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IFAC-PapersOnLine 48-21 (2015) 946–951

Condition-based maintenance for a two-component system with dependencies

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Abstract: The paper deals with a condition-based maintenance policy for a two-component system with dependencies. Two kinds of dependency are investigated. The first is state dependence whereby the deterioration speed of each component depends not only on its own state (deterioration level) but also on the state of the other. The second is economic dependence whereby combining maintenance activities is cheaper than performing maintenance on components separately. To select a component/group of components to be preventively maintained, adaptive preventive maintenance and opportunistic maintenance rules are proposed. A cost model is developed to find the optimal value of decision parameters. A numerical example is introduced to illustrate the use and the advantages of the proposed approach in the maintenance optimization framework.

Keywords: Condition-based maintenance, maintenance optimization, two-component system, state dependence, economic dependence.

1. INTRODUCTION

Maintenance involves preventive and corrective actions carried out to retain a system in or restore it to an operating condition. Optimal maintenance policies aim to determine an effective and efficient maintenance plan (e.g., inspection time, maintenance time and actions, required for maintenance,...) for each component of the system at lowest possible maintenance cost. In the literature, many policies have been developed for the maintenance of mono-component systems Wang [2002]. These maintenance policies may be applied for multi-component systems if the dependencies between components in the systems are neglected. This assumption may not be true for many industrial systems in which the dependencies often exist among the components. These dependencies can be divided into three types Nicolai and Dekker [2008]: "*(i)* economic dependence implies that the cost of joint maintenance of a group of components does not equal the total cost of individual maintenance of these components; *(ii)* stochastic dependence exists if the condition of components influences the lifetime distribution of other components; *(iii)* structural dependence occurs if components structurally form a part, so that maintenance of a failed component implies maintenance of working components, or at least dismantling them". Economic dependence has been investigated and integrated in a number of multi-component maintenance models, see for example Nicolai and Dekker [2008], Do Van et al. [2013], Liu et al. [2013], van der Duyn Schouten and Vanneste [1990]. Failure dependence between components (whereby the failure of a component can induce the failure of others) has been studied in the context of inspection by Golmakani and Moakedi [2012] and maintenance optimization by Scarf and Deara [2003] for two-component systems, for example. In the latter,several block replacement models considering both economic and failure interaction are proposed. Condition-based maintenance (CBM), in which the preventive maintenance decision is based on the observed system condition, has been introduced and becomes an efficient model in maintenance optimization framework. It has been also developed for two-component systems, see for example Barbera et al. [1999], Casternier et al. [2005], Liu et al. [2013]. However, in such maintenance models, only economic dependence has been considered. Thus, there is a need to consider stochastic and/or structural dependence in CBM; this presents not only for modeling and formulation but also for maintenance optimization problems.

With these issues in mind, in this paper a condition-based maintenance model with state dependence for a two-component system is proposed. In the state dependence, we suppose that the deterioration speed of each component depends not only on its own state but also on the state of the other one. This dependence phenomenon can be found in a number of industrial systems, e.g., the state (quality) of oil may directly impact upon the deterioration process of the crank and vice versa. The primary, novel contribution of the present paper is to propose a deterioration model taking into account the state dependence between components. In this model, we assume that due to inspection actions, the state of each component is identified at regular time intervals. Maintenance actions are then (optimally) planned based on the current state of the components. It is important to note that when considering state dependence between components, existing CBM policies may be suboptimal. Therefore, the second, novel contribution of the paper is to propose and develop a CBM policy in which adaptive preventive maintenance and opportunistic maintenance rules select a component or group of components to be maintained.

Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2015.09.648

The paper is organized as follows. Section 2 describes the system, the assumptions associated with inspection, maintenance operations and costs, and the deterioration model. Section 3 describes the proposed maintenance policy and the optimization process. To illustrate the use and the advantages of the proposed maintenance policy, an numerical example is considered in Section 4. The results therein include a sensitivity analysis. Finally, the last Section presents the conclusions drawn from this work.

