Preventive replacement with defaulting

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Abstract: This paper models age-replacement and block-replacement when there is the possibility of defaulting on the planned maintenance. A default occurs when a planned preventive replacement is not executed, and we discuss how defaults can arise in practice. Our aim is to study the robustness of block-replacement and age-replacement, bearing in mind that a) these policies are frequently used in practice, b) in the standard scenario (no defaulting) age-replacement has a lower economic cost-rate than block-replacement, and c) block-replacement is simple to manage because component age does not have to be monitored. We model defaults as independent Bernoulli trials. We prove that a cost-minimising critical-age for replacement in the age policy with defaulting exists if the time to failure distribution has an increasing failure rate. A numerical study of the policies indicates that: ~~if~~ age-replacement is ~~to be~~ effective ~~then~~ if maintenance control ~~must be~~ is good, that is, when there is only a small chance of defaulting; block-replacement is relatively robust to defaulting (postponement), but less so to lack of knowledge about component-reliability.

Keywords**:** Maintenance; reliability; replacement; quality; default.

1. Introduction

In this paper we describe generalizations of the classic preventive maintenance policies, ~~of~~ age-replacement and block-replacement, ~~in which~~ when a maintainer defaults on scheduled preventive replacements. In the case of age-replacement, the maintainer fails to execute replacement at all. In the case of block-replacement, replacement at a particular block-epoch is not executed. Many factors might explain such variations from a prescribed schedule such as: prioritisation of production, and thus revenue generation, over maintenance (Baglee et al., 2007; Budai et al., 2008; Ahmadi, 2019, Wu et al., 2019); an unpunctual operator (Wang et al., 2020); unavailability of spares (Scala et. al. 2014); lack of personnel (Safaei et al., 2011; George-Williams and Patelli 2017); short-term financial constraints (Litzka and Weninger-Vycudil, 2012); unpredictable events (Liu et al., 2018; Zhong et al., 2019; Finkelstein et al., 2020); maintenance time-constraints (Irawan, 2017; Yang et al., 2017); system mission-constraints (Khatab et. al. 2017; Diallo et al., 2018); poor communication and lack of information (Antonovsky et al., 2016); too much information (Budai et al., 2006); outsourcing of maintenance (Quinlan et al., 2013); system obsolescence (e.g. Dwight et al., 2012); and human error (Reason and Hobbs, 2017). We call the failure to execute a scheduled preventive replacement a default.

The broad purpose of our study is to consider the robustness of "time-based" maintenance policies to departures from schedule when optimal policies are not strictly followed. If When the policies are implemented precisely, and all else being is equal, it is well known that age-replacement is a more cost-efficient replacement policy than block-replacement (Barlow and Proschan, 1965). Nonetheless, However, defaulting may lead to the opposite. Also, block-replacement is easier to plan, and planning is also costly—13% of all maintenance activity is spent on planning (Alsyouf, 2009). Furthermore, the cost-efficiency of maintenance is an important issue because maintenance takes a large part of total spending (Zio and Compare, 2013) and resources (Waeyenberg and Pintelon, 2002), and industries are revising their maintenance plans as a result (Ruschel et. al. 2017; Cherkaoui et al., 2018). In addition, the simplest policies (periodic maintenance) are a major component of total maintenance activity (Alsyouf, 2009), and planned maintenance has well-known benefits (Lin et. al. 2019), even if much of it is planned qualitatively or imprecisely (Wang et al., 2011). We might expect that defaulting occurs much less frequently in the more regulated industries e.g. aviation (Safaei and Jardine, 2018) and nuclear (Khalaquzzaman et al., 2010).

Age-replacement and block-replacement themselves have been widely studied in the literature (de Jonge and Scarf, 2020). Generally, these models assume that replacement is executed at the scheduled moments, although it is known that maintenance policies are robust to small deviations from schedule (Baker and Scarf, 1995). Several works exist that consider sensitivity in relation to age-replacement and block-replacement (e.g. Wen et al., 2011; Fouladirad et. al, 2018). In De Jonge et al. (2015a), uncertainty in the input parameters of the lifetime distribution is modelled, and the notion of learning by postponement is articulated in de Jonge et al. (2015b). Li et al. (2016) consider advancement of the schedule due to production stops. However, defaulting on execution is little discussed in the literature, which is the focus of our paper.

We model defaults as independent Bernoulli trials, that is, a default occurs with probability p, and does not occur with probability $1-p$, independently of all other scheduled replacements and the state of the system. When a default is a cancellation, we suppose that then for age-replacement the occurrence of a default implies that the replacement cycle ends in failure with probability one, and for block-replacement under our model it a default implies the postponement of preventive replacement until at least the next scheduled replacement time (for which there may be a further default, and so on). Within this modelling environment, we are interested in determining the effect of the probability of default on the control parameters (decision variables), the long-run cost per unit time (cost-rate) and the system availability of the policies. We also consider the cost-efficiency of age-replacement with defaulting relative to block-replacement without defaulting. We also compare the policies and which policy is preferred when both are subject to defaulting, although this comparison requires careful interpretation because the nature of defaulting in our models of the two policies is different. We then discuss the practical implications of all this our findings for maintenance planning. This is important for practice because managers should know circumstances in which maintenance policies are not effective. Indeed, for a technical system with hundreds or thousands of parts it may be very difficult to apply age-replacement of at the level of these parts because the ages of parts become asynchronised. Block-replacements with their time-periodic (as opposed to age-periodic) schedules are easier to apply and to group (Wildeman et al., 1997; Do et al., 2013; Wang et. al. 2019). If there is a risk to default with age-replacement, then block-replacement may also be preferred on simple cost-rate grounds.

In summary, we think that the paper extends existing, well-known models in an interesting way. Broadly, it considers the robustness of the age-replacement and block-replacement maintenance policies. To our knowledge, this generalised setting of defaulting in periodic maintenance has not been studied before. This work is useful and important to practice because periodic maintenance is widely used.

