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The Non-Equivalence of Import Tariffs and Export Taxes in Trade Wars: Ad Valorem vs Specific Trade Taxes

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Abstract

Using perfectly competitive, general equilibrium models of international trade, specific import tariffs, specific export taxes, and *ad valorem* trade taxes are compared in a trade war. A trade war is modelled as a NE in trade policies, where each country can choose to use *ad valorem* trade taxes (import tariffs or export taxes, which are equivalent), or specific import tariffs, or specific export taxes. In the two-country case, where there is a negative terms of trade externality a specific export tax dominates a specific import tariff or *ad valorem* trade taxes. Hence, the Lerner Symmetry Theorem does not hold for specific trade taxes in a trade war. This result continues to hold when the model is extended to the case of many countries assuming that there is a negative terms of trade externality. In a trade policy game where two countries export the same good so there is a positive terms of trade externality in the trade policy game between these two countries, the results are reversed with a specific import tariff dominating a specific export tax or *ad valorem* trade taxes. Hence, again the Lerner Symmetry Theorem does not hold for specific trade taxes in a trade war.

JEL Classifications: F11; F13; C72; D51; H21

Keywords: *Ad Valorem* Trade Tax; Specific Trade Tax; Perfect Competition; General Equilibrium; NE in Trade Taxes; Lerner Symmetry Theorem.

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1. Introduction

Recent events such as the US-China trade conflict have made the analysis of trade wars a topical issue in international trade policy rather than just a theoretical curiosity that has interested economists for more than a century. In a perfectly-competitive, general equilibrium model with two countries, although free trade is Pareto-efficient for the global economy, it is well known that a large country with monopoly/monopsony power can improve its terms of trade and maximise its welfare by using an optimum trade tax. The Lerner (1936) Symmetry Theorem, which shows that an *ad valorem* import tariff is equivalent to an *ad valorem* export tax, implies that the optimum trade tax may be either an import tariff or an export tax.[1] Hence, the outcome in terms of the level of the optimum trade tax and in terms of the welfare of the country is the same whether the country uses an *ad valorem* import tariff or an *ad valorem* export tax.[2] Since the optimum trade tax is a beggar-my-neighbour policy as it worsens the terms of trade of the other country, it is likely that the other country will retaliate if a country pursues such a policy. The possibility of retaliation was first analysed by Johnson (1953) who modelled the resulting trade war as a Nash equilibrium (NE) in trade taxes and showed that, although it seems most likely that both countries will lose in a trade war, it was possible for one country to win the trade war if it had sufficient monopoly/monopsony power.[3] Again, the outcome in terms of the level of NE trade taxes and in terms of the NE welfare of the two countries is the same whether the countries use *ad valorem* import tariffs or *ad valorem* export

[1] For a modern treatment of the Lerner Symmetry Theorem, see Costinot and Werning (2019), who extend the theorem in a number of ways such as allowing for imperfect competition.

[2] The optimum import tariff rate will be equal to the optimum export tax rate if the import tariff is expressed as a proportion of the world price of the importable good and the export tax is expressed as a proportion of the domestic price of the exportable good.

[3] In a pure exchange economy, Kennan and Riezman (1988) showed that if a country was sufficiently large then it would win a trade war and this result was generalised by Syropoulos (2002).

taxes. Hence, the Lerner Symmetry Theorem clearly holds for *ad valorem* trade taxes in trade wars.

Although countries predominantly use *ad valorem* trade taxes, they do use specific (per-unit) trade taxes as well as other trade policies such as import quotas or export quotas.[4] Under perfect competition, *ad valorem* and specific taxes are generally regarded as equivalent and the same is true of trade taxes when a country unilaterally sets its trade tax. However, Horwell (1966) and Lockwood and Wong (2000) have shown that specific import tariffs are not equivalent to *ad valorem* import tariffs in trade wars. If the home country shifts from using a specific import tariff to using an *ad valorem* import tariff, then its offer curve will become more elastic and the foreign country will set a lower import tariff. Since the foreign country setting a lower import tariff will increase the welfare of the home country, choosing to use an *ad valorem* import tariff will dominate choosing to use a specific import tariff for the home country. Hence, since the same reasoning holds for the foreign country, both countries will choose to use *ad valorem* import tariffs and, in the symmetric case, both countries will have lower tariffs and higher welfare than when they both choose to use specific import tariffs.

This raises the question of how a specific export tax compares with an *ad valorem* export tax and whether the Lerner Symmetry Theorem holds for specific trade taxes. The approach used to answer this question will be similar to that used by Vives (1985) to compare the Cournot equilibrium (where the strategic variable is output) with the Bertrand equilibrium (where the strategic variable is price) in a differentiated product oligopoly model. He analysed the Cournot oligopolists as maximising profits by choosing their price subject to a constraint given by the demand function, and then comparing the prices set by the Cournot oligopolists with those set by the Bertrand oligopolists. Here, countries choosing to use specific trade taxes

[4] The use of quotas in a trade war has been analysed by Rodriguez (1974) and Tower (1975) who both show that trade will approach zero in a trade war.

will be analysed as maximising their welfare by choosing *ad valorem* trade taxes subject to a constraint that ensures the equivalence of the *ad valorem* and specific trade taxes. Then, the best-reply functions with specific trade taxes can easily be compared with the best-reply functions with *ad valorem* trade taxes. The advantage of this method is that it can be carried out in a general setting as in Dixit (1987) with the only significant assumption being about the sign of the terms of trade externality, which allows general results to be obtained and highlights the importance of the sign of the terms of trade externality for the results obtained.[5]

Section two considers a two-country trade policy game with a negative terms of trade externality, and it is shown that for both countries choosing to use specific export taxes dominates choosing to use *ad valorem* trade taxes or specific import tariffs. In the symmetric case, both countries set lower trade taxes and have higher welfare when they choose to use specific export taxes rather than *ad valorem* trade taxes or specific import tariffs. Hence, the Lerner Symmetry Theorem does not hold for specific trade taxes in trade wars. Section three extends the results to the many country case by assuming symmetry and keeps the assumption of a negative terms of trade externality. Section four considers a trade policy game between two countries that both export the same good in a many-country world, which implies that there is a positive terms of trade externality. In this case, it is shown that the results are reversed with countries choosing to use specific import tariffs dominating choosing to use *ad valorem* trade taxes or specific export taxes. Again, the Lerner Symmetry Theorem does not hold for specific trade taxes in a trade war. Section five presents some conclusions.

[5] According to Bagwell and Staiger (2016) the terms of trade externality has a critical role in the analysis of trade agreements.

2. Two-Country Trade Policy Game

Consider a conventional two-country trade policy game such as that analysed by Johnson (1953) and Dixit (1987) where there is a strategic interaction between the two countries due to the negative terms of trade externality. In this perfectly competitive, general equilibrium model, there are two large countries, labelled 1 and 2, and two goods, labelled 1 and 2, where country one exports good one and imports good two while country two exports good two and imports good one. Hence, the terms of trade for country one are: $\rho_1 = p_1/p_2$, and for country two are: $\rho_2 = p_2/p_1$, where p_1 and p_2 are the world prices of goods one and two, respectively. Obviously, the terms of trade of country one are the reciprocal of the terms of trade of country two, $\rho_1 = p_1/p_2 = 1/\rho_2$. For each country, an improvement (increase) in the terms of trade increases the relative price of its exports or, equivalently, decreases the relative price of its imports.

Since the countries are large, they can use trade policy to affect their terms of trade and welfare, and each country has two trade policy decisions to make in this model. First, each country has to choose whether to use an *ad valorem* trade tax, which can be either an import tariff or an export tax, and is denoted by T_j^{τ}; a specific import tariff, which is denoted by T_j^t; or a specific export tax, which is denoted by T_j^e, where $j = 1,2$ denotes the country. Then, each country, $j = 1,2$, has to decide the rate for the chosen type of trade tax with the *ad valorem* trade tax rate denoted by τ_j, the specific import tariff rate denoted by t_j, and the specific export tax rate denoted by e_j. In this general equilibrium model, specific trade taxes are expressed in terms of the untaxed good (the export good in the case of import tariffs and the import good in the case of export taxes).[6] As usual, it is assumed that the trade tax revenue of each country is remitted to the consumers of that country in a lump-sum manner, which prevents the occurrence of the Lerner (1936) paradox.

The domestic prices of goods one and two, respectively, are p_{11} and p_{21} in country one, and p_{12} and p_{22} in country two. Hence, the relative, domestic price of the importable good in the two countries in terms of their import tariffs (in the numerator) and export taxes (in the denominator) and world prices are:

$$\frac{p_{21}}{p_{11}} = \frac{p_2(1+\tau_1)+t_1 p_1}{p_1/(1+\tau_1)-e_1 p_2} \qquad \frac{p_{12}}{p_{22}} = \frac{p_1(1+\tau_2)+t_2 p_2}{p_2/(1+\tau_2)-e_2 p_1} \tag{1}$$

The first point to note from (1) is that if the specific taxes are equal to zero, $t_1 = e_1 = t_2 = e_2 = 0$ then it is clear that an *ad valorem* import tariff has exactly the same effect on the relative domestic price of the importable good as an *ad valorem* export tax at the same rate, and it can be shown that it raises exactly the same trade tax revenue in real terms. Hence, the equilibrium with an *ad valorem* import tariff is exactly the same as that with an *ad valorem* export tax at the same rate, which is the Lerner Symmetry Theorem. What is less clear is that if the *ad valorem* trade taxes are equal to zero, $\tau_1 = \tau_2 = 0$, then a specific import tariff t_1 is equivalent to an export tax $e_1 = t_1 p_1^2 / \left(p_2^2 + t_1 p_1 p_2\right)$ or $e_1 = t_1 \rho_1^2 / (t_1 \rho_1 + 1)$ since they both have the same effect on the relative domestic price of the importable good in country one and both raise the same trade tax revenue in real terms. Hence, although the equivalent specific export tax rate is not equal to the specific import tariff rate, the Lerner Symmetry Theorem still holds

[6] In a general equilibrium model, a specific tax cannot be expressed in nominal terms as this would imply that the real value of the tax would be affected by nominal prices so doubling nominal prices would halve the real value of the tax. Often, in the public economics literature, the untaxed good would be the numeraire good so the specific tax may seem to be expressed in nominal terms, but really it is expressed in terms of the untaxed good.

in the sense that a specific import tariff can be replaced by an equivalent specific export tax that results in the same relative domestic prices and the same welfare.[7] The second point is that a specific import tariff is equivalent to an *ad valorem* import tariff in terms of their effects on relative domestic prices if $\tau_1 p_2 = t_1 p_1$ or $t_1 = \tau_1/\rho_1$ for country one, and if $\tau_2 p_1 = t_2 p_2$ or $t_2 = \tau_2/\rho_2$ for country two. The third point is that a specific export tax is equivalent to an *ad valorem* export tax in terms of their effects on relative domestic prices if $\tau_1 p_1/(1+\tau_1) = e_1 p_2$ or $e_1 = \tau_1\rho_1/(1+\tau_1)$ for country one, and if $\tau_2 p_2/(1+\tau_2) = e_2 p_1$ or $e_2 = \tau_2\rho_2/(1+\tau_2)$ for country two.

Under perfect competition, since firms and consumers are price takers, their behaviour is unaffected by whether countries use *ad valorem* trade taxes or equivalent specific trade taxes. Therefore, equating demand and supply in the world market for one of the goods, yields the equilibrium terms of trade of the two countries: $\rho_1(\tau_1,\tau_2)$ and $\rho_2(\tau_1,\tau_2)$, as functions of the *ad valorem* trade taxes with specific trade taxes replaced by their *ad valorem* equivalent. Assuming that the Marshall-Lerner condition holds then the terms of trade of a country is increasing in its own trade tax, $\partial\rho_1/\partial\tau_1 > 0$ and $\partial\rho_2/\partial\tau_2 > 0$, and decreasing in the trade tax of the other country, $\partial\rho_1/\partial\tau_2 < 0$ and $\partial\rho_2/\partial\tau_1 < 0$. It is also assumed that the Metzler (1949) paradox does not occur so that the relative domestic price of the importable good in a country is increasing in its own trade tax. For country one, this requires that $(1+\tau_1)/\rho_1(\tau_1,\tau_2)$ is increasing in τ_1, which requires that $\rho_1 - (1+\tau_1)(\partial\rho_1/\partial\tau_1) > 0$ and this implies that $\rho_1 - \tau_1(\partial\rho_1/\partial\tau_1) > \partial\rho_1/\partial\tau_1 > 0$. A similar analysis for country two shows that

[7] In the terminology of Costinot and Werning (2019), this shows the neutrality of a tax reform where a specific import tariff is replaced with an equivalent specific export tax.

