A Case Against a Four Percent Inflation Target*

Engin Kara† and Tony Yates‡

January 21, 2020

Abstract

We reformulate the standard New Keynesian model to include heterogeneity in price stickiness suggested by micro-evidence on price changes and to allow for positive trend inflation. In the new model, higher trend inflation leads to a relatively greater long-run output loss and, consequently, a smaller determinacy region than in the standard model. When trend inflation is 4%, the determinacy region of the new model is almost non-existent, cautioning against increasing the inflation target to 4% as a means to avoid the zero lower bound in the future and pointing to the costs that high inflation may have had in the past.

Keywords: Trend inflation, determinacy, sticky prices, New Keynesian, Taylor Rules.

JEL classification: E52, E61, E66, C14, C18

---

*We are grateful to Ken West (the editor) and two anonymous referee for insightful comments and suggestions that helped to improve the paper. We benefited from comments by Dudley Cooke, Huw Dixon, Kul Luintel and seminar participants at Cardiff Business School and the Bank of England.

†Cardiff Business School, Colum Drive, Cardiff. CF10 3EU. E-mail: karae1@cardiff.ac.uk.

‡E-mail: anthony_yates@yahoo.com
1 Introduction

This paper adds both positive trend inflation and heterogeneity in the degree of price stickiness into an otherwise standard New Keynesian (NK) model. We use our model to study the determinacy of interest rate rules. Indeterminacy here refers to the property of rational expectations models whereby a single value for fundamental shocks like technology can, under some conditions, be consistent with multiple possible values for (in this context) inflation. In the NK model, indeterminacy is undesirable because it opens up the possibility of self-fulfilling disturbances to inflation expectations, which, through the normal channels induced by price stickiness (e.g. the costs of relative price distortion) are costly.

Our interest in determinacy derives from two prior strands of work: one normative and the other positive/empirical. The normative literature on determinacy of the price level under interest rate rules starts by considering the desirability and comparative performance of interest rate rules (see Bryant, Hooper and Mann (1993) and Taylor (1993)). A rigorous assessment of determinacy in NK models was first presented by Woodford (2003). Woodford’s focus is on the necessity for interest rate rules to be specified in terms of endogenous variables to generate determinacy and on the necessity/sufficiency, or lack of it, of the so called ‘Taylor Principle’. The Taylor Principle asserts that interest rates ought to respond more than one for one to fluctuations in the gap between inflation and its target value. Woodford’s conclusion is that the Taylor Principle is relevant in the NK model but the condition is different. The new condition is that “...at least in the long-run, nominal interest

---

1Formal analyses of determinacy in NK models has its antecedent in studies of the earlier generation of rational expectations monetary models (see, e.g., Sargent and Wallace (1975) and McCallum (1981)). Sargent and Wallace (1975) show that if interest rates are ‘pegged’ at a level invariant to conditions in the macro economy, there are multiple possible price levels consistent with given fundamentals under rational expectations. McCallum (1981) points out that an interest rate rule could be designed to mimic any desired path of the money stock - since with agents on their money demand curves these were duals - thus pinning down the price level.
rates should rise by more than the increase in the inflation rate" (Woodford (2003, p. 254)).

A related, positive, empirical monetary policy literature focuses on the question of whether central banks have in the past followed interest rate rules that satisfy appropriate determinacy conditions. For example, Clarida, Gali and Gertler (2000) estimate policy rules for the Fed and find coefficients that suggest policies that would have generated indeterminacy prior to Volcker but not after his tenure.

New light on both of these strands of work on indeterminacy is shed by Ascari and co-authors (see, e.g., Ascari and Ropele (2008) and Ascari and Sbordone (2014)). These authors develop a modification of the standard NK model, which is typically approximated around a zero inflation steady state. Allowing approximation around a non-zero inflation steady state (see also Bakhaski et al. (2007) and Kiley (2007)), they find that the range of parameters for which the model generates determinate rational expectations equilibria for inflation is narrowed as trend inflation increased. This finding is important for two reasons. First, it enumerates a possible, offsetting cost to consider, against the suggestion - by, for example Blanchard, Dell’Ariccia and Mauro (2014) and Ball (2014) - that the inflation targets of central banks should be raised to around 4% to combat the zero lower bound on interest rates on the grounds that higher inflation targets would provide more room

\[\text{\textsuperscript{2}}\text{Other papers that study determinacy using NK models include Bernanke and Woodford (1997), Bullard and Mitra (2007) and Bullard and Schaling (2009). Bernanke and Woodford (1997) consider determinacy under rules that involved a feedback from inflation forecasts rather than actual inflation (a practice some central banks described themselves as following). Bullard and Mitra (2007) consider rules with terms in lagged interest rates (which matched features of estimated central bank policy rules). Bullard and Schaling (2009) study determinacy in the open economy NK models.}\]

\[\text{\textsuperscript{3}}\text{This view is confirmed by Lubik and Schorfheide (2004) who present full-model estimates. Orphanides (2001) suggests that the inadequately inflation-responsive policy in the 1960s and 1970s may have been due to inaccurate information about the output-gap fed into an otherwise sound policy rule. Lubik and Matthes (2016) argue that policy leading to indeterminacy is also due in part to inadequate information about model structure.}\]
for cuts in nominal interest rates in response to future recessions.\footnote{This policy recommendation is for a permanent rise in the target; to be distinguished from the recommendations studied elsewhere that involve, in the pursuit of an unchanged inflation target, interest rates that are lower for longer, and have the corollary that inflation temporarily overshoots the target, with the compensating benefit of a smaller recession. We are also abstracting from the possibility that the target rise might not be believed; or, even if it is believed, might be reversed by some future policymaker with different objectives or incentives.} Second, it carries an empirical implication that it is more likely that high and volatile inflation in the post WW2-pre-Greenpan era was due to indeterminacy.\footnote{This inference is conditional on past inflation having been characterised by a fixed target. A literature has grown around the idea that past inflation is best characterized by a regime switching process. Such processes have their own implications for determinacy.}

Our paper revisits the impact of positive trend inflation on the determinacy region in a model with many sectors with different degrees of price stickiness. We find that a realistic calibration of the degree of heterogeneity greatly amplifies the extent to which trend inflation narrows the determinacy region for monetary policy rules. The implication that flows from this result is that there should be greater emphasis on the normative and positive implications of trend inflation than there has been hitherto.