This paper is organized as follows. In the next section, we present the assumptions and notation for the models. In Section 3, we formally describe age-replacement with defaulting and present a numerical study of cost and availability. In Section 4, we formulate and evaluate block-replacement with defaulting. We discuss the implications for practice in Section 5. Section 6 provides some concluding remarks.

2. Assumptions and notation

2.1. *Notation*

X	The lifetime of a component
f, F, $\bar{F}$, r	The density, distribution, reliability (survival) and failure-rate functions of X
μ	The mean of X
T	The critical age for replacement
~~Δ~~	~~The block-replacement interval~~
p	The probability of default
c	The cost of failure replacement relative to preventive replacement
Q	The long-run total cost per unit time (cost-rate)
d_{P}	The downtime during a preventive-replacement
d_{F}	Denoting the downtime during a failure-replacement

2.2. *Assumptions*

We consider a critical, non-repairable system S comprising a component C in a socket that together perform an operational function (Ascher and Feingold, 1984). The component C has two states, operating or failed, which determines exactly the system S state, operating or failed. We call replacement on failure a failure-replacement, and otherwise replacement is preventive-replacement. The cost of a preventive replacement is 1 unit. The cost of a failure replacement is $c > 1$. We assume that:

- A replacement of C corresponds to renewal of S;
- a replacement of C is instantaneous;
- a failure of C is immediately revealed (we know immediately when C enters the failed state);
- the age at failure (lifetime) of C is X, a positive-valued random variable with distribution function F and survival (reliability) function $\bar{F}$, continuous density function f, and mean μ.

3. Age-replacement with defaulting

3.1. *The model*

Let us suppose that preventive replacement of C is scheduled to occur at age *T*, and that failure-replacement occurs upon failure. The unit cost of the former is 1 and the latter is *c*. Furthermore, suppose that if there is a default on the scheduled preventive-replacement then preventive replacement will not be executed and failure replacement is inevitable, so that the policy "defaults" to failure-based maintenance. We suppose that the probability of a default is *p*. Then, the indicator variable for a default has a Bernoulli distribution with parameter *p*. This *p* (or more strictly $1-p$) quantifies the quality of maintenance management (Scarf and Cavalcante, 2012). It might be estimated subjectively or using observation of the relative frequency of defaults.

We shall denote this age-replacement policy with defaulting as the A*p* policy. Note, the model distinguishes between a *scheduled replacement* and an *executed replacement,* because scheduled replacements are not necessarily executed.

In an alternative model, we might suppose that: a system M monitors the time since last replacement of C; M can fail at age *Y* and these failures are unrevealed; and M is renewed when C is replaced. M bears some similarity to a protection or preparedness system (Alberti et. al. 2019), although M is not inspected but could be in an extension of this idea. If $Y < T$, then a default occurs. In this way, *p* can increase with *T*. We do not however discuss this model further in this paper.

3.2. *The cost-rate*

Let the cost of a renewal cycle be *U* and the length of a renewal cycle be *V*. Then, with probability *p*, failure of C is inevitable and so $V = X$ and $U = c$. With probability $1-p$, the age-replacement policy is implemented and so $V = \min(X,T)$ and $U = 1.I(X > T) + c.I(X < T)$, where $I(.)$ is an indicator function. Therefore

$$E(V) = pE(X) + (1-p)E(\min(X,T)) = p\mu + (1-p)\int_0^T \bar{F}(x)\mathrm{d}x,$$

and

$$E(U) = pc + (1-p)(\bar{F}(T) + cF(T)) = pc + 1 - p + (1-p)(c-1)F(T).$$

Therefore, the long run cost per unit time (the cost-rate) is

$$Q_{\mathrm{A}p}(T) = \frac{E(U)}{E(V)} = \frac{pc + 1 - p + (1-p)(c-1)F(T)}{p\mu + (1-p)\int_0^T \bar{F}(x)\mathrm{d}x}. \tag{1}$$

When $p = 1$ in (1), we obtain $Q_{\mathrm{A}}(T) = c/\mu$, which is the cost-rate of failure-based maintenance. When $p = 0$ we obtain the cost rate of standard age-replacement. With *F* specified as a Weibull distribution this is easy to study numerically, which we do in Section 3.3.

Proposition 1. For any $0 \le p < 1$, if F is IFR (*F* has a strictly increasing hazard-rate function) then there exists a unique, finite T^* such that $Q_{\mathrm{A}p}(T^*) < Q_{\mathrm{A}p}(T)$ for all $0 \le T \ne T^*$.

Proof. The result in the case $p = 0$ is well known (Barlow and Proschan, 1965, p.87). To prove the result for $0 < p < 1$, for shorthand, we will use the notation $q = Q_{\mathrm{A}p}$, $u = E(U)$, and $v = E(V)$. Now

$$q = (pc + 1 - p)/p\mu > c/\mu = \lim_{T\to\infty} q(T). \tag{2}$$

Since $q = u/v$, $qv = u$, therefore

$$v\mathrm{d}q/\mathrm{d}T + q\mathrm{d}v/\mathrm{d}T = \mathrm{d}u/\mathrm{d}T, \tag{3}$$

and $\mathrm{d}u/\mathrm{d}T = (1-p)(c-1)f(T)$ and $\mathrm{d}v/\mathrm{d}T = (1-p)\bar{F}(T)$ ("differentiating through the integral sign"). So, from (3),

$$\mathrm{d}q/\mathrm{d}T\big|_{T=0} = -q(0)\mathrm{d}v/\mathrm{d}T\big|_{T=0}/v(0) < 0 \tag{4}$$

