions during production
Q1.fx(i,k)$(ik(i,k) xor ic(i,k))=Q0(i,k);   //No entries between activity-commodity except those on diagonal
Q1.fx(c,k)=Q0(c,k);   //No entries between commodity-commodity
Q1.fx(g,k)=Q0(g,k);   //Fix GVA(SDLT)
Q1.fx(t,k)$(tk(t,k) xor tptk(t,k))=Q0(t,k);   //Only product taxes added on commodities
Q1.fx(n,k)$(nk(n,k) xor rukk(n,k))=Q0(n,k);   //Only imports added on commodities
Q1.fx(a,'c_R_ns')=Q0(a,'c_R_ns');   //For fixing real estate sectors' totals.
Q1.fx(a,'c_NR_ns')=Q0(a,'c_NR_ns');
Q1.fx(a,'c_R_rs')=Q0(a,'c_R_rs');
Q1.fx(a,'c_NR_rs')=Q0(a,'c_NR_rs');
Q1.fx(a,h)=Q0(a,h);   //Fix Corporate Tax payment, factor income distribution
Q1.fx(a,s)$(as(a,s) xor govtpt(a,s))=Q0(a,s); //Fix tax revenues for gov except product tax
Q1.fx(i,m)$(im(i,m) xor iruk(i,m))=Q0(i,m);   //Fix exports except RUK export
Q1.fx('i_R_ns',b)=Q0('i_R_ns',b);   //For fixing real estate sectors' totals.
Q1.fx('i_NR_ns',b)=Q0('i_NR_ns',b);
Q1.fx('i_R_rs',b)=Q0('i_R_rs',b);
Q1.fx('i_NR_rs',b)=Q0('i_NR_rs',b);

Q1.fx(c,m)=Q0(c,m);   //Fix all final demands including GFCF
Q1.fx(c,m)$(cm(c,m) xor cmanprfsav(c,m))=Q0(c,m);   //Fix all final demands except for GFCF of c_Man and c_prf, however total fixed
*Q1.fx(c,m)$(cm(c,m) xor csav(c,m))=Q0(c,m);   //Fix all final demands except for GFCF (investment demand), however it is fixed in total value

Q1.fx(g,m)=Q0(g,m);   //Fix HH rental exp
Q1.fx(t,m)=Q0(t,m);   //Fix HH tax exp

```
Q1.fx(n,m)$(nm(n,m) xor trf(n,m))=Q0(n,m);      //Fix inter-institutional transfers except the balancing entries

*Q1.fx('i_plc',b)=Q0('i_plc',b);
*Q1.fx(a,'c_plc')=Q0(a,'c_plc');

model DEMethod /all/;
solve DEMethod using nlp minimizing z1;

parameter sam1f(ac,ca) the balanced SAM WITH some fixed original value;
sam1f(a,b)=Q1.l(a,b);
sam1f('total',b)=ctotal1.l(b);
sam1f(a,'total')=rtotal1.l(a);

parameter z11 least square;
z11 = sum((a,b),power((Q1.l(a,b)-Q0(a,b)),2));

display Q1.l,z1.l,sam0,sam1f,z11;

execute_unload 'BalancedSAMf.gdx' sam1f
execute 'gdxxrw BalancedSAMf.gdx output=SAM1.xls par=sam1f'
*execute 'gdxxrw BalancedSAMf.gdx par=sam1f'
//The output file name is same as that of the gdx file by default. Can be otherwise specified.
```

IV-3 RAS Method

```
set ac     all accounts
           /i_agr,i_min,i_man,i_egy,i_con,i_r_ns,i_nr_ns,i_wr,i_trp,i_acm,i_inf,i_fin,
           i_r_rs,i_nr_rs,i_rtt,i_prf,i_adm,i_plc,i_edu,i_hlt,i_ent,i_oth,total,target/     //tgtt is short
for 'target total'
    i(ac)  row accounts
           /i_agr,i_min,i_man,i_egy,i_con,i_wr,i_trp,i_acm,i_inf,i_fin,i_rtt,i_prf,
           i_adm,i_plc,i_edu,i_hlt,i_ent,i_oth/
    j(ac)  column acounts
           /i_agr,i_min,i_man,i_egy,i_con,i_r_ns,i_nr_ns,i_wr,i_trp,i_acm,i_inf,i_fin,
           i_r_rs,i_nr_rs,i_rtt,i_prf,i_adm,i_plc,i_edu,i_hlt,i_ent,i_oth/
;

alias (ac,ca);                              //Note here the row accounts i not equal to column accounts j

table sam0(ac,ca)   put sam0 here for later comparison in display
$ondelim
$include Unbalanced 2013 Welsh SAM4_intermediate.csv
$offdelim
;

table sam1(ac,ca)   this sam1 would be evolved for balance
$ondelim
$include Unbalanced 2013 Welsh SAM4_intermediate.csv
$offdelim
;

display sam0,sam1;

parameter rowdis(i) discrepancy between row total and tgtt
          coldis(j) discrepancy between column total and tgtt
          maxdis    maximum among all discrepancies
          iter      number of iteration;
maxdis=0.1;
```

```
iter=1;

while (iter<5000 and maxdis>1e-10,
sam1('total',j)=sum(i,sam1(i,j));                    //update column totals
sam1(i,j)=sam1(i,j)*sam1('target',j)/sam1('total',j);    //column adjustment
*sam1(sec,secc)=sam0(sec,secc);                        //no need for the additional fixing condition
here, otherwise it is even
                                            //harder to have totals converging to target because it hinders
the contraction
                                            //process of next round's rowdis calculation

sam1(i,'total')=sum(j,sam1(i,j));                    //update row totals
sam1(i,j)=sam1(i,j)*sam1(i,'target')/sam1(i,'total');    //row adjustment
*sam1(sec,secc)=sam0(sec,secc);                        //the fixing condition can be put only here,
although no matter how many
                                            //iterations it cost, the totals just cannot hit the targets
                                            //BTW:500000 iterations cost 30 seconds!!
coldis(j)=abs(sam1('total',j)-sam1('target',j));
rowdis(i)=abs(sam1(i,'total')-sam1(i,'target'));
maxdis=smax((i,j),max(coldis(j),rowdis(i)));          //smax function could have (i,j) inside, not
only i
*maxdis=smax(i,max(coldis(j),rowdis(i)));             //wrong: uncontrolled set j
iter=iter+1;);
display sam0,sam1,maxdis,iter;                         //we can compare the original sam with the
balanced sam0 here

execute_unload 'BalancedSAMf_int.gdx' sam1
execute 'gdxxrw.exe BalancedSAMf_int.gdx par=sam1'
```

Appendix V Social Accounting Matrix 2013 Wales

SIC2007 Sector Section (Division. Group)	Order No.			Order No. / 2013 Welsh SAM	1	2	3	4	5	6	7
					Production Activities						
					Agriculture, forestry & fishing	Mining & quarrying	Manufacturing	Electricity, gas, water supply & waste management	Construction (excl. new ownership)	Construction - new residential ownership	Construction - new non-residential ownership
A	1	Production Activities		Agriculture, forestry & fishing							
B	2			Mining & quarrying							
C	3			Manufacturing							
D+E	4			Electricity, gas, water supply & waste management							
F	5			Construction (excl. new ownership)							
F - part	6			Construction - new residential ownership							
F - part	7			Construction - new non-residential ownership							
G	8			Wholesale & retail							
H	9			Transport & storage							
I	10			Accommodation & food service							
J	11			Information & communication							
K	12			Finance & insurance							
(68.20/1&	13			Real estate - residential rental							
L (68.3)	14			Real estate - agencies & management							
M	15			Professional, scientific & technical services							
N	16			Administrative & support services							
O	17			Public administration & defence services							
P	18			Education services							
Q	19			Human health & social work services							
R	20			Arts, entertainment & recreation							
S	21			Other services							
A	22	Commodities		Agriculture, forestry & fishing	217.78	0.06	1554.89	0.57	3.08	0.32	0.84
B	23			Mining & quarrying	11.16	157.32	769.96	441.57	232.58	24.38	63.56
C	24			Manufacturing	431.73	34.98	13643.36	89.53	495.04	51.88	135.28
D + E	25			Electricity, gas, water supply & waste management	45.55	13.67	2025.13	1671.31	32.78	3.44	8.96
F	26			Construction (excl. new ownership)	127.18	4.45	1126.92	113.95	1089.17	114.15	297.63
F - part	27			Construction - new residential ownership							
F - part	28			Construction - new non-residential ownership							
G	29			Wholesale & retail	62.97	6.85	3119.07	40.28	81.37	8.53	22.24
H	30			Transport & storage	12.32	18.74	1286.60	18.46	56.22	5.89	15.36
I	31			Accommodation & food service	10.31	0.98	249.07	4.15	12.36	1.30	3.38
J	32			Information & communication	12.48	1.01	360.98	15.28	29.86	3.13	8.16
K	33			Finance & insurance	218.48	25.32	5061.97	479.32	325.25	34.09	88.88
(68.20/1&	34			Real estate - residential rental							
L (68.3)	35			Real estate - agencies & management	20.82	0.43	157.18	9.97	52.54	5.51	14.36
M	36			Professional, scientific & technical services	4.46	0.18	155.45	22.26	4.58	0.48	1.25
N	37			Administrative & support services	2.83	24.22	1122.31	47.82	291.84	30.59	79.75
O	38			Public administration & defence services	5.06	0.25	35.27	18.37	0.95	0.10	0.26
P	39			Education services	2.78	0.13	156.08	7.95	5.31	0.56	1.45
Q	40			Human health & social work services	31.82	0.15	121.81	7.14	6.24	0.65	1.71
R	41			Arts, entertainment & recreation	49.34	0.25	274.43	7.12	7.93	0.83	2.17
S	42			Other services	5.94	0.01	7.24	29.95	2.06	0.22	0.56
	43	GVA	CoE	NIC (payable by employers)	2.51	5.15	401.29	56.75	112.21	14.22	37.08
	44			Labour income	32.21	66.12	5148.99	728.18	1439.79	182.49	475.80
	45		GOS	Land rental	50.00	2.00	23.00	8.00	117.35	14.87	38.78
	46			Other value added	220.82	117.34	2575.01	1144.76	31.32	103.25	269.19
	47	Taxes		Taxes less subsidies on production (excl. SDLT)	5.54	6.98	89.79	85.93	4.14	7.84	20.43
	48			Stamp duty land tax - residential							
	49			Stamp duty land tax - non-residential	0.46	0.02	0.21	0.07	1.09	0.14	0.36
	50			Taxes on products							
	51			Income tax	3.28	6.73	523.72	74.07	146.44	18.56	48.40
	52			NIC (payable by employees)							
	53			Council tax							
	54			Corporation tax	18.18	9.66	211.99	94.24	2.58	8.50	22.16
	55	Institutions		Households&NPISH							
	56			Government							
	57			Saving/Investment							
	58			RUK							
	59			ROW							
	60			**Total**	**1606.00**	**503.00**	**40201.71**	**5217.00**	**4584.09**	**635.91**	**1658.00**

8	9	10	11	12	13	14	15	16	17	18	19	20	21
Production Activities													
Wholesale & retail	Transport & storage	Accommodation & food service	Information & communication	Finance & insurance	Real estate - residential rental	Real estate - agencies & management	Professional, scientific & technical services	Administrative & support services	Public administration & defence services	Education services	Human health & social work services	Arts, entertainment & recreation	Other service activities
92.01	0.68	60.52	0.50	0.79		0.34	1.87	0.37	1.77	10.47	1.90	2.30	0.09
211.30	65.85	8.20	5.89	5.40		1.37	0.88	4.25	16.56	18.75	8.27	3.15	1.17
2537.47	180.55	589.63	203.20	212.42		11.62	44.90	147.36	349.22	233.03	244.00	133.45	25.87
372.41	23.43	85.25	26.75	33.31		2.32	24.63	51.23	93.00	65.77	59.23	34.65	10.74
978.23	111.46	67.10	99.26	103.88	225.80	76.62	14.93	303.49	348.04	105.74	42.68	29.41	12.81
361.18	21.58	42.85	88.41	34.70		2.11	6.61	24.66	53.59	30.80	38.90	27.14	6.26
2595.77	310.91	70.75	39.33	183.47		3.83	17.22	126.35	62.92	81.86	41.32	25.71	11.47
812.56	23.31	16.61	19.96	50.11		2.79	2.30	5.47	32.83	17.71	14.63	4.66	2.19
581.71	70.45	43.97	136.65	159.28		6.93	13.77	57.20	76.23	82.99	47.43	32.47	18.73
4255.05	294.43	267.60	435.22	1212.10	296.97	74.12	397.42	724.11	415.43	150.98	218.34	214.51	134.95
659.28	17.97	23.09	15.33	22.29	6.14	15.86	10.39	10.62	25.16	10.32	15.41	1.42	3.45
60.39	14.45	4.58	11.22	13.22		0.79	20.83	3.38	30.95	40.36	56.14	4.57	0.33
190.56	195.31	7.32	16.92	39.23		2.54	7.97	137.98	14.13	87.98	13.81	43.79	43.20
4.38	8.99	0.88	0.12	5.15		27.01	1.93	0.25	16.42	1.02	0.29	2.67	0.17
88.10	21.79	20.11	34.38	78.21		3.14	88.23	4.07	156.60	320.31	26.78	15.38	8.00
65.17	8.55	15.44	7.29	12.10		0.25	15.77	4.57	2.58	9.00	2845.84	3.88	10.19
127.45	31.19	59.49	58.25	66.17		2.63	5.31	14.62	68.61	43.87	44.24	400.88	35.16
45.04	8.09	21.62	5.33	15.93		0.12	6.16	40.71	14.79	7.84	12.95	39.25	18.22
273.85	90.31	74.19	58.87	63.62		22.73	94.74	83.38	198.40	252.38	315.54	37.33	61.38
3513.76	1158.82	951.98	755.31	816.34		291.61	1215.62	1069.81	2545.67	3238.25	4048.66	478.96	787.52
61.00	16.00	15.00	34.00	278.00	1553.05	5.71	23.00	37.00	0.00	4.00	13.00	7.00	4.00
928.56	342.78	403.76	437.02	747.46		136.74	550.67	360.34	1086.55	437.02	615.34	135.82	127.50
307.44	55.85	82.86	42.69	83.43		27.95	77.79	35.66	0.00	11.96	9.88	20.94	13.96
0.56	0.15	0.14	0.31	2.57		0.05	0.21	0.34	0.00	0.04	0.12	0.06	0.04
357.39	117.87	96.83	76.82	83.03		29.66	123.64	108.81	258.93	329.37	411.80	48.72	80.10
76.44	28.22	33.24	35.98	61.54		11.26	45.33	29.66	89.45	35.98	50.66	11.18	10.50
19557.06	**3219.00**	**3063.00**	**2645.00**	**4383.76**	**2081.96**	**760.09**	**2812.12**	**3385.70**	**5957.86**	**5627.80**	**9197.14**	**1759.29**	**1428.00**

22	23	24	25	26	27	28	29	30	31	32	33	34
Commodities												
Agriculture, forestry & fishing	Mining & quarrying	Manufacturing	Electricity, gas, water supply & waste management	Construction (excl. new ownership)	Construction - new residential ownership	Construction - new non-residential ownership	Wholesale & retail	Transport & storage	Accommodation & food service	Information & communication	Finance & insurance	Real estate - residential rental
1111.33												
	447.29											
		7770.18										
			4661.72									
				4181.28								
					635.91							
						1658.00						
							15751.76					
								2600.76				
									2724.64			
										2411.88		
											3476.45	
												2081.96
					9.42							73.71
						44.91						
139.35	2.51	3463.53	369.64	534.29	0.00	0.00	858.44	469.33	276.84	119.58	619.06	0.00
608.75	21.77	9062.43	1923.59	2357.83	0.00	0.00	65.23	2464.05	483.74	584.78	12496.76	0.00
91.72	1580.00	5868.28	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1951.16	**2051.56**	**26164.42**	**6954.95**	**7073.41**	**645.32**	**1702.91**	**16675.43**	**5534.15**	**3485.23**	**3116.24**	**16592.27**	**2155.67**

35	36	37	38	39	40	41	42	43	44	45	46
								Gross Value Added			
Commodities								Compensation of Employees & Self-employed		Gross Operating Surplus	
Real estate - agencies & management	Professional, scientific & technical services	Administrative & support services	Public administration & defence services	Education services	Human health & social work services	Arts, entertainment & recreation	Other service activities	NIC (payable by employers)	Labour income	Land rental	Other value added
609.78											
	2770.96										
		3348.92									
			5957.86								
				5546.81							
					7178.51						
						1644.98					
							136.26				
90.10	6.96	60.40	0.00	27.39	461.26	151.84	39.51				
									28945.91	1931.46	854.77
								2255.92		150.50	1939.75
										222.80	7976.73
423.82	13.85	308.59	1993.17	112.71	1775.12	761.22	106.27				
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00				
1123.70	**2791.77**	**3717.92**	**7951.02**	**5686.91**	**9414.89**	**2558.04**	**282.03**	**2255.92**	**28945.91**	**2304.76**	**10771.25**

47	48	49	50	51	52	53	54	55	56	57	58	59	60
Taxes								Institutions					
Taxes less subsidies on production (excl. SDLT)	Stamp duty land tax - residential	Stamp duty land tax - non-residential	Taxes on products	Income tax	NIC (payable by employees)	Council tax	Corporation tax	Households & NPISH	Government	Saving/Investment (GFCF)	RUK	ROW	**Total**
											229.14	265.53	**1606.00**
											2.71	53.00	**503.00**
											17965.06	14466.47	**40201.71**
											555.28	0.00	**5217.00**
											402.81	0.00	**4584.09**
											0.00	0.00	**635.91**
											0.00	0.00	**1658.00**
											3805.30	0.00	**19557.06**
											618.24	0.00	**3219.00**
											338.36	0.00	**3063.00**
											233.12	0.00	**2645.00**
											907.31	0.00	**4383.76**
											0.00	0.00	**2081.96**
											150.32	0.00	**760.09**
											41.16	0.00	**2812.12**
											36.78	0.00	**3385.70**
											0.00	0.00	**5957.86**
											80.99	0.00	**5627.80**
											2018.63	0.00	**9197.14**
											114.31	0.00	**1759.29**
											1291.74	0.00	**1428.00**
								0.00	0.00	0.00			**1951.16**
								0.00	0.00	0.00			**2051.56**
								0.00	0.00	6369.88			**26164.42**
								2271.40	0.00	0.00			**6954.95**
								963.89	0.00	716.61			**7073.41**
								0.00	0.00	645.32			**645.32**
								0.00	0.00	1702.91			**1702.91**
								12595.32	0.00	0.00			**16675.43**
								549.64	0.00	0.00			**5534.15**
								2198.55	0.00	0.00			**3485.23**
								1357.54	0.00	0.00			**3116.24**
								1267.73	0.00	0.00			**16592.27**
								2155.67	0.00	0.00			**2155.67**
								26.19	0.00	0.00			**1123.70**
								13.24	0.00	2328.67			**2791.77**
								1317.81	0.00	0.00			**3717.92**
								0.00	7821.50	0.00			**7951.02**
								735.06	3912.50	0.00			**5686.91**
								311.24	5933.50	0.00			**9414.89**
								629.10	629.00	0.00			**2558.04**
								0.00	0.00	0.00			**282.03**
													2255.92
													28945.91
													2304.76
													10771.25
													991.04
													83.13
													51.88
													7690.03
								1524.59					**4468.75**
								1562.83					**1562.83**
								1198.50					**1198.50**
													886.75
									13727.50				**45459.64**
991.04	83.13	51.88	7690.03	4468.75	1562.83	1198.50	886.75				10744.93		**32024.00**
								14781.35			-3972.49	-7245.00	**11763.40**
													35563.69
													7540.00
991.04	**83.13**	**51.88**	**7690.03**	**4468.75**	**1562.83**	**1198.50**	**886.75**	**45459.64**	**32024.00**	**11763.40**	**35563.69**	**7540.00**	**441473.98**

Appendix VI Simulation codes in GAMS

VI-1 Simulation in the short run

```
$Title  A CGE model for Wales - Short run

$eolcom //
*Definition of sets-----------------------------------------------------------
Set a    all accounts
        /agr,mnq,man,egy,con,r_ns,nr_ns,wnr,trp,acm,ict,fin,r_r,rtt,prf,adm,plc,
         edu,hth,ent,oth,
         nic1,lab,lnd,ova,t_pn,sdlt_nr_int,sdlt_r_fnl,sdlt_nr_fnl,t_pt,t_inc,nic2,t_cncl,t_cp,
         hh,gov,sav,ruk,row,total/
    aa   all accounts excluding the totals
        /agr,mnq,man,egy,con,r_ns,nr_ns,wnr,trp,acm,ict,fin,r_r,rtt,prf,adm,plc,
         edu,hth,ent,oth,
         nic1,lab,lnd,ova,t_pn,sdlt_nr_int,sdlt_r_fnl,sdlt_nr_fnl,t_pt,t_inc,nic2,t_cncl,t_cp,
         hh,gov,sav,ruk,row/
    i(a) all industrial sectors
        /agr,mnq,man,egy,con,r_ns,nr_ns,wnr,trp,acm,ict,fin,r_r,rtt,prf,adm,plc,
         edu,hth,ent,oth/
    inre(i) all sectors excluding the three real estate sale and rental sectors
        /agr,mnq,man,egy,con,wnr,trp,acm,ict,fin,rtt,prf,adm,plc,edu,hth,ent,oth/
    inr(i) all sectors excluding residential rental sector
        /agr,mnq,man,egy,con,r_ns,nr_ns,wnr,trp,acm,ict,fin,rtt,prf,adm,plc,edu,hth,ent,oth/
    ic(i) all sectors that produce goods for households¡¯consumption
        /egy,con,wnr,trp,acm,ict,fin,r_r,rtt,prf,adm,edu,hth,ent/
    ig(i) all sectors that produce goods for government consumption
        /plc,edu,hth,ent/
    ii(i) all sectors that produce goods for investment
        /man,con,r_ns,nr_ns,prf/
    d    variables presented in the simulation results summary table
        /d_YG,d_YG_DT,d_TSD_R,d_TSD_NR,d_YG_NDT,d_TP,d_TC,d_TY,d_TK,
```

```
d_TLR,d_TLE,d_YG_F,d_VTG,d_VTINT,d_VGVA,d_VTY,d_VTQ,d_GDP_B,d_GDP_C,
        d_YH,d_YH_D,d_VTC,d_S,d_PI,d_YF_NR,d_YT_NR,d_PT_NR,d_T_stock,d_YK,

d_YK_D,d_PK,d_K_stock,d_YL,d_PL,d_L_stock,d_VY_R_NS,d_PY_R_NS,d_Y_R_NS,

d_VY_NR_NS,d_PY_NR_NS,d_Y_NR_NS,d_VY_R_R,d_PY_R_R,d_Y_R_R,d_VERUK,
        d_VEROW,d_VMRUK,d_VMROW,d_VW,d_EV,d_CPI,d_PK_N,d_PL_N/
;
Alias (a,b),(aa,bb),(i,j),(inre,jnre),(inr,jnr);
*============================================================================
==============

*Loading SAM as the benchmark database-----------------------------------------
Table SAM0(a,b)
$ondelim
$include 2c_SAM_b_tkty_non0.csv
$offdelim
;
Display SAM0;
*============================================================================
==============

*Loading the base values for all the variables and calibration of parameters----
*  For endogenous variables (with suffix 0):
Parameter
C0(i)              households' consumption demand by commodity in benchmark
WD0(j)               regional production supplied to domestic market in benchmark
EROW0(i)               export supply to the ROW by sector in benchmark
ERUK0(i)              export supply to the RUK by sector in benchmark
G0(i)              fiscal expenditure demand by commodity in benchmark
ID0(i)              investment demand by commodity in benchmark
INT0(i,j)             intermediate inputs for regional productionin in benchmark
CINT0(j)              composite intermediate inputs of each sector in benchmark
```

MROW0(j) import demand from the ROW by commodity in benchmark

MRUK0(j) import demand from the RUK by commodity in benchmark

Q0(j) regional sales of composites combining regional production and all imports by commodity in benchmark

QW0(j) regional sales of domestic composites combining regional production and imports from RUK by commodity in benchmark

S0 total savings in benchmark

SH0 households¡¯savings in benchmark

SRUK0 inter-regional savings from RUK in benchmark

SROW0 foreign savings from ROW in benchmark

TC0 product tax revenue in benchmark

TCCL0 council tax revenue in benchmark

TK0 corporation tax revenue in benchmark

TLE0 NIC revenue payable by the employee in benchmark

TLR0 NIC revenue payable by the employer in benchmark

TP0 production tax revenue in benchmark

TRFG0 fiscal transfer received by regional government from central government in benchmark

TSD_R_NS0 Stamp Duty Land Tax revenue from residential properties new sale in benchmark

TSD_NR_NS0 Stamp Duty Land Tax revenue from non-residential properties new sale in benchmark

TSD_R_RS0 Stamp Duty Land Tax revenue from residential properties resale in benchmark

TSD_NR_RS0 Stamp Duty Land Tax revenue from non-residential properties resale in benchmark

TY0 income tax revenue in benchmark

VA0(j) value-added bundle of factors by sector in benchmark

W0(j) regional production supplied to regional market in benchmark

Y0(j) output of regional production by sector in benchmark

YG0 total fiscal revenue in benchmark

YG_DT0 devolved tax revenue in benchmark

YG_F0 factor income of regional government in benchmark

YG_NDT0 non-devolved tax revenue in benchmark

YH0 households' income in benchmark
YK0 factor income of capital in benchmark
YL0 factor income of labour in benchmark
YT0 factor income of land in benchmark
* For exogenous variables (first uppercase letter followed by lowercase letters):
Perow(i) price of export supply to the ROW by sector in domestic currency
Peruk(i) price of export supply to the RUK by sector in domestic currency
Pmrow(i) price of import demand from the ROW by sector in domestic currency
Pmruk(i) price of import demand from the RUK by sector in domestic currency
Q_nr_rs non-residential properties resale volume
Q_r_rs residential properties resale volume
Q_r_stock residential properties regional stock
Trfh social protection transfer received by households from regional government
Ks(j) capital demand by sector (factors fixed for each sector)
Ls(j) labour demand by sector (factors fixed for each sector)
Ts(j) land demand by sector (factors fixed for each sector)
Sruk extra-regional saving from RUK
Srow extra-regional saving from ROW
;
C0(i) =SAM0(i,'hh');
EROW0(i) =SAM0(i,'row');
ERUK0(i) =SAM0(i,'ruk');
G0(i) =SAM0(i,'gov');
ID0(i) =SAM0(i,'sav');
INT0(i,j) =SAM0(i,j);
CINT0(j) =sum(i,SAM0(i,j));
MROW0(j) =SAM0('row',j);
MRUK0(j) =SAM0('ruk',j);
S0 =SAM0('sav','total');
SH0 =SAM0('sav','hh');
TC0 =SAM0('gov','t_pt');
TCCL0 =SAM0('gov','t_cncl');
TK0 =SAM0('gov','t_cp');
TLE0 =SAM0('gov','nic2');

```
TLR0                  =SAM0('gov','nic1');
TP0                   =SAM0('gov','t_pn');
TRFG0                 =SAM0('gov','ruk');
TSD_R_NS0             =SAM0('sdlt_r_fnl','r_ns');
TSD_NR_NS0            =SAM0('sdlt_nr_fnl','nr_ns');
TSD_R_RS0             =SAM0('sdlt_r_fnl','r_r');
TSD_NR_RS0            =SAM0('sdlt_nr_int','total');
TY0                   =SAM0('gov','t_inc');
VA0(j)                =SAM0('lab',j)+SAM0('nic1',j)+SAM0('t_inc',j)        /////////////////////////////ty
                      +SAM0('lnd',j)+SAM0('sdlt_nr_int',j)
                      +SAM0('ova',j)+SAM0('t_cp',j);
Y0(j)                 =VA0(j)+sum(i,INT0(i,j));
WD0(j)                =Y0(j)+SAM0('t_pn',j)+SAM0('t_pt',j)+SAM0('sdlt_r_fnl',j)
                      +SAM0('sdlt_nr_fnl',j)-EROW0(j);
W0(j)                 =WD0(j)-ERUK0(j);
QW0(j)                =W0(j)+MRUK0(j);
Q0(j)                 =QW0(j)+MROW0(j);
YG0                   =SAM0('gov','total');
*For SDLT simulation-------------
YG_DT0                =TSD_R_NS0+TSD_NR_NS0+TSD_R_RS0+TSD_NR_RS0+TCCL0;
*--------------------------------
*For Corporation Tax simulation--
*YG_DT0
=TSD_R_NS0+TSD_NR_NS0+TSD_R_RS0+TSD_NR_RS0+TCCL0+TK0;
*--------------------------------
*For Income Tax simulation--
*YG_DT0
=TSD_R_NS0+TSD_NR_NS0+TSD_R_RS0+TSD_NR_RS0+TCCL0+TY0;
*--------------------------------
*For Cross simulation--Income&Corporation Tax
*YG_DT0
=TSD_R_NS0+TSD_NR_NS0+TSD_R_RS0+TSD_NR_RS0+TCCL0+TK0+TY0;
*--------------------------------
YG_F0                 =SAM0('gov','lnd')+SAM0('gov','ova');
```

```
YG_NDT0             =YG0-TRFG0-YG_F0-YG_DT0;
YH0                 =SAM0('hh','total')+sum(j,SAM0('t_inc',j));        ///////////////////////////////ty
YK0                 =SAM0('ova','total');
YL0                 =SAM0('lab','total');
YT0                 =SAM0('lnd','total');

Perow(i)            =1;
Peruk(i)            =1;
Pmrow(i)            =1;
Pmruk(i)            =1;
Q_nr_rs             =257;
Q_r_rs              =4977.667866;
Q_r_stock           =212046.417;
Trfh                =SAM0('hh','gov');
Ks(j)               =SAM0('ova',j);
Ls(j)               =SAM0('lab',j);
Ts(j)               =SAM0('lnd',j);
Sruk                =SAM0('sav','ruk');
Srow                =SAM0('sav','row');

Display
C0,EROW0,ERUK0,G0,ID0,INT0,CINT0,MROW0,MRUK0,Q0,QW0,S0,SH0,TC0,TCCL0
,TK0,TLE0,
TLR0,TP0,TRFG0,TSD_R_NS0,TSD_NR_NS0,TSD_R_RS0,TSD_NR_RS0,TY0,VA0,W0,
WD0,Y0,YG0,
YG_DT0,YG_F0,YG_NDT0,YH0,YK0,YL0,YT0,Ks,Ls,Perow,Peruk,Pmrow,Pmruk,Q_nr_r
s,Q_r_rs,
Q_r_stock,Ts,Trfh,Sruk,Srow
;

*  Calibration of parameters----------------------------------------------------
Parameter
mps                 marginal propensity of saving of households
skg                 share of factor income of capital distributed to regional government
```

