Operational Flexibility of Active Distribution Networks: Definition, Quantified Calculation and Application

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Abstract: With a high penetration of intermittent distributed generators (DGs), the uncertainties in active distribution networks (ADNs) are exacerbated and further coupled in the networks. It brings enormous challenges on system operation and puts forward a higher requirement for the operational flexibility of ADNs. However, due to the secure constraints and diverse operational requirements, the controllability of controllable resources (CRs) cannot fully facilitate the flexible operation of ADNs. Operational flexibility is seen as a link between diverse operational requirements and flexible adjustment capabilities of CRs, which also represents the ability of the network to deploy its CRs to respond to the change of operation states. Under the framework of operational flexibility, multiple types of operational optimization problems can be reinterpreted, which provides a new perspective for the operation of ADNs. In this paper, the definition and region-based mathematical formulation of operational flexibility for ADNs are proposed firstly. Then the quantified calculation method of operational flexibility is proposed to represent flexibility provision (FP) and flexibility availability (FA). The application of operational flexibility is analyzed from the perspective of diverse operation and improvement of flexibility. Finally, case studies are performed on the modified IEEE 33-node system to show the effectiveness of the proposed method.

Key words: operational flexibility, controllable resource, quantified calculation, region of operational flexibility, active distribution network.

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Nomenclature

Abbreviations

DG	Distributed generator
ADN	Active distribution network
CR	Controllable resource
ESS	Energy storage system
SOP	Soft open point
FP	Flexibility provision
FA	Flexibility availability
ROF	Region of operational flexibility
RE, RSO, RDO	Regions of equipment/secure operation/diverse operation

Sets

$\mathcal{N}$	Set of nodes
$\mathcal{L}$	Set of branches

Indices

h,i,j,k,l	Indexes of nodes
ij	Index of branches
t	Indexes of time periods

$\omega_{t,i}^{\mathrm{CR,P}}$, $\omega_{t,i}^{\mathrm{CR,Q}}$	Active/reactive power outputs of controllable resources at node i during period t (MW, Mvar)
$p_{t,i}^{\mathrm{LD}}$,$q_{t,i}^{\mathrm{LD}}$	Active/reactive load demand at node i during period t (MW, Mvar)
$p_{t,i}^{\mathrm{DG}}$,$q_{t,i}^{\mathrm{DG}}$	Active/reactive power outputs of uncontrollable distributed generators at node i during period t (MW, Mvar)
$P_{t,ij}^{0}$,$Q_{t,ij}^{0}$	Initial values of active/reactive power flow of branch ij during period t (MW, Mvar)
$v_{t,i}^{0}$	Initial value of squared voltage magnitude of node i during period t (kV•kV)
$S_{t,ij}^{0}$	Initial value of power flow of branch ij during period t (MVA)
P_{t}^{flex}, Q_{t}^{flex}	Total amount of adjustable active/reactive power during period t (MW, Mvar)
A_t	Verification index of Monte Carlo simulation during period t

Parameters

$\underline{\omega}_{i}^{\mathrm{CR,P}}$, $\overline{\omega}_{i}^{\mathrm{CR,P}}$	Lower/upper limit of active power output of controllable resource at node i (MW)
$\underline{\omega}_{i}^{\mathrm{CR,Q}}$, $\overline{\omega}_{i}^{\mathrm{CR,Q}}$	Lower/upper limit of reactive power output of controllable resource at node i (Mvar)
v_0	Squared voltage magnitude of the source node (kV•kV)
r_{ij},x_{ij}	Resistance/reactance of branch ij (Ω, Ω)
$\overline{S}_{ij}$	Upper limit of transmission capacity of branch ij (MVA)
$\underline{v}$, $\overline{v}$	Upper/lower limit of node voltage (kV, kV)
$\overline{p}_{t}^{\mathrm{loss}}$	Upper limit of power loss during period t (MWh)
$\overline{v}_{t}^{\mathrm{fluc}}$	Upper limit of sum of voltage fluctuation during period t (p.u.)

Variables

$C_{t,i}$	Equivalent storage capacity at node i during period t (MVA)
$\dot{e}_{t,i}$	Change of equivalent state of charge at node i during period t
$\zeta_{t,i}^{\mathrm{UD,P}}$, $\zeta_{t,i}^{\mathrm{UD,Q}}$	Active/reactive power outputs of uncontrollable devices at node i during period t (MW, Mvar)
$\underline{S}_{t,i}^{\mathrm{pene}}$	Lower limit of DG penetration at node i during period t (MWh)
$\underline{\zeta}_{t,i}^{\mathrm{UD,P}}$, $\overline{\zeta}_{t,i}^{\mathrm{UD,P}}$	Lower/upper limit of active power output of uncontrollable device at node i during period t (MW)
$\underline{\zeta}_{t,i}^{\mathrm{UD,Q}}$, $\overline{\zeta}_{t,i}^{\mathrm{UD,Q}}$	Lower/upper limit of reactive power output of uncontrollable device at node i during period t (Mvar)

1. Introduction

With a high penetration of intermittent distributed generators (DGs) in active distribution networks (ADNs) [1], the imbalance of power supply and demand is aggravated. The uncertainty and volatility are further exacerbated, which puts forward higher requirements for the flexible operation of ADNs [2]. To satisfy the diverse operational requirements of ADNs, multiple types of controllable resources (CRs) are integrated into the network. However, the operation strategies of CRs are limited by secure constraints and various operational requirements, so that the regulating capacity of CRs cannot be adequately utilized [3], [4], [5]. Thus, the controllability of CRs cannot fully facilitate the flexible operation of ADNs. Operational flexibility provides a link between diverse operational requirements and the flexible adjustment capacities of CRs. Multiple types of operational optimization problems can be reinterpreted under the framework of operational flexibility.

Previous studies have investigated the definitions of operational flexibility. Ref. [6] defined operational flexibility as the ability to maintain the power balance of system over a time period. In [7], it was defined as the insufficiency of adjusting ability, which reflects the imbalance of supply and demand in the system. However, the definitions in [6] and [7] are only suitable for the scenario when the power imbalance occurs. Ref. [8] proposed a definition of operational flexibility as the regulating capacity and ramping capacity that the network can provide. The authors of [9] and [10] defined it as the distance from the actual operating point to the flexible boundaries. The definitions in [8], [9] and [10] only apply to transmission networks, which are not appropriate for ADNs. The existing works mainly focus on the regulating ability of active power in the network, while the regulating ability of reactive power is ignored. In distribution networks, the power flow of the network can be regulated through the integration of reactive power compensation devices. Therefore, the definition of operational flexibility needs to reflect the regulating ability of both active and reactive power. Based on the analysis above, there is still a lack of definition of operational flexibility for flexible operation scenarios in ADNs.

Several flexibility metrics were adopted for quantifying operational flexibility. For the flexible regulation of units, the lack of ramping probability and insufficient ramping resource expectation were used to quantify flexibility in [11] and [12]. The regulating capacity and ramping capacity that the system could provide were adopted in [13]. Ref. [14] and [15] used the maximum of uncertainty that the distribution network can accommodate to quantify operational flexibility. The maximum penetration of DGs was adopted in [16] and [17]. These metrics are mainly used in quantifying the insufficiency of flexibility and long-term expansion planning of DGs. For quantified analysis in flexible scenarios, there is still a lack of quantified calculation method for the amount, change and improvement of operational flexibility in ADNs.

Besides, the visualization of operational flexibility is also vital for practical application. Several parameters were used to construct a multi-dimensional visualization for flexibility in [18]. Considering the ramping rate and ramping duration of controllable power plant, the authors in [19] proposed a three-dimensional probability box for visualizing operational flexibility. By taking the intersection of available operation strategies and flexible boundaries, the match of flexible resource and demand can be visualized in high-dimensional space in [20] and [21]. However, the existing methods only focus on the regulating capability of single CR. It is not effective when multiple CRs are coordinated in ADNs, since it is hard to visualize the operational flexibility in a high-dimensional space.

The thought of *region* is widely used in operational optimization of transmission and distribution networks, which can also be used in the process of quantification and visualization of operational flexibility. Ref. [22] proposed a static voltage security region for distribution networks considering the regulating capacities of controllable DGs and regulation devices, which can be used in voltage control of distribution networks. Further considering the uncertainties of power injections, the steady-state security regions were proposed in [23]. Based on region analysis, regions of operational flexibility (ROFs) are proposed in this paper, which can be seen as the mathematical formulation of operational flexibility in a multi-dimensional state space. Because of the nonlinearity in ADNs, it is hard to obtain the analytic representation of operational flexibility. Therefore, the numerical solutions are solved by Monte Carlo simulation [24]. To intuitively visualize the state of operational flexibility in different operation scenarios, *plane of operational flexibility* is proposed in this paper. ROFs in different scenarios are unidirectional mapped to the plane, where the operational flexibility of ADNs can be calculated and visualized at the same time.

