Design Curves for Stiffened Engineered Timber Wall Systems: A verified analytical approach

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Abstract

This manuscript introduces the mathematical modelling of efficient stiffened engineered timber wall systems and presents a collection of corresponding design curves. An analytical approach via the exact finite strip method through the Wittrick-Williams algorithm was used in this study to understand and predict the behaviour of such systems. Appropriate orthotropic material models and strength limits have been incorporated into the analytical method for the engineered timber panel and sawn cut timber stud stiffeners respectively. This is the first time that this approach has been used in this field of timber engineering and hence was verified through experimental testing and detailed Finite Element Analysis (FEA). Compared to FEA there is over a thousand-fold decrease in computational cost which allowed many parameters to be varied incrementally including: the thickness of the panel, number of stiffeners, height of the wall and applied load. The results of which has enabled a set of versatile and simple validated design curves to be developed and presented. With these design curves, for a desired load capacity, the optimal system configurations are given. Likewise, for a chosen configuration the allowable axial load is given.

Keywords: Stiffened Engineered Timber Wall Systems; Design Curves; Exact Finite Strip Method; Validated Analytical Modelling.

1. Introduction

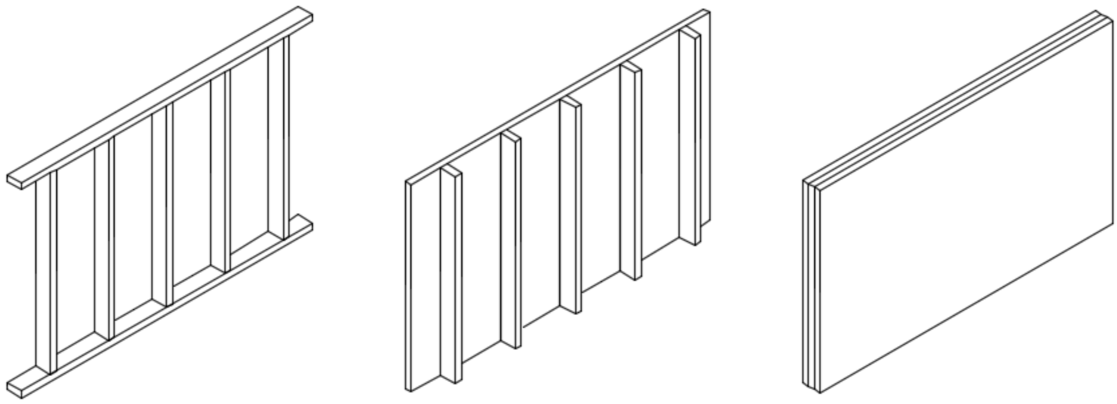
Prefabrication is progressively being adopted in construction, specifically with the increasing use of timber-based systems [1-4]. This developing industry is furthering its establishment for mid-rise construction and hence this industry is pushing forward with innovation and development [5-7]. Traditionally timber has been used in the form of open-panel lightweight frame construction [8, 9]. These are generally suitable for low rise 1-3 storey developments [10, 11]. Taller construction in timber can be achieved by what the International Building Code terms as 'massive timber systems', or 'mass wood construction' using large solid CLT (Cross-laminated timber) built up panels [12]. Buildings in this way have achieved 18 storeys or 53 meters in height with feasible developed plans to go up to 150 meters [13, 14]. However, a more suitable efficient material is required for mid-rise construction which is below the capabilities of CLT yet above the limits of lightweight-frame construction. The presented stiffened engineered timber wall systems offer an in-between for lightweight framing and massive wood construction.

There are many factors which relate to the viability and feasibility of a timber-based solution for a project. One such factor is the ability to undertake rapid modelling and preliminary design in the early stages. Time intensive purpose/project specific finite element modelling (FEM) and specialist knowledge at this early stage makes more traditional methods such as concrete and steel more attractive options in terms of perceived viability. There is no adequate method in Australian or International Standards to specifically cater for the design of Stiffened Engineered Timber Wall Systems with thicker wall panels that can transfer vertical loads. This contrasts with thin bracing sheets which are highly prone to local buckling. A tool for the rapid assessment of the capacity and optimal design of typical configurations of Stiffened Engineered Timber Wall Systems is needed.

2. Stiffened Engineered Timber Wall Systems

2.1 Comparison with current accepted systems

Lightweight timber framing and massive wooden construction are on opposing ends in terms of material usage and structural capacity. The proposed stiffened engineered timber walls are an intermediary balance between these two extremes as shown in **Figure 1.** For lateral resistance many lightweight framing systems also have either steel diagonal straps secured or thin (typically 6 mm) sheathing/braceboard [15, 16]. In these cases, the thin sheathing or braceboard is used only for shear resistance and is considered to provide no direct vertical resistance [17]. Indirectly though it does, as it braces the studs from in-plane buckling or tilting. These types of lightweight timber framing systems have many names such as diaphragm walls, shear walls and sheathed walls. However they all follow the same principle that the studs carry the vertical axial loads and the thin sheathing (affixed with staples, nails or screws) provides the lateral bracing [18, 19] . This is the principal difference that they have with the presented stiffened engineered timber wall. The stiffened walls have a comparably thick panel which is then integrally stiffened with an adhesive and pull-out resistant nail connection, so all elements are acting together to take axial loads much like blade stiffened walls in aerospace structures.



(a) (b) (c)

Figure 1. Isometric view of typical timber systems: **(a)** lightweight framing; **(b)** stiffened engineered timber wall; **(c)** massive wooden construction/CLT.

Stiffened engineered timber walls principally consist of stud stiffeners and engineered timber panels [20]. CLT has not been considered as the panel element of the proposed stiffened engineered timber wall system. This is due to the fact CLT is a massive wooden construction material with minimum thickness of 60 mm, that is 3 laminates of 20 mm each [21]. A stiffened engineered timber wall system whose panel thickness is lesser than this is designed to be a more efficient structural system for construction beyond that capabilities of lightweight framing, but below the high capacity which thick CLT panel buildings can achieve.

2.2 Materials and Material Properties

The orthotropic engineering constants of the material used for the finite element model can be found in section 6.2 Material Properties.

2.2.1 Panels / Sheathing

Panels/sheathing are flat engineered timber products, and common types include: oriented strandboard (OSB), plywood (PLY), particle board and fibreboard. Of these only OSB and PLY have been seriously considered for the panel component of stiffened engineered timber walls. This is due to their superior strength and durability over particleboard and fibreboard [22]. OSB was chosen for focus in this study due to its cost advantage over PLY. Additionally, OSB panels are the current market leader in terms of structural panels due to their cost to performance efficiency [23, 24]. Moreover, OSB is suitable for the intended application with a variety of thicknesses available currently in the common market and many thicker options also available when consulting with manufacturers. This engineered wood product efficiency will be taken advantage of for this application as the primary direction of the oriented strands are directed such that the higher bending capacity is along the height of the wall.

2.2.2 Stiffener / Stud

In Australia the standard stud sizes for interior walls are 70×35 mm and for exterior walls it is 90×45 mm and/or 90×35 mm [8, 25, 26]. These are generally spaced at 450 mm or 600 mm centres (cts) such that upon impact or touch the plasterboard does not excessively deflect, while matching the spacing of the joists of the floor above [8, 26]. The standard strength classification of these studs is MGP10, that is machine graded pine to meet a Young's modulus minimum of 10 GPa stiffness [27]. The typical species used for this application is radiata pine such as the case with the stiffened engineered timber walls presented in this study [8].