To introduce heterogeneity to the NK model, we employ the Multiple Calvo (MC) approach, as in Carvalho (2006) and Kara (2015), in which there are many sectors, each with a different Calvo-style contract. The share of each sector, which has a distinct expected contract duration, is calibrated according to the Bils and Klenow (2004) (BK) dataset, giving rise to a BK-MC. We reformulate the MC model to include a positive trend inflation rate, analogous to the way Ascari and Sbordone (2014) introduce trend inflation into the standard NK model. When sectors face the same probability of price adjustment, the model reduces to the Ascari and Sbordone (2014) model; when our model has identical probabilities of price adjustment and zero steady-state inflation, the model collapses further to the standard NK model.

Modifying the NK model to accommodate heterogeneity in price sticki-
ness is not merely an academic exercise: it is an important feature of the micro evidence on prices (see, e.g., Klenow and Malin (2011), Alvarez et al. (2006) and Hall, Walsh and Yates (2000)). Moreover, it has been shown that adding heterogeneity in price stickiness significantly improves the empirical performance of the Smets and Wouters (2007) (SW) model, which is considered to be state of the art in NK economics (see also Christiano, Eichenbaum and Evans (2005)). Kara (2015) replaces Calvo pricing in the SW model with MC pricing and estimates the resulting SW-MC model with Bayesian techniques using US data. He shows that while the SW-MC model fits the macroeconomic data as well as the SW model, two disturbing problems of the SW model disappear when heterogeneity in price stickiness is introduced. First, while the SW model requires large mark-up shocks to match inflation data (see Chari, Kehoe and McGrattan (2009)), the estimated variance of price mark-up shocks implied by the SW-MC is much smaller than that implied by the SW. Second, while the SW-MC matches the data on reset price inflation complied by Bils, Klenow and Malin (2012), the SW model cannot, suggesting that MC pricing is more consistent with that in reality than Calvo pricing.

To understand why heterogeneity in price stickiness aggravates the determinacy problem, first note that Woodford (2003, pages 252-261) suggests that the size of determinacy region depends on how much output changes in response to a change in inflation in the long-run (i.e. the slope of the Phillips curve in the long-run). In the standard NK model with no trend inflation, the determinacy region is large. Ascari and Sbordone (2014) show that in the version of the model with positive trend inflation, long-run output decreases with increasing trend inflation, resulting in a smaller determinacy region. This result is a consequence of the fact that increased trend inflation leads to larger price dispersion and a larger average price mark-up.

Heterogeneity in price stickiness magnifies this effect because it brings about a significantly larger fall in long-run output when there is an increase.
in trend inflation. This is due to the presence of sectors with low hazard rates in the BK-MC and the fact that long-run output in this model is mainly determined by these sectors. With a greater degree of price stickiness, such sectors have larger price dispersion and average price mark-ups than those with relatively flexible prices, reducing long-run output. Consequently, for a given trend inflation rate, average long-run output in the BK-MC is lower than in the Calvo model, resulting in a smaller determinacy region.

The remainder of the paper is organised as follows. Section 2 presents the model and discusses how trend inflation affects short-run inflation dynamics. Section 3 first studies long-run properties of the MC and then examines the implications of heterogeneity in price stickiness on the determinacy region. Finally, Section 4 concludes the paper.

2 Multiple Calvo (MC) with trend Inflation

The model presented here incorporates heterogeneity in price stickiness into the Ascari and Sbordone (2014) model using the assumption that there are multiple sectors, each with its own different Calvo price setting mechanism and corresponding probability of a price change. In this section, we first outline the main assumptions of our model and describe price setting. We then present the remaining equations of the model, which are identical to those in Ascari and Sbordone (2014) with logarithmic consumption utility.

There is a continuum of monopolistically competitive, profit-maximising firms indexed by \( f \in [0, 1] \), each producing a differentiated good \( Y_f \). Firms operate according to the following production function

\[
Y_{ft} = A_t N_{ft}
\]

where \( N_{ft} \) denotes labour and \( A_t \) denotes labour-augmenting technology. These goods are then combined, according to the Dixit-Stiglitz technology, to produce the final consumption aggregate. The final consumption aggregate
(Y_t), the corresponding price index (P_t) and the resulting demand function for firm’s i output (Y_{ft}) are all standard and are given by

\[ Y_t = \left[ \int_0^1 \frac{Y_{ft}}{Z_{10}} \, df \right]^{\frac{1}{1-\varepsilon}} \quad (2) \]
\[ P_t = \left[ \int_0^1 P_{ft}^{1-\varepsilon} \, df \right]^{\frac{1}{1-\varepsilon}} \quad (3) \]
\[ Y_{ft} = \left( \frac{P_{ft}}{P_t} \right)^{-\varepsilon} Y_t \quad (4) \]

where \( P_{ft}^* \) is the price level set by firm f, \( P_t \) is the general price level and \( \varepsilon \) is the elasticity of substitution between different goods. To introduce heterogeneity in price stickiness to the model, the unit interval of firms is divided into segments which are interpreted as sectors. There are \( N \) sectors, \( i = 1\ldots N \). Within each sector \( i \), there is a Calvo style contract. The share of sector \( i \) in the economy is \( \alpha_i \) and the sector-specific Calvo hazard rate is denoted by \( 1-\theta_i \). If we define the cumulative shares of sectors as \( \bar{\alpha}_i = \sum_{k=1}^{i} \alpha_k \), where \( k = 1\ldots i, \bar{\alpha}_0 = 0 \) and \( \bar{\alpha}_N = 1 \), the interval for sector \( i \) is \([\bar{\alpha}_{i-1}, \bar{\alpha}_i]\). With these assumptions, the general price index (\( P_t \)) can be rewritten in terms of sectors as follows.

