Optimal Inflation, Monetary Integration, and Asymmetric Sticky Prices

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Abstract

This paper explores the optimal rate of trend inflation in open economies with and without a monetary union, accounting for empirically observed differences in the degree of price stickiness across countries. In a closed economy, the optimal inflation rate is negative to offset the markup caused by imperfect competition. In an open economy there is a 'beggar-thy-neighbour' incentive and the optimal inflation is positive. Monetary union is globally welfare improving because it removes this externality. In both setups, as price stickiness increases, the degree of price dispersion increases, and the optimal inflation rate tends towards zero. Gains from monetary integration are higher for economies with more flexible prices.

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1 Introduction

The member states of the Eurozone are heterogeneous, yet they share the same monetary policy. The most debated are the core-periphery differences while different levels of price stickiness attract less interest in the literature. This paper studies the optimal rate of inflation in an open economy and gains from monetary integration when price stickiness differs across member economies. Dhyne et al. (2006) and Dhyne et al. (2009) estimate price change frequencies in the Eurozone and find that there is substantial degree of heterogeneity across both products and countries. Table 1 shows that these differences can be as high as double.

In previous work Cooley and Quadrini (2003) and Cooper and Kempf (2003) use models of perfect competition with flexible prices and commitment to show that monetary policy competition leads to higher long-term inflation: a consequence of the 'beggar-thy-neighbour' incentive. Monetary integration, even when it means losing an ability to react optimally to shocks, improves welfare by reducing losses from high trend inflation. Arseneau (2007) and Evans (2012) find that this inflationary bias of openness is significantly damped by the degree of imperfect competition. When this dampening effect is strong enough, monetary policy becomes inward looking: trend inflation and welfare gains from integration are significantly reduced. These results are derived in frameworks with flexible prices.

Sticky prices however, are an important feature of the economy. In the canonical New Keynesian model sticky prices constitute the main channel of monetary policy transmission. Most NK models used for policy analysis are approximated around a zero trend inflation (or, more precisely, zero-inflation steady-state). Thus, properly accounting for the effects of sustained trend inflation has become an important research agenda. Schmitt-Grohe and Uribe (2010), King and Wolman (2013), Ascari and Sbordone (2014), Kara and Yates (2017), and others show that when prices are sticky trend inflation endogenously increases the average markup and price dispersion in the economy which leads to large output and welfare losses. These results are derived in frameworks of a closed economy.

This paper contributes to the literature by bridging the gap between the two strands. I build a two-country general equilibrium model with money demand and sticky prices to study the welfare-maximizing rate of inflation under two regimes: local currencies and a monetary union. Households in both countries are infinitely lived and demand domestic and foreign money holdings via cash-in-advance constraints. Firms are imperfectly competitive and face Calvo staggered price setting. The solution to firms' problem is non-linear and generic for any given rate of trend inflation. A monetary authority commits to the rate of inflation that maximizes households' lifetime utility.

In the local currencies economy there are three effects that determine the optimal rate of inflation. The *spillover effect* works in the following way: inflation reduces the return on money holdings, which reduces labour supply. The gain from inflation, increased utility from leisure, lies entirely within the economy. The cost of inflation, the reduced return from money holdings, is spread across the two economies, as foreigners must hold the domestic currency to buy imports. This creates an inflationary pressure and a local monetary authority finds it optimal to create excessive inflation to manipulate the terms of trade. The second is the *markup effect*. The average markup increases in inflation and, because of the CIA constraint, works as a tax on labour. The higher the markup, the lower the labour supply and output. This creates a deflationary pressure in the economy as in Arseneau (2007). The third effect is the *price dispersion effect*. When prices are sticky any non-zero rate of inflation reduces output via the price dispersion. This creates incentives to pursue zero trend inflation. The combination of these three effects constitutes the first result of this paper: in local currencies when prices are flexible the first effect dominates and the optimal rate of inflation is positive. When price are sticky the third effect dominates and the optimal rate of inflation is close to zero.

In a monetary union however, the spillover effect is absent. The common monetary authority internalizes international spillovers and only cares about the average markup and the price dispersion. Hence, the second result: in a monetary union when prices are flexible, the first effect dominates and the optimal rate of inflation is negative. When prices are sticky the optimal rate of inflation is close to zero.

As a common monetary authority internalizes the spillover effect, a monetary union enjoys lower inflation, higher labour supply and higher consumption. After adopting a common currency joint welfare increases, even though a disutility from labour goes up. Welfare gains are the highest when both prices are flexible in both economies and, under conventional parametrization, are equal to 1.5% of lifetime domestic consumption. When prices are sticky gains from integration are close to zero. When price stickiness is asymmetric, the gains from a union are distributed unevenly and a country with more flexible prices benefits more. This constitutes the third result of this paper.

In the New Keynesian literature, where price stickiness is the main transmission channel of a monetary policy, the same level of stickiness is a common assumption.[1] Related literature that allows for asymmetric rigidities includes Liu and Pappa (2008), who study monetary policy coordination when sectoral composition is different in two countries and sectors may differ inter alia in their price stickiness. Gains from coordination are significant as a single social planner eliminates terms-of-trade externality. In this framework it corresponds to the elimination of the spillover effect. In a recent contribution Adam and Weber (2019) incorporate systematic differences in firm productivity trends to show that the optimal rate of inflation is positive. Within the traditional Mundell-Fleming model, De Grauwe (2014) shows that the cost of partici-

[1]See for example Clarida, Gali, and Gertler (2001), Clarida, Gali, and Gertler (2002),Benigno and Benigno (2003),Pappa (2004), Gali and Monacelli (2008).

Table 1: Price stickiness across selected Eurozone economies

Country	Frequency of price changes (in %)	Months between adjustments
Germany	13.5	7.4
Spain	13.3	7.5
France	20.9	4.8
Italy	**10.0**	10
Portugal	**21.1**	4.7

Notes: The first column represents the average monthly frequency of price adjustments based on a sample of 50 products representative of the CPI using country-specific weights. The second column presents pseudo-duration calculated as an inverse of the adjustment frequency.

Source: Author's calculations based on Dhyne et al. (2006).

pation in monetary union increases in the degree of differences in economic structures (eg. nominal rigidities or shocks asymmetries). Benigno (2004) considers structural differences in goods markets and demonstrates that optimal monetary policy should follow inflation targeting, where higher weight is given to the inflation rate in the country with higher degree of nominal rigidity. Andersen and Seneca (2010) show that in the case of country-specific shocks, a country with lower degree of nominal stickiness always benefits in terms of macroeconomic stability. Palivos (2005) uses the overlapping generations framework to find that the optimal rate of inflation is higher than the one implied by Friedman rule in a model with heterogeneous altruism of agents.