and is finite, since $\mathrm{d}u/\mathrm{d}T\big|_{T=0}=0$, $v(0)=p\mu$, and $\mathrm{d}v/\mathrm{d}T\big|_{T=0}=(1-p)$. To prove the result, (2) and (4) imply that it is sufficient to show that $q(T)$ has a unique turning point at a finite $T>0$ (because this turning point must then be a minimum point). Setting $\mathrm{d}q/\mathrm{d}T=0$ in (3) we obtain $q\mathrm{d}v/\mathrm{d}T=\mathrm{d}u/\mathrm{d}T$, that is, $q(T)(1-p)\bar{F}(T)=(1-p)(c-1)f(T)$, which after a little manipulation can be written as

$$r(T)\{p\mu+(1-p)\int_0^T \bar{F}(x)\mathrm{d}x\}-(1-p)F(T)=\frac{pc+1-p}{c-1}, \tag{5}$$

where $r(.)=f(.)/\bar{F}(.)$ is the failure-rate of *X*. The right hand side of (5) is a constant, and writing the left hand side as $g(T)$ we see that $\mathrm{d}g/\mathrm{d}T=\{p\mu+(1-p)\int_0^T \bar{F}(x)\mathrm{d}x\}\mathrm{d}r/\mathrm{d}T$, so that $g(T)$ is strictly increasing if and only if $r(T)$ is strictly increasing (if and only if *F* is IFR). Thus, if *F* is IFR, then (5) has a unique solution at a finite $T>0$. ☐

Proposition 2. For any $0<p<1$, the minimum cost-rate for the A*p* policy at a finite *T*, if it exists, is always strictly greater than the minimum cost-rate for the A*p* policy with $p=0$

Proof: The A*p* policy is a random interval policy in the sense of Barlow and Proschan (1965, p.86) and so the result follows immediately from Theorem 2.1 therein. ☐

These results suggest two important implications for practice. Firstly, even in the presence of defaulting, there exists an optimum age for preventive maintenance, and so the maintainer should persevere with the preventive maintenance plan. Secondly, the maintainer should seek to improve the capacity to comply with the maintenance plan, thus decreasing the probability of default, because the effect of defaults is deleterious for system performance. Thus, one might seek to improve maintenance control with more effective information management. Alternatively, the maintainer might compensate for inevitable defaults by using more reliable components at replacement. Numerical study of the increase in cost-rate could inform decision-making about potential investment in such improvements. We demonstrate this next.

3.3. *Numerical study of the cost-rate*

As typical of studies of this kind, we suppose that the lifetime of a component follows a Weibull $\mathrm{We}(\lambda,k)$ distribution: $\bar{F}_X=\exp(-(x/\lambda)^k), (x>0, \lambda>0, k\geq 1)$. Throughout, $\lambda=10$ in an arbitrary unit of time. We set $c=5$, recalling that the unit of cost is the cost of a preventive replacement. We show the cost-rate versus *T* in Figure 1 and the cost-rate versus *p* in Figure 2.

We can make some observations regarding Figure 1. The optimum *T* tends to increase with *p*, so that acting cautiously—making early preventive replacements—will tend to compound the problem if there is a high chance of default. This makes some intuitive sense because when *p* is large and *T* is small (relatively), the maintainer will tend to carry out preventive replacement at component ages that are rarely appropriate. Also, if default is more likely with a larger *T*, which is justifiable if there exists a monitoring system that itself ages, negative feedback may exacerbate the problem: managing defaulting by increasing *T* may itself increase the chance of default. The second broad point is that defaulting is a bigger issue—has a greater effect on the cost-rate—when there is less uncertainty about component lifetime. This makes sense because when there is greatest uncertainty in the component lifetime—$k=1$ the exponential case—defaulting cannot change the cost-rate of the optimum policy (failure-based maintenance).

Also shown in Figures 1 and 2 is the cost-rate for the standard block-replacement policy: replace C on failure and at times $kT, k = 1, 2, \ldots$. In this policy there is no defaulting. One can then observe the value of p for which ~~we prefer~~ block-replacement has a lower cost-rate than Ap ~~in cost-rate terms~~. This value of p is quite small, particularly for large k when there is less uncertainty about the time to failure (Figure 2c). Note, we are not directly claiming here that block-replacement is better policy than the Ap policy. Instead, the results indicate that, when there is a chance of default (in the manner described) in the age-replacement policy, a maintainer would be better not to use that policy, whence the block policy is the sensible alternative.

Notice also in Figure 1 that the block policy appears to be more sensitive to T than the Ap policy. This may manifest in a number of ways. Firstly, the block policy will be more cost-sensitive to postponement (or early replacement). Secondly, the age policy is more robust than the block policy to lack of knowledge about the values of parameters, in the manner studied in de Jonge and Jakobsons (2018). Note, these effects have motivated the study of risk-sensitive criteria for these policies (e.g. Wu et al, 2017; Jiang, 2019).

A maintainer might also default on replacement when operating a block-replacement policy. We consider this in section 4. Next, we consider system availability under the age-replacement policy with defaulting.

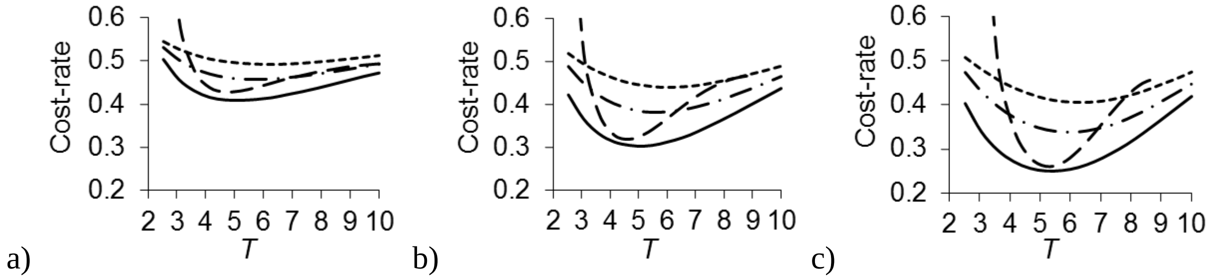

Fig. 1. Cost-rate versus T for Ap with $X \sim \mathrm{We}(10, k)$ and $c = 5$: $p = 0$ (—); $p = 0.2$ (–·–); $p = 0.4$ (---); and the block-replacement policy (— —). a) $k = 2$; b) $k = 3$; c) $k = 4$.

