A quasi-zero stiffness vibration isolator based on hybrid bistable composite laminate

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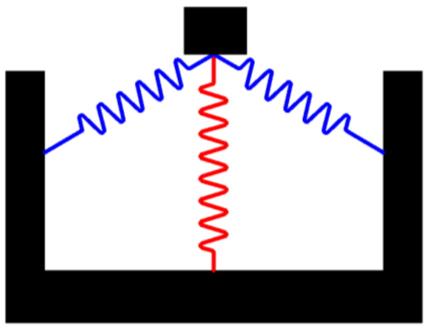
Abstract:
In many conceptual designs of quasi-zero stiffness (QZS) vibration isolators, the negative stiffness elements are very inefficient in terms of weight and volume. This is because they often need to attach additional structural components, and require external constraints or forces to pre-stress some particular components. As a consequence, the volume and weight of the QZS vibration isolators increase to unacceptable levels for many practical applications, such as space equipment and aviation crafts. In this study, a novel QZS vibration isolator that applies bistable composite laminates as the negative stiffness element is proposed and designed to meet the practical requirements. This novel design of the QZS vibration isolator has a greatly simplified structural geometry forms, due to the inherent negative stiffness of bistable composite laminates. The mechanism of this novel QZS vibration isolator mainly lies in the ingenious use of negative stiffness properties of bistable composite laminates. Its structural performance is analyzed and simulated using the finite element methods in Abaqus. The numerical simulation results successfully approve the design principle and outperformed characteristics of this proposed novel design of QZS vibration isolator.

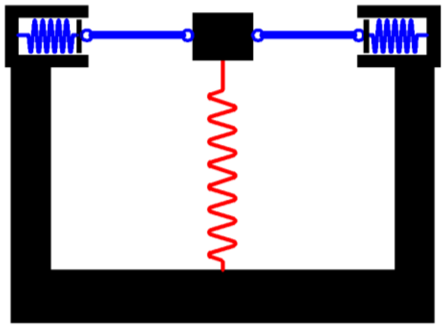
Keywords: quasi-zero stiffness; vibration isolator; negative stiffness; bistable laminate

1 Introduction

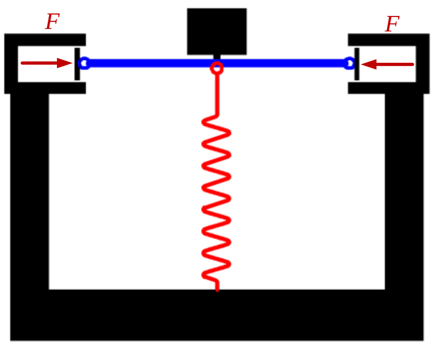
A linear passive vibration isolator can only function well when its natural frequency is much below the excitation frequency. Since a traditional linear passive isolator does not isolate vibrational behavior until the excitation frequency is greater than $\sqrt{2}$ times of its natural frequency, it is unable to isolate the low frequency and ultra-low frequency vibration [1]. Because of this reason, nonlinear vibration isolators are proposed and widely used in many engineering applications[1, 2]. The concept of quasi-zero stiffness (QZS) vibration isolator attracts a lot of research interests due to its distinct characteristics, e.g. high static stiffness and low dynamic stiffness. The low dynamic stiffness of a QZS vibration isolation system makes it quite efficient in isolating low frequency and ultra-low frequency vibration[3]. Practical applications of QZS vibration isolators range from space research, e.g., zero gravity simulation[4], to manufacturing machines, e.g. isolation of high precision machinery [5, 6]. A QZS vibration isolator usually consists of a positive stiffness element and a negative stiffness element which are connected in parallel. In last decades, many novel forms of QZS vibration isolators were proposed, and one representative form is presented in Fig. 1 [3], in which four different designs of QZS are illustrated. The theories and experiments of these techniques in the field of low frequency vibration isolation have been extensively studied in previous research works [7-13].



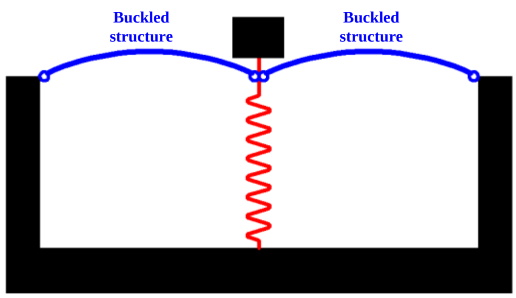
(a) QZS system achieved by combining a vertical spring acting as a positive stiffness element with two oblique springs acting as a negative stiffness element



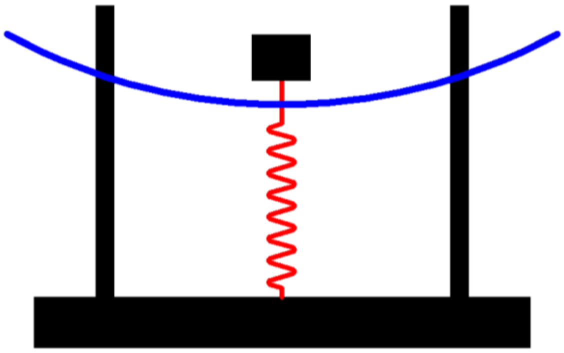
(b) QZS system achieved by combining a vertical spring acting as a positive stiffness element with symmetric horizontal springs and bars acting as a negative stiffness element



(c) QZS system achieved by combining a vertical spring acting as a positive stiffness element with a bucked beam acting as a negative stiffness element



(d) QZS system achieved by combining a vertical spring acting as a positive stiffness element with two inclined Euler columns as a negative stiffness element



(e) QZS system achieved by combining a vertical spring acting as a positive stiffness element with a Gospodnetic-Frisch-Fay beam as a negative stiffness element

Fig. 1 Sketch maps of representative QZS vibration isolators[3]

For the QZS vibration isolators presented in Fig. 1, in common external restraints or forces are needed to apply the pre-stresses to the negative stiffness elements. Meanwhile, the sizes of negative stiffness elements are difficult to be reduced due to their principles. As a consequence, this type of QZS vibration isolators is complicated and bulky, which results in increased weight and installation space. However, in many practical application circumstances, such as the space crafts, satellites and planes, the weight and installation space is extremely limited. Therefore, there remains continuous demand to design simple, small and light negative stiffness elements for the QZS vibration isolators.