2.3 Various typical configurations

The proposed engineered timber wall system consists of an OSB wall panel of varying thicknesses with a number of adjacent MGP10 studs (e.g. single, double, triple) nailed and glued at 450 mm cts. The term used 'number of studs/stiffeners' refers to the number of adjacent stiffeners per set spacing such that they are singular, double or triple per 450mm cts as shown in **Figure 2** [28]**.**

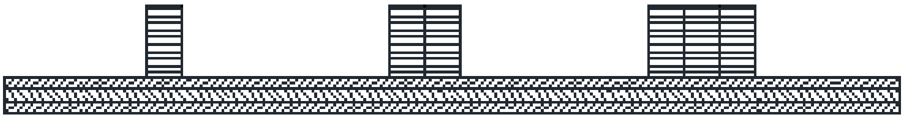


Figure 2. Number of stiffeners 1, 2 and 3 corresponds to single double or triple adjacent studs per set spacing.

The rationale behind this decision is to maximise the practical adoption of such systems through the use of common readily available materials, familiarity of methods and construction techniques along with cost effectiveness. Design curves will be presented which vary the thickness of the panel and the number of adjacent stiffeners/studs per 450 mm cts to meet a given load. Likewise, for a given load, the various wall configurations for consideration is presented. In this way manufacturers, builders and designers can quickly come to understand and grasp the capabilities and possibilities of an efficient timber solution for further investigation and verification.

3. Method

An analytical model based on the exact finite strip method (FSM) has been developed and applied to model the behaviour of stiffened engineered timber walls under axial compression. Since this is the first time such a method has been adapted to be used outside of aerospace engineering in thin laminate composite plates and into timber engineering in stiffened engineered timber walls, key validation was taken as per **Figure 3**. The finite element method (FEM) and finite strip method (FSM) are used to model a configuration which will be tested experimentally. An experimental program is run to which the results provide direct validation of the finite element model. Once validated, the FEM model is used to simulate several configurations to which a corresponding analytical model via the finite strip method will also be developed, then the results are compared. From this comparison the similarity and discrepancy between the analytical method and FEM model over a number of configurations can be seen. When good agreement is confirmed to be had, then the analytical method can be extended and applied to many more configurations. From the collection of results a set of design curves for stiffened engineered timber walls can be plotted and presented.

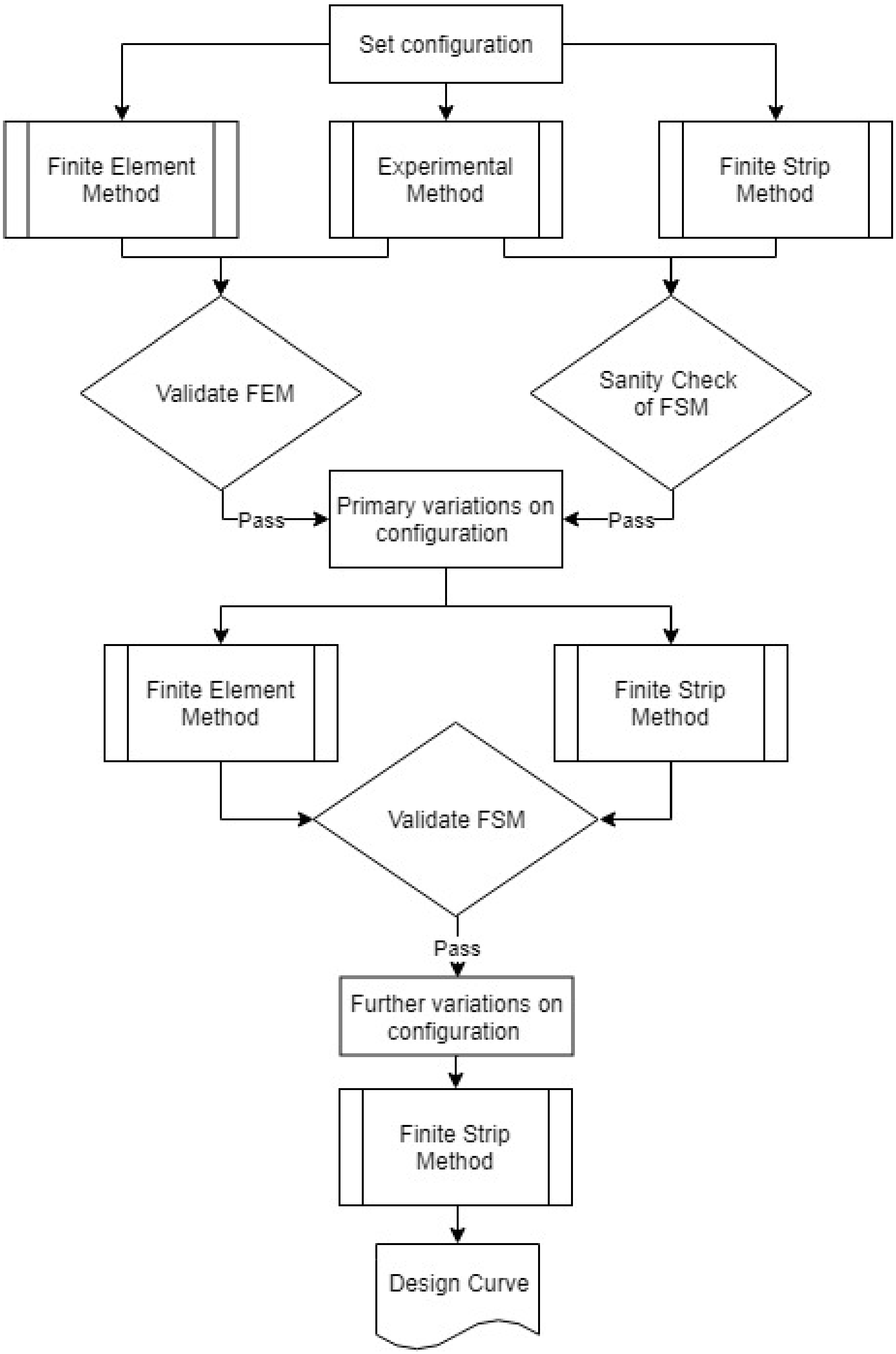


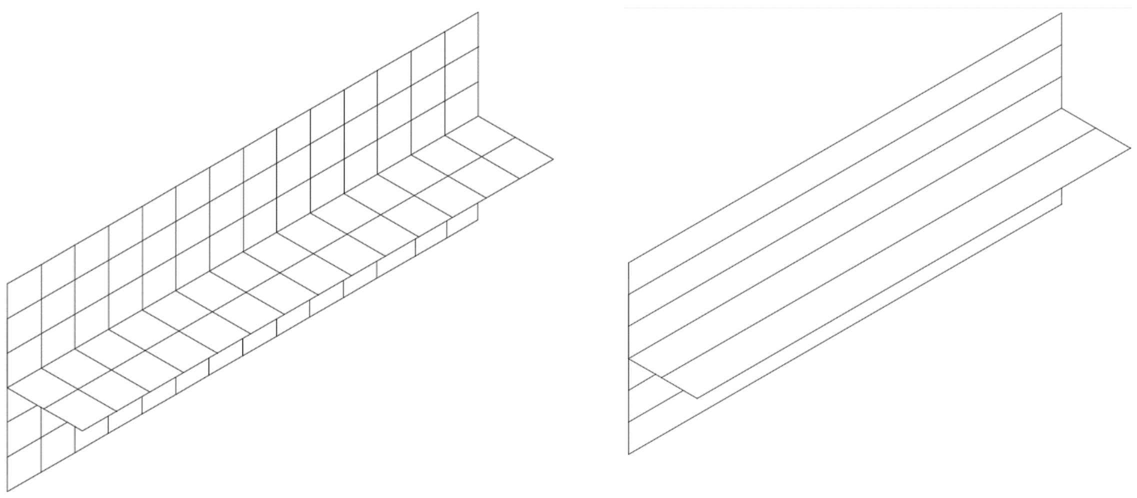
Figure 3. Method used to validate the FSM based on FEM and experimental results.