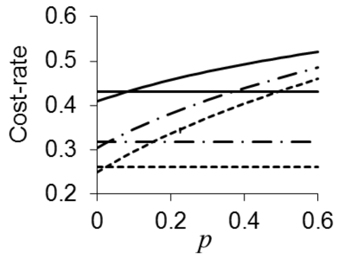

Fig. 2. Minimum cost-rate versus p for Ap with $X \sim \mathrm{We}(10, k)$ and $c = 5$: $k = 2$ (——); $k = 3$ (–·–); $k = 4$ (----). Horizontal lines: minimum cost-rate for the block-replacement policy.

3.4. *Average availability under the Ap policy*

Taking a multi-criteria approach (Munier, 2018), the average availability, defined by uptime/(uptime + downtime) (Badia and Berrade, 2009), can be calculated. The uptime is by definition $E(V) = p\mu + (1-p)\int_0^T \bar{F}(x)\mathrm{d}x$. Denoting the downtime during a failure replacement by d_{F} and the downtime during a preventive replacement by d_{P}, the downtime (the downtime in a renewal cycle) takes the value d_{P} if there is no default (with probability $1-p$) and $X > T$, and the value d_{F}

otherwise. Therefore, the downtime is $(1-p)\bar{F}(T)d_{\mathrm{P}}+\{1-(1-p)\bar{F}(T)\}d_{\mathrm{F}}$, and the average availability is thus

$$\mathrm{A}(T)=\frac{MTTF}{MTTF+MTTR}=\frac{p\int_T^{\infty}\bar{F}(x)\mathrm{d}x+\int_0^T\bar{F}(x)\mathrm{d}x}{\left(p\int_T^{\infty}\bar{F}(x)\mathrm{d}x+\int_0^T\bar{F}(x)\mathrm{d}x\right)+d_{\mathrm{F}}F(T)+(pd_{\mathrm{F}}+(1-p)d_{\mathrm{P}})\bar{F}(T)}.$$

The average availability is shown in Figures 3 and 4 for the parameter values used in section 3.3. Notice here that d_{F} and d_{P} are expressed in the same arbitrary unit of time as the mean time to failure (and therefore *k*). In a manner similar to the cost-rate, unavailability increases with *p*, and the critical age for replacement, *T*, that maximizes availability increases with *p*. Thus, a response to defaulting that decreases *T* will not help, and one which increases *T* may be problematic if the probability of default increases with *T*. Thus, the sensible response is improvement of maintenance control.

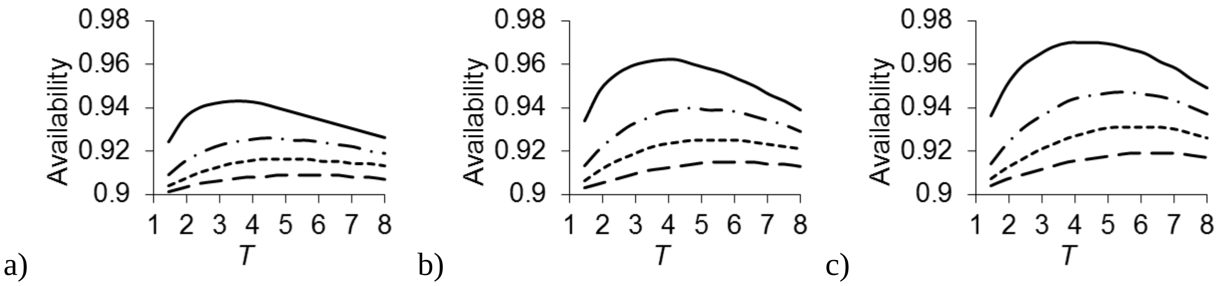

Fig. 3. Average availability versus T with $X\sim\mathrm{We}(10,k)$ and $d_{\mathrm{P}}=0.1$, $d_{\mathrm{F}}=1$: a) $k=2$; b) $k=3$; c) $k=4$; with $p=0$ (——); $p=0.2$ (–▪–); $p=0.4$ (----); $p=0.6$ (— —).

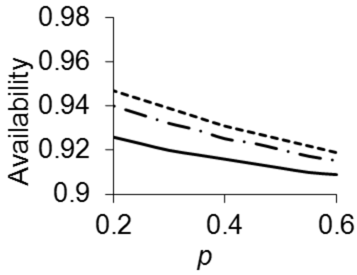

Fig. 4. Average availability versus *p* with $X\sim\mathrm{We}(10,k)$ and $d_{\mathrm{P}}=0.1$, $d_{\mathrm{F}}=1$ and $k=2$ (——); $k=3$ (–▪–); $k=4$ (----).

4. Block-replacement with defaulting

4.1. *The cost-rate*

Figure 2 suggests that when there is a chance of default in the age-replacement policy, block-replacement ~~would offer a cost advantage over age-replacement~~ is a sensible alternative to use in practice, particularly if there is good knowledge about component lifetime. Also, block-replacement does not require component age to be monitored. A natural question that then arises is how ~~does~~ might defaulting impact upon block-replacement. Therefore, in this section we will consider defaulting in the block-replacement policy.