skh share of factor income of capital distributed to households
stg share of factor income of land distributed to regional government
sth share of factor income of land distributed to households
tauccl effective council tax rate
tauc(j) effective product tax rate by commodity
tauk effective corporation tax rate
taule effective NIC rate payable by the employee
taulr effective NIC rate payable by the employer
taup(j) effective production tax rate by sector
tausd_nr_int(i) effective non-residential SDLT rate applied on intermediate non-residential land input
tausd_r_fnl(i) effective residential SDLT rate applied on final sectoral production of residential new sale
tausd_rr_fnl(i) effective residential SDLT rate applied on final sectoral production of residential rental
tausd_nr_fnl(i) effective non-residential SDLT rate applied on final sectoral production of non-residential new sale
tauy effective income tax rate
taumx(i) mixed tax rate combining net production and product tax rate & residential&non-residential SDLT final rates
ac(i) parameter with respect to households¡¯ demand for commodities
ag(i) parameter with respect to fiscal expenditure demand for commodities
aid(i) parameter with respect to investment demand for commodities
ak(j) share parameter with respect to capital demand in Cobb-Douglas production function by sector
al(j) share parameter with respect to labour demand in Cobb-Douglas production function by sector
at(j) share parameter with respect to land demand in Cobb-Douglas production function by sector
ava(j) Leontief parameter with respect to value-added bundle by sector
aint(i,j) Leontief parameter with respect to composite intermediate inputs by sector
;
mps =SH0/(YH0-TY0-TLE0-TCCL0);
skg =SAM0('gov','ova')/YK0;

```
skh                    =SAM0('hh','ova')/YK0;
stg                    =SAM0('gov','lnd')/YT0;
sth                    =SAM0('hh','lnd')/YT0;
tauccl                 =TCCL0/Q_r_stock;
tauc(j)                =SAM0('t_pt',j)/(Y0(j)+SAM0('t_pn',j));
tauk                   =TK0/(TK0+YK0);                         ////////////////////////////////tk
tauy                   =TY0/YH0;
taule                  =TLE0*(1-tauy)/YL0;                     ////////////////////////////////ty
taulr                  =TLR0*(1-tauy)/YL0;                     ////////////////////////////////ty
taup(j)                =SAM0('t_pn',j)/Y0(j);
tausd_r_fnl('r_ns')      =TSD_R_NS0/(Y0('r_ns')+SAM0('t_pn','r_ns'));
tausd_nr_fnl('nr_ns')    =TSD_NR_NS0/(Y0('nr_ns')+SAM0('t_pn','nr_ns'));
tausd_rr_fnl('r_r')      =TSD_R_RS0/Y0('r_r');
tausd_nr_int(inr)        =TSD_NR_RS0/(SAM0('lnd','total')-SAM0('lnd','r_r'));
taumx(i)
=(1+taup(i))*(1+tauc(i))*(1+tausd_r_fnl(i))*(1+tausd_nr_fnl(i))*(1+tausd_rr_fnl(i))-1;
ac(i)                  =C0(i)/(YH0-TY0-TLE0-TCCL0-SH0);
ag(i)                  =G0(i)/(YG0-Trfh);
aid(i)                 =ID0(i)/S0;
ak(j)                  =Ks(j)/(1-tauk)/VA0(j);                 ////////////////////////////////tk
al(j)                  =(1+taulr)*Ls(j)/(1-tauy)/VA0(j);         ////////////////////////////////ty
at(j)                  =1-ak(j)-al(j);
ava(j)                 =VA0(j)/Y0(j);
aint(i,j)              =INT0(i,j)/CINT0(j);

Parameter
sigmak(i)              elasticity of transformation in the CET function for RUK exports
sigmaw(i)              elasticity of transformation in the CET function for ROW exports
rhok(i)                parameter defined by elasticity of transformation in the CET function for
RUK exports
rhow(i)                parameter defined by elasticity of transformation in the CET function for
ROW exports
serow(i)               distribution parameter for export supply to the ROW by sector in the CET
function
```

```
seruk(i)                distribution parameter for export supply to the RUK by sector in the CET
function
thetak(i)               scaling coefficient in the CET function for RUK exports
thetaw(i)               scaling coefficient in the CET function for ROW exports
omegak(i)               elasticity of substitution in the Armington CES function for RUK imports
omegaw(i)               elasticity of substitution in the Armington CES function for ROW imports
etak(i)                 parameter defined by elasticity of substitution in the Armington CES function
for RUK imports
etaw(i)                 parameter defined by elasticity of substitution in the Armington CES function
for ROW imports
smrow(i)                distribution parameter for import demand from the ROW by sector in the
Armington CES function
smruk(i)                distribution parameter for import demand from the RUK by sector in the
Armington CES function
gammak(i)               scaling coefficient in the Armington CES function for RUK imports
gammaw(i)               scaling coefficient in the Armington CES function for ROW imports
;
sigmak(i)               =4;
sigmaw(i)               =2;
rhok(i)                 =(sigmak(i)+1)/sigmak(i);  //--> sigma=1/(rho-1) --> 1-rho= -1/sigma
rhow(i)                 =(sigmaw(i)+1)/sigmaw(i);
serow(i)                =EROW0(i)**(1-rhow(i))/(EROW0(i)**(1-rhow(i))+WD0(i)**(1-rhow(i)));
seruk(i)                =ERUK0(i)**(1-rhok(i))/(ERUK0(i)**(1-rhok(i))+W0(i)**(1-rhok(i)));
thetak(i)                                         =WD0(i)/(seruk(i)*ERUK0(i)**rhok(i)+(1-
seruk(i))*W0(i)**rhok(i))**(1/rhok(i));
thetaw(i)                                         =Y0(i)/(serow(i)*EROW0(i)**rhow(i)+(1-
serow(i))*WD0(i)**rhow(i))**(1/rhow(i));

omegak(i)               =4;
omegaw(i)               =2;
etak(i)                 =(omegak(i)-1)/omegak(i);  //--> omega=1/(1-eta) --> 1-eta= 1/omega
etaw(i)                 =(omegaw(i)-1)/omegaw(i);
smrow(i)                =MROW0(i)**(1-etaw(i))/(MROW0(i)**(1-etaw(i))+QW0(i)**(1-etaw(i)));
smruk(i)                =MRUK0(i)**(1-etak(i))/(MRUK0(i)**(1-etak(i))+W0(i)**(1-etak(i)));
```

```
gammak(i)                                   =QW0(i)/(smruk(i)*MRUK0(i)**etak(i)+(1-
smruk(i))*W0(i)**etak(i))**(1/etak(i));
gammaw(i)                                   =Q0(i)/(smrow(i)*MROW0(i)**etaw(i)+(1-
smrow(i))*QW0(i)**etaw(i))**(1/etaw(i));

* Below for sensitivity analysis regarding nest1 & nest2
*nest2:
Parameter
omega(i)            elasticity of substitution in the production function for GVA
eta(i)              parameter defined by elasticity of substitution in the production function for
GVA
sk(i)               distribution parameter of capital demand by sector in the production function
for GVA
sl(i)               distribution parameter of labour demand by sector in the production function
for GVA
st(i)               distribution parameter of land demand by sector in the production function for
GVA
gamma(i)            scaling coefficient in the production function for GVA
;

omega(i)            =1.4;
eta(i)              =(omega(i)-1)/omega(i);  //--> omega=1/(1-eta) --> 1-eta= 1/omega
sk(i)               =(1-tauy)*Ks(i)**(1-eta(i))/((1-tauy)*(1-tauk)*(1+tausd_nr_int(i))*Ts(i)**(1-
eta(i))+(1-tauk)*(1+taulr)*Ls(i)**(1-eta(i))+(1-tauy)*Ks(i)**(1-eta(i))); /////////ty
sl(i)=(1-tauk)*(1+taulr)*Ls(i)**(1-eta(i))/((1-tauy)*(1-tauk)*(1+tausd_nr_int(i))*Ts(i)**(1-
eta(i))+(1-tauk)*(1+taulr)*Ls(i)**(1-eta(i))+(1-tauy)*Ks(i)**(1-eta(i))); /////////ty
st(i)               =1-sk(i)-sl(i);
gamma(i)
=VA0(i)/(sk(i)*Ks(i)**eta(i)+sl(i)*Ls(i)**eta(i)+st(i)*Ts(i)**eta(i))**(1/eta(i));

*nest1:
Parameter
omegai(i)           elasticity of substitution between value added and composite intermediate
inputs
```

```
etai(i)                   parameter defined by elasticity of substitution between value added and
composite intermediate inputs
sva(i)                  distribution parameter of value added demand by sector
scint(i)                distribution parameter of composite intermediate demand by sector
gammai(i)                 scaling coefficient in the production function for regional output
;

omegai(i)               =0.13;
etai(i)                 =(omegai(i)-1)/omegai(i);  //--> omega=1/(1-eta) --> 1-eta= 1/omega
sva(i)                  =VA0(i)**(1-etai(i))/(VA0(i)**(1-etai(i))+CINT0(i)**(1-etai(i)));
scint(i)                =1-sva(i);
gammai(i)                 =Y0(i)/(sva(i)*VA0(i)**etai(i)+scint(i)*CINT0(i)**etai(i))**(1/etai(i));

Display
mps,skg,skh,stg,sth,tauccl,tauc,taumx,tauk,taule,taulr,taup,tausd_r_fnl,tausd_rr_fnl,
tausd_nr_int,tausd_nr_fnl,tauy,ac,ag,aid,aint,ak,al,at,ava,sigmaw,omegaw,sigmak,
rhok,rhow,seruk,serow,thetak,thetaw,omegak,etak,etaw,smrow,smruk,gammak,gammaw,
omega,eta,sk,sl,st,gamma,omegai,etai,sva,scint,gammai
;
*======================================================================
==============

*Defining model system----------------------------------------------------------
Variable
C(i)                    households' consumption demand by commodity
CPI                     CPI calculated using Fisher index
CPI_L                     CPI calculated using Laspeyres index
CPI_P                     CPI calculated using Paasche index
WD(j)                     regional production supplied to domestic market
EROW(i)                   export supply to the ROW by sector
ERUK(i)                   export supply to the RUK by sector
G(i)                    fiscal expenditure demand by commodity
ID(i)                   investment demand by commodity
INT(i,j)                 intermediate inputs for regional production
```

CINT(j) composite intermediate inputs for regional production

MROW(j) import demand from the ROW by commodity

MRUK(j) import demand from the RUK by commodity

PWD(i) price of regional production supplied to domestic market

PI average price of investment goods

PQ(i) price of regional sales of composites by commodity ¨C net of product taxes

PQW(i) price of regional sales of domestic composites by commodity ¨C net of product taxes

PVA(i) price of value added by sector

PW(i) price of regional production supplied to regional market

PY(i) price of output of regional production by sector ¨C basic price

Q(j) regional sales of composites combining regional production and all imports by commodity

QW(j) regional sales of domestic composites combining regional production and imports from RUK by commodity

S total savings

SH households¡¯savings

TC product tax revenue

TCCL council tax revenue

TK corporation tax revenue

TLE NIC revenue payable by the employee

TLR NIC revenue payable by the employer

TP production tax revenue

TRFG fiscal transfer received by regional government from central government

TSD_R_NS Stamp Duty Land Tax revenue from residential properties¡¯ new sale

TSD_NR_NS Stamp Duty Land Tax revenue from non-residential properties¡¯ new sale

TSD_R_RS Stamp Duty Land Tax revenue from residential properties¡¯ resale

TSD_NR_RS Stamp Duty Land Tax revenue from non-residential properties¡¯ resale

TY income tax revenue

U households¡¯ utility level

VA(j) value-added bundle of factors by sector

W(j) regional production supplied to regional market

Y(j) output of regional production by sector

YG	total fiscal revenue
YG_DT	devolved tax revenue
YG_F	factor income of regional government
YG_NDT	non-devolved tax revenue
YH	households' income
YK	factor income of capital
YL	factor income of labour
YT	factor income of land
PK(i)	capital return by sector (factors fixed for each sector)
PL(i)	labour wage by sector (factors fixed for each sector)
PT(i)	land return by sector (factors fixed for each sector)
PT_NR	economy-wide non-residential land return
PK_A_N	Pre-tax average capital return across sectors
PL_A_N	Pre-tax average labour wage across sectors
PK_A	average capital return across sectors
PL_A	average labour wage across sectors

;

Equation

eqva(j)	value added function
eqt(j)	land demand function
eqpt_nr	non-residential property rental price function
eqpk_a	average capital return across sectors function
eqpl_a	average labour wage across sectors function
eqpk_a_n	Pre-tax average capital return across sectors function
eqpl_a_n	Pre-tax average labour wage across sectors function
eqk(j)	capital demand function
eql(j)	labour demand function
eqyk	total capital income
eqyt	total land income
eqyl	total labour income
eqint(i,j)	intermediate demand function
eqcint(j)	composite intermediate demand function
eqy(j)	value added demand function

eqpva(j) regional production function

eqyh households income function

eqsh households savings function

eqc(i) households consumption demand function for non-residential-rental sectors

eqty income tax revenue function

eqtk corporation tax revenue function

eqtlr function for NIC revenue payable by the employer

eqtle function for NIC revenue payable by the employee

eqtccl council tax revenue function

eqtp net production tax revenue function

eqtc net product tax revenue function

eqtsd_r_ns function for SDLT revenue from residential properties¡¯ new sale

eqtsd_nr_ns function for SDLT revenue from non-residential properties¡¯ new sale

eqtsd_r_rs function for SDLT revenue from residential properties¡¯ resale

eqtsd_nr_rs function for SDLT revenue from non-residential properties¡¯ resale

eqyg_dt devolved tax revenue function

eqyg_ndt non-devolved tax revenue function

eqyg_f factor income of regional government function

eqtrfg function for fiscal transfer received by regional government from central government