$\rho_2 - \tau_2\left(\partial\rho_2/\partial\tau_2\right) > \partial\rho_2/\partial\tau_2 > 0$, and these two restrictions will be needed to sign some expressions later in the analysis.

Having solved for the equilibrium terms of trade of the two countries as functions of the trade taxes, in principle, it is possible to solve for consumption of the two goods and hence the utility or welfare of the two countries as functions of the trade taxes. The welfare of country one is $W_1\left(\tau_1,\tau_2\right)$ and the welfare of country two is $W_2\left(\tau_1,\tau_2\right)$, which are both assumed to be strictly quasi-concave in their own trade tax. Welfare under free trade of the two countries is obtained by setting all the trade taxes equal to zero, $W_1^F = W_1\left(0,0\right)$ and $W_2^F = W_2\left(0,0\right)$, and multilateral free trade is Pareto efficient for the world. In fact, any combination of trade taxes and subsidies such that $\left(1+\tau_1\right)\left(1+\tau_2\right)=1$ will equalise relative domestic prices in the two countries and yield Pareto efficiency. Although free trade is Pareto efficient for the world, a large country can improve its terms and increase its welfare by unilaterally introducing a small trade tax so $\partial W_1\left(0,0\right)/\partial\tau_1 > 0$ and $\partial W_2\left(0,0\right)/\partial\tau_2 > 0$. Since the terms of trade of each country are worsened by the trade tax of the other country, the welfare of each country will be reduced by the trade tax of the other country, $\partial W_1/\partial\tau_2 < 0$ and $\partial W_2/\partial\tau_1 < 0$, which is the negative terms of trade externality that results in the strategic interaction between the two countries. The trade taxes being strategic substitutes is commonly regarded as the central case according to Dixit (1987), and this would imply that $\partial^2 W_1/\partial\tau_2\partial\tau_1 < 0$ and $\partial^2 W_2/\partial\tau_1\partial\tau_2 < 0$. Although, *a priori*, the possibility that trade taxes are sometimes strategic complements cannot be ruled out, and this would imply that $\partial^2 W_1/\partial\tau_2\partial\tau_1 > 0$ and $\partial^2 W_2/\partial\tau_1\partial\tau_2 > 0$. For completeness, in the analysis that follows both possibilities will be considered.

Before analysing the Nash equilibria in trade taxes, it is useful to consider the optimum trade tax when a country unilaterally intervenes while the other country pursues a policy of free trade. If country one maximises its welfare while country two pursues a policy of free trade, then the first-order condition is:

$$\frac{\partial W_1\left(\tau_1^*, 0\right)}{\partial \tau_1} = 0 \qquad (2)$$

This defines the optimum trade tax that is either an *ad valorem* import tariff or export tax $\tau_1^* > 0$, or an equivalent specific import tariff $t_1^* = \tau_1^* \big/ \rho_1\left(\tau_1^*, 0\right) > 0$, or an equivalent specific export tax $e_1^* = \tau_1^* \rho_1\left(\tau_1^*, 0\right) \big/ \left(1 + \tau_1^*\right) > 0$.[8] At the optimum, regardless of the type of trade tax used, the welfare of country one is $W_1^* = W_1\left(\tau_1^*, 0\right)$, which is obviously higher than welfare under free trade, $W_1^* > W_1^F$. A similar analysis can be undertaken for country two, which would yield $\tau_2^* > 0$ and $W_2^* > W_2^F$.

2.1 Nash Equilibria in Trade Taxes

Now consider a trade war modelled as usual as the Nash equilibrium (NE) in *ad valorem* trade taxes (import tariffs or export taxes). In the NE, each country independently and simultaneously sets its trade tax to maximise its welfare given the trade tax set by the other country. Hence, assuming that there is an interior solution where trade occurs between the two countries, when both countries set *ad valorem* trade taxes, the first-order conditions for the NE in trade taxes are:

$$\frac{\partial W_1\left(\tau_1, \tau_2\right)}{\partial \tau_1} = 0 \qquad \frac{\partial W_2\left(\tau_1, \tau_2\right)}{\partial \tau_2} = 0 \qquad (3)$$

[8]The optimum *ad valorem* import tariff (or export tax) can be derived in terms of the foreign export supply elasticity using offer curves, which was the approach used by Horwell (1966) and Lockwood and Wong (2000), but this will not be necessary for the analysis used in this paper.

The equation on the left implicitly defines the best-reply function of country one, $\tau_1 = \tau_1\left(\tau_2, T_2^\tau\right)$, and the equation on the right implicitly defines the best-reply function of country two, $\tau_2 = \tau_2\left(\tau_1, T_1^\tau\right)$, where T_1^τ and T_2^τ denote that both countries have chosen to use *ad valorem* trade taxes. The best-reply functions are both shown in figure 1, and the intersection of the two best-reply functions is the NE in *ad valorem* trade taxes, which is assumed to be unique.[9] The welfare of each country is represented in figure 1 by the iso-welfare loci where $W_1^N\left(T_1^\tau, T_2^\tau\right)$ and $W_2^N\left(T_1^\tau, T_2^\tau\right)$ are the welfare of country one and country two, respectively, in the NE in *ad valorem* trade taxes. The shape of the iso-welfare loci for the two countries follows from the assumption that $\partial W_1/\partial \tau_2 < 0$ and $\partial W_2/\partial \tau_1 < 0$. Figure 1 is drawn for the central case where both countries are worse off in the NE than under free trade, $W_1^N < W_1^F$ and $W_2^N < W_2^F$, but as Johnson (1953) showed it is possible that one country (but not both countries) can be better off in the NE than under free trade when there are asymmetries. Free trade is Pareto efficient as are all the combinations of trade taxes or subsidies on the locus where $(1+\tau_1)(1+\tau_2) = 1$. The optimum trade taxes of each country when the other country pursues a policy of free trade, τ_1^* and τ_2^*, are shown in figure 1 together with welfare of the countries with these optimum trade taxes, W_1^* and W_2^*, where $W_1^* > W_1^F$ and $W_2^* > W_2^F$.

In this NE in *ad valorem* trade taxes, each country is indifferent between using an *ad valorem* trade tax and an equivalent specific trade tax given the *ad valorem* trade tax set by the other country. However, as Horwell (1966) and Lockwood and Wong (2000) have shown the type of trade tax chosen by the other country affects the best-reply function of a country. Now consider the best-reply function of country one when country two uses a specific import tariff

[9] Strictly speaking, it is assumed that there is a unique interior NE where there is still trade between the two countries as Dixit (1987) showed that autarky is also a Nash equilibrium in trade taxes.

so now country one sets its trade tax optimally given that country two sets a specific tariff t_2. From (1) the specific import tariff t_2 is equivalent to an *ad valorem* trade tax τ_2 if $t_2 = \tau_2 / \rho_2(\tau_1, \tau_2)$, which shows that the equivalent *ad valorem* trade tax of country two depends upon the *ad valorem* trade tax set by country one. Thus, when country two sets a specific import tariff, the constraint facing country one is different to when country two sets an *ad valorem* trade tax.[10] Country one now maximises its welfare $W_1(\tau_1, \tau_2)$ subject to the constraint $t_2 = \tau_2 / \rho_2(\tau_1, \tau_2)$ and, using the implicit function theorem, the first order condition is:

$$\frac{\partial W_1}{\partial \tau_1} + \frac{\partial W_1}{\partial \tau_2}\frac{d\tau_2}{d\tau_1} = 0 \qquad \text{where} \qquad \frac{d\tau_2}{d\tau_1} = \frac{\tau_2(\partial \rho_2 / \partial \tau_1)}{\rho_2 - \tau_2(\partial \rho_2 / \partial \tau_2)} < 0 \tag{4}$$

Regarding the expression for the slope of the constraint, $d\tau_2 / d\tau_1$, the denominator is positive given the assumption that rules out the Metzler paradox, and the numerator is negative since the tariff of country one has a negative effect on the terms of trade of country two. Since, it is assumed that $\partial W_1 / \partial \tau_2 < 0$ then $\partial W_1 / \partial \tau_1 < 0$ in (4), and quasi-concavity of the welfare function implies that the trade tax set by country one is higher when country two sets a specific import tariff. The situation is shown in figure 2 where the best-reply function of country one when country two uses an *ad valorem* trade tax is obtained by maximising welfare given the *ad valorem* trade tax of country two, so if $\tau_2 = \tau_2^\tau$ then the optimum is at α where $\tau_1 = \tau_1^\tau$.[11] When country two uses a specific import tariff then country one maximises welfare subject to

[10]In the same way that a duopolist faces a different constraint when its competitor sets output rather than price, and the approach used here will be similar to the method used by Vives (1985) to compare the Bertrand and Cournot equilibria in a differentiated products oligopoly model.

[11]The figure is similar to that used by Cheng (1985) to compare the Bertrand and Cournot equilibria where he shows how to describe the Cournot equilibrium in the price space rather than the quantity space.

the constraint $t_2 = \tau_2 / \rho_2(\tau_1, \tau_2)$, which from (4) is downward sloping and goes through α so $t_2 = \tau_2^\tau / \rho_2(\tau_1^\tau, \tau_2^\tau)$, so country one realises that when it increases its trade tax then there will be a decrease in the equivalent *ad valorem* trade tax of country two. Hence, the optimum is at β where $\tau_1 = \tau_2^t$ and $\tau_2 = \tau_2^t$, and this allows the best-reply function for country one when country two uses a specific import tariff, $\tau_1(\tau_2, T_2^t)$, to be derived.[12] Country two switching from using an *ad valorem* trade tax to using a specific import tariff leads the best-reply function to swivel clockwise around $(\tau_1^*, 0)$ and leads country one to set a higher trade tax. Although figure 2 is drawn for the case when trade taxes are strategic substitutes, the analysis is unchanged when trade taxes are strategic complements. A similar analysis can be used to derive the best-reply function of country two for the case when country one sets a specific import tariff $\tau_2(\tau_1, T_1^t)$.

Now consider the best-reply function of country one when country two uses a specific export tax so country one sets its trade tax optimally given that country two sets a specific export tax e_2. From (1) the specific export tax e_2 is equivalent to an *ad valorem* trade tax τ_2 if $e_2 = \tau_2 \rho_2(\tau_1, \tau_2)/(1+\tau_2)$, which acts as a constraint on country one when it sets its trade tax. Hence, country one maximises $W_1(\tau_1, \tau_2)$ subject to $e_2 = \tau_2 \rho_2(\tau_1, \tau_2)/(1+\tau_2)$ and the first order condition is:

$$\frac{\partial W_1}{\partial \tau_1} + \frac{\partial W_1}{\partial \tau_2}\frac{d\tau_2}{d\tau_2} = 0 \qquad \text{and} \qquad \frac{d\tau_2}{d\tau_1} = \frac{-\tau_2(1+\tau_2)(\partial \rho_2/\partial \tau_1)}{\rho_2 + \tau_2(1+\tau_2)(\partial \rho_2/\partial \tau_2)} > 0 \tag{5}$$

[12] The solution of the constrained maximisation problem would give the *ad valorem* trade taxes of the two countries as functions of the specific import tariff of country *B*, which would give a parametric representation of the best-reply function.

Regarding the expression for the slope of the constraint, $d\tau_2/d\tau_1$, the numerator is positive and the denominator is positive since the trade tax of country two has a positive effect on its terms of trade. Since, it is assumed that $\partial W_1/\partial\tau_2 < 0$ then $\partial W_1/\partial\tau_1 > 0$ in (5), quasi-concavity of the welfare function implies that the trade tax set by country one is lower when country two sets a specific export tax. The situation is shown in figure 3 where the best-reply function for country one when country two uses an *ad valorem* trade tax $\tau_1\left(\tau_2, T_2^{\tau}\right)$ is the same as in figure 2. When country two uses a specific export tax then country one maximises its welfare subject to the constraint $e_2 = \tau_2\rho_2\left(\tau_1,\tau_2\right)/\left(1+\tau_2\right)$, which is upward sloping in figure 3, and then the optimum is at γ where $\tau_1 = \tau_1^e$ and $\tau_2 = \tau_2^e$. Hence, the best-reply function for country one when country two uses a specific export tax, $\tau_1\left(\tau_2, T_2^e\right)$, can be derived. Country two switching from using an *ad valorem* trade tax to using a specific export tax leads the best-reply function to swivel anti-clockwise around $\left(\tau_1^*, 0\right)$ and leads country one to set a lower trade tax. Although figure 3 is drawn for the case when trade taxes are strategic substitutes, the analysis is unchanged when trade taxes are strategic complements. A similar analysis can be used to derive the best-reply function of country two for the case when country one uses a specific export tax $\tau_2\left(\tau_1, T_1^e\right)$.

