Foreword

Special issue of *Quantitative Finance* on 'Hawkes Processes in Finance'

Since the invention of Hawkes processes in the early 1970s, many researchers including the pioneer seismologists have found applications in a very wide range of research projects. However, these stochastic processes with incredible modelling advantage were only noticed in Finance research until 2005. Up to now, there has been a rapid expansion of papers applying them to diverse problems in Finance. But there is still great scope to make them become standard econometric tools and we, as the panel of guest editors of this issue, felt that it would be not only valuable but also timely to have an issue themed on a variety of financial applications all in one place at this time. Happily, when we approached the editors-in-chief of *Quantitative Finance* they were extremely supportive, and now we are having this great issue with a good range of excellent manuscripts.

We begin with an introduction by the author of Hawkes processes, **Professor Alan Hawkes**. He has be excited by the idea of the special issue and provided a brief history to describe the basic properties of Hawkes processes – essentially a class of stochastic models for series of events whose occurrence generally increases the probability of further events occurring, often described as a contagious effect. He concludes with a review of some recently published papers.

This is followed by a collection of nine papers, addressing many contemporary topics from both theoretical and practical aspects. **Achab, Bacry, Rambaldiy and Muzy** use a 12-dimensional mutually-exciting process to model interactions between different kinds of events in a single-asset high-frequency order book. A nonparametric method is used to estimate the branching ratio matrix directly, without considering the exact shape of the exciting kernels. The elements of this matrix measure the connectivity between event types. The process is extended to study interactions between two assets.

As mentioned above, Hawkes processes are usually positively exciting. However, **Chen, Hawkes and Khashanah** introduce a type of birth-death-immigration model. It turns out to be a special kind of two-dimensional mutually-exciting process and in it, the occurrence of some type of events can decrease the rate at which other events occur (so actually a bit depressing rather than exciting). This may be useful in modelling the decay of activities that is usually observed in a market with burst of motions. For example, burst of trading activities at the start of a trading day.

A number of studies of jumps in asset prices have assumed that the kernel of a self-exciting model has a simple exponential shape or perhaps a linear combination of two or three exponential components. **Chen, Hawkes, Scalas and Trinh** carry out a simulation study to compare the ability of various information criteria (AIC, BIC and HQ.) to decide on the best model to choose.

Calcagnile, Bormetti, Treccani, Marmi and Lillo study a portfolio with multiplicity and are concerned with the number of assets that jump at the same time (cojumps). It is suggested that some kind of mutually-exciting Hawkes process could be used to model the time-clustering of jumps. The authors also provide a novel approach to fit such a complicated model by simplifying assumptions, thus reducing the estimation of the model to be with only three parameters! Remarkably, it appears to fit data moderately well.

Lu and Abergel attempt to model the various events of an order book by a mutually-exciting Hawkes process with exponential kernels. On finding that some interactions

sometimes appear inhibitory rather than exciting, they replace the usual linear model by a non-linear model by allowing the usual linear form sometimes to result in a negative intensity that is then simply truncated to zero at that point.

Gao, Zhou and Zhu consider a self-exciting Hawkes process where the baseline intensity is time-dependent, the exciting function is a general function and the jump sizes of the intensity process are i.i.d. non-negative random variables. They obtain closed-form formulas for the Laplace transform, moments and the distribution of the process. The process is used to model the clustered arrival of trades in a dark pool and thereby analyze various performance metrics including time-to-first-fill, time-to-complete-fill and the expected fill rate of a resting dark order.

Schneider, Lillo and Pelizzon use a PoT method to identify abrupt liquidity drops from limit order book data utilising a mutually-exciting Hawkes process. Both the self-excitation of extreme changes of liquidity in the same asset (illiquidity spirals) and cross-excitation across different assets (illiquidity spillovers) can be quantified. The application of their method to the MTS sovereign bond market's show that the proportion of shocks explained by illiquidity spillovers roughly doubles from 2011 to 2015, suggesting an increased synchronization of extreme illiquidity across assets.

Chen, Hawkes Yang and Liu study the complex interaction between market prices, as measured by the S&P500 index, and investor sentiment, using Thomson Reuters News Sentiment data. A mutually-exciting Hawkes model with exponential kernels is fitted to four streams of events concerning returns and sentiment movements measured on a 15-minute time grid. The interactions among them are analysed by studying the mutually-exciting kernels of the model. This is currently the first application of Hawkes processes in Behaviour Finance literature.

The literature on Constant Proportion Portfolio Insurance (CPPI) typically assumes diffusive/Lévy dynamics for the risky asset. **Buccioli and Kokholm** introduce self-exciting Hawkes jumps to the model to account for contagion and derive the loss probability when trading continuously. They also estimate the risk involved of implementing discrete-time rebalancing. They show that failing to take contagion into account will significantly underestimate the risks of CPPI when rebalancing is performed less often than weekly.

We hope this special issue will encourage more scholars to contribute to the growing literature of Hawkes processes to financial research and commercial applications. We have received great number of high quality submissions and nine excellent manuscript finally form this great special issue. On behalf of all the guest editors, we thank contributing authors including those whose papers could not be accommodated in this issue, the referees and the Editors-in-Chief for their support.

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