2 The Model

Households and firms are infinitely lived and reside in two countries called Home and Foreign. Monopolistically competitive firms facing Calvo (1983) rigidities are optimizing as in Woodford (2011), King and Wolman (2013) and Ascari and Sbordone (2014). The model maintains the key insight from this literature, that in non-linear solution price markup is a function of trend inflation. Additionally, each firm sells its differentiated good both domestically and abroad. The two countries may differ in the level of price stickiness. Output is produced using immobile labour and there are no barriers to trade. Money enters the model in the form of two cash-in-advance constraints (for Home and Foreign consumption purchases) as in Arseneau (2007). At the beginning of each period monetary authority in each economy makes a lump-sum payment in domestic currency to domestic households. Foreign variables are denoted by an asterisk and the derivations for foreign variables are skipped when possible.

Total consumption by Home consumers is an aggregate of consumptions of Home and Foreign goods:

$$c_t = (c_t^h)^\theta (c_t^f)^{1-\theta}, \tag{1}$$

where θ is represents home bias in consumption. Consumption is an aggregate over differentiated goods produced across firms indexed by z:

$$c_t^i = \left(\int_0^1 c_t^i(z)^{\frac{\epsilon-1}{\epsilon}} dz \right)^{\frac{\epsilon}{\epsilon-1}} \qquad i = \{h, f\}, \tag{2}$$

where $\epsilon \geq 1$ is the elasticity of substitution among differentiated goods.

2.1 Firms

In each country there is a continuum of monopolistically competitive firms. Each firm faces two demand schedules given by Home and Foreign households' solution to their expenditure minimization problems:

$$c_t^h(z) = \left(\frac{P_t(z)}{P_t} \right)^{-\epsilon} c_t^h \tag{3}$$

$$c_t^{h^*}(z) = \left(\frac{P_t(z)}{P_t} \right)^{-\epsilon} c_t^{h^*} \tag{4}$$

where $c_t^h(z)$ and $c_t^{h^*}(z)$ are Home and Foreign demands for Home variety z. Firms are assumed to be able to change their price only in specific states of nature and must satisfy all demand at a quoted price. Each period a firm faces probability $1 - \lambda$ that it will be able to reset:

$$P_t(z) = \begin{cases} P_{t-1}(z) & \text{with prob. } \lambda \\ P_t^{\#}(z) & \text{with prob. } 1 - \lambda \end{cases},$$

where $P_t^{\#}(z)$ is the optimal price that a firm sets when it receives a signal to update. Firms discount profits j periods into the future by $\Delta_{t,j}\lambda^j$, where $\Delta_{t,j}$ is a discount factor and λ is the probability of not updating. When updating each firm maximizes the discounted sum of future real profits:

$$\max_{P_t^{\#}(z)} E_t \sum_{j=0}^{\infty} \Delta_{t,j}\lambda^j \left(\frac{P_t^{\#}(z) y_{t+j}(z)}{P_{t+j}} - \frac{W_{t+j} n_{t+j}(z)}{P_{t+j}} \right), \tag{5}$$

subject to (3), (4), a linear production function:

$$y_t(z) = n_t(z) \tag{6}$$

and imposing that output equals demand at both product ($y_t(z) = c_t^h(z) + c_t^{h^*}(z)$) and country level:

$$y_t = c_t^h + c_t^{h^*}. \tag{7}$$

From the first order condition each updating firm will choose the same reset price $P_t^{\#}$:

$$P_t^{\#} = \frac{\epsilon}{\epsilon - 1} \frac{E_t \sum_{j=0}^{\infty} \left(\Delta_{t+j} \lambda^j \Pi_{t+j}^{\epsilon} y_{t+j}\right) W_t}{E_t \sum_{j=0}^{\infty} \left(\Delta_{t+j} \lambda^j \Pi_{t+j}^{\epsilon-1} y_{t+j}\right)}, \tag{8}$$

where both prices and wages growth at the same rate and Π_{t+j} denotes gross cumulative inflation between periods t and $t + j$. It follows that a firm that is able to reset its price is charging a markup over a wage $P_t = \mu^{\#} W_t$. With sticky prices and constant trend inflation $P_{t+1} = P_t(1+\pi)$, this *marginal markup* is equal to:

$$\mu^{\#} = \frac{\epsilon}{\epsilon - 1} \frac{1 - \beta\lambda(1+\pi)^{\epsilon-1}}{1 - \beta\lambda(1+\pi)^{\epsilon}}. \tag{9}$$

Either with zero inflation or with flexible prices the marginal markup is equal to $\epsilon/(\epsilon - 1)$. With sticky prices, the marginal markup increases under positive trend inflation and is U-shaped under negative trend inflation.[2] Intuitively, when firms are able to re-optimize, they take into account that they may be stuck with the chosen price for many periods. Therefore, higher trend inflation is being overcompensated by setting higher markups. Firms' pricing decision is static, which it is a direct consequence of the CES preference structure. [3]

In the steady-state with trend inflation the *average markup* P_t/W_t is different from the marginal markup. The stochastic price setting results in a stationary distribution of firms in terms of time since the last price adjustment. The fraction of firms that last adjusted j periods ago is $\phi_j = (1 - \lambda)\lambda^j$. Thus, aggregate price level evolves according to $P_t = \left(\sum_{j=0}^{\infty} \phi_j \left(P_{t-j}{}^{\#}\right)^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}}$. Or, more simply:

$$P_t = \left((1-\lambda)(P_t^{\#})^{1-\epsilon} + \lambda(P_{t-1})^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}}. \tag{10}$$

The average markup μ_t is the ratio of the aggregate nominal price to the nominal wage:

$$\mu_t = \frac{P_t}{W_t} = \frac{P_t}{P_t^{\#}} \frac{P_t^{\#}}{W_t}, \tag{11}$$

where, following King and Wolman (2013), I label $P_t/P_t^{\#}$ the *price adjustment gap* and $P_t^{\#}/W_t$ the marginal markup. In an inflationary steady-state by evaluating (10) when P_t and $P_t^{\#}$ grow at the same rate π the

[2]The marginal markup admits minimum when $(\epsilon - 1)(\frac{1}{1+\pi} - \lambda(1+\pi)^{\epsilon-2}) = (\epsilon - 2)(1 - \lambda(1+\pi)^{\epsilon-2})$, that is strictly below 0 and for plausible parameter values close to -1.

[3]A notable example of departure from CES is Bergin and Feenstra (2000), who show that Calvo rigidity under translog demand structure implies a price-setting rule that is dynamic and gives weight to prices set by competitors.

price adjustment gap is:

$$\frac{P_t}{P_t^{\#}} = \left(\frac{1-\lambda\,(1+\pi)^{\epsilon-1}}{1-\lambda}\right)^{\frac{1}{\epsilon-1}}. \tag{12}$$

With sticky prices, trend inflation erodes relative prices of firms that are not adjusting. This results in the aggregate price being lower than the optimal reset price. Finally, the average markup consists of two parts: the marginal markup and the price adjustment gap and is equal to:

$$\mu = \frac{\epsilon}{\epsilon-1}\left(\frac{1-\beta\lambda(1+\pi)^{\epsilon-1}}{1-\beta\lambda(1+\pi)^{\epsilon}}\right)\left[\frac{1-\lambda\,(1+\pi)^{\epsilon-1}}{1-\lambda}\right]^{\frac{1}{\epsilon-1}}. \tag{13}$$

The average markup combines two counteracting effects. Positive inflation increases the marginal markup via resetting firms but decreases the price adjustment gap via non-resetting firms. The first effect dominates when inflation is positive, while the second effect dominates when inflation is negative, resulting in a U-shaped relationship between the rate of inflation and the average markup.[4] This result closely mirrors the detailed discussion of the two effects in Goodfriend and King (1997). Figure 1 depicts this relationship using $\epsilon = 8.14$ which corresponds to the modest and commonly used in the literature *static markup* of 1.14.