In this study, a novel QZS vibration isolator is proposed. In this new design, a bistable composite laminate is employed as the negative stiffness element. A bistable laminate has two stable configurations, and the stiffness of which exhibits negative when it is restrained at an unstable equilibrium saddle state [14-16]. The principle of this novel QZS vibration isolator is analyzed in details and its performance is numerically simulated using the finite element modelling.

2 Design of the quasi-zero stiffness vibration isolator

In this study, a hybrid metal-fiber bistable composite laminate is used and its layup is chosen to be [0/Al/90]. Previous studies have demonstrated that the [0/Al/90]] laminate has two stable cylindrical configurations due to the internal thermal strain, provided that the laminate dimensions are chosen, appropriately. For a square bistable laminate mounted at central, as illustrated in Fig. 2, the laminate transforms form a stable configuration to the other when loads are applied on the corner nodes. The classical strain energy curve and the force-displacement curve of a bistable laminate corresponding to the boundary conditions are presented in Fig. 3. Classical strain energy curve in Fig. 3 shows that a bistable laminate has two local potential energy wells, whereas the load-displacement curve demonstrates the negative stiffness property of the bistable laminate when it transforms form a potential energy well to the other.

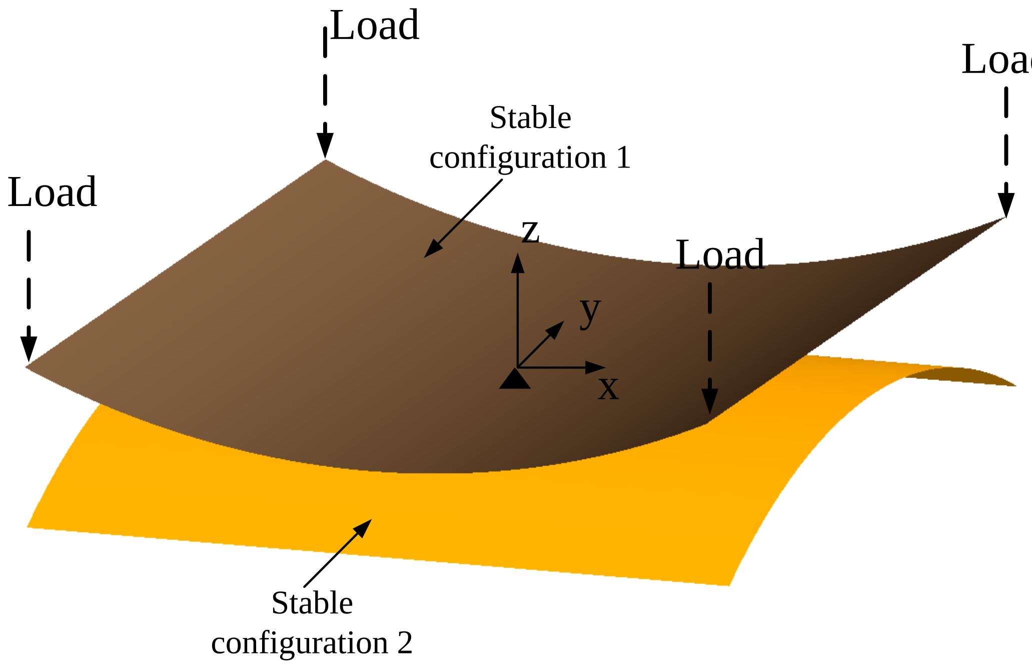


Fig. 2 Sketch map of a square bistable laminate mounted at central

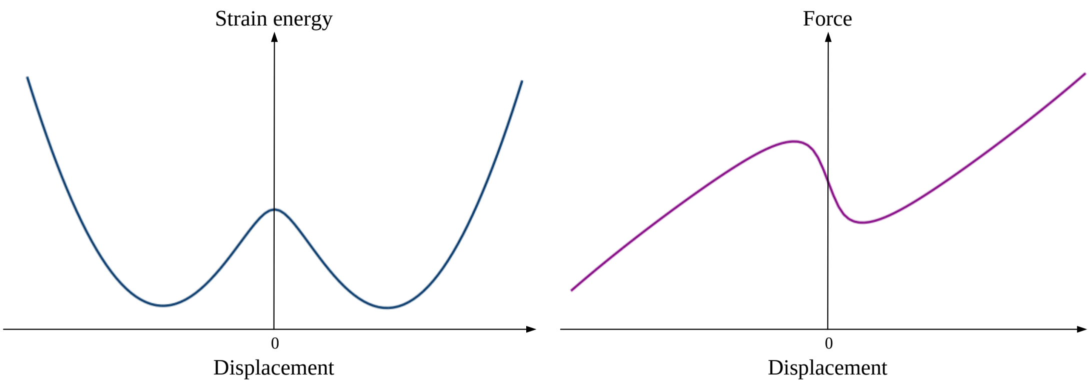


Fig. 3 Classical strain energy curve and load-displacement curves of a bistable laminate

In this study, a quasi-zero stiffness vibration isolator is proposed, by connecting the bistable [0/Al/90] to a linear spring in parallel, the principle of which is illustrated in Fig. 4. In this vibration isolator, the bistable laminate is restrained at the corner points by hinges in vertical direction. The laminate central is connected to the base via a linear spring. In this system, the bistable laminate is constrained to stay at an unstable saddle state, and functions as a negative stiffness spring. With an appropriate choose the linear spring, its linear positive stiffness can be fully counteracted by the negative stiffness of the bistable laminate. Consequently, the whole system exhibits zero stiffness at the equilibrium state.