2. SYSTEM MODELING AND ASSUMPTION

Consider a system consisting of two dependent components that are connected in series. When one or both components fail the system fails. It is assumed that a failure of a component is instantaneously revealed by the self-announcing mechanism. Each component i is subject to a continuous accumulation of wear in time which is assumed to be described by a scalar random variable X_t^i. The deterioration speed of a component depends not only its own characteristics but also on the state (deterioration level) of the other. Component i is failed if its deterioration level reaches a level L^i, namely the failure threshold. When a component is not operating for whatever reason, its deterioration level remains unchanged during the stoppage period if no maintenance is carried out.

It is assumed that continuous monitoring is impossible (e.g. monitoring equipment is not integrated in the system for whatever reason). Therefore, inspections are discrete. For maintenance activities, it is assumed also that both corrective and preventive maintenance are possible for each component. Note, by our definition, an inspection is not a maintenance activity.

2.1 Degradation modelling

Between two consecutive maintenance activities, it is assumed that the deterioration level of component i ($i = 1, 2$) at time $t+1$ can be expressed as follows:

$$X_{t+1}^i = X_t^i + f(X_t^j) + \Delta X^i, \qquad (1)$$

for $j = 1, 2$ and $j \neq i$ and where

- ΔX^i indicates the random increment in the deterioration level of component i during one time unit when the component is isolated from the system. It is assumed that ΔX^i follows a Gamma probability density (pdf) with shape parameter α^i and scale parameter β^i:

$$f_{\alpha^i,\beta^i}(x) = \frac{1}{\Gamma(\alpha^i)}(\beta^i)^{\alpha^i} x^{\alpha^i - 1} e^{-\beta^i x} \mathcal{I}_{\{x \geq 0\}},$$

  with $\mathcal{I}_{\{x\geq 0\}}$ is an indicator function $\mathcal{I}_{\{x\geq 0\}} = 1$ if $x \geq 0$, $\mathcal{I}_{\{x\geq 0\}} = 0$ otherwise;
- $f(X_t^j)$ is a function of the deterioration level of component j at time t and represents the impacts of component j on the deterioration speed of component i. It is assumed that $f(X_t^j)$ can be expressed as:

$$f(X_t^j) = \mu_*^j (X_t^j)^{\sigma^j}. \qquad (2)$$

  In this way, μ^j, σ^j are non-negative real number that quantify the influence of component j on the deterioration speed of component i ($i, j = 1, 2$ and $i \neq j$. When $\mu^j = 0$, component j does not have any influence on the deterioration behavior of component i. When $\mu^1 = 0$ and $\mu^2 = 0$, the two components are independently subject to gradual deterioration. When $\mu^j > 0$ and $\sigma^j = 0$, the impact of component j on the deterioration behavior of component i ($i \neq j$) does not then depend on the state (deterioration level) of component j ($f(X_t^j) = \mu^j$).

Fig. 1 illustrates the deterioration evolution of the two dependent components. Note that when the deterioration of a component reaches its failure threshold, the component fails and the deterioration level of the other component remains unchanged until the subsequent maintenance activity. We suppose that the failed component can be correctively maintained at only regular time intervals which co-incide with the inspection times. When a failed component is replaced (its deterioration level is reset to zero), the deterioration speed of the other component is thus reduced.

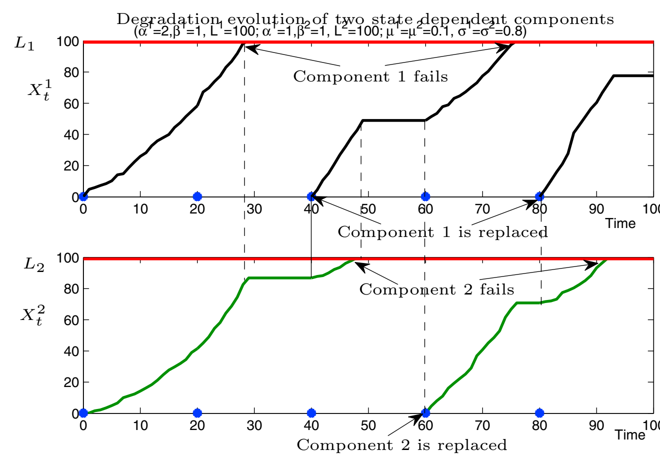

Fig. 1. Illustration of the deterioration evolution of two dependent components

2.2 Inspection, maintenance actions and associated costs

Inspection: Inspections provide information about the current state of components which can be used for decision-making on maintenance and related logistic support activities, [Barker and Newby, 2009, Grall et al., 2002, Do Van and Berenguer, 2010]. Inspections are an important part of condition-based maintenance. We assume as previously described that inspections are necessary and discrete. For simplicity we further suppose that inspections are periodic Non-periodic and periodic inspections are introduced and successfully applied in industry.