eqyg total fiscal revenue function

eqg(i) fiscal expenditure demand function

eqpyd(i) CET function for ROW exports and domestic goods

eqwds(i) domestic good supply function

eqerows(i) foreign export supply function

eqpwdd(i) CET function for RUK exports and regional goods

eqws(i) regional good supply function

eqeruks(i) RUK export supply function

eqpqws(i) Armington CES function RUK imports and regional goods

eqmrukd(i) RUK import demand function

eqwd(i) regional good demand function

eqpqs(i) Armington CES function ROW imports and domestic goods

eqmrowd(i) ROW import demand function

eqqwd(i) domestic good demand function

```
eqs                   total saving function
eqid(i)               investment demand function
eqcpi_l_b             CPI calculation function using Laspeyres index in base replication
eqcpi_p_b             CPI calculation function using Paasche index in base replication
eqcpi                 CPI calculation function using Fisher index
eqpi                  investment good price index calculation function
equ                   objective utility function
eqq(i)                market clearing condition for goods market
;

*------------------------production behaviour-----------------------------------
eqpva(j)..    PVA(j)*VA(j)=e=PK(j)*Ks(j)/(1-tauk)+(1+tausd_nr_int(j))*PT(j)*Ts(j)+(1+taulr)*PL(j)*Ls(j)/(1-tauy);    ///////////////////////////tkty
eqk(j)..      PK(j)*Ks(j)/(1-tauk)=e=ak(j)*PVA(j)*VA(j);    //for C-D nest2    /////////////////////////tk
eqt(j)..    (1+tausd_nr_int(j))*PT(j)*Ts(j)=e=at(j)*PVA(j)*VA(j);    //for C-D nest2
eql(j)..     (1+taulr)*PL(j)*Ls(j)/(1-tauy)=e=al(j)*PVA(j)*VA(j);    //for C-D nest2    ////////////////////ty
*eqk(j)..    PK(j)/(1-tauk)        =e=gamma(j)**eta(j)*sk(j)*PVA(j)*(VA(j)/Ks(j))**(1-eta(j));    //for CES nest2    //////////tk    ////////////////////////
*eqt(j)..       (1+tausd_nr_int(j))*PT(j)=e=gamma(j)**eta(j)*st(j)*PVA(j)*(VA(j)/Ts(j))**(1-eta(j));    //for CES nest2
*eql(j)..       (1+taulr)*PL(j)/(1-tauy)  =e=gamma(j)**eta(j)*sl(j)*PVA(j)*(VA(j)/Ls(j))**(1-eta(j));    //for CES nest2    //////////ty

eqyk..      YK=e=sum(j,PK(j)*Ks(j));
eqyt..      YT=e=sum(j,PT(j)*Ts(j));
eqyl..      YL=e=sum(j,PL(j)*Ls(j));
eqpt_nr..    PT_NR=e=sum(inr,PT(inr)*Ts(inr))/sum(inr,Ts(inr));
eqpk_a..     PK_A=e=sum(i,PK(i)*Ks(i))/sum(i,Ks(i));
eqpl_a..     PL_A=e=sum(i,PL(i)*Ls(i))/sum(i,Ls(i));
eqpk_a_n..   PK_A_N=e=sum(i,PK(i)*Ks(i)/(1-tauk))/sum(i,Ks(i));
eqpl_a_n..   PL_A_N=e=sum(i,PL(i)*Ls(i)/(1-tauy))/sum(i,Ls(i));
```

```
eqy(j)..     PY(j)*Y(j)=e=PVA(j)*VA(j)+PQ(j)*CINT(j);
eqva(j)..    VA(j)=e=Y(j)*(gammai(j)**etai(j)*sva(j)*PY(j)/PVA(j))**(1/(1-etai(j)));
eqcint(j)..  CINT(j)=e=Y(j)*(gammai(j)**etai(j)*scint(j)*PY(j)/PQ(j))**(1/(1-etai(j)));
eqint(i,j).. INT(i,j)=e=aint(i,j)*CINT(j);
*------------------------household behaviour------------------------------------
eqyh..       YH=e=skh*YK+sth*YT+YL/(1-tauy)+Trfh;                    //////////////////////ty
eqsh..       SH=e=mps*(YH-TY-TLE-TCCL);
eqc(i)..     PQ(i)*C(i)=e=ac(i)*(YH-TY-TLE-TCCL-SH);

*------------------------government behaviour-----------------------------------
eqty..       TY=e=tauy*YH;
eqtk..       TK=e=tauk*YK/(1-tauk);                                  //////////////////////////////tk
eqtlr..      TLR=e=taulr*YL/(1-tauy);                                //////////////////////ty
eqtle..      TLE=e=taule*YL/(1-tauy);                                //////////////////////ty
eqtccl..     TCCL=e=TCCL0;                                           ////////////////////////////////tccl
eqtp..       TP=e=sum(j,taup(j)*PY(j)*Y(j));
eqtc..       TC=e=sum(j,tauc(j)*PY(j)*(1+taup(j))*Y(j));
eqtsd_r_ns.. TSD_R_NS=e=sum(i,tausd_r_fnl(i)*PY(i)*(1+taup(i))*Y(i));
eqtsd_nr_ns..TSD_NR_NS=e=sum(i,tausd_nr_fnl(i)*PY(i)*(1+taup(i))*Y(i));
eqtsd_r_rs.. TSD_R_RS=e=sum(i,tausd_rr_fnl(i)*PY(i)*(1+taup(i))*Y(i));
eqtsd_nr_rs..TSD_NR_RS=e=sum(inr,tausd_nr_int(inr)*PT(inr)*Ts(inr));
*For SDLT simulation-------------
eqyg_dt..    YG_DT=e=TSD_R_NS+TSD_NR_NS+TSD_R_RS+TSD_NR_RS+TCCL;
eqyg_ndt..   YG_NDT=e=TY+TK+TLR+TLE+TC+TP;
*------------------------------------------
*For Corporation Tax simulation--
*eqyg_dt..    YG_DT=e=TSD_R_NS+TSD_NR_NS+TSD_R_RS+TSD_NR_RS+TCCL+TK;
*eqyg_ndt..   YG_NDT=e=TY+TLR+TLE+TC+TP;
*------------------------------------------
*For Income Tax simulation--
*eqyg_dt..    YG_DT=e=TSD_R_NS+TSD_NR_NS+TSD_R_RS+TSD_NR_RS+TCCL+TY;
*eqyg_ndt..   YG_NDT=e=TK+TLR+TLE+TC+TP;
*------------------------------------------
```

```
*For Cross simulation--Income&Corporation Tax
*eqyg_dt..
YG_DT=e=TSD_R_NS+TSD_NR_NS+TSD_R_RS+TSD_NR_RS+TCCL+TK+TY;
*eqyg_ndt..   YG_NDT=e=TLR+TLE+TC+TP;
*------------------------------------------
eqyg_f..     YG_F=e=skg*YK+stg*YT;
eqtrfg..    TRFG=e=YG0-YG_DT0-YG_F-YG_NDT;
eqyg..      YG=e=YG_DT+YG_F+YG_NDT+TRFG;
eqg(i)..    PQ(i)*G(i)=e=ag(i)*(YG-Trfh);

*-----------------------trade behaviour--------------------------------------
eqpyd(i)..                                  Y(i)=e=thetaw(i)*(serow(i)*EROW(i)**rhow(i)+(1-
serow(i))*WD(i)**rhow(i))**(1/rhow(i));
eqerows(i)..
EROW(i)=e=Y(i)*(thetaw(i)**rhow(i)*serow(i)*(1+taumx(i))*PY(i)/Perow(i))**(1/(1-
rhow(i)));
eqwds(i)..                                          WD(i)=e=Y(i)*(thetaw(i)**rhow(i)*(1-
serow(i))*(1+taumx(i))*PY(i)/PWD(i))**(1/(1-rhow(i)));

eqpwdd(i)..                                 WD(i)=e=thetak(i)*(seruk(i)*ERUK(i)**rhok(i)+(1-
seruk(i))*W(i)**rhok(i))**(1/rhok(i));
eqeruks(i)..          ERUK(i)=e=WD(i)*(thetak(i)**rhok(i)*seruk(i)*PWD(i)/Peruk(i))**(1/(1-
rhok(i)));
eqws(i)..    W(i)=e=WD(i)*(thetak(i)**rhok(i)*(1-seruk(i))*PWD(i)/PW(i))**(1/(1-rhok(i)));

eqpqws(i)..                                 QW(i)=e=gammak(i)*(smruk(i)*MRUK(i)**etak(i)+(1-
smruk(i))*W(i)**etak(i))**(1/etak(i));
eqmrukd(i)..  MRUK(i)=e=QW(i)*(gammak(i)**etak(i)*smruk(i)*PQW(i)/Pmruk(i))**(1/(1-
etak(i)));
eqwd(i)..             W(i)=e=QW(i)*(gammak(i)**etak(i)*(1-smruk(i))*PQW(i)/PW(i))**(1/(1-
etak(i)));

eqpqs(i)..                                  Q(i)=e=gammaw(i)*(smrow(i)*MROW(i)**etaw(i)+(1-
smrow(i))*QW(i)**etaw(i))**(1/etaw(i));
```

```
eqmrowd(i)..    MROW(i)=e=Q(i)*(gammaw(i)**etaw(i)*smrow(i)*PQ(i)/Pmrow(i))**(1/(1-
etaw(i)));
eqqwd(i)..          QW(i)=e=Q(i)*(gammaw(i)**etaw(i)*(1-smrow(i))*PQ(i)/PQW(i))**(1/(1-
etaw(i)));

*------------------------investment&saving behaviour----------------------------
eqs..                                    S=e=(1-skh-skg)*YK+(1-sth-stg)*YT+SH+Sruk+Srow;
/////////////////////////////////
eqid(i)..    PQ(i)*ID(i)=e=aid(i)*S;

*------------------------price equations----------------------------------------
eqcpi_l_b..  CPI_L=e=sum(inre,PQ(inre)*C0(inre))/sum(inre,C0(inre));
eqcpi_p_b..  CPI_P=e=sum(inre,PQ(inre)*C(inre))/sum(inre,C(inre));
eqcpi..      CPI=e=sqrt(CPI_L*CPI_P);
eqpi..       PI=e=sum(ii,PQ(ii)*aid(ii));

*------------------------market clearing conditions-----------------------------
eqq(i)..     Q(i)=e=C(i)+G(i)+ID(i)+sum(j, INT(i,j));

*------------------------objective function-------------------------------------
equ..        U=e=prod(ic, C(ic)**ac(ic));
*==============================================================
==============

*Initializing endogenous variables---------------------------------------------
C.l(i)       =C0(i);
CPI.l        =1;
CPI_L.l      =1;
CPI_P.l      =1;
WD.l(j)      =WD0(j);
EROW.l(i)    =EROW0(i);
ERUK.l(i)    =ERUK0(i);
G.l(i)       =G0(i);
ID.l(i)      =ID0(i);
```

```
INT.l(i,j)   =INT0(i,j);
CINT.l(j)    =CINT0(j);
MROW.l(j)    =MROW0(j);
MRUK.l(j)    =MRUK0(j);
PWD.l(i)     =1;
PI.l         =1;
PQ.l(i)      =1;
PQW.l(i)     =1;
PVA.l(i)     =1;
PW.l(i)      =1;
PY.l(i)      =1;
Q.l(j)       =Q0(j);
QW.l(j)      =QW0(j);
S.l          =S0;
SH.l         =SH0;
TC.l         =TC0;
TCCL.l       =TCCL0;
TK.l         =TK0;
TLE.l        =TLE0;
TLR.l        =TLR0;
TP.l         =TP0;
TRFG.l       =TRFG0;
TSD_R_NS.l   =TSD_R_NS0;
TSD_NR_NS.l  =TSD_NR_NS0;
TSD_R_RS.l   =TSD_R_RS0;
TSD_NR_RS.l  =TSD_NR_RS0;
TY.l         =TY0;
VA.l(j)      =VA0(j);
W.l(j)       =W0(j);
Y.l(j)       =Y0(j);
YG.l         =YG0;
YG_DT.l      =YG_DT0;
YG_F.l       =YG_F0;
YG_NDT.l     =YG_NDT0;
```

```
YH.l        =YH0;
YK.l        =YK0;
YL.l        =YL0;
YT.l        =YT0;
PK.l(i)     =1;
PL.l(i)     =1;
PT.l(i)     =1;
PT_NR.l     =1;
PK_A.l      =1;
PL_A.l      =1;
PK_A_N.l    =1/(1-tauk);
PL_A_N.l    =1/(1-tauy);
*===============================================================
==============

*Setting lower bounds for endogenous variables---------------------------------
C.lo(i)      =0.0000000001;
CPI.lo       =0.0000000001;
CPI_L.lo     =0.0000000001;
CPI_P.lo     =0.0000000001;
WD.lo(j)     =0.0000000001;
EROW.lo(i)   =0.0000000001;
ERUK.lo(i)   =0.0000000001;
G.lo(i)      =0.0000000001;
ID.lo(i)     =0.0000000001;
INT.lo(i,j)  =0.0000000001;
CINT.lo(j)   =0.0000000001;
MROW.lo(j)   =0.0000000001;
MRUK.lo(j)   =0.0000000001;
PWD.lo(i)    =0.0000000001;
PI.lo        =0.0000000001;
PQ.lo(i)     =0.0000000001;
PQW.lo(i)    =0.0000000001;
PVA.lo(i)    =0.0000000001;
```

```
PW.lo(i)      =0.0000000001;
PY.lo(i)      =0.0000000001;
Q.lo(j)       =0.0000000001;
QW.lo(j)      =0.0000000001;
S.lo          =0.0000000001;
SH.lo         =0.0000000001;
TC.lo         =0.0000000001;
TCCL.lo       =0.0000000001;
TK.lo         =0.0000000001;
TLE.lo        =0.0000000001;
TLR.lo        =0.0000000001;
TP.lo         =0.0000000001;
TRFG.lo       =0.0000000001;
TSD_R_NS.lo   =0.0000000001;
TSD_NR_NS.lo  =0.0000000001;
TSD_R_RS.lo   =0.0000000001;
TSD_NR_RS.lo  =0.0000000001;
TY.lo         =0.0000000001;
VA.lo(j)      =0.0000000001;
W.lo(j)       =0.0000000001;
Y.lo(j)       =0.0000000001;
YG.lo         =0.0000000001;
YG_DT.lo      =0.0000000001;
YG_F.lo       =0.0000000001;
YG_NDT.lo     =0.0000000001;
YH.lo         =0.0000000001;
YK.lo         =0.0000000001;
YL.lo         =0.0000000001;
YT.lo         =0.0000000001;
PK.lo(i)      =0.0000000001;
PL.lo(i)      =0.0000000001;
PT.lo(i)      =0.0000000001;
PT_NR.lo      =0.0000000001;
PK_A.lo       =0.0000000001;
```

```
PL_A.lo       =0.0000000001;
PK_A_N.lo     =0.0000000001;
PL_A_N.lo     =0.0000000001;
*============================================================================
==============

*Setting numeraire------------------------------------------------------
*CPI.fx       =1;
*============================================================================
==============

*Defining and solving the model & replicating the benchmark---------------------
Model WAGE_base /All/;
Solve WAGE_base maximizing U using nlp;

*  Show solutions for benchmark replication-------------
Display
C.l,CPI.l,CPI_L.l,CPI_P.l,WD.l,EROW.l,ERUK.l,G.l,ID.l,INT.l,MROW.l,MRUK.l,PWD.l,
PI.l,PQ.l,PQW.l,PVA.l,PW.l,PY.l,Q.l,QW.l,S.l,SH.l,TC.l,TCCL.l,TK.l,TLE.l,TLR.l,
TP.l,TRFG.l,TSD_R_NS.l,TSD_NR_NS.l,TSD_R_RS.l,TSD_NR_RS.l,TY.l,U.l,VA.l,W.l,Y.
l,
YG.l,YG_DT.l,YG_F.l,YG_NDT.l,YH.l,YK.l,YL.l,YT.l,PK.l,PL.l,PT.l,PT_NR.l,PK_A.l,
PL_A.l,CINT.l,PK_A_N.l,PL_A_N.l
;

*  Show how much the solutions deviate the benchmark----
*    Show percentage deviation against the benchmark----
Parameter
dC(ic),dCPI,dCPI_L,dCPI_P,dWD(j),dEROW(i),dERUK(i),dG(ig),dID(ii),dINT(i,j),dMRO
W(i),
dMRUK(i),dPWD(i),dPI,dPQ(i),dPQW(i),dPVA(i),dPW(i),dPY(i),dQ(j),dQW(j),dS,dSH,
dTC,dTCCL,dTK,dTLE,dTLR,dTP,dTRFG,dTSD_R_NS,dTSD_NR_NS,dTSD_R_RS,dTS
D_NR_RS,dTY,
```

```
dVA(j),dW(j),dY(j),dYG,dYG_DT,dYG_F,dYG_NDT,dYH,dYK,dYL,dYT,dPK(i),dPL(i),d
PT(i),
dPT_NR,dPK_A,dPL_A,dPK_A_N,dPL_A_N

*Below with prefix 'ad' is average deviations of each variable across sectors
adC,adWD,adEROW,adERUK,adG,adID,adMROW,adINT,adMRUK,adPWD,adPQ,adPQW
,adPVA,adPW,
adPY,adQ,adQW,adVA,adW,adY,ad,adPK,adPL,adPT
;

dC(ic)          =(C.l(ic)       /C0(ic)-1)*100;
dCPI            =(CPI.l         /1-1)*100;
dCPI_L          =(CPI_L.l       /1-1)*100;
dCPI_P          =(CPI_P.l       /1-1)*100;
dWD(j)          =(WD.l(j)       /WD0(j)-1)*100;
dEROW(i)$(EROW0(i) ne 0)        =(EROW.l(i)    /EROW0(i)-1)*100;
dERUK(i)$(ERUK0(i) ne 0)        =(ERUK.l(i)    /ERUK0(i)-1)*100;
dG(ig)          =(G.l(ig)       /G0(ig)-1)*100;
dID(ii)         =(ID.l(ii)      /ID0(ii)-1)*100;
dINT(i,j)$(INT0(i,j) ne 0)      =(INT.l(i,j)   /INT0(i,j)-1)*100;
dMROW(j)$(MROW0(j) ne 0)        =(MROW.l(j)    /MROW0(j)-1)*100;
dMRUK(j)$(MRUK0(j) ne 0)        =(MRUK.l(j)    /MRUK0(j)-1)*100;
dPWD(i)         =(PWD.l(i)      /1-1)*100;
dPI             =(PI.l          /1-1)*100;
dPQ(i)          =(PQ.l(i)       /1-1)*100;
dPQW(i)         =(PQW.l(i)      /1-1)*100;
dPVA(i)         =(PVA.l(i)      /1-1)*100;
dPW(i)          =(PW.l(i)       /1-1)*100;
dPY(i)          =(PY.l(i)       /1-1)*100;
dQ(j)           =(Q.l(j)        /Q0(j)-1)*100;
dQW(j)          =(QW.l(j)       /QW0(j)-1)*100;
dS              =(S.l           /S0-1)*100;
dSH             =(SH.l          /SH0-1)*100;
dTC             =(TC.l          /TC0-1)*100;
```

```
dTCCL        =(TCCL.l     /TCCL0-1)*100;
dTK          =(TK.l       /TK0-1)*100;
dTLE         =(TLE.l      /TLE0-1)*100;
dTLR         =(TLR.l      /TLR0-1)*100;
dTP          =(TP.l       /TP0-1)*100;
dTRFG        =(TRFG.l     /TRFG0-1)*100;
dTSD_R_NS    =(TSD_R_NS.l  /TSD_R_NS0-1)*100;
dTSD_NR_NS   =(TSD_NR_NS.l /TSD_NR_NS0-1)*100;
dTSD_R_RS    =(TSD_R_RS.l  /TSD_R_RS0-1)*100;
dTSD_NR_RS   =(TSD_NR_RS.l /TSD_NR_RS0-1)*100;
dTY          =(TY.l       /TY0-1)*100;
dVA(j)       =(VA.l(j)    /VA0(j)-1)*100;
dW(j)        =(W.l(j)     /W0(j)-1)*100;
dY(j)        =(Y.l(j)     /Y0(j)-1)*100;
dYG          =(YG.l       /YG0-1)*100;
dYG_DT       =(YG_DT.l    /YG_DT0-1)*100;
dYG_F        =(YG_F.l     /YG_F0-1)*100;
dYG_NDT      =(YG_NDT.l   /YG_NDT0-1)*100;
dYH          =(YH.l       /YH0-1)*100;
dYK          =(YK.l       /YK0-1)*100;
dYL          =(YL.l       /YL0-1)*100;
dYT          =(YT.l       /YT0-1)*100;
dPK(i)       =(PK.l(i)    /1-1)*100;
dPL(i)       =(PL.l(i)    /1-1)*100;
dPT(i)       =(PT.l(i)    /1-1)*100;
dPT_NR       =(PT_NR.l    /1-1)*100;
dPK_A        =(PK_A.l     /1-1)*100;
dPL_A        =(PL_A.l     /1-1)*100;
dPK_A_N      =(PK_A_N.l   /(1/(1-tauk))-1)*100;
dPL_A_N      =(PL_A_N.l   /(1/(1-tauy))-1)*100;

adC          =sum(ic,abs(dC(ic)))/card(ic);
adWD         =sum(i,abs(dWD(i)))/card(i);
adEROW       =sum(i,abs(dEROW(i)))/3;
```

```
adERUK          =sum(i,abs(dERUK(i)))/17;
adG             =sum(ig,abs(dG(ig)))/card(ig);
adID            =sum(ii,abs(dID(ii)))/card(ii);
adMROW          =sum(i,abs(dMROW(i)))/3;
adMRUK          =sum(i,abs(dMRUK(i)))/18;
adINT           =sum((i,j),abs(dINT(i,j)))/363;
adPWD           =sum(i,abs(dPWD(i)))/card(i);
adPQ            =sum(i,abs(dPQ(i)))/card(i);
adPQW           =sum(i,abs(dPQW(i)))/card(i);
adPVA           =sum(i,abs(dPVA(i)))/card(i);
adPW            =sum(i,abs(dPW(i)))/card(i);
adPY            =sum(i,abs(dPY(i)))/card(i);
adQ             =sum(i,abs(dQ(i)))/card(i);
adQW            =sum(i,abs(dQW(i)))/card(i);
adVA            =sum(i,abs(dVA(i)))/card(i);
adW             =sum(i,abs(dW(i)))/card(i);
adY             =sum(i,abs(dY(i)))/card(i);
adPK            =sum(i,abs(dPK(i)))/card(i);
adPL            =sum(i,abs(dPL(i)))/card(i);
adPT            =sum(i,abs(dPT(i)))/card(i);

ad
=(adC+adWD+adEROW+adERUK+adG+adID+adMROW+adINT+adMRUK+adPWD

+adPQ+adPQW+adPVA+adPW+adPY+adQ+adQW+adVA+adW+adY+abs(dCPI)+abs(dCP
I_L)
     +abs(dCPI_P)+abs(dPI)+abs(dS)+abs(dSH)+abs(dTC)+abs(dTCCL)+abs(dTK)

+abs(dTLE)+abs(dTLR)+abs(dTP)+abs(dTRFG)+abs(dTSD_R_NS)+abs(dTSD_NR_NS)

+abs(dTSD_R_RS)+abs(dTSD_NR_RS)+abs(dTY)+abs(dYG)+abs(dYG_DT)+abs(dYG_F)
     +abs(dYG_NDT)+abs(dYH)+abs(dYK)+abs(dYL)+abs(dYT)+adPK+adPL+adPT)
     /49;
```

```
Display
dC,dCPI,dCPI_L,dCPI_P,dWD,dEROW,dERUK,dG,dID,dMROW,dINT,dMRUK,dPWD,dP
I,dPQ,dPQW,
dPVA,dPW,dPY,dQ,dQW,dS,dSH,dTC,dTCCL,dTK,dTLE,dTLR,dTP,dTRFG,dTSD_R_NS
,dTSD_NR_NS,
dTSD_R_RS,dTSD_NR_RS,dTY,dVA,dW,dY,dYG,dYG_DT,dYG_F,dYG_NDT,dYH,dYK
,dYL,dYT,
dPK,dPL,dPT,dPT_NR,dPK_A,dPL_A,ad,dPK_A_N,dPL_A_N
;

*   Show Euclidean distance against benchmark----------
Parameter
edC,edCPI,edCPI_L,edCPI_P,edWD,edEROW,edERUK,edG,edID,edMROW,edINT,edMR
UK,edPWD,
edPI,edPQ,edPQW,edPVA,edPW,edPY,edQ,edQW,edS,edSH,edTC,edTCCL,edTK,edTLE,e
dTLR,
edTP,edTRFG,edTSD_R_NS,edTSD_NR_NS,edTSD_R_RS,edTSD_NR_RS,edTY,edVA,e
dW,edY,edYG,
edYG_DT,edYG_F,edYG_NDT,edYH,edYK,edYL,edYT,edPK,edPL,edPT
;

edC          =sqrt(sum(ic,sqr(C.l(ic)      -C0(ic))));
edCPI        =abs(CPI.l      -1);
edCPI_L      =abs(CPI_L.l    -1);
edCPI_P      =abs(CPI_P.l    -1);
edWD         =sqrt(sum(j,sqr(WD.l(j)     -WD0(j))));
edEROW       =sqrt(sum(i$(EROW0(i) ne 0),sqr(EROW.l(i)   -EROW0(i))));
edERUK       =sqrt(sum(i$(ERUK0(i) ne 0),sqr(ERUK.l(i)   -ERUK0(i))));
edG          =sqrt(sum(ig,sqr(G.l(ig)      -G0(ig))));
edID         =sqrt(sum(ii,sqr(ID.l(ii)     -ID0(ii))));
edINT        =sqrt(sum((i,j)$(INT0(i,j) ne 0),sqr(INT.l(i,j)   -INT0(i,j))));
edMROW       =sqrt(sum(j$(MROW0(j) ne 0),sqr(MROW.l(j)   -MROW0(j))));
edMRUK       =sqrt(sum(j$(MRUK0(j) ne 0),sqr(MRUK.l(j)   -MRUK0(j))));
edPWD        =sqrt(sum(i,sqr(PWD.l(i)    -1)));
```

```
edPI          =abs(PI.l        -1);
edPQ          =sqrt(sum(i,sqr(PQ.l(i)      -1)));
edPQW         =sqrt(sum(i,sqr(PQW.l(i)     -1)));
edPVA         =sqrt(sum(i,sqr(PVA.l(i)     -1)));
edPW          =sqrt(sum(i,sqr(PW.l(i)      -1)));
edPY          =sqrt(sum(i,sqr(PY.l(i)      -1)));
edQ           =sqrt(sum(j,sqr(Q.l(j)       -Q0(j))));
edQW          =sqrt(sum(j,sqr(QW.l(j)      -QW0(j))));
edS           =abs(S.l         -S0);
edSH          =abs(SH.l        -SH0);
edTC          =abs(TC.l        -TC0);
edTCCL        =abs(TCCL.l      -TCCL0);
edTK          =abs(TK.l        -TK0);
edTLE         =abs(TLE.l       -TLE0);
edTLR         =abs(TLR.l       -TLR0);
edTP          =abs(TP.l        -TP0);
edTRFG        =abs(TRFG.l      -TRFG0);
edTSD_R_NS    =abs(TSD_R_NS.l   -TSD_R_NS0);
edTSD_NR_NS   =abs(TSD_NR_NS.l  -TSD_NR_NS0);
edTSD_R_RS    =abs(TSD_R_RS.l   -TSD_R_RS0);
edTSD_NR_RS   =abs(TSD_NR_RS.l  -TSD_NR_RS0);
edTY          =abs(TY.l        -TY0);
edVA          =sqrt(sum(j,sqr(VA.l(j)      -VA0(j))));
edW           =sqrt(sum(j,sqr(W.l(j)       -W0(j))));
edY           =sqrt(sum(j,sqr(Y.l(j)       -Y0(j))));
edYG          =abs(YG.l        -YG0);
edYG_DT       =abs(YG_DT.l     -YG_DT0);
edYG_F        =abs(YG_F.l      -YG_F0);
edYG_NDT      =abs(YG_NDT.l    -YG_NDT0);
edYH          =abs(YH.l        -YH0);
edYK          =abs(YK.l        -YK0);
edYL          =abs(YL.l        -YL0);
edYT          =abs(YT.l        -YT0);
edPK          =sqrt(sum(i,sqr(PK.l(i)      -1)));
```

```
edPL          =sqrt(sum(i,sqr(PL.l(i)     -1)));
edPT          =sqrt(sum(i,sqr(PT.l(i)     -1)));

Display
edC,edCPI,edCPI_L,edCPI_P,edWD,edEROW,edERUK,edG,edID,edINT,edMROW,edMR
UK,edPWD,
edPI,edPQ,edPQW,edPVA,edPW,edPY,edQ,edQW,edS,edSH,edTC,edTCCL,edTK,edTLE,e
dTLR,
edTP,edTRFG,edTSD_R_NS,edTSD_NR_NS,edTSD_R_RS,edTSD_NR_RS,edTY,edVA,e
dW,edY,edYG,
edYG_DT,edYG_F,edYG_NDT,edYH,edYK,edYL,edYT,edPK,edPL,edPT
;

*  Record the replication solutions---------------------
Parameter
C1(i),CPI1,CPI_L1,CPI_P1,WD1(j),EROW1(i),ERUK1(i),G1(i),ID1(i),INT1(i,j),MROW1(i),
MRUK1(i),PWD1(i),PI1,PQ1(i),PQW1(i),PVA1(i),PW1(i),PY1(i),Q1(j),QW1(j),S1,SH1,
TC1,TCCL1,TK1,TLE1,TLR1,TP1,TRFG1,TSD_R_NS1,TSD_NR_NS1,TSD_R_RS1,TSD
_NR_RS1,TY1,
VA1(j),W1(j),Y1(j),YG1,YG_DT1,YG_F1,YG_NDT1,YH1,YK1,YL1,YT1,PK1(i),PL1(i),P
T1(i),
PT_NR1,PK_A1,PL_A1,CINT1(j),PK_A_N1,PL_A_N1
;

C1(i)          =C.l(i);
CPI1           =CPI.l;
CPI_L1         =CPI_L.l;
CPI_P1         =CPI_P.l;
WD1(j)         =WD.l(j);
EROW1(i)       =EROW.l(i);
ERUK1(i)       =ERUK.l(i);
G1(i)          =G.l(i);
ID1(i)         =ID.l(i);
INT1(i,j)      =INT.l(i,j);
```

```
CINT1(j)        =CINT.l(j);
MROW1(j)        =MROW.l(j);
MRUK1(j)        =MRUK.l(j);
PWD1(i)         =PWD.l(i) ;
PI1             =PI.l     ;
PQ1(i)          =PQ.l(i)   ;
PQW1(i)         =PQW.l(i)   ;
PVA1(i)         =PVA.l(i)    ;
PW1(i)          =PW.l(i)      ;
PY1(i)          =PY.l(i)      ;
Q1(j)           =Q.l(j)       ;
QW1(j)          =QW.l(j)      ;
S1              =S.l          ;
SH1             =SH.l         ;
TC1             =TC.l         ;
TCCL1           =TCCL.l       ;
TK1             =TK.l         ;
TLE1            =TLE.l        ;
TLR1            =TLR.l        ;
TP1             =TP.l         ;
TRFG1           =TRFG.l       ;
TSD_R_NS1       =TSD_R_NS.l   ;
TSD_NR_NS1      =TSD_NR_NS.l  ;
TSD_R_RS1       =TSD_R_RS.l   ;
TSD_NR_RS1      =TSD_NR_RS.l  ;
TY1             =TY.l         ;
VA1(j)          =VA.l(j)      ;
W1(j)           =W.l(j)       ;
Y1(j)           =Y.l(j)       ;
YG1             =YG.l         ;
YG_DT1          =YG_DT.l      ;
YG_F1           =YG_F.l       ;
YG_NDT1         =YG_NDT.l     ;
YH1             =YH.l         ;
```

```
YK1            =YK.l        ;
YL1            =YL.l        ;
YT1            =YT.l        ;
PK1(i)         =PK.l(i)     ;
PL1(i)         =PL.l(i)     ;
PT1(i)         =PT.l(i)     ;
PT_NR1         =PT_NR.l     ;
PK_A1          =PK_A.l      ;
PL_A1          =PL_A.l      ;
PK_A_N1        =PK_A_N.l    ;
PL_A_N1        =PL_A_N.l    ;
*==============================================================================
==============

*Simulation and results------------------------------------------------------

*Variation of residential SDLT rate
tausd_r_fnl('r_ns')     =1.1*TSD_R_NS0/(Y0('r_ns')+SAM0('t_pn','r_ns'));
tausd_rr_fnl('r_r')     =1.1*TSD_R_RS0/Y0('r_r');
*tausd_r_fnl('r_ns')     =0.9*TSD_R_NS0/(Y0('r_ns')+SAM0('t_pn','r_ns'));
*tausd_rr_fnl('r_r')     =0.9*TSD_R_RS0/Y0('r_r');

*Variation of non-residential SDLT rate
*tausd_nr_fnl('nr_ns')    =1.1*TSD_NR_NS0/(Y0('nr_ns')+SAM0('t_pn','nr_ns'));
*tausd_nr_int(inr)        =1.1*TSD_NR_RS0/(SAM0('lnd','total')-SAM0('lnd','r_r'));
tausd_nr_fnl('nr_ns')    =0.9*TSD_NR_NS0/(Y0('nr_ns')+SAM0('t_pn','nr_ns'));
tausd_nr_int(inr)        =0.9*TSD_NR_RS0/(SAM0('lnd','total')-SAM0('lnd','r_r'));

*Variation of corporation tax
*tauk                =1.05*TK0/(TK0+YK0);
*tauk                =0.95*TK0/(TK0+YK0);

*Variation of income tax
*tauy                 =1.05*TY0/YH0;
```

```
*tauy                =0.95*TY0/YH0;

option bratio=1;

Equation
eqcpi_l_s            CPI calculation function using Laspeyres index in simulation
eqcpi_p_s             CPI calculation function using Paasche index in simulation
;
eqcpi_l_s..  CPI_L=e=sum(inre,PQ(inre)*C1(inre))/sum(inre,PQ1(inre)*C1(inre));
eqcpi_p_s..  CPI_P=e=sum(inre,PQ(inre)*C(inre))/sum(inre,PQ1(inre)*C(inre));

Model WAGE_simulation
/eqva,eqt,eqpt_nr,eqk,eql,eqyk,eqyt,eqyl,eqint,eqy,eqpva,eqyh,eqsh,eqc,eqty,
eqtk,eqtlr,eqtle,eqtccl,eqtp,eqtc,eqtsd_r_ns,eqtsd_nr_ns,eqtsd_r_rs,eqtsd_nr_rs,
eqyg_dt,eqyg_ndt,eqyg_f,eqtrfg,eqyg,eqg,eqpyd,eqwds,eqerows,eqpwdd,eqws,eqeruks,
eqpqws,eqmrukd,eqwd,eqpqs,eqmrowd,eqqwd,eqs,eqid,eqcpi_l_s,eqcpi_p_s,eqcpi,eqpi,
equ,eqq,eqpk_a,eqpl_a,eqcint,eqpk_a_n,eqpl_a_n/
;

Solve WAGE_simulation maximizing U using nlp;

*  Show counter-factual solutions----------------------
Display
C.l,CPI.l,CPI_L.l,CPI_P.l,WD.l,EROW.l,ERUK.l,G.l,ID.l,INT.l,MROW.l,MRUK.l,PWD.l,
PI.l,PQ.l,PQW.l,PVA.l,PW.l,PY.l,Q.l,QW.l,S.l,SH.l,TC.l,TCCL.l,TK.l,TLE.l,TLR.l,
TP.l,TRFG.l,TSD_R_NS.l,TSD_NR_NS.l,TSD_R_RS.l,TSD_NR_RS.l,TY.l,U.l,VA.l,W.l,Y.
l,
YG.l,YG_DT.l,YG_F.l,YG_NDT.l,YH.l,YK.l,YL.l,YT.l,PK.l,PL.l,PT.l,PT_NR.l,PK_A.l,
PL_A.l,CINT.l,PK_A_N.l,PL_A_N.l
;

*  Show counter-factual changes against replication of benchmark---
Parameter
dC(ic),dCPI,dCPI_L,dCPI_P,dWD(j),dEROW(i),dERUK(i),dG(ig),dID(ii),dINT(i,j),
```

```
dMROW(i),dMRUK(i),dPWD(i),dPI,dPQ(i),dPQW(i),dPVA(i),dPW(i),dPY(i),dQ(j),dQW(j),
dS,dSH,dTC,dTCCL,dTK,dTLE,dTLR,dTP,dTRFG,dTSD_R_NS,dTSD_NR_NS,dTSD_R_
RS,dTSD_NR_RS,
dTSD_R,dTSD_NR,dTY,dVA(j),dW(j),dY(j),dYG,dYG_DT,dYG_F,dYG_NDT,dYH,dYH_
D,dYF_NR,
dYK,dYK_D,dYL,dYT,dVY(j),dVTY,dVQ(j),dVTQ,dVC(ic),dVTC,dVG(i),dVTG,dVID(i),
dTID,
dVTERUK,dVTEROW,dVTMRUK,dVTMROW,dVTWD,dVTW,dVTQW,dVVA(i),dVGV
A,dVINT(i,j),dVTINT,
dVRINT(i),dVCINT(j),dVERUK,dVEROW,dVMRUK,dVMROW,dVW,dGDP_B,dGDP_C,
dVY_R_NS,dPY_R_NS,
dY_R_NS,dVY_NR_NS,dPY_NR_NS,dY_NR_NS,dVY_R_R,dPY_R_R,dY_R_R,dYT_NR
,dPK(i),dPL(i),
dPT(i),dPT_NR,dPK_A,dPL_A,dCINT(j),dPK_A_N,dPL_A_N
;

dC(ic)          =(C.l(ic)       /C1(ic)-1)*100;
dCPI                          =(CPI.l        /CPI1-1)*100;
dCPI_L                         =(CPI_L.l      /CPI_L1-1)*100;
dCPI_P                         =(CPI_P.l      /CPI_P1-1)*100;
dWD(j)                         =(WD.l(j)      /WD1(j)-1)*100;
dEROW(i)$(EROW0(i) ne 0)         =(EROW.l(i)    /EROW1(i)-1)*100;
dERUK(i)$(ERUK0(i) ne 0)         =(ERUK.l(i)    /ERUK1(i)-1)*100;
dG(ig)          =(G.l(ig)       /G1(ig)-1)*100;
dID(ii)         =(ID.l(ii)      /ID1(ii)-1)*100;
dINT(i,j)$(INT0(i,j) ne 0)       =(INT.l(i,j)   /INT1(i,j)-1)*100;
dCINT(j)                        =(CINT.l(j)    /CINT1(j)-1)*100;
dMROW(j)$(MROW0(j) ne 0)         =(MROW.l(j)    /MROW1(j)-1)*100;
dMRUK(j)$(MRUK0(j) ne 0)         =(MRUK.l(j)    /MRUK1(j)-1)*100;
dPWD(i)         =(PWD.l(i)     /PWD1(i)-1)*100;
dPI             =(PI.l         /PI1-1)*100;
dPQ(i)          =(PQ.l(i)      /PQ1(i)-1)*100;
dPQW(i)         =(PQW.l(i)     /PQW1(i)-1)*100;
dPVA(i)         =(PVA.l(i)     /PVA1(i)-1)*100;
```

```
dPW(i)          =(PW.l(i)      /PW1(i)-1)*100;
dPY(i)          =(PY.l(i)      /PY1(i)-1)*100;
dQ(j)           =(Q.l(j)       /Q1(j)-1)*100;
dQW(j)          =(QW.l(j)      /QW1(j)-1)*100;
dS              =(S.l          /S1-1)*100;
dSH             =(SH.l         /SH1-1)*100;
dTC             =(TC.l         /TC1-1)*100;
dTCCL           =(TCCL.l       /TCCL1-1)*100;
dTK             =(TK.l         /TK1-1)*100;
dTLE            =(TLE.l        /TLE1-1)*100;
dTLR            =(TLR.l        /TLR1-1)*100;
dTP             =(TP.l         /TP1-1)*100;
dTRFG           =(TRFG.l       /TRFG1-1)*100;
dTSD_R_NS       =(TSD_R_NS.l   /TSD_R_NS1-1)*100;
dTSD_NR_NS      =(TSD_NR_NS.l  /TSD_NR_NS1-1)*100;
dTSD_R_RS       =(TSD_R_RS.l   /TSD_R_RS1-1)*100;
dTSD_NR_RS      =(TSD_NR_RS.l  /TSD_NR_RS1-1)*100;
dTY             =(TY.l         /TY1-1)*100;
dVA(j)          =(VA.l(j)      /VA1(j)-1)*100;
dW(j)           =(W.l(j)       /W1(j)-1)*100;
dY(j)           =(Y.l(j)       /Y1(j)-1)*100;
dYG             =(YG.l         /YG1-1)*100;
dYG_DT          =(YG_DT.l      /YG_DT1-1)*100;
dYG_F           =(YG_F.l       /YG_F1-1)*100;
dYG_NDT         =(YG_NDT.l     /YG_NDT1-1)*100;
dYH             =(YH.l         /YH1-1)*100;
dYH_D           =((YH.l-TY.l-TLE.l-TCCL.l)/(YH1-TY1-TLE1-TCCL1)-1)*100;
dYF_NR
=((YK.l+YL.l+sum(jnr,PT.l(jnr)*Ts(jnr)))/(YK1+YL1+sum(jnr,PT1(jnr)*Ts(jnr)))-1)*100;
dYK             =(YK.l         /YK1-1)*100;
dYK_D           =((YK.l-TK.l)  /(YK1-TK1)-1)*100;
dYL             =(YL.l         /YL1-1)*100;
dYT             =(YT.l         /YT1-1)*100;
dVY(j)          =(PY.l(j)*Y.l(j) /(PY1(j)*Y1(j))-1)*100;
```

```
dVTY          =(sum(j,PY.l(j)*Y.l(j)) /sum(j,PY1(j)*Y1(j))-1)*100;
dVQ(j)        =(PQ.l(j)*Q.l(j) /(PQ1(j)*Q1(j))-1)*100;
dVTQ          =(sum(j,PQ.l(j)*Q.l(j)) /sum(j,PQ1(j)*Q1(j))-1)*100;
dVC(ic)       =(PQ.l(ic)*C.l(ic) /(PQ1(ic)*C1(ic))-1)*100;
dVTC          =(sum(ic,PQ.l(ic)*C.l(ic)) /sum(ic,PQ1(ic)*C1(ic))-1)*100;
dVG(i)        =(PQ.l(i)*G.l(i) /(PQ1(i)*G1(i))-1)*100;
dVTG          =(sum(i,PQ.l(i)*G.l(i)) /sum(i,PQ1(i)*G1(i))-1)*100;
dVID(i)       =(PQ.l(i)*ID.l(i)/(PQ1(i)*ID1(i))-1)*100;
dTID          =(sum(ii,ID.l(ii)) /sum(ii,ID1(ii))-1)*100;
dTSD_R        =((TSD_R_NS.l+TSD_R_RS.l)/(TSD_R_NS1+TSD_R_RS1)-1)*100;
dTSD_NR       =((TSD_NR_NS.l+TSD_NR_RS.l)/(TSD_NR_NS1+TSD_NR_RS1)-1)*100;
dVVA(i)       =(PVA.l(i)*VA.l(i) /(PVA1(i)*VA1(i))-1)*100;
dVGVA         =(sum(i,PVA.l(i)*VA.l(i)) /sum(i,PVA1(i)*VA1(i))-1)*100;
dVINT(i,j)    =(PQ.l(i)*INT.l(i,j) /(PQ1(i)*INT1(i,j))-1)*100;
dVTINT        =(sum((i,j),PQ.l(i)*INT.l(i,j)) /sum((i,j),PQ1(i)*INT1(i,j))-1)*100;
dVRINT(i)     =(sum(j,PQ.l(j)*INT.l(i,j)) /sum(j,PQ1(j)*INT1(i,j))-1)*100;
dVCINT(j)     =(sum(i,PQ.l(j)*INT.l(i,j)) /sum(i,PQ1(j)*INT1(i,j))-1)*100;
dVERUK        =(sum(i,ERUK.l(i)) /sum(i,ERUK1(i))-1)*100;
dVEROW        =(sum(i,EROW.l(i)) /sum(i,EROW1(i))-1)*100;
dVMRUK        =(sum(i,MRUK.l(i)) /sum(i,MRUK1(i))-1)*100;
dVMROW        =(sum(i,MROW.l(i)) /sum(i,MROW1(i))-1)*100;
dVW           =(sum(i,PW.l(i)*W.l(i)) /sum(i,PW1(i)*W1(i))-1)*100;
dGDP_B        =((sum(i,PVA.l(i)*VA.l(i))+TP.l) /(sum(i,PVA1(i)*VA1(i))+TP1)-1)*100;
dGDP_C        =((sum(i,PVA.l(i)*VA.l(i))+TP.l+TC.l) /(sum(i,PVA1(i)*VA1(i))+TP1+TC1)-
1)*100;
dVY_R_NS      =(PY.l('r_ns')*Y.l('r_ns') /(PY1('r_ns')*Y1('r_ns'))-1)*100;
dPY_R_NS      =(PY.l('r_ns') /(PY1('r_ns'))-1)*100;
dY_R_NS       =(Y.l('r_ns') /Y1('r_ns')-1)*100;
dVY_NR_NS     =(PY.l('nr_ns')*Y.l('nr_ns') /(PY1('nr_ns')*Y1('nr_ns'))-1)*100;
dPY_NR_NS     =(PY.l('nr_ns') /(PY1('nr_ns'))-1)*100;
dY_NR_NS      =(Y.l('nr_ns') /Y1('nr_ns')-1)*100;
dPY_R_R       =(PY.l('r_r') /(PY1('r_r'))-1)*100;
dVY_R_R       =(PY.l('r_r')*Y.l('r_r') /(PY1('r_r')*Y1('r_r'))-1)*100;
dY_R_R        =(Y.l('r_r') /(Y1('r_r'))-1)*100;
```

```
dYT_NR          =(sum(jnr,PT.l(jnr)*Ts(jnr))/sum(jnr,PT1(jnr)*Ts(jnr))-1)*100;
dPK(i)          =(PK.l(i)      /PK1(i)-1)*100;
dPL(i)          =(PL.l(i)      /PL1(i)-1)*100;
dPT(i)          =(PT.l(i)      /PT1(i)-1)*100;
dPT_NR          =(PT_NR.l      /PT_NR1-1)*100;
dPK_A           =(PK_A.l      /PK_A1-1)*100;
dPL_A           =(PL_A.l      /PL_A1-1)*100;
dPK_A_N         =(PK_A_N.l    /PK_A_N1-1)*100;
dPL_A_N         =(PL_A_N.l    /PL_A_N1-1)*100;

Display
dC,dCPI,dCPI_L,dCPI_P,dWD,dEROW,dERUK,dG,dID,dMROW,dINT,dMRUK,dPWD,dP
I,dPQ,dPQW,
dPVA,dPW,dPY,dY,dQ,dQW,dS,dSH,dTC,dTCCL,dTK,dTLE,dTLR,dTP,dTRFG,dTSD_R
_NS,
dTSD_NR_NS,dTSD_R_RS,dTSD_NR_RS,dTSD_R,dTSD_NR,dTY,dVA,dW,dYG,dYG_
DT,dYG_F,dYG_NDT,
dYH,dYH_D,dYF_NR,dYK,dYK_D,dYL,dYT,dVY,dVTY,dVQ,dVTQ,dVC,dVTC,dVG,d
VTG,dVID,dVVA,
dVGVA,dVINT,dVTINT,dVRINT,dVCINT,dVERUK,dVEROW,dVMRUK,dVMROW,dV
W,dGDP_B,dGDP_C,
dVY_R_NS,dPY_R_NS,dY_R_NS,dVY_NR_NS,dPY_NR_NS,dY_NR_NS,dVY_R_R,dPY
_R_R,dY_R_R,
dPK,dPL,dPT,dPT_NR,dPK_A,dPL_A,dTID,dCINT,dPK_A_N,dPL_A_N
;
*==============================================================
==============

*Welfare measure: Hicksian equivalent variations--------------------------------
Parameter
U0              utility level in the Base Run Eq.
ep0             expenditure func. in the Base Run Eq.
ep1             expenditure func. in the Consumption function Eq.
EV              Hicksian equivalent variations
```

```
;

U0      =prod(ic, C0(ic)**ac(ic));
ep0     =U0 /prod(ic, (ac(ic)/1)**ac(ic));
ep1     =U.l/prod(ic, (ac(ic)/1)**ac(ic));
EV      =ep1-ep0;

Display ep0,ep1,EV;
*=================================================================
==============

*Simulation results summary & presentation-------------------------------------
Parameter
WAGE_s(d)     simulation results summary table
;
WAGE_s('d_YG')           =dYG;
WAGE_s('d_YG_DT')        =dYG_DT;
WAGE_s('d_TSD_R')        =dTSD_R;
WAGE_s('d_TSD_NR')       =dTSD_NR;
WAGE_s('d_YG_NDT')       =dYG_NDT;
WAGE_s('d_TP')           =dTP;
WAGE_s('d_TC')           =dTC;
WAGE_s('d_TY')           =dTY;
WAGE_s('d_TK')           =dTK;
WAGE_s('d_TLR')          =dTLR;
WAGE_s('d_TLE')          =dTLE;
WAGE_s('d_YG_F')         =dYG_F;
WAGE_s('d_VTG')          =dVTG ;
WAGE_s('d_VTINT')        =dVTINT;
WAGE_s('d_VGVA')         =dVGVA ;
WAGE_s('d_VTY')          =dVTY  ;
WAGE_s('d_VTQ')          =dVTQ  ;
WAGE_s('d_GDP_B')        =dGDP_B;
WAGE_s('d_GDP_C')        =dGDP_C;
```

```
WAGE_s('d_YH')          =dYH   ;
WAGE_s('d_YH_D')        =dYH_D ;
WAGE_s('d_VTC')         =dVTC  ;
WAGE_s('d_S')           =dS    ;
WAGE_s('d_PI')          =dPI   ;
WAGE_s('d_YF_NR')       =dYF_NR;
WAGE_s('d_YT_NR')       =dYT_NR;
*WAGE_s('d_PT_NR')      =dPT_NR;
*WAGE_s('d_T_stock')    =0;
WAGE_s('d_YK')          =dYK    ;
*WAGE_s('d_PK_N')       =dPK_A_N  ;
*WAGE_s('d_PK')         =dPK_A   ;
*WAGE_s('d_K_stock')    =0;
WAGE_s('d_YL')          =dYL    ;
*WAGE_s('d_PL_N')       =dPL_A_N  ;
*WAGE_s('d_PL')         =dPL_A   ;
*WAGE_s('d_L_stock')    =0;
WAGE_s('d_VY_R_NS')     =dVY_R_NS;
WAGE_s('d_PY_R_NS')     =dPY_R_NS;
WAGE_s('d_Y_R_NS')      =dY_R_NS ;
WAGE_s('d_VY_NR_NS')    =dVY_NR_NS;
WAGE_s('d_PY_NR_NS')    =dPY_NR_NS;
WAGE_s('d_Y_NR_NS')     =dY_NR_NS ;
WAGE_s('d_VY_R_R')      =dVY_R_R  ;
WAGE_s('d_PY_R_R')      =dPY_R_R  ;
WAGE_s('d_Y_R_R')       =dY_R_R   ;
WAGE_s('d_VERUK')       =dVERUK   ;
WAGE_s('d_VEROW')       =dVEROW   ;
WAGE_s('d_VMRUK')       =dVMRUK   ;
WAGE_s('d_VMROW')       =dVMROW   ;
*WAGE_s('d_VW')         =dVW     ;
*WAGE_s('d_EV')         =EV      ;
*WAGE_s('d_CPI')        =dCPI    ;
```

```
execute_unload 'WAGE_s.gdx',WAGE_s;
execute '=gdxviewer WAGE_s.gdx';
*=============================================================
==============
```