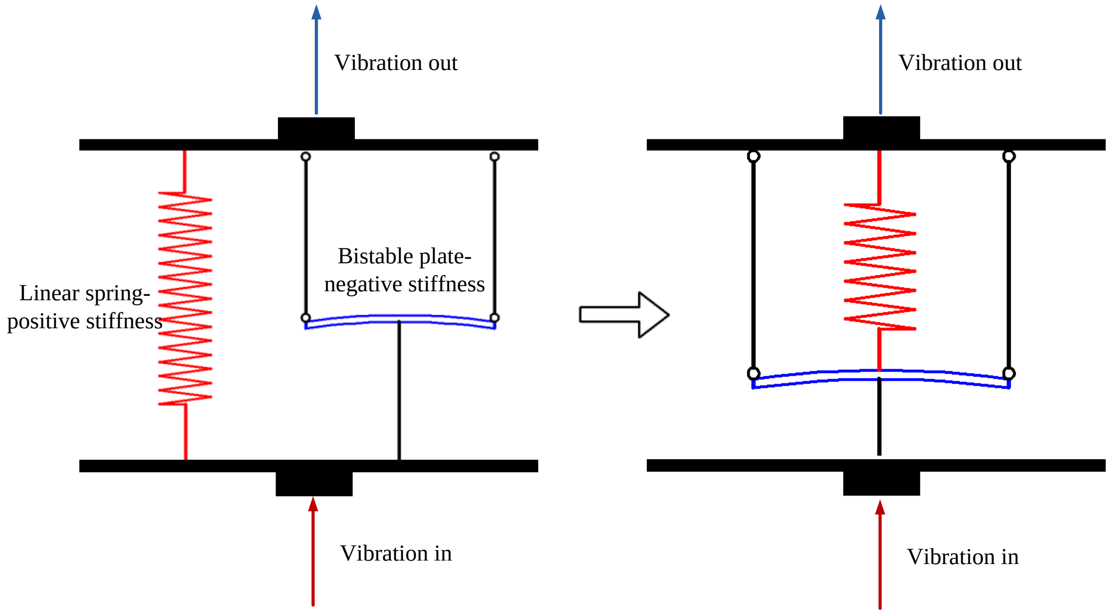


Fig. 4 Principle of the quasi-stiffness vibration isolator based on bistable laminate

Based on the principle illustrated in Fig. 4, a prototype of the quasi-stiffness vibration isolator is designed, and presented in Fig. 5. The bistable laminate is connected to the up platform at corners by four hinges, and is connected to the spring by threaded rod. The linear spring is installed in a columnar box which is bolted on the up platform.

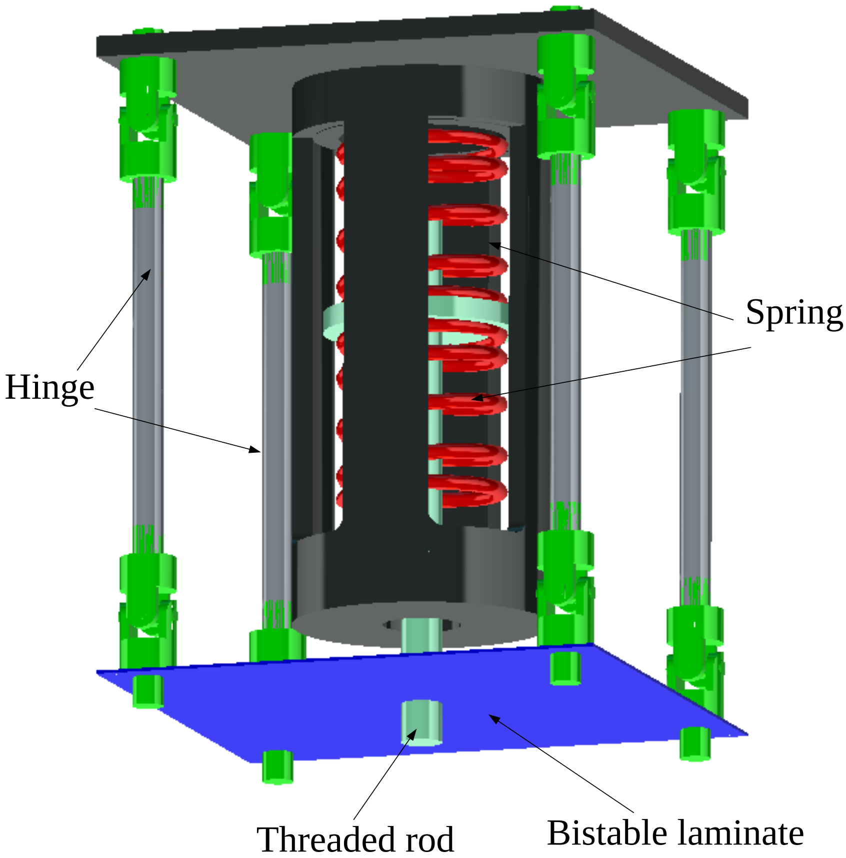


Fig. 5 Prototype design of the quasi-zero stiffness vibration isolator

3 Finite element analysis

This finite element analysis (FEA) of this novel QZS vibration isolator is performed using Abaqus. The material properties of the laminate are presented in Table. 1. The set-up of the finite element model is presented in Fig. 6. The bistable laminate dimension is 80mm×80mm, and modeled by shell elements S4R. A rigid platform is also established, the bistable laminate is connected to the rigid platform at corners by MPC-LINK constraints.