Furthermore, from the perspective of operational flexibility, multiple types of operational optimization problems can be reinterpreted under a unified framework. Operational requirements in ADNs include economic operation [25], improving voltage profile [26], [27], accommodating DG integration [13], etc. These requirements were regarded as different objectives previously. In this paper, operational flexibility is seen as a link between diverse operational requirements and the flexible adjustment capabilities of CRs. Taking the improvement of voltage profile as an example, the original operational flexibility to reduce network loss can be released and facilitate to reduce the voltage deviation. Specific requirements of ADNs are established as regions of diverse operation (RDO) in this paper. By taking intersection of regions of diverse operation and initial ROFs, it can be estimated whether the operational flexibility of ADNs can meet the diverse demand, which is the foundation for further quantitative analysis of operational flexibility.

To reinterpret the diverse operational problems from the perspective of operational flexibility, a unified framework for analyzing operational flexibility is proposed, including definition, quantified calculation and application. The major contributions are summarized as follows:

1) The definition of operational flexibility of ADNs for flexible operation scenarios is proposed. Flexibility provision (FP) and flexibility availability (FA) are introduced to represent the adjustable regulating capacity of ADNs. Region based mathematical formulation of operational flexibility is proposed, which can be expressed as ROF.

2) The quantified calculation method of operational flexibility of ADNs is proposed and the numerical solutions of operational flexibility are solved based on Monte Carlo simulation, and then are mapped to a two-dimensional plane. This plane of operational flexibility is proposed to intuitively visualize whether the state of operational flexibility can meet diverse operation requirements.

3) Applications of operational flexibility are elaborated under a unified framework of quantifying FP and FA in different operation scenarios. From the prospective of operational flexibility, diverse operation issues are reinterpreted. Furthermore, the influence factors of operational flexibility are analyzed.

The rest of this paper is organized as follows. In Section 2, the concept of operational flexibility of ADN is defined. The quantified calculation and visualization methods of operational flexibility are discussed respectively in Section 3. The applications of operational flexibility to address different issues are proposed in Section 4. Then case studies are given in Section 5 to quantify and visualize FP and FA of ADNs on the modified IEEE 33-node system. Finally, conclusions are drawn in Section 6.

2. Definition of operational flexibility for ADNs

Operational flexibility reflects the network to adapt to different operating states. For flexible operation scenarios in ADNs, the concept of operational flexibility is defined and the region-based mathematical formulation of operational flexibility is proposed in this section.

2.1. Definition of Operational Flexibility

For flexible operation scenarios in ADNs, operational flexibility can be seen as a link between diverse operational requirements and the flexible adjustment capabilities of CRs. The amount of operational flexibility depends on the regulating ability of CRs integrated into networks. Under different operation scenarios, the operation strategies of CRs are limited by the secure constraints and various operational requirements. Therefore, two concepts are introduced in detail to represent the amount of operational flexibility under different operation scenarios in ADNs.

(1) Flexibility provision

Flexibility provision (FP) is defined as the regulating capacity that ADNs can provide oriented to diverse operational requirements while satisfying secure constraints, which means all the operation constraints on power flow, node voltage and branch current are satisfied simultaneously.

(2) Flexibility availability

Flexibility availability (FA) is defined as the available regulating capacity that ADNs can further provide while secure constraints and diverse operational requirements are both satisfied. Note that FA is an essential portion of FP, in which diverse operational requirements are further considered. When FA is close to zero, there is a high possibility of insecure operation conditions.

In conclusion, operational flexibility of ADN is defined as the regulating ability of ADNs by the integration of CRs in order to meet diverse operational requirements.

2.2. Region-based Mathematical Formulation of Operational Flexibility

Because of the nonlinearity in ADNs, it is hard to obtain the explicit expressions of operation state and operation strategies of CRs. Linearization is seen as a compromise to solve nonlinear problems, if the fluctuation around the current operating state is small enough, we consider the following supposed conditions:

Assumption 1: The superposition theorem is satisfied by using linearized DistFlow model [4].

If the fluctuation around the current operating state is small enough, the accuracy of linearized power flow model can be guaranteed [28], [29]. By linearized DistFlow model, the explicit mathematical expressions of the constraints between operation strategies of CRs and operational requirements of ADNs can be established. If the operating state deviates from the initial operation point, re-linearization is needed.

Assumption 2: Three-phase imbalance in ADNs is ignored.

This paper focuses on the medium voltage distribution networks, in which the power flow is generally assumed to be balanced. When three-phase imbalance is considered in the network, the three-phase imbalanced DistFlow model [30] can be formulated. Thus, the proposed method is still applicable.

Assumption 3: There is no loop in the feeder.

Since distribution networks adopt closed-loop design and open-loop operation, it is assumed to be no loop in the feeder under normal operation scenarios. When the feeders have loops in ADNs, the difference of phase angle is approximate to 0 after traversing all the nodes in the loop. The DistFlow model can be modified by further adding loop-operation constraint [31]. Thus, the proposed method is still applicable.

Based on the above assumptions of distribution networks, the explicit region-based mathematical formulation of operational flexibility can be established.

(1) State space and state variables

In ADNs, outputs of uncontrollable devices (UDs) at nodes are predetermined but the operation strategies of CRs can be regulated to meet the several operational requirements. Based on power node model [18], Eq. (1) can be seen as the mathematical expression of operational flexibility of node i. (1.a) and (1.b) represent the active and reactive power exchange between devices and node i. (1.c) and (1.d) denote that the outputs of CRs are constrained by the capacities of equipment. When an energy storage system is connected to node, the equivalent storage capacity $C_{t,i} > 0$ in (1.e). $\dot{e}_{t,i}$ represents the change of equivalent state of charge at node i during period t.

$$C_{t,i}\dot{e}_{t,i} = \zeta_{t,i}^{\mathrm{UD,P}} + \omega_{t,i}^{\mathrm{CR,P}} - p_{t,i}^{\mathrm{LD}} + p_{t,i}^{\mathrm{DG}} \tag{1.a}$$

$$0 = \zeta_{t,i}^{\mathrm{UD,Q}} + \omega_{t,i}^{\mathrm{CR,Q}} - q_{t,i}^{\mathrm{LD}} + q_{t,i}^{\mathrm{DG}} \tag{1.b}$$

$$\underline{\omega}_{i}^{\mathrm{CR,P}} \le \omega_{t,i}^{\mathrm{CR,P}} \le \overline{\omega}_{i}^{\mathrm{CR,P}} \tag{1.c}$$

$$\underline{\omega}_{i}^{\mathrm{CR,Q}} \le \omega_{t,i}^{\mathrm{CR,Q}} \le \overline{\omega}_{i}^{\mathrm{CR,Q}} \tag{1.d}$$

$$0 \le C_{t,i} \tag{1.e}$$

$$-1 \le \dot{e}_{t,i} \le 1 \tag{1.f}$$

In this paper, the operation strategies of CRs are set as the state variables. The multi-dimensional state space consists of all the state variables. The dimension of state space is equal to the number of operation strategies.