VI-2 Simulation in the medium run

```
$Title  A CGE model for Wales - Medium run

$eolcom //
*Definition of sets------------------------------------------------------------
Set a    all accounts
        /agr,mnq,man,egy,con,r_ns,nr_ns,wnr,trp,acm,ict,fin,r_r,rtt,prf,adm,plc,
         edu,hth,ent,oth,
         nic1,lab,lnd,ova,t_pn,sdlt_nr_int,sdlt_r_fnl,sdlt_nr_fnl,t_pt,t_inc,nic2,t_cncl,t_cp,
         hh,gov,sav,ruk,row,total/
    aa   all accounts excluding the totals
        /agr,mnq,man,egy,con,r_ns,nr_ns,wnr,trp,acm,ict,fin,r_r,rtt,prf,adm,plc,
         edu,hth,ent,oth,
         nic1,lab,lnd,ova,t_pn,sdlt_nr_int,sdlt_r_fnl,sdlt_nr_fnl,t_pt,t_inc,nic2,t_cncl,t_cp,
         hh,gov,sav,ruk,row/
    i(a) all industrial sectors
        /agr,mnq,man,egy,con,r_ns,nr_ns,wnr,trp,acm,ict,fin,r_r,rtt,prf,adm,plc,
         edu,hth,ent,oth/
    inre(i) all sectors excluding the three real estate sale and rental sectors
        /agr,mnq,man,egy,con,wnr,trp,acm,ict,fin,rtt,prf,adm,plc,edu,hth,ent,oth/
    inr(i) all sectors excluding residential rental sector
        /agr,mnq,man,egy,con,r_ns,nr_ns,wnr,trp,acm,ict,fin,rtt,prf,adm,plc,edu,hth,ent,oth/
    ic(i) all sectors that produce goods for households¡¯consumption
        /egy,con,wnr,trp,acm,ict,fin,r_r,rtt,prf,adm,edu,hth,ent/
    ig(i) all sectors that produce goods for government consumption
        /plc,edu,hth,ent/
    ii(i) all sectors that produce goods for investment
        /man,con,r_ns,nr_ns,prf/
    d    variables presented in the simulation results summary table
        /d_YG,d_YG_DT,d_TSD_R,d_TSD_NR,d_YG_NDT,d_TP,d_TC,d_TY,d_TK,

d_TLR,d_TLE,d_YG_F,d_VTG,d_VTINT,d_VGVA,d_VTY,d_VTQ,d_GDP_B,d_GDP_C,
         d_YH,d_YH_D,d_VTC,d_S,d_PI,d_YF_NR,d_YT_NR,d_PT_NR,d_T_stock,d_YK,
```

```
d_YK_D,d_PK,d_K_stock,d_YL,d_PL,d_L_stock,d_VY_R_NS,d_PY_R_NS,d_Y_R_NS,

d_VY_NR_NS,d_PY_NR_NS,d_Y_NR_NS,d_VY_R_R,d_PY_R_R,d_Y_R_R,d_VERUK,
        d_VEROW,d_VMRUK,d_VMROW,d_VW,d_EV,d_CPI/
;
Alias (a,b),(aa,bb),(i,j),(inre,jnre),(inr,jnr);
*=========================================================================
==============

*Loading SAM as the benchmark database------------------------------------------
Table SAM0(a,b)
$ondelim
$include 2c_SAM_b_tkty_non0.csv
$offdelim
;
Display SAM0;
*=========================================================================
==============

*Loading the base values for all the variables and calibration of parameters----
*  For endogenous variables (with suffix 0):
Parameter
C0(i)           households' consumption demand by commodity in benchmark
WD0(j)          regional production supplied to domestic market in benchmark
EROW0(i)        export supply to the ROW by sector in benchmark
ERUK0(i)        export supply to the RUK by sector in benchmark
G0(i)           fiscal expenditure demand by commodity in benchmark
ID0(i)          investment demand by commodity in benchmark
INT0(i,j)       intermediate inputs for regional productionin in benchmark
CINT0(j)        composite intermediate inputs of each sector in benchmark
MROW0(j)        import demand from the ROW by commodity in benchmark
MRUK0(j)        import demand from the RUK by commodity in benchmark
```

Q0(j) regional sales of composites combining regional production and all imports by commodity in benchmark

QW0(j) regional sales of domestic composites combining regional production and imports from RUK by commodity in benchmark

S0 total savings in benchmark

SH0 households¡¯savings in benchmark

SRUK0 inter-regional savings from RUK in benchmark

SROW0 foreign savings from ROW in benchmark

TC0 product tax revenue in benchmark

TCCL0 council tax revenue in benchmark

TK0 corporation tax revenue in benchmark

TLE0 NIC revenue payable by the employee in benchmark

TLR0 NIC revenue payable by the employer in benchmark

TP0 production tax revenue in benchmark

TRFG0 fiscal transfer received by regional government from central government in benchmark

TSD_R_NS0 Stamp Duty Land Tax revenue from residential properties new sale in benchmark

TSD_NR_NS0 Stamp Duty Land Tax revenue from non-residential properties new sale in benchmark

TSD_R_RS0 Stamp Duty Land Tax revenue from residential properties resale in benchmark

TSD_NR_RS0 Stamp Duty Land Tax revenue from non-residential properties resale in benchmark

TY0 income tax revenue in benchmark

VA0(j) value-added bundle of factors by sector in benchmark

W0(j) regional production supplied to regional market in benchmark

Y0(j) output of regional production by sector in benchmark

YG0 total fiscal revenue in benchmark

YG_DT0 devolved tax revenue in benchmark

YG_F0 factor income of regional government in benchmark

YG_NDT0 non-devolved tax revenue in benchmark

YH0 households' income in benchmark

YK0 factor income of capital in benchmark

YL0 factor income of labour in benchmark

YT0 factor income of land in benchmark

K0(j) capital demand by sector

L0(j) labour demand by sector

T0(j) land demand by sector

* For exogenous variables (first uppercase letter followed by lowercase letters):

K_stock capital stock (factors fixed for total stock)

L_stock labour stock (factors fixed for total stock)

T_stock land stock (factors fixed for total stock)

Perow(i) price of export supply to the ROW by sector in domestic currency

Peruk(i) price of export supply to the RUK by sector in domestic currency

Pmrow(i) price of import demand from the ROW by sector in domestic currency

Pmruk(i) price of import demand from the RUK by sector in domestic currency

Q_nr_rs non-residential properties resale volume

Q_r_rs residential properties resale volume

Q_r_stock residential properties regional stock

Trfh social protection transfer received by households from regional government

Ks(j) capital demand by sector

Ls(j) labour demand by sector

Ts(j) land demand by sector

Sruk extra-regional saving from RUK

Srow extra-regional saving from ROW

;

C0(i) =SAM0(i,'hh');

EROW0(i) =SAM0(i,'row');

ERUK0(i) =SAM0(i,'ruk');

G0(i) =SAM0(i,'gov');

ID0(i) =SAM0(i,'sav');

INT0(i,j) =SAM0(i,j);

CINT0(j) =sum(i,SAM0(i,j));

K0(j) =SAM0('ova',j);

L0(j) =SAM0('lab',j);

```
MROW0(j)              =SAM0('row',j);
MRUK0(j)              =SAM0('ruk',j);

S0                    =SAM0('sav','total');
SH0                   =SAM0('sav','hh');
T0(j)                 =SAM0('lnd',j);
TC0                   =SAM0('gov','t_pt');
TCCL0                 =SAM0('gov','t_cncl');
TK0                   =SAM0('gov','t_cp');
TLE0                  =SAM0('gov','nic2');
TLR0                  =SAM0('gov','nic1');
TP0                   =SAM0('gov','t_pn');
TRFG0                 =SAM0('gov','ruk');
TSD_R_NS0             =SAM0('sdlt_r_fnl','r_ns');
TSD_NR_NS0            =SAM0('sdlt_nr_fnl','nr_ns');
TSD_R_RS0             =SAM0('sdlt_r_fnl','r_r');
TSD_NR_RS0            =SAM0('sdlt_nr_int','total');
TY0                   =SAM0('gov','t_inc');
VA0(j)                =SAM0('lab',j)+SAM0('nic1',j)+SAM0('t_inc',j)        ////////////////////////////ty
                      +SAM0('lnd',j)+SAM0('sdlt_nr_int',j)
                      +SAM0('ova',j)+SAM0('t_cp',j);
Y0(j)                 =VA0(j)+sum(i,INT0(i,j));
WD0(j)                =Y0(j)+SAM0('t_pn',j)+SAM0('t_pt',j)+SAM0('sdlt_r_fnl',j)
                      +SAM0('sdlt_nr_fnl',j)-EROW0(j);
W0(j)                 =WD0(j)-ERUK0(j);
QW0(j)                =W0(j)+MRUK0(j);
Q0(j)                 =QW0(j)+MROW0(j);
YG0                   =SAM0('gov','total');
*For SDLT simulation-------------
*YG_DT0               =TSD_R_NS0+TSD_NR_NS0+TSD_R_RS0+TSD_NR_RS0+TCCL0;
*--------------------------------
*For Corporation Tax simulation--
*YG_DT0
=TSD_R_NS0+TSD_NR_NS0+TSD_R_RS0+TSD_NR_RS0+TCCL0+TK0;
```

```
*For Income Tax simulation--
*YG_DT0
=TSD_R_NS0+TSD_NR_NS0+TSD_R_RS0+TSD_NR_RS0+TCCL0+TY0;
*--------------------------------
*For Cross simulation--Income&Corporation Tax
YG_DT0
=TSD_R_NS0+TSD_NR_NS0+TSD_R_RS0+TSD_NR_RS0+TCCL0+TK0+TY0;
*--------------------------------
YG_F0                 =SAM0('gov','lnd')+SAM0('gov','ova');
YG_NDT0               =YG0-TRFG0-YG_F0-YG_DT0;
YH0                   =SAM0('hh','total')+sum(j,SAM0('t_inc',j));          ///////////////////////////////ty
YK0                   =SAM0('ova','total');
YL0                   =SAM0('lab','total');
YT0                   =SAM0('lnd','total');

K_stock               =YK0;
L_stock               =YL0;
T_stock               =YT0;
Ks(j)                 =SAM0('ova',j);
Ls(j)                 =SAM0('lab',j);
Ts(j)                 =SAM0('lnd',j);
Perow(i)              =1;
Peruk(i)              =1;
Pmrow(i)              =1;
Pmruk(i)              =1;
Q_nr_rs               =257;
Q_r_rs                =4977.667866;
Q_r_stock             =212046.417;
Trfh                  =SAM0('hh','gov');
Sruk                  =SAM0('sav','ruk');
Srow                  =SAM0('sav','row');

Display
```

C0,EROW0,ERUK0,G0,ID0,INT0,CINT0,K0,L0,MROW0,MRUK0,Q0,QW0,S0,SH0,T0,TC0,TCCL0,TK0,

TLE0,TLR0,TP0,TRFG0,TSD_R_NS0,TSD_NR_NS0,TSD_R_RS0,TSD_NR_RS0,TY0,VA0,W0,WD0,Y0,

YG0,YG_DT0,YG_F0,YG_NDT0,YH0,YK0,YL0,YT0,Perow,Peruk,Pmrow,Pmruk,Q_nr_rs,Q_r_rs,

Q_r_stock,Trfh,K_stock,T_stock,L_stock,Ks,Ls,Ts,Sruk,Srow

;

* Calibration of parameters--

Parameter

mps marginal propensity of saving of households

skg share of factor income of capital distributed to regional government

skh share of factor income of capital distributed to households

stg share of factor income of land distributed to regional government

sth share of factor income of land distributed to households

tauccl effective council tax rate

tauc(j) effective product tax rate by commodity

tauk effective corporation tax rate

taule effective NIC rate payable by the employee

taulr effective NIC rate payable by the employer

taup(j) effective production tax rate by sector

tausd_nr_int(i) effective non-residential SDLT rate applied on intermediate non-residential land input

tausd_r_fnl(i) effective residential SDLT rate applied on final sectoral production of residential new sale

tausd_rr_fnl(i) effective residential SDLT rate applied on final sectoral production of residential rental

tausd_nr_fnl(i) effective non-residential SDLT rate applied on final sectoral production of non-residential new sale

tauy effective income tax rate

taumx(i) mixed tax rate combining net production and product tax rate & residential&non-residential SDLT final rates

ac(i) parameter with respect to households¡¯ demand for commodities

```
ag(i)                parameter with respect to fiscal expenditure demand for commodities
aid(i)               parameter with respect to investment demand for commodities
ak(j)                share parameter with respect to capital demand in Cobb-Douglas production
function by sector
al(j)                share parameter with respect to labour demand in Cobb-Douglas production
function by sector
at(j)                share parameter with respect to land demand in Cobb-Douglas production
function by sector
ava(j)               Leontief parameter with respect to value-added bundle by sector
aint(i,j)            Leontief parameter with respect to composite intermediate inputs by sector
;
mps                  =SH0/(YH0-TY0-TLE0-TCCL0);
skg                  =SAM0('gov','ova')/YK0;
skh                  =SAM0('hh','ova')/YK0;
stg                  =SAM0('gov','lnd')/YT0;
sth                  =SAM0('hh','lnd')/YT0;
tauccl               =TCCL0/Q_r_stock;
tauc(j)              =SAM0('t_pt',j)/(Y0(j)+SAM0('t_pn',j));
tauk                 =TK0/(TK0+YK0);
tauy                 =TY0/YH0;
taule                =TLE0*(1-tauy)/YL0;                  /////////////////////////////////ty
taulr                =TLR0*(1-tauy)/YL0;                  /////////////////////////////////ty
taup(j)              =SAM0('t_pn',j)/Y0(j);
tausd_r_fnl('r_ns')      =TSD_R_NS0/(Y0('r_ns')+SAM0('t_pn','r_ns'));
tausd_nr_fnl('nr_ns')    =TSD_NR_NS0/(Y0('nr_ns')+SAM0('t_pn','nr_ns'));
tausd_rr_fnl('r_r')      =TSD_R_RS0/Y0('r_r');
tausd_nr_int(inr)        =TSD_NR_RS0/(SAM0('lnd','total')-SAM0('lnd','r_r'));
taumx(i)
=(1+taup(i))*(1+tauc(i))*(1+tausd_r_fnl(i))*(1+tausd_nr_fnl(i))*(1+tausd_rr_fnl(i))-1;
ac(i)                =C0(i)/(YH0-TY0-TLE0-TCCL0-SH0);
ag(i)                =G0(i)/(YG0-Trfh);
aid(i)               =ID0(i)/S0;
ak(j)                =Ks(j)/(1-tauk)/VA0(j);
al(j)                =(1+taulr)*Ls(j)/(1-tauy)/VA0(j);           ///////////////////////////////ty
```

```
at(j)          =1-ak(j)-al(j);
ava(j)         =VA0(j)/Y0(j);
aint(i,j)      =INT0(i,j)/CINT0(j);

Parameter
sigmak(i)      elasticity of transformation in the CET function for RUK exports
sigmaw(i)      elasticity of transformation in the CET function for ROW exports
rhok(i)        parameter defined by elasticity of transformation in the CET function for RUK exports
rhow(i)        parameter defined by elasticity of transformation in the CET function for ROW exports
serow(i)       distribution parameter for export supply to the ROW by sector in the CET function
seruk(i)       distribution parameter for export supply to the RUK by sector in the CET function
thetak(i)      scaling coefficient in the CET function for RUK exports
thetaw(i)      scaling coefficient in the CET function for ROW exports
omegak(i)      elasticity of substitution in the Armington CES function for RUK imports
omegaw(i)      elasticity of substitution in the Armington CES function for ROW imports
etak(i)        parameter defined by elasticity of substitution in the Armington CES function for RUK imports
etaw(i)        parameter defined by elasticity of substitution in the Armington CES function for ROW imports
smrow(i)       distribution parameter for import demand from the ROW by sector in the Armington CES function
smruk(i)       distribution parameter for import demand from the RUK by sector in the Armington CES function
gammak(i)      scaling coefficient in the Armington CES function for RUK imports
gammaw(i)      scaling coefficient in the Armington CES function for ROW imports
;
sigmak(i)      =4;
sigmaw(i)      =1.5;
rhok(i)        =(sigmak(i)+1)/sigmak(i);   //--> sigma=1/(rho-1) --> 1-rho= -1/sigma
rhow(i)        =(sigmaw(i)+1)/sigmaw(i);
```

```
serow(i)           =EROW0(i)**(1-rhow(i))/(EROW0(i)**(1-rhow(i))+WD0(i)**(1-rhow(i)));
seruk(i)            =ERUK0(i)**(1-rhok(i))/(ERUK0(i)**(1-rhok(i))+W0(i)**(1-rhok(i)));
thetak(i)                                   =WD0(i)/(seruk(i)*ERUK0(i)**rhok(i)+(1-
seruk(i))*W0(i)**rhok(i))**(1/rhok(i));
thetaw(i)                                   =Y0(i)/(serow(i)*EROW0(i)**rhow(i)+(1-
serow(i))*WD0(i)**rhow(i))**(1/rhow(i));

omegak(i)            =4;
omegaw(i)            =2.5;
etak(i)              =(omegak(i)-1)/omegak(i);   //--> omega=1/(1-eta) --> 1-eta= 1/omega
etaw(i)              =(omegaw(i)-1)/omegaw(i);
smrow(i)           =MROW0(i)**(1-etaw(i))/(MROW0(i)**(1-etaw(i))+QW0(i)**(1-etaw(i)));
smruk(i)            =MRUK0(i)**(1-etak(i))/(MRUK0(i)**(1-etak(i))+W0(i)**(1-etak(i)));
gammak(i)                                   =QW0(i)/(smruk(i)*MRUK0(i)**etak(i)+(1-
smruk(i))*W0(i)**etak(i))**(1/etak(i));
gammaw(i)                                   =Q0(i)/(smrow(i)*MROW0(i)**etaw(i)+(1-
smrow(i))*QW0(i)**etaw(i))**(1/etaw(i));

* Below for sensitivity analysis regarding nest1 & nest2
*nest2:
Parameter
omega(i)             elasticity of substitution in the production function for GVA
eta(i)               parameter defined by elasticity of substitution in the production function for
GVA
sk(i)                distribution parameter of capital demand by sector in the production function
for GVA
sl(i)                distribution parameter of labour demand by sector in the production function
for GVA
st(i)               distribution parameter of land demand by sector in the production function for
GVA
gamma(i)             scaling coefficient in the production function for GVA
;

omega(i)             =0.5;
```

```
eta(i)                  =(omega(i)-1)/omega(i);  //--> omega=1/(1-eta) --> 1-eta= 1/omega
sk(i)                 =(1-tauy)*Ks(i)**(1-eta(i))/((1-tauy)*(1-tauk)*(1+tausd_nr_int(i))*Ts(i)**(1-
eta(i))+(1-tauk)*(1+taulr)*Ls(i)**(1-eta(i))+(1-tauy)*Ks(i)**(1-eta(i))); /////////ty
sl(i)=(1-tauk)*(1+taulr)*Ls(i)**(1-eta(i))/((1-tauy)*(1-tauk)*(1+tausd_nr_int(i))*Ts(i)**(1-
eta(i))+(1-tauk)*(1+taulr)*Ls(i)**(1-eta(i))+(1-tauy)*Ks(i)**(1-eta(i))); /////////ty
st(i)                   =1-sk(i)-sl(i);
gamma(i)
=VA0(i)/(sk(i)*Ks(i)**eta(i)+sl(i)*Ls(i)**eta(i)+st(i)*Ts(i)**eta(i))**(1/eta(i));

*nest1:
Parameter
omegai(i)               elasticity of substitution between value added and composite intermediate
inputs
etai(i)                   parameter defined by elasticity of substitution between value added and
composite intermediate inputs
sva(i)                  distribution parameter of value added demand by sector
scint(i)                distribution parameter of composite intermediate demand by sector
gammai(i)               scaling coefficient in the production function for regional output
;

omegai(i)               =0.5;
etai(i)                 =(omegai(i)-1)/omegai(i);  //--> omega=1/(1-eta) --> 1-eta= 1/omega
sva(i)                  =VA0(i)**(1-etai(i))/(VA0(i)**(1-etai(i))+CINT0(i)**(1-etai(i)));
scint(i)                =1-sva(i);
gammai(i)               =Y0(i)/(sva(i)*VA0(i)**etai(i)+scint(i)*CINT0(i)**etai(i))**(1/etai(i));

Display
mps,skg,skh,stg,sth,tauccl,tauc,taumx,tauk,taule,taulr,taup,tausd_r_fnl,tausd_rr_fnl,
tausd_nr_int,tausd_nr_fnl,tauy,ac,ag,aid,aint,ak,al,at,ava,sigmaw,omegaw,sigmak,
rhok,rhow,seruk,serow,thetak,thetaw,omegak,etak,etaw,smrow,smruk,gammak,gammaw,
omega,eta,sk,sl,st,gamma,omegai,etai,sva,scint,gammai
;
*==============================================================
==============
```

*Defining model system---

Variable

C(i) households' consumption demand by commodity

CPI CPI calculated using Fisher index

CPI_L CPI calculated using Laspeyres index

CPI_P CPI calculated using Paasche index

WD(j) regional production supplied to domestic market

EROW(i) export supply to the ROW by sector

ERUK(i) export supply to the RUK by sector

G(i) fiscal expenditure demand by commodity

ID(i) investment demand by commodity

INT(i,j) intermediate inputs for regional production

CINT(j) composite intermediate inputs for regional production

MROW(j) import demand from the ROW by commodity

MRUK(j) import demand from the RUK by commodity

PWD(i) price of regional production supplied to domestic market

PI average price of investment goods

PQ(i) price of regional sales of composites by commodity ¨C net of product taxes

PQW(i) price of regional sales of domestic composites by commodity ¨C net of product taxes

PVA(i) price of value added by sector

PW(i) price of regional production supplied to regional market

PY(i) price of output of regional production by sector ¨C basic price

Q(j) regional sales of composites combining regional production and all imports by commodity

QW(j) regional sales of domestic composites combining regional production and imports from RUK by commodity

S total savings

SH households¡¯savings

TC product tax revenue

TCCL council tax revenue

TK corporation tax revenue

TLE NIC revenue payable by the employee

TLR NIC revenue payable by the employer

TP production tax revenue

TRFG fiscal transfer received by regional government from central government

TSD_R_NS Stamp Duty Land Tax revenue from residential properties¡¯ new sale

TSD_NR_NS Stamp Duty Land Tax revenue from non-residential properties¡¯ new sale

TSD_R_RS Stamp Duty Land Tax revenue from residential properties¡¯ resale

TSD_NR_RS Stamp Duty Land Tax revenue from non-residential properties¡¯ resale

TY income tax revenue

U households¡¯ utility level

VA(j) value-added bundle of factors by sector

W(j) regional production supplied to regional market

Y(j) output of regional production by sector

YG total fiscal revenue

YG_DT devolved tax revenue

YG_F factor income of regional government

YG_NDT non-devolved tax revenue

YH households' income

YK factor income of capital

YL factor income of labour

YT factor income of land

K(j) capital demand by sector (factors fixed for total stock)

T(j) land demand by sector (factors fixed for total stock)

L(j) labour demand by sector (factors fixed for total stock)

PK economy-wide capital return (factors fixed for total stock)

PT_NR economy-wide non-residential land return (factors fixed for total stock)

PL economy-wide labour wage (factors fixed for total stock)

;