Again, it is important to distinguish between a *scheduled replacement* and an *executed replacement*, because in our model scheduled replacements are not necessarily executed. However, for the block-replacement policy there are two natural ways to consider defaulting. In the first, each preventive replacement in the periodic sequence of preventive replacements is subject to default whereby the preventive replacement is not executed with probability p and executed with probability $(1-p)$, independently of all other periodic replacements. We call this mode 1 defaulting. In the second, the maintainer defaults on the policy rather than individual scheduled replacements, so that with probability p the system is subject to failure-based maintenance and with probability $(1-p)$ it is subject to block-replacement with period T. This latter case (mode 2 defaulting) seems unlikely to occur in practice because of the relative ease of management of block-replacement. Also, the latter case does not accommodate postponements, whereby the maintainer postpones preventive replacement at kT to $(k+1)T$. Indeed, mode 1 defaulting is equivalent to the case in which the maintainer, at every scheduled replacement, postpones replacement to the next scheduled replacement time, independent of the history of postponements (so that multiple postponements are possible). Therefore, we consider the first mode of defaulting and not the second. Nevertheless, for comparison of the age- and block-replacement policies when both are subject to defaulting, which we consider later, mode 2 defaulting may be the more appropriate comparator. However, in this paper we shall compare the cost-rates of the two models that have the stronger practical justification.

We denote the block-replacement policy with mode 1 defaulting by B*p*. To develop the cost-rate for block-replacement with mode 1 defaulting, first note that the number of defaults until the first scheduled replacement that is executed has a geometric distribution. Denoting the event of an default on replacement by 0 and an executed replacement by 1, then, in a renewal cycle, the possible sequences of scheduled replacements until renewal are 1 and 01 and 001 and 0001 etc, and these occur with probabilities $(1-p)$, $(1-p)p$, $(1-p)p^2$,..., and the respective cycle lengths are $T, 2T, 3T, \ldots$, and the respective costs of the renewal cycles are $C_P + C_F H(T), C_P + C_F H(2T), C_P + C_F H(3T), \ldots$, where $H(t)$ is the renewal function, the expected number of failures in $[0,t]$, and C_P and C_F are the respective costs of a preventive and a corrective replacement.

Thus, for $0 \le p < 1$,

$$E(V) = \sum_{k=1}^{\infty}(1-p)p^{k-1}kT = T/(1-p),$$

and

$$E(U) = \sum_{k=1}^{\infty}(1-p)p^{k-1}(C_\text{P} + C_\text{F}H(kT)) = 1 + (1-p)c\sum_{k=1}^{\infty} p^{k-1}H(kT),$$

recalling that we set $C_P = 1$ and $C_F = c$.

These make intuitive sense. On average, due to defaults, the cycle length will be a little bigger than T. Also, the cost of preventive replacement will always be incurred exactly once no matter how many defaults, because the cycle always ends with the first executed preventive replacement.

So, we obtain

$$Q_{\text{B}p}(T) = \frac{E(U)}{E(V)} = \frac{1+(1-p)c\sum_{k=1}^{\infty} p^{k-1}H(kT)}{T/(1-p)} = \frac{1-p+(1-p)^2 c\sum_{k=1}^{\infty} p^{k-1}H(kT)}{T}. \quad (6)$$

When there are no scheduled replacements ($T \to \infty$), the policy is failure-based maintenance, and the cost-rate is $Q_{\text{B}p}(\infty) = c/\mu$.

We calculate the renewal function $H(t)$ using the discrete approximation given by

$$H(n) = \sum_{i=0}^{n-1}\{1 + H(n-i-1)\}\int_i^{i+1} f_X(t)\text{d}t. \quad (7)$$

(Jardine, 1973). We transform the timescale so that this discretization provides a good approximation, and we can obtain T^* to two decimal places reasonably quickly on a standard PC. We calculate *p* up to the value 0.6. Above this the computation time to obtain a reasonable approximation to the cost-rate is very long, because the sequences of possible scheduled replacements can be quite long. Nonetheless, we carried out a numerical study of the behaviour of $Q_{Bp}(T)$ in (6) as $p \to 1$ when $X \sim \mathrm{We}(10,3)$ and $c = 5$ for two instances, $T = 2$ and $T = 5$. In these cases, $\lim_{p\to 1} Q_{Bp}(T) = c/\mu = Q_{Bp}(\infty)$, as required. For these calculations, it was necessary that the sub-division of the timescale for the use of (7) to calculate $H(kT)$ was carried out carefully; the timescale was transformed (sub-divided) for small kT but not for large kT.

4.2. *Numerical study*

Figure 5 shows the cost-rate as a function of the block-replacement interval for the B*p* policy (with mode 1 defaulting) for the same parameter values that we use in section 3.3. It is apparent there that the straightforward response to the possibility of default (or postponement) is to increase the frequency of scheduled replacements.

Figure 6 compares the block policy with mode 1 defaulting (B*p*) with the age-replacement policy with defaulting (A*p*). In Figure 6, we can see that B*p* has a lower cost-rate except for very small values of *p*. Thus, in practice, if a maintainer was choosing between an age-replacement with the possibility of default on the scheduled replacement and a block-policy with the possibility of postponement of scheduled replacements, then block-replacement would nearly always offer two advantages—it would be less costly on average than age-replacement and it would be easier to manage. An advantage of age-replacement is that the cost-rate appears to be less sensitive to *T*, whence deviation from the true (unknown) optimum policy has a smaller effect on overall maintenance effectiveness. However, note, we cannot claim that when there is defaulting, block replacement is better than age replacement because the modes of default in the two polices here is different.