2.2 Households

Households and firms in both economies are infinitely lived. It is assumed that household make consumption purchases before receiving labour income. They have two means of savings: money (in forms of Home and Foreign currency M_t^h and M_t^f) and domestic currency bonds B_t. At the beginning of each period Home (Foreign) households receive lump-sum money transfer from their local monetary authority T_t (T_t^*) . Thus, this setup delivers money demand in the form of two cash-in-advance constraints for Home households:

$$P_t c_t^h = M_t^h + T_t + B_t - \frac{B_{t+1}}{R_t}, \tag{14}$$

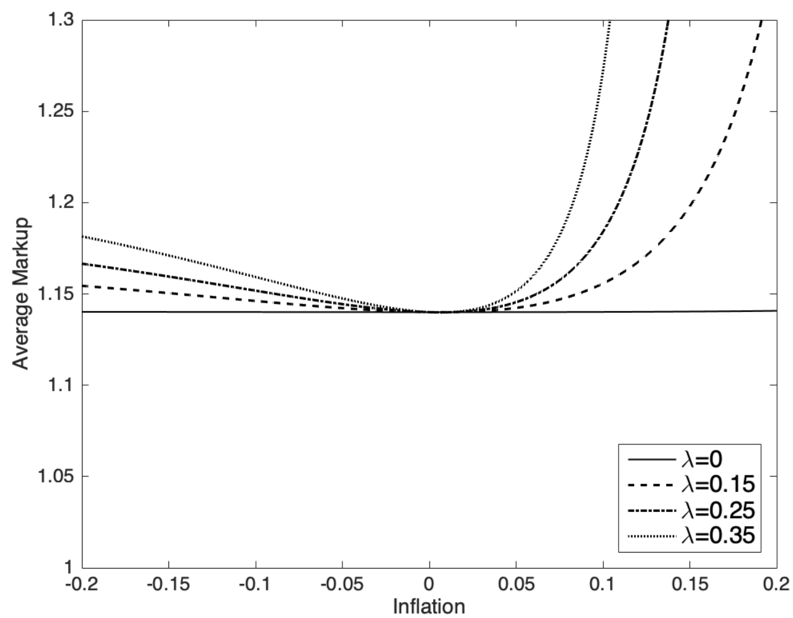
$$P_t^* c_t^f = M_t^f, \tag{15}$$

where R_t is the gross interest rate between periods t and $t+1$ and it is assumed that bond market closes before labour markets open. The CIA constraints of Foreign households are symmetric. This setup closely follows households side in Arseneau (2007).[5] This representation delivers dynamic household optimization

[4]The average markup is minimized when π is minimally greater than 0. For conventional parameters used later in the parametrized example the minimum is admitted for $\pi = 0.007$.

[5]An alternative approach to the households side in studies of optimal inflation and openness is the overlapping generations structure, where households supply labour in the first and consume in the second period as in e.g. Cooper and Kempf (2003), Cooper and Kempf (2004) and Evans (2012). After consumption purchases representative household receives labour income and dividends which are exchanged for new Home and Foreign money balances and new bond holdings. This alternative approach yields almost identical equilibrium conditions and was used in the previous versions of this paper. Derivations are available upon request.

Figure 1: Inflation and the average markup at different λ levels ($\beta = 0.95, \epsilon = 8.14$).



Source: Author's calculations.

and the demand for money, yet yields a model that is simple and analytically tractable with the minimum number of parameters. I denote real variables with lower case symbols and nominal variables with capital letters. Representative Home household maximizes:

$$\max_{c_t^h, c_t^f, l_t, M_{t+1}^h, M_{t+1}^f, B_{t+1}} \sum_{t=1}^{\infty} \beta^{t-1} \left(\theta log(c_t^h) + (1-\theta) log(c_t^f) - \frac{\eta}{1-\gamma} l_t^{1-\gamma} \right), \tag{16}$$

subject to two CIA constraints (14)-(15) and budget constraint:

$$P_t c_t^h + e_t P_t^* c_t^f + \frac{B_{t+1}}{R_t} + M_{t+1}^h + e_t M_{t+1}^f \leq M_t^h + e_t M_t^f + T_t + B_t + D_t + W_t l_t, \tag{17}$$

where the left-hand-side groups expenditures and the right-hand-side groups income, D_t are average dividends received from firms and e_t is the exchange rate. Taste preference for domestic goods θ is assumed to be constant and same for both countries $\theta \in [0, 1]$ with $\theta > 0.5$ representing the home bias. The first order conditions yield the following solution to the household's problem:

$$\frac{\theta}{P_t c_t^h} = \frac{1-\theta}{c_t^f e_t P_t^*}, \tag{18}$$

$$\frac{P_{t+1} c_{t+1}^h}{P_t c_t^h} = \beta R_t, \tag{19}$$

$$R_t \eta l_t^{-\gamma} c_t^h = \frac{\theta W_t}{P_t} \tag{20}$$

where (19) is a standard intertemporal Euler equation, (20) is an intratemporal condition linking consumption with labour supply and (18) implies that the money supply is held in constant proportions between currencies. This in turn implies, that representative household consumes a constant fraction of domestic output.

2.3 Aggregation and market clearing

In every period four markets in each country (goods, labour, money and bonds) and one international foreign exchange markets clear. Goods market clearing in Home country is given by (7). From individual firms' production function we derive aggregate labour demand:

$$l_t = \int_0^1 l_t(z)dz = \int_0^1 y_t(z)dz = y_t \int_0^1 \left(\frac{P_t(z)}{P_t}\right)^{-\epsilon} dz. \tag{21}$$

Following Ascari and Ropele (2009) and denoting by s_t the measure of *price dispersion* $s_t = \int_0^1 (\frac{P_t(z)}{P_t})^{-\epsilon} dz$ we get:

$$y_t = \frac{l_t}{s_t}. \tag{22}$$

Price dispersion is inertial variable evolving according to $s_t = (1-\lambda)\left(\frac{P_t^{\#}}{P_t}\right)^{-\epsilon} + \lambda(1+\pi_t)^{\epsilon} s_{t-1}$. Evaluated in the steady-state with trend inflation π we get:

$$s = \frac{1-\lambda}{1-\lambda(1+\pi)^{\epsilon}} \left(\frac{1-\lambda(1+\pi)^{\epsilon-1}}{1-\lambda}\right)^{\frac{\epsilon}{\epsilon-1}}. \tag{23}$$