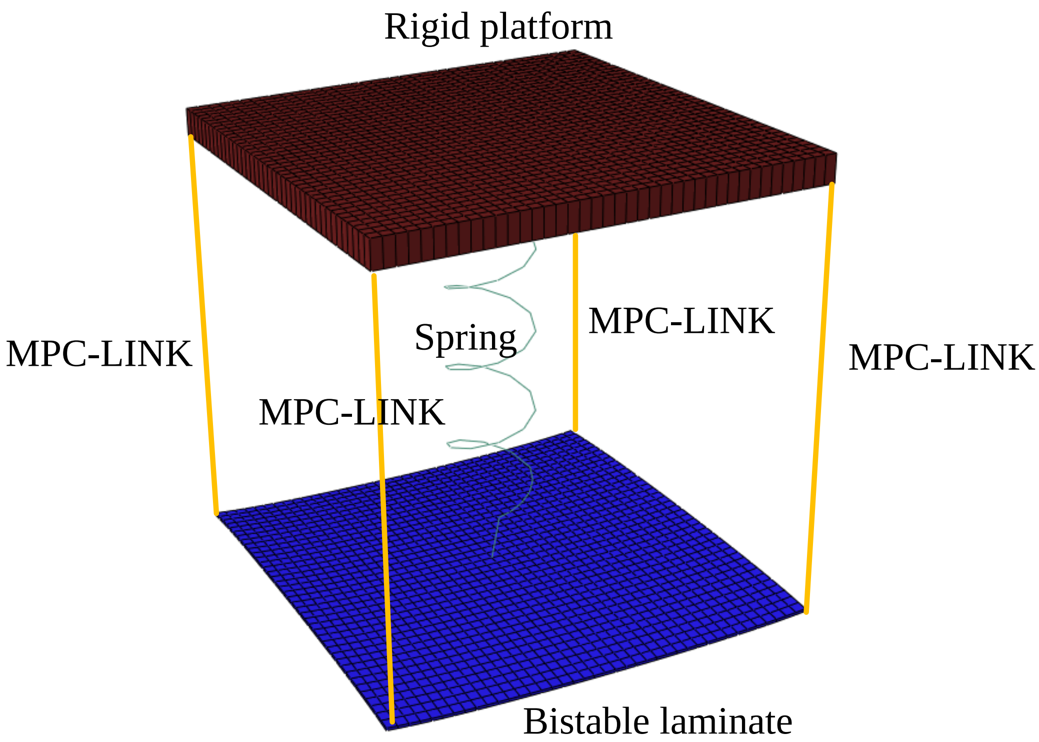


Fig. 6 Finite element model of QZS vibration isolator

Table. 1 Material properties of carbon fibre reinforced polymer(CFRP) and aluminum(Al)

CFRP CCF300/5428	E_{11}=145Gpa, E_{22}=9.75Gpa, G_{12}=5.69Gpa, ν_{12} =0.312, α_{11}= 0.4×10^{-6}/°C, α_{22} = 25×10^{-6}/°C, t=0.125mm Curing temperature 140 ℃
Al	E=70Gpa, ν_{12} =0.3, α_{11}= 23.3×10^{-6}/°C

The load-displacement relationships of a vibration isolator with different linear springs are simulated and plotted in Fig. 7. Concentrated load is applied on the laminate central along the vertical direction, and the rigid platform is restrained. In this isolator, the lay-up of the bistable laminate is $[0_{t=0.25mm}/Al_{t=0.2mm}/90_{t=0.25mm}]$, the stiffness of three selected different linear springs are 13KN/m, 16 KN/m and 20KN/m, respectively. FEA results show that, for a given bistable laminate, the stiffness of the isolator at the equilibrium point can be adjusted by the stiffness of spring. Theoretically, there is a critical value of spring stiffness, with which the isolator has zero stiffness at the equilibrium point. If the spring stiffness is larger than the critical value, the isolator has positive stiffness at the equilibrium. In contrast, the stiffness of the isolator at the equilibrium point is negative if the spring stiffness is smaller than the critical value. Therefore, the choice of the spring stiffness is critical for achieving the quasi-stiffness isolator.

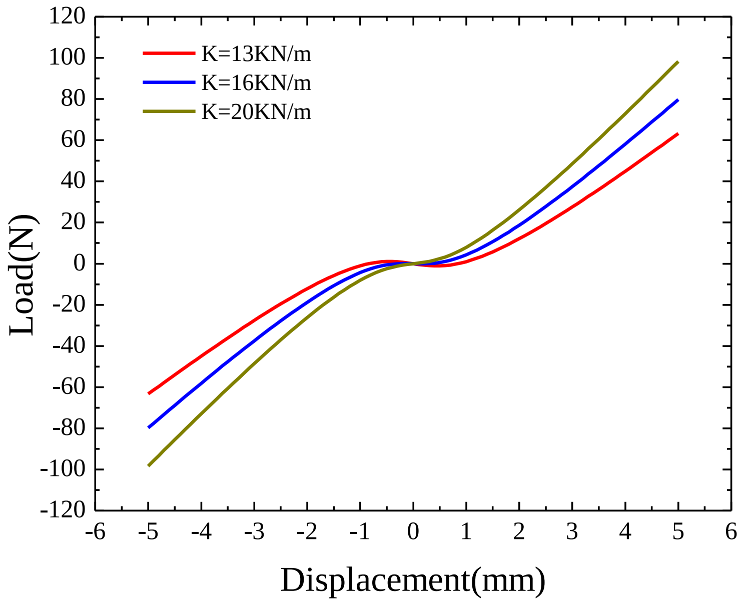


Fig. 7 Load-displacement curves of the vibration isolator, the lay-up of the bistable laminate is $[0_{t=0.25mm}/Al_{t=0.2mm}/90_{t=0.25mm}]$, K is the stiffness of linear srping

To counteract the negative stiffness of bistable laminate in the isolator, the critical stiffness of the spring equals to the maximum negative stiffness of the bistable laminate in magnitude. Applying FEA, it is easy to obtain the negative stiffness of a bistable laminate via the load-displacement curve. The critical spring stiffnesses corresponding to different bistable laminates are predicted and are presented in Fig. 8. It indicates that the critical stiffness of the spring varies nonlinearly with the thicknesses of CFRP ply and aluminum ply. In Fig. 8, there are top points for both curves, and this illustrates that for this type of QZS isolator, the stiffness off the equilibrium point cannot be increased endlessly by adjusting lay-up of the bistable laminate.

