A macro model for electroadhesive contact of a soft finger with a touchscreen

I.I. Argatov and F.M. Borodich

***Abstract*—A contact problem of electroadhesion for a conductive elastic body pressed against a rigid plane surface of a dielectric coating covering a conductive substrate is formulated applying the Johnsen–Rahbek approximation for the attractive surface stresses and the Derjaguin–Muller–Toporov (DMT) hypothesis about the influence of the adhesive stresses on the deformable shape of the elastic body. An approximate solution is obtained using the Winkler–Fuss deformation model with the equivalent (contact load dependent) stiffness coefficient evaluated according to the Xydas–Kao soft finger model. The friction force under applied voltage is evaluated as the product of the coefficient of friction and the integral of the macro contact pressure over the apparent contact area. The upper and lower estimates for the friction force are discussed in the case of absence of any external normal load.**

***Index Terms*—Electroadhesion, elastic contact, soft finger, friction force, Winkler–Fuss deformation model.**

I. Introduction

When two elastic bodies are pressed one against the other, both the apparent contact area, A, and the contact approach, δ, depend on the contact load, F_N, the elastic properties and the shape of the contacting solids. The phenomenon of adhesion exhibits itself in establishing contact between the bodies brought into contact even in the absence of external compressive loads. Physical causes of surface attractive forces can be different as well as approaches to their modeling [1, 2].

For instance, although both the Johnson–Kendall–Roberts (JKR) [3] and the Derjaguin–Muller–Toporov (DMT) [4] theories of the molecular attraction are based on explicit or implicit use of the Derjaguin approximation and the energy balance idea [5, 6], these theories differ by their assumptions about the influence of the surface forces acting between the contacting surfaces [7, 8].

In recent years (see, e.g., [9]), a considerable research interest has been paid to modeling of electroadhesion with application to contact between a finger and a touchscreen (see Fig. 1). Of a particular interest is the modulation (due to variable electric voltage) of friction between the finger and the display screen, which represents the mechanism of one of emerging haptic technologies [10, 11].

It is known that 'the arising electrical component of adhesion is especially important for the rapid processes of separation of two bodies' [12]. Therefore, to model the finger-screen contact one needs to describe properly electroadhesive interactions of the contacting pair.

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Electroadhesion between two nominally flat surfaces having different electric potentials has been modeled in a number of publications [10, 13]. The effect of surface roughness was recently investigated in [14], where, in particular, it is assumed that the effective loading pressure is represented as the sum $p+p_a$, where p_a is the electric attraction stress and $p = F_N/A$ is the applied pressure. However, the latter assumption (that is $p = \mathrm{const}$) is expected to be accurate at the micro scale, whereas at the macro scale the applied pressure is supposed to vary across the apparent contact area.

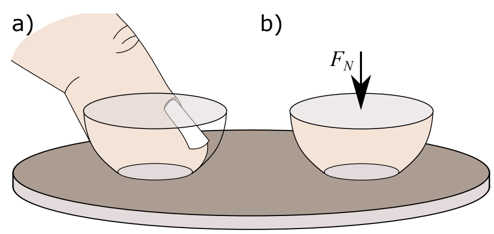

Fig. 1. Schematic of contact interaction between a human finger and a touchscreen (a) and its idealization for an equivalent hemispherical contact geometry in normal contact under an external load, F_N.

Very recently, a JKR-type macro model of electroadhesion was introduced in [15] via the equivalent specific work of adhesion. The JKR theoretical framework combined with the Maugis-Dugdale model of adhesion was used in [16] to describe the frictional contact between the finger and the screen under electrovibration. In the present paper, we develop a DMT-type macro model of electroadhesion by evaluating the attractive force taking into account the electrostatic attraction forces acting both inside the contact area and within the annular zone surrounding the circular contact area. The specificity of the finger contact deformation is accounted for by means of the Winkler–Fuss model and the soft finger contact model [17], which are implemented within a self-consistent framework. A special focus has been set on the evaluation of the friction force under electric voltage in the absence of external load. Since the model by Xydas and Kao [17] is mainly based on the analysis of contact deformation of anthropomorphic soft fingers in robotics, the models developed here are directly suitable only for describing the frictional contact between an "artificial" soft finger and a hard screen.