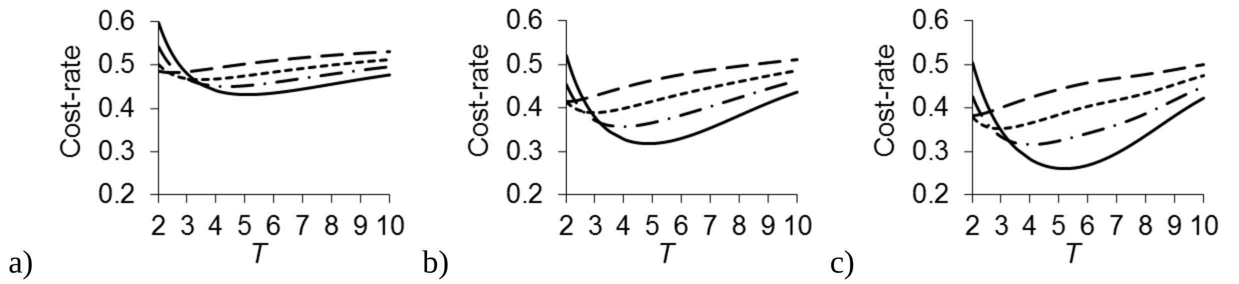

Fig. 5. Cost-rate versus *T* for B*p* with $X \sim \mathrm{We}(10,k)$ and $c = 5$: $p = 0$ (——); $p = 0.2$ (–·–); $p = 0.4$ (----); $p = 0.6$ (— —): a) $k = 2$; b) $k = 3$; c) $k = 4$.

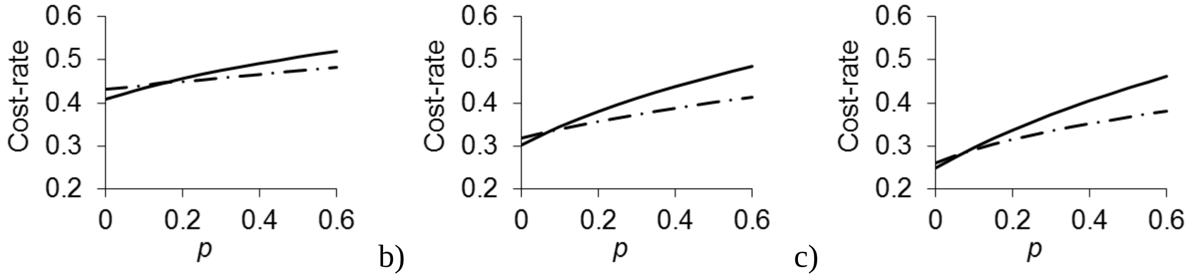

Fig.6. Minimum cost-rate versus p for Ap (——) and Bp (–·–) with $X \sim \text{We}(10, k)$ and $c = 5$: a) $k = 2$; b) $k = 3$; c) $k = 4$.

This comparison of the two policies supposes that such a choice exists for the maintainer. However, it may be argued that the age and block policies are applicable to different situations, for example, using age-replacement for relatively costly components, where the additional managerial requirements are justified, and block-replacement otherwise. In this case, Figure 5 remains informative while Figure 6 does not.

Finally, as we suggest above, the cost-advantage of block-replacement over age-replacement is arguably accounted for in the way defaulting is specified in the policies. In the latter, a default implies that failure (and corrective maintenance) is certain. The former does not, because default on one replacement does not preclude successful execution at another. Nonetheless, a policy in which default at a preventive replacement epoch implies default at every subsequent replacement epoch seems more like neglect, say of an obsolete system, than forgetfulness, say as a result of poor information-management. However, this does not preclude a study of a comparison of the policies given very particular circumstances of defaulting. Thus, one might conceive of a model of age-replacement that mimics forgetfulness, so that replacement is executed at some randomly chosen T (or failure whichever occurs earlier). This mode of defaulting might provide a fairer comparison to the mode of defaulting in the block policy that we use. This may provide an interesting avenue for further research, or investigation in the context of a real case study.

5. Implications for practice

Our findings that translate into implications for practice are broadly as follows:

- If there is a chance to default on the age-replacement policy in the way we describe, then a managerial response that changes the critical age for replacement is not a good one. Decreasing T would likely increase both the average cost of the policy and the average unavailability of the system. Increasing T would do likewise and also may incur the risk of a greater chance of default. Thus, it is better to seek to improve control of maintenance planning.
- If there is a chance to default on the block-replacement policy in the way we describe, then a simple managerial response is to increase the frequency of scheduled replacements (decrease T).
- Block-replacement is relatively more sensitive to departures from the prescribed optimal policy. Furthermore, maintainers should be aware that the true optimum policy cannot be known with certainty. Thus, a T obtained (estimated) through modelling is at best a guide for implementation of policy.

- If defaulting in the block-replacement policy arises in the manner of postponements, then block-replacement is cost-efficient relative to age-replacement except when the probability of a default is very small. Also, block-replacement is simpler to plan and to control.
- A practitioner might argue that the chance of default is small. However, for systems with many replicates and low-cost parts, e.g. windscreen wiper-blades on fleets of heavy goods vehicles, policy may be clear but managerial control may be weak. Also, where there are unrecorded deviations from plans (due to unavailability of resources or prioritisation of production), scheduled replacements may be forgotten rather than postponed. It is well-known that maintenance data recording is a significant issue in practice (Hodkiewicz et al., 2016). Also, maintenance management systems may not easily accommodate postponements (de Jonge et al., 2015b).
- Where there are particular circumstances for default that might arise in a particular, practical context, and which are not accommodated by the models we develop in this paper, then more detailed guidance might be achieved through a simulation study.

6. Conclusion

In this paper, we model defaulting in the age-replacement and block-replacement policies. These models are motivated by practical circumstances in which maintenance is not carried out according to the maintenance plan. We discuss circumstances in which managerial control of maintenance may be weak. Broadly, the results in the paper indicate that a) it is better to improve maintenance control than to seek a maintenance schedule that is robust to defaulting, and b) age-replacement is a sensible policy only ~~performs well~~ if the probability of default is small. We might claim a general insight that the occurrence of default will always result in higher costs and lower system availability. At a more detailed level, we find that even for small values of the probability of default block-replacement may be preferred to age-replacement both on cost-efficiency and planning-efficiency grounds, although block-replacement is more sensitive to knowledge about policy-parameters. This finding is however predicated on the notions that age- and block-replacement are competing policies and defaulting in the block policy arises in the manner of postponements of replacements. Therefore, we cannot claim that block-replacement is generally better than age-replacement.