Inspections are assumed to be instantaneous, perfect, and non-destructive. For each component i ($i = 1, 2$), an inspection incurs a cost $C_I^i = c_I^0 + c_I^i$. c_I^0 is the inspection set-up cost, which can be shared when performing simultaneous inspections on each component. c_I^i represents the specific inspection cost of component i which cannot be shared.

Maintenance activities: It is assumed that after a maintenance action (corrective or preventive maintenance) the maintained component becomes "as good as new" (the deterioration level after maintenance is assumed to be reset to zero). All necessary maintenance resources (such as spare parts, maintenance tools, repairmen, etc.) to execute maintenance actions are always available at a planned inspection time. It is also supposed that maintenance actions are carried out at discrete times and the maintenance duration can be neglected.

For preventive maintenance actions, as shown in Nicolai and Dekker [2008], Do Van et al. [2013], Wildeman et al. [1997] the preventive maintenance cost for component i, denoted C_p^i, can be divided into two parts ($C_p^i = c_p^0 + c_p^i$):

- a specific component cost c_p^i;
- a preventive setup-cost, denoted c_p^0, that represents the preparation cost (or logistic cost) and can be shared when several preventive maintenance actions are performed together since execution of a group of maintenance actions requires usually only one set-up;

In the same manner, when performing a corrective maintenance action on component i, a corrective maintenance cost, C_c^i, is incurred: $C_c^i = c_c^0 + c_c^i$ where c_c^0 ($c_c^0 \geq c_p^0$) and c_c^i ($c_c^i \geq c_p^i$) represent the corrective set-up cost and the specific corrective cost for component i, respectively. In addition, if a failed component is not immediately maintained, an additional cost c_d, a so-called downtime cost rate, is incurred for each time unit.

3. MAINTENANCE POLICY

3.1 Description of the proposed maintenance policy

It is assumed that the two components of the system are inspected at regular time intervals with inter-inspection interval ΔT. The inter-inspection interval ΔT is a decision parameter to be optimized. At inspections, the condition (deterioration level) of each component is measured. More precisely, for each component i ($i = 1, 2$), the deterioration level at inspection times $T_k = k\,\Delta T$ ($k = 1, 2, ...$) is $X^i_{T_k} = x^i_{T_k}$.

It is assumed also that a failure of a component is instantaneously revealed by the self-announcing mechanism. The maintenance policy is as follows:

- if component i fails between (T_{k-1}, T_k), then it is replaced at time T_k;
- if at time T_k, component i is still functioning, it is firstly inspected. Based on the inspection results, component is maintained or not at time T_k according to the preventive maintenance rules. Two kinds preventive maintenance rules, individual preventive maintenance and opportunistic preventive maintenance, are proposed.

Preventive maintenance at component level If the deterioration level of component i ($i = 1, 2$) at time T_k is greater or equal to the fixed level m_p^i, $x^i_{T_k} \geq m_p^i$, a component is immediately replaced. m_p^i, called the presentive threshold level of component i, is a decision parameter to be optimized.

Opportunistic maintenance The main idea of the proposed opportunistic maintenance is to take advantage of the positive economic dependence between the two components. To this end, for each component i, an opportunistic threshold, denoted m_o^i ($0 < m_o^i \leq m_p^i$), is introduced. The opportunistic maintenance decision rule is the following. If component j ($j = 1, 2$ and $j \neq i$) is selected to be correctively or preventively maintained at time T_k, component i is preventively replaced together with component j if the deterioration level of component i is such that $x^i_{T_k} \geq m_0^i$. The latter implies that the system is renewed at time T_k. It is important to note that m_o^i ($i = 1, 2$) are also decision parameters that must be optimized.

An illustration of the proposed maintenance policy is shown in Fig. 2.