4. Analytical Modelling

4.1 Theory

4.1.1 Finite Strip Method

The finite strip method (FSM) discretizes parts in a set of continuous planes, that is in strips. This can be contrasted to the finite element method (FEM) which discretizes parts in short and small parts, that is in elements as shown in **Figure 4.**



(a) **(b)**

Figure 4. Discretization method: **(a)** FEM; **(b)** FSM.

The result of this is a vast reduction in the computational cost. However, there are significant limitations on the geometry. Prismatic structures with regular rectangular sections of uniform cross-sectional area make the ideal 'strip' and hence the appropriate adoption of this method to stiffened panels in which stiffeners and the panel can be broken into strips for calculation. The first work utilising the finite strip method on structures was published by Cheung [29]. Following this the method has been studied extensively in flat plates [30], laminated panels [31], thin walled structures [32], curved plates [33], plates with combined load cases [34], composite shell structures [35], thin composite laminate stiffened panels [36-38], and composite panels with diaphragm ends [39]. The study and application of such methods beyond theoretical study has primarily been on thin laminate composite panels used for the aerospace industry which was sponsored and developed through NASA's Langley Research Center [40-46]. The finite strip method has proved to be an accurate and computationally cost-efficient means to model plates and thin stiffened composite laminate panels. This study utilizes and develops this method well outside of the areas which it was originated and used for in the past. The major developments are that the analytical modelling utilizing the finite strip method has been adapted and carried out to an entirely different application and industry. New areas of expertise presented and demonstrated are that of stiffened engineering timber walls whose panels and stiffeners are of different materials, are comparably thick, orthotropic in nature, and have limiting tensile and compressive strength in flexure which may govern during axial loading and buckling.

4.1.2 Wittrick-Williams algorithm

The Wittrick-Williams algorithm builds upon previous work on applying the finite strip method with classical plate theory for buckling and vibration investigations which include anisotropic assemblies under combined loading. The algorithm guarantees convergence of eigenvalues (λ) when using the stiffness matrix (**K**) approach to these problems [47]. That is the critical load factor (Q) can be found for buckling of plates in addition to the frequency of vibration which can be determined via an exact solution to the differential equations which govern them. The assumptions made have been that the panels are infinitely long, that the buckling mode produces displacements which are sinusoidal in nature along the length of the panel and no shear loading is present. This has led to 'exact' solutions and hence referred to as the exact finite strip analysis [48]. The transcendental eigenvalue problem in equation 1 forms the basis of the algorithm [34, 47, 49].

$$\mathbf{K}(Q)\,\mathbf{D} = \mathbf{P} \tag{1}$$

where,

K = global stiffness matrix

Q = critical load factor

D = displacement vector

P = perturbation forces

The elements of **K**(Q) are transcendental functions of the eigenparameter Q, and its eigenvalues may be found by solving equation 2.

$$\mathbf{K}(Q)\mathbf{D} = \mathbf{0} \tag{2}$$

Then equation 3 is solved to give the number of eigenvalues J which are lower than a trial value Q* of Q.

$$J(Q^*) = J_0(Q^*) + \text{sign}\,\{\mathbf{K}(Q^*)\} \tag{3}$$

The number of natural frequencies which Q* would still exceed if controls were implemented to make **D** the displacement vector null is termed as J_0. In addition to this a sign count is involved of the negative major diagonal elements of the upper triangular matrix $\mathbf{K}^{\Delta}(Q^*)$ which is obtained from the stiffness matrix **K** when Q = Q* through Gauss elimination [49-51]. The combination of free and clamped edges where **D** = 0 allows the Wittrick-Williams algorithm to be used to calculate the critical load factor Q or that of a higher buckling mode, for any half wavelength of response, including for stiffened plate buckling problems.

4.1.3 VIPASA

Developed by Wittrick and Williams, VIPASA (Vibration and Instability of Plate Assemblies including Shear and Anisotropy) is a command line based scripted executable which utilises the finite strip method, i.e. a stiffness matrix approach to solve buckling problems through the Wittrick-Williams algorithm up to 1000 times faster than those which use the FEM approach [44]. The plate differential equations are solved on regular prismatic structures, formed by defining the nodes at which rectangular sections can be placed, offset or rotated [44].

4.1.4 VICON

Lagrangian multipliers have been incorporated to fix inaccuracies when connecting to supporting structures and hence the development of VICON, which is VIpasa with CONstraints [44]. Material strength limits such as maximum tensile or compressive strain and stress and shear capacity can be included [52]. This is a very important functionality which is taken advantage of for stiffened engineered timber walls systems. Depending on the configuration, such as thickness of panel and depth and number of stiffeners, timber tensile failure or crushing is prone in the extreme fibres under high flexure derived from P- Δ effects.

4.1.5 VICONOPT

VICONOPT is the latest iteration of the algorithm which includes VICon with OPTimization, optimization being a minimum mass-based strategy for thin plates with stiffeners [31, 53, 54].

4.2 Inputs and assumptions

VICONOPT version 1.41 has been used to execute a purpose developed code which models pinned-supported stiffened wall configurations in full scale for 900 mm (two sets of 450 mm cts) section lengths of walls. An important part of the code in ensuring the appropriate adaption of this method to predict the maximum axial load of engineered timber stiffened walls, was the development of orthotropic and anisotropic stiffness matrices for the material models along with material strength limits. These required local orientation for each part to be assigned and modified accordingly, particularly if the corresponding strip is rotated. It is assumed that the stiffener and the panel are joined with the proper use of polyurethane adhesive with nails and nail spacing such that the bond doesn’t experience any major slip before material failure in the stud stiffener. The panel is considered thick and stiff enough such that local buckling will not govern but rather the global mode shape. Both assumptions used for the analytical study were verified by experimental investigation.

4.2.1 Configuration

The general configuration is shown in **Figure 5** is akin to thin blade stiffened panels used in aircraft and the aerospace industry. However, the strips which run between nodes along the length represent a considerably thicker member although the concept is the same. Additionally, the stiffeners and panel are both non-homogenous due to timber being used as the material. This technique assumes the stiffener and the panels are either integrally connected or else connected in such a way that the connection is stronger than the material itself [55]. This assumption is justified for the use in this application due to the fact that with appropriate preparation, glue, nail spacing and manufacturing conditions the connection is stronger than either material [56].

(a)

(b)

Figure 5. General input configuration used: **(a)** profile view; **(b)** plan view.

4.2.2 Material model

The material model takes form of a stiffness matrix along with constraints on the maximum stress and strain values. The direction along the length or height of the wall is termed ‘x’ and the direction along the width of the wall is termed 'y'. An anisometric and orthotropic material model is made through the use of stiffness matrices with the Kirchhoff hypothesis and shown in equation 4. A detailed derivation of the general matrix can be found in the literature [34, 57].