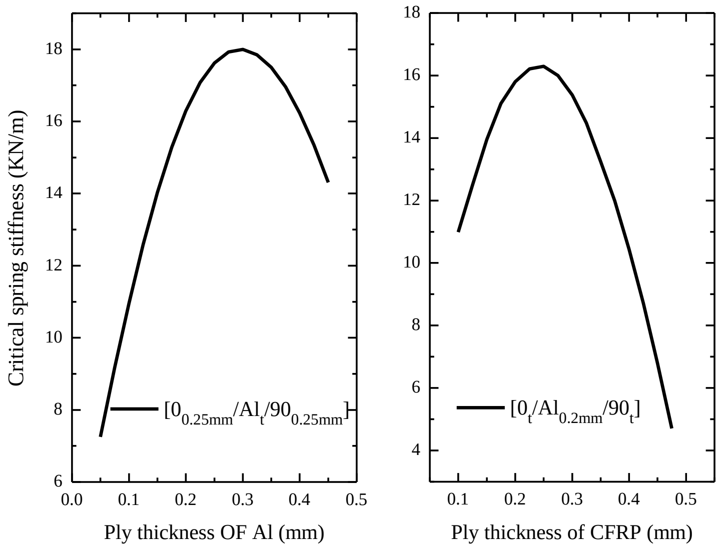


Fig. 8 Critical spring stiffnesses for the quasi-stiffness vibration isolators applying different bistable laminates

To verify the effectiveness of this QZS vibration isolator, a typical design is employed for illustration. In which, a 80mm×80mm, $[0_{0.25mm}/Al_{0.2mm}/90_{0.25mm}]$ bistable laminate is applied in the isolator; the matched stiffness of the linear spring is 16.295kN/m. The dynamic analysis of the isolator is conducted via the “Dynamic, Implicit” step in ABAQUS. In FEA, a concentrate mass is applied on the rigid platform, a period excitation is applied on the central of the bistable laminate in vertical direction, and the output acceleration of the rigid platform is monitored. The acceleration transmission rate is calculated by comparing the maximum acceleration magnitude of the rigid platform with the input acceleration. The predicted acceleration transmission rates are presented in Fig. 9, FEA indicates that the vibration isolator starts to function at quite low frequency. The acceleration transmission rate rapidly decreases to a small value and maintains as the excitation frequency grows. Due to the nonlinear stiffness, the initial working frequency of the QZS vibration isolator increases with the excitation acceleration while decreases with the isolated mass, and the maximum acceleration transmission rate increases with the isolated mass. The valid working frequency ranges of the QZS vibration isolator and a linear vibration isolator are compared in Table. 2. At valid working frequencies, the acceleration transmission rate of the isolator is negative. The QZS vibration isolator shows obvious advantage on the valid working frequency range over the linear vibration isolator.

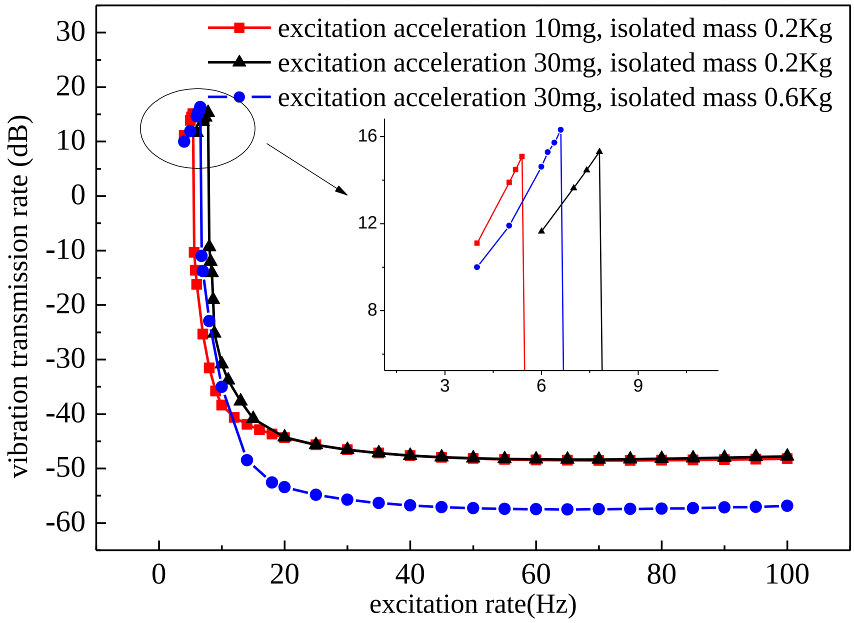


Fig. 9 Predicted acceleration transmission rates of a QZS vibration isolator, a 80mm×80mm, [$0_{0.25mm}$/$Al_{0.2mm}$/$90_{0.25mm}$] bistable laminate is applied in the isolator

Table. 2 Valid working frequency range of linear and QZS vibration isolators

excitation acceleration, isolated mass	10mg,0.2Kg	30mg,0.2Kg	30mg,0.6Kg
linear isolator, K=16.295KN/m	＞64.3Hz		＞37.2
QZS isolator	＞5.6Hz	＞8.0Hz	＞6.8