II. Contact problem of electroadhesion

With the aim of modeling contact between the human finger and a touchscreen under electroadhesion, we consider an axisymmetric contact problem for an elastic solid pressed against a flat rigid surface. Let the undeformed surface of the elastic solid be described by the paraboloidal shape function

$$\Phi(r) = \frac{r^2}{2R}, \tag{1}$$

where r is the radial coordinate of the cylindrical coordinate system (r, ϕ, z), R is the radius of curvature at the initial point of contact. We note that the solution constructed below does not depend on the angular coordinate ϕ due to the axisymmetry.

Under the action of an external vertical (normal) load, F_N, the elastic solid undergoes some contact approach towards the rigid surface, δ, thereby establishing a circular contact area, A, of radius a. Thus, inside the contact area the vertical surface displacement is given by

$$u_z(r) = \delta - \Phi(r), \quad r < a. \tag{2}$$

We note that Eqs. (1) and (2) are usual assumptions of the Hertzian contact mechanics about the local contact geometry and the kinematic condition of contact [18].

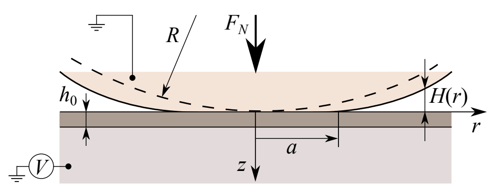

Fig. 2. Schematic of axisymmetric contact interaction between a soft finger and a rigid plane surface of a dielectric coating covering a conductive substrate. The dash line shows the position of the undeformed finger's surface.

Let us now additionally assume that the elastic solid is conducting, whereas the rigid counter-body is made of a conducting substrate (which is assumed to be rigidly fixed) and a dielectric coating of thickness h_0 with the relative permittivity ε. Under a voltage, V, applied between the conducting substrate and the conducting elastic solid, the surface of the elastic solid will experience distributed attractive forces, whose values depend on the normal gap, $H(r)$, between the solid's deformed surface and the surface of the dielectric coating (see Fig. 2).

In what follows, we apply the parallel-plate-capacitor model [10, 19] and express the normal component of attractive stress as follows [15]:

$$\sigma(H) = \frac{V^2 \varepsilon_0 \varepsilon^2}{2(h_0 + \varepsilon H)^2}. \tag{3}$$

Here, ε_0 is the vacuum permittivity. It is to note here [16] that a macroscopic model can approximately account for the discrete nature of contact due to the finger ridges by introducing into consideration an equivalent air gap. For the sake of simplicity, the dielectric constant of air can be taken equal to 1.

It is clear that inside the established contact area, $H = 0$, while outside the contact area, the normal stress at the deformed surface is

$$\sigma_{zz}(r) = \sigma\big(H(r)\big), \quad r > a, \tag{4}$$

where the normal gap between the contacting surfaces is defined as

$$H(r) = \Phi(r) - \delta + u_z(r). \tag{5}$$

Therefore, the contact approach δ and the contact radius a are related by the equation

$$\delta = \Phi(a) + u_z(a). \tag{6}$$

On the other hand, inside the contact area, we have

$$\sigma_{zz}(r) = \sigma_0 - p(r), \quad r < a, \tag{7}$$

where $p(r)$ is the contact pressure due to elastic deformation, and $\sigma_0 = \sigma(0)$, that is

$$\sigma_0 = \frac{V^2 \varepsilon_0 \varepsilon^2}{2h_0^2}. \tag{8}$$

The equation of equilibrium implies that

$$F_N = 2\pi \int_0^a p(r) r\, dr - \pi a^2 \sigma_0 - 2\pi \int_a^b \sigma\big(H(r)\big) r\, dr, \tag{9}$$

where b is the so-called external characteristic size of the elastic solid, where the electroadhesion is taken into consideration. (For instance, for an elastic sphere of radius R, we can take $b = R$.)