In the model with staggered price setting a positive price dispersion reduces output produced from a single unit of aggregate labour or, equivalently, increases amount of aggregate labour required to produce a unit of output (see a detailed discussion in Ascari and Sbordone (2014)). The price dispersion is U-shaped around zero and quantitatively acts similarly to the average markup. Comparing (13) and (23) for $\beta \approx 1$ we observe that:

$$\mu \approx \frac{\epsilon}{\epsilon - 1} s. \tag{24}$$

Since β is slightly below unity, we have that under negative (positive) inflation the average markup is slightly higher (lower) than $\frac{\epsilon}{\epsilon-1}s$. For conventional parametrization followed in the next sections the difference between μ and $\frac{\epsilon}{\epsilon-1}s$ is less than 5% for any $\pi < 15\%$ annually and any $\lambda < 0.35$.[6]

Money, foreign exchange and bonds market clearing are given by:

$$M_t = M_t^h + M_t^{h*}, \tag{25}$$

$$M_t^{h*} = e_t M_t^f, \tag{26}$$

$$B_t = 0, \tag{27}$$

where bonds are assumed to be in net zero supply. Transfers are paid for by new money supply, with time-invariant rate of money creation σ:

$$T_t = \sigma M_t \tag{28}$$

hence, the money stock law of motion reads:

$$M_{t+1} = M_t(1+\sigma). \tag{29}$$

2.4 Equilibrium

Definition 1. *A monetary stationary rational expectations equilibrium is given by the labour supply functions* (l_t, l_t^*), *the consumption functions* $(c_t^h, c_t^f, c_t^{h*}, c_t^{f*})$, *money and bond holdings* $(M_t^h, M_t^f, M_t^{h*}, M_t^{f*}, B_t, B_t^*)$ *and the system of prices* (P_t,P_t^*, e_t) *such that agents optimize and markets clear.*

In equilibrium Home consumption is the fixed proportion of Home output and domestic consumption of Foreign goods is a fixed proportion of Foreign output:

$$c_t^h = \theta y_t \tag{30}$$

$$c_t^f = (1-\theta)y_t^*. \tag{31}$$

This property of consumption combined with the first order condition of labour supply (20) implies that in equilibrium optimal labour supply is stationary, which in turn implies the stationary property of price evolution (see Online Appendix A for all derivations). In the steady-state with trend inflation where labour, output and consumption in both economies are constant, Home prices grow at the rate π and Home money stock grows at the rate σ we have the following:

$$M_{t+2} = M_{t+1}(1+\sigma) = P_{t+1}y = P_t(1+\pi)y = M_{t+1}(1+\pi)$$

$$\pi = \sigma \tag{32}$$

[6]See Online Appendix A for the derivation of the approximate equality. I would like to thank the anonymous referee for the helpful discussion regarding the role of price dispersion.

In the steady-state with trend inflation labour supply is decreasing in inflation (since s approximately equals μ up to a constant):

$$l = \left(\frac{\beta}{\eta(1+\pi)} \frac{s}{\mu} \right)^{\frac{1}{1-\gamma}}, \tag{33}$$

Inflation has a direct, negative effect on the labour supply. This is due to the fact that in the CIA economy, where consumption purchases are made before income from labour is cashed, expected inflation reduces the purchasing power of income in the next period. In this, it acts as a tax, hence this effect is later referred to as the *inflation tax*. Consistently, deflation acts as a labour subsidy increasing optimal labour supply. This effect echoes results in Cooper and Kempf (2003), where under flexible prices (albeit in a different setup), the sole inefficiency of inflation is the inflation tax. Unlike lump-sum tax, inflation tax is distortive.

The steady-state output is a decreasing function of inflation and a decreasing function price dispersion. The role of the average markup is minor due to its approximate equality to the price dispersion discussed before:

$$y = \left(\frac{\beta}{\eta(1+\pi)} \frac{s}{\mu} \right)^{\frac{1}{1-\gamma}} s^{-1}. \tag{34}$$

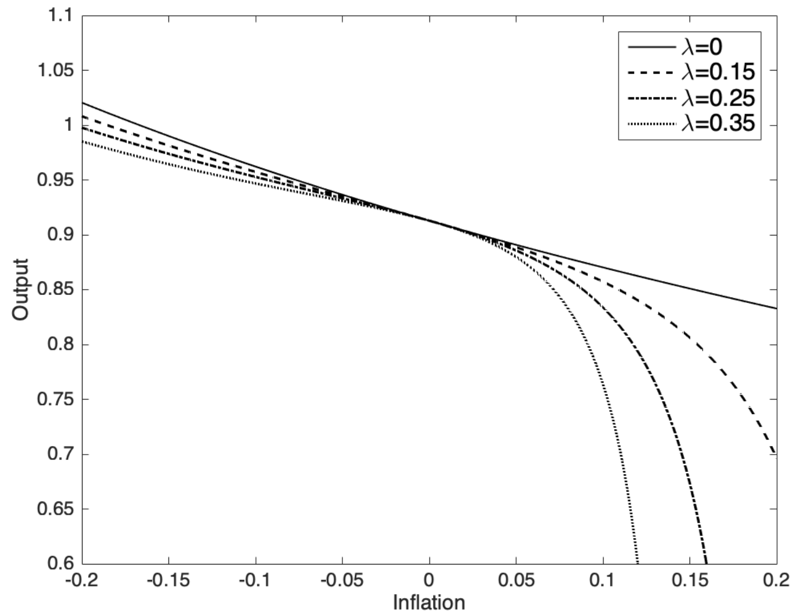
In the steady-state labour, output and consumption of Home and Foreign goods in both economies and exchange rate are constant and Home and Foreign prices grow at the rates σ and σ^*. The model manifests long run neutrality of money (as permanent changes in quantity of M only affect prices and not output and $\sigma = \pi$) but not superneutrality. There are real effects of a sustained long run inflation.

Proposition 1. *Inflation decreases output via both the positive inflation tax and the positive price dispersion. Deflation has two opposing equilibrium effects. It decreases output via the positive price dispersion and increases output via the negative inflation tax.*

Proof. The price dispersion effect derives from the equation (23) and the inflation tax effect follows from equation (34) assuming well-behaved labour supply ($\gamma < 0$). □

Proposition 1 is depicted graphically in Figure 2, which plots reaction of output to inflation and stickiness using parameter values introduced later. With flexible prices ($\lambda = 0$) only inflation tax effect is at play and output decreases with inflation (increases with deflation). The second effect comes from the price dispersion. When all prices are the same ($s = 0$), which happens with flexible prices or with zero inflation, aggregate output is equal to aggregate labour supply. When prices differ ($s > 1$) demand shifts from more expensive towards cheaper goods. This shift in demand, *ceteris paribus* (thus holding labour supply constant) results in a reduction of aggregate output (2). Price dispersion drives a wedge between aggregate labour and aggregate output, hence it also acts as a tax. Since price dispersion is similar to average markup, the implicit tax is

Figure 2: Inflation and output at different λ levels



Source: Author's calculations.

positive for both positive and negative inflation. As stickiness increases, the inflation tax effect is exacerbated by the price dispersion effect for positive inflation and mitigated for negative inflation. Yet still, with deflation the inflation tax effect dominates.