\[ P_t = \left[ \sum_{i=1}^{N} \int_{\bar{\alpha}_{i-1}}^{\bar{\alpha}_i} P_{ft}^{1-\varepsilon} \, df \right]^{\frac{1}{1-\varepsilon}} \quad (5) \]

A firm in sector \( i \) in period \( t \) choose the optimal price \( P_{ft}^* \) to maximise expected profits during the expected lifetime of the contract, subject to the demand curve and the production function. Solving the maximisation problem and log-linearising the resulting equation give the following pricing rule.
for the firms in sector $i$\textsuperscript{6}:

\[
\hat{p}_{it}^* = \hat{\psi}_{it} - \hat{\phi}_{it} \tag{6}
\]

\[
\hat{\psi}_{it} = (1 - \beta \theta_i \pi^\varepsilon) \hat{MC}_t + \beta \theta_i \pi^\varepsilon \left( \varepsilon E_t \hat{\pi}_{t+1} + E_t \hat{\psi}_{it+1} \right) \tag{7}
\]

\[
\hat{\phi}_{it} = \beta \theta_i \pi^{\varepsilon-1} \left( (\varepsilon - 1) E_t \hat{\pi}_{t+1} + E_t \hat{\phi}_{it+1} \right) \tag{8}
\]

Where $\pi_t$ is the inflation rate and $\hat{MC}_t$ denotes marginal costs. $\hat{MC}_t$ is

\[
\hat{MC}_t = \varphi \hat{s}_t + (1 + \varphi) \dot{Y}_t - (1 + \varphi) \hat{A}_t \tag{9}
\]

where $\varphi$ is the Frisch elasticity of labour supply, $\hat{s}_t = \sum_{i=1}^N c_i \hat{s}_{it}$ is the aggregate price dispersion and $\hat{s}_{it}$ is price dispersion within a sector. $\hat{s}_{it}$ is

\[
\hat{s}_{it} = (1 - \theta_i) (\hat{p}_{it}^*)^{-\varepsilon} + \theta_i \pi_i^{\varepsilon} \hat{s}_{it-1} \tag{10}
\]

The log-linearised sectoral (real) price ($\hat{p}_{it}$) is

\[
\hat{p}_{it} = (1 - \theta_i \pi^{\varepsilon - 1}) (\hat{p}_{it}^*) + \theta_i \pi_i^{\varepsilon - 1} (\hat{p}_{it-1} - \hat{\pi}_t) \tag{11}
\]

where $\hat{p}_{it}^*$ is the log-linearised (real) reset price in sector $i$. The aggregate price level in the economy is the weighted average of all prices in the economy. This relation implies that

\[
\sum_{i=1}^N \alpha_i \hat{p}_{it} = 0 \tag{12}
\]

and $\hat{p}_{it}$ can be expressed as

\[
\hat{p}_{it} = \hat{p}_{it-1} + \hat{\pi}_{it} - \hat{\pi}_t \tag{13}
\]

These equations can also nest the model in Ascari and Sbordone (2014)\textsuperscript{6}.

\textsuperscript{6}For a derivation of these equations see Appendix A.
by setting \( N = 1 \). The remaining equations are identical to those in Ascari and Sbordone (2014) with logarithmic consumption utility. Output is given by the standard Euler condition:

\[
\dot{Y}_t = E_t \dot{Y}_{t+1} - (i_t - E_t \hat{\pi}_{t+1})
\]  

(14)

Monetary policy is modelled as following a Taylor rule:

\[
\hat{i}_t = \phi_x \hat{\pi}_t + \phi_y \dot{Y}_t
\]  

(15)

where the \( \phi \)–coefficients are the parameters in front of the targeting variables.

### 2.1 Phillips Curve in the MC with trend inflation

Using Equations (6)-(13), inflation dynamics in sector \( i \) is given by\(^7\)

\[
\hat{\pi}_{it} = \beta E_t \hat{\pi}_{it+1} + \kappa_i \left( MC_t - \bar{p}_{it} \right) + d_i \left( \varepsilon E_t \hat{\pi}_{t+1} + E_t \hat{\psi}_{it+1} \right)
\]  

(16)

with

\[
\kappa_i = \frac{(1 - \theta_i \pi^{\varepsilon - 1}) (1 - \theta_i \beta \pi^{\varepsilon})}{\theta_i \pi^{\varepsilon - 1}} \quad \text{and} \quad d_i = \beta \varepsilon (\pi - 1) \left( 1 - \theta_i \pi^{\varepsilon - 1} \right)
\]

where \( d_i \) is the coefficient on expected economy-wide inflation and \( \hat{\psi}_{it+1} \), \( \kappa_i \) is the coefficient on marginal costs and relative prices in sector \( i \). Both \( d \) and \( \kappa \) coefficients depend on the degree of price stickiness in sectors as well as trend inflation. Setting \( N = 1 \) and dropping subscript \( i \) generates the Phillips curve in Ascari and Sbordone (2014). Assuming \( N > 1 \) but setting \( \pi = 1 \) gives sectoral inflation in the MC model without trend inflation.

An important implication of trend inflation for inflation dynamics is that inflation becomes less responsive (i.e. lower \( \kappa_i \)) to changes in marginal costs.

\(^7\)For a derivation of the sectoral Phillips Curve see Appendix B.
with increased trend inflation. There are two reasons for this result. First, as Equations (6)-(8) indicate, when resetting their prices, firms give more weight to the future, reducing the importance of current marginal cost in pricing decisions. They do so in order to protect their relative prices from increased trend inflation. Second, increased forward-lookingness implies that resetting firms price more aggressively than they otherwise would. As Coibion, Gorodnichenko and Wieland (2012) emphasize, such firms have an expenditure share which is decreasing with trend inflation. This in turn weakens the link between the average price level and reset prices and, consequently, the link between that inflation and marginal costs.