To conclude the contact problem formulation, an equation governing the relation between the surface displacements $u_z(r)$ and the contact pressure $p(r)$ is required. In the present paper, we develop a simple model based on a simplified description of elastic deformation in the framework of the generalized nonlinear Winkler–Fuss model.

III. Simple model of electroadhesive contact

Our first simplification is to employ the Winkler–Fuss model [20, 21] to evaluate the surface stress as follows:

$$\sigma_{zz}(r) = -k u_z(r), \quad r \leq a. \tag{10}$$

Here, k is the Winkler–Fuss stiffness coefficient.

Our second simplification is the DMT hypothesis [4, 7] that the contact deformations do not increase the adhesion force, so that outside the contact area Eq. (5) simplifies as

$$H(r) = \Phi(r) - \delta, \quad r > a. \tag{11}$$

Correspondingly, from Eqs. (4) and (11), it follows that

$$\sigma_{zz}(r) = \sigma\big(\Phi(r) - \delta\big), \quad r > a. \tag{12}$$

Now, from the continuity of surface stresses, we derive the condition

$$\sigma_{zz}(a) = \sigma_0, \tag{13}$$

which follows from Eq. (4).

Thus, from Eqs. (2), (7), and (10), we readily find that

$$p(r) = \sigma_0 + k\big(\delta - \Phi(r)\big), \quad r < a. \tag{14}$$

At the same time, Eqs. (10) and (13) yield

$$\delta = \Phi(a) - \frac{\sigma_0}{k}. \tag{15}$$

We note that from Eqs. (6) and (15), it follows that $u_z(a) = -\sigma_0/k$.

The substitution of (12) and (14) into Eq. (9) yields

$$F_N = \pi a^2 k\delta - 2\pi k \int_0^a \Phi(r) r\, dr - 2\pi \int_a^b \sigma\big(\Phi(r) - \delta\big) r\, dr. \tag{16}$$

In view of (3), three equations (8), (15), and (16) relate four physical quantities a, F_N, δ, and V. It is to emphasize that the value of the geometric parameter b is supposed to be known.

In the case of parabolic shape function (1), Eq. (15) takes the form

$$\delta = \frac{a^2}{2R} - \frac{\sigma_0}{k}. \tag{17}$$

Further, in light of (8), Eq. (3) can be represented as

$$\sigma(H) = \sigma_0 \Big(1 + \frac{\varepsilon H}{h_0}\Big)^{-2}. \tag{18}$$

Now, by using (17), we can exclude δ from Eq. (16). Moreover, by using formula (18) and carrying out the integration in Eq. (16), we obtain

$$\begin{aligned} F_N \quad = \quad & \frac{\pi a^4 k}{4R} - \pi a^2 \sigma_0 - \frac{2\pi\sigma_0 R h_0}{\varepsilon} \bigg\{ \Big(1 + \frac{\varepsilon\sigma_0}{h_0 k}\Big)^{-1} \\ & - \Big(1 + \frac{\varepsilon\sigma_0}{h_0 k} + \frac{\varepsilon(b^2 - a^2)}{2Rh_0}\Big)^{-1} \bigg\} \end{aligned} \tag{19}$$

The force of friction in the slip stage is assumed to be

$$F_T = 2\pi\mu \int_0^a p(r) r\, dr, \tag{20}$$

where μ is the coefficient of friction.

The substitution of (14) into Eq. (20) yields

$$F_T = \mu k \frac{\pi a^4}{4R}, \tag{21}$$

where Eq. (17) has been taken into account.