3 Optimal Monetary Policy with Local Currencies

In this section the optimal rate of inflation is calculated as a solution to the non-cooperative monetary authority optimization problem. Domestic monetary authority takes equilibrium outcomes and market clearing conditions as constraints and Foreign money growth rate as given and chooses domestic money growth rate. Said differently, it selects the steady state which yields highest welfare for households:

$$V(\sigma, \sigma^*) = \max_{\sigma} \sum_{t=1}^{\infty} \beta^{t-1} u\left(c, l\right) \tag{35}$$

subject to consumption decision (30), (31) and labour supply decision (33). Substituting in we have:

$$V(\sigma, \sigma^*) = \max_{\sigma} \frac{\theta\gamma log(s) - \theta log(1+\sigma) - \theta log(\mu) - \frac{\beta}{1+\sigma}\frac{s}{\mu} + C + F}{(1-\gamma)(1-\beta)}, \tag{36}$$

where price dispersion s and markup μ are functions of monetary policy instrument, C is a constant and F is a Foreign country variables block that depend on σ^* but not on σ:

$$C = (1-\gamma)\theta log(\theta) + \theta log(\beta) - \theta log(\eta) + (1-\gamma)(1-\theta)log(1-\theta),$$

$$F(\sigma^*) = (1-\theta)[log(\beta) + log(s^*) - log(\eta) - log(1+\sigma^*) - log(\mu^*)] - (1-\gamma)(1-\theta)log(s^*).$$

Foreign country monetary authority optimization is symmetric. The problem is highly non-linear (see derivations in Online Appendix A) therefore I proceed by solving two simplified benchmark cases first. The first benchmark is an economy with no frictions (perfect competition and flexible prices). The second benchmark is an economy with only one friction (monopolistic competition). The two benchmarks admit closed form solutions. The third benchmark is an economy with both frictions (monopolistic competition and sticky prices).

Benchmark 1. Open economy with flexible prices and perfect competition

This economy is similar to Cooper and Kempf (2003). Prices are flexible ($\lambda = 0$), the average markup and price dispersion are equal to unity.

Proposition 2. *In a two country economy with local currencies, flexible prices and perfect competition the optimal rate of inflation is equal to:*

$$\sigma^{B1} = \frac{\beta - \theta}{\theta} \tag{37}$$

Proof. See Online Appendix B. □

Trade openness $(1-\theta)$ measures the scope of exporting inflation abroad. In a closed economy ($\theta = 1$) the cost of inflation is fully borne by Home households and optimal inflation rate is small and negative $(\beta - 1)$. This is a straightforward consequence of the CIA constraint coupled with the Euler equation. The positive return on their money holdings compensates households for the discount they put on their future consumption purchases. With increasing openness (decreasing θ) the optimal rate of inflation increases. This is because the marginal benefit of consumption is lower than marginal cost of supplying labour. Home production is consumed only partially by Home households, while the remaining part is being exported. It is therefore optimal for the monetary authority to increase inflation. This decreases labour supply and hence decreases disutility from labour faster than decreasing the marginal benefit of consumption of Home produced goods. The optimal rate of inflation is obtained when the marginal benefit of consumption and marginal cost of labour are equalized. As a result, Home monetary authority pursues sustained, positive rate of inflation. This is later referred to as the *spillover effect*.

A local monetary authority exploits international spillovers to run inflationary, 'beggar-thy-neighbour' policy. Both labour supply and home output are lower than under zero inflation. As the fraction of output is consumed abroad, the inflation tax is partly exercised on foreign individuals. Home households enjoy more leisure without paying the full cost of lost output. Importantly, as Foreign monetary authority acts symmetrically, the Foreign inflation tax is exercised on Home households. The gains from inflation are thus wiped out. This is a classical prisoner's dilemma, where individual rationality of two monetary authorities leads to globally inferior outcome. Thus, it is a second friction, that introduces inflationary bias.

This echoes result in Evans (2012) where increased openness to international trade increases a country's long-run incentive to create inflation. It is immediate to notice, that if the two countries formed a monetary union (equivalent of a closed economy) all spillovers would be internalized by a common monetary authority. With only one player the prisoner's dilemma would vanish. There would be no possibility to exercise inflation tax on Foreign households, as there would be no neighbour to beggar.

Benchmark 2. Open economy with flexible prices and imperfect competition

In this economy there is no price dispersion $s = 0$ and the average markup is static and equal to $\mu = \mu^{\#} = \frac{\epsilon}{\epsilon - 1}$. It follows that:

Proposition 3. *In a two country economy with local currencies, flexible prices and monopolistic competition the optimal rate of inflation is equal to:*

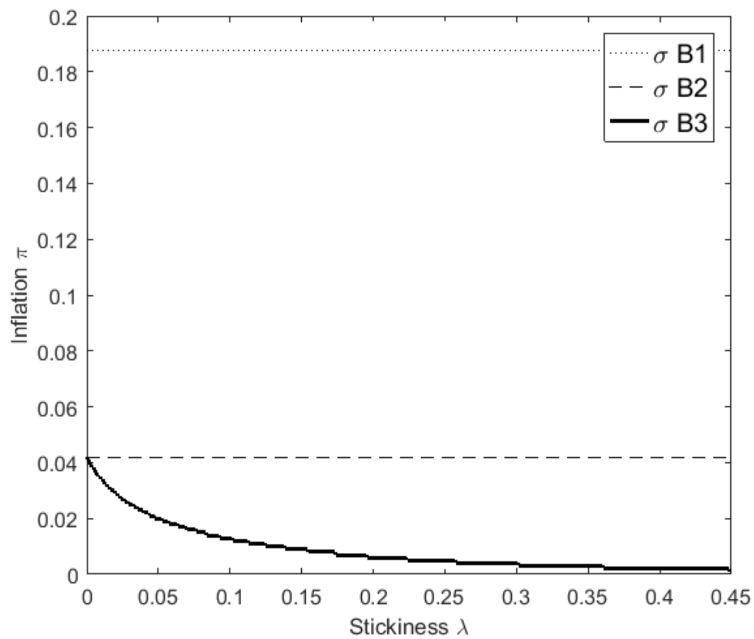
$$\sigma^{B2} = \frac{\beta - \theta\mu}{\theta\mu} \tag{38}$$

Proof. See Online Appendix B. □

In perfect competition economy one unit of labour produces one unit of consumption. The static markup present in imperfect competition economy drives a wedge between labour and consumption and hence, from households perspective is similar to a consumption (or labour) tax. The higher is the markup, the lower is the return from supplying labour and the optimal labour decreases (see equation 33). In this, the static markup acts in the same way as inflation tax discussed in Benchmark 1. From the households perspective the two instruments are substitutes. Since, in an open economy with local currencies, the monetary authority finds it optimal to decrease households labour supply, it does not need to impose inflation tax to its full extend as in Benchmark 1. In this, the static markup mitigates incentives to inflate.