The best-reply functions for both countries for all three types of trade tax are shown in figure 4 for the case of strategic substitutes and in figure 5 for the case of strategic complements. The three best-reply functions of country one intersect the three best-reply functions of country two nine times and are labelled from *i* to *ix*. In a static one-stage game where both countries independently and simultaneously choose the type of trade tax and the trade tax rate then all nine intersections are NE. Assuming that the countries are symmetric and use the same type of trade tax, $T^z = \left\{T_1^z, T_2^z\right\}$ where $z = \tau, t, e$, then the NE will be symmetric so $\tau_1^N\left(T^z\right)=\tau_2^N\left(T^z\right)=\tau^N\left(T^z\right)$ and $W_1^N\left(T^z\right)=W_2^N\left(T^z\right)=W^N\left(T^z\right)$. Then, comparing the Nash equilibria, the NE trade taxes are lowest when both countries use specific export taxes and highest when both countries use specific import tariffs, $\tau^N\left(T^e\right)<\tau^N\left(T^\tau\right)<\tau^N\left(T^t\right)$. If $\tau_1=\tau_2=\tau>0$ then the welfare of both countries is decreasing in the trade tax, $\partial W_j/\partial\tau<0$ for $j=1,2$. Hence, the NE welfare is highest when both countries use specific export taxes and lowest when both countries use specific import tariffs, $W^N\left(T^e\right)>W^N\left(T^\tau\right)>W^N\left(T^t\right)$.This leads to the following proposition:

Proposition 1: *In the NE in trade taxes when countries are symmetric and both countries choose to use the same type of trade tax, the NE trade taxes are lower and the NE welfare is higher when countries use specific export taxes than when they use specific import tariffs,* $\tau^N\left(T^e\right)<\tau^N\left(T^\tau\right)<\tau^N\left(T^t\right)$ *and* $W^N\left(T^e\right)>W^N\left(T^\tau\right)>W^N\left(T^t\right)$.

In a trade war, specific import tariffs are not equivalent to specific export taxes as the outcome in terms of trade tax rates and welfare is different. Although each country is indifferent between using a specific import tariff and a specific export tax given its conjecture about the trade tax of the other country, as implied by the Lerner (1936) Symmetry Theorem, the type of trade tax chosen by a country affects the best-reply function of the other country. When both countries choose to use specific import tariffs, both countries become more aggressive as the best-reply functions of both countries move outwards and the NE trade taxes are higher (compared to when they choose to use *ad valorem* trade taxes). Whereas, when both countries choose to use specific export taxes, both countries become less aggressive as their best-reply functions shift inwards and the NE trade taxes are lower (compared to when they choose to use *ad valorem* trade taxes). As a result of the lower trade taxes, the welfare of both countries is higher when they both choose to use specific export taxes rather than specific import tariffs.

The type of trade tax chosen by the two countries affects the outcome of the trade war, and specific export taxes are not equivalent to specific import tariffs, which shows that the Lerner Symmetry Theorem does not hold for specific trade taxes in a trade war.

In the symmetric case, although there are multiple NE, it seems reasonable to suggest that both countries will choose to use specific export taxes since this NE Pareto-dominates the other symmetric NE where both countries choose to use the same type of trade tax. However, changing the structure of the game between the two countries to a two-stage game will ensure that there is a unique outcome.

2.2 Trade Policy Game

Consider the two-stage trade policy game where each country independently and simultaneously chooses the type of trade tax (*ad valorem* trade tax, T_j^{τ}; specific import tariff, T_j^t; or specific export tax, T_j^e, where $j = 1,2$) to use at stage one and then sets the *ad valorem* equivalent trade tax rate τ_j at stage two. The nine possible NE of the second stage of the game are shown in figure 4 for the case of strategic substitutes and in figure 5 for the case of strategic complements, which depend upon the type of trade tax chosen by the two countries in the first stage. In the first stage, if country two chooses to use a specific import tariff then the best-reply function of country one will be $\tau_1\left(\tau_2, T_2^t\right)$ and the NE will be *iv* if country one chooses to use a specific import tariff, *v* if it chooses to use an *ad valorem* trade tax, and *vi* if it chooses to use a specific export tax. Country one will choose a specific export tax since the NE *vi* gives it a higher level of welfare since the trade tax set by country two is lower than in *iv* or *v*. A similar argument can be used to show that country one will choose to use a specific export tax if country two chooses to use an *ad valorem* trade tax or a specific export tax. Hence, choosing to use a specific export tax is a dominant strategy for country one, and by the same argument it is a dominant strategy for country two. Therefore, the subgame-perfect NE of this two-stage trade policy game is for both countries to choose to use specific export taxes and for the outcome to be given by *ix* in figure 4 and figure 5. In the symmetric case, as shown in Proposition 1, both countries are better off choosing to use specific export taxes rather than *ad valorem* trade taxes or specific import tariffs. This leads to the following proposition:

Proposition 2: *In the subgame-perfect NE of the two-stage trade policy game, both countries choose to use specific export taxes. In the symmetric case, both countries are better off than when they both choose to use specific import tariffs or ad valorem trade taxes.*

For each country, choosing to use a specific import tariff makes the other country more aggressive as its best-reply function shifts outwards whereas choosing to use a specific export tax makes the other country less aggressive as its best-reply function shifts inwards. A country choosing to use a specific export tax makes the other country set a lower trade tax in the second stage whatever type of trade tax the other country has chosen in the first stage. Therefore, choosing to use a specific export tax is a dominant strategy for both countries that results in lower NE trade taxes and higher welfare for both countries.

2.3 Sustaining Free Trade in an Infinitely-Repeated Game

A trade war will reduce world welfare compared to free trade and, in the symmetric case, it will reduce the welfare of both countries. The situation is a prisoners' dilemma since both countries use trade taxes in the NE, but both countries are better off under free trade. As is well known, this prisoners' dilemma can be avoided in an infinitely-repeated game where co-operation (free trade) can be sustained by the use of Nash-reversion trigger strategies.[13] In fact, as Grossman (2016) points out any formal trade agreement can only achieve those outcomes that are sustainable in an infinitely-repeated game. Suppose that the static trade

[13]The use of Nash-reversion trigger strategies to sustain co-operation in infinitely-repeated games was demonstrated by Friedman (1971), and Dixit (1987) provides an application to sustaining free trade.

policy game is repeated infinitely with the countries choosing the type of trade tax to use at the beginning of the game. With Nash-reversion trigger strategies, each country sets a zero trade tax as long as the other country does the same, but if a country deviates then the two countries revert to the NE in trade taxes forever afterwards. Then, if the discount factor is $\delta \in [0,1]$, free trade can be sustained if the present discounted value of welfare from free trade exceeds the present discounted value of welfare from deviation followed by the welfare in the NE forever afterwards. Assuming symmetry so that welfare under free trade is the same for both countries, $W_1^F = W_2^F = W^F$, which obviously does not depend upon the type of trade taxes chosen by the two countries. If a country deviates from free trade then its sets its optimum trade tax while the other country sets a zero trade tax as in (2), and its welfare is $W_1^* = W_2^* = W^*$, which again does not depend upon the type of trade taxes chosen by the two countries. In the trade war that follows any deviation, countries will receive the NE welfare, which does depend upon the type of trade taxes chosen by the two countries. There are multiple Nash equilibria so to limit the number of cases considered only the symmetric NE, where both countries use the same type of trade taxes, will be analysed so NE welfare is $W^N\left(T^z\right)$, where $z = \tau, t, e$. Hence, when the two countries are symmetric, free trade can be sustained in the infinitely-repeated game using Nash-reversion trigger strategies if:

$$\frac{1}{1-\delta}W^F > W^* + \frac{\delta}{1-\delta}W^N\left(T^z\right) \qquad z = \tau, t, e \tag{6}$$

Free trade is sustainable if the discount rate is greater than the critical value obtained by making the above expression an equality and solving for the discount factor, which depends upon the type of trade taxes chosen by the two countries:

$$\delta > \delta_N\left(T^z\right) = \frac{W^* - W^F}{W^* - W^N\left(T^z\right)} \qquad z = \tau, t, e \tag{7}$$

Clearly, the lower the welfare in the NE in trade taxes then the lower is the critical discount factor and the easier it is to sustain free trade. From above, welfare is higher when both countries use specific export taxes than when they use *ad valorem* trade taxes and this is higher than when they both use specific import tariffs, $W^N\left(T^e\right) > W^N\left(T^\tau\right) > W^N\left(T^t\right)$. Hence, the ranking of critical discount factors is: $\delta_N\left(T^e\right) > \delta_N\left(T^\tau\right) > \delta_N\left(T^t\right)$, so it is easier to sustain free trade when both countries threaten to use specific import tariffs than when they both threaten to use *ad valorem* trade taxes or specific export taxes.

Proposition 3: *In the NE in trade taxes when countries are symmetric, the critical discount factors when using Nash-reversion trigger strategies are such that:* $\delta\left(T^t\right) < \delta\left(T^\tau\right) < \delta\left(T^e\right)$.

It is easier to sustain co-operation (free trade) using Nash-reversion trigger strategies, the more severe is the punishment for deviation, which is the NE welfare. Since welfare in the NE is lowest when both countries choose to use specific import tariffs, it is easier to sustain free trade in this case. Hence, if the objective is to sustain free trade, the ranking of trade tax types is reversed in this infinitely-repeated game compared to the static game, but once again specific export taxes are not equivalent to specific import tariffs.

3. Multi-Country Trade Policy Game

The analysis can be extended to the case of many countries and many goods under the assumption that the countries are symmetric using a model loosely based upon Bond and Syropoulos (1996). To maintain symmetry, the analysis will be restricted to comparing the Nash equilibria where all countries use the same type of trade tax. The J countries and the J goods are labelled $j = 1,\ldots,J$, where $J \geq 2$, and the world price of the *j*th good is p_j. It is assumed that the *j*th country has a comparative advantage in the *j*th good so it exports this good and imports all the other goods. First, each country has to choose whether to use an *ad valorem* trade tax, which can be either an import tariff or an export tax, and is denoted by T_j^{τ}; a specific import tariff, which is denoted by T_j^{t}; or a specific export tax, which is denoted by T_j^{e}, where $j=1,\ldots,J$ denotes the country. Then, each country, $j=1,\ldots,J$, has to decide the rate for the chosen trade tax with the *ad valorem* trade tax rate denoted by τ_j, the (average) specific import tariff rate denoted by t_j, and the specific export tax rate denoted by e_j. The specific import tariff is expressed in terms of the export good and the specific export tax is expressed in terms of a bundle of the imported goods.

The domestic price of the *j*th good in the *h*th country is p_{jh} so the domestic price of the *j*th good relative to the price of the export good (the *h*th good) in the *h*th country in terms of the trade taxes and the world prices is:

$$\frac{p_{jh}}{p_{hh}} = \frac{p_j\left(1+\tau_h\right)+t_{jh}p_h}{p_h\big/\left(1+\tau_h\right)-e_h\overline{p}_{-h}} \qquad j \neq h,\ j,h=1,\ldots J \tag{8}$$

where t_{jh} is the specific tariff imposed on the *j*th good by the *h*th country and $\overline{p}_{-h}=\sum_{j\neq i}p_j\big/\left(J-1\right)$ is the average price of the imported goods since the specific export tax is expressed in terms of a bundle of imports. For the *j*th good, an imported good, a specific import tariff t_{jh} is equivalent to an *ad valorem* import tariff τ_h if $\tau_h p_j = t_{jh}p_h$. Then, summing over all the imported goods yields that $\tau_h\overline{p}_{-h}=t_h p_h$ or $t_h=\tau_h\big/\rho_h$, where $t_h=\sum_{j\neq h}t_{jh}$ is the average specific tariff and $\rho_h = p_h\big/\overline{p}_{-h}$ is the terms of trade of the *h*th country (the price of its exported good divided by the average price of its imported goods). In any symmetric equilibrium, the price of all imported goods will be the same and hence the specific import tariff will be same for all imported goods, $t_{jh} = t_h$ for all $j \neq h$.[14] A specific export tax is equivalent to an *ad valorem* export tax if $\tau_h p_h / (1+\tau_h) = e_h \overline{p}_{-h}$ or $e_h = \tau_h \rho_h / (1+\tau_h)$.