Equation

eqva(j) value added function

eqt_nr(j) non-residential land demand function

eqkstock factor market clearing condition for capital

eqtstock factor market clearing condition for land

eqlstock	factor market clearing condition for labour
eqk(j)	capital demand function
eql(j)	labour demand function
eqyk	total capital income
eqyt	total land income
eqyl	total labour income
eqint(i,j)	intermediate demand function
eqcint(j)	composite intermediate demand function
eqy(j)	value added demand function
eqpva(j)	regional production function
eqyh	households income function
eqsh	households savings function
eqc(i)	households consumption demand function for non-residential-rental sectors
eqty	income tax revenue function
eqtk	corporation tax revenue function
eqtlr	function for NIC revenue payable by the employer
eqtle	function for NIC revenue payable by the employee
eqtccl	council tax revenue function
eqtp	net production tax revenue function
eqtc	net product tax revenue function
eqtsd_r_ns	function for SDLT revenue from residential properties¡¯ new sale
eqtsd_nr_ns	function for SDLT revenue from non-residential properties¡¯ new sale
eqtsd_r_rs	function for SDLT revenue from residential properties¡¯ resale
eqtsd_nr_rs	function for SDLT revenue from non-residential properties¡¯ resale
eqyg_dt	devolved tax revenue function
eqyg_ndt	non-devolved tax revenue function
eqyg_f	factor income of regional government function
eqtrfg	function for fiscal transfer received by regional government from central government
eqyg	total fiscal revenue function
eqg(i)	fiscal expenditure demand function
eqpyd(i)	CET function for ROW exports and domestic goods
eqwds(i)	domestic good supply function
eqerows(i)	foreign export supply function

```
eqpwdd(i)            CET function for RUK exports and regional goods
eqws(i)              regional good supply function
eqeruks(i)           RUK export supply function
eqpqws(i)            Armington CES function RUK imports and regional goods
eqmrukd(i)           RUK import demand function
eqwd(i)              regional good demand function
eqpqs(i)             Armington CES function ROW imports and domestic goods
eqmrowd(i)           ROW import demand function
eqqwd(i)             domestic good demand function
eqs                  total saving function
eqid(i)              investment demand function
eqcpi_l_b            CPI calculation function using Laspeyres index in base replication
eqcpi_p_b            CPI calculation function using Paasche index in base replication
eqcpi                CPI calculation function using Fisher index
eqpi                 investment good price index calculation function
equ                  objective utility function
eqq(i)               market clearing condition for goods market
;

*------------------------production behaviour----------------------------------
eqpva(jnr)..                                             PVA(jnr)*VA(jnr)=e=PK*K(jnr)/(1-
tauk)+(1+tausd_nr_int(jnr))*PT_NR*T(jnr)+(1+taulr)*PL*L(jnr)/(1-tauy);
////////////////////////////tk ty
*eqk(j)..     PK*K(j)/(1-tauk)=e=ak(j)*PVA(j)*VA(j);                        //for C-D nest2
*eqt_nr(jnr)..(1+tausd_nr_int(jnr))*PT_NR*T(jnr)=e=at(jnr)*PVA(jnr)*VA(jnr);     //for C-D
nest2
*eql(j)..      (1+taulr)*PL*L(j)/(1-tauy)=e=al(j)*PVA(j)*VA(j);                    //for C-D nest2
/////////////////////ty
eqk(j)..              K(j)=e=VA(j)*(gamma(j)**eta(j)*sk(j)*(1-tauk)*PVA(j)/PK)**(1/(1-eta(j)));
//for CES nest2                                  /////////////////////////
eqt_nr(jnr)..T(jnr)=e=VA(jnr)*(gamma(jnr)**eta(jnr)*st(jnr)*PVA(jnr)/((1+tausd_nr_int(jnr)
)*PT_NR))**(1/(1-eta(jnr)));  //for CES nest2
eql(j)..          L(j)=e=VA(j)*(gamma(j)**eta(j)*sl(j)*(1-tauy)*PVA(j)/((1+taulr)*PL))**(1/(1-
eta(j)));                   //for CES nest2    ////////ty
```

```
eqyk..      YK=e=sum(j,PK*K(j));
eqyt..      YT=e=sum(jnr,PT_NR*T(jnr))+PVA('r_r')*VA('r_r');
eqyl..      YL=e=sum(j,PL*L(j));

eqy(j)..    PY(j)*Y(j)=e=PVA(j)*VA(j)+PQ(j)*CINT(j);
eqva(j)..   VA(j)=e=Y(j)*(gammai(j)**etai(j)*sva(j)*PY(j)/PVA(j))**(1/(1-etai(j)));
eqcint(j).. CINT(j)=e=Y(j)*(gammai(j)**etai(j)*scint(j)*PY(j)/PQ(j))**(1/(1-etai(j)));
eqint(i,j).. INT(i,j)=e=aint(i,j)*CINT(j);
*------------------------household behaviour------------------------------------
eqyh..      YH=e=skh*YK+sth*YT+YL/(1-tauy)+Trfh;                    //////////////////////ty
eqsh..      SH=e=mps*(YH-TY-TLE-TCCL);
eqc(i)..    PQ(i)*C(i)=e=ac(i)*(YH-TY-TLE-TCCL-SH);

*------------------------government behaviour-----------------------------------
eqty..      TY=e=tauy*YH;
eqtk..      TK=e=tauk*YK/(1-tauk);
eqtlr..     TLR=e=taulr*YL/(1-tauy);                   //////////////////////ty
eqtle..     TLE=e=taule*YL/(1-tauy);                   //////////////////////ty
eqtccl..    TCCL=e=TCCL0;
eqtp..      TP=e=sum(j,taup(j)*PY(j)*Y(j));
eqtc..      TC=e=sum(j,tauc(j)*PY(j)*(1+taup(j))*Y(j));
eqtsd_r_ns.. TSD_R_NS=e=sum(i,tausd_r_fnl(i)*PY(i)*(1+taup(i))*Y(i));
eqtsd_nr_ns..TSD_NR_NS=e=sum(i,tausd_nr_fnl(i)*PY(i)*(1+taup(i))*Y(i));
eqtsd_r_rs.. TSD_R_RS=e=sum(i,tausd_rr_fnl(i)*PY(i)*(1+taup(i))*Y(i));
eqtsd_nr_rs..TSD_NR_RS=e=sum(inr,tausd_nr_int(inr)*PT_NR*T(inr));
*For SDLT simulation-------------
*eqyg_dt..   YG_DT=e=TSD_R_NS+TSD_NR_NS+TSD_R_RS+TSD_NR_RS+TCCL;
*eqyg_ndt..   YG_NDT=e=TY+TK+TLR+TLE+TC+TP;
*------------------------------------------
*For Corporation Tax simulation--
*eqyg_dt..   YG_DT=e=TSD_R_NS+TSD_NR_NS+TSD_R_RS+TSD_NR_RS+TCCL+TK;
*eqyg_ndt..   YG_NDT=e=TY+TLR+TLE+TC+TP;
*------------------------------------------
```

```
*For Income Tax simulation--
*eqyg_dt..   YG_DT=e=TSD_R_NS+TSD_NR_NS+TSD_R_RS+TSD_NR_RS+TCCL+TY;
*eqyg_ndt..   YG_NDT=e=TK+TLR+TLE+TC+TP;
*-------------------------------------------
*For Cross simulation--Income&Corporation Tax
eqyg_dt..
YG_DT=e=TSD_R_NS+TSD_NR_NS+TSD_R_RS+TSD_NR_RS+TCCL+TK+TY;
eqyg_ndt..   YG_NDT=e=TLR+TLE+TC+TP;
*-------------------------------------------
eqyg_f..     YG_F=e=skg*YK+stg*YT;
eqtrfg..     TRFG=e=YG0-YG_DT0-YG_F-YG_NDT;
eqyg..       YG=e=YG_DT+YG_F+YG_NDT+TRFG;
eqg(i)..     PQ(i)*G(i)=e=ag(i)*(YG-Trfh);

*-----------------------trade behaviour---------------------------------------
eqpyd(i)..                                  Y(i)=e=thetaw(i)*(serow(i)*EROW(i)**rhow(i)+(1-
serow(i))*WD(i)**rhow(i))**(1/rhow(i));
eqerows(i)..
EROW(i)=e=Y(i)*(thetaw(i)**rhow(i)*serow(i)*(1+taumx(i))*PY(i)/Perow(i))**(1/(1-
rhow(i)));
eqwds(i)..                                             WD(i)=e=Y(i)*(thetaw(i)**rhow(i)*(1-
serow(i))*(1+taumx(i))*PY(i)/PWD(i))**(1/(1-rhow(i)));

eqpwdd(i)..                                 WD(i)=e=thetak(i)*(seruk(i)*ERUK(i)**rhok(i)+(1-
seruk(i))*W(i)**rhok(i))**(1/rhok(i));
eqeruks(i)..         ERUK(i)=e=WD(i)*(thetak(i)**rhok(i)*seruk(i)*PWD(i)/Peruk(i))**(1/(1-
rhok(i)));
eqws(i)..    W(i)=e=WD(i)*(thetak(i)**rhok(i)*(1-seruk(i))*PWD(i)/PW(i))**(1/(1-rhok(i)));

eqpqws(i)..                              QW(i)=e=gammak(i)*(smruk(i)*MRUK(i)**etak(i)+(1-
smruk(i))*W(i)**etak(i))**(1/etak(i));
eqmrukd(i)..  MRUK(i)=e=QW(i)*(gammak(i)**etak(i)*smruk(i)*PQW(i)/Pmruk(i))**(1/(1-
etak(i)));
```

```
eqwd(i)..        W(i)=e=QW(i)*(gammak(i)**etak(i)*(1-smruk(i))*PQW(i)/PW(i))**(1/(1-
etak(i)));

eqpqs(i)..                   Q(i)=e=gammaw(i)*(smrow(i)*MROW(i)**etaw(i)+(1-
smrow(i))*QW(i)**etaw(i))**(1/etaw(i));
eqmrowd(i)..   MROW(i)=e=Q(i)*(gammaw(i)**etaw(i)*smrow(i)*PQ(i)/Pmrow(i))**(1/(1-
etaw(i)));
eqqwd(i)..      QW(i)=e=Q(i)*(gammaw(i)**etaw(i)*(1-smrow(i))*PQ(i)/PQW(i))**(1/(1-
etaw(i)));

*------------------------investment&saving behaviour----------------------------
eqs..      S=e=(1-skh-skg)*YK+(1-sth-stg)*YT+SH+Sruk+Srow;
eqid(i)..   PQ(i)*ID(i)=e=aid(i)*S;

*------------------------price equations----------------------------------------
eqcpi_l_b..  CPI_L=e=sum(inre,PQ(inre)*C0(inre))/sum(inre,C0(inre));
eqcpi_p_b..  CPI_P=e=sum(inre,PQ(inre)*C(inre))/sum(inre,C(inre));
eqcpi..     CPI=e=sqrt(CPI_L*CPI_P);
eqpi..      PI=e=sum(ii,PQ(ii)*aid(ii));

*------------------------market clearing conditions-----------------------------
eqq(i)..    Q(i)=e=C(i)+G(i)+ID(i)+sum(j, INT(i,j));
eqkstock..   sum(j,K(j))=e=K_stock;
eqtstock..   sum(jnr,T(jnr))=e=T_stock-Ts('r_r'); //Non-residential land use is fixed
eqlstock..   sum(j,L(j))=e=L_stock;

*------------------------objective function-------------------------------------
equ..      U=e=prod(ic, C(ic)**ac(ic));
*===============================================================
==============

*Initializing endogenous variables---------------------------------------------
C.l(i)      =C0(i);
CPI.l       =1;
```

```
CPI_L.l      =1;
CPI_P.l      =1;
WD.l(j)      =WD0(j);
EROW.l(i)    =EROW0(i);
ERUK.l(i)    =ERUK0(i);
G.l(i)       =G0(i);
ID.l(i)      =ID0(i);
INT.l(i,j)   =INT0(i,j);
CINT.l(j)    =CINT0(j);
MROW.l(j)    =MROW0(j);
MRUK.l(j)    =MRUK0(j);
PWD.l(i)     =1;
PI.l         =1;
PQ.l(i)      =1;
PQW.l(i)     =1;
PVA.l(i)     =1;
PW.l(i)      =1;
PY.l(i)      =1;
Q.l(j)       =Q0(j);
QW.l(j)      =QW0(j);
S.l          =S0;
SH.l         =SH0;
TC.l         =TC0;
TCCL.l       =TCCL0;
TK.l         =TK0;
TLE.l        =TLE0;
TLR.l        =TLR0;
TP.l         =TP0;
TRFG.l       =TRFG0;
TSD_R_NS.l   =TSD_R_NS0;
TSD_NR_NS.l  =TSD_NR_NS0;
TSD_R_RS.l   =TSD_R_RS0;
TSD_NR_RS.l  =TSD_NR_RS0;
TY.l         =TY0;
```

```
VA.l(j)      =VA0(j);
W.l(j)       =W0(j);
Y.l(j)       =Y0(j);
YG.l         =YG0;
YG_DT.l      =YG_DT0;
YG_F.l       =YG_F0;
YG_NDT.l     =YG_NDT0;
YH.l         =YH0;
YK.l         =YK0;
YL.l         =YL0;
YT.l         =YT0;
K.l(j)       =K0(j);
T.l(j)       =T0(j);
L.l(j)       =L0(j);
PK.l         =1;
PT_NR.l      =1;
PL.l         =1;
*=================================================================
==============

*Setting lower bounds for endogenous variables---------------------------------
C.lo(i)       =0.0000000001;
CPI.lo        =0.0000000001;
CPI_L.lo      =0.0000000001;
CPI_P.lo      =0.0000000001;
WD.lo(j)      =0.0000000001;
EROW.lo(i)    =0.0000000001;
ERUK.lo(i)    =0.0000000001;
G.lo(i)       =0.0000000001;
ID.lo(i)      =0.0000000001;
INT.lo(i,j)   =0.0000000001;
CINT.lo(j)    =0.0000000001;
MROW.lo(j)    =0.0000000001;
MRUK.lo(j)    =0.0000000001;
```

```
PWD.lo(i)     =0.0000000001;
PI.lo         =0.0000000001;
PQ.lo(i)      =0.0000000001;
PQW.lo(i)     =0.0000000001;
PVA.lo(i)     =0.0000000001;
PW.lo(i)      =0.0000000001;
PY.lo(i)      =0.0000000001;
Q.lo(j)       =0.0000000001;
QW.lo(j)      =0.0000000001;
S.lo          =0.0000000001;
SH.lo         =0.0000000001;
TC.lo         =0.0000000001;
TCCL.lo       =0.0000000001;
TK.lo         =0.0000000001;
TLE.lo        =0.0000000001;
TLR.lo        =0.0000000001;
TP.lo         =0.0000000001;
TRFG.lo       =0.0000000001;
TSD_R_NS.lo   =0.0000000001;
TSD_NR_NS.lo  =0.0000000001;
TSD_R_RS.lo   =0.0000000001;
TSD_NR_RS.lo  =0.0000000001;
TY.lo         =0.0000000001;
VA.lo(j)      =0.0000000001;
W.lo(j)       =0.0000000001;
Y.lo(j)       =0.0000000001;
YG.lo         =0.0000000001;
YG_DT.lo      =0.0000000001;
YG_F.lo       =0.0000000001;
YG_NDT.lo     =0.0000000001;
YH.lo         =0.0000000001;
YK.lo         =0.0000000001;
YL.lo         =0.0000000001;
YT.lo         =0.0000000001;
```

```
K.lo(j)      =0.0000000001;
T.lo(j)      =0.0000000001;
L.lo(j)      =0.0000000001;
PK.lo        =0.0000000001;
PT_NR.lo      =0.0000000001;
PL.lo        =0.0000000001;
*=====================================================================
==============

*Setting numeraire-------------------------------------------------------
*CPI.fx      =1;
*=====================================================================
==============

*Defining and solving the model & replicating the benchmark---------------------
Model WAGE_base /All/;
Solve WAGE_base maximizing U using nlp;

*  Show solutions for benchmark replication-------------
Display
C.l,CPI.l,CPI_L.l,CPI_P.l,WD.l,EROW.l,ERUK.l,G.l,ID.l,INT.l,MROW.l,MRUK.l,PWD.l,
PI.l,PQ.l,PQW.l,PVA.l,PW.l,PY.l,Q.l,QW.l,S.l,SH.l,TC.l,TCCL.l,TK.l,TLE.l,TLR.l,
TP.l,TRFG.l,TSD_R_NS.l,TSD_NR_NS.l,TSD_R_RS.l,TSD_NR_RS.l,TY.l,U.l,VA.l,W.l,Y.
l,
YG.l,YG_DT.l,YG_F.l,YG_NDT.l,YH.l,YK.l,YL.l,YT.l,K.l,T.l,L.l,PK.l,PT_NR.l,PL.l,
CINT.l
;

*  Show how much the solutions deviate the benchmark----
*    Show percentage deviation against the benchmark----
Parameter
dC(ic),dCPI,dCPI_L,dCPI_P,dWD(j),dEROW(i),dERUK(i),dG(ig),dID(ii),dINT(i,j),
dMROW(i),dMRUK(i),dPWD(i),dPI,dPQ(i),dPQW(i),dPVA(i),dPW(i),dPY(i),dQ(j),dQW(j),
```

```
dS,dSH,dTC,dTCCL,dTK,dTLE,dTLR,dTP,dTRFG,dTSD_R_NS,dTSD_NR_NS,dTSD_R_
RS,dTSD_NR_RS,
dTY,dVA(j),dW(j),dY(j),dYG,dYG_DT,dYG_F,dYG_NDT,dYH,dYK,dYL,dYT,dK(j),dT(j)
,dL(j),
dPK,dPT_NR,dPL,

*Below with prefix 'ad' is average deviations of each variable across sectors
adC,adWD,adEROW,adERUK,adG,adID,adMROW,adINT,adMRUK,adPWD,adPQ,adPQW
,adPVA,adPW,
adPY,adQ,adQW,adVA,adW,adY,ad,adK,adT,adL
;

dC(ic)          =(C.l(ic)     /C0(ic)-1)*100;
dCPI          =(CPI.l      /1-1)*100;
dCPI_L        =(CPI_L.l     /1-1)*100;
dCPI_P        =(CPI_P.l     /1-1)*100;
dWD(j)        =(WD.l(j)     /WD0(j)-1)*100;
dEROW(i)$(EROW0(i) ne 0)      =(EROW.l(i)    /EROW0(i)-1)*100;
dERUK(i)$(ERUK0(i) ne 0)      =(ERUK.l(i)    /ERUK0(i)-1)*100;
dG(ig)          =(G.l(ig)     /G0(ig)-1)*100;
dID(ii)         =(ID.l(ii)     /ID0(ii)-1)*100;
dINT(i,j)$(INT0(i,j) ne 0)     =(INT.l(i,j)   /INT0(i,j)-1)*100;
dMROW(j)$(MROW0(j) ne 0)      =(MROW.l(j)    /MROW0(j)-1)*100;
dMRUK(j)$(MRUK0(j) ne 0)      =(MRUK.l(j)    /MRUK0(j)-1)*100;
dPWD(i)       =(PWD.l(i)    /1-1)*100;
dPI          =(PI.l      /1-1)*100;
dPQ(i)        =(PQ.l(i)     /1-1)*100;
dPQW(i)       =(PQW.l(i)    /1-1)*100;
dPVA(i)       =(PVA.l(i)    /1-1)*100;
dPW(i)        =(PW.l(i)     /1-1)*100;
dPY(i)        =(PY.l(i)     /1-1)*100;
dQ(j)         =(Q.l(j)      /Q0(j)-1)*100;
dQW(j)        =(QW.l(j)     /QW0(j)-1)*100;
dS           =(S.l       /S0-1)*100;
```

```
dSH            =(SH.l        /SH0-1)*100;
dTC            =(TC.l        /TC0-1)*100;
dTCCL          =(TCCL.l      /TCCL0-1)*100;
dTK            =(TK.l        /TK0-1)*100;
dTLE           =(TLE.l       /TLE0-1)*100;
dTLR           =(TLR.l       /TLR0-1)*100;
dTP            =(TP.l        /TP0-1)*100;
dTRFG          =(TRFG.l      /TRFG0-1)*100;
dTSD_R_NS      =(TSD_R_NS.l   /TSD_R_NS0-1)*100;
dTSD_NR_NS     =(TSD_NR_NS.l  /TSD_NR_NS0-1)*100;
dTSD_R_RS      =(TSD_R_RS.l   /TSD_R_RS0-1)*100;
dTSD_NR_RS     =(TSD_NR_RS.l  /TSD_NR_RS0-1)*100;
dTY            =(TY.l        /TY0-1)*100;
dVA(j)         =(VA.l(j)     /VA0(j)-1)*100;
dW(j)          =(W.l(j)      /W0(j)-1)*100;
dY(j)          =(Y.l(j)      /Y0(j)-1)*100;
dYG            =(YG.l        /YG0-1)*100;
dYG_DT         =(YG_DT.l     /YG_DT0-1)*100;
dYG_F          =(YG_F.l      /YG_F0-1)*100;
dYG_NDT        =(YG_NDT.l    /YG_NDT0-1)*100;
dYH            =(YH.l        /YH0-1)*100;
dYK            =(YK.l        /YK0-1)*100;
dYL            =(YL.l        /YL0-1)*100;
dYT            =(YT.l        /YT0-1)*100;
dK(j)$(K0(j) ne 0)         =(K.l(j)      /K0(j)-1)*100;
dT(j)$(T0(j) ne 0)         =(T.l(j)      /T0(j)-1)*100;
dL(j)$(L0(j) ne 0)         =(L.l(j)      /L0(j)-1)*100;
dPK            =(PK.l        /1-1)*100;
dPT_NR         =(PT_NR.l     /1-1)*100;
dPL            =(PL.l        /1-1)*100;

adC            =sum(ic,abs(dC(ic)))/card(ic);
adWD           =sum(i,abs(dWD(i)))/card(i);
adEROW         =sum(i,abs(dEROW(i)))/3;
```

```
adERUK          =sum(i,abs(dERUK(i)))/17;
adG             =sum(ig,abs(dG(ig)))/card(ig);
adID            =sum(ii,abs(dID(ii)))/card(ii);
adMROW          =sum(i,abs(dMROW(i)))/3;
adMRUK          =sum(i,abs(dMRUK(i)))/18;
adINT           =sum((i,j),abs(dINT(i,j)))/363;
adPWD           =sum(i,abs(dPWD(i)))/card(i);
adPQ            =sum(i,abs(dPQ(i)))/card(i);
adPQW           =sum(i,abs(dPQW(i)))/card(i);
adPVA           =sum(i,abs(dPVA(i)))/card(i);
adPW            =sum(i,abs(dPW(i)))/card(i);
adPY            =sum(i,abs(dPY(i)))/card(i);
adQ             =sum(i,abs(dQ(i)))/card(i);
adQW            =sum(i,abs(dQW(i)))/card(i);
adVA            =sum(i,abs(dVA(i)))/card(i);
adW             =sum(i,abs(dW(i)))/card(i);
adY             =sum(i,abs(dY(i)))/card(i);
adK             =sum(i,abs(dK(i)))/20;
adT             =sum(i,abs(dT(i)))/20;
adL             =sum(i,abs(dL(i)))/20;
ad
=(adC+adWD+adEROW+adERUK+adG+adID+adMROW+adINT+adMRUK+adPWD

+adPQ+adPQW+adPVA+adPW+adPY+adQ+adQW+adVA+adW+adY+abs(dCPI)+abs(dCP
I_L)
    +abs(dCPI_P)+abs(dPI)+abs(dS)+abs(dSH)+abs(dTC)+abs(dTCCL)+abs(dTK)

+abs(dTLE)+abs(dTLR)+abs(dTP)+abs(dTRFG)+abs(dTSD_R_NS)+abs(dTSD_NR_NS)

+abs(dTSD_R_RS)+abs(dTSD_NR_RS)+abs(dTY)+abs(dYG)+abs(dYG_DT)+abs(dYG_F)
    +abs(dYG_NDT)+abs(dYH)+abs(dYK)+abs(dYL)+abs(dYT)+adK+adT+adL+abs(dPK)
    +abs(dPT_NR)+abs(dPL))/52;

Display
```

```
dC,dCPI,dCPI_L,dCPI_P,dWD,dEROW,dERUK,dG,dID,dMROW,dINT,dMRUK,dPWD,dP
I,dPQ,dPQW,
dPVA,dPW,dPY,dQ,dQW,dS,dSH,dTC,dTCCL,dTK,dTLE,dTLR,dTP,dTRFG,dTSD_R_NS
,dTSD_NR_NS,
dTSD_R_RS,dTSD_NR_RS,dTY,dVA,dW,dY,dYG,dYG_DT,dYG_F,dYG_NDT,dYH,dYK
,dYL,dYT,
dK,dT,dL,dPK,dPT_NR,dPL,ad
;

*    Show Euclidean distance against benchmark----------
Parameter
edC,edCPI,edCPI_L,edCPI_P,edWD,edEROW,edERUK,edG,edID,edMROW,edINT,edMR
UK,edPWD,
edPI,edPQ,edPQW,edPVA,edPW,edPY,edQ,edQW,edS,edSH,edTC,edTCCL,edTK,edTLE,e
dTLR,
edTP,edTRFG,edTSD_R_NS,edTSD_NR_NS,edTSD_R_RS,edTSD_NR_RS,edTY,edVA,e
dW,edY,edYG,
edYG_DT,edYG_F,edYG_NDT,edYH,edYK,edYL,edYT,edK,edT,edL,edPK,edPT_NR,edP
L
;

edC          =sqrt(sum(ic,sqr(C.l(ic)      -C0(ic))));
edCPI        =abs(CPI.l       -1);
edCPI_L      =abs(CPI_L.l     -1);
edCPI_P      =abs(CPI_P.l     -1);
edWD         =sqrt(sum(j,sqr(WD.l(j)      -WD0(j))));
edEROW       =sqrt(sum(i$(EROW0(i) ne 0),sqr(EROW.l(i)    -EROW0(i))));
edERUK       =sqrt(sum(i$(ERUK0(i) ne 0),sqr(ERUK.l(i)    -ERUK0(i))));
edG          =sqrt(sum(ig,sqr(G.l(ig)      -G0(ig))));
edID         =sqrt(sum(ii,sqr(ID.l(ii)      -ID0(ii))));
edINT        =sqrt(sum((i,j)$(INT0(i,j) ne 0),sqr(INT.l(i,j)   -INT0(i,j))));
edMROW       =sqrt(sum(j$(MROW0(j) ne 0),sqr(MROW.l(j)    -MROW0(j))));
edMRUK       =sqrt(sum(j$(MRUK0(j) ne 0),sqr(MRUK.l(j)    -MRUK0(j))));
edPWD        =sqrt(sum(i,sqr(PWD.l(i)     -1)));
```

```
edPI            =abs(PI.l        -1);
edPQ            =sqrt(sum(i,sqr(PQ.l(i)      -1)));
edPQW           =sqrt(sum(i,sqr(PQW.l(i)     -1)));
edPVA           =sqrt(sum(i,sqr(PVA.l(i)     -1)));
edPW            =sqrt(sum(i,sqr(PW.l(i)      -1)));
edPY            =sqrt(sum(i,sqr(PY.l(i)      -1)));
edQ             =sqrt(sum(j,sqr(Q.l(j)       -Q0(j))));
edQW            =sqrt(sum(j,sqr(QW.l(j)      -QW0(j))));
edS             =abs(S.l         -S0);
edSH            =abs(SH.l        -SH0);
edTC            =abs(TC.l        -TC0);
edTCCL          =abs(TCCL.l      -TCCL0);
edTK            =abs(TK.l        -TK0);
edTLE           =abs(TLE.l       -TLE0);
edTLR           =abs(TLR.l       -TLR0);
edTP            =abs(TP.l        -TP0);
edTRFG          =abs(TRFG.l      -TRFG0);
edTSD_R_NS      =abs(TSD_R_NS.l   -TSD_R_NS0);
edTSD_NR_NS     =abs(TSD_NR_NS.l  -TSD_NR_NS0);
edTSD_R_RS      =abs(TSD_R_RS.l   -TSD_R_RS0);
edTSD_NR_RS     =abs(TSD_NR_RS.l  -TSD_NR_RS0);
edTY            =abs(TY.l        -TY0);
edVA            =sqrt(sum(j,sqr(VA.l(j)      -VA0(j))));
edW             =sqrt(sum(j,sqr(W.l(j)       -W0(j))));
edY             =sqrt(sum(j,sqr(Y.l(j)       -Y0(j))));
edYG            =abs(YG.l        -YG0);
edYG_DT         =abs(YG_DT.l     -YG_DT0);
edYG_F          =abs(YG_F.l      -YG_F0);
edYG_NDT        =abs(YG_NDT.l    -YG_NDT0);
edYH            =abs(YH.l        -YH0);
edYK            =abs(YK.l        -YK0);
edYL            =abs(YL.l        -YL0);
edYT            =abs(YT.l        -YT0);
edK             =sqrt(sum(j$(K0(j) ne 0),sqr(K.l(j)       -K0(j))));
```

```
edT             =sqrt(sum(j$(T0(j) ne 0),sqr(T.l(j)      -T0(j))));
edL             =sqrt(sum(j$(L0(j) ne 0),sqr(L.l(j)      -L0(j))));
edPK            =abs(PK.l        -1);
edPT_NR         =abs(PT_NR.l     -1);
edPL            =abs(PL.l        -1);

Display
edC,edCPI,edCPI_L,edCPI_P,edWD,edEROW,edERUK,edG,edID,edINT,edMROW,edMR
UK,edPWD,
edPI,edPQ,edPQW,edPVA,edPW,edPY,edQ,edQW,edS,edSH,edTC,edTCCL,edTK,edTLE,e
dTLR,
edTP,edTRFG,edTSD_R_NS,edTSD_NR_NS,edTSD_R_RS,edTSD_NR_RS,edTY,edVA,e
dW,edY,edYG,
edYG_DT,edYG_F,edYG_NDT,edYH,edYK,edYL,edYT,edK,edT,edL,edPK,edPT_NR,edP
L
;

*  Record the replication solutions---------------------
Parameter
C1(i),CPI1,CPI_L1,CPI_P1,WD1(j),EROW1(i),ERUK1(i),G1(i),ID1(i),INT1(i,j),MROW1(i),
MRUK1(i),PWD1(i),PI1,PQ1(i),PQW1(i),PVA1(i),PW1(i),PY1(i),Q1(j),QW1(j),S1,SH1,TC
1,
TCCL1,TK1,TLE1,TLR1,TP1,TRFG1,TSD_R_NS1,TSD_NR_NS1,TSD_R_RS1,TSD_NR_
RS1,TY1,VA1(j),
W1(j),Y1(j),YG1,YG_DT1,YG_F1,YG_NDT1,YH1,YK1,YL1,YT1,K1(j),T1(j),L1(j),PK1,P
T_NR1,
PL1,CINT1(j);

C1(i)           =C.l(i);
CPI1            =CPI.l;
CPI_L1          =CPI_L.l;
CPI_P1          =CPI_P.l;
WD1(j)          =WD.l(j);
EROW1(i)        =EROW.l(i);
```

```
ERUK1(i)        =ERUK.l(i);
G1(i)           =G.l(i);
ID1(i)          =ID.l(i);
INT1(i,j)       =INT.l(i,j);
CINT1(j)        =CINT.l(j);
MROW1(j)        =MROW.l(j);
MRUK1(j)        =MRUK.l(j);
PWD1(i)         =PWD.l(i) ;
PI1             =PI.l     ;
PQ1(i)          =PQ.l(i)   ;
PQW1(i)         =PQW.l(i)   ;
PVA1(i)         =PVA.l(i)    ;
PW1(i)          =PW.l(i)      ;
PY1(i)          =PY.l(i)      ;
Q1(j)           =Q.l(j)       ;
QW1(j)          =QW.l(j)      ;
S1              =S.l          ;
SH1             =SH.l         ;
TC1             =TC.l         ;
TCCL1           =TCCL.l       ;
TK1             =TK.l         ;
TLE1            =TLE.l        ;
TLR1            =TLR.l        ;
TP1             =TP.l         ;
TRFG1           =TRFG.l       ;
TSD_R_NS1       =TSD_R_NS.l   ;
TSD_NR_NS1      =TSD_NR_NS.l  ;
TSD_R_RS1       =TSD_R_RS.l   ;
TSD_NR_RS1      =TSD_NR_RS.l  ;
TY1             =TY.l         ;
VA1(j)          =VA.l(j)      ;
W1(j)           =W.l(j)       ;
Y1(j)           =Y.l(j)       ;
YG1             =YG.l         ;
```

```
YG_DT1          =YG_DT.l     ;
YG_F1           =YG_F.l      ;
YG_NDT1         =YG_NDT.l    ;
YH1             =YH.l        ;
YK1             =YK.l        ;
YL1             =YL.l        ;
YT1             =YT.l        ;
K1(j)           =K.l(j)    ;
T1(j)           =T.l(j)    ;
L1(j)           =L.l(j)    ;
PK1             =PK.l     ;
PT_NR1          =PT_NR.l  ;
PL1             =PL.l     ;
*===========================================================================
==============

*Simulation and results------------------------------------------------------

*Variation of residential SDLT rate
*tausd_r_fnl('r_ns')     =1.1*TSD_R_NS0/(Y0('r_ns')+SAM0('t_pn','r_ns'));
*tausd_rr_fnl('r_r')     =1.1*TSD_R_RS0/Y0('r_r');
*tausd_r_fnl('r_ns')     =0.9*TSD_R_NS0/(Y0('r_ns')+SAM0('t_pn','r_ns'));
*tausd_rr_fnl('r_r')     =0.9*TSD_R_RS0/Y0('r_r');

*Variation of non-residential SDLT rate
*tausd_nr_fnl('nr_ns')    =1.1*TSD_NR_NS0/(Y0('nr_ns')+SAM0('t_pn','nr_ns'));
*tausd_nr_int(inr)        =1.1*TSD_NR_RS0/(SAM0('lnd','total')-SAM0('lnd','r_r'));
*tausd_nr_fnl('nr_ns')    =0.9*TSD_NR_NS0/(Y0('nr_ns')+SAM0('t_pn','nr_ns'));
*tausd_nr_int(inr)        =0.9*TSD_NR_RS0/(SAM0('lnd','total')-SAM0('lnd','r_r'));

*Variation of corporation tax
*tauk                    =1.05*TK0/(TK0+YK0);
tauk                    =0.95*TK0/(TK0+YK0);
```

```
*Variation of income tax
*tauy                    =1.05*TY0/YH0;
tauy                    =0.95*TY0/YH0;

option bratio=1;

Equation
eqcpi_l_s              CPI calculation function using Laspeyres index in simulation
eqcpi_p_s              CPI calculation function using Paasche index in simulation
;
eqcpi_l_s..  CPI_L=e=sum(inre,PQ(inre)*C1(inre))/sum(inre,PQ1(inre)*C1(inre));
eqcpi_p_s..  CPI_P=e=sum(inre,PQ(inre)*C(inre))/sum(inre,PQ1(inre)*C(inre));

Model WAGE_simulation
/eqva,eqt_nr,eqkstock,eqtstock,eqlstock,eqk,eql,eqyk,eqyt,eqyl,eqint,eqy,eqpva,
eqyh,eqsh,eqc,eqty,eqtk,eqtlr,eqtle,eqtccl,eqtp,eqtc,eqtsd_r_ns,eqtsd_nr_ns,
eqtsd_r_rs,eqtsd_nr_rs,eqyg_dt,eqyg_ndt,eqyg_f,eqtrfg,eqyg,eqg,eqpyd,eqwds,
eqerows,eqpwdd,eqws,eqeruks,eqpqws,eqmrukd,eqwd,eqpqs,eqmrowd,eqqwd,eqs,eqid,
eqcpi_l_s,eqcpi_p_s,eqcpi,eqpi,equ,eqq,eqcint/
;

Solve WAGE_simulation maximizing U using nlp;

*  Show counter-factual solutions----------------------
Display
C.l,CPI.l,CPI_L.l,CPI_P.l,WD.l,EROW.l,ERUK.l,G.l,ID.l,INT.l,MROW.l,MRUK.l,PWD.l,
PI.l,PQ.l,PQW.l,PVA.l,PW.l,PY.l,Q.l,QW.l,S.l,SH.l,TC.l,TCCL.l,TK.l,TLE.l,TLR.l,
TP.l,TRFG.l,TSD_R_NS.l,TSD_NR_NS.l,TSD_R_RS.l,TSD_NR_RS.l,TY.l,U.l,VA.l,W.l,Y.
l,
YG.l,YG_DT.l,YG_F.l,YG_NDT.l,YH.l,YK.l,YL.l,YT.l,K.l,T.l,L.l,PK.l,PT_NR.l,PL.l,
CINT.l
;

*  Show counter-factual changes against replication of benchmark---
```

```
Parameter
dC(ic),dCPI,dCPI_L,dCPI_P,dWD(j),dEROW(i),dERUK(i),dG(ig),dID(ii),dINT(i,j),
dMROW(i),dMRUK(i),dPWD(i),dPI,dPQ(i),dPQW(i),dPVA(i),dPW(i),dPY(i),dQ(j),dQW(j),
dS,dSH,dTC,dTCCL,dTK,dTLE,dTLR,dTP,dTRFG,dTSD_R_NS,dTSD_NR_NS,dTSD_R_
RS,
dTSD_NR_RS,dTSD_R,dTSD_NR,dTY,dVA(j),dW(j),dY(j),dYG,dYG_DT,dYG_F,dYG_N
DT,dYH,
dYH_D,dYF_NR,dYK,dYK_D,dYL,dYT,dVY(j),dVTY,dVQ(j),dVTQ,dVC(ic),dVTC,dVG(
i),dVTG,
dVID(i),dVTERUK,dVTEROW,dVTMRUK,dVTMROW,dVTWD,dVTW,dVTQW,dVVA(i
),dVGVA,dVINT(i,j),
dVTINT,dVRINT(i),dVCINT(j),dVERUK,dVEROW,dVMRUK,dVMROW,dVW,dGDP_B,
dGDP_C,dVY_R_NS,
dPY_R_NS,dY_R_NS,dVY_NR_NS,dPY_NR_NS,dY_NR_NS,dVY_R_R,dPY_R_R,dY_R
_R,dYT_NR,dK(j),
dT(j),dL(j),dPK,dPT_NR,dPL,dCINT(j)
;

dC(ic)          =(C.l(ic)       /C1(ic)-1)*100;
dCPI            =(CPI.l         /CPI1-1)*100;
dCPI_L          =(CPI_L.l       /CPI_L1-1)*100;
dCPI_P          =(CPI_P.l       /CPI_P1-1)*100;
dWD(j)          =(WD.l(j)       /WD1(j)-1)*100;
dEROW(i)$(EROW0(i) ne 0)        =(EROW.l(i)     /EROW1(i)-1)*100;
dERUK(i)$(ERUK0(i) ne 0)        =(ERUK.l(i)     /ERUK1(i)-1)*100;
dG(ig)          =(G.l(ig)       /G1(ig)-1)*100;
dID(ii)         =(ID.l(ii)      /ID1(ii)-1)*100;
dINT(i,j)$(INT0(i,j) ne 0)      =(INT.l(i,j)    /INT1(i,j)-1)*100;
dCINT(j)                        =(CINT.l(j)     /CINT1(j)-1)*100;
dMROW(j)$(MROW0(j) ne 0)        =(MROW.l(j)     /MROW1(j)-1)*100;
dMRUK(j)$(MRUK0(j) ne 0)        =(MRUK.l(j)     /MRUK1(j)-1)*100;
dPWD(i)         =(PWD.l(i)      /PWD1(i)-1)*100;
dPI             =(PI.l          /PI1-1)*100;
dPQ(i)          =(PQ.l(i)       /PQ1(i)-1)*100;
```

```
dPQW(i)         =(PQW.l(i)     /PQW1(i)-1)*100;
dPVA(i)         =(PVA.l(i)     /PVA1(i)-1)*100;
dPW(i)          =(PW.l(i)      /PW1(i)-1)*100;
dPY(i)          =(PY.l(i)      /PY1(i)-1)*100;
dQ(j)           =(Q.l(j)       /Q1(j)-1)*100;
dQW(j)          =(QW.l(j)      /QW1(j)-1)*100;
dS              =(S.l          /S1-1)*100;
dSH             =(SH.l         /SH1-1)*100;
dTC             =(TC.l         /TC1-1)*100;
dTCCL           =(TCCL.l       /TCCL1-1)*100;
dTK             =(TK.l         /TK1-1)*100;
dTLE            =(TLE.l        /TLE1-1)*100;
dTLR            =(TLR.l        /TLR1-1)*100;
dTP             =(TP.l         /TP1-1)*100;
dTRFG           =(TRFG.l       /TRFG1-1)*100;
dTSD_R_NS       =(TSD_R_NS.l   /TSD_R_NS1-1)*100;
dTSD_NR_NS      =(TSD_NR_NS.l  /TSD_NR_NS1-1)*100;
dTSD_R_RS       =(TSD_R_RS.l   /TSD_R_RS1-1)*100;
dTSD_NR_RS      =(TSD_NR_RS.l  /TSD_NR_RS1-1)*100;
dTY             =(TY.l         /TY1-1)*100;
dVA(j)          =(VA.l(j)      /VA1(j)-1)*100;
dW(j)           =(W.l(j)       /W1(j)-1)*100;
dY(j)           =(Y.l(j)       /Y1(j)-1)*100;
dYG             =(YG.l         /YG1-1)*100;
dYG_DT          =(YG_DT.l      /YG_DT1-1)*100;
dYG_F           =(YG_F.l       /YG_F1-1)*100;
dYG_NDT         =(YG_NDT.l     /YG_NDT1-1)*100;
dYH             =(YH.l         /YH1-1)*100;
dYH_D           =((YH.l-TY.l-TLE.l-TCCL.l)/(YH1-TY1-TLE1-TCCL1)-1)*100;
dYF_NR
=((YK.l+YL.l+sum(jnr,PT_NR.l*T.l(jnr)))/(YK1+YL1+sum(jnr,PT_NR1*T1(jnr)))-1)*100;
dYK             =(YK.l         /YK1-1)*100;
dYK_D           =((YK.l-TK.l)  /(YK1-TK1)-1)*100;
dYL             =(YL.l         /YL1-1)*100;
```

```
dYT          =(YT.l       /YT1-1)*100;
dVY(j)       =(PY.l(j)*Y.l(j) /(PY1(j)*Y1(j))-1)*100;
dVTY         =(sum(j,PY.l(j)*Y.l(j)) /sum(j,PY1(j)*Y1(j))-1)*100;
dVQ(j)       =(PQ.l(j)*Q.l(j) /(PQ1(j)*Q1(j))-1)*100;
dVTQ         =(sum(j,PQ.l(j)*Q.l(j)) /sum(j,PQ1(j)*Q1(j))-1)*100;
dVC(ic)      =(PQ.l(ic)*C.l(ic) /(PQ1(ic)*C1(ic))-1)*100;
dVTC         =(sum(ic,PQ.l(ic)*C.l(ic)) /sum(ic,PQ1(ic)*C1(ic))-1)*100;
dVG(i)       =(PQ.l(i)*G.l(i) /(PQ1(i)*G1(i))-1)*100;
dVTG         =(sum(i,PQ.l(i)*G.l(i)) /sum(i,PQ1(i)*G1(i))-1)*100;
dVID(i)      =(PQ.l(i)*ID.l(i)/(PQ1(i)*ID1(i))-1)*100;
dTSD_R       =((TSD_R_NS.l+TSD_R_RS.l)/(TSD_R_NS1+TSD_R_RS1)-1)*100;
dTSD_NR      =((TSD_NR_NS.l+TSD_NR_RS.l)/(TSD_NR_NS1+TSD_NR_RS1)-1)*100;
dVVA(i)      =(PVA.l(i)*VA.l(i) /(PVA1(i)*VA1(i))-1)*100;
dVGVA        =(sum(i,PVA.l(i)*VA.l(i)) /sum(i,PVA1(i)*VA1(i))-1)*100;
dVINT(i,j)   =(PQ.l(i)*INT.l(i,j) /(PQ1(i)*INT1(i,j))-1)*100;
dVTINT       =(sum((i,j),PQ.l(i)*INT.l(i,j)) /sum((i,j),PQ1(i)*INT1(i,j))-1)*100;
dVRINT(i)    =(sum(j,PQ.l(j)*INT.l(i,j)) /sum(j,PQ1(j)*INT1(i,j))-1)*100;
dVCINT(j)    =(sum(i,PQ.l(j)*INT.l(i,j)) /sum(i,PQ1(j)*INT1(i,j))-1)*100;
dVERUK       =(sum(i,ERUK.l(i)) /sum(i,ERUK1(i))-1)*100;
dVEROW       =(sum(i,EROW.l(i)) /sum(i,EROW1(i))-1)*100;
dVMRUK       =(sum(i,MRUK.l(i)) /sum(i,MRUK1(i))-1)*100;
dVMROW       =(sum(i,MROW.l(i)) /sum(i,MROW1(i))-1)*100;
dVW          =(sum(i,PW.l(i)*W.l(i)) /sum(i,PW1(i)*W1(i))-1)*100;
dGDP_B       =((sum(i,PVA.l(i)*VA.l(i))+TP.l) /(sum(i,PVA1(i)*VA1(i))+TP1)-1)*100;
dGDP_C       =((sum(i,PVA.l(i)*VA.l(i))+TP.l+TC.l) /(sum(i,PVA1(i)*VA1(i))+TP1+TC1)-
1)*100;
dVY_R_NS     =(PY.l('r_ns')*Y.l('r_ns') /(PY1('r_ns')*Y1('r_ns'))-1)*100;
dPY_R_NS     =(PY.l('r_ns') /(PY1('r_ns'))-1)*100;
dY_R_NS      =(Y.l('r_ns') /Y1('r_ns')-1)*100;
dVY_NR_NS    =(PY.l('nr_ns')*Y.l('nr_ns') /(PY1('nr_ns')*Y1('nr_ns'))-1)*100;
dPY_NR_NS    =(PY.l('nr_ns') /(PY1('nr_ns'))-1)*100;
dY_NR_NS     =(Y.l('nr_ns') /Y1('nr_ns')-1)*100;
dPY_R_R      =(PY.l('r_r') /(PY1('r_r'))-1)*100;
dVY_R_R      =(PY.l('r_r')*Y.l('r_r') /(PY1('r_r')*Y1('r_r'))-1)*100;
```

```
dY_R_R          =(Y.l('r_r') /(Y1('r_r'))-1)*100;
dYT_NR          =(sum(jnr,PT_NR.l*T.l(jnr))/sum(jnr,PT_NR1*T1(jnr))-1)*100;
dK(j)$(K0(j) ne 0)          =(K.l(j)      /K1(j)-1)*100;
dT(j)$(T0(j) ne 0)          =(T.l(j)      /T1(j)-1)*100;
dL(j)$(L0(j) ne 0)          =(L.l(j)      /L1(j)-1)*100;
dPK             =(PK.l        /PK1-1)*100;
dPT_NR          =(PT_NR.l     /PT_NR1-1)*100;
dPL             =(PL.l        /PL1-1)*100;

Display
dC,dCPI,dCPI_L,dCPI_P,dWD,dEROW,dERUK,dG,dID,dMROW,dINT,dMRUK,dPWD,dP
I,dPQ,dPQW,
dPVA,dPW,dPY,dY,dQ,dQW,dS,dSH,dTC,dTCCL,dTK,dTLE,dTLR,dTP,dTRFG,dTSD_R
_NS,
dTSD_NR_NS,dTSD_R_RS,dTSD_NR_RS,dTSD_R,dTSD_NR,dTY,dVA,dW,dYG,dYG_
DT,dYG_F,dYG_NDT,
dYH,dYH_D,dYF_NR,dYK,dYK_D,dYL,dYT,dVY,dVTY,dVQ,dVTQ,dVC,dVTC,dVG,d
VTG,dVID,dVVA,
dVGVA,dVINT,dVTINT,dVRINT,dVCINT,dVERUK,dVEROW,dVMRUK,dVMROW,dV
W,dGDP_B,dGDP_C,
dVY_R_NS,dPY_R_NS,dY_R_NS,dVY_NR_NS,dPY_NR_NS,dY_NR_NS,dVY_R_R,dPY
_R_R,dY_R_R,
dYT_NR,dK,dT,dL,dPK,dPT_NR,dPL,dCINT
;
*================================================================
==============

*Welfare measure: Hicksian equivalent variations--------------------------------
Parameter
U0            utility level in the Base Run Eq.
ep0           expenditure func. in the Base Run Eq.
ep1           expenditure func. in the Consumption function Eq.
EV            Hicksian equivalent variations
;
```

```
U0      =prod(ic, C0(ic)**ac(ic));
ep0     =U0 /prod(ic, (ac(ic)/1)**ac(ic));
ep1     =U.l/prod(ic, (ac(ic)/1)**ac(ic));
EV      =ep1-ep0;

Display ep0,ep1,EV;
*=============================================================================

*Simulation results summary & presentation-------------------------------------
Parameter
WAGE_m(d)     simulation results summary table
;

WAGE_m('d_YG')            =dYG;
WAGE_m('d_YG_DT')         =dYG_DT;
WAGE_m('d_TSD_R')         =dTSD_R;
WAGE_m('d_TSD_NR')        =dTSD_NR;
WAGE_m('d_YG_NDT')        =dYG_NDT;
WAGE_m('d_TP')            =dTP;
WAGE_m('d_TC')            =dTC;
WAGE_m('d_TY')            =dTY;
WAGE_m('d_TK')            =dTK;
WAGE_m('d_TLR')           =dTLR;
WAGE_m('d_TLE')           =dTLE;
WAGE_m('d_YG_F')          =dYG_F;
WAGE_m('d_VTG')           =dVTG ;
WAGE_m('d_VTINT')         =dVTINT;
WAGE_m('d_VGVA')          =dVGVA ;
WAGE_m('d_VTY')           =dVTY  ;
WAGE_m('d_VTQ')           =dVTQ  ;
WAGE_m('d_GDP_B')         =dGDP_B;
WAGE_m('d_GDP_C')         =dGDP_C;
WAGE_m('d_YH')            =dYH   ;
```

```
WAGE_m('d_YH_D')         =dYH_D ;
WAGE_m('d_VTC')          =dVTC  ;
WAGE_m('d_S')            =dS    ;
WAGE_m('d_PI')           =dPI   ;
WAGE_m('d_YF_NR')        =dYF_NR;
WAGE_m('d_YT_NR')        =dYT_NR;
*WAGE_m('d_PT_NR')       =dPT_NR;
*WAGE_m('d_T_stock')     =0.0000000001;
WAGE_m('d_YK')           =dYK     ;
*WAGE_m('d_PK')          =dPK     ;
*WAGE_m('d_K_stock')     =0.0000000001;
WAGE_m('d_YL')           =dYL     ;
*WAGE_m('d_PL')          =dPL     ;
*WAGE_m('d_L_stock')     =0.0000000001;
WAGE_m('d_VY_R_NS')      =dVY_R_NS;
WAGE_m('d_PY_R_NS')      =dPY_R_NS;
WAGE_m('d_Y_R_NS')       =dY_R_NS ;
WAGE_m('d_VY_NR_NS')     =dVY_NR_NS;
WAGE_m('d_PY_NR_NS')     =dPY_NR_NS;
WAGE_m('d_Y_NR_NS')      =dY_NR_NS ;
WAGE_m('d_VY_R_R')       =dVY_R_R  ;
WAGE_m('d_PY_R_R')       =dPY_R_R  ;
WAGE_m('d_Y_R_R')        =dY_R_R   ;
WAGE_m('d_VERUK')        =dVERUK   ;
WAGE_m('d_VEROW')        =dVEROW   ;
WAGE_m('d_VMRUK')        =dVMRUK   ;
WAGE_m('d_VMROW')        =dVMROW   ;
*WAGE_m('d_VW')          =dVW      ;
*WAGE_m('d_EV')          =EV       ;
*WAGE_m('d_CPI')         =dCPI     ;

execute_unload 'WAGE_m.gdx',WAGE_m;
execute '=gdxviewer WAGE_m.gdx';
*=====================================================================
```