In an imperfect competition economy with flexible prices the optimal level of inflation is strictly below the one in an economy with perfect competition and flexible prices $\bar{\sigma}^{B2} < \bar{\sigma}^{B1}$. Inflationary bias of openness is dampened by the degree of imperfect competition and monetary policy becomes more inward looking, as in Arseneau (2007) and Evans (2012). In extreme cases, for either a high markup or low openness, the optimal

Figure 3: Optimal Inflation with Local Currencies



Source: Author's calculations.

level of inflation is negative. Specifically, in this benchmark a single monetary authority in a monetary union ($\theta = 1$) would choose a strictly negative level of inflation $\sigma^{B2} = \frac{\beta - \mu}{\mu}$. This is later referred to as the markup effect.

Benchmark 3. Open economy with sticky prices

In Benchmark 3 inflation affects the size of the average markup and the price dispersion. As the problem does not have an analytical solution, I derive the solution numerically. I discipline the model with a standard parametrization. The period in the model is one year. The elasticity of substitution ϵ is equal to 8.14, which results in the static markup $\epsilon/(\epsilon - 1)$ of 1.14. The home bias parameter θ is set to 0.8, as in Pappa (2004). Frisch elasticity of labour $-1/\gamma$ is set to 1, resulting in a quadratic disutility function. The weighting parameter η is set to unity. The stickiness parameter λ is varied between 0 and 0.45 to asses how the model diverges from the first two benchmark cases. An important middle case value of $\lambda = 0.32$ is a yearly equivalent of a quarterly 0.75; a standard stickiness value assumed in a quantitative literature.

Figure 3 plots parametrized solution of all three benchmark economies with varying stickiness parameter

λ. With $\lambda = 0$ Benchmark 3 economy has flexible prices and is therefore the same as Benchmark 2 economy; previously discussed. With increasing stickiness the optimal level of inflation decreases asymptotically to zero. The intuition behind this result is the role of price dispersion. As noted earlier, for conventional values of time discount factor β, the average markup and price dispersion are approximately equal up to a constant. Therefore, the average markup drops out from expressions for the optimal labour supply (33) and equilibrium output (34), leaving equilibrium being highly and negatively dependent on price dispersion. Since price dispersion is U-shaped in inflation, the monetary authority has strong incentives to impose zero inflation to avoid output losses. This is later referred to as the price dispersion effect.

On the other hand, since the economy is open, the spillover effect, discussed in Benchmark 1, creates incentives to inflate (even when it is dampened by the markup effect discussed in Benchmark 2). As stickiness increases the price dispersion effect becomes stronger. The spillover effect and markup effect do not depend on stickiness. This results in a negative relationship between stickiness and optimal inflation.

A monetary authority chooses the optimal rate of inflation to solve a trade-off among three distortions: the spillover effect, the markup effect and the price dispersion effect. The first one incentivizes to inflate, the second one to deflate. The third one incentivizes the monetary authority to set inflation rate equal to zero and becomes stronger with increasing stickiness. Hence, the first result follows:

Result 1. *In a two country economy with local currencies, when prices are flexible the spillover effect dominates and the optimal inflation is positive. When prices are sticky the price dispersion effect dominates and the optimal inflation is zero.*

Even though the model is stylized, quantitative results of the optimal rate of inflation are plausible and within the observable bounds ranging from 4% in flexible prices open economy to 0% when stickiness reaches $\lambda \approx 0.4$, a value above those typically estimated in the literature.

4 Optimal Monetary Policy in a Monetary Union

A common monetary authority chooses one money supply growth rate equal for both economies in order to maximize the joint welfare of households in the two countries. Let σ^U be the growth rate of the money stock in a union. Joint welfare is:

$$V^U(\sigma^U) = wV(\sigma^U, \sigma^U) + (1 - w)V^*(\sigma^U, \sigma^U), \tag{39}$$

where w is a weight assigned to the Home representative household. The monetary authority maximizes (39) subject to the consumption decisions (30), (31) and the labour supply decisions (33) in both economies.[7]

[7] This paper compares optimal inflation in a local currencies setting with a monetary union setting. One could also think of

Substituting in and assuming equal weights the problem can be rewritten (see Online Appendix A for derivations) in the following form:

$$V^U(\sigma^U) = \max_{\sigma} \frac{\gamma log(s) - log(\mu) + \gamma log(s^*) - log(\mu^*) - 2log(1+\sigma) - \frac{\beta}{1+\sigma}\left(\frac{s}{\mu} + \frac{s^*}{\mu^*}\right) + C}{2(1-\gamma)(1-\beta)} \tag{40}$$

where C is a constant. It can be already seen, that in a symmetric union the level of openness does not affect the optimal level of inflation. With perfect competition and flexible prices ($s = 1, \mu = 1$) the optimal level of inflation in a monetary union is equal to:

$$\sigma^{U1} = \beta - 1 \tag{41}$$

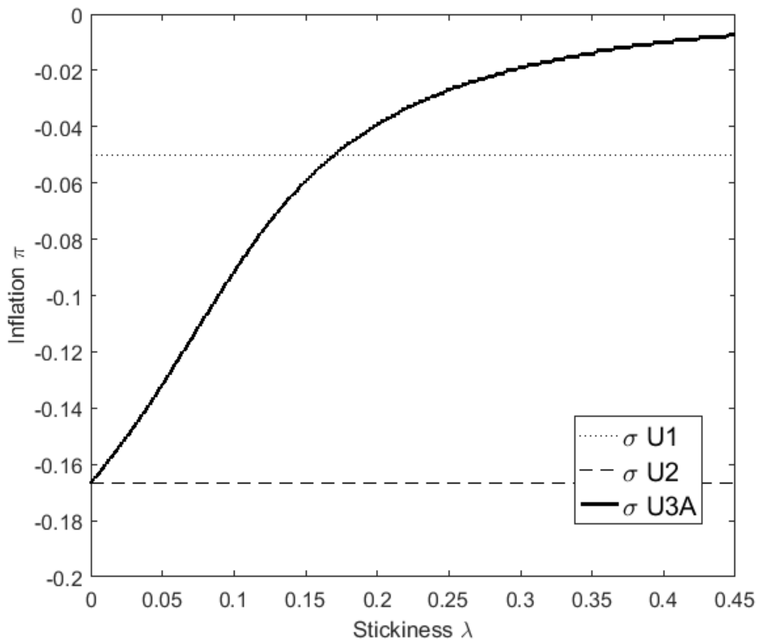
which can be immediately derived from (37) setting $\theta = 1$. Similarly, with imperfect competition and flexible prices ($s = 1, \mu = \frac{\epsilon}{\epsilon - 1}$) the optimal level of inflation in a monetary union is equal to:

$$\sigma^{U2} = \frac{\beta - \mu}{\mu} \tag{42}$$

In both settings the union is welfare improving, since negative externality in the prisoners dilemma game between monetary authorities is removed and a common monetary authority fully internalizes the spillovers created by the inflation tax.[8] As incentives to impose inflation tax are removed in a monetary union, the optimal level of inflation is always lower than in the local currencies economy in each respective benchmark. With imperfect competition it is always negative.