3 Equilibrium Determinacy

Woodford (2003) suggests that to achieve determinacy the nominal interest rate should increase by more than the increase in inflation in the long-run. This suggestion implies that equilibrium is determinate if

$$\frac{\partial i}{\partial \pi} \Big|_{LR} = \phi_\pi + \phi_y \frac{\partial Y}{\partial \pi} \Big|_{LR} > 1$$

Equation (17) makes it clear that the key to understanding the impact of trend inflation on the determinacy region, and the corresponding required long-run interest rate response to an inflation change, is understanding the impact of trend inflation on long-run output. In the next section, we turn to analyse the long-run properties of the model. In doing so, we use the approach in Ascari and Sbordone (2014), who use a non-linear formulation of the model to study the long-run properties of their model.
3.1 Long run properties of the MC

Long-run output in the MC is given by \(^8\):

\[
Y = \left( \frac{1}{s\sigma \mu} \right)^{\frac{1}{1+\sigma}} \tag{18}
\]

where \(\mu\) is a real price mark-up and \(s\) denotes price dispersion. This equation shows that long-run output depends on long-run values of price dispersion and the real price mark-up. We now turn to analyse each component of long-run output in detail. We start by considering price dispersion. Price dispersion is the weighted average of sectoral price dispersion \((s_i)\) and is given by

\[
s = \sum_{i=1}^{N} \alpha_i s_i \tag{19}
\]

Using Equations (6)-(15), after some tedious algebra\(^9\), price dispersion in sector \(i\) can be expressed as follows

\[
s_i = \frac{1 - \theta_i}{1 - \theta_i \pi^\varepsilon} \left( \frac{1 - \theta_i \pi^{\varepsilon - 1}}{1 - \theta_i} \right)^{\frac{\varepsilon}{\varepsilon - 1}} \tag{20}
\]

Equation (19) indicates that price dispersion in sector \(i\) depends on the degree of price stickiness in sector \(i\) \((\theta_i)\), trend inflation \((\pi)\) and the elasticity of substitution between different goods \((\varepsilon)\). Sectors have a different degree of price dispersion since they face a different degree of price stickiness. In the sector with fully flexible prices (i.e. \(i = 1\)), there is no price dispersion. Sectors with stickier prices have larger price dispersion. Next, we consider the effect of trend inflation on price dispersion. Price dispersion within each sector increases with trend inflation. This is true because higher trend infla-

---

\(^8\)This expression is the same as in Ascari and Sbordone (2014) for the Calvo model when the labour disutility parameter \((d_a)\) and the risk-aversion parameter \((\sigma)\) are both normalised to unity.

\(^9\)See Appendix C for a derivation of equations in this section.
tion results in a larger difference between reset prices and the average price level. Finally, a higher \( \varepsilon \) has the same effect as trend inflation, as it exacerbates distortions induced by relative price changes, resulting in higher price dispersion.\(^\text{10}\)

We now study the price mark-up. The price mark-up is the weighted average of sectoral real price mark-ups \( (\mu_i) \) and is given by

\[
\mu = \sum_{i=1}^{N} \alpha_i \mu_i 
\]  

(21)

Following Ascari and Sbordone (2014), we decompose the price mark-up in each sector \( i \) into two components – a price adjustment gap and a marginal mark-up:

\[
\mu_i = \frac{P_i - P_i^*}{MC} 
\]  

(22)

where \( \frac{P_i}{P_i^*} \) is the sectoral price adjustment gap and \( \frac{P_i^*}{MC} \) is the sectoral marginal mark-up. The marginal mark-up in sector \( i \) is the ratio of reset price in sector \( i \) to marginal cost, while the price adjustment gap in sector \( i \) is the ratio of the average price level in sector \( i \) to reset price in sector \( i \). The sectoral marginal mark-up is given by

\[
\frac{P_i^*}{MC} = \frac{\varepsilon}{\varepsilon - 1} \frac{1 - \beta \theta_i \pi^{\varepsilon - 1}}{1 - \beta \theta_i \pi^\varepsilon} 
\]  

(23)

This equation suggests that marginal mark-ups differ across sectors since each sector has a different degree of price stickiness. It is easy to verify that, with reasonable parameter values, sectors with greater price stickiness have larger marginal mark-ups. Equation (23) further suggests that marginal mark-ups increases with trend inflation. To understand these results, first

\(^{10}\)How much price dispersion affects long-run output depends crucially on the Frisch elasticity of labour supply (\( \varphi \)). While in general it increases with \( \varphi \), if we follow Hansen (1985) and assume that labour is indivisible (i.e. \( \varphi = 0 \)), long-run output will not be directly affected by changes in price dispersion and will solely be determined by the real price mark-up.
note that firms with sticky prices, when resetting their prices, increase their prices more aggressively than otherwise to protect their relative prices from increased inflation. Firms in sectors with longer-term contracts do so more than those in sectors with relatively flexible prices. Consequently, in sectors with longer-term contracts, marginal mark-ups are higher than those with relatively flexible prices.

Finally, we consider sectoral price adjustment gaps, which can be expressed as

\[
\frac{P^*_i}{P} = \left( \frac{1 - \theta_i}{1 - \theta_i \pi^{t-1}} \right)^{\frac{1}{\tau}}
\]

Equation (24) shows that sectoral price adjustment gaps decrease with trend inflation. As we have discussed, with trend inflation, firms in the sticky-price sector set prices more aggressively, when given the opportunity to do so, leading to larger marginal mark-ups. As a result, the expenditure share of such firms is decreasing with trend inflation, meaning that prices of such firms become less important in the economy. This mechanism reduces the average mark-up in the economy. Although the price adjustment gap decreases with trend inflation, numerical tests we performed (not reported here) suggest that except for very low values of trend inflation (less than 0.5%), the increase in the marginal mark-up is always sufficiently large that the average mark-up in the model increases with trend inflation.

