# An Axiomatic Approach to Truth Discovery

Extended Abstract

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## KEYWORDS

Truth discovery; Axioms; Trust and reputation; Social choice theory

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## 1 INTRODUCTION

There is an increasing amount of data available in today's world, particularly from the web, social media platforms and crowdsourcing systems. The openness of such platforms makes it simple for a wide range of users to share information quickly and easily, potentially reaching a wide international audience. It is inevitable that amongst this abundance of data there are *conflicts*, where data sources disagree on the truth regarding a particular object or entity. This can be caused by low-quality sources mistakenly providing erroneous data, or by malicious sources aiming to misinform.

Truth discovery methods have emerged to resolve such conflicts and find the true facts by considering the *trustworthiness* of sources [9, 10]. The general principle is that believable facts are those claimed by trustworthy sources, and trustworthy sources are those that claim believable facts. Application areas include real-time traffic navigation [7], crowdsourcing and social sensing [11, 16].

In this paper we initiate work on formal foundations for truth discovery by providing a general framework which allows algorithms to be evaluated in a principled way based on their theoretical properties. To do so we pose truth discovery as a social choice problem and apply the *axiomatic approach*, as has been successfully done for closely related problems such as judgment aggregation [8], voting theory [17] and ranking systems [1, 2, 13].

## 2 A FRAMEWORK FOR TRUTH DISCOVERY

We consider fixed finite and mutually disjoint sets $\mathcal{S}$, $\mathcal{F}$ and $\mathcal{O}$, called the *sources*, *facts* and *objects* respectively. Sources represent providers of information (e.g. social media users, websites), facts represent (potentially false) pieces of information, and objects represent real-world entities or questions to which the facts relate.

A core definition of the framework is that of a *truth discovery network*, which represents the input to a truth discovery problem. In keeping with the literature, we model this as a tripartite graph with certain constraints [9, 15].

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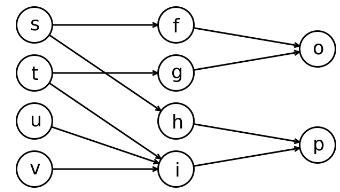


**Figure 1: Example TD network with sources $\{s, t, u, v\}$, facts $\{f, g, h, i\}$ and objects $\{o, p\}$.**

*Definition 2.1.* A *truth discovery network* (hereafter a *TD network*) is a directed graph $N = (V, E)$ where $V = \mathcal{S} \cup \mathcal{F} \cup \mathcal{O}$, and $E \subseteq (\mathcal{S} \times \mathcal{F}) \cup (\mathcal{F} \times \mathcal{O})$ has the following properties:

(1) For each $f \in \mathcal{F}$ there is a unique $o \in \mathcal{O}$ with $(f, o) \in E$, denoted $\text{obj}_N(f)$. That is, each fact is associated with exactly one object.
(2) For $s \in \mathcal{S}$ and $o \in \mathcal{O}$, there is at most one directed path from $s$ to $o$. That is, sources cannot claim multiple facts for an object.
(3) $(\mathcal{S} \times \mathcal{F}) \cap E$ is non-empty. That is, at least one claim is made.

We say that $s$ *claims* $f$ when $(s, f) \in E$. Let $\mathcal{N}$ denote the set of all TD networks. Note that the special case of our framework where every object has exactly two associated facts is close to the setting studied in judgment aggregation [8] and binary aggregation with abstentions [5, 6]. As an example, consider the network $N$ shown in Fig. 1. Here sources $s$ and $t$ disagree on the true fact for object $o$ (claiming $f$ and $g$ respectively). Sources $u$ and $v$ do not comment on object $o$, but agree with $t$ for object $p$.

To simplify the notation in what follows, for a network $N = (V, E)$ we write $\text{facts}_N(s) = \{f \in \mathcal{F} : (s, f) \in E\}$ for the set of facts claimed by a source $s$, and $\text{src}_N(f) = \{s \in \mathcal{S} : (s, f) \in E\}$ for the set of sources claiming a fact $f$.

We now turn to the output of truth discovery. Contrary to many approaches in the literature, we consider the output to be *rankings* of the sources and facts. This is because for the theoretical analysis we wish to perform it is only important that a source is *more trustworthy* than another; the particular numeric scores produced by an algorithm are irrelevant. This point of view is common across the social choice literature. Formally, let $\mathcal{L}(X)$ denote the set of all total preorders on a set $X$. We define a *truth discovery operator* as follows.