However, in a monetary union with imperfect competition and sticky prices the level of inflation affects two average markup and, more importantly, two price dispersions. Since monetary union is a closed economy, the 'beggar-thy-neighbour' distortion is removed and a common monetary authority trades off deflationary incentives created by monopolistic competition and zero-inflation incentives from price dispersion distortion. Moreover, since the members of a union may differ in their stickiness, the same level of inflation will affect the two economies differently. To disentangle the role of closed economy from the role of price dispersion two cases are studied: symmetric and asymmetric monetary union.

Figure 4: Optimal Inflation in a Union with Symmetric Sticky Prices $\lambda = \lambda^*$



Source: Author's calculations.

Case A. Monetary union with symmetric sticky prices

Figure 4 plots the optimal rate of inflation in a monetary union when both members, Home and Foreign economies, have the same level of price stickiness λ. The graph plots three monetary union economies: with flexible prices and perfect competition (σ^{U1} discussed before), with flexible prices and imperfect competition (σ^{U2} discussed before) and with symmetric sticky prices and imperfect competition (σ^{U3A} discussed here). When stickiness parameter is equal to zero the economy is equivalent to a closed economy with flexible prices and imperfect competition discussed before ($\sigma^{U3A} = \sigma^{U2}$). A monetary authority faces strong deflationary bias: deflation helps to overcome the CIA distortion and the imperfect competition distortion. With in-

a third setting, the coordinated solution, where a single monetary authority can choose different money growth rates for each economy. Mathematically, the coordinated solution is the same as in local currencies when $\theta = 1$, which is also equivalent to a monetary union Case A solution discussed below, albeit the economic interpretation is different. For the ease of exposition I do not discuss this setting in the main text, but provide derivations and short intuition in Online Appendix A. I would like to thank the anonymous referee for making this point.

[8] See discussion in Benchmark 1 and 2. This argument is also a result of Cooper and Kempf (2003). Detailed proofs are available upon request.

creasing level of stickiness however, the price dispersion distortion increases. Similarly to the open economy setting, for high levels of stickiness the price dispersion becomes dominant resulting in a zero-optimal-inflation policy.

Result 2. *In a two country economy with common currency the spillover effect is absent. When prices are flexible the average markup effect dominates and the optimal inflation is negative. When prices are sticky the price dispersion effect dominates and the optimal inflation is zero.*

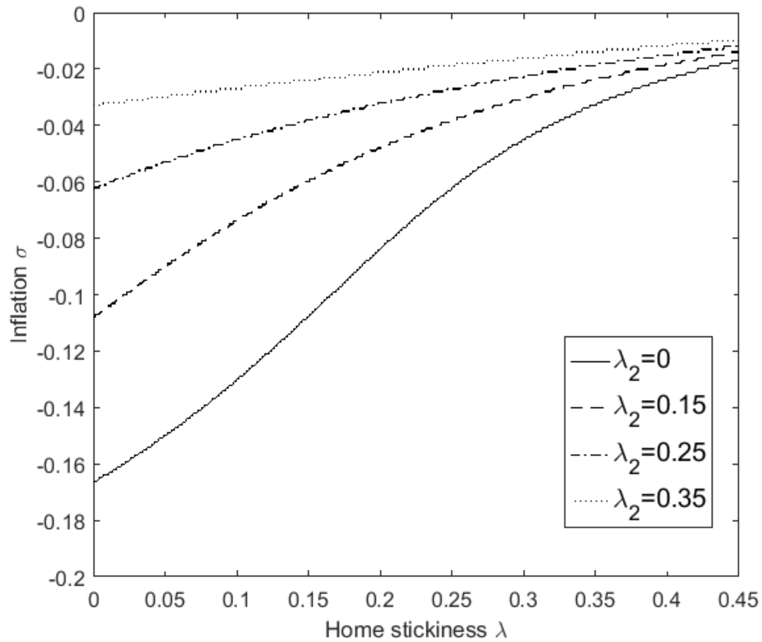
Given that a monetary union with symmetric sticky prices is equivalent to a big closed economy it can be compared with the seminal contribution of Schmitt-Grohe and Uribe (2010) (Chapter 6.3), where in a closed economy with monopolistic competition and sticky prices the optimal level of inflation is zero. The Case A economy nests their result when stickiness parameter is high. The discrepancy for low levels of stickiness derives from the fact than in a CIA economy considered here, unlike in benchmark NK model, trend inflation also works as a labour tax (and trend deflation as a labour subsidy). Hence, for low levels of stickiness, when price dispersion is low, the inflation tax (deflation subsidy) motive dominates. The average markup also works as a labour tax, which decreases labour supply. Hence, the monopolistic distortion is mitigated by a monetary authority by setting a negative rate of inflation, which increases labour supply back to efficient level.

Case B. Monetary union with asymmetric sticky prices

With $\lambda \neq \lambda^*$ Foreign and Home labour supplies are not equal. The average markup and price dispersion is a function of a common trend inflation and individual stickiness. Figure 5 plots numerical solution of the Case B economy for the chosen ϵ and θ, four different values of foreign stickiness parameter λ^* and varying the stickiness parameters λ on the horizontal axis.

The optimal rate of inflation in an asymmetric union is negative and increasing in both Home and Foreign stickiness - up to zero. For high levels of either ($\lambda > 0.4$ or $\lambda^* > 0.4$) it is close to zero. This is because with increasing stickiness in either member economy the losses associated with increasing price dispersion dominate the optimal policy. The optimal rate of inflation varies substantially across the range of possible stickiness asymmetries, between -16% and 0%. Individually, each country with a local currency would optimally set its inflation between 4% and 0% (see Figure 3). For high levels of stickiness in each economy forming a union brings little change in the optimal level of trend inflation. This is because price dispersion is the dominant distortion in both institutional settings. For low levels of stickiness however, a reduction in the optimal rate of trend inflation is substantial. This is because price dispersion is low and inflation tax is a dominant distortion. With reduced trend inflation in a union the optimal levels of labour

Figure 5: Optimal Inflation in a Union with Asymmetric Stickiness $\lambda \neq \lambda^*$



Source: Author's calculations.

supply and output increase in both economies. Even though the higher labour supply increases disutility, the increase in utility from consumption more than compensates for that.