The equilibrium terms of trade of the *h*th country is $\rho_h(\boldsymbol{\tau})$ where $\boldsymbol{\tau} = (\tau_1, \tau_2, \ldots, \tau_J)$ is the vector of *ad valorem* trade taxes and, as usual, it is assumed that the terms of trade are increasing in a country's own trade tax, $\partial \rho_h / \partial \tau_h > 0$. It is also assumed that the terms of trade are decreasing in the trade taxes of all the other countries, $\partial \rho_h / \partial \tau_j < 0$ with $j \neq h$, which implies a negative terms of trade externality. As in the two-country model, the possibility of the Metzler paradox will be ruled out, which implies that $\rho_h - \tau_h (\partial \rho_h / \partial \tau_h) > \partial \rho_h / \partial \tau_h > 0$. Furthermore, another useful result can be obtained from symmetry and the definition of the terms of trade, which implies that:

$$\frac{\sum_{j=1}^{J} p_j}{p_h} = \frac{J-1+\rho_h}{\rho_h} \tag{9}$$

Then, summing the reciprocal of (9) over all the countries yields:

$$\sum_{h=1}^{J} \frac{\rho_h}{J-1+\rho_h} = 1 \tag{10}$$

Differentiating with respect to the trade tax of the *j*th country and imposing symmetry yields:

$$\frac{\partial \rho_j}{\partial \tau_j} = -(J-1)\frac{\partial \rho_h}{\partial \tau_j} \qquad h \neq j\ h, j = 1, \ldots, J \tag{11}$$

[14] Technically, when a country chooses to use specific import tariffs then its strategic variable is the average specific tariff. In any deviation from a symmetric equilibrium, the country sets the average specific tariff and the individual specific tariffs are adjusted so that the equivalent *ad valorem* tariff rate is the same for all imported goods.

The welfare of the *j*th country is $W_j(\boldsymbol{\tau})$, which is assumed to be strictly quasi-concave in its own trade tax and decreasing in the trade taxes of all the other countries, $\partial W_j/\partial \tau_h < 0$ for $h \neq j$, due to the negative terms of trade externality. Since all countries are large, they have a unilateral incentive to use a trade tax if all other countries are pursuing a policy of free trade, $\partial W_j(\mathbf{0})/\partial \tau_j > 0$. If all countries set a common trade tax, $\tau_j = \tilde{\tau}$ for all j, then the welfare of the *j*th country is a function of this common trade tax: $\tilde{W}_j(\tilde{\tau}) = W_j(\tilde{\boldsymbol{\tau}})$ where $\tilde{\boldsymbol{\tau}} = (\tilde{\tau}, \tilde{\tau}, \ldots, \tilde{\tau})$, which is assumed to be strictly quasi-concave in $\tilde{\tau}$. Since free trade is Pareto-efficient, the welfare of every country, $\tilde{W}_j(\tilde{\tau})$ is maximised when $\tilde{\tau} = 0$ and strict quasi-concavity implies that welfare is decreasing in the common trade tax, $\partial \tilde{W}/\partial \tilde{\tau} < 0$ for all $\tilde{\tau} > 0$.

When all countries use *ad valorem* trade taxes, $T^{\tau} = \{T_1^{\tau}, T_2^{\tau}, \ldots, T_J^{\tau}\}$, the first-order conditions for the symmetric (interior) Nash equilibrium, which is assumed to be unique, are:

$$\frac{\partial W_j\left(\boldsymbol{\tau}^N\left(T^{\tau}\right)\right)}{\partial \tau_j} = 0 \qquad j = 1, \ldots, J \tag{12}$$

where $\boldsymbol{\tau}^N\left(T^{\tau}\right) = \left(\tau_1^N\left(T^{\tau}\right), \tau_2^N\left(T^{\tau}\right), \ldots \tau_J^N\left(T^{\tau}\right)\right)$ is the vector of NE trade taxes when all countries use *ad valorem* trade taxes with $\tau_j^N\left(T^{\tau}\right) = \tau_h^N\left(T^{\tau}\right) = \tau^N\left(T^{\tau}\right)$ for $j, h = 1, \ldots, J$.

When all countries use specific import tariffs, $T^t = \{T_1^t, T_2^t, \ldots, T_J^t\}$, each country maximises its welfare given the specific import tariffs set by the other $J-1$ countries. In the symmetric NE, the *j*th country sets τ_j to maximise its welfare $W_j(\boldsymbol{\tau})$ subject to the $J-1$ constraints that $t_h = \tau_h/\rho_h(\boldsymbol{\tau})$ where $t_h = t_i$ for all $h, i \neq j$. Hence, the first-order condition for the *j*th country is:

$$\frac{\partial W_j}{\partial \tau_j} + \sum_{h \neq j} \frac{\partial W_j}{\partial \tau_h} \frac{d\tau_h}{d\tau_j} = 0 \tag{13}$$

Totally differentiating the constraint $t_h = \tau_h / \rho_h(\boldsymbol{\tau})$ then imposing symmetry, $d\tau_i = d\tau_h$ for all $h, i \neq j$, and using (11) yields:

$$\frac{d\tau_h}{d\tau_j} = \frac{(J-1)\tau_h\left(\partial\rho_h/\partial\tau_j\right)}{(J-1)\rho_h - \tau_h\left(\partial\rho_h/\partial\tau_h\right)} < 0 \tag{14}$$

The numerator is negative due to the negative terms of trade externality and the denominator is positive as the Metzler paradox is ruled out. Note that if there are two countries, $J = 2$, then this derivative is the same as in (4). Since the derivative (14) is negative and there is a negative terms of trade externality, $\partial W_h / \partial \tau_j < 0$, $\partial W_j\left(\boldsymbol{\tau}^N\left(T^t\right)\right) / \partial \tau_j$ is negative in the symmetric NE where $\tau_j^N\left(T^t\right) = \tau_h^N\left(T^t\right) = \tau^N\left(T^t\right)$ for all j, h. Since there is assumed to be a unique symmetric NE and $\partial W_j(\mathbf{0}) / \partial \tau_j > 0$, $\partial W_j(\tilde{\boldsymbol{\tau}}) / \partial \tau_j > (<) 0$ if the common trade tax $\tilde{\tau} < (>) \tau^N\left(T^\tau\right)$. Hence, $\partial W_j\left(\boldsymbol{\tau}^N\left(T^t\right)\right) / \partial \tau_j$ being negative implies that the NE trade taxes are higher when all countries use specific import tariffs than when all countries use *ad valorem* trade taxes, $\tau^N\left(T^t\right) > \tau^N\left(T^\tau\right)$. Since welfare is decreasing in the common trade tax, $\partial \tilde{W}_j / \partial \tilde{\tau} < 0$, the welfare of all countries is lower with a specific import tariff than with an *ad valorem* trade tax, $W^N\left(T^t\right) \equiv \tilde{W}_j\left(\tau^N\left(T^t\right)\right) < \tilde{W}_j\left(\tau^N\left(T^\tau\right)\right) \equiv W^N\left(T^\tau\right)$.

When all countries use specific export taxes, $T^e = \left\{T_1^e, T_2^e, \ldots, T_J^e\right\}$, each country maximises its welfare given the specific export taxes set by the other $J-1$ countries where $e_h = e_i$ for all $h, i \neq j$ in the symmetric NE. The *j*th country sets τ_j to maximise its welfare

$W_j(\boldsymbol{\tau})$ subject to the $J-1$ constraints that $e_h = \tau_h \rho_h(\boldsymbol{\tau})/(1+\tau_h)$ for all $h,i \neq j$. Hence, the first-order condition for the *j*th country is:

$$\frac{\partial W_j}{\partial \tau_j} + \sum_{h \neq j} \frac{\partial W_j}{\partial \tau_h} \frac{d\tau_h}{d\tau_j} = 0 \tag{15}$$

Totally differentiating the constraint $e_h = \tau_h \rho_h(\boldsymbol{\tau})/(1+\tau_h)$ then imposing symmetry, $d\tau_i = d\tau_h$ for all $h,i \neq j$, and using (11) yields:

$$\frac{d\tau_h}{d\tau_j} = \frac{-\tau_h(1+\tau_h)(J-1)(\partial \rho_h/\partial \tau_j)}{(J-1)\rho_h + \tau_h(1+\tau_h)(\partial \rho_h/\partial \tau_h)} > 0 \tag{16}$$

The numerator is positive due to the negative terms of trade externality and the denominator is clearly positive. Note that if there are two countries, $J = 2$, then this derivative is the same as in (5). Since the derivative (16) is positive and there is a negative terms of trade externality, $\partial W_h/\partial \tau_j < 0$, $\partial W_j(\boldsymbol{\tau}^N(T^e))/\partial \tau_j$ is positive in the symmetric NE in trade taxes where $\tau_j^N(T^e) = \tau_h^N(T^e) = \tau^N(T^e)$ for all j,h. Since there is assumed to be a unique symmetric NE and $\partial W_j(\mathbf{0})/\partial \tau_j > 0$, $\partial W_j(\tilde{\boldsymbol{\tau}})/\partial \tau_j > (<) 0$ if the common trade tax $\tilde{\tau} < (>) \tau^N(T^\tau)$. Hence, $\partial W_j(\boldsymbol{\tau}^N(T^e))/\partial \tau_j$ being positive implies that the NE trade taxes are lower when all countries use specific export taxes than when all countries use *ad valorem* trade taxes, $\tau^N(T^e) < \tau^N(T^\tau)$. Since welfare is decreasing in a common trade tax, $\partial \tilde{W}_j/\partial \tilde{\tau} < 0$, the welfare of all countries is higher with specific export taxes than with an *ad valorem* trade taxes, $W^N(T^e) \equiv \tilde{W}_j(\tau^N(T^e)) > \tilde{W}_j(\tau^N(T^\tau)) \equiv W^N(T^\tau)$. This leads to the following proposition:

Proposition 4: *In the NE in trade taxes when countries are symmetric and all countries choose to use the same type of trade tax, the NE trade taxes are lower and the NE welfare is higher* *when countries use specific export taxes than when they use specific import tariffs,* $\tau^N\left(T^e\right)<\tau^N\left(T^\tau\right)<\tau^N\left(T^t\right)$ *and* $W^N\left(T^e\right)>W^N\left(T^\tau\right)>W^N\left(T^t\right)$.

As in the case of two countries, a specific export tax is not equivalent to a specific import tariff in a trade war as the outcome is different in terms of the level of trade taxes and welfare. Hence, again the Lerner Symmetry Theorem does not hold for specific trade taxes in a trade war. It would be straightforward to use Proposition 4 to extend Proposition 3 to the case of many countries.

4. Trade Policy Game with Positive Terms of Trade Externalities

Until now it has been assumed that there is a negative terms of trade externality, but there may be a positive terms of trade externality when some countries export or import the same good as in Panagariya and Schiff (1994, 1995) and Zissimos (2009). In this perfectly competitive model, there are three countries, one, two, and three, and two goods, one and two, where country one and two both export good one to country three and import good two from country three. Hence, the terms of trade for both country one and two are: $\rho = p_1/p_2$, where p_1 and p_2 are the world prices of goods one and two, respectively. The analysis will only consider the trade policies of countries one and two with country three assumed to be passively pursuing a policy of free trade. First, countries one and two each have to choose whether to use an *ad valorem* trade tax, which is denoted by T_j^τ; a specific import tariff, which is denoted by T_j^t; or a specific export tax, which is denoted by T_j^e. Then, each country, $j=1,2$, has to set the rate for the chosen type of trade tax with the *ad valorem* trade tax rate denoted by τ_j, the specific import tariff rate denoted by t_j, and the specific export tax denoted by e_j. The domestic prices of goods one and two, respectively, are p_{1j} and p_{2j} for $j=1,2$, and the relative domestic price of the importable good in terms of the world prices and trade taxes is:

$$\frac{p_{2j}}{p_{1j}} = \frac{p_2\left(1+\tau_j\right)+t_j p_1}{p_1\big/\left(1+\tau_j\right)-e_j p_2} \tag{17}$$

A specific import tariff is equivalent to an *ad valorem* import tariff if $t_j p_1 = \tau_j p_2$ or $t_j = \tau_j/\rho$, and a specific export tax is equivalent to an *ad valorem* export tax if $e_j p_2 = \tau_j p_1/\left(1+\tau_j\right)$ or $e_j = \tau_j \rho/\left(1+\tau_j\right)$ for $j = 1,2$. Equating demand and supply in the world market would yield the equilibrium terms of trade for countries one and two, $\rho\left(\tau_1,\tau_2\right)$, as a function of the *ad valorem* trade taxes of the two countries, and where $\partial\rho/\partial\tau_j > 0$ as an increase in either trade tax will improve the terms of trade of both countries so there is a positive terms of trade externality. The welfare of the two countries can be written as a function of the *ad valorem* trade taxes of the two countries, $W_j\left(\tau_1,\tau_2\right)$, where it is assumed that W_j is strictly quasi-concave in τ_j, and that $\partial W_j/\partial\tau_h > 0$ due to the positive terms of trade externality, where $h \neq j$ and $j,h = 1,2$.