(2) Regions of operational flexibility

The thought of *region* is widely used in operational optimization of transmission and distribution networks [32], [33]. Based on the methodology of region, the mathematical formulation of operational flexibility can be obtained, which is defined as the region of operational flexibility

(ROF). The internal operation points of ROF can satisfy the requirements of flexible operation while the external operation points do not meet the operational requirements. When ROF is not empty, it means that CRs can find available operation strategies to keep the network flexible. Namely, the network has enough regulation capacity. Otherwise, when ROF is an empty set, it indicates that the network does not have enough regulation ability to adjust the operation state, which means the network is not flexible. ROF can be clarified as follows:

The basis of operational flexibility is from the CRs integrated into the networks. The operation strategies of CRs are constrained by the capacities of equipment, which can be defined as region of equipment (RE). The region of equipment of node k in the distribution networks can be expressed as (2):

$$\begin{gathered} \Phi_{\mathrm{RE}} = \{(\omega_{t,k}^{\mathrm{CR,P}}, \dot{e}_{t,k}, \omega_{t,k}^{\mathrm{CR,Q}})| \\ \underline{\omega}_{k}^{\mathrm{CR,P}} \leq \omega_{t,k}^{\mathrm{CR,P}} \leq \overline{\omega}_{k}^{\mathrm{CR,P}}, \underline{\omega}_{k}^{\mathrm{CR,Q}} \leq \omega_{t,k}^{\mathrm{CR,Q}} \leq \overline{\omega}_{k}^{\mathrm{CR,Q}}, -1 \leq \dot{e}_{t,k} \leq 1\} \end{gathered} \tag{2}$$

The region of secure operation (RSO) represents the safe operation region, which means there is no line overload or voltage deviation. The overall line resistance R_{ij} and line reactance X_{ij} are introduced:

$$\begin{aligned} R_{ij} &:= \textstyle\sum_{(h,k)\epsilon\mathcal{L}_i \cap \mathcal{L}_j} r_{hk} \\ X_{ij} &:= \textstyle\sum_{(h,k)\epsilon\mathcal{L}_i \cap \mathcal{L}_j} x_{hk} \end{aligned} \tag{3}$$

$\mathcal{L}_i \subseteq \mathcal{L}$ is defined as a line set of the unique path from node i to the source node. The region of secure operation of node k in the distribution networks can be expressed as (4):

$$\Phi_{\mathrm{RSO}} = \{(\omega_{t,k}^{\mathrm{CR,P}}, \dot{e}_{t,k}, \omega_{t,k}^{\mathrm{CR,Q}})| \ \ \mathrm{s.t.}\,(5)\} \tag{4}$$

The secure constraints of ADNs include the line power transmission constraints and node voltage constrains. The power constraint of line ij and voltage constraint of node i can be expressed as (5). $\beta(j)$ represents a set of all son nodes including node j itself, namely $\beta(j) = \{i | \mathcal{L}_j \subseteq \mathcal{L}_i\}$.

The initial operation point of ADNs is solved by power flow calculation. The initial values of power flow and node voltage including $P_{t,ij}^0$, $Q_{t,ij}^0$ and $v_{t,i}^0$ can be obtained.

$$\begin{aligned}&\left[P_{t,ij}^0-\sum_{k\epsilon\beta(j)}\left(\omega_{t,k}^{\mathrm{CR,P}}-C_{t,i}\dot{e}_{t,k}\right)\right]^2+\left(Q_{t,ij}^0-\sum_{k\epsilon\beta(j)}\omega_{t,k}^{\mathrm{CR,Q}}\right)^2\le\overline{S}_{ij}^2\\&\underline{v}^2\le v_{t,i}^0+2\sum_{k\in\mathcal{N}}\left(\omega_{t,k}^{\mathrm{CR,P}}-C_{t,i}\dot{e}_{t,k}\right)R_{ik}+2\sum_{k\in\mathcal{N}}\left(\omega_{t,k}^{\mathrm{CR,Q}}\right)X_{ik}\le\overline{v}^2\end{aligned}\tag{5}$$

In ADNs, region of equipment and secure operation are essential ROFs. Further considering the optional requirements, the region of diverse operation (RDO) is established, which is the relation constraints of diverse operational requirements and operation strategies of CRs.

1) Reducing network losses

The available operation points which satisfy the upper limit of power loss $\overline{p}_t^{\mathrm{loss}}$ are shown in (6).

$$\begin{aligned}&\Phi_{\mathrm{RDO}}=\{\left(\omega_{t,k}^{\mathrm{CR,P}},\dot{e}_{t,k},\omega_{t,k}^{\mathrm{CR,Q}}\right)|\\&\sum_{ij}r_{ij}*\left[\left(S_{t,ij}^0\right)^2-2P_{t,ij}^0\sum_{k\epsilon\beta(j)}\left(\omega_{t,k}^{\mathrm{CR,P}}-C_{t,k}\dot{e}_{t,k}\right)-2Q_{t,ij}^0\sum_{k\epsilon\beta(j)}\omega_{t,k}^{\mathrm{CR,Q}}\right]/v_{t,i}^0\le\overline{p}_t^{\mathrm{loss}}\}\end{aligned}\tag{6}$$

2) Improving voltage profile

The available operation points which satisfy the upper limit of sum of voltage fluctuation $\overline{v}_t^{\mathrm{fluc}}$ are shown in (7).

$$\begin{aligned}&\Phi_{\mathrm{RDO}}=\{\left(\omega_{t,k}^{\mathrm{CR,P}},\dot{e}_{t,k},\omega_{t,k}^{\mathrm{CR,Q}}\right)|\\&\sum_k\left|1-\left(2\sum_{j\in\mathcal{N}}\left(\omega_{t,k}^{\mathrm{CR,P}}-C_{t,k}\dot{e}_{t,k}\right)R_{kj}+2\sum_{j\in\mathcal{N}}\omega_{t,k}^{\mathrm{CR,Q}}X_{kj}+v_{t,k}^0\right)^{0.5}\right|\le\overline{v}_t^{\mathrm{fluc}}\}\end{aligned}\tag{7}$$

3) Accommodating DG integration

The available operation points which satisfy the lower limit of DG penetration $\underline{S}_{t,l}^{\mathrm{pene}}$ are shown in (8).

$$\begin{aligned}&\Phi_{\mathrm{RDO}}=\{\left(\omega_{t,k}^{\mathrm{CR,P}},\dot{e}_{t,k},\omega_{t,k}^{\mathrm{CR,Q}}\right)|\\&\left(P_{t,kj}^0-\sum_{k\epsilon\beta(j)}\left(\omega_{t,k}^{\mathrm{CR,P}}-C_{t,k}\dot{e}_{t,k}\right)-\sum_{l\epsilon\beta(j)}\underline{S}_{t,l}^{\mathrm{pene}}\right)^2+\left(Q_{t,kj}^0-\sum_{k\epsilon\beta(j)}\omega_{t,k}^{\mathrm{CR,Q}}\right)^2\le\overline{S}_{kj}^2,\\&2\sum_{j\in\mathcal{N}}\left(\omega_{t,k}^{\mathrm{CR,P}}-C_{t,k}\dot{e}_{t,k}\right)R_{kj}+2\sum_{j\in\mathcal{N}}\left(\omega_{k}^{\mathrm{CR,Q}}\right)X_{kj}+2\sum_{j\in\mathcal{N}}\left(\underline{S}_{t,l}^{\mathrm{pene}}\right)R_{lj}+v_{t,k}^0\le\overline{v}^2\}\end{aligned}\tag{8}$$

4) Adapting to uncertainty

Further considering the uncertainties in ADNs, the real load demand and DG outputs deviate from the predicted operation points with fluctuations, which can be seen in Eq. (9).

$$\begin{gathered}\Phi_{\mathrm{RDO}}=\{(\omega_{t,k}^{\mathrm{CR,P}},\dot{e}_{t,k},\omega_{t,k}^{\mathrm{CR,Q}})| \\ \mathrm{s.t.}\,(1),p_{t,i}^{\mathrm{LD}}=(1\pm\alpha_1)p_{t,i}^{\mathrm{LD,0}},q_{t,i}^{\mathrm{LD}}=(1\pm\alpha_3)q_{t,i}^{\mathrm{LD,0}}, \\ p_{t,i}^{\mathrm{DG}}=(1\pm\alpha_2)p_{t,i}^{\mathrm{DG,0}},q_{t,i}^{\mathrm{DG}}=(1\pm\alpha_4)q_{t,i}^{\mathrm{DG,0}}\}\end{gathered} \tag{9}$$

In Eq. (9), $p_{t,i}^{\mathrm{LD,0}}$, $p_{t,i}^{\mathrm{DG,0}}$, $q_{t,i}^{\mathrm{LD,0}}$and $q_{t,i}^{\mathrm{DG,0}}$ represent the predicted load demands and uncontrollable DG outputs. To represent the magnitudes of fluctuations, α_1, α_2, α_3, α_4 are introduced.

3. Quantified calculation and visualization methods of operational flexibility

Based on the analysis above, the operational flexibility of ADNs can be mathematically defined. However, the amount, change and improvement of operational flexibility need to be quantified and visualized. A unified framework for quantified calculation and visualization of operational flexibility is proposed in this section.

3.1. Quantified Calculation and Visualization Method of Operational Flexibility

Because of the nonlinearity in ADNs, it is hard to analytical represent operational flexibility. Therefore, the operation points of CRs within different regions are solved by Monte Carlo simulation, which can be seen as numerical solutions of operational flexibility.