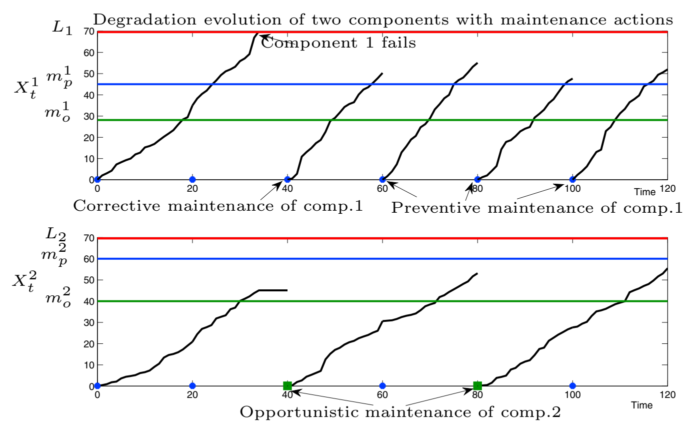

Fig. 2. Illustration of the proposed maintenance policy

3.2 Optimization of the proposed maintenance policy

As mentioned above ($\Delta T, m_p^1, m_o^2, m_p^2, m_o^2$) are the decision parameters of the proposed maintenance policy. They must be optimized. For this purpose, a maintenance cost model is developed in this section. In fact, the long-run expected maintenance per unit of time including the unavailability cost is used and considered as the main criterion in order to find the optimal decision parameters $\Delta T, m_p^1, m_o^1, m_p^2$ and m_o^2.

The long-run expected total cost per unit of time is defined as:

$$C^{\infty}(\Delta T, m_p^1, m_o^1, m_p^2, m_o^2) = \lim_{t \to \infty} \frac{C^t(\Delta T, m_p^1, m_o^2, m_p^2, m_o^2)}{t - t_{down}}, \tag{3}$$

where $C^t(\Delta T, m_p^1, m_o^2, m_p^2, m_o^2)$ and t_{down} are respectively the cumulative total cost and the cumulative downtime of the system within the period $(0\ t]$. According to the renewal theory Ross [1996], Eq. (3) can be rewritten as follows:

$$C^{\infty}(\Delta T, m_p^1, m_o^2, m_p^2, m_o^2) = \frac{\mathbb{E}[C^{T_{re}}(\Delta T, m_p^1, m_o^2, m_p^2, m_o^2)]}{\mathbb{E}[T_{re} - T_{down}]}, \tag{4}$$

where $\mathbb{E}[.]$ is mathematical expectation and T_{re} is the length of the first life cycle of the system, i.e., all components of the system are replaced at time T_{re}. T_{down} represents the cumulative downtime within the first lifetime cycle. It is supposed that $T_{re} = \Delta T.m$ (m is a positive integer), and so one gets:

$$C^{T_{re}}(\Delta T, m_p^1, m_o^2, m_p^2, m_o^2) = \frac{\sum_{k=1}^{m}(C_{ins}^k + C_{main}^k) + T_{down}.C_d}{m \cdot \Delta T - T_{down}},$$

where C_d is downtime cost of the system per time unit; C_{ins}^k, C_{main}^k are respectively the total inspection, maintenance cost at discrete time T_k. Furthermore,

- $C_{ins}^k = \sum_{i=1}^{u} c_I^i + \mathcal{I}_{\{u \geq 1\}} c_I^0$ with u $(u = 0, 1, 2)$ being the number of components inspected at T_k;
- $C_{main}^k = c_p^1 + c_p^2 + c_p^0$ if two components are preventively replaced; $C_{main}^k = c_p^i + c_p^0$ if only component i is preventively replaced; $C_{main}^k = c_p^i + c_c^j + c_c^0$ if component i is preventively maintained and component j $(j \neq i)$ is correctively replaced; $C_{main}^k = c_c^i + c_c^0$ if only component i is correctively replaced and $C_{main}^k = 0$ if no maintenance is performed at T_K.