4 Experimental verification and discussion

A prototype of the QZS vibration isolator is fabricated, as presented in 错误!未找到引用源。. In this study, the effectiveness of the isolator is verified by the sweeping frequency experiment. The load displacement curve of the bistable laminate is measured by a test machine. The bistable laminate is supported at corners and vertical displacement load is applied on the laminate central by the test machine. The measured load-displacement curve of bistable laminate is presented in 错误!未找到引用源。. The measured maximum magnitude of the negative stiffness of bistable laminate is 19.18KN/m, which is larger than the FEA value. The error of FEA with respect to experiment may be results of imprecise material properties of CFRP, imprecise ply thickness of the specimen and inaccurate curing temperature control in the manufacturing process. The ideal spring in the isolator should have the stiffness of 19.18KN/m. However, in experiment it is difficult to precisely match the stiffness of the spring. Instead, a spring with the stiffness of 20KN/m is installed in the isolator.

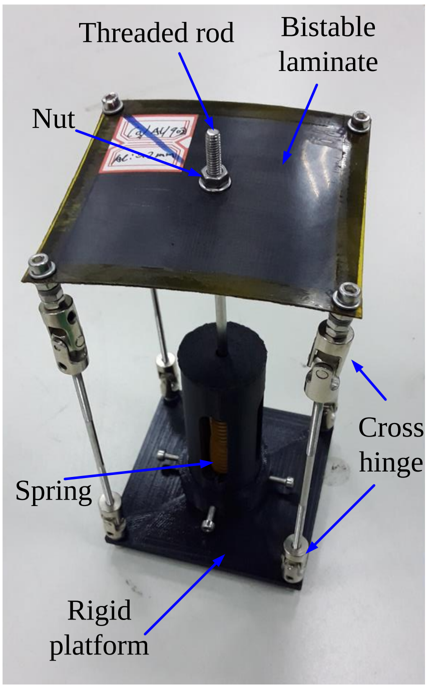


Fig. 10 Prototype of a quasi-stiffness vibration isolator, the dimension and lay-up of the bistable laminate are 80mm×80mm, [$0_{0.25mm}$/$Al_{0.2mm}$/$90_{0.25mm}$]

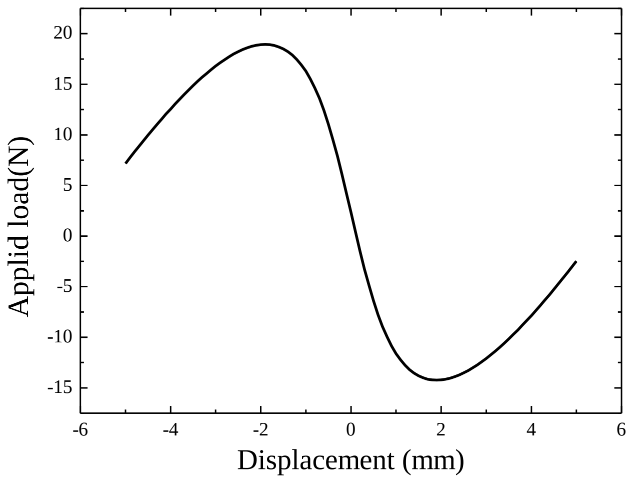


Fig. 11 Measured load-displacement curve of the QZS vibration isolator

The settlement of the sweeping frequency experiment is illustrated in 错误!未找到引用源。. In experiment, the magnitude of excitation acceleration is maintained at 30mg, the sweep speed is 2oct/min. The acceleration transmission rate of the linear spring is also tested in experiment by removing the bistable laminate, and is compared with the QZS vibration isolator. The experimental results are presented in 错误!未找到引用源。. Experiment shows that the QZS vibration isolator has improved acceleration transmission rate with respect to a linear spring. The maiximum acceleration transmission rate is about 5dB for the QZS vibration isolator, which is much lower than that of the linear spring, 12.5dB. The vibration isolating frequency range of the QZS vibration isolator starts form 39Hz, while the linear spring starts to isolate vibration at 44Hz. As the excitation frequency increases, the acceleration transmission rate of the QZS vibration isolator is similar with the linear spring. Nevertheless, the performance of the QZS vibration isolator is not as excellent as predicted by FEA.