*Definition 2.2.* A *truth discovery operator* $T$ (hereafter *TD operator*) is a mapping $T : \mathcal{N} \to \mathcal{L}(\mathcal{S}) \times \mathcal{L}(\mathcal{F})$. We shall write $T(N) = (\sqsubseteq_N^T, \preceq_N^T)$, i.e. $\sqsubseteq_N^T$ is a total preorder on $\mathcal{S}$ and $\preceq_N^T$ is a total preorder on $\mathcal{F}$. We interpret $s_1 \sqsubseteq_N^T s_2$ to mean $s_2$ is *at least as trustworthy* as $s_1$ (and similarly for facts).

# 3 AXIOMS FOR TRUTH DISCOVERY

We now introduce axioms for truth discovery. Many axioms are adapted from the social choice literature, although modifications are necessary in places to match the semantics of truth discovery.

A guiding principle of many truth discovery approaches is that sources claiming highly believed facts are trustworthy, and that facts backed by highly trusted sources are believable; the source and fact rankings should *cohere* in this sense. We capture this intuition for specific cases in our first two axioms, which are inspired by *transitivity* axioms for ranking systems [2, 13].

First, a preliminary definition is required. For $Y, Y' \subseteq \mathcal{F}$, we say $Y$ is *less believable* than $Y'$ with respect to a TD network $N$ and operator $T$ if there is a bijection $\phi : Y \to Y'$ such that $f \preceq_N^T \phi(f)$ for each $f \in Y$, and $\hat{f} \prec_N^T \phi(\hat{f})$ for some $\hat{f} \in Y$. For $X, X' \subseteq \mathcal{S}$ we define $X$ *less trustworthy* than $X'$ with respect to $N$ and $T$ similarly.

**Axiom 1** (Source Coherence). *If* $\mathsf{facts}_N(s_1)$ *is less believable than* $\mathsf{facts}_N(s_2)$ *with respect to N and T, then* $s_1 \sqsubset_N^T s_2$.

**Axiom 2** (Fact Coherence). *If* $\mathsf{src}_N(f_1)$ *is less trustworthy than* $\mathsf{src}_N(f_2)$ *with respect to N and T, then* $f_1 \prec_N^T f_2$.

For the next axiom, we say TD networks $N$ and $N'$ are *equivalent* if there is a graph isomorphism $\pi$ between them that preserves sources, facts and objects, i.e., $\pi(s) \in \mathcal{S}$, $\pi(f) \in \mathcal{F}$ and $\pi(o) \in \mathcal{O}$. In such case we write $\pi(N)$ for $N'$.

**Axiom 3** (Symmetry). *If N and* $N' = \pi(N)$ *are equivalent networks, then* $s_1 \sqsubseteq_N^T s_2$ *iff* $\pi(s_1) \sqsubseteq_{N'}^T \pi(s_2)$ *and* $f_1 \preceq_N^T f_2$ *iff* $\pi(f_1) \preceq_{N'}^T \pi(f_2)$.

The next axiom states that when $f$ receives extra support from a new source $s$, its ranking should receive a strictly positive boost. Note this axiom assumes sources do not have 'negative' trust levels; otherwise extra support could be reason for *decreased* belief in $f$.

**Axiom 4** (Monotonicity). *Suppose* $N \in \mathcal{N}, s \in \mathcal{S}, f \in \mathcal{F} \setminus \mathsf{facts}_N(s)$. *Write E for the set of edges in N, and let N′ be the network with edges* $E' = \{(s, f)\} \cup E \setminus \{(s, g) : g \neq f, \mathsf{obj}_N(g) = \mathsf{obj}_N(f)\}$ *Then for all* $g \neq f$, $g \preceq_N^T f$ *implies* $g \prec_{N'}^T f$.

An important idea in social choice is *independence*. In voting, a core axiom is IIA [4], which says the social ranking of $a$ and $b$ depends only on the individual preferences of $a$ vs $b$. In the truth discovery context, an analogous axiom would state that the ranking of facts $f_1$ and $f_2$ for some object $o$ depends only on the sources which make a claim for $o$.