3.2 The quantitative implications of trend inflation

In this section, we quantitatively examine the effect of trend inflation on long-run output and its determinants in the MC. Before presenting the results for the MC, it is useful to discuss what is already known from prior work based on the Calvo model. Assuming \( N = 1 \) and dropping subscript \( i \)'s, the expressions in equations (19)-(24) reduce to those in the Calvo model. Panel A of Figure 1 plots long-run output in the Calvo model as a function of
trend inflation. Panels B and C of Figure 1 plot the determinants of long-run output (i.e. price dispersion and the average price mark-up) against trend inflation. As is evident from the figures, both price dispersion and the average price mark-up increase with trend inflation, resulting in lower long-run output. However, the fall in long-run output, given the increase in the trend inflation, appears to be small. Even at 4% trend inflation, the fall in output, relative to the case without trend inflation, is less than 0.5%.

Turning to the MC, calculating long-run output in the MC requires calibrating the share of each sector (or each frequency) \((\alpha_i)\). We calibrate the share of each sector according to the micro evidence provided by Bils and Klenow (BK) (2004), who report this frequency for around 300 product categories, which covers 70% of the US CPI. The distribution suggested by the BK dataset is plotted in Figure 2. The mean frequency of price adjustment \((1 - \theta)\) across the whole economy is around 0.4. As the figure shows, there are quite a few flexible contracts. The share of flexible contracts is around 35%. But the distribution has a long tail.

Figure 1 shows that when trend inflation increases, long-run output falls a lot more in the BK-MC than it does in the Calvo model. This is because increased trend inflation leads to a larger increase in the average mark-up and price dispersion in the BK-MC than in the Calvo model. Figure 1 in particular shows that in the case of the BK-MC, when trend inflation increases from 2% to 4%, the output loss due to the increase in trend inflation is larger than that suggested by the Calvo model and is around 2.5%. These findings...

---

11 Following Ascari and Sbordone (2014), we assume \(\varepsilon = 10\) and \(\beta = 0.99\). We assume that \(\theta = 0.6\), a value suggested by the Bils and Klenow dataset (2004).

12 Following Kara (2015), we aggregate up from their 300 sectors so that we have just 10 sectors with distinct price reset probabilities. The aggregation is performed by forming probability focal points in increments of 0.1 percentage points [thus: 0, 0.1, 0.2, 0.3...etc.]. We then round the Bils-Klenow reset probabilities to 0.1 percentage point, and allocate the 300 BK sectors to these 10 focal points. The sectors are scaled by the share in expenditure that is allocated to each focal point.

13 In calculating this figure, following Ascari and Sbordone (2014), we calibrate the Frisch elasticity of labor supply \(\varphi\) to one. This parameter is important, since it determines how
provide a reason why the Calvo model understates the cost of raising the inflation target relative to the benefit in terms of avoiding the liquidity trap.

These results bring up a question: Why is it that a higher trend inflation rate is more disruptive in the BK-MC? Trend inflation is more disruptive in the BK-MC because of the presence of sectors with hazard rates lower than the mean hazard rate. With a greater degree of price rigidity, these sectors have significantly larger price dispersion and average mark-up than those with relatively flexible prices. As a consequence, these sectors have a disproportionate contractionary effect on long-run output, leading to a lower long-run output in the MC.

To illustrate the role of longer-term contracts in our results, we use the simplest version of the MC with only two sectors (i = 1, 2). Specifically, we consider three 2-sector MCs. In all the three cases, the mean hazard rate is the same as that in the BK-MC but each case has a different relative price stickiness (RS). We assume sectors have equal shares (i.e. $\alpha_1 = \alpha_2 = 0.5$) and define $RS$ as $RS = (1 - \theta_1) / (1 - \theta_2)$. In the first case, we assume an economy with the common assumption that sectors have the same degree of price stickiness (i.e. $1 - \theta_1 = 1 - \theta_2 = 0.4$) and, therefore, $RS = 1$. In the second case, we have $\theta_1 = 0.4$, $\theta_2 = 0.7$ and $RS = 2$. Finally, in the third case, we consider a limiting case in which prices are fully flexible in sector 1, while sticky in sector 2 ($\theta_2 = 0.75$), implying that $RS = 4$. Overall, in this experiment, an increase in $RS$ means that prices in the sector 2 become stickier, while those in sector 1 become more flexible. The case with $RS = 4$ is especially useful since it helps to isolate the effect of longer-term contracts on the economy. This is true because in this case, sector 1 prices are fully flexible, meaning that in sector 1, there is no price dispersion and the average price mark-up is independent of trend inflation ($\mu_1 = \varepsilon / (1 - \varepsilon)$). This implies that any change in economy-wide price dispersion or in the much price dispersion affects long-run output. Gali (2013) suggests that this parameter is smaller and calibrates it to 0.2. If we use Gali’s calibration, the loss is only slightly lower and is around 2%. 

15
average price mark-up would arise through sector 2.

Figures 3 and 4 show the results from this experiment. Figure 3 reports price dispersion and the average price mark-up at the aggregate level in the three economies and Figure 4 at the sectoral level. Figure 3 also plots long-run output. Two results stand out from the figures. First, the magnitude of price dispersion and the average price mark-up within each sector is highly sensitive to the degree of price stickiness in that sector and trend inflation. When there is an increase in $RS$, in sector 2, given trend inflation, both price dispersion and the average price mark-up become increasingly larger, while those in sector 1 decrease. Second, consistent with the findings we have reported so far, a more dispersed economy has lower long-run output. Figure 3 shows that the reason for lower long-run output is that an increase in $RS$ results in a larger price dispersion and the average price mark-up.