The cost-rate presented in Equation (4) can be calculated, given $\Delta T, m_p^1, m_o^1, m_p^2, m_o^2$, using Monte Carlo simulation. By varying the values of the decision parameters $(\Delta T, m_p^1, m_o^1, m_p^2, m_o^2)$ and performing a crude search, the minimum cost rate can be identified. The optimal value of the decision parameters are obtained when the minimum cost rate is reached. i.e,

$$C^{\infty}(\Delta T^*, m_p^{1*}, m_o^{1*}, m_p^{2*}, m_o^{2*}) = \min\{C^{\infty}(.)_{\Delta T>0, 0 < m_p^1 \leq L^1, 0 < m_o^1 \leq m_p^1, 0 < m_p^2 \leq L^2, 0 < m_o^2 \leq m_p^2}\}. \quad (5)$$

4. NUMERICAL EXAMPLE

The purpose of this section is to show how the proposed maintenance policy can be used in maintenance optimisation through an example whose characteristics are described in Section 2.

Consider a two-dependent component system with downtime cost rate $c_d = 70$, and setup costs $c_I^0 = 2$, and $c_p^0 = c_c^0 = 10$. Table 1 reports the data of deterioration behavior and maintenance costs (all costs are given in arbitrary units) associated with each individual component.

Table 1. Data of two-dependent component system.

Component i	α^i	β^i	μ^i	σ^i	L^i	c_I^i	c_p^i	c_c^i
1	2	1	0.1	0.5	30	4	50	60
2	1	1	0.1	0.5	30	4	60	70

4.1 Optimum maintenance policy

To evaluate the cost-rate, a very large number of simulation realizations are done. In order to find the optimal decision parameters $(\Delta T, m_p^1, m_o^1, m_p^2, m_o^2)$, the cost-rate $C^{\infty}(\Delta T, m_p^1, m_o^1, m_p^2, m_o^2)$ is evaluated for different values of ΔT $(\Delta T > 0)$, m_p^1 $(0 < m_p^1 \leq L^1)$, m_o^1 $(0 < m_o^1 \leq m_p^1)$, m_p^2 $(0 < m_p^2 \leq L^2)$ and m_o^2 $(0 < m_o^2 \leq m_p^2)$ using Equation (4). The optimum values of the decision parameters are $\Delta T^* = 10$, $m_p^{1*} = 7$, $m_o^{1*} = 6$, $m_p^{2*} = 18$ and $m_o^{2*} = 16$ with the minimum cost rate $C^{\infty}(\Delta T^*, m_p^{1*}, m_o^{1*}, m_p^{2*}, m_o^{2*}) = 10.82$. Fig. 3 shows the cost-rate for different values of the preventive maintenance thresholds m_p^1 and m_p^1.

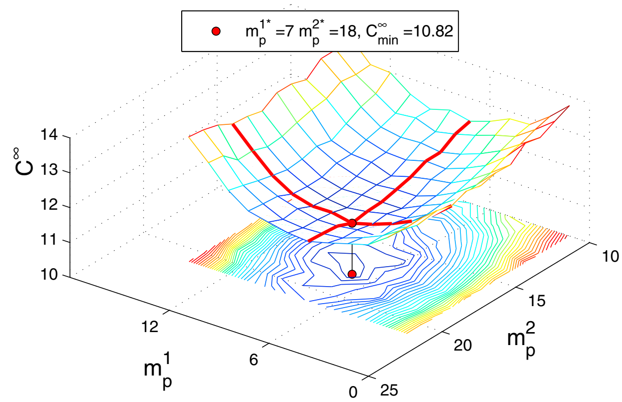

Fig. 3. Maintenance cost rate as a function of m_p^1 and m_p^2 with $\Delta T = 10$

Fig. 4 shows the relationships between the cost-rate and the inter-inspection interval ΔT when $m_p^1 = 7$, $m_o^1 = 6$, $m_p^2 = 18$ and $m_o^2 = 16$.

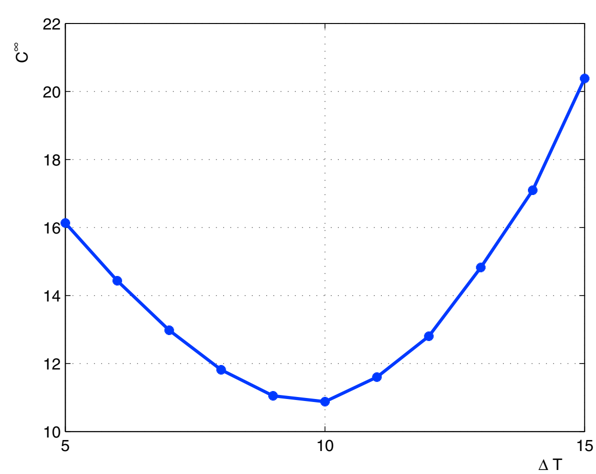

Fig. 4. Maintenance cost rate as a function of ΔT with $m_p^1 = 7$, $m_o^1 = 6$, $m_p^2 = 18$ and $m_o^2 = 16$