IV. Soft finger model and the equivalent Winkler–Fuss stiffness coefficient

In the papers [17, 22], the following constitutive equations have been used for incompressible nonlinear elastic materials (see also [23]):

$$\epsilon_{ij} = \Big(\frac{\sigma_e}{\epsilon_0 E}\Big)^n \frac{\partial \sigma_e}{\partial \sigma_{ij}}. \tag{22}$$

Here, σ_e is the Von Mises stress, ϵ_{ij} are the infinitesimal strain components, ϵ_0 is a characteristic elastic strain, E is Young's modulus (a material constant with stress unit), and n is the stress exponent ($0 \leq n \leq 1$).

The soft finger model [17, 24] predicts a power-law nonlinear relation $a = cF_N^\gamma$ between the contact radius a and the normal contact force F_N, where c is a dimensional constant that, in particular, depends on the size and curvature of the soft finger, and

$$\gamma = \frac{n}{2n+1}. \tag{23}$$

Because the local contact geometry is characterized by the paraboloid (see Eq.(1)), we have $a^2 \sim \delta R$. Now, taking into account that $F_N \sim a^{1/\gamma}$, our dimensionless analysis of the soft finger model yields the following form of the force-displacement relationship:

$$F_N = B_\gamma E R^2 \Big(\frac{\delta}{R}\Big)^{1/(2\gamma)}. \tag{24}$$

Here, B_γ is a dimensionless constant, which can be obtained experimentally (see, e.g., [25]).

On the other hand, the Winkler–Fuss foundation model predicts the relation

$$F_N = \pi k R^3 \Big(\frac{\delta}{R}\Big)^2. \tag{25}$$

It is easy to check that Eqs. (24) and (25) will coincide, provided the equivalent Winkler–Fuss stiffness coefficient is taken to be

$$k = B_\gamma^{4\gamma} \frac{E}{\pi R} \Big(\frac{ER^2}{F_N}\Big)^{4\gamma - 1}, \tag{26}$$

where γ is determined by formula (23).

V. Touchscreen-finger friction under voltage

Let us apply the developed model to the case of contact in the absence of the external normal load, i.e., when $F_N = 0$.

A. *Upper bound for the friction force*

For the paraboloidal contact geometry (1), it makes sense to consider the limit situation as $b \to \infty$. Then, according to Eqs. (19) and (21), the upper bound for the friction force is given by

$$F_T = \mu\pi a^2 \sigma_0 + 2\pi\mu\sigma_0 R h_0 \varepsilon^{-1} \Big(1 + \frac{\varepsilon\sigma_0}{h_0 k}\Big)^{-1}, \tag{27}$$

where the contact radius a solves the equation

$$\frac{ka^4}{4R} = \sigma_0 a^2 + 2\sigma_0 R h_0 \varepsilon^{-1} \Big(1 + \frac{\varepsilon\sigma_0}{h_0 k}\Big)^{-1}. \tag{28}$$

It is suggested to make use of Eqs. (27) and (28) within a self-consistent framework by requiring that the Winkler–Fuss modulus k is determined by Eq. (26), where F_N is replaced with F_T/μ, that is

$$k = B_\gamma^{4\gamma} \frac{E}{\pi R} \Big(\frac{\mu E R^2}{F_T}\Big)^{4\gamma - 1}. \tag{29}$$

We recall that σ_0 is proportional to the voltage squared (see Eq. (8)). Note also that Eq. (27) was derived from Eq. (19) under the simplifying assumption that $R/b \ll 1$.