The work is of practical importance because time-based (periodic) maintenance policies are in common use—about 33% of planned maintenance activity (Alsyouf, 2009)—and the execution of scheduled maintenance on-plan is a great challenge not only because of the pressure to reduce costs and increase production but also because maintenance resource is often over-extended by unplanned activity.

Other researchers consider the effect of deviations from schedule, e.g. postponements, and sensitivities, e.g. to parameters that typically must be estimated from ~~typically~~ scarce data, but none to date to our knowledge consider defaulting in the manner of this paper. Nonetheless, and as we suggest above, the models of defaulting perhaps favour block-replacement somewhat. This is because, in our models, if a default occurs in age-replacement then failure is a consequence whereas for block-replacement there remains the possibility to prevent failure at a subsequent scheduled replacement. However, this point to an extent illustrates qualitatively how block-replacement may be robust to defaulting.

More complicated models of default could be considered in the case of age-replacement. These might include: allowing the probability of a default to be increasing function of critical age for replacement T; or modelling the lifetime Z of a monitoring system M, so that the renewal-cycle length is $\min(X,T)$ if $Z \geq T$ and X if $Z < T$, recalling that X is the component-lifetime. These models would be interesting to study in further work. Their consideration in the case of block-replacement would be more difficult to justify because it is hard to see how they could arise in practice. Defaulting in the context of the modified block-replacement policy (Berg and Epstein, 1976) or inspection-maintenance might also make interesting studies.

In contexts where component replacements in multi-component systems are combined into a preventive maintenance schedules, postponement (or advancement) of replacement may be possible but cancellation may not. Then, other models of defaulting may be appropriate. So, again, we cannot claim the models of defaulting that we propose (and the findings that arise from them) are universal. Indeed, it would be interesting to study defaulting in a multi-component context, where, for example, defaulting and resource-limitations interact.

References

Ahmadi R (2019) An integrated approach to maintenance scheduling of multi-state production systems subject to deterioration. *IMA Journal of Management Mathematics* 30, 235–264.

Alberti AR, Cavalcante CAV, Scarf PA, Silva ALO (2019) Modelling inspection and replacement quality for a protection system. *Reliability Engineering & System Safety* 176, 145-153.

Alsyouf I (2009) Maintenance practices in Swedish industries: survey results. *International Journal of Production Economics* 121, 212-223.

Antonovsky A, Pollock C, Straker L (2016) System reliability as perceived by maintenance personnel on petroleum production facilities. *Reliability Engineering & System Safety* 152, 58-65.

Badía FG, Berrade MD (2009) Optimum maintenance policy of a periodically inspected system under imperfect repair. *Advances in Operations Research* 2009, Article ID 691203.

Baglee D, Trimble R, MacIntyre J (2007) Maintenance strategy development within sme's: the development of an integrated approach. In *ASME. International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, Volume 1: 21st Biennial Conference on Mechanical Vibration and Noise, Parts A, B, and C*, pp.1967-1976.

Baker RD, Scarf PA (1995) Can models fitted to small data samples lead to maintenance policies with near-optimum cost?, IMA Journal of Mathematics Applied in Business & Industry 6, 3-12.

Berg M, Epstein B (1976) A modified block replacement policy. *Naval Research Logistics* 23, 15-24.

Budai G, Dekker R, Nicolai RP (2008) Maintenance and production: A review of planning models. In Kobbacy K., Murthy D.N.P. (eds.) *Complex System Maintenance Handbook*, Springer, pp.321-344.

Budai G, Huisman D, Dekker R (2006) Scheduling preventive railway maintenance activities. *Journal of the Operational Research Society* 57, 1035-1044.

Bunea C, Bedford T (2002) The effect of model uncertainty on maintenance optimization. *IEEE Transactions on Reliability* 51, 486-493.

Cherkaoui H, Huynh K, Grall A (2018) Quantitative assessments of performance and robustness of maintenance policies for stochastically deteriorating production systems. *International Journal of Production Research* 56, 1089-1108.

De Jonge B, Jakobsons E (2018) Optimizing block-based maintenance under random machine usage, *European Journal of Operational Research* 265, 703-709.

De Jonge B, Klingenberg W, Teunter R, Tinga T (2015a) Optimum maintenance strategy under uncertainty in the lifetime distribution. *Reliability Engineering & System Safety* 133, 59-67.

De Jonge B, Dijkstra AS, Romeijnders W (2015b) Cost benefits of postponing time-based maintenance under lifetime distribution uncertainty. *Reliability Engineering & System Safety* 140, 15-21.

De Jonge B, Scarf PA (2020) A review on maintenance optimisation. *European Journal of Operational Research* https://doi.org/10.1016/j.ejor.2019.09.047.

Diallo C, Venkatadri U, Khatab A, Liu Z (2018) Optimal selective maintenance decisions for large serial k-out-of-n: G systems under imperfect maintenance. *Reliability Engineering & System Safety* 175, 234-245.

Do P, Barros A, Bérenguer A, Bouvard K, Brissaud F (2013) Dynamic grouping maintenance with time limited opportunities. *Reliability Engineering & System Safety* 120, 51-59.

Dwight R, Scarf PA, Gordon P (2012) Dynamic maintenance requirements analysis for asset management. *In Advances in Safety, Reliability, and Risk Management* (Berenguer, Gralland, and Guedes Soares, eds), Taylor and Francis, London, pp. 847-852.

Fouladirad M, Paroissin C, Grall A (2018) Sensitivity of optimal replacement policies to lifetime parameter estimates. *European Journal of Operational Research* 266, 963-975.

Finkelstein M, Cha JH, Levitin G (2020) A hybrid preventive maintenance model for systems with partially observable degradation. *IMA Journal of Management Mathematics* (in press).