VI-3 Simulation in the long run

```
$Title  A CGE model for Wales - Long run

$eolcom //
*Definition of sets-----------------------------------------------------------
Set a    all accounts
        /agr,mnq,man,egy,con,r_ns,nr_ns,wnr,trp,acm,ict,fin,r_r,rtt,prf,adm,plc,
         edu,hth,ent,oth,
         nic1,lab,lnd,ova,t_pn,sdlt_nr_int,sdlt_r_fnl,sdlt_nr_fnl,t_pt,t_inc,nic2,t_cncl,t_cp,
         hh,gov,sav,ruk,row,total/
    aa   all accounts excluding the totals
        /agr,mnq,man,egy,con,r_ns,nr_ns,wnr,trp,acm,ict,fin,r_r,rtt,prf,adm,plc,
         edu,hth,ent,oth,
         nic1,lab,lnd,ova,t_pn,sdlt_nr_int,sdlt_r_fnl,sdlt_nr_fnl,t_pt,t_inc,nic2,t_cncl,t_cp,
         hh,gov,sav,ruk,row/
    i(a) all industrial sectors
        /agr,mnq,man,egy,con,r_ns,nr_ns,wnr,trp,acm,ict,fin,r_r,rtt,prf,adm,plc,
         edu,hth,ent,oth/
    inre(i) all sectors excluding the three real estate sale and rental sectors
        /agr,mnq,man,egy,con,wnr,trp,acm,ict,fin,rtt,prf,adm,plc,edu,hth,ent,oth/
    inr(i) all sectors excluding residential rental sector
        /agr,mnq,man,egy,con,r_ns,nr_ns,wnr,trp,acm,ict,fin,rtt,prf,adm,plc,edu,hth,ent,oth/
    ic(i) all sectors that produce goods for households¡¯consumption
        /egy,con,wnr,trp,acm,ict,fin,r_r,rtt,prf,adm,edu,hth,ent/
    ig(i) all sectors that produce goods for government consumption
        /plc,edu,hth,ent/
    ii(i) all sectors that produce goods for investment
        /man,con,r_ns,nr_ns,prf/
    d    variables presented in the simulation results summary table
        /d_YG,d_YG_DT,d_TSD_R,d_TSD_NR,d_YG_NDT,d_TP,d_TC,d_TY,d_TK,

d_TLR,d_TLE,d_YG_F,d_VTG,d_VTINT,d_VGVA,d_VTY,d_VTQ,d_GDP_B,d_GDP_C,
```

```
d_YH,d_YH_D,d_VTC,d_S,d_TID,d_PI,d_YF_NR,d_YT_NR,d_PT_NR,d_T_stock,d_YK,

d_YK_D,d_PK,d_K_stock,d_YL,d_PL,d_L_stock,d_VY_R_NS,d_PY_R_NS,d_Y_R_NS,

d_VY_NR_NS,d_PY_NR_NS,d_Y_NR_NS,d_VY_R_R,d_PY_R_R,d_Y_R_R,d_VERUK,

d_VEROW,d_VMRUK,d_VMROW,d_VW,d_EV,d_CPI,d_GDP_B_pc,d_GDP_C_pc,d_YH
_pc,d_YH_D_pc/
;
Alias (a,b),(aa,bb),(i,j),(inre,jnre),(inr,jnr);
*=================================================================
==============

*Loading SAM as the benchmark database-----------------------------------------
Table SAM0(a,b)
$ondelim
$include 2c_SAM_b_tkty_non0.csv
$offdelim
;
Display SAM0;
*=================================================================
==============

*Loading the base values for all the variables and calibration of parameters----
*  For endogenous variables (with suffix 0):
Parameter
C0(i)                 households' consumption demand by commodity in benchmark
WD0(j)                regional production supplied to domestic market in benchmark
EROW0(i)              export supply to the ROW by sector in benchmark
ERUK0(i)              export supply to the RUK by sector in benchmark
G0(i)                 fiscal expenditure demand by commodity in benchmark
ID0(i)                investment demand by commodity in benchmark
INT0(i,j)             intermediate inputs for regional productionin in benchmark
CINT0(j)              composite intermediate inputs of each sector in benchmark
```

MROW0(j) import demand from the ROW by commodity in benchmark

MRUK0(j) import demand from the RUK by commodity in benchmark

Q0(j) regional sales of composites combining regional production and all imports by commodity in benchmark

QW0(j) regional sales of domestic composites combining regional production and imports from RUK by commodity in benchmark

S0 total savings in benchmark

SH0 households¡¯savings in benchmark

SRUK0 inter-regional savings from RUK in benchmark

SROW0 foreign savings from ROW in benchmark

TC0 product tax revenue in benchmark

TCCL0 council tax revenue in benchmark

TK0 corporation tax revenue in benchmark

TLE0 NIC revenue payable by the employee in benchmark

TLR0 NIC revenue payable by the employer in benchmark

TP0 production tax revenue in benchmark

TRFG0 fiscal transfer received by regional government from central government in benchmark

TSD_R_NS0 Stamp Duty Land Tax revenue from residential properties¡¯ new sale in benchmark

TSD_NR_NS0 Stamp Duty Land Tax revenue from non-residential properties¡¯ new sale in benchmark

TSD_R_RS0 Stamp Duty Land Tax revenue from residential properties¡¯ resale in benchmark

TSD_NR_RS0 Stamp Duty Land Tax revenue from non-residential properties¡¯ resale in benchmark

TY0 income tax revenue in benchmark

VA0(j) value-added bundle of factors by sector in benchmark

W0(j) regional production supplied to regional market in benchmark

Y0(j) output of regional production by sector in benchmark

YG0 total fiscal revenue in benchmark

YG_DT0 devolved tax revenue in benchmark

YG_F0 factor income of regional government in benchmark

YG_NDT0 non-devolved tax revenue in benchmark

YH0 households' income in benchmark
YK0 factor income of capital in benchmark
YL0 factor income of labour in benchmark
YT0 factor income of land in benchmark
K0(j) capital demand by sector
L0(j) labour demand by sector
T0(j) land demand by sector
* For exogenous variables (first uppercase letter followed by lowercase letters):
T_stock total land stock
Perow(i) price of export supply to the ROW by sector in domestic currency
Peruk(i) price of export supply to the RUK by sector in domestic currency
Pmrow(i) price of import demand from the ROW by sector in domestic currency
Pmruk(i) price of import demand from the RUK by sector in domestic currency
Q_nr_rs non-residential properties resale volume
Q_r_rs residential properties resale volume
Q_r_stock residential properties regional stock
Trfh social protection transfer received by households from regional government
Sruk extra-regional saving from RUK
Srow extra-regional saving from ROW
Ks(j) capital demand by sector
Ls(j) labour demand by sector
Ts(j) land demand by sector
Pk(i) capital return by sector
Pl(i) labour wage by sector
;

C0(i) =SAM0(i,'hh');
EROW0(i) =SAM0(i,'row');
ERUK0(i) =SAM0(i,'ruk');
G0(i) =SAM0(i,'gov');
ID0(i) =SAM0(i,'sav');
INT0(i,j) =SAM0(i,j);
CINT0(j) =sum(i,SAM0(i,j));
K0(j) =SAM0('ova',j);

```
L0(j)                  =SAM0('lab',j);
MROW0(j)               =SAM0('row',j);
MRUK0(j)               =SAM0('ruk',j);
S0                     =SAM0('sav','total');
SH0                    =SAM0('sav','hh');
T0(j)                  =SAM0('lnd',j);
TC0                    =SAM0('gov','t_pt');
TCCL0                  =SAM0('gov','t_cncl');
TK0                    =SAM0('gov','t_cp');
TLE0                   =SAM0('gov','nic2');
TLR0                   =SAM0('gov','nic1');
TP0                    =SAM0('gov','t_pn');
TRFG0                  =SAM0('gov','ruk');
TSD_R_NS0              =SAM0('sdlt_r_fnl','r_ns');
TSD_NR_NS0             =SAM0('sdlt_nr_fnl','nr_ns');
TSD_R_RS0              =SAM0('sdlt_r_fnl','r_r');
TSD_NR_RS0             =SAM0('sdlt_nr_int','total');
TY0                    =SAM0('gov','t_inc');
VA0(j)                 =SAM0('lab',j)+SAM0('nic1',j)+SAM0('t_inc',j)        /////////////////////////////ty
                      +SAM0('lnd',j)+SAM0('sdlt_nr_int',j)
                      +SAM0('ova',j)+SAM0('t_cp',j);
Y0(j)                  =VA0(j)+sum(i,INT0(i,j));
WD0(j)                 =Y0(j)+SAM0('t_pn',j)+SAM0('t_pt',j)+SAM0('sdlt_r_fnl',j)
                      +SAM0('sdlt_nr_fnl',j)-EROW0(j);
W0(j)                  =WD0(j)-ERUK0(j);
QW0(j)                 =W0(j)+MRUK0(j);
Q0(j)                  =QW0(j)+MROW0(j);
YG0                    =SAM0('gov','total');
*For SDLT simulation-------------
*YG_DT0                 =TSD_R_NS0+TSD_NR_NS0+TSD_R_RS0+TSD_NR_RS0+TCCL0;
*--------------------------------
*For Corporation Tax simulation--
*YG_DT0
=TSD_R_NS0+TSD_NR_NS0+TSD_R_RS0+TSD_NR_RS0+TCCL0+TK0;
```

```
*--------------------------------
*For Income Tax simulation--
YG_DT0
=TSD_R_NS0+TSD_NR_NS0+TSD_R_RS0+TSD_NR_RS0+TCCL0+TY0;
*--------------------------------
*For Cross simulation--Income&Corporation Tax
*YG_DT0
=TSD_R_NS0+TSD_NR_NS0+TSD_R_RS0+TSD_NR_RS0+TCCL0+TK0+TY0;
*--------------------------------
YG_F0          =SAM0('gov','lnd')+SAM0('gov','ova');
YG_NDT0        =YG0-TRFG0-YG_F0-YG_DT0;
YH0            =SAM0('hh','total')+sum(j,SAM0('t_inc',j));     ///////////////////////////////ty
YK0            =SAM0('ova','total');
YL0            =SAM0('lab','total');
YT0            =SAM0('lnd','total');

T_stock        =YT0;
Perow(i)       =1;
Peruk(i)       =1;
Pmrow(i)       =1;
Pmruk(i)       =1;
Q_nr_rs        =257;
Q_r_rs         =4977.667866;
Q_r_stock      =212046.417;
Trfh           =SAM0('hh','gov');
Pk(j)          =1;
Pl(j)          =1;
Ks(j)          =SAM0('ova',j);
Ls(j)          =SAM0('lab',j);
Ts(j)          =SAM0('lnd',j);
Sruk           =SAM0('sav','ruk');
Srow           =SAM0('sav','row');

Display
```

C0,EROW0,ERUK0,G0,ID0,INT0,CINT0,K0,L0,MROW0,MRUK0,Q0,QW0,S0,SH0,T0,TC0,TCCL0,TK0,
TLE0,TLR0,TP0,TRFG0,TSD_R_NS0,TSD_NR_NS0,TSD_R_RS0,TSD_NR_RS0,TY0,VA0,W0,WD0,Y0,
YG0,YG_DT0,YG_F0,YG_NDT0,YH0,YK0,YL0,YT0,Perow,Peruk,Pmrow,Pmruk,Q_nr_rs,Q_r_rs,
Q_r_stock,Trfh,T_stock,Ks,Ls,Ts,Sruk,Srow
;