Let us introduce dimensionless variables

$$f = \frac{F_T}{\mu\pi R^2 \sigma_0}, \quad \alpha = \frac{a}{R}. \tag{30}$$

Then, the substitution of (30) into Eq. (29) yields

$$\frac{kh_0}{\sigma_0} = \frac{B_\gamma^{4\gamma}}{\pi^{4\gamma}} \Big(\frac{\sigma_0}{E}\Big)^{-4\gamma} \frac{h_0}{R} f^{1-4\gamma}. \tag{31}$$

Therefore, in light of (30) and (31), Eqs. (27) and (28) imply

$$f = \frac{B_\gamma}{\pi} \Big(\frac{\sigma_0}{E}\Big)^{-1} \Big(\frac{\alpha}{\sqrt{2}}\Big)^{1/\gamma}. \tag{32}$$

Hence, the substitution of (30)–(32) into Eq. (27) results in the following equation for the relative contact radius:

$$\alpha^{1/\gamma} = \frac{\pi 2^{1/(2\gamma)}\sigma_0}{B_\gamma E}\biggl\{\alpha^2 + \frac{2h_0}{\varepsilon R}\biggl(1 + \frac{\pi\varepsilon\sigma_0 R}{B_\gamma E h_0}\Bigl(\frac{\alpha}{\sqrt{2}}\Bigr)^{(4\gamma-1)/\gamma}\biggr)^{-1}\biggr\}. \tag{33}$$

For a given value of voltage drop across the contact interface, Eqs. $(30)_2$ and (33) determine α as a function of V, whereas Eqs. $(30)_1$ and (32) produce the sought for relation between V and F_T. It is interesting to note that the same result can be recovered by first assuming that $b = R$ in Eq. (19) and second by simplifying the resulting equation under the condition that R is much larger than h_0.

Finally, let us consider the friction under voltage in the Winkler–Fuss model-based framework (27), (28). It is readily seen that Eqs. (27) and (28) yield

$$F_T = \mu\frac{\pi k a^4}{4R}, \tag{34}$$

where Eq. (28) can be rewritten in the form of bi-quadratic equation

$$\Bigl(\frac{a}{R}\Bigr)^4 - \frac{4\sigma_0}{Rk}\Bigl(\frac{a}{R}\Bigr)^2 - \frac{8\sigma_0 h_0}{\varepsilon R^2 k}\Bigl(1 + \frac{\varepsilon\sigma_0}{h_0 k}\Bigr)^{-1} = 0, \tag{35}$$

from where it follows that

$$\frac{a^2}{R^2} = \frac{2\sigma_0}{Rk}\biggl\{1 + \sqrt{1 + \frac{2h_0 k}{\varepsilon\sigma_0}\Bigl(1 + \frac{\varepsilon\sigma_0}{h_0 k}\Bigr)^{-1}}\biggr\}, \tag{36}$$

and the substitution of (36) into Eq. (34) results in the voltage-force relation.

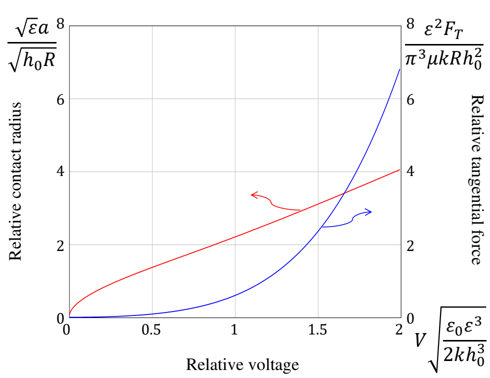

Fig. 3. Variation of the contact radius and contact friction force in the absence of external load as functions of the voltage.

It is interesting that, in view of (8), each of Eqs. (34) and (36) can be characterized by a single core line, provided an appropriate scaling procedure was applied (see Fig. 3).