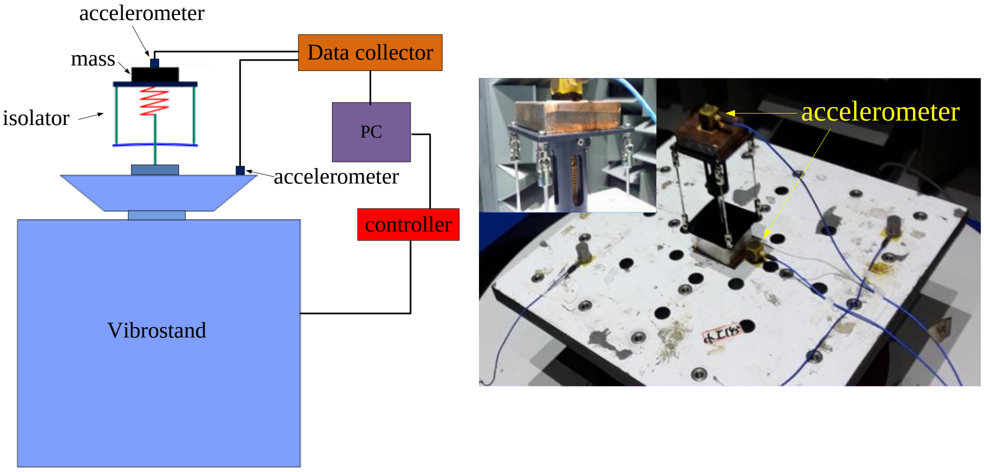


Fig. 12 Sweeping frequency experiment settlement

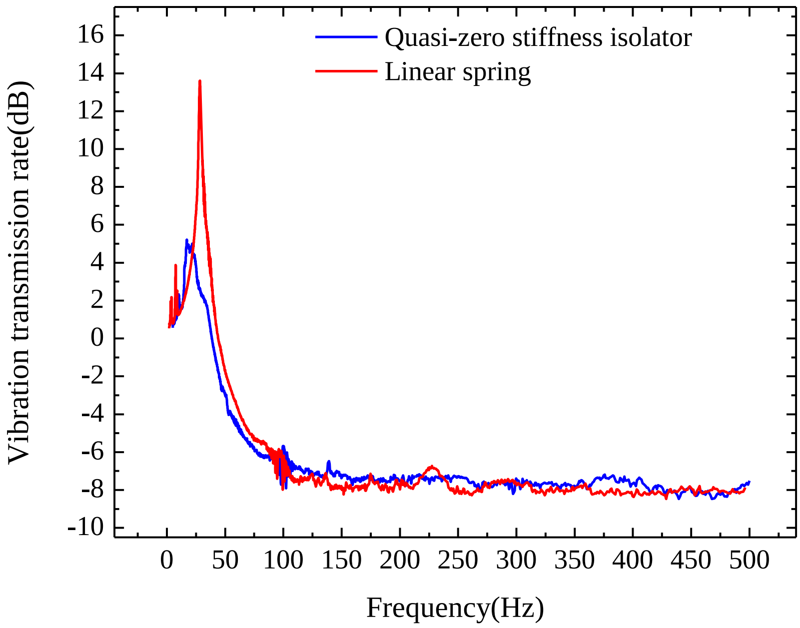


Fig. 13 Sweeping frequency experiment results of the linear spring and the QZS vibration isolator

To explain the performance degradation of the QZS vibration isolator, two possible reasons are investigated, including the error of linear spring stiffness and the assembly error. In experiment, the measured negative stiffness of the bistable laminate is -19.18KN/m. However the stiffness of the spring in the isolator is 20KN/m, which is larger than the ideal value, i.e. 19.18KN/m. the influence of larger spring stiffness is investigated by FEA, and the predicted results are presented in 错误!未找到引用源。. FEA shows that the error of spring stiffness has significant negative influence on the performance of the isolator. With only +5% error of the spring stiffness, the snap phenomenon of the acceleration transmission rate after the peak point disappears. Both the acceleration transmission rate and the isolation starting frequency increase with the error of spring stiffness.

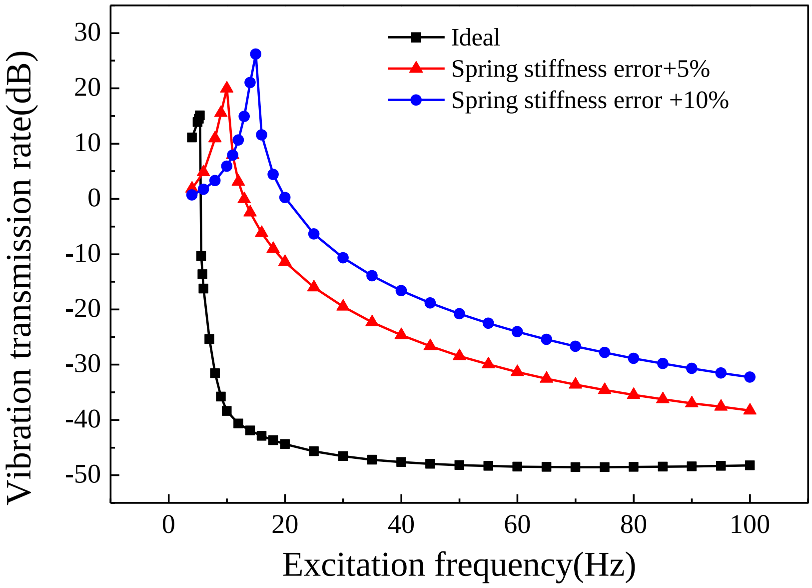


Fig. 14 Predicted influence of spring stiffness error on the performance of QZS vibration isolator, the 80mm×80mm, [$0_{0.25mm}$/$Al_{0.2mm}$/$90_{0.25mm}$] bistable laminate is applied in the isolator, the excitation acceleration magnitude is 10mg, and the isolated mass is 0.2kg

Theoretically, at the static equilibrium state, the stiffness of the QZS vibrator is zero. In experiment, to balance the gravity of the isolated mass and the rigid platform, the distance between the bistable laminate central and the linear spring is adjusted by a nut, which is presented in 错误!未找到引用源。. The adjusting amplitude equals to the compressing of the linear spring under gravity. However, in experiment, it is unlikely to minimize the assembly error to zero. If there is an unbalanced gravity of 0.02N, for instance, FEA shows that the bistable laminate central is 0.144mm off the equilibrium point, and therefore at the static state the stiffness of the vibration isolator is nonzero. The acceleration transmission rate curve of a QZS vibration isolator with assembly error is compared with an ideal isolator is presented in 错误!未找到引用源。. It indicates that the assembly error also has significant negative influence on the performance of the QZS vibration isolator. Further, the performance of a QZS vibration isolator with both spring stiffness error and assemble error is predicted by FEA, and is presented in 错误!未找到引用源。. FEA result indicates that the spring stiffness error and the assembly error have accumulative negative influences on the performance of the vibration isolator.