(1) Plane of operational flexibility

Since the active power outputs and reactive power outputs of CRs are both controllable, the number of dimensions in the state space is equal to the number of operation strategies of CRs. If too many CRs are integrated into ADNs, it is hard to realize the visualization and calculation of operational flexibility in multi-dimensional state space.

Based on the analysis above, the operational flexibility is calculated and visualized in a two-dimensional plane. In this paper, to obtain the adjustable regulating capacity in ADNs, a 2D plane is used to quantify the operational flexibility of the whole distribution network, which is called the *plane of operational flexibility*. The x-axis of the plane stands for active power while y-axis stands for reactive power. The area in plane of operational flexibility represents the total amount of adjustable regulating capacity that the distribution network can provide. The operational flexibility can be quantified in plane of operational flexibility by unidirectional mapping:

$$
\begin{aligned}
P_t^{\text{flex}} &= \sum_{i\in\mathcal{N}} \omega_{t,i}^{\text{CR,P}} \\
Q_t^{\text{flex}} &= \sum_{i\in\mathcal{N}} \omega_{t,i}^{\text{CR,Q}}
\end{aligned}
\tag{10}
$$

In the plane of operational flexibility, the coordinate (P_t^{flex}, Q_t^{flex}) of each point stands for the sum of active and reactive regulating power that CRs within the network can provide. The area of points in the plane of operational flexibility can be used to quantify the total amount of operational flexibility. Furthermore, the variety of area in the plane of operational flexibility represents the change of flexibility.

(2) Calculation method for FP and FA

Through the union and intersection between ROFs, the operational flexibility under different operational requirements can be obtained. FP refers to the regulating capacity under secure operation conditions, which can be quantified by intersection of region of equipment Φ_{RE} and region of secure operation Φ_{RSO}, as shown in Eq. (11). FA can be seen as the available regulating capacity after satisfying the operational requirements, which also can be seen as the intersection of region of equipment Φ_{RE}, region of secure operation Φ_{RSO} and region of diverse operation Φ_{RDO}, as shown in Eq. (12).

$$\Phi_{\text{FP}} = \Phi_{\text{RE}} \cap \Phi_{\text{RSO}} \tag{11}$$

$$\Phi_{\text{FA}} = \Phi_{\text{RDO}} \cap \Phi_{\text{FP}} = \Phi_{\text{RE}} \cap \Phi_{\text{RSO}} \cap \Phi_{\text{RDO}} \tag{12}$$

Additionally, the flexibility improvement (FI) of ADNs can be quantified by subtraction of ROFs, as shown in Eq. (13):

$$\Phi_{\text{FI}} = \Phi'_{\text{FP}} - \Phi_{\text{FP}} \tag{13}$$

3.2. Verification of the Proposed Method

In this paper, the operational flexibility of ADNs is directly quantified through the calculation with linearized DistFlow model and Monte Carlo simulation. Thus, the deviation of the obtained results is mainly from the linearized model and Monte Carlo sampling.

Linearized DistFlow model and Monte Carlo simulation can meet the practical requirement with acceptable accuracy. If the fluctuation around the current operating state is small enough, the accuracy of linearized DistFlow model can be guaranteed, which has been verified in [28] and [29]. If the sampling number of Monte Carlo simulation is large enough, the accuracy can also be guaranteed [34], [35].

To further verify the accuracy of the proposed method, a verification index has been elaborated in this paper, as can be seen in Eq. (14).

$$\begin{aligned} A_t = 1 - \max\ \{&\left|\overline{P}_t^{\mathrm{flex}} - P_t^{\max}\right|/|P_t^{\max}|, \quad \left|\underline{P}_t^{\mathrm{flex}} - P_t^{\min}\right|/|P_t^{\min}|, \\ &\left|\overline{Q}_t^{\mathrm{flex}} - Q_t^{\max}\right|/|Q_t^{\max}|, \quad \left|\underline{Q}_t^{\mathrm{flex}} - Q_t^{\min}\right|/|Q_t^{\min}|\} \end{aligned} \tag{14}$$

In Eq. (14), A_t represents the accuracy of linearization, which can be expressed as one minus the maximum value of relative error between the actual and accurate values. Four operation points at the boundary of the region are chosen to validate the accuracy of linearization. The actual values of maximum and minimum outputs of controllable resources including $\overline{P}_t^{\mathrm{flex}}$, $\underline{P}_t^{\mathrm{flex}}$, $\overline{Q}_t^{\mathrm{flex}}$ and $\underline{Q}_t^{\mathrm{flex}}$ are obtained by Monte Carlo simulation. The accurate values of maximum and minimum outputs of controllable resources including $P_t^{\max}$, $P_t^{\min}$, $Q_t^{\max}$ and $Q_t^{\min}$ are solved by Internal Point OPTimizer (IPOPT), which is implemented based on the primal-dual interior point method. The lower limit of A_t is set according to different accuracy requirements. When the limit is reached, re-linearization at a new operation point is needed and A_t is recalculated.

3.3. Unified Framework of Quantifying and Visualizing Operational Flexibility of ADNs

Based on the analysis above, the process of quantified calculating operational flexibility for ADNs is expressed as follows and shown in Fig. 1. From the perspective of operational flexibility, multiple types of operational optimization problems can be reinterpreted under the proposed framework.

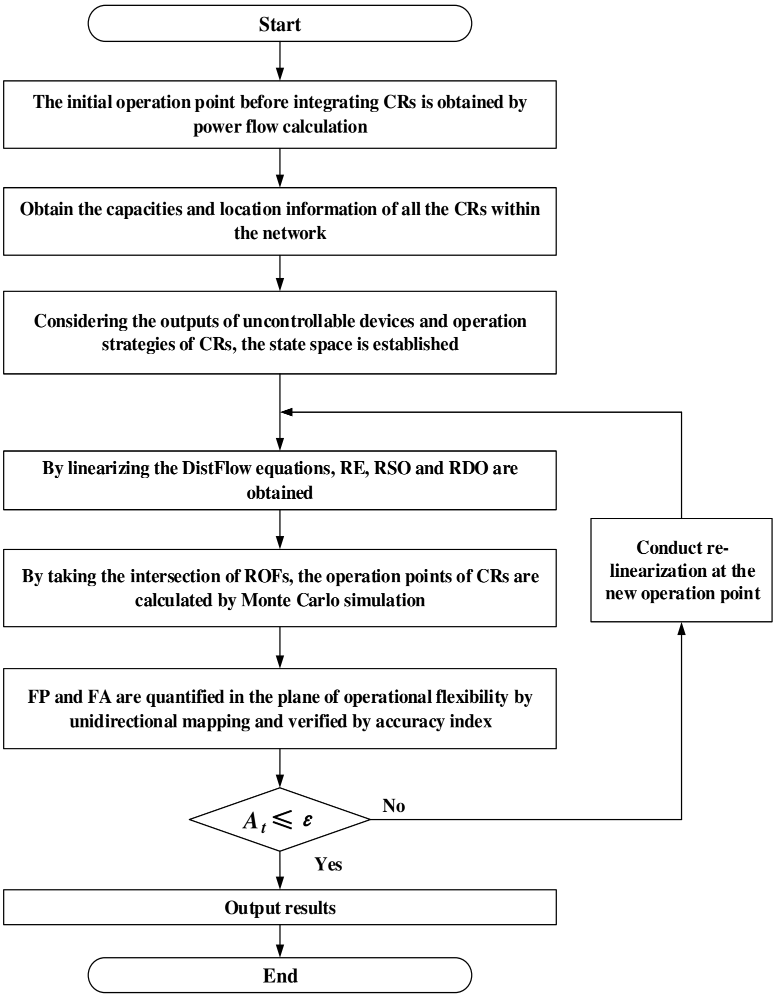

Fig. 1 Unified framework of quantifying and visualizing operational flexibility of ADNs.

1) The initial operation point before integrating CRs is obtained by power flow calculation.

2) Capacities and location information of all the CRs within the network are obtained.

3) Considering the outputs of uncontrollable devices and operation strategies of CRs, the state space for quantifying and visualizing operational flexibility is established.

4) The linearization is conducted around the above initial operation point. The mathematical expressions of regions of operational flexibility can be explicitly obtained. Essential ROFs including region of equipment and secure operation are established. Region of diverse operation is optionally established based on diverse requirements.

5) By taking the intersection of ROFs, the operation points within different regions are solved by Monte Carlo simulation, which can be seen as numerical solutions of operational flexibility.