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & B_{11} & B_{12} & B_{13} \\ A_{12} & A_{22} & A_{23} & B_{12} & B_{22} & B_{23} \\ A_{13} & A_{23} & A_{33} & B_{13} & B_{23} & B_{33} \\ B_{11} & B_{12} & B_{13} & D_{11} & D_{12} & D_{13} \\ B_{12} & B_{22} & B_{23} & D_{12} & D_{22} & D_{23} \\ B_{13} & B_{23} & B_{33} & D_{13} & D_{23} & D_{33} \end{bmatrix} = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ -\kappa_x \\ -\kappa_y \\ -\kappa_{xy} \end{bmatrix} \tag{4}$$

where A_{ij}, B_{ij} and D_{ij} are respectively the in-plane, coupling and out-of-plane stiffnesses, N and M are the stress and moment resultants at the ends of the wall, ε is the mid surface strain, γ is the shear strain and κ is the change in curvature. For the balanced and symmetrical case, the [**B**] matrix is the zero matrix [**0**] and $A_{12} = A_{23} = 0$. The individual remaining coefficients are calculated through equations 6 to 13 where v is Poisson's ratio, E is Young's modulus, G is the shear modulus and h the thickness, the results of which are presented in **Table 1**.

$$v_{yx} = v_{xy} \frac{E_y}{E_x} \tag{5}$$

$$A_{11} = \frac{E_x h}{1 - v_{xy} v_{y,x}} \tag{6}$$

$$A_{12} = A_{21} = \frac{v_{y,x} E_x h}{1 - v_{xy} v_{y,x}} = \frac{v_{y,x} E_y h}{1 - v_{xy} v_{y,x}} \tag{7}$$

$$A_{22} = \frac{E_y h}{1 - v_{xy} v_{y,x}} \tag{8}$$

$$A_{33} = G_{xy} h \tag{9}$$

$$D_{11} = \frac{E_x h^3}{12(1 - v_{xy} v_{y,x})} \tag{10}$$

$$D_{12} = D_{21} = \frac{v_{yx} E_x h^3}{12(1 - v_{xy} v_{y,x})} = \frac{v_{xy} E_x h^3}{12(1 - v_{xy} v_{y,x})} \tag{11}$$

$$D_{22} = \frac{E_y h^3}{12(1 - v_{xy}v_{y,x})} \quad (12)$$

$$D_{33} = \frac{G_{xy} h^3}{12} \quad (13)$$

Table 1. Stiffness coefficients calculated in MPa used for the **A** and **D** matrices

	A_{11} (MN/m)	A_{12} (MN/m)	A_{22} (MN/m)	A_{33} (MN/m)	D_{11} (MNm)	D_{12} (MNm)	D_{22} (MNm)	D_{33} (MNm)	D_{13} (MNm)	D_{23} (MNm)
Panel	144.85	52.40	154.12	47.46	0.0174	0.0063	0.0185	0.0057	0.0041	0.0042
Stiffener	352.30	6.98	21.14	23.45	0.0360	0.0007	0.0022	0.0024	0.0011	0.0010

Strength limits have been incorporated in the code developed which takes the calculated coefficients as input. The key implemented limits which were the restriction of the maximum tensile stress of the extreme fibre and likewise the maximum compressive stress to be those of the respective materials. These and the full set of orthotropic engineering constants are outlined in the FEM section 6.2 Material Properties.

4.3 Graphs / Output

4.3.1 Buckling Modes

Under axial load the engineered timber stiffened panels undergo buckling due to their slender nature. The first buckling mode is when the half-wavelength (Z) is equal to that of the height (H) of the wall and for all cases and configurations this is the most critical mode. **Figure 6** displays the first two modes of buckling, with Z = H governing. Note that the loading connections are pinned, and the direction of bow is towards the stiffeners as observed and replicated in the experimental program.

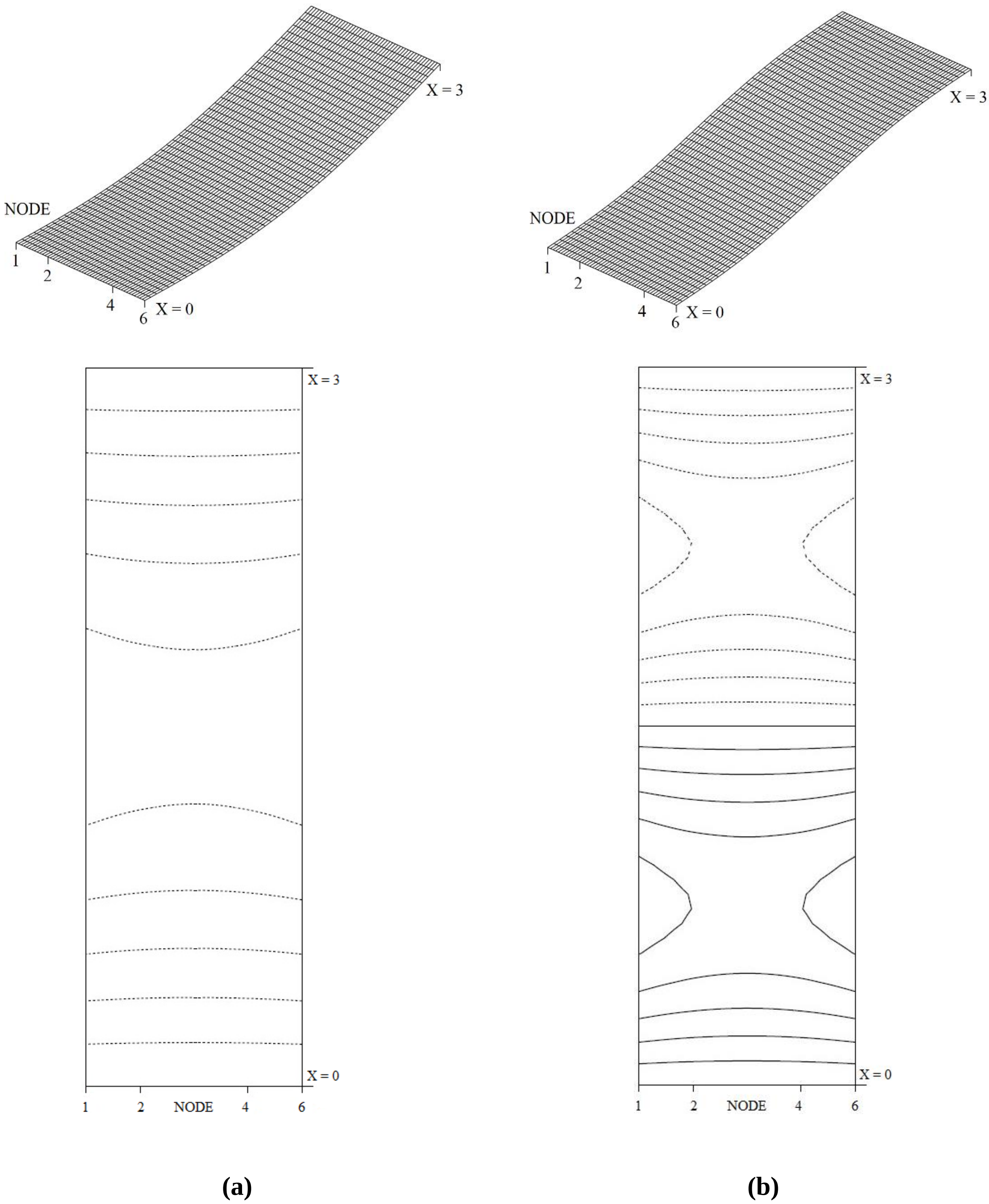


Figure 6. Buckling modes in isometric and contour drawings: **(a)** Z = H; **(b)** Z = H/2.