George-Williams H, Patelli E (2017) Maintenance strategy optimization for complex power systems susceptible to maintenance delays and operational dynamics. *IEEE Transactions on Reliability* 66, 1309-1330.

Hodkiewicz M, Ho M T-W (2016) Cleaning historical maintenance work order data for reliability analysis. *Journal of Quality in Maintenance Engineering* 22, 146-163.

Irawan C, Ouelhadj D, Jones D, Stalhane M, Sperstad I (2017) Optimisation of maintenance routing and scheduling for onshore wind farms. *European Journal of Operational Research* 256, 76-89.

Jardine AKS (1973) *Maintenance, Replacement and Reliability*. Pitman, London.

Jiang R (2019) Risk-sensitive cost models of age replacement policy: Comments and proposals. *Journal of the Operational Research Society* 70, 548-554.

Khalaquzzaman M, Kang HG, Kim MC, Seong PH (2010) Quantification of unavailability caused by random failures and maintenance human errors in nuclear power plants. *Nuclear Engineering and Design* 240, 1606-1613.

Khatab A, Aghezzaf EH, Djelloul I, Sari Z (2017) Selective maintenance optimization for systems operating missions and scheduled breaks with stochastic durations. *Journal of Manufacturing Systems* 43, 168-177.

Li P, Wang W, Peng R (2016) Age-based replacement policy with consideration of production wait time. *IEEE Transactions on Reliability* 65, 235-247.

Lin B, Wu J, Lin R, Wang J, Wang H, Zhang X (2019) Optimization of high-level preventive maintenance scheduling for high-speed trains. *Reliability Engineering & System Safety* 183, 261-275.

Litzka J, Weninger-Vycudil A (2012) The effect of restricted budgets for road maintenance. *Procedia - Social and Behavioral Sciences* 48, 484-494.

Liu Q, Dong M, Chen FF, Lv W, Ye C (2019) Single-machine-based joint optimization of predictive maintenance planning and production scheduling. *Robotics and Computer-Integrated Manufacturing* 55, 173-182.

Liu Y, Chen Y, Jiang T (2018) On sequence planning for selective maintenance of multi-state systems under stochastic maintenance durations. *European Journal of Operational Research* 268, 113-127.

Munier BR (2018) On bespoke decision-aid under risk: the engineering behind preference elicitation. *IMA Journal of Management Mathematics* 29, 253–273.

Quinlan M, Hampson I, Gregson S (2013) Outsourcing and offshoring aircraft maintenance in the US: implications for safety. *Safety Science* 57, 283-292.

Reason J, Hobbs A (2017) *Managing Maintenance Error: A Practical Guide* (eBook). CRC Press, London.

Ruschel E, Santos EAP, Loures EFR (2017) Industrial maintenance decision-making: A systematic literature review. *Journal of Manufacturing Systems* 45, 180-194.

Safaei N, Banjevic D, Jardine AKS (2011) Bi-objective workforce-constrained maintenance scheduling: a case study. *Journal of the Operational Research Society*, 62, 1005-1018.

Safaei N, Jardine AKS (2018) Aircraft routing with generalized maintenance constraints. *Omega* 80, 111-122.

Scala NM, Rajgopal J, Needy KL (2014) Managing nuclear spare parts inventories: a data driven methodology. *IEEE Transactions on Engineering Management* 61, 28-37.

Scarf PA, Cavalcante CAV (2012) Modelling quality in replacement and inspection maintenance. International Journal of Production Economics 135, 372-381.

Waeyenberg G, Pintelon L (2002) A framework for maintenance concept development. *International Journal of Production Economics* 77 , 299-313.

Wakiru JM, Pintelon L, Muchiri PN, Chemweno PK (2019) A simulation-based optimization approach evaluating maintenance and spare parts demand interaction effects. *International Journal of Production Economics* 208, 329-342.

Wang Q, Liu W, Zhong X, Yang J, Yuan Q (2011) Development and application of equipment maintenance and safety integrity management system. *Journal of Loss Prevention in the Process Industries* 24, 321-332.

Wang X, Li L, Xie M (2020) An unpunctual preventive maintenance policy under two-dimensional warranty. *European Journal of Operational Research* 282, 304-318.

Wang X, Zhang Y, Wang L, Wang J, Lu J (2019) Maintenance grouping optimization with system multi-level information based on BN lifetime prediction model. *Journal of Manufacturing Systems* 50, 201-211.

Wen L, Jia X, Wu S, Li B (2011) Robustness of age replacement strategy in maintenance management. *International Conference on Quality, Reliability, Risk, Maintenance, and Safety Engineering* 614-619.

Wildeman RE, Dekker R, Smit ACJM (1997) A dynamic policy for grouping maintenance activities. *European Journal of Operational Research* 99, 530-551.

Wu S, Coolen FP, Liu B (2017) Optimization of maintenance policy under parameter uncertainty using portfolio theory. *IISE Transactions* 49, 711–721.

Wu T, Ma X, Yang L, Zhao Y (2019) Proactive maintenance scheduling in consideration of imperfect repairs and production wait time. *Journal of Manufacturing Systems* 53, 183-194.

Yang C, Remenyte-Prescott R, Andrews J (2017) Road maintenance planning using network flow modelling. *IMA Journal of Management Mathematics* 28, 387–402.

Yang L, Ye Z-S, Lee C-G, Yang S-F, Peng R (2019) A two-phase preventive maintenance policy considering imperfect repair and postponed replacement. *European Journal of Operational Research* 274, 966-977.

Zio E, Compare M (2013) Evaluating maintenance policies by quantitative modeling and analysis. *Reliability Engineering and System Safety* 109, 53-65.

Zhong S, Pantelous AA, Goh M, Zhou J (2019) A reliability-and-cost-based fuzzy approach to optimize preventive maintenance scheduling for offshore wind farms. *Mechanical Systems and Signal Processing* 124, 643-663.