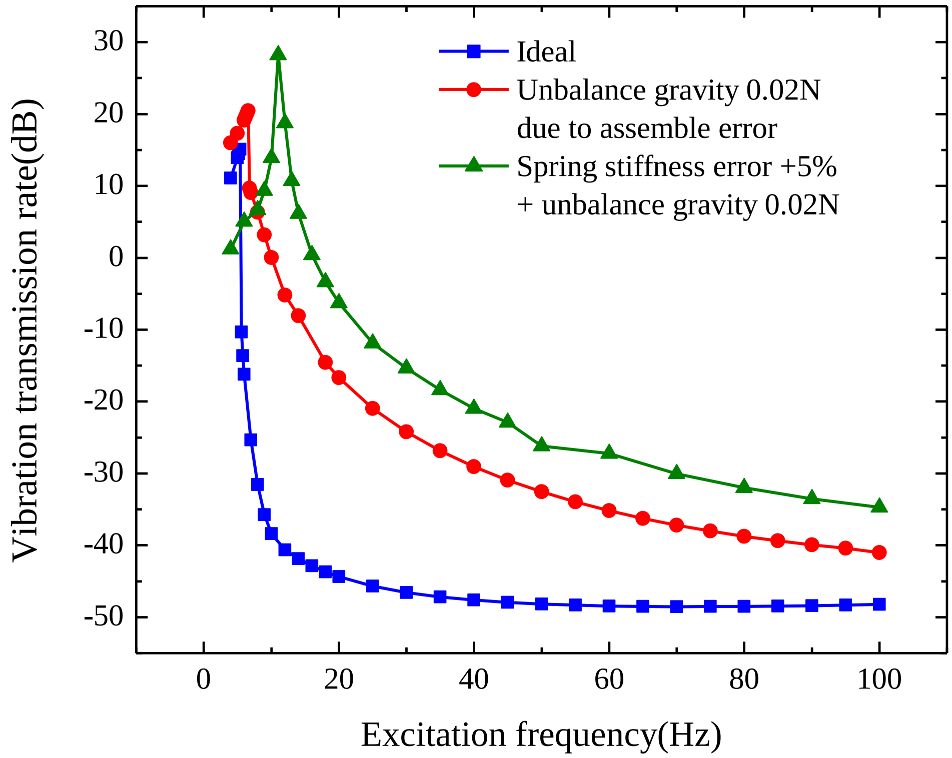


Fig. 15 Predicted influence of assemble error on the performance of QZS vibration isolator, the 80mm×80mm, [$0_{0.25mm}$/$Al_{0.2mm}$/$90_{0.25mm}$] bistable laminate is applied in the isolator, the excitation acceleration magnitude is 10mg, and the isolated mass is 0.2kg

Table. 3 Predicted major characteristics of QZS vibration isolator with spring stiffness error and assembly error

	Maximum acceleration transmission rate	Valid working frequency range
0%	15.08dB	＞5.6Hz
Spring stiffness error (+5%)	19.97dB	＞13Hz
Spring stiffness error (+10%)	26.16db	＞21Hz
Assembly error (unbalanced gravity 0.02N)	20.46dB	＞11Hz
Spring stiffness error(+5%)+ Assembly error(unbalanced gravity 0.02N)	28.22dB	＞17Hz

FEA indicates that the QZS vibration isolator in ideal condition is quite efficient in isolating vibration from quite low frequency. Although in experiment the performance of the isolator is better than a linear spring, the measured efficiency is far below the designed purpose. The spring stiffness error and the assembly error are demonstrated to have significant negative influence on the performance of the isolator. However, the errors can not be fully eliminated in the fabricating process. Meanwhile, some other imperfections such as the imprecise ply thickness of bistable laminate, material defects and inaccurate curing procedure of bistable laminate, etc., may also have negative influences on the isolator's performance. If the isolated mass varies, the isolator has to be tuned simultaneously to balance the gravity. Thus, the robustness of this QZS isolator is poor and is needed to be improved, and this is a common problem for other types of QZS isolators[8]. Nevertheless, by applying bistable laminates, the proposed QZS vibration isolator has advantages including the simplicity in structure and light weight, and these advantages are highly valued in the application on weight controlled equipments, such as satellites and spaceships.Experimental verification and discussion

A prototype of the QZS vibration isolator is fabricated, as presented in 错误!未找到引用源。. In this study, the effectiveness of the isolator is verified by the sweeping frequency experiment. The load displacement curve of the bistable laminate is measured by a test machine. The bistable laminate is supported at corners and vertical displacement load is applied on the laminate central by the test machine. The measured load-displacement curve of bistable laminate is presented in 错误!未找到引用源。. The measured maximum magnitude of the negative stiffness of bistable laminate is 19.18KN/m, which is larger than the FEA value. The error of FEA with respect to experiment may be results of imprecise material properties of CFRP, imprecise ply thickness of the specimen and inaccurate curing temperature control in the manufacturing process. The ideal spring in the isolator should have the stiffness of 19.18KN/m. However, in experiment it is difficult to precisely match the stiffness of the spring. Instead, a spring with the stiffness of 20KN/m is installed in the isolator.

5 Summary

A novel QZS vibration isolator which takes advantages of the negative stiffness properties of the bistable composite laminates is proposed. The design concept is based on the inherent negative stiffness of bistable laminates, the system exhibits zero stiffness at the equilibrium point. Principle of this QZS vibration isolator is introduced and its structural performance is simulated and verified by finite element analysis. The FEA analysis results show that the proposed isolator starts from quite low frequency within the working frequency range. The acceleration transmission rate drops immediately to a low value as long as the excitation frequency exceeds a critical value. A prototype of QZS vibration isolator is fabricated and verified in the experiment. Although the measured performance of the isolator is much better than a linear isolator, it does not achieve the numerical prediction given by the FEA analysis. The negative influences of spring stiffness error and the assembly error is demonstrated by FEA, which explains the dissatisfactory performance that was measured in the experiment. Both experiment measurement and FEA simulation results indicate that the robustness of the proposed QZS vibration isolator needs to be improved. Nevertheless, the distinct advantages of applying bistable laminates for constructing high efficient, simple structural form, and lightweight vibration isolators had been clearly approved.

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