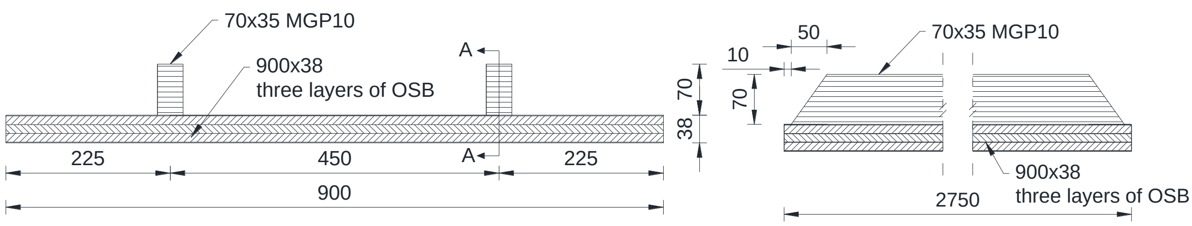
5. Experimental Verification

5.1 Purpose

The purpose of the experimental verification was to confirm assumptions such as the governing mode of buckling to be of a singular semi sinusoidal shape, mode of failure, lack of local buckling due to the thickness of the panel, direct validation of the finite element model (FEM) and sanity check for analytical results.

5.2 Experimental setup

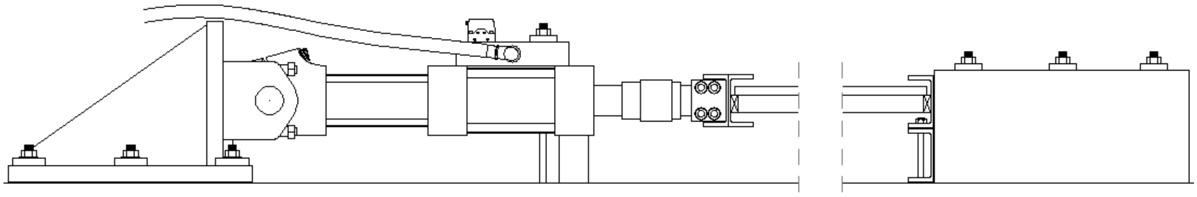
Five stiffened engineered timber walls were tested with pinned supports under axial compression. An interior style wall with 70×35 mm MGP10 studs at 450 mm cts was chosen for the experimental testing. This is due to the lower bending capacity compared to the 90×45 mm studs common for exterior walls and allowed results to be comfortably within the limits of the load cell and actuator used [28]. Specimens as shown in **Figure 7** consist of two sets of single studs at 450 mm cts due to the stability over a single stud with 450 mm of panel as the stiffness of the stiffener/stud is approximately 3 times that of the panel [28]. The studs are glued with polyurethane adhesive and nailed with glue coated pull-out resilient ring shank nails at 200 mm cts of 3.75 mm diameter and 95 mm length.



(a) **(b)**

Figure 7. Details of the engineered stiffened wall: **(a)** cross section; **(b)** section A-A.

The testing apparatus is depicted in **Figure 8** shows the orientation of the tested walls being horizontal. This is due to the increased versatility and capacity of testing specimens of greater heights in a laboratory setting [28]. Load from the actuator head was distributed through two 200 mm parallel flange channel (PFC) welded back to back, and a similar method was used on the butting end. Both sides were securely anchored to the strong floor with M24 threaded rods and to mitigate dynamic effects a displacement loading rate of 5 mm/min was used.



(a)

(b)

Figure 8. Experimental setup: **(a)** schematic; **(b)** laboratory picture.

5.3 Experimental results

The strength of the specimens under axial compression is captured by the force vs axial deflection graph as shown in **Figure 9.** Some slack was taken up in the first test which yielded the largest axial displacement. This was due to the bolted connections, but it did not affect the ultimate capacity due to the slow loading rate.

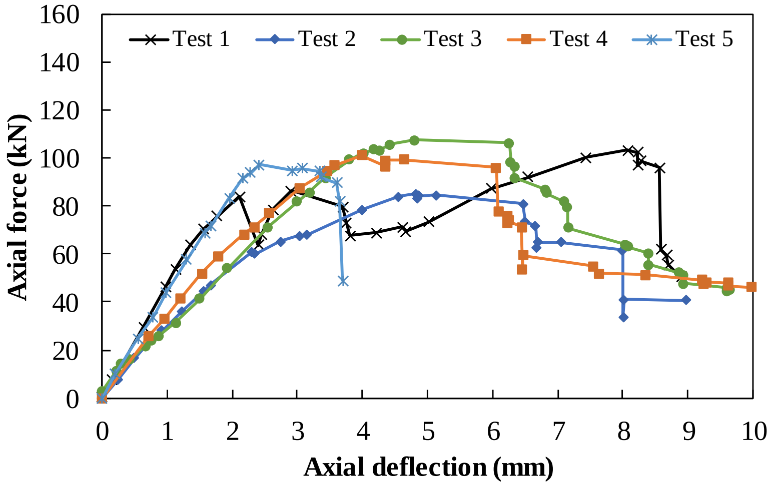


Figure 9. Axial force versus axial deflection.

The average experimental result for the ultimate capacity was found to be 99.0 kN. This occurred with a flexural failure mode which the extreme fibres failed in tension as shown in **Figure 10.** The somewhat large standard deviation of 8.5 kN for the ultimate capacity was found and expected. This is explained due to the nature of the material in question and the mode of failure being flexural. That is the studs are a sawn cut timber material which is non-homogeneous and can contain knots. If these knots happen to occur at mid-height, then a lower capacity is reached due to their reduced strength in tension.

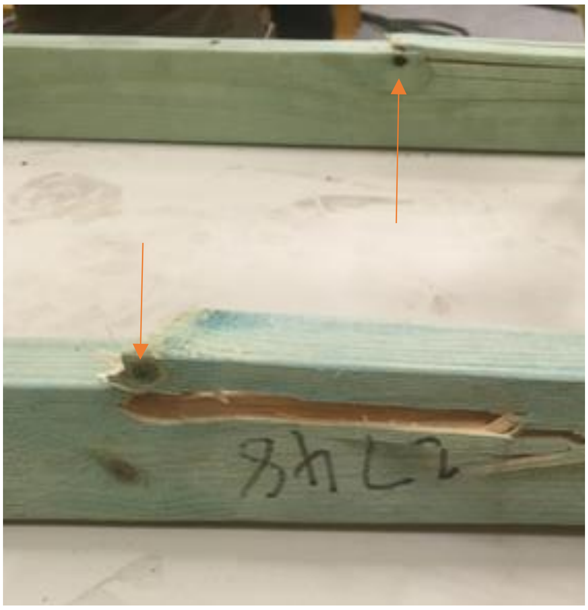


Figure 10. Flexural failure of the stud at mid-height.