* Calibration of parameters---
Parameter
mps marginal propensity of saving of households
skg share of factor income of capital distributed to regional government
skh share of factor income of capital distributed to households
stg share of factor income of land distributed to regional government
sth share of factor income of land distributed to households
tauccl effective council tax rate
tauc(j) effective product tax rate by commodity
tauk effective corporation tax rate
taule effective NIC rate payable by the employee
taulr effective NIC rate payable by the employer
taup(j) effective production tax rate by sector
tausd_nr_int(i) effective non-residential SDLT rate applied on intermediate non-residential land input
tausd_r_fnl(i) effective residential SDLT rate applied on final sectoral production of residential new sale
tausd_rr_fnl(i) effective residential SDLT rate applied on final sectoral production of residential rental
tausd_nr_fnl(i) effective non-residential SDLT rate applied on final sectoral production of non-residential new sale
tauy effective income tax rate
taumx(i) mixed tax rate combining net production and product tax rate & residential&non-residential SDLT final rates
ac(i) parameter with respect to households¡¯ demand for commodities

ag(i)   parameter with respect to fiscal expenditure demand for commodities

aid(i)   parameter with respect to investment demand for commodities

ak(j)   share parameter with respect to capital demand in Cobb-Douglas production function by sector

al(j)   share parameter with respect to labour demand in Cobb-Douglas production function by sector

at(j)   share parameter with respect to land demand in Cobb-Douglas production function by sector

ava(j)   Leontief parameter with respect to value-added bundle by sector

aint(i,j)   Leontief parameter with respect to composite intermediate inputs by sector

```
;
mps                  =SH0/(YH0-TY0-TLE0-TCCL0);
skg                  =SAM0('gov','ova')/YK0;
skh                  =SAM0('hh','ova')/YK0;
stg                  =SAM0('gov','lnd')/YT0;
sth                  =SAM0('hh','lnd')/YT0;
tauccl               =TCCL0/Q_r_stock;
tauc(j)              =SAM0('t_pt',j)/(Y0(j)+SAM0('t_pn',j));
tauk                 =TK0/(TK0+YK0);
tauy                 =TY0/YH0;
taule                =TLE0*(1-tauy)/YL0;            ////////////////////////////////ty
taulr                =TLR0*(1-tauy)/YL0;            ////////////////////////////////ty
taup(j)              =SAM0('t_pn',j)/Y0(j);
tausd_r_fnl('r_ns')      =TSD_R_NS0/(Y0('r_ns')+SAM0('t_pn','r_ns'));
tausd_nr_fnl('nr_ns')    =TSD_NR_NS0/(Y0('nr_ns')+SAM0('t_pn','nr_ns'));
tausd_rr_fnl('r_r')      =TSD_R_RS0/Y0('r_r');
tausd_nr_int(inr)        =TSD_NR_RS0/(SAM0('lnd','total')-SAM0('lnd','r_r'));
taumx(i)
=(1+taup(i))*(1+tauc(i))*(1+tausd_r_fnl(i))*(1+tausd_nr_fnl(i))*(1+tausd_rr_fnl(i))-1;
ac(i)                =C0(i)/(YH0-TY0-TLE0-TCCL0-SH0);
ag(i)                =G0(i)/(YG0-Trfh);
aid(i)               =ID0(i)/S0;
ak(j)                =Ks(j)/(1-tauk)/VA0(j);
al(j)                =(1+taulr)*Ls(j)/(1-tauy)/VA0(j);        //////////////////////////////ty
```

```
at(j)                  =1-ak(j)-al(j);
ava(j)                 =VA0(j)/Y0(j);
aint(i,j)              =INT0(i,j)/CINT0(j);

Parameter
sigmak(i)              elasticity of transformation in the CET function for RUK exports
sigmaw(i)              elasticity of transformation in the CET function for ROW exports
rhok(i)                parameter defined by elasticity of transformation in the CET function for
RUK exports
rhow(i)                parameter defined by elasticity of transformation in the CET function for
ROW exports
serow(i)               distribution parameter for export supply to the ROW by sector in the CET
function
seruk(i)               distribution parameter for export supply to the RUK by sector in the CET
function
thetak(i)              scaling coefficient in the CET function for RUK exports
thetaw(i)              scaling coefficient in the CET function for ROW exports
omegak(i)              elasticity of substitution in the Armington CES function for RUK imports
omegaw(i)              elasticity of substitution in the Armington CES function for ROW imports
etak(i)                parameter defined by elasticity of substitution in the Armington CES function
for RUK imports
etaw(i)                parameter defined by elasticity of substitution in the Armington CES function
for ROW imports
smrow(i)               distribution parameter for import demand from the ROW by sector in the
Armington CES function
smruk(i)               distribution parameter for import demand from the RUK by sector in the
Armington CES function
gammak(i)              scaling coefficient in the Armington CES function for RUK imports
gammaw(i)              scaling coefficient in the Armington CES function for ROW imports
;
sigmak(i)              =4;
sigmaw(i)              =1.5;
rhok(i)                =(sigmak(i)+1)/sigmak(i);    //--> sigma=1/(rho-1) --> 1-rho= -1/sigma
rhow(i)                =(sigmaw(i)+1)/sigmaw(i);
```

```
serow(i)             =EROW0(i)**(1-rhow(i))/(EROW0(i)**(1-rhow(i))+WD0(i)**(1-rhow(i)));
seruk(i)              =ERUK0(i)**(1-rhok(i))/(ERUK0(i)**(1-rhok(i))+W0(i)**(1-rhok(i)));
thetak(i)                                                  =WD0(i)/(seruk(i)*ERUK0(i)**rhok(i)+(1-
seruk(i))*W0(i)**rhok(i))**(1/rhok(i));
thetaw(i)                                                   =Y0(i)/(serow(i)*EROW0(i)**rhow(i)+(1-
serow(i))*WD0(i)**rhow(i))**(1/rhow(i));

omegak(i)              =4;
omegaw(i)               =2.5;
etak(i)                 =(omegak(i)-1)/omegak(i);    //--> omega=1/(1-eta) --> 1-eta= 1/omega
etaw(i)                  =(omegaw(i)-1)/omegaw(i);
smrow(i)             =MROW0(i)**(1-etaw(i))/(MROW0(i)**(1-etaw(i))+QW0(i)**(1-etaw(i)));
smruk(i)               =MRUK0(i)**(1-etak(i))/(MRUK0(i)**(1-etak(i))+W0(i)**(1-etak(i)));
gammak(i)                                                =QW0(i)/(smruk(i)*MRUK0(i)**etak(i)+(1-
smruk(i))*W0(i)**etak(i))**(1/etak(i));
gammaw(i)                                                 =Q0(i)/(smrow(i)*MROW0(i)**etaw(i)+(1-
smrow(i))*QW0(i)**etaw(i))**(1/etaw(i));

* Below for sensitivity analysis regarding nest1 & nest2
*nest2:
Parameter
omega(i)               elasticity of substitution in the production function for GVA
eta(i)                 parameter defined by elasticity of substitution in the production function for
GVA
sk(i)                  distribution parameter of capital demand by sector in the production function
for GVA
sl(i)                  distribution parameter of labour demand by sector in the production function
for GVA
st(i)                  distribution parameter of land demand by sector in the production function for
GVA
gamma(i)               scaling coefficient in the production function for GVA
;

omega(i)               =0.48;
```

```
eta(i)                  =(omega(i)-1)/omega(i);  //--> omega=1/(1-eta) --> 1-eta= 1/omega
sk(i)                 =(1-tauy)*Ks(i)**(1-eta(i))/((1-tauy)*(1-tauk)*(1+tausd_nr_int(i))*Ts(i)**(1-
eta(i))+(1-tauk)*(1+taulr)*Ls(i)**(1-eta(i))+(1-tauy)*Ks(i)**(1-eta(i))); /////////ty
sl(i)=(1-tauk)*(1+taulr)*Ls(i)**(1-eta(i))/((1-tauy)*(1-tauk)*(1+tausd_nr_int(i))*Ts(i)**(1-
eta(i))+(1-tauk)*(1+taulr)*Ls(i)**(1-eta(i))+(1-tauy)*Ks(i)**(1-eta(i))); /////////ty
st(i)                  =1-sk(i)-sl(i);
gamma(i)
=VA0(i)/(sk(i)*Ks(i)**eta(i)+sl(i)*Ls(i)**eta(i)+st(i)*Ts(i)**eta(i))**(1/eta(i));

*nest1:
Parameter
omegai(i)               elasticity of substitution between value added and composite intermediate
inputs
etai(i)                  parameter defined by elasticity of substitution between value added and
composite intermediate inputs
sva(i)                 distribution parameter of value added demand by sector
scint(i)               distribution parameter of composite intermediate demand by sector
gammai(i)               scaling coefficient in the production function for regional output
;

omegai(i)               =0.5;
etai(i)                =(omegai(i)-1)/omegai(i);  //--> omega=1/(1-eta) --> 1-eta= 1/omega
sva(i)                 =VA0(i)**(1-etai(i))/(VA0(i)**(1-etai(i))+CINT0(i)**(1-etai(i)));
scint(i)               =1-sva(i);
gammai(i)               =Y0(i)/(sva(i)*VA0(i)**etai(i)+scint(i)*CINT0(i)**etai(i))**(1/etai(i));

Display
mps,skg,skh,stg,sth,tauccl,tauc,taumx,tauk,taule,taulr,taup,tausd_r_fnl,tausd_rr_fnl,
tausd_nr_int,tausd_nr_fnl,tauy,ac,ag,aid,aint,ak,al,at,ava,sigmaw,omegaw,sigmak,
rhok,rhow,seruk,serow,thetak,thetaw,omegak,etak,etaw,smrow,smruk,gammak,gammaw,
omega,eta,sk,sl,st,gamma,omegai,etai,sva,scint,gammai
;
*===============================================================
==============
```

*Defining model system--

Variable

C(i) households' consumption demand by commodity

CPI CPI calculated using Fisher index

CPI_L CPI calculated using Laspeyres index

CPI_P CPI calculated using Paasche index

WD(j) regional production supplied to domestic market

EROW(i) export supply to the ROW by sector

ERUK(i) export supply to the RUK by sector

G(i) fiscal expenditure demand by commodity

ID(i) investment demand by commodity

INT(i,j) intermediate inputs for regional production

CINT(j) composite intermediate inputs for regional production

MROW(j) import demand from the ROW by commodity

MRUK(j) import demand from the RUK by commodity

PWD(i) price of regional production supplied to domestic market

PI average price of investment goods

PQ(i) price of regional sales of composites by commodity ¨C net of product taxes

PQW(i) price of regional sales of domestic composites by commodity ¨C net of product taxes

PVA(i) price of value added by sector

PW(i) price of regional production supplied to regional market

PY(i) price of output of regional production by sector ¨C basic price

Q(j) regional sales of composites combining regional production and all imports by commodity

QW(j) regional sales of domestic composites combining regional production and imports from RUK by commodity

S total savings

SH households¡¯savings

TC product tax revenue

TCCL council tax revenue

TK corporation tax revenue

TLE NIC revenue payable by the employee

TLR	NIC revenue payable by the employer
TP	production tax revenue
TRFG	fiscal transfer received by regional government from central government
TSD_R_NS	Stamp Duty Land Tax revenue from residential properties¡¯ new sale
TSD_NR_NS	Stamp Duty Land Tax revenue from non-residential properties¡¯ new sale
TSD_R_RS	Stamp Duty Land Tax revenue from residential properties¡¯ resale
TSD_NR_RS	Stamp Duty Land Tax revenue from non-residential properties¡¯ resale
TY	income tax revenue
U	households¡¯ utility level
VA(j)	value-added bundle of factors by sector
W(j)	regional production supplied to regional market
Y(j)	output of regional production by sector
YG	total fiscal revenue
YG_DT	devolved tax revenue
YG_F	factor income of regional government
YG_NDT	non-devolved tax revenue
YH	households' income
YK	factor income of capital
YL	factor income of labour
YT	factor income of land
K(j)	capital demand by sector
T(j)	land demand by sector
L(j)	labour demand by sector
PT_NR	economy-wide non-residential land return

;

Equation

eqva(j)	value added function
eqt_nr(j)	non-residential land demand function
eqk(j)	capital demand function
eql(j)	labour demand function
eqtstock	factor market clearing condition for land
eqyk	total capital income