B. Lower bound for the friction force

Let us now consider the case where $b = a$, so that the attractive forces outside the contact area are neglected. Then, Eq. (19) simplifies to

$$F_N = \frac{\pi a^4 k}{4R} - \pi a^2\sigma_0, \tag{37}$$

from where, in the absence of external normal load ($F_N = 0$), it follows that

$$a^2 = \frac{4R\sigma_0}{k}. \tag{38}$$

At the same time, Eq. (21) reduces to

$$F_T = \mu\pi a^2\sigma_0. \tag{39}$$

Hence, in view of Eq. (26) for the Winkler–Fuss stiffness coefficient, Eqs. (39) and (38), respectively, yield

$$F_T = \mu(2\pi\sigma_0)^{1/(1-2\gamma)}R^2(B_\gamma E)^{-2\gamma/(1-2\gamma)}, \tag{40}$$

$$\pi a^2 = (2\pi)^{1/(1-2\gamma)}R^2\Bigl(\frac{\sigma_0}{B_\gamma E}\Bigr)^{2\gamma/(1-2\gamma)}. \tag{41}$$

We note that, in view of (23), we have $1/(1-2\gamma) = 2n+1$ and $2\gamma/(1-2\gamma) = 2n$.

On the other hand, for the Winkler–Fuss model, Eqs. (39) and (38) simply imply that

$$F_T = \mu\frac{4\pi R}{k}\sigma_0^2, \tag{42}$$

because the stiffness coefficient k is assumed to be constant.

C. Touchscreen-finger friction in the Winkler–Fuss model

In many practical situations we have $h_0 \ll R$, and therefore, Eq. (19) can be simplified as follows:

$$F_N = \frac{\pi a^4 k}{4R} - \pi a^2\sigma_0 - \frac{2\pi\sigma_0 R h_0}{\varepsilon}\Bigl(1 + \frac{\varepsilon\sigma_0}{h_0 k}\Bigr)^{-1}. \tag{43}$$

Let us introduce the dimensionless normal force as

$$f_N = \frac{F_N}{kRh_0^2}, \quad \alpha = \frac{a}{R}. \tag{44}$$

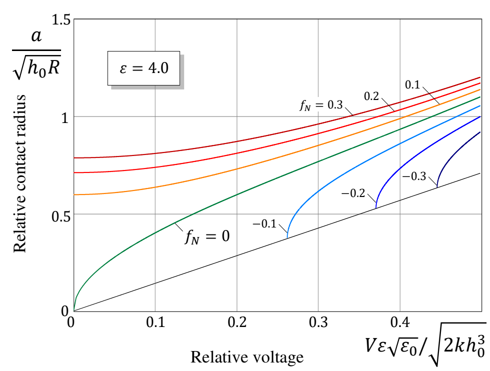

Fig. 4. Variation of the contact radius as a function of the voltage for different values of the relative normal force.

The behavior of the solution to Eqs. (21) and (44) is shown in Figs. 4 and 5 both for positive and negative values of the relative normal force (44).

VI. Discussion

In this section, we overview the proposed approach and discuss its assumptions and limitations.

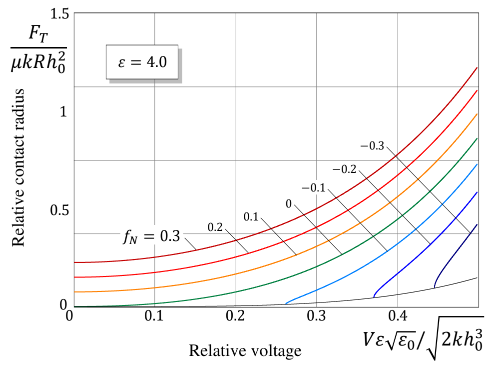

Fig. 5. Variation of the contact friction force as a function of the voltage for different values of the relative normal force.

A. Some generalizations

First of all, observe that, strictly speaking, the Winkler–Fuss model is applicable to thin elastic compressible layers (see, e.g., [26]). The Winkler–Fuss model allows analytical treatment of contact problems even for non-convex indenters [27]. The JKR-type adhesive contact problem for thin elastic layers have been considered in [28–32]. The Winkler–Fuss modeling framework has been used in a number of papers (see, e.g., [33]) for the purpose of modeling the finger contact deformation. In our analysis, we made use of the Winkler–Fuss model not only to simplify the solution of the electroadhesion contact problem (1)–(9), but also to utilize it as an auxiliary solution for incorporating the soft finger model [17] via a self-consistent approach.