6. FEA

6.1 Modelling technique

Explicit analysis in the finite element software Abaqus was conducted with locally orientated orthotropic materials defined for the OSB and MGP10 sawn cut radiata pine studs as used for the panel and the stiffener respectively [58, 59]. Each of the parts was made of an eight-node linear brick with reduced integration (C3D8R) to model them as three-dimensional elements. The connection between the panel and the stiffeners was made by gluing and nailing and thus both features were considered and modelled. A cohesive interaction was used to model the adhesion in accordance to tests as outline in section 6.2.2 Adhesive. Finally an embedded 2 -

node linear beam element (B31) was used to model the nail with the maximum allowable stress in accordance to AS/NSZ 1720.1 [60].

6.2 Material Properties

6.2.1 Timber

The set of values used for the orthotropic materials for the timber panel and stiffener are displayed in **Table 2**. Tests in accordance to the Australian Standards [61-63] and from additional studies [64-68] were used to determine the constants for OSB. For the radiata pine studs AS1720.1 was used to obtain the relevant corresponding constants [54-61].

Table 2. Elastic constants for orthotropic materials.

Element	E_1 **(MPa)**	E_2 **(MPa)**	E_3 **(MPa)**	v_{12}	v_{13}	v_{23}	G_{12} **(MPa)**	G_{13} **(MPa)**	G_{23} **(MPa)**
Stiffener (MGP10)	10000	600	600	0.33	0.50	0.40	670	670	50
Panel (OSB)	4100	2950	3450	0.31	0.32	0.34	1370	1250	130

The stiffeners are MGP10 radiata pine studs which have a parallel to the grain tensile and compressive strength limit of 7.7 MPa and 18 MPa respectively. This in accordance to AS/NZS1720 [69]. The panel has tensile and compressive strength limit of 11.9 MPa and 12.5 MPa respectively in accordance to manufacturer's specification as per AS/NZS 2269.1 and AS/NZS 4063.1 [70, 71].

6.2.2 Adhesive

Sikaflex-221, a 1C PUR (one component polyurethane) glue with a tensile capacity of 1.8 MPa was chosen due to its mainstream proven use and suitability for bonding timber [72]. The elasticity and the fracture energy needed to model the glue was obtained through scale testing on samples equivalent to the wall engineered stiffened wall panels but at 150 mm lengths to minimize the bond area. The method used is in keeping with previous studies [73-75] and attained an elasticity (E) of 7.8 MPa and fracture energy (G_f) of 0.9 N/mm which were then used in the cohesive interaction property.

6.3 FEA results

The ultimate loads obtained through FEM for the corresponding 70×35mm single, double, triple and quadruple configurations at 450 mm centres (cts) are: 106.5 kN, 153.9 kN, 193.2 kN and 232.4 kN respectively. The FEM result of 106.5 kN corresponds to the experimental configuration of single studs at 450 mm cts. Although this value is 7.6% above the average experimental result for the same configuration, due to the large variation in the experimental results, it is still within 1 standard deviation. Further to this the FEM result is below the maximum experimental result obtained of 107.5 kN. For these reasons the FEM is considered validated with the recommendation that the panels be carefully manufactured with any knots in the timber studs to be located away from centre height of the wall panel.

7. Results

7.1 Verification and Validation

Verification of the FEM is made by comparing the FEM model result of 96.21 kN for the configuration of the single 70×35 mm stud at 450 mm cts with wall height of 2.75 m with the corresponding analytical results of 95.46 kN. Additionally, the analytical result can be compared to the experimental result of 99 kN to provide validation as per Error! Reference source not found. . This allowed the justified development of additional FEM models which were then compared to analytical results. The comparison is shown in **Table 4** where it can be seen that the analytical method provides a good agreement with the FEM results, particularly for walls with a greater number of stiffeners.

Table 3. Analytical to experimental comparison and validation. 70×35 mm studs at 450 mm cts with wall height 2.75 m.

	F_u Axial load (kN)		
Stud configuration	**Experiment**	**Analytical**	**Discrepancy of Analytical Method**

Single	$\overline{x}$ = 99.0, σ = 8.5	95.47	3.6% (to experimental)

Table 4. Analytical to FEM comparison of results. 70×35 mm studs at 450 mm cts with wall height 2.75 m.

	F_u Axial load (kN)		
Stud configuration	**FEM**	**Analytical**	**Discrepancy of Analytical Method**
Double	153.9	148.74	3.35% (to FEM)
Triple	193.2	191.75	0.75% (to FEM)
Quadruple	232.4	229.40	1.29% (to FEM)

7.2 Design Curve

The validated analytical method has been used on the proposed engineered stiffened wall in an array of 81 different configurations in order to generate simple and versatile design curves for various scenarios. The term ‘number of stiffeners’ represents the variation of the number of adjacent stiffeners per the set standard spacing of 450 mm, that is, single, double and triple stud scenarios. This time a slightly larger stud dimension is used such that it is in keeping with that used in load bearing exterior wall applications rather than the slightly smaller studs used in interior walls. However, the strength category of MPG10 is the same. The design curves shown in **Figure 11** vary the thickness of the panel, number of stiffeners, height of the wall and applied load. As a result, for a desired load capacity, the optimal system configurations are given. Likewise, for a chosen configuration the maximum axial load is given.