Now consider the NE in *ad valorem* trade taxes (import tariffs or export taxes) of the trade policy game between country one and country two. Assuming that there is an interior solution, the first-order conditions for the NE in trade taxes are:

$$\frac{\partial W_1\left(\tau_1,\tau_2\right)}{\partial\tau_1} = 0 \qquad\qquad \frac{\partial W_2\left(\tau_1,\tau_2\right)}{\partial\tau_2} = 0 \tag{18}$$

The equation on the left implicitly defines the best-reply function of country one, $\tau_1 = \tau_1\left(\tau_2, T_2^{\tau}\right)$, and the equation on the right implicitly defines the best-reply function of country two, $\tau_2 = \tau_2\left(\tau_1, T_1^{\tau}\right)$. The best-reply functions are shown in figure 6, and the intersection of the two best-reply functions is the NE in *ad valorem* trade taxes, which is assumed to be unique. The welfare of each country is represented by the iso-welfare loci where

$W_1^N\left(T_1^\tau, T_2^\tau\right)$ and $W_2^N\left(T_1^\tau, T_2^\tau\right)$ are the welfare of country one and two, respectively, in the NE. The shape of the iso-welfare loci follows from the assumption that there is a positive terms of trade externality, $\partial W_1/\partial\tau_2 > 0$ and $\partial W_2/\partial\tau_1 > 0$, since the slope of the iso-welfare loci of country one is $d\tau_2/d\tau_1 = -\left(\partial W_1/\partial\tau_1\right)/\left(\partial W_1/\partial\tau_2\right)$. The best-reply functions are drawn under the assumption that trade taxes are strategic complements as this is regarded as the central case when there is a positive terms of trade externality, but this is not important for the current analysis. The optimum trade taxes when one country unilaterally deviates from free trade are: $\tau_1^* = \tau_1\left(0, T_2^\tau\right)$ and $\tau_2^* = \tau_2\left(0, T_1^\tau\right)$. The aggregate welfare of the two countries (but not the world due to the presence of the third country) is maximised at *C*, which is on the diagonal in the symmetric case shown in figure 6. This implies that welfare of the two countries in the symmetric case is increasing along the diagonal from the origin *0* to *C*.

Now consider the best-reply function of country one when country two chooses to use a specific import tariff, T_2^t. From (17), a specific import tariff t_2 is equivalent to an *ad valorem* trade tax τ_2 if $t_2 = \tau_2/\rho\left(\tau_1, \tau_2\right)$, which shows that the equivalent *ad valorem* trade tax of country two depends upon the trade tax set by country one. Country one now maximises its welfare $W_1\left(\tau_1, \tau_2\right)$ subject to the constraint $t_2 = \tau_2/\rho\left(\tau_1, \tau_2\right)$, which yields the first-order condition:

$$\frac{\partial W_1}{\partial \tau_1} + \frac{\partial W_1}{\partial \tau_2}\frac{d\tau_2}{d\tau_1} = 0 \qquad \text{where} \qquad \frac{d\tau_2}{d\tau_1} = \frac{\tau_2\left(\partial\rho/\partial\tau_1\right)}{\rho - \tau_2\left(\partial\rho/\partial\tau_2\right)} > 0 \tag{19}$$

The denominator of the derivative is positive given the assumption that rules out the Metzler paradox, and the numerator is positive since the tariff of country one has a positive effect on the terms of trade of both countries.[15] Since it is assumed that $\partial W_1/\partial\tau_2 > 0$ then $\partial W_1/\partial\tau_1 < 0$ in (19) so strict quasi-concavity of the welfare function implies that the tariff set by country one is higher when country two uses a specific import tariff. The situation is shown in figure 7 where the best-reply function of country one when country two uses an *ad valorem* trade tax is obtained by maximising welfare given the *ad valorem* trade tax of country two so if $\tau_2 = \tau_2^\tau$ then the optimum is at α where $\tau_1 = \tau_1^\tau$. When country two uses a specific import tariff then country one maximises welfare subject to the constraint $t_2 = \tau_2/\rho(\tau_1,\tau_2)$, which from (19) is upward sloping in figure 7 so country one realises that when it increases its trade tax then there will be an increase in the equivalent *ad valorem* trade tax of country two. Hence, the optimum is at β where $\tau_1 = \tau_1^t$ and $\tau_2 = \tau_2^t$, and this allows the best-reply function of country one when country two uses a specific import tariff $\tau_1(\tau_2, T_2^t)$ to be derived. Country two switching from using an ad valorem trade tax to using a specific import tariff leads the best-reply function to swivel clockwise around $(\tau_1^*, 0)$, and leads country one to set a higher trade tax. Although figure 7 is drawn for the case when trade taxes are strategic complements, the analysis is unchanged when trade taxes are strategic substitutes. A similar analysis can be used to derive the best-reply function of country two for the case when country one uses a specific import tariff $\tau_2(\tau_1, T_1^t)$.

Now consider the best-reply function of country one when country two uses a specific export tax, T_2^e. From (17) a specific export tax e_2 is equivalent to an *ad valorem* trade tax τ_2 if $e_2 = \tau_2\rho(\tau_1,\tau_2)/(1+\tau_2)$, which acts as a constraint when country one sets its trade tax.

[15] For later reference, see footnote 16, it is assumed that the Metzler paradox does not hold in a hypothetical situation (customs union) where both countries simultaneously increase their import tariffs. This implies that the derivative in (19) is less than one.

Hence, country one maximises its welfare $W_1(\tau_1,\tau_2)$ subject to $e_2 = \tau_2\rho(\tau_1,\tau_2)/(1+\tau_2)$ and the first order condition is:

$$\frac{\partial W_1}{\partial \tau_1}+\frac{\partial W_1}{\partial \tau_2}\frac{d\tau_2}{d\tau_1}=0 \qquad \text{and} \qquad \frac{d\tau_2}{d\tau_2}=\frac{-\tau_2(1+\tau_2)(\partial\rho/\partial\tau_1)}{\rho+\tau_2(1+\tau_2)(\partial\rho/\partial\tau_2)}<0 \tag{20}$$

The denominator of the derivative is clearly positive as $\partial\rho/\partial\tau_2 > 0$, and the numerator is clearly negative as $\partial\rho/\partial\tau_1 > 0$. Since it is assumed that $\partial W_1/\partial\tau_2 > 0$ then $\partial W_1/\partial\tau_1 > 0$ in (20) so quasi-concavity of the welfare function implies that the trade tax set by country one is lower when country two sets a specific export tax. The situation is shown in figure 8 where the best-reply function for country one when country two uses an *ad valorem* trade tax $\tau_1(\tau_2,T_2^\tau)$ is the same as in figure 7.When country two uses a specific export tax then country one maximises its welfare subject to the constraint $e_2 = \tau_2\rho(\tau_1,\tau_2)/(1+\tau_2)$, which is downward sloping in figure 8, and the optimum is at γ where $\tau_1=\tau_1^e$ and $\tau_2=\tau_2^e$. Hence, the best-reply function of country one when country two uses a specific export tax $\tau_1(\tau_2,T_2^e)$ can be derived. Country two switching from using an *ad valorem* trade tax to using a specific export tax leads the best-reply function to swivel anti-clockwise around $(\tau_1^*,0)$ and leads country one to set a lower trade tax. Although figure 8 is drawn for the case when trade taxes are strategic complements, the analysis is unchanged for the case of strategic substitutes. A similar analysis can be used to derive the best-reply function for country two when country one uses a specific export tax $\tau_2(\tau_1,T_1^e)$.

The best-reply functions of both countries and for all three types of trade taxes are shown in figure 9 for the case of strategic complements. The three best-reply functions of country one intersect the three best-reply functions of country two nine times and the intersections are labelled from *i* to *ix*. In a static one-stage game where both countries independently and simultaneously choose the type of trade tax and the trade tax rate then all nine intersections are NE. Assuming that the countries are symmetric and that both countries use the same type of trade tax, $T^z = \{T_1^z, T_2^z\}$ where $z = \tau, t, e$, then the NE will be symmetric with the NE trade taxes: $\tau_1^N(T^z) = \tau_2^N(T^z) = \tau^N(T^z)$ and the NE welfare $W_1^N(T^z) = W_2^N(T^z) = W^N(T^z)$. Then comparing the symmetric NE, the NE trade taxes are lowest when both countries use specific export taxes and highest when both countries use specific import tariffs, and the NE welfare is lowest when both countries use specific export taxes and highest when both countries use specific import tariffs.[16] This leads to the following proposition:

Proposition 5: *In the NE in trade taxes when countries are symmetric and both countries choose to use the same type of trade tax, the NE trade taxes are higher and the NE welfare is higher when countries use specific import tariffs than when they use specific export taxes,* $\tau^N(T^e) < \tau^N(T^\tau) < \tau^N(T^t)$ *and* $W^N(T^t) > W^N(T^\tau) > W^N(T^e)$.

As in the case of negative terms of trade externalities, a specific export tax is not equivalent to a specific import tariff in a trade war as the outcome is different in terms of the trade tax rates and welfare of the countries.

The analysis can be extended as in section 2.2 for the case of a two-stage game where countries first choose the type of trade tax and then choose the trade tax rates. It can be shown that choosing to use a specific import tariff is a dominant strategy for both countries. Similarly, the analysis can be extended as in section 2.3 to consider co-operation in an infinitely-repeated game. In this case co-operation between the two countries would be at *C* in figure 6 rather than

[16] Note that the slope of the constraint (19) being less than one implies that the NE with specific import tariffs, *iv* in figure 9, is closer to the origin than the co-operative outcome *C* in figure 6.

free trade. It can be shown that it is easier to sustain co-operation if both countries threaten to use specific export taxes.

5. Conclusions

The most significant result of this paper is that the Lerner Symmetry Theorem does not hold for specific trade taxes in trade policy games such as trade wars. The result has been obtained in a perfectly competitive, general equilibrium model using a general analysis that avoids unnecessary complexity by starting from welfare and the terms of trade as functions of the equivalent *ad valorem* trade taxes. This yields general results that depend only upon the sign of the terms of trade externality, and perhaps surprisingly do not depend upon whether trade taxes are strategic substitutes or strategic complements. When there is a negative terms of trade externality as in the standard two-country model of trade policy games, choosing to use specific export taxes is a dominant strategy for all countries. In the symmetric case, all countries will set lower trade taxes and have higher welfare if they choose to use specific export taxes rather than specific import tariffs or *ad valorem* trade taxes. When there is a positive terms of trade externality as when two countries export the same good, choosing to use specific import tariffs is a dominant strategy for these two countries.

When the objective is to sustain free trade in an infinitely-repeated game with a negative terms of trade externality, the discount factor is lowest and hence it is easiest to sustain free trade when countries threaten to use specific import tariffs rather than specific export taxes or *ad valorem* trade taxes. This is because specific import tariffs yield the worst outcome in terms of NE welfare and so they are better at deterring deviation from free trade.

Hence, in all the trade policy games analysed, it has been shown that the Lerner Symmetry Theorem does not hold and that a specific trade tax (an import tariff or an export tax depending upon the game) is superior to an *ad valorem* trade tax. Clearly, this is a significant result for the analysis of trade policy games where trade taxes have usually been assumed to be *ad valorem*.

References

Bagwell, Kyle, and Robert W. Staiger. 2016. "Chapter 8 - The Design of Trade Agreements." In *Handbook of Commercial Policy*, edited by Kyle Bagwell and Robert W. Staiger, 435-529. North-Holland. doi: https://doi.org/10.1016/bs.hescop.2016.04.005.

Bond, Eric W., and Constantinos Syropoulos. 1996. "The size of trading blocs Market power and world welfare effects." *Journal of International Economics* 40 (3):411-437. doi: https://doi.org/10.1016/0022-1996(95)01412-8.

Cheng, Leonard. 1985. "Comparing Bertrand and Cournot Equilibria: A Geometric Approach." *The RAND Journal of Economics* 16 (1):146-152. doi: https://doi.org/10.2307/2555596.

Costinot, Arnaud, and Iván Werning. 2019. "Lerner Symmetry: A Modern Treatment." *American Economic Review: Insights* 1 (1):13-26. doi: https://doi.org/10.1257/aeri.20170006.

Dixit, Avinash. 1987. "Strategic aspects of trade policy." In *Advances in Economic Theory: Fifth World Congress*, edited by Truman Fassett Bewley, 329-362. Cambridge: Cambridge University Press. doi: https://doi.org/10.1017/CCOL0521340446.009.

Friedman, James W. 1971. "A Non-cooperative Equilibrium for Supergames." *The Review of Economic Studies* 38 (1):1-12. doi: https://dx.doi.org/10.2307/2296617.

Grossman, Gene M. 2016. "Chapter 7 - The Purpose of Trade Agreements." In *Handbook of Commercial Policy*, edited by Kyle Bagwell and Robert W. Staiger, 379-434. North-Holland. doi: https://doi.org/10.1016/bs.hescop.2016.04.016.