If we look at Figure 4, we see that the source of the increase in the economy-wide price mark-up and price dispersion is sector 2. It is interesting to note that when $RS = 4$, even though one of the sectors has fully flexible prices, economy-wide price dispersion, the average price mark-up and the consequent fall in long-run output are larger than the other cases with $RS = 1$ and $RS = 2$. These results highlight how long-term contracts can be disruptive in a high trend inflation environment.\footnote{In an alternative setting, we fix $\theta_1$ to be 0.4, as in $RS = 2$ and increase $\theta_2$ from 0.7 to 0.9 in increments of 0.1. In each case, we adjust the sectors’ shares in a way to ensure that the mean hazard rate is equal to that in the BK-MC. An increase in $\theta_2$ means that the share of sector 2 in the economy is lower. We then consider the effect of increasing trend inflation in such a setting. We find that that our conclusions remained unchanged. Despite the decreasing share of sector 2, increasing the degree of nominal rigidity in sector 2 leads to a significant increase in price dispersion, the average mark-up and, consequently, a large fall in long-run output.}

Finally, it is important to note that if we compare Figures 1 and 3, we see that the fall in long-run output in the BK-MC when there is an increase in trend inflation is larger than that in the 2-sector MCs. In two-sector MCs, the fall in long-run output is largest in the limiting case with $RS = 4$. Even
in this case, when trend inflation is 4%, the fall in long-run output is around one-fifth of what it is in the BK-MC. The reason for this should be clear. The BK-MC has a wide range of contracts and a few contracts are longer than those in 2 sectors MCs.

3.3 The determinacy region in the MC

With some vital bits of intuition now in hand, we can now turn to our main question: What are the implications of higher trend inflation on the determinacy region in the MC? The determinacy region here is the two dimensional space defined by the parameters on the inflation rate and the output gap in a Taylor rule for which the model is determinate.

The discussion so far suggests that the determinacy region should shrink faster in the MC than in the Calvo model, as trend inflation increases, because long-run output in the MC decreases a lot more with an increase in trend inflation than in the Calvo model. Indeed, Figures 5-7 confirm this suggestion.

We start by discussing the determinacy region in the Calvo model. Figure 5 shows that the determinacy region shrinks in the Calvo model, as trend inflation increases\(^{15}\). At zero inflation, the case that has been studied extensively in the literature, the determinacy region is large. All the area to the right of the almost vertical line beginning at \((\phi_x = 1, \phi_y = 0)\) and heading ‘North East’ are determinate. As we increase trend inflation, the region shrinks. The shrinkage can be seen by the gradual clockwise rotation of the line separating determinacy region (below and to the right) from indeterminacy region (above and to the left). For positive trend inflation, the figure suggests that greater responses to inflation can ‘buy’ a higher value of the

\(^{15}\) When computing determinacy regions, we employ the Blanchard and Kahn method. The condition is checked at points at intervals of 2 decimal points. Given the nature of the determinacy condition derived algebraically, we invoke a continuity argument that points between those computed explicitly can be assumed to yield a result equal to those either side of them.
output gap response coefficient for which the model is still determinate. The slope of the determinacy border falls with each increment to trend inflation, suggesting that if we increase the output gap coefficient, we have to increase the inflation coefficient by more if we want to preserve determinacy.

Next, we consider 2-sector MCs. Figure 6 plots the determinacy region in 2-sector MCs with $RS = 2$ and $RS = 4$. The determinacy region for zero trend inflation is unchanged by the switch to 2-sector MCs and is the same as that for the Calvo model, which is plotted in Figure 5. But as we raise the trend inflation rate by the same increments of 2 percentage points, as before, now the determinacy region is smaller, confirmed by noting that the determinacy border is tilted clockwise relative to the same border for the single sector Calvo model above. The shrinkage becomes faster with increasing $RS$ since the more dispersed economy has a lower long-run output.

Having illustrated the influence of heterogeneity in price stickiness using simple 2-sector MCs, we turn now to present results from the BK-MC. Figure 7 plots the determinacy region in the BK-MC. We see from this figure that, reflecting the fact that increments of trend inflation lower long-run output in the BK-MC significantly more than in 2-sector MCs, an increase in trend inflation shrinks the determinacy region in the BK-MC more dramatically than in 2-sector MCs. At 4%, in the BK-MC model, the determinacy region is almost non-existent.

4 Conclusions

We have developed our own approach to evaluate the policy proposal of increasing the inflation target to 4%. Our approach modifies and extends the standard New Keynesian model in two ways. First, it is linearised around empirically relevant, positive rates of inflation, instead of the more normal and convenient 0%. Second, we allow the degree of price stickiness to vary across sectors, encoding observations made in Bils and Klenow (2004) result-
ing in a Multiple Calvo (MC) model. Our results articulate a reason why raising inflation to 4% might be cautioned against.

We have shown that in the MC the region for which monetary policy rules render rational expectations equilibria indeterminate is enlarged as trend inflation increases, relative to that in the standard NK model with Calvo pricing. At the commonly proposed target of 4%, in the MC, the indeterminacy region is very small, whereas in the corresponding Calvo model the determinacy region is quite large and is not too different from the region when the target is 2%.

The determinacy region shrinks more with increasing trend inflation in the MC because long-run output decreases more with trend inflation in this model. This is a consequence of the presence of sectors with a high degree of price rigidity. In such sectors, an increase in trend inflation results in larger price dispersion and price mark-ups than those with relatively flexible prices. These sectors have disproportionately negative effect on long-run output. This reduction in long-run output shrinks the determinacy region.

As well as heightening a concern about the option of a 4% target over the future, a corollary of our work is that historically high inflation rates in economies like the US and the UK are much more likely to have led to indeterminacy than researchers may previously have been aware. This lends credence to the argument that sunspot shocks to inflation - the possibility for which is opened up by indeterminacy - were part of the explanation for past inflation volatility.