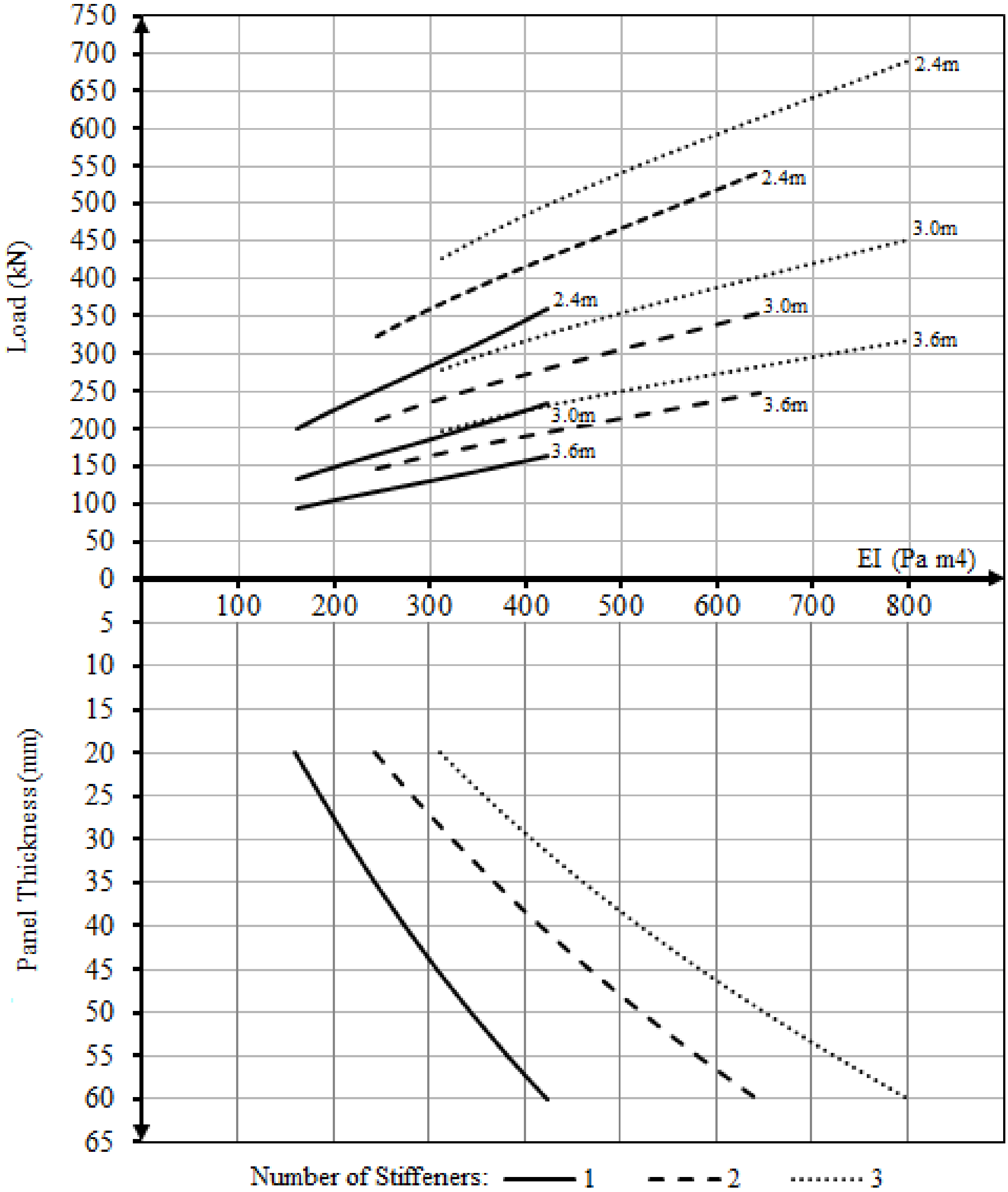


Figure 11. Design Curves for OSB wall panels stiffened with 90 by 45 mm MGP10 studs.

8. Discussion

To the best knowledge of the authors this is the first time that this method has been adapted for practical use outside of the aircraft and aerospace field and into timber engineering. The analytical approach of the exact strip method through the Wittrick-Williams algorithm has shown encouraging results for timber applications when used with the appropriate material stiffness coefficients and limits. This work shows that this analytical method can be adapted to timber engineering as a preliminary and/or alternative means to predict the axial load capacity of engineered stiffened timber panels walls. This unlocks huge time saving capabilities in the design of these structural systems as the typical computation CPU time on a standard office laptop with an Intel Core i7 6600 @2.6GHz is 1.89 seconds but can be as low as 1.65 seconds. Compare this to an hour, or for that matter a number of hours that it takes on the same machine for an FEM analysis on a stiffened timber wall system. One can conclude that computationally this is easily thousands of times faster if not tens of thousands of times faster. This starkly clear benefit for an exact analytical solution has been exploited in this study to very efficiently run 81 scenarios to create design curves for such systems.

Although the proposed stiffened engineered timber wall system utilizes stud sizes and strengths which are the most common to Australia, this sustainable system and the method can be easily adapted and applied to any specific need or any country's context [76]. For example, in America and Canada the standard stud are '2 by 4' that is 89 mm by 38 mm [77] in dimension and not the 90 mm by 45 mm used in Australia. As for the stiffness and strength of the wood, these are also slightly different too. However, this does not change the validity of lack of applicability to use this method to efficiently create design curves for very similar stiffened engineered timber wall systems specific for those regions too.

The structural and material efficiency, along with the widespread applicability for this proposed system and the simple and versatile design curves which it produces, is an important fundamental step in promoting the increase use of timber. This is due to providing an easily accessible level of understanding on the potential capabilities of such a timber system to go beyond the limits of lightweight framing without needing to resort to massive wooden CLT construction or the use of steel and concrete materials. The design curves can be read starting from many perspectives. However only a couple will be discussed to convey the broad scope and importance. Take for example an engineer who is determining the feasibility of such a timber system for a project. They may start with the required load and trace right and match with a certain height for single, double and triple stiffeners then trace down to the corresponding curve and left to obtain the required thickness of panel. An example of this is shown in **Figure 12** where beginning with a 150 kN axial load requirement for a 3.6 m high wall yields the understanding that this only needs to be achieved with either a single or double stud configuration as triple stud is not necessary as it does not intersect. From the two possibilities of number of studs, the corresponding thickness of panel is obtained, 24 mm minimum thickness for the single stud configuration and 55 mm minimum thickness for the double stud configuration. Both configurations are suitable for the input load and wall height required. These two configurations can be examined further and costed for material price and manufacturing cost to which one would be deemed more suitable. This can be done for higher or lower levels in the building too, where the result would be tapering off from ground level up in terms of lesser number of stiffeners and thinner panels hence providing greatest efficiency of material usage. This is done without disturbing the spacing to keep the load paths direct. Moreover, the floor joists above are in uniform keeping and hence leading the entire structural system beyond just the walls is highly suitable for rapid repeatable manufacturing through automated prefabrication.

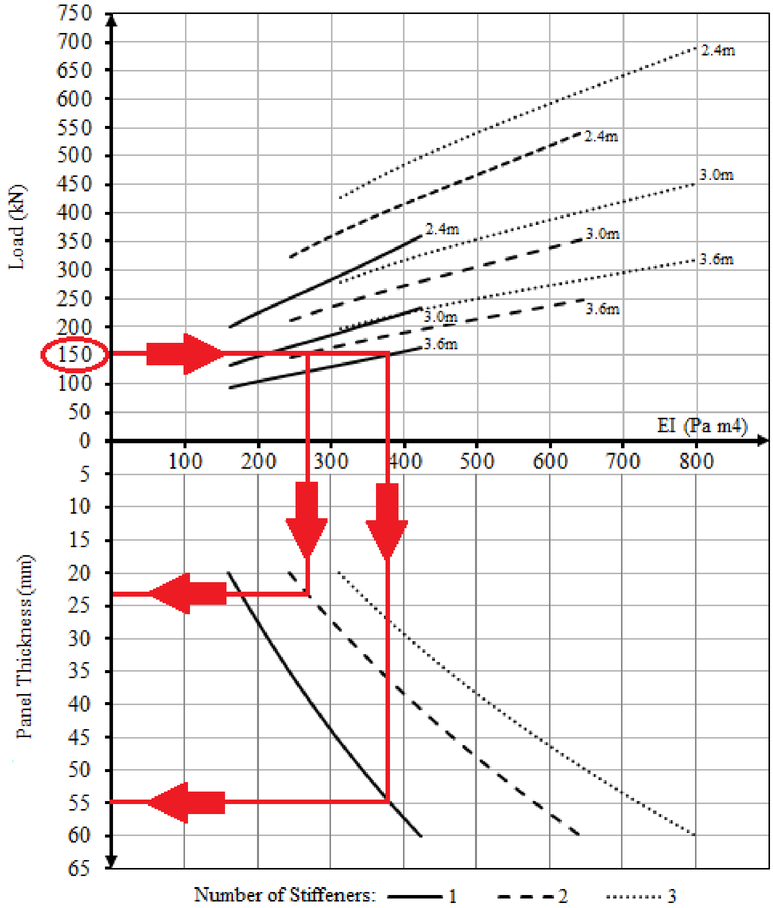


Figure 12. Possible satisfactory configurations for required load.

From another perspective, say a group affiliated with a timber panel producer or distributor wants to add value to a specific panel product by offering design information to potential buyers or possible uses of their product to increase sale and market share. This can be obtained through the reverse direction of the design curve for stiffened engineered timber wall systems. An example of this is shown in **Figure 13** where an engineered timber panel manufacturer or mass distributor produces or holds vast stock of a decently thick 40 mm OSB panel and wishes to provide general design guide to builders and potential buyers of their product on how best to use it. It is shown that for an average double stud configuration this wall panel can achieve axial loads of 195 kN, 275 kN and 420 kN for the corresponding heights of 3.6 m, 3.0 m and 2.4 m respectively.