Horwell, David J. 1966. "Optimum Tariffs and Tariff Policy." *The Review of Economic Studies* 33 (2):147-158. doi: https://doi.org/10.2307/2974438.

Johnson, Harry G. 1953. "Optimum Tariffs and Retaliation." *The Review of Economic Studies* 21 (2):142-153. doi: https://doi.org/10.2307/2296006.

Kennan, John, and Raymond Riezman. 1988. "Do Big Countries Win Tariff Wars?" *International Economic Review* 29 (1):81-85. doi: https://doi.org/10.2307/2526808.

Lerner, Abba P. 1936. "The Symmetry between Import and Export Taxes." *Economica* 3 (11):306-313. doi: https://doi.org/10.2307/2549223.

Lockwood, Ben, and Kar-yiu Wong. 2000. "Specific and ad valorem tariffs are not equivalent in trade wars." *Journal of International Economics* 52 (1):183-195. doi: https://doi.org/10.1016/S0022-1996(99)00053-7.

Metzler, Lloyd A. 1949. "Tariffs, the Terms of Trade, and the Distribution of National Income." *The Journal of Political Economy* 57 (1):1-29. doi: https://doi.org/10.1086/256766.

Panagariya, Arvind, and Maurice Schiff. 1994. "Can revenue maximizing export taxes yield higher welfare than welfare maximizing export taxes?" *Economics Letters* 45 (1):79-84. doi: https://doi.org/10.1016/0165-1765(94)90062-0.

Panagariya, Arvind, and Maurice Schiff. 1995. "Optimun and Revenue Maximizing Trade Taxes in a Multicountry Framework." *Revista de Análisis Económico – Economic Analysis Review* 10 (1):19-35.

Rodriguez, Carlos Alfredo. 1974. "The non-equivalence of tariffs and quotas under retaliation." *Journal of International Economics* 4 (3):295-298. doi: https://doi.org/10.1016/0022-1996(74)90048-8.

Syropoulos, Constantinos. 2002. "Optimum Tariffs and Retaliation Revisited: How Country Size Matters." *The Review of Economic Studies* 69 (3):707-727. doi: https://doi.org/10.1111/1467-937X.t01-1-00028.

Tower, Edward. 1975. "The Optimum Quota and Retaliation." *The Review of Economic Studies* 42 (4):623-630. doi: https://doi.org/10.2307/2296799.

Vives, Xavier. 1985. "On the Efficiency of Bertrand and Cournot Equilibria with Product Differentiation." *Journal of Economic Theory* 36 (1):166-175. doi: http://dx.doi.org/10.1016/0022-0531(85)90086-9.

Zissimos, Ben. 2009. "Optimum tariffs and retaliation: How country numbers matter." *Journal of International Economics* 78 (2):276-286. doi: https://doi.org/10.1016/j.jinteco.2009.04.003.