Finally, we have only focussed on studying determinacy issues after the proposed policy is already in place without discussing how such a policy can be implemented. Considering the current low level of interest rates, the transition to a 4% inflation target is an daunting task. We leave the implementation of the policy as a matter for further research.
References


Firm $f$ in sector $i$ solves the following profit maximisation problem:

$$
E_t \sum_{j=0}^{\infty} \beta^j \frac{\lambda_{t+j} \theta^j}{\lambda_0} \left[ \frac{P^*_f t}{P_{t+j}} Y_{fit+j} - \frac{W_{t+j} Y_{fit+j}}{P_{t+j} A_{t+j}} \right]
$$

subject to the demand function faced by a firm $f$ in sector $i$ (Equation (4)). Substituting Equation (4) into Equation (25) and solving the maximisation problem, we obtain

$$
\frac{P^*_{it}}{P_t} = \frac{E_t \sum_{j=0}^{\infty} \beta^j \lambda_{t+j} \theta^j \left[ \frac{1}{\Pi_{t,t+j}} \right]^{-\varepsilon} Y_{t+j}}{E_t \sum_{j=0}^{\infty} \beta^j \lambda_{t+j} \theta^j \left[ \frac{1}{\Pi_{t,t+j}} \right]^{-1-\varepsilon} Y_{t+j}}
$$

where $\Pi_{t,t+j} = \frac{P_{t+j}}{P_t}$, $\lambda_{t+j} = C_{t+j}^{-1} = Y_{t+j}^{-1}$, $w_{t+j} = \frac{W_{t+j}}{A_{t+j}}$ and $MC_t = \frac{w_t}{A_t}$. Note that subscript $f$ is dropped in the above equation, as all the firms that reset their prices in sector $i$ set the same price. Define $\psi_{it}$ and $\phi_{it}$.
\[
\psi_{it} = E_t \sum_{j=0}^{\infty} \beta^j \lambda_{t+j} \theta_{i}^{j} MC_{t+j} \left[ \frac{1}{\Pi_{t+j}} \right]^{-\varepsilon} Y_{t+j} = MC_t + \beta \theta_i E_t \left[ \pi_{t+1}^{\varepsilon} \psi_{it+1} \right] \\
\phi_{it} = E_t \sum_{j=0}^{\infty} \beta^j \lambda_{t+j} \theta_{i}^{j} \left[ \frac{1}{\Pi_{t+j}} \right]^{1-\varepsilon} Y_{t+j} = 1 + \beta \theta_i E_t \left[ \pi_{t+1}^{\varepsilon-1} \phi_{it+1} \right]
\]

(27)

Log-linearsing Equations (26)-(28) gives Equations (6)-(8) in the main text. The average price level in sector \( i \) is

\[
\frac{P_{it}}{P_t} = \left[ \theta_i \left( \frac{P_{it-1}}{P_{t-1}} \right)^{1-\varepsilon} + (1 - \theta_i) \left( \frac{P_{it}}{P_t} \right)^{1-\varepsilon} \right]^{1/\varepsilon}
\]

(29)

The log-linearsed version of this equation is reported in Equation (11) in the main text. To calculate price dispersion, we use the production function (Equation (1)). Aggregating over firms within the same sector gives: \( N_{it} = \frac{Y_{it}}{A_t} \). Substituting the sectoral demand function \( Y_{it} = (P_{it}/P_t)^{-\varepsilon} Y_t \) into this equation and aggregating across sectors give

\[
N_t = \sum_{i=1}^{N} \alpha_i \frac{Y_{it}}{A_t} = \frac{Y_t}{A_t} \sum_{i=1}^{N} \alpha_i \left[ \frac{P_{it}}{P_t} \right]^{-\varepsilon} \underbrace{\text{\( s_{it} \)}}
\]

where \( s_{it} \) is the relative price dispersion measure in sector \( i \) and captures the cost of relative price dispersion in that sector due to positive trend inflation. This measure can be rewritten as
\begin{align*}
s_{it} &= (1 - \theta_i) \left[ \frac{P_{it}}{P_t} \right]^{-\varepsilon} + \theta_i (1 - \theta_i) \left[ \frac{P_{it-1} P_{t-1}}{P_{t-1} P_t} \right]^{-\varepsilon} + \theta_i^2 (1 - \theta_i) \left[ \frac{P_{it-2} P_{t-2} P_{t-1}}{P_{t-2} P_{t-1} P_t} \right]^{-\varepsilon} + \ldots \\
\end{align*}

Equation (10) reports the log-linearised version of Equation (31).

\section*{Appendix B}

Sectoral price index (Equation (11)) can be rewritten as

\[ \hat{p}_{it} = \frac{\theta_i \pi^{t-1}_i}{1 - \theta_i \pi^{t-1}_i} \tilde{p}_t + \frac{1}{1 - \theta_i \pi^{t-1}_i} [\hat{p}_t - \theta_i \pi^{t-1}_i \hat{p}_{it-1}] \]  

(32)

Substituting this equation into (6) for \( \hat{p}_{it} \) and rearranging gives

\[ \hat{\phi}_{it} = \hat{\psi}_{it} - \left\{ \frac{\theta_i \pi^{t-1}_i}{1 - \theta_i \pi^{t-1}_i} \tilde{p}_t + \frac{1}{1 - \theta_i \pi^{t-1}_i} [\hat{p}_t - \theta_i \pi^{t-1}_i \hat{p}_{it-1}] \right\} \]  

(33)

Substituting Equation (33) into Equation (8) and rearranging yields

\[ \hat{\psi}_{it} = \left\{ \frac{\theta_i \pi^{t-1}_i}{1 - \theta_i \pi^{t-1}_i} \tilde{p}_t + \frac{1}{1 - \theta_i \pi^{t-1}_i} [\hat{p}_t - \theta_i \pi^{t-1}_i \hat{p}_{it-1}] \right\} \] 

\[ + \beta \theta_i \pi^{t-1}_i \left( (\varepsilon - 1) E_t \hat{\pi}_{it+1} + E_t \hat{\phi}_{it+1} \right) \]  

(34)