We would like to emphasize that the power-law relation (24) is of the same form as that obtained in [6, 34] in the problems of frictionless and non-adhesive contact for an anisotropic physically nonlinear elastic medium, using both the self-similarity technique and the Hertzian half-space approximation. Indeed, from Eq. (22) it follows that the elastic energy potential is a homogeneous function of degree $(1+n)/n$ with respect to the strain components ϵ_{ij}. Therefore, for a more general case of a power-law description of the finger shape (cf. Eq. (1))

$$\Phi(r) = \Lambda r^{\lambda}, \tag{45}$$

where $1 \le \lambda$ is a real number, and Λ is a dimensional constant, the self-similar solution of Borodich [6, 34] predicts that

$$a(F_N) = a(1)F_N^{n/[2n+\lambda-1]}, \tag{46}$$

$$\delta(F_N) = \delta(1)F_N^{\lambda n/[2n+\lambda-1]}, \tag{47}$$

where $a(1)$ and $\delta(1)$ are the contact radius and the contact approach under action of unit load.

In the case $\lambda = 2$, when Eq. (45) reduces to Eq. (1) with $\Lambda = (2R)^{-1}$, Eq. (47) implies $F_N \sim \delta^{(2n+1)/(2n)}$, which, in view of (23), exactly corresponds to Eq. (24). By the way, the self-similarity formulas (46) and (47) are recommended for the use in the soft finger model.

Further, it is to note again that the upper bound solution presented above for the special case $F_N = 0$ assumes that $R \ll b$. However, this simplification can be easily removed by replacing the factor $(1+\varepsilon\sigma_0/(h_0 k))^{-1}$ in Eqs. (27), (28), and (35) with the expression from the curly braces in (19). This will modify Eqs. (33) and (35), so that the simple formula (36) is not valid any more.

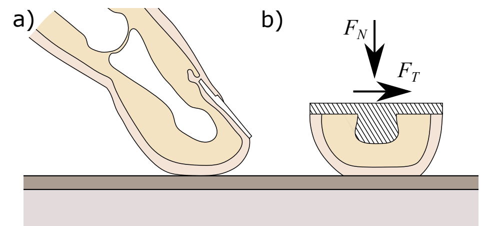

Fig. 6. Schematic of the human finger structure, including skin, fat, bone (distal phalanx), and nail (a) and its idealization for an equivalent hemispherical contact geometry and a layered tissue inhomogeneity in normal and tangential contact under external loads, F_N and F_T.

It is interesting to compare the lower bounds for the friction force (40) and (42). The soft finger model-based Eq. (40) predicts that $F_T \sim \sigma_0^{2n+1}$, whereas the Winkler–Fuss model-based Eq. (34) implies that $F_T \sim \sigma_0^2$ as $\sigma_0 \to 0$. On the other hand, in view of (8), the Popov–Heß model [15] suggests that $F_T \sim \sigma_0^{5/3}$, which is quite close to the last case, whereas $2n+1 \in (1, 3]$ as $n \in (0, 1]$.

It should be emphasized that the developed macro model suffers from several drawbacks. First, formula (3) for the electric attraction stress does not account for the micro-gap due to the skin surface roughness. However, following [9], formula (3) can be generalized to account for the air film as well. Also, using the multi-scale modeling approach [35, 36], the effect of surface roughness can be incorporated into the analysis by constructing a hierarchy of mathematical models, which links the macro model with the corresponding micro model.

Second, the friction force is calculated by the simple formula (20), using the concept of the coefficient of friction. At the same time, it is known [37] that the friction of human skin against smooth surfaces can be more accurately described by the adhesion model of friction, which takes into account the real contact area and the interfacial shear strength [38]. The corresponding generalization can be achieved via the multi-scale modeling approach [39, 40].