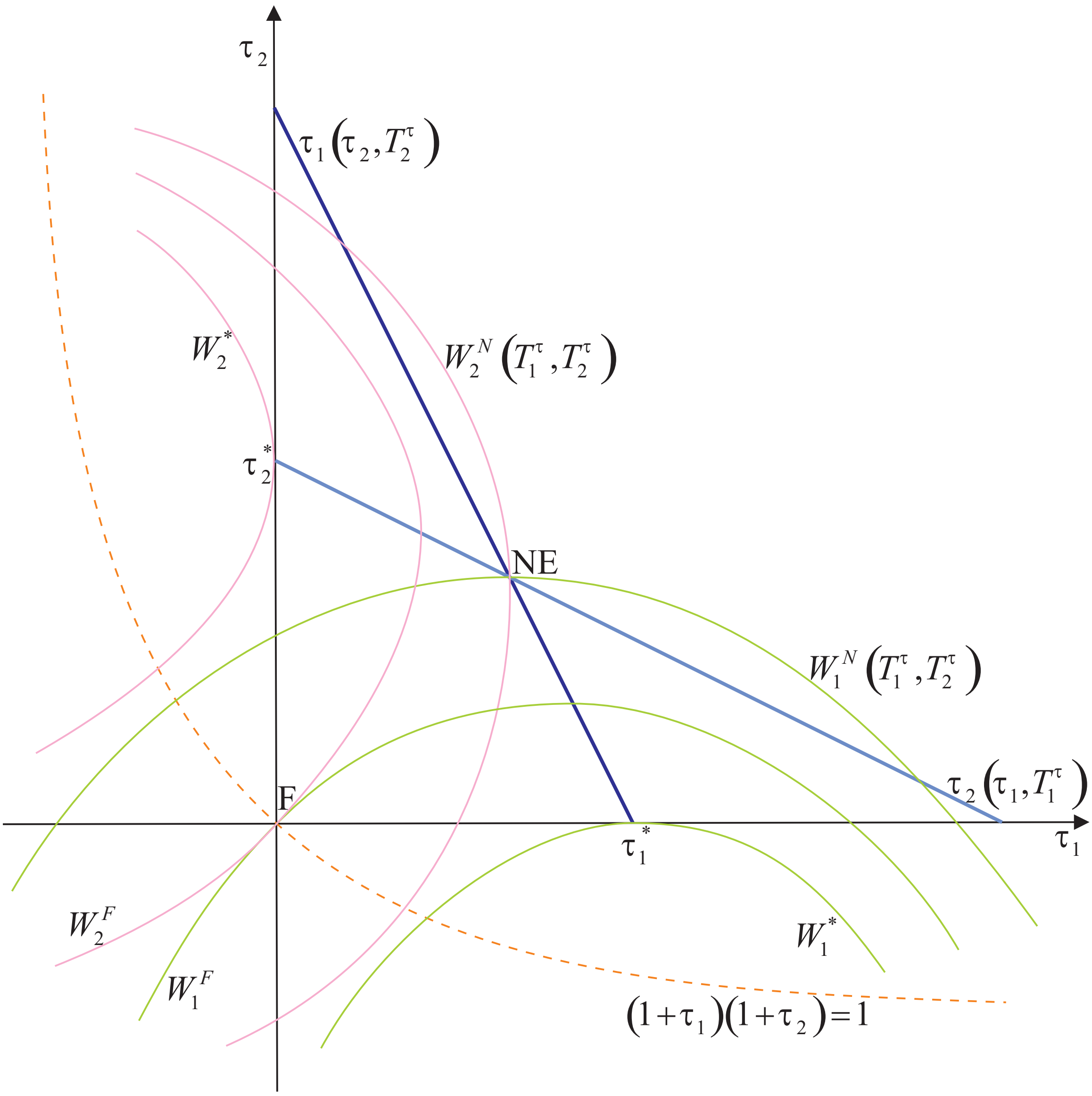

Figure 1: Nash Equilibrium in *Ad Valorem* Trade Taxes

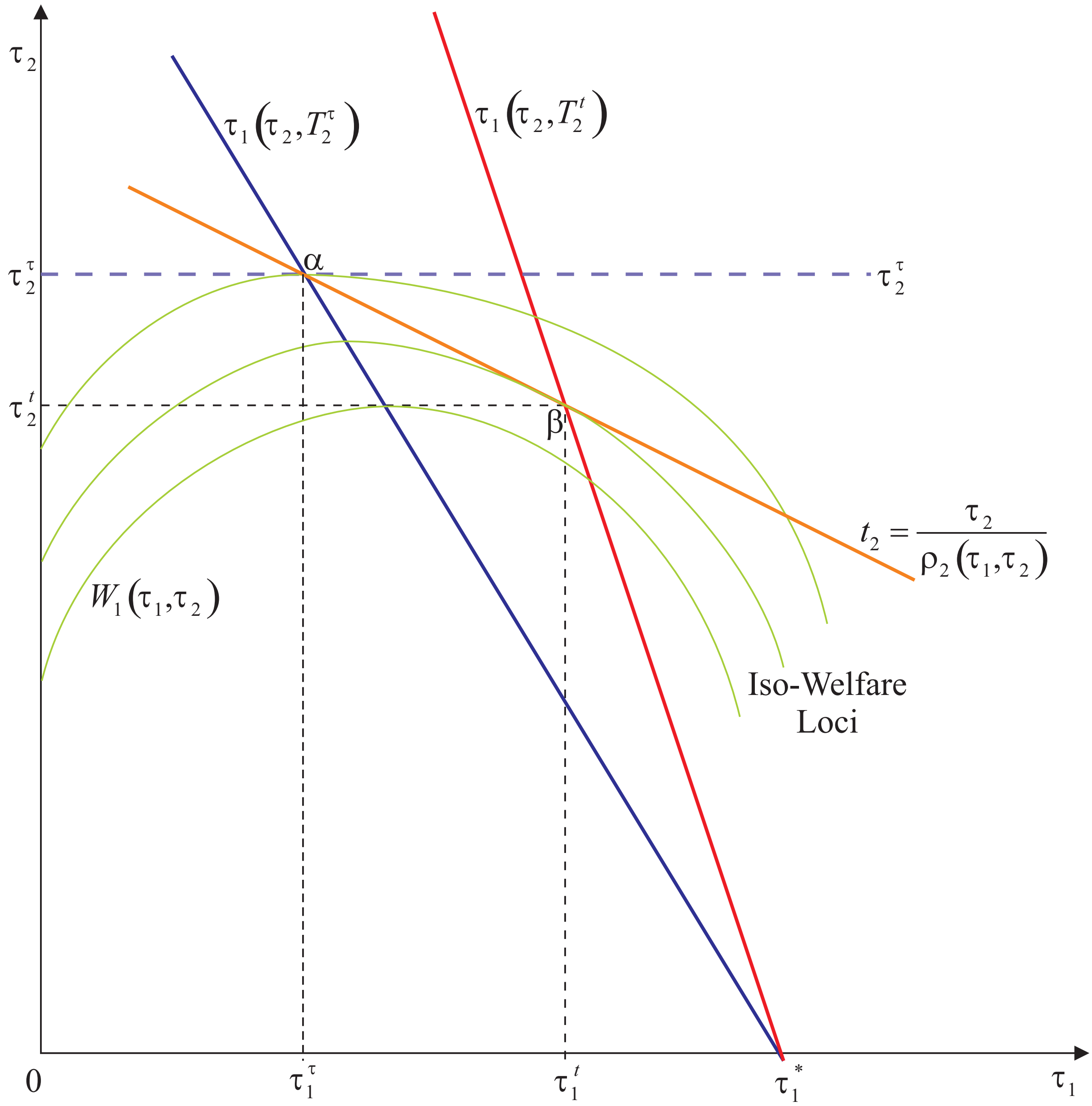

Figure 2: Best Reply for Country One to a Specific Import Tariff

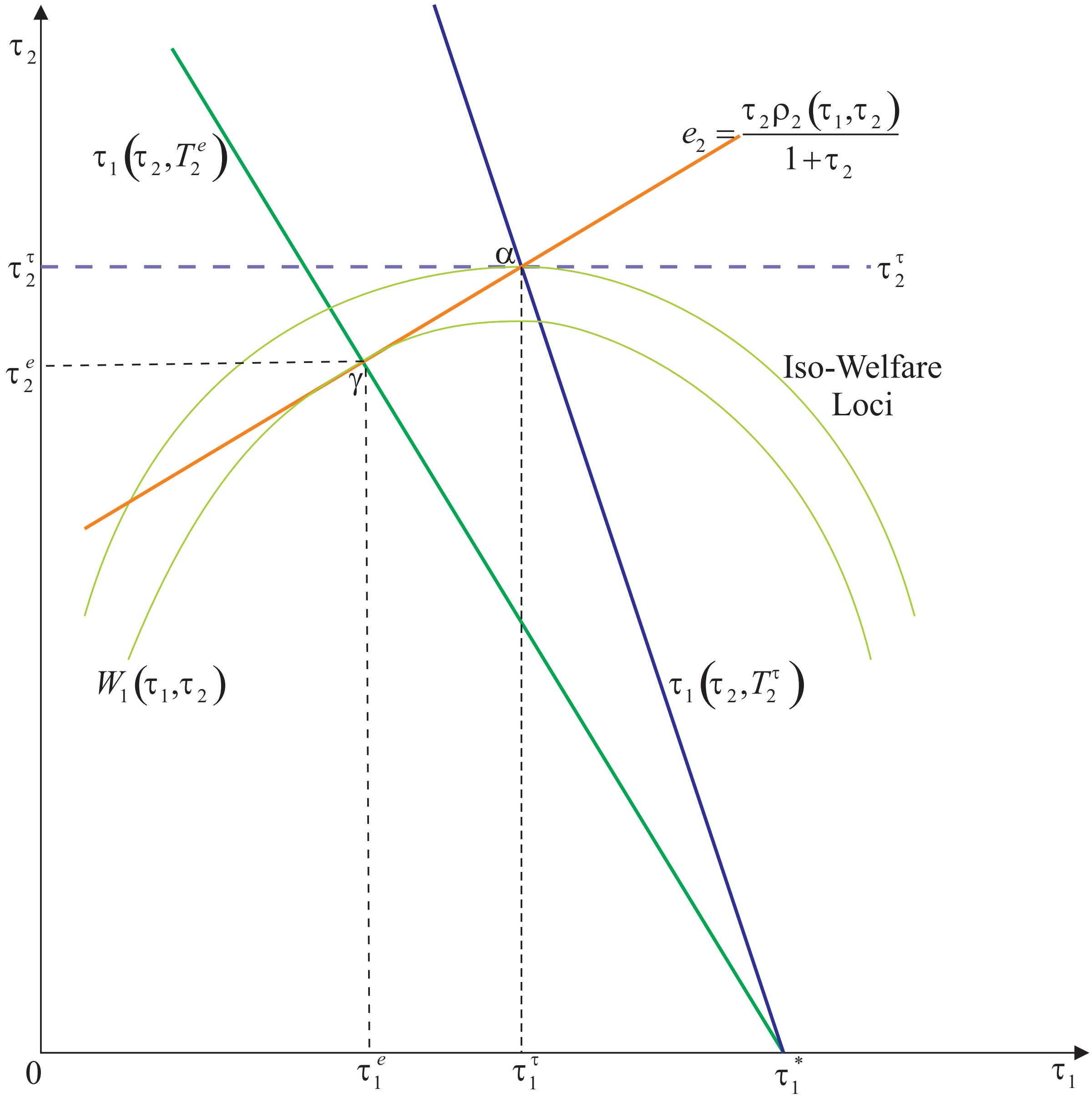

Figure 3: Best Reply for Country One to a Specific Export Tax

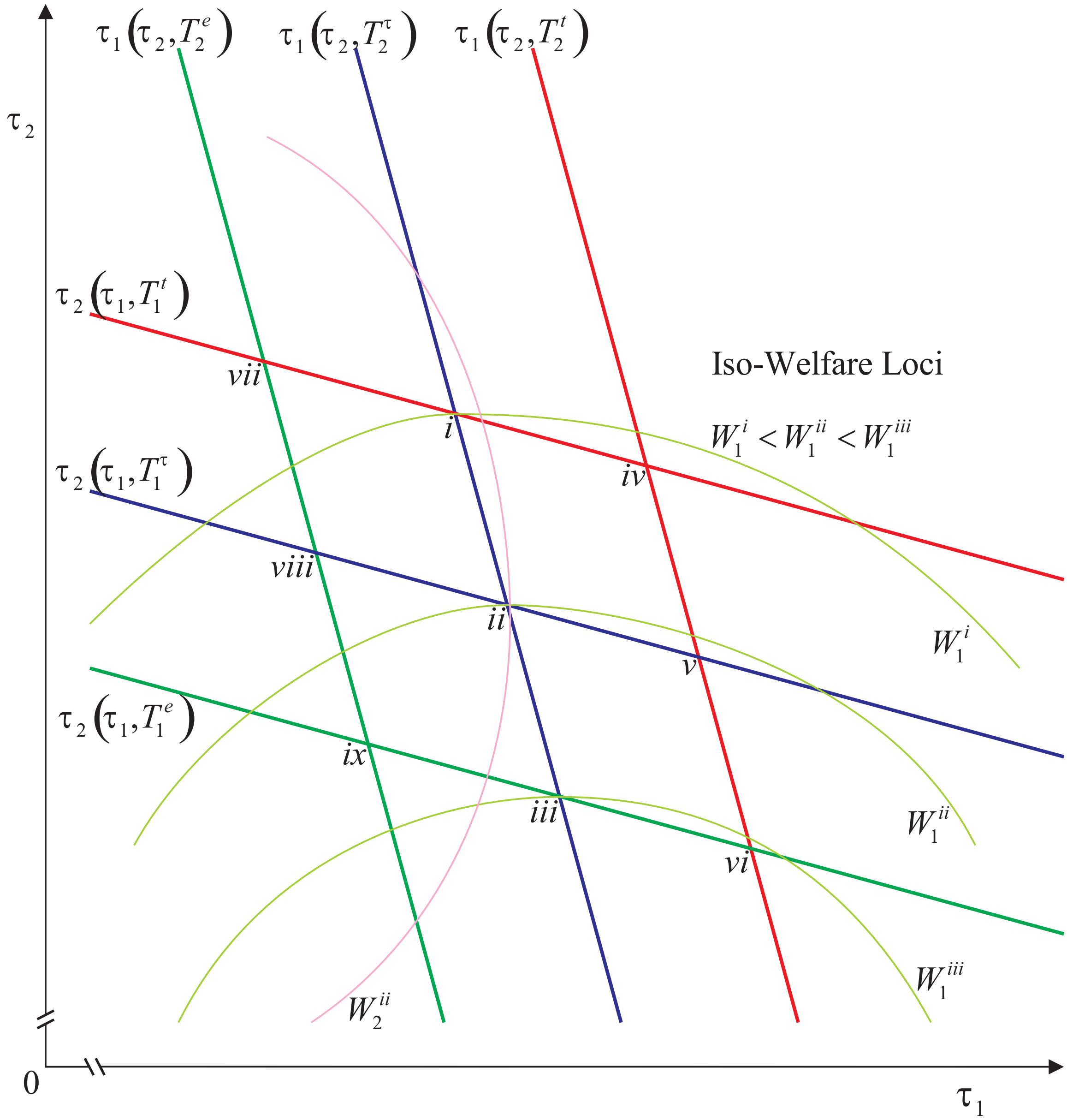

Figure 4: *Ad Valorem* Trade Taxes vs Specific Trade Taxes: Strategic Substitutes

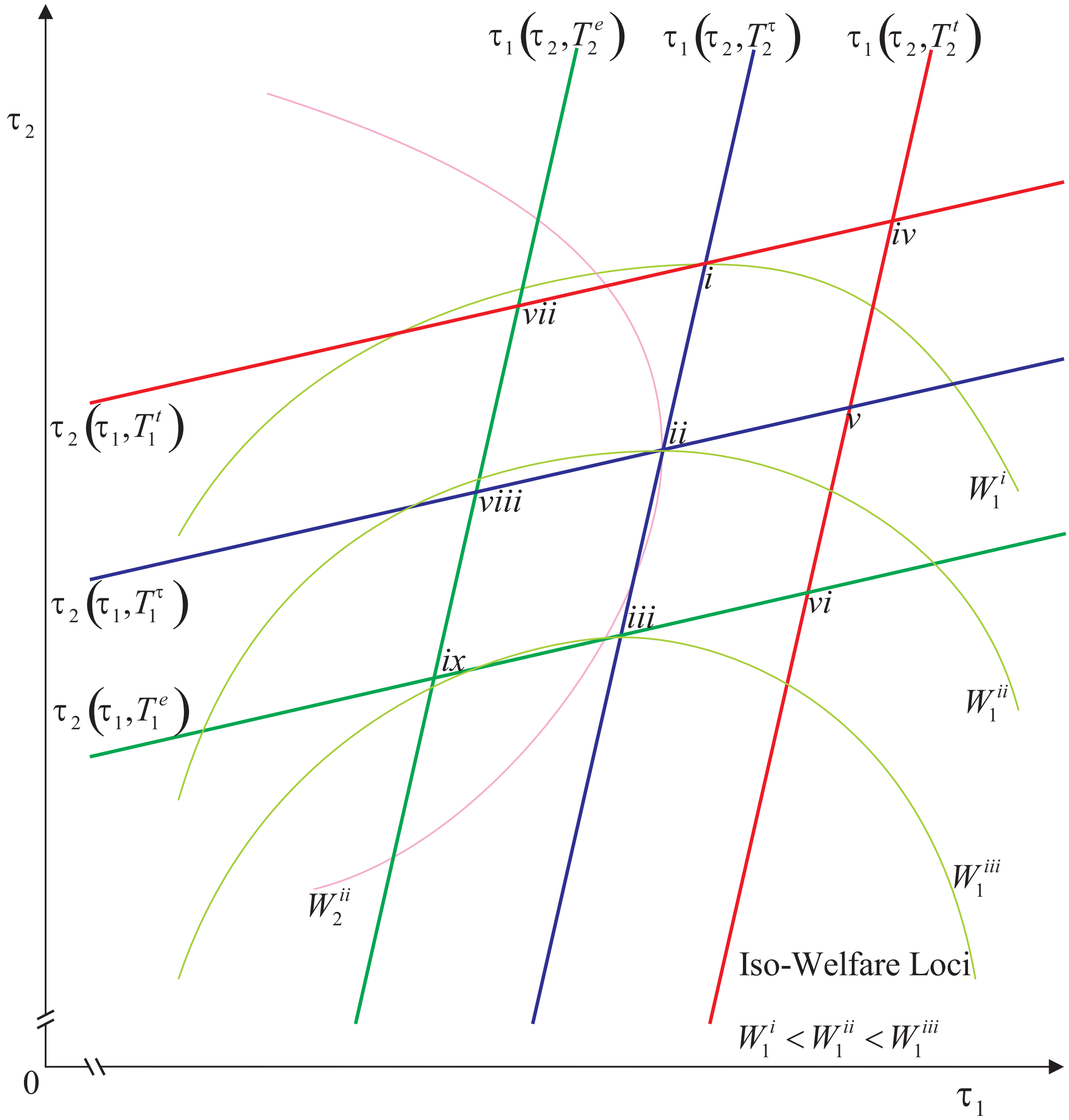

Figure 5: *Ad Valorem* Trade Taxes vs Specific Trade Taxes: Strategic Complements

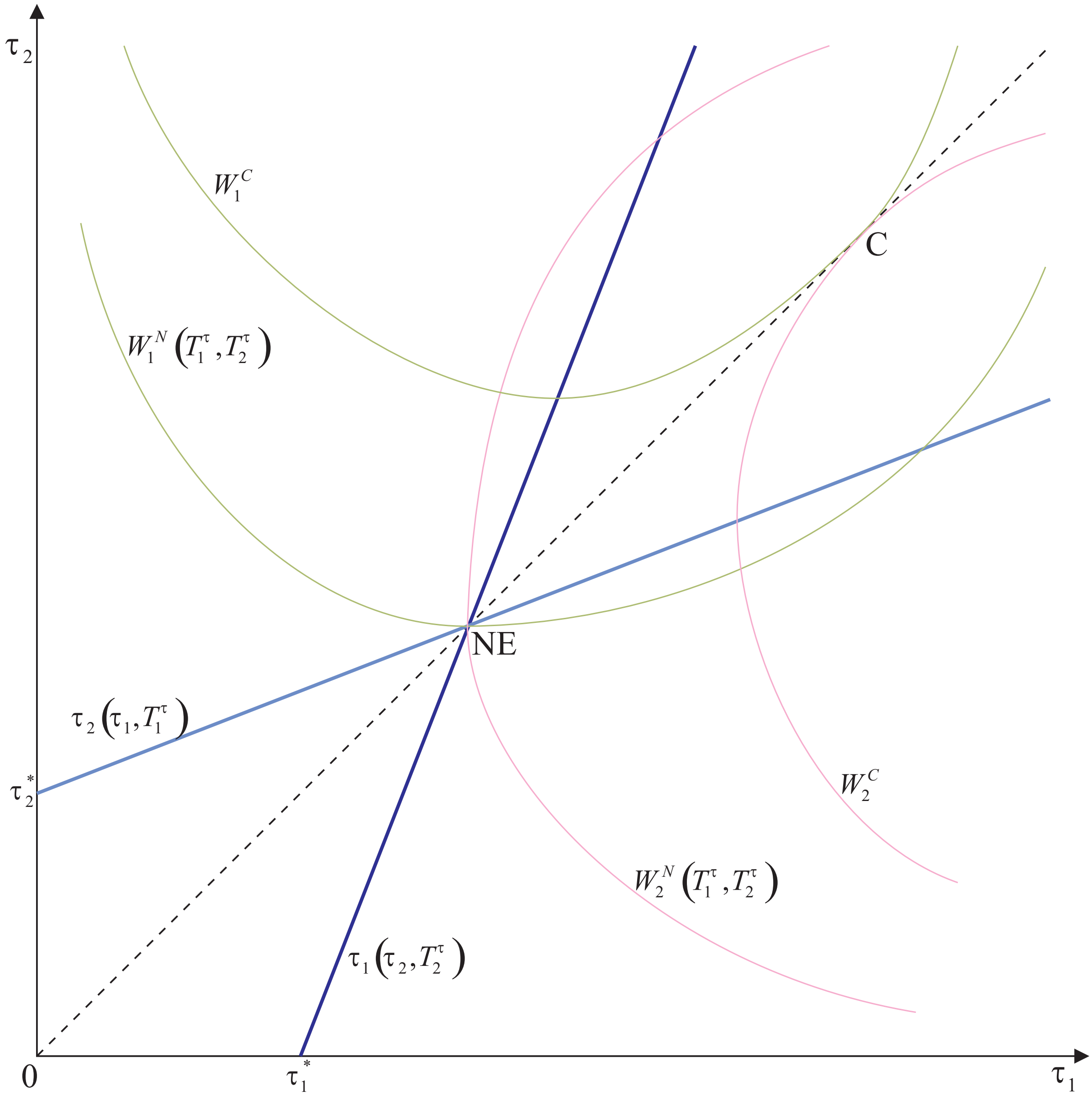

Figure 6: Nash Equilibrium in *Ad Valorem* Trade Taxes with Positive Terms of Trade Externalities