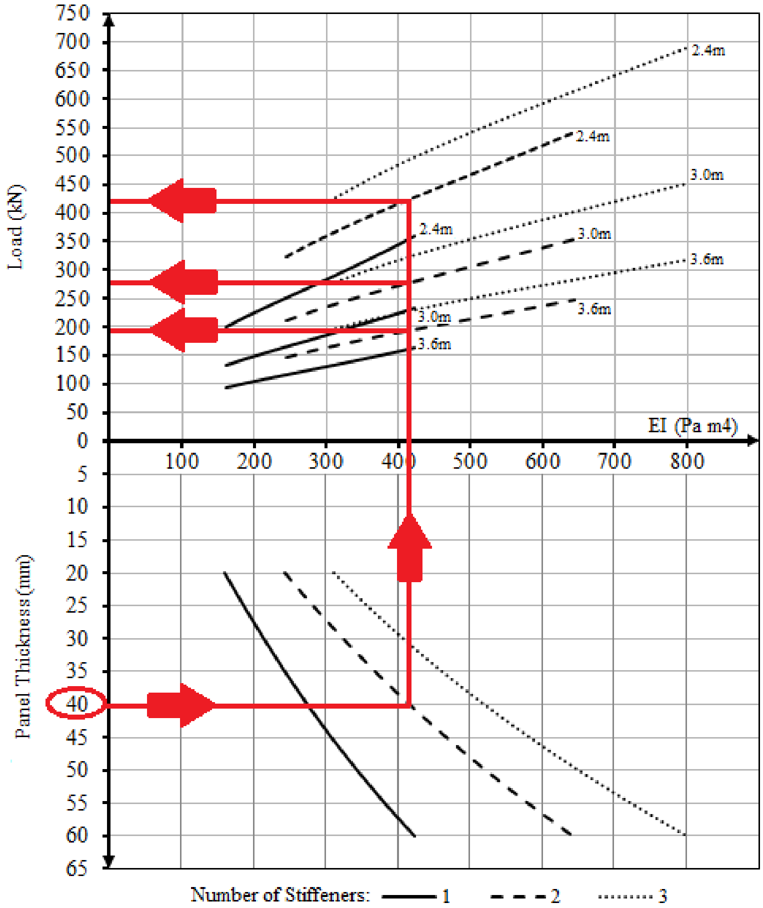


Figure 13. Load from any particular configuration.

Lastly, consider a concrete or steel engineer who is not as familiar with timber, whose company is looking into tendering for a government project which highlights the preferred use of renewable and sustainable materials. The engineer may intuitively know the approximate required EI value for the proposed building for loading and serviceability if it was to be made of concrete but has no idea whether a timber system would be suitable instead, let alone what type of configuration and how much timber would be required. To get an appreciable feel for the capacity and configuration of a suitable engineered timber wall system with equivalent EI value they can start from the horizontal axis and trace up and down for the corresponding load and configurations. An example of this is shown in **Figure 14** in which an EI of 600 Pa m^4 is chosen for a wall of 3.0 m resulting in the determination that only double and triple stud configurations are suitable. They are to have corresponding minimum panel thicknesses of 46 mm and 56 mm respectively with axial load capacities of 340 kN and 390 kN respectively.

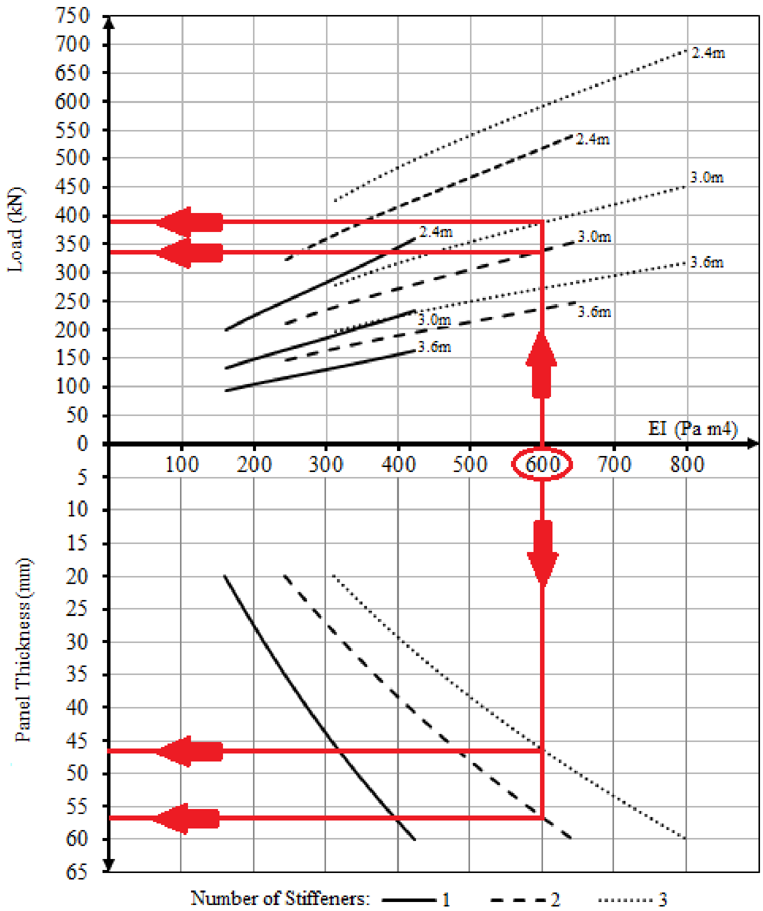


Figure 14. Load and configuration from flexural capacity.

In this way it has been shown that the proposed design curves for stiffened engineered timber walls are highly versatile and impactful. The results show the positive relationship between load capacity, the number of stiffeners per spacing and the thickness of the panel. Furthermore, for many scenarios it can be seen that for a given load there are a number of satisfactory solutions. That is, a wall with a single stiffener per 450 mm cts with a thicker panel can have the same capacity of a wall with a thinner panel but double stiffeners. This method provides the user with refined options which they can investigate further. This would include estimating costs of the options in terms of materials and manufacturing in order to decide the most suitable case for their context.

9. Conclusion

This study has introduced the mathematical modelling of stiffened engineered timber wall systems which offer a material efficient solution for timber buildings. These go beyond the limits of lightweight framing but below the capacity of massive wooden construction from CLT. A collection of corresponding simple and versatile design curves has been presented to aid in the initial feasibility analysis of the capabilities of such systems. This is applicable for numerous configurations which can be used from several perspectives to meet the specific need of the user. The results were obtained through an analytical approach via the exact finite strip method based upon the Wittrick-Williams algorithm with appropriate orthotropic material models and strength limits to capture the behaviour of the timber elements. This is the first time such a method has been adapted to be used outside of aerospace engineering in thin laminate composite plates and into timber engineering in stiffened engineered timber walls. Hence, key validation through experimental testing and finite element analysis was taken, which proved a successful result. The computational cost efficiency of this analytical method is thousands if not tens of thousands of times greater than that of corresponding FEM analysis of stiffened engineering timber walls. This allowed for 81 models to be easily executed and thus facilitated this method to be efficiently applied to any specific design need or country context to generate similar design curves. The parameters which were varied include: the thickness of the panel, number of stiffeners, height of the wall and applied load. Using the created design curves, for a desired load capacity, the optimal system configurations are given. Likewise, for a chosen configuration the allowable axial load is given.

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