Third, it is well known that the structure of the human finger is complicated (see Fig. 6), and, so far as possible, the issue of macro inhomogeneity should be accounted for upon prescribing the finger's deformation response. In particular, the developed model can be extended to describe more realistically the contact between the human finger and screen by taking into account the effect of skin, including the deformation of the stratum corneum as well as the presence of the finger ridges and roughness, which influence the air gap thickness.

B. Why apparent contact area is reduced during sliding?

Let us now discuss the main limitation of the proposed approach, and this is the infinitesimal strain theory, which is the basis of the Hertzian contact mechanics and commonly adopted in the JKR and DMT theories of adhesive contact.

Indeed, consider a weightless elastic spring of stiffness k and length l (see Fig. 7) with its upper end fixed and a gap of

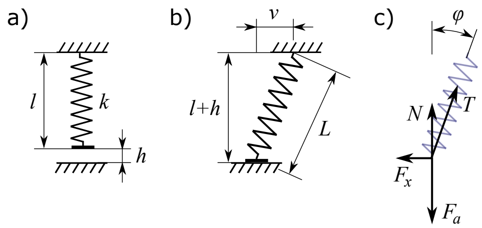

Fig. 7. Schematic of the tangential deformation of the soft tissue: (a) a spring of stiffness k in the undeformed state; (b) the deformed state of the spring, which is brought in contact with a flat base by an attractive force, F_a; (c) the force diagram in the slip state (limit state of equilibrium).

width h between the lower end and a rigid base. As the first step, under the action of a vertical force, F_a, the lower end can be brought into contact with the initial support reaction $N_0 = F_a - kh$. As the second step, the upper spring end is displaced horizontally to some distance, v, while the lower spring end is assumed to be restricted due to friction. Let φ denote the current angle between the spring axis and the vertical, so that $\sin\varphi = v/L$ and $\cos\varphi = (h+l)/L$, where L is the length of the deformed spring. The spring tension force, T, is proportional to the elongation $(L-l)/l$, that is $T = k(L-l)$. The equilibrium equations yield $N = F_a - T\cos\varphi$ and $F_x = T\sin\varphi$.

Finally, in the limit state before the onset of slip, we have $F_x = \mu N$. Therefore, the maximum displacement will be $v_* = h(1+\chi^{-1})\tan\varphi_*$, where $\chi = h/l$ and φ_* is the maximum value of φ. By excluding the variables N and T from the above equations, we find that φ_* is the root of the equation

$$\frac{F_a}{kh} = \frac{1}{\chi}\Big(\frac{1}{\mu}\tan\varphi + 1\Big)(\chi + 1 - \cos\varphi). \tag{48}$$

It can be shown that $N_* < N_0$, so that the shear deformation results in diminishing the contact pressure as well as in detachment of those elements, where the initial contact pressure is relatively low. Thus, the initial decrease in the contact area at the beginning of slip can be explained by the nonlinear shear deformation of the soft tissue.

It is to note that the issue of contact area reduction under tangential loading was observed in [41–43] in experiments with a soft rubber with high adhesive hysteresis and a rough elastomer block. Very recently, this phenomenon was modeled theoretically in [44–46] based on the linear elasticity fracture mechanics and using the concept of mixed-mode interfacial fracture [47]. Here we emphasize the effect of nonlinear shear deformation.

VII. Conclusion

To conclude, in the present work we have emphasized the importance of the macro contact pressure distribution across the apparent contact area in the electroadhesive contact between a soft finger and a touchscreen. The main result is the formulation of the DMT-type electroadhesive contact problem (2)–(9) and its approximate solution obtained using the Winkler–Fuss deformation model with the equivalent stiffness coefficient, which can be determined experimentally.

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