eqyt	total land income
eqyl	total labour income
eqint(i,j)	intermediate demand function
eqcint(j)	composite intermediate demand function
eqy(j)	value added demand function
eqpva(j)	regional production function
eqyh	households income function
eqsh	households savings function
eqc(i)	households consumption demand function for non-residential-rental sectors
eqty	income tax revenue function
eqtk	corporation tax revenue function
eqtlr	function for NIC revenue payable by the employer
eqtle	function for NIC revenue payable by the employee
eqtccl	council tax revenue function
eqtp	net production tax revenue function
eqtc	net product tax revenue function
eqtsd_r_ns	function for SDLT revenue from residential properties¡¯ new sale
eqtsd_nr_ns	function for SDLT revenue from non-residential properties¡¯ new sale
eqtsd_r_rs	function for SDLT revenue from residential properties¡¯ resale
eqtsd_nr_rs	function for SDLT revenue from non-residential properties¡¯ resale
eqyg_dt	devolved tax revenue function
eqyg_ndt	non-devolved tax revenue function
eqyg_f	factor income of regional government function
eqtrfg	function for fiscal transfer received by regional government from central government
eqyg	total fiscal revenue function
eqg(i)	fiscal expenditure demand function
eqpyd(i)	CET function for ROW exports and domestic goods
eqwds(i)	domestic good supply function
eqerows(i)	foreign export supply function
eqpwdd(i)	CET function for RUK exports and regional goods
eqws(i)	regional good supply function
eqeruks(i)	RUK export supply function
eqpqws(i)	Armington CES function RUK imports and regional goods

```
eqmrukd(i)           RUK import demand function
eqwd(i)              regional good demand function
eqpqs(i)             Armington CES function ROW imports and domestic goods
eqmrowd(i)           ROW import demand function
eqqwd(i)             domestic good demand function
eqs                  total saving function
eqid(i)              investment demand function
eqcpi_l_b            CPI calculation function using Laspeyres index in base replication
eqcpi_p_b            CPI calculation function using Paasche index in base replication
eqcpi                CPI calculation function using Fisher index
eqpi                 investment good price index calculation function
equ                  objective utility function
eqq(i)               market clearing condition for goods market
;

*------------------------production behaviour-----------------------------------
eqpva(j)..                                                        PVA(j)*VA(j)=e=Pk(j)*K(j)/(1-
tauk)+(1+tausd_nr_int(j))*PT_NR*T(j)+(1+taulr)*Pl(j)*L(j)/(1-tauy);    ///////////////////////////tk
ty
*eqk(j)..     Pk(j)*K(j)/(1-tauk)=e=ak(j)*PVA(j)*VA(j);             //for C-D nest2
*eqt_nr(j)..  (1+tausd_nr_int(j))*PT_NR*T(j)=e=at(j)*PVA(j)*VA(j);   //for C-D nest2
*eql(j)..           (1+taulr)*Pl(j)*L(j)/(1-tauy)=e=al(j)*PVA(j)*VA(j);          //for   C-D   nest2
///////////////////////////ty
eqk(j)..          K(j)=e=VA(j)*(gamma(j)**eta(j)*sk(j)*(1-tauk)*PVA(j)/Pk(j))**(1/(1-eta(j)));
//for CES nest2                                          /////////////////////////
eqt_nr(j)..
T(j)=e=VA(j)*(gamma(j)**eta(j)*st(j)*PVA(j)/((1+tausd_nr_int(j))*PT_NR))**(1/(1-eta(j)));
//for CES nest2
eql(j)..       L(j)=e=VA(j)*(gamma(j)**eta(j)*sl(j)*(1-tauy)*PVA(j)/((1+taulr)*Pl(j)))**(1/(1-
eta(j)));   //for CES nest2    ///////ty

eqyk..       YK=e=sum(j,Pk(j)*K(j));
eqyt..       YT=e=sum(j,PT_NR*T(j));
eqyl..       YL=e=sum(j,Pl(j)*L(j));
```

```
eqy(j)..     PY(j)*Y(j)=e=PVA(j)*VA(j)+PQ(j)*CINT(j);
eqva(j)..    VA(j)=e=Y(j)*(gammai(j)**etai(j)*sva(j)*PY(j)/PVA(j))**(1/(1-etai(j)));
eqcint(j)..  CINT(j)=e=Y(j)*(gammai(j)**etai(j)*scint(j)*PY(j)/PQ(j))**(1/(1-etai(j)));
eqint(i,j).. INT(i,j)=e=aint(i,j)*CINT(j);
*------------------------household behaviour-----------------------------------
eqyh..       YH=e=skh*YK+sth*YT+YL/(1-tauy)+Trfh;                    /////////////////////ty
eqsh..       SH=e=mps*(YH-TY-TLE-TCCL);
eqc(i)..     PQ(i)*C(i)=e=ac(i)*(YH-TY-TLE-TCCL-SH);

*------------------------government behaviour----------------------------------
eqty..       TY=e=tauy*YH;
eqtk..       TK=e=tauk*YK/(1-tauk);
eqtlr..      TLR=e=taulr*YL/(1-tauy);                                /////////////////////ty
eqtle..      TLE=e=taule*YL/(1-tauy);                                /////////////////////ty
eqtccl..     TCCL=e=TCCL0;
eqtp..       TP=e=sum(j,taup(j)*PY(j)*Y(j));
eqtc..       TC=e=sum(j,tauc(j)*PY(j)*(1+taup(j))*Y(j));
eqtsd_r_ns.. TSD_R_NS=e=sum(i,tausd_r_fnl(i)*PY(i)*(1+taup(i))*Y(i));
eqtsd_nr_ns..TSD_NR_NS=e=sum(i,tausd_nr_fnl(i)*PY(i)*(1+taup(i))*Y(i));
eqtsd_r_rs.. TSD_R_RS=e=sum(i,tausd_rr_fnl(i)*PY(i)*(1+taup(i))*Y(i));
eqtsd_nr_rs..TSD_NR_RS=e=sum(inr,tausd_nr_int(inr)*PT_NR*T(inr));
*For SDLT simulation-------------
*eqyg_dt..    YG_DT=e=TSD_R_NS+TSD_NR_NS+TSD_R_RS+TSD_NR_RS+TCCL;
*eqyg_ndt..   YG_NDT=e=TY+TK+TLR+TLE+TC+TP;
*-------------------------------------------
*For Corporation Tax simulation--
*eqyg_dt..    YG_DT=e=TSD_R_NS+TSD_NR_NS+TSD_R_RS+TSD_NR_RS+TCCL+TK;
*eqyg_ndt..   YG_NDT=e=TY+TLR+TLE+TC+TP;
*-------------------------------------------
*For Income Tax simulation--
eqyg_dt..    YG_DT=e=TSD_R_NS+TSD_NR_NS+TSD_R_RS+TSD_NR_RS+TCCL+TY;
eqyg_ndt..   YG_NDT=e=TK+TLR+TLE+TC+TP;
*-------------------------------------------
```

```
*For Cross simulation--Income&Corporation Tax
*eqyg_dt..
YG_DT=e=TSD_R_NS+TSD_NR_NS+TSD_R_RS+TSD_NR_RS+TCCL+TK+TY;
*eqyg_ndt..   YG_NDT=e=TLR+TLE+TC+TP;
*-------------------------------------------
eqyg_f..     YG_F=e=skg*YK+stg*YT;
eqtrfg..     TRFG=e=YG0-YG_DT0-YG_F-YG_NDT;
eqyg..       YG=e=YG_DT+YG_F+YG_NDT+TRFG;
eqg(i)..     PQ(i)*G(i)=e=ag(i)*(YG-Trfh);

*-----------------------trade behaviour---------------------------------------
eqpyd(i)..                              Y(i)=e=thetaw(i)*(serow(i)*EROW(i)**rhow(i)+(1-
serow(i))*WD(i)**rhow(i))**(1/rhow(i));
eqerows(i)..
EROW(i)=e=Y(i)*(thetaw(i)**rhow(i)*serow(i)*(1+taumx(i))*PY(i)/Perow(i))**(1/(1-
rhow(i)));
eqwds(i)..                                             WD(i)=e=Y(i)*(thetaw(i)**rhow(i)*(1-
serow(i))*(1+taumx(i))*PY(i)/PWD(i))**(1/(1-rhow(i)));

eqpwdd(i)..                              WD(i)=e=thetak(i)*(seruk(i)*ERUK(i)**rhok(i)+(1-
seruk(i))*W(i)**rhok(i))**(1/rhok(i));
eqeruks(i)..          ERUK(i)=e=WD(i)*(thetak(i)**rhok(i)*seruk(i)*PWD(i)/Peruk(i))**(1/(1-
rhok(i)));
eqws(i)..    W(i)=e=WD(i)*(thetak(i)**rhok(i)*(1-seruk(i))*PWD(i)/PW(i))**(1/(1-rhok(i)));

eqpqws(i)..                          QW(i)=e=gammak(i)*(smruk(i)*MRUK(i)**etak(i)+(1-
smruk(i))*W(i)**etak(i))**(1/etak(i));
eqmrukd(i)..  MRUK(i)=e=QW(i)*(gammak(i)**etak(i)*smruk(i)*PQW(i)/Pmruk(i))**(1/(1-
etak(i)));
eqwd(i)..             W(i)=e=QW(i)*(gammak(i)**etak(i)*(1-smruk(i))*PQW(i)/PW(i))**(1/(1-
etak(i)));

eqpqs(i)..                           Q(i)=e=gammaw(i)*(smrow(i)*MROW(i)**etaw(i)+(1-
smrow(i))*QW(i)**etaw(i))**(1/etaw(i));
```

```
eqmrowd(i)..    MROW(i)=e=Q(i)*(gammaw(i)**etaw(i)*smrow(i)*PQ(i)/Pmrow(i))**(1/(1-
etaw(i)));
eqqwd(i)..          QW(i)=e=Q(i)*(gammaw(i)**etaw(i)*(1-smrow(i))*PQ(i)/PQW(i))**(1/(1-
etaw(i)));

*------------------------investment&saving behaviour----------------------------
eqs..       S=e=(1-skh-skg)*YK+(1-sth-stg)*YT+SH+Sruk+Srow;
eqid(i)..    PQ(i)*ID(i)=e=aid(i)*S;

*------------------------price equations----------------------------------------
eqcpi_l_b..  CPI_L=e=sum(inre,PQ(inre)*C0(inre))/sum(inre,C0(inre));
eqcpi_p_b..  CPI_P=e=sum(inre,PQ(inre)*C(inre))/sum(inre,C(inre));
eqcpi..      CPI=e=sqrt(CPI_L*CPI_P);
eqpi..       PI=e=sum(ii,PQ(ii)*aid(ii));

*------------------------market clearing conditions-----------------------------
eqq(i)..     Q(i)=e=C(i)+G(i)+ID(i)+sum(j, INT(i,j));
eqtstock..   sum(j,T(j))=e=T_stock;
*------------------------objective function-------------------------------------
equ..        U=e=prod(ic, C(ic)**ac(ic));
*=============================================================
==============

*Initializing endogenous variables----------------------------------------------
C.l(i)       =C0(i);
CPI.l        =1;
CPI_L.l      =1;
CPI_P.l      =1;
WD.l(j)      =WD0(j);
EROW.l(i)    =EROW0(i);
ERUK.l(i)    =ERUK0(i);
G.l(i)       =G0(i);
ID.l(i)      =ID0(i);
INT.l(i,j)   =INT0(i,j);
```

```
CINT.l(j)    =CINT0(j);
MROW.l(j)    =MROW0(j);
MRUK.l(j)    =MRUK0(j);
PWD.l(i)     =1;
PI.l         =1;
PQ.l(i)      =1;
PQW.l(i)     =1;
PVA.l(i)     =1;
PW.l(i)      =1;
PY.l(i)      =1;
Q.l(j)       =Q0(j);
QW.l(j)      =QW0(j);
S.l          =S0;
SH.l         =SH0;
TC.l         =TC0;
TCCL.l       =TCCL0;
TK.l         =TK0;
TLE.l        =TLE0;
TLR.l        =TLR0;
TP.l         =TP0;
TRFG.l       =TRFG0;
TSD_R_NS.l   =TSD_R_NS0;
TSD_NR_NS.l  =TSD_NR_NS0;
TSD_R_RS.l   =TSD_R_RS0;
TSD_NR_RS.l  =TSD_NR_RS0;
TY.l         =TY0;
VA.l(j)      =VA0(j);
W.l(j)       =W0(j);
Y.l(j)       =Y0(j);
YG.l         =YG0;
YG_DT.l      =YG_DT0;
YG_F.l       =YG_F0;
YG_NDT.l     =YG_NDT0;
YH.l         =YH0;
```

```
YK.l        =YK0;
YL.l        =YL0;
YT.l        =YT0;
K.l(j)      =K0(j);
T.l(j)      =T0(j);
L.l(j)      =L0(j);
PT_NR.l     =1;
*==============================================================================

*Setting lower bounds for endogenous variables----------------------------------
C.lo(i)       =0.0000000001;
CPI.lo        =0.0000000001;
CPI_L.lo      =0.0000000001;
CPI_P.lo      =0.0000000001;
WD.lo(j)      =0.0000000001;
EROW.lo(i)    =0.0000000001;
ERUK.lo(i)    =0.0000000001;
G.lo(i)       =0.0000000001;
ID.lo(i)      =0.0000000001;
INT.lo(i,j)   =0.0000000001;
CINT.lo(j)    =0.0000000001;
MROW.lo(j)    =0.0000000001;
MRUK.lo(j)    =0.0000000001;
PWD.lo(i)     =0.0000000001;
PI.lo         =0.0000000001;
PQ.lo(i)      =0.0000000001;
PQW.lo(i)     =0.0000000001;
PVA.lo(i)     =0.0000000001;
PW.lo(i)      =0.0000000001;
PY.lo(i)      =0.0000000001;
Q.lo(j)       =0.0000000001;
QW.lo(j)      =0.0000000001;
S.lo          =0.0000000001;
```

```
SH.lo        =0.0000000001;
TC.lo        =0.0000000001;
TCCL.lo      =0.0000000001;
TK.lo        =0.0000000001;
TLE.lo       =0.0000000001;
TLR.lo       =0.0000000001;
TP.lo        =0.0000000001;
TRFG.lo      =0.0000000001;
TSD_R_NS.lo   =0.0000000001;
TSD_NR_NS.lo  =0.0000000001;
TSD_R_RS.lo   =0.0000000001;
TSD_NR_RS.lo  =0.0000000001;
TY.lo        =0.0000000001;
VA.lo(j)     =0.0000000001;
W.lo(j)      =0.0000000001;
Y.lo(j)      =0.0000000001;
YG.lo        =0.0000000001;
YG_DT.lo     =0.0000000001;
YG_F.lo      =0.0000000001;
YG_NDT.lo    =0.0000000001;
YH.lo        =0.0000000001;
YK.lo        =0.0000000001;
YL.lo        =0.0000000001;
YT.lo        =0.0000000001;
K.lo(j)      =0.0000000001;
T.lo(j)      =0.0000000001;
L.lo(j)      =0.0000000001;
PT_NR.lo     =0.0000000001;
*============================================================
=============

*Setting numeraire----------------------------------------------------
*CPI.fx      =1;
*============================================================
```

```
==============

*Defining and solving the model & replicating the benchmark---------------------
Model WAGE_base /All/;
Solve WAGE_base maximizing U using nlp;

*  Show solutions for benchmark replication-------------
Display
C.l,CPI.l,CPI_L.l,CPI_P.l,WD.l,EROW.l,ERUK.l,G.l,ID.l,INT.l,MROW.l,MRUK.l,PWD.l,
PI.l,PQ.l,PQW.l,PVA.l,PW.l,PY.l,Q.l,QW.l,S.l,SH.l,TC.l,TCCL.l,TK.l,TLE.l,TLR.l,
TP.l,TRFG.l,TSD_R_NS.l,TSD_NR_NS.l,TSD_R_RS.l,TSD_NR_RS.l,TY.l,U.l,VA.l,W.l,Y.
l,
YG.l,YG_DT.l,YG_F.l,YG_NDT.l,YH.l,YK.l,YL.l,YT.l,K.l,L.l,T.l,PT_NR.l,CINT.l
;

*  Show how much the solutions deviate the benchmark----
*    Show percentage deviation against the benchmark----
Parameter
dC(ic),dCPI,dCPI_L,dCPI_P,dWD(j),dEROW(i),dERUK(i),dG(ig),dID(ii),dINT(i,j),dMRO
W(i),
dMRUK(i),dPWD(i),dPI,dPQ(i),dPQW(i),dPVA(i),dPW(i),dPY(i),dQ(j),dQW(j),dS,dSH,
dTC,dTCCL,dTK,dTLE,dTLR,dTP,dTRFG,dTSD_R_NS,dTSD_NR_NS,dTSD_R_RS,dTS
D_NR_RS,dTY,
dVA(j),dW(j),dY(j),dYG,dYG_DT,dYG_F,dYG_NDT,dYH,dYK,dYL,dYT,dK(i),dL(i),dT(i
),dPT_NR,

*Below with prefix 'ad' is average deviations of each variable across sectors
adC,adWD,adEROW,adERUK,adG,adID,adMROW,adINT,adMRUK,adPWD,adPQ,adPQW
,adPVA,adPW,
adPY,adQ,adQW,adVA,adW,adY,ad,adK,adL,adT
;

dC(ic)          =(C.l(ic)       /C0(ic)-1)*100;
dCPI            =(CPI.l        /1-1)*100;
```

```
dCPI_L          =(CPI_L.l      /1-1)*100;
dCPI_P          =(CPI_P.l      /1-1)*100;
dWD(j)          =(WD.l(j)      /WD0(j)-1)*100;
dEROW(i)$(EROW0(i) ne 0)        =(EROW.l(i)    /EROW0(i)-1)*100;
dERUK(i)$(ERUK0(i) ne 0)        =(ERUK.l(i)    /ERUK0(i)-1)*100;
dG(ig)          =(G.l(ig)       /G0(ig)-1)*100;
dID(ii)         =(ID.l(ii)      /ID0(ii)-1)*100;
dINT(i,j)$(INT0(i,j) ne 0)      =(INT.l(i,j)   /INT0(i,j)-1)*100;
dMROW(j)$(MROW0(j) ne 0)        =(MROW.l(j)    /MROW0(j)-1)*100;
dMRUK(j)$(MRUK0(j) ne 0)        =(MRUK.l(j)    /MRUK0(j)-1)*100;
dPWD(i)         =(PWD.l(i)     /1-1)*100;
dPI             =(PI.l         /1-1)*100;
dPQ(i)          =(PQ.l(i)      /1-1)*100;
dPQW(i)         =(PQW.l(i)     /1-1)*100;
dPVA(i)         =(PVA.l(i)     /1-1)*100;
dPW(i)          =(PW.l(i)      /1-1)*100;
dPY(i)          =(PY.l(i)      /1-1)*100;
dQ(j)           =(Q.l(j)       /Q0(j)-1)*100;
dQW(j)          =(QW.l(j)      /QW0(j)-1)*100;
dS              =(S.l          /S0-1)*100;
dSH             =(SH.l         /SH0-1)*100;
dTC             =(TC.l         /TC0-1)*100;
dTCCL           =(TCCL.l       /TCCL0-1)*100;
dTK             =(TK.l         /TK0-1)*100;
dTLE            =(TLE.l        /TLE0-1)*100;
dTLR            =(TLR.l        /TLR0-1)*100;
dTP             =(TP.l         /TP0-1)*100;
dTRFG           =(TRFG.l       /TRFG0-1)*100;
dTSD_R_NS       =(TSD_R_NS.l   /TSD_R_NS0-1)*100;
dTSD_NR_NS      =(TSD_NR_NS.l  /TSD_NR_NS0-1)*100;
dTSD_R_RS       =(TSD_R_RS.l   /TSD_R_RS0-1)*100;
dTSD_NR_RS      =(TSD_NR_RS.l  /TSD_NR_RS0-1)*100;
dTY             =(TY.l         /TY0-1)*100;
dVA(j)          =(VA.l(j)      /VA0(j)-1)*100;
```

```
dW(j)           =(W.l(j)       /W0(j)-1)*100;
dY(j)           =(Y.l(j)       /Y0(j)-1)*100;
dYG             =(YG.l         /YG0-1)*100;
dYG_DT          =(YG_DT.l      /YG_DT0-1)*100;
dYG_F           =(YG_F.l       /YG_F0-1)*100;
dYG_NDT         =(YG_NDT.l     /YG_NDT0-1)*100;
dYH             =(YH.l         /YH0-1)*100;
dYK             =(YK.l         /YK0-1)*100;
dYL             =(YL.l         /YL0-1)*100;
dYT             =(YT.l         /YT0-1)*100;
dK(j)$(K0(j) ne 0)          =(K.l(j)       /K0(j)-1)*100;
dT(j)$(T0(j) ne 0)          =(T.l(j)       /T0(j)-1)*100;
dL(j)$(L0(j) ne 0)          =(L.l(j)       /L0(j)-1)*100;
dPT_NR          =(PT_NR.l      /1-1)*100;

adC             =sum(ic,abs(dC(ic)))/card(ic);
adWD            =sum(i,abs(dWD(i)))/card(i);
adEROW          =sum(i,abs(dEROW(i)))/3;
adERUK          =sum(i,abs(dERUK(i)))/17;
adG             =sum(ig,abs(dG(ig)))/card(ig);
adID            =sum(ii,abs(dID(ii)))/card(ii);
adMROW          =sum(i,abs(dMROW(i)))/3;
adMRUK          =sum(i,abs(dMRUK(i)))/18;
adINT           =sum((i,j),abs(dINT(i,j)))/363;
adPWD           =sum(i,abs(dPWD(i)))/card(i);
adPQ            =sum(i,abs(dPQ(i)))/card(i);
adPQW           =sum(i,abs(dPQW(i)))/card(i);
adPVA           =sum(i,abs(dPVA(i)))/card(i);
adPW            =sum(i,abs(dPW(i)))/card(i);
adPY            =sum(i,abs(dPY(i)))/card(i);
adQ             =sum(i,abs(dQ(i)))/card(i);
adQW            =sum(i,abs(dQW(i)))/card(i);
adVA            =sum(i,abs(dVA(i)))/card(i);
adW             =sum(i,abs(dW(i)))/card(i);
```

```
adY            =sum(i,abs(dY(i)))/card(i);
adK            =sum(i,abs(dK(i)))/card(i);
adL            =sum(i,abs(dL(i)))/card(i);
adT            =sum(i,abs(dT(i)))/card(i);

ad
=(adC+adWD+adEROW+adERUK+adG+adID+adMROW+adINT+adMRUK+adPWD

+adPQ+adPQW+adPVA+adPW+adPY+adQ+adQW+adVA+adW+adY+abs(dCPI)+abs(dCP
I_L)
      +abs(dCPI_P)+abs(dPI)+abs(dS)+abs(dSH)+abs(dTC)+abs(dTCCL)+abs(dTK)

+abs(dTLE)+abs(dTLR)+abs(dTP)+abs(dTRFG)+abs(dTSD_R_NS)+abs(dTSD_NR_NS)

+abs(dTSD_R_RS)+abs(dTSD_NR_RS)+abs(dTY)+abs(dYG)+abs(dYG_DT)+abs(dYG_F)

+abs(dYG_NDT)+abs(dYH)+abs(dYK)+abs(dYL)+abs(dYT)+adK+adL+adT+abs(dPT_NR))
      /50;

Display
dC,dCPI,dCPI_L,dCPI_P,dWD,dEROW,dERUK,dG,dID,dMROW,dINT,dMRUK,dPWD,dP
I,dPQ,dPQW,
dPVA,dPW,dPY,dQ,dQW,dS,dSH,dTC,dTCCL,dTK,dTLE,dTLR,dTP,dTRFG,dTSD_R_NS
,dTSD_NR_NS,
dTSD_R_RS,dTSD_NR_RS,dTY,dVA,dW,dY,dYG,dYG_DT,dYG_F,dYG_NDT,dYH,dYK
,dYL,dYT,
dK,dL,dT,dPT_NR,ad
;

*    Show Euclidean distance against benchmark----------
Parameter
edC,edCPI,edCPI_L,edCPI_P,edWD,edEROW,edERUK,edG,edID,edMROW,edINT,edMR
UK,edPWD,
edPI,edPQ,edPQW,edPVA,edPW,edPY,edQ,edQW,edS,edSH,edTC,edTCCL,edTK,edTLE,e
```

```
dTLR,
edTP,edTRFG,edTSD_R_NS,edTSD_NR_NS,edTSD_R_RS,edTSD_NR_RS,edTY,edVA,e
dW,edY,edYG,
edYG_DT,edYG_F,edYG_NDT,edYH,edYK,edYL,edYT,edK,edL,edT,edPT_NR
;

edC          =sqrt(sum(ic,sqr(C.l(ic)      -C0(ic))));
edCPI        =abs(CPI.l       -1);
edCPI_L      =abs(CPI_L.l     -1);
edCPI_P      =abs(CPI_P.l     -1);
edWD         =sqrt(sum(j,sqr(WD.l(j)      -WD0(j))));
edEROW       =sqrt(sum(i$(EROW0(i) ne 0),sqr(EROW.l(i)    -EROW0(i))));
edERUK       =sqrt(sum(i$(ERUK0(i) ne 0),sqr(ERUK.l(i)    -ERUK0(i))));
edG          =sqrt(sum(ig,sqr(G.l(ig)      -G0(ig))));
edID         =sqrt(sum(ii,sqr(ID.l(ii)     -ID0(ii))));
edINT        =sqrt(sum((i,j)$(INT0(i,j) ne 0),sqr(INT.l(i,j)   -INT0(i,j))));
edMROW       =sqrt(sum(j$(MROW0(j) ne 0),sqr(MROW.l(j)    -MROW0(j))));
edMRUK       =sqrt(sum(j$(MRUK0(j) ne 0),sqr(MRUK.l(j)    -MRUK0(j))));
edPWD        =sqrt(sum(i,sqr(PWD.l(i)     -1)));
edPI         =abs(PI.l        -1);
edPQ         =sqrt(sum(i,sqr(PQ.l(i)      -1)));
edPQW        =sqrt(sum(i,sqr(PQW.l(i)     -1)));
edPVA        =sqrt(sum(i,sqr(PVA.l(i)     -1)));
edPW         =sqrt(sum(i,sqr(PW.l(i)      -1)));
edPY         =sqrt(sum(i,sqr(PY.l(i)      -1)));
edQ          =sqrt(sum(j,sqr(Q.l(j)       -Q0(j))));
edQW         =sqrt(sum(j,sqr(QW.l(j)      -QW0(j))));
edS          =abs(S.l         -S0);
edSH         =abs(SH.l        -SH0);
edTC         =abs(TC.l        -TC0);
edTCCL       =abs(TCCL.l      -TCCL0);
edTK         =abs(TK.l        -TK0);
edTLE        =abs(TLE.l       -TLE0);
edTLR        =abs(TLR.l       -TLR0);
```

```
edTP            =abs(TP.l        -TP0);
edTRFG          =abs(TRFG.l      -TRFG0);
edTSD_R_NS      =abs(TSD_R_NS.l   -TSD_R_NS0);
edTSD_NR_NS     =abs(TSD_NR_NS.l  -TSD_NR_NS0);
edTSD_R_RS      =abs(TSD_R_RS.l   -TSD_R_RS0);
edTSD_NR_RS     =abs(TSD_NR_RS.l  -TSD_NR_RS0);
edTY            =abs(TY.l        -TY0);
edVA            =sqrt(sum(j,sqr(VA.l(j)      -VA0(j))));
edW             =sqrt(sum(j,sqr(W.l(j)       -W0(j))));
edY             =sqrt(sum(j,sqr(Y.l(j)       -Y0(j))));
edYG            =abs(YG.l        -YG0);
edYG_DT         =abs(YG_DT.l      -YG_DT0);
edYG_F          =abs(YG_F.l       -YG_F0);
edYG_NDT        =abs(YG_NDT.l     -YG_NDT0);
edYH            =abs(YH.l        -YH0);
edYK            =abs(YK.l        -YK0);
edYL            =abs(YL.l        -YL0);
edYT            =abs(YT.l        -YT0);
edK             =sqrt(sum(i,sqr(K.l(i)      -K0(i))));
edL             =sqrt(sum(i,sqr(L.l(i)      -L0(i))));
edT             =sqrt(sum(i,sqr(T.l(i)      -T0(i))));
edPT_NR         =abs(PT_NR.l      -1);

Display
edC,edCPI,edCPI_L,edCPI_P,edWD,edEROW,edERUK,edG,edID,edINT,edMROW,edMR
UK,edPWD,
edPI,edPQ,edPQW,edPVA,edPW,edPY,edQ,edQW,edS,edSH,edTC,edTCCL,edTK,edTLE,e
dTLR,
edTP,edTRFG,edTSD_R_NS,edTSD_NR_NS,edTSD_R_RS,edTSD_NR_RS,edTY,edVA,e
dW,edY,edYG,
edYG_DT,edYG_F,edYG_NDT,edYH,edYK,edYL,edYT,edK,edL,edT,edPT_NR
;

*  Record the replication solutions---------------------
```

```
Parameter
C1(i),CPI1,CPI_L1,CPI_P1,WD1(j),EROW1(i),ERUK1(i),G1(i),ID1(i),INT1(i,j),MROW1(i),
MRUK1(i),PWD1(i),PI1,PQ1(i),PQW1(i),PVA1(i),PW1(i),PY1(i),Q1(j),QW1(j),S1,SH1,
TC1,TCCL1,TK1,TLE1,TLR1,TP1,TRFG1,TSD_R_NS1,TSD_NR_NS1,TSD_R_RS1,TSD
_NR_RS1,TY1,
VA1(j),W1(j),Y1(j),YG1,YG_DT1,YG_F1,YG_NDT1,YH1,YK1,YL1,YT1,K1(i),L1(i),T1(i
),
PT_NR1,CINT1(j)
;

C1(i)          =C.l(i);
CPI1           =CPI.l;
CPI_L1         =CPI_L.l;
CPI_P1         =CPI_P.l;
WD1(j)         =WD.l(j);
EROW1(i)       =EROW.l(i);
ERUK1(i)       =ERUK.l(i);
G1(i)          =G.l(i);
ID1(i)         =ID.l(i);
INT1(i,j)      =INT.l(i,j);
CINT1(j)       =CINT.l(j);
MROW1(j)       =MROW.l(j);
MRUK1(j)       =MRUK.l(j);
PWD1(i)        =PWD.l(i) ;
PI1            =PI.l     ;
PQ1(i)         =PQ.l(i)   ;
PQW1(i)        =PQW.l(i)   ;
PVA1(i)        =PVA.l(i)    ;
PW1(i)         =PW.l(i)      ;
PY1(i)         =PY.l(i)      ;
Q1(j)          =Q.l(j)       ;
QW1(j)         =QW.l(j)      ;
S1             =S.l          ;
SH1            =SH.l         ;
```

```
TC1            =TC.l        ;
TCCL1          =TCCL.l      ;
TK1            =TK.l        ;
TLE1           =TLE.l       ;
TLR1           =TLR.l       ;
TP1            =TP.l        ;
TRFG1          =TRFG.l      ;
TSD_R_NS1      =TSD_R_NS.l  ;
TSD_NR_NS1     =TSD_NR_NS.l ;
TSD_R_RS1      =TSD_R_RS.l  ;
TSD_NR_RS1     =TSD_NR_RS.l ;
TY1            =TY.l        ;
VA1(j)         =VA.l(j)     ;
W1(j)          =W.l(j)      ;
Y1(j)          =Y.l(j)      ;
YG1            =YG.l        ;
YG_DT1         =YG_DT.l     ;
YG_F1          =YG_F.l      ;
YG_NDT1        =YG_NDT.l    ;
YH1            =YH.l        ;
YK1            =YK.l        ;
YL1            =YL.l        ;
YT1            =YT.l        ;
K1(i)          =K.l(i)      ;
L1(i)          =L.l(i)      ;
T1(i)          =T.l(i)      ;
PT_NR1         =PT_NR.l     ;
*=================================================================
==============

*Simulation and results-----------------------------------------------------------

*Variation of residential SDLT rate
*tausd_r_fnl('r_ns')     =1.1*TSD_R_NS0/(Y0('r_ns')+SAM0('t_pn','r_ns'));
```

```
*tausd_rr_fnl('r_r')      =1.1*TSD_R_RS0/Y0('r_r');
*tausd_r_fnl('r_ns')      =0.9*TSD_R_NS0/(Y0('r_ns')+SAM0('t_pn','r_ns'));
*tausd_rr_fnl('r_r')      =0.9*TSD_R_RS0/Y0('r_r');

*Variation of non-residential SDLT rate
*tausd_nr_fnl('nr_ns')    =1.1*TSD_NR_NS0/(Y0('nr_ns')+SAM0('t_pn','nr_ns'));
*tausd_nr_int(inr)        =1.1*TSD_NR_RS0/(SAM0('lnd','total')-SAM0('lnd','r_r'));
*tausd_nr_fnl('nr_ns')    =0.9*TSD_NR_NS0/(Y0('nr_ns')+SAM0('t_pn','nr_ns'));
*tausd_nr_int(inr)        =0.9*TSD_NR_RS0/(SAM0('lnd','total')-SAM0('lnd','r_r'));

*Variation of corporation tax
*tauk                     =1.05*TK0/(TK0+YK0);
*tauk                     =0.95*TK0/(TK0+YK0);

*Variation of income tax
*tauy                     =1.05*TY0/YH0;
tauy                      =0.95*TY0/YH0;

option bratio=1;

Equation
eqcpi_l_s                 CPI calculation function using Laspeyres index in simulation
eqcpi_p_s                 CPI calculation function using Paasche index in simulation
;
eqcpi_l_s..  CPI_L=e=sum(inre,PQ(inre)*C1(inre))/sum(inre,PQ1(inre)*C1(inre));
eqcpi_p_s..  CPI_P=e=sum(inre,PQ(inre)*C(inre))/sum(inre,PQ1(inre)*C(inre));

Model WAGE_simulation
/eqva,eqt_nr,eqk,eql,eqtstock,eqyk,eqyt,eqyl,eqint,eqy,eqpva,eqyh,eqsh,eqc,eqty,
eqtk,eqtlr,eqtle,eqtccl,eqtp,eqtc,eqtsd_r_ns,eqtsd_nr_ns,eqtsd_r_rs,eqtsd_nr_rs,
eqyg_dt,eqyg_ndt,eqyg_f,eqtrfg,eqyg,eqg,eqpyd,eqwds,eqerows,eqpwdd,eqws,eqeruks,
eqpqws,eqmrukd,eqwd,eqpqs,eqmrowd,eqqwd,eqs,eqid,eqcpi_l_s,eqcpi_p_s,eqcpi,eqpi,
equ,eqq,eqcint/
;
```

```
Solve WAGE_simulation maximizing U using nlp;

*  Show counter-factual solutions----------------------
Display
C.l,CPI.l,CPI_L.l,CPI_P.l,WD.l,EROW.l,ERUK.l,G.l,ID.l,INT.l,MROW.l,MRUK.l,PWD.l,
PI.l,PQ.l,PQW.l,PVA.l,PW.l,PY.l,Q.l,QW.l,S.l,SH.l,TC.l,TCCL.l,TK.l,TLE.l,TLR.l,
TP.l,TRFG.l,TSD_R_NS.l,TSD_NR_NS.l,TSD_R_RS.l,TSD_NR_RS.l,TY.l,U.l,VA.l,W.l,Y.
l,
YG.l,YG_DT.l,YG_F.l,YG_NDT.l,YH.l,YK.l,YL.l,YT.l,K.l,L.l,T.l,PT_NR.l,CINT.l
;

*  Show counter-factual changes against replication of benchmark---
Parameter
dC(ic),dCPI,dCPI_L,dCPI_P,dWD(j),dEROW(i),dERUK(i),dG(ig),dID(ii),dINT(i,j),
dMROW(i),dMRUK(i),dPWD(i),dPI,dPQ(i),dPQW(i),dPVA(i),dPW(i),dPY(i),dQ(j),dQW(j),
dS,dSH,dTC,dTCCL,dTK,dTLE,dTLR,dTP,dTRFG,dTSD_R_NS,dTSD_NR_NS,dTSD_R_
RS,dTSD_NR_RS,
dTSD_R,dTSD_NR,dTY,dVA(j),dW(j),dY(j),dYG,dYG_DT,dYG_F,dYG_NDT,dYH,dYH_
D,dYF_NR,
dYK,dYK_D,dYL,dYT,dVY(j),dVTY,dVQ(j),dVTQ,dVC(ic),dVTC,dVG(i),dVTG,dVID(i),
dTID,
dVTERUK,dVTEROW,dVTMRUK,dVTMROW,dVTWD,dVTW,dVTQW,dVVA(i),dVGV
A,dVINT(i,j),dVTINT,
dVRINT(i),dVCINT(j),dVERUK,dVEROW,dVMRUK,dVMROW,dVW,dGDP_B,dGDP_C,
dVY_R_NS,dPY_R_NS,
dY_R_NS,dVY_NR_NS,dPY_NR_NS,dY_NR_NS,dVY_R_R,dPY_R_R,dY_R_R,dYT_NR
,dK(i),dL(i),
dT(i),dKS,dLS,dPT_NR,dGDP_B_pc,dGDP_C_pc,dYH_pc,dYH_D_pc,dCINT(j)
;

dC(ic)          =(C.l(ic)       /C1(ic)-1)*100;
dCPI                            =(CPI.l        /CPI1-1)*100;
dCPI_L                            =(CPI_L.l      /CPI_L1-1)*100;
```

```
dCPI_P                         =(CPI_P.l      /CPI_P1-1)*100;
dWD(j)                         =(WD.l(j)      /WD1(j)-1)*100;
dEROW(i)$(EROW0(i) ne 0)         =(EROW.l(i)    /EROW1(i)-1)*100;
dERUK(i)$(ERUK0(i) ne 0)         =(ERUK.l(i)    /ERUK1(i)-1)*100;
dG(ig)           =(G.l(ig)       /G1(ig)-1)*100;
dID(ii)          =(ID.l(ii)      /ID1(ii)-1)*100;
dINT(i,j)$(INT0(i,j) ne 0)       =(INT.l(i,j)   /INT1(i,j)-1)*100;
dCINT(j)                        =(CINT.l(j)    /CINT1(j)-1)*100;
dMROW(j)$(MROW0(j) ne 0)         =(MROW.l(j)    /MROW1(j)-1)*100;
dMRUK(j)$(MRUK0(j) ne 0)         =(MRUK.l(j)    /MRUK1(j)-1)*100;
dPWD(i)          =(PWD.l(i)     /PWD1(i)-1)*100;
dPI              =(PI.l         /PI1-1)*100;
dPQ(i)           =(PQ.l(i)      /PQ1(i)-1)*100;
dPQW(i)          =(PQW.l(i)     /PQW1(i)-1)*100;
dPVA(i)          =(PVA.l(i)     /PVA1(i)-1)*100;
dPW(i)           =(PW.l(i)      /PW1(i)-1)*100;
dPY(i)           =(PY.l(i)      /PY1(i)-1)*100;
dQ(j)            =(Q.l(j)       /Q1(j)-1)*100;
dQW(j)           =(QW.l(j)      /QW1(j)-1)*100;
dS               =(S.l          /S1-1)*100;
dSH              =(SH.l         /SH1-1)*100;
dTC              =(TC.l         /TC1-1)*100;
dTCCL            =(TCCL.l       /TCCL1-1)*100;
dTK              =(TK.l         /TK1-1)*100;
dTLE             =(TLE.l        /TLE1-1)*100;
dTLR             =(TLR.l        /TLR1-1)*100;
dTP              =(TP.l         /TP1-1)*100;
dTRFG            =(TRFG.l       /TRFG1-1)*100;
dTSD_R_NS        =(TSD_R_NS.l   /TSD_R_NS1-1)*100;
dTSD_NR_NS       =(TSD_NR_NS.l  /TSD_NR_NS1-1)*100;
dTSD_R_RS        =(TSD_R_RS.l   /TSD_R_RS1-1)*100;
dTSD_NR_RS       =(TSD_NR_RS.l  /TSD_NR_RS1-1)*100;
dTY              =(TY.l         /TY1-1)*100;
dVA(j)           =(VA.l(j)      /VA1(j)-1)*100;
```

```
dW(j)          =(W.l(j)      /W1(j)-1)*100;
dY(j)          =(Y.l(j)      /Y1(j)-1)*100;
dYG            =(YG.l        /YG1-1)*100;
dYG_DT         =(YG_DT.l     /YG_DT1-1)*100;
dYG_F          =(YG_F.l      /YG_F1-1)*100;
dYG_NDT        =(YG_NDT.l    /YG_NDT1-1)*100;
dYH            =(YH.l        /YH1-1)*100;
dYH_D          =((YH.l-TY.l-TLE.l-TCCL.l)/(YH1-TY1-TLE1-TCCL1)-1)*100;
dYF_NR
=((YK.l+YL.l+sum(jnr,PT_NR.l*T.l(jnr)))/(YK1+YL1+sum(jnr,PT_NR1*T1(jnr)))-1)*100;
dYK            =(YK.l        /YK1-1)*100;
dYK_D          =((YK.l-TK.l)  /(YK1-TK1)-1)*100;
dYL            =(YL.l        /YL1-1)*100;
dYT            =(YT.l        /YT1-1)*100;
dVY(j)         =(PY.l(j)*Y.l(j) /(PY1(j)*Y1(j))-1)*100;
dVTY           =(sum(j,PY.l(j)*Y.l(j)) /sum(j,PY1(j)*Y1(j))-1)*100;
dVQ(j)         =(PQ.l(j)*Q.l(j) /(PQ1(j)*Q1(j))-1)*100;
dVTQ           =(sum(j,PQ.l(j)*Q.l(j)) /sum(j,PQ1(j)*Q1(j))-1)*100;
dVC(ic)        =(PQ.l(ic)*C.l(ic) /(PQ1(ic)*C1(ic))-1)*100;
dVTC           =(sum(ic,PQ.l(ic)*C.l(ic)) /sum(ic,PQ1(ic)*C1(ic))-1)*100;
dVG(i)         =(PQ.l(i)*G.l(i) /(PQ1(i)*G1(i))-1)*100;
dVTG           =(sum(i,PQ.l(i)*G.l(i)) /sum(i,PQ1(i)*G1(i))-1)*100;
dVID(i)        =(PQ.l(i)*ID.l(i)/(PQ1(i)*ID1(i))-1)*100;
dTID           =(sum(ii,ID.l(ii)) /sum(ii,ID1(ii))-1)*100;
dTSD_R         =((TSD_R_NS.l+TSD_R_RS.l)/(TSD_R_NS1+TSD_R_RS1)-1)*100;
dTSD_NR        =((TSD_NR_NS.l+TSD_NR_RS.l)/(TSD_NR_NS1+TSD_NR_RS1)-1)*100;
dVVA(i)        =(PVA.l(i)*VA.l(i) /(PVA1(i)*VA1(i))-1)*100;
dVGVA          =(sum(i,PVA.l(i)*VA.l(i)) /sum(i,PVA1(i)*VA1(i))-1)*100;
dVINT(i,j)     =(PQ.l(i)*INT.l(i,j) /(PQ1(i)*INT1(i,j))-1)*100;
dVTINT         =(sum((i,j),PQ.l(i)*INT.l(i,j)) /sum((i,j),PQ1(i)*INT1(i,j))-1)*100;
dVRINT(i)      =(sum(j,PQ.l(j)*INT.l(i,j)) /sum(j,PQ1(j)*INT1(i,j))-1)*100;
dVCINT(j)      =(sum(i,PQ.l(j)*INT.l(i,j)) /sum(i,PQ1(j)*INT1(i,j))-1)*100;
dVERUK         =(sum(i,ERUK.l(i)) /sum(i,ERUK1(i))-1)*100;
dVEROW         =(sum(i,EROW.l(i)) /sum(i,EROW1(i))-1)*100;
```

```
dVMRUK          =(sum(i,MRUK.l(i)) /sum(i,MRUK1(i))-1)*100;
dVMROW          =(sum(i,MROW.l(i)) /sum(i,MROW1(i))-1)*100;
dVW             =(sum(i,PW.l(i)*W.l(i)) /sum(i,PW1(i)*W1(i))-1)*100;
dGDP_B          =((sum(i,PVA.l(i)*VA.l(i))+TP.l) /(sum(i,PVA1(i)*VA1(i))+TP1)-1)*100;
dGDP_C          =((sum(i,PVA.l(i)*VA.l(i))+TP.l+TC.l) /(sum(i,PVA1(i)*VA1(i))+TP1+TC1)-
1)*100;
dVY_R_NS        =(PY.l('r_ns')*Y.l('r_ns') /(PY1('r_ns')*Y1('r_ns'))-1)*100;
dPY_R_NS        =(PY.l('r_ns') /(PY1('r_ns'))-1)*100;
dY_R_NS         =(Y.l('r_ns') /Y1('r_ns')-1)*100;
dVY_NR_NS       =(PY.l('nr_ns')*Y.l('nr_ns') /(PY1('nr_ns')*Y1('nr_ns'))-1)*100;
dPY_NR_NS       =(PY.l('nr_ns') /(PY1('nr_ns'))-1)*100;
dY_NR_NS        =(Y.l('nr_ns') /Y1('nr_ns')-1)*100;
dPY_R_R         =(PY.l('r_r') /(PY1('r_r'))-1)*100;
dVY_R_R         =(PY.l('r_r')*Y.l('r_r') /(PY1('r_r')*Y1('r_r'))-1)*100;
dY_R_R          =(Y.l('r_r') /(Y1('r_r'))-1)*100;
dYT_NR          =(sum(j,PT_NR.l*T.l(j))/sum(j,PT_NR1*T1(j))-1)*100;
dK(j)$(K0(j) ne 0)          =(K.l(j)      /K1(j)-1)*100;
dT(j)$(T0(j) ne 0)          =(T.l(j)      /T1(j)-1)*100;
dL(j)$(L0(j) ne 0)          =(L.l(j)      /L1(j)-1)*100;
dKS             =(sum(i,K.l(i)) /sum(i,K1(i))-1)*100;
dLS             =(sum(i,L.l(i)) /sum(i,L1(i))-1)*100;
dPT_NR          =(PT_NR.l      /PT_NR1-1)*100;
dGDP_B_pc
=(((sum(i,PVA.l(i)*VA.l(i))+TP.l)/sum(i,L.l(i)))/((sum(i,PVA1(i)*VA1(i))+TP1)/sum(i,L1(i)
))-1)*100;
dGDP_C_pc
=(((sum(i,PVA.l(i)*VA.l(i))+TP.l+TC.l)/sum(i,L.l(i)))/((sum(i,PVA1(i)*VA1(i))+TP1+TC1)
/sum(i,L1(i)))-1)*100;
dYH_pc          =((YH.l/sum(i,L.l(i)))/(YH1/sum(i,L1(i)))-1)*100;
dYH_D_pc                        =(((YH.l-TY.l-TLE.l-TCCL.l)/sum(i,L.l(i)))/((YH1-TY1-TLE1-
TCCL1)/sum(i,L1(i)))-1)*100;

Display
dC,dCPI,dCPI_L,dCPI_P,dWD,dEROW,dERUK,dG,dID,dMROW,dINT,dMRUK,dPWD,dP
```

```
I,dPQ,dPQW,
dPVA,dPW,dPY,dY,dQ,dQW,dS,dSH,dTC,dTCCL,dTK,dTLE,dTLR,dTP,dTRFG,dTSD_R
_NS,
dTSD_NR_NS,dTSD_R_RS,dTSD_NR_RS,dTSD_R,dTSD_NR,dTY,dVA,dW,dYG,dYG_
DT,dYG_F,dYG_NDT,
dYH,dYH_D,dYF_NR,dYK,dYK_D,dYL,dYT,dVY,dVTY,dVQ,dVTQ,dVC,dVTC,dVG,d
VTG,dVID,dVVA,
dVGVA,dVINT,dVTINT,dVRINT,dVCINT,dVERUK,dVEROW,dVMRUK,dVMROW,dV
W,dGDP_B,dGDP_C,
dVY_R_NS,dPY_R_NS,dY_R_NS,dVY_NR_NS,dPY_NR_NS,dY_NR_NS,dVY_R_R,dPY
_R_R,dY_R_R,
dK,dL,dT,dTID,dKS,dLS,dPT_NR,dGDP_B_pc,dGDP_C_pc,dYH_pc,dYH_D_pc,dCINT
;
*=====================================================================
==============

*Welfare measure: Hicksian equivalent variations--------------------------------
Parameter
U0          utility level in the Base Run Eq.
ep0         expenditure func. in the Base Run Eq.
ep1         expenditure func. in the Consumption function Eq.
EV          Hicksian equivalent variations
;

U0     =prod(ic, C0(ic)**ac(ic));
ep0    =U0 /prod(ic, (ac(ic)/1)**ac(ic));
ep1    =U.l/prod(ic, (ac(ic)/1)**ac(ic));
EV     =ep1-ep0;

Display ep0,ep1,EV;
*=====================================================================
==============

*Simulation results summary & presentation-------------------------------------
```

```
Parameter
WAGE_l(d)     simulation results summary table
;
WAGE_l('d_YG')            =dYG;
WAGE_l('d_YG_DT')         =dYG_DT;
WAGE_l('d_TSD_R')         =dTSD_R;
WAGE_l('d_TSD_NR')        =dTSD_NR;
WAGE_l('d_YG_NDT')        =dYG_NDT;
WAGE_l('d_TP')            =dTP;
WAGE_l('d_TC')            =dTC;
WAGE_l('d_TY')            =dTY;
WAGE_l('d_TK')            =dTK;
WAGE_l('d_TLR')           =dTLR;
WAGE_l('d_TLE')           =dTLE;
WAGE_l('d_YG_F')          =dYG_F;
WAGE_l('d_VTG')           =dVTG ;
WAGE_l('d_VTINT')         =dVTINT;
WAGE_l('d_VGVA')          =dVGVA ;
WAGE_l('d_VTY')           =dVTY  ;
WAGE_l('d_VTQ')           =dVTQ  ;
WAGE_l('d_GDP_B')         =dGDP_B;
WAGE_l('d_GDP_C')         =dGDP_C;
WAGE_l('d_YH')            =dYH   ;
WAGE_l('d_YH_D')          =dYH_D ;
WAGE_l('d_VTC')           =dVTC  ;
WAGE_l('d_S')             =dS    ;
WAGE_l('d_PI')            =dPI   ;
WAGE_l('d_YF_NR')         =dYF_NR;
WAGE_l('d_YT_NR')         =dYT_NR;
*WAGE_l('d_PT_NR')        =dPT_NR;
*WAGE_l('d_T_stock')      =0.0000000001;
WAGE_l('d_YK')            =dYK     ;
*WAGE_l('d_PK')           =0.0000000001;
*WAGE_l('d_K_stock')      =dKS;
```

```
WAGE_l('d_YL')            =dYL     ;
*WAGE_l('d_PL')           =0.0000000001;
*WAGE_l('d_L_stock')      =dLS;
WAGE_l('d_VY_R_NS')       =dVY_R_NS;
WAGE_l('d_PY_R_NS')       =dPY_R_NS;
WAGE_l('d_Y_R_NS')        =dY_R_NS ;
WAGE_l('d_VY_NR_NS')      =dVY_NR_NS;
WAGE_l('d_PY_NR_NS')      =dPY_NR_NS;
WAGE_l('d_Y_NR_NS')       =dY_NR_NS ;
WAGE_l('d_VY_R_R')        =dVY_R_R  ;
WAGE_l('d_PY_R_R')        =dPY_R_R  ;
WAGE_l('d_Y_R_R')         =dY_R_R   ;
WAGE_l('d_VERUK')         =dVERUK   ;
WAGE_l('d_VEROW')         =dVEROW   ;
WAGE_l('d_VMRUK')         =dVMRUK   ;
WAGE_l('d_VMROW')         =dVMROW   ;
WAGE_l('d_GDP_B_pc')      =dGDP_B_pc;
WAGE_l('d_GDP_C_pc')      =dGDP_C_pc;
WAGE_l('d_YH_pc')         =dYH_pc;
WAGE_l('d_YH_D_pc')       =dYH_D_pc;

*WAGE_l('d_VW')           =dVW      ;
*WAGE_l('d_EV')           =EV       ;
*WAGE_l('d_CPI')          =dCPI     ;

execute_unload 'WAGE_l.gdx',WAGE_l;
execute '=gdxviewer WAGE_l.gdx';
*===================================================================
```