We eliminate \( \hat{\phi}_{it+1} \) in Equation (34) by updating Equation (33) one period and substituting the resulting expression for \( \hat{\phi}_{it+1} \) into Equation (34). Doing so results in the following expression...
\[ \hat{\psi}_{it} = \left( \frac{\theta_i \pi^{\varepsilon-1}}{1 - \theta_i \pi^{\varepsilon-1}} \right) \hat{\pi}_t + \left( \frac{1}{1 - \theta_i \pi^{\varepsilon-1}} \right) [\hat{p}_{it} - \theta_i \pi^{\varepsilon-1} \hat{p}_{it-1}] \] (35) 

\[ + \theta_i \beta \pi^{\varepsilon-1} [\hat{\psi}_{it+1} - \left( \frac{\theta_i \pi^{\varepsilon-1}}{1 - \theta_i \pi^{\varepsilon-1}} \right) E_t \hat{\pi}_{t+1} + (\varepsilon - 1) E_t \hat{\pi}_{t+1} - \left( \frac{1}{1 - \theta_i \pi^{\varepsilon-1}} \right) [\hat{p}_{it+1} - \theta_i \pi^{\varepsilon-1} \hat{p}_{it}] \]

Substituting Equation (7) into this equation and rearranging gives

\[ \frac{1}{\theta_i \pi^{\varepsilon-1}} [\hat{p}_{it} - \theta_i \pi^{\varepsilon-1} \hat{p}_{it-1}] = \beta [E_t \hat{p}_{it+1} - \theta_i \pi^{\varepsilon-1} \hat{p}_{it}] + \kappa_i \hat{MC}_t \]

\[ + \beta \left[ 1 + \varepsilon (\pi - 1) (1 - \theta_i \pi^{\varepsilon-1}) \right] E_t \hat{\pi}_{t+1} - \hat{\pi}_t \]

\[ - \beta (1 - \varepsilon) (1 - \theta_i \pi^{\varepsilon-1}) E_t \hat{\psi}_{it+1} \]

Using Equation (13), we get

\[ \frac{1}{\theta_i \pi^{\varepsilon-1}} [\hat{p}_{it} - \theta_i \pi^{\varepsilon-1} (\hat{p}_{it} - \hat{\pi}_t + \hat{\pi}_t)] = \beta [\hat{p}_{it} + \hat{\pi}_{it+1} - \hat{\pi}_{t+1} - \theta_i \pi^{\varepsilon-1} \hat{p}_t] + \kappa_i \hat{MC}_t \]

\[ + \beta \left[ 1 + \varepsilon (\pi - 1) (1 - \theta_i \pi^{\varepsilon-1}) \right] E_t \hat{\pi}_{t+1} - \hat{\pi}_t \]

\[ - \beta (1 - \varepsilon) (1 - \theta_i \pi^{\varepsilon-1}) E_t \hat{\psi}_{it+1} \]

Simplifying this equation gives Equation (16) in the main text.

**Appendix C**

Dropping subscript \( t \) in Equation (31) and solving for \( s \), we obtain an expression for long-run price dispersion

\[ s_i = \frac{1 - \theta_i}{1 - \theta_i \pi^{\varepsilon}} \left( \frac{P_i^*}{P} \right)^{-\varepsilon} \] (36)

\( P_i^* / P \) is the sectoral price adjustment gap. Using Equation (29), we obtain the following expression for the sectoral price adjustment gap
Substituting Equation (37) into Equation (36) gives Equation (20) in the main text. In a steady-state with constant inflation, Equation (26) becomes

$$\frac{p_i^*}{P} = \left( \frac{1 - \theta_i \pi^{\varepsilon - 1}}{1 - \theta_i} \right)^{\frac{1}{\varepsilon^*}}$$  \hspace{1cm} (37)

Rearranging this expression gives the sectoral marginal mark-up in the main text (Equation (23)).

$$p_i^* = \frac{\varepsilon}{\varepsilon - 1} \frac{1 - \beta \theta_i \pi^{\varepsilon - 1}}{1 - \beta \theta_i \pi^{\varepsilon - 1}} MC$$  \hspace{1cm} (38)
Figure 1: Effects of trend inflation on long-run output, price dispersion and the average price mark-up in Calvo and Multiple Calvo models. Note: Variables are expressed as percentage deviation from the zero inflation steady-state.
Figure 2: The Bils and Klenow (2004) distribution of price spells

Note: Using the US CPI data, Bils and Klenow (2004) report the frequency of price changes for around 300 product categories, which covers 70% of the US CPI. These frequencies are rounded to one decimal point and resulting numbers aggregated up so, leading to 10 distinct price reset probabilities, which are reported in this figure.
Figure 3: Effects of trend inflation on long-run output, price dispersion and the average price mark-up in two-sector Multiple Calvo models with different relative price stickiness (RS).

Note: The mean contract length is the same as that suggested by the Bils and Klenow distribution plotted in Figure 2. The figure shows that long-run output falls more in a more dispersed economy when there is an increase in trend inflation. Variables are expressed as percentage deviation from the zero inflation steady-state.
Figure 4: Sectoral price dispersion and average price mark-ups in two-sector and Multiple Calvo models with different relative price stickiness ($RS$).

Note: Prices in sector 1 are more flexible than those in sector 2. This figure shows that when there is an increase in $RS$, price dispersion and average price mark-ups in Sector 1 decrease, while those in Sector 2 increase. This is because with an increase in $RS$, prices in sector 1 become more flexible, while those in sector 2 get stickier. Variables are expressed as percentage deviation from the zero inflation steady-state.
Figure 5: Determinacy regions for alternative trend inflation rates in the standard Calvo model

Note: Consistent with the findings reported in Ascari and Sbordone (2004), this figure shows that increasing trend inflation shrinks the determinacy region.
Figure 6: Determinacy regions in two-sector Multiple Calvo models with different relative price stickiness ($RS$).

Note: The case with $RS = 1$ is the same as the standard Calvo model plotted in Figure 5. The figure shows that, given trend inflation, determinacy regions shrinks more with increasing RS.
Figure 7: Determinacy regions in the MC with Bils and Klenow (2004) distribution model for alternative trend inflation rates
Notes: This figure shows that, given trend inflation, increments of trend inflation shrinks the determinancy region more than it does in the Calvo model.