6) By unidirectional mapping, the operation points are mapped to the plane of operational flexibility. The area of points represents the total amount of adjustable regulating capacity that the distribution network can provide. FP or FA is directly calculated and the amount of operational flexibility is quantified.

7) To ensure the accuracy of the calculation, a verification index is proposed in this paper. When the limit is reached, re-linearization at a new operation point is needed.

4. Applications of operational flexibility of ADNs

In this section, the applications of operational flexibility are proposed while operational flexibility is used in addressing different issues. Furthermore, the mechanism of improving operational flexibility is elaborated.

4.1. Applications of Quantifying and Visualizing FP and FA in Different Operation Scenarios

In the operation dispatching of ADNs, several optimization objectives are usually considered simultaneously. Diverse requirements were regarded as different objectives previously. In this paper, specific requirements of ADNs are established as optional regions of diverse operation. By taking intersection of regions of diverse operation and initial ROFs, it can be estimated whether the operational flexibility of ADNs can meet the diverse demand. From the prospective of operational flexibility, the optimization problem is reinterpreted.

A simple network with two branches and three nodes (including one source node) is used as an example. The adjustable regulating active power at end node is set as 3.0 MW while reactive power is set as 1.0 Mvar. The lower and upper limits of system voltage are set from 0.9 p.u. to 1.1 p.u. The power transmission limits of branches are set as 2.5 MVA. Take reducing line load rate as an example, the schematic diagrams of plane of operational flexibility are shown from Fig. 2 to Fig.4.

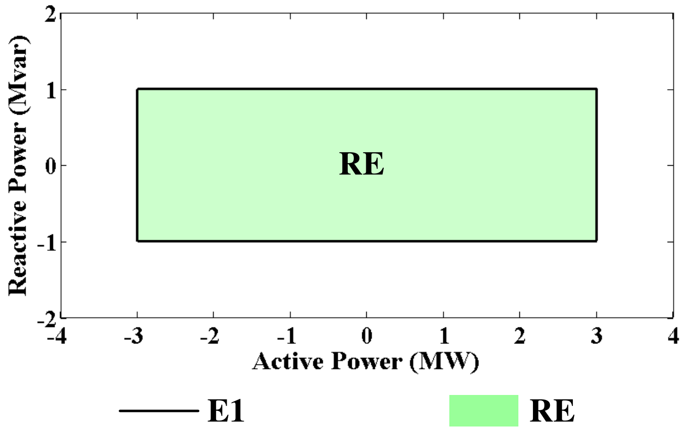

Fig. 2 RE in the plane of operational flexibility.

In Fig.2, the black solid lines stand for the equipment boundaries (E1) of CRs. The light green part in these figures is defined as RE.

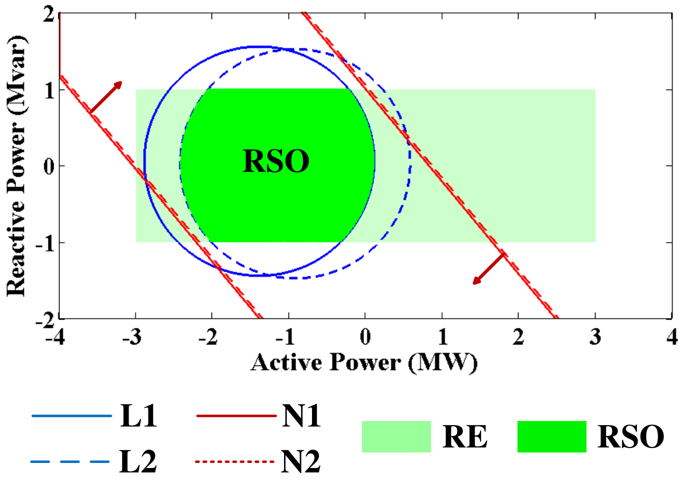

Fig. 3 RE, RSO in the plane of operational flexibility.

The blue curves show the power constraints (L1 and L2) of the two lines in Fig. 3. Since the voltage of source node is determined, the red lines represent the voltage constraints of two nodes (N1 and N2). These two lines form boundaries of RSO, so that the green part represents RSO.

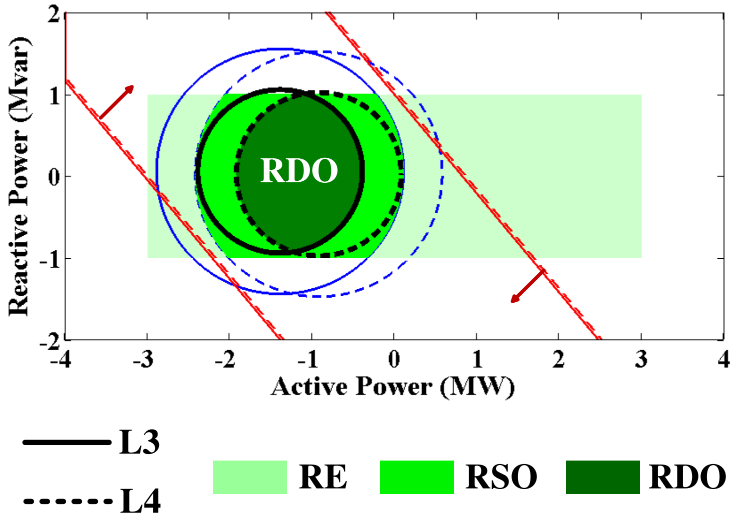

Fig. 4 RE, RSO and RDO in the plane of operational flexibility.

The heavy black curves in Fig. 4 show the operational requirement (L3 and L4). The upper limit of line load rate is set as 60% (1.5 MVA). The dark green part in Fig.4 represents region of diverse operation. By taking the intersection of ROFs, the area of operational flexibility is reduced. Furthermore, regions of diverse operation can be changed to satisfy diverse operational requirements, such as reducing voltage deviation and improving penetration of DGs.

4.2. Influence Factors of Improving Operational Flexibility

In ADNs, more accurate parameters and more flexible power flow regulation will improve flexibility, which are defined as observability and controllability [17]. Operational flexibility of ADNs can be effectively improved by enhancing observability and controllability.

In this paper, the parameters are considered to be accurate, which means the influence of information communication and the effect of observability on flexibility are not considered. Controllability refers to the ability to adjust the operating state of the network through effective control of its structure and devices in multiple time scales. CRs in ADNs can be separated into two types:

(1) Equivalent power supply resource

Equivalent power supply resources can regulate the active and reactive power flow through adjustment of operation strategies, which are the basis of controllability. By integrating CRs into the network, operational flexibility can be improved, such as energy storage system (ESS) and static var compensator (SVC). Base on soft open point (SOP), power transmission between feeders is further realized [36]. As mentioned in Section II, if an equivalent power supply resource is integrated into the network, the dimensions of state space are increased. As a result, the area of operation points in the plane of operational flexibility is increased at the same time, which means the operational flexibility has been improved.

(2) Network device

Supporting device can improve the utilization of CRs, so that the operational flexibility can be improved. For example, due to the operation of switching lines and tie switches, the relationship between lines and nodes will be changed. The range of CR can be increased by network reconfiguration. Meanwhile, the plane of operational flexibility is increased. In this way, the utilization of CRs in ADNs can be efficiently improved.

5. Case studies and analysis

In this section, the modified IEEE 33-node distribution system is used to quantify and visualize the FP, FA of ADNs in different operation scenarios. And the operational flexibility considering the time-series characteristics and improvement of operational flexibility are further analyzed.

5.1. Modified IEEE 33-node System

The modified IEEE 33-node distribution system with 32 branches is shown in Fig. 5, whose voltage level is 12.66 kV. The total active and reactive power loads of the system are 3.715 MW and 2.3 Mvar, respectively. The detailed parameters are provided in [37]. The locations of DGs and CRs are referred to the previous study [6], [16] while the capacities of DGs and CRs are further modified based on the load level of the network and the requirement of flexible operation.