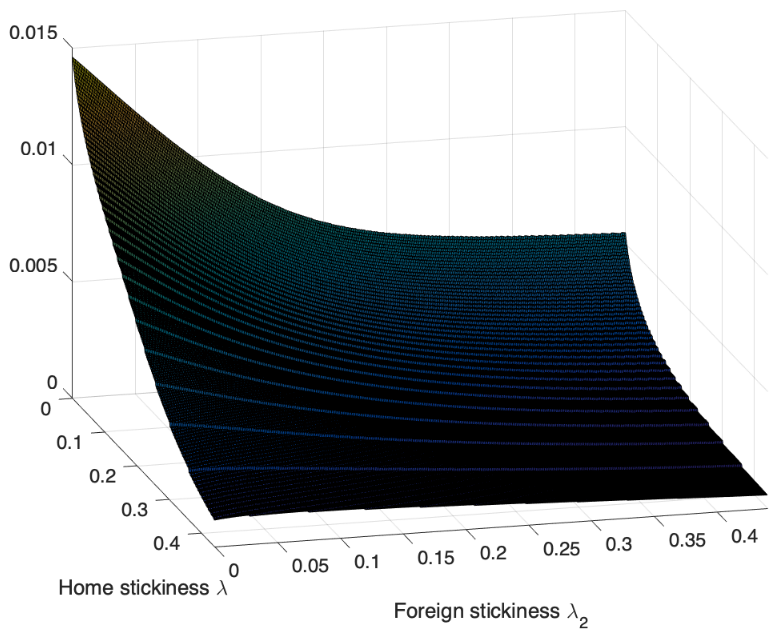
Even when the difference between the optimal trend inflation with local currencies and in a union is high, adopting a common currency is welfare improving. This is because a union removes incentives to print money, even with asymmetries in price rigidities. In this case benefits from joining a union are unequally distributed, but are positive for each member.

Result 3. *Monetary union improves global welfare even when stickiness is different in two countries. The economy with more flexible prices benefits more.*

Result 3 is visualized in Figure 6, which plots welfare gains from joining a monetary union in lifetime domestic consumption equivalent. Gains from joining a union are as high as 1.5% of lifetime domestic consumption equivalent when both economies have flexible prices. Gains are decreasing in stickiness - optimal trend inflation becomes inward looking, as output loss motive dominates the 'beggar-thy-neighbour' motive. Gains are close to zero when both economies have very sticky prices.

What is usually observed is that countries with higher inflation rates have more flexible prices. This paper

Figure 6: Welfare gains from monetary integration in lifetime domestic consumption equivalent $\lambda \neq \lambda^*$



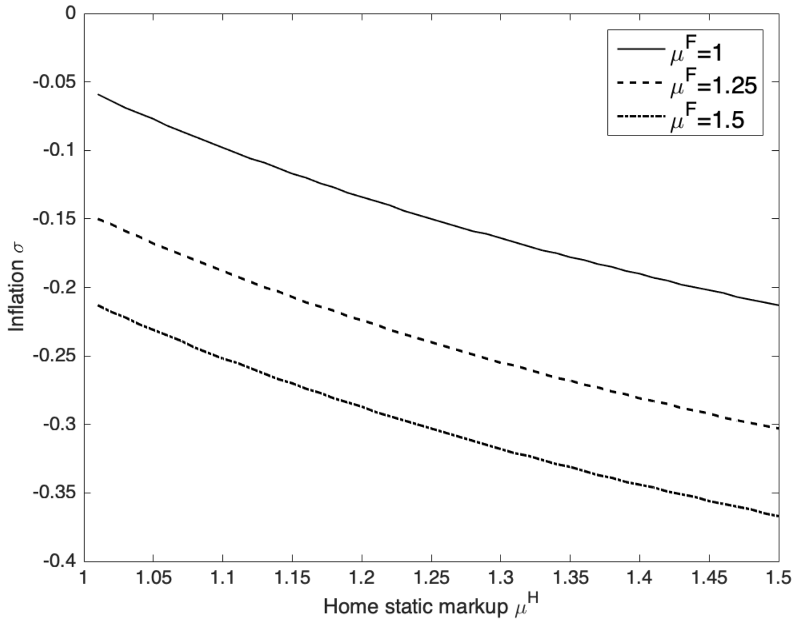
Source: Author's calculations.

studies causality channel from stickiness to optimal inflation, but a reverse relationship is also possible. When firms adjust their prices more frequently in regimes characterized by high inflation rates, then trend inflation is treated as exogenous and stickiness is an endogenous choice. In such framework joining a monetary union is also especially beneficial for an economy with high inflation and highly flexible prices due to gains from commitment, as in Alesina and Barro (2002).

Case C. Monetary union with asymmetric demand elasticities

In previous analysis it was assumed that demand elasticities (and the resulting static markups) are identical in both economies. As an extension, I study the case where $\epsilon \neq \epsilon^*$. Asymmetry in demands elasticities leads to an asymmetry in markups: the higher is the demand elasticity ϵ the lower is the static markup μ. This effect increases the (optimally positive) level of inflation in local currencies (see equation 38) and increases the (optimally negative) level of inflation in a union (see equation 42). This relationship is graphically visualized in Figure 7. It plots the relationship among Home markup μ on the horizontal axis, the Foreign markup μ^* plotted as different lines and the resulting optimal level of inflation in a union on the vertical axis. For simplicity the assumption is that prices are flexible in both economies ($\lambda = \lambda^* = 0$). The optimal

Figure 7: Optimal Inflation in a Union with Asymmetric Markups $\mu \neq \mu^*$ (assuming $\lambda = \lambda^* = 0$)



Source: Author's calculations.

level of inflation in a union decreases in both Home and Foreign markup, hence it increases in the level of demand elasticity.

When prices are sticky, the optimal level of inflation decreases down to zero in local currencies and increases up to zero in a union regardless of the level of demand elasticity. With high stickiness the price dispersion distortion is the dominant one and a monetary authority seeks to minimize it. The price dispersion is minimized when inflation is equal to zero for all levels of demand elasticity ϵ.

5 Conclusions

Are monetary unions welfare improving? A positive answer is usually backed by three arguments: they reduce transaction costs, promote international trade and a common central bank finds it easier to commit. In this paper I study another dimension. Thanks to the common central bank, countries commit not to tax each other with inflation and hence monetary union improves welfare. An important policy takeaway is that monetary integration is beneficial even for economies that are highly asymmetric in terms of their price rigidities. Equally importantly, benefits are spread unevenly. An economy with more flexible prices benefits more from the integration.

The results of the paper show that optimal rates of inflation in a union are negative, which is not usually observed empirically. This reinforces the puzzle identified in papers such as Schmitt-Grohe and Uribe (2010), that available theories consistently imply that the optimal rates of inflation lay below inflation targets observed around the world.

The framework presented in this paper can be further developed in the direction of new literature on the dominant currency pricing. It is conceivable that member states could benefit from collectively establishing a dominant currency for trade and thus exploit the 'beggar-thy-neighbour' effect with the rest of the world. The exploration of this channel is left for future research.

Supplementary Material

Supplementary material is available on the OUP website. These are the replication file and the Online Appendix.

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