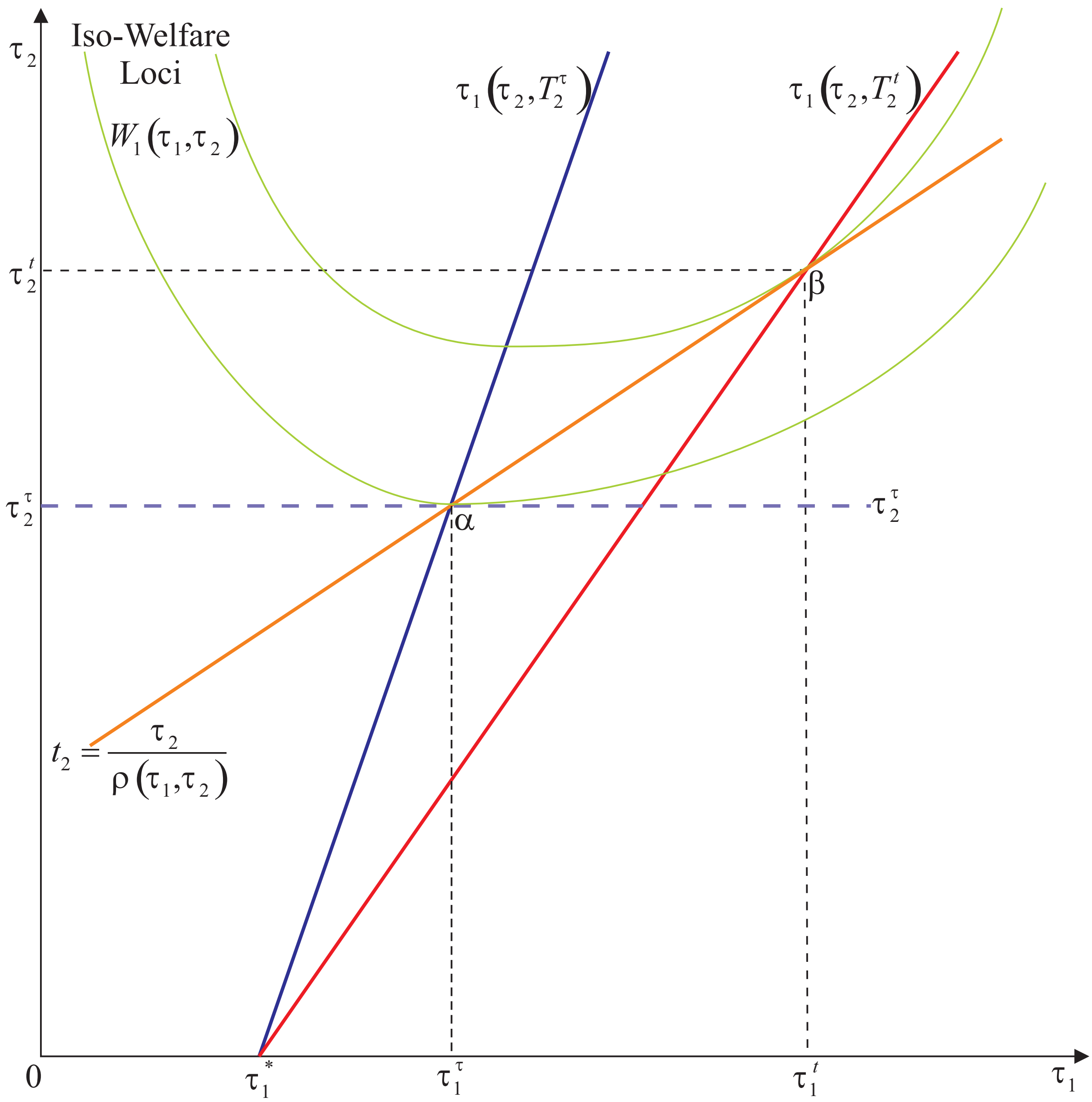

Figure 7: Best Reply for Country One to a Specific Import Tariff with a Positive Terms of Trade Externality

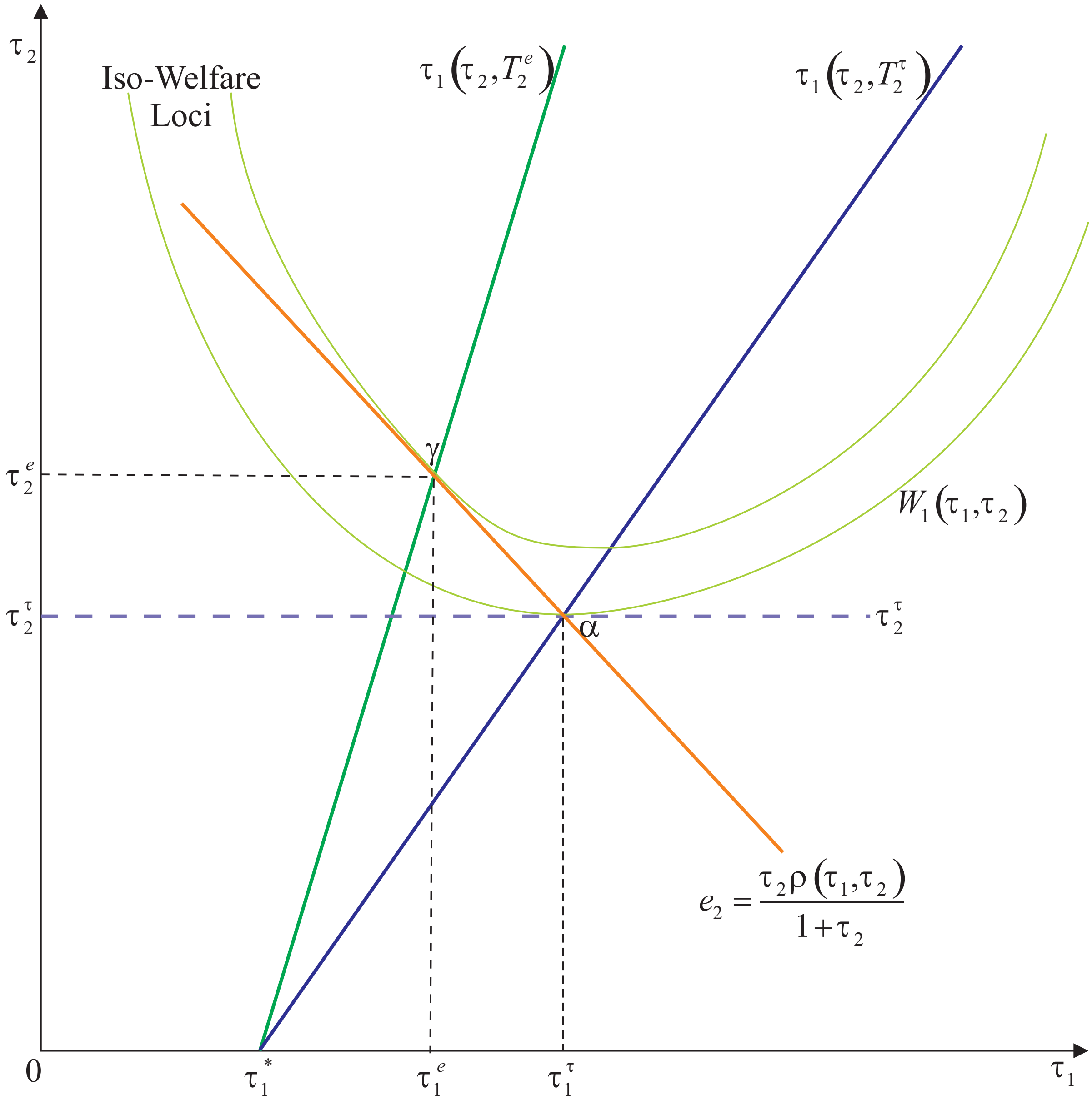

Figure 8: Best Reply for Country One to a Specific Export Tax with a Positive Terms of Trade Externality

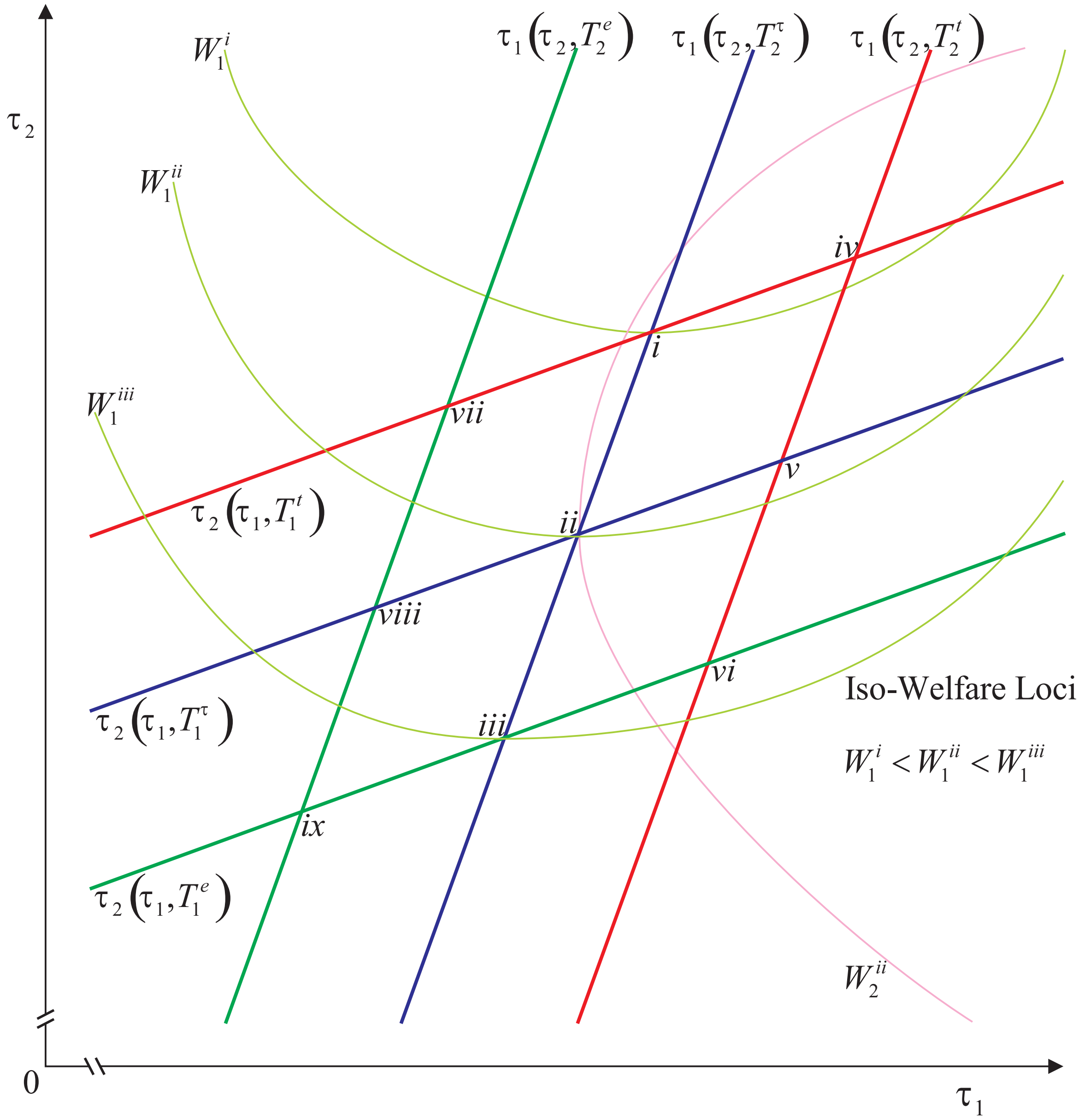

Figure 9: *Ad Valorem* Trade Taxes vs Specific Trade Taxes:
Positive Terms of Trade Externalities