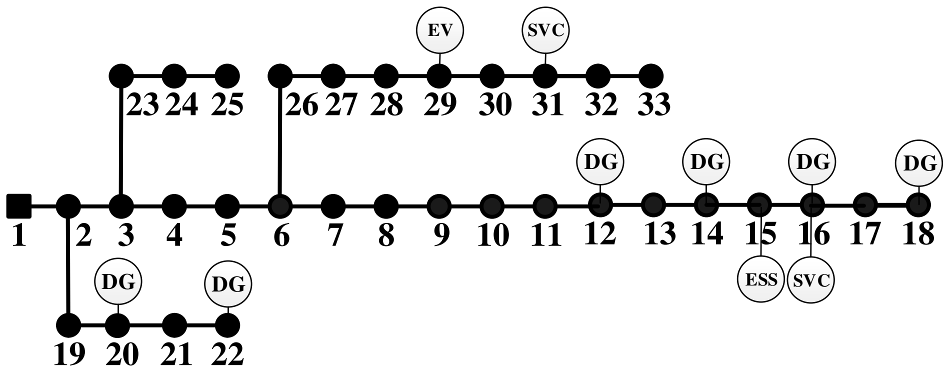

Fig. 5 Structure of the modified IEEE 33-node system.

To fully consider the impact of high penetration of DGs, six uncontrollable DGs of 1.0 MVA are integrated into network at node 12, 14, 16, 18, 20, 22. Three types of CRs are considered in the system, including ESS, charging station of electric vehicles (EV) [38], [39] and SVCs, as shown in Table 1. In distribution networks, the branches near the substation have greater power transmission limits than those far from the substation. Table 2 lists the power transmission limits of branches. The lower and upper limits of system voltage are set from 0.9 p.u. to 1.1 p.u..

Table 1 Locations and capacities of CRs.

Location of CRs	Type of CRs	Capacity of CRs
15	ESS	3.0 MVA
16	SVC	1.0 Mvar
29	EV	3.0 MW

Table 2 Branch power limitation configurations.

Rated capacity (MVA)	Capacity in normal operation (MVA)
1.5	< 0.5
2.5	0.5 – 1.2
5	1.2 – 2.5
10	2.5 - 4
12	≥ 4

The initial number of tests in Monte Carlo simulation is set as 10^6 to ensure the accuracy of simulation and efficiency of the calculation. The proposed method is implemented using MATLAB R2014a on a computer with an Intel Xeon CPU processor running at 2.36 GHz and 32 GB of RAM.

5.2. Analysis of FP under Secure Constraints

To quantify the operational flexibility under various constraints, three scenarios are adopted to evaluate the FPs of ADNs. The load demand is set as 0.55 p.u. and the DG output is set as 0.64 p.u., obtained from the daily curve of load and DG at hour 16:00 in [26].

Scenario I: Without secure constraints, the initial FP of CRs is obtained.

Scenario II: Node voltage constraint in distribution network is considered.

Scenario III: Line power transmission constraint is further considered.

The FP in each scenario is shown in Table 3. It can be seen that the initial operational flexibility of CRs cannot be fully translated into the FP of the whole system because of the secure constraints, including node voltage constraint and line power transmission constraint. The operational flexibility in each scenario is visualized from Fig. 6 to Fig. 8. The area of nodes in plane of operational flexibility is decreased after node voltage constraint is considered. The area is even smaller after line power transmission constraints are further considered.

In conclusion, the FP in Fig. 8 is defined as the total amount of operational flexibility of ADN, where the essential ROFs including RE and RSO are considered.

Table 3 FPs in each Scenarios.

Scenario	FP (MVA)
I	127.1984
II	72.8110
III	16.8159

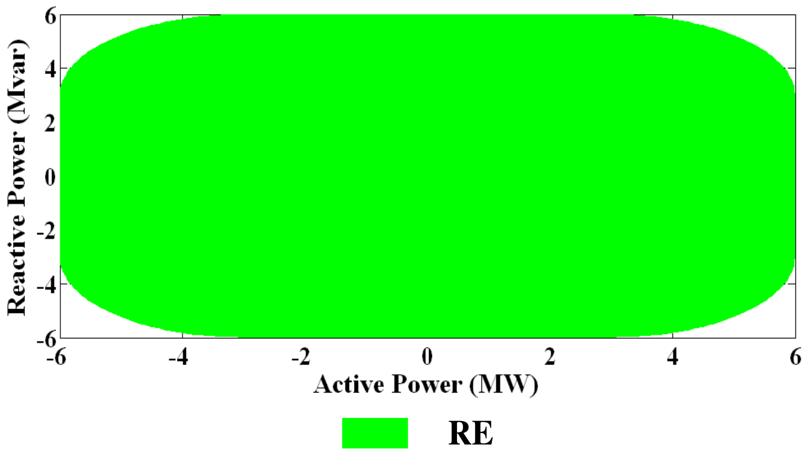

Fig. 6 Operational flexibility of ADNs in Scenario I.

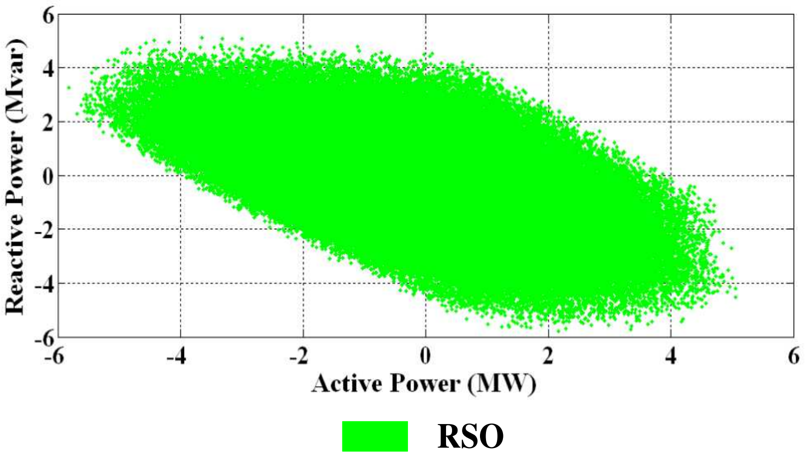

Fig. 7 Operational flexibility of ADNs in Scenario II.

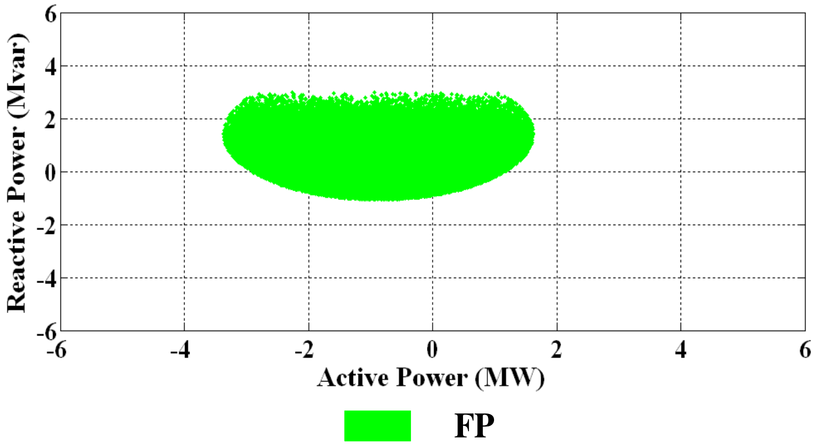

Fig. 8 Operational flexibility of ADNs in Scenario III.

5.3. Verification of the case study

To verify the accuracy of the proposed method, IPOPT and the proposed method are both used to obtain the four specific operation points in Scenario III. The lower limit of A_t is set as 95%. The four operation points represent the specific boundary points in plane of operational flexibility, as can be seen as the red crosses in Fig. 9. The detailed results of the verification including the actual values and accurate values of the outputs of controllable resources are shown in Table 4. As can be seen from Fig. 9 and Table 4, the accuracy of the proposed method is acceptable.

Table 4 Comparison of proposed method and IPOPT.