However, one may claim this property is *undesirable*, since truth discovery aims to infer the believability of the $f_i$ based on the trustworthiness of their sources, which may in turn depend on facts for objects other than $o$. This undesirability can be made precise: in combination with Symmetry and Monotonicity, it implies $T$ ranks facts for the same object by a simple *voting* procedure, where each source votes for the facts it claims (c.f. [5]). That is, source trustworthiness is not considered at all. Formally:

**Axiom 5** (Per-object Independence (POI)). *Let* $o \in \mathcal{O}, N_1, N_2 \in \mathcal{N}$. *Suppose* $\mathsf{obj}_{N_1}^{-1}(o) = \mathsf{obj}_{N_2}^{-1}(o) = F_o$ *and* $\mathsf{src}_{N_1}(f) = \mathsf{src}_{N_2}(f)$ *for each* $f \in F_o$. *Then* $f_1 \preceq_{N_1}^T f_2$ *iff* $f_1 \preceq_{N_2}^T f_2$ *for all* $f_1, f_2 \in F_o$.

THEOREM 3.1. *Let T be any operator satisfying Symmetry, Monotonicity and POI, and suppose* $\mathsf{obj}_N(f_1) = \mathsf{obj}_N(f_2)$. *Then* $f_1 \preceq_N^T f_2$ *iff* $|\mathsf{src}_N(f_1)| \leq |\mathsf{src}_N(f_2)|$.

| | Voting | SC-Voting | Sums [12] | U-Sums |
|---|---|---|---|---|
| S. Coherence | X | ✓ | ✓ | ✓ |
| F. Coherence | ✓ | X | ✓ | ✓ |
| Symmetry | ✓ | ✓ | ✓ | ✓ |
| Mon. | ✓ | ✓ | X | ? |
| POI | ✓ | ✓ | X | X |
| PCI | ✓ | X | X | ✓ |

**Table 1: Satisfaction of the axioms for the various operators.**

Recall that the Coherence axioms encode the idea that source trustworthiness should inform the fact ranking and vice versa. Clearly the voting procedure implied by Thm. 3.1 does not meet these criteria, so we should expect Source and Fact Coherence to fail. Indeed, we have the following impossibility result.

THEOREM 3.2. *There is no TD operator satisfying Source Coherence, Fact Coherence, Symmetry, Monotonicity and POI.*

Since the Coherence axioms represent the core intuition behind truth discovery, Thm. 3.2 provides further justification that POI is undesirable. Fig. 1 provides the counterexample used in the proof: applying the voting procedure to $h, i$ and $f, g$ gives $h \prec_N^T i$ and $f \approx_N^T g$, so by Source Coherence we get $s \sqsubset_N^T t$; but then Fact Coherence gives $f \prec_N^T g$ – a contradiction.

The problem with POI is that the indirect links – such as between $f$ and $h$ in Fig. 1 – cannot be considered in the rankings. Our final rendering of independence allows such links to be used, while still ensuring the ranking of facts (and sources) does not depend on irrelevant details of the input network.

**Axiom 6** (Per-component Independence (PCI)). *Suppose* $N_1$ *and* $N_2$ *have a common connected component G. Then for all* $s_1, s_2 \in G \cap \mathcal{S}$ *and* $f_1, f_2 \in G \cap \mathcal{F}$, *we have* $s_1 \sqsubseteq_{N_1}^T s_2$ *iff* $s_1 \sqsubseteq_{N_2}^T s_2$ *and* $f_1 \preceq_{N_1}^T f_2$ *iff* $f_1 \preceq_{N_2}^T f_2$.[1]

Table 1 shows the satisfaction of our axioms for some example operators. *Voting* is a trivial baseline operator which ranks all sources equally and ranks facts according to the number of claims received. It is possible to modify *Voting* to achieve Source Coherence at the expense of Fact Coherence (which *Voting* only satisfies vacuously) and PCI; we call this operator *SC-Voting*. For a non-trivial example, we considered *Sums* [12], which is well-known in the literature. It satisfies both Coherence axioms, but surprisingly fails PCI and Monotonicity. In some sense the cause of PCI failure is a numeric normalisation step at each iteration of the *Sums* algorithm; removing this step and modifying the convergence criteria accordingly we obtain a new operator we call *U-Sums*, which retains Coherence whilst satisfying PCI.

# 4 CONCLUSION

In this paper we formalised a mathematical framework for truth discovery and initiated axiomatic study of the problem. We provided an impossibility result and brief axiomatic analysis of example operators. Future work in this area will investigate state-of-the-art algorithms with respect to the axioms, and determine whether existing methods actually find the *truth* as opposed to simply *consensus* amongst sources.

[1]We have found that real-world datasets with multiple connected components do exist: for example the *Book* and *Restaurant* datasets found at the following web page each contain two connected components: http://lunadong.com/fusionDataSets.htm

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