4.2 Sensitivity to setup cost

To analyze the impact of the proposed maintenance opportunity, the proposed maintenance policy and a variant, namely the same policy but without opportunity, are studied. This is carried out by considering different values of the maintenance setup cost c_m^0 $(c_m^0 = c_p^0 = c_c^0)$. The policy without opportunity can be easily obtained from the proposed policy by setting the preventive opportunity thresholds equal to the preventive thresholds, that is by setting $m_o^1 = m_p^1$ and $m_o^2 = m_p^1$. For each value of c_m^0 (c_m^0 is varied from 0 to 40), the minimum cost-rate of the proposed policy and that of the policy without opportunity are determined. The obtained results are shown in Fig. 5.

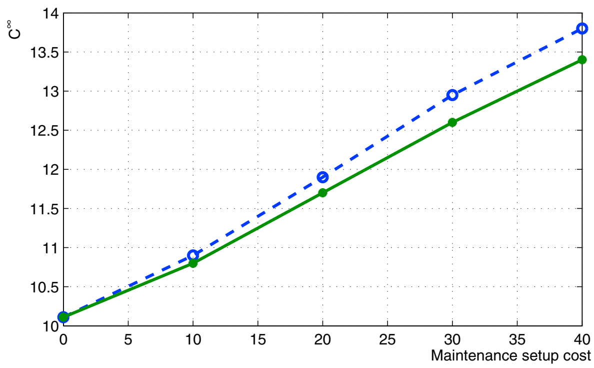

Fig. 5. Sensitivity analysis to the maintenance setup cost

The figure shows that the opportunity maintenance can significantly reduce the total maintenance cost when the setup cost is high. It is important to note that the policy without opportunity is simpler than the policy with opportunity since the number of decision parameters is fewer (three decision parameters instead of five).

4.3 Optimum maintenance policy without considering state dependence

To study the impact of state dependence between components on the optimum maintenance policy, it is assumed that the deterioration processes of the two components are described by two independent processes. This can be easily done by setting $\mu^1 = 0, \mu^2 = 0$ in the proposed deterioration models presented in Equation (1). The proposed maintenance policy is then applied. In that way, we obtained the optimal decision parameters $\Delta T^* = 12$, $m_p^{1*} = 10$, $m_o^{1*} = 8$, $m_p^{2*} = 16$ and $m_o^{2*} = 15$. When compared the results obtained in Section 4.1, these optimal values are significantly different. This means that not considering the state dependence between two components can lead to a sub-optimum maintenance policy. In addition, if we apply these optimal decision parameters for the case considering the state dependence between components, the maintenance cost rate is then $C^\infty(\Delta T^*, m_p^{1*}, m_o^{1*}, m_p^{2*}, m_o^{2*}) = 12.55$ which is significantly higher than the one obtained when the state dependence is considered in deterioration modeling ((12.55-10.82)/10.82)x100=16% higher). Of course, the difference depends on the “dependence degree” between the components.

5. CONCLUSIONS

In this work, a condition-based maintenance policy for a two-dependent component system is proposed. State dependence, which implies that the deterioration speed of each component depends not only on its state but on the state of the other one, is modeled and integrated in a maintenance model. To select a component or components to be preventively maintained at each regular time interval, adaptive preventive maintenance and opportunistic maintenance rules are proposed. A cost model taking into account the economic dependence between components is developed to find the optimal value of decision parameters. The performance of the proposed policy is illustrated and discussed through some numerical results for the two-deteriorating component system. The numerical results show that *(i)* the state dependence between components has a significant impact on the maintenance cost and therefore should be considered in deterioration modeling; *(ii)* the proposed policy with maintenance opportunity appears more efficient than the policy without opportunity.

Our future research work will focus on the development of (i) an analytical calculation of the cost-rate in order to to find quickly the maintenance decision parameters and (ii) the estimation of model parameters describing the state dependence among components. Furthermore, we hope to develop the proposed models for multi-component systems.

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