	Proposed method	IPOPT	Relative Error (%)
Minimum active power output of CRs (MW)	-3.3630	-3.3670	0.12
Maximum active power output of CRs (MW)	1.6191	1.6224	0.20
Minimum reactive power output of CRs (Mvar)	-1.0358	-1.0643	2.68
Maximum reactive power output of CRs (Mvar)	2.9734	3.0000	0.89
A_t (%)	97.32	/	/

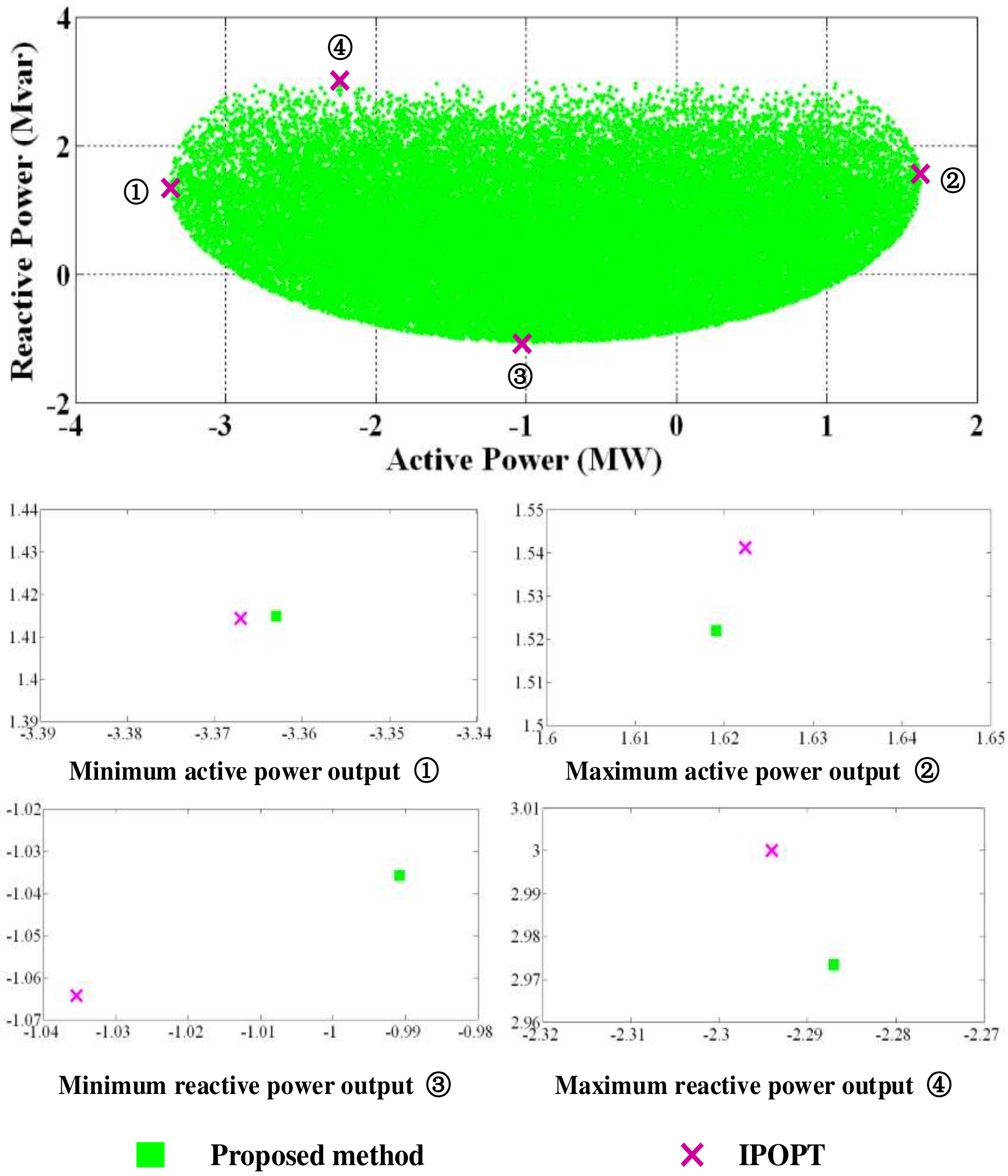

Fig. 9 Comparison of four specific operation points.

5.4. Analysis of FA under Operational Requirements

Based on the analysis above, Fig. 8 represents the initial FP of the network, which is set as the light green part in the following analysis. To further quantify the FA under different operational requirements, three scenarios are adopted to evaluate the FA of the network. The FAs in different scenarios are shown in Table 5.

Scenario IV: The upper bound of operational loss in ADNs is considered, which is set as 70%

of the initial operational loss.

Scenario V: The upper bound of sum of voltage violation is considered, which is set as 65% of the initial voltage violation.

Scenario VI: The lower bound of DG penetration at node 14 is considered, which is set as 200% of the initial DG capacity.

Scenario VII: The uncertainties of load demand and DG outputs are considered, in which α_1, α_2, α_3, α_4 are set as 20%.

The FA in each scenario is shown in Table 5. Under different operational requirements, the operational flexibility is decreased. The visualizations of operational flexibilities in the four scenarios are shown from Fig. 10 to Fig. 13. The area of dark green part represents FAs under diverse operational requirements. In Fig. 10, the operational loss is more related to the line power transmission constraint, the boundaries of FA approximate circles in plane of operational flexibility. While in Fig. 11, the boundaries approximate lines in plane of operational flexibility. In Fig. 12, two factors are both related so that the boundaries are complex. By taking the intersection of FPs under uncertain scenarios, operation strategies of CRs can be obtained to meet uncertainty demand. The most conservative FPs are shown Fig. 13. The dark green area represents the FP under 20% uncertainty in Scenario VII while the light green part represents the initial FP under deterministic parameters. The operation strategies of CRs are more conservative.

Table 5 FAs in each Scenarios.

Scenario	Reduction of operational flexibility (MVA)	FA (MVA)
IV	9.7072	7.1087
V	10.0707	6.7452
VI	5.9308	10.8851
VII	4.0527	12.7632

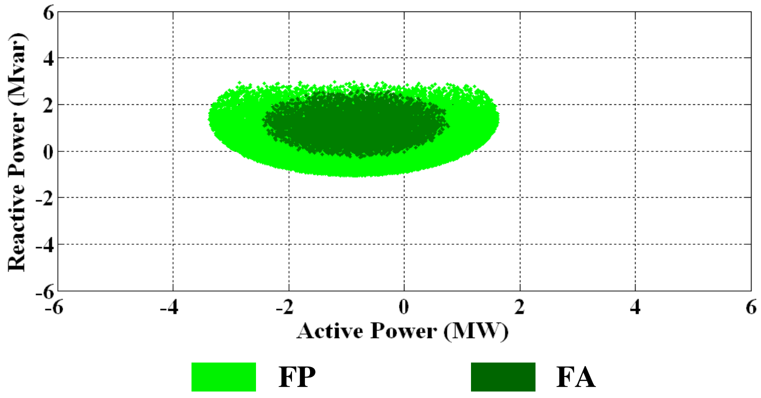

Fig. 10 Operational flexibility in Scenario IV.

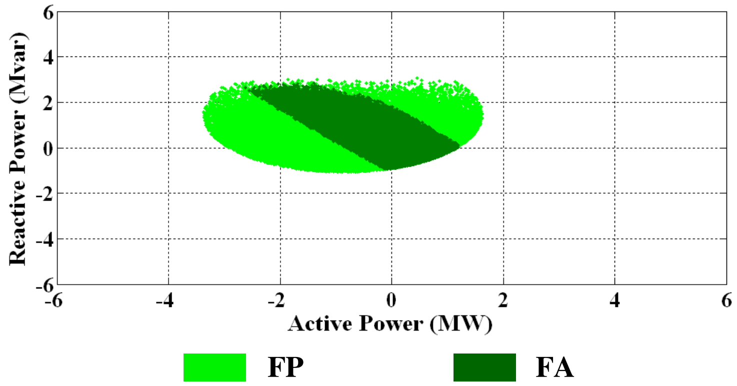

Fig.11 Operational flexibility in Scenario V.

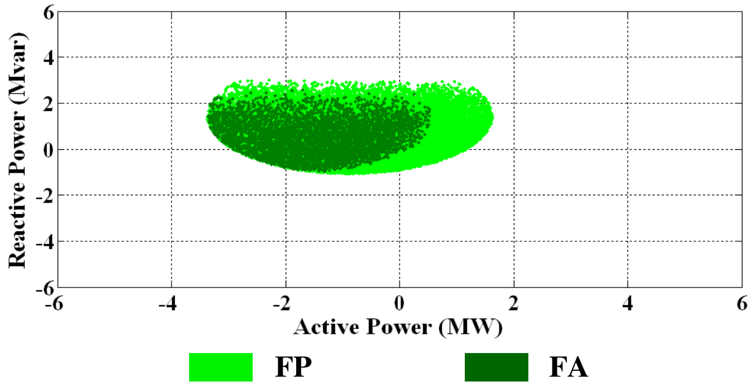

Fig.12 Operational flexibility in Scenario VI.

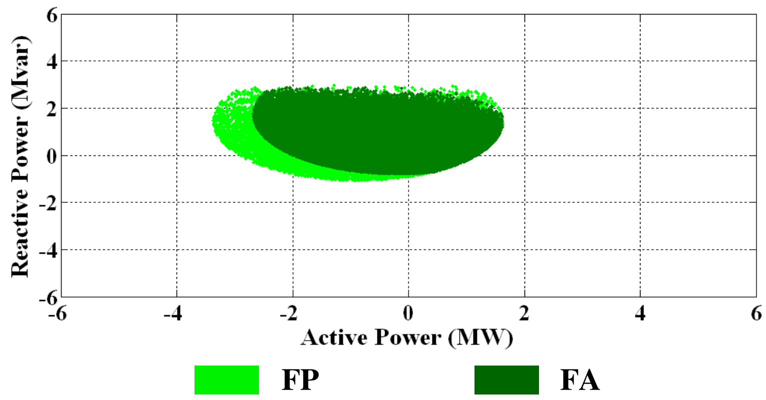

Fig.13 Operational flexibility in Scenario VII

5.5. Time-series Analysis of Operational Flexibility

Further considering the time-series characteristics of load demand and DG output, the quantification and visualization are considered for any hourly characteristic in this subsection. The predicted daily DGs and loads operation curves are shown in [26]. The analysis step is set to 2 hours. To ensure the accuracy of the proposed method, the linearization is used in each operation point.

The visualization of FPs with two hour intervals is shown in Fig. 14. It can be seen from Fig. 13 that the minimum value of FPs occurs at time 13:00, because of the high load current caused by increment of DG outputs. The maximum value of FPs occurs at time 5:00, since the power supply and demand are nearly balanced.

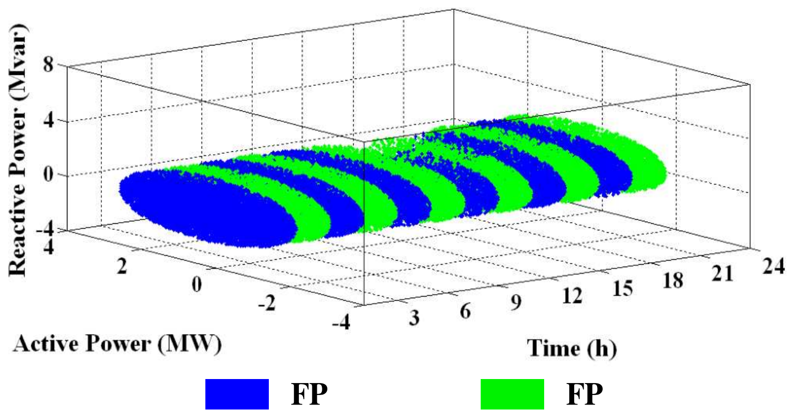

Fig.14 FPs with two hour intervals.

5.6. Operational Flexibility Improvement Based on CRs

With integration of CRs in ADNs, the operational flexibility of the network can be improved. To fully consider the impact of CRs, multiple types of CRs are integrated into the system, as can be seen in Fig. 15. Two scenarios are set to quantify the improvement of flexibility by integrating CRs.

Scenario VIII: Tie switch between node 25 and node 29 is integrated into the network. To keep the network radial, line 28-29 is disconnected.

Scenario IX: An SOP with capacity of 1.0 MVA between node 25 and node 29 is integrated to replace the tie switch.

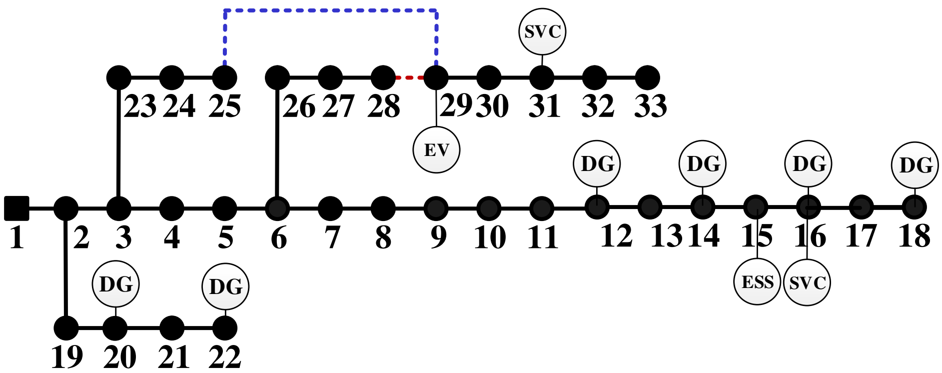

Fig.15 Structure of the modified IEEE 33-node system integrated with CRs.

The FPs and improvement of operational flexibility are shown in Table VI. The visualization of FIs in the two scenarios are shown in Fig. 16 and Fig. 17. The areas of light green part in these figures represent the initial operational flexibility and the area of dark green part represents FI. It can be seen that SOP can realize connectivity between feeders and provide reactive power to the network, so that the operational flexibility is improved. By replacing tie switch with SOP, the radial operation of ADNs can be ensured so that line 28-29 is not needed to be disconnected.

Table 6 Improvement of operational flexibility.

Scenario	FP (MVA)	FI (MVA)
VIII	31.9877	15.1718
IX	33.3139	16.4980

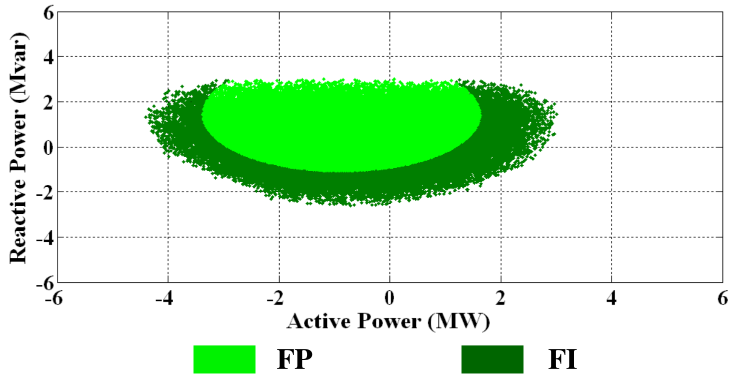

Fig.16 Improvement of operational flexibility in Scenario VIII.

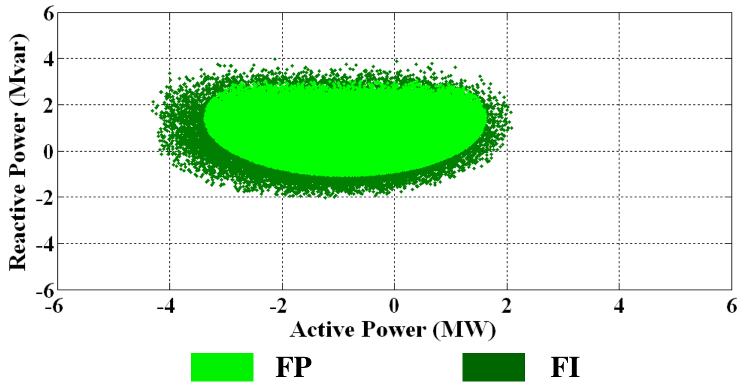

Fig.17 Improvement of operational flexibility in Scenario IX.

6. Conclusion

Operational flexibility represents the ability of the network to deploy its CRs to respond to diverse operational requirements. Multiple types of optimization problems can be reinterpreted from the perspective of operational flexibility. In this paper, a unified framework of analyzing operational flexibility is proposed including the definition, quantified calculation and application. Operational flexibility is defined as the regulating ability of ADNs by the integration of CRs, which can be seen as a link between diverse operational requirements and flexible control of CRs. The mathematical formulation of operational flexibility is proposed based on region analysis. Then the quantified calculation and visualization of operational flexibility are realized base on Monte Carlo simulation. Plane of operational flexibility is used to intuitively quantify and visualize the amount, change and improvement of operational flexibility. After that, the applications of operational flexibility are proposed and the mechanism of improving flexibility based on CRs is discussed. Finally, case studies are performed on the modified IEEE 33-node system to quantify and visualize the operational flexibility in ADNs. Results show that the FP, FA can be quantified and visualize in the plane of operational flexibility. It can be used in estimating whether the operational flexibility of ADNs can meet diverse operational requirements. Through verification method of operational flexibility, the proposed method is effective and efficient.

Under the proposed framework, the FPs and FAs of ADNs can be quantified and visualized to estimate whether the operational requirements are satisfied. In the future work, the optimal operation strategies of CRs are needed to be considered into the proposed method. Furthermore, to improve operational flexibility by integration of CRs in ADNs, the optimal location planning of CRs can be obtained under the proposed method.

Acknowledgements

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